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HOUSEHOLD SAVING, TIME ALLOCATION AND TAXATION

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Abstract

Current discussions of tax policy and pension reform are very much concerned with the implications of policy changes for household saving. This paper analyses these issues in the context of a model which recognises that households typically consist of two individuals who jointly take decisions on saving and labour supply, and that an important issue in evaluating the impact of policy change arises out of the fact that labour supplies of secondary earners vary substantially across households. We show that, when we take account of this, the conventional wisdom about the effects on household saving of tax changes which increase inequality of after-tax incomes may not hold. Given certain plausible stylised facts about labour supply elasticities, reductions in the higher tax rates, financed by increases in the lower tax rates, can reduce saving, as well as welfare overall.

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1 Introduction

An issue of some significance for public policy in a number of countries is that of the effects of changes in tax rates and pension arrangements on household saving. Among the changes in tax policy currently under consideration is a flattening of the progressive tax rate structure, mainly through reductions in the higher marginal tax rates. Given revenue neutrality, this must be associated with increases in the lower marginal rates of tax or in indirect tax rates. Overall, therefore, such reforms can be expected to be regressive in impact, but the resulting increase in inequality of disposable incomes, insofar as it is not in itself regarded as desirable, is considered an acceptable price to pay for the expected improvements in incentives to work and save. The resulting increases in labour supply and capital accumulation, it is argued, will increase welfare overall. In addition, in the light of the expected increases in the ratio of retired to economically active households in the populations of virtually all the developed economies, an issue of considerable concern in many of them is the question of whether state pensions financed by "pay as you go" arrangements, that is, out of current taxation, should be at least partly replaced by pension funds built up out of private saving. To facilitate the latter, tax incentives for saving are often advocated.

The theoretical analysis of such measures as these is typically conducted in the framework of a two-period model of individual consumer choice, in which current labour supply and intertemporal consumption allocation decisions are made optimally in the light of the prevailing tax and pension parameters[1]. Thus labour supply and saving are determined jointly by net of tax wage and interest rates. In this paper we want to introduce a new element into the analysis of these decisions, that we think is of considerable relevance for the evaluation of the policy proposals in practice. A central feature of the real economy, excluded from the standard analysis, and yet posing a fundamental problem for tax policy, is the fact that households typically contain two adult members and that, at least as importantly, while the labour supply of the primary earner (typically male) is fairly constant across households, that of the secondary earner (typically female) varies quite considerably. We want to show that this, in conjunction with what appear to be established facts about labour supply and saving behaviour, has important implications for the analysis of the effects of taxation and pensions policy

[1] For a thorough and comprehensive exposition of this analysis see Sandmo (1985).

on saving and on individual welfares. In order to sharpen the theoretical results, we make considerable use of empirical data on two-parent families with employed head of household, drawn from the 1992 UK Family Expenditure Survey. Rather than presenting the theoretical results in their full, but usually inconclusive, generality, we use this data to derive what appear to be the empirically relevant results.

We find it useful to make a broad distinction between what we term "traditional" and "non-traditional" households. In the former, the secondary earner, typically female, specialises in household work and supplies very little labour to the market, while in the latter there is a substantial market labour supply by the secondary earner. Operationally, we take non-traditional households to be those in which the secondary earner works at least 10 hours per week outside the home and earns an income of at least £40 per week. On this basis the secondary earner in the average traditional household works about 3.5 hours per week outside the home for an average income of just under £10. In our data sample, non-traditional households represent 53% of the total and traditional households the remainder.

It does not appear to be generally recognised that much the greater proportion of household saving in the economy is in fact done by non-traditional households. We estimate that the marginal propensity to save out of the secondary earner's income is about half as large again as that out of primary earner's income. Thus, in spite of the fact that there will normally be "fixed costs of work" such as travel costs, associated with the secondary earner's labour supply, this second income makes a considerable contribution to the saving of the household and of the economy as a whole.

It is not hard to understand why saving by non-traditional households would be so much higher than that by traditional households. The latter are specialising more in the production of goods that are inherently non-saveable, whereas market income can be saved. By their labour supply and saving behaviour non-traditional households are substituting future consumption of market goods for current consumption of household goods, traditional households are doing the reverse.

Table 1 about here

The empirical basis for these propositions is illustrated in Table 1. Households are classed into quartiles on the basis of total personal income of the "head of household" (subsequently referred to as "primary earner"). On the basis of a regression analysis of household saving, set out in the Appendix to this paper, we can show that, controlling for net disposable income of the primary earner as well as for demographic characteristics[2], replacing the average secondary income in a traditional household with the average secondary income in a non-traditional household makes a very large difference to the household's saving, both in absolute terms and in terms of the saving rate. Overall, a non-traditional household saves more than twice as much as a traditional household with the same primary-earner income and demographic characteristics, and saving rates are also significantly higher. The regression analysis suggests that the marginal propensity to save out of primary earner income is about 0.2, while that out of secondary earner income is almost 0.3.

It follows from these empirical observations that tax and pension policies that have a significant impact on secondary earner labour supply also have important implications for saving in the economy, and the purpose of this paper is to explore these issues. In the next section we set out the basic model of intertemporal choice of consumption, household production and labour supply for the 2-person household. We use this in section 3 to analyse the effects of various kinds of tax reform and pension reform policies on household welfares and saving. A central feature of this analysis, which is of considerable analytical usefulness, is the assumption that all individuals face the same market wage rate. Section 4 considers the extent to which the results of the analysis are robust in the more realistic case in which there is a distribution of market wage rates. Section 5 concludes.

2 The Household Model

The household maximises its intertemporal utility[3]

$$u = u_0 + \delta u_1 \tag{1}$$

where

$$u_t = u(x_t, y_t, z_{1t}, z_{2t}) \quad t = 0, 1 \tag{2}$$

[2] In particular the number of young children and family size.

