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Brief Paper

Variable structure control method to the output tracking control of cascade non-linear switched systems

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Abstract: This study is concerned with the output tracking control problem for a class of cascade non-linear switched systems with external disturbances under some average dwell-time based switching laws. The problem is solved by the variable structure control technique. The variable structure controllers and the average dwell time are designed under which the output of the closed-loop switched system can follow the desired output exactly after a finite time interval and all the states remain globally bounded. The effectiveness of the proposed design approach is illustrated with simulation results.

1 Introduction

Variable structure control with sliding model has developed into a general design method being examined for a wide spectrum of system types, which is characterised by a discontinuous control that changes structure on reaching a sliding surface. The objective of variable structure control has been greatly extended from stabilisation to other control problems. This control method can make the system completely insensitive to parametric uncertainty and external disturbances. Today, research and development continue to apply variable structure control to a wide variety of engineering systems.

On the other hand, a large class of natural and man-made systems are often governed by several dynamical modes. The interchange between modes is often determined by time t and state x or based on some environmental factors that are not predicted a priori. Such a system is called a switched system, which is a special kind of hybrid system that consists of a family of continuous-time or discrete-time dynamical systems and a rule called the switching signal to control the switching between modes. An important qualitative property of such systems is stability [1–10]. The challenge to analyse the stability of switched systems lies partly in the fact that even if the individual systems are stable, the switched system might be unstable. Morse [2] showed that when all modes are exponentially stable the entire switched system is exponentially stable under any switching signal if the time between two successive switchings, called the dwell-time, is sufficiently large. Later, Hespanha and Morse [3] extended the dwell-time approach to the concept of average dwell-time. Then, Zhai *et al.* [4] used this average dwell-time approach to achieve the same stability result where the family of modes was enlarged to include unstable modes. Colaneri *et al.* [5] used the concept to analyse the stability of general non-linear switched systems. Besides this

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method, many other methods have been reported, such as common Lyapunov function method [6, 7], multiple Lyapunov function method [8], switched Lyapunov function method [9], convex combination method [10] and so on. All these methods are summarised in the books [11, 12].

In addition to the stability analysis problem for switched systems, many other problems such as controllability and reachability problems [13, 14], robust H_∞ control problems [15, 16], passivity [17] have been extensively investigated. The tracking control for switched systems has extensive applications in robot tracking control [18], underactuated vehicle tracking control [19], guided missile tracking control and so on. Moreover, many engineering systems can be described by cascade switched systems. All of this motivated us to study the tracking control problem for cascade non-linear switched systems. As far as we know, no results have been reported on tracking control for cascade switched systems.

In fact, switched systems are a certain kind of variable structure systems. Therefore sliding modes may exist on the switching surfaces, which we usually prefer them not to happen. But, there still have existing results enforcing sliding mode occurring for switched systems such that the systems possess certain desirable properties such as insensitivity to parameter variations and external disturbances [20, 21]. A few results using the variable structure control with sliding model to deal with problems of the switched system have been reported [22–24].

In this paper, we consider the output tracking control problem for a class of cascade non-linear switched systems via variable structure control strategy under some average dwell-time based switching laws. A common sliding surface is constructed, the variable structure controllers and the average dwell-time are designed. Under the designed variable structure controllers and the average dwell-time the output of the system can follow the desired signal exactly after a finite time interval while all the states remain globally bounded.

The rest of this paper is organised as follows. In Section 2, a non-linear switched cascade model is given, together with the output tracking control problem formulation. Section 3 gives the main result of output tracking control. An example is given to illustrate effectiveness of the proposed design procedures in Section 4. Finally, conclusions are included in Section 5.

Notation: We use standard notations throughout this paper. Given a real matrix M, M^T denotes the transpose of M. I is the identity matrix. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of a symmetric matrix P. $\|\cdot\|$ denotes the Euclidean norm. R^n denotes the n-dimensional real Euclidean space. $R^{m\times n}$ is the set of all real $m \times n$ matrix.

