Transport and stability of two-dimensional matter-wave solitons in a driven optical lattice.

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Abstract
We study the transport of quasi-two-dimensional matter-wave solitons in a Bose-Einstein condensate with attractive atomic interactions in a two-dimensional driven optical lattice potential. In order to describe the dynamics of 2D matter-wave soliton analytically we use the effective Hamiltonian theory. We also present numerical studies of the soliton evolution in the system.

Keywords: matter-wave solitons, Bose-Einstein condensate

Introduction
The investigation of matter-wave solitons in a Bose-Einstein condensate have attracted considerable attention in recent years. Recent theoretical studies have shown that the stabilisation of the two-dimensional (2D) matter-wave solitons can be achieved either in optical lattices, see, e.g., Efremidis et. al. (2003), or in a presence of long-range non-local interactions, as studied by Skupin et. al (2006). Another way of soliton stabilisation was achieved recently in time-dependent potentials, Satio et. al.(2007).

In this work, we aim to demonstrate the existence of a stable 2D matter-wave soliton in a BEC with a local, contact interaction between the atoms, which is supported by a 2D optical lattice. Moreover, we show that such a soliton can be moved across the lattice by application of an additional weak optical lattice potential which is asymmetric in space and rapidly varying in time.

Model
We start our analysis with a BEC described by the dimensionless mean-field time dependent Gross-Pitaevsky (GPE) equation for a condensate wave function.

$$iu_t = -\frac{1}{2}u_{xx} - \frac{1}{2}u_{yy} + V(x,y,t)u - |u|^2u. \tag{1}$$

We assume that the condensate is trapped within the optical lattice potential of the following form:

$$V(x,y,t) = V_x \cos(x - X(t) + \phi_1) + V_y \cos(y - Y(t) + \phi_2) \tag{2}$$

where V_x, V_y,ϕ_1,ϕ_2 are strength and phases in the given directions and $X(t) = X_0 \cos(\omega t)$, $Y(t) = Y_0 \cos(\omega t)$ are the time varying components of the lattice. The time-independent "backbone" component of the two dimensional optical potential is shown in Fig.1 (a).

For large values of driving frequency, $\omega \gg 1$, the effective particle approach can be used to describe the motion of the soliton's center of mass. Choosing a Gaussian trial function and using the effective Hamiltonian theory [see Poletti et. al. (2008)], one can easily derive the equations of motion for the center of mass in (x, y) plane.

$$\frac{d^2x_0}{dt^2} = V_x e^{-\frac{a^2}{4}} \sin(x_0 + \phi_1 - X(t)),$$

$$\frac{d^2y_0}{dt^2} = V_y e^{-\frac{b^2}{4}} \sin(y_0 + \phi_2 - Y(t))$$

These equations allow us to have an intuitive insight into the transport properties of the matter-wave soliton in a 2D lattice.

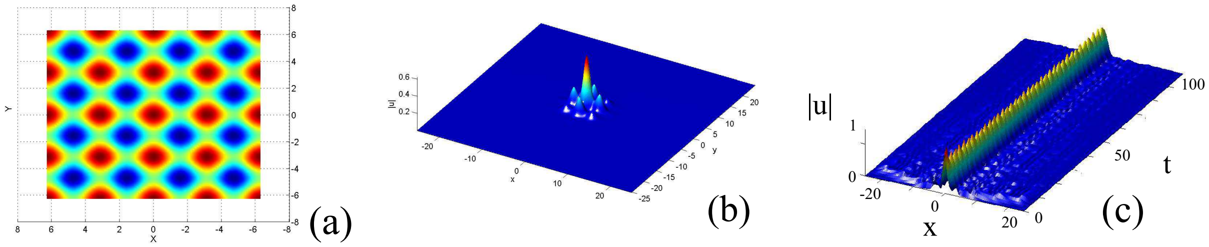

Figure 1: a) Optical lattice field distribution with energy levels varying from low (blue) to high (red). b) The ground state of the soliton in optical lattice at the $t = 0$. The parameters of the potential are $V_x = V_y = 3$ and $\phi_1 = \phi_2 = X(t) = Y(t) = 0$ c) Lattice soliton propagation profile in potential with $V_x = V_y = 3$, $Y(t) = \phi_1 = \phi_2 = 0$, $X(t) = \pi/2\cos(\omega t)$ here $\omega = 20$

Results and Discussion

To study the problem numerically we solve the Gross-Pitaevskii equation (2) with the two-dimensional optical potential shown in Fig.1 (a) by relaxation method, which allows to get a stationary soliton solution and establish its stability and evolution properties. The typical spatial structure of the soliton wavefunction, obtained numerically, is shown in Fig.1 (b).

For time varying potential $V(x, y, t)$, see Eq. (1), the GPE has the solution in the form of a moving soliton. By performing numerical simulations, we observe time-stable lattice soliton propagation, shown in Fig.1 (c) for the harmonically time driven lattice, $X(t) = \pi/2\cos(\omega t)$ with normalized frequency $\omega = 20$. Small losses due to soliton radiation are present in the system, thus the amplitude of soliton is decreasing with time, causing the broadening of the soliton. However, the latter effect does not affect the dynamical stability of the soliton.

Conclusions

In conclusion, we have studied analytically and numerically the the transport and stability of the 2D matterwave solitons formed in an attractive Bose-Eistien condensate trapped in a 2D optical lattice. Our analysis showed that, while the time-independent lattice potential provides for the stability of the 2D soliton, the transport of the soliton across the lattice can be achieved and controlled dynamically with the aid of the time varying potential. The numerical simulations of the matter-wave evolution show that the soliton remains dynamically stable for physically meaningful evolution times.

Acknowledgments

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