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Exchange-enhanced spin-splitting in a two-dimensional electron gas in the presence of the Rashba spin–orbit interaction

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Abstract

We present a theoretical study on how many-body effects can affect the spin-splitting of a two-dimensional electron gas in the presence of the Rashba spin–orbit interaction. The standard Hartree–Fock approximation and Green's function approach are employed to calculate the energy spectrum and density of states of a spin-split two-dimensional electron gas (2DEG). We find that the presence of the exchange interaction can enhance significantly the spin-splitting of a 2DEG on top of the Rashba effect. The physical reasons behind this important phenomenon are discussed.

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Keywords: Spin–orbit interaction; Two-dimensional electron gas; Rashba effect; Exchange interaction

1. Introduction

At present, one important aspect in the field of spintronics is to study electronic systems with a finite spin-splitting realized from narrow-gap semiconductor quantum wells in the absence of an external magnetic field. It is known that in such systems, the spontaneous spin-splitting of the carriers can be achieved by the inversion asymmetry of the microscopic confining potential due to the presence of the heterojunction [1]. This corresponds to an inhomogeneous surface electric field and, hence, is electrically equivalent to the Rashba spin-splitting or Rashba effect. The published experimental results [2] have indicated that in InGaAs/InAlAs-based two-dimensional electron gas (2DEG) systems, the value of the Rashba parameter can reach up to $(3-4)\times10^{-11}$ eV m and can even be higher to 8×10^{-11} eV m [3]. These experimental data are much larger (at least a factor of 2) than those obtained from theoretical calculations such as the $\mathbf{k}\cdot\mathbf{p}$ band-structure calculation [1] and self-consistent calculation on the basis of the $\mathbf{k}\cdot\mathbf{p}$ results [4]. It should be noted that the $\mathbf{k}\cdot\mathbf{p}$ calculation is basically a single-particle approach. Although the Hartree potential induced by electron distribution along the growth direction has been taken into account, the self-consistent calculation [4] normally cannot give a full consideration of the many-body effects. Therefore, it is valuable and important to examine how electron–electron (e–e) interaction can affect the spin–orbit interaction (SOI) in a 2DEG in the presence of the Rashba effect. And this is the prime motivation of the present theoretical study.

2. Outline of theoretical approaches

We consider a 2DEG in which the SOI is induced by the Rashba effect. In such a case, the single-particle Schrödinger equation including the lowest-order of SOI can be solved analytically. Applying the electron wave functions to the e–e interaction induced by the Coulomb potential, we can study the bare e–e interaction. Using the energy

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spectrum of a spin-split 2DEG, the electron density–density correlation function can be obtained [5]. Thus, under the random-phase approximation (RPA), the dielectric function matrix can be obtained [6]. For a spin-split 2DEG, there are four channels for electronic transition, i.e., $v = (\sigma', \sigma) = (+,+)$, $(+,-)$, $(-,+)$ and $(-,-)$ with $\sigma = \pm$ referring to different spin branches. Hence, the dielectric function is a 4×4 matrix [6]. On the other hand, using the single–electron wave functions to form the two-particle Slater wave functions, we can derive the electrostatic energy induced by e–e interaction under the standard Hartree–Fock approximation and obtain the energies induced by Hartree and exchange (or Fock) interactions in the absence of e–e screening. With the inverse RPA dielectric function matrix, the Hartree and exchange energies in the presence of e–e screening then can be calculated. Thus, the self-energy induced by e–e interaction can be obtained. We find that for a spin-split 2DEG, the Hartree interaction does not affect the self-energy for inter-SO transition. It has been shown that the inverse RPA screening length induced by intra-SO transition $K_+(q) \to \infty$ when $q \to 0$ [6], where q is the change of electron wave vector during an e–e scattering event. As a result, in the presence of e–e screening the Hartree interaction does not contribute to self-energy for intra-SO transition neither. Therefore, the self-energy comes from exchange interaction alone, which, at low temperatures (i.e., $T \to 0$), reads

$$\Sigma_\sigma^{\sigma'}(k) = -\sum_{\mathbf{k}'<\mathbf{k}_F^{\sigma'}} \frac{2\pi e^2}{\kappa[q + K_{\sigma'\sigma}(q)]}\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}. \tag{1}$$

