g factor of the $2_1^+$ state of $^{168}\text{Hf}$

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The g factor of the $2_1^+$ state of $^{168}\text{Hf}$ was measured using the perturbed angular correlation technique in a static external magnetic field. The result, $g(2_1^+) = 0.17(3)$, is discussed in relation to the systematics of the previously reported g factors in the Hf isotopes and compared to the predictions of several models. An interesting outcome of the analysis presented in this paper has to do with the relatively small result for the g factor. This indicates that in the Hf isotopes, a minimum in the $g(2_1^+)$ dependence on $N$ occurs at $N \approx 98$ and not at midshell, as expected from IBA-2 or large-scale shell-model calculations. The pairing plus quadrupole model of Kumar and Baranger predicts a minimum at $N = 98$ and gives the best description of the experimental data. The present result clearly shows the importance of g-factor measurements in “fine-tuning” among different models.

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Systematic measurements of nuclear g factors provide useful information on nuclear structure evolution across the Periodic Table. In particular, studies of g factors of first excited $2^+$ states in even-even nuclei have been used to better understand the collective motion of protons and neutrons and the proton-neutron interaction. An important outcome from these studies is the introduction of the concept of effective proton or neutron numbers [1], i.e., in quite a few collective states, such a correlation is shifted by about five degrees. The experiment was performed at the tandem accelerator of the Wright Nuclear Structure Laboratory (WNSL) at Yale University. A beam of 15 pA, 130 MeV $^{16}$O was used to produce $^{169}\text{Ta}$ nuclei via the $^{159}\text{Tb}(^{16}\text{O}, n)^{168}\text{Ta}$ reaction. $^{168}\text{Ta}$ subsequently $\beta$-decays to $^{168}\text{Hf}$ with a half-life of 2 min. The $^{168}\text{Ta}$ activity was deposited on an aluminized tape and then moved in accumulation-transport-counting cycles of 5 min to the center of a superconducting dipole magnet. A coplanar set of eight coaxial hyperpure germanium detectors was positioned around the coil at about 11 cm from its center. Coincidence events between all 28 pairs of detectors were recorded. The detectors were arranged in order to maximize the number of pairs for which the separation angle between the detectors was 145° or 35°. At these angles, the maximum perturbation effect of the $\gamma-\gamma$ angular correlation is obtained for $0 \rightarrow 2 \rightarrow 0$ cascades. Six pairs of detectors were at 35°, six pairs were at 145°, and four pairs were at 180°.

The data acquisition system enabled the sorting of all $\gamma-\gamma$ coincidence events originating from the same angle between the respective detectors to be sorted in the same two-dimensional coincidence matrix. More details of the experimental setup were published elsewhere [3]. In Fig. 1, we present the total $\gamma$ projection spectrum obtained with the magnetic field oriented in the upward direction for all angles. The $\gamma$-ray energies of the $0 \rightarrow 2 \rightarrow 0$ cascade in the decay of $^{168}_{\text{Hf}}$ to $^{168}_{\text{Hf}}$ are 818–124 keV. In Fig. 1, we see several lines that belong to the decay of $^{169}_{\text{Lu}}$ to $^{169}_{\text{Yb}}$, $^{166}_{\text{Yb}}$, $^{166}_{\text{Lu}}$ being formed in the $(2\gamma n)$ channel of the reaction. In particular, very close to the 818 keV line of $^{168}_{\text{Hf}}$, there is a doublet composed of two $\gamma$ lines, 811 and 814 keV, which belong to the decay of $^{166}_{\text{Lu}}$. However, in the gated spectra the interference of these lines is completely eliminated.

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For example, in Figs. 2(a) and 2(b), we present the spectrum obtained by gating on the $2^+ \rightarrow 0^+$, 124 keV line of $^{168}$Hf. We note that the 811 and 814 keV doublet of $^{166}$Yb does not show up in this spectrum. Furthermore, when we place a gate on the doublet itself, we do not see the $2^+ \rightarrow 0^+$, 124 keV line of $^{168}$Hf, but we do see, as expected, the $2^+ \rightarrow 0^+$, 102 keV line of $^{166}$Yb [Fig. 2(c)]. Therefore, we are confident that the analysis of the $0^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$ cascade of $^{168}$Hf is free of any interference from lines in the decay of $^{166}$Lu to $^{166}$Yb.

