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A Model of Seller Holdout*

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Abstract:
We model a buyer who wishes to combine objects owned by two separate sellers in order to realize higher value. Sellers are able to avoid entering into negotiations with the buyer, so that the order in which they negotiate is endogenous. Holdout occurs if at least one of the sellers is not present in the first round of negotiations. We demonstrate that complementarity of the buyer's technology is a necessary condition for equilibrium holdout. Moreover, a rise in complementarity leads to an increased likelihood of holdout, and an increased efficiency loss. Applications include patents, the land assembly problem, and mergers.

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1 Introduction

A significant class of economic activity requires that separately owned objects be combined. Drug development can require use of separate patents. A shopping mall developer may need to purchase multiple blocks of land. Firms often purchase the assets of other firms. A key common factor of these examples is that the production technology exhibits complementarity: The value of output exceeds the stand-alone value of the separate objects. The existence of complementarity suggests that an important strategic effect may be present. A seller who wishes to obtain a high value for its object might find it advantageous to delay negotiations with the buyer in the hope of receiving a share of a large residual surplus. We refer to the problem of sellers delaying negotiations as "the holdout problem". The purpose of our analysis is to model this problem, and investigate the effect of changes in complementarity on the tendency of sellers to holdout for the best deal.

There are several strands of literature that examine the problem of delay in related contexts. Fernandez and Glazer (1991) examine inefficiencies in bargaining to explain delay in labor negotiations. We assume that bargaining is efficient once parties begin negotiations. The possibility of holdout in our model comes from the ability of sellers to strategically delay entry into a bargain with the buyer.

The land assembly problem (for example, Eckart (1985), Asami (1988) and O'Flaherty (1994)), has some features that are common with our setup. In this literature, a developer wants to assemble several parcels of land. The project undertaken by the developer delivers positive externalities to surrounding land owners. When the developer cannot make a credible all-or-nothing offer, inefficiency can occur: Existing owners will wait in order to capture the external benefits resulting from completion of the project.[1] In our model, there are no external benefits to sellers who do not relinquish their land. The only potential gain from delay in our framework is the possibility of a large residual surplus. An analysis related to ours is Cai (2000), who shows that delay can occur when land owners are approached sequentially and in exogenously fixed order by another player who requires every plot of land in order to produce. In our model, the order in which sellers approach the buyer is endogenous, and production is possible with only one object. In any case, our main focus is on the comparative statics of changes in complementarity on the incentive to delay, and the resulting welfare effects, rather than the phenomena of delay itself.

The possibility of delay in models with asymmetric information has been examined by Fuden-

[1] Grossman and Hart argue that these externalities could be avoided if the developer could hide his intentions from the lot owners. There is, however, a large literature on land assembly. Recent papers eliminate the externality problem either by assuming that lot owners can make final offers above their reservation prices, as in Eckart (1985), or by taking a cooperative approach, as in Asami (1988). O'Flaherty (1994) studies urban renewal – when a public authority has the power to buy the lots and resell them to the developer – and shows that it is not a good remedy for the externality problem.

berg and Tirole (1983) (among others). Fudenberg, Levine, and Tirole (1985) survey the literature on these models. The incentive to delay is present in bargaining models with asymmetric information, because the offer and acceptance game reveals valuable information. Busch and Wen (1995) show that delay can occur in bargaining with perfect information if there are multiple equilibria in the disagreement game. We assume that information is symmetric. In addition, the incentive to delay in our model comes from production complementarity rather than the outcome of the disagreement game.

We develop an infinite horizon game, where in any period each of two sellers is able to avoid entering into negotiations with a sole buyer. The sellers simultaneously and non-cooperatively choose a probability of entry. Each seller may end up being the first to enter, or the last. Sellers may find they have entered simultaneously, or that neither has entered, in which case the entry game is repeated. We assume that any bargaining that occurs after entry, follows variants Rubinstein's (1982) alternating offers game. Our main results are centered on the effect of complementarity on the likelihood of holdout. We find that holdout—in the form of both pure and mixed strategy equilibria—always occurs when complementarity is sufficiently high (Proposition 3). Rises in complementarity lead to a rise in the symmetric mixed strategy equilibrium probability of holdout (Proposition 4). When a rise in complementarity is due to a fall in the stand-alone value of one of the objects, holdout happens in a larger measure of cases regardless of whether it occurs as a mixed or pure strategy equilibrium (see the discussion in section 2.2.2). Importantly, we demonstrate that when the objects become more complementary, the deadweight loss generated by delay increases (Proposition 5 and Corollary 6).

