



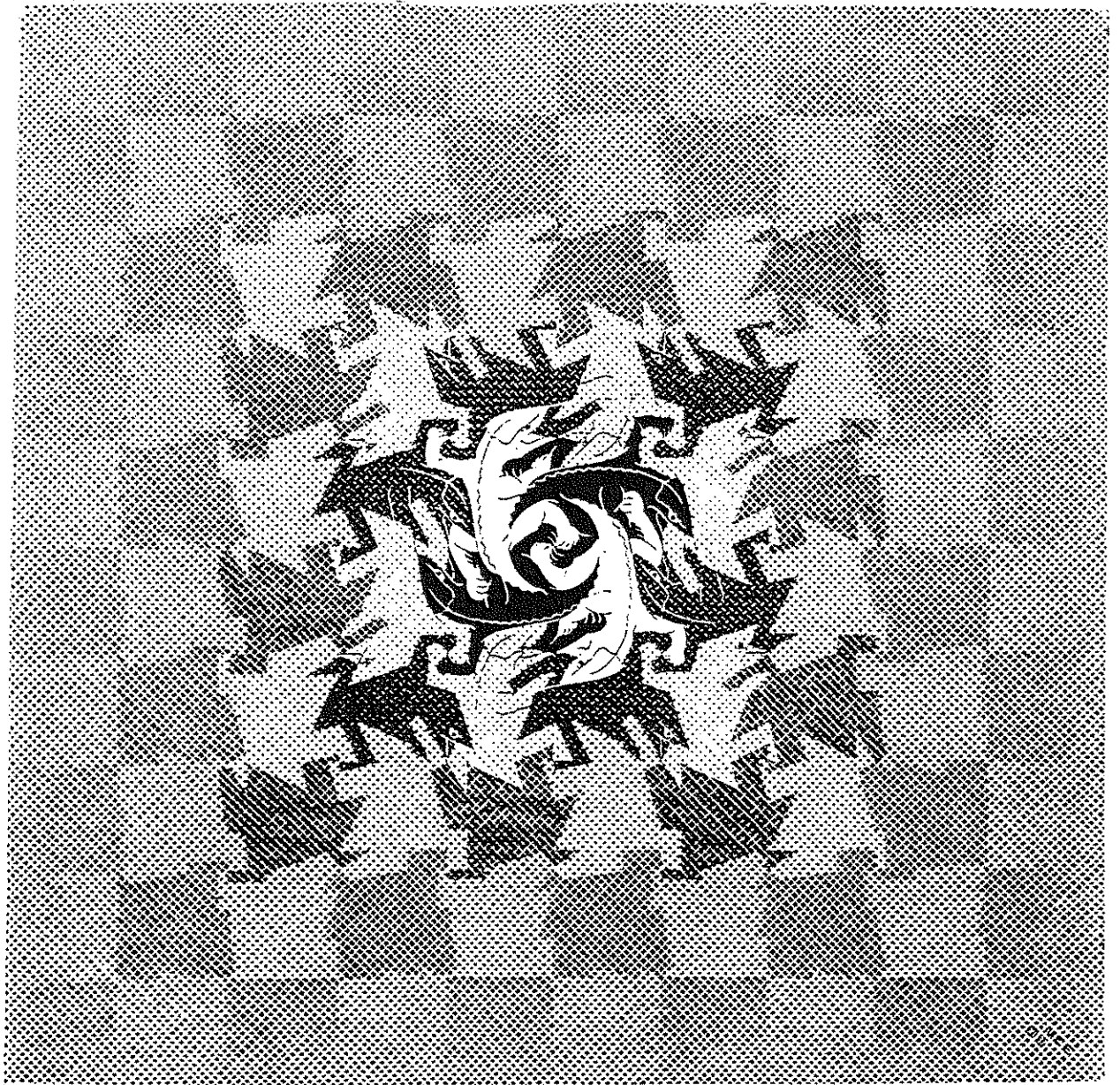
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BEING COHERENTLY VAGUE
The Logic and Metaphysics of Vagueness

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A thesis submitted for the degree of Doctor of Philosophy
of The Australian National University, November 1993.

Declaration

This dissertation is my own work,
except where otherwise acknowledged.

A handwritten signature in black ink that reads "Dominic Hyde". The signature is written in a cursive style with a prominent flourish at the end of the word "Hyde".

Dominic Hyde

- *If you hit a rock hard enough and often enough with an iron hammer, some mollycules of the rock will go into the hammer and contrariwise likewise.*

- *That is well known, he agreed.*

- *The gross and net result of it is that people who spend most of their natural lives riding iron bicycles over the rocky roadsteads of the parish get their personalities mixed up with the personalities of their bicycles as a result of the interchanging of the mollycules of each of them, and you would be surprised at the number of people in country parts who are nearly half people and half bicycles.*

Mick made a little gasp of astonishment ...

- *Good Lord, I suppose you're right.*

- *And you would be unutterably flibbergasted if you knew the number of stout bicycles that partake serenely of humanity.*

Here the sergeant produced his pipe ...

- *Are you sure about the humanity of bicycles? Mick enquired of him. Does it not go against the doctrine of original sin? Or is the Molecule Theory as dangerous as you say?*

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My early inquiries into the problems that vagueness begets, typified by the sorites paradox, were aided and abetted by Graham Priest; his rigour, patience and dissatisfaction with the *status quo* provided fertile ground for inquiry. During two visits to Lund, Sweden, Bertil Rolf generously gave of his time as an advisor. His standards of critical inquiry continue to serve as a benchmark and his influence permeates this thesis, along with that of Richard Sylvan.

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Introduction

The concept of 'vagueness' has many differing senses. The sense of 'vagueness' with which this thesis deals is not that colloquial sense whereby something counts as vague if it is unclear, inexact or hazy; we are concerned with a more technical sense to be found in the philosophical literature, according to which the hallmark of vagueness is the presence of "borderline cases". Vagueness, in this sense, is primarily an attribute of terms of natural language and manifests itself in apparent semantic indeterminacy; that indeterminacy, for example, that arises when asked where to draw the line between the red and the non-red, or the tall and the non-tall.

The aim of this thesis is to show how this concept of vagueness can also be applied to the world — that which vague natural language seeks to describe.

In Chapter One we focus in on this technical sense, clarifying and disambiguating in the process, in an attempt to arrive at a systematic description of the phenomenon of vagueness. The exercise is not merely descriptive however; the phenomenon gives rise to a logical puzzle, commonly attributed to the ancient Greek philosopher Eubulides of Miletus. This puzzle, more usually considered in the form of a paradox — the sorites paradox, presents vagueness as more than just a challenge for orthodox semantic theory; it is a challenge for orthodox logical theory as well.

The task then is to provide an analysis of the phenomenon of vagueness constrained by the need to satisfactorily dispel puzzlement surrounding the sorites paradox. We shall look in detail at the three theories of vagueness which dominate the analytic approach to this phenomenon; the Epistemic Theory; the Representational Theory; and the Ontological Theory.

The Epistemic Theory discussed in Chapter Two seeks to find a place for vagueness within the orthodoxy. Vagueness, says the epistemic theorist, merely *appears* to have its source in semantic indeterminacy; in fact, natural language is semantically determinate. Claims to the contrary are rejected in favour of an account according to which the purported semantic indeterminacies are in fact epistemological. Vagueness in natural language is a manifestation of the unknowability of certain semantic facts.

Vagueness on this account presents no challenge to orthodox semantics or logic; the sorites paradox has a solution which leaves classical logic intact.

The widespread view that vagueness is properly a semantic phenomenon has left most philosophers dissatisfied with this account (so much so that the epistemic theory is

often ruled out by definition). The remainder of this thesis — Chapters Three to Six — will be taken up with an analysis of vagueness as a semantic phenomenon.

Those theorists advocating a semantic approach to vagueness can be further distinguished by attending to matters metaphysical. Some agree with the epistemic theorist that vagueness is in no way attributable to "the world"; though language contains terms whose meaning *is* indeterminate, this does not reflect any indeterminacy in that which language describes. Such theorists, in claiming vagueness to be a merely semantic phenomenon, endorse what I have termed a Representational Theory of vagueness. It is by far the most popular approach and, consequently, the many variations on the representationalist's theme will occupy us throughout Chapters Three, Four and Five.

Their response to the challenge posed by vagueness varies — some declaring vague language beyond the scope of any semantic and logical theory, and some admitting that orthodox logic and semantics requires extension. At worst, orthodox metaphysics is retained whilst classical logic and semantics are conservatively extended.

However, a small minority, myself included, see vagueness as at least sometimes ontologically grounded; some semantic vagueness is due to indeterminacies in that which is described. Such a view — the Ontological Theory of vagueness — seeks to show how it is that the world could be vague and to subsequently show how classical reasoning in the context of vagueness leads to puzzles which can be avoided if logic is revised appropriately. The task of Chapter Six is to establish this account as a viable and desirable alternative.

This thesis, therefore, will attempt to show how vagueness can constitute grounds for a deviant metaphysics, semantics and logic.

Chapter One

The Concept of Vagueness

Lying along the eastern edge of the great deserts of Central Australia is a varied and arid landscape stretching some 2,000 kilometres. From the south you traverse the dry open woodlands of the mallee country, across sandy creek beds and occasional ranges, then gibbers, spinifex and massive flood-plains bordered, towards the Interior, by high rolling red sand dunes. Occasionally, as if by magic, one chances upon permanent water-holes where pelicans fish and where complex, ancient communities of Australia's indigenous inhabitants once thrived. Continuing north over flat, lightly wooded grasslands you cross a final range before meeting the tropical shores of the northern oceans. This seemingly harsh country, where the first whites, misguided sons of the British Empire, paid for their "grim discoveries" with their lives (and, tragically, those of numerous communities of one of the most ancient cultures) has long been viewed by subsequent European-culture as barren and lifeless. Yet, ironically, intermittent localised rains sustain an ephemeral lakes system within this magnificent country which is, in fact, what ecologists have recently described as an extensive and diffuse wetlands.¹ Where precisely do these particular wetlands begin and end; what are their precise boundaries? When does a region count as a 'wetlands' area?

These questions appear difficult, even impossible, to answer. There would seem to be no sharp line delimiting this particular wetlands from the surrounding country; there are regions that would seem to be part of the wetlands and regions not part of the wetlands, yet intuitively there is no sharp line that separates them from each other. Analogously, there would seem to be no sharp line delimiting situations where the term 'wetlands' itself applies, from situations where it does not. The extensive region described above was not always a wetlands area, yet it seems impossible to identify any precise moment in its evolutionary history as that moment at which the term 'wetlands' was first applicable.

We are confronted with vagueness.

¹ CSIRO report, cited on *Quantum*, ABC television, Wed. March 15, 1989, and ABC radio's *The Science Show*, Sat. April 24, 1993.

What exactly is vagueness and why is its analysis of interest? This chapter starts with a discussion of vagueness in its most common setting, natural language, and proceeds initially by discussing the paradigmatic concept of vagueness as applied to predicates. Then, having described a sufficiently thick notion of predicate-vagueness, we are in a position to consider that ancient conundrum which presents vagueness as a problem — the sorites paradox. Various kinds of predicate-vagueness are subsequently distinguished and the concept is extended beyond predicates to other parts of language, necessitating, in due course, some further qualification of the notion of predicate-vagueness.

A plausible, elegant and general account of what it means to speak of vagueness in language is offered. With this taxonomy of vagueness to hand we can pursue our investigations into its cause and logic with broad agreement as to the phenomenon being addressed and the nature of the challenge it presents.

1.1 *Vagueness Introduced*

'Vague' is an ambiguous term yet this thesis is only concerned with vagueness in one specific sense. In this first section therefore I shall try to delimit that sense of vagueness around which the thesis revolves. Much of it is expository and draws on the work of others in the field. In this way it is to be hoped that a general consensus will begin to emerge, not only for the sake of this thesis but for the discussion of the problem by philosophers in general.

1.1.1 *What is Vagueness?*

Apparent lack of sharp boundaries is prevalent in our use of natural language. Consider Jo Bloggs going to a party stone-cold sober. She sits down to talk to a friend and fills her glass from what appears to be a large whiskey supply. After this one glass we would be inclined to say she is tipsy; after one bottle we would almost certainly say she is drunk and after four bottles we may suspect she is dead. Which sip made her tipsy, which made her drunk and which made her die?

Likewise we may ask, seemingly to no avail, at what instant did the autumn leaves turn brown or did that person become rich, famous, bald, tall or an adult. These predicates — 'is tipsy', 'is drunk', 'is brown', 'is rich', etc. — are all examples of predicates whose limits of application seem essentially indefinite or indeterminate, and they are typical examples of what are termed *vague predicates*. Take the predicate 'is tall' for instance. We might line up a crowd of people starting with the shortest and progressing monotonically

to the tallest. The crowd seems not to be clearly partitioned into two mutually exclusive and exhaustive sets of those to whom the predicate applies and those to whom it fails to apply. The transition from one set to the other would seem not to be precise and one might ask rhetorically, as Diogenes Laërtius is reputed to have done, "Where do you draw the line?".²

The most common instances of vague predicates are those for which the applicability of the predicate just seems to fade off, as in the above examples, and it consequently appears that no sharp boundary could conceivably be drawn separating the predicate's positive extension from its negative extension. The behaviour of vague predicates is thus contrasted with such precise predicates as 'is tall*' (defined to mean over two metres in height), or 'is greater than two' defined on the natural numbers. With enough effort we could presumably draw a sharp line between those people who are determinately tall* and those who are determinately not. Similarly, we can divide the domain of natural numbers, \mathbb{N} , into two sharp sets: $P^- = \{0,1,2\}$ and $P^+ = \{3,4,5, \dots\}$; the set P^- comprising those natural numbers determinately failing to satisfy the predicate 'is greater than two' and the set P^+ comprising those natural numbers that determinately satisfy it.

The sense of vagueness we shall be working with then can already be distinguished from another sense in which language is often said to be vague — vague in the sense of inexact, unspecific or general. Take for example the claim that there are between two hundred and one thousand species of *Eucalyptus* trees. It might be responded that this claim is very "vague" and one could be a lot more "precise". However, it is easy to see that vagueness in this sense is quite different from vagueness as I have described it above. A thoroughgoing discussion is to be found in Chapter Three where problems arise for Russell due to his confusion of the two phenomena, but the distinction can be made with enough intuitive force for present purposes as follows. Being between 1.01 and 3.24 metres is an inexact description of someone's height but it is not vague in the sense of there being indeterminate limits to its application — it will be true if their height lies between these two figures and false otherwise; now I can make a much more exact estimation of their height which nonetheless is more vague, e.g. approximately 2 metres. Increasing exactness is consistent with a decrease in precision whilst a decrease in exactness is consistent with an increase in precision. It might be thought that vagueness always involves some inexactness (if for instance one takes vagueness to be a semantic phenomenon that does not have its source in that described) but they are nonetheless

² Diogenes Laërtius, *Lives of Prominent Philosophers*, (translated and edited by R.D. Hicks) London (1925), vii. 82.

distinct. When speaking of vagueness this thesis will be concerned with the sense outlined above, reserving the more explicit terms like 'inexactness' to describe this other sense.³

The symptom of vagueness alluded to above, our inability to draw a sharp line between those things in the predicate's positive extension and those in its negative extension, is tantamount to there being *border* (*borderline*, or *penumbral*) cases for the predicate in question — cases which jointly constitute the *border region* (*borderline region* or *penumbra*) for the vague predicate.⁴ Intuitively, such cases are where there are objects to which the predicate meaningfully applies (i.e. an object in the predicate's domain of significance) yet for which it appears essentially indeterminate whether the predicate or its negation truthfully applies. That is to say, there are situations where a language user, having carried out all the empirical and conceptual research possible concerning the case to hand, will nonetheless still be unable either to apply the predicate determinately to an object to which the predicate may be said to apply meaningfully or to apply its negation determinately. This apparent indeterminacy or indefiniteness, taken as the *sine qua non* of this, and most other, discussions of vagueness, is not due to the lack of knowledge of facts or of meanings that one could in principle come to know — hence the use of "essential" above.

Notice that I have characterised border cases in terms of an agent's ability to apply predicates, rather than adopting the common characterisation in terms of the semantic properties of the predicate. This is necessary since we are required to leave open the possibility that the vagueness may have its source in us rather than the semantic properties of the predicate itself. In this way we do not foreclose on that response to vagueness which claims it to be an epistemic phenomenon.

I have however prejudged the issue of whether or not border cases might be partly characterised in paraconsistent terms; that is, in terms of an agent's *applying both the predicate and its negation*. The dual notions of over- and underdetermination both seem *prima facie* evident in contexts of vagueness. Situations typically described using locutions associated with vagueness like 'It's neither raining nor not raining' are often alternately described as 'It's raining and it isn't'; more generally, 'Neither' is often recast as

³ Roy Sorensen has found the prevalence of equivocation common enough to warrant an article explaining the distinction, urging that philosophers revise their language and put aside the inexactness sense once and for all, thereby making "the streets of speculation just a little bit safer for the philosophers of tomorrow". See: Sorensen, R.A., 'The Ambiguity of Vagueness and Precision', *Pacific Philosophical Quarterly* 70 (1989): 174-83. This equivocation is further discussed, with examples cited, in Chapter Three, §3.3.1 and Chapter Four, §4.4.

⁴ I shall speak of *border* cases, instead of the more usual *borderline* cases, to remove any prejudicial suggestion that there is still a line between determinate cases and determinate non-cases. *Borderline* cases are typically exemplified by such cases as X's exam paper being a borderline case between a credit (75% - 84%) and a distinction (85%+) where it is marginal whether it should fall one side of the (sharp) line or the other. Our terminology ought to be as neutral as possible on whether there are any sharp lines of categorisation or not since it at least appears that there are none.

'Both'. This overdetermination aspect is almost universally derided as a mere *façon de parler* yet this far from satisfactory. In the absence of any principled reason for the differential treatment of the two pieces of evidence, we seem required to treat both locutions in a like manner — both require accounting for in our theory of vagueness or both are to be counted as a mere *façon de parler*. Yet I suspect that the underlying motivation for discounting evidence for overdetermination is simply that contradictions are to be avoided at all costs, which, as it stands, does not constitute a reasoned defence. More would need to be said.

Moreover, it should be noted that attempts to approach vagueness from a paraconsistent perspective have been made; it is no straw man. In Marxist philosophy key examples of dialectical situations are provided by focussing on what to say about the application of vague predicates to border cases. A seedling in a process of becoming a tree is said to be both a seedling (by virtue of what it was) and not a seedling (by virtue of what it will become); a man growing a beard is at some stage both bearded and not bearded. Terms involved like 'seedling' and 'bearded' are examples of what we would call vague predicates, yet for classical Marxists they are characteristically dialectical, issuing in contradictions.⁵

A more recent discussion, by McGill and Parry, that explicitly incorporates vagueness (as characterised by Russell, Black and Hempel — so well within the analytic tradition's usage of 'vague') into the Marxist tradition makes the claim that "In any concrete continuum there is a stretch where something is both A and ~A. ... There is a sense in which the ranges of application of red and non-red [in so far as 'red' is vague] overlap, and the law of non-contradiction does not hold."⁶ In agreeing with McGill and Parry that vagueness constitutes a case for the failure of the law of non-contradiction, Newton Da Costa and Robert Wolf suggest that one requirement of a paraconsistent dialectical logic "is that the proposed logic be interpretable as a logic of vagueness."⁷

This explicit interest in vagueness from a paraconsistent perspective is not uncommon. A paraconsistent approach to vagueness has been pursued within analytic philosophy by a number of non-classical logicians and philosophers.⁸

Though I am sympathetic to the paraconsistent approach to defining vagueness, I feel that the tasks I have set myself in this thesis are already sufficiently problematic without taking on board this dispute as well. So I shall retain the above underdetermination approach to defining vagueness.

⁵ See: Plekhanov, G., *Fundamental Problems of Marxism*, (ed. by D. Ryazanov) Lawrence and Wishart (1941), esp. pp. 114 ff.

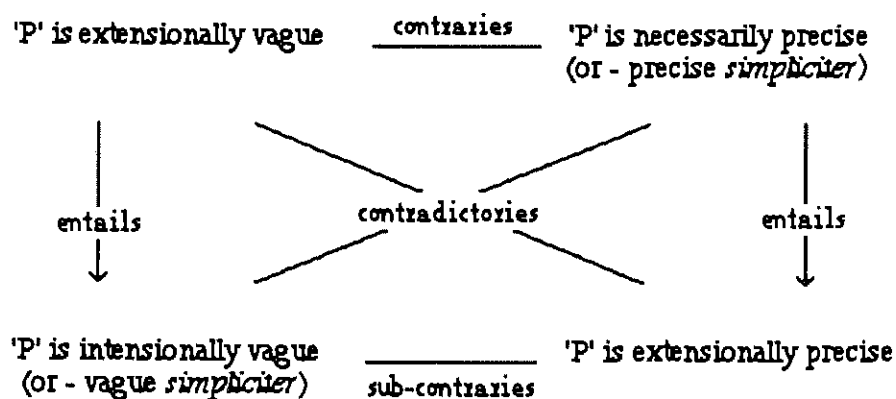
⁶ McGill, V.J. & Parry, W.T., 'The Unity of Opposites: A Dialectical Principle', *Science and Society* 12 (1948): 418-44, esp. pp. 428 ff.

⁷ Da Costa, N.C.A. & Wolf, R.G., 'Studies in Paraconsistent Logic I: The Dialectical Principle of the Unity of Opposites', *Philosophia* 9 (1980): 189-217, esp. p. 194.

⁸ Cf. Chapter Five, n. 22 for references to further work making such claims.

A further caveat must also be added. Though we commonly speak of a predicate's vagueness in terms of there (actually) being border cases for the predicate, there is a weaker sense of vagueness that does not depend on mere contingencies regarding what actually exists — namely, the very possibility of there being border cases. In this weaker sense a predicate does not cease to be vague simply because its border cases cease to exist; the logical possibility of their existence is enough to guarantee the conceivability of border cases and it is this, rather than the actual existence of border cases, that is of logical interest. This weak sense is more exactly described as *intensional vagueness* and depends only on the conceptual possibility of border cases as opposed to the stronger *extensional vagueness* which requires the actual existence of a border case.⁹ Analogously, though *extensional precision* — the actual absence of border cases — may sometimes be of interest (e.g. in assessing claims to actual truth) we shall generally be more interested in the logically necessary absence of border cases, *necessary precision*. With this in mind we shall often speak of vagueness and precision *simpliciter*, where these are to be understood respectively as intensional vagueness and necessary precision.

The following square of opposition describes the logical interrelatedness of the four concepts.



In subsequent chapters we shall inquire further into the *cause* or source of the essential indeterminacy underpinning vagueness — Is it perhaps epistemic, due to the in principle unknowability of certain semantic facts? Or, if a semantic property of language, could this be due to vagueness somehow 'in the world'? The exact causes of these border cases — the reason for the failure of any research to resolve such cases one way or the other — is unquestionably the major watershed amongst analytic philosophers writing on the topic of vagueness; it is therefore not something that can be settled at this stage. The reason for

⁹ Though the terms 'intensional' and 'extensional' are overworked, they have become part of the technical jargon in the area and so I shall follow the established convention.

this is that vagueness has, like so many other issues in contemporary analytic philosophy, entered the arena as a puzzle any resolution of which seems to raise problems somewhere in the delicate fabric of epistemology, semantics, metaphysics and logic. This thesis can be seen, in part, as an attempt to address this issue, bound up as the issue is in how one analyses the phenomenon of vagueness in general. Yet however one goes on to diagnose the cause or source of vagueness, the presence of border cases — this border region — is in principle irremovable. If the indeterminacy turns out to be analysed as epistemic, for example, then it will be an epistemic gap that is unbridgeable in principle.

For the moment we need to focus more clearly on its *content*.

Cashing out our intuitions regarding border cases even further then, we must distinguish border cases from cases where one is at a loss what to say due to ambiguity or context sensitivity. Take Mark Sainsbury's example using the ambiguous word 'bank' — meaning a financial institution or a river's edge. It may be essentially indeterminate whether or not to agree to the claim 'Jo went to the bank yesterday' since the single sentence may be used to assert more than one thing. An audience is unable to determinately answer one way or the other since they firstly need to determine *which* proposition is being asserted. Border cases are different. We may be unable to say whether or not Jo is rich but this is not due to our being unable to determine *what* is meant by 'rich' — that is presumably understood. The word 'rich' has a single vague meaning as opposed to having two or more distinct meanings; border cases can arise in our use of unambiguous words and ambiguous words need have no border cases.

Context sensitivity is similarly distinct in so far as context sensitive words might be vague, even if a context is fixed, and precise words can be context sensitive. Take the word 'tall' for example. What counts as tall can vary from context to context; tall pygmies are usually short basketball players. Now even when this relativity to context is fixed, e.g. tall relative to basketball players, there might still be border cases — Is he a tall basketball player or not? There may also be situations where one is unable to say whether or not the word applies simply because the correct ascription depends on the context and this phenomenon occurs regardless of the vagueness of 'tall'; that is, this inability does not constitute a border case. Suppose 'tall' were to mean 'above average in height' which intuitively makes it a precise term; nonetheless there may be situations where I am uncertain whether to apply the term or not since it depends on whether the average is taken over pygmies or basketball players.

So intuitively vagueness is to be distinguished from meaninglessness, ambiguity and context sensitivity.

Thus vagueness is introduced as a phenomenon affecting predicates and, as such, is characterised by the presence of border cases as they have been described above. This initial focus on predicate-vagueness reflects one aspect of what we might describe as *the*

paradigmatic concept of vagueness :— vagueness as applied to predicates and characterised by the presence of border cases. Alston for example, like so many others describing vagueness in the literature, commences his *Encyclopedia of Philosophy* entry, 'Vagueness', with the following:

To say that a word is vague is to say that there are cases in which there is no definite answer to whether it applies to something. Thus "middle-aged" is vague, for it is not clear whether a person aged 40 or a person aged 59 is middle-aged.¹⁰

Apart from the fact that Alston, again like so many others, seems to have prejudged the issue of whether vagueness could be epistemic (i.e. whether the essential indeterminacy could be due to there being a determinate answer though an unknowable one) negatively, he typifies the paradigmatic approach.

Peirce, Black, and Church all characterise vagueness in this way in their various entries in dictionaries of philosophy, as do Russell and Quine in their writings on vagueness and Alston in his influential *Philosophy of Language*.¹¹ To this extent the name 'the paradigmatic concept of vagueness' is descriptive as well as prescriptive; there really is no other characterisation of vagueness available as yet.

It is little wonder that discussions of vagueness have tended to centre around the concept of vagueness as applied to predicates since this is where issues surrounding vagueness originate historically. Problems with vagueness arose in antiquity in the context of the sorites puzzles, puzzles understood as exploiting the apparent lack of sharp boundaries for certain predicates. In subsequent sections we'll consider the sorites puzzles in detail (§1.2), look at various types of predicate-vagueness (§1.3), and see how one might extend the concept of vagueness beyond predicates to other semantic categories (§1.4), but right now I want to consider more carefully that other aspect of the paradigmatic concept of vagueness — the relationship between vague predicates and their border cases.

A predicate having no border cases is precise; border cases are therefore necessary for vagueness, as already remarked. However, are they sufficient? Is a predicate's vagueness really *characterised* or *defined* by its having border cases? According to a naïve paradigmatic concept of vagueness we might suppose it is :— a predicate is vague if and

¹⁰ Alston, W.P., 'Vagueness' in P. Edwards (ed.), *The Encyclopedia of Philosophy*, MacMillan (1967), p. 218.

¹¹ Peirce, C.S., 'Vague' in J.M. Baldwin (ed.), *Dictionary of Philosophy and Psychology*, London (1902), p. 748; Black, M., 'Vagueness' and Church, A., 'Vague' in D. Runes (ed.), *Dictionary of Philosophy*, Philosophical Library, New York (1960), p. 329; Russell, B., esp. 'Vagueness', *Australasian Journal of Philosophy and Psychology*, 1 (1923), pp. 84-92; Quine, W.V.O., *Word and Object*, MIT (1960), Ch. 4; Alston, W.P., *Philosophy of Language*, Prentice-Hall (1964), pp. 84-5.

only if it has border cases. On this understanding of vagueness, the presence of border cases is both necessary *and sufficient*.

However, even having made refinements to the notion of a border case so as to exclude indeterminacy due to meaninglessness, ambiguity, and context specificity, meeting the criteria for being a border case still might seem insufficient for vagueness. Russell, Alston, and many recent theorists have further qualified the paradigmatic concept of vagueness, expanding on the notion of a border case.

We can illustrate the worry using an example described by Sainsbury — the predicate 'child*'. It is to count as true of all those people who have not yet reached their sixteenth birthday, false of all those who have reached their eighteenth birthday, and neither true nor false of all other people. Now, for a seventeen year old it is neither determinately true that they are a child* nor is it determinately false that they are a child* — they do not determinately satisfy the predicate nor do they determinately satisfy its negation. So it would seem that a seventeen year old counts as a border case for the predicate 'child*' thereby making it vague, even though, intuitively, the predicate is perfectly precise. It is indeed the case that the predicate fails to draw a single sharp boundary between its positive and negative extensions, thus there are cases of indeterminacy, however the fact that it draws two sharp boundaries, one between its positive and border cases and one between its border and negative cases, seems to disqualify it as a candidate for vagueness proper; it is rather an incomplete predicate.¹²

Similar cases can be manufactured at will if incomplete objects are admitted as candidate satisfiers for predicates. Some philosophers are prepared to treat such a name as 'Hamlet' as a name for the incomplete object Hamlet. This way of dealing with fictional entities involves a commitment to incomplete objects and as such seems to make the predicate 'wears size 8 shoes' vague — Hamlet neither determinately satisfies the predicate nor determinately satisfies its negation and so would appear to be a border case for the predicate. Yet such indeterminacy again only seems to lead to a sharp tripartite division of the predicate's range of significance: those things of which we can truly predicate 'wears size 8 shoes', those of which we can falsely predicate 'wears size 8 shoes', and those objects for which the predicate is neither true nor false.

What has gone wrong is that we seem to have again confused vagueness with incompleteness. Incomplete predicates effect a sharp tripartite division of their range of significance, whereas vague predicates appear to draw no sharp boundaries. Of course,

¹² Crispin Wright, in his 'Further Reflections on the Sorites Paradox', *Philosophical Topics* 15 (1987): 227-90, esp. pp. 244 ff, has argued that the vague predicate 'red' *does* effect a tripartite division of its range of significance; it has a penumbra of border cases but this penumbra is itself precise. Sainsbury convincingly rebuts Wright's argument by denying Wright's criterion for the correct use of 'red'; cf. Sainsbury, R.M., 'Is there Higher-Order Vagueness?', *Philosophical Quarterly* 41 (1991): 167-82, esp. pp. 176 ff. Even were Wright's argument successful, there is no reason to think that it would hold in general.

there is no reason why predicates cannot be both vague and incomplete, but now their vagueness will be evidenced by border cases for some of the three categories.

What is being gestured at or sought when we dismiss cases like those above cannot yet be fully explained by means of the simple notion of a border cases between the positive and negative extensions of a predicate; something more needs to be said. Russell acknowledged an extra ingredient.

The fact is that all words are attributable without doubt over a certain area, but become questionable within a penumbra, outside of which they are again certainly not attributable. *Someone might seek to obtain precision in the use of words by saying that no word is to be applied in the penumbra, but fortunately the penumbra itself is not accurately [precisely] definable, and all the vaguenesses [sic] which apply to the primary use of words apply also when we try to fix a limit to their indubitable applicability.* [my italics]¹³

And Alston more recently echoed this in the *Encyclopedia* when, after having endorsed the border case conception (as we noted previously), he continues:

..."middle-aged" is vague, for it is not clear whether a person aged 40 or a person aged 59 is middle-aged. Of course there are uncontroversial areas of application and nonapplication. At age 5 or 80 one is clearly not middle-aged, and at age 45 one clearly is. But on either side of the area of clear application there are *indefinitely* bounded areas of uncertainty. [my italics]¹⁴

The now common response to the vagueness of the penumbra itself is simply to say that the penumbra has border cases. Thus, in an attempt to get at this extra ingredient by means of the notion of a 'border case', that is, from within the paradigmatic concept of vagueness, talk moves to a hierarchy of border cases and the paradigmatic concept is iterated.

Since the mid-seventies this phenomenon of *higher-order vagueness* (HOV) has come to the fore in discussions of vagueness. Higher-order vagueness arises, as we have seen, because vague predicates typically fail to draw *any* apparent sharp boundaries within their range of significance. The paradigmatic concept we have been discussing initially attempts to accommodate the intuition that there is no apparent sharp boundary between the positive and negative extension of a predicate by describing the presence of a penumbra or border region (or border cases). So, for example, with the predicate 'red' the absence of any apparent sharp boundary between the red and the non-red is initially

¹³ Russell, *op. cit.*, p. 87.

¹⁴ Alston, W.P., 'Vagueness', *op. cit.*, p. 218.

described by reference to border cases. The requirement that there be no apparent sharp boundary is thus satisfied by eliminating the sharp boundary and replacing it with a border region. Yet the region itself might paradoxically appear sharply bounded — unless it too has border cases.

There is no more an apparent sharp boundary between the positive extension and the border region than there was between the positive extension and the negative extension. So, if the presence of a penumbral region is taken as definitive of vagueness then it is not itself characterised merely by the existence of border cases; there must be border cases of border cases. But why stop here? There appears to be no more reason to suppose that there is a sharp boundary between the determinately determinately red and the vaguely determinately red than there was to suppose a sharp boundary between the determinately red and vaguely red. "At no point does it seem natural to call a halt to the increasing orders of vagueness"¹⁵, as Kit Fine puts it, so the iteration seems endless — border cases echo up through the hierarchy.

The real lesson of higher-order vagueness is that vague predicates *draw no apparent sharp boundaries*, not merely that they apparently fail to draw a sharp boundary at the first level, or the first and second levels, or ...

After having said all of the above one might think, like Sainsbury¹⁶, that the iterative conception of vagueness is both inescapable on the paradigmatic approach to vagueness and misguided, thus motivating a search for a entirely new approach which avoids the so-called "problem of higher-order vagueness". Or, one might think that the iterative conception, with qualifications *ad infinitum*, is adequate and unproblematic. (Wright's attempt, in 'Further Reflections...', to argue that sometimes, at least, there are no higher-orders of vagueness is both unsuccessful and too weak in any case). These two responses presuppose there to be a real problem to be addressed.

I want to argue that the iterative conception captures a feature of vagueness that is real enough — the phenomenon of higher orders of vagueness — but that this phenomenon ultimately is an echo of a more basic feature of border cases. There is no problem here really. Recourse to an infinite string of qualifications, like those above, betrays an ignorance of the ambiguity of 'border case'.

We shall not pursue these further issues surrounding the problem of higher-order vagueness just yet however. Such a discussion must wait until the end of §1.4 where I shall argue that HOV is a pseudo-problem — by that stage we shall be in a position to appreciate that the border case characterisation of vagueness enables us to distinguish vagueness from incompleteness, as it stands.

¹⁵ Fine, K., 'Vagueness, Truth and Logic', *Synthese* 30 (1975), p. 297.

¹⁶ Sainsbury, R.M., 'Is there Higher-Order Vagueness?', *op. cit.*, p. 173.

In spite of these apparent problems to do with HOV, the paradigmatic concept of vagueness nonetheless provides us with a provisional border case definition of predicate-vagueness which is sufficient to get on with. Further refinements to predicate-vagueness will be made later in this chapter to take account of deviant border cases but the spirit of the border case approach will remain unchanged.

1.1.2 *How Pervasive is Vagueness?*

Now that we have a grasp of *what* predicate-vagueness amounts to, *where* does it arise — just how pervasive is it? Well, once we start looking we see vagueness everywhere. The language of the social sciences, for example, seems ineradicably vague. Terms like 'neighbourhood', 'community' and 'state of recession' all exhibit an apparent lack of sharp boundaries of application thus giving rise to border cases. Menges and Skala describe the situation thus:

In the field of social sciences one must ... pull down the natural barriers between things and thrust the 'real' into 'artificial' new units. ... *Social science concepts*, due to the specific formation process, *are in principle vaguer than natural science concepts*. To that extent the problem of vagueness is particularly important in the social sciences and it is necessary to draw upon suitable formalized theories.¹⁷

This is sometimes given as reason for supposing the social sciences to be among the "second-grade" disciplines. Vagueness, seen as a flaw to be eliminated, is a suspect and second-grade feature of language and thus any discipline relying on such language is itself second-grade.

The behavioural sciences also employ terms that are vague like 'depressed', or 'schizophrenic', and as such they too are often described as second-grade.

This entails, of course, that the physical sciences, in so far as they are a paradigm of a "first-grade" discipline, are free of vagueness — as, indeed, they are generally supposed to be. In the language of the physical sciences vagueness is generally seen as a cancer to be eradicated; precision is often seen as one of the aims of a language for such science, though it is questionable whether the language of mathematics, physics or chemistry could ever achieve perfect precision.¹⁸

Beyond the hard sciences, the languages of ecology, ethics and law all exhibit the phenomenon of vagueness. In ecology, terms such as 'environment' and 'habitat' would

¹⁷ Menges, G. & Skala, H.J., 'Vagueness in the Social Sciences' in G. Menges (ed.), *Information, Inference and Decision*, Reidel (1974), p. 55.

¹⁸ This will be discussed in more detail in Chapter Four, esp. §4.4.

seem to be vague and this is sometimes seen as problematic for the demarcation of areas of special interest as "protected".¹⁹

In ethics, terms such as 'voluntary euthanasia' are vague and their use has been criticised on the grounds that the ensuing indeterminacy of application will (or could) lead down "slippery slopes", from cases of legitimate application in situations where the term is applied to cases where it clearly ought not to apply; the slippery ethical slope is then the slide from the ethical legitimisation of the former cases to the legitimisation of the latter cases. Again, the underlying thought is that vagueness must be eliminated, in this case from ethical theories, and in this way we will be able to draw a sharp line demarcating legitimate from illegitimate ethical practices. Vagueness infuses the theory with an instability that leads to untoward consequences and thus one must reject the theory.²⁰

In law, courts try to precisify language in an attempt to provide sharp distinctions between the legal and the illegal, border cases being seen as problematic for the instigation of legal action. Yet lengthy debates before a judge as to the possibility of, say, counting an act as one of 'legally justifiable defence' (and thus permissible by law) are at least sometimes due to the vagueness of the legal terms involved; if we are to draw a line somewhere and thus resolve the vagueness of the offending term for future legal use, the argument concerns where in the border area it will be drawn.²¹

Of course, the fact that language is vague does not mean that we cannot resolve border cases by invention or stipulation. In the case of artificial languages we, as stipulators, may attempt to eradicate vagueness by using only precise predicates. However, whilst we are free to precisify wherever and whenever we choose, the transformed predicate is more than a mere refinement in sense of the original vague term, it is a redefinition. Redefined legal concepts such as 'drunk' or 'alive' and scientific concepts such as 'blue' (interpreted as 'reflecting 445-500 nanometres wavelength of light') may diverge considerably from many natural language users' applications of them. Abortion debates and IVF-program controversies are cases in point. The question of their admissibility as surrogates for the original vague predicate will be discussed in future chapters.

The simple fact is that vagueness is ubiquitous. I might add that, on the account of vagueness I shall eventually endorse, it is also generally seen as a perfectly respectable feature of natural language.

19 A good example of this is the very practical problem of defining boundaries for protected areas discussed in *Biological Conservation of the South-East Forests*, Joint Scientific Committee Report to Commonwealth and NSW State Governments, Government Printers (1990), p. 16 & n. 31.

20 A recent book devoted to this problem is Douglas Walton's *Slippery Slope Arguments*, OUP (1992).

21 Hart discusses this problem in: Hart, H.L.A., 'Theory and Definition in Jurisprudence', *Proceedings of the Aristotelian Society Supplementary Volume 29* (1955): 239-64, esp. §V; also: Hart, H.L.A., *The Concept of Law*, OUP (1961), pp. 120 ff.

1.2 *The Sorites Paradox*

I said earlier that the fact that the paradigmatic concept of vagueness centres on predicates is of little wonder given the historical roots of the problem. The reason for this is that vagueness comes to us through history as more than a mere curio; our attempts to arrive at an understanding of the concept of vagueness arise quite naturally from a series of puzzles — puzzles that are generally understood as depending crucially on the lack of sharp boundaries for the predicates involved. In modern philosophy, these puzzles are usually presented as a class of paradoxes known as *the sorites paradoxes*. In coming to appreciate the puzzlement that these paradoxes engender, we will see why the widespread presence of vagueness has been seen as such a problem.

1.2.1 *From Puzzle to Paradox*

Diogenes Laërtius attributed seven puzzles to the logician Eubulides of Miletus, a contemporary of Aristotle.²² These include, amongst others,

The Liar: A man says that he is lying. Is what he says true or false?

The Hooded Man: You say that you know your brother. Yet that man who just came in with his head covered is your brother and you did not know him.

The Bald Man or The Heap: Would you describe a man with one hair as bald? Yes. Would you describe a man with two hairs as bald? Yes. Would you describe ... You must refrain from describing a man with a million hairs as bald, so where do you draw the line? OR: Would you describe a single grain of wheat as a heap? No. Would you describe two grains of wheat as a heap? No. Would you ... You must admit the presence of a heap sooner or later, so where do you draw the line?

This last puzzle, when presented as a series of questions about the application of the predicate 'bald', was known as the *falakros* puzzle and, when presented as a series of questions about the application of the predicate 'heap', was known as the *sorites* puzzle (from the Greek word 'soros' meaning a heap). Later, the whole class of puzzles of this type became known as sorites puzzles.

It is not known whether Eubulides actually invented the sorites puzzles. Some scholars have attempted to trace its origins back to Zeno of Elea, claiming his paradox of the Millet Seed as a sorites puzzle. However, the evidence seems to point to Eubulides as the first to employ the sorites. Nor is it known just what he had in mind when he

²² Diogenes Laërtius, ii 108.

formulated this puzzle. Many targets have been suggested, however he is said to have been exclusively interested in logic and the general consensus is that it was for its delightful puzzlement alone that he proffered such a conundrum.²³

It was, however, employed by later Greek philosophers to attack various positions; most notably by the Sceptics against the Stoics' claims to knowledge. Describing this early history is a fascinating scholarly exercise in itself, but to repeat here what has already been done in this area seems pointless. With the important early texts translated by Long and Sedley, excellent discussions by Myles Burnyeat and especially Jonathan Barnes, and a forthcoming discussion by Timothy Williamson which covers the history of the sorites from antiquity to the twentieth century, I see no reason to add to the already sufficient writings.²⁴

These puzzles of antiquity are now more usually described as *paradoxes*, that is, as apparently valid arguments with apparently true premises and an apparently false conclusion. Though the conundrum can be presented informally as a series of questions whose puzzling nature gives it dialectical force, it can be, and was, presented as having logical structure as well.

The following *argument* form of the sorites was common²⁵:

If a man with 1 hair on his head is bald then a man with 2 is.

If a man with 2 hairs on his head is bald then a man with 3 is.

.
.
.

If a man with 9,999 hairs on his head is bald then a man with 10,000 is.

But A man with 1 hair on his head is bald.

∴ A man with 10,000 hairs on his head is bald.

²³ For more on this see: Barnes, J., 'Medicine, Experience and Logic' in J. Barnes, *et al* (eds) *Science and Speculation*, CUP (1982), pp. 35 ff.

²⁴ Long, A.A., & Sedley, D.N., *The Hellenistic Philosophers*, CUP (1987), Vol. 1; Burnyeat, M.F., 'Gods and Heaps' in M. Schofield & M.C. Nussbaum (eds), *Language and Logos*, CUP (1982); Barnes, J., *op. cit.*; Williamson, T., *Vagueness*, Routledge (forthcoming). See also: Mignucci, M., 'The Stoic Analysis of the Sorites', *Proceedings of the Aristotelian Society* 93 (1993): 231-45.

²⁵ Some Stoic presentations of the argument also used a form which replaced all the conditionals, 'If P then Q', with 'Not(P and not-Q)' to stress that the conditional should not be thought of as being a strong one, but rather the weak Philonian conditional (the modern material conditional) according to which 'If P then Q' was equivalent to 'Not(P and not-Q)'. Such emphasis was deemed necessary since there was a great deal of debate in Stoic logic regarding the correct analysis for the conditional. In thus judging that a connective as weak as the Philonian conditional underpinned this form of the paradox they were forestalling resolutions of the paradox that denied the truth of the conditionals based on a strong reading of them. However, in judging that a connective as *strong* as the Philonian conditional *really* was employed in the sorites arguments, since the argument thus interpreted *was* valid, they reduced their range of possible responses to the paradox to denying either the apparent truth of the premises or the apparent truth of the conclusion. They opted for the former.

It is paradoxical since, though it appears valid, its premises appear true whilst the conclusion seems false. (If a man with 10,000 hairs *does* still seem bald then, since it is arbitrarily chosen, pick any n — say, a million — for which he does not and extend the argument until we can infer, contrary to appearances, that a man with n hairs is bald.)

Innumerable sorites paradoxes can be expressed in this form. For example, one can present the original paradox which gave its name to the whole family. Since one grain of sand is not describable as a 'heap' and if one is not then two is not, so two grains of sand do not constitute a heap. Again, if two is not then three is not, so three grains do not constitute a heap, ... etc. So, for any number of grains of sand n , n grains of sand do not constitute a heap yet we rightly feel that there are piles of sand describable as 'heaps'. Similarly, if one is prepared to admit that there are piles of sand describable as 'heaps', then one could prove that one (or even zero) grains of sand count as heaps since the removal of one grain at any stage cannot make the relevant difference! The falakros paradox leads, as we just saw, to the conclusion that any number of hairs still makes for baldness if 1 hair does, but can also be run using a negative form — so if anyone is not bald then everyone is not bald! Thus the argument comes in both a positive and negative form. The predicates 'is bald', 'is a heap', as well as 'is small', 'is a child', etc. are all *soritical* — i.e. can figure in sorites, or soritical arguments, also known as 'little-by-little' arguments.²⁶

This standard form of the sorites can be schematically represented as follows:

Standard Sorites

$Fa_1, Fa_1 > Fa_2, Fa_2 > Fa_3, \dots, Fa_{n-1} > Fa_n / Fa_n$ for arbitrarily large n

— where '>' represents the connective 'If ... then ...'; 'F' represents the soritical predicate, e.g. 'is bald', or 'is a heap', etc.; and the series $\langle a_1, \dots, a_n \rangle$ represents the sequence of subjects with regard to which F is soritical, e.g. \langle a man with 1 hair on his head, ..., a man with n hairs on his head \rangle or \langle 1 grain of wheat, ..., n grains of wheat \rangle , and so on.

Having described this standard form, we can state conditions under which *any* argument of this form is soritical.²⁷ Initially, the series $\langle a_1, \dots, a_n \rangle$ must be ordered; for example, scalps ordered according to number of hairs, heaps ordered according to number of grains of wheat, and so on. Secondly, the predicate 'F' must satisfy the following three constraints: (i) it must appear true of a_1 , the first item in the series; (ii) it must appear false of a_n , the last item in the series; (iii) each adjacent pair in the series, a_i and a_{i+1} , must be

²⁶ For more versions of the sorites paradox see: Cargile, J., 'The Sorites Paradox', *Journal of the Philosophy of Science* 20 (1969): 193-202.

²⁷ Conditions described by Barnes, *op. cit.*, pp. 30-2.

sufficiently similar as to appear indiscriminable in respect of F — that is, both a_i and a_{i+1} appear to satisfy F or neither do.

Under these conditions 'F' will be soritical relative to the series $\langle a_1, \dots, a_n \rangle$ and any argument of the above form using 'F' and $\langle a_1, \dots, a_n \rangle$ will be soritical.²⁸ It is easy to see now that the argument has both positive and negative forms — what the ancients described as sorites by adding or subtracting; 'F' will be soritical relative to $\langle a_1, \dots, a_n \rangle$ if and only if 'not-F' is soritical relative to $\langle a_n, \dots, a_1 \rangle$.

That key feature of soritical predicates which drives the paradoxes, constraint (iii), is described by Crispin Wright as "tolerance".²⁹ Predicates such as '*.is a heap*' or '*.is bald*' appear tolerant of sufficiently small changes in the relevant respects — namely number of grains or number of hairs. The degree of change between adjacent members of that series relative to which 'F' is soritical would seem too small to make any difference to the application of the predicate 'F'.³⁰ Yet large changes in the relevant respects will make a difference, even though large changes are the accumulation of small ones which don't seem to make a difference. This is the very heart of the conundrum which has delighted and perplexed so many for so long.

How might we respond then to the paradoxical nature of standard sorites arguments? Slaney describes the options thus.³¹ One can:

- (a) Deny that the problem can legitimately be set up in the first place; that is, logic does not apply to soritical expressions.
- (b) Assume logic does apply to soritical expressions but deny that the argument is valid. Iterated *modus ponens* is not valid for the conditional '>'. Since the argument is valid by the canons of classical logic this response amounts to a refutation of classical logic.
- (c) Assume logic does apply to soritical expressions and that the argument is valid, but deny one of the premises.

²⁸ Some logic texts describe multi-premise syllogisms, polysyllogisms, as sorites arguments; cf. Copi, I., *Introduction to Logic*, Macmillan (1972), §7.5; Luce, A.A., *Logic*, English Universities Press (1958), Ch. VIII. Polysyllogistic arguments are similar to sorites as we have defined them in so far as they are chain-arguments, however polysyllogisms need not be paradoxical and sorites as we have defined them need not be syllogistic in form. The usage we have adopted is the more usual these days.

²⁹ Wright, C., 'On the Coherence of Vague Predicates', *Synthese* 30 (1975), pp. 333-4.

³⁰ Naturally, if the degree of change between adjacent members of some other series $\langle b_1, \dots, b_m \rangle$ exceeds the limits of tolerance of the predicate 'F' then condition (iii) above is not met, and though 'F' may be soritical relative to the series $\langle a_1, \dots, a_n \rangle$ it will not be soritical relative to the series $\langle b_1, \dots, b_m \rangle$. For example the predicate '*.is small*' is soritical relative to the series $\langle 1, 2, 3, \dots, 10,000 \rangle$ since it appears true of '1', false of '10,000', and seems tolerant of a difference of 1; however it is (arguably) not soritical relative to the series $\langle 1, 100, 200, 300, \dots, 10,000 \rangle$. The predicate '*.is small*' is (arguably) not tolerant of such large changes.

³¹ See: Slaney, J., 'Vagueness Revisited', Technical Report TR-ARP-15/88 ARP, Australian National University (1989).

- (d) Assume logic applies, and that the argument is both valid and has true premises and so accept its conclusion — thus embracing the incoherence of soritical terms.

Option (a) is what Haack describes as the 'no-item' strategy; soritical expressions are beyond the scope of logic.³² Well known advocates of this approach include Frege and Russell. Frege thought that predicates with fuzzy boundaries of application, vague predicates, lack sense and hence cannot figure in sentences having truth conditions.³³ Russell, whose views will be considered in more detail in Chapter Three, claimed that "all traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life .." where vague language abounds.³⁴ Since soritical expressions are vague (a point we shall return to shortly) then logic does not apply to them. This response to the sorites paradoxes was based on ideal language doctrines popular earlier this century and associated with the demand for logically perfect languages. Ordinary language, in so far as it fell short of perfection, was deemed unfit for serious consideration and vagueness, like so many other phenomena in natural language, was seen as a defect to be eliminated.

The obvious problem with this approach is that logic is relegated to a "celestial realm" (Russell), and the fact that we *do* logically evaluate everyday discourse speaks against such an approach. The fading of ideal language doctrines and respect for ordinary ways of talking have meant that this approach is no longer viewed as tenable. A closely related approach still persists however and will be discussed in Chapter Four. Though ordinary ways of talking ought to be respected and vagueness *need* not be eliminated, it *can* nonetheless be eliminated. Vague language is not explicitly relegated to the rubbish bin but regimented so that problems arising from the vagueness of some expressions simply do not arise. This is Quine, Carnap and Haack's preferred approach; rather than solving the paradox it simply attempts to avoid it. As we shall see in Chapter Four this approach is beset with problems.

Options (b) and (c) both presuppose natural language to be in order as it is and attempt to describe how it is that a logic of vagueness, in particular a logic of soritical predicates, shows us a way out of the paradoxes that beset such language. Having accepted that the conclusion of soritical arguments is false, opponents of the paradox are then divided into what Barnes has described as the *radical opponents* — those who endorse option (b) and claim that the argument is invalid, and the *conservative opponents*

³² Haack, S., *Deviant Logic*, CUP (1974), Ch. 6, §4. This approach, though given considerable space, is not endorsed by Haack.

³³ See: Frege, G., *Grundgesetze der Arithmetik*, Vol ii (1903), §56; translated in: P. Geach & M. Black (eds), *Translations from the Philosophical Writings of Gottlob Frege*, 3rd edn, Blackwell (1980), p. 139.

³⁴ Russell, *op. cit.*, p. 88-9.

— those who endorse option (c) and claim that the argument, though valid, has false premises.³⁵

Option (b) is seen as radical since it requires a reformation of classical logic. This is the approach that I shall try to spell out in the final chapter of this thesis, Chapter Six. One task of this thesis, then, will be to explain how it is that an argument form endorsed by classical logic (and, it must be said, some non-classical logics like intuitionist logic) is invalid even though many take it to be a paradigm of good reasoning; surely, it will be objected, any connective identified as a conditional must satisfy *modus ponens*, iterated or not.

The incredulity with which option (b) is generally met is a measure of the popularity of option (c), given that most theorists these days are concerned to develop a logic of vagueness rather than take the more hard-nosed options presented by (a) and (d). The conservative opponent naturally picks the conditional premises as the place to attack the argument. Though it appears that adjacent subjects in the sorites series are sufficiently similar in respect of 'F' to treat them alike in this regard, appearances are deceiving. We shall consider, in due course, two accounts of vagueness which take up this option: the ultra-realist epistemic view of vagueness according to which classical logic remains completely intact, even in the context of vagueness (Chapter Two); and the supervaluations account which extends classical logic in a conservative way to include a vagueness operator, preserving iterated *modus ponens*, and all classical theorems — not unlike modal logics (Chapter Five).

Option (d) eschews any such resolution of the paradox; soritical arguments are valid and have only true premises, so the conclusion is true. No collection of grains of wheat counts as a heap and every such collection does; no-one is bald and everyone is; etc. Soritical predicates are radically incoherent. They're not simply inconsistently applicable in the sense of admitting some isolated item as being both in their extension and in their anti-extension; every member of the series with regard to which the predicate 'F' is soritical is in the extension of 'F', whilst by a negative sorites (using the predicate 'not-F') every member of the series is in the extension of 'not-F'. This option is argued for by Dummett who thinks that "Frege appears to be vindicated, and the use of vague predicates [soritical ones at least] ... is intrinsically incoherent".³⁶

This extraordinary conclusion is forced upon us by the failure, in Dummett's view, of options (a)-(c); it is, in other words, an option of "last resort". Since I think that option (b) can be shown to succeed, I think that Dummett's argument is blocked. It would indeed

³⁵ Of course these options are by no means exclusive; it may be that the argument is both invalid and has false premises thus the sorites paradox is doubly dissolved. No-one has, to my knowledge, pursued such a course and the reasons are obvious. It is difficult enough to resolve the paradox by either route alone; since either will itself be enough to resolve the matter there is no perceived need to engage in the doubly difficult task of convincing an audience that orthodox logical theory has gotten it wrong on two counts.

³⁶ Dummett, M., 'Wang's Paradox', *Synthese* 30 (1975), p. 319.

be surprising if, contrary to all appearances, soritical predicates were so radically defective; they certainly don't appear so.

Matters are further complicated by the fact that soritical arguments can take other forms. So, though we might initially focus on the standard form, what is really required is a response to the sorites paradox in all its forms. Three other forms (at least) can be discerned.

One version interprets the a_i 's as simply the natural numbers 1, 2, ..., i , ..., n and replaces the 9,999 conditional premises with a universally quantified premise of the form ' $\forall a_i(Fa_i > Fa_{i+1})$ ', representing the sorites as proceeding by mathematical induction:

Mathematical Induction Sorites

$Fa_1, \forall a_i(Fa_i > Fa_{i+1}) / Fa_n$ for arbitrarily large n .

So, for example, it is argued that since a man with 1 hair on his head is bald and since the addition of one hair cannot make the difference between being bald and not bald (for any number x , if a man with x hairs is bald then so is a man with $x+1$ hairs), a man with n hairs on his head is bald — where n is arbitrarily large, 10,000 say.

Yet another form, due to Bertil Rolf³⁷, is a variant of this Inductive form. Assume a man with n hairs on his head is not bald. Then by the least number principle (equivalent to the principle of mathematical induction) there must be a least number $\leq n$, $i+1$ say, such that a man with $i+1$ hairs on his head is not bald; since a man with 1 hair on his head is bald it follows that $i+1$ must be greater than 1, so $1 < i+1 \leq n$. So, there must be some number i ($1 \leq i < n$) such that a man with i hairs counts as bald whilst a man with $i+1$ does not. Thus it is argued that though a_1 is bald, not everyone is, so there must be some point at which baldness ceases.

Line-drawing Sorites

$Fa_1, \sim Fa_n / \exists a_i(Fa_i \& \sim Fa_{i+1})$ for $1 \leq i < n$.

More recently versions of sorites paradoxes have been set up using phenomenal predicates like 'looks red' in a conditional-free form, using a relation of F-indiscriminability, ' \sim_F ' which appears to permit substitution much as an identity relation would. Suppose we have to hand a series of 10,000 colour-patches $\langle a_1, \dots, a_{10,000} \rangle$ gradually (and monotonically) becoming more yellow as we progress along the series, where a_1 is clearly red and $a_{10,000}$ is clearly not (it's clearly yellow, say). Furthermore,

³⁷ Rolf, B., 'Sorites', *Synthese* 58 (1984): 219-50.

suppose that just by looking we cannot discriminate between any two adjacent patches a_i and a_{i+1} with respect to colour, i.e. $a_i \sim_C a_{i+1}$, for all i . But then a_1 looks red and a_2 looks the same colour as a_1 , so a_2 looks red, a_3 looks the same colour as a_2 so a_3 looks red, ... and $a_{10,000}$ looks the same colour as $a_{9,999}$ so, paradoxically, $a_{10,000}$ looks red.

More generally, for a phenomenal predicate 'F', if a_1 is F and adjacent members of the series $\langle a_1, \dots, a_n \rangle$ are indiscriminable in respects relevant to F, then it seems a_n is F.

Phenomenal Sorites

$Fa_1, a_1 \sim_F a_2, a_2 \sim_F a_3, \dots, a_{n-1} \sim_F a_n / Fa_n$ for phenomenal 'F'.

Any argument having the form of any of the above three sorites will be soritical just if conditions similar to those set down for the standard sorites are met.

Now obviously, given that sorites arguments have been presented in these four forms, "the sorites paradox" will not be solved by merely claiming, say, iterated *modus ponens* to be invalid; *all* forms need to be addressed one way or another. One would hope to solve it, if at all, by revealing some general underlying fault common to all forms of the paradox. No such general solution could depend on the diagnosis of a fault peculiar to any one form. On the other hand, if no general solution seems available then "the sorites paradox" will only be solved when each of its forms separately have been rendered toothless. In either case, to single out something peculiar to one of the forms will not, *by itself*, solve the problem. Of course, some solution peculiar to one of the forms might lie at the heart of the matter in so far as this feature is implicitly involved in all the other forms but then we will only have a general solution once we have pointed out how this feature underlies all forms of the sorites.

My contention will be that there is indeed a general solution which has as one of its consequences that *modus ponens* for the conditional is not unrestrictedly valid.

1.2.2 Soriticality and Vagueness

Having thus set out and commented upon the nature of sorites puzzles and paradoxes it is easy to see intuitively that soritical predicates are vague; soritical predicates appear tolerant so it would seem that there cannot be any sharp boundaries within the predicate's range of significance.

More exactly, soritical predicates are vague according to the border case characterisation of vagueness. Assume that there were no border cases for the soritical

predicate 'F'. Then everything of which 'F' can be significantly predicated, in particular everything in the series $\langle a_1, \dots, a_n \rangle$ relative to which 'F' is soritical, appears either determinately describable as F or determinately describable as non-F. By the criteria for soricality, a_1 appears to be a clear case of F and a_n appears to be a clear case of not-F. Given that the series can be ordered we can say that there must be a last a_i to which F appears determinately applicable and, since it is the last, a_{i+1} must appear determinately not-F. (So a_i and a_{i+1} are on either side of an apparent sharp boundary.) But this contradicts the fact that F, by its soritical nature, must appear tolerant of a difference as small as that between a_i and a_{i+1} thereby validating condition (iii) on 'F'. So, by *reductio*, if 'F' is soritical then there must be border cases for 'F'.

An analogous argument can then be mounted to show that the absence of border cases between the predicate's determinate cases and border cases, in so far as this presents an apparent sharp boundary, would also contradict the apparent tolerant nature of 'F'. And so on, up the supposed hierarchy ...

So soritical predicates are vague according to both the intuitive understanding of what vagueness amounts to and the border case concept of vagueness. Are all vague predicates soritical though?

The phenomenon of vagueness is typified by soricality. The reasons for this presumably are to be found in the fact that vagueness is historically rooted in sorites puzzles and because this is where the challenge presented by vagueness seems most forcefully presented. Yet, some suggest, the lack of apparent sharp boundaries or presence of border cases can arise for predicates that are not soritical.

If one thinks of vague predicates as predicates which fail to sharply partition, in any way, their range of significance then questions of ordering the range and doing so in such a way as to make adjacent members apparently indiscriminable in respects relevant to the application of the predicate are simply left wide open. One could of course *define* vagueness in such a way that all and only soritical predicates are vague. However there are broader issues concerning the apparent deviant semantic behaviour of vague predicates (famously exemplified by Frege's worries about concepts that have "fuzzy boundaries") that motivate our treating some non-soritical predicates on a par with soritical ones — that is, for our treating some non-soritical predicates as vague.

Rolf, for example, cites the possibility of a mathematical predicate having border cases without there necessarily being any ordered series of objects in the predicate's range of significance with regard to which it could be said to be soritical.³⁸ It may be that an ordering is unobtainable or, given some ordering, it may be that adjacent members of any such series are sufficiently dissimilar to falsify any claims to tolerance on behalf of the predicate. Mark Sainsbury has also suggested, in conversation, that there could be border

³⁸ Rolf, B., *Topics on Vagueness*, PhD Thesis, Lund (1981), p. 98.

cases for a predicate which also seem not to require any ordering of objects, each marginally different from the last, on which a sorites could be run.

Before delving into this further with examples, it will help if we look to the kinds of vagueness already described in the literature.

1.3 *Kinds of Vagueness*

The supposed distinction between soritical vagueness and non-soritical vagueness can be further amplified by discussing a distinction drawn by Alston in the *Encyclopedia* between what he calls *degree vagueness* and *combinatory vagueness*.

According to Alston, *degree vagueness* consists of those cases in which the vagueness stems from the lack of precise boundaries between application and nonapplication — or at least their apparent lack (as we noted earlier Alston prejudices this matter) — along some dimension. This is the standard kind of vagueness alluded to in most discussions, furnishing examples of soritical terms. 'Bald' fails to draw any sharp boundaries along the dimension of hair quantity; 'heap' fails to draw any sharp boundaries along the dimension of grain quantity; etc.

This kind of vagueness is contrasted with *combinatory vagueness* and since it is combinatory vagueness that importantly provides examples of non-soritical vagueness, I shall quote Alston at length.

Another, more complex, source of indeterminacy of application is to be found in the way in which a word may have a number of logically independent conditions of application. A significant example is the word "religion." If we consider clear cases of religions, such as Roman Catholicism and Orthodox Judaism, we find that they exhibit certain striking features, each of which seems to have something to do with making them religious. These include:

- (1) Beliefs in supernatural beings (gods).
- (2) The demarcation of certain objects as sacred.
- (3) Ritual acts focused around sacred objects.
- (4) A moral code believed to be sanctioned by the gods.
- (5) Characteristic feeling, such as awe and a sense of mystery, which tend to be aroused in the presence of sacred objects and which are associated with the gods.
- (6) Prayer and other forms of communication with the gods.
- (7) A world view, that is, a general picture of the world as a whole, including a specification of its over-all significance, and a picture of the place

of the individual in the world.

(8) The individual's more or less total organisation of his life based on the world view.

(9) A social organisation bound together by the preceding characteristics.

The existence of a plurality of distinguishable conditions of application does not in itself render a term vague. We can distinguish two conditions of application of the word "square": being a rectangle and having all sides equal. Here there is a definite answer to the question of what combination of these conditions is necessary and what combination is sufficient for the application of the term, the answer being that each of the conditions is necessary but not sufficient and that their combination is sufficient. With "religion" it seems clear that the combination of all the features listed above would be sufficient to guarantee application of the term. But what feature, or combination thereof is necessary? And is any subset of the features sufficient? There do not seem to be definite answers to these questions.³⁹

This indeterminacy (paraphrasing now) is further fleshed out by noting that, though some people are inclined to take beliefs in supernatural beings as necessary for religion, the term is nonetheless applied to systems, such as humanism, Hinayana Buddhism and Communism, where belief in supernatural beings is lacking. Ritual too seems unnecessary considering that it has disappeared from the Quaker movement though the movement is still sometimes described as a religion. Similarly controversial are all claims to the effect that some subset of conditions (1)-(9) is sufficient to make something a religion. Communism for example could be said to have sacred objects in the form of Lenin's body and the writings of Karl Marx, a definite world view, and a way of life based on it, yet it is not clear whether this is sufficient for calling it a religion.

In such cases we have a variety of conditions, all of which have something to do with the application of the term, yet are not able to make any sharp discriminations between those combinations which are, and those which are not, sufficient and/or necessary for application. There will be certain combinations, like the one exemplified by the case of Communism, in which we get uncertainties and disagreements among fluent speakers of the language as to whether the word is applicable. We may call this "combinatory vagueness."⁴⁰

³⁹ Alston, W.P., 'Vagueness', *op. cit.*, p. 219.

⁴⁰ *Ibid.*

Terms that are combinatorially vague then are vague by virtue of the apparent indeterminacy surrounding what to count as the set of necessary and sufficient conditions, regardless of whether or not the considered conditions themselves are vague. Importantly, this category then furnishes us with examples of non-soritical vagueness. In cases where none of the candidate conditions are themselves vague there can be no soriticality along any one dimension associated with the relevant conditions. Furthermore, when we come to consider the conditions collectively, we note that no soritical argument can be mounted since there is no linearly ordered sequence along which to run such an argument. To put the point another way — were a combinatorially vague term to be soritical then it would have to be soritical relative to some linearly ordered subset of its domain of significance; yet if all the candidate conditions are precise then any such subsets are sharply partitioned by the relevant condition and thus are not susceptible to soritical argument.