Here, $\mathbf{k} = (k_x, k_y)$ is the electron wave vector along the 2D-plane, $\mathbf{k}_F^{\pm}$ is the Fermi wave vector in the $\pm$ spin branch, κ is the dielectric constant of the material, and $K_{\sigma'\sigma}(q)$ is the inverse screening length induced by electronic transition in different spin channels [6]. For intra- ($\sigma'\sigma = +$) and inter-SO ($\sigma'\sigma = -$) transition

$$K_\pm(q) = \frac{16e^2 m^*}{\pi\hbar^2\kappa q}\sum_\sigma \int_0^{\sqrt{4\pi n_\sigma}} \mathrm{d}k\, H_\sigma^\pm(k,q) \times \frac{k\,(k+q)}{(2k+q+2\sigma k_\alpha)(k+q+|k-q|)}, \tag{2}$$

where n_σ is the electron density in the σ spin branch, m^* is the electron effective mass, $k_\alpha = \alpha m^*/\hbar^2$ with α being the Rashba parameter, and

$$H_\sigma^\pm(k,q) = \frac{-1 \pm 1}{2}K(A) + \Pi(AB_\pm, A) + \frac{q\,(q+2\sigma k_\alpha)}{4k\,(k+\sigma k_\alpha)}[\Pi(AC_\pm, A) - \Pi(AB_\pm, A)]$$

with $K(x)$ and $\Pi(n,x) = \Pi(\pi/2, n, x)$ being, respectively, the complete elliptic integral of the first and third kind, $A = (k+q-|k-q|)/(k+q+|k-q|)$, $B_\pm = [(2k+q)/q]^\pm$, and $C_\pm = [(q-2\sigma k_\alpha)/(2k+q+2\sigma k_\alpha)]^\pm$. These results indicate that for a spin-split 2DEG, the self-energy differs for different transition channels.

Applying the self-energy induced by e–e interaction into the diagrammatic techniques, the Green's function for a spin-split 2DEG in the presence of e–e interaction can be represented by

$$G_{\sigma\sigma'}(E,k) = [G_\sigma^{-1}(E,k) - \Sigma_\sigma^{\sigma'}(k)]^{-1}, \tag{3}$$

where $G_\sigma(E,k) = [E - E_\sigma(k) + i\delta]^{-1}$ is the diagonal Green's function for a spin-split 2DEG in the absence of e–e interaction, and

$$E_\sigma(k) = \hbar^2k^2/2m^* + \sigma\alpha k \tag{4}$$

is the energy spectrum of a spin-split 2DEG without e–e interaction. In a form of matrix, we have

$$G_{\sigma\sigma'}(E,k) = \frac{1}{\Delta_0 + \mathrm{i}\Delta_1\delta}\begin{bmatrix} E_- + \mathrm{i}\delta & -\Sigma_+^-(k) \\ -\Sigma_-^+(k) & E_+ + \mathrm{i}\delta \end{bmatrix}. \tag{5}$$

Here $E_\sigma = E - E_\sigma(k) - \Sigma_\sigma^\sigma(k)$, $\Delta_0 = E_+E_- - \Sigma_+^-(k)\Sigma_-^+(k)$, and $\Delta_1 = E_+ + E_-$. The corresponding electron density-of-states (DoS) is then given by

$$D_{\sigma\sigma'}(E) = -\frac{1}{\pi}\sum_{\mathbf{k}} \mathrm{Im}G_{\sigma\sigma'}(E,k). \tag{6}$$

Thus, we obtain a live many-body quasi-particle with a lifetime $\tau_k = 1/\delta = \infty$ and an energy dispersion law determined by $\Delta_0(E,k) = 0$, which reads

$$E_\sigma^*(k) = \frac{\hbar^2k^2}{2m^*} + \frac{\Sigma_+^+(k) + \Sigma_-^-(k)}{2} + \sigma\alpha_k^* k \tag{7}$$

with again $\sigma = \pm 1$ and

$$\alpha_k^* = \alpha + \frac{\Sigma_+^+(k) - \Sigma_-^-(k)^2}{2k} + \frac{\Sigma_+^-(k)\Sigma_-^+(k)^{1/2}}{k^2}$$

being the effective Rashba parameter in the presence of e–e interaction. In the presence of e–e interaction, the electron density in the σ spin branch is determined by the diagonal elements of the DoS through

$$n_\sigma = \int_{-\infty}^{\infty} \mathrm{d}E f(E)\, D_{\sigma\sigma}(E), \tag{8}$$

with $f(x)$ being the Fermi–Dirac function, which gives, for $T \to 0$,

$$n_\sigma = \frac{n_e}{2} - \sigma\int_{k_F^+}^{k_F^-} \mathrm{d}k\,\frac{2\alpha k + \Sigma_+^+(k) - \Sigma_-^-(k)}{8\pi\alpha_k^*}. \tag{9}$$

Here k_F^σ is the solution for k resulting from $E_F = E_\sigma^*(k)$ with E_F being the Fermi energy and

$$n_e = n_+ + n_- = [(k_F^+)^2 + (k_F^-)^2]/4\pi \tag{10}$$

is the total electron density. We have considered that a 2DEG is in equilibrium so that a single Fermi level is present in the system.