[3] We depart from our previous work in this area by taking a single household utility function rather than separate individual utility functions for each household member. As we have shown elsewhere, see Apps and Rees (1988), this is innocuous as long as we are not interested in *within household* distributional effects, as is the case in this paper. It is equivalent to assuming that the policy-maker and the household have identical preferences over the distribution of income within the household.

x_t is market good consumption in period $t = 0, 1$; y_t is consumption of the household good produced by the secondary earner in period t; z_{it} is i's pure leisure in period t; $\delta \in (0, 1)$ is a time preference discount factor.

Household production is described by the production functions

$$y_0 = k(T - l_2 - z_{20}) \quad (3)$$

$$y_1 = (T - z_{21}) \quad (4)$$

with T as total time, and l_2 as the secondary earner's market labour supply in period 0. The productivity parameters may vary over time, being normalised at 1 in the second period, with $k \gtreqless 1$ in the first period. In period 1, the retirement period, market labour supply is constrained to be zero. The budget constraint in period 0 is:

$$x_0 = \alpha + \sum_{i=1}^{2} \beta_i w_i l_i - s \quad (5)$$

where s is saving and α and β_i are the parameters of a linear tax system, the marginal rates, $(1 - \beta_i)$, of which may differ across the individuals in the household. Under an individual progressive marginal rate tax system we would typically think of the primary earner, who has a wage rate w_1, as facing a higher marginal rate than the secondary earner, who earns the wage rate $w_2 \leqq w_1$, that is, $\beta_1 < \beta_2$, but if we are considering a joint taxation system, in which income splitting takes place, then the rates would be equal. Given the production function (3) we can write the full income period 0 budget constraint as

$$x_0 + py_0 + \sum_i \beta_i w_i z_{i0} = \alpha + T \sum_i \beta_i w_i - s \quad (6)$$

where $p = \beta_2 w_2 / k$ is the implicit price of the household good in period 0.

In period 1 the budget constraint is:

$$x_1 = P + s(1 + \theta r) \quad (7)$$

where P is the (untaxed) state pension payment, r the market interest rate and $1-\theta$ is the tax rate on interest income. Note that there is no wage income in this period. Substituting for s in (5) then gives the wealth constraint

$$x_0 + py_0 + \sum_i \beta_i w_i z_{i0} + \frac{x_1}{1 + \theta r} = \alpha + T \sum_i \beta_i w_i + \frac{P}{1 + \theta r} \quad (8)$$

The household's problem is then to maximise the intertemporal utility in (1) subject to the wealth constraint (8) and the production constraint (4). Now notice that if x_1 is fixed, we can solve the problem of maximising $u(x_1, y_1, z_{11}, z_{21})$ subject to the constraint in (4) to obtain optimal values of y_1 and z_{i1} as functions of x_1. We can write the optimised period 1 utility as $\phi(x_1)$ and so the household's problem becomes, by the Envelope Theorem,

$$\max u_0 + \delta\phi(x_1) \tag{9}$$

subject only to the wealth constraint in (8). The parameters of this problem are r,θ, p, $\beta_i w_i$, α, and P, but we are interested in this paper only in changes in the marginal tax rates and pension payment P. Hence we write the household's indirect utility as

$$v = v(\theta, \beta_1, \beta_2, P) \tag{10}$$

and application of the Envelope Theorem gives

$$\frac{\partial v}{\partial \theta} = \lambda r(x_1 - P)/(1 + \theta r)^2 = \lambda rs/(1 + \theta r) \tag{11}$$

$$\frac{\partial v}{\partial \beta_1} = -\lambda w_1(z_{10} - T) = \lambda w_1 l_1 \tag{12}$$

$$\frac{\partial v}{\partial \beta_2} = -\lambda[w_2(z_{20} - T) + w_2 y_0/k_0] = \lambda w_2 l_2 \tag{13}$$

$$\frac{\partial v}{\partial P} = \lambda/(1 + \theta r) \tag{14}$$

where λ is the marginal utility of household wealth.

In what follows we shall be considering households with different labour supplies of the secondary earner. In the present model, if preferences and wage rates facing households are identical, such differences can only be generated by allowing k, the productivity of the secondary earner in household production, to vary across households. However, as we show elsewhere[4], the effects of varying k on secondary earner market labour supply are ambiguous. On the one hand, raising this productivity reduces the implicit price of the domestic good, increases its demand and so increases the demand for time within the household and reduces market labour supply. On the other hand, this increase in productivity also reduces the amount of time that has to

[4]See Apps and Rees (1999b).

be supplied within the household to meet any given level of demand for the domestic good. The net effect on market labour supply could therefore be positive or negative. This is important because, on the one hand, the amount of labour the secondary earner supplies to the market determines the effect on the welfare of the household of a change in the marginal tax rate: an increase in the marginal tax rate on the secondary earner reduces household utility more, the greater the secondary earner's market labour supply. And, on the other hand, the higher is k, other things being equal, the higher are the utility possibilies of the household, and so the relationship between k and secondary earner market labour supply determines whether better- or worse-off households are made worse off by an increase in the marginal tax rate on secondary earners. Unfortunately, nothing is known empirically about the relationship between productivity in household production and market labour supply. In what follows therefore we shall consider all possibilities.

3 Tax Reform.[5]

There are two household types, differentiated according to the value of k, which determines the secondary earner's market labour supply. For concreteness we assume[6]

$$\frac{l_{11}}{l_{21}} > \frac{l_{12}}{l_{22}} \geq 1 \tag{15}$$

and so we refer to household 1 as the "traditional household", the one with relatively lower labour supply from the secondary earner, and household 2 as the "non-traditional" household.

We now go on to consider the *across-household welfare effects* and the *effects on household saving* of several types of revenue-neutral tax reform. We regard the initial position as one of proportional taxation, in which all individuals are taxed at the same rate. We then consider first a move in the direction of individual progressive taxation, which involves raising the tax rate on primary earners (since they work longer hours and possibly have

[5] The general theory of tax reform is set out comprehensively and rigorously in its most modern form in Guesnerie (1995). An important early development and application of the theory is given in Ahmad and Stern (1984).