Rolf's example of the mathematical term (that he cites elsewhere as an example of vagueness in mathematics), 'polyhedron', is a good example of such non-soritical vagueness. Lakatos discusses it in his *Proofs and Refutations* as being a term that was, for a long time, vague.⁴¹ There was a long history, dating back to Descartes and Euler, of attempts at providing some combination of precise conditions that were necessary and sufficient for the concept but each conjectured definition faced counterexamples. 'Polyhedron' is, or was, vague in just the way that 'religion' is — combinatorially vague. Moreover, though (combinatorially) vague, it is clearly non-soritical.

Of course, the candidate conditions themselves might be soritical, as is the case with the term 'religion'. I am uncertain as to whether this makes for the soriticality of the term 'religion'. More generally, though some combinatorially vague terms are undoubtedly non-soritical, it is not clear to me whether all such vague terms are non-soritical.

Another distinction due to Burks is that between *linear* and *multi-dimensional* vagueness. The former is described as follows:

Linear vagueness is best illustrated by means of qualities which form a continuum, such as the colours of the colour spectrum. One may define blue ostensibly by presenting objects of various shades of blue, and counterinstances of various shades of purple and green. Since it is impossible to detect color differences beyond a certain point, borderline cases, such as blue-greens, will arise.⁴²

There is a mistaken (though common and usually harmless enough) assumption here that colours vary along one dimension whereas, as Rolf quite rightly points out, colours may

⁴¹ Lakatos, I., *Proofs and Refutations*, CUP (1976), esp. Ch. 1, §8.

⁴² Burks, A.W., 'Empiricism and Vagueness', *The Journal of Philosophy* 43 (1946), pp. 481-2. Cited in Rolf's *Topics on Vagueness*.

vary according to hue, brightness and saturation; colours can vary in a multi-dimensional way. The idea of linear vagueness is simple enough though. A better example than 'blue' would be 'small number'; here there seems only one dimension of variation — up and down the number series.

Multi-dimensional or non-linear vagueness, on the other hand, is said to arise

in connection with qualities which are such that their instances cannot be ordered linearly. The concept of chair is an example ... How much of a back does a chair need in order to be a chair rather than a stool? At what point does an article of furniture cease being a chair and become a chaise longue? Various features are involved in being a chair and a piece of furniture may be a borderline case because it lacks one or another of these, or because it has them all but not to a sufficient extent. Since one cannot give these various features weighting factors (it would be arbitrary to say that having a back is one-third as important as having a certain length of seat, for instance), it is clearly impossible to arrange chairs and non-chairs in a linear sequence.⁴³

One can of course arrange some subset of them in a sequence; take that subset that varies along only one dimension, e.g. the height of the back. A soritical argument could be run using the term 'chair' in this way so 'chair' is soritical; however its multi-dimensional vagueness means that it is not soritical relative to its whole range of significance, but only to some linear subset.

Linear predicates totally order their range of significance whereas non-linear ones, multi-dimensional ones, only partially order their range providing a plurality of linear sub-orderings or chains. This is why multi-dimensionally vague predicates could only ever be soritical relative to a subset of their range — soriticality is relative to some chain or linear, totally ordered sequence.

One might well wonder now whether the distinction between degree vagueness and combinatory vagueness coincides with Burks' distinction between linear and multi-dimensional vagueness. Contra Rolf, I think not. Terms that exhibit linear vagueness are degree-vague since their vagueness stems from lack of sharp boundaries along some dimension. And combinatory vagueness is obviously multi-dimensional. However it would seem that multi-dimensionally vague terms can be degree vague (so multi-dimensionally vague terms needn't be combinatory vague and degree vague terms needn't be linearly vague).

Generally we can say that, given vague predicates of degree 'G' and 'H' (say), their conjunction 'G & H' will be a multi-dimensionally vague predicate of degree. 'Knowledge' provides a good example of this non-linear degree vagueness in so far as it is defined as

⁴³ *Ibid.*, p. 482.

justified, true belief; beliefs can come in degrees, as can justification. Thus, though p may be true and justified, if it counts as a border case for 'belief' it will be a border case for knowledge. On the other hand, we could fix on some true belief q that is a border case for 'justified' and thereby generate another border case for knowledge. (The Sceptics concentrated on this justificatory aspect to show the soriticality of 'knowledge', thereby presenting the Stoics with a problem in so far as soritical concepts were seen as problematic.) The fact that p and q are both in the significance range of 'knowledge' though neither can be said to be any more nor less a case of knowledge than the other shows 'knowledge' to be multi-dimensionally vague. Yet these border cases for 'knowledge' are clearly not due to combinatory vagueness; they do not arise as a result of any indeterminacy regarding what to count as necessary and sufficient conditions for knowledge.

Yet another distinction bisects that already drawn between combinatory and degree vagueness — *vagueness of application* versus *vagueness of individuation*.⁴⁴ We began this chapter by describing the no-apparent-sharp-boundaries phenomenon typical of vague predicates and then proceeded to focus more carefully on this signature of vagueness via cases where there appeared to be no sharp boundary to the application of the predicate. Such cases present us with vagueness in the predicate's application conditions, i.e. vagueness of application. The term 'wetlands' for example has vagueness of application in so far as there are regions for which it is indeterminate whether or not the term applies. Yet as we noted at the outset, in addition to the predicate itself seeming indeterminate in extension, almost any particular wetlands region would seem indeterminate in spatio-temporal extent. So 'wetlands' also exhibits vagueness in regard of how much land to include in something that is determinately described as a wetlands. Quine describes the two types of vagueness thus:

Commonly a general term true of physical objects will be vague in two ways: as to the several boundaries of all its objects [what Alston has dubbed vagueness of individuation] and as to the inclusion or exclusion of marginal objects [what Alston has dubbed vagueness of application]. Thus take the general term 'mountain': it is vague on the score of how much terrain to reckon into each of the indisputable mountains, and it is vague on the score of what lesser eminences to count as mountains at all. To a lesser degree 'organism' has both sorts of vagueness. Thus under the first heading there is the question at what stage of ingestion or digestion to count food a part of the organism ...

⁴⁴ Cf. Alston, W.P., 'Vagueness', *op. cit.*, p. 219.

Under the second heading there is the question whether to count filterable vira as organic at all.⁴⁵

'Mountain' exhibits vagueness of application in so far as there are topographical features which are border cases for the term 'mountain'. It may seem essentially indeterminate whether or not some particular topographic feature *is* a mountain. And 'mountain' exhibits vagueness of individuation in so far as there may be spatio-temporal regions which are border cases for being part of that mountain; or, as Alston explains it, the term exhibits vagueness of individuation since there is an indeterminacy as to what we are to count as one mountain or two. It may seem essentially indeterminate whether or not some particular topographic feature, which *is* a mountain, is *a* mountain.

So predicates may exhibit vagueness of individuation as well as vagueness of application. (Remember of course that the vagueness might be extensional or intensional depending on whether or not the relevant indeterminacy obtains in our world, in which case the vagueness is extensional, or only in some possible world, in which case the vagueness is intensional.) Moreover, according to Quine, singular terms like 'Mount Rainier' exhibit vagueness of individuation.

Insofar as it is left unsettled how far from the summit of Mount Rainier one can be and still count as on Mount Rainier, 'Mount Rainier' is vague. Thus vagueness affects not only general terms but singular terms as well. A singular term naming a[n] ... object can be vague in point of the boundaries of that object ...⁴⁶

In other words, the vagueness of a singular term N consists in its being vague what is denoted by N — there are spatio-temporal points which count as border cases for being part of the denotation of N. Does 'Mount Rainier' denote the spatio-temporal region including that point on the lower ridge to the north-east? — for instance. Most proper names exhibit this kind of vagueness which Russell also describes in his 1923 article, focussing on the gradual processes of a person's birth and death as generating temporal border cases for the denotation of the name 'Ebenezer Wilkes Smith'.

⁴⁵ Quine, W.V.O., *Word and Object*, MIT (1960), p. 126.

⁴⁶ *Ibid.*

1.4 *Distinguishing Vagueness in Different Grammatical Categories*

The extension of the concept of vagueness to include names as well as predicates quite naturally leads one to inquire after vagueness in other grammatical categories as well. Since the task ahead is already looming large enough and the issues to be addressed can be discussed using only predicate-, name- and sentence-vagueness I shall, in the main, restrict our attention to these categories. Some brief remarks will serve to indicate how vagueness extends recursively to any category of expressions and this should suffice for the purposes of seeing how the vagueness of an expression does or does not infect any other expression in which it is embedded.

It is common to explicitly describe the basic vagueness of a sentence — for example, its being vague whether the sun is a hot star — in terms of its appearing to be neither determinately true nor determinately false. The idea is simple. The sentence 'The sun is a hot star' describes a state-of-affairs which appears to neither determinately obtain nor determinately not obtain; that state-of-affairs which actually does obtain in our world counts as a border case for the denotation of the sentence 'The sun is a hot star'. The vague sentence is a border case for the truth predicate, appearing to be neither determinately true nor determinately false. (Since an *actual* state-of-affairs is the border case the vagueness is extensional. An intensionally vague sentence is one for which there could conceivably be a border case.)

Earlier remarks attempting to distinguish border cases from other types of indeterminacy apply here in so far as the notion of border cases is employed (just as, of course, they apply when we speak of the vagueness of a name in terms of there being border cases for its denotation). So too do remarks concerning higher-order vagueness. Thus the apparent semantic indeterminacy is not due to relativity, ambiguity, meaninglessness, incompleteness, etc. And the apparent truth-value gap is not itself sharp. Again, just as we could only really gesture at the notion of a border case for a predicate rather than give any strict reductive definition, this account of sentence vagueness only points towards an intuitive understanding of what sentence-vagueness consists in. But it's enough to get on with.

Given the basic recognition of a sentence as vague, it is this sentence-vagueness which leads us to say that the denotation of the subject term counts as a border case for the predicate. In other words, I take it that vagueness is primarily (though not paradigmatically) an attribute of sentences, the recognition of which leads us to point out that the predicate involved has a border case and subsequently leads us to say that the predicate itself is vague. The vagueness of a sentence is seen as giving rise to the vagueness of the predicate involved.

However, the leap from sentence-vagueness to predicate-vagueness might be contested. Whilst sentence-vagueness does generate border cases for predicates, it is

questionable whether this presence of border cases is sufficient for predicate-vagueness. Having recognised that singular terms or names can be vague, mightn't a sentence be vague by virtue of our predicating a precise predicate of a vague singular term? If so, then whilst the vagueness of the sentence entails that the predicate has a border case, it would not necessarily mean that the predicate itself was vague. Thus it appears we may have to amend our account of predicate-vagueness offered in §1.1.

Bertil Rolf, in his paper 'A Theory of Vagueness'⁴⁷, invites us to consider the following two sentences:

- (1) The sun is a hot star.
- (2) The sun has a diameter of exactly 1.39×10^9 metres.

Both are assumed to be vague and the sun is therefore to be considered a border case for both predicates involved.⁴⁸ However, whilst the vagueness of sentence (1) seems correctly attributable to indeterminacies surrounding what to count as a hot star — that is, predicate-vagueness — sentence (2) seems vague by virtue of the vagueness of individuation of 'the sun'. The predicate 'has a diameter of exactly 1.39×10^9 m' seems, intuitively, precise, even though the sun constitutes a border case for the predicate.⁴⁹

Like Rolf, I take this intuition to be well-founded and on this assumption I shall spell out an account of predicate-vagueness which employs a revised, stricter notion of a border case which can service this intuition. We cannot, however, simply help ourselves to Rolf's means for making the requisite revision since his account depends on our having to hand some notion of ontological vagueness or fuzziness and it is hotly contested whether this notion is even coherent. (Though I think it is, it will not be until Chapter Six that such an opinion will be argued for.) A much less question-begging, and thus more widely acceptable, account will be proffered, replacing talk of the fuzziness of an object with the vagueness of a singular term.

Sentence (1) is vague by virtue of the vagueness of the predicate; any vagueness in the subject term is passive. Assume all apparent indeterminacies in delimiting that thing denoted by 'the sun' to be resolved determinately one way or the other. So, for example, that outer solar flare is to be considered either a determinate part or a determinate non-part of the object denoted by 'the sun', and so on. This provides us with a range of *admissible delimitations* of the sun. Now the temperature variation across the various admissible delimitations is too small to make any difference to the application or non-application of the predicate 'is a hot star'; any admissible delimitation of the sun is of such a temperature

47 Rolf, B., 'A Theory of Vagueness', *Journal of Philosophical Logic* 9 (1980): 315-25.

48 The use of 'exactly' in sentence (2) should not be treated as synonymous with 'determinately' which would of course render sentence (2) false, rather than indeterminate.

49 It should be said that though Michael Tye (whose position will be discussed more in Chapter Six) and I follow Rolf in this amendment (which is also consistent with remarks made by Kit Fine some years earlier in his 'Vagueness, Truth and Logic', *op. cit.*, pp. 266-7), Mark Sainsbury has begun to investigate the line which simply denies these intuitions, maintaining instead that all sentence vagueness *does* amount to predicate vagueness.

to count as a border case for being a hot star. So, even were 'the sun' considered precise, denoting any of the admissible delimitations, its referent would be a border case for the predicate 'is a hot star'. The vagueness of the singular term is thus irrelevant to the vagueness of the sentence; it is passive vagueness. The source of the vagueness in sentence (1) is to be found in the vagueness of the predicate.

Sentence (2), on the other hand, is vague specifically because it is vague just what counts as the sun. The vagueness of the subject term is, in this case, active. Were it precise, that is, if 'the sun' clearly denoted any one of the admissible delimitations, then the sentence would appear determinately true or determinately false; the object denoted would either determinately have the relevant diameter or determinately not. The sun is, therefore, only a border case for the precise predicate 'has a diameter ...' *because* we are unable to determine precisely what counts as the sun. There's the source of our sentence-vagueness.

The general picture can be put like this: given the above accounts of sentence- and name-vagueness, we shall say that the vagueness of an atomic sentence 'Pa' is due to the vagueness of the predicate 'P' if and only if any vagueness in 'a' is irrelevant, or passive in the context of 'P'. In other words — if and only if, were *a* to count as a border case for the predicate 'P', it would count as a border case no matter what admissible delimitation of *a* was considered. Of course, if the singular term figuring in the vague sentence is precise then trivially any vagueness in the singular term is passive — there is none — so in all such cases the vagueness is due to the vagueness of the predicate.

Predicates we now prefer to describe as precise have (or could have) border cases in the sense of a border case we have hitherto been using, as witnessed by sentence (2) above, but we can now see our way clear of such deviant cases. A stricter notion of predicate-vagueness, which takes account of cases like these, whilst preserving the relation between predicate-vagueness and the (possible) presence of border cases, can be described by further constraining what counts as a "border case". Let us characterise something as a *resilient border case* for a predicate 'P' just if it is a border case for the predicate and remains a border case no matter what admissible delimitation is considered (i.e., no matter how one chooses to resolve any apparent indeterminacy regarding what to count as that thing). Then, though the sun constitutes a border case for the predicate 'has a diameter of exactly 1.39×10^9 m', it is not a resilient border case — there are ways of determinately individuating the sun as having exactly that diameter and ways of determinately individuating the sun as not. On the other hand, the sun is a resilient border case for the predicate 'is a hot star' — it counts as a border case no matter how it is individuated.

Similarly deviant cases of indeterminacy surrounding what to count into the extension of a name can be conceived of, however, having carefully chosen an account of name-vagueness, they do not give rise to deviant cases of name-vagueness as we have defined it. The deviancy that arose with regard to predicates — something's being a border

case only because we were unable to determine precisely what to count as that thing — underlies cases where a certain spatio-temporal region is a border case for being part of the denotation of some name N only because it appears indeterminate precisely what to count as that spatio-temporal region. For example, suppose the name 'Quine National Park' denotes a precisely delimited spatio-temporal region one of whose spatial boundaries runs along Russell River. It could appear indeterminate whether 'Quine National Park' denotes that spatial region including Russell River, but the border case, Russell River, will only give rise to such apparent indeterminacy by virtue of apparent indeterminacy surrounding what to count as the extension of 'Russell River' — it will not be a resilient border case.

However, having defined the vagueness of a singular term as we did (a singular term N is vague if and only if there are spatio-temporal *points* which count as border cases for being part of the denotation of the N), though Russell River is a border case for being part of the denotation of 'Quine National Park', it is not a *point* (but rather, a possibly indeterminate collection of points), and so does not make for the vagueness of 'Quine National Park'. The trick, of course, is that the existence of any region as a border case for being part of the denotation of a term N will make for the vagueness of N just if it is a resilient border case (in which case the indeterminacy must be due to the vagueness of N) and spatio-temporal points are necessarily resilient border cases if they are border cases at all.

With the basic conception of sentence-vagueness and the carefully worded account of name-vagueness, we are thus able to amend our account of predicate-vagueness via the notion of 'resilience', thereby excluding deviant cases arising from the vagueness of names. Rather than carrying the qualified notion of 'resilient border cases' along with us in subsequent chapters though, for convenience I shall simply speak of border cases as constitutive of vagueness. Border cases *are* generally resilient and, where they are not and this is of import, we can then help ourselves to the distinction we have just been describing; it is there to be used where required though I will not always explicitly acknowledge it.

We have now to hand conceptions of predicate-, name-, and sentence-vagueness which serve to underpin the intuitions described earlier regarding how the categories of vagueness interact. Assume that some atomic sentence, 'Pa', is vague; it appears essentially indeterminate whether the denotation of 'a' satisfies 'P'. So the denotation of 'a' is a border case for 'P'. But then if 'a' is precise the denotation of 'a' is a resilient border case for 'P', so 'P' is vague. On the other hand if 'P' is precise then the denotation of 'a' is not a resilient border case for 'P' so, since it is non-resilient though a border case nonetheless, the vagueness in predicating 'P' of the denotation of 'a' is attributable to the vagueness of the denotation of 'a'.

In other words, given a vague sentence, if the predicate is precise then the name is vague and if the name is precise then the predicate must be vague. The vagueness of the whole must be attributed to the vagueness of some part; there must be some vague part which is active in the sense that it causes the vagueness of the whole sentence. Of course, just because the vagueness of the sentence is blamed on the predicate it does not follow that the name must be precise, witness sentence (1); in such a case what we *can* say is that any vagueness in the subject term is passive.

This is a restricted theory of vagueness in the sense that vagueness is only defined for predicates, names and sentences. In so far as it goes, this theory is in the spirit of that articulated by Rolf in his paper cited above. Any phrase in one of these categories can be said to be vague and when quizzed on what this means the vagueness is explained via the notion of a border case. Ultimately, of course, worries attend this explanation (higher-order vagueness seems a problem for border case accounts of vagueness in general) but there is a fairly broad consensus that we are pointing in the right direction. *Intuitively, the vagueness of phrases in these three grammatical categories consists in apparent essential indeterminacies regarding what they denote or represent — vagueness of extension.* Does 'hot star' denote a property including the sun? Does 'Mt Rainier' denote something that includes that lower point on the marginal ridge? Does 'The sun has a diameter of exactly 1.39×10^9 m' denote that state-of-affairs which obtains in our world — what fact does it represent?

Such rhetorical questions however cannot be posed for phrases in categories which do not purport to represent anything.⁵⁰ It makes no sense to speak of border cases for logical notions like 'all', 'some', 'many', 'few', 'and', 'not', or for modifiers like 'roughly', 'rapidly', 'heavy', and 'large'.

In generalising our theory of vagueness to encompass these categories as well, we shall take the vagueness of the three denoting categories as basic and outline a general recursive account of vagueness, going beyond phrases which purport to represent, beyond sentence-, name-, and predicate-vagueness. The idea is that a non-denoting phrase will count as vague if and only if it causes the vagueness of a phrase of which it is a part; that is, just if it occurs properly within a vague phrase all of whose other components are precise. We must locate the source of the phrase's vagueness and, if all other components are precise, there is only one candidate.

Thus we shall simply expand upon the notion of vagueness already arrived at whereby a sentence's vagueness, if not attributable to the subject term, must be blamed on the remaining predicate. The underlying principle being followed is that *precision is inherited*. It is already encapsulated in our discussion of the denoting phrases; if a precise

⁵⁰ What follows is adapted from Rolf's paper cited earlier: Rolf, B., 'A Theory of Vagueness', esp. §§3-4, and parts of Ch. 1 of his *Topics on Vagueness*, also cited above.

predicate is predicated of a precise subject term then the resulting sentence will be precise. Precision is inherited to this extent. Now the idea driving the generalisation amounts to a generalisation of this principle to all phrases, denoting and non-denoting. The general principle is that: *if all but one of the constituent sub-phrases of a complex phrase are precise then, if the complex phrase is vague, so is that one remaining constituent sub-phrase*. In other words, if all but one of the constituent sub-phrases of a complex phrase are precise then if the one remaining constituent sub-phrase is precise so must the complex phrase be precise. This principle then can be used to recursively define the vagueness of a phrase of an arbitrary non-denoting grammatical category:

DEF (1) A non-denoting phrase P is extensionally vague iff there is an extensionally vague complex phrase containing P as a sub-phrase all of whose other constituents are extensionally precise.

As usual:

DEF (2) A phrase P is (intensionally) vague iff it is logically possible that P is extensionally vague.⁵¹

(As always, when we speak of something's being vague *simpliciter* we shall interpret this as the weaker claim that it is intensionally vague, unless specified otherwise. The relations that hold between intensional vagueness, extensional vagueness, extensional precision and necessary precision described in the square of opposition on page 6, can now be said to hold for any phrase P in general.)

Of course, the concept of vagueness thus generalised is *derivative upon* the vagueness of denoting phrases in the sense that when we say of a phrase that it is vague, ultimately this is to be understood in terms of its making for or causing the vagueness of some denoting phrase (sentence, name, or predicate). Hence vagueness, when analysed, is ultimately to be analysed in terms of this vagueness of extension and any explanation as to the source of vagueness rests with an explanation of this vagueness (of denoting phrases). Since sentence-, name-, and predicate-vagueness underpin the recursive

⁵¹ A non-recursive variant one could always make use of is the following:

DEF (1') A non-denoting phrase P is extensionally vague iff there is an extensionally vague sentence containing P as a sub-phrase all of whose other constituents are extensionally precise.

All this talk of grammatical categories presupposes that there is some categorial grammar that gives the logical deep structure of English. Our definition will succeed to the extent that this assumption can be founded. A categorial grammar is given in David Lewis's 'General Semantics', *Synthese* 22 (1970): 18-67. Lewis takes the categories of sentences and names as basic, defining the other categories by means of them. Similarly we could have taken the vagueness of sentences and names as basic and defined the vagueness of all other categories in terms of them; however, rather than taking this approach we have effectively defined the vagueness of predicates via the vagueness of sentences and names then taken the vagueness of all three denoting categories as basic.

generalisation of vagueness to all grammatical categories, it is their vagueness which does the explanatory work and begs for further analysis.

As a worked example using the above definition, consider the vague sentence 'Many photons passed through the slide S between t_1 and t_2 '. Assume that the predicate 'passed through the slide S between t_1 and t_2 ' is precise. Then the quantifier phrase 'many photons' is vague. Suppose further that the predicate 'is a photon' is precise. It follows then that the quantifier 'many' is vague.

In this way modifiers can also be shown to be vague. The predicate 'is a large rectangle' is vague, in fact it's soritical. Yet 'rectangle' seems precise, at least if applied to ideal bodies. So it must be the modifier 'large' which is vague. We can also show that many of the logical constants are precise. Even if the constants include vague sentences in their argument range, it is generally agreed that the classical propositional connectives retain their classical functional roles when only precise sentences are considered as arguments; i.e., given only the possibility of determinate truth and falsity as inputs, the outputs of the truth-functions for ' \wedge ', ' \vee ', ' \supset ' and classical ' \sim ' are always either determinate truth or falsity. So, every truth-functional compound of precise sentences is precise; or, putting it the other way, if a truth-functional compound of sentences is vague then one of the constituent sentences is vague. Vagueness comes in, ultimately, via the atomic sentences, not via any of these constants. There are no surprises here; this view of the constants simply reflects the view that vagueness is only ever a problem in the propositional calculus if the propositional parameters are permitted to range over vague (as well as precise) sentences — that is, if the formal language is taken to include vague sentences.

The universal and existential quantifiers can also be shown to be precise. The precision of these constants contrasts with the vagueness of constants peculiar to epistemic logics like 'It is known that ..' which can be shown to be vague.⁵² Take a perfectly precise true justified sentence 'A' about which I have only vague beliefs; then it might be vague whether I know that A.

The counterfactual ' $\Box \rightarrow$ ' (and, by virtue of an appropriate equivalence, ' $\Diamond \rightarrow$ ') is also vague. To begin with, the vagueness of counterfactual *sentences* is explicitly acknowledged in the Lewis account.⁵³ The fact that counterfactuals may, on his theory, sometimes appear essentially indeterminate is taken to reflect the fact that they would ordinarily be thought of as vague anyway. Where, according to our theory of the invasion of vagueness, is vagueness in the sentence ' $A \Box \rightarrow B$ ' to be located? It can be "blamed

52 This vagueness once prompted Dummett to reject epistemic logics; he thought that the vagueness of these notions renders them unfit for logical treatment. See: Dummett, M., *Frege, Philosophy of Language*, Duckworth (1973), pp. 285-8.

53 Lewis, D., *Counterfactuals*, Basil Blackwell (1973), §1.2.

upon" the logical connective. The connective can be shown to be vague; it may be vague whether B would be the case were A the case even though 'A' and 'B' are precise.

Lewis's analysis also treats the constant itself as vague, analysing it in terms of the vague notion of 'similarity'. To this extent his analysis is in exactly the right direction — employing vague terms to analyse a vague logical constant. Rolf himself mentions in a number of places in his *Topics* that, since precision is inherited, it follows that one cannot define a vague phrase or concept in purely precise terms. This claim, following from our theory of the invasion of vagueness, is of crucial importance to satisfactorily dissolving worries about higher-order vagueness and underpins the view that we cannot hope to define vague natural language terms by means of a purely precise ideal language. The latter point will be taken up again in Chapter Four; we shall return to the former point shortly.

Before following up the consequences of the inheritance of precision however, I want to consider that other aspect of the relation between the vagueness or precision of parts and the vagueness or precision of wholes — how the vagueness of the parts affects the vagueness of the whole. According to the above theory of the invasion of vagueness, precision is inherited.

THE INHERITANCE OF PRECISION: If all the constituent phrases of a complex phrase are precise then the complex phrase is precise.

What of the converse claim though? If a complex phrase is precise then are all of its constituent phrases precise?

In short, the answer is no. This negative response conflicts with what Rolf has dubbed "the infection theory of vagueness". As a typical expression of the infectious or active nature of vagueness, Rolf cites Rosenberg's claim that "If the meaning of one term is connected to that of another, then the vagueness of either will infect the other."⁵⁴ Russell made the most of this idea of infectious vagueness claiming that all language is vague. His argument to this effect can be seen to depend on two crucial premises:⁵⁵

- (I) *The Weak Infection Theory of Vagueness:* If all the constituent phrases of a complex phrase P are vague then so is P;
- and
- (II) *The Strong Infection Theory of Vagueness:* If P is a phrase taking Q as an argument then, if Q is vague so is P.

⁵⁴ Rosenberg, A., 'The Virtues of Vagueness in the Languages of Science', *Dialogue* 14 (1975), p. 309; cited in Rolf's *Topics on Vagueness*, p. 91.

⁵⁵ See Rolf's analysis of Russell's argument in his *Topics on Vagueness*, Ch. 1, 'Russell's Theses on Vagueness', esp. pp. 5-12.

(II) can be seen to be false by means of the fact that the classical conjunction connective ' \wedge ' is precise (necessarily) though it takes vague sentences as arguments. (Using the same counterexample, it is easy to see that (II) is false even if the vagueness of Q is extensional.) We also saw earlier that there could be precise predicates taking vague names as arguments. So, (II) is false. As a consequence, contra Russell, it does not follow from the fact that the truth predicate takes vague sentences as arguments that the truth predicate is itself vague.⁵⁶

(I) can also be counterexemplified. Consider the sentence 'This red hill is red'. Both the subject term and predicate are vague. Nonetheless the sentence is true and necessarily so. Thus the sentence is precise though all its constituent phrases are vague. The vagueness of each of the constituent phrases, when combined to form a complex phrase, logically cancel each other out, as it were. This cancelling effect has an analogue in cases where extensionally vague phrases combine to form an extensionally precise complex phrase. For instance, the sentence 'Caesar attained much power' is actually true and thus is extensionally precise in spite of the fact that both 'Caesar' and 'attained much power' are extensionally vague. This example points to the falsity of an extensional variant of (I), namely: if all the constituent phrases of a complex phrase P are extensionally vague then so is P. Appreciating the falsity of this variant involves an appreciation of the fact that we can truthfully describe aspects of our world in purely vague terms; embracing vagueness does not mean that we have to wallow in it to our detriment.

In so far as our theory of the spread of vagueness and precision (largely Rolf's) is right then vagueness is not unqualifiedly infectious. One consequence is that, though Russell's account of the nature of vagueness is satisfactory (as we shall see in Chapter Three), his account of the invasion and spread of vagueness is wrong. Though precision is inherited and vague phrases cannot be defined in purely precise terms, precise phrases *can* be defined in purely vague terms.

So, we now have to hand a characterisation of the vagueness of language, applicable to any grammatical category, and an account of the interaction of vague and precise language in terms of which passive vagueness can be distinguished from active vagueness. The general theory is adapted from the work of Rolf, though some important modifications were required.

Now, as promised, I want to return to the problem of higher-order vagueness (HOV) with a view to explaining away its apparently puzzling nature. My claim will be that though that aspect of the nature of vagueness which higher-order vagueness seeks to describe is real enough, an endless description stating the presence of border cases at each level of the hierarchy is not required in characterising vagueness, and the so-called problem of HOV

⁵⁶ Russell, *op. cit.*, p. 88.

is thus a pseudo-problem. It is transformed into a serious problem only for those who have, in the course of characterising the nature of vagueness, failed to rid themselves of the prejudice in favour of the precise. As we shall see, vagueness cannot be defined in precise terms, nonetheless we can (all other things being equal) characterise vagueness quite simply in terms of the notion of a border case — a notion which is itself vague. The realisation that 'border case' is itself vague ought to defuse any concern associated with this simple characterisation that has commonly led people up the garden path of ascending orders of vagueness.

The first thing we need to acknowledge is that 'vague' is itself vague — it is an homological (or autological) term. It may sometimes appear indeterminate whether to count something as a border case for a predicate or not, thus it might be vague whether the predicate has any border cases or not. In such circumstances the predicate would constitute a border case for vagueness. A convincing argument to this effect is provided by Sorensen, so let's rehearse his argument.⁵⁷

Consider the sequence of predicates '1-small', '2-small', '3-small', and so on, defined on the natural numbers. The n^{th} predicate on the list is defined in such a way as to apply to only those integers that are either small or less than n . Using these disjunctive predicates as arguments we are able to construct a sorites paradox for the predicate 'vague'. Namely:

'1-small' is vague.
If 'n-small' is vague then 'n+1-small' is vague.
 \therefore '10⁶-small' is vague.

The predicate '1-small' is as vague as 'small' since both predicates clearly apply to 0 and both apply in exactly the same way to all other integers. The same holds for '2-small' and '3-small'. Each of these two predicates apply to the integers in exactly the same way as 'small' does, with the exception that '2-small' has 0 and 1 as clear instances whilst '3-small' has 0, 1 and 2 as clear instances; since 0, 1 and 2 are all clearly small it follows that '2-small' and '3-small' are as vague as 'small' itself. However, we eventually reach predicates where the 'less than n ' clause has the effect of making some integers clear instances of the predicate 'n-small' whereas they were border cases for the predicate 'small'. Some border cases are eliminated. Still further down the series of predicates — at 'q-small', say — we find that all border cases for 'small' have been eliminated from the border region of 'q-small' and thus the predicate 'q-small' has no border cases at all and is therefore precise. For example, it is clear cut whether or not to apply the predicate '10⁶-small' to any integer; if the integer is less than 10⁶ then the predicate clearly applies and if the integer is 10⁶ or

⁵⁷ What follows is taken from Sorensen's 'An Argument for the Vagueness of "Vague"', *Analysis* 45 (1985): 134-7.

greater than, since it is clearly neither small nor less than 10^6 , the predicate clearly does not apply. Yet, as Sorensen points out, it is a vague matter where along the sequence the predicates with border cases end and the ones without border cases begin. Consequently 'vague' is vague.

Since 'vague' is vague, it cannot be defined in purely precise terms; this follows from the fact that precision is inherited — no vague phrase can be defined in purely precise terms. Somewhere in our characterisation of vagueness we are forced to employ vague terms. I take this to be something we can all agree on. So, thus enlightened, let us turn to the so-called "problem of HOV".

Recall how the ascent up the hierarchy — requiring that vague predicates not only possess border cases but border cases of border cases and border cases of border border cases, etc. — was motivated. The worry was that the simple requirement that vague predicates have border cases could not be sufficient for vagueness since incomplete predicates seemed to have border cases (cases to which the predicate did not appear to determinately apply nor determinately not apply). 'Child*' was cited as an example of a predicate which is precise though it has border cases. However, given a characterisation of vague predicates in terms of their possessing border cases, the recognition that 'child*' could not be said to possess border cases whilst remaining precise inspired a modification in the characterisation of predicate-vagueness. This modification involved the requirement that there be border border cases, etc. In other words, an adequate characterisation of the notion of predicate-vagueness via border cases was seen to be available only if it was explicitly specified that 'border case' is vague, and 'border border case' is vague, and so on — resulting in the inadequacy of anything short of an infinite iteration within the characterisation.

Now I think that there *are* higher orders of vagueness, but that this is already entailed by the paradigmatic conception and can be seen to follow when the notion of 'border case' employed therein is properly understood.

Enter the vagueness of 'vague'. When the concept being analysed is vagueness we need some vagueness in the *analysans* to capture the vagueness in the *analysandum*. Hence, implicitly, when characterising predicate-vagueness in terms of border cases we must suppose the notion of a border case (and associated notions like penumbra and border region) to be vague itself. Why? Well, because there is no other candidate for vagueness in the *analysans*. Predicate-vagueness is characterised by "there being border cases", yet the existential quantifier is precise so the notion of a border case must be vague. There are border border cases, border border border cases, and so on.

In passing, we should note that this puts paid to any attempt to avoid the supposed difficulties associated with HOV by either denying there to be any higher orders of vagueness (as Wright has tried to suggest — cf. n. 12), or by claiming it to be vague whether there are higher orders of vagueness — a line of defence presumably endorsed

by Michael Tye who, seeing the admission of higher orders of ontological vagueness as problematic, claims it to be vague whether there are any such higher orders.⁵⁸ The phenomenon of higher-order vagueness is definitely real.

What are we to say then with regard to the predicate 'child*'? It is precise since it lacks higher-order border cases, yet appears to have border cases and so would appear to render the existence of border cases insufficient for vagueness. The answer is simply that, contrary to appearances, it *does not* possess a border case at all, at least in the sense of 'border case' used in the characterisation of vagueness — the vague sense of 'border case'. That is, 'child*' does have apparently indeterminate instances, however the term 'indeterminacy' here is precise, whereas the apparent indeterminacy required for vagueness is itself vague. (It would be preferable, I think, if some other terms could be used to distinguish the precise and vague sense of 'determinate'. Given that border case terminology is now quite firmly entrenched in the literature on vagueness, a new term could be used to play the role of 'determinate' in the precise sense and then the term 'border case' could be defined by means of 'determinacy' in the vague sense thus ensuring that when one speaks of border cases one is speaking vaguely.)

In requiring that the indeterminacy characteristic of vagueness is vague, one might be tempted to think that a worrisome circularity is lurking in the background. I think this worry is misplaced. Were we to characterise 'vague' using the term 'vague' then a vicious circularity would indeed arise, just as it would were we to characterise 'meaningful' using the term 'meaningful'. However we are not characterising 'vague' using the term 'vague'; rather we are characterising 'vague' using vague terms and this is no more a problem than characterising 'meaningful' in meaningful terms. In fact, far from being viciously circular, it is (as I have already pointed out) required of us that we characterise 'vague' in vague terms for exactly the same reason that we are required to characterise 'meaningful' in meaningful terms — both 'meaningful' and 'vague' are homological expressions. The apparent circularity is simply the misplaced recognition of this homological aspect of vagueness. It is a misplaced recognition of the fact that any characterisation of what it is for a term to be vague will itself partially characterise the term 'vague' since 'vague' is a vague term; the analysis catches itself within its scope, as it were.

Summing up then: the problem of higher orders of vagueness arises when one tries to explicitly state something about the nature of vagueness that manifests itself in the characterisation anyway — that "something" is the phenomenon of higher-order vagueness. It is not necessary to explicitly state any extra conditions in one's characterisation of vagueness to ensure compatibility with the phenomenon *unless* one thinks that the notion of a border case is precise-unless-stated-otherwise... and this is simply false! Ignorance of the lurking ambiguity in the term 'border case' (ultimately as a

⁵⁸ Cf. M. Tye, 'Vague Objects', *Mind* 99 (1990), p. 535. This is further discussed in the context of ontological vagueness; cf. §6.1.

result of the ambiguity of 'determinately') creates unnecessary difficulties. There are border border cases for vague predicates but this need not be stated as part of the analysis of the concept of predicate-vagueness any more than, having said that 'red' is a predicate, one must then go on and state that "'red' is a predicate" is a predicate, and that "'red' is a predicate" is a predicate' is a predicate, and so on. One is simply repeating oneself and adding nothing new. The vagueness of 'vague' can be seen to be built in right from the very start, as it were, when we characterised vagueness in terms of border cases, once we have conceded (as indeed we must since 'vague' is vague) that the sense of border case employed in the characterisation was the vague sense. In the rest of this thesis the term 'border case' is always to be understood as vague.

1.5 *The Task Ahead*

Turning now to the task ahead. Having characterised the concept of vagueness, the next question to emerge is 'What is the cause or source of such vagueness in natural language?' Remembering that the notions basic to our generalised concept of vagueness were the vagueness of sentences, names, and predicates (themselves further characterised in terms of border cases), this question then amounts to our inquiring after the cause of our inability to describe sharp boundaries to the extension of sentences, names and predicates. Where does the associated apparent indeterminacy have its source?

Is it perhaps epistemic, due to the in principle unknowability of certain semantic facts? Advocates of an epistemic account of the source of vagueness think so. They maintain that the apparent essential indeterminacy regarding the application of, say, 'red' to some object is *merely* apparent. In every case where the predicate may be said to be meaningfully applicable, there is a semantic fact as to whether the predicate determinately applies or determinately doesn't; it is simply that we are in principle barred from coming to know the relevant determinate semantic fact in certain situations — namely, where one is close to the sharp semantic boundary — and so it *appears* that there is no determinate fact of the matter. For example, it appears that there is no sharp boundary to the extension of the predicate 'red', yet there is one, though we cannot know where this boundary lies. This analysis of vagueness clears the way for a solution to the sorites paradox which centres on the rejection of one of the conditional premises in the standard sorites — option (c) — which we earlier described as a conservative solution (the sorites is valid though unsound).

Other theorists eschew the extreme semantic realism inherent in such an account maintaining rather that our inability to know where the boundary lies reflects a semantic

fact about the predicate 'red'. It is true that there is no knowable sharp boundary but this is because there is, as a matter of semantic fact, no sharp boundary. Vagueness is properly a semantic phenomenon.

Theorists taking this line, by far the majority, can be further distinguished from one another. Most it would seem take vagueness to be *merely* semantic. That is, though they accept that there is no determinate semantic fact of the matter as to whether 'red' applies to some border case, there is no sense in which this can be said to be grounded in the world. The vagueness is not, in any sense, ontological; it is purely semantic. Various responses to the sorites paradox attend this analysis of vagueness. Some theorists, in accepting that vagueness is really semantic, go on to conclude that vague language is beyond the scope of logic, thus solving the paradox by denying the problem legitimacy in the first place — option (a). Others, whilst accepting that logic applies to vague language, deny that the paradoxical argument is sound — option (c); like the epistemic theorist they opt for a conservative solution though they do admit that vagueness necessitates some revision of classical logic and semantics — a conservative extension of classical logic is proffered.

The purely semantic approach contrasts with a theory of vagueness which admits of ontological indeterminacies, analogous to semantic indeterminacies which they are sometimes taken to underpin. The vagueness of the predicate 'red' (for instance) might, on this ontological account, reflect the vagueness (or something analogous) of the property denoted by 'red', namely redness. More controversially, the vagueness of terms like 'Mount Rainier' might, on this account, reflect the vagueness (or something analogous) of the object referred to by the name, namely Mount Rainier itself. Theorists taking this approach sometimes opt for a conservative solution to the sorites and sometimes opt for a radical solution. (Recall that a radical solution is one which denies the *validity* of soritical arguments.) In arguing for this approach in at least some cases, I shall suggest a radical solution — option (b).

Of course, mixed accounts of vagueness are also possible (though, as we shall see, giving way to semantic or ontological analyses of vagueness in some limited cases may open the floodgates to these types of analysis where previously it was denied). Some border cases may be epistemic, some semantic and perhaps in some cases it is ontologically grounded. Or we may get all three kinds of "vagueness" at once; epistemic, semantic and ontological indeterminacies might all contribute simultaneously to make for the vagueness of a word or phrase.

Typically though, theorists offer one account of vagueness with a view to forestalling attempts to get a more radical account off the ground. Thus advocates of the epistemic account commonly offer an epistemological analysis of vagueness as *universal* and subsequently claim it as a virtue of their theory that the phenomenon of vagueness requires no revision of classical semantics, logic, or metaphysics. Advocates of the purely semantic account, whilst being able to help themselves to the epistemic account at no extra

cost, typically universalise at least to the extent that all vagueness is said to be epistemic or purely semantic. In thus denying the ontological account any room in the theoretical landscape they are relieved of the need to revise their metaphysics.

The rest of this thesis will be taken up with the description and evaluation of these accounts of the source or cause of vagueness. I shall argue that vagueness is at least sometimes ontologically grounded and that, as a result, its presence in natural language necessitates a revision of logic, semantics and metaphysics.

Chapter Two

Epistemic Theories of Vagueness

We have been using a conception of vagueness that is characterised by *apparent* indeterminacy; for all that we could ever come to know there would seem to be a blurring of boundaries — for example there are things that are apparently neither determinately red nor determinately non-red. Beyond this general conception theories diverge. The first major bifurcation I want to consider separates those who take the apparently semantic feature of some vague phrase to be just that — *merely apparent*, from those who take the appearance to reflect an underlying *actual* semantic feature — a property of the meaning of the phrase said to be vague. Both approaches take the vagueness of the phrase to be characterised by the unknowability of certain facts, yet advocates of the former view maintain that the relevant facts nonetheless obtain though epistemological limitations bar our access to them.

This account, whereby the vagueness of a phrase may be said to be a property of our epistemic states rather than the more traditional conception of vagueness as a semantic property, has some support in the philosophical community, and a lineage that can be traced back as far as the Stoics, though it is ruled out as an account of vagueness by many definitions invoked in the literature on the subject. In recent times such an account has been advocated most notably by J. Cargile, Roy Sorensen, and Timothy Williamson.¹

As we shall see, epistemic approaches to vagueness may differ as regards their scope. The most ambitious variant makes the claim that *all* vagueness is epistemic. I shall refer to this as *the strong epistemic theory of vagueness*. According to this general theory, vagueness *per se* is a pseudo-problem. There is no semantic vagueness in natural language; its apparent presence is always to be attributed to epistemological

¹ More comprehensively, epistemic approaches are to be found in: Stoic theories of vagueness discussed in Barnes, J., *et al* (eds), *Science and Speculation*, CUP (1982), esp. pp. 52-3, and Burnyeat, M.F., 'Gods and Heaps' in M. Schofield & M.C. Nussbaum (eds), *Language and Logos*, CUP (1982), esp. pp. 335-6 (cited in: Williamson, T., 'The Early History of the Sorites', typescript (1991), itself an excellent discussion of the Stoic account); Cargile, J., 'The Sorites Paradox', *British Journal for the Philosophy of Science* 20 (1969): 193-202, and more recently, *Paradoxes, A Study in Form and Predication*, CUP (1979), § 37; Campbell, R., 'The Sorites Paradox', *Philosophical Studies* 26 (1974): 175-91; Podosky, M.P., *Vagueness, Bivalence and the Sorites Paradox*, PhD thesis (1985), Monash University, Melbourne; Sorensen, R.A., *Blindspots*, Clarendon Press, Oxford (1988), esp. Ch. 6, pp. 239 ff; Williamson, T., *Identity and Discrimination*, Basil Blackwell (1990), esp. §6.4., 'Inexact Knowledge', *Mind* 101 (1992): 217-42, and 'Vagueness and Ignorance', *Proceedings of the Aristotelian Society Supplementary Volume* 66 (1992): 145-62.

considerations. As such, vagueness in natural language fails to signify any structural or semantic indeterminacy. On the contrary, it merely signifies the presence of cases of epistemic undecidability in the application of language to the world and as such requires no revision of classical logic or semantics (let alone metaphysical revision). This conservative aspect of the epistemic theory is often cited as its major virtue.

A less comprehensive variant, *the weak epistemic theory*, merely claims that *some* vagueness is epistemic. Though vagueness may sometimes be a matter of semantic indeterminacy, this weaker theory has it "that such indeterminacy is not the root of all vagueness". This position faces the problem that it incurs the major difficulty associated with the epistemic account — justifying the semantic realism inherent in such an account, whilst having none of the supposed benefits that accrue to the strong theory by virtue of its logical, semantic and metaphysical conservatism. Once semantic indeterminacy is admitted, classical semantics would seem to be threatened and consequently the epistemic theorist's plea that we accept their account of vagueness in spite of its counter-intuitive implications since we can thereby save classical logic is undermined. If we are prepared to accept a limited semantic theory of vagueness (that is, for *some* vague terms) then there can be little reason for our nonetheless maintaining that we should endorse a classical solution for other vague terms where this seems counter-intuitive. Non-classical (semantic) solutions have been offered precisely on the grounds that we can thereby avoid the apparent excesses of an epistemic account.

My claim will be that the stronger theory is highly implausible. The weaker theory is coherent yet, having opened up the possibility that vagueness is at least sometimes a semantic property, the choice between a semantic and an epistemic analysis will come down to a case-by-case study. Since many find the epistemic account counter-intuitive in most cases, they will presumably opt for the semantic analysis in most cases. In other words, though often unjustly ruled out by definition, the extent to which the epistemic account makes a comeback given a more tolerant approach to vagueness is still insufficient to settle or stifle questions surrounding vagueness as a semantic phenomenon — for example: What are we to say in response to the sorites? Could semantic vagueness be grounded in a vague ontology? etc.

In what follows therefore I want to defend the view that vagueness is not always an epistemic phenomenon. I shall be rather brief in my treatment. This thesis aims to survey the landscape of vagueness but an important objective is to describe an ontological view. For this reason I really just want to describe the epistemic view in passing, saying why I find it incomplete as an account of vagueness, and then move on to the what I and many others see as the major watershed in the area.

2.1 *An Epistemological Approach to the Sorites*

The sorites paradox in antiquity did not remain an isolated curio or pedantic conundrum; it had an edge which the Sceptics hoped to use against the Stoic theory of knowledge in particular, by showing that the Stoics' conception of knowledge, in being soritical, was incoherent.

The Stoics' response, exemplified by Chrysippus, amounted to the claim that the major premise of the standard sorites argument form was false and thus the Sceptics' argument was considered unsound. 'Knowledge', though soritical relative to an appropriately chosen series, is semantically determinate so there is a sharp cut-off point to its application. In the imperceptible slide from cognitive impressions (analogous to the clear and distinct ideas of Descartes and the foundation for knowledge on the Stoic account) to non-cognitive impressions there comes a point where two seemingly indistinguishable impressions are such that one serves to ground claims to knowledge whilst the other does not — even though they are, as just remarked, apparently indistinguishable.

The inclination to validate all the premises of the argument (along with the inference pattern employed) was to be explained via the unknowable nature of the sharp semantic boundary. The Sceptics were, in effect, taken to confuse our inability to know the sharp boundaries of knowledge with the absence of a sharp boundary. Though everyone agreed that no sharp boundary could be known, according to the Stoic defence this was as deep as the problem went. The conundrum was an epistemological one.²

Thus the Stoics rejected the threat of wholesale epistemological scepticism (there could be no coherent claims to knowledge) in favour of the limited scepticism arising from our inability to know the precise boundaries to knowledge. "Nothing can be known" was rejected in favour of "the precise boundaries to knowledge itself cannot be known" — wholesale ignorance was replaced by ignorance of precise boundaries.

This quite specific response to the soritical, and hence paradoxical, nature of 'knowledge' generalises, of course. One might respond to the paradoxicality of any soritical predicate by denying one of the premises of the standard sorites argument involving that predicate (thus pursuing option (c) of §1.2.1.) and the conditional premises are the natural target. In answer to the apparent incoherence of soritical terms *per se* due to their apparently unbounded application it is claimed that there are bounds, precise bounds, but that they are unknowable.

² For more on this cf. Frede, M., 'Stoics and Sceptics on Clear and Distinct Ideas' in M.F. Burnyeat (ed.), *The Skeptical Tradition*, University of California Press (1983); cited in: Williamson, T., 'The Early History of the Sorites', typescript (1991).

In thus requiring that there be a determinate fact of the matter as to whether the predicate applies to any given case in its range of significance such an account is committed to the view that determinate semantic facts may transcend our ability to know whether or not they obtain. This is a strong expression of semantic realism and, as such, is vulnerable to the usual scepticism.

Many think the response runs counter to our intuitions on the matter.³ We feel that one hair cannot make the difference between being described as bald and being described as hirsute; that two colour-patches indiscriminable in colour cannot be described respectively as red and orange; and in so doing we are echoing the more time-worn view of Galen (c. 129 - c. 199):

If you do not say with respect to any of the numbers, as in the case of 100 grains of wheat for example, that it now constitutes a heap, but afterwards when a grain is added to it, you say that a heap has now been formed, consequently this quantity of corn becomes a heap by the addition of the single grain of wheat, and if the grain is taken away the heap is eliminated. And I know of nothing worse and more absurd than that the being and non-being of a heap is determined by a grain of corn.⁴

«« There is a point of clarification required here. Many discussions of the sorites (particularly the fragments from antiquity) present the problem of soriticality as one having essentially to do with the principle of bivalence ('Every sentence is either true or false'). If, as the Stoics thought, bivalence were to hold then, it is said, the existence of a sharp boundary or cut-off point for the application of a term follows. In this way a semantic approach to vagueness, in particular a semantic approach to soriticality, is seen as entailing the failure of bivalence. Yet, as Tim Williamson has pointed out, the denial of bivalence seems incoherent.⁵ The argument for this claim is essentially that given by Susan Haack in her *Deviant Logic*.⁶ The denial of bivalence, assuming 'truth' is taken to satisfy Tarski's condition of material adequacy (that is, where 'truth' satisfies Tarski's disquotational schema: T'A' iff A), seems committed to incoherence in so far as the denial appears to lead to contradiction.

There is a lot one might say in response to this argument. Options include: (1) admitting the characterisation of vagueness in terms of the failure of bivalence, and either

³ King, J., 'Bivalence and the Sorites Paradox', *American Philosophical Quarterly* 16 (1979): 17-25; Sanford, D., 'Competing Semantics of Vagueness', *Synthese* 33 (1976), p. 197; Parfit, D., *Reasons and Persons*, OUP (1984), p. 239; Forbes, G., *The Metaphysics of Modality*, Clarendon Press (1985), p. 168.

⁴ Galen, *On Medical Experience*, 16.1-17.3; translated in A.A. Long & D.N. Sedley, *The Hellenistic Philosophers*, CUP (1987), Vol. 1, p. 223.

⁵ Williamson, T., 'Vagueness and Ignorance', *op. cit.*, pp. 145-9.

⁶ Haack, S., *Deviant Logic*, CUP (1974), pp. 66-8.

(i) accepting that vagueness leads to contradictions — cf. Chapter One, pp. 4 f, or (ii) rejecting the T-Schema; or (2) rejecting the characterisation of vagueness in terms of the failure of bivalence. Given the characterisation of vagueness adopted in Chapter One, option (2) is the appropriate response; the above argument simply shows, at best, that the expressive resources of the language of classical logic are too weak to permit the coherent expression of actual (as opposed to apparent) semantic indeterminacy. Discussions of vagueness this century have generally avoided the problem by making use of locutions like 'clearly', 'definitely' or 'determinately'. The increase in expressive power afforded by such locutions makes it *prima facie* coherent to speak of semantic vagueness. With vagueness characterised in terms of apparent indeterminacy, as we have done, semantic vagueness amounts to there being cases where the relevant term is neither determinately applicable nor is its negation. Thus Galen's incredulity can now be expressed as the view that the determinate being and determinate non-being of a heap cannot be determined by a single grain of corn.

Thus re-expressed, the absence of a sharp semantic boundary for a term is not tantamount to a denial of bivalence, rather it involves a denial of a closely related principle:— *the principle of strong bivalence* — every sentence is either determinately true or determinately false.

For the epistemic theorist, of course, strong bivalence is equivalent to the weaker simple principle of bivalence — every sentence is either true or false — since determinacy is a redundant notion from their point of view. Thus they accept both the weak and strong principle, avoiding incoherent vagueness (as described above) by interpreting "apparent indeterminacy" as "unknowability". >>>

Returning to Galen's comments then... Absurd as it seems, the existence of a precise cut-off point follows from the epistemic theory which uses precisely this feature to evade the Sorites; classical logic is not threatened since precise cut-off points exist and we may therefore claim the major premise of the Sorites as false.

Let us look more closely then to the nature of the epistemic gap, characteristic of soritical expressions, on the epistemic account. The first quick point we ought to be clear on is that Galen, in expressing reservations concerning an epistemic analysis, ought to (and presumably would) agree that one cannot know the sharp boundaries of vague terms but claim that this is because there is none to be known. The reservation is not that one *can* know the semantic boundaries to soritical terms but that, though one cannot, there is one nonetheless.

Secondly, and equally trivially perhaps, the gap is one that is unbridgeable *in principle*. (Cf. Chapter One, pp. 4, 6-7.) It's not that I simply don't currently know where particular semantic boundaries for soritical expressions lie. Wheeler seems a bit ambivalent on the matter when he says:

Cases where there is an answer as to whether a predicate is true of a case where the speaker 'doesn't know what to say', but this answer depends on data such that, if it were in, the speaker *would* know what to say, are cases of epistemological vagueness. Epistemological vagueness ... is a matter of ... not having 'total information' about the case [though] total information is, perhaps in principle, unavailable⁷

As it stands, this concept of vagueness is too general. Any predicate could be such that, due to my current state of ignorance, I am unable to determinately apply the predicate or determinately apply its negation. Were vagueness to depend on current contingencies regarding my epistemic state then precision could be increased in many cases by my getting myself into a better epistemic position. But cases of vagueness are cases where, *even having gotten myself into the best epistemic position possible*, I am still unable to determinately apply the relevant predicate either way.

Epistemic vagueness is a matter of *necessary* ignorance. The cutting edge of the concept is restored by insisting that cases of vagueness due to lack of knowledge be restricted to those in which total information is *in principle* unattainable. In the case of vague predicates, there is still a determinate answer as to whether or not the predicate applies, it is just that it is impossible to know the answer — it is unknowable.

The claim that vagueness amounts to nothing more than an epistemic gap is generally met with incredulity. What could possibly be the cause for such a gap? Williamson's recently elaborated diagnosis, premised on the adequacy of the epistemic response, is the most forceful and coherent I have encountered to date. He offers us "one line of thought [that] may rescue the epistemic theory".⁸

He claims that a margin-of-epistemic-error principle precludes knowledge of the boundary of a predicate's application over a sorites series. In this way he hopes to undermine our incredulity by providing an explanation as to *why* we cannot know where the boundary is. That is, *if* you want to claim that there is a sharp boundary then the margin-of-error principle will help to explain its unknowability.

The specific epistemological problem, as Williamson sees it, briefly is this: we cannot know the semantic determinations on the sharp boundary of a soritical term's application since this violates an error margin principle that is required for knowledge.⁹ So what is this error principle? The general claim is that in order to know that A one must at least be reliably right about it; knowing A entails our being reliably right in supposing

⁷ Wheeler, S. C., 'Reference and Vagueness', *Synthese* 30 (1975), p. 369-70.

⁸ Williamson, T., *Identity and Discrimination*, OUP (1990), p. 104.

⁹ Much of what follows is drawn from §6.4 of Williamson's *Identity and Discrimination*, *op. cit.* and §2 of his 'Inexact Knowledge', *Mind* 101 (1992).

A to be the case. Being reliably right in supposing A to be the case in turn entails A's being the case in sufficiently similar circumstances. (Of course, as Williamson points out, the dimensions of similarity depend on 'A'.) Thus the constraint that knowledge be reliable results in the following (vague but non-trivial) general principle:

<i>THE MARGIN FOR ERROR</i>	If 'It is known that A' is true then 'A' is true
<i>PRINCIPLE</i>	in all sufficiently similar cases.

In other words, if a proposition is true whilst there are sufficiently similar cases in which it is false, it is not available to be known.

How does this help to explain the unknowability of the sharp semantic boundaries to soritical terms? Well, the above general principle has, as a particular consequence that: if a_i is (determinately) P whilst a_{i+1} is (determinately) not P then one cannot know that a_i is (determinately) P.¹⁰ In order to know that a predicate applies to a particular case one must at least be reliably right about it; knowing that a_i is P entails our being reliably right in supposing a_i to be P. Being reliably right in supposing a_i to be P in turn entails things sufficiently close to a_i being P (the dimensions of closeness depending on P). Now each adjacent pair in the series, a_i and a_{i+1} , must appear indiscriminable in respect of P (cf. Chapter One, pp. 16-17; condition (iii) for the soriticality of 'P') and, in so far as they are indiscriminable, they are taken to be sufficiently close. So, the constraint of reliability in effect says that one can know a_i to be P only if adjacent members of the series with regard to which P is supposed soritical, namely a_{i+1} and a_{i-1} , are also P.

Thus the reliability constraint on knowledge results in the following specific principle governing what one can say about a P-soritical series $\langle a_1, \dots, a_n \rangle$:

If a_i is known to be P then a_{i+1} is P.

Applying this principle at P's (supposedly) sharp semantic boundary then explains the unknowability of the boundary. To see this, suppose the boundary divides a_i and a_{i+1} ; that is, a_i is (determinately) P whilst a_{i+1} is (determinately) not P. But then, since a_i is truly P whilst the sufficiently similar case a_{i+1} is not, the requirement on knowledge that there be a margin for error precludes knowledge of the fact that a_i is (determinately) P. Similarly, we can show that the margin-for-error requirement precludes knowledge of the fact that a_{i+1} is (determinately) not P. So, were there a sharp boundary to the application of a soritical term within a series relative to which the term in question is soritical, it would necessarily be unknowable.¹¹

¹⁰ Obviously, as pointed out earlier, any talk of determinacy as distinct from unknowability is redundant for the epistemic theorist.

¹¹ The epistemic blindspot, postulated by the margin-for-error-principle, on the boundary of a term's application will, of course, only serve as a reasonable explanation of our reticence to acknowledge any boundary if the KK-principle is rejected — as Williamson himself is eager to stress, 'Inexact Knowledge', *op. cit.*, pp. 224-5.

To see this, assume the KK-principle. Since a_{i+1} is (determinately) not P, it follows by the margin-for-error-principle, that we do not know that a_i is (determinately) P. But, if we do not

What we have been discussing is, in effect a weak epistemic theory of vagueness. The margin-for-error principle explains how it is that some vague terms — soritical ones — could, contrary to appearances, be semantically determinate.

All other things being equal, we might be able to live with an epistemic account of soriticality. The burden of proof that weighs against a purely epistemic analysis can be lifted to some degree by paying attention to the margin-for-error principle. (Let's be straight about this though; the principle only serves to explain, and thus, as Williamson says, make *available* an epistemic analysis — it does not make such an analysis *compulsory*.) Furthermore, however heavy the burden of proof that remains, however counterintuitive we feel the existence of a precise cut-off point to be, gains in simplicity afforded by the retention of classical logic for the class of soritical terms will go some way to offset such a burden.

Of course, this latter point is crucial. The extent of the gains depends crucially on what we go on to say about vague non-soritical terms. If vague non-soritical terms are given a semantic analysis then the gains will be marginal at best. Though the particular class of soritical terms might be governed by simple classical semantic and logical principles, this would not hold true of natural language in general; as a consequence one might well think that the retention of classical theory is a minor victory, or worse, a pyrrhic one in so far as its retention for soritical terms in the face of its abandonment for non-soritical vague terms amounts to a drawback since the phenomenon common to both classes — vagueness *simpliciter* — would not, on this mixed account, be given unified treatment.

It is all well and good to plug up holes that threaten to destroy a theory, but who would plug up the dyke with their thumb when the floods are streaming over the top! But are the supposedly threatening waters of non-classicism breaching the defences? Can the weak epistemic theory be strengthened to a strong one; can an epistemic analysis be given for vague terms in general, thereby preserving classical logic and semantics in the face of vagueness?

know this, then we do not know that we know that a_{i-1} is (determinately) P — since, by the error-principle, knowing that we know that a_{i-1} is (determinately) P entails knowing that a_i is (determinately) P, which we just said was impossible. Now, by the KK-principle, if we do not know that we know that a_{i-1} is (determinately) P then we do not know that a_{i-1} is P, *simpliciter*. ... We can continue arguing in this way until we are able to conclude, absurdly, that we do not know that a_1 is (determinately) P.

What the KK-principle effectively does is extend the blindspot endlessly; yet we know that, if there is a blindspot, it is only locally spread around the supposed sharp boundary. The KK-principle must be abandoned.

2.2 *The Strong Epistemic Theory of Vagueness*

The problem of non-soritical vagueness did not arise in antiquity. Vagueness *per se* was not an issue. However those who would endorse (and perhaps, as Williamson does, expand upon) the Stoics' epistemic account of the sorites often do so with a view to arriving at a general account of vagueness in purely epistemic terms. So let's look at the plausibility of the resulting strong epistemic theory of vagueness.

According to this general conservative resolution of the problem of vagueness *all* apparent indeterminacy giving rise to border cases is to be attributed to epistemic considerations.

Even supposing the much vaunted fact that classical logic is not challenged by vagueness counts in its favour, how are we to explain the unknowability of sharp semantic boundaries with regard to non-soritical terms? The problem with the extended strong epistemic theory is that no generally applicable reason is given as to why we cannot know the precise semantic boundaries of vague phrases. It is true that in Williamson's most recent publication 'Vagueness and Ignorance' he spends much of his time diffusing arguments as to why *any* epistemic analysis must fail due to its incoherence (arguments from supervenience, meaning as use, etc.¹²) thereby attempting to make the epistemic theory more palatable. Nonetheless, even supposing it to be coherent, many (myself included) still find the account highly counterintuitive.

Williamson himself concedes elsewhere that "[i]f one cannot find any barrier to knowledge of a proposition, it is not unreasonable to conclude that it falls outside the class of unknowable truths."¹³ It was to this end that the margin for error principle was invoked; it provided a barrier to knowledge. The trouble is that the principle cannot help with cases of non-soritical vagueness. Thus this principle will not, contra Williamson, "rescue the [strong] epistemic theory of vagueness".¹⁴ The margin for error model does *not* "make available .. the idea that vagueness depends on the unlocatability of [existent] cut-off points".¹⁵ At best, it only makes available the idea that *soriticality* depends on the unlocatability of existent cut-off points.

But hang on, I hear you say, "just about any vague predicate gives rise to a sorites series".¹⁶ So aren't we making a great deal out of very little in focussing on problems concerning non-soritical vagueness?

12 Questions regarding the success of these arguments are not relevant here. I am making the weaker claim that, even if coherent, anything but a very weak epistemic theory is highly implausible thereby motivating the search for some semantic analysis.

13 Williamson, T., *Identity and Discrimination*, *op. cit.*, p. 104.

14 *Ibid.*

15 Williamson, T., 'Inexact Knowledge', *op. cit.*, p. 236.

16 Williamson, T., *Identity and Discrimination*, *op. cit.*, p. 104.

Well firstly, near enough is not, in this case, good enough. Even were it the case that there are very few non-soritical vague terms, it is precisely the existence of these few that, for many, constitutes the thin end of the wedge. More importantly though, the class of terms whose vagueness is combinatory will presumably provide numerous examples of non-soritical predicates.

Thirdly, it has been implicitly assumed throughout the discussion in this chapter that soritical predicates are such that *all* their border cases are embedded within soritical series. However, to say that a predicate is soritical is simply to say that there is some series by means of which a sorites argument can be constructed using that predicate; this does *not* mean that all of its border cases are to be found embedded a sorites series. We may remain sceptical about the existence of unknowable semantic facts concerning those border cases which are not so embedded — our worries concerning such cases cannot be dispelled by attending to margins for error.

Thus I think we can safely assume that there remains a substantial amount of vagueness in natural language which cannot be attributed to the presence of border cases in a sorites series; the unknowability of facts concerning these cases awaits an explanation.

So now when we assess this strong epistemic theory there is nothing to offset our worries that this approach just gets it wrong. To be sure, there may be gains in simplicity, but I for one (along with many others) think that any such gains are outweighed by the thought that the theory just gets it wrong in many cases. "[N]ot every anomaly falsifies a theory"¹⁷ but in the face of certain anomalies one begins to look elsewhere for more elegant accounts.

This, naturally enough, has repercussions for the weak epistemic theory outlined earlier. Its counter-intuitiveness remember was lessened by the provision of an explanation as to why we are barred from knowledge of the boundary — an explanation not available for vagueness in general. If, as a consequence, one opts for a semantic construal of the apparent indeterminacy we encounter with non-soritical vagueness then one either accepts a mixed account (vagueness is sometimes to be seen as involving semantic indeterminacy and sometimes merely epistemic) or one abandons the epistemic account altogether. The worry is that the mixed account appears unstable, degenerating into a very weak epistemic theory — one much weaker than that outlined above according to which all soritical vagueness was seen as epistemically based.