2 Problem statement

We consider the cascade non-linear switched systems described by

$$\begin{cases} \dot{z} = f_\sigma(z, \xi) \\ \dot{\xi} = A_\sigma \xi + B[G_\sigma(z, \xi)u_\sigma + F_\sigma(z, \xi, t)] \\ y = C\xi \end{cases} \tag{1}$$

where $z \in R^{n-d}$, $\xi \in R^d$ are the states, $y \in R^m$ is the measurable output, $\sigma : [0, \infty] \to I_N = \{1, \dots, N\}$ is the switching signal that will be determined later and $\sigma(t) = i$ means that the ith subsystem is activated, $u_i(t) \in R^m$ is the control input, $F_i(z, \xi, t)$ represent the time-varying external disturbances, A_i, B are known matrices, $f_i(z, \xi)$, $G_i(z, \xi)$ are known smooth vector fields with appropriate dimensions. Further, $\det(G_i(z, \xi)) \neq 0$ for $\forall\, [z^T, \xi^T]^T \in R^n$, $f_i(0, 0) = 0$.

In this paper, we need the following assumptions.

Assumption 1: Matrix B is full of column rank, and $m < d$.

Assumption 2: The matrix CB is non-singular.

Assumption 3: $\|F_i(z, \xi, t)\| \leq \rho_i(t)$, $i \in I_N$ for some known continuous and uniformly bounded functions $\rho_i(t)$.

The basic assumption on the reference trajectory $y_d(t)$ is as follows.

Assumption 4: The reference trajectory $y_d(t)$ is piecewise differentiable. Additionally, there exist known constants Y_1 and Y_2 such that

$$\|y_d(t)\| \leq Y_1, \quad \|\dot{y}_d(t)\| \leq Y_2, \quad \forall t \in [0, \infty)$$

Remark 1: Assumptions 1–3 are assumptions that are usually made in the variable structure control literature. Similarly, Assumption 4 is a common assumption made when the tracking control problem is considered.

Many practical systems with switchings can be described in the form of system (1), for instance, switching control of power converter systems, switching control of many industry produce processes. More specifically, the planar Cartesian haptic display system (a two DOF gantry mechanism) can be described by the switched system (1) [25, 26].

We now state the output tracking control problem for switched system (1).

The output tracking control problem: Find, if possible, a switching law σ and a variable structure feedback controller u_σ such that the following two facts are true:

1. $\lim_{t\to+\infty} (y(t) - y_d(t)) = 0$;

2. the state $(z^T, \xi^T)^T$ of the closed-loop system (1) is globally bounded.

Our objective is to solve the output tracking control problem with variable structure feedback.

The following definition and lemma will be used in the development of our results.

Consider the general non-linear system

$$\dot{x} = f(x, u) \tag{2}$$

where $x \in R^n$ is the state, $u \in R^p$ is the input, $f(x, u)$ is a smooth vector and satisfies $f(0, 0) = 0$. Denote $||| \cdot |||$ as the essential supremum norm in the functional space L_∞^p, that is

$$||| u ||| = \sup\{\|u(t)\|, t \geq 0\} < \infty$$

Definition 1 [27]: System (2) is input-to-state stable if and only if there exist a proper, positive definite and radially unbounded function $V(x)$ such that for some class $\mathcal{K}_\infty$ functions γ, η we have

$$\frac{\partial V(x)}{\partial x} f(x, u) \leq -\eta(\|x\|) + \gamma(\|u\|), \quad \forall x, \ u$$

Consider the non-linear switched system

$$\dot{x} = f_\sigma(x, v) \tag{3}$$

where σ is the switching signal as given in system (1), $f_i(x, v)$ are smooth vector fields, the set of measurable functions $v: [0, \infty) \to R^l$ is the input.

Lemma 1 [28]: Suppose that there exist continuous differentiable functions $V_p: R^n \to [0, \infty)$, $p \in I_N$, class $\mathcal{K}_\infty$ functions α_1, α_2, γ and numbers λ_0, $\mu \geq 1$ such that for $\forall x \in R^n$, $v \in R^l$, and $\forall p$, $q \in I_N$, we have

$$\alpha_1(\|x\|) \leq V_p(x) \leq \alpha_2(\|x\|) \tag{4}$$

$$\frac{\partial V_p}{\partial x} f_p(x, v) \leq -\lambda_0 V_p(x) + \gamma(\|v\|) \tag{5}$$

$$V_p(x) \leq \mu V_q(x) \tag{6}$$

Let σ be a switching signal having average dwell-time τ_a. Then, the non-linear switched system (3) is input-to-state stable if $\tau_a > (\ln \mu / \lambda_0)$.