1.1 An Example

Some insight into our model can be gained using a simple two period game with fixed bargaining outcomes. While our model is different, some features can be captured. Sellers simultaneously and non-cooperatively choose the probability that they will enter into a bargain with the buyer immediately. With the complementary probability, they delay until the second period. Consider the following payoffs. Entry by both sellers leads to a trilateral bargain between two sellers and the buyer, with a payoff of u^t to each seller. Delay by one seller and entry by the other yields u^d to the delaying party, and u^e to the entering party. If both parties delay, the payoff is u^c each. (This outcome is intended to be a reduced form representation of the continuation payoff to the players from repeating the game in the true version of the model.) Seller 1's expected payoff is

$$EU_1 = p_1 p_2 \cdot u^t + p_1 (1 - p_2) \cdot u^e + (1 - p_1) p_2 \cdot u^d + (1 - p_1)(1 - p_2) \cdot u^c,$$

where p_1 is seller 1's choice of entry probability, and p_2 is seller 2's choice. Equilibria can be found by comparing the payoff to seller 1 from entering ($p_1 = 1$); $p_2 \cdot u^t + (1 - p_2) \cdot u^e$, with the payoff from delaying ($p_1 = 0$); $p_2 \cdot u^d + (1 - p_2) \cdot u^c$. The difference between these payoffs is

$$p_2[u^t - u^d] + (1 - p_2)\,[u^e - u^c]. \tag{1}$$

If $u^t > u^d$, and $u^e > u^c$, then both sellers will enter immediately. The bargain with three exceeds the payoff to a single party from delay, and the payoff from entering and being alone with the buyer is larger than the continuation payoff: Immediate entry is a dominant strategy. However, if $u^t < u^d$ and $u^e > u^c$, expression 1 can be used to find a mixed strategy (symmetric) equilibrium with $p_1 = p_2 = p < 1$ may be chosen. The sellers may choose to delay with some probability, with the hope of capturing u^d or u^e. An alternative equilibrium is where one of the sellers chooses to enter, and receives u^e, and the other delays and receives u^d. Note that for either type of equilibrium there is seller holdout.[2] In this example, an important condition is whether or not u^t –the payoff from the trilateral bargain – exceeds, or falls below u^d, the delay payoff. This feature is also important in the generic version of our model below. Of greater interest is our development of an extensive form bargaining game that allows us to find the difference between u^t and u^d *endogenously*; depending on primitives of the bargaining game. And an important primitive is the degree of complementarity between the sellers' objects.

2 Model

There are three players $i \in \{0, 1, 2\}$ in the game. Player 0 is the sole buyer, and players 1 and 2 are sellers of two distinct objects that can be combined by the buyer to produce an output. We let $x_i \in \{0, 1\}$, $i \in \{1, 2\}$ denote whether or not seller i's object has been purchased by the buyer. $x_i = 1$ indicates that the buyer has purchased seller i's object. $x_i = 0$ indicates that the buyer does not hold seller i's object. We think of the pair (x_1, x_2) as an input profile for the buyer's production process. The surplus generated by profile (x_1, x_2) is denoted $s\,(x_1, x_2)$. For example, if the buyer has not purchased seller 1's object, but has purchased seller 2's, then surplus $s\,(0, 1)$ is generated by production. Similarly $s\,(1, 1)$ is the surplus generated when both objects are owned by the buyer and used as inputs.[3] We will be working with the case where the technology is

[2] It is not obvious which type of equilibrium –mixed or pure–is more salient than the other. The pure strategy equilibrium could deliver a low payoff to the entering party, and a high payoff to the delaying party. Neither party would wish to be the entering party. If there is some external characteristic to indicate which party should enter, and which should delay, the pure strategy is more likely. Otherwise, the mixed strategy equilibrium is more likely. See Myerson (1991)

[3] The surpluses can be thought of as the present value of a stream of returns generated by the object. For example, if object 1 is purchased immediately, and object 2 is not, then $s\,(1, 0)$ is generated in present value terms.

symmetric, since this case is far less notationally cumbersome and little insight into the nature of the holdup problem is lost. Here, we denote $s(1,1) = S$, and $s(0,1) = s(1,0) = s$. We normalize the value each seller places on its object to zero.