We used a magnetic field of 5.55 T, and the total counting time was about 120 h with field up and 140 h with field down. In order to determine the $g$ factor, one should measure the shift of the angular correlation due to the interaction of the external field and the magnetic moment of the $2^+_1$ state. Another way to determine the $g$ factor is to use the double ratio:

$$R(\theta, B) = \left[ \frac{I(\theta, B)}{I(\theta, -B)} / \frac{I(-\theta, B)}{I(-\theta, -B)} \right]^{1/2},$$

where $I(\theta, B)$ is the coincidence intensity at angle $\theta$ and external field $B$. The use of this relation has the advantage that it eliminates the need for normalization for the total integrated count on target for field up and field down, and the different relative efficiencies of detector pairs cancel out and need not be accounted for. The coincidence intensity $I(\theta, B)$ for a given spin sequence can be calculated using the formalism given by Frauenfelder and Steffen [8]. Comparison of the calculated double ratios for a range of $g$ factors with the experimental value provides the experimental value of the $g$ factor. In the calculation of the double ratio $R(\theta, B)$, we used the usual convention for positive and negative angles as given in Ref. [9], i.e., clockwise rotation goes in the direction of the field. In particular, according to this rule we have the following simple relation: $I(35^\circ, +B) = I(145^\circ, -B)$. Since, as mentioned before, the maximum perturbation effect of the angular correlation is obtained for $0^+ \rightarrow 2^+ \rightarrow 0^+$ cascades at $145^\circ$ and $35^\circ$, only the experimental values $R(145, 5.55T)$ of this cascade were used to extract the $g$ factor, and all other experimental values of the double ratio were used as consistency checks, as described in previous publications [3–5]. The data at $35^\circ$ were used in the calculation of $R(145, 5.55T)$, using the above relation between $I(35, B)$ and $I(145, -B)$.

In Table I, we present the results of this experiment. In columns 1–4, the energies of the $\gamma$ rays, the spin sequences, the angles, and the experimental values of the respective double ratios are given. For all the experimental values of $R$ that were not used to determine the $g$ factor, we present in column 5 the values of $R(\theta, B)$ calculated using the deduced value of the $g$ factor. The good agreement, within the experimental errors, between the experimental and calculated values in columns 4 and 5 can be considered as evidence of the lack of systematic errors in the experiment.

In Fig. 3, we present the calculated values of the double ratio for a $0^+ \rightarrow 2^+ \rightarrow 0^+$ cascade as a function of the $g$ factor. In the calculation of the double ratio, we used the recently reported value of the half-life of the $2^+_1$ state, $T_{1/2} = 1.237(10)$ ns [10]. The experimental value of $R(\theta, B)$ with its error bar is shown as a horizontal line. The deduced experimental value from Fig. 3 is

$$g(2^+_1)_{\text{exp}} = 0.17(3).$$

We now proceed to discuss the significance of the result within the framework of several models. In Ref. [5], we compared the known experimental $g$ factors of $2^+_1$ states in Hf isotopes with calculations using four different models: the hydrodynamic model of Greiner [11], the IBA-2 model [12], the semiempirical model of Zhang et al. [6], which uses the concept of effective valence particles, and the large-scale shell-model calculation of Bao-An Bian et al. [13]. In Fig. 4,
TABLE I. The values of the double ratio $R(\theta, B)$ obtained from the coincidence data for several cascades of $^{168}$Hf (see text).