3 Main result

In this section, we solve the output tracking control problem by variable structure control approach.

Theorem 1: Suppose Assumptions 1–4 are satisfied and that

1. there exist smooth positive definite functions W_i, $i \in I_N$, class $\mathcal{K}_\infty$ functions β_1, β_2, γ_i and positive numbers λ_{0i}, $\mu_z \geq 1$ such that for $\forall z \in R^{n-d}$, $\xi \in R^d$, and $\forall i$, $j \in I_N$, we have

$$\beta_1(\|z\|) \leq W_i(z) \leq \beta_2(\|z\|) \tag{7}$$

$$\frac{\partial W_i(z)}{\partial z} f_i(z, \xi) \leq -\lambda_{0i} W_i(z) + \gamma_i(\|\xi\|) \tag{8}$$

$$W_i(z) \leq \mu_z W_j(z) \tag{9}$$

2. there exist a positive definite matrix Q, matrix N and a positive scalar ϑ such that the following inequalities

$$A_i Q + Q A_i^T + BN + N^T B^T + \vartheta Q + I < 0 \tag{10}$$

$$B^T Q^{-1} = C \tag{11}$$

hold.

Let

$$e(t) = (CB)^{-1}[y(t) - y_d(t)] \tag{12}$$

and an arbitrary switching law satisfying the average dwell time

$$\tau_a \geq \tau_a^* = \frac{\ln \mu_z}{\lambda} \quad \text{and} \quad t_2 \geq t_d \tag{13}$$

where $\lambda \in (0, \lambda_0)$, $\lambda_0 = \min\{\lambda_{0i} \mid i = 1, 2, \ldots, N\}$ and t_d is a certain positive constant that will be calculated later. The variable structure controller

$$u_i = -G_i^{-1}(z, \xi)[(CB)^{-1} CA_i \xi + \kappa_1 e + (\kappa_2 + \rho(t)) \operatorname{sgn} e - (CB)^{-1} \dot{y}_d(t)] \tag{14}$$

will solve the output tracking control problem for the corresponding closed-loop system (1), where κ_1 and κ_2 are two positive constants, $\rho(t) = \max\{\rho_i(t) \mid i \in I_N\}$, $\rho_i(t)$ is defined as in Assumption 3.

Proof: The proof is divided into two parts. First of all, we will show that the output $y(t)$ of (1) can follow exactly the desired signal $y_d(t)$ after a finite time interval. Then, we will show that the state of (1) is globally bounded under the average dwell-time based switching laws.

We first give the proof for the first part. The derivative of $e(t)$ along the trajectory of system (1) with (14) is

$$\begin{aligned}\dot{e}(t) &= (CB)^{-1}[\dot{y}(t) - \dot{y}_d(t)] \\ &= (CB)^{-1}[C\dot{\xi} - \dot{y}_d(t)] \\ &= (CB)^{-1} CA_i \xi + [G_i(z, \xi) u_i + F_i(z, \xi, t)] - (CB)^{-1} \dot{y}_d(t) \\ &= -\kappa_1 e - (\kappa_2 + \rho(t)) \operatorname{sgn} e + F_i(z, \xi, t)\end{aligned}$$

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For any $p = 1, 2, \ldots, m$. When $e_p(t) > 0$ we can obtain

$$\begin{aligned}\dot{e}_p(t) &= -\kappa_1 e_p - (\kappa_2 + \rho(t))\operatorname{sgn} e_p + (F_i(z, \xi, t))_p \\ &\le -\kappa_1 e_p - \kappa_2 - \rho(t) + \|(F_i(z, \xi, t))_p\| \\ &\le -\kappa_1 e_p - \kappa_2 - \rho(t) + \|(F_i(z, \xi, t))\| \\ &\le -\kappa_1 e_p - \kappa_2 \end{aligned} \tag{15}$$

