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Citation: Review of Scientific Instruments 86, 093504 (2015); doi: 10.1063/1.4929873
View online: http://dx.doi.org/10.1063/1.4929873
View Table of Contents: http://scitation.aip.org/content/aip/journal/rsi/86/9?ver=pdfcov
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(Received 3 June 2015; accepted 19 August 2015; published online 10 September 2015)

This paper presents the development and testing of the prototype Imaging Motional Stark-Effect (IMSE) diagnostic, designed for ASDEX upgrade. A detailed description of the core hardware, theory of operation, and application to complex MSE spectra are presented and analytical evaluation methods suitable for the required accuracy are developed. The diagnostic is tested with a MSE-like polarised spectrum to assess the accuracy of different modulation modes suggested in previous works. Each is found to have small systematic errors due to non-ideal effects of the components, which must be carefully examined. In particular, the effect of intrinsic contrast that results from imperfect parallelism of the birefringent plates is found to have a strong effect. Methods to mitigate and correct for this are discussed. With the necessary corrections and calibrations, the accuracy of polarisation orientation is shown to be within ±0.2◦. The effect of finite ellipticity is examined and the possibility to measure this to an accuracy of ±2.0◦ is demonstrated. The system is shown to be insensitive to broadband polarised background light, temperature variations, and critically to variations in the details of the MSE spectrum. © 2015 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4929873]

I. MOTIONAL STARK EFFECT (MSE)

The study of fusion relevant plasmas requires accurate knowledge of the internal magnetic field and current distribution. External measurements provide rapidly diminishing accuracy towards the plasma centre, so internal measurements, such as those made by MSE diagnostics, are essential. MSE diagnostics observe Do emission from a beam of neutral hydrogen/deuterium particles injected at high energy. The Do line is Stark-splitted by the local magnetic field experienced as an electric field in the rest frame of the particle. The polarisations of the resulting multiplet components are oriented according to this field, so they carry information about the magnetic field pitch angle.

While the principle of MSE is well proven and MSE polarimeters are routinely used on many tokamaks,1-4 reproducible measurements based on independent calibrations are rare. Despite considerable effort, the pitch angles inferred from first principles typically appear inconsistent with what is already known about the plasma magnetic field. Reliable measurements are usually achieved, but always involve reference to a plasma with a current profile that is considered known.5,6 The assumptions used to infer that current profile, e.g., via an equilibrium solver, will unavoidably have some influence on the calibration, so further development is required before MSE can truly be considered a routine independent diagnostic. One significant complexity of MSE measurements is that the integral of the multiplet over wavelength is unpolarised. Typical MSE polarimeters, such as the existing MSE system at ASDEX Upgrade (AUG),5 must employ independent narrow spectral filters, detectors, and digitisers for each desired spatial point, limiting the quantity of available data and requiring an individual calibration for each spatial point.

II. IMAGING MSE (IMSE)

The recent development of IMSE10 presents several significant advances. A 2D image of the neutral beam emission modulated by interference patterns that encode the polarisation state is captured using a CCD/CMOS camera. The system is designed to produce interferograms with a strongly wavelength dependent phase that ensures all of the Stark multiplet transitions add constructively, despite the orthogonal polarisation of the σ and π components.

This removes the need for narrow optical filters and allows the use of all of the available light. Coupled with the high speed, sensitivity, and useful area of modern CCDs, the 2D image provides superior signal/noise and at least an order of magnitude more data than a typical MSE diagnostic. Several variants of the IMSE have been proposed,10,11 which encode the polarisation information in different ways. One method was tested on the Textor tokamak12 and while the polarimeter appeared to function correctly, issues with the optical head precluded the accurate inference of pitch angles. A variant of the system described here has been installed at K-Star13 with initial results submitted to Journal of Instrumentation.

This paper concerns the development of a prototype IMSE diagnostic for AUG that was temporarily installed on the viewing optics of the existing MSE system in 2013/2014. With the typical problems of calibrating MSE systems in mind, the IMSE was developed with an emphasis on flexibility, robustness, and independence of the spectral details, in order to facilitate isolation and mitigation of any unanticipated effects in the forward viewing optics or the MSE emission physics. Prior to installation, it was thoroughly tested with a bench top MSE spectrum mock-up, to ensure that the system can reliably measure the polarisation angle to the required accuracy of ±0.2◦. In this regard, the different IMSE operating modes were...
each evaluated. This paper presents the setup of the diagnostic, the diagnosis theory and the results, and consequences of the tests. It is found that many non-ideal effects of the components used are important and either needed to be included in the mathematical treatment of the image evaluation or mitigated by developing a new calibration procedure.