[6] We now need to introduce a subscript for the household type. This is $h = 1, 2$. Then l_{ih} is labour supply of individual i in household of type h, s_h is household saving, and so on.

higher wage rates, and therefore have higher wage income) and lowering that on secondary earners. Accordingly, primary earners can be assumed to be taxed at the marginal rate $(1 - \beta_1)$ and secondary earners at the rate $(1 - \beta_2)$, where these are equal initially. We want to explore the conditions under which such a move, away from the kind of joint taxation system that prevails in the USA and Germany, and toward the kind of system prevailing in the UK, Canada and Australia, where primary and secondary earners in a household may be taxed at different rates, can increase both aggregate household saving and household welfares.

For the second type of tax reform, we take the case of a move in the direction of joint progressive taxation, in which both household members are taxed at the same rate and this rate increases with total household income. It follows that, relative to the initial situation of proportional taxation, traditional households experience a reduction in their tax rate, since they have a lower joint income, and non-traditional households an increase. In this case traditional households can be assumed to be taxed at the marginal rate $(1-\beta_1)$ and non traditional at the rate $(1-\beta_2)$, where these are equal initially. The effects on household welfares are in this case obvious, and the point of interest is to examine the effects on saving of increasing the tax burden on non-traditional households while reducing that on traditional households.

As the final kind of reform considered in this section, we take a reduction in the pension payment P, representing a movement away from the "pay as you go" type of scheme. We compare the effects of using the proceeds of this, on the one hand, to reduce the tax rate (pay a subsidy on) interest income, and, on the other hand, to reduce the marginal tax rate on secondary earners' income. The idea here is to identify the relative effects on saving and household welfares of these two measures.

For simplicity we at first set $\theta = 1$, and ignore taxation of interest income, and also assume initially no wage dispersion across households. The wage rate w measures the exogenous labour productivity in market work of all individuals. The interest rate r is also exogenously given. After considering the results for this case, we then allow positive taxation of saving. In the following section we introduce wage dispersion, though only for one of the tax reforms just defined, since this is enough to bring out the general implications of allowing this.

3.1 Tax reform with no taxation of interest income: from proportional to individual progressive taxation

In the first case of tax reform, in which there is a move from a proportional to an individual progressive tax system, the government budget constraint can be written as

$$\sum_{h=1}^{2} n_h \sum_{i=1}^{2} (1-\beta_i) w l_{ih} - n(\alpha + P) = R \tag{16}$$

where R is a given revenue requirement and $n = \sum_h n_h$ is the total number of households. Notice that the pension, P, plays the same role as a lump sum transfer in the government budget constraint, though in the household budget constraint it is discounted because the household receives it in the second period.

Using (12) and (13), the effects on household utilities of an increase in the tax rate on primary earners and reduction of that on secondary earners, implying $d\beta_1 < 0 < d\beta_2$, are given by

$$dv_h = \lambda_h w \sum_{i=1}^{2} l_{ih} d\beta_i \gtreqless 0 \quad h = 1, 2 \tag{17}$$

The sign of this expression depends on the relative sizes of the tax rate changes and the relative labour supplies of primary and secondary earners. It simply says that household utility rises if and only if after-tax household income, evaluated at the pre-tax-change labour supplies, increases[7]. To characterise the relative sizes of the tax rate changes we need to differentiate the government budget constraint.

The balanced budget requirement implies

$$d\beta_2 = \mu d\beta_1 \tag{18}$$

where

$$\mu \equiv -\frac{\sum_h n_h [w l_{1h} - \sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial \beta_1}]}{\sum_h n_h [w l_{2h} - \sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial \beta_2}]} \tag{19}$$

[7] For further interpretation of this appealingly simple condition see Apps and Rees (1999a).

This expression reflects the fact that the effect of a change in a tax rate on the government budget constraint depends first of all on the size of the tax base to which it is applied, $\sum n_h w l_{ih}$, and secondly on the changes in incomes induced by the resulting changes in market labour supplies, which can be expressed in terms of labour supply elasticities. Intuitively, the tax base corresponding to primary earners is much larger than that of secondary earners, while the labour supply elasticities of secondary earners are generally held to be much larger than those of primary earners. This means that, in absolute value, $d\beta_2$ will tend to be much larger than $d\beta_1$, and this drives the main results of this section. We now go on to make this more precise.

It is reasonable to assume $\mu < 0$, so that a reduction in one marginal tax rate requires an increase in the other[8]. Thus, given $d\beta_1 < 0$, household $h = 1, 2$ is worse off if and only if

$$l_{1h} + \mu l_{2h} > 0 \tag{20}$$

or

$$\frac{l_{1h}}{l_{2h}} > -\mu \tag{21}$$

Now let

$$\delta_{ih} = 1 - \frac{1}{wl_{ih}} \sum_{j=1}^{2} (1 - \beta_j) w \frac{\partial l_{jh}}{\partial \beta_i} \tag{22}$$

$$= 1 - \frac{1 - \beta_i}{\beta_i} \frac{\beta_i w}{l_{ih}} \frac{\partial l_{ih}}{\partial \beta_i w} - \frac{1 - \beta_j}{\beta_i} \frac{\beta_i w}{l_{ih}} \frac{\partial l_{jh}}{\partial \beta_i w} \quad i, j, h = 1, 2 \;\; i \neq j \tag{23}$$

We will now argue that on the stylised facts about labour supply elasticities of primary and secondary earners, we can make

Assumption 1: $\delta_{1h} > \delta_{2j}$, for $h, j = 1, 2$.

In words, the expression in (23) is larger for any primary earner than for any secondary earner, regardless of household. To support this assumption, we rely on the following stylised facts:

- labour supply elasticities of secondary earners are substantially larger than labour supply elasticities of primary earners, regardless of household, so that

$$\frac{1 - \beta_2}{\beta_2} \frac{\beta_2 w}{l_{2h}} \frac{\partial l_{2h}}{\partial \beta_2 w} > \frac{1 - \beta_1}{\beta_1} \frac{\beta_1 w}{l_{1j}} \frac{\partial l_{1j}}{\partial \beta_1 w} \quad h, j = 1, 2 \tag{24}$$

[8] Otherwise a reduction in both tax rates could increase welfare. We are therefore assuming that the tax rates are efficient in the sense that this is ruled out.