For a given total electron density n_e and a given Rashba parameter α, the Fermi wave vector $k_F^\pm$ can be determined by solving Eq. (10) and an equation $E_+^*(k_F^+) = E_-^*(k_F^-)$. The Fermi energy can be calculated by $E_F = E_+^*(k_F^+) = E_-^*(k_F^-)$ and the electron density in the σ spin branch by Eq. (9).

Thus, the physical quantities, such as quasi-particle energy spectrum $[E^*_\sigma(k)]$, DoS $[D_{\sigma\sigma'}(E)]$, Fermi energy (E_F) and Fermi wave vector in different spin branches (k_F^σ), electron density in different spin branches (n_σ), etc., can be calculated self-consistently.

3. Results and discussions

In this paper, we consider an InGaAs/InAlAs-based 2DEG in which a strong Rashba SOI has been observed experimentally [2]. The typical sample parameters (such as $n_e \sim 10^{11}\,\text{cm}^{-2}$ and $\alpha \sim 10^{-11}\,\text{eV m}$) are taken within the numerical calculations. The dependence of the electron distribution in different spin branches on total electron density n_e and the Rashba parameter α is shown, respectively, in Figs. 1 and 2. Here we compare the results obtained from inclusion and exclusion of e–e interaction. One can see that the presence of the exchange interaction can enhance significantly the spin-splitting in a 2DEG over a wide regime of n_e and α. With increasing α or decreasing n_e, the density difference between different spin–orbits increases. This implies that a stronger spin polarization occurs in a sample with a larger α and/or lower n_e. A full spin polarization (i.e., $n_- = n_e$ and $n_+ = 0$) can be achieved via e–e interaction at a smaller α and/or larger n_e, in comparison with the case where the e–e interaction is absent.

To understand the physical reason behind an exchange-enhanced spin-splitting in a 2DEG, in Fig. 3 we show the energy dispersion relation at a fixed n_e and a fixed α for cases with and without inclusion of the e–e interaction. Here the Fermi levels (E_F) and the energy spectra $[E_\sigma(k)$ and $E^*_\sigma(k)]$ are obtained by using Eqs. (4) and (7), respectively, for cases without and with e–e interaction. Firstly, we note that similar to the case of a spin-degenerate 2DEG, the exchange interaction lowers the energy and the Fermi level of the system. Secondly, the presence of the e–e interaction results in a split energy bands even at $k = 0$, in sharp contrast to the case without e–e interaction where $E_+(0) = E_-(0) = 0$. It is known that small-k states are most likely occupied by electrons. An energy gap $E^*_+(0) - E^*_-(0) \neq 0$ implies that more DoS below E_F are induced in the '−' spin branch so that more electrons can stay in the '−' spin branch than in the '+' branch. This is one of the main reasons why spin-splitting can be enhanced by e–e interaction. Thirdly, we know that the intersections of the curves for $E_\pm(k)$ or $E^*_\pm(k)$ with E_F, projected onto the k-axis, give the Fermi wave vector $k_F^\pm$ for different spin branches. The difference $k_F^- - k_F^+$ leads to a difference in k-space area: $\pi(k_F^-)^2 > \pi(k_F^+)^2$. Accordingly, the electron densities in the ± branches are different. From Fig. 3, we see that the presence of the exchange interaction can enlarge significantly the difference between k_F^- and k_F^+, and this becomes another major reason why many-body effect can enhance spin-splitting in a 2DEG.

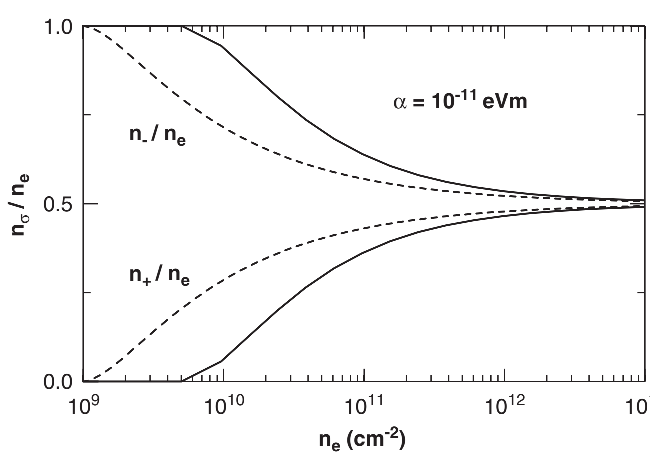

Fig. 1. Electron distribution in different spin branches as a function of total electron density n_e at a fixed Rashba parameter α for cases with (solid curves) and without (dotted curves) e–e interaction. Here n_σ is the electron density in the σ spin branch.