III. MEASUREMENT PRINCIPLE

A. Optical arrangement

The complete optics for IMSE are shown in Figure 1(a), but it is helpful to first consider a simple imaging polarimeter that would require only a single displacer plate, the polariser, and lenses. The displacer plate is a birefringent crystal with an optic axis tilted at 45° away from the surface plane. It introduces a phase shift $\phi_d(x)$ between the ordinary and extraordinary components, which depends on the incident angle $\alpha_i$ and hence the image position in the plane of the optic axis (in this case, $x$). The polariser recombines the components, creating interference fringes with an amplitude dependent on the input polarisation $\theta$ relative to the plate’s optic axis, as $I \propto 1 + \sin(2\theta)$. The MSE multiplet (as in Figure 4(a)) has two orthogonal components ($\sigma$ and $\pi$) of equal intensity that would require only a single displacer plate, the polariser, but it is helpful to first consider a simple imaging polarimeter used are important and either needed to be included in the mathematical treatment of the image evaluation or mitigated by developing a new calibration procedure.

$$S = M_{polariser} \cdot M_{(displacer+delay)} \cdot M_{savart} \cdot S$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\phi_0 + \phi_0) & \sin(\phi_0 + \phi_0) \\ 0 & 0 & 1 & 0 \\ 0 & -\sin(\phi_0 + \phi_0) & 0 & \cos(\phi_0 + \phi_0) \end{pmatrix} \cdot \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

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$$2I = s_0 + s_1 \cos(\phi_0 + \phi_0) - s_2 \sin(\phi_0 + \phi_0) \sin \phi_s$$

$$+ s_3 \sin(\phi_0 + \phi_0) \cos \phi_s.$$
is rotated by 90° so that its phase is subtracted and $\delta \rightarrow \delta - \frac{\pi}{2}$, giving the total Savart phase delay,

$$\phi_s = \frac{\omega L_i}{c/N^2} \sin \left( \delta' - \frac{\pi}{4} \right) \sin \alpha_i, \quad (6)$$

where $\delta'$ is defined relative to the optic axis of the first plate. In the IMSE system, the Savart plate is orientated at 45° to the x-axis, so that $\delta' = \delta + \frac{\pi}{4}$, giving a $\sin \delta \sin \alpha_i$ dependence.

The angles $\alpha_i$ and $\delta$ relate to the physical CCD position $(x, y)$ via the focal length of the imaging lens $f_i$,

$$\cos \delta \sin \alpha_i = \frac{x}{f_i}, \quad \sin \delta \sin \alpha_i = \frac{y}{f_i}. \quad (7)$$

The phase terms are now given by

$$\phi_s = \alpha \omega y, \quad (\phi_0 + \phi_d) = \beta \omega x + \gamma \omega, \quad (8)$$

with the three system constants,

$$\alpha = \frac{L_i N^2}{c f_i N^2}, \quad \beta = \frac{L_d N^2}{c f_i}, \quad \gamma = \frac{\Delta N}{c} \left( L_0 + \frac{1}{2} L_d \right). \quad (9)$$

The neglected $O(\alpha^2)$ terms, dispersion, optical distortion, and other non-ideal effects will introduce small offsets and non-linear terms to these; however, the values are suitable for development of the evaluation algebra which will be shown to be independent of their precise values.

The AUG prototype IMSE system observes blue-shifted $D_\alpha$ light in the range 651 nm $< \lambda < 655$ nm using $\alpha$-phase Barium Borate crystals ($n_\alpha = 1.666, n_\beta = 1.549$) with thicknesses: $L_\alpha = 7.6$ mm, $L_d = 5.4$ mm, $L_0 = 1.2$ mm, and the focal length is $f_i = 50$ mm. The Savart and displacer plate thicknesses are chosen to give $\alpha \omega / 2 \pi = \beta \omega = 12 \times 10^3$ fringes m$^{-1}$ which is approximately 75 fringes over the CCD area $(l, y) < 3$ mm. The choice of $L_0$, giving $\gamma \omega / 2 \pi = 700$ fringes, is discussed in Section III D.

C. Demodulation

Together, Equations (2), (8), and (9) give the image intensity in terms of the CCD position $(x, y)$,

$$4I = 2s_0 + 2s_1 \cos (\beta \omega x + \gamma \omega) - s_2 \cos (\beta \omega x + \gamma \omega + \alpha \omega y)$$

$$- s_3 \sin (\beta \omega x + \gamma \omega + \alpha \omega y) + s_2 \sin (\beta \omega x + \gamma \omega - \alpha \omega y)$$

$$- s_3 \sin (\beta \omega x + \gamma \omega - \alpha \omega y). \quad (10)$$

Figure 3 shows a typical image and its Fourier transform on which the four independent Fourier components are indicated. The fringes vary much faster than the Stokes vector in $(x, y)$, so the carriers encode images of the stokes vector components $s(x, y)$.