- the derivative of the primary earner's labour supply with respect to the secondary earner's net wage is just about zero, implying

$$\frac{1-\beta_1}{\beta_2}\frac{\beta_2 w}{l_{2h}}\frac{\partial l_{1h}}{\partial \beta_2 w} \approx 0 \quad h = 1,2 \tag{25}$$

- the derivative of the secondary earner's labour supply with respect to the primary earner's net wage is close to zero and negative, so that

$$\frac{1-\beta_2}{\beta_1}\frac{\beta_1 w}{l_{1h}}\frac{\partial l_{2h}}{\partial \beta_1 w} < 0 \quad h = 1,2 \tag{26}$$

It is easy to see that these stylised facts imply Assumption 1. Then we have

Proposition 1 *Given Assumption 1, at worst only traditional households are made worse off by the tax reform and otherwise both household types are made better off.*

Proof: If all households are made worse off, then from (19), (20) and (22) we have

$$\frac{\sum_h n_h w l_{1h}}{\sum_h n_h w l_{2h}} > -\mu = \frac{\sum_h n_h w l_{1h}\delta_{1h}}{\sum_h n_h w l_{2h}\delta_{2h}} \tag{27}$$

But given Assumption 1 this is a contradiction and so not all households can be worse off.

Since

$$\frac{l_{11}}{l_{21}} > \frac{l_{12}}{l_{22}} \tag{28}$$

if any household is worse off it must from condition (21) be a traditional household. However since by Assumption 1, $-\mu > 1$, it is possible, for l_{21} sufficiently close to l_{11}, that

$$-\mu > \frac{l_{11}}{l_{21}} > \frac{l_{12}}{l_{22}} \tag{29}$$

so that all households are made better off.

The intuition for this result is simply that if the labour supply elasticities of secondary earners are sufficiently higher than those of primary earners, and their tax base sufficiently smaller, then the increase in the marginal tax rate of the primary earners has to be so much lower than the reduction in marginal tax rate of the secondary earners, in order to replace the lost tax revenue, that at least the households with the higher secondary earner labour supply have higher net income, and it is possible that all households have higher net income. This, in the context of tax reform, is basically similar to the Ramsey result on the relation between marginal tax rates and labour supply elasticities. Reducing tax on the more elastic labour supply and raising it on the less elastic so increases the aggregate tax base that, with a given revenue requirement, at least some households and possibly all households are made better off.

If we assume that the traditional households are the ones with lower domestic productivity and hence lower utility possibilities, we would characterise the tax reform as regressive if it makes only non-traditional households better off. On the other hand we have no particular empirical basis for this assumption. The converse case, in which non-traditional households have lower domestic productivity, is *a priori* just as possible, in which case the tax reform woud improve the welfare position of the worse off households. This underlines the importance of the empirical relationship between domestic productivity and secondary earner market labour supply, something about which virtually nothing seems to be known.

We now turn to the effects on aggregate saving, and show that, on the assumptions so far made, the proposed tax reform increases aggregate net labour income. As we have just seen, it also certainly increases the net income of the non-traditional households. In the light of the empirical evidence presented earlier, it does not seem plausible to argue that the savings propensity of traditional households is sufficiently higher than that of non-traditional households that, in the case in which net income of the traditional households falls, aggregate saving would fall. We make this explicit in

Assumption 2: Aggregate saving varies positively with aggregate household after-tax income for all tax reforms considered in this paper.

Then we have:

Proposition 2 *Given Assumptions 1 and 2, a revenue neutral tax reform that raises the marginal tax rate on primary earners and reduces that on secondary earners increases aggregate saving.*

Proof*:* Given Assumption 2, we just have to show that the tax reform increases aggregate net income. That is, we have to show that

$$\sum_{h=1}^{2} n_h \sum_{i=1}^{2} (wl_{ih} + \sum_{j=1}^{2} \beta_j w \frac{\partial l_{jh}}{\partial \beta_i}) d\beta_i > 0 \tag{30}$$

Again using (19), (20) and (23) this becomes the condition

$$\frac{\sum_h n_h w l_{1h} \gamma_{1h}}{\sum_h n_h w l_{2h} \gamma_{2h}} < -\mu = \frac{\sum_h n_h w l_{1h} \delta_{1h}}{\sum_h n_h w l_{2h} \delta_{2h}} \tag{31}$$

where

$$\gamma_{ih} \equiv 1 + \sum_{j=1}^{2} \frac{\beta_j w}{l_{ih}} \frac{\partial l_{jh}}{\partial \beta_i w} \quad h, i = 1, 2 \tag{32}$$

But our earlier assumption about elasticities implies that

$$\gamma_{2j} > \gamma_{1h} \quad h, j = 1, 2 \tag{33}$$

This in turn implies the inequalities

$$\frac{\gamma_{1h}}{\gamma_{2j}} < 1 < \frac{\delta_{1h}}{\delta_{2j}} \quad h, j = 1, 2 \tag{34}$$

which are sufficient to give the condition in (31). To see this let

$$\eta_{ih} = n_h w l_{ih} \quad i, h = 1, 2 \tag{35}$$

and note that cross-multiplying in (31) gives

$$\sum_{h=1}^{2} \sum_{j=1}^{2} \eta_{1h} \eta_{2j} (\gamma_{1h} \delta_{2j} - \gamma_{2j} \delta_{1h}) < 0 \tag{36}$$

as required, since every bracketed term is negative.