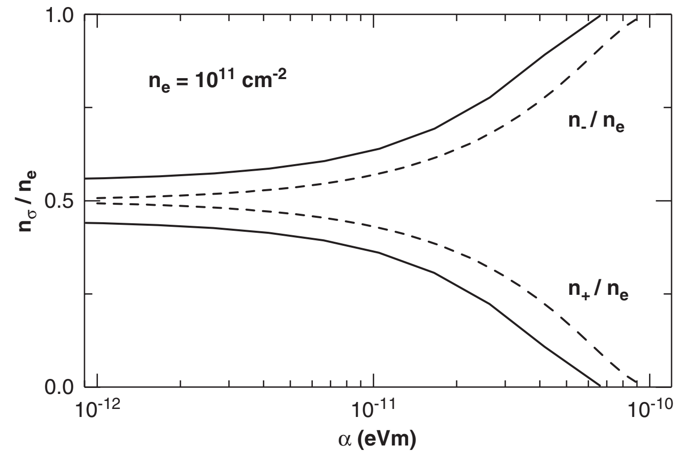

Fig. 2. Electron density in different spin branches n_σ as a function of the Rashba parameter α at a fixed total electron density n_e for cases with (solid curves) and without (dotted) e–e interaction.

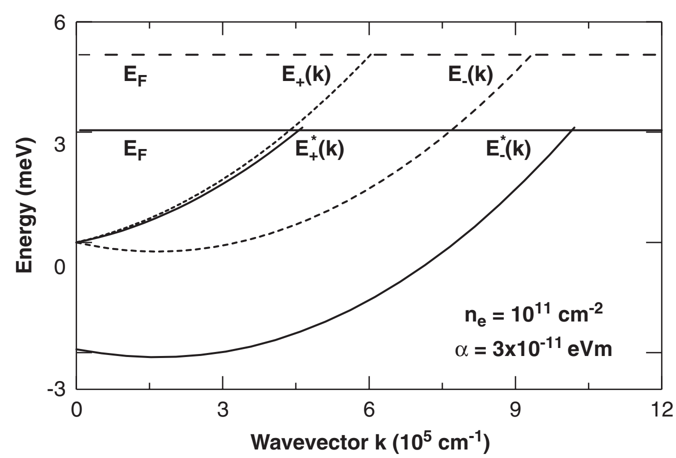

Fig. 3. Dispersion relation for a spin-split 2DEG without (dotted curves and Eq. (4)) and with (solid curves and Eq. (7)) inclusion of e–e interaction for fixed electron density n_e and the Rashba parameter α. E_F is the Fermi energy.

With increasing α, the SOI in the system increases and the difference between n_- and n_+ increases. With decreasing n_e, the Fermi level of the system decreases so that more electrons are in the lower energy '−' branch. As a result, the difference between n_- and n_+ increases with decreasing n_e. We see these interesting features clearly in Figs. 1 and 2.

From the results obtained from present theoretical study, we can draw an important conclusion that the many-body effects such as exchange interaction can enhance spin-splitting markedly in a Rashba spintronic system. In a 2DEG in which the Rashba SOI is present, the exchange interaction can play the following roles. (1) It can lower the Fermi level of the system, which pushes more electrons to the lower energy '−' spin branch which has more DoS for electrons. (2) It can create an energy gap between the $\pm$ branches even at $k = 0$ and can increase the gap in the $\pm$ energy states. (3) It can enlarge the difference of the Fermi wave vector in different spin branches. All these many-body induced effects favor on increasing of spin-splitting of a 2DEG. Consequently, exchange-enhanced spin-splitting is another important origin for which a strong Rashba SOI can be observed in InGaAs-based 2DEG systems. Finally, we hope these theoretical findings can at least partly interpret why the values of α determined experimentally are larger than those obtained theoretically using the $\mathbf{k} \cdot \mathbf{p}$ calculation (single-particle approach) [1] and the self-consistent calculation on the basis of the $\mathbf{k} \cdot \mathbf{p}$ results (exchange interaction is not included) [4].

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