Similarly, when $e_p(t) < 0$, we have

$$\begin{aligned}\dot{e}_p(t) &= -\kappa_1 e_p - (\kappa_2 + \rho(t))\operatorname{sgn} e_p + (F_i(z, \xi, t))_p \\ &\ge -\kappa_1 e_p + \kappa_2 + \rho(t) - \|(F_i(z, \xi, t))_p\| \\ &\ge -\kappa_1 e_p + \kappa_2 + \rho(t) - \|(F_i(z, \xi, t))\| \\ &\ge -\kappa_1 e_p + \kappa_2 \end{aligned} \tag{16}$$

It can be seen from (15) and (16) that all $e_p(t), p = 1, 2, \ldots, m$, will arrive at zero in finite time interval and be kept there thereafter. Denote the time instant that all e_p hit zero as t_d.

Now, we proceed to prove the second part. Firstly, we will prove that the state ξ of the second part of system (1), that is

$$\dot{\xi} = A_\sigma \xi + B[G_\sigma(z, \xi)u_\sigma + F_\sigma(z, \xi, t)] \tag{17}$$

is globally bounded under switching laws satisfying the average dwell-time (13).

View $y_{nd} = [y_d^T, \dot{y}_d^T]^T$ as the new input for the closed-loop system (14), (17). Let $P = Q^{-1}$, $K = NP$. It is easy to verify that (10) is equivalent to

$$PA_i + A_i^T P + PBK + K^T B^T P + \vartheta P + P^2 < 0 \tag{18}$$

Choose

$$V(\xi) = \xi^T P \xi \tag{19}$$

as the common Lyapunov function candidate for system (17), where P is the common solution P of (18).

Then, based on (11) and (18), when $\sigma = i$, the derivative of $V(\xi)$ along the trajectory of (17) is

$$\begin{aligned}\dot{V} &= \xi^T (A_i^T P + PA_i)\xi + 2\xi^T PB[G_i(z, \xi)u_i + F_i(z, \xi, t)] \\ &= \xi^T (A_i^T P + PA_i + PBK + K^T B^T P)\xi \\ &\quad + 2\xi^T C^T [G_i(z, \xi)u_i + F_i(z, \xi) - K\xi] \\ &\le -\vartheta \xi^T P \xi - \xi^T P^2 \xi + 2y^T [G_i(z, \xi)u_i + F_i(z, \xi, t) - K\xi]\end{aligned}$$

Let $\delta = \lambda_{\min}(P^2)$, then, from (14), we have

$$\begin{aligned}\dot{V} \le &- \vartheta V(\xi) - \delta\|\xi\|^2 + 2y^T[-(CB)^{-1}CA_i\xi - \kappa_1 e \\ &- (\kappa_2 + \rho(t))\operatorname{sgn} e + (CB)^{-1}\dot{y}_d(t) + F_i(z, \xi, t) - K\xi]\end{aligned}$$

Based on $\|y_d\| \le \|y_{nd}\|$, $\|\dot{y}_d\| \le \|y_{nd}\|$, Assumption 3 and $y = y_d$ when $t \ge t_d$, we can find positive constants ϖ_1, ϖ_2, ϖ_3, such that

$$\begin{aligned}\dot{V} &\le -\vartheta V(\xi) - \delta\|\xi\|^2 + \varpi_1\|y_{nd}\|\|\xi\| + \varpi_2\|y_{nd}\|^2 + \varpi_3\|y_{nd}\| \\ &\le -\vartheta V(\xi) - \delta\left(\|\xi\| - \frac{\varpi_1}{2\delta}\|y_{nd}\|\right)^2 + \left(\varpi_2 + \frac{\varpi_1^2}{4\delta}\right) \\ &\quad \times \|y_{nd}\|^2 + \varpi_3\|y_{nd}\| \\ &\le -\vartheta V(\xi) + \left(\varpi_2 + \frac{\varpi_1^2}{4\delta}\right)\|y_{nd}\|^2 + \varpi_3\|y_{nd}\|, \quad \forall t \ge t_d\end{aligned}$$