3.2 Tax reform with no taxation of interest income: from proportional to joint progressive taxation

We now turn to the case of a move from proportional taxation in the direction of joint progressive taxation, which implies a reduction in the tax rate paid by traditional and an increase in that paid by non-traditional households, that is, $d\beta_1 > 0 > d\beta_2$ (note that the tax rates are now indexed on households). The across-household welfare effects are of course obvious, and the question of interest is the effect on aggregate saving. In fact we show that on the previous assumptions, augmented by the simplifying assumption that all secondary earners' labour supply elasticities are equal, as are those of all primary earners, aggregate saving falls.

The government budget constraint is now written as

$$\sum_{h=1}^{2} n_h(1-\beta_h)\sum_{i=1}^{2} wl_{ih} - n(\alpha + P) = R \tag{37}$$

giving, from the revenue neutrality requirement,

$$d\beta_2 = \hat{\mu} d\beta_1 \tag{38}$$

where

$$\hat{\mu} \equiv -\frac{n_1 \sum_i [wl_{i1} - (1-\beta_1)w\frac{\partial l_{i1}}{\partial \beta_1}]}{n_2 \sum_i [wl_{i2} - (1-\beta_2)w\frac{\partial l_{i2}}{\partial \beta_2}]} \tag{39}$$

Now define

$$\hat{\delta}_{ih} = 1 - \frac{1}{wl_{ih}}(1-\beta_h)w\frac{\partial l_{ih}}{\partial \beta_h} \tag{40}$$

$$= 1 - \frac{1-\beta_h}{\beta_h}\frac{\beta_h w}{l_{ih}}\frac{\partial l_{ih}}{\partial \beta_h w} \tag{41}$$

We now replace Assumption 1 by

Assumption 1': $\hat{\delta}_{11} = \hat{\delta}_{12} > \hat{\delta}_{21} = \hat{\delta}_{22}$

which extends Assumption 1 by assuming that all primary earners have the same elasticities, as do all secondary earners. We then have

Proposition 3 *Given Assumptions 1' and 2, a tax reform which raises the joint marginal tax rate on non-traditional households and reduces that on traditional households reduces aggregate saving.*

Proof: Again, given Assumption 2, we just have to show that aggregate income falls, that is, we have to show that

$$\sum_{h=1}^{2} n_h \sum_{i=1}^{2}(wl_{ih} + \beta_h w\frac{\partial l_{ih}}{\partial \beta_h})d\beta_h < 0 \tag{42}$$

Defining now

$$\hat{\gamma}_{ih} \equiv 1 + \frac{\beta_h w}{l_{ih}} \frac{\partial l_{ih}}{\partial \beta_h w} \quad h, i = 1, 2 \tag{43}$$

and using (39), this requires us to show that

$$\frac{n_1 w \sum_i l_{i1} \hat{\gamma}_{i1}}{n_2 w \sum_i l_{i2} \hat{\gamma}_{i2}} < -\hat{\mu} = \frac{n_1 w \sum_i l_{i1} \hat{\delta}_{i1}}{n_2 w \sum_i l_{i2} \hat{\delta}_{i2}} \tag{44}$$

Now Assumption 1' also implies

$$\hat{\gamma}_{21} = \hat{\gamma}_{22} > \hat{\gamma}_{11} = \hat{\gamma}_{12} \tag{45}$$

Using this in (44), cross-multiplying, rearranging and cancelling terms then gives

$$\frac{n_1 w \sum_i l_{i1} \hat{\gamma}_{i1}}{n_2 w \sum_i l_{i2} \hat{\gamma}_{i2}} < \frac{n_1 w \sum_i l_{i1} \hat{\delta}_{i1}}{n_2 w \sum_i l_{i2} \hat{\delta}_{i2}} \Longleftrightarrow \frac{l_{11}}{l_{21}} > \frac{l_{12}}{l_{22}} \tag{46}$$

as required.

3.3 Tax Reform with Taxation of Interest Income

Beginning with a situation of progressive individual taxation, in which primary earners pay a higher marginal rate of tax than secondary earners, and with interest income now taxed at a non-zero rate $1 - \theta$, we consider the effects of a reduction in the rate of tax on interest income, financed by an increase in the tax rate on secondary earners. Thus $d\theta > 0 > d\beta_2$. We show that, under reasonable assumptions, at best only traditional households are made better off and otherwise all households are worse off, and that aggregate saving (given Assumption 2 earlier) will fall. These results are essentially due to the fact that the adverse labour supply effects of the tax reform outweigh the net interest rate effects. The point of this analysis is to show that the simple intuition, that would lead one to expect a reduction of taxation on (increased subsidy to) income from saving to increase saving, ignores the fact that this must be financed by an increase in taxation somewhere. Since, given the current thrust of tax reform policy, this is unlikely to take the form of increases in the higher tax rates, it is likely to result in increases in the lower tax rates, paid by secondary earners. The resulting changes in labour supply and household income then cause saving to fall, contrary to the initial intuition.

The effect of the tax reform on household welfare is now given by

$$dv_h = \lambda_h(wl_{2h}d\beta_2 + \frac{rs_h}{1+\theta r}d\theta) \gtreqless 0 \quad h = 1,2 \tag{47}$$

Thus, the larger the labour supply of the secondary earner, the more likely is welfare to fall, while the higher the household saving, the less likely is welfare to fall. Since non-traditional households have the larger savings as well as the larger secondary earner labour supplies, we might think that the effects on their welfare would be ambiguous, while traditional households, as long as they save, would be clearly better off. However this intuition is incorrect. Taking account of effects on the government budget constraint, and again applying the stylised facts, we find that non-traditional households are worse off, and tradional households may also be.

