Dipole-fiber systems: radiation field patterns, effective magnetic dipoles, and induced cavity modes

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ABSTRACT

We study the radiation patterns produced by a dipole placed at the surface of a nanofiber and oriented perpendicular to it, either along the radial (r -oriented) or azimuthal (ϕ -oriented) directions. We find that the dipole induces an effective circular cavity-like leaky mode in the nanofiber. The first radiation peak of the ϕ -oriented dipole contributes only to TE radiation modes, while the radiation of the r -oriented dipole is composed of both TE and TM radiation modes, with a relative contribution depending on the refractive index of the nanofiber. We reveal that the field pattern of the first resonance of a ϕ -oriented dipole is associated with a magnetic dipole mode and strong magnetic response of an optical nanofiber.

Keywords: dipole-fiber system, whispering gallery modes, induced magnetic dipole

1. INTRODUCTION

The interaction between quantum emitters and electromagnetic waves in cavities, recognized as cavity quantum electrodynamics (CQED),^{1,2} has been explored for weak and strong coupling regimes for different cavities,³ such as microspheres,^{4–6} microtoroids^{7,8} and microdisks.⁹ The modification of different aspects of the emission of quantum emitters, such as radiation peaks, in the vicinity of dielectric waveguides have also been considered.^{10–18} Radiation peaks in the emission of quantum emitters in the vicinity of optical fibers have been observed experimentally¹⁰ and studied theoretically being associated with the whispering gallery modes (WGMs) of fibers.^{15,17,18} The coupling of the emission from quantum emitters into guided and radiation modes of optical fibers has been studied,^{10,13,15–17,19–24} and identified as a potential platform for developing devices for quantum information science.^{20,23,24}

Here, we investigate the radiation modes and field patterns of a dipole-fiber system where the dipole is placed perpendicular to the fiber with two different orientations, either along the radial (the so-called r -oriented dipole) or azimuthal (the so-called ϕ -oriented dipole) directions. We show that the field pattern clearly indicates the formation of induced radial cavity modes which overlap with the positions of TM-polarized whispering gallery modes. In addition, we find that the first radiation peak of the ϕ -oriented dipole has contribution only from TE radiation mode, while the first peak of the r -oriented dipole-fiber system has strong contribution of both TE and TM radiation modes. The relative contribution of TE and TM radiation modes in the system depends on the refractive index of the fiber. We reveal that the field pattern of the first resonance of a ϕ -oriented dipole is associated with a magnetic dipole mode and strong magnetic response of an optical nanofiber.

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2. ANALYTICAL RESULTS

We consider an optical system consisting of an electric dipole and air-clad fiber, as shown in Fig. 1. The electric dipole is located on the core-clad interface and the fiber is oriented along z -axis. We assume that the dipole emits at a single wavelength ($\lambda = 700\text{nm}$) and telluride ($n = 2.02$) is the hosting material of the fiber. It was shown that a z -oriented dipole-fiber system have strong resonance peaks in radiated power, which are associated with the TE whispering gallery modes (WGMs) formed in the cross section of the fiber.¹⁸

Figure 1. Schematic diagram of electric dipole and fiber system.

To find the total radiated power, one can start from the total electric field of the system.¹⁸ The total electric field of the dipole-waveguide system can be expanded using z -propagating plane waves as:¹⁸

$$\mathbf{E}(x, y, z) = \sum_j a_j \mathbf{e}_j(x, y) e^{i\beta_j z} + \sum_j \int_0^{Q_{\max}} a_j(Q) \mathbf{e}_j(x, y, Q) e^{i\beta(Q)z} dQ + BK, \quad (1)$$

in which n_{co} and n_{cl} are the refractive indices of the core and cladding, respectively, a_j , $a_j(Q)$ and $\mathbf{e}_j(x, y)$, $\mathbf{e}_j(x, y, Q)$ are coupling coefficients and modal vector fields for forward guided and radiation modes, respectively, β_j 's are the corresponding propagation constants, $Q = (D/2)(k^2 n_{cl}^2 - \beta^2)^{1/2}$, D is the core diameter, $k = 2\pi/\lambda$, and BK represents the contribution of backward guided and radiation modes.^{23,25} In general, guided and radiation modes are orthogonal to each other in the sense that $\int_{A_\infty} (\mathbf{e}_i \times \mathbf{h}_j^*) \cdot \hat{z} dA = 2N_i \delta_{ij}$ and the discrete and continuous values of the propagation constants β of these modes, respectively, are bounded by:²⁵ $k_{cl} \leq \beta_j \leq kn_{co}$ for guided modes and $0 \leq \beta \leq kn_{cl}$ for radiation modes. The coupling coefficients a_j are:

$$|a_j|^2 = \frac{\omega^2}{16N_j^2} |\mathbf{e}_j^*(\mathbf{r}_0) \cdot \mathbf{p}_0(\mathbf{r}_0)|^2, \quad (2)$$

with a similar equation for $a_j(Q)$. Here ω is the transition frequency of the dipole, $\mathbf{p}_0$ is the dipole moment with the strength of p_0 , and $N_j = (1/2) \int_{A_\infty} (\mathbf{e}_j \times \mathbf{h}_j^*) \cdot \hat{z} dA$ is a normalization term. The power captured in each mode is $P_j = |a_j|^2 N_j$ and hence the total emitted power of the dipole-waveguide system is the sum of the power captured into all guided and radiation modes, i.e.

$$P_{\text{total}} = \sum_j P_j + \sum_\nu \int_0^{Q_{\max}} P_\nu(Q) dQ. \quad (3)$$

The normalized radiated power for a ϕ - and r -oriented dipole excitations (cylindrical coordinates) are shown in Figs. 2 (a) and (b). The total radiated power is normalized to radiated dipole power in telluride glass, $P_{Te} = \mu_0 p_0^2 \omega^4 n / 12\pi c$. Due to the normalization to P_{Te} , the results shown in Fig. 2 are independent of the dipole strength p_0 . Note that the ratio of the radiation power to P_{Te} is equal to the Purcell factor. It can be seen that for an r -oriented dipole the enhancement is very strong (more than 2 for the first radiation peak).

We investigate the physics behind the radiation peaks by using an independent method that explains the formation of WGMs in a 2D circular disk, which is a representation of a thin cross section of the fiber.¹⁸ For such a disk, TE (with non-zero components; $\mathbf{E}_z$, $\mathbf{H}_r$, and $\mathbf{H}_\theta$) and TM (with non-zero components; $\mathbf{H}_z$, $\mathbf{E}_r$, and

Figure 2. Normalized total radiated power of a dipole-fiber system with (a) ϕ -, and (b) r -oriented excitation. All the radiated powers are normalized to the dipole radiation power in bulk tellurite glass. The radius of the $(m, 1)$ resonances of 2D TM-WGMs are shown with red triangles.

$\mathbf{E}_\theta$) solutions of Maxwell's equations, for electromagnetic fields propagating around the disk, have the dispersion relations;

$$J'_m(n_{co}x)H_m(n_{cl}x) - q_s H'_m(n_{cl}x)J_m(n_{co}x) = 0, \quad (4)$$

where $x = kD/2$, J_m and H_m are the Bessel and Hankel functions, and $q_s = (n_{cl}/n_{co})$ for TE and $q_s = (n_{co}/n_{cl})$ for TM modes. Solutions of Eq.(4) determine the location of WGM resonances ($kD/2$ values) that can be labeled by (m, p) , where m and p are the azimuthal and radial mode numbers, respectively.

We observe that the fundamental $(m, 0)$ TM-WGM resonances at $\lambda = 700$ nm overlap nicely with the radiation peaks of the ϕ -oriented dipole-fiber system, Fig.2 (a), which have been obtained independently through Eqs. (4). While, for an r -oriented dipole-fiber system it nicely overlaps for $m > 4$ resonances. To investigate the reason for discrepancy of the location of the peaks and TM-WGM resonance, we looked into the contribution of TM and TE radiated components. The radiation modes are divided into TE- and TM-like elementary modes,²⁵ modes where respectively the longitudinal electric and magnetic fields are zero or negligible. The total radiation power is the summation of the TE and TM radiation mode powers.^{23,25}

The normalized radiated power and the contribution of TE and TM components form a ϕ - and r -oriented dipole excitation are shown in Figs. 3 (a)-(c). When the electric dipole source is oriented along ϕ direction, the total radiation has a dominant TE component, which defines the position of the resonance peaks [see Fig. 3(a)]. A ϕ -oriented dipole has a magnetic field oriented along z direction in the xy -plane leading to excitation of the radiated modes with z -component of magnetic field, i.e. mainly the TE modes. When the electric dipole source is oriented along r direction (Figs. 3 (b) and (c)), the four first radiation peaks have both TE and TM radiation contribution. An r -oriented dipole has a magnetic field oriented along z direction in the xy -plane and an electric field along z direction in the xz -plane leading to excitation of both TE and TM radiated modes respectively. As a result, the four first radiation peaks do not align with the position of the WGM resonances due to existence of both components. Comparison of Figs. 3 (b) and (c), where the refractive index of the fiber is different, shows that the contribution of TE and TM components depend on the refractive index of the fiber for the r -oriented dipole. This is due to higher concentration of electric field component in the higher dielectric medium.