Let $\chi(\|y_{nd}\|) = (\varpi_2 + (\varpi_1^2/4\delta))\|y_{nd}\|^2 + \varpi_3\|y_{nd}\|$, we know that $V(\xi)$, $\chi(\|y_{nd}\|)$ are class $\mathcal{K}_\infty$ functions. It is easy to know that the closed-loop system (14), (17) is input-to-state stable with respect to the new input y_{nd} when $t \ge t_d$ under arbitrary switching laws. So, under the switching laws satisfying the average dwell-time (13), it is also input-to-state stable with respect to the new input y_{nd} when $t \ge t_d$. By virtue of Assumption 4, we can conclude that y_{nd} is bounded. Thus, the global boundedness of ξ under the switching laws satisfying the average dwell-time (13) when $t \ge t_d$ follows from the property of input-to-state stability.

When $t \le t_d$, noticing that all the switched subsystems of the closed-loop system (14), (17) are globally Lipschitz continuous for a given bounded y_{nd}, therefore the state ξ has no finite escape time when $t \in [0, t_d]$ for arbitrary switched subsystems. So, for the closed-loop system (14), (17), the state ξ has no finite escape time under arbitrary switching laws satisfying the average dwell-time (13) no matter which subsystem is activated first.

The above analysis shows that the state ξ is globally bounded for the closed-loop switched subsystem (14), (17) under arbitrary switching laws satisfying the average dwell-time (13).

From (7) to (9) and Lemma 1, it is easy to see that the z-part of system (1) is input-to-state stable with regard to ξ under arbitrary switching laws satisfying the average dwell time

$$\tau_{a_1} \ge \tau_{a_1}^* = \frac{\ln \mu_z}{\lambda_0}, \quad \lambda_0 = \min\{\lambda_{0i} \mid i = 1, 2, \ldots, N\} \tag{20}$$

As the designed average dwell-time (13) is a special case of (20), it is easy to know that the z-part of system (1) is also input-to-state stable with regard to ξ under arbitrary switching laws satisfying the average dwell-time (13). Similarly, from the input-to-state stable theory, we know that the state z is also globally bounded under the designed switching law.

Remark 2: It is pointed out in the proof of Theorem 1 that the state ξ has no finite escape time for arbitrary switched subsystems of system (1) when $t \le t_d$. However, if switchings occurs during $0 \le t \le t_d$, the state ξ of the switched system (1) would have finite escape time. So, we let $t_2 \ge t_d$ in order to guarantee the boundedness of the state ξ of the switched system (1).

4 Example

Consider the switched system

$$\begin{cases} \dot{z} = f_\sigma(z, \xi) \\ \dot{\xi} = A_\sigma \xi + B[G_\sigma(z, \xi)u_\sigma + F_\sigma(z, \xi, t)] \\ y = C\xi \end{cases} \tag{21}$$

where $f_1 = -z^5 - 2z + \xi_2, f_2 = -z^3 - z + \xi_1$, $G_1 = 6 + \xi_1^2$ $G_2 = 2 + z^2$, $F_1 = 0.05 \sin t$, $F_2 = 0.06 \cos t$

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -4 & 0 \\ -2 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 4 \end{bmatrix},$$
$$C = [0.5, \ 1.5]$$

Let $\vartheta = 1$. Solving the matrix inequalities (10) and (11), we can obtain

$$Q = \begin{pmatrix} 133.4 & -38.2 \\ -38.2 & 14.3 \end{pmatrix}, \quad N = [23.7, \ \ -50.4]$$

Then, we have

$$P = Q^{-1} = \begin{pmatrix} 0.03 & 0.09 \\ 0.09 & 0.3 \end{pmatrix}$$

Choosing $y_d = \cos 10t$, $\rho(t) = 0.06$ and the initial state be $[1\ 1\ 2]^T$, we can calculate that $e = 0.05\xi_1 + 0.15\xi_2 - 0.1y_d$, $t_d = 0.53$.