Spectral leakage caused by the finite oscillation at the images edges manifests as vertical and horizontal spreads of the Fourier components. The CCD is intentionally not aligned to $(x, y)$ coordinates of the crystals to limit cross-contamination of this leakage between components. Isolating each of the components using the windows shown and performing the inverse Fourier transform on each extract the positive frequency oscillations. Denoting the components $A(+, 0)$ and $A(+, \pm)$ according to the sign of their frequencies in $x$ and $y$, these terms are

$$8A(+, 0) = 2s_1 \exp(\text{i} \omega \left[ \beta x + \gamma \right]),$$

$$8A(+, +) = -(s_2 + is_3) \exp(\text{i} \omega \left[ \beta x + \gamma + \alpha y \right]),$$

$$8A(+, -) = (s_2 - is_3) \exp(\text{i} \omega \left[ \beta x + \gamma - \alpha y \right]). \quad (11)$$

Small changes to $\alpha, \beta$, or $\gamma$ or non-linear terms in $x$ and $y$ will not significantly move the components in Fourier space, so provided the windows are large enough, the polarisation information can be extracted in various ways from the amplitudes and/or phases of these complex images. For instance, in the simplest form and where there is no ellipticity ($s_3 = 0$), the image of polarisation direction $\theta$ can be obtained from the ratio of two of the amplitudes,

$$\tan 2 \theta = \frac{2 |A(+, +)|}{|A(+, 0)|}. \quad (12)$$

The spatial resolution of $\theta$ is determined by the size of the windows and the maximum theoretical resolution would be obtained by choosing a fringe period of three pixels. The three windows would then span one third of the Fourier transform, covering it edge to edge. In practice, however, such high frequency fringes are affected by the focus performance of the optical system which reduces the amplitude of the slightly higher frequencies $A(+, \pm)$ by more than it reduces the $A(+, 0)$ amplitudes. A longer fringe period avoids this and smaller windows reduce the noise level in any case.

D. Simplified MSE spectrum response

Equations (10) and (11) give the system response to a single polarisation state at a single frequency. To account for the MSE spectrum, it is useful to first consider the ideal form, which consists of a central $\sigma$ component with frequency $\omega_0$ polarised at some angle $\theta$ (relative to the Savart plate) and two $\pi$ wings polarised at $\theta + 90^\circ$ at $\omega_0 \pm \Delta \omega$ (see bars in Figure 4(a)). The $\sigma$ also has an unpolarised fraction which can be ignored here as it contributes only to $s_0$ and does not produce interference fringes. The Stokes vectors can be defined relative to the polarisation of the $\sigma$ component as

$$s_{\sigma} = I (1, s_1, s_2, 0), \quad \omega_{\sigma} = \omega_0,$$

$$s_{\sigma^*} = I \left( \frac{1}{2}, -s_1, -s_2, 0 \right), \quad \omega_{\sigma^*} = \omega_0 \pm \Delta \omega. \quad (13)$$

The phase of individual emission events are independently random, so the image recorded is simply the sum of
Equation (10) for each component \( I = I_\sigma + I_{\pi^+} + I_{\pi^-} \). The result has the same form and same 4 carriers as Equation (10) but with modified amplitudes,

\[
\begin{align*}
8A(+,0) &= 2s_1\zeta_{+0}\exp(i\omega_0[\beta x + \gamma]), \\
8A(+,+) &= -s_2\zeta_{++}\exp(i\omega_0[\beta x + \gamma + a y]), \\
8A(+,-) &= s_2\zeta_{+-}\exp(i\omega_0[\beta x + \gamma - a y]). \\
\end{align*}
\]

Equation (11) multiplied by the distinct factors \( \zeta \)

These are the single component amplitudes of Equation (11) multiplied by the distinct factors \( \zeta \),

\[
\begin{align*}
\zeta_{+0} &= 1 - \cos\left(\left[\beta x + \gamma\right]\Delta\omega\right), \\
\zeta_{++} &= 1 - \cos\left(\left[\beta x + \gamma + a y\right]\Delta\omega\right), \\
\zeta_{+-} &= 1 - \cos\left(\left[\beta x + \gamma - a y\right]\Delta\omega\right).
\end{align*}
\]

At the centre of the image \((x, y = 0)\), the 3 factors are equal and Equation (12) can be used. The common factor \( \zeta \) \( \sim [1 - \cos(\gamma\Delta\omega)] \) is called the nett contrast and is produced by the fixed phase terms of the displacer and delay plates. Without this fixed phase offset \((\gamma = 0)\), \( \zeta = 0 \) and the fringe patterns vanish entirely. This occurs because the MSE multiplet is nett-unpolarised and shows that some spectral discrimination is required. The contrast is maximized when the phase is completely reversed over the interval between the \( \sigma \) and \( \pi \) frequencies \((\gamma\Delta\omega = 180^\circ)\), cancelling the phase reversal caused by their perpendicular polarisations and allowing the interference patterns to sum constructively. The contrast is controlled by the delay plate thickness \( L_0 \) and for the AUG IMSE, where \( 0.35 \text{ nm} < \Delta\lambda < 0.45 \text{ nm} \), this requires \( L_0 \sim 1.4 \text{ mm} \).