The government budget constraint is now written as

$$\sum_{h=1}^{2} n_h[\sum_{i=1}^{2}(1-\beta_i)wl_{ih} + (1-\theta)rs_h] - n(\alpha + P) = R \tag{48}$$

and so the changes in tax rates are related, through the revenue neutrality requirement, by

$$d\beta_2 = \mu_2 d\theta \tag{49}$$

where

$$\mu_2 \equiv -\frac{\sum_h n_h[rs_h - \sum_i(1-\beta_i)w\frac{\partial l_{ih}}{\partial\theta} - (1-\theta)r\frac{\partial s_h}{\partial\theta}]}{\sum_h n_h[wl_{2h} - \sum_i(1-\beta_i)w\frac{\partial l_{ih}}{\partial\beta_2} - (1-\theta)r\frac{\partial s_h}{\partial\beta_2}]} \tag{50}$$

The welfare effects are then given by

$$dv_h = \lambda_h(\mu_2 wl_{2h} + \frac{rs_h}{1+\theta r})d\theta \gtreqless 0 \quad h = 1,2 \tag{51}$$

Now define

$$\sigma_h \equiv 1 - \frac{1}{rs_h}[\sum_i(1-\beta_i)w\frac{\partial l_{ih}}{\partial\theta} + (1-\theta)r\frac{\partial s_h}{\partial\theta}] \tag{52}$$

$$= 1 - [\sum_i \frac{(1-\beta_i)}{\theta}\frac{wl_{ih}}{rs_h}\frac{\theta r}{l_{ih}}\frac{\partial l_{ih}}{\partial\theta r} + \frac{(1-\theta)}{\theta}\frac{\theta r}{s_h}\frac{\partial s_h}{\partial\theta r}] \quad h = 1,2 \tag{53}$$

and

$$\delta_h \equiv 1 - \frac{1}{wl_{2h}}[\sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial \beta_2} + (1-\theta) r \frac{\partial s_h}{\partial \beta_2}] \quad (54)$$

$$= 1 - [\sum_i \frac{(1-\beta_i)}{\beta_2} \frac{\beta_2 w}{l_{2h}} \frac{\partial l_{ih}}{\partial \beta_2 w} + \frac{(1-\theta)}{\beta_2} \frac{r s_h}{w l_{2h}} \frac{\beta_2 w}{s_h} \frac{\partial s_h}{\partial \beta_2 w}] \quad h = 1, 2 \quad (55)$$

Thus the value of σ_h depends on the elasticity of labour supplies with respect to the after-tax interest rate and on the interest elasticity of saving, while δ_h depends on the elasticities of labour supplies and saving with respect to the secondary earner's net wage. We now make:

Assumption 3: $\sigma_1 = \sigma_2$ and $\delta_1 = \delta_2$

Thus as a simplifying assumption we take the effects of a change in the net interest rate to be the same across households of different types, and similarly a change in the net wage of the secondary earner has the same effect across households. Though this is special, we do not regard it as crucial to the results. We denote the common values of σ_h and δ_h by σ and δ respectively.

Finally, it is empirically plausible to make

Assumption 4: It is the case that

$$\frac{n_2 s_2}{\sum_h n_h s_h} < \frac{n_2 l_{22}}{\sum_h n_h l_{2h}} \quad (56)$$

That is, the data show that although non-traditional households account for a large proportion of the total saving in the economy, they account for an even larger proportion of the total secondary earner labour supply.

We then have

Proposition 4 *Given Assumptions 3 and 4, if $\sigma(1+\theta r) > \delta$ then a revenue neutral tax reform that reduces the tax rate on interest income and increases that on wage income of secondary earners at best makes only traditional households better off, and otherwise makes all households worse off.*

Proof: From (51), the condition under which a household is better off is given by

$$\frac{r s_h/(1+\theta r)}{w l_{2h}} > -\mu_2 = \frac{\sum_h n_h r s_h \sigma}{\sum_h n_h w l_{2h} \delta} \quad (57)$$

or

$$\frac{s_h / \sum_h n_h s_h}{l_{2h} / \sum_h n_h l_{2h}} > \frac{\sigma(1+\theta r)}{\delta} \quad (58)$$

But from Assumption 4 we know that this cannot hold for non traditional households if $\sigma(1+\theta r)/\delta > 1$, and moreover may not hold for traditional households, if $\sigma(1+\theta r)/\delta$ is sufficiently greater than 1.

The condition in this proposition, $\sigma(1+\theta r) > \delta$, is essentially a condition on the relative values of the interest elasticity of saving on the one hand and the wage elasticity of secondary earner labour supply on the other, as a comparison of (53) and (55) shows. If the secondary earner labour supply elasticity is larger than the interest elasticity of saving, which seems reasonably to be the case, then this condition is almost sure to be satisfied, since the cross elasticities are likely to be small, and the effect of the net interest rate on labour supply could well be negative, due to an income effect on consumption of the domestic good for households with positive saving. In other words it is plausible to take $(\delta/\sigma) < 1$.

Finally, we consider formally the effects of the contemplated tax reform on aggregate household saving. Regarding household saving simply as a function of the tax rates β_2 and θ, we have

$$d\sum_h n_h s_h = \sum_h n_h\left(\frac{\partial s_h}{\partial \beta_2} d\beta_2 + \frac{\partial s_h}{\partial \theta} d\theta\right) \tag{59}$$

Now define the elasticities

$$\zeta_h \equiv \frac{\beta_2 w}{s_h}\frac{\partial s_h}{\partial \beta_2 w} \tag{60}$$

$$\varphi_h \equiv \frac{\theta r}{s_h}\frac{\partial s_h}{d\theta r} \tag{61}$$

as the elasticies of saving with respect to the net of tax wage rate and the net of tax interest rate respectively. As a benchmark case, suppose the ζ_h are equal across households, as are the φ_h. Finally, define

$$\rho \equiv \frac{\sum_h n_h \theta r s_h}{\sum_h n_h \beta_2 w l_{2h}} \tag{62}$$

as the ratio of total net interest income to total secondary earner net wage income. Then using (59) we can write the condition under which the tax reform actually reduces aggregate saving as

$$\varphi < \rho\frac{\sigma}{\delta}\zeta \tag{63}$$

It is of course entirely an empirical question whether this inequality holds, but it is certainly *a priori* possible that it does. In other words it is quite possible that a reduction in taxation of interest income actually reduces aggregate saving, because it implies an increase in taxation of secondary earners with sufficiently adverse effects on net wage income that saving falls as a result. Empirical estimates of the interest elasticity of saving are typically close to zero, while we saw in the introduction that secondary earner income appears to have a strong positive effect on saving. A reasonable estimate of ρ would be about 0.3, while we suggested earlier that σ/δ could reasonably be taken to be greater than 1. An elasticity of saving with respect to secondary earner net wage that is roughly three times as high as the interest elasticity of saving would then satisfy this condition.