The former (mixed) option appeared to be that preferred by Williamson a few years ago. "One can have an epistemic theory of vagueness without holding that all vague predicates are semantically determinate. Indeed, it is hard to deny that semantic

¹⁷ Williamson, T., 'Vagueness and Ignorance', *op. cit.*, p. 162.

indeterminacy can occur."¹⁸ So not only *can* one have a mixed account, it is hard not to. But now that we are prepared to accept semantic indeterminacy with all its attendant revisions to classical logic and semantics, why should we bother to defend an epistemic analysis in those cases where it seems wildly at odds with our intuitions on the matter? Gains in simplicity afforded by the retention of classical logic can no longer be cited as a motive — we have just said that we're prepared to abandon such a theory.

My point then is that many cases where we formerly opted for an epistemic analysis (cases of soritical vagueness) can, without loss, be redescribed as involving semantic indeterminacy. Note that this does not entail that we forgo an epistemic analysis in all cases. When the epistemic theorist invoked the margin-for-error principle I think they were on to something; there may well be cases of soritical vagueness where the choice between epistemic and semantic indeterminacy swings in favour of an epistemic analysis once this principle has been acknowledged. Nonetheless, there is no compulsion to claim all soritical vagueness as epistemic. Even the weak epistemic theory of §2.1 seems too strong.

In more recent articles Williamson appears to have hardened his stance.¹⁹ He has shifted to the more stable, though less plausible, strong epistemic theory with Sorensen as company. I don't think that either of them have yet provided compelling reasons why we should follow them down what looks like an heroic but misguided course (though, as the dialectic proceeds, more and more sweeteners are being offered). Sorensen does however cite a reason which, though mistaken, warrants some discussion.

2.3 *Semantic Realism, Vagueness and Bivalence*

Sorensen has suggested that any non-epistemic analysis of vagueness commits one to being a semantic anti-realist across the board. A denial of the strong epistemic theory "can only be motivated by a prior resolve to allow no unknowable truths".²⁰ If correct, this would be an important factor in our choice of theory. However it is simply not true. It is further encouraged by Dummett's high redefinition of 'realism' so we shall, in turn, consider Dummett's views on the matter. Contrary to both, a commitment to semantic indeterminacy is not a commitment to anti-realism — semantic or metaphysical.

¹⁸ *Ibid.*, p. 107.

¹⁹ In his 'Vagueness and Ignorance' he supports "the epistemic view of vagueness" which can be seen, from the surrounding text, to amount to the strong epistemic view.

²⁰ Sorensen, R., *Blindspots*, *op. cit.*, p. 245.

Let me firstly reiterate the point made earlier that the strong epistemic theory of vagueness carries with it a commitment to realism.

Realism about a particular class of statements is, for the purpose of this paper, the view that we so understand those statements as to render intelligible to ourselves the possibility that any of them be true without our being able to recognize that it is so. If the statements in question are not effectively decidable, then, obviously, we commit ourselves to realism in this sense if we hold of each of them that either it, or its negation, is determinately true. So one who endorses the Principle of [Strong] Bivalence for non-effectively decidable statements of a particular kind commits himself — provided he intends a truth-distributive interpretation of disjunction — to realism about them.²¹

Though there may be problems with the above characterisation of 'realism', the thesis expressed is true enough (the clause requiring a truth-distributive interpretation of disjunction ensures our being able to infer the determinate truth or determinate falsity of any claim A from the determinate truth of it or its negation).²² Advocating the strong epistemic theory commits one to realism with regard to statements about the physical world.

But what of the converse relation? Is a commitment to realism tantamount to a commitment to the Principle of Strong Bivalence for vague statements, and therefore a commitment to a strong epistemic theory of vagueness? If the answer to this question were in the affirmative then Sorensen would be right and *the advocate of an epistemic account would have a new argument for the strong epistemic theory of vagueness that could only be avoided by adopting anti-realism*. The answer, however, is a resounding 'No'. Realism, as characterised above, carries with it no claims as to the correctness of strong bivalence.

²¹ Wright, C., *Realism, Meaning and Truth*, Basil Blackwell (1987), p. 85; Wright mentions in a footnote that this later seemed less obvious to him but his thoughts on this later in the book (cf. §1 of essays 10 and 11) seem not to affect the point made in regard of *in principle* undecidable statements. See also: Rasmussen, S.A., & Ravnkilde, J., 'Realism and Logic', *Synthese* 52 (1982): 379-437, where they argue for Wright's original claim quoted above.

²² This account of 'realism' is rather weak but this does not affect the argument that a semantic analysis involves no commitment to realism. In fact, if we can show that we are not committed to 'realism' even in this weak sense then we will thereby have established that we are not committed to 'realism' in any stronger, more reasonable sense.

Why is Wright's notion too weak? Well (paraphrasing Sylvan, R., *Deep Plurallism* [sic], Edinburgh University Press (forthcoming), Ch. 7, p. 259.) if, as seems preferable, we characterise 'realism' as some elaboration of the position that

- there exists a unique actual world or reality (the world), and
- it enjoys certain prized properties, including both externality and as well independence of mind, cognitive agents, and such like.

then Wright's notion is defective since it does not exclude a superidealism, strictly incompatible with realism; for the problematic truths may be recognised and certified by other creatures superior in such divination to us present humans (perhaps extra-terrestrials, perhaps future terrestrials, etc.). Independence of cognitive agents is not duly guaranteed.

As Williamson himself admits, there is nothing to stop the believer in semantic vagueness admitting the existence of unknowable truths in general.²³ For example, it can be freely admitted that there may be logical barriers to knowledge of truths as in the case of the sentence 'The number of books in TW's room on 1 July 1989 is even and no one will ever know that it is'. Similarly, one might adopt a realist position with regard to the undecidable statements of mathematics, or the past, and so on.

The semantic theorist might simply think that there seems no general reason to suppose vague statements to be of this type; no general barrier to knowledge of determinate semantic relations is discernible.

And even where a barrier is discernible, thereby making available a realist response in those cases (e.g. soritical predicates), this needn't generalise to a wholesale acceptance of strong bivalence for all vague terms — even if it leads to the acceptance of strong bivalence for that limited class (itself a questionable move, as we saw at the end of §2.2!).

The rejection of the Principle of Strong Bivalence need only be motivated by the limited acceptance of vagueness as a semantic property of some vague terms. This needn't extend to all vague terms, and certainly needn't extend to all other areas of discourse where the issue of the existence of unknowable truths arises. So Sorensen's claim that one can only avoid the strong epistemic theory by "a prior resolve to allow no unknowable truths" is false.

Dummett's is a slightly more subtle though equally fallacious way of endorsing the idea that a commitment to realism is tantamount to a commitment to the Principle of Strong Bivalence for vague statements, and therefore a commitment to a strong epistemic theory of vagueness. Such a commitment is explicitly guaranteed by Dummett's view of what is constitutive of realism:

Realism I characterise as the belief that statements of the disputed class possess an objective truth-value, independently of our means of knowing it: they are [determinately] true or [determinately] false in virtue of a reality existing independently of us. ... That is, the realist holds that the meanings of statements of the disputed class are not directly tied to the kind of evidence for them that we can have, but consist in the manner of their determination as true or false by states of affairs whose existence is not dependent on our possession of evidence for them.²⁴

So, for Dummett, realism with regard to statements about the physical world entails our endorsing strong bivalence for statements about the physical world and so leads to the

²³ Williamson, T., *Identity and Discrimination*, *op. cit.*, p. 104.

²⁴ Dummett, M., *Truth and Other Enigmas*, Duckworth (1978), p. 146.

view that there is some objective determinate fact of the matter as to whether a vague term applies to a case or not, vagueness is just a matter of its being essentially unknowable which is the case.

Putnam, in considering the relation between realism and vagueness, supports Dummett arguing that, faced with the phenomenon of vagueness which Putnam implicitly assumes to be semantic, something seems to be wrong with realism.²⁵ To consistently maintain realism would be to deny the possibility of this type of vagueness.

In fact, faced with the phenomenon of *semantic* vagueness, something seems to be wrong with *Dummettian* realism. Though I agree with Putnam that a semantic analysis of vagueness is incompatible with *Dummettian* realism, and I take vagueness to result (at least sometimes) in semantic indeterminacy, I nonetheless agree with Wright in thinking that such vagueness can be absorbed into a realist position.²⁶

Let us distinguish the following non-equivalent features (amongst others identified by Putnam²⁷) in Dummett's notion of 'realism' in discourse about the physical world:

- (1) endorsement of Strong Bivalence for statements about the physical world which are not effectively decidable; and
- (2) admission of the possibility of verification-transcendent truth for statements about the physical world.

A realist, though committed to truth holding in virtue of what is objectively the case, is not committed to feature (1) of Dummett's characterisation of realism. In short, there can be realism without Dummettian realism; (2) does not entail (1).²⁸ Dummett and Putnam have both succumbed to the fallacy of redefinition.

What is required is that one distinguish Dummettian realism from realism *per se*. Following Wright, let us define *bare realism* as a realism that assumes there to be no important objection to the idea that we can understand what it is for statements about the physical world to be true in a manner transcending our capacities for knowledge; and let *classical realism* be a realism that endorses the principle of Strong Bivalence for non-effectively decidable statements about the physical world. Dummettian realism is just classical realism. If one is a classical realist then vagueness will be epistemic but there is no reason why one cannot account for vagueness along semantic lines and accept bare realism. *A commitment to realism is not a commitment to an epistemic account of vagueness unless one endorses classical realism.*

25 Putnam, H., 'Vagueness and Alternative Logic', *Realism and Reason*, CUP (1983), pp. 272-4. Note that Putnam, in implicitly assuming vagueness to be that which it is commonly described as being — a semantic phenomenon — fails to consider that option whereby the realist accounts for vagueness along epistemic lines. It would seem that epistemic vagueness is simply a non-contender as far as Putnam is concerned.

26 Wright, C., *op. cit.*, p. 319.

27 Cf. Putnam, *op. cit.*, p. 272.

28 John McDowell has also argued for this. Cf. §6 of McDowell's 'Truth-conditions, Verificationism and Bivalence' in J. McDowell & G. Evans, *Truth and Meaning: Essays in Semantics*, OUP (1976), pp. 54-5.

Wright makes the point thus:

In Dummett's writings, the cardinal realist thesis is that of the unrestricted acceptability of the principle of [Strong] Bivalence, that every statement — so long as it is not too vague — is determinately either true or false. Undeniably this is an important element in the thought of many realists about pure mathematics. ... Vagueness is, however, a pervasive and, arguably, an ineliminable feature of the greater part of our non-mathematical discourse. To suggest that [Strong] Bivalence is, or should be the hallmark of realism everywhere is accordingly to be committed to claiming either that there is no such thing as realism about vague discourse, or that the vagueness of a statement ... is a feature consistent with its possession of a determinate truth-value [i.e., an epistemic feature]. ... The obvious response would be to suggest that [Strong] Bivalence is merely the natural form for an acceptance of the possibility of evidence-transcendent truth to take when we are concerned with statements which are not vague; and that it is, accordingly, the status of such a conception of truth which Dummett's proposal, generalized, would make the crucial issue.²⁹

So the realist is free to accept vagueness as that which it is commonly understood to be, a semantic phenomenon; the argument that they are committed to an epistemic account fails. Realism involves no commitment to either account of vagueness; a semantic analysis of vagueness involves no commitment to anti-realism.³⁰

Semantic vagueness, though it is a watershed for the bare-realism / classical-realism debate, is not a watershed for the realism / anti-realism debate. That schism is properly to do with rival conceptions of 'truth' irrespective of strong bivalence.

2.4 Summary

So we see that the strong epistemic theory is implausible. This is not because an epistemic analysis of vagueness is necessarily incoherent. A weak theory seems tenable but both

²⁹ Wright, C., *op. cit.*, p. 4. It is worth pointing out that vagueness is also a feature of our mathematical discourse though perhaps not part of the 'hard core' of mathematics. Cf. Chapter One, p. 25 and Lakatos, I., *Proofs and Refutations*, CUP (1981), pp. 99 ff.

³⁰ In effect then, Dummett's redefinition of 'realism' is far too high, unduly constraining the realist. This compares with Wright who joins us in accepting a less restrictive, lower definition but offers one that is probably too low — cf. n. 22.

soritical and non-soritical vague terms are nonetheless to be analysed in terms that admit of vagueness as a semantic property.

With this in mind I now want to focus on vagueness as a semantic phenomenon. As such, it poses many of the problems commonly associated with vagueness. Rivals of the strong epistemic theory are generally thought to "involve revisions of classical logic and semantics"³¹ (and some, myself included, would have it that metaphysical revision is also required);

- semantic revision, in as much as the *apparent* lack of sharp boundaries to the extension of vague predicates, for instance, is now *real*.
- logical revision, in as much as some classical principles may seem threatened and, more forcefully, the conservative response to the sorites resolving the paradoxicality of vague terms offered by the epistemic theorist is no longer generally available.

Of course, to think that a semantic analysis of vagueness constitutes a threat to classical logic presupposes such vague natural language to be within the scope of logic at all. Classical logic can always be saved by declaring the apparently falsifying evidence beyond the pale, beyond the scope of the theory of classical logic.

Russell, whose views we shall consider next, would object to the strong epistemic theory on the grounds that it mistakenly supposes logical theory to apply to natural language. In effect he pursues a generalisation of option (a) cited in Chapter One as a response to the sorites. If logic applies to language at all it properly applies to some ideal language and natural-language semantics must be distinguished from pure ideal semantics. Whatever we say about his defence of classical logic his semantic analysis of vagueness is of special interest constituting, I shall argue, a paradigm for the *purely* semantic analysis of vagueness involving no metaphysical revision.

³¹ Williamson, T., 'Vagueness and Ignorance', *op. cit.*, p. 162.

Chapter Three

Russell's Representational Theory of Vagueness

Whilst admitting that there may be cases of epistemic vagueness, there seems insufficient support for the claim that all vagueness is epistemic. Some instances of apparent semantic indeterminacy would seem to be real.

Of course, whether this semantic indeterminacy is analysable as a *purely* semantic phenomenon, or is perhaps seen as a reflection of some underlying ontological indeterminacy, is independent issue. That constitutes a further, major watershed in approaches to vagueness, as we saw in §1.5, and will be taken up later. In either case vagueness is admitted as a semantic phenomenon; *contra* the epistemic theorist, vagueness is (at least) sometimes an essential semantic feature of specific terms in natural language.

For the record, this is certainly the majority view. Like so many philosophical positions, the semantic view of vagueness has dominated twentieth century philosophy (so much so that the epistemic view is often precluded by definition — as we saw with Alston, for example, in Chapter One). Bertrand Russell speaks for many when, by way of an initial explanation of vagueness, he asks us to

consider the various ways in which common words are vague, and let us begin with such a word as "red." It is perfectly obvious, since colours form a continuum, that there are shades of a colour concerning which we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word "red," but because it is a word the extent of whose application is essentially doubtful. This, of course, is the answer to the old puzzle about the old man who went bald. It is supposed that at first he was not bald, that he lost his hairs one by one, and that in the end he was bald; therefore, it is argued, there must have been one hair the loss of which converted him into a bald man. This, of course, is absurd. Baldness is a vague conception; some men are certainly bald, some are certainly not bald, while

between them there are men of whom it is not true to say they must either be bald or not bald.¹

Russell's conception not only typifies the view of vagueness as semantic but exemplifies what Bertil Rolf has described as *the traditional concept of vagueness*: as well as describing vagueness as a semantic phenomenon of natural language, this concept endorses the paradigmatic conception of vagueness as applied to predicates and characterised by the presence of border cases — identified in Chapter One.²

In much of the rest of this thesis I shall speak simply of "vagueness", restricting the discussion to that *semantic* conception of vagueness presupposed in the traditional account — a conception which, whilst admitting vagueness as semantic, does not necessarily conceive of vagueness as *merely* semantic. Our moving beyond the strong epistemic theory means that an analysis of the semantics of vagueness is sought; classical semantics seems threatened.

Moreover we cannot avail ourselves of the epistemic solution to the sorites; classical logic seems threatened as well. After all, mightn't it be right to think, as many people do, that the admission of vagueness as semantic gives rise to some sort of semantic tolerance sufficient to justify all the conditional premises of sorites arguments? If the premises are all true (forestalling option (c) of §1.2) then aren't we compelled to take issue with the reasoning involved (and thus pursue option (b) of §1.2)?

Well there is of course another option; anyone wishing to preserve the validity of classical logic may still deny there to be any tension between vagueness and classical logic. Vagueness, they may claim, does not constitute a challenge to classical logical theory since the said theory is taken either: (i) to be a theory dealing with set-theoretical structures hence a study of structures like the world, which is not vague; or (ii) a theory dealing with symbols hence a study of structures like language which the classicist might claim to be precise in essence or in all respects necessary to describe the world. A view according to which the vagueness of natural language is seen as irrelevant to logic is expressed by both Russell and Quine and amounts to a defence of classical logic by pursuing option (a) of §1.2.

¹ Russell, B., 'Vagueness', *Australian Journal of Philosophy* 1 (1923), pp. 85-6.

² Rolf, B., *Topics on Vagueness*, PhD Thesis, Lund (1981), p. 74.

One further, and final, feature of the traditional concept is its failing to distinguish resilient border cases from border cases *simpliciter*. Thus there are predicates which are vague according to the traditional concept of vagueness though precise according to the account offered in Chapter One, §1.4. The traditional concept then amounts to a naïve paradigmatic conception in so far as simple border cases are the defining characteristic, in conjunction with the view that the relevant vagueness is to be seen as a semantic property of the term involved. Most discussions of vagueness in the literature approach the phenomenon via this concept of vagueness — hence its descriptive title as 'traditional'. Issues surrounding the naïveté of this concept, in so far as resilience is ignored, will become important later — in §3.3.2.2. For the moment we can ignore this complication.

This position, in advocating the retention of classical logic, leads to the view that the world is precise. *Either* logic describes the structure of the world and so, since classical logic makes no room for vague objects, the world is precise *or* logic describes the structure of language. Thus, the vagueness of natural language must be a merely contingent feature that can be eliminated from any language used to describe the world; the ideal language is precise.³ Therefore the world is precise. So, by dilemma, the world is precise.

The view according to which vagueness is a semantic but not ontological phenomenon — vagueness as *merely* semantic — I shall refer to as *the representational theory*. Vagueness is recognised as a feature of representations of the world, contra the strong epistemic theorist, but not of that represented — the world itself.

The reasons for its popularity are, I think, the following. It rightly recognises that feature of natural language that initiated the problem in the first place — vagueness — as semantic, whilst denying that this has any import at all for metaphysics and promises to be of no more than superficial import for logical theory. (Of course, some account is required of the relation between vague natural language and the world but this will be constrained by the fact that the world is precise.) The means described above for retaining classical logic in the face of semantic vagueness amounts to a representational theory of vagueness, a classical-representational theory.

In this chapter I want to consider the classical-representational theory as outlined by Bertrand Russell. For Russell, as with some later representational theorists (e.g. Quine) the retention of classical metaphysics is sufficient for the retention of classical logic without extension or revision. The 'deep-structure' or 'canonical regimentation' of natural language, Ideal Language, is said to mirror the world whose structure remains completely classical. Logic is parasitic on this Ideal and thus the logic of *Principia Mathematica* remains intact. The 'superficial' structure of natural language should not misguide us as to the true logical structure of language, just as the 'superficial' structure of the world (Appearance) should not misguide us as to its true logical structure (Reality). The classical logical theory can then no longer be viewed as a normative theory of natural language argumentation since natural language has been shown to exhibit features that that theory forbids; its adequacy is defended by means of language of a suitable ideal type.

In the following chapter we shall look at the Quinean variant on this theme, and thereafter a more recent representational account that takes natural language seriously as a

³ The ideal language must of course be subject to the constraint that it is expressively complete lest the claim that classical logic be grounded in a precise language becomes the trivial claim that logic can be grounded in a fragment of language that is precise; the existence of a precise language must be non-trivial in the sense that this language can express all that can be expressed in natural language about the world (i.e., there is preservation of information content). In other words, expressive completeness is preserved in the move from natural language to a precise language and the world can be described in purely precise terms.

structure to be modelled, yet nonetheless makes use of the underlying metaphysical conservatism to arrive at a conservative modal extension of classical logic as the logic of vague natural language.

3.1 *The World is Not Vague*

Let us assume now that 'vagueness' as a feature of terms in natural language *is* an indispensable semantic feature of those terms. The world is vaguely described in natural language. Might it be the case that the world is vague? The answer given by the advocate of the representational account of vagueness is 'No'. One is not forced to look very far to find claims to this effect. In what is considered by many to be the *locus classicus* on vagueness, Bertrand Russell claims:

There is a certain tendency in those who have realized that words are vague to infer that things also are vague. ... This seems to me precisely a case of the fallacy of verbalism — the fallacy that consists in mistaking the properties of words for the properties of things. Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example. *They have to do with the relation between a representation and that which it represents.* Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; *things are what they are, and there is an end of it. Nothing is more or less what it is, or to a certain extent possessed of the properties which it possesses.*⁴ [my italics]

More recently, Margalit suggests:

Things are what they are. They are not what they are in grades, shades, or degrees. It is only relative to our ways of classifying them that they are subjected to gradations. The ascription of vagueness to objects may yield the quantities-turn-qualities kind of 'logic' ('dialectical' or otherwise), which commit the fallacy of *verbalism*, i.e., the ascription of properties of words to objects.⁵

⁴ Russell, B., *op. cit.*, pp. 84-5.

⁵ Margalit, A., 'Vagueness in Vogue', *Synthese* 33 (1976), p. 213. An approach to vagueness via a "quantities-turn-qualities kind of 'logic'" referred to by Margalit is endorsed by some Marxists; Plekhanov especially uses phrases like "quantity-turn-quality" in his discussion of what we would now describe as vagueness. Margalit presumably has these theorists in mind. Cf. Chapter One, p. 5.

Once upon a time Dummett held this position too maintaining that "...the notion that things might actually *be* vague, as well as vaguely described, is not properly intelligible."⁶ He has since recanted by suggesting that this view, according to which reality cannot be vague, is nothing more than deep-seated metaphysical prejudice.⁷ What I shall argue in later chapters is that it is more substantive than mere prejudice (though, ultimately, no more defensible); it rests upon adherence to classical logic which, though able to make room for vague language by suitable scope restrictions (Russell, Quine) or extension (supervaluation theories), makes no room for vague objects.

3.2 *Agnosticism*

A bit of ground clearing is required before proceeding further. I have described the representationalist as maintaining, in part, that "the world is not vague" in the sense that they are to be understood to be making a substantive metaphysical claim. This I take to be the substance of both Russell's and Margalit's claims above. However, it may be said that by this it is simply meant that it is *meaningless* to ascribe vagueness to the world; the world's not being vague does not entail its being precise, agnosticism is a tenable alternative. Then the question remains: in what sense might the world be said to be 'other-than-precise'? — it must be this that the representationalist denies.

One way in which it may be thought meaningless to ascribe vagueness to the world is by one's assuming 'the world' to be some noumenal world. Thus a Kantian might argue that the world cannot be said to be vague since nothing can be said of the noumenal realm (not even that it is precise). This position is simply not interesting. We can and regularly do speak about cats, dogs and mountains and it make no difference to the positions of interest in the current debate whether or not these 'things' must be said to be things in the phenomenal world or not. The question simply becomes one of the vagueness or precision of the phenomenal world.

A second way in which it may be thought meaningless to ascribe vagueness to the world is by assuming that, though the world being represented in language can be talked about, 'vague' and its opposite 'precise' are, as Russell has said above, relational attributes that cannot be applied to the world as opposed to the represented world. On this view it is akin to a category mistake to ascribe vagueness or precision to that which language represents; they are features that can only be applied in the realm of representations. If we accept this terminological point then, though 'vague' is not applicable to the world, there

⁶ Dummett, M., 'Wang's Paradox', *Synthese* 30 (1975), p. 314.

⁷ Dummett, M., *The Interpretation of Frege's Philosophy*, Duckworth, London (1981), p. 440.

may nonetheless be a sense in which we may want to meaningfully inquire as to whether or not that feature of representations under discussion in the context of vagueness has any analogous correlate in discussions of things or the world.

Is there an analogue of 'vague' that can be meaningfully applied to the world? Well yes; 'fuzziness' understood to mean indeterminacy-in-extension will function as a meaningful analogue of 'vagueness' in discussion about the world, where this is simply intended as an ontological analogue of semantic indeterminacy of extension.

So there is a sense in which the world might be said to be vague, namely its being fuzzy (though many think that ultimately this is logically incoherent) and a sense in which the world might be said to be precise, its not being fuzzy (which many think is logically necessary), which we can express as its being *sharp*.⁸

To sum up then. I have characterised the representationalist account as maintaining, amongst other things, that the world is "precise". In characterising the view in this way I have used the terminology that has been prevalent in the literature on the subject which supposes such a feature to be meaningfully attributable to the world; it is to be understood as the claim that though vagueness is a feature of natural language there is no sense in which this phenomenon may be attributed to features of the world. In light of the above, however, we should distinguish vagueness (an attribute of representations) from fuzziness (an attribute of ontological items) and can now characterise the representationalist account as follows: though vagueness is a feature of natural language the describable world (including cats, dogs, mountains, etc.) is sharp; there is no fuzziness in the world.

However, you may object, Russell never said the world couldn't be fuzzy; only that it couldn't be vague and, given his terminology, he is right. What we shall see is that, in the course of charitably interpreting Russell's definition of vagueness, he must be understood as using 'vague' to mean 'vague or fuzzy'. This is because, as we shall see in spite of opinions to the contrary, he has (wittingly or not) provided a blueprint for representational vagueness. A charitable interpretation then sees him as committed to a representational account, including a sharp world.

It is to this task I now turn.

3.3 *Russell on Vagueness*

The Russellian position on vagueness is described most notably in his 1923 paper 'Vagueness', with further remarks to be found scattered through his other writings. These

⁸ I have adopted the term 'sharp' in preference to that adopted by Bertil Rolf who uses the term 'hard' in his *Topics on Vagueness*, p. 13.

More will be said in developing this idea of "indeterminacy-of-extension" later, when required.

other remarks, found for example in *The Analysis of Mind* and *An Inquiry Into Meaning and Truth*, do not represent a departure from the 1923 treatise. His view — vagueness is a semantic feature of natural language having its source in the representational nature of language and not in that which is represented, the world, which is sharp — remained unchanged. For this reason we can concentrate on the 1923 work, deferring to other remarks if and when required.

3.3.1 *Incompleteness, Lack-of-Specificity and the Source of Vagueness*

Before presenting his 1923 definition of vagueness, it will help to look at some examples Russell presents as being examples of vagueness to see just where he takes the source of vagueness to be. We have already seen above that he sees vagueness as applicable to representations in general, be they "cognitive or mechanical", so linguistic and perceptual representations are both items which can exhibit the phenomenon. Moreover his account of the source is primarily concerned with examples from cases of perception.

He says, for instance:

What is clear is that the knowledge that we can obtain through our sensations is not as fine-grained as the stimuli to those sensations. We cannot see with the naked eye the difference between two glasses of water of which one is wholesome while the other is full of typhoid bacilli. In this case a microscope enables us to see the difference, but in the absence of a microscope the difference is only inferred from the differing effects of things which are sensibly indistinguishable. It is this fact that things which our senses do not distinguish produce different effects — as, for example, one glass of water gives you typhoid while the other does not — that has led us to regard the knowledge derived from the senses as vague. And the vagueness of the knowledge derived from the senses infects all words in the definition of which there is a sensible element.⁹

This feature of perception whereby the perceived item (that item having all and only those properties perceived of the thing in question) lacks properties possessed by the thing perceived is more commonly described as the incompleteness or selectivity of perception.¹⁰ The more selective or incomplete a perception, the less specific the perceived item or representation is — where 'specificity' is defined as follows:

⁹ Russell, *op. cit.*, p. 87.

¹⁰ Notice that it would follow then that, if Russell is consistent in his use of what counts as vague, the vagueness of perception can be used in arguments for a representative theory of perception and scepticism, assuming the validity of more traditional arguments from the incompleteness of perception or, in certain cases, from the composition of matter (cf. Moore and H.H. Price).

A representation B is *less specific* than a representation A iff A specifies 'more' about the world than B does; i.e., the information content of A properly includes that of B.

For example, a photograph of a man from a great distance will be less specific (*ceteris paribus*) than one taken at short range since there will be features discernible in the latter not discernible in the former, e.g., his having a tear on his left cheek.

On Russell's analysis, lack-of-specificity and vagueness share a common source.

Vagueness in our knowledge is, I believe, merely a particular case of a general law of physics, namely, the law that what may be called the appearances of a thing at different places are less and less differentiated as we get further away from the thing. When I speak of "appearances" I am speaking of something purely physical — the sort of thing, in fact, that, if it is visual, can be photographed. From a close-up photograph it is possible to infer a photograph of the same object at a distance, while the contrary inference is much more precarious. That is to say, there is a one-many relation between distant and close-up appearances... *I think all vagueness in language and thought is essentially analogous to this vagueness which may exist in a photograph.* [my italics]¹¹

And at times he goes so far as to equate vagueness with lack-of-specificity.

... it is obvious that what you see of a man who is 200 yards away is vague [less specific] compared to what you see of a man who is 2 feet away; that is

Taking the latter first. Suppose that Simple Simon were directly acquainted through the senses with (i.e., directly sees) a mangosteen, then, since a mangosteen is a collection of elementary particles, he would directly see a collection of elementary particles, but this is not what Simple Simon sees so, by reductio, he does not directly see a mangosteen. To see how vagueness leads to scepticism one need only present Simple Simon with the typhoid-infected glass of water and ask him what he sees. By the vagueness of knowledge derived from the senses he does not see water infected with typhoid bacilli and yet the presented sample was just that, so it is not the sample that Simple Simon directly sees. Vagueness, as described above by Russell, leads to a representative theory of perception and so to scepticism.

Alternatively, arguments from the selectivity or relative incompleteness of perception are often taken, by similar reasoning, to lead to the same conclusion. Take, for example, the speckled hen argument. Assume I now see before me a speckled hen and the hen in question in fact has 999 speckles, yet I cannot see exactly how many speckles it has; my perception is a perception of a hen with an indeterminate number of speckles (cf. F. Jackson, *Perception*, CUP (1977), p. 115) and thus is not direct perception of the speckled hen itself. The directness of perception is denied and scepticism results. Note however that, for Russell, this would be an instance of the "vagueness" of perception since it is a case where our senses fail to distinguish *all* distinguishing characteristics of an object and so will be an instance of perception where we would be unable to distinguish, on the basis of perception alone, two objects that differ on some property not given in the perceptual event.

That I take these arguments to be invalid is beside the point. For Russell, these arguments hold water and so he is committed to whatever consequences might ensue. That such arguments for scepticism *are* invalid and that classicists such as Russell *are* committed to their validity is argued for in R. Routley's *Exploring Meinong's Jungle and Beyond*, Departmental Monograph #3, Dept. of Philosophy, RSSH, Australian National University (1980), Ch. 8, §10.

¹¹ Russell, *op. cit.*, p. 91.

to say, many men who look quite different when seen close at hand look indistinguishable at a distance, while men who look different at a distance never look indistinguishable when seen close at hand... there is less vagueness [more specificity] in the near appearance than in the distant one...¹²

This claim according to which the incompleteness of perception relative to that represented constitutes the vagueness of the representation is, I think, a mistake on Russell's part (as we shall see later on in §3.3.2, it is a point Russell himself gives conflicting accounts of) and one he can abandon. Though incompleteness and distilled Russellian vagueness *are* in some respects analogous as to their source, they can be seen to be distinct once a coherent exposition of Russell's often very confusing account is achieved.

It is worthwhile digressing here to point out that this confusion regarding the notions of vagueness and lack of specificity may be found in the writings of numerous other authors as well (often resulting in the *misconception* that precise propositions are harder to verify than vague ones). Cases in point are A. Rosenberg's article, 'The Virtues of Vagueness in the Languages of Science' and, perhaps more significantly given the nature of the text, Alston's discussion of vagueness in his *Philosophy of Language*; Duhem and Austin are similarly confused.¹³

Rosenberg's synonymous use of 'vague' and 'inexact' can be seen in the following passage:

In the face of the inexactness of observation terms one might boldly suggest ... that we might be able to provide an observation language which ... would be purged of vague terms, and would be perfectly exact.¹⁴

This confusion of Rosenberg's regarding 'vague' / 'inexact' can be traced back to Körner's use of the term 'inexact' for what is more commonly described as 'vague'.¹⁵ Körner however, unlike Rosenberg, appears to have consistently used the non-standard term 'inexact' in place of the more usual 'vague'; in this sense there is no confusion in Körner's work on vagueness, merely terminological difference. In discussing some of Körner's ideas though, Rosenberg has not consistently used this new term; sometimes he appears to have the non-standard sense as used by Körner in mind (as indeed one might expect when referring to and discussing Körner's work) and, at other times he has slipped

¹² *Ibid.*

¹³ Rosenberg, A., 'The Virtues of Vagueness in the Languages of Science', *Dialogue* 14 (1975): 281-311; Alston, W.P., *Philosophy of Language*, Prentice-Hall (1964), esp. pp. 84-5; Duhem, P., *The Aim and Structure of Scientific Theories*, (transl. and ed. by P.P. Wiener) Princeton University Press (1954), esp. pp. 178-9; Austin, J. L., *Sense and Sensibilia*, OUP (1962), esp. pp. 125-31.

¹⁴ Rosenberg, A., *op. cit.*, p. 283.

¹⁵ Körner, S., *The Philosophy of Mathematics*, Hutchinson, London (1960), esp. pp. 160-1.

into the common use of the term where 'inexact' is synonymous with 'lack of specificity'. For example,

A theory can predict the length of an observable rod to 10^6 decimal places, and this may be an exact observational consequence. And though we can only assert that the length of the rod is observed to (usually) lie *between* two points respectively of considerably greater and lesser magnitude, this inexact proposition does provide observations which test the exact predictions of the theory.¹⁶

This equivocation on the use of the term 'inexact' results in lack of specificity being, at least sometimes, equated with vagueness — an unacceptable equation, having as a consequence (as I said above) the misconception that "a theory couched in inexact [or vague] terms, ... has a much higher probability of being true than any one of the exact [or precise] theories with which it is incompatible" or that "... unfalsifiability ... follows in the train of vagueness...".¹⁷ Vagueness is (unjustly) portrayed as an enemy of Popperian scientific theorizing, echoing Russell's claim that

a vague belief has a much better chance of being true than a precise one ... If I believe so-and-so is tall, I am much more likely to be right than if I believe that his height is between 6ft. 2in. and 6ft. 3in. ... Science is perpetually trying to substitute more precise beliefs for vague ones; this makes it harder for scientific propositions to be true than for the vague beliefs of uneducated persons to be true, but makes scientific truth better worth having if it can be obtained.¹⁸

The scientific "second-gradeness" of vague beliefs or propositions, that makes them a target for elimination in the present philosophical climate (a view shared by some notable philosophers of late), is based upon this confusion between vagueness and lack-of-specificity. Though it is true that I am much more likely to be right if I believe so-and-so is tall (a vague belief) than if I believe that his height is between 6ft. 2in. and 6ft. 3in. (a more precise belief), *it is not true in general that vaguer beliefs or propositions have a better chance of being true than more precise ones* (unless by 'vague' one means 'less specific') as evidenced by my being much more likely to be right if I believe so-and-so to be between 1ft. and 10ft. in height (a relatively precise belief) than if I believe that he is tall (a vaguer belief).

¹⁶ Rosenberg, A., *op. cit.*, p. 298.

¹⁷ *Ibid.*, pp. 299-300.

¹⁸ Russell, *op. cit.*, p. 91.

It is not that more precise propositions are harder to verify than vague ones, but that more specific propositions are harder to verify than less specific ones. Following Rolf¹⁹ I shall say that, for propositions A and B:

A is more precise than B if A's border cases are also B's but not conversely;
furthermore, whenever B is clearly (not) applicable to a situation A is (not).

So precisifications preserve determinate truth and falsity of application of propositions to situations and reduces the number of border cases. Note that A need not entail B since A might determinately apply to a situation where B is only doubtfully applicable (as the above counter-example shows).

What is true in general is that a less specific belief or proposition has a much better chance of being true than a more specific one — as a moment's reflection on the (aforementioned) definition of 'specificity' will reveal. Recall that:

A is more specific than B if A specifies more about the world than B does.

As a consequence A entails B, and this explains why less specific propositions are easier to verify than more specific ones.

Having thus diagnosed a quite common confusion in discussions of vagueness that is also apparent in some of Russell's thinking on the matter, let us move on to his definition and see how it (or at least a coherent interpretation of what he had in mind) captures the notion of vagueness.

3.3.2 *Russell's Definition of Vagueness*

In order to understand his theory of vagueness, a great deal of unravelling and clarification of his "definition" is required. The definition which I shall eventually put forward may seem remote from Russell's; however, my claim is that whilst it charitably interprets his remarks, thus making them more coherent and plausible, it nonetheless remains faithful to the spirit of Russell's views. Thus I see the ensuing reading as constrained by the following two principles of interpretation: Charity:— choose that interpretation which maximises the interest of the text; and Textual Fidelity:— choose that interpretation which most naturally fits the text. Of course these principles may pull against one another and we should aim, as I have done, for reflective equilibrium; there is the presumption that authors will mean what they say yet, if we respect their intelligence, we should hesitate to attribute foolish or trivial doctrines to them.²⁰

Russell presented the following "definition" of vagueness:

¹⁹ Rolf, B., *op. cit.*, p. 76.

²⁰ Paraphrasing Peter Carruthers who cites these principles as guiding his interpretation of Wittgenstein's *Tractatus*. Cf. Carruthers, P., *The Metaphysics of the Tractatus*, CUP (1990), p. xii.

[A] representation is *vague* when the relation of the representing system to the represented system is not one-one, but one-many. For example, a photograph which is so smudged that it *might* equally represent Brown or Jones or Robinson is vague. A small-scale map is usually vaguer than a large-scale map, because it does not show all the turns and twists of the roads, rivers etc., so that various slightly different courses are compatible with the representation that it gives ... Passing from representation in general to the kinds of representation that are specially interesting to the logician, the representing system will consist of words, perceptions, thoughts, or something of the kind, and the would-be one-one relation between the representing system and the represented system will be *meaning*. In an accurate [precise] language, meaning [denotation] would be a one-one relation; no word would have two meanings ... In actual languages, as we have seen, meaning is one-many. [my italics]²¹

Exactly what this definition amounts to, in detail, is far from clear. What's more, it has been little discussed and this, I think, is unfortunate. To some extent I am in agreement with Rolf when he says that there is more to be learnt from Russell's theory than from "those present-day theories which take vagueness only as a pretext for indulging in the game of funny logics". That is not to say that all present-day theories of vagueness that invoke "funny logics" do so *only* to indulge the authors or that *any* such theorising is less informative on the issue than Russell's theory, it is just that his theory is undervalued. This is of particular relevance now as there has been something of an explosion of work on vagueness appearing in the literature in recent times. It will be my contention that, given a proper understanding of Russell's notion of vagueness, one can view all representational theories as either ensuant upon or equivalent to Russell's theory, and thus by arriving at an understanding of his definition and the presuppositions it involves we are better able to understand what is implicit in many current accounts of vagueness.

3.3.2.1 *Unravelling Russell's Definition*

The most thoroughgoing treatment of Russell's account is, to my knowledge, that presented by Bertil Rolf in his 1981 thesis, *Topics On Vagueness*. Therein Rolf presents two interpretations of Russell's definition;

- R1 The representation *r* is *vague* if and only if there are two different entities, *x* and *y*, such that it is logically possible that *r* represents *x* and it is logically possible that *r* represents *y*.

²¹ Russell, *op. cit.*, pp. 89-90.

or

R2 The representation r is *vague* if and only if it is logically possible that there are two different entities, x and y , such that it is logically possible that r represents x and it is logically possible that r represents y .

The distinction between R1 and R2 is just that between intensional and extensional vagueness as described in Chapter One. In the context of the current discussion this distinction is of no import and I shall speak of *an* interpretation.

This interpretation may be thought inadequate as regards its use of mere 'logical possibility'. Representations that are intuitively precise are such that it is *logically* possible that they might represent a whole range of things; mere logical possibility is too extensive. In fact (and this seems to be what Rolf has in mind anyway) the logical possibilities are constrained. As Rolf explains it, the right-hand sides of R1 and R2 are true just when a representation r has certain "essential properties" but that these do not determine whether or not r represents x or y — for two, perhaps non-actual but logically possible, entities x and y . Both alternatives are possible in as much as x and y only differ on some property that is not determined either way by the properties essential to the representation; that is, *both alternatives are logically possible to the extent that they are consistent with the essential properties of the representation*. What then is this notion of an "essential property"?

Let us say that an *essential property* of a representation r is any property constitutive of r itself — qua representation; that is, one that could, in principle, be directly given to or known by an agent to be a property of r on the basis of the representation r itself (which might be a painting or photo, knowledge, beliefs, propositions, or other linguistic items depending on whether the representation is mechanical, cognitive or linguistic). For example, a colour photo, p , showing a blue ball has as one of its essential properties that what is represented is represented as being blue and, though it may, as it happens, in fact be a photo of a green ball this is simply an accidental property of p .

As another example, consider a predicate. Properties essential to a predicate (qua representation) are just the semantic properties of the predicate; for example, that the predicate ' P ' denotes the property α , or applies to some object a , say.

In focussing on the essential properties of a representation we are interested merely in how that which is actually represented is represented as being — something internal to the representation itself. In the above example of the photograph, the ball that is actually photographed is green yet represented as being blue. In this case the represented — the ball — is represented as being a colour which it in fact is not and to this extent the photograph is a *misrepresentation*, however such a feature has no bearing on whether or not the representation is vague — external considerations figure in determining whether or not a representation misrepresents. A representation of x counts as a misrepresentation

just in case it represents x as being something it is not — it is a property of the representation which obtains on the basis of the relation between x and the essential properties of the representation of x ; a representation is vague, on the other hand, by virtue of features internal to the representation itself — the representation's essential properties.

Consider Russell's smudged photo. Its essential properties do not have the distinctness which would entail either that the photo does not represent Brown or that it does not represent Robinson. It is therefore possible relative to the essential properties of the photo that it is a photo of Brown (the essential properties do not rule this out) or of Robinson (the essential properties do not rule this out either). The photo underdetermines its referent and it is this, according to Russell, which constitutes its being vague.

So, in the interpretation of Russell's definition above, x and y are not just logically possible referents of the representation r ; we can be more specific. They are logical possibilities constrained by the fact that they must be consistent with what is already represented as being the case or *logical possibilities relative to the essential properties of r* . Letting Σ_r stand for the set of essential properties of the representation r , this type of possibility can be written as ' \Diamond_{Σ_r} ' (and by means of it we can define $\Box_{\Sigma_r}B =_{df} \sim\Diamond_{\Sigma_r}\sim B$ and $\nabla_{\Sigma_r}B =_{df} \Diamond_{\Sigma_r}B \ \& \ \sim\Box_{\Sigma_r}B$).²²

Returning (finally) then to the Russellian definition of vagueness, x and y are logically possible referents of r subject to the restriction that they be consistent with what is already represented as being the case.

Thus we may now interpret Russell's definition more adequately as follows:

R3 The representation r is (*extensionally*) vague if and only if there are two different entities, x and y , such that it is logically possible relative to Σ_r that r represents x and it is logically possible relative to Σ_r that r represents y .

or

R4 The representation r is (*intensionally*) vague if and only if it is logically possible that there are two different entities, x and y , such that it is logically possible relative to Σ_r that r represents x and it is logically possible relative to Σ_r that r represents y .

Less formally we can say that a representation is vague just if there are, or could be, various different referents compatible with the representation given. On the basis of the representation itself one cannot determine its referent — various possibilities remain open,

²² Obviously 'consistency' is logic relative however, since nothing turns upon the use of different logics here, for simplicity I shall speak of 'consistency' *simpliciter*. Russell of course would use 'consistency' in the sense of 'classical-consistency'. Furthermore, Russell himself was hostile to the idea of modal logics and so would have objected to treating ' \Diamond_{Σ_r} ' as a properly logical notion so we must be careful not to attribute such a view to him. Nonetheless we can make use of these notions — Russell would just have added that, in so doing, we are not engaging in any new logic.

the representation lacking the distinctness which would entail any particular referent as being represented.

Comparing Russell's all too brief "definition" cited at the beginning of §3.3.2 with that given above, I claim that the definition of vagueness given above is faithful to Russell, thereby satisfying the interpretive principle of Textual Fidelity. It spells out in more detail the sense in which Russell took representations to be vague — when the relation of the representing system to the represented system (which, in the case where the representing system is linguistic, will be the relation of *meaning* or *denotation*) is one-many. Moreover, by the time we reach the end of §3.3.2 it will become clear that we have, in the above definition, the essence of an account of representational vagueness and so, in accord with the interpretive principle of Charity, Russell can be seen as the paradigmatic representationalist endorsing these definitions.

3.3.2.2 *The Adequacy of Russell's Definition*

Russell's definition remains problematic for the moment however, even given the above refinement. The problem concerns the logical type of the 'entities' referred to in R3 and R4. The narrow construal of them as items of the lowest logical type, i.e. individuals — items of type 0, causes problems for the definition(s). It is to this problem that I now turn.

Rolf discusses Russell's definition of vagueness as he understands it and concludes that "Russell's definition of vagueness as denotation being one-many is not an attempt to account for ordinary usage of the word 'vague'".²³ Russell has in effect, says Rolf, confused vagueness with lack-of-specificity from the very outset (cf. my comments at the end of §3.3.1) and therein lies the root of the problem whereby he cannot make the requisite distinctions one requires of a definition of vagueness, e.g. that between vagueness and generality. Similarly, Max Black and Marvin Kohl seem to take this line in their discussions of Russell's theory, as will become apparent.

I think, however, that Rolf has only considered one of (at least) two possible interpretations of Russell's confused and conflicting claims (namely — he has assumed the 'entities' to be of type 0) and that, with appropriate repair, Russell's definition can make good those distinctions that seem necessary to an adequate account of vagueness. Thus it is worthwhile considering Rolf's objections and to see how Russell's definition needs filling out in order that it characterise 'vagueness' — at least as traditionally understood.

At this point a methodological remark is called for. I am going to use cross-textual evidence to try to explain alleged difficulties with Russell's account by drawing on remarks made about vagueness at roughly the same time in his *The Analysis of Mind*.²⁴ This might be thought problematic in general, especially when trying to discredit

²³ Rolf, *op. cit.*, p. 19.

²⁴ Russell, B., *The Analysis of Mind*, Allen and Unwin (1921).

someone's theory (e.g., via an *ad hominem* by virtue of inconsistency). However we are trying to *reconstruct* a confused account to make it plausible, so I think the cross-textual usage is warranted. Moreover, one could argue (convincingly) that Russell's theory of vagueness is unchanged across texts.²⁵

So what problems does Rolf see for Russell's definition?

The first thing that Rolf notes about Russell's definition of vagueness (as it stands) is that *it apparently fails to distinguish vagueness from generality*. This charge is echoed by Black when he asserts that "Russell's definition of vagueness as constituted by a one-many relation between symbolising and symbolised systems is held to confuse vagueness with generality" whereas vagueness is a "feature of the boundary of a term's extension and is not constituted by the extension itself".²⁶ And Kohl objects to Russell's view of vague language in part "because it confuses vagueness with generality".²⁷ It is then, it would seem, a fairly common (and, I would assert, misguided) critique of Russell's definition.

What then is generality? Taking that type of representation that is of primary interest in this debate, language, we may say that: a term is *general* (not to be confused with 'ambiguous'²⁸) when it is understood to be applicable to a number of different objects in virtue of some common property, or, alternatively, when it designates or denotes a class that is its extension. It is important to note that, in this sense, a term denotes its extension and not the members of its extension so, though a general term is said to *apply* to a number of different objects, it does not *denote* a number of different objects (the members of its extension, items of type 0) but denotes the class having those objects as members. For example, 'man' is a general term since it denotes the class, mankind, whose members (to which the term is applicable but which the term does not denote) all possess the property of 'masculinity'. It is this crucial and rather elementary distinction, whereby a general term applies to a number of different objects but denotes the class comprising those objects, that paves the way for a suitable interpretation of Russell's definition.

Rolf argues that if *r* is a general word like 'man', it can represent — denote — many different men and would therefore count as vague on Russell's definition. But a general word can, at least *prima facie*, be precise. So "Russell's definition of vagueness cannot by itself distinguish generality from vagueness".²⁹ However, we have just seen how the denotation relation between a general term and its denotatum is one-one. Rolf is confused about the denotative properties of general terms and consequently argues fallaciously.

²⁵ For example, on page 182 he claims that "a memory is "vague" when it is appropriate to many different occurrences", a remark that follows from the definition of vagueness offered in his paper 'Vagueness'.

²⁶ Black, M., 'Vagueness: An Exercise in Logical Analysis', *Philosophy of Science* 4 (1937); reprinted (including a reply to Hempel) in M. Black, *Language and Philosophy*, Cornell University Press (1949). This reference is from the latter, p. 29.

²⁷ Kohl, M., 'Bertrand Russell on Vagueness', *Australian Journal of Philosophy* 47 (1969), pp. 37 ff.

²⁸ Cf. Quine, W.V.O., *Methods of Logic*, 2nd edn RKP (1962), p. 203.

²⁹ Rolf, *op. cit.*, pp. 3-4.

In Rolf's defence, one might be excused for thinking 'man' is "vague" to Russell's way of thinking since Russell himself sometimes seemed confused as to whether general terms were vague. In *The Analysis of Mind* for instance he does say "'I met a man' is vague, since any man would verify it"³⁰ but then goes on to contradict himself by claiming that "[a] vague word is not to be identified with a general word, though in practice the distinction may often be blurred."³¹ Nevertheless, in text preceding his definition of vagueness he makes the following declaration: when we speak of a proposition having a certain degree of vagueness,

there is not one definite fact necessary and sufficient for its truth, but a certain region of possible facts, any one of which would make it true. And this region is itself ill-defined: *we cannot assign to it a definite boundary*. This is the difference between vagueness and generality. A proposition involving a general concept — e.g. "This is a man" — will be verified by a number of facts, such as "This" being Brown or Jones or Robinson. But if "man" were a precise idea, the set of possible facts that would verify "this is a man" would be quite definite. Since, however, the conception "man" is more or less vague, *it is possible to discover prehistoric specimens concerning which there is not, even in theory, a definite answer to the question, "Is this a man?"* As applied to a such specimens, the proposition "this is a man" is neither definitely true nor definitely false. [my italics]³²

So Russell *does* informally distinguish vagueness from generality; lack of determinate or definite boundaries and border cases are invoked to do this, as the above italicised claims show. Now, whether the definition is successful or not depends on its ability to formally incorporate these insights regarding the distinction.

This brings us to a second criticism levelled at Russell's definition by Rolf: *the definition makes no use of the notion of a border case*. As we have just seen though, Russell invokes the notion informally in order that a requisite distinction concerning vagueness and generality can be made; furthermore he explicitly says, in his initial consideration of just what one means when one speaks of vagueness, that common words such as 'red' are vague since "there are shades of colour concerning which we shall we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word 'red', but because it is a word the extent of whose application is essentially doubtful [indeterminate]."³³ So Russell is certainly working with the notion of a border case. My claim will be that it also does in fact figure implicitly in his definition

³⁰ Russell, B., *The Analysis of Mind*, p. 182.

³¹ *Ibid.*, p. 184.

³² Russell, 'Vagueness', *op. cit.*, p. 88.

³³ *Ibid.*, p. 85.

and to this extent is "used". This traditional signature of vagueness — the presence of border cases — is absorbed into Russell's definition and, though transformed in the process, it is nonetheless operative thus enabling Russell's definition to distinguish vagueness from generality.

This is the dialectical position then. Initially Rolf and others argued that Russell's definition of vagueness as one-manyness couldn't make a necessary distinction between vagueness and generality. In the process of rebutting Rolf's second criticism — in showing *that* the definition makes use of border cases — his first criticism will be seen to be not only unwarranted (since based upon a confusion, as described above) but false, since once we understand *how* the existence of border cases relates to the one-manyness of denotation we will see that the definition is able to distinguish vagueness from generality. So how do border cases figure in the definition?

We have already seen that the one-manyness of the representation relation in Russell's definition arises, not as a result of the one vague representation r *actually* representing many referents, but as a result of each of a plurality of distinct referents being *possible relative to the essential (semantic) properties of r — Σ_r* . What I propose is that this modal relation — 'what r might represent given Σ_r ' — is one-many just when reference varies depending whether or not the representation's border case(s) is (are) to be included in or excluded from the range of the representation function. In other words, that characterising feature of vague representations on Russell's account has its source in exactly that which characterises representations as vague on the traditional account — namely, border cases. The only way in which his definition differs from the more common account in terms of border cases is that, as I said earlier, his definition incorporates the concept of border cases into his specific (representational) theory of vagueness; it thereby becomes subsumed into the more specific account and no longer features explicitly.

How then are border cases responsible for the one-manyness of the representation relation and *vice versa*? This question is best answered, I think, by contrasting the traditional conception of vagueness, border case (semantic) vagueness, with another notion employed by Russell — "accuracy". In the preamble to his definition of vagueness he says the following:

One system of terms related in various ways is an *accurate* representation of another system of terms related in various other ways if there is a one-one relation of the terms of the one to the terms of the other, and likewise a one-one relation of the relations of the one to the relations of the other, such that, when two or more terms in the one system have a relation belonging to that system, the corresponding terms of the other system have the corresponding

relation belonging to the other system. Maps, charts, photographs, catalogues, etc., all come within this definition in so far as they are accurate. [my italics]³⁴

In other words, one structured system is an accurate representation of another if there is a one-one relation correlating elements of one to elements of the other by means of which the former can be interpreted as a model of the latter. When such a structure-preserving correlation obtains between a system A and another system B, A is said to be isomorphic to B.

Consider, for example, a map which consists of various marks on paper related in various ways, a legend saying what each type of mark represents, a scale, contours, and so on. If this map is accurate or isomorphic to that which it purports to represent then each mark correlates one-one with (denotes, represents) something in the region mapped, and the relations that obtain on the map, when interpreted and scaled, correspond to relations that obtain in the region mapped. If for instance a dot designated 'Black Mtn' is 1.5 cms from another dot designated 'ANU' in a direction designated as 'west' on a 1:100,000 map of Canberra then, using the intended or canonical one-one correlation, the map represents the elevation Black Mountain as 1.5kms west of the Australian National University in the city of Canberra. What's more, the map is then accurate in so far as this is the case.

If our geographical world is however not as described then the correlation, though one-one, is not structure-preserving and we may suppose the map to be inaccurate in this regard. The map is a *factually inaccurate* representation of our world because it *misrepresents* our world, nonetheless there is some world it can be taken to accurately represent — namely a world which, were it to exist, would be as the map describes. Though factually inaccurate, the map is isomorphic to some possible world.

Thus, factually inaccurate representations are contrasted with accurate ones. But there is another contrast we can draw. When we come to consider vague natural language descriptions we are, on the representationalist view, forced to concede that such descriptions could never be isomorphic to the world. There is a different type of inaccuracy inherent in vague language — *logical inaccuracy*.

Were vague language ever isomorphic to the world, say, then the world would have to exhibit that feature analogous to vagueness yet this, according to representationalists (and, I shall argue, Russell in particular) is impossible. As a matter of necessity, that represented by language is never vague or fuzzy so any possible candidate referent for a vague term will fail to share that logical property analogous to vagueness — fuzziness; any candidate for reference, anything in the range of the denotation function, is sharp. So vague descriptions can never accurately represent precise or sharp referents and hence don't stand in a one-one relation to that represented.

34 *Ibid.*, p. 89.

Russell's smudged-photo example is an apt illustration of this inaccuracy and the paradigm for vagueness, in particular for vagueness in language. "[A]ll vagueness in language and thought is essentially analogous to this vagueness."³⁵ Being smudged is, like vagueness on Russell's view, a property a photograph possesses by virtue of the nature of its relation to that photographed. The indistinctness in the photograph amounts to the necessary inaccuracy of the representation rather than the (impossible) indeterminacy of the represented. Just as the represented is never smudged in itself, so too the represented can never be vague, nor anything analogous, in itself. Smudged or vague representations are inaccurate in a way that mere factually inaccurate representations are not.

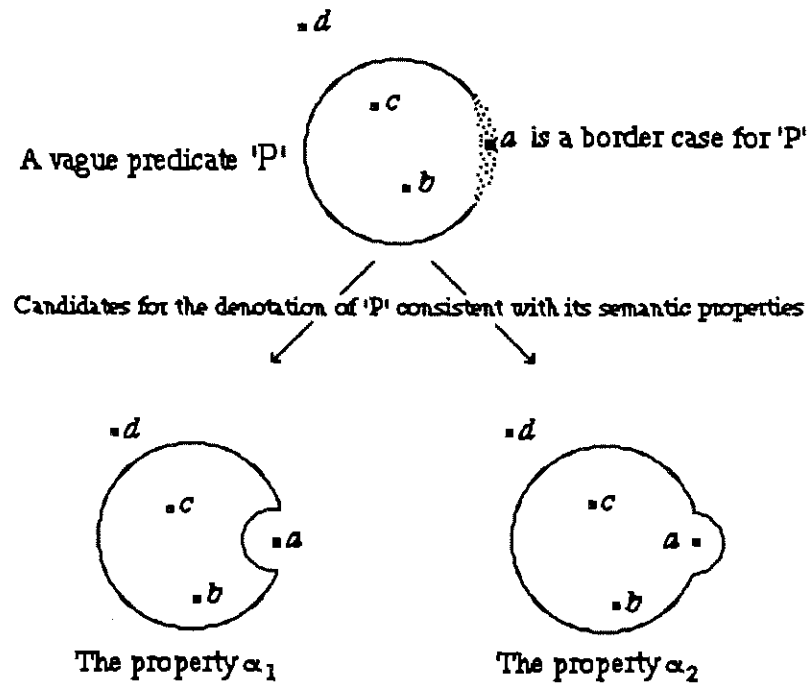
Using that example of vagueness where our intuitions seem strongest — the case of a vague predicate 'P' — we can show, more specifically and firstly, that representational vagueness results in one-manyness. One-manyness is a necessary condition for representational vagueness.

To say that the predicate is vague as traditionally understood is to say that there is some border case for 'P', *a* say. Suppose further that this is the predicate's sole border case, with *d* being such that 'P' determinately does not apply, and objects *b* and *c* being such that the predicate determinately does apply. So what property might 'P' denote? Since its denotation must be sharp there are *two* candidates consistent with its semantic properties, that is, consistent with the fact that *b* and *c* must determinately possess the property denoted by 'P' whilst *d* must determinately not possess the property denoted by 'P'. Either the border case *a* determinately possesses the property denoted by 'P' or it determinately does not. The predicate's border case must be resolved at the ontological level, one way or the other.³⁶

³⁵ *Ibid.*, p. 91.

³⁶ Thus, for the sake of the argument, we are simply assuming that property-sharpness is the ontological analogue of predicate-precision — there are no border cases for the property.

Again, I am setting aside the distinction between resilient and non-resilient border cases for the moment (cf. n. 2, this chapter). So, having already admitted earlier that our current concerns with vagueness are really concerns with the traditional, naïve conception of vagueness, we now note that talk of fuzziness, in so far as it is simply an analogue of vagueness, is really talk of a naïve conception of fuzziness. More sophistication will be added in due course, but to do so now would simply confuse things.



The vague predicate's denotation might consistently be said to be the property α_1 or the property α_2 . Moreover, this one-manyness regarding what the predicate might represent is one-manyness about whether or not to include the border case in the property denoted. Since the vagueness in the representation has no analogue in what is represented the vagueness can (must) be resolved one of the two ways described, each being consistent with the essential properties of the representation (in this case, the semantic properties of 'P') though neither being necessary.

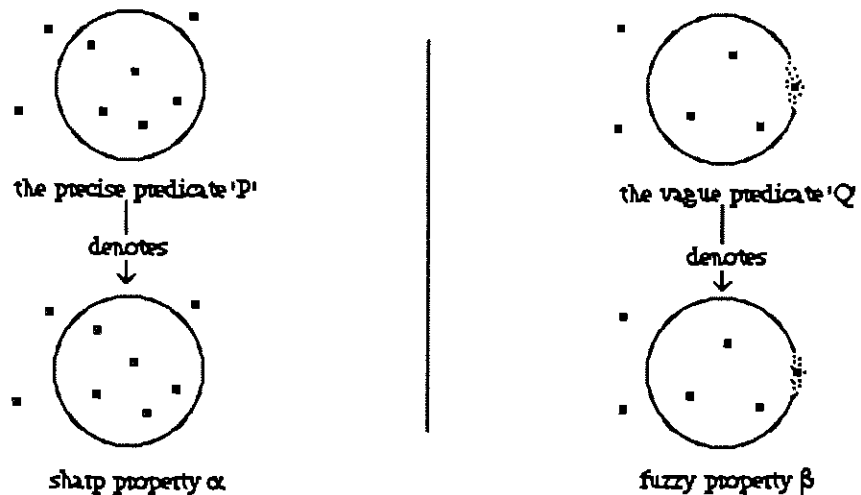
In the case of predicate vagueness, then, we can see that not only does the presence of *some* border case give rise to *some* one-manyness, thereby contrasting vagueness with accuracy; the very border case *itself* is the source of the referential indeterminacy.

Secondly, with predicates at least, we can do more than merely contrast vagueness with accuracy by showing that border cases give rise to one-manyness — the contrasting notion instantiated, one-manyness, is instantiated *solely* by terms that are border case vague. That is to say, we can do more than merely show that vagueness is contrasted with accuracy by showing that one-manyness is a necessary condition for vagueness — we can also show that one-manyness is sufficient for vagueness.

The simplest way to show this is to establish that precise predicates can stand in a one-one correlation with a sharp property, so precision is sufficient for one-oneness. A one-one correlation (that will also be factually accurate) between a predicate 'P' and a sharp property can be obtained by simply taking the property, α , to be that which all objects describable as 'P' have in common; hence the set of objects having the property α (this set often being identified with the property α itself) is the set of objects satisfying 'P'. So by definition the property α is correlated one-one with the predicate and α is sharp since the

fact that everything either determinately satisfies the predicate or determinately doesn't entails that all objects instantiate α determinately or determinately not — there are no border cases for α . (See diagram, below left.)

So, conjoining the two claims arrived at so far, one-manyness of denotation is both necessary *and* sufficient for traditional predicate-vagueness on the assumption that the represented is sharp; hence, Russell's definition is a definition of traditional predicate-vagueness *if* the represented is sharp.



On Russell's account the above mentioned one-one correlation is not possible with vague predicates because they could only ever be so correlated with a fuzzy property and there are none according to his view. However, if there could be fuzzy properties then the same one-one correlation could then be employed. (See diagram, above right.) In other words, if the represented could be fuzzy then not all vagueness need result in one-manyness; a vague predicate 'Q' could accurately represent or denote some fuzzy property β , say. So we can further add that Russell's definition is a definition of traditional predicate-vagueness *only if* the represented is sharp.

Extending the picture beyond mere predicate-vagueness to the vagueness of denoting phrases (predicates, singular terms and sentences) in general, the following claims begin to seem plausible: (i) If the represented is sharp then all traditional vagueness of denoting phrases results in one-manyness and *vice versa*; and (ii) if all traditional vagueness of denoting phrases results in one-manyness and *vice versa* then the represented is sharp. So, conjoining these two claims, it would follow that:

CONJECTURE: Russell's definition of vagueness and the traditional definition of vagueness for denoting phrases are equivalent if and only if the represented is sharp.

If this conjecture is correct (as I think it is) then Russell has successfully defined traditional vagueness for all and only those denoting phrases where vagueness is semantic but not ontological; that is, he has given a definition of traditional *representational vagueness* for denoting phrases.

If, furthermore, the definition can be amended to avoid that traditional aspect of vagueness whereby some intuitively precise predicates are counted as vague by virtue of their possessing border cases (discussed in Chapter One)³⁷ and if it can also be extended beyond merely denoting phrases then Russell's definition provides the foundation for *any* adequate representational account of vagueness.

I think all these conditions can be met. This is a far cry from charges of confusion often levelled at the Russell account; with a few minor adjustments to avoid some counter-intuitive consequences of vagueness as traditionally conceived (which, it should be said, most theories stand in equal need of) Russell has gotten the semantic analysis of representational vagueness exactly right.

Actually proving such a result, though satisfying in its generality, would, I think, be unnecessarily complex. I shall content myself with proving Russell's definition as an adequate definition of traditional representational vagueness for predicates. A subsequent sketch of how to extend the result to vague denoting phrases in general ought to be convincing enough grounds for accepting the Conjecture. Then, with the addition of the recursive extension of §1.4, Russell's definition can be used to generalise to a definition of representational vagueness *simpliciter*.

So, let's begin with the restricted proof for the special case of traditional predicate-vagueness. That is, let's firstly prove:

PRELIMINARY CONJECTURE: Russell's definition of predicate-vagueness and the traditional definition of predicate-vagueness are equivalent if and only if properties are sharp.

A Digression: Formulating The Notion Of 'Sharpness'

In order to appreciate the proof in support of this restricted claim, some formulation is required of what it is for properties to be 'sharp'. Of course this is not easily done without opening up a veritable Pandora's Box. Any account of 'sharpness' will, by implication, commit one, at least partially, to a theory of ontological vagueness (as coherent thus-and-so, or as defective thus-and-so, or ...) and as such is bound to be controversial — there being few, if any, agreed principles in this area. However, some understanding is currently required of what it is for properties to be sharp. It is this task to which we now turn.

³⁷ Cf. n. 2, this chapter.

At present all that is meant by the ascription of sharpness is the *ontological* analogue of the ascription of precision. So, taking the case of properties, to say that a property is sharp is just to ascribe to the property that ontological analogue of ascriptions of precision to predicates; that is (ignoring problems to do with resilience for the moment — cf. n. 2), a property is sharp if and only if it has no border cases.

Let us characterise the sharpness of properties then as follows: for any property ψ and any object x , either x determinately instantiates ψ or determinately does not.³⁸ Call this principle P(ropery) S(harpness).

$$(PS) \quad \forall \psi \forall x (D\psi x \vee D\sim\psi x)$$

Of course, for any property ψ and object x , if it is determinately the case that ψx , then ψx .