Our goal is to analyze a situation where sellers can delay sale of their object in order to improve their bargaining position vis-a-vis the buyer. Therefore, we make the following basic assumption:

Entry Assumption *Sellers can avoid entering into a bargain if they choose.*

In particular, sellers are able to choose the probability with which they will enter into bargaining. This assumption is intended to capture the idea that the seller is able to make it difficult for the buyer to make contact, and so difficult to begin a bargaining process. We model entry to bargaining as an extensive form infinite horizon model as described below.

2.1 The Generic Entry Game

We call the entry game with arbitrary payoffs the 'generic' entry game, and derive results that obtain for particular configurations of its reduced-form payoffs. This approach has the advantage of generality. We can check whether these configurations are possible for *any* specific extensive form bargaining game we might develop, provided it generates stationary payoffs. In section 2.2, we examine an extensive form in which bargaining is based on variants of Rubinstein's (1982) alternating offers game.

Let $\tau = 0, 1, 2..., ...$ represent time. At date 0 the two sellers simultaneously and non-cooperatively choose the probability with which they will enter into a bargain. The probability that seller 1 enters at date 0 is denoted p_1, and the probability that seller 2 enters at date 0 is denoted p_2. The time subscript is suppressed, because we examine only stationary equilibria. Diagram 1 represents the date 0 entry game, and shows that there are four possible outcomes:

1. With probability $p_1 \cdot p_2$ both sellers enter and are committed to engage in a trilateral bargaining game with the buyer. Seller payoffs in this subgame are u^t, where t denotes that the bargaining game is trilateral. The subgame is called the 'trilateral' subgame.

2. With probability $p_1 \cdot (1 - p_2)$ seller 1 enters and seller 2 delays. Realization leads to the entry subgame. The payoff to seller 1 is denoted u^e, where e stands for 'enter', and the payoff to seller 2 is u^d, where d denotes 'delay'.

3. With probability $(1 - p_1) \cdot p_2$, seller 2 enters and seller 1 does not enter. Seller 2 receives u^e and seller 1 receives u^d in the delay subgame.

If object 2 is purchased in the subsequent period, this adds present value $\delta[s(1,1) - s(1,0)]$ to total surplus, which is discounted due to the delay. The total net present value is therefore $s(1,0) + \delta[s(1,1) - s(1,0)]$.

4. With probability $(1-p_1)\cdot(1-p_2)$ neither seller ends up entering into negotiations; they both delay. The payoffs in this subgame are explained below.

In the last case sellers find themselves at date 1, facing a similar entry decision to that of date 0. Each seller subsequently chooses probability p_i of entering at date 1, and faces the analogous four possible outcomes. Payoffs are discounted at rate $\delta \in [0,1]$ between any two periods. The expected payoff for a seller at the beginning of a period is denoted as EV_i, $i=1,2$. With this notation the payoff from outcome 4 above, where neither seller enters at date 0, is δEV_i the discounted expected payoff from the game.

Sellers' expected payoffs are defined recursively as probability-weighted sums of the payoffs that each subgame delivers. For seller 1 we have

$$EV_1 = p_1 p_2 u^t + p_1 (1-p_2) u^e + (1-p_1) p_2 u^d + (1-p_1)(1-p_2)\delta EV_1 \tag{2}$$

Player 2's expected payoff is the same, although with subscripts interchanged. These expressions allow us to find equilibria conditional on the payoffs u^t, u^e and u^d.

2.1.1 Equilibria in the Generic Entry Game

Mixed strategy equilibria are of particular interest in our model of holdout, because they allow us to find the comparative statics effects of changes in complementarity. Mixed strategy equilibria are found by equating players' payoffs conditional on not entering, with their payoffs conditional on entering. Substitution of $p_1 = 0$ into (2) yields $EV_{1|_{p_1=0}} = p_2 u^d + (1-p_2)\delta EV_{1|_{p_1=0}}$ which can be written as

$$EV_{1|_{p_1=0}} = \frac{p_2 u^d}{1-(1-p_2)\delta} \tag{3}$$

for $\delta < 1$. Substitution of $p_1 = 1$ into (2) gives

$$EV_{1|_{p_1=1}} = p_2 u^t + (1-p_2) u^e. \tag{4}$$

Symmetric equilibria are found by setting $p_2 = p$, and equating (3) and (4). This yields the following quadratic equation defining the symmetric equilibrium value of p:

$$f(p;\theta) \equiv \delta\left(u^e - u^t\right)\cdot p^2 + \left[(1-\delta)\left(u^e-u^t\right) + u^d - \delta u^e\right]\cdot p - (1-\delta) u^e = 0 \tag{5}$$

In a specific bargaining game that generates stationary payoffs, the returns u^t, u^e and u^d will depend on the the parameters S, s and δ. Thus (with a slight abuse of notation) θ is used to represent one of the parameters S, s or δ. The quadratic form of the equation defining mixed strategy equilibria is fortunate: We can take advantage of the relationship between the coefficients of p^2 and p and the intercept term, to determine existence and deduce properties of these equilibria.