While sufficient as a first approximation, the assumption that \( \zeta_{++}/\zeta_{+0} = 1 \) leads to an error of \( \delta\theta \sim 0.5Y \) degrees, where \( Y = y/y_{\text{max}} \), when applying Equation (12). An alternative expression that uses information from all of the carriers to cancel out the first order effect of the \( \pm x, y \) terms in \( \zeta \) is given by Equation (16),

\[
\tan^22\theta \sim \frac{4A(+,+)}{A(+,0)^2}.
\]

Equation (16) gives a measurement that is periodic on \( 90^\circ \) and reverses at \( 45^\circ \). Since the MSE polarisation angle is expected to cover a range of less than \( 20^\circ \), the system can be rotated to have a working range that avoids the ambiguity every \( 45^\circ \).

### E. Full MSE spectrum response

For the ideal MSE spectrum, the \( \sigma \) and \( \pi \) fringes are summed perfectly, but the observed MSE spectrum is in reality much more complex and some aspects of the physics remain difficult to model accurately. To illustrate this, the spectrum produced by forward modelling of the neutral beam emission is shown in Figure 4(b). The model includes beam attenuation, Stark sub-level splitting, non-statistical population of excited states, \( \text{the three beam energy components, integration over the beam width, and Doppler broadening due to beam divergence. It does not include the small Stark-Zeeman coupling effects} \) but does broadly match measurements made of the AUG MSE spectrum. \( \text{17} \)

It is essential to show that the measured \( \theta \) is independent of the details of the spectrum, for any such realistic case. The general response to any Stokes-spectrum \( s(\omega) \) can be found by integrating Equation (10) over frequency. For any MSE spectrum, \( s(\omega) \) changes only in intensity and sign since the polarisation changes only by \( 90^\circ \) changes in \( \theta \). It can therefore be split into a fixed Stokes vector and a signed intensity function of frequency \( s(\omega) = sI(\omega) \). This leads to the recovery of Equation (14), with the following form for the component contrasts:

\[
\begin{align*}
\zeta_a &= \Gamma(\beta x + \gamma), \\
\zeta_{b/c} &= \Gamma(\beta x + \gamma \pm a y).
\end{align*}
\]

Equation (18) is the Fourier transform of the spectrum onto the spatial variable of each component. To the first approximation, this remains \([1 - \cos(\gamma\Delta\omega)]\) and the optimal choice of \( L_0 \) is not modified. It is now clear that spectral components far from the central wavelength will lead to the fastest variations in \( Y \). For AUG, the spectrum can span up to a few \text{nm} which gives variations in \( Y \) almost significant on the scales of the CCD. To check this, the forward model for the AUG spectrum was used to calculate summed images directly from Equation (10) and the polarisation angles inferred from (16) compared to the original. In this case, the modelling work confirms that the effect is satisfactorily compensated but generally, care must always be taken when designing IMSE systems to avoid this issue.

Equation (16) is therefore theoretically valid for recovering \( \theta \) provided that only the two linear polarisation directions are present. This is a strong advantage of this form of the IMSE system, as details of the spectrum that cannot be known or that may not be stable should not affect the result. These might include the \( \pi/\sigma \) intensity ratio, \( \Delta\omega \), beam energy, beam divergence, smearing by line-of-sight integration, and effects of non-statistically distributed beam excitation. \( \text{16} \)

### F. Phase encoded measurement

An alternative form of the IMSE system can be realised by adding a quarter-wave plate to the front of the system with its axes orientated at \( 45^\circ \) to the Savart. \( \text{11} \) This transforms the input Stokes vector as \( s \to (s_0, -s_3, s_2, s_1) \). Denoting amplitudes in this state with \( A' \), the \( A(+,+) \) component becomes

\[
8A'(+,+) = -(s_2 + is_1)\exp(i\xi),
\]

where \( \xi = \omega[\beta x + \gamma + a y] \).

\[
(19)
\]
and the polarisation angle can be extracted from the phase
\[
\tan \Phi'(+,+) = \frac{\Im[A(+,+)]}{\Re[A(+,+)]} = \frac{s_2 \sin \xi + s_1 \cos \xi}{s_2 \cos \xi - s_1 \sin \xi} = \tan(2\theta + \xi). \tag{20}
\]

Integration over the MSE spectrum proceeds as before and results in Equation (19) multiplied by some complex contrast term. This can be combined with the anyway unknown $\xi(x, y)$ into one effective phase reference image. If that could be measured in a calibration, then $\theta$ can be determined. Unlike the amplitude system, the assumption of $s_3 = 0$ is not required, so the measurement is valid for finite ellipticity. However, to calibrate out $\xi$, a measurement with known polarisation state and exactly the same spectrum as the plasma is required and this is difficult to achieve for plasma measurements. Beam-into-gas experiments were considered for this purpose but it has been theoretically predicted and experimentally observed\textsuperscript{16-18} that the spectrum can change significantly between beam-into-gas and plasma measurements due to the difference in the atomic physics governing the excited state populations of the beam neutrals. In any case, the measurement requires that the spectrum be perfectly stable. This is not the case for the AUG IMSE, as it observes all three beam energy components, so changes to the electron density profile may also change the phase reference.