Call this principle D.

$$(D) \quad \forall \psi \forall x (D\psi x \rightarrow \psi x)$$

Given D, PS is equivalent³⁹ to the claim that every property is instantiated determinately if at all. I shall refer to this as PS*.

$$(PS^*) \quad \forall \psi \forall x (\psi x \rightarrow D\psi x)$$

So, if properties are sharp then principles D and PS* jointly hold and they are jointly equivalent to its being the case that a property is instantiated *if and only if* it is instantiated determinately. Call this principle DPS*:

$$(DPS^*) \quad \forall \psi \forall x (\psi x \leftrightarrow D\psi x).$$

38 The variable ' ψ ' is taken to range over properties.

39 The equivalence between PS and PS* can be established as follows.

Assume PS: every property is either determinately instantiated by an object or determinately not. Now assume some arbitrary property α is instantiated by a . Then it is not the case that a determinately does not instantiate α (given consistency and D); so, by PS, a must determinately instantiate α . And this is universally generalisable.

I.e.:

(1) αa	for arbitrary α and a .
(2) $\sim D\sim\alpha a$	D, instantiated with ' $\sim\alpha a$ ', (1) and Modus Tollens.
(3) $D\alpha a$	PS, instant. with ' αa ', (2) and Disj. Syll.
(4) $\alpha a \rightarrow D\alpha a$	' \rightarrow ' intro., discharging (1).
(5) $\forall \psi \forall x (\psi x \rightarrow D\psi x)$	(4) and univ. generalisation.

Conversely, to show that every property is instantiated by every object either determinately or determinately not, let's assume PS*: that every property is instantiated determinately if at all. In particular, assume that for some property α that, if α is instantiated by anything then it is determinately instantiated by that thing; furthermore assume that α is not determinately instantiated by a . But then it is not instantiated by a at all; i.e., $\sim\alpha a$ is the case. But then, by PS* again, it ($\sim\alpha$) must be determinately instantiated by a . In other words, given PS*, if α is not determinately instantiated by a then it is determinately not instantiated by a , or, if you prefer, either α is determinately instantiated or it is determinately not. And this is universally generalisable since both α and a were arbitrary.

I.e.:

(1) $\sim D\alpha a$	for arbitrary α and a .
(2) $\sim\alpha a$	PS*, instantiated with ' αa ', (1) and Modus Tollens.
(3) $D\sim\alpha a$	PS* instantiated with ' $\sim\alpha a$ ', (2) and Modus Ponens.
(4) $\sim D\alpha a \rightarrow D\sim\alpha a$	' \rightarrow ' intro., discharging (1).
(5) $D\alpha a \vee D\sim\alpha a$	(4) and interdefinability of ' \vee ' and ' \rightarrow '.
(6) $\forall \psi \forall x (D\psi x \vee D\sim\psi x)$	(5) and univ. generalisation.

QED.

Which is just to say that with regard to property instantiation we have a 'D'-Redundancy Thesis; though, of course, with regard to properties-as-represented we don't have such a thesis — that predicates are vague is one of the reasons why the 'D' operator was invoked in the first place.

The extension of the concept of 'sharpness' to other ontological categories will simply mirror the extension of 'vagueness' as described in Chapter One.

Having digressed to formulate the sharpness of properties, we are ready to attempt a proof of the Preliminary Conjecture above.

The proof will contain two sub-proofs: (I) If properties are sharp then a predicate is vague on the traditional conception if and only if it is vague on the Russellian conception; and (II) If every predicate that is vague on the traditional conception is vague on the Russellian conception and *vice versa* then properties are sharp.

(I) If properties are sharp then a predicate is vague on the traditional conception if and only if it is vague on the Russellian conception.

Informally we can argue as follows. Assume a predicate, \underline{P} say, is vague as traditionally understood $[\exists x \underline{P}x]$.⁴⁰ That is, there is some (possible) border case, a say, such that it is neither determinately the case that a satisfies \underline{P} $[\sim D \underline{P}a]$ nor determinately not the case that a satisfies \underline{P} $[\sim D \sim \underline{P}a]$. This indeterminacy is an intrinsic semantic feature of the predicate \underline{P} so it follows that the semantic properties of the predicate do not entail that a determinately instantiates the property denoted by \underline{P} $[\sim \square_{\Sigma \underline{P}} D(a \text{ i } \underline{\psi}(\underline{\psi} = \text{den} \underline{P}))]$, which I shall shorten to: $\sim \square_{\Sigma \underline{P}} D \underline{P}a$ nor whether it determinately does not instantiate the property denoted by \underline{P} $[\sim \square_{\Sigma \underline{P}} D \sim(a \text{ i } \underline{\psi}(\underline{\psi} = \text{den} \underline{P}))]$, which I shall shorten to: $\sim \square_{\Sigma \underline{P}} D \sim \underline{P}a$. Yet, by sharpness (DPS*), any property which \underline{P} actually denotes is determinately instantiated by a if and only if it is instantiated by a *simpliciter* $[D(a \text{ i } \underline{\psi}(\underline{\psi} = \text{den} \underline{P})) \equiv a \text{ i } \underline{\psi}(\underline{\psi} = \text{den} \underline{P})]$, i.e. $\underline{P}a \equiv D \underline{P}a$. So, it can't follow from the semantic properties of \underline{P} that \underline{P} denotes a property instantiated by a $[\sim \square_{\Sigma \underline{P}} \underline{P}a]$ and consequently \underline{P} might be consistently said to denote a property not instantiated by a $[\diamond_{\Sigma \underline{P}} \sim \underline{P}a]$. Similarly, it might denote a property that is instantiated by a $[\diamond_{\Sigma \underline{P}} \underline{P}a]$. The semantic properties of the predicate leave it undetermined. That is to say, it is essentially contingent, relative to those properties of the predicate \underline{P} determined by its semantic properties, whether the denotation

⁴⁰ I shall proceed using a one-place predicate. The result for any n -place predicate is an obvious generalisation. Furthermore, for formatting reasons, in this section of the thesis I shall adopt the convention whereby predicates are written underlined when named (e.g. \underline{P}) and not when used (e.g. P); when particular properties are named I shall continue to use Greek letters (e.g. α , β , ...).

of \underline{P} is instantiated by a or not $[\forall \Sigma_P Pa]$. In this sense the denotation of \underline{P} is one-many — that is, not unique — and thus satisfies Russell's definition.

Conversely, if the denotation of \underline{P} is one-many in the above sense — for all that is determined by its semantic properties it might denote the property instantiated by a yet it might not $[\diamond \Sigma_P Pa \ \& \ \diamond \Sigma_P \sim Pa]$ — then the semantic properties of \underline{P} could never warrant our claiming a as a determinate satisfier of \underline{P} nor as a determinate non-satisfier of \underline{P} $[\sim \square \Sigma_P DP a \ \& \ \sim \square \Sigma_P D \sim Pa]$. And so, argues the representationalist, it is essentially semantically indeterminate whether a satisfies \underline{P} and a is therefore a border case for the predicate \underline{P} . QED.

Given that DPS* holds, then the formal equivalence between the traditional conception of vagueness and Russell's definition can be established — that is, we can establish

$$(DPS^*) \vdash (\exists x \underline{I}Px \leftrightarrow \exists x \forall \Sigma_P Px)$$

— as follows:

The predicate \underline{P} is vague as traditionally understood iff $\exists x \underline{I}Px$.

Assume \underline{P} 's border case is a ;

that is, $\underline{I}Pa$.

iff	$\sim DP a \ \& \ \sim D \sim Pa$	(by definition of 'I')
iff	$\sim \square \Sigma_P DP a \ \& \ \sim \square \Sigma_P D \sim Pa$	(by meaning of ' $\square \Sigma_P$ ')
iff	$\diamond \Sigma_P \sim DP a \ \& \ \diamond \Sigma_P \sim D \sim Pa$	(by definition of ' $\diamond \Sigma_P$ ')
iff	$\diamond \Sigma_P \sim Pa \ \& \ \diamond \Sigma_P Pa$	(by DPS*)
iff	$\forall \Sigma_P Pa$	(by definition of ' $\forall \Sigma_P$ ').
So,	$\forall x (\underline{I}Px \text{ iff } \forall \Sigma_P Px)$	(by universal generalisation).

I shall refer to this claim as '(RUSS)'.⁴¹ (RUSS) says effectively that a predicate \underline{P} is vague as traditionally understood if and only if, for all that we could know, it might include

41 The equivalence (RUSS), $\forall x (\underline{I}Px \text{ iff } \forall \Sigma_P Px)$, can also be expressed using the variant:

$$\forall x (DPx \text{ iff } \square \Sigma_P Px).$$

That this latter formulation is a variant of the former is proven as follows:

(i)	Assume	(1)	$DP a$.	
	Hence	(2)	$\sim \underline{I}Pa$	(by definition and 1).
	Hence	(3)	$\square \Sigma_P \sim Pa \vee \square \Sigma_P Pa$	(by (RUSS) and 2).
	Yet	(4)	$\underline{P}a$	(by 1).
	And assuming	(5)	$\square \Sigma_P \sim Pa$	
	then	(6)	$\sim \underline{P}a$	(by 5).
	So	(7)	$\sim \square \Sigma_P \sim Pa$	(by <i>reductio</i> on 4 and 5).
	Hence	(8)	$\square \Sigma_P Pa$	(by Disj. Syll., 3 and 7).
	Hence	(9)	$DP a \rightarrow \square \Sigma_P Pa$	(by ' \rightarrow Intro', 1 and 8).
	Hence		$\forall x (DPx \rightarrow \square \Sigma_P Px)$	(by ' \forall Intro' on 9).
(ii)	Assume	(1)	$\square \Sigma_P Pa$	
	Hence	(2)	$\sim \forall \Sigma_P Pa$	(by definition and 1).
	Hence	(3)	$D \sim Pa \vee DP a$	(by (RUSS) and 2).

its border case in its extension and it might not; that is, \underline{P} might denote the property instantiated by a and it might denote some other property not instantiated by a . In this sense its denotation relation is "one-many".

(RUSS) entails $\exists x I \underline{P}x$ iff $\exists x \nabla \Sigma_P Px$.

And so, given the sharpness of properties, we can establish the slightly weaker claim that any predicate vague on the traditional conception is vague according to Russell's definition. It is weaker because unlike (RUSS) it does not assert that the border case is *the* case giving rise to semantic contingency; merely that if \underline{P} has some border case then it has some case giving rise to semantic contingency.

Having presented the proof, it is worth noting that the following sub-proof is valid:

(D) $\vdash \exists x \nabla \Sigma_P Px \rightarrow \exists x I \underline{P}x$.

This holds independently of the truth of DPS*. A proof of the right-hand-side only requires the conditional ' $\forall \psi \forall x (D\psi x \rightarrow \psi x)$ ' — something's determinately instantiating a property implies its instantiating the property *simpliciter*; that is, principle D. (DPS*) is an unnecessary premise.

In other words, whilst epistemic and representational theorists are committed to claiming *all* semantic vagueness as instancing one-manyness of denotation (vacuously so for strong epistemic theorists), every theorist is committed to instances of Russellian one-manyness being instances of traditional predicate-vagueness. Hence any ontological account of vagueness, whilst claiming that there can be cases of semantic vagueness without one-manyness of denotation, must admit instances of Russellian vagueness (if there are any) as cases of traditional vagueness proper and advocates of a strong epistemic account, since they deny that there is any such thing as traditional (semantic) vagueness, are compelled to deny that there are any instances of Russellian vagueness.

(II) If every predicate that is vague on the traditional conception is vague on the Russellian conception and *vice versa* then properties are sharp.

Yet	(4)	$\underline{P}a$	(by 1).
And assuming	(5)	$D \sim \underline{P}a$	
then	(6)	$\sim \underline{P}a$	(by 5).
So	(7)	$\sim D \sim \underline{P}a$	(by <i>reductio</i> on 4 and 5).
Hence	(8)	$D \underline{P}a$	(by Disj. Syll., 3 and 7).
Hence	(9)	$\Box_{\Sigma_P} \underline{P}a \rightarrow D \underline{P}a$	(by ' \rightarrow Intro', 1 and 8).
Hence		$\forall x (\Box_{\Sigma_P} Px \rightarrow D \underline{P}x)$	(by ' \forall Intro' on 9).

Combining (i) and (ii) then: $\forall x (D \underline{P}x \leftrightarrow \Box_{\Sigma_P} Px)$. \blacksquare

To show this I shall argue that if a property were fuzzy then any predicate uniquely or "accurately" denoting the property would be vague, in which case not all predicate vagueness need result in one-manyness of denotation.

Assume then that some property, α say, is fuzzy $[\exists x(\sim D\alpha x \ \& \ \sim D\sim\alpha x)]$ with border case a say. Then it is neither determinately the case that a instantiates the property α nor determinately not the case that a instantiates α $[\sim D\alpha a \ \& \ \sim D\sim\alpha a]$. Assume further that \underline{P} 's semantic properties are such as to make it the case that it accurately denotes α $[\Box_{\Sigma_P}(\iota\psi(\psi = \text{den}\underline{P}) = \alpha)]$, which I shall shorten to: $\Box_{\Sigma_P}(P = \alpha)$. Now if \underline{P} 's semantic properties are such that it accurately denotes α then (by # — see below) it follows from \underline{P} 's semantic properties that it denote something determinately (not) instantiated by a if and only if α is determinately (not) instantiated by a $[\Box_{\Sigma_P}D(a \ \iota\psi(\psi = \text{den}\underline{P})) \equiv D\alpha a]$, and $\Box_{\Sigma_P}D\sim(a \ \iota\psi(\psi = \text{den}\underline{P})) \equiv D\sim\alpha a$, which I shall shorten to: $\Box_{\Sigma_P}DPa \equiv D\alpha a$, and $\Box_{\Sigma_P}D\sim Pa \equiv D\sim\alpha a$. Since α is neither determinately nor determinately not instantiated by a then it neither follows from \underline{P} 's semantic properties that it denote something determinately instantiated by a $[\sim\Box_{\Sigma_P}DPa]$ nor that it denote something determinately not instantiated by a $[\sim\Box_{\Sigma_P}D\sim Pa]$. But this is just to say that \underline{P} neither determinately applies to a nor determinately doesn't $[\sim DP a \ \& \ \sim D\sim Pa]$. So a is a border case for \underline{P} and \underline{P} is vague (as traditionally understood). So, if Russellian predicate-vagueness is equivalent to traditional predicate-vagueness then properties must be sharp. QED.

(#) is a special case of a substitution schema which says that if the semantic properties of a representation r determine that what is represented is A — if r is an accurate representation of A — then, for a range of contexts ϕ , the semantic properties of the representation r determine that what is represented ϕ 's if and only if A ϕ 's.

$$\Box_{\Sigma_r}(\iota\psi(\psi = \text{den}(r)) = A) / \Box_{\Sigma_r}\phi(\iota\psi(\psi = \text{den}(r))) \equiv \phi(A), \text{ for a range of contexts } \phi.$$

As an example, consider a map of Canberra. If it follows from the map that Canberra has an area of n^2 kms then it follows from the map that Canberra has a large area if and only if n^2 is a large area. (Of course, if the map is also accurate then Canberra is as described by the map.)

In cases where ' r ' is a predicate \underline{P} and ' A ' is the property α , the substitution schema can be instantiated and, using the usual abbreviations, shortened to:

$$\Box_{\Sigma_P}(P = \alpha) / \phi(\alpha) \equiv \Box_{\Sigma_P}\phi(P), \text{ for a range of contexts } \phi.$$

In the special case where ϕ is of the form ' \cdot is determinately instantiated by a ' or ' \cdot is determinately not instantiated by a ' we can say:

$$\text{\#} \quad \Box_{\Sigma_P}(P = \alpha) / D\alpha a \equiv \Box_{\Sigma_P}DPa \quad \text{and} \quad \Box_{\Sigma_P}(P = \alpha) / D\sim\alpha a \equiv \Box_{\Sigma_P}D\sim Pa.$$

Having said all this we can now argue more formally as follows:

Assume	(1)	$\sim D\alpha a \ \& \ \sim D\sim\alpha a$	(by def'n of fuzziness)
and	(2)	$\Box_{\Sigma_P}(P = \alpha)$	(by def'n of one-oneness).
Then	(3)	$\Box_{\Sigma_P}DPa \equiv D\alpha a$	(by 2 and #).
and	(4)	$\Box_{\Sigma_P}D\sim Pa \equiv D\sim\alpha a$	(by 2 and #).
Yet	(5)	$\sim D\alpha a$	(by 1 and &-elim.).
Hence	(6)	$\sim\Box_{\Sigma_P}DPa$	(by 3 and 5)
and so	(7)	$\sim DPa$	(by 6 and meaning of ' \Box_{Σ_P} ').
Also	(8)	$\sim D\sim\alpha a$	(by 1 and &-elim.).
Hence	(9)	$\sim\Box_{\Sigma_P}D\sim Pa$	(by 4 and 7)
and so	(10)	$\sim D\sim Pa$	(by 9 and meaning of ' \Box_{Σ_P} ').
Therefore	(11)	$\exists xIPx$	(by 7, 10, def'n of 'I' and \exists -intro).
So		If $(\exists\psi\forall_{\Sigma_P}(P = \psi) \text{ iff } \exists xIPx)$ then properties are sharp.	
That is,		If $(\exists x\forall_{\Sigma_P}Px \text{ iff } \exists xIPx)$ then properties are sharp. ⁴²	

Consequently, if Russell is right in characterising predicate-vagueness as he does then properties are sharp. **I**

What I claim then, via (I) and (II), is that the Preliminary Conjecture cited earlier is established — the Russellian conception of predicate-vagueness and the traditional conception of predicate-vagueness are equivalent if and only if properties are sharp.

That is, conjoining (I) and (II), we have the following:

Lemma 3.1 Russell's definition of predicate-vagueness and the traditional definition of predicate-vagueness are equivalent if and only if properties are sharp.

Whether Russell intended to or not, what he has characterised — in this limited case for predicates at least — is representational predicate-vagueness as standardly conceived, traditional predicate-vagueness. Given this fact then, we could say that he was providing an alternative characterisation of predicate-vagueness were we to suppose properties to be sharp — that is, on the assumption that all predicate-vagueness was representational. Thus, charitably interpreted, we should take Russell's remarks concerning the impossibility of vagueness in the world to mean that properties (at least) are not vague or fuzzy; the

⁴² This last move is warranted by the following equivalence: there is some property ψ such that it is contingent relative to Σ_P whether or not the denotation of the predicate P is ψ if and only if there is some object x (instantiating the property ψ) such that it is contingent relative to Σ_P whether or not the denotation of the predicate P is a property instantiated by x .

It is simply the analogue of the equivalence between the existence of a property which counts as a border case for the denotation of the predicate and the existence of a border case for the predicate.

complete agnosticism of §3.2 is not a coherent option, and Russell is a representationalist about traditional predicate-vagueness at least.

So can we generalise on this result to establish the Conjecture proper and in the process show both: (i) that a charitable interpretation of Russell has him committed to the absence of *any* vagueness or fuzziness in the world — thereby suggesting that he was not agnostic at all about ontological vagueness or fuzziness and truly intended a representationalist account of vagueness; and (ii) more strongly, that all and only those so committed should endorse Russell's definition as providing necessary and sufficient conditions for vagueness *simpliciter*? In other words, can we show that Russell's definition can be extended to provide a definition of representational vagueness in general? I think we can but, as I said earlier, I shan't prove the extended result; rather I shall sketch how such a proof might proceed.

The first point we need to address, prior to extending beyond the category of predicates, is whether Russell's definition can be modified to provide necessary and sufficient conditions for predicate-vagueness *simpliciter* as opposed to traditional predicate-vagueness.⁴³ The point of difference, remember, was that traditional vagueness is evidenced by the presence of border cases rather than the presence of *resilient* border cases required for vagueness proper. So can the above proofs, (I) and (II), be reworked using resilient border cases as opposed to the simple border cases employed therein? Of course they can. What it amounts to is our claiming that, when we cite something as a border case in the proofs (*a* say, as we did), we must now suppose there to be no vagueness in identifying what counts as that thing. This caveat is then carried on through the proofs. Nothing in the proofs depended on the border case being non-resilient, so resilience can be added without jeopardising any steps in the proofs.

In other words, Russell's account of what it is for a predicate to be vague, though it falls foul of the same objection that besets most accounts of predicate-vagueness (not distinguishing resilient from non-resilient border cases), can take on board the required distinction — the one-manyness of reference is one-manyness about possible candidates which are themselves precisely individuated.

Having added the required caveats to Russell's definition so that it captures predicate-vagueness as characterised in Chapter One, it remains to be seen whether this characterisation of predicate-vagueness can be extended to any grammatical category in general. The obvious way to proceed with such an investigation is to see if the generalisation can be had by mimicking the extension used in Chapter One when we moved from predicate-vagueness to vagueness in general.

⁴³ Remember that when we speak of vagueness *simpliciter* we intend this to be understood as semantic vagueness, having put aside epistemic analyses in Chapter Two.

The first and crucial extension was that from predicate-vagueness to vagueness of denoting phrases — predicates, singular terms and sentences. The unifying aspect of all these categories of vagueness is vagueness of extension, which Russell's conception of vagueness is easily able to capture. Though I proceeded with an intuitive description, then an argument and proof for predicate-vagueness only, Russell's account dealt more generally with vagueness of denoting phrases. It is not hard to see that the equivalence between the presence of border cases and the underdetermination of a referent is quite general.

In the discussion (with diagrams) on pages 79-81 we saw how a border case object for the predicate 'P' gave rise to sharp properties α_1 and α_2 which could be said to be border cases for the denotation of the relevant predicate. Assuming the necessary sharpness of properties, the presence of border cases for the denotation of 'P' could be shown to be equivalent to the underdetermination of any particular property as *the* referent by the semantic properties of the predicate. So too, we can see that a border case for the denotation of a singular term 'N' is equivalent to the underdetermination of any particular sharp object by the semantic properties of 'N'.

The diagrams and surrounding discussion of pages 79-81 serve as the key to this insight. By redescribing the predicate-extension of the first diagram as the extension of the relevant vague singular term we see that the fact that some spatio-temporal point *a* is neither determinately in the extension of 'N' nor determinately not in the extension of 'N' means that there are two possible sharp candidates for the denotation of 'N' consistent with 'N's determinate semantic properties (that is, consistent with *b*'s being a determinate part of the extension of 'N', *d*'s being a determinate non-part, and so on).

Moreover, redescribing the second diagram, if the presence of border points in the denotation of 'N' is tantamount to there being a plurality of possible referents then objects must be sharp since were fuzzy objects logically possible then it would be logically possible that a vague phrase 'N' determinately denotes a fuzzy object having as an indeterminate part that spatio-temporal point that was neither determinately nor determinately not in the extension of 'N'. Indeterminacy in extension makes for a plurality of possible sharp referents and a plurality of sharp referents entails there being some indeterminacy of extension.

A similar reworking of the discussion on pages 79-81 can be given for sentences. Now we can redescribe the extension of 'P' as the extension of the sentence 'S' consisting of the set of possible worlds where 'S' determinately obtains (in substituting talk of states-of-affairs with talk of a set of possible worlds I am following the common convention whereby a state-of-affairs is equated with the set of possible worlds where that state-of-affairs obtains). If 'S' is vague then there could be worlds where it is neither determinately true nor determinately false. That is, there is some possible world, *a*, such that it is indeterminate whether or not it is in the extension of 'S', the set of worlds where 'S'

determinately obtains. The extension of 'S' has a border case, world a . Yet if states-of-affairs are sharp then, given the equivalence between a state-of-affairs and the set of worlds wherein it obtains, any set of possible worlds that is a candidate for the extension of 'S' is sharp; that is, every world, a included, is either a determinate member of the set or a determinate non-member. Hence there are two possible candidates for the extension of 'S' — the set determinately including world a , or the set determinately excluding a .

Moreover, redescribing the second diagram, if the presence of border cases for the denotation of 'S' is tantamount to there being a plurality of possible extensions (sets of worlds) then states-of-affairs must be sharp since were fuzzy states logically possible then it would be logically possible that a vague phrase 'S' determinately denotes a fuzzy state-of-affairs, having as its extension an indeterminate set of worlds that neither determinately included world a nor determinately excluded it.

In this way I think it can be shown that Russell's account of the (semantic) vagueness of denoting phrases gets it exactly right (given fine-tuning to take account of resilience) if and only if properties, objects, and states-of-affairs — that is, the world — are sharp. The definitions R3 and R4 of §3.3.2.1 adequately characterise extensional and intensional representational vagueness of denoting phrases. That is,

Theorem 3.1 Russell's definition of vagueness adequately characterises the (semantic) vagueness of denoting phrases if and only if the world is sharp.

Now given the claimed success of Russell's characterisation of the representational vagueness of denoting phrases let's see how it can be further extended to characterise the representational vagueness of any grammatical category; that is, how it can be extended to characterise the representational vagueness of language in general.

Russell's definition only characterises the representational vagueness of *denoting* phrases — predicates, singular terms and sentences. He relied on an infection theory of vagueness to extend to talk of vagueness for non-denoting phrases like the logical constants. In thus extending via a theory which we saw to be fallacious in Chapter One, §1.4, his generalised account (implicit in his article 'Vagueness') is misguided. However, supposing that we are correct in claiming Theorem 3.1 as established, then to generalise we need only make use of the (supposedly correct) recursive extension described in §1.4 and encapsulated in DEF (1) and DEF (2) of §1.4. If Russell has gotten things right for the basic cases (of denotational vagueness) then, since the recursive extension will preserve this adequacy, extending via DEF (1) and DEF (2) will result in a characterisation of

vagueness in language appropriate for all and only those cases of representational vagueness.⁴⁴

Let us pause to consider what has been said in this section, §3.3.2. It has been suggested by a number of philosophers that Russell's definition of vagueness is defective for a variety of reasons. I believe they are mistaken to the extent that, though Russell himself may perhaps have had something else in mind, the reconstruction above is consistent with the spirit of his proposal for denoting phrases and preserves the essential ingredient of "one-manyness". Moreover, far from being an inadequate definition of vagueness, his definition (or, if you prefer, my detailed interpretation of his definition) is both sufficient and necessary to the task of characterising representational vagueness when it is conjoined with an appropriate theory extending the concept of vagueness from denoting phrases to any grammatical category in general.

Thus Rolf's criticism (cited at the beginning of §3.3.2.2) that Russell's definition of vagueness in terms of the one-manyness of denotation cannot account for ordinary usage of the word 'vague' is seen to be false. Russell has accounted for the most popular use of 'vague', the representationalist's use, if not the most general and theory-free use.

3.3.3 *Vagueness and the World*

The developed Russellian account of vagueness *per se* depends upon an account whereby the vagueness of representations is tantamount to their logical inaccuracy in relation to that which is being represented. With regard to the central issue of this thesis, vagueness in natural language, the vagueness of denotative linguistic items (e.g., predicates) arises, for Russell, as a result of the logically inaccurate representation of that aspect of the world (e.g., properties) that we, the linguistic community in general, purport to represent when we employ that linguistic item in speaking about that aspect. It originates from the relation between words and the world about which these words convey certain information.

This relation is itself mediated by another class of representations, epistemic ones. It is the vagueness of these epistemic representations — in other words, the vagueness of knowledge — that "infects" the linguistic representations which express this knowledge of ours.⁴⁵ Vagueness is a characteristic of the relation between a cognising subject and the

44 We have only discussed Russell's theory of vagueness for linguistic items and language. But I see no reason to think that we could not extend his account as analysed above to include all forms of representation, linguistic or otherwise and thus finally arrive at a complete explanation of his theory of the vagueness of representations in general. This is not a task I shall pursue in this thesis.

45 Thus, for Russell, epistemic indeterminacy as described by the epistemic theorist of Chapter Two is real enough and in fact is *the* source of vagueness. However, contrary to the epistemic theorist, it gives rise to semantic indeterminacy.

external world, or as Russell expresses it, "[v]agueness in a cognitive occurrence is a characteristic of its relation to that which is known, not a characteristic of the occurrence in itself."⁴⁶ That feature of the relation giving rise to this characteristic is its being one-many in nature. The vagueness of a cognitive occurrence, its inaccuracy, is nothing more nor less than the failure of the occurrence to uniquely determine what it is that is known and thus "is a matter of degree, depending upon the extent of the possible differences between different systems represented by the same representation."⁴⁷ That there are instances of vagueness is simply due to the fact that there are cases where it is in principle impossible to decide which of the different possible systems represented by the epistemic representation is to be designated as *that which is known*. To paraphrase Rolf, any attempt to come to know everything that is the case in the world (problems of a changing world aside) will result in vague knowledge compatible with several possible states of the world; our knowledge of the world will *never* pin down the world in a unique state.

For Russell then, appreciating the ineliminability of epistemic indeterminacy is the key to a correct analysis of (semantic) vagueness in natural language. Though the world is sharp, our knowledge of it is irremediably vague; or, if you prefer, the-world-as-represented-to-us is indeterminate in a way that the world-itself can never be. But this is nothing short of the old Appearance\Reality distinction. Appearance, the way the world is represented to us as being, is distinct from Reality, the world itself (or, more generally, what *is*), if for no other reason than that the former is vague whereas the latter simply is what it is, there can be no analogous feature in Reality.⁴⁸ This wedge that Russell drives between Appearance and Reality constitutes the very essence of a representationalist account of vagueness.

The attraction of such an account for one such as Russell is that a wedge is readily available: that of scepticism. This is, I think, most easily seen when we come to look at the theory of perception that underlies Russell's account of vagueness, a representative theory of perception. (This point has already been made by Rolf.⁴⁹) Representative theories of perception assert, in part, that what I stand in direct relation to when I perceive something (the *immediate* object of perception) is *never* identical with the thing perceived; there is always some mediating object of perception (in veridical cases, simply a representation of the thing perceived) generally thought to be some mental item — for example, ideas (Locke), impressions (Hume), sense-data (Moore), percepts, images, or qualia. Identity theories on the other hand, including Naive Realism and various formulations of Direct

⁴⁶ Russell, *op. cit.*, p. 85.

⁴⁷ *Ibid.*, p. 90.

⁴⁸ Of course for Russell there are numerous other reasons why they should be distinguished. There are, in his opinion, many aspects of appearances that cannot be features of reality; for example: appearances may be incomplete though the world itself cannot be, appearances may be relative though the world itself is not, etc. In short, what is may be other than it appears to be. A common theme in post-Cartesian philosophy.

⁴⁹ Rolf, *op. cit.*, pp. 15 ff.

Realism, assert that perception is a *direct* relation, there are no mediating objects of perception; the thing perceived is *identical* with the object of my perception, and in veridical cases, it exists and is as it appears to be.

For example, where I (veridically) see a white sheet of paper, representative theories have it that I stand in a relation to some "representative" of the external object whereas identity theories maintain that I in fact see (directly) the white sheet. Of course, given that the paper might slowly turn from white to yellow, my perception of the sheet of paper as white could be vague (that is, the perception, what I see, is intensionally vague) in the sense that it may be essentially indeterminate whether or not, at some intermediary stage of the colour transition, my perception (of the sheet as white) is veridical or non-veridical. That is, my perception of the yellowish sheet of paper as white neither determinately represents the facts nor determinately misrepresents the facts. Now on Russell's account of vagueness this is just to say that when I see a white sheet of paper what I see might be a veridical representation of the yellowish sheet and might be a non-veridical representation of that yellowish sheet. Thus what I see cannot be the independent object itself. This is usually thought to be enough to establish the claim that the underlying theory of perception is representational yet, it may be objected, the claim follows only on the condition that 'what I see' be understood as 'the object of my perception' rather than 'my perception of the object'. For Russell this condition is satisfied since what I see is simply the thing's appearance and, says Russell,

[w]hen I speak of "appearances" I am speaking of something purely physical
— the sort of thing, in fact, that, if visual, can be photographed.⁵⁰

So what I see is purely physical, that which the eye 'photographs', yet this is not the independent object itself; it is a vague representation of the independent external object. QED.

So a wedge has already been driven between Appearance and Reality (with the additional claim that the former mediates between the agent and Reality in epistemic relations).

Making use of the above proof, we can construct a more direct argument for the representative theory of perception; one which must be admitted as sound if the conventional arguments from illusion, etc. are. Since Russell took these as sound, he is committed to the following argument; thus we can present yet another argument that he must accept, based on premises that follow from his definition of vagueness.

The argument proceeds as follows: things that exist in the external world can appear vague in certain respects. That is to say, the apparent object has some indeterminate quality, Q say. But, by the above proof, the world is sharp which is just to say that the

⁵⁰ Russell, *op. cit.*, p. 91.

external object has all its qualities determinately, so it cannot have any such indeterminate quality Q. Therefore, since the apparent object has some quality not possessed by any external object, by Leibnitz's Law, the apparent object is distinct from every external object; that is, the apparent object is not an external object. Hence, proceeds the argument from vagueness, what we are directly aware of cannot be the external object but must be something else — for example, sense-data.

Of course, this new addition to the arguments for a representative theory of perception is just grist to Russell's sceptical mill; vagueness is just another feature of perception that leads us to infer that that to which we are directly related in instances of perception is not the external world itself but representations thereof. Let it be said here and now, however, that I do not think a representational account of perception is a necessary condition for accepting a representational account of vagueness; one can consistently maintain that appearances are vague and the world sharp whilst not claiming that appearances mediate perception. Nonetheless, the representational account of vagueness invites scepticism to exactly that degree to which the arguments from illusion invite scepticism; to borrow a phrase from Richard Routley, "[t]he invitation is a bit like one from the mafia or the state to join its latest insurance scheme."

Though the scepticism of the strong epistemic theorist is rejected, scepticism of another type is wheeled out to save the day.

3.3.4 *Vagueness and Classical Logic*

We have detailed an account of vagueness that distinguishes Appearance from Reality and claims that vagueness is merely a property of Appearance (and so semantic, by virtue of the relationship between natural language and appearances). In throwing the veil of Appearance around the world, the world itself is saved from non-classical features that one might otherwise attribute to it. And, of course, for Russell, this precise world (our world) is describable in an ideal language that is precise. Vagueness, on his view, just represents a particular type of epistemic indeterminacy with regard to the facts in conjunction with the claim that perfect knowledge of the world is unattainable. What is an *essential* limitation for us humans, that gives rise to the essential indeterminacy of border cases, is eliminable given perfect knowledge and the language of "perfect knowers", the ideal language, is therefore free of the "defect" of vagueness.

Russell, though admitting vagueness to be an essential feature of natural language, postulates the existence of an ideal language whose role is analogous to that of natural language for the strong epistemic theorist of Chapter Two. They claimed natural language was precise and were sceptical about our ability to know the precise set of linguistic rules governing such language. Russell denies this: "It is perfectly obvious ... that there are

shades of colour concerning which we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word 'red,' but because it is a word the extent of whose application is essentially doubtful."⁵¹ He's sceptical about our ability to know the precise set of ontological relations that obtain in the world, and so sceptical about our ability to know the precise set of semantic relations that obtain in the ideal language, but natural language is the expression of what we know so we should not be linguistic sceptics with regard to natural language. Nonetheless he holds a view very close to theirs with regard to his ideal language which *is* precise though unknowably so and which is able to support classical logic. Were we able to occupy the epistemic high-ground (indeed heaven, the epistemically highest!) all would be clear ... and classical.

All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial existence. Where, however, this celestial existence would differ from ours, so far as logic is concerned, would be not in the nature of what is known, but only in the accuracy [precision] of our knowledge.⁵²

Arguing in this way for the retention of classical logic whilst satisfying the intuition that natural language is vague, Russell has, one might think, distanced logic so much as to make it virtually useless; it is no longer appropriate for the assessment of the validity of many arguments in natural language — in particular, the sorites paradox. Moreover, since the heady days of atomism and positivism, ideal language theories have come under a great deal of criticism and current philosophical fashions are such as to make one suspicious of theories that make essential reference to such ideals (though we may have no more faith these days in the ability of ordinary language to act as final arbiter either and have moved on from the antithesis to Russell offered to us in the nineteenfifties by Oxford). Perhaps what we should concern ourselves with is natural language as it is; after all, if logic is a theory of valid argumentation then surely a logic of those terms or concepts used in argument is what is of primary interest to us.

By virtue of such considerations it might be suggested that what is of interest here is a logic of natural language or, more particularly, a logic of vagueness as understood by the representationalist. This post-Russellian development of the representational account will be taken up in following chapters.

51 Russell, *op. cit.*, p. 85.

52 *Ibid.*, p. 89.

3.4 Summary

The representational account is so popular these days as to be almost accepted as *the* account of vagueness. This, coupled with the fact that Russell's semantic analysis of vagueness is the paradigm of a representational account (even to the extent that it provides a canonical interpretation of supervaluation logics of vagueness — discussed in Chapter Five), means that understanding Russell's conception of vagueness is a key to understanding the dominant approach to vagueness in natural language. For this reason Russell's views are especially deserving of our attention.

What I have been at pains to show is that logic can remain unaffected by the data we are faced with in admitting that natural language contains semantically vague terms only by committing ourselves to a classical metaphysical view according to which the world cannot itself be fuzzy or indeterminate. (Even then however, one might have qualms about the role logic is left to play, as we have just seen.) Such an account therefore simply shifts the burden of proof into metaphysics. Claims encountered in the beginning of this chapter (§3.1) to the effect that 'the world is what it is' do not constitute the required proof since one may simply ask: 'What is it that it is, sharp or fuzzy?' In other words, such claims simply miss the mark.

Taking a closer look, one is left with the feeling that Russell, and similarly Margalit, have succumbed to just the kind of prejudice identified by Dummett — mentioned in §3.1. To say that "things are what they are" is, presumably, tautologous; I may claim something to be what it is even though what it is is vague. Assertions of self-identity need not (perhaps even cannot) be indeterminate or false, and if this is what Margalit means when he says that "they are not what they are in degrees" one might agree and yet claim that he has missed the mark. Things are what they are, but what counts as that, beyond the tautological reply already proffered? To have any consequences for the advocate of a fuzzy world the above statements must be read not as denying the indeterminacy of being-self-identical, but as denying the indeterminacy of being- ψ , for some property ψ .⁵³

Russell's claim that nothing is to a certain extent possessed of the properties it possesses surely only holds for those properties a thing possesses determinately, of which, self-identity may be thought to be one. Is it not possible that something may be to a certain extent possessed of a property if it possesses that property to a certain extent? Given that things are what they are, they are only precise if they are precise. (Hardly

⁵³ A notable exception to this would be Peña's arguments to the effect that the fuzziness of something entails that thing's being distinct from itself to some extent. Peña, L., 'Identity, Fuzziness and Contradiction', *Nous* 18 (1984): 227-59.

Note that, according to Kathinka Evers (in her *Plurality of Thought*, Library of Theoria #18 (1991), p. 97), A. Hägerström also makes just this assimilation of determinacy to self-identity in his *Die Philosophie der Gegenwart in Selbstdarstellungen*, VII, Raymund Schmidt (1929).

controversial metaphysics!) From this, the thesis that they are precise can only figure as the conclusion given the premise that they are precise.

Thus, as yet, the metaphysical issue in question remains either question-beggingly assumed or simply asserted. As we shall see, more recent advocates of the representational account have attempted to provide arguments for this claim, thus bolstering a representational view.

In addition, even if the metaphysical basis for the representational approach can be secured and one accepts Russell's analysis of the semantics of vague terms, his means for defending classical logic — grounding logic in some celestial realm — are generally unacceptable these days. The implications for logic — in other words, what the representationalist goes on to say about logic — have changed somewhat with the development of modal logics.

These are the issues to which we now turn.

Chapter Four

Recent Developments of the Representational Theory — I Descriptive Representationalism

Jointly, Russell's account of vagueness coupled with his view of the nature of logic would diffuse any tension between (semantic) vagueness and classical logic; that is, it would provide a defence of classical logic against some deviant logic-of-vagueness. We noted two key elements in this defence that emerged from the previous chapter: an *a priori* acceptance of the precision of the world or ideal world-description, and a view of logic that relegates it to the realm of the celestial, mirroring the structure of some Ideal Language or corresponding Reality behind Appearances. Both of these elements have, since Russell, been rejected by various authors in giving representational accounts of natural language vagueness, and in the next two chapters I want to take up these variations on the representationalist's theme with a view to their implications for classical logic.

Beginning with the former point, let us now turn our attention to attempts to defend classical logic by means of a variant account of vagueness that eschews *a priori* arguments in favour of arguments for the actual existence of a descriptively complete, precise ideal language. If sound, these arguments establish the precision of the world, thereby characterising it as a representational account. I shall refer to this type of representationalism as *descriptive representationalism* since it is from our supposed ability to actually describe the world precisely that we are entitled to infer its precision.

The appearance/reality or representation/represented distinction coupled with the view that vagueness is an inevitable feature of appearances or representations forced Russell to maintain that though all representations — epistemic and semantic — are vague, the represented — the world — can nonetheless be known to be precise by means of *a priori* reasoning; he advocated what we might term '*a priori* representationalism'. But

what if vagueness is not thought to be an inevitable feature of representations? Then it is conceptually possible that the world be precisely describable (unless as a matter of conceptual necessity the world is vague; a view which has, to my knowledge, no advocates). Arguments might then be sought to establish whether or not the world is, *as a matter of fact*, completely describable in some precise ideal language and thereby precise. Arguments establishing such an ideal language description would provide a case for a representationalist account of vagueness, though distinct from Russell's, and provide a defence of classical logic.

Let us be clear on how this project is to proceed. The preceding Russellian account dealt with vague terms in natural language considered as in themselves ineliminably vague (vagueness was taken to be a semantic feature), moreover (and more importantly in the present context) the class of vague terms was itself considered an essential part of any complete description of the world. What is now being suggested is that, although vague terms are irremediably vague — there being no disagreement as to the semantic nature of vagueness — these terms can, *as a class*, be dispensed with in providing such a description; that is, language as a system of representation can be *purged* of its vague elements — whilst of course preserving the language's descriptive power, thereby ensuring the non-triviality of the claim.

What will we count as "a complete description of the world" though? Obviously it depends in part on what we take to be "the world". If we are prepared to wield Ockham's Razor with gusto and accept a minimal ontology then a complete description is perhaps more easily had. It also depends on how rigidly one construes the completeness constraint. Is close enough good enough? This issue is the source of a distinction between: (i) those descriptive representationalists who maintain that the class of vague terms can as a whole be dispensed with, and this without *any* loss of descriptive completeness — I take Carnap as making this claim; and (ii) those pragmatically minded descriptive representationalists, such as Haack and Quine, who maintain that the class of vague terms can as a whole be dispensed with at perhaps some cost to descriptive completeness, but a cost that is nonetheless worth paying for the simplicity it affords — namely, our thereby being able to defend classical logic with what Quine describes as its "sweet simplicity".¹

So the representationalist might agree that natural language is vague in its description of a precise world but claim that this vague language makes no essential contribution to the completeness of the description; it is in some sense eliminable — §4.1, redundant — §4.2, or superficial — §4.3.

¹ Quine, W.V.O., 'What Price Bivalence?', *Journal of Philosophy* 78 (1981), p. 91.

4.1 *Eliminativism*

A very common move is to advocate the complete elimination of vague terms. The suggestion is that one could, in principle, reconstruct an ideal or regimented language containing only precise terms by means of which a precise world could be described. Quine for instance suggests that though the common person's ontology is vague and untidy, it really does not matter so long as we can think of ourselves as approximating a scientific language free of vagueness.² Carnap and Haack have also endorsed this ideal language approach.³

Carnap and Haack however, it should be noted, have not explicitly declared any interest in the ontological issue that presently concerns us in our attempt to arrive at a satisfactory analysis of vagueness. They are implicitly or explicitly concerned to address the issue of whether vagueness constitutes a threat to classical logic, as we are in this thesis, however they effectively attempt to defuse the issue at the outset by claiming the class of vague terms as a whole to be eliminable and thus of no concern to anyone, let alone logic. Our concern with ontology is motivated by Russell's analysis of vagueness as representational and is a broader more complex issue; if the program of regimentation is thought to be unsuccessful then further logical and ontological issues naturally arise in relation to the problem of vagueness on which Carnap and Haack are silent.

For the eliminativist the ontological question is effectively trivial or irrelevant — one would hardly go to all the trouble of purging language of vagueness only to admit it into one's ontology nor, arguably, could one maintain the precision of an ideal description whilst accepting that the world is vague. Metaphysical issues need only arise for those who see vague language as inescapable whilst denying any need for logical innovation, for then the only defence seems to be via metaphysics. Nonetheless, since their very silence on metaphysical matters may be taken as a sign of their acceptance of the *status quo*, I shall attribute to them the claim that the world is precise.

This eliminativist response to the problem of vagueness is described by Putnam as the "'what me worry' response".⁴ Bearing in mind the distinction noted above, we might describe such a response more fully as either: (i) 'What, me worry about vagueness? We

² Quine, W.V.O., 'Facts of the Matter' in R. Shahan & K. Merrill (eds), *American Philosophy from Edwards to Quine*, Oklahoma (1977), pp. 176-96.

³ See, for instance: Carnap, R., *Logical Foundations of Probability*, Chicago University Press (1950), Ch. 1, and *Philosophical Foundations of Physics*, (ed. by M. Gardner) Basic Books, New York (1966), Part II; Haack, S., *Deviant Logics*, CUP (1974), Ch. 6 and *Philosophy of Logics*, CUP (1978), Ch. 9.

⁴ Putnam, H., 'Vagueness and Alternative Logic' in *Realism and Reason*, CUP (1983), pp. 274-6.

can eliminate it.' or (ii) 'What, me worry about vagueness? We can afford to eliminate it.'

Thus, ultimately, the eliminativist, though committed to a precise world that is not precisely describable in *natural* language, can answer the question 'Why suppose the world to be precise if it is not completely describable as such?' by suggesting that it is indeed, as a matter of fact, completely describable in an *ideal* language or that our interests are best served by assuming this to be the case. The burden of proof confronting the representationalist with regard to the assumption of a precise world vaguely described by natural language would thus be relieved.⁵ Equally so, the burden of proof regarding the appropriateness of classical logic; the precision of the world or language used to justify classical logic is seen as a matter of fact to obtain or at least be affordable.

The claim that the world is completely describable in an ideal language free of vagueness is thus crucial in shifting the burden of proof back onto advocates of ontological accounts of vagueness and non-classical logics of vagueness. It is my contention that such a claim does not withstand close scrutiny.

An obvious way in which this program might initially be problematic is that the replacement of vague terms by precise surrogates requires that there be some such surrogates. It has, as I have already indicated, been contested by Russell that no such terms are available: *all* language is vague. However his arguments for this claim were found wanting (in Chapter One) and so we may grant that there may be a non-empty class of precise terms available as possible replacements for vague terms.

So how then are vague terms to be eliminated without loss?

4.1.1 *The Limit Argument*

A first thought might be that we can suppose the existence of a complete ideal language description since any vague description of the world can always be replaced by successive descriptions that are more and more precise and thus, in the limit, is replaceable by a precise description. To use an example of Alston's: we may try to remove the vagueness of the term 'city' by replacing it with 'a community of more than 50,000 inhabitants'. The problem that now arises is that the term 'inhabitant' is vague. We might now try to substitute a more precise phrase or term in its place... And so on. Finally, a precise substitute will emerge.

Notice the form of this 'limit argument': natural language is vague, but we can approximate, more and more finely, precise descriptions in some ideal language. Thus

⁵ Note that relief could come from the claim that, on pain of incoherence, the world must be sharp, i.e. a fuzzy ontology is essentially incoherent. Such attempts (witness Evans' purported proof) will be discussed in Chapter Five.

we obtain a (possibly) infinite sequence of descriptions where description $i+1$ succeeds description i just if $i+1$ is more precise than i and less precise than $i+2$. (Recall that description Q is 'more precise than' description P if and only if each border case for Q is also a border case for P but not vice versa.⁶) Now in the limit, as n tends to infinity, the description n of some state of affairs is precise. However, to conclude from this that there exists a precise description is to assume the existence of the limit. One may only infer the existence of the limit when employing limit arguments if one is guaranteed by some other assumption that the limit of the relevant sequence exists. That is to say, the construction of a sequence tending to a limit is itself not enough to guarantee the existence of the limit.

These arguments are most commonly employed in areas where some sort of completeness assumption has already been made — completeness in the sense that just such a 'thing' (number, description, etc.) is supposed to exist lest the domain under consideration lack an 'object' which we may get infinitely close to, as it were. An example of this would be attempts to describe the real numbers in terms of descriptions employing decimal expansions. The assumption is that the infinite sequence of rational numbers tends to a limit in a space whose topology is such that it is complete in the topological sense of containing no "holes"; if the sequence can be 'reasonably' said to converge to some point then that point exists. Thus we may assume in such circumstances that, where we can construct infinite sequences that converge like {'the # described by the decimal expansion 2.1', 'the # described by the decimal expansion 2.11', ...}, the limit of the sequence (some real number) exists.

In the context to hand however the assumption of the existence of a precise description by virtue of its being the limit of an infinite sequence of less and less vague descriptions is exactly what is in question, so it is circular to assume its existence and hence such an argument showing the possibility of eliminating vague natural language in favour of some ideal precise description seems questionable. The circularity amounts to implicitly *assuming* there to be a precise ideal language for talking about the world, whilst arguing that we may assume the world is precise since there is a precise language in terms of which it can be described. The circularity is obvious. Furthermore, in Putnam's words, "if one thinks that language which is 'vague' in the sense of not having the unimaginable precision that the ... ideal language would have, can be philosophically, ontologically and metaphysically kosher then one may throw the challenge back with the counter-challenge: 'Why *should* it be possible to account for ordinary language in a perfectly precise language?'"⁷ In other words, having questioned the assumption of the existence of an ideal language we may be inclined to ask why we

⁶ Rolf, B., *Topics on Vagueness*, PhD Thesis, Lund (1981), p. 76.

⁷ Putnam, *op. cit.*, p. 275.

should suppose it to exist in the first place. Alston puts the point clearly when he says: "At this point we may feel that the removal of all vagueness from a given term is an unrealistic goal; the most we can hope to do is to approach it asymptotically."⁸

4.1.2 Comparative Descriptions

It has sometimes been suggested that vagueness can be eliminated without loss by replacing attributive expressions with precise comparative ones. Briefly, the thought is this: where vagueness arises, for example with predicates like 'red', one might attempt to replace the vague attributive expressions with corresponding comparative expressions, for example 'is redder than'. If comparatives are free of vagueness and all vague terms are replaceable by comparatives without loss of descriptive power then it would seem that vagueness can be eliminated without loss. Both Carnap and Christopher Peacocke have made suggestions along these lines.⁹

Such a manoeuvre is hopelessly misguided it seems to me. In the first instance, vague singular terms cannot be eliminated in this way. Suppose it to be vague whether Black Mountain is high and suppose that the vagueness of the predicate 'is high' can be eliminated in favour of the comparative 'is higher than' — Black Mountain is higher than Mt Ainslie. Nonetheless, this latter claim might be vague by virtue of the vagueness of the singular terms involved — it might depend on how one delimits Black Mountain and Mt Ainslie, and that will be a vague matter.¹⁰ Even if such replacement seemed promising in the case of predicative terms it does not in the case of vague singular terms.

Secondly, this systematic replacement of even vague predicates requires that the predicates being replaced be predicates of degree where the degrees are totally ordered. A predicate 'P' is a predicate of degree, according to Wright, if the comparative 'is less/more P than' makes sense and iteration of one of these relations may transform something P into something not P or vice versa (iteration is required to ensure the non-triviality of the notion of being a predicate of degree — every predicate 'P' admits of at least two degrees 'P' and 'non-P').¹¹ And the degrees are totally ordered if the degree to which one object satisfies 'P' must be either greater, less or the same as the degree to which some other object is 'P'. Now, if the vagueness of a predicate 'P' is to be avoided

⁸ Alston, W.P., *Philosophy of Language*, Prentice Hall (1964), p. 91.

⁹ Carnap, *op. cit.*; Peacocke, C., 'Are Vague Predicates Incoherent?', *Synthese* 46 (1981), pp. 133-35.

¹⁰ NB: The vagueness of the comparative sentence 'Black Mountain is higher than Mt Ainslie' does not show the comparative relational expression to be vague (a result which would completely undercut the position being considered). The vagueness here may be said to be due to the vagueness of the singular terms involved.

¹¹ Wright, C., 'On the Coherence of Vague Predicates', *Synthese* 30 (1975), p. 348.

by the use of comparatives then obviously the comparative locution must make sense; furthermore, if the only way in which the comparative 'being less/more P than' makes sense is in there being two degrees — non-P (so in being less P than something that is P) or P (so in being more P than something that is non-P) then the use of comparatives will not avoid problems of vagueness since this use of comparatives is merely elliptical for describing something as P or non-P and the problem of vagueness will simply re-arise for the comparative locution. So 'P' must be a predicate of degree in the above sense. The demand that the ordering be total is required by Peacocke to avoid the possibility of its being vague whether one thing is as P as / more P than / less P than some other thing; it must always be the case, for any pair $\langle a, b \rangle$ in the range of significance of the relevant predicate P, that one of the three comparative relations determinately holds.

Now many vague predicates do admit of degrees (indeed, Wright takes this as an essential feature of soritical predicates and Rayme Engel claims that every predicate, or "virtually" every predicate admits of differences of degree¹²) yet many of these — such as colour predicates, or predicates like 'is a chair' — exhibit non-linear or multi-dimensional vagueness; in other words, their degrees are not totally ordered. So far as Peacocke is concerned then, vagueness is not eliminated by the substitution of comparative relations; comparatives might be indeterminate as well.

Thirdly, even if subject terms were eliminable in this way and demands for total ordering were met, what justifies this restriction in our description of the world to comparative locutions? The substitution of a vague description in terms of kinds for a description of how things are relative to each other leaves out most of our ordinary talk about the world. Many properties we take to be properties of things in the world are left out of such a description — descriptive completeness is not preserved by such a revision of ordinary talk.

4.1.3 *The Language of Science*

Perhaps the most popular means of resolving the problem of identifying the ideal language is by way of that class of terms that seem undoubtedly precise and thus are perfect candidates — quantitative terms. Now the program seems to be that one can either precisify vague qualitative terms by devising precise criteria for their application using quantitative terms or simply remove any qualitative terms in favour of quantitative ones. In Quine's words, "what had been observation terms are arbitrarily reconstrued ... as theoretical terms whose application may depend in marginal cases on

¹² Engel, R.E., 'On Degrees', *The Journal of Philosophy* 86 (1989), p. 24.

protracted tests and indirect inferences".¹³ *A complete description of the world can be given in the language of an ideal physics.* Consider the term 'hot' for example; this qualitative notion may either be precisified by defining a new term 'hot*' to be 'more than 300° Kelvin' (say) or eliminated altogether in favour of quantified descriptions of the entropy of a system, that is, 'temperature of x° Kelvin'. Such was the programme of Carnap in his *Philosophical Foundations of Physics* for instance, more recently endorsed by Haack and Quine.¹⁴

As both Haack and Alston point out, the indeterminacy due to vagueness is now replaced by an indeterminacy due to limitations in our power to discriminate as finely as the quantitative scale might demand thus giving rise to inexactness of measurement of these quantities, however this latter problem is an epistemic one, not semantic and as such classical semantic and logical theories are not threatened.

It has been suggested that there is an associated problem though. We might be inclined to respond, as do Dummett and Sainsbury, that we would have no use for terms such as 'hot*' on the grounds that we cannot in general tell just by the use of our unaided senses whether or not something is hot*.¹⁵ These new theoretical terms require, as Quine suggested, protracted tests for their application and thus are not applicable on the basis of (casual) observation as the vague observational terms were. The general criticism being raised here is that, as a matter of practical everyday concern we require terms whose applicability or otherwise depends solely on casual observation and as the precise theoretical terms lack this feature we would have no *practical* use for them; it is simply impractical to require speakers of a language to push around wheel-barrow of colour charts or to defer application of a term until the requisite tests can be carried out to determine the precise temperature of an object.

This response however does not constitute a telling criticism. Quine, Carnap and Haack might agree that it is practically cumbersome to defer to protracted tests (Haack for instance speaks of "the advantages that vague ways of speaking undoubtedly possess") but this practical difficulty is not a difficulty *in principle*. Competent speakers endowed with complex measuring devices could in principle speak such a language containing precise substituends for their vague counterparts.

So, the claim being made is that, having identified the language of science as ideal (that is, precise), it is feasible to suppose that we could "tidy up" (as Haack puts it) or "regiment" (Quine) ordinary language by means of this ideal. (Whether or not the language of science *is* precise will be discussed in §4.4.)

13 Quine, 'What Price Bivalence?', *op. cit.*, p. 92.

14 Quine, 'Facts of the Matter', *op. cit.*; also reference cited in previous footnote.

15 Dummett, M., 'Wang's Paradox', *Synthese* 30 (1975), pp. 314-16; Sainsbury, R.M., *Paradoxes*, CUP (1988), p. 28.

4.1.3.1 *The Language of Science and Descriptive Completeness*

But is it really feasible to suppose that whatever can be said in the vague qualitative language of ordinary discourse can either, in principle be eliminated in favour of talk in the (purportedly) precise quantitative language of science or eliminated altogether without loss? In other words, having identified a *prima facie* candidate for the ideal language, the language of science, the question remains — is it sufficient to the task of providing a complete description of the world? Can we say all we need to say in precise terms?

Physical-quality terms already considered like 'hot', 'heap' or 'bald', which are absent from the ideal description, are said by Quine to depend on mere verbal conventions, not matters of fact, so regimentation (that is, their elimination in favour of precise substituends) is not problematic, descriptive completeness is not threatened. "Where to draw the line ... is not determined by the distribution of microphysical states, known or unknown; it remains an open option."¹⁶ So, according to Quine, we are free to resolve indeterminacies any way we like so long as the proposed resolution does not conflict with the distribution of microphysical states — an expression of his physicalism.

Similarly for terms describing intentional states; moral and aesthetic terms such as 'good' or 'beautiful'; and terms describing people such as 'noble' or 'slovenly'. Language describing the contents of beliefs, desires, intentions, indeed any psychological state, might be essentially vague by virtue of the vagueness of the content of the relevant state (a point suggested by Williamson) and the notions of 'belief', 'desire', etc. themselves are essentially vague.¹⁷ (Of course intensional language is also problematic for reasons other than its vagueness and the responses are varied.) Moral and aesthetic terms such as 'good' or 'beautiful' also seem incapable of precise regimentation, along with terms describing people as 'noble', or 'slovenly', etc.

Physicalists like Quine will argue that we are free to resolve any indeterminacies that might arise in their application any way we like; though there are costs to common sense, these costs are not ones we need worry about. For those of us who take common sense to constitute a partial constraint on philosophical theories, the cost of such a programme may seem excessively high; no complete ideal language description of the world will include anything's being hot or tall or red. Nor, it would appear, can it include anything's being beautiful, good, noble, believed or desired, and so on.

¹⁶ Quine, W.V.O., 'What Price Bivalence?', *Journal of Philosophy* 78 (1981), p. 94.

¹⁷ It was this vagueness that prompted Dummett's rejection of epistemic logics; the essential vagueness of the notions render them unfit for logical treatment. See: Dummett, M., *Frege, Philosophy of Language*, Duckworth (1973), pp. 285-8.

Perhaps more surprisingly though, descriptions of things as tables, chairs, persons, stones or indeed as any ordinary thing, are all eliminated — all the respective terms are vague and as a consequence are absent from the ideal language. These consequences bear some discussion; after all, it's rather surprising to find that not only are you not reading a thesis but I never wrote one, nor ever could have sat on a chair or typed onto a computer. In fact, neither I nor you could ever have done anything — there are no persons!

These latter consequences, though considered briefly by Quine in a rather confusing passage, are most clearly brought to light by considering the sceptical arguments of Peter Unger and Samuel Wheeler whereby the extent of the linguistic revision entailed by the regimentation program is made obvious.¹⁸ Unger and Wheeler argue that all soritical terms, since incoherent by the soundness of classical sorites reasoning, must be eliminated with corresponding ontological consequences. The impact of these arguments comes from their showing just how much of language is soritical and consequently just how far common sense- or folk-ontology needs to be revised. Now, the bearing these arguments have on our present concern with the regimentation program is as follows: those terms that must be eliminated, according to Unger and Wheeler, due to their soritical nature must be eliminated from Quine's regimented language since any term is vague if soritical, and absent from the precise regimented language if vague. Unger and Wheeler then, in showing just how much of everyday language is soritical, show just how far-reaching the regimentation program is. (It should be noted that Unger and Wheeler, far from seeing the extent of the revision as a possibly troubling consequence, whole-heartedly embrace the extensive scepticism which ensues.)

Let us diverge then briefly to consider their position. Their sceptical arguments proceed by considering the sorites of Eubulides (Wheeler), or some variation thereof (Unger) — e.g. the sorites-of-decomposition, the sorites-of-slicing-and-grinding and the

¹⁸ Unger, P., 'I Do Not Exist' in G.F. Macdonald (ed.), *Perception and Identity*, Macmillan (1979), pp. 235-51; 'There are no Ordinary Things', *Synthese* 41 (1979): 117-54; 'Why there are no People', *Midwest Studies in Philosophy* IV (1979): 177-222; 'Skepticism and Nihilism', *Noûs* 14 (1980): 517-45; Wheeler, S.C., 'On that Which is Not', *Synthese* 41 (1979): 155-73. Patrick Grim has argued that the consequences of accepting the line of argument advocated by Unger and Wheeler are far more extensive than even they recognise. (Grim, P., 'What Won't Escape Sorites Arguments', *Analysis* 42 (1982): 38-43; the debate is then joined by Abbott, W.R., 'A Note on Grim's Sorites Arguments', *Analysis* 43 (1983): 163-65 and continued by Grim, 'Is this a Swizzle Stick Which I See Before Me?', *Analysis* 43 (1983): 165-67.) If Grim's line of argument is accepted as sound then the point I go on to make is strengthened, however I think Grim's argument is flawed.

Quine mentions Unger's argument in his article 'What Price Bivalence?', *op. cit.*, pp. 92 ff, but goes on to discuss it in the context of the problem of the One and the Many, on which Unger has also written (Unger, P., 'The Problem of the Many', *Midwest Studies in Philosophy* 5 (1980): 411-67). Now, I do not deny that issues surrounding vagueness can be employed to motivate this latter problem (cf. Lewis, D., 'Many, But Almost One' in J. Bacon, K. Campbell, & L. Reinhardt (eds), *Ontology, Causality and Mind: Essays on the Philosophy of D.M. Armstrong*, CUP (1993), pp. 23-38) but Quine's digression here is simply confused in my opinion.

sorites-of-cutting-and-separating. By way of example, consider the sorites-of-decomposition which purports to establish the non-existence of stones.¹⁹

Unger invites us to entertain the following three propositions:

- (1) There is at least one stone.
- (2) For anything there may be, if it is a stone, then it consists of many atoms but a finite number.
- (3) For anything there may be, if it is a stone (which consists of many atoms but a finite number), then the net removal of one atom, or only a few, in a way which is most innocuous and favourable, will not mean the difference as to whether there is a stone in the situation.

It is then argued that they form an inconsistent set. The reasoning here is simple, says Unger. Consider a stone, composed of a certain finite number of atoms, n say. (That there is such a thing is guaranteed by (1) and (2) above.) If we or some physical process should remove one atom (any piece of matter just below the threshold of perceptibility would do), without replacement, then there is left that number minus one, presumably constituting a stone still (by (3) above). Now, after another atom is removed, there is that original number minus two; and so on. After n atoms have been removed in similar stepwise fashion there are no atoms left at all, yet we must still be supposing that there is a stone there (since again, by (3) above, no one removal could make any difference, so there is no difference at any stage). But this contradicts (2). There is then, claims Unger, a blatant inconsistency in our thought. "However discomfoting it may be, I suggest any adequate response to this contradiction must include a denial of the first proposition, that is, the denial of the existence of even a single stone." The term 'stone' is an inconsistent or incoherent notion — by virtue of its vagueness rendering it susceptible to sorites-contradictions — and so there are no stones.

Thus we have, according to Unger, an argument for the claim that there are no stones — an argument which can be generalised to refute the existence of all ordinary things.²⁰ In showing just how many terms are susceptible to sorites-contradictions, the arguments of Unger and Wheeler highlight the extent of the regimentation program.

¹⁹ Discussed by Unger in his 'There are no Ordinary Things', *op. cit.*, pp. 120 ff.

²⁰ What remains, you might well wonder? Some physical objects remain impervious to Unger's argument — "certain particulars which are prominent in the physical sciences, ... [e.g.] electrons, hydrogen atoms, and water molecules" (Unger, P., 'There are no Ordinary Things', *op. cit.*, p. 122). Presumably mereologically-simple objects, those objects that are not composed of any finer objects, are also immune; "certain sub-atomic particles may provide an example." (Unger, P., 'I Do Not Exist', *op. cit.*, p. 241) In addition, Unger leaves open the possible existence of physical (though not ordinary) objects — even middle-sized ones (Unger, P., 'There are no Ordinary Things', *op. cit.*, pp. 150-1), in contrast to Wheeler who bolsters his argument against ordinary things by arguing against all middle-sized objects (Wheeler, 'On that Which is Not', *op. cit.*, p. 164).

For both Unger and Wheeler, this sceptical conclusion is expressible both in the material and formal mode; that is, it is at one and the same time a claim about language and the world, applying equally well to the terms denoting ordinary things (e.g. 'stone') and to the things thereby denoted (e.g. a stone);

It is true that we have shown that ... terms for ordinary things are incoherent. In that this is so, those terms cannot apply to anything real. And from that it follows that there are no such ordinary things as those words purport to designate. Accordingly, our results concern words and things alike.²¹

And so too for Quine. The soritical nature of terms describing ordinary things, which renders them incoherent according to the above argument, precludes them from figuring as terms in the ideal regimented language (since, as I noted above, soritical terms are vague) and so, since ontological accounting only makes sense relative to this regimentation, there are no ordinary things.

Such consequences of regimenting natural language have long been known to obtain. It is reported by Galen (c. 129-199) in his *On Medical Experience*, that the Empirical Doctors, in attempting to defend their use of the vague notion of 'sufficiently many' in their account of inductive inference, replied to the attack of the Dogmatic Doctors as follows:

according to what is demanded by the analogy [that is, by parity of reasoning], there must not be such a thing in the world as a heap of grain, a mass or satiety, neither a mountain nor strong love, nor a row, nor strong wind, nor city, nor anything else which is known from its name and idea to have a measure of extent or multitude, such as the wave, the open sea, a flock of sheep and herd of cattle, the nation and the crowd.²²

The consequences of banishing such vagueness (whether this is because they are vague as Quine is forced to do or because they are soritical as the Dogmatists would have it) seem unacceptable. As Quine himself acknowledges: "At this point, if not before, the creative element in theory building may be felt to be getting out of hand, and second thoughts ... may arise".²³

²¹ Unger, P., 'There are no Ordinary Things', *op. cit.*, p. 147.