The results below characterize equilibria in the generic game. Define a 'hybrid' equilibrium as an equilibrium where one seller chooses $p_i \in \{0,1\}$, and the other chooses some $p_{-i} \in (0,1)$. The existence and type of pure strategy equilibria depend directly on the comparison between the trilateral and delay payoffs.

Lemma 1 .

(i) *Suppose* $u^t > u^d$, $u^e > 0$ *and* $\delta < 1$.

- **(a)** *The choice* $(p_1, p_2) = (1,1)$ *is the unique* pure strategy *equilibrium* [4] *, and no hybrid equilibria exist.*
- **(b)** *If* $u^d < \min[u^t, u^e]$*, a* mixed strategy *equilibrium does not exist.*
- **(c)** *If* $u^t > u^d > u^e$*, any* mixed strategy *equilibria that may exist are Pareto dominated by the pure strategy equilibrium* $(1,1)$.

(ii) *Suppose* $u^t < u^d$, $u^e > 0$ *and* $\delta < 1$.

- **(a)** *The choice* $(p_1, p_2) \in \{(1,0), (0,1)\}$ *is the unique set of* pure strategy *equilibria, and no hybrid equilibria exist.*
- **(b)** *A unique symmetric* mixed strategy *equilibrium* p^* *exists and satisfies* $0 < p^* < 1$*, and* $f'(p;\theta) > 0$ *locally.*

Proof. For the pure strategies, see the appendix section 4.0.3. For the mixed strategies, see the appendix section 4.0.4. ■

The advantage of working with a reduced form becomes immediately apparent when considering these results. Equilibria are characterized for any extensive form that has stationary payoffs and satisfies the conditions of the Lemma. For (i), the trilateral payoff exceeds the delay payoff. Both parties would therefore prefer to bargain at the same time than to delay. The lemma indicates that this can happen with pure strategy equilibria. In (a), $(1,1)$ is an equilibrium because if one party deviates, it receives the lower payoff u^d. Mixed strategies are also possible, however in (b) $u^d < \min[u^t, u^e]$, i.e. the delay payoff is lower than any possible payoff from entry, so that symmetric mixed strategy equilibria do not exist. For (c), mixed strategy equilibria are possible because if one seller believes that the other one will delay with some probability, it may have an incentive to delay to coordinate with the other in order to capture the higher trilateral payoff $u^t > u^d > u^e$.

For (ii), the trilateral payoff is lower than the delay payoff. Both sellers therefore have an incentive to avoid being in a bargain when the other is present. One way to achieve this is for the sellers to coordinate to enter at separate times. If one party enters immediately and the other

[4] *We have been somewhat loose in our description of equilibrium in these results. Referring to* $(\hat{p}_1, \hat{p}_2)$ *as an equilibrium means that in any period* τ *in which neither player has entered, the players choose* $(p_{1\tau}, p_{2\tau}) = (\hat{p}_1, \hat{p}_2)$*. For a period* τ *in which the realization has one player entering and the other not entering, it is understood that the other player enters with probability 1 in period* $\tau + 1$ *after the bargain between the buyer and the first player.*

delays, the entering party receives $u^e > 0$, and the delaying party gets u^d. Deviation by the entering party leads to another round of play, and both parties receive the discounted value of the payoff they could have earned earlier.[5] Deviation by the delaying party leads to the lower trilateral payoff u^t. The other way for sellers to avoid bargaining together is to use a mixed strategy, as in part (b). The condition that $f'(p;\theta) > 0$ locally is useful for finding comparative statics effects.

The Lemma above makes a key prediction about the nature of equilibria: the comparison between trilateral and delay payoffs is crucial. Our goal is not to emphasize this result, but rather use it for a specific extensive form. We are interested in whether extensive form bargaining games can deliver holdout as an *endogenous* phenomena, in order to find the effect of changes in model parameters on the degree of holdout and on welfare. Specifically, the main aim of the paper is to find the effects of changes in the degree of complementarity. To this end we adopt the usual definition that a technology is complementary if the surplus from both objects exceeds the sum of the 'stand alone' surpluses, i.e. $S > 2s$. Complementarity rises if S rises or if s falls.