3. NUMERICAL MODELING

In order to verify the formation of whispering gallery modes in the system, we modelled our structure using CST microwave studio. We use an electric dipole excitation as the source. We use open boundary conditions (waves passing through the boundary with minimum reflection, i.e. perfectly matched layer) in the z -direction and "open (add space)" boundary conditions (similar to open with an extra space) in x - and y -directions, to represents infinitely long fiber along z -axis. We examine the dipole-fiber system at the position of the eight

Figure 3. Normalized total radiated power (black dashed line) and the contribution of TM (blue dotted line) and TE (red solid line) radiated power of a dipole-fiber system with (a) ϕ -, (b) and (c) r -oriented excitation. The refractive index of the fiber is $n_{\text{core}} = 2.05$ for parts (a) and (b), while it is $n_{\text{core}} = 4$ for part (c). The radius of the $(m, 0)$ resonances of 2D TM-WGMs are shown with red triangles.

radiation peak, shown with blue arrows in Fig. 2. The position of these radiation peaks overlaps with TE WGM (8,0) for z orientation and TM WGM (8,0) for ϕ and r orientations. Figure 4 (a) shows the z -component of electric field in the cross-section of the fiber. It confirms the formation of TE(8,0) whispering gallery mode in the fiber cross-section. Figures 4 (b) and (c) shows the z -component of magnetic field in the cross-section of the fiber. It confirms the formation of TM(8,0) whispering gallery mode in the fiber cross-section when dipole is oriented along ϕ and r , respectively. Figures 4 (d) and (e) show the 3D electric field patterns for z -oriented dipole system respectively at the position of the second and eighth radiation peak. These verifies that for higher order modes (Fig. 4 (e)) the radiation is localized within a transverse plane normal to the fiber axis and at the position of the dipole, i.e. the peak is mainly due to contributions from a very small range of non-transverse propagating modes (skew modes) compared to that of low order resonances (Fig. 4 (d)). Note that the dipole is located on the core-clad interface on the right hand side of the fiber, shown with black circle in Figs. 4 (a)-(c).

We also investigate the magnetic and electric fields at the position of the first peak for each orientation of dipole. We noticed that when electric dipole is along ϕ direction the magnetic field inside the fiber makes a loop as shown in Fig. 5. This indicates formation of a magnetic dipole along the fiber, which is induced due to z -component of the magnetic field of the excitation.

4. CONCLUSIONS

We have studied the radiation patterns of an r - and ϕ -oriented dipole-fiber system and observed that the dipole induces radial cavity modes which overlaps with position of whispering gallery modes. Moreover, we have found that the first radiation peak of the ϕ -oriented dipole has contribution only from TE radiation modes, while that of the r -oriented dipole-fiber system has strong contribution from both TE and TM radiation modes. Furthermore, we have demonstrated the formation of WGM using numerical modelling and showed that the field pattern of the first resonance of a ϕ -oriented dipole-fiber system corresponds to an effective magnetic response.

Figure 4. (a) E_z component of the electric field in $z = 0$ plane, when dipole is oriented along the z direction. (b) and (c) H_z component of the magnetic field in $z = 0$ plane, when dipole is oriented along the ϕ and r directions, respectively. The black circle represents the core-clad interface. The diameter of the fiber is $1.1 \mu\text{m}$, $1.285 \mu\text{m}$ and $1.285 \mu\text{m}$ respectively for parts (a), (b) and (c), which overlaps with (8, 0) resonances of 2D TE-WGM for part (a) and TM-WGM for parts (b) and (c). The dipole is located on the core-clad interface on the right hand side of the fiber, shown with black circle. The 3D electric field patterns for z -oriented dipole system at the position of the (d) second and (e) eight radiation peaks.

Figure 5. H_z component of the magnetic field in $y = 0$ plane, when dipole is oriented along the ϕ directions. The induced magnetic dipole is highlighted with red elliptical shapes.

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