G. Temporally switched mode

One method to eliminate $\xi$ is to use a quarter-wave Ferroelectric Liquid crystal (FLC). The FLC can be driven into one of the two states: in the nominally “off” state, its axes are aligned to those of the Savart which adds only a constant 90° to $\phi_0$ and has no significant effect on the measurement. In the “on” state, the FLC axes are aligned at 45° and act as the quarter wave plate in Section III. F. The FLC can be switched rapidly ($<50 \mu s$) so that images in both states are recorded with the same spectrum and polarisation. The “off” state then carries the phase reference,
\[
\tan \Phi(+,+) = \frac{\Im[A(+,+)]}{\Re[A(+,+)]} = \frac{s_2 \sin \xi + s_3 \cos \xi}{s_2 \cos \xi - s_3 \sin \xi} = \tan(\xi) \quad \text{for} \quad s_3 = 0. \tag{21}
\]

For zero ellipticity ($s_3 = 0$), $\theta$ can be isolated from the phase difference between the states,
\[
2\theta = \Phi'(+,+) - \Phi(+,+). \tag{22}
\]

Since only a single component is needed, a single displacer plate could be used, producing only one set of spatial fringes. This allows high spatial resolution in one direction and was chosen for the IMSE system on K-Star.\textsuperscript{19}

H. Effective ellipticity

There are two possible sources of ellipticity in the IMSE system. The presence of a small phase shift in one of the optical elements would introduce a small ellipticity to both components and add a fixed offset to $\theta$. Regardless of the orientation or phase shift of the hypothetical element, the circular component $s_3$ will be the same magnitude with opposite sign for the $\pi$ components as for the orthogonally polarised $\sigma$. It is therefore summed by the spectral integration, giving an $s_3$ term in the result exactly as if only $\sigma$ is measured.

Ellipticity may also be present in the beam emission. The MSE situation lies in reality somewhere between a pure Stark effect due to the Lorenz electric field and the Zeeman effect caused directly by the magnetic field. A detailed examination can be found elsewhere\textsuperscript{6} but an important feature is that the circular contribution of the two $\pi$ lines should have equal magnitude with opposite handedness. With opposite signs of $s_3$, these should approximately cancel in the spectral integral, leaving only $s_3$ of the sigma component.

Generally, all circular contributions to the spectrum will be integrated into a single effective $s_3$ in the $A(+, \pm)$ Fourier components, although the relation between this and the individual component ellipticities is not trivial. The effective $s_3$ appears in the amplitude modulated measurement (Equation (16)) as
\[
\frac{4A(+,+)}{A(+,0)^2} = -\frac{s_3^2 + s_3^2}{s_1^2} = -\frac{\tan^2\theta - \tan^2\chi}{\cos^22\theta}. \tag{23}
\]

where $\chi$ is the ellipticity angle and $\theta$ the orientation of the major axis. Even for small $\chi$, the second term can be large at very small $\theta$. It is therefore prudent to choose a working range nearer $\theta = 45^\circ$ than $\theta = 0$. Ellipticity has no effect on the simple phase measurement (FLC-on, Equation (20)) but does appear in the phase reference of the switched system, where Equation (21) becomes
\[
\tan \Phi(+,+) = \tan(\xi + \kappa) \quad \text{and} \quad \tan \kappa = \frac{\tan 2\chi}{\sin 2\theta}. \tag{24}
\]

This linear addition is much stronger than for the amplitude modulation, where $\tan \chi$ is added in quadrature.

Fortunately, the effective ellipticity itself can be measured and eliminated from Equation (23) or (24). In the FLC-on state, the effective ellipticity is held in the amplitude modulation and Equation (16) becomes
\[
\frac{4A(+,+)}{A(+,0)^2} = -\tan^2\left(\frac{\pi}{4} - \chi\right). \tag{25}
\]

There are in general many possible IMSE variants, each with particular advantages and disadvantages. The FLC was included in the AUG IMSE prototype primarily to measure the effective ellipticity but also to assess the performance of the phase encoded and switched measurements. At the vest least, the phase information can be used to eliminate one of the 45° ambiguity points from the amplitude modulation.