Questions about complementarity can be answered by analyzing the effect a change in a parameter $\theta \in \{S, s\}$ on $f(p;\theta)$ when there is holdout, i.e. when $u^d > u^t$ holds, as in lemma 1 part (ii)(b). We have shown that under these conditions, $f'(p;\theta) > 0$ in the region near the equilibrium. This is represented in diagram 2. Suppose a small change in θ to some other value θ' leads to a shift upwards in f (i.e. $f(p;\theta') > f(p;\theta)$), and suppose that the conditions in lemma 1 part (ii)(b) are satisfied at both θ and θ' so that f is upwards sloping. Then p falls, and the probability of holdout rises, as the diagram indicates. Differentiation of f with respect to θ allows us to make a general statement about the relationship between holdout and changes in stationary payoffs:

Lemma 2 *Suppose that $u^d > u^t$ at some value of S, s and δ, and players adopt mixed strategies. The probability of holdout rises if*

$$\frac{df(p;\theta)}{d\theta} = -(1-p)(1-\delta+\delta p)\frac{\partial u^e}{\partial \theta} - p(1-\delta+\delta p)\frac{\partial u^t}{\partial \theta} + p\frac{\partial u^d}{\partial \theta} > 0. \tag{6}$$

Proof. Differentiation of (5) with respect to θ yields the result. ■

As an example, players are enticed to delay when the delay payoff rises relative to the trilateral and entry payoffs. This happens if $\frac{\partial u^d}{\partial \theta} > 0$,$\frac{\partial u^t}{\partial \theta} < 0$, and $\frac{\partial u^e}{\partial \theta} < 0$ in (6). Our goal is to use this lemma to make more precise statements about the effect of complementarity in the specific bargaining game in section 2.2 below, where u^d, u^t and u^e depend directly on S and s.

Key results in the generic game can be summarized as follows:

[5] Recall that the seller who holds out does not enter until bargaining ends (in the next period). If the entering party were to deviate and delay too, then the entry game begins anew.

Summary 1 *Holdout occurs as a pure strategy equilibrium or as a mixed strategy symmetric equilibrium if $\delta < 1$, the trilateral payoff falls below the delay payoff, and the entry payoff is positive. The probability of holdout increases with a change in an exogenous parameter if the delay payoff increases sufficiently relative to trilateral and entry payoffs.*

This summary, and the existence of both pure and mixed strategy equilibria gives rise to the question of which of these equilibria will obtain. The literature on salience of equilibria can give some insights. Schelling (1960) pointed out that, in a game with multiple equilibria, anything that tends to focus the players' attention on one equilibrium may cause all players to expect it, and hence this equilibrium outcome might obtain. Schelling referred to this as the focal-point effect. In this context, Myerson (1991, p. 109) argued that with respect to the Battle of the Sexes game, if society was such that women had to defer to their husbands for any decision, then the particular equilibrium where the husband gets his preferred outcome becomes focal and hence more likely to be implemented. Another way that players could become focused on a particular equilibrium is by some process of preplay communication (see, for example, Farrel (1988) and Myerson (1989).

From this literature, we can say that which equilibrium is more likely to be chosen depends on the specific application, and whether there are salient features of the identity of each sellers that allow them to coordinate in that particular application. If one player has a characteristic that somehow gives it seniority or dominance over the other, then we might predict the asymmetric equilibria, with the dominant player choosing the strategy that admits the highest payoff. If such characteristics are absent, and players are homogenous, salience considerations would lead us to predict the mixed strategy equilibrium.

2.2 The Extensive Form Bargaining Game

We use variants of Rubinstein's 1982 game to model bargaining in each subgame, because it is a well known and well understood protocol. Diagram 3 represents the extensive form. In the trilateral subgame (box A) bargaining follows the variant analyzed by Shaked and Sutton (1984): As in Cai (2000), and the general literature on extensive form bargaining games, we assume that strategies are restricted to immediate cash offers. One player makes an offer, and the other players decide sequentially whether to accept or reject the offer. Shaked demonstrates that there is a unique *stationary* subgame perfect equilibrium.[6] We make a small modification to his game, by assuming that the order of first offer is random, and that each player faces a probability $\frac{1}{3}$ of being the offerer. This removes asymmetries and substantially simplifies the analysis. The players