<table>
<thead>
<tr>
<th>Cascade (keV)</th>
<th>Spin sequence</th>
<th>Angle (deg)</th>
<th>$R_{\text{exp}}(\theta, 5.55T)$</th>
<th>$R_{\text{calc}}(\theta, 5.55T)^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>818–124</td>
<td>$0^+_2 - 1^+_2 - 0^+_1$</td>
<td>145</td>
<td>1.26(5)</td>
<td></td>
</tr>
<tr>
<td>262–124</td>
<td>$4^+_1 - 2^+_1 - 0^+_1$</td>
<td>145</td>
<td>1.006(8)</td>
<td>1.03(1)</td>
</tr>
<tr>
<td>752–124</td>
<td>$2^+_2 - 1^+_2 - 0^+_1$</td>
<td>145</td>
<td>1.08(2)</td>
<td>1.09(1)$^b$</td>
</tr>
<tr>
<td>907–124</td>
<td>$3^+_1 - 2^+_1 - 0^+_1$</td>
<td>145</td>
<td>0.96(2)</td>
<td></td>
</tr>
<tr>
<td>818–124</td>
<td>$0^+_2 - 1^+_2 - 0^+_1$</td>
<td>180</td>
<td>0.97(4)</td>
<td>1.00</td>
</tr>
<tr>
<td>262–124</td>
<td>$4^+_1 - 2^+_1 - 0^+_1$</td>
<td>180</td>
<td>1.02(1)</td>
<td>1.00</td>
</tr>
<tr>
<td>752–124</td>
<td>$2^+_2 - 1^+_2 - 0^+_1$</td>
<td>180</td>
<td>1.01(3)</td>
<td>1.00</td>
</tr>
<tr>
<td>907–124</td>
<td>$3^+_1 - 2^+_1 - 0^+_1$</td>
<td>180</td>
<td>1.004(34)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*aThe values of $R_{\text{calc}}$ and its error bars, where given, were obtained using the value $g(2^+_1)_{\text{exp}} = 0.17(3)$ (see text).

$b$The double ratio was calculated assuming pure $E2$ character for the first transition of the cascade.

we present again the results of these calculations, as well as the experimental $g$ factors including the result of the present experiment at $N = 96$. We see that none of the aforementioned models reproduces the clear trend indicated by the result for $^{168}$Hf, namely, an increase of the $g$ factors as a function of neutron number beyond $N = 96$. The IBA-2 predicts a minimum of the $g$ factors at midshell, and consequently an increase of the $g$ factors only beyond $N = 104$, as expected for any valence model. The shell model also predicts a minimum at $N = 104$, but, in addition, shows a shallow minimum at $N = 98$. The systematics of the experimental $g$ factors, with the significantly low value for $^{168}$Hf from this experiment, clearly indicates that a minimum in the $g$-factor dependence on $N$, or at least a sharp drop in their values, should be at $N = 96$ or less. None of the four models reproduces this dependence. A fifth calculation, using the pairing plus quadrupole model of Kumar and Baranger [14], is also presented in Fig. 4. This calculation predicts only one minimum in the $g$-factor dependence on $N$, at $N = 98$, and the absolute value of the predicted $g$ factor for $^{168}$Hf (at $N = 96$) is the closest of any of the models to our experimental result, and in fact agrees with it within two standard deviations of the experimental error. The other experimental values in Fig. 4 do not disagree with the predictions of the Kumar and Baranger model at the two-standard-deviations level. In fact, the predictions of this model give the best agreement with the experimental results. An interesting outcome of this work is that it indicates that in the Hf isotopes, there seems to be a minimum in the $g$-factor dependence on $N$, not at midshell, as expected, but at a lower value of $N$. The only theoretical model that predicts only one minimum in the $g$ factors, and at an $N$ value significantly lower than midshell, is that of Kumar and Baranger. In order to confirm these statements, more experimental results are needed, especially for Hf isotopes with $N < 96$. Also, more theoretical work is warranted in order to explain the difference in the predictions of the models discussed here, and to find out why the pairing plus quadrupole model of Kumar and Baranger predicts a minimum at $N = 98$. Finally, we reiterate the importance of $g$-factor measurements to help “fine-tune” among different theoretical models.

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FIG. 4. Systematics of $g(2^+_1)$ data for the Hf isotopes. The point at $N = 96$ is from the present work. The results for $N = 98$, 100 are from our previous measurements [4,5], and the other three experimental points are from Stone’s tabulation in [15]. The data are compared with the predictions of several models: the rotational and vibrational limits of the hydrodynamical model [11], the IBA-2 [12], the phenomenological model of Zhang et al. [6], the results of a large-scale shell model microscopic calculation [13], and the pairing plus quadrupole calculations of Kumar and Baranger [14].
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