3.4 Pension and tax reform

We can model a (marginal) move away from a "pay as you go" pension system by setting $dP < 0$. The question then is: how should the resulting saving in public expenditure be used? An obvious associated policy would be to reduce taxation on (pay a subsidy to) interest income from saving, implying $d\theta > 0$, to induce an increase in private saving that will hopefully at least compensate for the reduction in the public pension. A not so obvious alternative would be to reduce the tax rate on secondary earners, implying $d\beta_2 > 0$, thus achieving an increase in saving through the induced changes in labour supplies and household income. We now compare the effects on saving of these two policies.

The effects on aggregate saving of the two alternative policies are given respectively by

$$d\sum_h n_h s_h = \sum_h n_h (\frac{\partial s_h}{\partial \theta} d\theta + \frac{\partial s_h}{\partial P} dP) \tag{64}$$

$$d\sum_h n_h s_h = \sum_h n_h (\frac{\partial s_h}{\partial \beta_2} d\beta_2 + \frac{\partial s_h}{\partial P} dP) \tag{65}$$

while differentiating the government budget constraint gives the revenue neutrality condition in each case

$$d\theta = \mu_3 dP \tag{66}$$

$$d\beta_2 = \mu_4 dP \tag{67}$$

with

$$\mu_3 = -\frac{n - \sum_h n_h[\sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial P} - (1-\theta) r \frac{\partial s_h}{\partial P}]}{\sum_h n_h [r s_h - \sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial \theta} - (1-\theta) r \frac{\partial s_h}{\partial \theta}]} \tag{68}$$

$$\mu_4 = -\frac{n - \sum_h n_h[\sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial P} - (1-\theta) r \frac{\partial s_h}{\partial P}]}{\sum_h n_h [w l_{2h} - \sum_i (1-\beta_i) w \frac{\partial l_{ih}}{\partial \beta_2} - (1-\theta) r \frac{\partial s_h}{\partial \beta_2}]} \tag{69}$$

It follows that the difference in the effect on aggregate saving is given by

$$D = \sum_h n_h (\frac{\partial s_h}{\partial \theta} \mu_3 - \frac{\partial s_h}{\partial \beta_2} \mu_4) dP \tag{70}$$

$$= \sum_h n_h (\frac{\partial s_h}{\partial \theta} - \frac{\partial s_h}{\partial \beta_2} \frac{\mu_4}{\mu_3}) d\theta \tag{71}$$

But notice that $\mu_4/\mu_3 = -\mu_2$, as defined in the previous subsection. Thus, using the same definitions and arguments as in the previous subsection, we can obtain (63) as the condition under which there is a larger positive effect on saving from reducing the tax rate on secondary earners as from reducing that on saving. We conclude that if the elasticity of saving with respect to the secondary earner's net wage is at least about three times as large as the interest elasticity of saving, then the proceeds of a reduction in the state pension would be better applied to reducing the tax rate on secondary earners income.

4 Tax reform with wage dispersion

The analysis in the preceding section relied heavily on the assumption that the low-income lower-tax rate group consisted entirely of secondary earners, who have the higher labour supply elasticities, while the higher-income higher-tax rate group consisted only of primary earners with much lower labour supply elasticities. This was consistent with the idea that there was only one market wage rate, so that a lower labour supply necessarily implied lower income. In reality of course there is a distribution of market wage rates, and it is quite possible that high wage secondary earners could be earning higher incomes and paying higher marginal tax rates than low wage primary earners. It is then entirely an empirical question, as to whether the actual distribution of primary and secondary earner incomes and marginal tax rates is sufficiently close to the polar case analysed above for the results to be realistic. We now show that, for the UK at least, this seems to be the case.

In Table 2 we show the quartile distribution of wage income for the sample of UK households used to contruct Table 1. The lowest 25% of individual wage earners had an average weekly income of £15, corresponding closely to the female income of £14 since 99% of the individuals in this group were women. The highest 25% of individuals had an average weekly wage income of £539, which is close to the male weekly wage income of £552, since only 9% of this group were women. The last column of the table shows the estimated marginal tax rates paid on average by each member of the respective income quartile. We can interpret the "low" tax rate as the 18% paid by the second quartile, and the "high" tax rate as the 31% paid by the highest two quartiles. The results in the previous section essentially depend on the absolute value of μ being well above unity. We now calculate the value of μ implied by the figures in Table 2.

Table 2 about here

Taking the marginal tax rates in Table 2 as the tax rates $1-\beta_i$ in a linear tax system, we can formulate a "government budget constraint" by calculating the tax revenue T raised from the individuals recorded in the table[9]. Thus let $q = 1,..,4$ denote the quartile, and $i = 1,2$ the individual, with $i = 1$ male and $i = 2$ female. n_{iq} is the number of type-i individuals in quartile q, y_{iq} the corresponding wage income, $1-\beta_1 = 0.31$ is the higher marginal tax rate and $1-\beta_2 = 0.18$ the lower. Then we have

$$T = (1-\beta_2)\sum_{i=1}^{2} n_{i2}y_{i2} + (1-\beta_1)\sum_{q=3}^{4}\sum_{i=1}^{2} n_{iq}y_{iq}$$

where of course y_{iq} is the product of a wage rate and a labour supply. If now the marginal tax rates are changed so that we have

$$d\beta_2 = \mu d\beta_1 \tag{72}$$

we can write

$$\mu \equiv -\frac{\sum_{q=3}^{4}\sum_{i=1}^{2} n_{iq}y_{iq}\delta_{iq}}{\sum_{i=1}^{2} n_{i2}y_{i2}\delta_{i2}} \tag{73}$$