[6] The stationary equilibrium payoffs for a pie of size 1 are

$$\left(\frac{1}{1+\delta+\delta^2}, \frac{\delta}{1+\delta+\delta^2}, \frac{\delta^2}{1+\delta+\delta^2}\right).$$

bargain over the surplus S, making the stationary equilibrium payoff in the trilateral subgame

$$u^t = \frac{S}{3}.$$

Now suppose seller 1 enters at date 0, and seller 2 does not; the 'entry' subgame as depicted in box B in diagram 3. We assume that the buyer and seller 1 engage in Rubinstein bargaining, with each player having a 50% probability of being first offerer. To find a perfect equilibrium in the entry subgame, we first solve the delay subgame (represented by box C), and use backwards induction.

The delay subgame proceeds as follows. Seller 2 enters once the buyer and seller 1 have concluded their bargain.[7] Then seller 2 and the buyer engage in a Rubinstein bargaining game with equal chance of a player being first offerer. The largest available surplus at this point is $S - s$, i.e. the amount that seller 2's object adds to the value of the buyer's production. The bargain yields the buyer and seller 2 half of this surplus each in expectation, and in date 0 value terms we have

$$u^d = \frac{1}{2}\delta(S - s).$$

We can now determine subgame-perfect equilibrium payoffs in the entry subgame (box B) by backwards induction: we simply calculate the maximum surplus available, and note that players receive half of this each in expected value. The maximum surplus is the sum of the surpluses available, to the buyer–seller 1 pair, currently and in the future. At date 0, Seller 1's object adds value s. At date 1 the buyer subsequently earns $\frac{1}{2} \cdot \delta \,(S - s)$ in its dealings with seller 2. Therefore the total surplus over which the buyer and seller 1 bargain at date 0 is $s + \frac{1}{2} \cdot \delta \cdot (S - s)$. This implies an entry payoff

$$u^e = \frac{1}{2}\left[s + \frac{1}{2} \cdot \delta \cdot (S - s)\right].$$

The subgame where player 2 enters first and player 1 delays, is depicted in boxes D and E. It is directly analogous to the immediately preceding discussion. We present proofs, and a more detailed discussion of the structure of the bargaining game in the appendix section 4.0.5.[8]

[7] Seller 2 is assumed not to be able to enter bargaining until the buyer and seller 1 have concluded their negotiations. We discuss alternative assumptions in the appendix. Specifically, we discuss a game where the seller who delays is able to enter after one round of bargaining, and explain why it yields similar results to the present game.

[8] Note that we have only presented sellers' payoffs. This is partly because it is only the sellers who are able to choose whether or not to enter negotiations each period, and the focus of the paper is on these choices. In any case, it is immediate that the buyer makes a positive surplus in the game, because it earns a share of a positive total surplus in each subgame. Therefore, the buyer will always be willing to participate.The buyer makes $\frac{S}{3} > 0$ in the trilateral subgame; $\frac{1}{4}\left(\delta S + (2 - \delta)\, s\right) > 0$ in the subgame where 1 enters first; and $\frac{1}{4}\left(\delta S + (2 - \delta)\, s\right) > 0$ in the subgame where 2 enters first.

2.2.1 Results

Results from the generic game can be applied directly to the payoffs derived above. The inequalities $u^t \lesseqgtr u^d$ give rise to some of regions represented in diagram 4, and to results that link the values of primitives to the question of holdout:

Proposition 3 .

(i) *All equilibria in region A (i.e.* $\left\{(S, s, \delta) : S > \frac{3\delta}{3\delta-2}s; 1 > \delta > \frac{2}{3}\right\}$ *) involve delay: There exist pure strategy equilibria* $(p_1, p_2) \in \{(1, 0), (0, 1)\}$*, and symmetric mixed strategy equilibria* $p_1 = p_2 = p < 1$*.*

(ii) *In region* $B \cup C$ *(i.e.* $\left\{(S, s, \delta) : 0 \leq S < \frac{3\delta}{3\delta-2}s\right\}$*) there exist pure strategy equilibria with no delay, i.e.* $(p_1, p_2) = (1, 1)$*. Mixed strategy equilibria with delay may exist, but these are Pareto dominated by* $(1, 1)$*.*

Proof. Application of Lemma 1. See the appendix at the bottom of section 4.0.6. ■

Intuition is aided by diagram 4, which represents pertinent values of S and δ for some fixed positive value of s. Consider part (i). The curved border of Region A is defined by $S = \frac{3\delta}{3\delta-2}s$, and $\delta > \frac{2}{3}$. Holdout occurs because the delay payoff $u^d = \frac{1}{2}\delta(S - s)$ exceeds the trilateral payoff $u^t = \frac{S}{3}$ in this region. Players' incentive to delay is motivated by the potential to capture a larger fraction of a total surplus S that significantly exceeds the surplus s generated by a single object. Region A is characterized by a high discount factor, so that players do not downgrade the potential gain $\frac{1}{2}\delta (S - s)$enough to remove the incentive to delay.