²² Galen, *On Medical Experience*, XVI 1, p. 114 W; cited in: Barnes, J., 'Medicine, Experience and Logic' in J. Barnes, J. Brunschwig, M. Burnyeat & M. Schofield (eds), *Science and Speculation*, CUP (1982), p. 34. Cf. also Galen, quoted in Long, A.A. & Sedley, D.N., *The Hellenistic Philosophers*, CUP (1987), Vol. 1, pp. 222-3.

²³ Quine, 'What Price Bivalence?', *op. cit.*, pp. 94-5.

Why? Well, because such consequences conflict *so markedly* with common sense. Note that this response does not entail the unqualified acceptance of common sense views — what Unger calls the "Mooreian gambit", perhaps better named the Reid Gambit ("When a man suffers himself to be reasoned out of the principles of common sense, by metaphysical arguments, we may call this *metaphysical lunacy* ...")²⁴; though it may not *always* be appropriate to clutch onto common sense at the expense of anything else, including philosophical reasoning, it might *sometimes* be appropriate and this might well be one of those times. We might reasonably claim that we are more certain that there are ordinary things (or that terms describing ordinary things are coherent) than that classical logic is correct.

After "second thoughts" on the matter however, Quine nonetheless opts for a defence of the regimentation program and classical logic. Regimentation is costly, though cost-effective; this is the Quinean defence. Though it seems that regimentation is not possible without some descriptive incompleteness, nonetheless, the resulting loss in utility is offset by gains in simplicity afforded by classical logical theory; all things considered, our adherence to classical logic optimises the drive for evidence and the drive for system, i.e. utility and simplicity.

The adequacy of this defence depends, amongst other things, upon how one evaluates the cost-benefit equation. (The further critical issue of whether or not regimentation itself is even possible will be taken up in the next section.) Is the loss in descriptive completeness less than the gain in simplicity afforded by classical logic? Well, that depends firstly on how little the loss in descriptive completeness is deemed to be (for physicalists like Quine who have a rather minimal ontology to begin with, the loss will be less than for those with more populated ontologies) and secondly, on how great the gain in simplicity afforded by classical logic is deemed to be. The first issue is well beyond the scope of this thesis (and probably irresolvable anyway); the second issue surely cannot be addressed until competing logical theories are available for evaluation as regards their complexity.²⁵ It is a conjunction of these two points that, in my opinion renders this defence inadequate; it is simply not clear whether the gains in simplicity outweigh the loss engendered by the program of regimentation. How could it be, even to Quine who is able to fix the debit column (the cost of descriptive loss) at a level favourable to his case (that is, low)? Balancing the books with unknown credit can only be done by making presumptions — which here amount to Quine's supposing that classical logic is far and away simpler than any rival that may appear. Ultimately Quine,

²⁴ Cf. Unger, P., 'There are no Ordinary Things', *op. cit.*, p. 124; Reid, T., *An Inquiry into the Human Mind on the Principles of Common Sense* (1764), (ed. by T. Duggan) University of Chicago Press (1970), pp. 268-9. I am indebted to Dereck Brookes for this reference to Reid.

²⁵ Avishai Margalit, whilst endorsing the Quinean program, admits that this latter concern prohibits evaluation of the cost-benefit equation; cf. Margalit, A., 'Vagueness in Vogue', *Synthese* 33 (1976), pp. 216-7.

having appeared to open the door to rival logics ... on certain conditions, slams it shut again claiming no rival could meet these conditions — an open-minded sort of prejudice prevails.²⁶

Let's take stock of our position then. The representationalist has suggested that classical logic is not threatened by vagueness since, though the vagueness of a term is ineliminable, the class of vague terms as a whole can be eliminated from a complete description of the world; a suitably regimented or ideal language purged of its vague elements, e.g. the language of science, is sufficient to the task. However, vagueness is such a pervasive feature of natural language that the program of regimentation seems to threaten descriptive completeness for all but those with the sparsest of ontologies. Thus it seems likely that regimentation is not possible without some loss. Can this loss of descriptive completeness be offset by a gain in the simplicity of that logical theory deemed adequate after regimentation — classical logic? Quine's inclination is to answer in the affirmative "for the simplicity of theory it affords", but this cost-benefit analysis is premature, other alternatives not being presented for evaluation.

Of course, what I have argued so far does not constitute a knock-down argument against the regimentation view but it may make one feel some disquiet about such a program.

4.1.4 *Is Regimentation Possible?*

We have been discussing the regimentation program as if it were in principle possible though its consequences may be deemed unfavourable. Yet the very idea of regimenting vague ordinary language faces serious problems according to Hilary Putnam.²⁷

Whatever one's Quinean ideal language, the regimentation from ordinary to ideal language is not unique; there is no non-arbitrary way to regiment or precisify vague terms. The regimentation to a language containing only precise terms is not specified; there are at least as many regimentations as there are ways of precisifying vague terms, one for each precisification. (This position resembles in some ways the stance adopted

²⁶ Of course, the Unger/Wheeler position (even were it coherent, which is denied by Hilary Putnam in his 'Vagueness and Alternative Logic', *op. cit.*, p. 277, and Rolf in his *Topics on Vagueness*, *op. cit.*, p. 135, on the grounds that the language used to state the position contains vague terms which are, by the position's own lights, true of nothing) cannot be used to defend the claim we are considering — that, since regimentation is possible, classical logic is not threatened by vagueness — nor do they attempt to do so. It might be thought that their argument could be adapted as follows: we can give a precise complete description of the world (i.e. regimentation is possible) since we can *only* give such a description, on pain of incoherence, (i.e. regimentation is necessary). But of course, as Unger and Wheeler would no doubt themselves concede, in the context to hand such a defence is circular since the necessity of regimentation follows from the supposed soundness of classical reasoning. One can hardly defend classical logic against the challenge posed by vagueness by assuming its correctness in the first place.

²⁷ Putnam, H., *Realism and Reason*, CUP (1983), pp. 274-6.

by the supervenient with regard to vague language, with the crucial difference that the supervenient theorist takes logic to model ordinary language rather than any one of the precise regimentations. I shall have more to say on this in Chapter Five.) As far as Quine is concerned there is no fact of the matter as to which is the correct regimentation of ordinary language:

Ordinary language is only loosely referential, and any ontological accounting makes sense only relative to an appropriate regimentation of language. The regimentation is not a matter of eliciting some latent but determinate ontological content of ordinary language. It is a matter rather of freely creating an ontology-oriented language that can supplant ordinary language in serving some particular purpose one has in mind.²⁸

Now though, according to Putnam, a problem arises which compromises Quine's position — the problem of the status of the regimentation itself. In the absence of any one correct translation we must decide what is to count as an "appropriate regimentation". The argument then proceeds via the following dilemma: either the notion 'appropriate' is a notion in the ideal language or a notion in ordinary language. If the latter, then the notion only loosely refers and stands in need of regimentation itself; if the former, then an "acceptability predicate" is available for ordinary language, which Putnam seems to implicitly suppose contradicts Quine's view that statements in ordinary language aren't true or false.

Taking the latter horn first then. If ordinary language is just so much noise awaiting regimentation into a language that is factually significant (the ideal language) then the metalanguage in which the regimentation is carried out, and in terms of which an "appropriate regimentation" is to be specified, had better be the ideal language lest we try to solve the problem of regimenting vague language by means which themselves stand in need of regimentation. To put the point another way: if the regimentation manual(s) by means of which Quine proposes to make cognitive sense of ordinary language is itself written in ordinary language — in particular: if it contains vague language — then we can make no cognitive sense of *it*... Unless of course the manual can itself be understood by means of appropriate regimentation; but this would require another manual and the problem repeats itself. Regress ensues.

So, an "appropriate regimentation" must be specified in the ideal language, and hence is a precise notion. However, there is then a precise predicate in the ideal language, 'true under an appropriate regimentation', (or some slight variation on this which Putnam seems to assume will have the same devastating effect) taking as arguments ordinary language propositions. This, says Putnam, is a bivalent

²⁸ Quine, *ibid.*, p. 165.

acceptability predicate for ordinary language, and the damning consequences are implicitly assumed to follow.

The argument is rather too brief and, I think, fallacious. Presumably, the supposed worry is that an acceptability predicate for ordinary language will act something like a truth predicate and, if precise, will provide a precise truth predicate which is exactly what Quine denies is possible. But of course the presence of a truth predicate for any regimented proposition, $R(p)$, does not constitute a truth predicate for the unregimented proposition, p , unless the regimentation R is a (meaning-preserving) translation; if regimentation was simply translation then p would have truth conditions given by the truth conditions for $R(p)$ which are precise (by assumption), hence p would have precise truth conditions. But regimentation is not translation and herein may lie the source of Putnam's confusion. Throughout his argument he uses 'translation' as synonymous with 'regimentation' — for example, he says that in Quine's view "statements in ordinary language *aren't* true or false; they are only true or false relative to a translation scheme (or 'regimentation') ..." ²⁹ Now there is a fallacy of equivocation; on the one hand 'translation' is synonymous with 'regimentation' and so translations are not meaning-preserving and his argument breaks down, on the other hand translations are apparently meaning preserving (since only in this way would it appear that Putnam has any argument at all) and so the term is not synonymous with 'regimentation' and Putnam has misrepresented Quine's position.

Putnam's argument is unclear, and even charitably construed, fallacious. Quine's ideal language program seems able to avoid charges levelled at it by Putnam by claiming that the theory's use of 'appropriate regimentation' is precise and there's an end to it. Quine's program is nonetheless problematic for the reasons outlined above involving loss of descriptive power.

4.2 *The Irreducibility of Vague Terms*

One might be tempted to respond to the failure of the elimination program by claiming that, though a complete description of the world must include the aforesaid language since such terms are, as a class, ineliminable, their apparent resistance to regimentation is illusory in the following sense — they are reducible to talk in the ideal language. Such an account of vagueness would count as representational since a precise ontology, the ontology of the reducing theory, can be retained; just as the semantic and logical

²⁹ Putnam, *op. cit.*, p. 274.

challenge would be met by the reduction program, so too the ontological challenge. This *prima facie* plausible position does not withstand scrutiny however.

Let's begin at the beginning: what do we mean by reduction? Reduction is generally understood as a relation between theories — in the case to hand, between a vague natural language theory of the world, T_V , and a precise ideal language theory typically said to be provided by the language of science, T_P . Now to say that T_V can be reduced to T_P is to say that one can define the terms of T_V as logical complexes of terms of T_P ; and one can use those definitions to derive within T_P all the statements of T_V as suitably re-written statements of T_P .

Intuitively then the idea is this: vagueness is no threat to classical logic since vague talk is translatable into talk in some precise language without loss of descriptive power; we cannot make do without the descriptive ability made possible by vague language, but this ability can be underwritten by precise language via translations generated by the definitions.

Yet, as Bertil Rolf has pointed out, *no vague phrase can be defined solely by means of precise ones*.³⁰ As we saw in Chapter One, §1.4, precision is inherited; that is, if all the constituent parts of a complex phrase Q are precise then Q is also precise. So it follows that if we have a class of precise phrases we cannot define any vague phrase by means of them (though, in defiance of the fallacious 'infection theory', there is nothing in principle to stop us from defining precise phrases by means of vague ones, so reduction in the opposite direction could be possible!). As an example (varying slightly from that given by Rolf): If all the primitive terms of physics are precise, a sentence such as 'That is red' (suitably contextualised) cannot, by virtue of its vagueness, be defined in terms of physics.

An associated stronger criticism follows from the fact that these definitions or bridge laws are subject to the following requirement:

Each primitive predicate P of the theory being reduced, T_V , is connected with a co-extensive predicate Q of the reducer, T_P , in a biconditional law of the form:

'For all x , Px iff Qx '.

and similarly for all relational predicates.³¹ If the requirement on the bridge laws is not met then reduction fails and the requirement *cannot* be met. Let P be some vague predicate (e.g. 'is red') and Q be a predicate of the precise language (e.g. 'is in microstate m '). The biconditional law must fail since a vague term cannot be co-extensive with a precise term; a term's being vague entails its having border cases

³⁰ Rolf, *Topics on Vagueness*, *op. cit.*, p. 92.

³¹ Tennant, N., *Deep Thought*, manuscript (1991), Ch. 14, p. 16; Nagel, E., *The Structure of Science*, Harcourt (1961).

which is exactly what fails to be the case for precise terms. Co-extension (and consequently attempts at definition too) fails.

Essentially, these two criticisms arise from the fact that vague predicates cannot have sharp translations since translations must preserve vagueness. It seems therefore that vague talk cannot be reduced to talk in some precise language — be it the language of physics, or anything else.

4.3 *The Supervenience of the Vague on the Precise*

Having beaten a retreat from eliminativist and reductionist claims (discretion being the better part of valour for most people), the representationalist might attempt a final stand resting their hopes on supervenience claims.

The use of supervenience to justify a parsimonious ontology is neither new nor uncommon. The paradigm example of supervenience is that relation that is said to obtain between the mental and the physical, and examples of ontological reduction on the basis of this relation, in favour of a physicalist ontology, are not hard to find. Quine, for instance, alludes to the existence of this relation when he says:

If a man were twice in the same physical state, then, the physicalist holds, he would believe the same things both times, he would have the same thoughts, and he would have all the same unactualized dispositions to thought and action. ... It is not a reductionist doctrine of the sort sometimes imagined. ... what it does say about the life of the mind is that there is no mental difference without a physical difference.

Having made what amounts to a supervenience claim, he then continues:

It is a way of saying that the fundamental objects are the physical objects. It accords physics its rightful place as the basic natural science without venturing any dubious hopes of reduction of other disciplines. ... If there is no mental difference without a physical difference, then there is a pointless ontological extravagance in admitting minds as entities over and above bodies ... Thus it is that the physicalist comes out with an ontology of just

physical objects, together with the sets or other abstract objects of mathematics; no minds as additional entities.³²

Adapting this justification of the physicalist's ontology to current circumstances then, the thought is this: if vague natural language could be shown to supervene on some precise language then it would be ontologically superfluous to admit vague entities over and above the precise. It would thus be pointless to admit the world to be vague.

This route to a precise ontology has been suggested by Christopher Peacocke:

Suppose we have a language L containing [all] vague expressions. Then the suggestion that the world itself is not vague is the suggestion that there will be some conceivable language L¹ which contains no vague expressions and which has the following property: it is *a priori* that if two situations agree in all respects describable using the language L¹, then they agree in all respects describable using the language L. This is a form of supervenience.

Given this form of supervenience, Peacocke goes on to define what it is for vagueness to be 'superficial'.

I shall say that the vagueness of a vague expression E is *superficial* if for any language L whose sole vague expression is E, there is some language L¹ containing only sharp [precise] expressions, and such that the descriptions of L supervene on those of L¹ in the sense just explained. ... On this construal, then, the thesis that the world itself is vague would be the thesis that not all (possible) vague expressions have merely superficial vagueness.³³

In other words, *the world is precise if and only if all vagueness is "superficial"* — that is, the world is precise if and only if all possible natural language descriptions supervene (in the above sense) on precise descriptions.

Quite clearly, such a thesis coupled with the view that all vagueness is "superficial" would count as a representational account of vagueness, moreover it might be thought to provide a defence of classical logic. Logic, it might be urged, deals with the fundamental structures of language or the world; it is concerned with 'deep-structure' and not misleading superficial features. Vagueness, in being superficial, is therefore irrelevant to logical issues. The plausibility of this line of defence depends on our preparedness firstly to accept the "superficiality" of vagueness in the technical sense

³² Quine, 'Facts of the Matter', *op. cit.*, pp. 186-8.

³³ Peacocke, C., 'Are Vague Predicates Incoherent?', *Synthese* 46 (1981), pp. 132-3.

described above, and then to accept the move from the "superficiality" of vagueness (that is, the supervenience of the vague on the precise) to the superficiality of vagueness in the everyday sense of a surface feature of language and the world without depth, a *mere* surface feature.

I don't think these ontological and logical claims can be sustained. One objection is that the move from the "superficiality" of vagueness to its superficiality is questionable. Furthermore, the "superficiality" (that is, the supervenience as it is characterised above) of vague language can be contested. If these counter-claims can be established then both this defence of classical logic and this defence of a representational account of vagueness prove to be ineffective.

4.3.1 *Is all Vagueness Superficial?*

Taking the former point first: assuming vagueness to be "superficial", does it follow that it is a mere surface feature of natural language? Well, the motivation for claiming vagueness to be superficial given its supervenience on the precise is most plausibly by way of the biconditional cited above:

(+) The world is precise iff all vagueness is "superficial".

The assumption that vagueness is "superficial" (that is, supervenes in the way described above) in conjunction with (+), establishes the world as precise. The presence of semantic vagueness does not indicate ontological vagueness; vagueness is superficial in so far as it is not ontological.

The supposed superficiality might then be further bolstered by the belief that a precise world is precisely describable, though perhaps only in some ideal language whose construction is guaranteed in principle by the precision of the world. We are now back to a Russellian or Quinean defence of classical logic — the world is precise and ideally describable in precise terms so logic, in being grounded either in the world or an ideal language description thereof, is not threatened by vagueness.

This particular defence of classical logic is weakened by noticing that the claim that a precise world can be precisely described is highly debatable. Brian Garrett, for example, thinks it highly contentious since our conceptual apparatus may be such as to render impossible a precise representation of a precise world. Michael Dummett has expressed a similar qualification, saying that: there being no vagueness in the world "presumably means that the world cannot be such as to require vague ... [language] to describe it, although *we* may be, for various reasons, forced to employ them."³⁴

³⁴ Brian Garrett's point was made in conversation. Dummett, M., 'Common Sense and Physics' in G. Macdonald (ed.), *Perception and Identity*, Macmillan (1979), p. 9.

The claim might be saved by invoking a very attenuated sense of 'can'; if one is prepared to talk not only of an ideal language, but an ideal agent as well then it becomes almost tautological that a precise world can be precisely described (by an ideal agent using an ideal language). All that is being suggested then is that, if the world is precise then there must be a language, no matter how rarefied or celestial, isomorphic in logical structure to the world — echoing the now silent cries of the Logical Atomists. The question then remaining is whether or not a logic grounded in such a language is worth having; that is, given the sense of 'can' invoked to preserve the truth of the claim, it turns out that (all other things being equal) one *can* retain classical logic, but the possibility seems too remote to be of any value. Even Quine, though grounding logic in an ideal language, took his ideal to be of this earth!

The retreat to an ontological grounding of classical logic may prove to be equally problematic since (+) can be challenged. By suggesting that the world is precise since all semantic vagueness supervenes on the precise, one is in effect displaying a view of ontology whereby all that there is is what there is at the subvening level; more particularly, it is to suggest that ontological accounting need only take place at that level of description sufficient to fix the way the world is.

However, assume the supervenience relation holds; thus (it is *a priori* that) there is no change in the vague description of the world without change in the precise description. There is a sense, then, in which the ontology of the precise description might be said to be "fundamental"; the way the world is at this level fixes the way it is at higher supervenient levels. Yet, even though the higher level is fixed — even though we can then say *that* it is determined, we cannot say *how* it is. A description of the world at the subvening level is, I claim, not a complete description of the world and, for this reason, to say what the "determiners" are is not to exhaust what there is, unless one redefines what is meant by 'ontology'.

Ontological accounting need, arguably, only take place at the level of a *complete* description, however descriptions at the subvening level are incomplete.

The view that I am opposing surfaces in different guises in many current discussions in analytic philosophy. It is the general view that, were β -talk to supervene on α -talk, then any complete α -description is thereby complete in regard to β -facts (and so β -ontology is redundant).

Take, for instance, the following claim of Kripke's:

Although the statement that England fought Germany in 1943 perhaps cannot be *reduced* to any statement about individuals, nevertheless in some sense it is not a fact 'over and above' the collection of all facts about persons and their behaviour over history. The sense in which facts about nations are

not facts 'over and above' those about persons can be expressed in the observation that a description of the world mentioning all facts about persons but omitting those about nations can be a *complete* description of the world, from which facts about nations follow. Similarly, perhaps, facts about material objects are not facts 'over and above' facts about their constituent molecules.³⁵

The old reductionist programs of the past, having met with failure, are unable support these claims for completeness. Recent, more sophisticated apologists, have sought to shore up Kripke's claim by means of supervenience — facts about nations are said to supervene, in some sense, on facts about people; facts about material objects are said to supervene on facts about their constituent molecules. That is how completeness is said to be guaranteed — by means of some notion of supervenience.

My objection is that, though the fate of nations is determined by what people do, a nation's fate is *not* described by describing the fate of its individuals; a description mentioning only persons is incomplete. Similarly, a description mentioning only facts about molecules is incomplete.

If, indeed, facts about nations were deducible from a description including all facts about persons, then we could agree that a description at the level of persons was complete; that is, facts about nations would not be facts over and above those about persons. In stating all the facts about persons no new content is given by stating facts about nations if all the facts about persons jointly entail all the facts about nations. Conversely, if no new content arises in stating facts about nations over and above those already stated about persons, then facts about people entail facts about nations; that is, if a complete description can be given solely in terms of persons then facts about nations must be deducible from facts about those persons.

This is simply an application of the following principle, which I take to express a preanalytic truth about the relationship between entailment and content,

E: A entails B if and only if the content of B is included in the content of A.³⁶

³⁵ Kripke, S., *Naming and Necessity*, Basil Blackwell (1980), p. 50.

³⁶ Thus, however one subsequently analyses 'entailment' and 'content', the analysis should respect this equivalence.

For more details see: Routley, R., *Exploring Meinong's Jungle and Beyond*, Departmental Monograph #3, Philosophy Department, Research School of Social Sciences, Australian National University (1980), pp. 935 ff. (This material is a reprint of his 'Ultralogic as Universal?', *Relevance Logic Newsletter* 2 (1977), pp. 50-97 & 138-75.)

The principle is also endorsed by Ackermann who analyses it as a claim about his "rigorous implication" rather than entailment *simpliciter*; cf. Ackermann, W., 'Begründung einer strenger Implikation', *Journal of Symbolic Logic* 21 (1956): 113-28.

And Frank Jackson implicitly endorses E in describing the *entry by entailment* thesis: the one and only entry ticket into a story is by being entailed by that story. As a special case of some interest to Jackson, "a putative psychological fact has a place in the materialist's world view if and only if it is entailed by the physical story about the world. The one and only way of getting a place is by entailment." Cf. Jackson, F., 'Armchair Metaphysics', pp. 5 & 13-14, to appear in: Neander, K. & Ravenscroft, I. (eds), *Prospects For Intentionality*, Working Papers in Philosophy — II,

For example, given that 'p & q' entails 'p', no new content is given in asserting 'p' over and above that given in asserting 'p & q'. Any description including 'p & q' cannot be incomplete in respect of 'p'.

Of course, we may not know that B merely by knowing that A, since we may not know the relevant entailment; thus, a description including 'p & q' may appear incomplete in respect of 'p', were one blind to the fact that the former entails the latter. However, the extent of such epistemic blindspots will merely reflect the extent to which we fail to be logically omniscient; this epistemological point is something over and above claims regarding completeness.

So, given E, claims to completeness require that the supervening facts follow by means of entailments — entailments which need to be established.

(Of course, there is an opening here for someone to say that, though the relevant supervenience claim warrants the claim for completeness, this warrant does not proceed via entailments. That is, they might reject E. Since I am not aware of anyone pursuing this line, I shall foreclose on this option here and now; principle E, regardless of how one goes on to analyse the constituent notions of 'entailment' and 'content', is to be taken as given.³⁷)

Can supervenience ground entailments? Well, according to one prominent school of thought it can. Advocates of the modern functionalist account of mind (e.g., Frank Jackson and David Lewis) argue that the sense in which the psychological supervenes on the physical is sufficient to show that the psychological story about our world is entailed by the physical story about our world. Their argument in support of this generalises to an argument which purports to show that, wherever β -talk supervenes on α -talk, the totality of β -truths are entailed by the totality of α -truths — so long as the supervenience relation entails necessary coextension (which most do³⁸).

The argument proceeds as follows. Suppose that any β -description of a situation supervenes on the α -description of that situation, at least in the sense of entailing the necessary coextension of the α -description with the β -description. But then, for any β -description, B, there is some α -description, A, such that, necessarily, B is coextensive with A. As a consequence, every world at which A is true is a world at which B is true. So, by the classical conception of entailment (the impossibility of A's being true whilst B is false), A entails B.

Dept. of Philosophy, RSSH, ANU (1993); cf. also 'Finding the Mind in the Natural World', typescript (1993), read to the *Wittgenstein Conference*, Austria, 1993, pp. 8 ff.

Of course, Routley, Ackermann and Jackson all have something different in mind when they speak of entailment and content, nonetheless principle E is accepted.

37

To this extent, Jackson, Routley and I are in agreement.

38

Cf. Bacon, J., 'Supervenience, Necessary Coextension, and Reducibility', *Philosophical Studies* 49 (1986): 163-76, esp. p. 172.

What this argument serves to show is that a classical conception of entailment is sufficient to show that most supervenience relations can warrant the required entailments.³⁹

Truths about persons may fix truths about nations, but it remains an open question whether or not this relation is sufficient to ground the required entailments; similarly, it is an open question whether or not truths about molecules entail truths about material objects. Those, like myself, who see the classical account of entailment as unduly weak (for all the familiar reasons — e.g., any necessarily false claim classically entails anything whatsoever, which seems highly counterintuitive) will remain unconvinced by the above argument that supervenience *is* sufficient to ground entailments; the burden of proof will not be relieved by use of the above argument.⁴⁰ Those classically minded will see supervenience as sufficient. However, until this dialectic has run its course the issue remains unresolved; positive claims for completeness based on supervenience are conditional upon this important logical debate being decided in favour of classical entailment. No reason will have been given as to why the believer in non-classical entailment should accept the subvening description as complete.⁴¹

Let us turn then from general considerations of descriptive completeness to the particular case at issue — the claim that the world is precise since a complete description can be given in purely precise terms, and this since the vague supervenes, in Peacocke's sense, on the precise.

Assuming vague language supervenes on precise language in the way described by Peacocke, then a complete description can be had at the level of the (subvening) precise language if and only if the required entailments between the vague and the precise can be established. Now, the supervenience relation under consideration certainly entails necessary coextension and so the above considerations to do with entailment apply here; that is, the entailments themselves can be established if the entailment relation is as classically conceived.

³⁹ It also seems highly plausible that any reasonable non-classical account of entailment, stepping up to some variety of relevant entailment (i.e., by E, working with a more intensional account of content), will be strictly stronger than any supervenience relation. If this intuition is well founded then, not only is the classical conception sufficient to show that supervenience can ground the required entailments, it is *necessary*. In this event, all and *only* those who accept the classical conception will be in a position to justify the claim that a complete description of the world can be had at the subvening level (and thus claim ontological reduction of the supervenient).

⁴⁰ The parsimonious metaphysics would depend crucially on one's choice of logic. Thus, though Jackson and I agree that claims for the completeness of a story or theory amount to claims regarding the truth of the relevant entailments, I do not think that supervenience grounds the required entailments and thus cannot ground claims about completeness, whereas Jackson does since he accepts the classical conception of entailment.

⁴¹ For this reason, I also think that arguments for a physicalist ontology via supervenience are lacking. The ontological reduction simply doesn't follow. If we want to do more than just provide a description which fixes the way the world is and actually give a complete description of how it is then, lacking the appropriate entailments, any ontological commitment present at the supervenient level remains.

Yet, in the absence of any further argument, those who think that objections to the classical conception of entailment are compelling are left wondering how supervenience can ground the required entailments.⁴² Lacking these entailments, a precise description cannot be claimed to be complete, and so ontological accounting at the supposed subvening level cannot be said to provide an exhaustive list of what there is.

Those of us, then, who remain sceptical about (+) will be sceptical of the move from the "superficiality" of vagueness to its superficiality, and will therefore deny that classical logic has been defended via supervenience; furthermore, the claim, via (+), that a supervenience account of vagueness is representationalist, is found wanting.

4.3.2 *Is all Vagueness "Superficial"?*

From the foregoing considerations, we see that those who take the supervenience account to offer an acceptable defence of classical metaphysics and/or logic seem committed to a classical account of entailment — even if all vagueness *did* supervene, in Peacocke's sense, on precise language (that is, even if all vagueness *was* "superficial") further argument would be needed to show that the world was precise, unless 'entailment' is as classically described. The supervenience claim as it stands is, therefore, of limited use in the defence of classical logic or a representational account of vagueness.

Though this restriction on the ontological reduction to a purely precise world may be thought unproblematic by many, the supervenience claim, as defined by Peacocke, is itself a cause for concern.

Recall what the supervenience claim amounted to: all vague language supervenes on precise language. That is: suppose we have a language L containing all vague expressions (e.g. natural language, including all possible extensions thereof). Then the suggestion is that there will be some conceivable language L¹ which contains no vague expressions and which has the following property:

⁴² If the intuition expressed in n. 39 is correct, then no further argument could establish the entailments; they could not to be had unless entailment is as classically conceived. As a consequence, non-classical entailment theorists are not left wondering how supervenience could ground the required entailments ... they will argue that it *cannot*.

Of course, all can agree that, if such entailments were available, then not only would the relevant claims to descriptive completeness be validated, but so too would the claims to supervenience. If two situations agree in all respects describable using precise language and if being in a particular precisely described state *entails* being in some particular vaguely described state, then it is *a priori* that the two situations agree in all respects describable using vague language. For example, if particular state-descriptions using supposedly precise micro-physical predicates like 'reflects light of wavelength *n* nanometres' entail particular state-descriptions using vague colour-predicates then colour-talk supervenes on wavelength-talk. This result holds in general; that is, if α -talk entails β -talk then β -talk supervenes on α -talk.

it is *a priori* that if two situations agree in all respects describable using the language L^1 , then they agree in all respects describable using the language L .

The supervenience claim is thus an existential claim, asserting the existence of a precise subvening language.

Just what this precise language will be is left open. To convince the sceptic that there will be some conceivable language which contains no vague expressions and which can underwrite a supervenience relation, advocates of such a claim must either argue for this as a matter of conceptual necessity, that is, that to assume otherwise leads to incoherence, or argue for it by generalizing from some particular instance. It is hard to imagine an argument as strong as the former which is not circular or question-begging in some way. An argument of the latter type is, I think, the argument most usually endorsed, if only implicitly. The almost universally agreed candidate to instance the existential claim is the language of science. If this is not how the claim is supported then we are owed some other reason for accepting it as even plausible.

Suppose then that the existential claim is instanced by a language of science. Is it the case that it has the properties described above? In other words: (i) Is a language of science precise? and (ii) Is it *a priori* that, if two situations agree in all scientifically describable respects then they agree in all respects describable in vague language?

Assuming for the present that a language of science *is* precise, let's begin with the second question: Is it *a priori* that, if two situations agree in all scientifically describable respects then they agree in all respects describable in vague language? The answer is 'No'. The first argument for this negative conclusion is by way of counterexample; the second, inconclusive argument is by way of a *reductio*.

Let's begin with the counterexample. Consider the vague predicate 'is blue', and assume that we have two objects a_1 and a_2 that agree in all respects describable in the language of science (e.g. both a_1 and a_2 reflect light of 480 nanometres wavelength, etc). Assume also that a_1 is blue. The question we need to address is whether or not it is *a priori* that, given these assumptions, a_2 is blue. Does it follow that anyone who understands the concept 'blue' must describe a_2 as blue? Well obviously not. It is not impossible to suppose that, having understood the concept 'blue', one might still wonder whether these two objects, both of which reflect the same wavelength of light, etc., may have turned out to be different colours. It is not part of the meaning of 'blue' that any two things described as blue have the same specific microphysical property; that is something we have learnt from scientific enquiry — it is nomologically necessary, not *a priori*.

Peacocke offered 'many' as an example of a vague expression whose vagueness is "superficial"; it supervenes on cardinality quantifiers. There cannot, he said, for

example, be two situations with respect to one of which some sentence of the form 'Many F's are G' is true and with respect to the other is false, if the two situations have the same number of F's being G.⁴³ I agree that 'many' certainly does fit the supervenience account. The concept of number or cardinality *is* caught up in the analytic explication of or the understanding of 'many'. 'Tall' may also fit this account — it seems that height has a role to play in the analytic explication of the meaning of the term 'tall', so it is *a priori* that any two persons of the same height (a physical quantity) are both tall or neither is. So I agree with Peacocke that the vagueness of *some* expressions supervene on precise ones; that is, that the vagueness of some expressions is (as Peacocke puts it) "superficial". However, not all vague expressions are like this. Colour concepts, as we have just seen, are not. In general, any term which is connected to physical terms via physical laws will not supervene.

(Thus we have what amounts to an argument against the view that a complete description of the world can be given in precise scientific terms. If it is conceptually possible that colour ascriptions, for example, could vary independently of micro-physical ascriptions then there cannot be entailment relations from micro-physical descriptions to colour descriptions, and so descriptive completeness fails. A more general way to see the point is as follows: if a precise scientific description of the world entails some vague description then vague language must supervene on precise scientific language. But vague language does not supervene on precise scientific language, so there cannot be the required entailments needed to ensure claims of descriptive completeness.)

A further problem that might beset those who claim that all vague language supervenes on precise language is that it would need to be shown that supervenience of the type described by Peacocke does not entail reduction of the vague to the precise. If this could not be established then the possibility would arise that the supervenience claim was incoherent by virtue of the impossibility of reducing the vague to the precise (this latter claim regarding the impossibility of reduction having been established in §4.1.2.) This worry is not unmotivated either. There has been considerable debate in the ever-growing literature on supervenience as to whether or not reduction does follow from supervenience.⁴⁴

Note that I am not claiming here that the supervenience relation does *not* obtain; merely that those who claim it *does* are obliged, via the impossibility of reducing the vague to the precise, to establish that one can have supervenience without reduction.

⁴³ Peacocke, *op. cit.*, p. 133.

⁴⁴ See, for instance: Kim, J., 'Supervenience as a Philosophical Concept', *Metaphilosophy* 21 (1990): 1-27, esp. pp. 17 ff; and Bacon, J., 'Supervenience, Necessary Coextension, and Reducibility', *op. cit.*

A further point to note is that I have only been evaluating supervenience *as characterised by Peacocke*. It may be that, amongst the many different relations qualifying as supervenience, one can be found that does not lead to reducibility and which is such that the vague does, in this specific sense, supervene on the precise. However, even were some amended version of supervenience successful, it would be a hollow victory unless one could show that this supervenience relation justified the claims of ontological parsimony discussed in §4.3.1.

Another way in which the instanced supervenience claim could be undermined is if the answer to (i) was anything short of a resounding 'Yes'; that is, if the supposed precision of scientific language was questionable. In short, my claim will be that there may be doubts as to the precision of scientific language and so, for those who depend on an affirmative answer, the burden of proof remains.

The implications for such a result however go well beyond supervenience accounts of vagueness. We have seen assumptions regarding the precision of scientific language invoked in both the eliminativist and reductionist accounts. It seems to me that it is the perceived precision in our best theories about the world that often, as a matter of psychological fact, motivates defences of the kinds considered so far in this chapter, and so raising doubts concerning science as a safe refuge from vagueness has far reaching effects.

4.4 *The Precision of Scientific Language*

The world is completely describable in an ideal language containing only precise terms. The paradigm of such a language is a language of science. These are the claims that we have heard echo throughout the preceding discussion. It is certainly commonly, if not universally, accepted that a language of science aims ideally to be precise. But *is* a language of science precise, and if not, could it ever *be* completely precise?

In discussing these questions I do not claim to have any bold new insights to offer, yet the bringing together of various lines of argument already extant in the literature may serve to describe the current state of play surrounding this issue.

An initial problem for the above thesis arises from Russell's claim that *all language* is vague. This very strong claim would deliver a negative verdict on the above questions but, as we have seen, this claim cannot be sustained.

Further doubts concerning the precision of scientific discourse might arise from the following argument due to Benjamin and Burks. They argue that *all empirical*

language is necessarily vague, so, since science must have an empirical vocabulary, the language of science is vague.⁴⁵ Must empirical vocabulary be vague and must it be considered part of the language of science?

The claim that empirical vocabulary must be vague is premised on the idea that empirical vocabulary is ostensively defined and, since ostensively defined terms are necessarily vague, empirical terms are necessarily vague. Ostensively defined terms are necessarily vague since they are defined by reference to finite positive and negative samples of objects which are taken respectively to instance the correct application of the term and to counter-instance its application, yet between the instances and counter-instances there are or could be cases which will count as border cases. For example, 'red' it is claimed is taught ostensively by our being presented with some determinate instances and counter-instances as paradigms and our then applying the term to future cases by means of their resemblance to these paradigms. In this way border cases will arise since it may be indeterminate whether or not some particular case resembles any of the paradigm cases. Stephen Körner and Quine also argue for the vagueness of ostensively defined terms in this way.⁴⁶ The indeterminacy of future reference makes for vagueness.

Rolf's response is that this process of ostensive definition does not necessarily lead to vagueness — as is Haack's, though her response is less convincing⁴⁷. He offers the following counterexample. Consider a number-theoretic predicate 'twiddle' ostensively defined by offering 1, 4, 9, 16, 25, 36, and 49 as twiddles (to which we could add the numbers 2, 5, 10, 15 and 82 as non-twiddles). "If the student is intelligent enough, he will guess that 64, 81 and 100 are also twiddles. ... The guess may be right or wrong. What is important is that he guesses 'at' a precise content which he may be unable to express."⁴⁸ Guesses as to the correct application of the ostensively defined term to future cases go beyond the cases presented yet the guesses might "aim at perfectly precise contents". Though lots of ostensively defined terms are vague this is not necessarily the case.

This counterexample also puts paid to attempts by Wright to establish the vagueness of ostensively defined terms via the notion of 'observationality'. Wright's general argument is that ostensively defined terms are observational, observational terms are tolerant and terms that are tolerant are vague; thus ostensively defined terms are vague. ('Tolerance', perhaps the most important property underlying the sorites paradox, is explained as follows: a term is tolerant if there is some degree of change in

45 Benjamin, A.C., 'Science and Vagueness', *Philosophy of Science* VI (1939): 421-31; Burks, A.W., 'Empiricism and Vagueness', *Journal of Philosophy* 43 (1946): 477-86.

46 Körner, S., *The Philosophy of Mathematics*, Hutchinson (1960), p. 167; Quine, W.V.O., *Word and Object*, MIT (1960), pp. 125-6. These references are due to Rolf.

47 Haack, *Deviant Logics*, *op. cit.*

48 Rolf, *op. cit.*, p. 96.

that to which the term applies, which is too small to make any difference to the application of the term; for example, the predicate 'bald' is tolerant with respect to the loss of a single hair, 'red' is tolerant with respect to a change of 1Å in the light-reflective property, and so on.⁴⁹) Rolf concedes that the notion 'observational', though notoriously difficult to pin down, can be given a reasonable interpretation so that any observational term is vague, however it is then patently false that ostensive terms are observational — the above example of defining 'twiddle' ostensively will be a counterexample.

So not all ostensively defined vocabulary is vague. However, since some is, the way seems clear for Benjamin and Burks to argue that science must be infected with vagueness since either: it must make use of empirical terms some of which are vague or (using the Wright variation) it must make use of observational terms all of which are vague. Haack, though not explicitly considering this move, seems to advocate a retreat to the quantitative, theoretical terms of mature science.

Such a retreat seems necessary if a precise language of science is to be had since it seems unavoidable that the practice of science makes use of at least some vague language in its experimental phase and during phases of conceptual innovation, whether this be innovation in mathematical concepts — whose language is considered as part of the language of science — or innovation in physical concepts. Thus, says Rolf:

Platonistically speaking, conceptual innovation involves the invention of new predicates and names for newly discovered properties and entities. In this process, it is more important to classify the clear cut cases than to delimit the predicates of the classificational system so that they have no borderline cases. It would be a happy coincidence if such a process gave rise to precise predicates [and names]. Therefore, as long as discovery and conceptual innovation are among the practices of a science and as long as there is more for it to discover, precisification will always lag behind.

This echoes the views of Benjamin, expressed more than five decades ago:

If the place of vague ideas in science is denied, the mystery of scientific discovery becomes more than ever a mystery. For discovery occurs only as a result of the firm conviction that no scientific advancement is possible without an unwavering faith in the cognitive value of hunches, vague insights and intuitions.⁵⁰

Thus we see an obvious defence of the claim that the world can be completely described in a language of science — exclude those parts of scientific discourse that

⁴⁹ Cf. Wright, C., 'On the Coherence of Vague Predicates', *Synthese* 30 (1975), pp. 337 ff.
⁵⁰ Rolf, *op. cit.*, p. 101-2; Benjamin, *op. cit.*, p. 430.

seem vague, whilst maintaining descriptive completeness. This narrowing of what we mean by 'a language of science' presumably is not thought to threaten us with scepticism (a position one can always adopt to that degree required to maintain descriptive completeness — though obviously at some cost; cf. §4.1) since what is jettisoned is merely the preamble or 'rough draft' required for a full blown scientific description of what is. However, this suggests that what is being offered in defence of the claim that there is a precise complete scientific description of the world is a promissory note to the effect that *when science is complete* there will be a precise complete scientific description of the world. On what grounds does this defence rest other than sheer (question-begging) faith in precision?

There is another conceivable, though, to my mind, misguided line of attack on the claim that a language of science provides the means for completely describing the world in precise terms which threatens even the weakest of defences that completed science will eventually provide a precise description of the world. It is a line taken by Kathinka Evers concerning Quantum Mechanics, and its critical evaluation serves as a lesson both in the danger of misunderstanding what QM has to say and in the need to clearly disambiguate the many senses of 'vague' and 'precise'.⁵¹

The threat does not come from showing that the precision claim is false; it is far more modest than that, and, in this regard, would be far easier to establish. It merely aims to raise sceptical doubts concerning the possibility of a precise scientific description from within a contested part of science itself — doubts, though, which advocates of the precision claim *must* quash since they are the ones making the dogmatic assertion. It is, in other words, merely an attempt to shift the burden of proof back onto those asserting the above claim; as such, it does not depend on any particularly strong claims though its consequences, were it successful, would be very strong.

The sceptical attack is, as I said, motivated by considerations in perhaps the most disputed current scientific theory — Quantum Mechanics. The fact that the theory *is* so disputed legitimately plays into the hands of those sceptics challenging the claim that a language of science is precise. *If* any of the disputes surrounding QM involve the questioning of the assumption of precision then the sceptics case would seem sound; Quine *et al* could not then assume a language of science to be precise without begging the question against those disputing this assumption. Evers certainly sees QM this way (though she is interested in the ontological aspect — cf. n. 51); having defined her use

⁵¹ Evers uses QM to motivate a discussion of ontological vagueness in *Plurality of Thought*, Library of Theoria #18 (1991), §6.2.1. Her concern with the issue of precision at the ontological level can either be seen as a direct challenge to the view that the world is precise or, as I am suggesting here, an indirect challenge to the view that the world can be completely described in a precise language of science and thereby a challenge to the belief in ontological precision.

of the terms 'determinate' and 'indeterminate' as synonymous with 'precise' and 'vague' respectively, she says:

The classical ... picture of the material universe depicts it as essentially determinate. ... If waves are fundamentally, materially real, then indeterminacy could be regarded as a fundamental feature of material reality by virtue of the "unsharp" and "boundless" nature of waves. Similarly, ontological indeterminacy is arguably introduced if quantum mechanics posits imprecise positions and momenta of particles. ... According to Heisenberg, it follows from the wave-particle duality that it is impossible to ascertain simultaneously the determinate position and the determinate momentum of quantum objects, such as photons and fields. In some interpretations, this impossibility is neither a consequence of ignorance, nor a problem of measurement, or definability: some quantum objects do not *have* both a definite [determinate] momentum and a definite [determinate] position — their values are unsharp and fuzzy.⁵²

But Evers' case doesn't withstand scrutiny. In so far as 'determinacy' is synonymous with 'precision', any threat of QM indeterminacy undermines claims to the precision of scientific theory; yet, in so far as QM involves indeterminacy, it is not synonymous with vagueness. There is a hopeless equivocation in her discussion. Firstly, the "unsharp" and "boundless" nature of waves refers to Hanson's entry in *The Encyclopedia of Philosophy* under 'Quantum Mechanics' which describes particles as entities "with ideally sharp coordinates — that is, in one place at one time"; i.e. particles are sharp in the sense of their having an exact point-valued position, whereas a wave "essentially lacks sharp coordinates. It spreads boundlessly ..." But what does this have to do with vagueness; waves are described in QM by their wave functions (probability amplitudes) which are precise descriptions; the sense in which waves are unsharp or boundless is in their lacking exact point-valued position, but this is not a matter of vagueness.

Secondly, and more interestingly, some discussions of QM certainly include claims that *suggest* that the theory requires vague concepts in order to resolve perceived problems to do with the descriptive incompleteness or predictive incompleteness of the theory. For example, Paul Teller says:

[In the case of continuous quantum-mechanical quantities] quantum mechanics provides only descriptions with spread, not point-valued descriptions. ... If quantum-mechanical systems have exact point values, the

⁵² Evers, *op. cit.*, pp. 154-56.

dispersed quantum-mechanical descriptions seriously misrepresent the nature of these systems. ... Physicists sometimes deal with this situation by tacitly giving up the point-value theory and acquiescing in the orthodox theorist's talk of imprecise or "imperfectly defined" positions and momenta.⁵³

He then goes on to say that:

One who believes that continuous quantities take on precise values must judge quantum mechanics to be descriptively incomplete.⁵⁴

And, as Evers points out, Lockwood discusses QM saying that "according to quantum mechanics, particles do not possess precise positions or momenta", whilst Redhead is acknowledged as mentioning the terms 'unsharp' and 'fuzzy' — "What can one say about the value of an observable, call it Q, in QM when the state of the system is not in an eigenstate of Q [a state where measurement of Q yields an outcome that is an eigenvalue]? We will distinguish three general sorts of answer that have been given to this question:

View A: Q has a sharp but unknown value.

View B: Q has an unsharp or 'fuzzy' value.

View C: The value of Q is undefined or 'meaningless'." ⁵⁵

However, further reading through these discussions reveals that "precise" is used in the sense of exact. In the above quotation from Teller, we see him equating the abandonment of "exact point values" with the acceptance of "imprecise" values, which themselves are associated with "descriptions with spread". Redhead goes on to say, with regard to View B, that "the terminology of unsharp or 'fuzzy' values is prevalent in some elementary textbooks on QM. But what is really intended is that the observable Q does not possess a *value* at all. What the QM system does in reality possess is a *propensity* or *potentiality* to produce various possible results on measurement [from some range of values], in respect of the observable Q." It is the dispersion or inexactness of values that makes for the absence of precise values. What is being discussed is the idea that particles have position, etc., that can at best be described to lie within some interval — that is, they take on inexact values with the potential to take on some exact value on measurement — though, of course, this interval itself is precisely describable. Quine himself feels no threat from QM: "[p]resumably regions are always wanted rather than single points — sometimes because of indeterminacy at the quantum

⁵³ Teller, P., 'Quantum Mechanics and the Nature of Continuous Physical Quantities', *The Journal of Philosophy*, 76 (1979), pp. 349-50.

⁵⁴ *Ibid.*, p. 353.

⁵⁵ Lockwood, M., *Mind, Brain and the Quantum*, Blackwell, Oxford (1989), p. 339; Redhead, M., *Incompleteness, Nonlocality, and Realism*, Clarendon, Oxford (1987), p. 45.

level and sometimes for more obvious reasons ..."⁵⁶ These disputes over quantum mechanical indeterminacy do not have to do with vagueness as we have defined it.⁵⁷

Thus, I think, Evers is mistaken when she sees QM as constituting a threat to the claim that the world is precise; there is no implicit threat to the claim that the language of science is precise. QM has yet again been used where it had nothing to say and the popular confusion between vagueness and inexactness has again surfaced.⁵⁸

Another argument suggested by remarks of Richard Routley regarding intensionality would however, if sound, show that, even if we only considered the language of mature scientific theory, excluding the observational base on which theories are assessed, some scientific language would be vague. We start by noticing that intensional statements are often vague. They are therefore in need of replacement analysis or junking (in the various ways canvassed previously in this chapter).

Carried through this [analysis or junking] in fact appears to throw out not only most but virtually all of science, so far as science, according to empiricist belief, depends upon observation. For predicates of observation, functors like 'sees', 'observes', 'hears [that]' etc. are ... vague, imprecise, suspect, etc. But true statements about what people have observed ... are the major, and according to empiricists, the entire evidence for the statements of science. *If those statements ... are vague and imprecise and suspect, then what they are evidence for ... can hardly be much better.*⁵⁹ [my italics]

The suggestion is then that the language of mature scientific theory (that is, the statements of scientific theory independent of the statements that constitute evidence for it) is infected with vagueness via the vague grounding of science and so to claim that the world can be completely described in a language free of vagueness "is, at bottom when followed through, a scientifically dressed up form of scepticism".⁶⁰

I don't think this argument is valid. It simply does not follow that, if statement A is vague, then what it is evidence for, B, is vague. Jill's being roughly the same height as me constitutes evidence for her being between 165cm and 175cm in height. Though the evidential basis may be vague, that which it is evidence for is not. But what if the

⁵⁶ Quine, W.V.O., 'Facts of the Matter' in R. Shaban & K. Merrill (eds), *American Philosophy from Edwards to Quine*, Oklahoma (1977), p. 191.

⁵⁷ Other discussions where 'vague' is used in the sense of 'inexact' or 'unspecific' in interpreting QM indeterminacy include: Glymour, C., 'Determinism, Ignorance and Quantum Mechanics', *Journal of Philosophy*, 68 (1971), esp. p. 746; Krips, H., *The Metaphysics of Quantum Theory*, Clarendon Press, Oxford (1987), esp. Ch 1, §6.

⁵⁸ The confusion, discussed in Chapter One, is so prevalent that Roy Sorensen has devoted a whole article to disambiguating 'vague'; see his 'The Ambiguity of Vagueness and Precision', *Pacific Philosophical Quarterly*, 70 (1989): 174-83.

⁵⁹ Routley, R., *Exploring Meinong's Jungle*, *op. cit.*, p. 682, n. 2.

⁶⁰ *Ibid.*

entire evidential basis for B is (everywhere) vague? If A is vague and A is the *entire* evidence for B does it follow that B is vague? Again I think the answer is 'no'. The entire basis for claiming Jill to be between 165cm and 175cm in height might be the two claims consisting of my being very close to 170cm in height and her being roughly the same height as me.

What Routley is asking us to accept is the following general schema:

(ϵ) If A is suspect and A is the entire evidence for B then B is suspect.

The relation between A and B — 'is the entire evidence for' — is an epistemic one; as such, epistemic deficiencies in A carry over to B. For example, if A is suspect in the sense of being an unreliable claim to knowledge and A is the entire evidence for B then yes, B is suspect in the sense of being itself an unreliable claim to knowledge. Yet the "suspect" nature of A that is of interest to us in this context is its vagueness and this is a logical rather than epistemological feature. Schema (ϵ) is ambiguous and, though it seems plausible when construed epistemically, I think it fails when construed more logically.⁶¹

What the argument does bring into sharp relief, regardless of its validity, is that the unqualified rejection of vagueness is devastating. In science, vague observational bases have at least instrumental value *qua* vague observational bases, just as in philosophy where arguments that make use of vague language have at least instrumental value in so far as they might support some conclusion. Of course, where that conclusion is to the effect that vague language itself is valueless, as in the arguments of Unger and Wheeler, a self-defeating incoherence ensues. The fact that science and philosophy are underpinned by language which is oftentimes vague shows just how absurd it is to ban such talk outright. Even were a final complete description of the world possible in precise terms, to relegate logic to the realm of the precise on these grounds undermines the very possibility of reasoning logically to support one description over another. *A logic of vagueness is at the very least instrumentally valuable, even by the lights of those who maintain that a complete description of the world can be given in precise terms.*

⁶¹ The argument might go through given a now discredited theory known as 'operationalism' described by A. Flew in his *Dictionary of Philosophy* as "the theory in the philosophy of science that holds that all physical entities, processes, and properties are definable in terms of the set of operations and experiments by which they are apprehended ... first propounded by P.W. Bridgman in *The Logic of Modern Physics* (1927)".

4.5 *Summary*

In this chapter I have tried to evaluate the following response to the problem of vagueness: though vagueness is a semantic phenomenon it does not entail any revision of the classical metaphysical view that the world is not vague, nor does it necessitate any revision of classical logic, including classical metatheory. The view of vagueness as having to do only with representations was supported by arguments that purported to show that the world was in fact completely describable in a precise ideal language or, at least, that our interests were best served by supposing this to be the case. The precision of the world then followed from the availability of a complete precise description thereof, and the superfluous nature of vague language as a whole was taken to relieve any tension between the vagueness of natural language and classical logic.

The retention of classical logic and classical metaphysics thus all depended on the claim that there exists a complete precise ideal language description of the world. My conclusion is that such an existence claim can only be defended if one is prepared to pay the price — the abandonment of many common-sense ways of talking and thinking about the world. If much of the data of ordinary discourse is declared irrelevant and one digs one's heels in, such a view can perhaps be maintained but I think the costs are simply too high in any case. When all is said and done the canonical language, with all the logic and metaphysics that hangs on it, would be no closer than the celestial realm to which Russell banished it. This chimerical "ideal language" is too remote to have any use other than to support a theoretical stance from which some will never waver.

Chapter Five

Recent Developments of the Representational Theory — II Enlightened Representationalism and a Logic of Vagueness

In this chapter I want to turn to a post-Russellian representational account that, whilst accepting *a priori* arguments for the precision of the world as sound, is nonetheless critical of ideal language theories and proposes a logic of natural language encompassing vague terms; just how serious a revision of logical theory is required?

Russell thought that vagueness demonstrated the inapplicability of logic to natural language. Indeed, it demonstrates the inapplicability of classical two-valued logic to natural language but Russell, rather than revise classical two-valued logic or natural language, simply restricted the scope of logic to an ideal precise language that was unobtainable in principle and located in some 'celestial realm'. Russell's view of logic as utterly divorced from natural language analysis has, as already noted, been viewed with suspicion in more recent times and two distinct responses have dominated:

— retain classical two-valued logic, both classical logic and its metatheory, and regiment natural language to obtain an ideal language that is constructible in principle; that is, revise the data;

— accept natural language as being in order as it is and modify classical two-valued logic to enable the evaluation of arguments involving vague language; that is, revise classical metatheory and/or classical logic.¹

The first of these responses has just been described in Chapter Four. It is to the second that we now turn. This general response is due, in part at least, to the questioning of 'ideal language' doctrines. As a result of seeing natural language as being in order as it

¹ I have, up to now, been using 'classical logic' to cover a multitude of sins including both classical logical theory (characterised by the classical account of validity and logical consequence) and classical metatheory. Distinguishing them becomes important in the ensuing discussion so I shall now reserve the term 'classical logic' for just the logical theory; its metatheory will be considered as separate.

is, logicians have been more willing to attempt to systematise natural language argument. In so doing they have been forced to take vagueness (a feature of natural language) more seriously and describe how logic can accommodate reasoning with vague terms.

This developing acceptance of vague language as being in order as it is was exemplified by the rise of ordinary language philosophy, in particular Wittgenstein's *Philosophical Investigations*.

One might say that the concept 'game' is a concept with blurred edges.—"But is a blurred concept a concept at all?"—Is an indistinct photograph a picture of a person at all? Is it even always an advantage to replace an indistinct picture by a sharp one? Isn't the indistinct one often exactly what we need?²

With this in mind I now want to investigate a logic which advertises itself as being a logic of representational vagueness. Though Russell, himself a representationalist, thought a logic of natural language to be ill-conceived, more recent theorists who are in agreement with Russell as to the nature of vagueness have, contra Russell, described such a logic. To the extent that the representational view now being considered accepts vague language as being within the scope of logic I shall describe it as an *enlightened representational theory of vagueness*.

5.1 *Supervaluationism*

The bulk of this chapter will be taken up with an explication and analysis of that revision of classical two-valued logic advocated by supervaluation theorists — that is, theorists who maintain that a logic of vagueness can be obtained by an appropriate reinterpretation of the "presuppositional languages" of Van Fraassen.³

We shall be focussing on supervaluationism as a logic of representational vagueness for two reasons: firstly because advocates of enlightened representationalism commonly claim supervaluationism as their preferred logic (e.g., David Lewis); and secondly because without the metaphysical thesis that the world is sharp a supervaluationist logic of vagueness would seem to have little or no philosophical support. That is to say, there seem to be positive grounds for discussing supervaluationism in the context of enlightened

² Wittgenstein, L., *Philosophical Investigations*, Basil Blackwell (1958), § 71.

³ Van Fraassen, B.C., 'Singular Terms, Truth-Value Gaps, and Free Logic', *Journal of Philosophy* 63 (1966); 'Presupposition, Implication and Self-reference', *Journal of Philosophy* 65 (1968); 'Presuppositions, Supervaluations and Free Logic' in K. Lambert (ed.), *The Logical Way of Doing Things*, Yale (1969).

representationalism — namely, enlightened representationalists in fact often do adopt supervaluationism; and there are negative grounds for discussing supervaluationism in the context of enlightened representationalism — namely, if a supervaluationist approach to a logic of vagueness is to work anywhere it will be in the context of an enlightened representational approach to vagueness.

If, after having read all I have to say on the supervaluations approach, one thinks that the association of supervaluationism with enlightened representationalism is misguided then one might well seek an alternative logic of representational vagueness. However, any threat to the underlying metaphysical thesis that the world is not vague will constitute a challenge to *any* such logic, supervaluationist or otherwise, in so far as it depends on vagueness being merely semantic. So, though I have chosen supervaluationism as my target, any other which advertises itself as a logic of representational vagueness would serve equally as well since it is ultimately (in Chapter Six) this metaphysical thesis that I want to go on to challenge.

As we shall see, there are two approaches the supervaluationist might adopt, the second being an extension of the first. The first and most conservative approach is to suggest that the logic of vagueness is classical though its metatheory is not — that is, the theory of classical logic (its theorems and consequence relation) is adequate though its metatheory, e.g. the Principle of Bivalence, needs revision. The second approach, whilst agreeing with the first on the need to revise classical metatheory, also extends the syntax of classical logic to include a determinacy operator 'D', thereby incorporating classical logic within a conservative extension thereof.

5.1.1 *Motivating and Describing the Supervaluation Theory*

So what is the theory of S(uper)V(aluations) and how does it propose to deal with problems that arise in the context of vagueness? The theory of SV is a theory incorporating both logical and metatheoretical principles and has been proposed most noticeably by M. Przelecki (1969), David Lewis (1970), Michael Dummett (1975), J.A.W. Kamp (1975) and Kit Fine (1975).⁴ A rigorous model-theoretic account of the theory can be found in Fine's paper so I shall simply offer an informal account of the theory sufficient for the understanding of its key features. I want to spend some time trying to motivate the account first and as we proceed the SV account will emerge.

⁴ Przelecki, M., *The Logic of Empirical Theories*, Routledge (1969); Lewis, D., 'General Semantics', *Synthese* 22 (1970): 18-67, reprinted in D. Davidson & G. Harman (eds), *Semantics of Natural Language*, Reidel (1972); Dummett, M., 'Wang's Paradox', *Synthese* 30 (1975): 301-24; Kamp, J.A.W., 'Two Theories About Adjectives' in E. Keenan (ed.), *Formal Semantics of Natural Languages*, CUP (1975); Fine, K., 'Vagueness, Truth and Logic', *Synthese* 30 (1975): 265-300.

Our first assumption will be that every natural language predicate P has: (i) a clear or determinate extension (the set of objects of which P is determinately true); and (ii) a clear or determinate anti-extension (the set of objects of which P is determinately false) that are exclusive. Hence, no natural language sentence is both determinately true and determinately false. Moreover, every precise predicate is such that every object is either in the determinate extension or determinate anti-extension of the predicate — that is, every precise predicate is such that its determinate extension and determinate anti-extension are exhaustive; hence, every precise sentence in natural language is either determinately true or determinately false. So, if we are concerned only with that fragment of natural language which is precise then determinate predicate extensions, and determinate truth and falsity, are exclusive and exhaustive.

However, vague predicates are such that their determinate extension and anti-extension are not exhaustive; there are objects which are in neither. And thus, assuming a full complement of names in the language, there are sentences for which the possibility arises of their being neither determinately true nor determinately false.

The SV theorist then equates 'determinate truth' with 'truth' *simpliciter*, or "supertruth" as they sometimes call it, thus defining a concept of truth by means of which bivalence may be coherently denied.⁵

Given this account of 'truth' the metatheory for the precise fragment of natural language is still classical since, in this limited fragment, truth and falsity are both exclusive and exhaustive. The assumptions cited above which serve to ground this claim are not essential. A supervaluation model structure could be based upon an underlying metatheory that was nonclassical, e.g. intuitionist or paraconsistent — in this sense a supervaluations approach offers a superstructural logic to be used in conjunction with one's preferred logic, but things are made considerably simpler by assuming that where vagueness does not arise the default metatheory is classical.

The metatheory for natural language as a whole, on the other hand, can be seen to be non-classical to the extent that some sentences are neither true nor false. Now given this non-classical aspect to the metatheory we may ask whether or not classical logic (the classical theorems and consequence relation) remains intact. For instance, if a sentence S and its negation $\sim S$ are indeterminate what of their conjunction and disjunction? Are they likewise indeterminate; are the classical laws of Excluded Middle and Non-Contradiction still theorems?

⁵ Problems associated with its denial were discussed in §2.1 — the epistemic theorist arguing that bivalence must hold for any truth-predicate satisfying the T-Schema. Fine himself acknowledges that the equating of truth with determinate truth and falsity with determinate falsity means bivalence fails for this SV conception of truth and so the T-Schema fails for this conception of truth. As he points out though, the T-Schema does hold for that more primitive conception of truth by means of which SV-truth is defined. The SV theory is bivalent at bottom though a non-bivalent conception of truth is defined. Cf. Fine, *op. cit.*, pp. 296 ff.

What Fine *et al* want to do is to respect what they describe as "penumbral connections". To paraphrase Fine: suppose that a certain blob is a border case of 'red' and let S be the sentence 'the blob is red'. Though we may agree that S is indeterminate as is its negation, $\sim S$, nonetheless their conjunction is false since they are contradictories: the boundary of the one shifts, as it were, with the boundary of the other. (Notice that the sentence $S \& S$ is indeterminate since it is equivalent to plain S , whilst $S \& \sim S$ is false so, since a conjunction with indeterminate conjuncts is sometimes indeterminate and sometimes not, '&' is not truth-functional according to SV.) Similarly, since S and $\sim S$ are complementary over the given colour range, their disjunction, $S \vee \sim S$, is true (so ' \vee ' is not truth-functional either according to SV). Now *penumbral connection* is the possibility that logical relations hold among indeterminate sentences. The supervaluationist's claim then is that penumbral truths must be respected (and, as a consequence, some non-truth-functional approach must be sought). They are insisting, in effect, that classical theorems, in so far as they reflect penumbral connections, must be respected. They also think that classical consequence relations ought to be respected. So, although the SV theorist is prepared to abandon classical metatheory, classical logic remains intact.⁶

The question is: how are we to construct a semantics for vague statements which would justify the retention of the laws and consequence relation of classical logic? The supervaluationist logic of vagueness attempts to answer just this question, but rather than simply presenting the SV logic and metatheory I want to approach the question from the perspective of enlightened representationalism.

It is my contention that the representationalist is committed to a specific conception of vagueness by virtue of Theorem 3.1 of Chapter Three. It follows from this theorem that any representational account of vagueness is such that vagueness is definable in terms of one-manyness. Since the account of vagueness for which we are now seeking a logical characterisation (enlightened representationalism) is of this class, it too must work with this or some logically equivalent conception of vagueness. It is this philosophical conception, I believe, that may be taken to underpin the formal SV logic of vagueness and, as such, it provides a canonical interpretation of SV as applied to the problem of describing a logic of vagueness. It also provides a plausible motivation for the enlightened representationalist to adopt the SV logic, so let me try to describe how I think this conception of vagueness leads to supervaluationism.

The Russellian conception of vagueness as 'one-manyness of denotation' is the key insight (though historically Russell's theory seemed to play no part in this development). Russell defined a denoting phrase to be vague if and only if its denotation relation was not one-one but one-many, in the following sense — there are, at least, two possible

⁶ Fine, *op. cit.*, pp. 269 ff.

candidates for denotation consistent with the truths involving that linguistic item; any referent capable of serving as the denotation of the linguistic item in question must be sharp yet the presence of border cases makes it essentially indeterminate what this referent is. Take the predicate 'is red', for example, and suppose there to be only three possible objects a_1 , a_2 , and a_3 , where a_1 is determinately red, a_3 is determinately not red and a_2 is the predicate's sole resilient border case. Preservation of the truths involving a_1 and a_3 requires that any (sharp) property in the denotation range of the predicate must have a (sharp) extension that includes a_1 , excludes a_3 , and might include a_2 or might exclude a_2 ; it must either include a_2 or exclude a_2 (since the world is sharp, a_2 is either in the extension of the property denoted by the predicate or it isn't), yet it is indeterminate which. There are therefore two possible candidates for denotation — the set $\{a_1, a_2\}$ or the set $\{a_1\}$ — consistent with a_1 's being red and a_3 's being not red. Any possible referent for the predicate is sharp, but the presence of a resilient border case makes for two distinct possibilities.

By associating each of the two respective possible denotations with linguistic items — precise predicates 'is red¹' (with satisfiers a_1 and a_2) and 'is red²' (with satisfier a_1) — we can shift talk of vagueness in terms of denotation relations to intra-linguistic talk of a relation between vague and precise predicates. We can say that the vagueness of the predicate 'is red' amounts to there being two ways of making the predicate precise consistent with a_1 's being determinately red and a_3 's being determinately not red. Let's call this 'making precise subject to the consistency constraints' *admissible precisifying*. Then an *admissible precisification* of some vague predicate P is a precise predicate P^i such that any determinate satisfier for P is a satisfier for P^i and any determinate satisfier for not- P is a satisfier for not- P^i . Now the vagueness of the predicate 'is red' amounts to there being two admissible precisifications, 'is red¹' and 'is red²'; moreover, the determinateness of a_1 's being red now corresponds to its being both red¹ and red², the determinateness of a_3 's being not red corresponds to its being both not red¹ and not red², whilst a_2 's being a resilient border case corresponds to its being red¹ but not red². Generalising to any predicate: an object a is determinately (not) in the extension of predicate P if and only if a is in the (anti-)extension of every admissible precisification of P ; an object a is a resilient border case for a predicate P if and only if a is in the extension of some admissible precisification and in the anti-extension of some admissible precisification. So, a predicate P is vague with resilient border case a if and only if there are (at least) two admissible precisifications P^1 and P^2 such that a satisfies one but not the other. Similar accounts can be given for the vagueness of n -place relations generally and names.