IV. PERFORMANCE TESTS

A. Setup of tests

The setup shown in Figure 5(a) was created to mimic the basic MSE spectrum. Two semi-silvered mirrors are used
to superimpose the images of three intensity controlled bulbs representing the $\sigma$ and $\pi\pm$ components. Before combination, each path passes through an interference filter and polariser to create the desired polarised spectrum. The filters have approximately the same spectral width as the MSE components and are tilt-tuned to mimic the expected splitting $\Delta\omega$. All other components are centralised and set perpendicular to the system optical axis to within ±0.5°. The $\sigma$ and $\pi$ polarisers are oriented orthogonally to within ±0.05° and approximately vertical and horizontal to mitigate differences between $s$ and $p$ transmittance and reflectance at the 2nd combining mirror. The combined test image is focused onto a virtual image plane similar to that created by the AUG MSE viewing optics, to which the complete IMSE diagnostic is coupled. A spectrometer, with a resolution of ~0.5nm, is attached behind the 2nd mirror to monitor the on-axis (image centre) spectrum, which is shown in Figure 4(a). The filter blue-shift with incidence angle causes the spectrum to change across the field of view and while this is also true of the real MSE spectrum due to the varying Doppler shifts of the beam particles, it is not possible to simulate the true variation with the available filters.

B. Check of delay optimisation

Before assembling the full crystal set, the correct optimisation of $\phi_0$ was confirmed. Figure 5(b) shows image cross sections for each bulb individually and together using only the zero nett delay Savart plate. As expected, the orthogonal polarisations of $\sigma$ and $\pi\pm$ almost entirely cancel. Figure 5(c) shows the same, with the full optimised delay $\phi_0$ in which the interference patterns add as intended.

C. Amplitude encoded $\theta$—Intrinsic contrast

The test setup includes stepper-motor controlled half and quarter wave plates to rotate and add ellipticity to the simulated spectrum. Figure 6 shows the result of the initial amplitude modulated measurements, evaluated by Equation (16), in which a systematic discrepancy of up to 7° can be seen.

It was determined that the discrepancy is caused by a large intrinsic contrast reduction in the Savart plate. Inhomogeneities in the crystal plates can cause reductions in the contrast when a large fraction of the plate volume is illuminated. Savart plate inhomogeneities give a reduction by some factor $\mu$ of the amplitude of the $\phi_1$ terms in Equation (2) and produce a measurement of $\tan 2\theta = \mu \tan 2\theta$, exactly as observed in Figure 6. The delay and displacer plates also have significant intrinsic contrast but since the factor appears equally in all terms of Equation (2), it does not affect $\theta$.

To further investigate the problem, a separate experiment was performed where a neon lamp is imaged onto a 2 mm point on the surface of the Savart plate. This is defocused onto the CCD to provide an interference image from the full range of incidence angles passing through the plate at one location. The plate is then scanned using two translation stages to produce a measurement of the phase $\phi$ and local contrast across the plate dimensions. The local contrast is around 0.70 everywhere on the surface. This could be an inherent contrast of the rest of the system, e.g., imperfect focusing of the imaging lens or could be due to very small scale inhomogeneities inside the 2 mm illumination point. In either case, it contributes a fixed value to $\mu$ that can be easily measured in the lab and calibrated out.

Figure 7 shows the measured phase, the strong variation of which presents a serious issue. In the normal IMSE setup, a large area (preferably all) of the plate surface area is illuminated for each point on the CCD. The variation of phase within that area blurs the fringes and reduces the contrast. For a linear slope covering 60°, the contrast would be reduced by 5%, causing an error of up to $\delta\theta = 1.0°$. The phase is a complex function of the surface, so any factor which changes the illuminated area of the crystal will change $\mu$. First, the image position has this effect due to vignetting, which makes $\mu$ itself a 2D quantity $\mu(x, y)$. The most critical implication, however,
is that $\mu$ now depends on the input light cone which can be influenced by all lenses and apertures between the plasma and the CCD. To calibrate $\mu$, a measurement like in Figure 6 must be made for a light source at the neutral beam position using the same observation optics as for the measurement. Care must then be taken to separate the intrinsic contrast effect from those introduced by those optics, such as birefringence.

The non-switched (without FLC) phase encoded IMSE measurement will be affected in the sense that the phase calibration that is already required is dependent on the collection optics, as is the case for Doppler coherence imaging systems.\textsuperscript{20} The switched phase encoded system (e.g., Ref. 11) is however not affected, as the phase reference is inherently taken with the true light source on each frame pair.

Measurements with a Fizeau interferometer confirm the phase variation and isolate non-zero parallelism of the plate surfaces as the dominant cause. Crystal manufacturers are typically able to polish the crystals to a parallelism of 20 arc-sec, which would allow a 3 $\mu$m variation in thickness across the crystal. Figure 7 includes an axis showing the thickness deviation correspondence to the measured $\phi$. Mechanical stress on the crystal from its holder also causes a significant variation of phase although this is easily mechanically solved.