In regions B and C, pure strategy equilibria with no delay exist. The union of these regions is characterized by the delay payoff $u^d = \frac{1}{2}\delta(S - s)$ falling below the trilateral $u^t = \frac{S}{3}$. The region ensures that combinations of discount factor and the residual surplus $S - s$ from delay are sufficiently low that a player will not deviate from immediate entry if the other player chooses to enter. In the appendix, results are further refined: We show that in region C, $(1, 1)$ is the unique equilibrium. This region is the union of two cases: $u^t > \max\{u^d, u^e\}$ and $u^d < u^t < u^e$. In these regions, $S - s$ is sufficiently small that play other than entry is strictly dominated. In region B, $u^e < u^d < u^t$, admitting moderate values of the residual surplus $S - s$. Players may choose to delay if they believe the other delays with some probability, in order to capture a share of S rather than a share of a modest residual surplus. As discussed previously, these mixed strategy equilibria are of less interest as they are Pareto dominated by $(1, 1)$.

2.2.2 Complementarity and Holdout

The important characteristic of region A is that there is holdout in equilibrium. Players do not sell their objects immediately, and this leads to inefficiency because the buyer discounts the future.

Note that if there is (Pareto undominated) holdout in equilibrium (i.e. if we are in Region A), then $S > 3s$, so the technology is certainly complementary. In other words, complementarity is a necessary condition for holdout.

There is another immediate reason why complementarity is important. In addition to equilibrium delay in Region A, the set of cases where holdout occurs—i.e. the size of Region A relative to the other regions—is increasing as the production technology becomes more complementary in the following way: If s declines *ceteris paribus*, then the objects become more complementary in production for any $S > 2s$. We would expect a rise in complementarity through a fall in s to make delay more attractive, because the potential gain from delay rises. By inspection, it is clear that the boundary $S = \frac{3\delta}{3\delta-2}s$ of A becomes flatter, and the bottom of the region $S = 3s$ also falls as s declines. The sum of the areas of Regions B and C decline, and the area of Region A increases. Therefore, even in the situation where for some salience reason pure strategy asymmetric equilibria are predicted, a rise in complementarity means there is a larger measure of cases where delay occurs.[9]

Note that $\delta < 1$ implies there are efficiency consequences of pure strategy equilibrium holdout. The surplus S from combining both objects cannot be attained in period 0. In the pure strategy equilibrium with delay, the surplus s is realized at date 0, and the remainder $S - s$ is realized at date 1. Social welfare is therefore $W = s + \delta (S - s)$. Defining the degree of complementarity in production as $S - 2s$, note that a rise in complementarity through a rise in S increases welfare. However, a rise in complementarity through a fall in s reduces welfare. The situation is different when we considered the impact of these changes on the deadweight loss from holdout. Efficiency loss is the difference between the first-best level of welfare, S, and the second-best W, i.e.

$$DWL \equiv S - W = (1 - \delta)(S - s).$$

We conclude that the deadweight loss increases with a rise in complementarity regardless of the source.

Since pure strategy equilibria do not necessarily obtain, depending on salience issues, important questions remain concerning mixed strategy equilibria. First, what is the effect of an increase in complementarity on the mixed strategy equilibrium probability of holdout? Second, what is the effect on welfare of a rise in complementarity, whether it be through a rise in S or a fall in s? The proposition below answers the first of these questions.

[9] It is also obvious that even if players switch between pure and mixed strategies as s changes, one is able to make the statement that an increase in complementarity will always eventually lead to more holdout.