For the z-part of system (21), select $G_1(z) = (1/2)z^2$, $G_2(z) = z^2$. A simple calculation shows that $\lambda_{01} = \lambda_{02} = 1.5$, $\gamma_{01}(\|\xi\|) = \xi_2^2$, $\gamma_{02}(\|\xi\|) = \xi_1^2$, $\mu_z = 2$. Choose $\lambda = 1 < \lambda_0 = 1.5$, the average dwell time that the switching laws satisfied is

$$\tau_a \ge \tau_a^* = 0.65 \tag{22}$$

Design the variable structure controller for system (21) as (see (23))

where $\kappa_1 = 1$, $\kappa_2 = 0.01$.

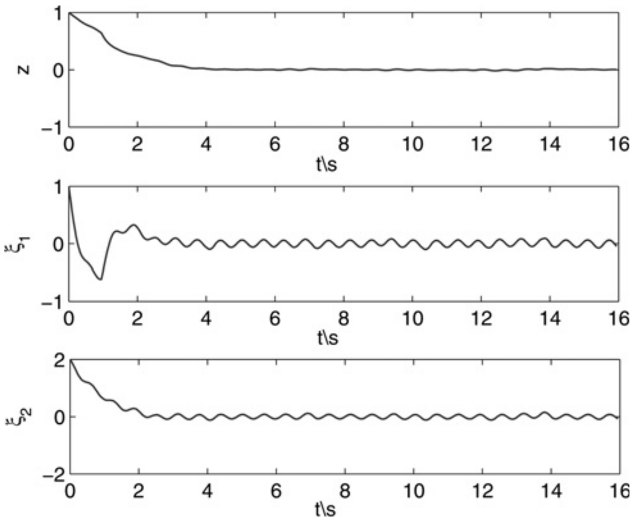

Figure 1 *State response of the closed-loop switched system*

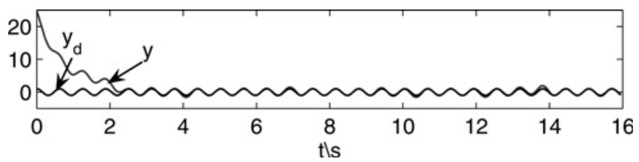

Figure 2 *Output of the system and the reference trajectory*

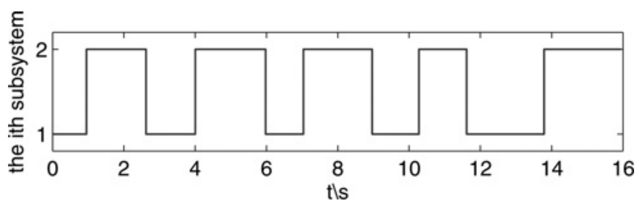

Figure 3 *Switching signal*

Figs. 1–3. show the simulation results using the proposed method. Figs. 1 and 2 indicate that the output of system (21) can track the desired signal y_d reasonably and all the states of system (21) remain uniformly bounded under the designed average dwell-time-based switching law. Fig. 3 shows that the length of the first switching time interval of the designed switching law is greater than 0.53, and the average time interval between consecutive switchings is not less than 0.65, which is in accordance with (13).

5 Conclusions

In this paper, we have studied the output tracking control problem for a class of cascade non-linear switched systems with external disturbances under some average dwell-time

$$u_i = \begin{cases} -\dfrac{1}{6 + \xi_1^2}[-0.1\xi_1 + 0.15\xi_2 + e + 0.07\text{sgn}\, e + \sin 10t], & i = 1 \\ -\dfrac{1}{2 + z^2}[-0.5\xi_1 - 0.75\xi_2 + e + 0.07\text{sgn}\, e + \sin 10t], & i = 2 \end{cases} \tag{23}$$

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based switching law. Based on Lyapunov function method, a sliding surface and variable structure controllers are designed. The average dwell time is constructed explicitly based on the structural characteristic of the system. An example is given to demonstrate the design procedure of our approach. Simulation results show that the goal of output tracking can be achieved by the approach.

Output tracking control problem for non-linear switched systems is very difficult and many problem needs to be addressed. For example, how to remove the matching condition deserves further study.

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