Analogously, an admissible precisification of an atomic sentence A is a constrained way of making all the constituent expressions of A precise; it is another atomic sentence which is precise (is either true or false), subject to the constraint that it is true (false) in all those circumstances in which A is true (false). An atomic sentence then is true (false) if

and only if it is true (false) for all admissible precisifications; it is indeterminate in truth value if and only if it has (at least) two admissible precisifications, one of which is true and one of which is false.

For example, the sentence 'My lighter is red' may be vague and so neither true nor false because there are two admissible precisifications of the sentence 'My lighter is red¹' and 'My lighter is red²' (assuming the subject term to be precise) one of which is true and the other false. We could draw a sharp line between red and non-red in various ways, thereby making the sentence precise in ways that makes for a difference in truth value. For many atomic sentences of course, how we draw the line will have no effect on the truth value; e.g., any determinately red cigarette lighter will, by virtue of the constraint on what counts as an admissible precisification, count as red no matter how one precisifies. Various admissible precisifications of an atomic sentence will only differ on truth value ascriptions if the sentence ascribes a predicate to its border case.

Thus we can rewrite Russell's account of vagueness in terms of a relation between a linguistic item and its admissible precisifications. This account, since it applies generally to all representational accounts of vagueness, must underpin the enlightened representationalist's account as well.

Yet nothing we have said so far suffices to justify or explain the preservation of classical laws and classically valid inferences. Extending the account of vagueness of atomic sentences to complex ones involving logical constants in a straight forward truth-functional manner might seem to threaten classical laws. Take the Law of Excluded Middle for instance. Is it always true that $A \vee \sim A$? One might think not; you might see vagueness as a threat to classical logic perhaps because you think that if A and $\sim A$ were indeterminate their disjunction would be too via some truth-functional account of ' \vee '. However, there is another alternative suggested by the above account of vagueness for atomic sentences which would permit the retention of classical laws. Rather than evaluating *atomic* sentences by means of their admissible precisifications and then extending to all sentences of the language via recursive clauses for the logical constants one might evaluate *all* sentences in the same way, regardless of their logical complexity. So: *any* sentence of the language is true (false) if and only if it is true (false) for all admissible precisifications and indeterminate if and only if there are (at least) two admissible precisifications thereof, one of which is true and the other false.

Now consider a particular instance of the Law of Excluded Middle — say, 'My lighter is red or my lighter is not red'. Precisify the constituent expressions any way you like. Regardless of how this is done, the precisification of 'My lighter is red' comes out true or false, so it or its negation is true. So, assuming the recursive clauses giving the truth conditions of complexes *in an admissible precisification* are just the classical clauses (that is, where no vagueness is present, classical logic applies by default), then the disjunction of 'My lighter is red' with its negation must come out true, *no matter what the*

precisification, and so comes out true in every admissible precisification. In this way, contra the above reservations, all instances of the Law of Excluded Middle turn out to be (determinately) true.⁷

If we now define logical truth or validity for single formulae as follows — a formula A is valid if and only if, in all models, A is (determinately) true — then the classical Law of Excluded Middle turns out to be valid, as do all other classical laws. To see this, assume A to be classically valid — so, true in all classical models; then A is true in all admissible precisifications no matter how they are interpreted (since an interpreted admissible precisification is just a classical model) and so A is (determinately) true in all models. Conversely, suppose A is not classically valid — so there is a classical model falsifying A; then in some model A is false and so not (determinately) true.

In generalising to validity for arguments, classical consequence could be retained in at least two ways. Firstly, just as a sentence of the language was evaluated as (determinately) true if it was true for all admissible precisifications, one might analogously define an inference to be valid if and only if it is valid in all admissible precisifications. So:

B is a *valid consequence* of A if and only if, in all admissible precisifications, B is true whenever A is.

All and only the classically valid inferences are valid in all such precisifications, and so all and only the classically valid inferences are valid according to this definition of 'semantic consequence'. This is the account suggested by Dummett.⁸ Now, as Dummett points out, it follows that an inference valid on this account will lead from (determinately) true premises to a (determinately) true conclusion. This latter, strictly weaker claim to do with the preservation of (determinate) truth is used by Fine to define an alternative consequence relation which also preserves classical consequence.

Fine's account is as follows: an inference is valid if and only if whenever the premises are (determinately) true the conclusion is (determinately) true. So:

B is a *valid consequence* of A if and only if whenever A is true in all admissible precisifications, B is true in all admissible precisifications.

This account of 'semantic consequence' also admits as valid all and only those inferences that are classically valid. Suppose that an argument is classically valid and suppose the premises to be true on all admissible precisifications. Admissible precisifications are classical, so the conclusion is true on all admissible precisifications. Conversely, suppose an argument is not classically valid. Then there is an admissible precisification wherein the

⁷ Russell does say that he thinks the "law of excluded middle" fails in the presence of vagueness (cf. Russell, *op. cit.*, pp. 85-6) however this is best understood as a rejection of the Principle of Bivalence. The distinction between the two was often blurred.

⁸ Dummett, *op. cit.*, p. 311.

premises are true whilst the conclusion is false. The conclusion therefore is not (determinately) true yet the premises could be, so the argument is not valid by the above account.

Though the first account of consequence is strictly stronger than the latter, they are materially equivalent on the language of the classical predicate calculus. The easiest way to see this is simply to note that both accounts validate all and only those inferences validated by the classical account of consequence. (We shall see later — in §5.3.1 — that, if the language is extended to include the expression of determinateness in the object language, the two accounts come apart materially. The former account is stronger than the latter and invalidates inferences validated by the latter account.) Moreover, each simplifies to the exact same definition of validity for single formulae, logical truth.

Thus we see how Russell's account of vagueness at the simple or atomic level seems able to be generalised, thereby generating a logic that is nonclassical in the metatheory whilst preserving classical logic.

In fact, what I have outlined above using the notion of an 'admissible precisification', is just the SV account of vagueness.⁹ We can speak of a sentence's being neither (determinately) true nor (determinately) false, or predicates whose determinate extension and determinate anti-extension are not exhaustive, whilst respecting what Fine calls 'penumbral connections'. For the SV theorist 'determinate truth', 'truth' *simpliciter*, or "supertruth" is 'truth in all admissible precisifications' just as, in modal logic, 'necessary truth' is 'truth in all possible worlds'. Now, just as not all sentences in modal logic are necessary or impossible — some may be merely contingent, so too in SV logic not all sentences are (determinately) true or (determinately) false — some may be neither to accommodate vagueness. Moreover, just as the modal logician might deny the necessary truth of A and the necessary truth of $\sim A$ whilst asserting the necessary truth of their disjunction $A \vee \sim A$, the SV theorist may deny the (determinate) truth of A and the (determinate) truth of $\sim A$ whilst asserting the (determinate) truth of their disjunction $A \vee \sim A$.

Of course, unlike the modal logician who distinguishes 'truth' *simpliciter* from 'necessary truth', the SV theorist, in associating 'truth' *simpliciter* with 'determinate truth', is committed to a non-truth-functional disjunction and conjunction as was pointed out earlier.

The (possibly non-classical) truth value of any sentence is given by the behaviour of that sentence under (classical) valuations in all admissible precisifications. SV does not take a trivalent interpretation function or trivalent truth function as primitive. Rather, these

⁹ Fine's conditions of "stability", "fidelity", "completeability" and "resolution", discussed respectively in Fine, *op. cit.*, pp. 268 & 272, pp. 268 & 272, p. 272 and pp. 278-9, are all implicitly assumed in the discussion above.

non-classical functions on the domain of natural language expressions are defined in terms of the corresponding classical functions relativised to elements of a set of 'admissible precisifications'.

5.2 *Supervaluations and Enlightened Representationalism*

So what does this SV account of vagueness have to do with an enlightened representational account? Well, in contradistinction to Quine and others who aim to recast vague discourse in precise terms, the enlightened representationalist sees a need to accommodate reasoning involving vague language; to ignore vague ways of talking and reasoning requires an inflated acceptance of 'ideal' language and betrays too little respect for ordinary language. This perceived need to accommodate vague language within the scope of logic, the demand for *a logic of vagueness*, requires the abandonment of bivalence and hence requires some modification of classical metatheory; language within the scope of logic is vague so our reasoning *about* it had better reflect this — the logical metatheory had better be non-classical. To this extent the SV account is a logic of enlightened representational vagueness. Furthermore, to the extent that vagueness is thought to be merely semantic and not undermine classical metaphysics, the enlightened representationalist is in agreement with Quine *et al* in assuming the world to be precise and it is this presumably that justifies the retention of classical logic; the world is not vague so our reasoning *about* it had better reflect this — the logical theory need not be revised in the face of vagueness.

Both take precisified or regimented language to have privileged status — the logical structure of the world is properly reflected in a precise language. In the case of the Quineans this is obvious; they rest with one, any one, of these regimentations and ground logic and its metatheory therein thus retaining classical logic and classical metatheory. In the case of the enlightened representationalist the regimented language is not privileged in the sense of grounding logical metatheory (the metatheoretic features of vague natural language, e.g. the failure of bivalence, necessitate their not doing this) yet, no matter how the world is, some one of the precisifications of natural language will precisely describe it or "cuts it at its joints". Thus one might argue that anything true in *all* precisifications is true of the world; no matter how one attempts to precisely describe the world, certain "truths" shine through. For example, everything is red or not red because that's the way the world is, though I may be unable to say precisely whether that thing is red or not. Contra Quine, vagueness cannot be ignored yet, its source does not lie so deep as to

warrant a change in logic (logic ought not to be sidetracked by the vagaries of natural language like vagueness).

Thus it would seem that the enlightened representationalist, whilst agreeing that classical metatheory stands in need of minor revision to accommodate reasoning involving vague language, might nonetheless assume the revisions required to accommodate vagueness do not go so far as to undermine classical logic.

Though this view justifying the appropriateness of SV as a logic of vagueness has to my knowledge never been quite so explicitly stated, I think it is this which is implicitly taken to provide a *general* justification of SV. (Some justification *is* required — what justifies the retention of classical logic in the face of vagueness?). Remarks by Dummett seem to speak in favour of this understanding of the SV account. "[I]f we suppose that all vagueness has its source in the vagueness of certain primitive predicates, relational expressions and quantifiers, we may stipulate that a statement, atomic or complex, will be definitely true just in case it is true under every sharpening of the vague expressions of these kinds which it contains."¹⁰ And so classical laws are preserved. As we shall see shortly when we come to evaluate *specific* consequences of the SV account, Fine seems similarly motivated.

5.2.1 *In Defence of SV*

So, the enlightened representationalist might suppose the logic of vagueness is classical though the metatheory is not. Van Fraassen's supervaluation logic can be reinterpreted, as Fine and others have shown, to provide just such a logic, SV. Yet endorsing classical logic whilst admitting vague language to be within the scope of logic raises an important question: How is the supposed tension between vagueness and classical logic relieved? Such a question was quite intentionally avoided by earlier theorists who relieved any tension by banning vague language, and hence soritical language, from the scope of logic. Classical machinery, both at the level of logic and metatheory remained unchanged precisely because vagueness was banished. But the enlightened theorist being considered here acknowledges the presence of vagueness on the one hand whilst retaining classical logic (both classical validity and consequence) on the other.¹¹ This gives rise to two sources of tension: between vagueness and classical laws, and between vagueness and classical consequence. Is it not the case that the presence of vague language within the

¹⁰ Dummett, *op. cit.*, p. 311.

¹¹ Since the motivation suggested earlier for adopting SV was by no means watertight there may be room for an enlightened representational account which adopts some logic other than SV, in particular one which does *not* preserve classical validity and consequence. Such theories will be implicitly addressed when we come to the more general criticism of representational accounts of vagueness; at this stage I am only discussing one logic — SV — that has been much talked about and falls easily into the enlightened representationalist's camp.

scope of logic threatens the validity of some classical laws, e.g. the Law of Excluded Middle? Furthermore, is it not the case that the presence of vague language within the scope of logic threatens the validity of some classical inferences?

5.2.1.1 *In Defence of Classical Consequence*

So what of the latter source; can a logic of vagueness reasonably be supposed to preserve classical consequence? The threat to the classical account of the consequence relation is most perspicuously highlighted in the case of the ancient paradox of the sorites. So, how is the sorites paradox resolved?

Let's remind ourselves of the sorites paradox by considering the following version:

A man with no hairs on his head is bald.

If a man with n hairs on his head is bald then a man with $(n+1)$ hairs on his head is bald.

∴ A man with a million hairs on his head is bald.

The premises seem *prima facie* acceptable, whilst the conclusion seems unacceptable yet the reasoning is valid by classical lights. So, is the SV theorist, who also endorses the classical account of consequence, to accept the argument as not only valid but sound (that is, to embrace the conclusion) or, to accept the argument as valid though unsound (that is, to deny one of the premises)? The latter option is that taken up. They deny the truth of 'If a man with n hairs on his head is bald then a man with $(n+1)$ hairs on his head is bald'. Does it then follow from the mere non-truth of the premise that it is false? Well, not according to the SV theorist; remember that bivalence has been rejected. Nonetheless for the SV theorist the premise *is* false; it is true (*simpliciter*) that there is some n for which a man with n hairs on his head is bald whilst a man with $(n+1)$ hairs on his head is not! This is so because for every admissible precisification of the vague predicate 'bald' it is true that there is some such falsifying n , therefore it is determinately true that there is some such n and so true (*simpliciter*).¹²

That it is true that there is some such falsifying n however is not to be confused with the following claim which in SV is quite distinct: there is some n for which it is true that a man with n hairs on his head is bald and a man with $(n+1)$ hairs on his head is not bald. The truth of there being a hair-splitting n no more entails there being an n of which it is true that *it* is hair-splitting than the truth of $A \vee \sim A$ entails the truth of A or the truth of $\sim A$. There is no n which is the hair-splitter in every precisification (though, as we have just

¹² This SV response must be presumed to generalise to a solution to all the sorites forms to count as a defence of classical consequence. The rejection of Bivalence obviously underpins solutions to all but the phenomenal form.

seen, in every precisification there is a hair-splitting n); the distinguished n shifts for different admissible precisifications — what Fine refers to as "the truth-value shift".

So what are we to make of all this? Well, there are two points worth taking up here. Firstly, in claiming the major premise is false, SV might seem committed to the precision of the predicate 'bald'. Were this charge sustainable, the enlightened representationalist would appear to retain classical consequence in the face of the sorites by denying there ever was any semantic vagueness to worry about in the first place; classical consequence would be retained only by ignoring that which was thought to be a threat. Hardly an enlightened position — the enlightened representationalist would then have simply side-stepped the whole issue of vagueness rather than boldly confronting it with classical logic as they claimed to be doing.

The charge is that in rejecting the second premise as false the SV account, as we saw, accepted the existence of a hair-splitting n , and this seems to imply the semantic precision of 'bald'. Dummett himself, whilst advocating the SV approach to vagueness, responds that "[the SV] solution may, for the time being, allay our anxiety over identifying the source of the paradox. It is, however, gained at the cost of not really taking vague predicates seriously, as if they were vague only because we had not troubled to make them precise"; after all, it's simply that the semantic decision as to which is the correct precisification remains unmade.¹³ Fine's view is that one need not, indeed should not, take semantic vagueness "seriously" in the sense of denying the existence of a hair-splitting n , but that one should deny the implication from the existence of a hair-splitting n to the precision of the predicate 'bald'. He does not elaborate other than to suggest that due attention to the "truth-value shift" falsifies any such implication. Presumably precision would only follow if one could be in a position to say *which* n is the hair-splitter, yet SV, whilst asserting that there is some hair-splitting n , nonetheless refrains from asserting of any n that *it* is the hair-splitting case.

Thus it seems that Fine avoids the implication by assuming that precision follows from the existence of a cut-off point only if the existence claim is one for which an instance could be given, yet the sense of 'there is' employed by the SV theorist when they claim that there is a cut-off point is not the sense in which it is reasonable to ask what the cut-off point is. The use of 'there is' in this sense has led some philosophers, e.g. Rolf, Sainsbury and Williamson, to claim that it is not used in its usual sense in SV. This is the second point of dispute over the SV solution to the sorites. We shall return to both points below.

¹³ Dummett, *op. cit.*, p. 312.

Before considering these two points any further I want to digress to consider a related though much more general objection to SV as a logic of vagueness. This more general criticism supports the charge that SV ignores the very problem it seeks to describe as a logic of vagueness — not just in its solution to the sorites paradox, but in its whole conception. It proceeds by arguing that, according to the general theory of supervaluations, all language within the scope of SV is precise. For example, regardless of whether the precision of the predicate 'bald' follows from the existence of a cut-off point, Fine's preferred logic of vagueness might nonetheless seem to make it independently true that the predicate 'bald' is precise since the predicate is precise in every admissible precisification. In fact, it might seem that — just as it is determinately true that one hair can make the difference between being bald and hirsute and that one grain of sand can make the difference between a heap and a non-heap — it is determinately true that every SV predicate is precise (more generally, it is determinately true that all SV language is precise); and it is determinately true that all SV sentences are true or false (more generally, it is determinately true that the SV metatheory is classical).

Were the objection well-founded in the first place then it would amount to the claim that supervaluationism works too well; SV is a logical account of how it that we can reason using vague language but the solution offered by SV would entail there being no vagueness within the scope of SV to worry about — all language within its scope is precise.

I think that the objection is flawed from the very outset and requires no counter-move by the SV theorist; the problem is therefore to be dissolved rather than resolved. It exploits use/mention confusion in a harmful way. It is true, according to SV, that one hair can make the difference between being bald and non-bald; it is also true according to SV that one grain of sand can make the difference between being a heap and being a non-heap, and so on. In making such claims SV *uses* the predicate 'bald', etc., and shows that what it refers to, baldness, etc., is precise. But when one attempts to argue that the predicate is precise by claiming that *it* is precise in every admissible precisification, one is identifying *it* (the predicate 'bald') with one of its admissible precisifications, as opposed to identifying the referent of it with the referent of one of its admissible precisifications. This identification is simply fallacious; the vague predicate 'bald' is *related* to each of its admissible precisifications (trivially, the latter is a precisification of the former) but an admissible precisification of the predicate should not be *identified* with the predicate itself. The predicate 'bald' is not to be found in any admissible precisification; only precisified correlates. This charge that SV is inconsistent with the vagueness of any language within its scope can be diffused.

An alternative way to phrase the objection is as follows: SV theorists agree that 'bald' is vague. But then, by their own lights, there are at least two admissible precisifications of 'bald'. Yet the claim that there are at least two admissible precisifications of 'bald' would

seem to be (super>false; so it is not the case that there are at least two admissible precisifications of 'bald' and hence 'bald' is not vague after all. There is then, according to the SV analysis of vagueness, no vagueness to be accounted for! SV works *too* well. The thought behind this phrasing of the objection seems to be that no matter how we precisify the claim 'there are at least two admissible precisifications of 'bald'', the vague term 'bald' is precisified and so, since precisifications of precise terms are unique, the claim 'there are at least two admissible precisifications of 'bald'' is false in every admissible precisification and so is (super>false.

However, this way of phrasing the objection is no less fallacious than it was before — the claim 'there are at least two admissible precisifications of 'bald'' is not (super>false but (super>true. In order to determine the (super>true value of the claim we are indeed required by SV to consider the (classical) truth values of all admissible precisifications of the claim. Yet 'bald' here figures as a name. It is not vague what the name denotes, it denotes the predicate 'bald'; *though the predicate 'bald' is vague, the name "bald" is precise*.¹⁴ So, when we come to consider the admissible precisifications of the claim by precisifying all the constituent expressions, the name "bald" remains unaffected. It still names a vague predicate and so it is still true after precisifying the claim itself that there are at least two admissible precisifications of the predicate; so the claim *is* (super>true. (Perhaps it would be helpful to recast the claim as 'there are at least two admissible precisifications of *that* predicate' — accompanied by a precise pointing gesture. Any urge to treat the name as if it were the predicate named would thereby be removed.) The objection rests upon a confusion.

Though the name "bald" is precise the predicate 'bald' is vague, and arguments to the effect that SV is committed to the precision of the predicate 'bald' simply confuse the name of the predicate with the predicate itself.

Interestingly, this charge against supervaluationism as a means of resolving Geach's "paradox of 1,001 cats" and Unger's "problem of the many" is considered by Lewis in his 'Many, But Almost One'.¹⁵ Lewis suggests van Fraassen's method of supervaluations as a plan for coping with the problem. Take Geach's problem as described by Lewis. Cat Tibbles is alone on the mat. Tibbles has hairs $h_1, h_2, \dots, h_{1,000}$. Let c be Tibbles including all these hairs; let c_1 be all of Tibbles except for h_1 ; and similarly for $c_2, \dots, c_{1,000}$. Each of these c 's is a cat. So instead of one cat on the mat, Tibbles, we have at least 1,001 cats — which is absurd. After considering a number of resolutions Lewis suggests the

¹⁴ One has to be a bit careful here; there is a sense in which "bald" is vague. The name might pick out a number of distinct tokens of the predicate, each marginally different from the other, and so the class of tokens to which the name refers may end up being vague. Yet, in so far as this is a possibility, it amounts to the name being vague in a different dimension to that being considered here; vagueness in this dimension leaves the ensuing argument unaffected.

¹⁵ Geach, P.T., *Reference and Generality*, edn. 3, Cornell University Press (1980), pp. 215 ff; Unger, P., 'The Problem of the Many', *Midwest Studies in Philosophy* 5 (1980): 411-67; Lewis, D., 'Many, But Almost One' in J. Bacon, K. Campbell & L. Reinhardt (eds), *Ontology, Causality and Mind: Essays in Honour of D.M. Armstrong*, CUP (1993), pp. 23-38.

supervaluation approach; no matter how we pick an admissible referent for 'Tibbles', there is only one cat, so it is supertrue that there is just one cat. There is a semantic decision as to what we mean by 'Tibbles' that remains unmade, but no matter how we make it there is just one cat — so there is just one cat, though it is undecided which possible candidate that is.

He then goes on to consider objections, one of which (apparently suggested by remarks made by Kripke) is that the plan works too well; supervaluationism stops one from ever stating the problem in the first place. His response to this objection is to claim that "fanatical supervaluationism is mistaken"; the supervaluationist rule should be considered as a defeasible presumption to be suspended if it leads to nonsense.¹⁶ As Frank Jackson has pointed out in conversation, such a response, suitably spelt out, could be invoked by the SV theorist in the context of vagueness, yet, as it is described by Lewis, this response sounds *ad hoc*; apply SV where this offers the hope of a solution, yet refrain from applying it where it doesn't — the response sounds contentless. What we want is a principled reason for the application of SV in some situations and not others.

In fact, I think that the foregoing analysis in the context of vagueness shows us a way to redescribe the Lewis-counter-move that avoids the charge of *ad hocness* (and perhaps was what Lewis had in mind all along). What Lewis describes as "fanatical supervaluationism" is better described as "fallacious supervaluationism"; supervaluationism is a way of analysing our *use* of certain semantically deviant expressions. But this does not necessarily mean that supervaluationism has any significant contribution to make at all in analysing our *mention* of semantically deviant expressions (that is, in analysing our use of semantically standard expressions which may themselves refer to semantically deviant expressions). The Lewis defence is not *ad hoc* at all in spirit, though his explanation may make it seem so by selling the defence short; it can be given a principled justification via the use/mention distinction. SV is an account of how to analyse specific cases of deviant reference (e.g. vague object-language talk about a sharp world). It simply reduces to standard analyses where reference is not deviant (e.g. in precise metalinguistic talk about vague language) and so its fanatical use is not so much misguided as excessive and irrelevant. Suspension of its use is not required to avoid nonsensical consequences — no nonsense ensues; rather, suspension is permitted by virtue of the irrelevance of its use. Lewis was wrong to think that, where supervaluationism seems appropriate, fanatical supervaluationism leads to any nonsensical outcome; where it is fallaciously thought to cause trouble it is simply ineffectual. To this extent its use is sometimes fanatical; valuations *simpliciter* (as opposed to *supervaluations*) will usually do.

¹⁶ Lewis, D., *op. cit.*, pp. 29-30.

So the general objection to SV — that all language within its scope is precise — is dissolved. But what of our original concerns regarding SV's retention of classical consequence; firstly that the SV solution to the sorites paradox in particular seems to imply that the vague predicate involved is precise? Does it follow from its being true that there is some n such that a man with n hairs on his head is bald whilst a man with $(n+1)$ hairs on his head is not bald, that the predicate 'bald' is precise? In short the SV theorist's answer is 'No'.

Fine's rather brief defence, remember, was that due attention to the "truth-value shift" will convince us of this. It seems that he avoids the implication by assuming that precision follows from the existence of a cut-off point only if the existence claim is one for which an instance could be given, yet the sense of 'there is' employed by the SV theorist when they claim that there is a cut-off point is not the sense in which it is reasonable to ask what the cut-off point is. Fine's defence is sound, but requires elaboration.

Consider the following two claims:

- (1) It is true that there is some n such that a man with n hairs on his head is bald whilst a man with $(n+1)$ hairs on his head is not bald.

I.e. $\top \exists n(B(n) \ \& \ \sim B(n+1))'$.

- (2) For some n , it is true that a man with n hairs on his head is bald whilst a man with $(n+1)$ hairs on his head is not bald.

I.e. $\exists n(\top B(n) \ \& \ \sim B(n+1))'$.

Now the enlightened representationalist certainly ought to endorse (1) since it simply says that baldness is precise, and of course SV does endorse (1) — it is the foundation of the SV resolution of the sorites paradox. It reflects the representationalist belief that the world is precise. What does SV say with regard to (2)? They deny it of course; asserting (1) whilst denying (2) is what the "truth-value shift" amounts to in this context. It's not the case for any number n that *it* is the cut-off point between all admissible precisifications of 'bald' and 'non-bald'; the cut-off point shifts from admissible precisification to admissible precisification. We can say that there is some cut-off point without being able to specify precisely what the cut-off point is.

So, having duly noted the effect the "truth-value shift" has — asserting (1) whilst denying (2) — how does this remove any commitment to the precision of the predicate 'bald'? Well, because (2) is equivalent to claiming that the predicate 'bald' is precise, and (2) is rejected as we have just seen. This equivalence can be established by reasoning as follows: assume for some n that it is (super)true that a man with n hairs on his head is bald whilst a man with $(n+1)$ hairs on his head is not bald. In other words, assume (2). According to SV this is the case if and only if, for some n , every admissible precisification

of 'bald', 'baldⁱ', is such that a man with n hairs on his head is baldⁱ whilst a man with $n+1$ hairs on his head is not; that is, if and only if every admissible precisification of 'bald' is equivalent to every other. Yet to say, in SV, that the predicate 'bald' has a unique precisification is equivalent to claiming that the predicate 'bald' is precise — there is no truth-value shift across border cases.

So SV accepts (1), but this does not imply that the predicate 'bald' is precise unless one accepts the implication from (1) to (2) — that is, unless one thinks that claiming the existence of a cut-off point implies one can specify precisely what this cut-off point is. SV simply denies the implication.

Yet, as we have seen above, one might then query the account of the English quantifier 'there is' described in SV — this was the second point noted earlier in reference to the SV solution to the sorites paradox. SV makes it true that there is a hair the addition of which makes one non-bald, whilst denying that there is a hair such that it is true that *it* makes the difference between being bald and non-bald (thereby avoiding the above objection just considered). Rolf objects that SV therefore does not give a true description of the truth-conditions of the English phrase 'there is'.¹⁷ The enlightened representationalist seems compelled to respond either by rejecting SV as the preferred logic, by defending this use as descriptively accurate or by suggesting that, whilst it may seem to conflict with our unreformed intuitions, it is prescribed by the enlightened representational account of vagueness.

The first alternative will not be taken up here. My primary concern in this chapter is enlightened representationalism and we are presently discussing SV since it seems the preferred logic of such representationalists. To go on and consider alternative logics is beyond the scope of this thesis and, to the extent that they presuppose a representationalist account of vagueness, they will be implicitly addressed when we come to the general criticism of representational accounts of vagueness.

The second alternative — that the SV account of 'there is' is descriptively accurate — seems hard to swallow. The common incredulity at the SV analysis of the sorites stems precisely from the belief that in denying its major premise, that is, in affirming (1), one is committed to something's being such that *it* is the falsifier ... which everyone agrees is not the case. This commonly supposed commitment itself bespeaks an understanding of the existential quantifier as something other than that offered by SV.¹⁸

Let's consider the third alternative. If vagueness, considered representationally, is within the scope of logic then the logic of 'there is' might seem to be exactly that described

¹⁷ Rolf, *op. cit.*, pp. 129-30. Rolf's objection is made independently of the objection we have just been considering.

¹⁸ Sainsbury, in his *Paradoxes*, Ch. 2, p. 39, n. 15, takes this descriptive inaccuracy as conclusive grounds for rejecting the SV analysis of the sorites paradox.

by SV — some true existential generalisations have true instances, as one would intuitively expect, yet some true generalisations in the form of (1), namely those involving vague predicates, do not have true instances. What makes a sentence like (1) above acceptable, enlightened representationalists might argue, is there being something which is a cut-off point between any possible denotation of 'bald' and of 'non-bald'. Yet were I able to truly instance (1) and say precisely what the cut-off point was then I would be in a position to precisely describe that feature of the (precise) world and from that extract the precise conditions (or rules) for the application of the predicate in question, and it is exactly this that the enlightened representationalist denies. We can say *that* the world is precise, whilst being unable to say precisely *how* it is.

(This may sound like an epistemic approach to vagueness and indeed, in so far as they both take the world to be precise, they agree that there is a cut-off point; but the epistemic account has it that our inability to say precisely how things are is due to merely epistemic limitations — it adopts the view that there is no semantic vagueness, i.e. there is also a cut-off point in the application of the predicate — whereas the enlightened representationalist denies this.)

According to this response it's not so much that SV theorists are committed to finding some explanation of their account of 'there is', no matter how counter-intuitive — to look at it this way is to put the cart before the horse. Rather, it follows from the very understanding of the enlightened representational account underpinning SV that 'there is' works in this way, and so, in so far as the representational position is plausible, it (in and of itself) provides a perfectly good explanation of how 'there is' could work in this way.

This appears to be Lewis's defence against the charge that the supervaluationist account of 'there is' is "peculiar". Recall that Lewis advocated supervaluations as a means of resolving Geach's paradox of the 1,001 cats. In so doing he was committed to the same account of 'there is' whereby although it is (super>true that something is a cat on the mat, there is nothing such that it is (super>true that *it* is a cat on the mat. Lewis's response is that it is peculiar (presumably, in so far as it conflicts with ordinary intuitions on the matter) but "once you know the reason why, you can learn to accept it."¹⁹

That the SV logic may not seem to yield an accurate description of how 'there is' is used in English would only count against enlightened representationalism if it purported to be a *purely* descriptive theory of the logic of natural language. However this is not necessarily what is claimed; we might use natural language phrases with a non-SV logic but, given what has just been said, it is simply a matter of our linguistic use failing to reflect the fact that the world is precise. SV may be claimed as a logical theory of natural language, not as it is actually used, but as it ought to be used given certain constraints that

¹⁹ Lewis, 'Many, But Almost One', *op. cit.*, p. 29.

must be respected — e.g. that the world is precise (for which some argument is required, of course, especially if our use of language fails to reflect this claim).

In other words, SV need not necessarily be assessed solely for its descriptive accuracy; it might be seen as a partially prescriptive theory. One takes the ordinary use of natural language as a starting point for semantic theory and then subjects the putative model to further constraints such as consistency with an underlying metaphysic.²⁰

Of course, since SV is seen as descriptively inaccurate our attention must turn to those constraints that purportedly make it prescriptively accurate to see how sound they are (cf. Chapter Six). The SV theorist's *modus ponens* (since one must accept the enlightened representationalist's account of vagueness, one must accept the consequences) might well be seen by others as a possible *modus tollens* (since the consequences are unacceptable, what gives rise to them must also be). If there is any reasonable way to remove those constraints that purportedly force one to revise ordinary usage then the preferable option might arise of our being able to offer a descriptively accurate account of vague language.

In conclusion then it seems that the supervaluationist's retention of classical consequence brings with it some apparently unpalatable features, however, were the defensive line of reasoning suggested above valid, then this would be offset by the realisation that such consequences attach to the acceptance of the most popular account of vagueness in natural language. Thus the dialectical stage is set: one can either accept the enlightened representationalist's account of vagueness (as many theorists seem to do) and simply bite the bullet when it comes to conflict with ordinary usage or accept ordinary usage and deny the representationalist's assumption that the world is precise. If the arguments I have offered in support of SV's retention of classical consequence are found to be invalid then so much the worse for SV. What I have tried to offer is a best case scenario for SV and even then there is room for dialectical dispute.

20 So it seems debatable whether Sainsbury is justified in rejecting SV on the grounds of descriptive inaccuracy. Williamson might also judge SV prematurely when he says:

The supervaluational treatment of the sorites argument is formally elegant. The question is whether it defuses the intuitive backing for the major premise. Many people have found the major premise plausible just because it seemed to them that there could not be a number n such that $n+1$ grains make a heap and n do not. Supervaluationism makes the very claim they find incredible. Nor should the supervaluationist say that the claim does not mean what they think it means. *The point of the enterprise is to give semantic descriptions of the vague sentences we use. If supervaluationism delivers a meaning for the existential claim other than its ordinary one, the enterprise fails.* [my italics]

The claim is made in a typescript, 'Supervaluations'.

5.2.1.2 *In Defence of Classical Laws*

Similarly, problems regarding the non-truth-functional account of the English connective 'or' set forth in SV can be diffused. This brings us back to the former source of tension identified earlier — the tension engendered by endorsing a classical logic of vagueness, between vagueness and classical laws. Isn't it the case that the presence of vague language within the scope of logic threatens the validity of some classical laws, e.g. the Law of Excluded Middle?

Again, 'No'. Since we cannot precisely describe the precise world we cannot in general say precisely *how* it is, though we can say *that* 'A \vee \sim A' is the case if we can say *that* the world is precise. When the objector argues that, for vague 'A', 'A \vee \sim A' fails because neither A nor \sim A (e.g. 'Tim is tall or Tim is not tall' fails because Tim is neither tall nor not tall), the SV theorist responds by admitting that neither 'A' nor its negation are (super)true, and so were ' \vee ' truth-functional, 'A \vee \sim A' would fail to be (super)true *but* ' \vee ' is *not truth-functional*. In other words, though the SV theorist accepts what amounts to the failure of the metatheoretic Principle of Bivalence, this would only entail the failure of the Law of Excluded Middle were disjunction to be truth-functional, which it is not.

The objection is based on the supposition that acceptance of LEM commits one to semantic precision (if everything's red or it isn't then 'red' is precise), however, *one should not confuse the Law of Excluded Middle with the Principle of Bivalence*. One should not confuse the claim that 'A \vee \sim A' is (super)true with the claim that 'A' is (super)true or ' \sim A' is (super)true. SV accepts the former whilst denying the latter. That is to say, for vague 'A', the SV theorist accepts

(3) T'A \vee \sim A' yet denies

(4) T'A' or T' \sim A'.

The latter is denied by virtue of the vagueness of 'A' (just as (2) above was denied by virtue of the vagueness of the predicate 'B') yet (3) is accepted (just as (1) above was) by virtue of the precision of that described.²¹

²¹ As a side issue it is worth pointing out that the argument SV offers in defence of classical consequence is arguably a special instance of the argument just used in its defence of the Law of Excluded Middle. (1)'s failure to imply (2) can be analysed in terms of (3)'s failure to imply (4) if one accepts an analysis of ' \exists ' in terms of ' \vee '. Were one to accept the following equivalence:

$$\exists xPx \equiv Pa \vee Pb \vee Pc \vee \dots \text{ etc.}$$

then (1) can be re-expressed as

(1') T'(B(0) & \sim B(1)) \vee (B(1) & \sim B(2)) \vee ... \vee (B(999,999) & \sim B(1,000,000))'

whilst (2) can be re-expressed as

(2') T'B(0) & \sim B(1)' or T'B(1) & \sim B(2)' or ... or T'B(999,999) & \sim B(1,000,000)'.
Now the implication from (1) to (2) is expressed as an implication from (1') to (2') which amounts to the distribution of 'T' over disjunction. Such distribution, presupposing the truth-

functionality of disjunction as it does, is invalid according to SV when some disjuncts are vague as indeed they are in the case to hand — namely when n is a borderline case for the predicate 'B' — 'A man with ... hairs on his head is bald'.

A dual account can be given of the (super)truth of the Law of Non-Contradiction, that is, the (super)falsity of all contradictions. That is, though neither of the two conjuncts 'A' and '~A' may be (super>false, nonetheless their conjunction 'A & ~A' is (super>false.

Interestingly, some of those who criticise SV for its non-truth-functional account of disjunction and retention of LEM (often deferring to ordinary usage to support their claim that LEM fails) do not go on to criticise SV for its non-truth-functional account of conjunction; they accept that, though 'A' and '~A' are not (super>false, their conjunction is (super>false (even in the face of ordinary usage to the contrary — e.g. 'Tim is tall and he isn't' — which now gets dismissed as a *façon de parler*). It would seem that an overriding desire to reject all contradictions as (super>false leads them to accept the SV account on this score even though it implies the non-truth-functionality of conjunction. (Notable exceptions to this endorsement of LNC in the face of vagueness include: paraconsistentists such as Lorenzo Peña, Graham Priest, Richard Sylvan né Routley, Ayda Arruda; some advocates of a truth-functional many-valued approach to vagueness; and perhaps C.S. Peirce.²²) Parity of reasoning (if not the strict equivalence of LEM and LNC, which might be thought to be up for grabs) commits the rational objector to treat each case alike.²³

Lloyd Humberstone and John Burgess — in their paper entitled "Natural Deduction Rules For A Logic Of Vagueness"²⁴ — have suggested that we accept LNC whilst rejecting LEM so as to respect our "intuitions" on the matter. I think that, though they have performed an interesting exercise in formal logic, there is little philosophical grounding for such a logic of vagueness. What is the philosophical account of the nature of vagueness underpinning such a logic? Letting one's intuitions regarding what to say when confronted with vagueness serve as a constraint on logic has little to recommend it philosophically when these intuitions conflict with ordinary usage. In short, some explanation, presumably grounded in the nature of vagueness, is required as to why LEM and LNC should not be treated alike.

Of course, the retention of the classical laws themselves, especially LEM, in conjunction with the resulting non-truth-functional account of disjunction and conjunction

²² Peña, L., 'Verum et ens Convertuntur'; Priest, G. & Routley, R., 'Applications of a Paraconsistent Logic'; Arruda, A., 'Aspects of the Historical Development of Paraconsistent Logic'; all appear in G. Priest, R. Routley & J. Norman (eds), *Paraconsistent Logic: Essays on the Inconsistent*, Analytica (1989). Advocates of the many-valued approach abandoning the Law of Non-Contradiction include Goguen, J., 'The Logic of Inexact Concepts', *Synthese* 19 (1969); Machina, K., 'Truth, Belief and Vagueness', *Journal of Philosophical Logic* 5 (1976); and Zadeh, L., 'Fuzzy Logic and Approximate Reasoning', *Synthese* 30 (1975).

C.S. Peirce seems to endorse such an approach when he claims that "anything is ... vague in so far as the principle of contradiction does not apply to it." (Peirce, C.S., *Collected Papers*, Vol. 5 (edited by C. Hartshorne & P. Weiss), Harvard University Press (1935), §5.448) though his usage seems idiosyncratic. For more discussion see: Engel-Tiercelin, C., 'C.S. Peirce et le projet d'une "logique du vague"', *Archives de Philosophie* 52 (1989): 553-79.

²³ Peña *does* treat them as strictly equivalent thereby ensuring parity. See his 'Partial Truth, Fringes, and Motion: Three Applications of a Contradictorial Logic', *Studies in Soviet Thought* 39 (1990), esp. p. 300.

²⁴ Burgess, J.A. & Humberstone, I.L., 'Natural Deduction Rules for a Logic of Vagueness', *Erkenntnis* 27 (1987): 197-229.

again raises problems concerning the descriptive adequacy of SV. Nonetheless SV theorists might again respond by defending their logical theory on prescriptive grounds rather than descriptive ones — such a logic *follows from* an enlightened representational account of vagueness — and so, once again the dispute shifts to questions concerning the overall adequacy of an enlightened representational account.

This seems to be the line taken by Fine. He does not wish to deny that LEM is counter-intuitive in the context of vagueness, yet the merely semantic nature of vagueness does not impugn LEM.

Suppose I press my hand against my eyes and 'see stars'. Then LEM *should* hold for the sentence S = 'I see many stars', if it is taken as a vague description of a precise experience. [my italics]

If vagueness is merely semantic, as the enlightened representationalist takes it to be, then LEM is prescribed and so defensible.

Fine seems to go further though; he also says that

[t]here is however, a good *ontological* reason for disputing LEM. ... LEM should fail for S if it is taken as a precise description of an intrinsically vague experience.²⁵

I think this additional and quite distinct claim by Fine is correct. SV, appropriately interpreted as a logic of vagueness, works with a conception of vagueness as Russellian "one-manyness", as I tried to show in §5.1.1. Furthermore, SV must be seen as doing so essentially, it seems, if it is to justify its use of admissible precisifications as philosophically adequate. The particular aspect of SV to hand, namely LEM, is defended by means of the "truth-value shift" which is, after all, a particular case of Russellian "one-manyness". Yet, we know, by Theorem 3.1, that if all vagueness is equivalent to "one-manyness" then there is no ontological vagueness. So SV, when considered in conjunction with what I take to be its canonical philosophical interpretation, if advocated as a logic of vagueness, would seem to commit one to the view that there can be no ontological vagueness.²⁶ There can be good ontological reasons for disputing LEM.

It would seem then that an enlightened representationalist approach to vagueness might not only be *sufficient* for the retention of LEM and (arguably and more generally) the legitimacy of employing SV as a logic of vagueness, but also *necessary*. The failure of bivalence in SV is to ensure the ability to encompass semantic vagueness within the scope

²⁵ Fine, *op. cit.*, p. 285.

²⁶ This should not be confused with the false claim, denied by Fine in conversation, that supervaluationism *entails* that there can be no ontological vagueness; the formal theory SV involves no such metaphysical commitment. What I am suggesting is that SV, *canonically interpreted*, involves such a commitment.

of logic, whilst, if Fine is correct, the retention of LEM (amongst other things) entails that vagueness is not ontological. SV is the appropriate logic for an enlightened representationalist account of vagueness and *only* an enlightened representationalist account.

Of course, it is only the sufficiency-claim that is relevant to the issue currently under discussion — the legitimacy of retaining LEM. LEM is defensible in so far as vagueness is not ontological and so, since the enlightened representationalist assumes vagueness not to be ontological, LEM is defensible. However the further necessity-claim, that LEM is defensible *only* in so far as vagueness is not ontological, has serious repercussions if vagueness sometimes turns out to be ontological and representationalism is not universally correct.

If this then was all there was to say about the logic of vagueness and its metatheory then from the point of view of the enlightened representationalist SV might appear justified as the prescribed logical choice. Moreover, it could be justified *simpliciter* if arguments could be given as to why one should adopt an enlightened representational perspective; from what has already been said in support of vagueness being semantic and in support of an enlightened approach to vagueness encompassing vagueness as it does within the scope of logic, there remains only the need to establish vagueness as *merely* semantic — that is, to establish the world as precise. However, before looking at arguments that purport to establish just such a conclusion (§5.4) there are further aspects of SV to consider.

5.3 *Extending Supervaluationism — SV+*

The above defences of SV — the SV defence of its resolution of the sorites paradox and of the preservation of classical laws — were couched in metatheoretic terms. This should not come as any surprise since it is in the metatheory that the logical revision required to accommodate vagueness takes place; one would expect that any explanations as to why *prima facie* objectionable object-language claims are in fact acceptable *would* require ascent into the metalanguage.

However, Rolf objects that since the paradox can be framed in the object-language and since it is in the object-language alone that any classical resolution seems paradoxical, any theory which rests with a classical consequence relation is stuck with the paradoxical nature of the reasoning no matter how much explanation is offered at the level of metatheory. To paraphrase Rolf: the problem about accepting the major premise of the

sorites of §5.2.1.1 as false is simply that it runs counter to our conviction that a man can cease being bald by the addition of one hair; but this conviction can be expressed in the object-language, so why should the elaborate metalinguistic theory be relevant here? *If the paradox can be framed in the object-language then it should be met in the object-language.* This point is made even more emphatic by Rolf's suggestion that one need have no metatheory; if someone formulates a sorites in the object-language, using only those logical principles of the object-language as the SV theorist accepts, it appears completely beside the point to accuse that person of some sort of error in their metatheory.²⁷

Similarly one might object to the SV defence of LEM on the grounds that it too demands ascent into the metalanguage.

In each case it is an explanation of (super)truth and its interaction with the logical constants that supposedly diffuses any tension thought to be implicit in the SV account. Yet, if the explanatory role played by (super)truth could be filled by an object-language expression then this objection could be met; no ascent into metatheory would be required. One obvious way in which (super)truth could be mirrored in the object-language is by denying Tarski's prohibition on semantic closure; in other words, if one admits that a language can contain its own truth predicate then the object-language/metalanguage distinction collapses and (super)truth is mirrored in the object-language by itself. A less controversial way of capturing the notion of (super)truth within the object-language is by means of an expression which is not itself a truth-predicate. Many accounts of the supervaluationist response to vagueness include just such an expression: 'It is determinately (or definitely) the case that ...'.

One might think that such an expression (or, if semantic closure is deemed acceptable, the (super)truth predicate) is independently necessary on the grounds of expressive completeness. Presumably, the fact that it might be vague whether some colour counts as red or whether so-and-so is tall is a fact that ought to be expressible in the object-language. To this end, the SV object-language is extended to include just such an expression subject to the following constraint:

Determinately A if and only if 'A' is (super>true.

'Determinately' can then be given a semantics analogous to the possible worlds semantics for a language including the expression 'necessarily'. Now, by way of analogy to the definition of contingency in modal logic, one can define what it is for something to be indeterminate:

Indeterminately A if and only if not determinately A and not determinately not A.

²⁷ Rolf, *op. cit.*, pp. 129 & 123 respectively.

In the now extended language, SV^+ , one can express the idea of its being vague whether Tim is tall as: It is indeterminate whether Tim is tall.²⁸

Using 'D' and 'I' as operators to represent these expressions we can now answer Rolf's criticism. In the absence of any metatheoretic notions one can now express the supervaluationist resolution of the Sorites in the object-language. When it is objected that a man cannot cease being bald by the addition of one hair (there is no cut-off point), the SV^+ theorist can respond by saying that it is indeed the case that one hair cannot make the difference between being determinately bald and determinately non-bald (there is no determinate cut-off point). In other words SV^+ endorses: $\sim\exists n(DB(n) \ \& \ D\sim B(n+1))$; which, given $D(A \ \& \ B)$ iff $DA \ \& \ DB$, is equivalent to the denial of

$$(2^+) \quad \exists nD(B(n) \ \& \ \sim B(n+1))$$

— the SV^+ object-language formulation of SV 's metalinguistic expression (2). However, this does not impugn the fact that there determinately is some cut-off point; it does not imply the denial of

$$(1^+) \quad D(\exists n(B(n) \ \& \ \sim B(n+1))).$$

Just as SV justified their resolution of the (standard) sorites by endorsing (1) whilst denying (2), so too does SV^+ endorse (1⁺) whilst denying (2⁺); moreover, in recognition of Rolf's criticism of SV , SV^+ has no need of metatheory to explain its position. Furthermore, the objection to the supervaluation resolution of the Sorites which, as we saw in the case of SV , was based upon a fallacious inference from (1) to (2), is even more clearly seen as fallacious in the context of SV^+ involving as it does an inference from (1⁺) to (2⁺).

Such an inference is analogous to the modal inference from $\Box\exists xPx$ to $\exists x\Box Px$. An example would be: 'Necessarily something is the top card in the pack so something is necessarily the top card in the pack'; or, 'Necessarily some number is the number of the planets so some number is necessarily the number of the planets'. Such inferences are now commonly said to be fallacious modal inferences since the terms 'the top card in the pack' and 'the number of the planets' are (paradigmatically) non-rigid designators varying their denotation across possible worlds. Now the SV^+ theorist can offer an analogous diagnosis of the fallacy of inferring (2⁺) from (1⁺), speaking as some do of "imprecise designators" (cf. n. 42 & 43). What the objector is effectively doing, the SV^+ theorist may say, is inferring that something is *determinately* the cut-off point between baldness and non-baldness from the fact that it is indeterminate that *something* is the cut-off point. Yet the

²⁸ This way of expressing vagueness is ultimately that which I have attributed to Russell in discussing his definition in Chapter Three, however Russell was hostile to the suggestion that vagueness ought to be encompassed within the scope of logic and to the general notion of modality. Thus he did not go on to develop any logical account, unlike advocates of SV^+ .

term 'the cut-off point' varies its denotation across precisifications and, as such, the inference is invalid. This is just to rephrase Fine's "truth-value shift" explanation of the fallacy of inferring (2) from (1).

Similarly, the SV defence of the retention of classical laws can be re-expressed in SV⁺ without recourse to metatheoretic notions like (super)truth. When it is objected that LEM fails since, for example, Tim may be neither tall nor not tall, the SV⁺ theorist can respond by admitting that it is indeed the case that Tim may be neither determinately tall nor determinately not tall but that this does not impugn the fact that he must determinately be one or the other. That is to say, for vague 'A', the SV⁺ theorist accepts

(3⁺) $D(A \vee \sim A)$ yet denies

(4⁺) $DA \vee D\sim A$.

Just as SV justified their retention of classical laws by denying the implication from (3) to (4), SV⁺ justifies the retention of classical laws by denying the implication from (3⁺) to (4⁺). One should not think that (3⁺) implies (4⁺) any more than one should accept the analogous modal inference from $\Box(A \vee \sim A)$ to $(\Box A \vee \Box \sim A)$.²⁹ That SV⁺ *does* thereby retain all classical laws can be seen from the fact that, given any classical law L, the corresponding SV⁺ formula DL remains valid in SV⁺ and the inference from DL to L is valid in SV⁺; hence L is valid in SV⁺.

Of course, Rolf might still object that his criticism has not been met. Instead of making essential reference to the metatheoretic notion of (super)truth, SV⁺ now makes essential reference to the object-language operator 'D'; yet why should one suppose that a speaker of English, say, has 'D' as part of their vocabulary anymore than they have metatheoretic notions as a part thereof? Well, in short, I think the answer is because without some such notion the idea that natural language is vague cannot be coherently expressed, and without the ability to express or describe vagueness their original objections to SV as a defective account of this phenomenon cannot get off the ground. More generally, whatever one uses to express the very idea of semantic vagueness — be it an essentially metalinguistic notion of (super)truth, or a notion of (super)truth in a semantically closed language, or a 'D' operator — can be invoked in the explanation of the supervaluation defence of classical logic.

²⁹ This defence of LEM is mentioned by Kamp in n. 4 of his 'The Paradox of the Heap' in U. Mönnich (ed.), *Aspects of Philosophical Logic*, Dordrecht (1981).

5.3.1 *Approximating the Logic of SV⁺*

What of the logic of the newly extended language? Well-formed formulae may include the operator 'D', so a formal semantics is required for 'D'. By equating admissible precisifications with possible worlds a semantics analogous to those for modal languages can be given (cf. Fine for details).

As Williamson has pointed out, one can initially model a semantics for SV⁺ on a model structure analogous to that for S5, though, as we shall see in the next section, problems arising from the phenomenon of higher-order vagueness necessitate restrictions analogous to those imposed on semantic structures for modal logics weaker than S5. Let's proceed initially by considering only first-order vagueness (not taking higher-order vagueness into account), and look at the supervaluation logic of a first-order vague language including 'D'.

The definition of logical truth is the same as that for the unextended language, namely — a formula A is valid if and only if, in all models, A is (determinately) true. Now it turns out that, as well as recognising as valid all those formulae of the unextended language that are classically valid and hence SV-valid, the following SV⁺ formulae are logical truths:

$$\models_{SV^+} D(A \supset B) \supset (DA \supset DB)$$

$$\models_{SV^+} DA \supset A$$

$$\models_{SV^+} \sim DA \supset D\sim DA$$

and, if $\models_{SV^+} A$ then $\models_{SV^+} DA$.

The second-last of these being analogous to the characteristic axiom for the modal logic S5.

The definition of argument validity however is not so straightforward. Remember that two definitions were offered, a strong and weak notion of semantic consequence:

- (i) B is a valid SV-consequence of A ($A \models_{SV} B$) if and only if, in all admissible precisifications, B is true whenever A is.
- (ii) B is a valid SV-consequence of A ($A \models_{SV} B$) if and only if whenever A is true in all admissible precisifications, B is true in all admissible precisifications.

The first and stronger definition entails the second but not vice versa. As it turned out, in SV the two definitions were materially equivalent, both being coextensive with classical validity yet, when extended to SV⁺ in the obvious way, they come apart; there are arguments valid by the second definition that are not valid by the first. Though both definitions agree that:

$$DA \models_{SV^+} A,$$

the following claim:

$$(*) \quad A \models_{SV^+} DA.$$

is endorsed by (ii), whereas definition (i) denies it.

Now the reason this *is* of interest is because of the following unequivocal fact. In some admissible precisifications 'A' may be true yet 'determinately A' false, since 'A' may not be true for all admissible precisifications — e.g. if 'A' is vague, so:

$$\not\models_{SV^+} A \supset DA.$$

This is simply a consequence within the theory SV^+ of the fact that 'D' is a non-redundant operator on vague sentences. Therefore, any definition of semantic consequence which endorses (*) will be such that the Deduction Theorem will fail for it. In particular, since definition (ii) endorses (*), the Deduction Theorem will fail for it. Fine, who defines consequence in this way, recognises and accepts this, offering the following relationship between logical truth and consequence:

$$A \models_{SV^+} B \quad \text{iff} \quad \models_{SV^+} DA \supset B.^{30}$$

The above counter instance to the Deduction Theorem is a counter instance to Conditional Proof — $\{A, X_1, \dots, X_n\} \models B \Rightarrow \{X_1, \dots, X_n\} \models A \supset B$ — in particular; so Conditional Proof fails if the consequence relation is defined as (ii).

So too does Contraposition — $\{A, X_1, \dots, X_n\} \models B \Rightarrow \{\sim B, X_1, \dots, X_n\} \models \sim A$. The reasons given above for the invalidity of ' $A \supset DA$ ' serve to show that, even on the weaker definition (ii) of consequence,

$$\sim DA \not\models_{SV^+} \sim A;$$

yet definition (ii) endorses (*). So Contraposition fails.

Proof by Cases, or \vee -Elimination — $\{A, X_1, \dots, X_n\} \models C$ and $\{B, Y_1, \dots, Y_m\} \models C \Rightarrow \{A \vee B, X_1, \dots, X_n, Y_1, \dots, Y_m\} \models C$ — also fails, given (ii). It is counterexemplified as follows: according to definition (ii) both $A \models_{SV^+} DA \vee D\sim A$ and $\sim A \models_{SV^+} DA \vee D\sim A$, yet $A \vee \sim A \not\models_{SV^+} DA \vee D\sim A$. (Definition (ii) endorses $A \vee \sim A \models_{SV^+} D(A \vee \sim A)$ as a special case of (*), however, neither definition (i) nor definition (ii) validates the unacceptable inference $D(A \vee \sim A) \models_{SV^+} DA \vee D\sim A$.)

³⁰ Fine, *op. cit.*, p. 290. This equivalence is then modified to take account of higher-order vagueness:
 $A \models_{SV^+} B$ iff the set $\{\sim A, B, DB, DDB, \dots\}$ is not satisfiable.

Finally, the acceptance of (ii) also invalidates *Reductio ad Absurdum* — $\{A, X_1, \dots, X_n\} \models B$ and $\{A, Y_1, \dots, Y_m\} \models \neg B \Rightarrow \{X_1, \dots, X_n, Y_1, \dots, Y_m\} \models \neg A$. According to definition (ii), $\{A \& \neg DA\} \models_{SV^+} DA$ and $\{A \& \neg DA\} \models_{SV^+} \neg DA$, yet $\not\models_{SV^+} \neg(A \& \neg DA)$ since this latter formula is equivalent to 'A \supset DA' and we have already established that $\not\models_{SV^+} A \supset DA$.

Drawing all this together then we can see that the acceptance of (ii) as an account of semantic consequence in SV^+ leads to the failure of: the Deduction Theorem, in particular Conditional Proof; Contraposition; Proof by Cases; and *Reductio*. These failures do not invalidate any inferences in the object language of SV^+ that are classically valid; as we saw earlier, any classically valid inferences are SV -valid and so SV^+ valid, and any classical logical truths are SV logical truths and so SV^+ logical truths. It is only in the extended fragment of the language that such inferential failures occur and then only in the metatheory.

Definition (i), on the other hand, does not have such consequences attaching to it. The problematic inference $A \models_{SV^+} DA$ does not count as valid using definition (i). It is easy to see that the Deduction Theorem (and hence Conditional Proof) holds; moreover, the counterexamples to Contraposition, Proof by Cases and *Reductio ad Absurdum* all depend on the validity of this inference and so are similarly defused by using (i) to define SV^+ -consequence. Williamson (who was kind enough to let me look at a typescript on supervaluations, and from whom a number of the foregoing points were taken) suggests that perhaps the supervaluationist should revert to a definition of semantic consequence like (i) above. This, he notes, would restore the problematic inference patterns but at the cost of the identification of truth with supertruth.

The point here is a good one: if truth is to be identified with supertruth then it might seem that, to be consistent with this identification, one should adopt that variant of the classical definition of consequence which is obtained by substituting 'supertrue' for 'true', that is, definition (ii) above. However, I think that there is an equally good motivation for adopting (i). One might well argue that underlying the supervaluationist's identification of truth with supertruth in the metalanguage is the more basic idea that one ought to endorse, assert, or accept as true *any claim* (not just any object language claim) which is true regardless of how any vagueness therein is resolved, because vagueness is a defect in language to be resolved, like ambiguity, and so nothing should be counted as anything less than true if it is true no matter how the defect is removed. That is to say, one might well think that what is right about supervaluationism is the fact that claims are super-evaluated (evaluated for all possible resolutions of their defective vagueness) in general.

Even in the object language one takes the claim being assessed *as a whole* to see if it's true regardless of how any vagueness therein is resolved. One does not, for example, identify truth with supertruth at some designated level of complexity such as the level of atomic sentences of the language, super-evaluating only at this level, and then extend to

more complex claims via some recursive semantic clauses. If a claim, simple or complex, is true no matter how any vagueness therein is resolved then it should count as true *simpliciter*. Now why should this only be adhered to with regard to claims made in the object language; why is it only the evaluation of sentences in the object language that are to be super-evaluated? Why shouldn't the complexity be taken to include, not only compounds of atomic object language claims, but compound metatheoretic claims like consequence relations? If a (complex) disjunction is counted as true *simpliciter* since when evaluated as a whole it is true regardless of how any vagueness within is resolved (i.e. since it is true when super-evaluated), it seems plausible to suppose that complex claims about consequence relations should count as true *simpliciter* if they, regarded as a whole, are true regardless of how any vagueness therein is resolved. However, definition (ii) takes the level of the object language as the basic level at which the identification takes place and proceeds classically from there rather than super-evaluating any claim to reach a verdict.

The more general supervvaluations approach I am suggesting validates definition (i) above since it is in the spirit of such wholesale super-evaluation. The consequence claim in the metatheory to be assessed, $\{X_1, \dots, X_n\} \models C$, is of the form $\forall x(\text{if } Fx \text{ then } Gx)$ where ' Fx ' = 'Premises X_1, \dots, X_n are true in model x ' and ' Gx ' = ' C is true in model x '. Now, since predicates ' F ' and ' G ' could be vague, we might super-evaluate in accord with the general SV approach to vague sentences I am advocating and, moreover, super-evaluate the entailment claim as a whole. Thus we would then count the entailment claim as true just if it was true no matter how we precisified the vague components therein, resulting in definition (i).

Of course nothing too grand hinges on the choice. The SV theorists have already accepted a non-classical metatheory so failures of classical metatheoretic principles do not add any qualitatively new objections. However, in accord with a principle of conservatism ('Don't scratch what doesn't itch') definition (i) has the upper hand; the contested principles in and of themselves are no problem in vague contexts so why not leave them well alone?

In summation then, even though, on at least one account of semantic consequence, the metatheory becomes radically nonclassical, all classical tautologies and all classically valid inferences are preserved and so the logic of vagueness SV^+ , though no longer classical, is merely an *extension* of classical logic. Furthermore, on a first approximation — ignoring problems to do with higher-order vagueness — its axioms correspond to those of the modal logic S5 though its valid inferences may not, depending on exactly how consequence is defined.

5.3.2 *Higher-Order Vagueness and the Logic of SV⁺*

Matters are complicated somewhat by the phenomenon of (H)igher-(O)rder (V)agueness though. An excellent discussion is to be found in Williamson's typescript on supervaluationism (parts of which I shall again make use of here), whilst Fine includes a discussion at the end of his 'Vagueness, Truth and Logic'. I simply want to mention some aspects relevant to the logic of vagueness, SV⁺.

Higher orders of vagueness arise because, as Fine puts it, the vague may itself be vague, or vaguely vague, etc. The three-valued semantics of SV⁺ divides the sentences of the language into the true, the false, and the neither true nor false, yet there is no more a sharp boundary between the true and the neither than there was between the true and the false or the red and the non-red.

In their efforts to recognise the absence of a first-order boundary between the red and the non-red, supervaluationists claim that ascriptions of redness to some things (border cases) may be neither true nor false; now, in order to recognise the absence of any second-order boundary between the true and the neither, or the neither and the false, supervaluationists claim that ascriptions of truth to sentences may themselves be neither true nor false. The admissibility of a precisification may itself be indeterminate since, to paraphrase Dummett, the boundaries between which admissible precisifications of a term range are themselves indeterminate — 'admissibility' is itself a vague notion. Some object *a*, say, may be a border case of a border case of red and thus it will be neither true nor false to say that it is neither true nor false that *a* is red.

Second-order vagueness thus shows itself in the failure of bivalence of the metametalanguage. In general, *n*th order vagueness will, in this way, manifest itself in the failure of bivalence of the metaⁿlanguage. Higher orders of vagueness can also manifest themselves in the object language of SV⁺ containing 'D' and 'I'; if *a* is a border case of a border case of red, then it is indeterminate that it is indeterminate that *a* is red — IIR*a*. In general, *n*th order vagueness is expressible in the object language with the help of the operators; if *a* is an *n*th order border case of red then IⁿR*a* expresses this.

So far, in ignoring HOV, the logic of SV⁺ has been very S5 like. The axioms and valid inferences mimic those of S5 if consequence is defined as (i), though if consequence is defined as (ii) the following disanalogy appears:

$$A \models_{SV^+} DA.$$

In particular, if vagueness were only a first-order phenomenon then the logic of SV⁺ would validate the analogue of the S5 principle for 'D':

$$\models_{SV^+} \sim DA \supset D\sim DA.$$

This result was implicitly stated earlier when I claimed that, ignoring HOV (i.e. assuming all vagueness to be merely first-order), the logic of 'D' was such as to validate the analogue of the S5 principle for 'D'. We can prove it as follows. Assume all vagueness *is* first-order. Then the application of the operator 'D' would eliminate vagueness — that is, 'DA' would be precise and so hold determinately or determinately not. So,

$$\models_{SV^+} DDA \vee D\sim DA.$$

But we also have the uncontroversial:

$$\models_{SV^+} DDA \supset DA$$

and so, if $\sim DA$ then $\sim DDA$, hence $D\sim DA$. That is:

$$\models_{SV^+} \sim DA \supset D\sim DA.$$

Now however, taking into account the phenomenon of HOV, the analogue of the S5 principle fails for 'D', and as a consequence, SV^+ theoremhood is strictly weaker than S5 theoremhood. This is because we can prove the converse of the above claim — namely that, if the analogue of the S5 principle for 'D' is valid in SV^+ then all vagueness is first-order; and, since not all vagueness is first-order, the result follows. To see this, note that the analogue of the S5 principle for 'D' entails the analogue of the S4 principle for 'D':

$$\models_{SV^+} DA \supset DDA.$$

This analogue of the S4 principle in conjunction with the analogue of the S5 principle and the fact that:

$$\models_{SV^+} DA \vee \sim DA$$

entails $\models_{SV^+} DDA \vee D\sim DA$.

Yet, given second-order vagueness this latter claim fails, so the analogue of the S5 principle fails.

It follows therefore from the presence of HOV (in particular, from the presence of second-order vagueness) that S5-like models cannot be used to model 'D' in just the same way that S5 models cannot be used to model \Box ' where the type of necessity at issue does not satisfy the S5 principle. Of course, constrained accessibility relations are used in modal semantics to get around just this problem and a similar idea could be introduced into the semantic models for SV^+ . The semantics will have to be such as to invalidate even

the analogue of the S4 principle according to Dummett³¹ and Williamson has made the point that without any conditions on the relation the logic will be analogous to the weak modal logic T, admitting as theorems just those listed in §5.3.1 except the analogue of the S5 principle.

I shall not attempt to describe the semantics for SV^+ in the light of HOV; it is a considerable task in itself and my major concern with SV^+ is the underlying philosophical view of vagueness that supports it, however a few general comments on the phenomenon of HOV are in order.

The presence of HOV makes the logic of representational vagueness I have been considering, SV^+ , extremely complex. Contrary to the results of §1.4, attempts might be made to show that all vagueness extends only to the first order.

A line of argument to that effect has been suggested (though not thereby endorsed) by Jackson and is suggested by Fine's comment that "[a]nything that smacks of being a borderline case is treated as a clear borderline case."³² Just what Fine means by "smacks of being a borderline case" is unclear. If he means anything which might be suspected (correctly or incorrectly) of being a border case then, as Williamson has pointed out, we might be forced by this suggestion to treat as a clear border case something which never was a border case yet was mistakenly suspected of being one. If, on the other hand, Fine means something which is similar to a border case, then (as Williamson has again noted), anything very close to the boundary between clear cases and border cases, since similar to

31 A point worth mentioning in the light of this is the following: were any logic system S to accept the following principle

$$(**) \quad \vDash_S A \supset DA$$

it would be equivalent to the claim that there is no vague language, not even first-order vagueness, within the scope of the logic S; one way to see this is to note that 'D' would be redundant or, alternatively, to note that the principle (**) is equivalent to the principle

$$\vDash_S DA \vee D\sim A$$

i.e. all sentences of S are precise. Now it might be thought that the absence of higher-order vagueness can be characterised by a particular second-order instance of the former principle, namely the S4 principle

$$\vDash_S DA \supset DDA$$

This seems to be the view of Dummett when he says ['Wang's Paradox', p. 311] that a logic of HOV will be weaker than S4. What we have just shown is that the stronger S5 principle

$$\vDash_S \sim DA \supset D\sim DA$$

is required to eliminate HOV since it is the S5 principle which is equivalent to

$$\vDash_S DDA \vee D\sim DA.$$

So, the acceptance of first-order vagueness within the scope of S entails the denial of (**) in S; however the acceptance of HOV does not thereby entail the denial of its higher-order analogue, the S4 principle. It is the stronger S5 principle that must be denied.