Efforts to polish the crystals to low enough parallelism are ongoing, but in the meantime, our attention returns to calibrating away the issue. The half-wave plate scan already provides the required calibration and is easy to perform in the lab. Figure 8 shows the value of $\theta$ obtained for 4 image positions after correcting for the intrinsic contrast. Within the expected operating range of $\theta$, the error is below the desired 0.2° for all except two corners of the image.

For the plasma measurements, the situation is more difficult as access inside the vessel, while the diagnostic installed is not possible for the AUG prototype IMSE. It is however possible to use the neutral beam itself as the light source. During a normal plasma shot, a polariser is placed in the IMSE system and rotated. If a large enough scan can be completed during a plasma shot, the $\mu$ image can be recovered. In practice, the method works well enough to achieve ∼0.5° accuracy, but is not entirely satisfactory because the polariser is now selecting parts of the spectrum by polarisation state and polarising them in the same direction. At some angles, equal intensities of $\pi$ and $\sigma$ are allowed to pass and now with the same polarisation destructively interfere due to the delay optimisation that normally ensures constructive interference when they are orthogonal. It should be possible to alleviate this problem by adding a static polariser to isolate just the $\sigma$ component and then a quarter wave plate to linearly polarise it, just before the rotating polariser.

D. Phase encoded $\theta$ and temporally switched mode

Figure 9(a) shows the residual of the phase-based measurement from Equation (20). A large deviation as in the uncorrected amplitude modulated system is not present, as expected since the plate surface variation should not distort the variation of the phase with polarisation angle when the optical setup is not changed. There is however a systematic variation of ∼0.8°. This results from the non-ideal behaviour of the FLC that provides the function of the quarter-wave plate in this case. Subsequent measurements showed that the FLC has a phase shift of 84° instead of 90° and axis orientation of 41° instead of 45° when switched on. The FLC could easily be replaced with a precise, properly aligned zero-order wave plate but this would leave no possibility of calibrating the reference phase, without being sensitive to changes in the spectrum.

The FLC-off state in principle provides this phase reference but it is strongly affected by ellipticity, as seen in Equation (24). In the test setup, the half-wave plate used to rotate the spectrum has a phase shift of 177° at the test wavelength and the small ellipticity generated greatly distorts the phase reference. The switched FLC response, shown in Figure 9(b), is therefore significantly distorted.

The non-ideal behaviour of the FLC is a significant problem for the switched systems and has been examined more thoroughly elsewhere, but an additional discovery here is that the axis rotation can be adjusted by controlling the temperature of the FLC. In this case, heating it to 47 °C achieves the correct 45°. The phase shift however remains an issue.
It should also be noted that the phase modulated systems have a higher observed noise level than the amplitude modulation. This is in qualitative agreement with the theoretical prediction made elsewhere.\textsuperscript{22}

E. Ellipticity

Quarter and half-wave plates in the test setup can be used to add a known ellipticity to the components and to rotate the result arbitrarily. Unfortunately, the wave plates used here have phase shifts of 177° and 83.2° at the target wavelength, so the generated polarisation does not relate exactly to the plate angles as if they were ideal wave plates. The created polarisation state can however be accurately modelled. Figure 10 shows this calculation and the measured ellipticity for a scan of the half-wave plate angle at two positions of the quarter-wave plate. The measured ellipticity has been compensated for the intrinsic contrast effect. There is a systematic disagreement of up to 0.8° which is readily explained by the non-ideal behaviour of the FLC described previously and this implies that the ellipticity measurements cannot be considered accurate to more than a few degrees. This is however sufficient to identify any ellipticity strong enough to affect the amplitude modulated \( \theta \) measurements.

F. Background and contamination

The strongest source of background contamination comes from \( H\alpha \) emission from high temperature plasma ions neutralised by charge exchange with the beam. This produces a wide spectral distribution that extends out into the edge of the MSE signal. To mitigate this, a very steep low-pass interference filter is added to the AUG IMSE to almost completely exclude the \( H\alpha \) charge exchange light. The filter was added to the forward model and to the test setup, in order to ensure that the abrupt cut of MSE spectrum does not affect the amplitude based measurements by creating large differences in the contrast terms (Equation (18)).

Any unpolarised background light clearly cannot affect the measurement as it does not produce an interference pattern. Wide-band sources such as Bremsstrahlung or FIDA (Fast-Ion D-alpha) will also not produce strong fringes, even when polarised, due to the large delay \( \phi \), reducing its contrast. To test this, a strong wide-band light source was introduced to the test setup on top of the test polarisation image. Figure 11 shows the result, in which no effect beyond the desired \( \delta \theta \sim 0.2^\circ \) can be seen.