[9]There are around 370 individuals in each quartile, and so there are 266 women in the second quartile, 70 in the third, and 37 in the fourth.

where, analogously with the earlier definition

$$\delta_{iq} = 1 - \frac{1-\beta_i}{\beta_i} \frac{\beta_i w_{iq}}{l_{iq}} \frac{\partial l_{iq}}{\partial \beta_i w_{iq}}, \quad q = 2,3,4, \; i = 1,2, \tag{74}$$

We have departed from the earlier definition of the $\delta's$ by setting the cross-elasticities to zero, which will slightly reduce the estimated value of μ. Plausible empirical values for male and female labour supply elasticities are respectively 0.1 and 0.5. This gives the following estimates:

$\delta_{12} = 0.989$
$\delta_{22} = 0.890$
$\delta_{1q} = 0.955 \;\; q = 3,4$
$\delta_{2q} = 0.775 \;\; q = 3,4$

Then, using these values together with the values for n_{iq} and y_{iq} from Table 2 gives

$$\mu = -\frac{[(300\times276)+(333\times552)]0.955+[(70\times232)+(37\times407)]0.775}{(104\times169)0.989+(266\times104)0.890} = -6.65$$

This implies that in the substantial majority of non-traditional households, where the primary earner is taxed at the higher marginal rate and the secondary earner at the lower, a revenue neutral change in the marginal tax rates, reducing the lower and raising the higher rate, will increase household welfare and saving, given that, recalling the condition in (17) earlier, the primary earner's gross wage income is not more than about six to seven times greater than that of the secondary earner. This seems to be an eminently reasonable condition in practice[10].

5 Conclusions

In this paper we have tried to derive the effects on household saving and welfare of some currently advocated reforms to tax and pension systems. We have emphasised that this must be done in the context of models of households containing two adult members, taking into account also the fact that there is considerable variation across households in the labour supply

[10] To check the robustness of this, we have calculated the μ -values for male labour supply elasticities in the range 0.0 - 0.2, and for female labour supply elasticities in the range 0.2 - 1.0. The resulting range of μ -values is 6.218 to 7.080.

of the secondary earner. This latter fact is important in relation both to the effects of tax changes and to household saving behaviour. A central result is that a reduction in the degree of progressivity of the individual tax system, implying an increase in inequality of individual after-tax incomes, can be expected to reduce saving. We have also argued that, under plausible assumptions, reducing marginal tax rates on secondary earners can be a more effective way of stimulating saving than reducing tax rates on income from saving. To obtain these results we have made a number of assumptions about empirical relationships, which we have tried to substantiate by the data we have presented and analysed. The basic analytical approach can however also be used to derive the implications of any alternative set of empirical assumptions.

Appendix

In this appendix we give the details of the regression model used to control for demographic variation and primary earner disposable income in the computation of saving by the two household types reported in Table 1.

The model is estimated on a sample of two-parent families selected from the 1992 UK FES on the following criteria: "head of household" (referred to in text as "primary earner") with usual hours of work of at least 10 p.w. and earning a normal gross wage/salary income of at least £40 p.w. and reporting a non-negative total personal income; at least one child aged 0-17 years; head of household and "wife of head of household" (referred to in text as "secondary earner") aged 20-60 years.

The dependent saving variable is computed as the difference between disposable household income and total household expenditure.

Regression model results:

Regressors	Parameters	Std. errors
No of children 0-2 yrs	-29.656	12.31
No of children 3-4 years	-2.477	9.85
Household size	-4.976	4.76
Age of primary earner	2.065	0.566
Disposable income of primary earner £pw	0.210	0.043
Disposable income of secondary earner £pw	0.294	0.056
Constant	-115.96	32.99

$R^2 = 0.1145$

References

[1] Ahmad, E. and Stern, N., 1984. The theory of reform and Indian indirect taxes. *Journal of Public Economics* 25, 259-298.

[2] Apps, P. F., and Rees, R., 1988. Taxation and the household. *Journal of Public Economics* 35, 355-369.

[3] Apps, P. F., and Rees, R., 1999a. Joint vs individual taxation in models of household production. *Journal of Political Economy* 107, 393-403

[4] Apps, P. F., and Rees, R., 1999b. On the taxation of trade within and between households. *Journal of Public Economics* 73, 241-263.

[5] Guesnerie, R., 1995. A Contribution to the Pure Theory of Taxation, Cambridge University press, Cambridge.

[6] Sandmo, A., 1985, The effects of taxation on saving and risk taking. Ch 5 of *Handbook of Public Economics*, vol 1, A. S. Auerbach and M. S. Feldstein (eds), Elsevier Science Publications, North-Holland.

Tables

Table 1

Q'tile	TPI	PDI	SDI T	SDI NT	Sav T	Sav NT	Kids	Hrs T	Hrs NT
1	197	161	40	122	-13.13	10.87	1.99	5	27
2	293	227	40	130	1.84	28.28	1.93	4	27
3	389	294	43	149	16.06	47.37	1.87	4	26
4	722	520	50	177	69.66	107.00	1.97	2	26
All	400	300	44	124	18.37	42.01	1.94	3	27

TPI: total personal income of head of household, £ p.w.

PDI: primary earner net disposable income (includes nonwage income) £ p.w.

SDI: secondary earner net disposable income, £ p.w.

T: traditional households

NT: non-traditional households

Sav: saving, £ p.w.

Kids: number of dependent children in household

Hrs: average hours worked per week.

Table 2

Quartile	GWI	GWIM	CWIF	%F	AvMTR
1	15	69	14	99	-
2	122	169	104	72	0.18
3	268	276	232	19	0.31
4	539	552	407	9	0.31
All	236	386	86	50	0.23

GWI: gross weekly wage income, £ p.w.

GWIM: gross weekly wage/salary income, £ p.w., primary earner.

GWIF: gross weekly wage/salary income, £ p.w., secondary earner

%F: percentage female

AvMTR: average of marginal tax rates (NI contribution plus income tax)