In general then we can say that $(n+1)^{\text{th}}$ -order vagueness is characterised by the failure of

$$\vDash_S DD^n A \vee D\sim D^n A$$

rather than failures of the weaker

$$\vDash_S D^n A \supset DD^n A$$

though at the first-order they are equivalent.

32 Fine, *op. cit.*, p. 297.

a border case, will be counted as a clear border case and anything similar to it will likewise be counted a clear border case, and so on ... There may be no end to the assimilation of determinate cases to the category of clear border cases until everything counts as a border case.

What Fine *might* have meant, and Jackson *does* mean, is the following idea. Consider an object *a* for which it is indeterminate whether it is a border case of 'red', say. Then, since it *is* a border case, it should count as border case *simpliciter*; border cases of border cases are border cases *simpliciter*, as are border cases of determinate instances and border cases of determinate counter-instances. So everything that 'red' can be significantly predicated of is either a determinate instance, a determinate counter-instance, or a border case. As a consequence all vagueness is first order vagueness.

In particular, 'vague' is not vague; either there determinately are border cases for a linguistic item or there determinately are not since, if it is ever indeterminate whether or not there are border cases for it, then there determinately *are* border cases for it and so it is vague.

Given that 'vague' *is* vague, established in §1.4, something must be wrong with this line of argument.

The first thing to notice is that the argument (perhaps better described as a suggestion) is fallacious. The notion of a 'border case' is treated as if it were an absolute notion, which it is not. Any border case is a border case relative to something. (If you need to be convinced of this just consider how you'd respond to someone who said, pointing, 'That's a border case' or 'That's an indeterminate instance'. A border case *of what?*) So, when spelt out, what is really being suggested is that any border case of a border case of 'red' is a border case of 'red'. Obviously a border case of a border case of 'red' is a border case of something (*simpliciter*), but being a border case of something does not make it a border case of 'red'. It is in fact a border case of the predicate 'indeterminately red'. To suppose that border cases of 'indeterminately red' are border cases of 'red' simply begs the question.

Secondly, the consequence for the supervenientist of adopting such a view undermines the very plausibility of there being border cases in the first place. To account for the apparent lack of a sharp (first order) boundary between, say, a predicate's extension and anti-extension the supervenientist admits first order vagueness and models it by means of a sharp tripartite division. To go on and defend this tripartite division's commitment to sharp (second order) boundaries where there appear to be none undermines the grounds for abandoning the classical view of there being a sharp bipartite division in the first place. How can one, at one level of vagueness (the first), claim that the apparent lack of hidden boundaries is more than merely apparent yet then go on to claim that at the next level of vagueness (the second) this apparent lack is merely apparent? There would seem to be no qualitative difference between the vagueness at each level —

each urge to deny the presence of hidden though distinguished points in a soritical series should be treated alike. There is no better ground for claiming there to be a distinguished point between determinate instances of 'red' and border instances than there was for making such a claim with regard to determinate instances and determinate counter instances, yet the former claim is accepted whilst the latter is rejected.

Having once started to model vagueness by effecting a sharp tripartite division, the SV theorist seems compelled to make room for any higher-order vagueness, that is vagueness around these sharp boundaries, by effecting yet another tripartite division around each. "At no point does it seem natural to call a halt to the increasing orders of vagueness", so the iteration seems endless.³³

5.4 *The World is Not Vague — a Defence of Representationalism*

We have seen that SV⁺ was defended as a logic of vagueness by claiming that it followed (even with its counter-intuitive aspects — LEM, etc.) from an enlightened representational account. SV⁺ would only seem justifiable in so far as the enlightened representational account of vagueness can be justified; on what grounds ought one accept LEM or the SV account of 'there is'? — on the grounds that an enlightened representational account prescribes such results. Even if such a defence was thought to be adequate then we would be entitled to ask for the grounds on which an enlightened representational account is prescribed. And prescribed it must be in order that its consequences be prescribed — so that they have the right modal force attaching to them — because, remember, there were doubts as to their descriptive accuracy which were offset by claiming them to be necessary in some sense.

It is to this task we now turn. Why should one adopt an enlightened representational account of vagueness? This question can be narrowed; if we accept that vague language should be encompassed within logical theory then we need only ask why one should adopt a representational account? Representationalist accounts already considered seemed to depend either on *a priori* prejudice to the effect that the world is precise (Russell) or on the supposition that it can be precisely described and can thereby be assumed to be precise (Quine, *et al*). The former is obviously an inadequate defence, whilst the latter was seen to be sustainable only at great ontological cost. The supervaluationist however can look to *a priori* arguments like that put forward by Gareth Evans in the late 1970's in support of their representational view.

33 *ibid.*

Ontological vagueness, or *fuzziness*, has been criticised on the grounds that *de re* vague (indeterminate) identity is incoherent; identity cannot be vague *de re* and thus there can be no fuzzy objects.³⁴

Perhaps the best known attempt to argue for the impossibility of vague objects is that of Gareth Evans, put forward in his 1978 article entitled "Can There Be Vague Objects?".³⁵ This publication was seen as offering a substantive argument where previously there had only been prejudice: "[w]here previously those of us sympathetic to the view that there might be vague objects had nothing to engage with except the unreasoned intonement of a mantra ... Evans was now presenting, perhaps for the first time, what appeared to be a cogent argument against that view."³⁶ It *appeared* to be cogent, however debate has raged since whether anything more substantive than the mere appearance can be gotten from the Evans Argument.

In the light of the flurry of papers and discussion on the matter in the last decade it is very difficult to uncontroversially state the Evans Argument and simultaneously show how it was supposed to offer a *reductio* of the view that there could be vague objects. The very basic belief shared by many that ontological vagueness simply does not make sense has been the overriding fact uniting representationalists; no univocal agreement has been reached as to how the Evans Argument supports this belief — the argument has, in a sense, been all things to all people. Nonetheless, after stating the argument I shall try to show how it at least appeared to resolve the debate. The representationalist certainly seemed to many to be in possession of a powerful argument in favour of their view. What I want to do, then, is tread the narrow path between giving a final assessment of the argument (which will concern us in the next chapter) and saying nothing about it at all.

So, heeding the words of Lewis who claims that the Evans Argument, whilst being well known, is frequently misunderstood³⁷, we should determine just *what* the argument is and *how* it aims to achieve its end.

Evans begs us to consider the possibility ...

³⁴ Though the terms '*de re*' and '*de dicto*' are contentious I shall use them as a convenient shorthand in the following specific sense: let's call the vagueness of an expression '*de re* vagueness' just if its vagueness is not attributable to (merely) semantic vagueness (i.e. it is not representational vagueness) but must be thought of as being due to the vagueness of that to which the expression refers; I shall call the vagueness of an expression '*de dicto* vague' just if its vagueness is attributable to the semantic vagueness of the terms involved (i.e. if it is attributable to representational vagueness).

³⁵ Evans, G., 'Can there be Vague Objects?', *Analysis* 38 (1978): 208. Nathan Salmon presents a similar argument in his *Reference and Essence*, Princeton University Press (1981), pp. 243-4; so too does David Wiggins in his 'On Singling out an Object Determinately' in P. Pettit & J. McDowell (eds), *Subject, Thought and Context*, OUP (1986), Ch. 6, esp. pp. 173-6.

³⁶ Burgess, J.A., 'Vague Identity: Evans Misrepresented', *Analysis* 49 (1989), p. 113.

³⁷ Lewis, D., 'Vague Identity: Evans Misunderstood', *Analysis* 48 (1988).

... that the world might itself *be* vague. Rather than vagueness being a deficiency in our mode of describing the world, it would then be a necessary feature of any true description of it. It is also said that amongst the statements which may not have a determinate truth value as a result of their vagueness are identity statements. Combining these two views we would arrive at the idea that the world might contain certain objects about which it is a *fact* that they have fuzzy boundaries. But is this idea coherent?³⁸

The question is subjected to the ensuing logical proof — the Evans Proof — which proceeds as follows: assume '*a*' and '*b*' are singular terms and that the sentence '*a* = *b*' has indeterminate truth value. Let the operator ' ∇ ' be that sentential operator expressing the notion of indeterminacy. Then:

$$(1) \quad \nabla(a = b).$$

(1) tells us that we may ascribe to *b* the property ' $\lambda x[\nabla(x = a)]$ ':

$$(2) \quad \lambda x[\nabla(x = a)] b.$$

And yet:

$$(3) \quad \neg\nabla(a = a)$$

thus:

$$(4) \quad \neg\lambda x[\nabla(x = a)] a.$$

From (2), (4) and Leibnitz's Law:

$$(5) \quad \neg(a = b)$$

which Evans claims contradicts the initial assumption that the identity statement '*a* = *b*' has indeterminate truth value. If we are not yet convinced of this Evans invites us to strengthen the conclusion, (5), as follows: let ' Δ ' ('Determinately') be the dual of ' ∇ '. On the assumption that ' Δ ' and ' ∇ ' generate a modal logic as strong as S5 we can prefix (1)-(4) and Leibnitz's Law with the 'Determinately' operator and so strengthen the conclusion to:

$$(5') \quad \Delta\neg(a = b).$$

This, according to Evans, "... is straightforwardly inconsistent with (1)."³⁹

The most plausible (and charitable) interpretation of the Evans Argument will make this last remark of his proof bear upon the "coherence" of the notion of 'objects having fuzzy boundaries'. The argument is of *reductio* form having as its implicit conclusion some statement involving fuzziness.⁴⁰

38 Evans, *op. cit.*

39 *Ibid.*

40 As Richard Sylvan has pointed out, this argument-form can also be instanced with properties P_1 and P_2 using second-order property-abstraction, and a variant of Leibnitz's Law for property-identity.

I.e.: assume ' P_1 ' and ' P_2 ' are predicates and that the sentence ' $P_1 = P_2$ ' has indeterminate truth value. Let the operator ' ∇ ' be that sentential operator expressing the notion of indeterminacy. Then:

$$(1) \quad \nabla(P_1 = P_2).$$

(1) tells us that we may ascribe to P_2 the property ' $\lambda X[\nabla(X = P_1)]$ ':

$$(2) \quad \lambda X[\nabla(X = P_1)] P_2.$$

I think that it is pretty clear just how this formal proof might be thought to bear upon the notion of fuzziness. In order that a *reductio* proof might (relevantly) entail anything at all about fuzziness it must engage the notion at some point in the proof (and then go on to show that the assumption involving fuzziness entails absurdity, i.e., is absurd). The only steps in the proof reasonably interpreted as doing this are the assumption of (1) in addition to the move from (1) to (2). If this is not obvious (and it would seem not judging from some of the commentaries on Evans' argument) let me suggest why.

The move from (1) to (2) involves a change in scope of the indeterminacy operator, '∇', which has sometimes been criticised as fallacious. Its status is not so easily determined however; it depends on the sense in which the indeterminacy is thought to apply, *de dicto* or *de re*. If the indeterminacy apparent in (1) were thought to be mere semantic indeterminacy, arising from our fixing the reference of 'a' and/or 'b' by vague descriptive means, then the attribution of a *property* ('being indeterminately identical to x', for some appropriate instance of x) to one or t'other *is* fallacious; the indeterminacy is merely semantic and not attributable to any object. In the words of Lewis:

If vagueness is semantic indeterminacy, then wherever we have vague statements, we have several alternative precisifications of the vague language involved, all with equal claim to being 'intended'. These alternative precisifications play a role analogous to alternative worlds in modal logic. The operator 'it is vague whether ...' is analogous to an operator of contingency, and means 'it is true on some but not all of the precisifications that ...'. A term like 'Princeton' that denotes different things on different precisifications is, analogically speaking, non-rigid. When 'a' is non-rigid, the ... [change of scope] ... is fallacious. It is analogous to the fallacious modal equivalence

And yet:

$$(3) \quad \neg \nabla (P_1 = P_1)$$

thus:

$$(4) \quad \neg \lambda X [\nabla (X = P_1)] P_1.$$

From (2), (4) and "Leibnitz's Law":

$$(5) \quad \neg (P_1 = P_2)$$

which, presumably, Evans would claim as contradicting the initial assumption that the identity statement ' $P_1 = P_2$ ' has indeterminate truth value. If we are not yet convinced of this we will presumably be invited to strengthen the conclusion, (5), as follows: let 'Δ' ('Determinately') be the dual of '∇'. On the assumption that 'Δ' and '∇' generate a modal logic as strong as S5 we can prefix (1)-(4) and the Axiom of Extensionality with the 'Determinately' operator and so strengthen the conclusion to:

$$(5') \quad \Delta \neg (P_1 = P_2).$$

The Evans Argument counts, therefore, as a successful denial of the coherence of object-fuzziness if and only if it counts as a successful denial of the coherence of property-fuzziness. This forestalls any attempt to reject the idea of object-fuzziness on the basis of Evans' proof whilst nonetheless accepting that there could be (paraphrasing Evans) certain properties concerning which it is a *fact* that they have fuzzy extension. Object- and property-fuzziness stand or fall together.

between 'It is contingent whether the number of planets is nine' (true) and 'The number of planets is such that it is contingent whether it is nine' (false), ...⁴¹

However, if the vagueness is assumed to be ontological indeterminacy no scope fallacy obtains.

If the names 'a' or 'b' are representationally vague, shifting their reference from precisification to precisification (in just the same way as representationally vague predicates shift extension from precisification to precisification) — if they are not what Thomason describes as *precise designators*⁴² — then the indeterminacy may be characterised as representational. So we could identify 'Δ' and '∇' respectively with 'D' and 'T' as described by SV⁺. Now the fallacious move alluded to by Lewis — our being able to quantify into the scope of '∇' — is exactly what SV⁺ ought to deny in accord with the modal analogy alluded to by Lewis, where quantification into the scope of modal operators is sanctioned only if the names are rigid (cf. the invalidity of moving from (1⁺) to (2⁺) in §5.3). So, if the Evans-operators expressed representational (or, as Lewis puts it, semantic) determinacy/indeterminacy then the proof is fallacious as Lewis suggests. However this result is hardly surprising; it simply leaves open the possibility of there being semantically indeterminate identity statements — any supposed proof to the effect that there can be no semantically indeterminate identity statements is blocked. On the other hand, if the names are indeed taken to be precise designators, then '∇(a = b)' must be taken to precisely describe a vague state-of-affairs, and as such quantification across '∇' and 'Δ' is legitimate. The ability to read '∇' as having narrow scope in (1) — that is, the ability to infer (2) from (1) — is sanctioned.⁴³

This is a charitable interpretation to the extent that, though the analogy with modal logics needs explaining (what counts as the analogue of the base or actual world, for

41 Lewis, D., *op. cit.*, pp. 128-9. Thomason makes essentially the same point in reference to the move in the Evans argument from (3) to (4); cf. Thomason, R., 'Identity and Vagueness', *Philosophical Studies* 42 (1984), p. 331.

42 A designator is precise just if its referent is not fixed by any vague description or vague ostension and it is not vague *what* the designator picks out (though, if the world could be other than sharp, what it picks out might itself be fuzzy); cf. Thomason, *op. cit.*, p. 331.

43 There are other versions of the Evans Argument, that due to Wiggins for example, which do not make use of λ-abstraction (property formation) but simply employ a substitution principle like the following: $a = b \supset (\nabla(c = a) \supset \nabla(c = b))$. Now it might be wondered, as Rasmussen does in his 'Vague Identity', *Mind* 95 (1986), esp. 82-3, how any such argument can reduce to absurdity the supposition that there can be vague identities *due to ontological vagueness* since that Evans-inference, which guarantees that the vagueness of the identity claim is not semantic, is not employed in such proofs. However, the substitution principle itself is commonly restricted, being said to be valid only if the names 'a' and 'b' are precise designators. Just as the inference from a wide to a narrow scope reading of (1) is said, by analogy with modal contexts, to be legitimate only if the names in (1) precisely designate, it is argued, by analogy with modal contexts, that the principle of substitution is only valid if the names 'a' and 'b' precisely designate. If they did not then the substitution made would be analogous to the following in a modal context: '(the number of the planets = 9) \supset (it is contingent that the number of the planets = 9 \supset it is contingent that 9 = 9)' which is false. If the substitution principle is taken to guarantee substitutivity within the scope of '∇' then 'a' and 'b' are precise designators. Either proof of the incoherence of vague identity is taken to go through (if at all) only on the assumption that 'a' and 'b' are precise designators — that is, on the assumption that any indeterminacy in identity is ontological.

example) and the inference from (1) to (2) needn't, on some views, be a commitment to *de re* indeterminate identity (Sainsbury has recently suggested in conversation that representationalists can accept the inference⁴⁴), without this the proof seems unable to succeed. So let's continue with this (perhaps excessively charitable) line of thought.

The negative conclusion that Evans goes on to draw is then neutral on the more general issue of whether or not there are indeterminate identity statements. What is denied by Evans is the possibility of their being indeterminate identity statements that are interpretable *de re*. One need not further attribute to Evans the claim that there *are* indeterminate identity relations. His proof is silent on this issue. His claim is simply that *if* there are, they must (on pain of incoherence) be read *de dicto*. In this sense then claims made in the literature to the effect that Evans is not committed to the vagueness of identity statements are justified.⁴⁵ The stronger claim made by Lewis⁴⁶ to the effect that Evans assumes some identity statements to be vague is not supported by the textual evidence under consideration.

Nonetheless, such an assumption may be thought to be easily justified by merely concocting or citing an example using a name that is vague by virtue of having its reference fixed by vague descriptive means. "Example: 'Princeton = Princeton Borough'. (It is unsettled whether the name 'Princeton' denotes just the Borough, the Borough plus the surrounding Township, or one of the countless somewhat larger regions.)"⁴⁷ The acceptability (by most) of the belief in the existence of vague identity statements is further evidenced by the response Evans' argument is often said to elicit: namely that he has endorsed a fallacious proof of the absurd conclusion that there can be no vague identity.⁴⁸ This response is misguided as we have seen, he is not committed to the impossibility of vague identity statements; on the other hand, nor is he committed to their existence, though to do so seems hardly controversial.

Having now given an interpretation of what is going on in the initial part of the Evans Proof, we see that (1) in conjunction with the validity of the inference from (1) to (2) is equivalent to the claim that a *de re* vague identity relation obtains between *a* and *b*. This is the reading of (1) required for the Evans Argument to get off the ground. Now, is

44 This would entail that *any* indeterminate identity statement was incoherent if the Evans Argument were successful — even identity statements construed *de dicto*. In response, Sainsbury remarked that this might indeed be the case, serving to show that there are no vague singular terms (cf. n. 49 Chapter One). In the light of what was said above in n. 40 however, this line of thought quickly runs into difficulties; if the Evans Argument is found telling against the vagueness of singular terms, then the analogous argument involving predicates will serve to show that there can be no vague predicates. The simple absurdity of such a conclusion would mean that anyone holding this position is forced to reject the Evans' Argument as unsound.

45 See for example: Garrett, B.J., 'Vagueness and Identity', *Analysis* 48 (1988), p. 131.

46 Lewis, D., *op. cit.*, p. 129.

47 *Ibid.*, p. 128. Garrett cites another example in his 'Vague Identity and Vague Objects', *Noûs* 25 (1991), p. 341.

48 See: Lewis' comment, *op. cit.*, p. 129, first paragraph; Rolf, B., *op. cit.*, pp. 72-3; and Noonan, H., 'Vague Objects', *Analysis* 42 (1982), pp. 3-6.

it this possibility that Evans considers necessary for the claim that there can be fuzzy objects? The short answer is that we just don't know; Evans never explicitly said this. The next obvious question then is: *should* one consider the possibility of there being *de re* vague identity relations as being necessary for the claim that there can be fuzzy objects? Though I shall not offer a definitive answer until the notions of a fuzzy object and identity in the context of fuzziness have been analysed (in Chapter Six), we *can* say that it was, for a long time after the publication of Evans' article, generally assumed to be necessary. That is to say, it was generally agreed that the impossibility of a *de re* reading of (1) was sufficient to prove vagueness in language to be merely semantic and rule out any possibility of there being ontological vagueness.

It is certainly an inviting construal of the argument. Evans was *apparently* trying to prove a *de re* reading of (1) to be inconsistent, thereby proving fuzziness to be incoherent. This suggests that he took *de re* indeterminate identity to be a necessary condition for fuzziness; that it is a sufficient condition is relatively uncontroversial. (Of course, it is only the identity clause as a *necessary* condition for fuzziness that is crucial.) Many have certainly taken this line or assented to claims that could plausibly lead to this view. Fine, for instance, in his 1975 article (thus predating Evans' argument) wrote that "for any vague name *a* there is a uniquely referring [precise] name *b* for which the identity-sentence *a = b* is neither true nor false." This is of course a thoroughly semantic claim but its ontological analogue is precisely the claim at issue, linking vagueness and identity. David Wiggins also endorses the claim, identifying as he does "the existence of objects that are indeterminate in respect of identity" with "objects such that it is indeterminate which things they are". And Sainsbury, in his 1988 book *Paradoxes*, supports the idea "that if an object were vague, it would be a vague matter what object it is identical with".⁴⁹

In this spirit, Brian Garrett, apparently speaking for all, stated in 1988 the following *Vague-Identity Thesis*:

[t]he thesis that there can be vague [fuzzy] objects is the thesis that there can be identity statements which are indeterminate in truth-value (i.e. neither true nor false) as a result of vagueness ... *the singular terms of which do not have their references fixed by vague descriptive means.*⁵⁰

Now that we know where fuzziness was generally considered to engage with the proof we can look to see what the *reductio* purports to show regarding this fuzziness.

⁴⁹ Fine, *op. cit.*, p. 266; Wiggins, *op. cit.*, p. 173; Sainsbury, R.M., *Paradoxes*, CUP (1988), Ch. 2, p. 47.

⁵⁰ Garrett, B.J., 'Vagueness and Identity', *Analysis* 48 (1988), p. 130. As we shall see in the next chapter, this equivalence between the possibility of *de re* vague identity statements and the possibility of there being vague objects is but one way to cache out the idea that the world is or could be vague; there are other alternatives that free us from the need to establish the coherence of *de re* vague identities. This is also Garrett's view of the matter; he no longer considers the identity criterion as necessary.

Assume there to be some object, b , similar in some respects but not identical to some other object, a ; b possesses the property ' $\lambda x[\forall(x = a)]$ '. By the law of self-identity, ' $a = a$ '. Moreover,

even if ... a were a vague [fuzzy] object, we still ought to be able to obtain a (so to speak) perfect case of identity, provided we were careful to mate a with exactly the right object. And surely a is exactly the right object to mate with a . There is a complete correspondence. All their vagueness matches exactly.⁵¹

So we may reasonably suppose it to be determinately the case that ' $a = a$ '.

The collective "we" here will not include everyone. Hegelians might disagree and Lorenzo Peña has argued that (paraphrasing), from the highly plausible hypothesis that some propositions are as true as they are false we seem to be compelled to conclude that every entity is both, and in the same degree, identical to itself and distinct from itself. The upshot is (still paraphrasing) that we have a good argument in support of the principle of distinction, viz.: *everything is rather distinct from itself*.⁵²

Those who, contra Peña, are prepared to accept the determinacy of self-identity will be committed to its not being the case that a is indeterminately identical to a , ' $\neg\forall(a = a)$ '. Thus (given λ -abstraction theory) a may be said not to possess the property ' $\lambda x[\forall(x = a)]$ ', i.e., ' $\neg\lambda x[\forall(x = a)]$ a '. Now, in accordance with identity criteria specified by Leibnitz's Law, we may say that, since a and b differ on some property, they are not identical, ' $\neg(a = b)$ '.

This, according to Evans, "contradict[s] the assumption, with which we began, that the identity statement ' $a = b$ ' is of indeterminate truth value."⁵³ In other words, having effectively proven that if $a = b$ then determinately $a = b$ (the Wiggins/Sainsbury contraposed version of the Evans Argument), it is suggested that indeterminate identity is incoherent. Both Sainsbury and Wiggins accept this consequence in the works cited; Sainsbury, for instance, claims that showing that identity is not a vague relation is equivalent to showing "that questions of the form 'Is this thing... the same as that thing...?' have definite [determinate] answers. The suggestion is that, quite generally: if β is α , then β is definitely [determinately] α ."⁵⁴

However, it is not obvious how a proof of the latter claim (or, equivalently, the claim which Evans has supposedly proven) addresses the former point. To say that questions of identity have determinate answers presumably means that answers like 'Determinately yes' or 'Determinately no' are the only ones possible, but the suggestion is only to the effect that if the answer is affirmative then the answer 'Determinately yes' is required. What

51 Wiggins, *op. cit.*, p. 175.

52 Peña, L., 'Identity, Fuzziness and Noncontradiction' *Noûs* 18 (1984): 227-59, esp. p. 230.

53 Evans, *op. cit.*

54 Sainsbury, *op. cit.*

about answers in the negative? If it is *not* the case that one thing is the same as another is it determinately different? This is a general problem for the Evans Proof seen as an attempt to reduce a *de re* reading of (1) to absurdity. No contradiction appears derivable in the object language, as yet.⁵⁵

Some have responded at this point by claiming, in effect, that no object language inconsistency is required; a metatheoretic inconsistency is already to hand. Nathan Salmon and Terence Parsons take this line and recently Garrett has opted for this approach.⁵⁶ (This line of defence for Evans' *reductio* proof will be explored in Chapter Six.)

Others, given the lack of outright inconsistency, claim it is still possible to coherently maintain that, though *a* is not identical to *b*, ' $\neg(a = b)$ ', it is similar to *b*, ' $\nabla(a = b)$ '. Such a position is unstable according to Evans because, on the assumption that ' ∇ ' and its dual ' Δ ' generate a modal logic as strong as S5, we may prefix all assumptions with ' Δ ' and so may infer the stronger conclusion ' $\Delta\neg(a = b)$ '. *Ex hypothesi*, its being determinately not the case that *a* is identical to *b* entails its not being the case that *a* is indeterminately identical to *b*, ' $\neg\nabla(a = b)$ ', which contradicts our initial assumption that ' $\nabla(a = b)$ '.

The first thing to note with Evans' response is that the duality assumption had better be false; that is, it had better not be the case that: $\nabla A \equiv \sim\Delta\sim A$ and $\Delta A \equiv \sim\nabla\sim A$. Were one to accept the duality principles then ' ∇ ' and ' Δ ' would stand in the same relation to each other as ' \diamond ' and ' \square ' respectively. Then (1) would amount to the claim that it is not determinately not the case that $a = b$, for arbitrary *a* and *b* — in which case the Evans Proof would supposedly show that all *de re* identities determinately don't hold! Of course, such a proof will not go through in any case on the duality assumption since (3) would be false if the duality assumption were true. What (3) would say on this reading is that it is not even *at least* vaguely the case that $a = a$; that is, it is neither vaguely the case that $a = a$ nor determinately the case that $a = a$. So, were one to assume duality then both (1) and (3) would have readings contrary to those intended by Evans; the operators ' ∇ ' and ' Δ ' are not duals. Though ' Δ ' is analogous to ' \square ', ' ∇ ' is analogous to contingency not ' \diamond '.

Even though duality is unacceptable, this in fact does not affect what Evans goes on to argue for — he does not actually need to make use of the duality assumption. He need only make use of the (questionable) analogy between ' Δ ' and ' \square ' (an analogy which holds

55 This contrasts with discussions of contingent identity — being identical but only contingently so. A contradiction is immediately obtained in the object language by assuming there to be cases of contingent identity given the supposed truth of the claim that all identity is necessary identity.

This serves to highlight a further *prima facie* disanalogy between contingency and vagueness. Whilst we might, contra Kripke, entertain the idea of an identical pair whose identity is not a necessary matter, there seems to be no corresponding thought in the case of vagueness; the target of the Evans Proof is not the possibility of an identical pair whose identity is nonetheless a vague matter — one might of course think that it *should* have been his target (reinforcing the modal analogy) but this would require further argument.

56 Cf. Salmon, *op. cit.*, p. 244, n. 24; Parsons, T., 'Entities Without Identity' in J. Tomberlin (ed.), *Philosophical Perspectives, 1, Metaphysics*, Ridgeview Publishing Co. (1987), pp. 12-13; Garrett, B., 'Vague Identity and Vague Objects', *op. cit.*, pp. 347-8.

independently of duality) and the assumption that ' Δ ' generates a modal logic as strong as S5. Evans requires the following strengthened premises:

$$(1') \Delta \nabla(a = b) \quad , \quad (3') \Delta \sim \nabla(a = a) \quad \text{and} \quad \Delta((a = b) \supset \forall F(Fa \equiv Fb)).$$

The second and third of these will follow from the fact that: if $\models A$ then $\models \Delta A$, whilst the first would be available if ' Δ ' generated a logic as strong as S5, given ' $\nabla(a = b)$ '.

However, the assumption that ' Δ ' generates a modal logic as strong as S5 is false — this is the second, telling criticism of his strengthened argument against indeterminate identity. Arguments against the logic of (representational) vagueness, SV^+ , being S5-like discussed in §5.3.2 will apply equally well against a logic of (ontological) vagueness involving ' Δ '.

It has been suggested by Garrett (in conversation) that even the general assumption that the logic of ' Δ ' is S5-like is unnecessary. Leibnitz's Law and premise (3) seem able to be strengthened as much as one likes given their status as theorems, and (1) can be strengthened to (1') if appropriate a and b are chosen. The problem with this attempt to derive an outright inconsistency by getting ourselves into a position where we are able to prove $\Delta \sim(a = b)$, and thus conclude $\sim \nabla(a = b)$, is that the assumption to be contradicted is now $\Delta \nabla(a = b)$ — we have not *inferred* $\Delta \nabla(a = b)$ from $\nabla(a = b)$, but rather *replaced* the latter with the former, stronger premise and so we would now require a conclusion at least as strong as $\sim \Delta \nabla(a = b)$ for *reductio*. The Evans Proof is not strengthened by such a move, rather it is replaced by a new proof as fraught with difficulties as that which it replaces.

After critical examination, Evans' proof appears to fall short of its intended mark — its validity is questionable. We must wait until the next chapter to see if it can be salvaged via the Salmon/Parsons/Garrett approach mentioned a few paragraphs previously.

In summary then, *if* the Evans Proof is sound, there can be no *de re* vague identities. (As I said however, this negative conclusion concerning *de re* vague identities is neutral on the more general issue of whether or not there are vague identity statements.) The general argument that there can be no fuzzy objects would then be sound *if* the possibility of there being fuzzy objects was taken to at least *entail* the possibility of there being *de re* vague identity statements — an entailment guaranteed by the acceptance of the Vague-Identity Thesis. So we can say that the representationalist's case is secured by the Vague-Identity Thesis in conjunction with the soundness of the Evans Proof. In that case it would follow in particular that the notion of spatio-temporally fuzzy objects was incoherent.

Whether or not the Evans Proof *is* sound is, however, a controversial matter. Those who claim it *is* disagree as to exactly how the conclusion is established, whilst those, like

myself, who claim it is *not* disagree on where to locate the fallacy. My views on just how it is that the proof, though valid, fails to be sound will be taken up in the next chapter.

Further problems arise with the more general evaluation of Evans' overall argument since it seems to assume that the coherence of *de re* vague identity is a necessary condition for the possibility of there being ontological vagueness and this is not at all obvious. With acceptance of the Vague-Identity Thesis on the wane, the Evans Proof appears more and more peripheral to the primary issue of the possibility of ontological vagueness. That is to say, with a weakening of the bridge that links Evans' proof to the representationalist's position the proof has less importance in the overall discussion than it has previously enjoyed. Of course, it is still crucial to issues surrounding identity in the context of an ontological account of vagueness (necessarily) and to this extent still deserves consideration, however a more constructive approach will be adopted towards ontological vagueness in the next chapter than that of proceeding via the perceived need to defuse the Evans Proof first.

5.5 Summary

Our primary concern in this chapter was to investigate that account of vagueness which I have termed enlightened representational vagueness — though vagueness is merely representational it is not thereby excluded from the scope of logic. It distinguished itself from the Russellian and Quinean accounts by denying that vagueness was beyond the scope of logic though it, like Russell's account, supposed ontological vagueness to be impossible on *a priori* grounds.

On the assumption that ontological vagueness could be shown to be *a priori* impossible, we proceeded to investigate the consequences for logic of adopting such an account. What I have concentrated on is the popular supervaluations approach, suggesting a line of defence for those advocating that logical approach — though a logic of representational vagueness is deviant in its metatheory, the fact that vagueness is taken to be merely semantic can be used to defend the retention of one's preferred logical theory (e.g. classical, intuitionistic, relevant, and so on). Whether or not the theory also requires extension depends on whether or not one extends the object language to include the expression of vagueness within it; at worst, the revised logical theory turns out to be an extension of the original logical theory atop of which the supervaluational superstructure is added.

I have further tried to suggest that not only is the supervaluations approach defensible on an enlightened representationalist account of vagueness but that it or

something very much like it might seem prescribed by an enlightened representational account. The defence of a supervaluations approach against charges of descriptive inadequacy would seem to require just such a claim; it is because the vagueness within the scope of logic is merely semantic that those who were prepared to endorse, say, classical logic in the absence of vagueness (the default logic I have worked with here) ought to endorse the preservation of classical laws and classical inferences, regardless of whether or not such consequences conflict with actual practices.

This joint claim — that if one is an enlightened representationalist then one can and should endorse SV or SV⁺ or some close relative — obviously does not entail that those endorsing a supervaluation logic of vagueness must be representationalists. However it is difficult to see what philosophical defence there could be for, say, the retention of classical laws (e.g. LEM) with the associated loss of truth-functionality other than an acceptance of the view that the world is necessarily precise (thus committing supervaluationists to representationalism). Fine himself thought that there could be good ontological grounds for rejecting LEM — namely, if the world could be vague. If he is right, as I think he is, then its retention implies the necessary precision of the world. More generally, the fact that supervaluationism analyses vague language in terms of precise language suggests that precision is in some sense logically primitive. Why should we think that the logical properties of language in general depend upon the logical properties of precise language unless precise language is privileged? Yet if the world were vague, if precision was not metaphysically privileged or metaphysically primitive, then precise language would itself seem not to be either.

So a supervaluation logic of vagueness might seem to go hand in hand with an enlightened representational account of vagueness; enlightened representationalists ought to be supervaluationists and supervaluationists would seem to have to be (enlightened) representationalists.

Now, as we have seen, the logic is exceedingly complex in light of higher-order vagueness. This in conjunction with the charges of descriptive inadequacy might well lead one to doubt the virtues of adopting such a logic as the logic of vagueness. However, if the Evans Argument, or any other having the same conclusion, is sound then, given the semantic nature of vagueness and the need to encompass vagueness within the scope of logic, we would seem to have little choice. Revisionist and complex though it is, SV⁺ would be prescribed by the necessity for an enlightened representational approach to vagueness. If, on the other hand, the conclusion can be falsified then the supervaluationist theory will fail to adequately describe a general logic of vagueness.

If my attempt to link the supervaluationist with a representationalist account of vagueness is judged unsuccessful then some alternative logic will be sought and, to the extent that it presupposes a representational account of vagueness, it too will fail in its attempt to generally describe a logic of vagueness given the falsity of Evans' conclusion.

My claim in the next chapter will be that Evans' conclusion *is* false. We can coherently describe what it is for objects to be vague. Of course, if one goes on to accept the Evans Proof for the incoherence of *de re* vague identity as sound then one will be forced to reject the Vague-Identity Thesis. On the other hand, one might reject the Evans Proof as unsound, accepting the coherence of *de re* vague identity, thereby leaving open the possibility of endorsing the Vague-Identity Thesis. In either case vague objects are seen as metaphysically kosher.

As a consequence, one might accept vagueness as (at least) a semantic phenomenon and think that vague language ought to be within the scope of logic whilst nonetheless rejecting an enlightened representational account.

Of course, this doesn't mean that there's no room for a representational account or that there is no role for SV+ semantics (or some as yet unspecified alternative) to play; it's simply that the enlightened representationalist is wrong in so far as they are committed to the incoherence of any ontological account. Though the way would be open for an analysis of semantic vagueness in terms of ontological vagueness, some semantic vagueness might still be thought not to have its source in ontology. Some semantic vagueness might also be thought to exceed that which can reasonably be accounted for in terms of ontological vagueness thereby requiring a mixed analysis. In each of these two senses it might be thought that natural language is vaguer than the world.

Were either of these possibilities to be countenanced then the need would arise for a logic capable of accommodating representational vagueness though the superstructural addition of supervaluations would now be laid atop a logic that was presumably already non-classical and the way would be open for a theory of vagueness that contained within it the possibility of doing better justice to the natural language data.

Chapter Six

Towards a Theory of Ontological Vagueness

Can the world be vague or fuzzy? As already noted in Chapter Three, Dummett once believed that "the notion that things might actually *be* vague, as well as vaguely described, is not properly intelligible."¹ He has since recanted by suggesting that this view, according to which reality cannot be vague, is nothing more than deep-seated metaphysical prejudice.² This metaphysical prejudice in favour of a precise ontology was, earlier this century, enough to rule out an ontic account of vagueness.

More recently this prejudice has been replaced by the Evans Argument and subsequent variants, discussed in the previous chapter, which aim to establish the incoherence of the notion of fuzzy objects; the proofs involved yielding what might well be referred to as 'the paradoxes of indeterminacy', analogous to 'the paradoxes of intensionality'.

The problem is that those theorists opposed to ontological indeterminacy have set up the terms of the debate and so their arguments, if sound, would only show that fuzziness *as they have characterised it* is incoherent. As we saw in the previous chapter their account proceeds via the notion of indeterminate identity and, as a result, arguments about the coherence of fuzzy objects have almost exclusively centred on the coherence of this notion. It is this coherence that the paradoxes of indeterminacy attempt to undermine.

In what follows, another more explanatory account of fuzziness is sought that does not proceed via the notion of indeterminate identity — §6.1. Having detailed this rival account we are then free to consider, as an independent issue, what theory of identity we might reasonably adopt and to what extent the theory of fuzziness advocated conforms to the characterisation proffered by opponents — §6.2. Finally, we shall describe how the ontological account resolves the class of sorites paradoxes — §6.3.

The default logical theory throughout this chapter will again be classical.

¹ Dummett, M., 'Wang's Paradox', *Synthese* 30 (1975), p. 314.

² Dummett, M., *The Interpretation of Frege's Philosophy*, Duckworth, London (1981), p. 440.

6.1 *Outlining a Fuzzy Ontology*

The means for developing a theory of ontological vagueness or fuzziness are to be found in the characterisation of semantic vagueness. The signature of vagueness there was, as we saw, indeterminacy of extension — border cases of extension. Mimicking this in the ontological realm will serve as the key to an account of fuzziness.

Properties, objects, and states-of-affairs, in so far as they can all be represented through their extensions, will be caught within this very broad conception of fuzziness. So, what is it for an extended item to be fuzzy?

Items can be specified topologically by means of their extensions. Classically, an item's extension will be fully specified when we have specified those elements determinately included in its extension, since any element not determinately included in an item's extension is determinately not in its extension, and *vice versa*. A classical item is such that those elements determinately in the item's extension — the item's *interior*, and those determinately not in the item's extension — its *exterior*, are exclusive and jointly exhaustive of the super-region being considered; all and only those elements within the super-region that are not in the item's interior are part of its exterior.

In classical set-theory, for example, a set is completely specified when we have specified all those elements (determinately) in the set; all and only those other elements of the superset under consideration are (determinately) not in the set.

These classical assumptions of exclusivity and exhaustiveness are, however, to be challenged. In the case of some items, there may be elements which are neither determinately included in, nor determinately excluded from, their extension; that is, there may be elements which are neither in the item's interior nor in its exterior. Typically, these elements comprise the item's border region — its *terior*, the possession of which characterises an item as fuzzy.³

Take Mt Rainier, for example. Standing on its summit we will fairly take ourselves to be determinately *on* the mountain, and beginning our descent, we will start confident in the knowledge that we are still on the mountain. Eventually, Zeno notwithstanding, we will find ourselves determinately *off* the mountain. However, since there is no sharp boundary separating the mountain from its surrounds, there must have been a time when it was

³ Were we to take a paraconsistent approach to the characterisation of vagueness, perhaps in addition to the underdetermination approach adopted in this thesis, then fuzzy items would, by analogy, be such as to both determinately include and determinately exclude some element; that is, there would be elements that belonged to both the item's interior and its exterior. Thus, the dual of underdetermination, overdetermination, would be included in the characterisation of fuzziness, and 'neither' would be supplemented with 'both'.

indeterminate whether we were still on the mountain or not. The mountain's possession of a terior, that region between being determinately on the mountain and determinately off the mountain, is what makes for the fuzziness of Mt Rainier.

Let us begin then with a general account of fuzziness for such simple cases — those items extended in a space of points — objects.

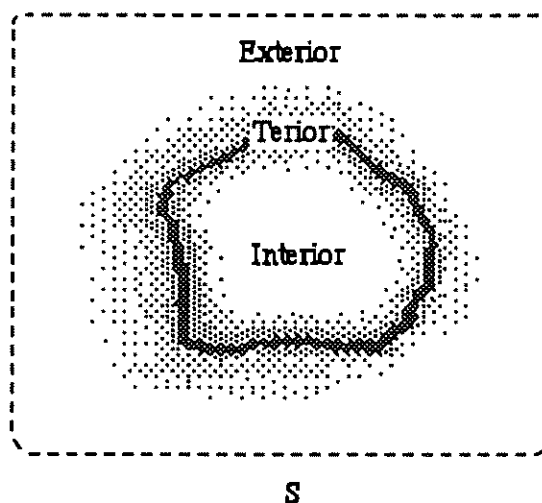
Definition 6.1 Let S be a non-empty space of points. An object, x , is a subspace whose interior, $I(x)$, consists of all those points determinately within the subspace and exterior, $E(x)$, consists of all those points determinately not within the subspace.

The terior (or border region) of x , $T(x)$, consists of all remaining points in S — that is, $T(x) = \bar{E}(x) \cup \bar{I}(x)$, where $\bar{E}(x)$ and $\bar{I}(x)$ are, respectively, the compliment of $E(x)$ and $I(x)$.

Now, x is *fuzzy* iff $T(x)$ is non-empty;

and x is *sharp* iff $T(x)$ is empty — that is, iff $I(x) \cup E(x) = S$.

It follows, by definition, that any point in the interior of x is not in the exterior. A fuzzy object can, therefore, be pictorially represented thus:



The definition covers concrete objects, real and imaginary, existent and non-existent, if S consists of space-time points.⁴ Thus:

⁴ I have heard it said that this cannot be what one means by a fuzzy object since the phenomenon of indeterminate spatio-temporal extension is so pervasive (most if not all material objects, would on this account, be considered fuzzy) as to reduce the concept of fuzziness to virtual vacuity. This objection is misguided, I think, for the following reason. Indeterminacy of material extension is simply the ontological analogue of indeterminate semantic extension therefore the pervasiveness of fuzzy objects on this account merely mimics the pervasiveness of vague names. If this pervasiveness results in vacuity for objects then the same will be true for names; but this is not the case. It is never, to my knowledge, suggested that Quine and Alston's characterisation of the

A concrete object o is fuzzy iff there are space-time points in $T(o)$.

Clouds, rainbows, persons, and puddles are all counted as fuzzy objects on this definition. An interesting example is the extensive wetlands of Eastern Australia with which we began this thesis; it provides a case of an object so diffuse and fuzzy as to be almost beyond belief.

Note that, were S reinterpreted as a space whose points were possible worlds or interpretations, then the definition could be reinterpreted to provide an account of what it is for a state-of-affairs to be fuzzy.

Given the above definition, it now makes sense to speak of the precisification of an object. Precisifications are just what one might presume them to be — namely they are what results from making extended items (e.g. objects) precise or sharp. In other words, a precisification of an extended item x , is an extended item x' which results from specifying a sharp item that (determinately) includes all those regions of S that are determinately included in x — that is, the interior of x' includes the interior of x — and (determinately) excludes all those regions of S that are determinately excluded from x — that is, the exterior of x' includes the exterior of x .

Definition 6.2 An object x' is a *precisification* of an object x iff $I(x') \cup E(x') = S$ and $I(x) \subseteq I(x')$ & $E(x) \subseteq E(x')$. Extensionless points count as degenerate cases whose precisifications are just themselves. The precisification-functor maps objects to sharp objects — in the special case of points, it is just the identity-functor.

For example, the sun counts as a fuzzy object by Definition 6.1, with precisifications that are sharply specified portions of space-time lying somewhere between the outermost regions of that fuzzy object and the innermost regions. All such precisifications have roughly the same temperature, colour, etc.

Note that:

Theorem 6.1 An object is sharp iff it has a unique precisification — itself.

Part-whole relations between objects also need spelling out in the context of fuzzy objects. Intuitively, x will count as a determinate part of y just if anything determinately a part of x is determinately a part of y and anything determinately not a part of y is determinately not a part of x ; x will count as a determinate non-part of y just if some

vagueness of names is vacuous because it is so pervasive. And what of vague predicates? They are certainly pervasive on the standard account of what it is for a predicate to be vague but this is never cited as grounds for rejecting the characterisation.

determinate part of x is a determinate non-part of y ; and x will count as an indeterminate part of y otherwise.

Definition 6.3 Given extended items x and y , we shall say that x is:

a *determinate part of* y iff $I(x) \subseteq I(y) \ \& \ E(y) \subseteq E(x)$;
 a *determinate non-part of* y iff $I(x) \not\subseteq \bar{E}(y)$;
 an *indeterminate or border part of* y otherwise.

For example, assume all of Russell River is within the sharp boundaries of Quine National Park except one of its vague boundaries (its southern bank, say) which overlaps the sharp southern boundary of Quine National Park. Then Russell River is an indeterminate or border part of Quine National Park since its interior does not include anything in the exterior of Quine National Park ($I(x) \subseteq \bar{E}(y)$) whilst its exterior fails to include everything in the exterior of Quine National Park ($E(y) \not\subseteq E(x)$).

Before continuing towards an account of item-fuzziness in general, an immediate concern regarding Definition 6.1 demands attention. Just as there was an apparent problem of higher-order vagueness, there may be thought to be an analogous problem of *higher-order fuzziness*. Again, I think this is a pseudo-problem (cf. §1.4).

It is indeed true that fuzzy objects are not merely fuzzy to the first order; there is no more a sharp boundary between the interior of an object and its exterior than there was between its interior and exterior. Walking down the slopes of Mt Rainier there is no single point at which one ceases to be determinately on the mountain and finds oneself determinately off the mountain; this is just to say that one passes through a penumbral region — the mountain's exterior or border region. However, the exterior itself is not sharply bounded either; there is no single point at which one ceases to be determinately determinately on the mountain and finds oneself determinately in the border regions; the border regions, the mountain's exterior, is itself fuzzy. And so on for the exterior's border regions — the exterior's exterior, etc. Objects that are fuzzy are totally lacking in sharp boundaries at any level.

The reason that this higher-order phenomenon is deemed problematic is because, as with the case of higher-order vagueness, once admitted, it seems to require of us that we specify this as an explicit part of our definition of object-fuzziness. Thus, Tye reacts by effectively adopting a position which has it that it is vague whether there are any higher orders of fuzziness at all. To the question 'Are there higher orders of ontological indeterminacy?' one might say 'determinately yes', 'determinately no', or 'it is indeterminate whether there are any'. Tye chooses the third.

Consider Mount Everest. It seems obvious that there is no line which sharply divides the matter composing Everest from the matter outside it. Everest's

boundaries are fuzzy. Some molecules are inside Everest and some molecules outside. But some have an indefinite status: there is no objective, determinate fact of the matter about whether they are inside or outside. Are there any remaining molecules? To suppose that it is true that this is the case is to postulate more categories of molecules than are demanded by our ordinary, everyday conception of Everest and hence to involve ourselves in gratuitous metaphysical complications. It is also to create the need to face an endless series of such questions one after the other as new categories of molecules are admitted. On the other hand, to suppose that it is false that there are any remaining molecules is to admit that every molecule fits cleanly into one of the three categories so that there are sharp partitions between the molecules inside Everest, the molecules on the border, so to speak, and the molecules outside. And, intuitively, pretheoretically it is not true that there are any sharp partitions here. What, I think, we should say is that it is objectively indeterminate as to whether there are any remaining molecules.⁵

I simply do not see that the metaphysical consequences of admitting higher orders of fuzziness are either gratuitous or complicated. The consequences are not gratuitous since the phenomenon of higher-order fuzziness is real, for reasons exactly analogous to those offered in Chapter One in favour of higher orders of vagueness; and they are not complicated unless one falls into the trap of thinking that higher-order fuzziness is a problem. We need not "postulate", in any strong sense, any categories in addition to those postulated in Definition 6.1; we need only recognise that their existence follows from the definition.

Similarly, it is simply not true that we face a potentially endless series of questions regarding the existence of higher orders of fuzziness. That there *are*, unquestionably, higher orders, follows from the fact that the operator 'It is indeterminate that...' is itself vague, as was argued in Chapter One. As a consequence, the terior of an object can only be vaguely defined, and so admits of a terior itself, which is only vaguely defined, etc. Tye is reticent to admit to this because, as he says, this would entail an infinite iteration of orders of fuzziness. Yet, to the extent that this is true, it is unproblematic.

So, clever though his response is, Tye has been caught in the trap of thinking higher-order fuzziness to be a problem at all, and then tried to avoid it by claiming the matter to be vague.

Rolf, on the other hand, appears committed to the determinate absence of higher orders of fuzziness. His definition of object-fuzziness, presented in his 'A Theory of

⁵ Tye, *op. cit.*, p. 535.

Vagueness', makes use of two-tuples of classical sets to specify what effectively amounts to the interior and exterior of an object — in this way he then counts an object as fuzzy iff the interior and exterior are not exhaustive, just as we do.⁶ However, the fact that he appears to equate a classical set with each of the interior and the exterior, means that the interior or border region itself (the set of remaining elements of the space) would seem to be a classical set and will therefore be sharply bounded — it cannot itself possess a interior. This unnecessary use of classical sets as a base from which to construct fuzzy objects, as opposed to the use of set-theoretic notions in conjunction with a (vague) determinacy-functor as primitive — as we have done, results therefore in the determinate absence of any higher-orders of indeterminacy.

It should be noted that Rolf makes no mention of higher-order vagueness or anything analogous (e.g. higher-order fuzziness) anywhere in his writings on vagueness. Whether this is because he (mistakenly) thinks there is no such phenomenon or simply because he has not thought about it at all, is unclear. His defective account could easily be amended in line with Definition 6.1 above. What Rolf should not do is persist with his account of object-fuzziness which characterises fuzzy objects as determinately lacking higher orders of fuzziness.

The correct resolution of the apparent problem of higher-order fuzziness is to acknowledge, as with higher-order vagueness, that though the phenomenon is real enough, there is no problem here. It is a pseudo-problem.

The problems with Tye's characterisation of object-fuzziness do not stop, even were higher-order fuzziness to be properly accounted for. Higher-order problems aside, he has fallen into the trap of thinking border-parts sufficient for object-fuzziness since, as he characterises fuzziness, an object will count as first-order fuzzy just if "it has border spatio-temporal parts — that is, just if there is no determinate fact of the matter about whether some region is a spatio-temporal part of that object."⁷

As was noted in Chapter One, pp. 31-2, the existence of border spatio-temporal parts for a singular term's denotation cannot be sufficient for the vagueness of that singular term. Analogously, the existence of a border spatio-temporal part for an object is insufficient for object-fuzziness, as the following example shows. Suppose Quine National Park is such that it is indeterminate whether to count it as that spatio-temporal region including Russell River; Russell River counts as a border-part of Quine National Park. Setting aside issues to do with higher-order fuzziness then, Quine National Park counts as fuzzy on Tye's account since it has just such a border spatio-temporal part.

⁶ Rolf, B., 'A Theory of Vagueness', *Journal of Philosophical Logic* 9 (1980), p. 316.

⁷ Cf. Tye, M., 'Vague Objects', *Mind* 99 (1990), pp. 535-536.

Nonetheless, Quine National Park might be a *sharply* delimited region of space-time. The source of indeterminacy here may well be due to the fuzziness Russell River — it may be indeterminate whether it is a part because it is itself indeterminate in extent. Part-whole indeterminacies do not, of themselves, make for the indeterminacy of the whole — it would need to be shown that the source of the indeterminacy did not reside in the part.

By analogy with the vagueness of singular terms, discussed in Chapter One, we must ensure that there are *resilient* border parts to establish the fuzziness of the whole. As it happens, Definition 6.1 is not only free of the above deviant example of fuzziness (since it defines fuzziness by reference to border parts that are points rather than parts in general), we can show that objects are fuzzy, in the sense of the definition, if and only if there are border-parts that *are* resilient.

To do this, we firstly need an account of what it is for something to be a resilient border part.

Definition 6.4 An object x is a *resilient border part* of y iff x is a border part of y and any precisification x' is also a border part of y .

On the basis of these definitions we can show:

Theorem 6.2 A object is fuzzy iff it has a resilient border part.

Proof. Assume x to be a fuzzy object. Then x 's border region is non-empty and so it contains at least one point, p , say. Moreover, p is sharp so any precisification of p is simply p itself; hence any precisification of p is also a border part of x . So p is a resilient border part of x . Conversely, assume there is some object y that is a resilient border part of a simple object x , yet (for *reductio*) x is nonetheless sharp. Then y is a border part of x and so is any precisification of y . Let y' be some particular precisification; it will be an indeterminate part of x . So, by Definition 6.3, (i)— $I(y') \not\subseteq I(x)$ or $E(x) \not\subseteq E(y')$, whilst (ii)— $I(y') \subseteq \bar{E}(x)$. Yet both x and y' are sharp, so (iii)— $I(x) = \bar{E}(x)$ and (iv)— $I(y') = \bar{E}(y')$. Consider (i). Either $I(y') \not\subseteq I(x)$, which is impossible since we can show (using (iii) for substitution into (ii)) $I(y') \subseteq I(x)$; or $E(x) \not\subseteq E(y')$. This latter possibility is equivalent to $\bar{E}(y') \not\subseteq \bar{E}(x)$ ⁸, and so is equally impossible since we can show (using (iv) for substitution into (ii)) $\bar{E}(y') \subseteq \bar{E}(x)$. By *reductio* then, if there is some object y that is a resilient border part of x then x must be fuzzy. ▮

⁸ This follows from the fact that $E(x) \subseteq E(y')$ iff $\bar{E}(y') \subseteq \bar{E}(x)$, which can itself be seen to hold as follows. $E(x) \subseteq E(y')$ iff anything determinately not in x is determinately not in y' . However, that is the case iff anything not determinately not in y' is not determinately not in x , which is, in turn equivalent to saying $\bar{E}(y') \subseteq \bar{E}(x)$.

Thus, we have shown that the usual classical equivalence — $A \subseteq B$ iff $\bar{B} \subseteq \bar{A}$ — one would expect *might* extend to vague sets, *does*.

So, Definition 6.1 is free of *any* such counter-examples that would exploit the fuzziness of parts to generate deviant cases of fuzziness for the whole. Tye's account of object-fuzziness is defective both in its characterisation of first-order fuzziness and in its response to higher-order fuzziness.

In moving to an account of property-fuzziness, note that these items are extended in a space S whose elements might themselves be extended items. Thus we face potential problems like those encountered by Tye's defective definition just considered, since now it is possible that the elements of S are themselves fuzzy. These worries are just the ontological analogue of those we faced in Chapter One when we realised that we could not simply specify the vagueness of a predicate in terms of its possessing border cases (cf. Chapter One, pp. 29 ff). It might, after all, be indeterminate whether the sun is an element of the extension of the property of having a diameter of exactly 1.39×10^9 m simply because the sun is itself fuzzy, whilst the property is intuitively sharp.

In our discussion of vagueness in the first chapter we avoided this problem, again, by invoking the notion of 'resilience'. A predicate was said to be vague if and only if there is (or could be) some object which is a resilient border case for that predicate. Pursuing the intended analogy between vagueness and fuzziness, we shall follow this amendment in giving an account of property-vagueness.

Definition 6.5 Let S be the space whose elements are objects. An element of S is a *border element* of a property P extended in S iff it is a border case for being an element of the extension of P . It is a *resilient border element* of P iff it is a border element of P and any precisification is also a border element of P .

For example, the sun is a border element for the extension of the property of having a diameter of exactly 1.39×10^9 metres, and for the extension of the property of being a hot star. However, the sun is only a resilient border element for the latter. *Any* precisification of the sun will count as a border element for the extension of the property of being a hot star, if the sun itself does — the heat-difference between the sun and any precisification thereof is too small to make any difference. Yet *no* precisification of the sun will count as a border element for extension of the property of having a diameter of exactly 1.39×10^9 metres.

Before finally defining the fuzziness of properties, notice that there are two senses in which properties might be thought to be fuzzy. The first, stronger sense has to do with the way the world *actually is* — extensional-fuzziness. A property is fuzzy in this sense just in case the extent of its *actual* instances is indeterminate or fuzzy. A second weaker sense has to do with the way the world *could be* — intensional-fuzziness. A property is fuzzy in this sense just in case the extent of its *possible* instances is indeterminate or

fuzzy. I agree with Tye in taking the latter as constituting the fuzziness of properties *simpliciter*, for the same reason that we took talk of predicate-vagueness *simpliciter* to be talk of intensional predicate-vagueness; that is, because in talking of vagueness or fuzziness *per se* we are, after all, interested in a purely logical feature of predicates or properties which is independent of the contingencies of our world.

We can now say what it is for a property, extended in a space S whose elements may themselves be extended, to be fuzzy.

Definition 6.6 A property is *fuzzy* if and only if it could have resilient border elements. A property which can have no resilient border elements is *sharp*.

For example, the property of being a hot star is fuzzy with the sun as a resilient border element.

If we now extend the concepts of an interior, exterior and terior to be applicable to properties:

Definition 6.7 The *interior* of a property P , $I(P)$, consists of all those objects determinately in the extension of P ; its *exterior*, $E(P)$, consists of all those objects determinately not in the extension of P ; and its *terior*, $T(P)$, consists of all those objects that are resilient border elements of P .

then trivially:

Theorem 6.3 A property P is fuzzy iff the terior of P , $T(P)$, is non-empty. It is sharp iff $T(P)$ is empty.

A couple of examples may help to breathe life into the foregoing discussion. The set of persons exactly 2m tall is sharp, even though it has border members — for instance, Jenny, who is roughly 2m in height; whereas, using an example of Tye's, the set of tall men is fuzzy since Jack, a bit under 2m in height, counts as a border element and a resilient one at that. Tye cites the property of being 2000 feet in height as being sharp and indeed it is according to the above definitions, since any border element for the extension of the property will not be resilient — its precisifications will fail to be border elements.

The foregoing amounts to an account of property-fuzziness for first-order properties. The fuzziness of higher-order properties (e.g. properties of properties of objects, etc.) can be characterised by simply extending our talk of resilient border elements to include properties themselves; that is, to extend Definition 6.5 to allow S to be a space whose elements are themselves objects or properties. Now we can define a property of any order to be fuzzy iff it could have resilient border elements, thereby providing a general recursive account of property-fuzziness in general.

The fuzziness of sets can obviously be characterised by a minor reinterpretation of the foregoing account of object- and property-fuzziness. Sets of the simplest order correspond to points and, as such, are degenerate cases of extended items, being necessarily sharp. A set α of any complexity can then be defined as fuzzy if and only if it has resilient border members — members which, in general, might themselves be fuzzy. Recursively ascending the Zermelo-Fraenkel hierarchy, the fuzziness of a set will be well-defined for any such set in the hierarchy. Abstracting in this way from objects and properties to sets in general, the resulting fuzzy or vague set theory can subsequently be viewed as the explanatory paradigm in terms of which object- and property-fuzziness are characterised. It will, moreover, provide the canonical model for a semantics and logic of vagueness.

Tye also offers an account of the fuzziness (or, as he puts it, the vagueness) of sets following on, quite naturally, from his account of object-fuzziness. As such, it is defective for the same reason his account of object-fuzziness was. Again, higher-order problems aside, a set α is fuzzy, on Tye's account, just if it has border members.⁹ Now, whilst he admits that the property of being 2000 feet in height is sharp, he is forced to admit that the corresponding set is fuzzy. That there are objects which are neither determinately 2000 feet in height nor determinately not is, as he admits, insufficient grounds for thinking the property fuzzy — the border objects themselves might be the source of the indeterminacy — however, it will make for the fuzziness of the corresponding set, on his account, since the set has border members. The fact that he effectively includes resilience as a condition for the fuzziness of properties but not sets, thereby permitting sets defined by means of sharp properties to be fuzzy, serves to underline the problem with his account of fuzzy sets.¹⁰

⁹ Tye, *op. cit.*, p. 536.

¹⁰ Another point on which I disagree with Tye's theory of vague or fuzzy sets is his response to supposed worries concerning the Axiom of Extensionality — for any sets A and B, A is identical to B if and only if any object x belongs to A if and only if it belongs to B. Trouble arises with this axiom for Tye since his three-valued logic of vagueness has connectives whose truth-tables are given by those for Kleene's strong connectives (cf. Tye, *op. cit.*, p. 544). In particular, for indeterminate 'p' and 'q', 'p → q' is indeterminate — hence, for indeterminate 'p', 'p → p' is not true but indeterminate (moreover, this is maintained in the face of 'p's being counted a logical consequence of 'p', so the Deduction Theorem no longer holds); similarly, for indeterminate 'p' and 'q', 'p ↔ q' is indeterminate. These counterintuitive results are premised on the need to preserve the equivalence between 'p → q' and '¬p ∨ q' (*ibid.*, pp. 544-5), an equivalence which I think fails; their classical provable equivalence depends upon the Law of Excluded Middle after all, and the status of this law is questionable in vague circumstances.

The Axiom of Extensionality now faces the following problem. Consider a fuzzy set A. It is truly identical to itself; yet, since it has border members, the above account of '↔' has it that it is indeterminate whether any object x belongs to A if and only if it belongs to A. The Axiom is such that its left-hand side can be true whilst its right-hand side is indeterminate. Accepting the consequences Tye takes this only to show that this axiom is what he calls a "quasi-tautology" — it is at least not (ever) false (*ibid.*, pp. 546-7).

All this can be avoided with an improved semantics for '→', admitting 'p → p' as always true. The Axiom of Extensionality remains true in such a fuzzy set theory and the Deduction Theorem is also retained.

Whilst the notions of 'being a determinate-element' or 'border-element of the set α ' are well-defined, we so far lack any account of what it is for something to be an element of α *simpliciter*.

Definition 6.8 We shall say that a is an element (*simpliciter*) of the set α ($a \in \alpha$) if and only if a is sufficiently similar to some b such that b is determinately an element of α ; i.e. iff a is sufficiently similar to some b such that b is an element of the interior of α , $I(\alpha)$.

The notion of 'sufficient similarity' employed in this definition is just that relation described by Williamson in his discussion of THE MARGIN FOR ERROR PRINCIPLE of §2.1.¹¹ As he points out, the degree and kind of the required similarity depend on the circumstances; the appropriate dimension of closeness depends on α . For example, if α were the set of red things then the kind of similarity required would be colour-similarity, and the degree of similarity required (i.e. what would count as being "sufficiently similar" in colour) would be a vague matter depending on the particular context to hand. The most obvious example of object-pairs exhibiting this relation will, of course, be adjacent pairs in a soritical series; *ex hypothesi*, these pairs are sufficiently similar in relevant respects — that is, in respects relevant to that with regard to which the series is soritical.

As a consequence of this account of set-membership it follows that:

Theorem 6.4 If $a \in I(\alpha)$ then $a \in \alpha$.

Proof. Assume $a \in I(\alpha)$. By the reflexivity of sufficient similarity, a is sufficiently similar to a . But then, by hypothesis, a is sufficiently similar to some b such that $b \in I(\alpha)$. So, by Definition 6.8, $a \in \alpha$. ■

By means of the identification of a sentence A with the (now possibly fuzzy) set of points (e.g. worlds or interpretations) at which it obtains, $[A]$, we can say:

- (i) A is true at a if and only if $a \in [A]$; in particular, DA is true at a iff $a \in [DA]$.

In addition:

- (ii) A is determinately true at a if and only if $a \in I[A]$;

and, since A is determinately true at a if and only if DA is true at a :

- (iii) $a \in I[A]$ if and only if $a \in [DA]$.¹²

With a standard account of the material conditional (assumed, since the background framework is classical, to satisfy the Deduction Theorem):

- (iv) $A \supset B$ is true at a if and only if, if $a \in [A]$ then $a \in [B]$,

¹¹ Cf. Williamson, T., *Identity and Discrimination*, OUP (1990), pp. 104 ff.; also his 'Inexact Knowledge', *Mind* 101 (1992), §2; and 'Vagueness and Ignorance', *The Aristotelian Society Supplementary Volume* 66 (1992), § VI.

¹² 'Satisfaction' is similarly redefined in the model where predicates are identified with possibly fuzzy sets comprising their satisfiers. E.g. a determinately satisfies 'P' iff $a \in I(P)$ iff $a \in DP$, etc.

Theorem 6.4 then provides the condition on the canonical model for a logic of vagueness, V , required to establish the following:

$$\models_V DA \supset A.$$

Perhaps a little more surprisingly, Definition 6.8 also entails:

Theorem 6.5 If $a \in I(\alpha)$ then all sufficiently similar elements are elements of α .¹³

I.e. if $a \in I(\alpha)$ then $s(a) \subseteq \alpha$ — where $s(a) = \{b: b \text{ is suff. similar to } a\}$.

Proof. Assume $a \in I(\alpha)$. Then everything sufficiently similar to a is sufficiently similar to something — namely, a itself — determinately an element of α . Hence, by Definition 6.8, everything sufficiently similar to a is an element of α ; i.e. $s(a) \subseteq \alpha$. \blacksquare

As with Theorem 6.4, Theorem 6.5 can be taken to provide a condition on the canonical model for a metaphysics and logic of vagueness. As such, it constitutes a constraint on the model sufficient to warrant the ontological and semantic analogues of Williamson's margin-of-epistemic-error — THE ONTOLOGICAL MARGIN PRINCIPLE (OMP) and THE SEMANTIC MARGIN PRINCIPLE (SMP).