The remaining concern is that metal components in the visual background might reflect and polarise narrow-band (e.g., MSE) light which could disturb the measurement. Although little can be done to remove such reflections, they should be easy to identify as they will appear with sharp changes on the object edges in the \( \theta \) image. To demonstrate the effective spatial resolution of the IMSE system, a polarising mask was added to the test setup at the virtual image location. Figure 12 shows this mask and the resulting \( \theta \) image. Where the polarising mask is present, the intensity is greatly reduced and the noise level is higher. Despite this, the sharp changes are clearly seen with no contamination of the neighbouring regions. The transition regions include faster changes than the highest frequencies of the component Fourier windows which causes the calculation of \( \theta \) to break down. This produces imaginary values from Equation (16) which are depicted white in the image.

G. Temperature sensitivity

The refractive indices of the static crystals and both the phase difference and rotation angle of the FLC all have significant temperature dependence. In order to minimise the resulting drifts and systematic uncertainties, the crystal plates and FLC are housed within a sealed temperature control cell which maintains the temperature to within ±0.25 °C. To examine the effect of the temperature variations within this uncertainty, a fixed input polarisation was measured while the cell was heated from 32 °C to 47 °C while switching the...
FLC. The various inferred quantities are shown in Figure 13. Also shown are the steepest gradients from which the worst-case uncertainty can be assessed. The non-switched phase measurement is effectively unusable for plasma measurements with a systematic uncertainty of ±3.75°. The switched system is almost within the acceptable range, at ±0.3° and for the amplitude based measurements of θ and χ, temperature instability can be entirely neglected, with a value of less than ±0.02°.

H. Spectrum sensitivity

Changes to the externally imposed toroidal magnetic field between shots will change the multiplet splitting (Δω) and changes of the beam energy will change both this and the Doppler shift (ω0). During each shot, these should be stable but the plasma diamagnetic field could dynamically change Δω and changes to the density profile will change the relative weight of the 3 energy components. Further subtle changes could be introduced by changes to the σ vs π intensities due to non-equilibrium distribution of the excited states of the beam neutrals. It must therefore be demonstrated that the IMSE is independent to small changes in Δω and ω0. A wide enough variation of the component wavelengths was difficult to achieve with the tilt tuning of the filters in the test setup, so a separate experiment was conducted using a temperature controlled diode laser. A set of images were recorded while scanning the laser wavelength with a polariser fixed at 22°.

VI. SUMMARY

A thorough examination of the performance and robustness of the prototype IMSE has been made, examining each of the possible operation modes and measurements. It was found that several non-ideal effects of the components and assumptions in the data evaluation can easily cause systematic artefacts much greater than the desired resolution.

Of the different modes, the best performance is achieved by amplitude modulation, once the intrinsic contrast effect is corrected. It has less statistical noise, is less sensitive to ellipticity, and is satisfactorily insensitive to changes in the spectrum. The intrinsic contrast issue remains critical but the
outlook for improving or even eliminating this appears good, as the crystals can be independently polished to arc-second precision. Once this is solved, in order to push to $\delta \theta < 0.1^\circ$, a more systematic investigation of the spectral integration and contrast terms (Equation (15)) will be needed, followed by a more precise laboratory test of the spectrum sensitivity and of the ellipticity.

Although noisier, the switched FLC system is also promising if sufficiently accurate FLCs could be manufactured. At present, this appears unlikely as sourcing the FLCs has become difficult, particularly at large apertures. Solutions with two FLCs that minimise these effects and simultaneously isolate ellipticity have been proposed.\(^{21}\) Combined with the discovery that the phase shift of one state and the axis rotation in the other state can be corrected by controlling the temperature, a sufficiently accurate system might yet be possible. Although the amplitude based measurement is preferred here, the FLC will remain in order to test for ellipticity and correct it if necessary. The ellipticity measurement is sufficient for this, although not as good as expected.

The final conclusion of this work is that the AUG prototype IMSE is ready for operation using amplitude modulated operation. It can provide measurements of the polarisation orientation to within $\pm 0.2^\circ$ and the effective ellipticity to $\pm 2^\circ$ within an $\sim 30^\circ$ operating range, with an intrinsic contrast calibration and using Equation (16). This is robust against temperature changes of less than $\pm 2^\circ$ C, for multiplet splitting in the range $0.35 \text{ nm} < \Delta \lambda < 0.45 \text{ nm}$ and with unpolarised or wideband background.

**ACKNOWLEDGMENTS**

The author would like to thank Dr. S. Bozhenkov and Dr. A. Mlynek for many helpful discussions and Dr. M. Reich and the ASDEX upgrade team for providing information and support regarding application to AUG. This project has received funding from the European Union’s Horizon 2020 research and innovation programme under Grant Agreement No. 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.


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