OMP If P is a soritical property relative to the series $\langle a_1, \dots, a_n \rangle$ then, if a_i determinately instantiates P , a_{i+1} instantiates P .

SMP If ' P ' is a soritical predicate relative to the series $\langle a_1, \dots, a_n \rangle$ then, if a_i determinately satisfies ' P ', a_{i+1} satisfies ' P '.

I.e. if ' P ' is soritical relative to the series $\langle a_1, \dots, a_n \rangle$ then,
 $\models_V DP a_i \supset P a_{i+1}$ — if ' $P a_i$ ' is determinately true then ' $P a_{i+1}$ ' is true.

Now one might well wonder why such principles should be verified by a logic of vagueness construed either ontologically or semantically. Why should the fact that a_{i+1} is sufficiently similar to a_i — a determinate satisfier of ' P ', i.e. a satisfier of ' DP ' — count as a sufficient condition for a_{i+1} 's being a satisfier of ' P ' *simpliciter*. Why is sufficient similarity to a determinate P a sufficient condition for being a P ? The answer lies in an adequate understanding of determinacy.

Williamson has argued in a number of places that claims to knowledge of p must be reliable in the sense that p must be sufficiently distant from the likelihood of being false as to warrant its being reliably true. Reliable truth is a necessary condition for knowledge and vague judgements (e.g. the judgement that a_i is P) are reliably true if and only if they are true in sufficiently similar circumstances. Thus, were one to know that a_i is P then a_i

¹³ I am also in favour of the converse of Theorem 6.5, though, as yet, lack a proof to that effect.

counts as P in all sufficiently similar circumstances. He thereby sought to explain how sharp boundaries between the true and the false — were they supposed to exist — could be unknowable, thus relieving the burden of proof accompanying his epistemic account of vagueness.¹⁴ My concern here is not whether the burden is lifted by such an argument (I suggested in Chapter Two that it did so only for an epistemic account of soriticality, leaving the existence of unknowable boundaries for vague non-soritical terms as much a mystery as ever before) but rather to put the argument-form to use in explaining the plausibility of the above tolerance principles.

Let us suppose along with Williamson that judgements count as reliable if and only if they are correct in all sufficiently similar circumstances. Thus, given a soritical term P, the judgement that a_i is P is reliable only if a_{i+1} is P. Now were reliability a necessary condition for determinate ascriptions then one could judge that a_i is determinately P only if a_{i+1} is P (along any other objects considered sufficiently similar to a_i). And reliability is a necessary condition. The claim that something is determinately red is generally cached out in terms of its being a clear, definite or certain instance of redness; yet, this is just to say that determinate instances of redness are those instances where (all other things being equal) reliable judgements can be made. If something were sufficiently similar to a non-red thing, and thus could not be reliably judged red, then it could not be clearly red since possibly imperceptible differences separate it from a non-red thing.

According to this line of reasoning then the principles OMP and SMP simply reflect the fact that determinate ascriptions, either ontological or semantic, must be reliable in the sense outlined above. Thus it is a virtue of the theory of fuzzy sets outlined so far that it can ground these plausible principles.¹⁵

A special case of Theorem 6.5 is where α is itself the interior of some set β . In such a case we have the following:

Corollary If $a \in I(I(\beta))$ then all elements sufficiently similar to elements sufficiently similar to a are elements of β .

As a consequence, for P soritical relative to the series $\langle a_1, \dots, a_n \rangle$:

$$\models_{\forall} \text{DDPa}_i \supset \text{Pa}_{i+2}.$$

The result generalises: $\models_{\forall} \text{D}^n \text{Pa}_i \supset \text{Pa}_{i+n}.$

¹⁴ *Ibid.*

¹⁵ Williamson comments at one point ('Vagueness and Ignorance', *op. cit.*, p. 161) that rejection of the epistemic account might make it look as if vague terms are governed by the sorites inducing tolerance principle: $\models \text{Pa}_i \supset \text{Pa}_{i+1}$. Notice that this principle is strictly stronger than any endorsed in the foregoing discussion. When we come to discuss the sorites paradox in §6.3, a principle syntactically similar to this will be endorsed, however subsequent analysis will show that no paradoxical conclusions result from it.

This is simply a manifestation of the fact that determinately determinate ascriptions require a doubly wide tolerance margin and, in general, every additional claim to determinateness increases the required margin for tolerance.

Once established, SMP itself serves to show why a soritical predicate's semantic boundary must be indeterminate — even if it were the case that, for some j , Pa_j and $\sim Pa_{j+1}$, a possibility which is left open in the fuzzy set theory outlined so far. Suppose that DPa_j . Then, by SMP, Pa_{j+1} . Were it the case that $D\sim Pa_{j+1}$ then it would follow that $\sim Pa_{j+1}$, so since Pa_{j+1} it follows that $\sim D\sim Pa_{j+1}$. In other words, there can be no determinate semantic boundary in respect of P between sufficiently similar possible satisfiers.

Kit Fine thought that "[p]hilosophers have been unduly dismissive over intrinsically vague entities." By way of explanation he suggested that "[t]his attitude may derive, in part, from the view that any piece of empirical reality is isomorphic to a mathematical structure; since the structure is precise, so is the reality. Thus, the blurred outline becomes isomorphic to a set of points in Euclidean space." As he points out however, it is not clear that all mathematical entities *are* precise, suggesting that one could perhaps develop an intuitive theory of vague sets. "Hopefully, it would not even be interpretable within standard set theory; so that the sceptic could not then treat vague sets on the onion-model, as a 'façon de parler'."¹⁶ The above account of ontological vagueness in terms of the indeterminacy of extension is underwritten by just such a theory of vague or fuzzy sets — a theory which will not be interpretable within standard set theory without extension, since it makes essential reference to the vague notions of 'indeterminacy' (thereby avoiding apparent problems to do with higher-order vagueness or fuzziness) and 'sufficient similarity' and these cannot be defined in the purely precise language of the standard (classical) theory of sets without violating THE INHERITANCE OF PRECISION principle described in Chapter One, page 36.

The popular line of attack on fuzzy ontology is however via considerations on identity, about which we have so far said nothing. So let us turn to identity.

¹⁶ Fine, K., 'Vagueness, Truth and Logic', *Synthese* 30 (1975), n. 10.

In contrast to this radically non-classical proposal, compare Sainsbury's view that, though there may be vague objects (he seems agnostic as yet on this matter), *sets* are not vague since sets are sharp objects (Sainsbury, R.M., 'Concepts Without Boundaries', typescript, Department of Philosophy, King's College, London (1990), pp. 7-8); so there is no set of red things, nor of truths, etc.

6.2 *Fuzziness and Identity*

Since the appearance of Quine's dictum "No entity without identity" one is at least rhetorically compelled in any discussion of a fuzzy ontology to consider its coherence in relation to the notion of 'sameness' or 'identity'. More than this though, we may reasonably require a theory of identity so that we may sensibly re-identify objects. Having detailed a theory of fuzzy ontology it is necessary now to analyse this relation between fuzziness and identity to arrive at an acceptable account of identity.

As we have already seen in the previous chapter, much of the recent work appearing in the literature regarding ontological vagueness concerns itself with problems about this notion. However most, if not all, of these discussions are motivated by concerns different to ours. Most theorists start from the assumption that fuzziness is to be *defined* in terms of identity and are concerned to show that fuzzy objects are or are not coherent depending on whether or not they can argue for the coherence of the defining conditions. Garrett typified this approach when he claimed that

[t]he thesis that there can be vague [fuzzy] objects is the thesis that there can be identity statements which are indeterminate in truth-value (i.e. neither true nor false) as a result of vagueness ... *the singular terms of which do not have their references fixed by vague descriptive means*. (If this is *not* what is meant by the thesis that there can be vague [fuzzy] objects, it is not clear what is meant by it.)¹⁷

The thought here seems to be that the indeterminacy of identity constitutes the vagueness of some singular term in the identity statement; moreover, if the vagueness of the singular term is not due to semantic indeterminacy, i.e. is not due to their references being fixed by vague means, then the vagueness must be inherent in that thing named by the singular term — a fuzzy object. If, on the other hand, there can be fuzzy objects then there can be singular terms that are vague which do not however have their references fixed by vague descriptive means and these terms can give rise to identity statements that are indeterminate.

For advocates of this Vague-Identity Thesis, problems regarding identity are central to their being able to get an account of fuzzy objects off the ground *at all*. Our position is different; we *already* have an account of what it is for an object to be fuzzy. Recalling Garrett's final parenthetical remark we may say that it is quite clear what is meant by 'fuzzy' without any mention of identity. What we require is a theory of identity to accompany our

¹⁷ Garrett, B.J., 'Vagueness and Identity', *Analysis* 48 (1988), p. 130.

already developed account — and, if it subsequently turns out that fuzzy objects satisfy the Vague-Identity Thesis, this will be a derived result rather than an axiomatic one.

6.2.1 *Some Consequences of Fuzziness*

Let's explore some consequences of the thesis that there can be fuzzy objects a little further with a view to its implications for identity.

An object x , as defined in §6.1, includes some space-time points as determinate parts — all those in $I(x)$; it may also include some as determinate non-parts — all those in $E(x)$; finally, there may be some space-time points which are only vaguely a part — all those in its terior or border region, $T(x)$ (which will be empty in the special case when x is sharp).

Let us say now that:

— two objects determinately agree on a part z if and only if, if z is a determinate part of one then it's a determinate part of the other and if z is a determinate non-part of one then it's a determinate non-part of the other and if z is an indeterminate part of one then it's an indeterminate part of the other. (I.e. two objects determinately agree on some part z iff z is a determinate part of both objects, a determinate non-part of both or an indeterminate part of both.)

— two objects determinately differ on a part z iff z is a determinate part of one but a determinate non-part of the other,

and

— two objects indeterminately agree on z otherwise.

As it turns out, the thesis that there are objects that are fuzzy (in the sense of §6.1) is equivalent to the claim that two objects may fail to determinately differ on any part whilst they may nonetheless fail to determinately agree on all parts.¹⁸ To see this assume x to be a fuzzy object. Let x' be a precisification of x . By definition x and x' don't determinately differ on any part, yet nor do they determinately agree on all parts (since x 's indeterminate parts are either determinate parts or determinate non-parts of x'). Conversely assume there to be two objects, x and y , that neither determinately agree on all parts nor determinately disagree on any part. If x and y were both sharp then their failing to determinately agree on all parts would entail their determinate disagreement on some part, but this is not the case, so x and y cannot both be sharp; one or the other is fuzzy.

¹⁸ Assuming identity satisfies the Axiom of Extensionality, two objects x and y fail to determinately agree on all part iff $I(x) \neq I(y)$ or $E(x) \neq E(y)$. Two objects x and y fail to determinately disagree on any part iff $I(x) \cap E(y) = \emptyset$ & $E(x) \cap I(y) = \emptyset$.

The point I am trying to get across here is the following: firstly, all objects determinately agree with themselves on all parts — I take this to be as evident as the law of self-identity. Yet fuzzy objects may stand in a special relation *R* to another object, paradigmatically exemplified by the relation between a fuzzy object and its precisification — lack of determinate agreement without determinate disagreement on parts.

x is fuzzy if and only if it is indeterminate whether or not *x* and its precisification *x'* share all and only the same parts.¹⁹

This indeterminacy is constitutive of what it is for an object to be fuzzy.

We might now try to *define* an identity relation in terms of 'sameness of parts' which will turn out to hold indeterminately between a fuzzy object and some precisification thereof. Such an "identity" relation will be sensitive to fuzziness in the sense that distinct modes of "identity" obtain between: (i) two objects determinately agreeing on all parts (determinate identity — as typically instantiated by the two-tuple consisting of an object and itself); (ii) between two objects exhibiting indeterminate disagreement on some part without any determinate disagreement on any part (indeterminate identity); and (iii) between two objects exhibiting determinate disagreement on some part (determinate difference).

This *sensitivity feature* of identity as defined above, namely the possibility of its holding indeterminately in appropriate situations, I take to be a desirable feature of a general account of identity by virtue of the ready availability of situations where 'indeterminate' seems the natural response to questions of identity — in this way our theory of identity can respect the evidence and can, to this extent at least, be counted as descriptively accurate.

Consider the following variation on the case of The Ship of Theseus due to Garrett. A certain ship, *A*, is composed of 100 planks (keel, ribs, rudder and sails being ignored — truly a philosopher's ship!).

CASE 1 At time *t*₁ one plank is removed from *A* and quickly replaced with another plank. Call the ship at *t*₂, thus altered, '*B*'. It seems uncontentious (in the absence of any other changes) that it's *true* that *A* is identical to *B*.

CASE 2 At time *t*₁ two planks are removed from *A* ...

.

.

CASE 99 At time *t*₁ ninety-nine planks are removed from *A* ...

¹⁹ On the basis of the formal definitions provided in n. 18 we can say:
x is fuzzy iff [*I*(*x*) ≠ *I*(*x'*) or *E*(*x*) ≠ *E*(*x'*)] & [*I*(*x*) ∩ *E*(*x'*) = ∅ & *E*(*x*) ∩ *I*(*x'*) = ∅].

CASE 100 At time t_1 all A's planks are instantly removed and replaced with a new set. The ship thus altered at t_2 , B, is completely materially discontinuous with A. It seems therefore uncontentious that it's *false* that A is B.

Garrett then invites us to consider CASE 50 where half of A's planks are replaced to create B. Here it is thought plausible to suggest that it is indeterminate whether A is B. There are insufficient continuities for determinate sameness and insufficient discontinuities for determinate difference. There is, says Garrett, simply no fact of the matter as to whether or not A is B. The sentence 'A is B' in CASE 50 seems to be an example of indeterminate identity due to the fuzziness of the objects referred to therein.²⁰

Further examples due to Terence Parsons are those of the identity relation between the puddle outside now and the puddle I stepped in yesterday (considered in the obviously relevant circumstances) and the identity relation between the pile of trash I (he) swerved around yesterday and the pile of trash by the roadside today. Both are taken to be examples where one seems required to describe the relation as indeterminate. David Wiggins discusses more examples which he describes as "impressive-seeming examples" for the claim that the identity relation is sometimes indeterminate.²¹

In light of these examples, one might plausibly *require* that the identity relation admit of indeterminacy. That is, one might take the view, as I do, that any reasonable account of identity be sensitive to such cases. The sensitivity feature would then amount to a *sensitivity requirement* — the identity relation must admit of indeterminacy.

Now, there can be no criticism of my simply defining such a sensitive relation, however my right to claim it to be an identity relation will be queried. The concern might arise from two quite distinct views. The first "hard-line" view maintains that nothing is identity unless it satisfies Leibnitz's Law; I must go on and verify that this inviolate constraint is met, they believe, before I am entitled to speak of identity. A second and more liberal view maintains that, though some violation is permitted in extending identity to non-classical situations, the relation ought to conservatively extend the traditional conception. Rather than go on and define a relation and then investigate its Leibnitzian properties (its substitution features) I shall characterise it in a neo-Leibnitzian manner from the outset. I think this route, proceeding from the classical account of identity, will help us better assess the strength of the charge that when I speak of identity I mean something different to the traditional conception as governed by Leibnitz's Law.

²⁰ Garrett, B., 'Vague Identity And Vague Objects', *Nous* Vol. XXV (1991), p. 342.

²¹ Parsons, T., 'Entities Without Identity' in J. Tomberlin (ed.), *Philosophical Perspectives, I, Metaphysics*, Ridgeview Publishing Co. (1987), pp. 2-3; Wiggins, D., 'On Singling Out an Object Determinately' in P. Pettit & J. McDowell (eds), *Subject, Thought and Context*, OUP (1986), pp. 172-173.

6.2.2 *Leibnitz's Law in Fuzzy Contexts*

If identity is constrained by Leibnitz's Law (LL) then classically we may say that

LL: $(a = b) / \forall F(Fa \equiv Fb)$.

Objects are to count as indiscernible if they're identical. What the unstrengthened Evans Proof (and the Wiggins-variant) sought to show was that, assuming the determinate reflexivity of identity (DR) and Leibnitz's Law (LL), *de re* identity cannot be indeterminate. But this conclusion would forestall any attempt to arrive at an account of identity in fuzzy contexts satisfying the sensitivity requirement of §6.2.1.

Should we then accept the unstrengthened Evans Proof as sound and simply reject this requirement as a logical impossibility (a move which, it should be emphasised, will not entail the impossibility of fuzzy objects or fuzzy properties since we are not compelled to endorse the Vague-Identity Thesis)? I think not. Whilst I think that identity as governed by LL and DR *does* imply that identity cannot be *de re* indeterminate, I shall argue that LL requires modifying in fuzzy contexts; a neo-Leibnitzian account of identity will emerge that is capable of satisfying the sensitivity requirement.

So, how does it follow from LL and DR that identity cannot be *de re* indeterminate? Well, recall the state of play with the Evans Proof in §5.4. Any statement of *de re* indeterminate identity, $I(a = b)$, was — given LL, DR and the fact that a *de re* construal legitimated λ -abstraction — seen to entail $\sim(a = b)$. Whilst this alone was, *contra* Evans, considered insufficient to establish an outright syntactic inconsistency in supposing there to be *de re* indeterminate identity statements, a range of possible semantics for a logic of vagueness *can* expose an inconsistency.

In other words, accepting the premises of Evans' unstrengthened argument *does* entail the rejection of the possibility of there being *de re* indeterminate identity statements, though not in the manner envisaged by Evans.

Tye's semantics for a logic of vagueness, provides one route to the inconsistency sought by Evans.²² Suppose it to be true that $I(a = b)$. Then, according to the proposed semantics, the claim that $(a = b)$ is not true, since it is neither true nor false.²³ If, furthermore, this indeterminacy is *de re* and LL and DR hold then $I(a = b)$ entails $\sim(a = b)$.

²² Tye, *op. cit.*, pp. 544 ff.

²³ Of course, in rejecting the principle of bivalence, we are now forced to face the Williamson argument (cited in Chapter Two, §2.1) according to which any rejection of bivalence entails absurdity, given the T-Schema.

As Graham Priest has suggested in conversation however, one might wonder why the T-Schema has any force in such circumstances. After all, for a sentence A which is neither true nor false, it is false that $T'A$ since A is not true, whilst it is not false that A ; but then, since $T'A$ is false and A is not, it is not true that $T'A \equiv A$ — the T-Schema fails in non-bivalent contexts. Hence, though Williamson's argument (a variant of Haack's argument in *Deviant Logic, op. cit.*, pp. 66-8) may be valid, it is not sound and we can consistently deny bivalence.

Yet if it is not true that $(a = b)$ then neither is it true that $\sim(a = b)$; and so we have a truth entailing a non-truth. However, since only truth is designated, we have shown that a designated claim entails a non-designated claim — which is impossible. Therefore, if the indeterminacy in question is *de re*, and LL and DR are accepted, then — according to Tye's semantics — identity claims *cannot* be indeterminate.

Alternatively, a contradiction can be established by means of the semantics outlined in §6.1. Tye's countermodel makes use of the fact that his semantics permits ' $I(a = b)$ ' to be true whilst ' $\sim(a = b)$ ' is nonetheless not true; a stronger countermodel available in the theory of fuzzy sets outlined allows ' $I(a = b)$ ' to be true whilst ' $\sim(a = b)$ ' is false (i.e. whilst ' $(a = b)$ ' is true). Let ' $(a = b)$ ' be true at x ; that is, let x be sufficiently similar to some y at which ' $D(a = b)$ ' is true. Since $x \in s(y)$ does not guarantee in the model that x is y , ' $D(a = b)$ ' need not be true at x . Hence ' $\sim D(a = b)$ ' can be true at x and, since ' $(a = b)$ ' is true at x (thereby making ' $\sim D\sim(a = b)$ ' true at x), it follows that ' $I(a = b)$ ' can be true at x . Thus we have a countermodel for the claim that $I(a = b)$ entails $\sim(a = b)$, yet if the indeterminacy is *de re* and we accept DR and LL then just such an entailment obtains — as the Evans Proof shows. According to the semantics of §6.1 then there can be no *de re* indeterminate identity given DR and LL.

What Evans was really after (and thought he could derive) was an inconsistency in the object language itself: namely that, given DR and LL, ' $I(a = b)$ entails $\sim I(a = b)$ '.²⁴ But this is not provable without the further, false, S5-like principle,

$$\sim(a = b) / D\sim(a = b) \quad \text{or, equivalently,} \quad I(a = b) / (a = b).$$

If what I have said above is correct, however, the answer to the problem that has generated so much discussion since Evans' article is that, though his hoped for "straightforward inconsistency" in the object language is not derivable without further false assumptions, *there is an inconsistency nonetheless in supposing identity to be de re indeterminate if DR and LL hold.*²⁵

Given DR then, there is tension in the joint assumption that identity can sometimes be *de re* indeterminate and is nonetheless governed by LL. Whilst DR is not universally endorsed — Peña dissenting, as we saw in §5.4 — I stand with the majority in accepting it. Thus we are left with two choices: deny that identity can be *de re* indeterminate; or reject LL.

²⁴ Parsons wrongly presents Evans' proof as being explicitly metalinguistic, thereby sidestepping the whole issue of how it is that Evans' proof constitutes a *reductio*; cf. Parsons, T., *op. cit.*, pp. 12-13.

²⁵ Wiggins (using his variant of the Evans Proof) also claimed, in effect, that the unstrengthened Evans Proof constituted a straightforward *reductio* of the view that the identity relation can be *de re* indeterminate; as did Sainsbury, in his book *Paradoxes*, p. 47. Given what we have just said, they too are mistaken; additional semantic argument is required for a metatheoretic *reductio*.

Now I have already given reasons for maintaining that identity statements *can* be *de re* indeterminate in §6.2.1 and so shall pursue the latter option, rejecting LL. (Of course if the reasons are not thought to be compelling then the former option is open, though the descriptive inaccuracies inherent in any such account — denying the common-sense description of those cases cited in §6.2.1 as cases of *de re* indeterminate identity — require explanation.) But what alternative account of identity should we adopt? The answer is suggested by looking to similar problems that beset attempts to get an account of contingent identity off the ground.

The account of identity I want to endorse for a logic of vagueness is strictly analogous to that offered by Routley in his analysis of the paradoxes of intensionality, of which Quine's modal paradox is, perhaps, the most renowned.²⁶ Though, as has been noted, the analogy between modality and vagueness *per se* is not as strong as has sometimes been supposed, the paradox of indeterminacy — there can be no indeterminate identity — and the modal paradox — there can be no contingent identity — do share a common form. In describing an account of identity which defuses the modal paradox and admits of contingency, both *de dicto* (i.e. contingent statements of identity) and *de re* (i.e. contingent identity between individuals), an account which defuses the paradox of indeterminacy and admits of indeterminate identities, both *de dicto* (i.e. indeterminate statements of identity) and *de re* (i.e. indeterminate identity between individuals) will, by analogy, emerge. Similar problems, similar solution.²⁷

The paradox of indeterminacy will thus be dissolved in such a way as to yield an account of identity able to satisfy the sensitivity requirement.

Let us begin by considering the most basic form that the paradoxes take; that is, when they are presented, not as arguments against *de re* contingent or indeterminate identity, but as arguments against *any* contingent or indeterminate identity.

<u>Paradox of Indeterminacy</u>		<u>Paradox of Modality</u>	
$a = b$	(Hyp)	$a = b$	(Hyp)
$D(a = a)$	(Truism - DR)	$\Box(a = a)$	(Truism)
$D(a = b)$	(LL, \forall -elim., MP)	$\Box(a = b)$	(LL, \forall -elim., MP)
\therefore		\therefore	
$(a = b) \supset D(a = b)$	— (1)	$(a = b) \supset \Box(a = b)$	— (2)

²⁶ Routley, R., *Exploring Meinong's Jungle and Beyond*, Departmental Monograph #3, Philosophy Department, RISS, Australian National University (1980), §1.11.

²⁷ In thus endorsing the obvious analogy between the paradox of indeterminacy and that of modality, we are relieved of the challenge facing those like Noonan who think there can be instances of *de re* contingent identity whilst denying there can be cases of *de re* indeterminate identity. See: Noonan, H., 'Indeterminate Identity, Contingent Identity and Abelardian Predicates', *The Philosophical Quarterly* 41 (1991): 183-93.

Paradoxically though, there *are* many indeterminate and/or contingent identity claims.

The most popular response in the modal case is well known, consisting of "a ... strategy for hanging onto full substitutivity, which ... appeals to a division of subjects into logically proper subjects, e.g. proper names of some kind — for such subjects there are no failures of substitutivity — and remaining subjects, e.g. descriptions ..."28 LL is quarantined off from resilient counterexamples by being claimed to only be applicable to a restricted class of subject expressions. In modern parlance the claim is that LL is only applicable where 'a' and 'b' are rigidly referring terms — rigid designators. In this way, a distinction can be drawn between a *de re* reading of the paradox of modality which is still considered valid (though now, it is urged, there is nothing paradoxical about (2) at all — identity between individuals is necessary, if it obtains at all) and a *de dicto* reading which is no longer valid (thereby eliminating any threat of paradox in recognising a limited class of identity statements for what they are — namely, contingent). Substitution on the basis of LL within *de dicto* modal contexts is seen as illegitimate, whereas if the modalities are *de re* then use of LL is sanctioned.

By analogy, so long as 'a' and 'b' are precise designators thereby enabling a *de re* reading of 'D', LL is taken to be legitimate, thus the *de re* version of the paradox of indeterminacy goes through and the consequences must simply be accepted. It is therefore still incoherent to claim that there can be *de re* indeterminate identity claims. However, faced with the incontrovertible evidence of some indeterminate identity claims, it is suggested that if 'a' and 'b' are imprecise designators then 'D' must be read *de dicto* and so LL cannot be legitimately employed in such circumstances; there can therefore be *de dicto* indeterminate identity claims.²⁹

This common restriction on LL contrasts with another cited by Routley and advocated by Parsons. Parsons' is the orthodox "strategy of suitably narrowing the application of 'property', 'condition' or 'trait' ... so that sentence contexts or sentential functions containing intensional or modal [or (in)determinacy] operators do not specify properties or traits."³⁰ In this way LL is implicitly qualified by a restriction on the class of expressions that can be substituted for 'F'. He sees

no objection to the use of property abstraction when it is suitably restricted.

But it is well known that paradoxes result in a variety of ways from the

28 Routley, *op. cit.*, p. 99.

29 This was the position Evans was described as being in in the previous chapter. He was presented as holding a position whereby the paradox of indeterminacy was invalid for imprecise designators 'a' and 'b' (i.e. when the indeterminacy was read as *de dicto*), and valid for precise designators (i.e. when the indeterminacy was read as *de re*). The latter was taken to be a result we must simply learn to live with.

30 Routley, *op. cit.*, p. 99.

combination of a completely unrestricted property abstraction with a host of expansions of the language beyond classical quantification theory; ... abstraction in epistemic and modal languages must be carefully crafted so as to avoid trouble.³¹

The class of F's over which the universal quantifier in LL ranges is, for Parsons (as it was for Evans), given by the class of well-formed property abstracts and, in his view, abstraction is illicit in the presence of epistemic and modal operators. To this extent Parsons follows Quine — quantification within the scope of intensional operators is banned.³² And so too with the operators 'D' and 'T'.

The problem for Parsons with the first strategy (discussed above) is that it still suffers from a deficiency analogous to that which results in Routley rejecting it as a solution to the modal paradox. Just as it forces one, incorrectly in Routley's view, to accept that there can be no contingent identity between individuals, it also forces one, incorrectly in Parsons' view to accept that there can be no indeterminate identity between individuals. Parsons, like myself, thinks there can be such identities. Property abstraction (or, alternatively, substitution) must be restricted so as to exclude contexts involving 'D' and 'T'.

Now, whilst I agree with Parsons that the first strategy for restricting LL does not go far enough — recognising the coherence of *de dicto* indeterminate identity claims, whilst still rejecting *de re* indeterminate identity claims (indeterminate identity between individuals) — Parsons' own strategy is unduly restrictive. There is a third strategy for restricting LL which admits that, as regards the possibility of indeterminate identity, there is no difference between *de re* and *de dicto* contexts (i.e. it does not matter whether the designators are imprecise or not, identity can still be indeterminate) and it does so without restricting what is to count as a property or legitimate quantification.

Let us distinguish properties, or property abstracts involving intensional operators and/or the operators 'D' and 'T' — call them *complex* properties — from properties free of any such operators — call them *simple* properties. Parsons' reasoning that complex properties not be counted as properties at all, since in this way one can "satisfactorily" (according to our shared desideratum) resolve the paradoxes of intensionality and indeterminacy, is undercut by a third strategy: namely, qualifying substitutivity.

The full, or unrestricted indiscernibility of identicals is replaced by appropriately qualified substitutivity. LL is rejected in favour of an account of identity which restricts LL as follows:

³¹ Parsons, *op. cit.*, p. 14.

³² See, for instance: Quine, W.V.O., 'Three Grades of Modal Involvement' in *Ways of Paradox*, Random House (1966).

Parsons, however, need not (and, I think, would not) agree with Quine that the prohibition arises from the fact that all modality is *de dicto*.

LLR: $(a = b) / \forall F_{\text{simple}}(Fa \equiv Fb)$.

Simple identity entails agreement on all simple properties and permits intersubstitutivity within simple sentence contexts only.

The virtue of this restricted substitutivity, in Routley's view, is that it "removes at once certain traditional and modern puzzles about identity." For example, we can explain how identity can have (logical) importance and be other than trivial. "The solution ... is simply that *de facto* truths of identity do not legitimate replacement within intensional sentence frames such as those formed with functors such as 'it is trivial that'. For instance the truth, Cicero = Tully does not legitimate single replacement of 'Cicero' by 'Tully' in the statement 'It is trivial that Cicero = Cicero'." More importantly in the present context though, such a restriction also "eliminates paradoxes that emerge as soon as classical identity logic is grafted onto quantified modal logic."³³

And, by analogy, we eliminate the paradox of indeterminacy that emerges as soon as classical identity logic (including LL) is grafted onto a quantified logic of vagueness. By means of LLR, the derivation of ' $\sim(a = b)$ ' is blocked; a and b , though they differ on a complex property, do not differ on any simple property and so the conclusion that $\sim(a = b)$ is not warranted. We may consistently assume statements of identity to be sometimes indeterminate in truth value — including those where the indeterminacy is considered *de re*.

To sum up the argument so far. We have been looking at what happens if we simply accept the classical account of identity as characterised by LL. We have seen that a consequence of this would be that, on pain of inconsistency, we would then be forced to deny the possibility of there being identity statements that are *de re* indeterminate. Assuming there to be an option between the classical account of identity and the account characterised by LLR, I have opted for the LLR-account since the ensuing account of identity tolerates the possibility of such indeterminate identity statements thereby enabling the sensitivity requirement to be met, whilst, at the same time, leaving complex properties intact.

But is LLR an option? The hard-liner identified at the end of §6.2.1 will think not — rejecting the possibility of restricting LL in both fuzzy and modal contexts — since now, contrary to the classical account, identity does not tolerate substitutivity in all contexts (only simple ones).

I have not yet encountered any rational defence of such a view. Dogmatic adherence to the classical conception requires supplementation with some *independent argument* as

³³ Routley, *op. cit.*, pp. 99 ff.

to why, when extending identity to non-classical contexts, we should suppose it to hold unchanged. In fact, as we have seen, the idea that LL can be retained unrestrictedly in extended languages only makes sense anyway if one already accepts implicit restrictions thereby enabling the explicit form of LL to be retained.

The more tolerant position, which simply requires that the account of identity for a logic of vagueness not conflict with LL in precise or sharp (e.g. classical) set-ups, must admit LLR as an option since it does conservatively extend classical identity. In classical circumstances 'D' is redundant and hence the distinction between simple and complex properties is redundant — any property involving 'D' is exemplified by all and only those objects exemplifying the corresponding property not involving 'D', i.e. a simple property.

The account of identity offered above either compels or permits (depending on the force of "the sensitivity-requirement" of §6.2.1) the conjoining of LLR to a quantified logic of vagueness, thereby yielding an account of identity that can be indeterminate.

Note that, given the (determinate) reflexivity of identity, we can prove:

$$\forall F_{\text{simple}}(Fa \equiv Fb) / (a = b),^{34}$$

This, in conjunction with the validity of LLR, entails the validity of:

$$(a = b) \equiv \forall F_{\text{simple}}(Fa \equiv Fb).$$

Assuming the plausible principle:

$$\vdash_{\forall} A \Rightarrow \vdash_{\forall} DA,^{35}$$

it would then follow that:

$$\vdash_{\forall} D[(a = b) \equiv \forall F_{\text{simple}}(Fa \equiv Fb)].$$

Hence:

$$\text{LLR(1)} \quad \vdash_{\forall} D(a = b) \equiv D\forall F_{\text{simple}}(Fa \equiv Fb).$$

$$\text{LLR(2)} \quad \vdash_{\forall} I(a = b) \equiv I\forall F_{\text{simple}}(Fa \equiv Fb).$$

$$\text{LLR(3)} \quad \vdash_{\forall} D\sim(a = b) \equiv D\sim\forall F_{\text{simple}}(Fa \equiv Fb).$$

³⁴ Assume $\forall F_{\text{simple}}(Fa \equiv Fb)$. Then, by \forall -elimination, letting 'F' be '(a = x)', $(a = a) \equiv (a = b)$. Since $D(a = a)$, it follows that $(a = a)$. So we can infer that $(a = b)$. That is:
 $\forall F_{\text{simple}}(Fa \equiv Fb) / (a = b)$.

³⁵ This presumed *admissibility* of the rule 'A / DA' — i.e. $\vdash_{\forall} A \Rightarrow \vdash_{\forall} DA$ — would fail were one to countenance the existence of vague theorems since then the logical truth of A would not necessarily entail the logical truth of DA. Even were the rule deemed inadmissible on these grounds there is no reason to think the theorem at issue to be vague.

It should be added that even if the rule is considered admissible the stronger claim that DA is deducible from A need not be accepted and in fact I take it to fail. (This is analogous to the epistemic theorist's rejection of the deducibility of KA from A, and the supervaluationist's rejection of the deducibility of DA from A, at least on Dummett's definition of deducibility — definition (1) of §5.3.1 — which I have suggested the SV theorist ought to adopt.)

6.2.3 *Fuzziness and Indeterminate Identity*

Returning now to question of fuzziness and indeterminate identity. What is the relationship between our claiming an object to be fuzzy and its possible identity relations? The question is significant since, as we have seen, the claim that some object is fuzzy has been asserted in terms of indeterminate identity relations holding *de re* — between individuals.

What I claim is the following:

Theorem 6.6 An object x is fuzzy iff x stands in a relation of indeterminate identity to a precisification, x' , of itself; in other words, if and only if, for some precisification x' of x , $I(x = x')$.

Proof. Let x' be a precisification of x and assume it to be indeterminate whether or not $x = x'$. Then x is not determinately identical to its precisification x' . But if x were sharp then x would be determinately identical with any precisification of itself [by Theorem 6.1 and DR]. So, by modus tollens, x is fuzzy.

Conversely, suppose for *reductio* that x is fuzzy, yet it is not indeterminate whether $x = x'$; it is therefore either determinately the case that $x = x'$ or determinately not the case. The latter horn is impossible; an object and its precisification cannot determinately differ on any simple property. So, it must be determinately the case that $x = x'$, in which case x and x' determinately share all simple properties — including spatio-temporal ones. But then they must be determinately the same spatio-temporally extended object, in which case x must be sharp, which is impossible. So, if x is fuzzy then it is indeterminate whether $x = x'$. **I**

We can therefore expand on Quine's dictum and claim:

No fuzzy entity without indeterminate identity.

Now one might ask oneself why I have opted for the above dictum rather than the much catchier "No fuzzy entity without *fuzzy* identity". The explanation is not simply that I lack a poetic disposition! There is a substantive point here; one discussed by Sainsbury, in his article 'What is a Vague Object?'.³⁶

Recall from our earlier discussion of properties that the mere fact that a property had indeterminate exemplars or entered into an indeterminate state-of-affairs did not make it fuzzy; the ensuing indeterminacy may have been due to the fuzziness of the object involved. Well similar remarks will apply to identity qua relation. The indeterminacy of identity detailed above, is a case where the two-tuple $\langle x, x' \rangle$ is a border case for the identity

³⁶ Sainsbury, R.M., 'What Is A Vague Object?', *Analysis* 49 (1989): 99-103.

relation. It does follow that identity is fuzzy without the further fact that $\langle x, x \rangle$ is a resilient border case — it merely shows that it has cases of indeterminate application. However the indeterminacy in these cases is explicable in terms of the fuzziness of the objects involved. As Sainsbury puts it: "If you blame the objects for the vagueness [fuzziness] then you need not blame the relation."³⁷ We can accept Sainsbury's slogan — "vague [fuzzy] objects without vague [fuzzy] identity"³⁸ — without contravening Theorem 6.6 since the latter equates the fuzziness of objects with cases of indeterminate identity rather than the fuzziness of the relation itself.

6.2.4 *The Vague-Identity Thesis*

Returning finally now to the Vague-Identity Thesis. My claim is that, to the extent that identity is as I have characterised it — thereby admitting of *de re* indeterminacy, the Vague-Identity Thesis holds; to the extent that identity does not admit of *de re* indeterminacy, the Vague-Identity Thesis fails. In effect then I am presupposing firstly that the Thesis is ambiguous. This will be contested for two distinct reasons (both of which I reject): (i) my rival account of identity is not really an account of identity at all (e.g. Wiggins, Tye); (ii) my rival account, whilst entitled to be counted an identity relation, is equivalent to the classical account (e.g. Parsons).

My response to the first objection is the same as it was at the end of §6.2.2 — there must be some independent argument in support of the assumption that LL is constitutive of the notion of identity.

The second criticism gains a foothold as follows. I claim identity to hold between objects *a* and *b* if and only if all simple properties are shared by *a* and *b*; this differs from the classical account of identity since such an account claims identity to hold between objects *a* and *b* if and only if *all* properties are shared. Now, of course, if all properties are simple then the two accounts are equivalent and there is no ambiguity as to what one means by identity; restricted substitution is pre-empted and made redundant by restricting what is to count as a 'property'. This is Parsons' strategy. Whilst I don't think Parsons' position can be convincingly refuted, it's just that the reasons he gives for restricting property abstraction aren't compelling; we can adopt the methodologically preferable option of admitting unrestricted property abstraction *without* thereby embroiling ourselves in paradoxical results. Identity, and hence the Vague-Identity Thesis, is ambiguous.

Thus I claim that the Vague-Identity Thesis holds, *where 'identity' is as I have described it above*. But this does not follow immediately from Theorem 6.6. Theorem 6.6

³⁷ Sainsbury, *op. cit.*, p. 102.

³⁸ *Ibid.*, p. 103.

is concerned with some *particular* object's fuzziness whereas the Vague-Identity Thesis concerns the generalised claim that "there can be fuzzy objects". However the following theorem is derivable given Theorem 6.6.

Theorem 6.7 There can be fuzzy objects if and only if there can be identity relations such that it is indeterminate whether or not they obtain between some objects *a* and *b*.

Proof. Left to right follows from Theorem 6.6. Conversely, suppose that for some objects *a* and *b*, it is indeterminate whether or not *a* is identical to *b*. Then, given my account of identity, it is indeterminate whether or not *a* and *b* share all simple properties. Suppose now that there can be no fuzzy objects; then *a* and *b* are both sharp. So they determinately agree or determinately differ on their simple spatio-temporal properties. If the latter, then they determinately do not share all simple properties contrary to assumptions. If the former, then *a* and *b* are one and the same object and hence determinately share all properties, in particular all simple properties — contrary to assumptions. Hence, by *reductio*, *a* and *b* are not both sharp; at least one is fuzzy. ▮

Now Theorem 6.7 is just the Vague-Identity Thesis expressed in the material mode as opposed to Garrett's metalinguistic formulation. It is quite clear however that Garrett endorses the Vague-Identity Thesis with Leibnizian identity in mind — he endorses LL. However the commitment to fuzzy objects (as defined in §6.1) is in no way a commitment to the indeterminacy of Leibnizian identity and so now the Vague-Identity Thesis fails (one means something else by the "Vague-Identity Thesis" given the classical sense of identity).

We are finally in a position to comment on the two alternate ways of defining fuzziness.

A — The means of defining 'fuzzy' I have adopted:

- (1) can be coupled with a neo-Leibnizian theory of identity such that the Vague-Identity Thesis holds — as I have done and I endorse this account;
- (2) can be coupled with a Leibnizian theory of identity in conjunction with unrestricted property abstraction, though now the Vague-Identity Thesis fails — Tye;
- (3) can be coupled with a Leibnizian theory of identity in conjunction with restricted property abstraction such that the Vague-Identity Thesis holds — Parsons.

B — The means of defining 'fuzzy' via the Vague-Identity Thesis (following Evans, Wiggins, Sainsbury, Garrett and Parsons):

- (4) can be coupled with a Leibnizian theory of identity in conjunction with unrestricted property abstraction — Evans, Wiggins, Sainsbury, Garrett;
- (5) can be coupled with a Leibnizian theory of identity in conjunction with restricted property abstraction — Parsons.

Account (4), in terms of which the debate over the logical coherence of fuzzy objects has raged, is incoherent. Yet it was high time a more explanatory account was offered anyway. Account (3)/(5) (they are equivalent) is coherent but objectionable by virtue of its postulate restricting property abstraction; account (2) is coherent but includes an account of identity unable to satisfy the sensitivity requirement; and account (1), the same as (2) except as regards identity, is coherent, permits unrestricted abstraction and has an acceptable account of identity. (It should be noted that this debate between Tye — account (2) — and myself — account (1) — does not concern the hard-core of the theory of fuzziness herein presented; we are arguing about what theory of identity to add to the core account of ontological fuzziness.)

What I claim in the final analysis then is that there is a fuzzy variant of classical ontology and logic that remains coherent for all that has been said about it so far. The ever expanding literature on the coherence of fuzzy ontologies, in focussing on indeterminate identity, either falls wide of the target I have presented in §6.1 (if the identity in question is classical) or hits its target (if restricted identity, LLR, is considered) though in a benign way.

Even were we to agree with Pelletier's description of theories suggesting vagueness as an ontological feature as neither The Good nor The Bad but The Ugly, then we may yet want to say that The Ugly, having received the kiss of tolerance, turns out to be the handsome, though admittedly young, prince.³⁹

6.3 *The Sorites Paradox Revisited*

Christopher Peacocke once asked whether the world's being vague would undermine his solution to the sorites paradox.⁴⁰ However, after deciding that the world could not be vague, he did not go on to consider what connections there might be between ontological vagueness and the sorites paradox. It is this that I now want to consider.

The epistemic theorist's response to the sorites remember was that, though there is a determinate cut-off point, we are precluded from ever having knowledge of its exact whereabouts by virtue of the necessary margin for epistemic error; the representationalist

³⁹ Cf. Pelletier, J., 'The Good, the Bad and the Ugly', typescript (1988).

⁴⁰ Peacocke, C., 'Are Vague Predicates Incoherent?', *Synthese* 46 (1981), p. 132.

Should we so willingly accept the logic involved? The challenge is to explain how the reasoning, which after all only amounts to the iterated use of MP and thus is classically valid to the core, could lead us into error.

Some indication of the non-standard behaviour of a conditional capable of rendering the premises true can already be found in the fact that the transitivity of '>' — i.e.

$$Fa_1 > Fa_2, \dots, Fa_{n-1} > Fa_n / Fa_1 > Fa_n$$

— fails. Though a_i may be sufficiently similar in relevant respects to a_{i+1} to warrant ' $Fa_i > Fa_{i+1}$ ', a_1 is sufficiently different from a_n to falsify ' $Fa_1 > Fa_n$ '. Of course, the fact that a conditional is non-transitive, though initially a little surprising perhaps, does not show it to be non-possible in any way. There are modal conditional logics, for example, in which transitivity fails though MP is still fully reliable.⁴² However, there are weaker non-transitive conditional logics in which even MP fails — for example, Chellas's basic conditional logic CK.⁴³

What I take the standard sorites to show is that there is another conditional whose logic is analogously weak — the conditional '>' of the sorites. A vague conditional logic ought to invalidate transitivity *and* unrestricted MP.

Why MP? Well because iterated MP, by hypothesis, is not truth-preserving; it leads us from (determinate) truth to (determinate) falsity. Since it is not truth-preserving neither is what yields it, MP. Its failure to preserve truth, apparent under iteration, is (to use a metaphor of Sylvan's) analogous to the unreliability of some cars, apparent on long journeys. However, just as some cars might nonetheless be *locally* reliable though unreliable for distant purposes, so too, MP may be only locally reliable. Vague conditional logic can admit MP as truth-preserving in nearby applications whilst denying its reliability *per se* — that is, for any amount of iteration whatsoever.

The standard sorites, according to this metaphor, commits the fallacy of supposing that, since any long journey can be broken up into a series of successive short journeys, a car reliable for short trips can also be relied on for long ones. (A fallacy which, I might add, has brought many a traveller to grief!) This does not mean that one always requires a "roadster" capable of taking one anywhere anytime; it is often an unnecessary extravagance. The shopping and carrying needs of most can be met by a more modest

42 For instance, the Lewis-Stalnaker account of modal conditionals; cf. Stalnaker, R.C., 'A Theory of Conditionals', *Studies in Logical Theory*, American Philosophical Quarterly Monograph Series #2, Blackwell (1968), also in W.L. Harper, *et al* (eds), *Ifs: Conditionals, Belief, Decision, Chance and Time*, Reidel (1981), pp. 41-55; Lewis, D., 'Counterfactuals and Comparative Possibility', *Journal of Philosophical Logic* 2 (1973): 418-46, also in W.L. Harper, *et al* (eds), *Ifs, op. cit.*, pp. 57-85.

43 See Chellas, B., 'Basic Conditional Logic', *Journal of Philosophical Logic* 4 (1975): 133-53, esp. §7.

form of transport. In fact, modesty in this regard is sometimes preferable in so far as it is more easily obtainable.

The claim that, even from a classical perspective, MP may be only locally satisfactory for '>', is similar to that offered by the entailment theorist for material conditionals. The material conditional is fully ponible according to classical conditional logic, yet it is claimed that there is an even weaker conditional which is subject to restrictions. Similarly, a relevant conditional '→' is fully ponible according to entailment theory, yet it is recognised that the weaker material conditional '⊃' must be restricted.

So what about the semantics for '>'? The required features of a vague conditional emerge from the semantics defined by the evaluation rule:

$$(\kappa) \quad v_x(A > B) = 1 \text{ if and only if, if } s(x) \subseteq [A] \text{ then } x \in [B]$$

— where '[A]' is $\{a: v_a(A) = 1\}$ and 's' is a selection function from points (or worlds) to sets of points. As in modal conditional logics, 's' is interpreted in terms of similarity. It is that function which maps a point onto the set of sufficiently similar points; that is: $s(x) = \{y: y \text{ is sufficiently similar to } x\}$ — where the relation of 'sufficient similarity' is that reflexive, symmetric, non-transitive relation explained in discussion of Definition 6.8. The degree and kind of similarity required depending on the relevant circumstances.⁴⁴

Recalling that:

$$v_x(A \supset B) = 1 \text{ if and only if, if } x \in [A] \text{ then } x \in [B],$$

it follows that:

$$v_x(A > B) = 1 \Rightarrow v_x(DA \supset B) = 1.⁴⁵$$

$$\text{Also: } v_x(A \supset B) = 1 \Rightarrow v_x(A > B) = 1;$$

$$\text{and so: } \models_v (A \supset B) \supset (A > B).⁴⁶$$

⁴⁴ In fact the semantics for '>' bears a close resemblance to the Lewis semantics for the conditional; namely:

$$v_x(\text{if } A \text{ then } B) = 1 \text{ if and only if } s(x, [A]) \subseteq [B]$$

— where 's' is a selection function from worlds and sets of worlds to sets of worlds. Cf. Lewis, D., 'Counterfactuals and Comparative Possibility', *op. cit.*, p. 422.

⁴⁵ If we were prepared to accept the converse of Theorem 6.5 — If all elements of the universal set sufficiently similar to a are elements of α then $a \in I(\alpha)$ — (support for which could come from the view that reliability is not only a necessary condition for determinate ascriptions, as discussed in §6.1, but also *sufficient*) then we would have the stronger claim that:

$$v_x(A > B) = 1 \Leftrightarrow v_x(DA \supset B) = 1.$$

⁴⁶ The converse claim: $(A > B) \supset (A \supset B)$,

Given the above semantics for '>' we can verify that $Fa > Fb$ whenever a is sufficiently similar to b ; that is, we can show that ' $Fa > Fb$ ' is true at any point at which ' $b \in s(a)$ ' is true. Assume ' Fa ' is true at any point sufficiently similar to x , i.e. $s(x) \subseteq [Fa]$. Since something counts as a at a point sufficiently similar to x if and only if it is sufficiently similar to a at x , it follows that, for all c sufficiently similar to a at x , ' Fc ' is true at x . Assume further that ' $b \in s(a)$ ' is true at x ; i.e., b is sufficiently similar to a at x . As a consequence then ' Fb ' is true at x ; i.e. $x \in [Fb]$. Hence we have shown that, whenever it is true that $b \in s(a)$, it is true that $Fa > Fb$.

This enables the proper expression of Wright's intuition that it is the "tolerance" of many vague predicates that underlies their susceptibility to paradoxical reasoning (cf. §4.4, where 'tolerance' was discussed). Objects which, given the context, count as sufficiently similar — that is, those which are sufficiently similar in respects relevant to F , say — are such that if one is F then the other is also F . Where ' F ' is a soritical predicate, objects which differ marginally in respect of F can nonetheless be sufficiently similar, and so ' F ' is tolerant — in Wright's sense. As a consequence, the conditional premises of the standard sorites are shown to be true since, for any i , a_i is sufficiently similar (in respects relevant to F — naturally) to a_{i+1} . However, this does not result in contradiction unless the conditional is fully ponible.

The condition required in the model to ensure that MP for '>' is truth-preserving is:

(†) if $x \in \alpha$ then $s(x) \subseteq \alpha$.⁴⁷

For suppose ' A ' is true at x ; i.e. $x \in [A]$. Then, by (†), $s(x) \subseteq [A]$. Assuming ' $A > B$ ' is true at x then, by (κ), $x \in [B]$, and so ' B ' is true at x . That is, given (†):

$A > B, A / B$.

But (†) does not generally obtain in the model described in §6.1, the theory of fuzzy sets. MP for '>' is only truth-preserving in the following restricted sense:

$A > B, DA / B$.

would be valid in V , as one would expect, only if the converse of Theorem 6.4 were to hold. That is only if:

$$v_x(A) = 1 \Leftrightarrow v_x(DA) = 1,$$

since then we would have: $\models_V A \supset DA$.

Under such conditions *modus ponens* would be valid for '>'.

⁴⁷ Compare this to the modelling condition required to ensure the modal conditional as truth-preserving:

$$\text{if } x \in \alpha \text{ then } x \in s(x, \alpha).$$

See: Chellas, *op. cit.*, p. 142.

Starting from the determinate truth of 'Fa₁', as we are invited to do in soritical arguments, the truth of 'Fa₂' is ensured. Whether or not a₃ is F depends; it is possible but logically guaranteed only if a₃ ∈ s(a₁). Unreliability accumulates so that after sufficient iteration reliability gives way to unreliability. How far down the soritical chain one can go and still be sufficiently close to one's starting point is a vague matter; where truth gives out in the soritical argument is, consequently, a vague matter — as expected.⁴⁸

In precise contexts, of course, MP is unrestrictedly truth-preserving since for precise 'A', A / DA. The modelling condition (†) obtains for sharp α.

Obviously, '>' is not generally transitive either (as expected) since 'sufficient similarity', though reflexive and symmetric, is non-transitive in general. What need not be contested is the general admissibility of MP for '>'. That is, where 'A' and 'A > B' are provable, so is 'B'.

As a result of the foregoing account of the vague conditional '>' and the account of vague identity described in §6.2, we have the following substitution principles governing simple 'F':

$$\begin{aligned} a \approx_F b & / Fa > Fb \\ a = b & / Fa \supset Fb \\ D(a = b) & / D(Fa \supset Fb) \end{aligned}$$

where ' $a \approx_F b$ ' represents a 's being sufficiently similar to b in respects relevant to F — a 's being sufficiently F-similar to b .

Acceptance of vague conditionals dissolves the standard sorites paradox; the relevant similarity of adjacent elements of a sorites series is sufficient to support the conditional premises, but only if the conditional is vague in which case the reasoning is invalid. Naturally, such reasoning is valid if a fully possible conditional were employed, but since the resulting conditional premises must be underwritten by something stronger than mere similarity they fall short of truth.

Puzzlement arising from the inductive form the paradox can take (cf. §1.2.1):

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A nice feature of fuzzy logics, often proposed to deal with vagueness, is that they offer an effective measure suitable for evaluating reliability. A reliability measure can be independently added to the semantic theory outlined so far above, without introducing degrees of truth. For more on this see: Sylvan, R. & Hyde, D., 'Ubiquitous Vagueness Without Embarrassment', *op. cit.*, §1.

So we *can* have this desirable feature without the full semantics of fuzzy logic; moreover, we should, I think, reject fuzzy logic as a logic for vagueness. Firstly, a fuzzy logic with a totally ordered truth-set will fail to account for non-linear vagueness. Secondly, even a partially ordered truth-set is inadequate; non-soritical vagueness, or more generally, any case of vagueness that cannot be embedded within an ordered sequence, is ignored by such a theory.

$Fa_1, \forall a_i(Fa_i > Fa_{i+1}) / Fa_n$ for arbitrarily large n

is now not long-lived. The principle of mathematical induction, when framed with the vague conditional, is invalid. Induction amounts to iterated *modus ponens* on the conditional involved, which, when the conditional is the vague conditional '>', is not universally truth-preserving; induction fails for the same reason MP does. A quantified conditional premise which *would* validate the induction uses a conditional at least as strong as a material conditional, but in that case the paradoxicality is dispelled by noticing that the premise is not true; 'sufficient similarity' is too weak a relation to support such a strong conditional in the premise.

In both forms of the sorites using conditionals, paradoxicality is fostered by a fallacious equivocation. Where the premises are framed using a fully ponible conditional capable of supporting induction, they are too strong to be true; when framed with a weaker, vague conditional — thereby ensuring their truth — they are not fully ponible and do not support induction.

Obviously, any proposed solution to the line-drawing form of the sorites (cf. §1.2.1):

$Fa_1, \sim Fa_n / \exists a_i(Fa_i \& \sim Fa_{i+1})$ for $1 \leq i < n$

should already be available from claims implicit in dissolving the inductive form, in so far as the line-drawing form makes use of the least number principle (LNP). The fallacy exposed in the paradoxical application of the principle of mathematical induction ought to have its analogue in the paradoxical application of the equivalent LNP. It is application of this principle remember which, in conjunction with the assumption that $\sim Fa_n$, entitles us to infer that (relative to the well-ordered sorites series) there must be a least j such that $\sim Fa_j$. This amounts to the claim that, for some $j \leq n$, $\sim Fa_j$ and for all $i < j$, Fa_i ; that is (rearranging a little), for some $i < n$, $Fa_i \& \sim Fa_{i+1}$. Given that Fa_1 , it would then follow that $Fa_i \& \sim Fa_{i+1}$, for $1 \leq i < n$. So, there must be a line between the F's and the non-F's.

With the truth of the premises beyond question, is the argument sound and the conclusion actually false?

There is a sense in which the conclusion is false; namely, were it the slightly stronger claim that there is some line between the determinate-F's and the non-F's — that is, for some a_i , $DFa_i \& \sim Fa_{i+1}$.⁴⁹ We already know that, given the relevant sufficient similarity of a_i and a_{i+1} , $Fa_i > Fa_{i+1}$, so $DFa_i \supset Fa_{i+1}$ (cf. also SMP, §6.1); yet if $DFa_i \&$

⁴⁹ The falsity of this claim is sufficient to falsify the claim that there is some sharp, determinate line between the F's and the non-F's (though the truth of the former is insufficient for the truth of the latter) since were there a determinate line between F's and non-F's then there would be a line between determinate-F's and non-F's — which there isn't.

$\sim Fa_{i+1}$ then $\sim(DFa_i \supset Fa_{i+1})$, so paradoxically $\sim(Fa_i > Fa_{i+1})$. And such a conclusion *would* be derivable given a version of LNP, the determinate least number principle, which maintained that:

DLNP For any F defined on an ordered series $\langle a_1, \dots, a_n \rangle$, if $\sim Fa_n$ then there is a *determinately* least j such that $\sim Fa_j$

— in the sense that anything less than j is determinately not a non-F index; that is, for some j , $\sim Fa_j$ and for all $i < j$, $D\sim\sim Fa_i$.

However, in the context of vagueness there is good reason to reject such a "least" number principle; it is fallacious to suppose that there must be a "least" element of the sorites series, in this strong sense. DLNP is valid relative to LNP if modelling condition (†) is met, but (†) does not generally obtain — as we saw above. The use of DLNP to arrive at a false conclusion merely shows DLNP to be invalid and the version of the line-drawing sorites which employs it to be unsound.

The principle LNP need not itself be contested. Such line-drawing sorites are sound but only validate the benign conclusion that there must be a line between the F's and non-F's which can be accepted as (vaguely) true so long as adjacent members of the series are not identical.

Again, the fallacy amounts to one of equivocation.⁵⁰ To the extent that the reasoning is valid, thereby making the line-drawing sorites sound, its conclusion is acceptable; to the extent that the conclusion is unacceptable, the reasoning required to support it is invalid.

All that remains is the phenomenal form the paradox can take (cf. §1.2.1):

$Fa_1, a_1 \sim_F a_2, a_2 \sim_F a_3, \dots, a_{n-1} \sim_F a_n / Fa_n$ for phenomenal 'F'.

This argument-form involves no conditional and does not explicitly proceed by *modus ponens*, mathematical induction or any equivalent principle; it proceeds via the rule:

(\sim) $a \sim_F b, Fa / Fb$ for phenomenal 'F'

— if a is indiscriminable (or indiscernible) from b in respects relevant to F and a is F then b is F.

⁵⁰ There is another version of the line-drawing sorites which employs the Law of Excluded Middle and Disjunctive Syllogism to generate the conclusion. Again, the fallacy is one of equivocation. Either a strong version of LEM, $DA \vee D\sim A$, is employed to validly derive the false conclusion that for some i , $1 \leq i < n$, DFa_i & $D\sim Fa_{i+1}$ — but in this case the assumption that $DA \vee D\sim A$ should be challenged; or a weak version of LEM is employed — but in this case no paradoxical conclusion can be derived.

The first thing to notice is that the relation of F-indiscriminability, ' \sim_F ', is non-transitive and so is weaker than identity. In fact, such a relation is highly subjective and admits of *non*-identity. Of course, one might treat ' \sim_F ' on a par with identity, at least to the extent of admitting (\sim) as truth-preserving — even when iterated. In this case one is forced to accept, paradoxically, that Fa_n . Were such a logic to govern our use of phenomenal terms then, they must be deemed incoherent; though admittedly this only becomes manifest under testing iteration.

On the other hand, in an attempt to give a consistent account of phenomenal terms, one could seek to restrict (\sim), so that, though locally reliable, it is not considered universally so. The theory outlined earlier already contains just such a relation, the vague relation of 'sufficient F-similarity' — ' \approx_F ', so no new logic is required. Reinterpreted, ' \approx_F ' provides an appropriate model for 'F-indiscriminability'.

Thus modelled, (\sim) is not a universally reliable substitution principle. It is certainly truth-preserving whenever MP is reliable for ' $>$ ' — i.e., locally. This is because we have already established that:

$$a \approx_F b \ / \ Fa > Fb$$

and, if MP is permitted, (\sim) follows as a consequence of the reinterpretation. However, substitution on the basis of (\sim) is not truth-preserving under iteration since this again presumes the overly strong modelling condition (\dagger).

Given the above verdict for the class of paradoxes collectively referred to as the sorites paradoxes, how would we answer Peacocke's question with which we began §6.3? The answer is a resounding 'Yes'. An ontological account of vagueness enables the development of a logic of vagueness capable of diffusing the sorites by challenging the logic involved. In Barnes' terminology it makes one a "radical opponent" of the sorites, and that Peacocke is not.⁵¹

6.4 Summary

In this chapter we have witnessed the beginnings of a non-classical metaphysics of vagueness. At its most abstract it amounts to a theory of fuzzy sets which serve as the

⁵¹ Barnes, J., 'Medicine, Experience and Logic' in J. Barnes, *et al* (eds), *Science and Speculation*, CUP (1982), p. 47.

paradigm by means of which ontological vagueness or fuzziness can be defined. The ontological account was developed so as to mimic semantic vagueness thereby permitting the common-sense correlation of vague names with fuzzy objects and vague predicates with fuzzy properties.

Epistemic and representational accounts of vagueness are united in the view that such an account is misguided. The strongest support for this view comes from the claim that ontological vagueness is logically impossible; constraints on identity have been cited as the much sought *a priori* grounds for barring any such account. However, the claim that identity cannot possess the logical features required by a theory of ontological vagueness was viewed with scepticism from two angles.

Firstly, it was questioned whether that feature which identity purportedly could not possess — *de re* indeterminacy — actually *was* required by an ontological theory. Prior to our having such a theory, it is hard to see how such a necessary condition could be justifiably placed on such a theory; that is, the Vague-Identity Thesis, which imposes just such a condition, seems unsupported. The possibility of *de re* indeterminate identity seems an independent issue.

Secondly, independent arguments in favour of identity exhibiting just that feature led to our questioning that classical account of identity which denied its possession. In response, to supplement the already existing metaphysics, a non-classical account of identity was endorsed which was capable not only of exhibiting *de re* indeterminacies but also diffused modal and other puzzles surrounding identity. It was subsequently shown that the disputed feature *is*, from this position on identity, required by the theory; the Vague-Identity Thesis is proven. It was admitted however that, if one insisted on retaining the classical account of identity, then this feature which identity lacks is *not* required by an ontological account of vagueness. Given a classical account of identity the Vague-Identity Thesis fails.

To the extent that the Vague-Identity Thesis does constitute a constraint on ontological vagueness it obtains; to the extent that it does not obtain it does not constrain the ontological account. Identity, however one reasonably accounts for it, is no bar to a theory of vague objects and beyond.

The final test for an ontological account, as indeed it is for any account of vagueness, is whether it can provide an adequate response to the ancient sorites paradoxes. The account outlined tackled the paradoxes by challenging the logic involved; it is not only metaphysically non-classical, but only constitutes grounds for logical deviance. The apparent tension between our classical logical theory and our desire to ascribe truth where we do (for example, to accept the premises of the standard sorites, say, as true) was resolved in favour of our truth ascriptions; classical theory is enriched with vague features which serve to explain how paradox can be avoided, whilst making it apparent how paradox might so easily be thought to ensue.

Conclusion

To conclude then, let us be clear about what is and is not being claimed in this thesis. Firstly, in advocating an ontological account of vagueness I am not suggesting that *all* vagueness is ontologically grounded. An Ontological Theory need not, and indeed should not, be construed as providing an account of vagueness *per se*; it should be understood as endorsing the common-sense response to vagueness according to which talk is *often* vague by virtue of the vagueness of that being talked about, without requiring that vagueness *always* be analysed in this way.

We saw in Chapter Two that Epistemic Theories could be distinguished — strong from weak; weak versions offer an epistemological analysis of *some* vagueness, whereas strong versions attempt to account for *all* vagueness epistemologically. Similar variations in the scope of the theory in question can be discerned within Representational and Ontological Theories; we can distinguish weak from strong Representational accounts of vagueness, and weak from strong Ontological accounts. In light of this distinction, where an Epistemic Theory is argued for, in part or solely on the basis of simplicity — as indeed it often is, as we have seen — then the theory in question must be a strong Epistemic one. Any restriction in the scope of an epistemic analysis of vagueness entails abandonment of a thoroughly classical treatment since there will be cases where the epistemic theory does not apply. However, strong variants of both the Representational and Ontological Theories are unnecessarily constricting. Either account of vagueness can admit cases of epistemic vagueness at no extra cost — no new, more deviant analysis is required. Naturally though, a Representational Theory is generally advocated with a view to excluding ontological accounts, weak or strong; so, whilst weak representationalism seems the most rational position for Representationalists to hold, it will generally be a weak variant excluding even a weak ontological account.

A strong Ontological Theory would be doubly constricting in so far as it would exclude cases of epistemological vagueness *and* cases of representational vagueness. A more liberal, mixed account is correspondingly easier to defend — an account, that is, which gives primacy to the philosophically deviant, though often intuitively compelling analysis of vagueness as ontological, whilst admitting some cases of epistemic and representational vagueness. Moreover, vague sentences like 'The greatest ruler is the wisest ruler' — commonly cited as an undeniable example of representational (i.e.

purely semantic) vagueness — do seem to provide a strong argument for the existence of some vagueness that is not ontologically grounded. One need not, and should not, claim all vagueness to be ontologically grounded.

Secondly, having clarified problems concerning the scope of the Ontological Theory, we should be clear about the logical status of the claim, inherent in such a theory, that the world is vague. I am certainly not suggesting that the world necessarily contains vague objects; that is a contingent matter. There are possible worlds containing only sharp objects. What I *am* suggesting is that our world, the actual world, does contain a plethora of vague objects which give rise to semantic vagueness.

The case of properties is a little more complicated. Property-vagueness is an intensional attribute, depending only on the logical possibility of there being resilient border cases for that property. Extensional property-vagueness, the actual existence of such cases, is obviously a contingent attribute, but property-vagueness *simpliciter* is necessary if the S5-principle 'Possibly p therefore necessarily possibly p' governs the use of logical possibility.

The general methodology employed in this thesis also warrants mention. A deviant metaphysics and logic of vagueness was arrived at by letting the theory and logic be constrained by the data. In preference to condemning vague discourse, and therewith much of ordinary language, to radical incoherence, or restricting the scope of logic and ontological accounting, I have followed a course according to which the "recalcitrant" data — vague language — can be accommodated for in our general philosophical theory by amending orthodox metaphysics and logic. This methodological principle extends well beyond analyses of vagueness. It provides the means for keeping epistemology and reference as simple as possible in the face of initially recalcitrant data without ignoring the data in any way.

In pursuing this course, we have seen how one can be coherently vague — not only epistemically and semantically, but, more contentiously, ontologically. Being itself can be coherently vague.

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