Proposition 4 *For the region A (i.e.* $\left\{(S,s,\delta) : S > \frac{3\delta}{3\delta-2}s; 1 > \delta > \frac{2}{3}\right\}$*) a rise in complementarity through either an increase in S or a decrease in s leads to a rise in the probability of holdout in the symmetric mixed strategy equilibrium.*

Proof. See the appendix, section 4.0.7. ■

The region defined in the proposition has $u^t - u^d = \frac{S}{3} - \frac{\delta}{2}(S-s) < 0$. Recall the discussion of comparative statics in the generic game in lemma 1 If s falls then $u^d = \frac{\delta}{2}(S-s)$ rises, increasing the temptation to delay. Alternatively, when S rises, both $u^t = \frac{S}{3}$ and $u^d = \frac{\delta}{2}(S-s)$ increase: When $\delta > \frac{2}{3}$, the rise in u^d exceeds the rise in u^t, also increasing the temptation to delay.

Rises in the tendency of sellers to hold-out with complementarity, begs the question of whether there is a negative impact on welfare of this behavior. To examine this question, note that social welfare is defined recursively by

$$EW(p;\theta) = p^2 S + (1-p)p \cdot 2(s + \delta(S-s)) + (1-p)^2 \delta EW(p;\theta). \tag{7}$$

That is, with probability p^2 both sellers are present at date 0, and the full surplus S is realized. With probability $(1-p)p$ the surplus from delay, i.e. $2[s + \delta(S-s)]$, obtains. And with probability $(1-p)^2$, players move to the next round and receive the discounted expected welfare $\delta EW(p;\theta)$. A change in one of the parameters (S or s) affects EW directly, and also indirectly through a change in the equilibrium probability p. As well as the effect on expected welfare, we are interested in the impact of parameter changes on the *deadweight loss* from delay. Since S is the first-best surplus, the deadweight loss is defined as

$$S - EW(p;\theta) \tag{8}$$

The following proposition summarizes the welfare impact of parameter changes.

Proposition 5 *Suppose that p is increasing in s, decreasing in S, and that* $S > 2s$. *A rise in complementarity through a fall in s leads to a fall in expected welfare, and increased deadweight loss. A rise in complementarity through a rise in S has an indeterminate impact on expected welfare, but increases the deadweight loss.*

Proof. See the appendix section 4.0.4. ■

The following is a rough intuition of these results. Suppose that s falls, and consider equations (7) and (8). The term $s + \delta(S-s)$ in (7) is clearly lower. Since p falls by assumption, there is a decrease in the probability that the full surplus S is realized, i.e. a decrease in p^2. The probability of the other events is $1-p^2$, which clearly rises. However, the payoffs in these other events involve delay, and clearly fall below S. Moreover, the payoffs in the events with delay are lower because s has fallen. With a higher probability of lower payoffs, expected welfare must decline. The welfare loss (8) clearly also increases.

The effect of an rise in complementarity through an increase in S is different. Even if a rise in S induces a lower value of p, there is a larger surplus to be gained in the event that both players enter immediately. A fall in p might be offset by a rise in S through the term p^2S in (7), depending of the magnitude of the changes in p and S. The effect on expected welfare is therefore indeterminate. However, efficiency loss increases because the probability of the event where the now higher S is obtained is less than unity.

We can now combine the results on welfare, with the results on holdout above:

Corollary 6 *For region A (i.e.$\left\{(S, s, \delta) : S > \frac{3\delta}{3\delta-2}s; 1 > \delta > \frac{2}{3}\right\}$) a rise in complementarity through either an increase in S or a decrease in s leads to higher deadweight loss. A rise in complementarity through a decrease in s leads to a fall in expected welfare.*

Proof. Direct from Propositions 4 and 5. ■

3 Conclusion

We construct a model in which sellers are able to delay their entry into bargaining with a single buyer. General results can be deduced with a model that considers arbitrary payoffs for each outcome of the entry game. With stationary equilibria, we demonstrate that holdout occurs if the payoff from trilateral bargaining falls below the payoff from delay. Otherwise, holdout cannot occur as a Pareto undominated equilibrium. This result holds true for any extensive form bargaining game that satisfies this condition, and which is able to yield stationary symmetric payoffs. To show that the conditions for holdout can emerge endogenously, we used bargaining protocols based on variants of Rubinstein's classic model. The trilateral payoff falls below the delay payoff if, *ceterus-parabus*, the surplus generated from use of both objects is sufficiently large, and the discount factor is sufficiently high. Players delay because of an incentive to capture a large residual surplus.

The specific bargaining game allows us to answer other key motivating questions of the paper, i.e. the impact that changes in complementarity has on the likelihood that sellers will hold out. Our model makes strong and robust predictions about the impact of complementarity. We show that holdout cannot occur without complementarity (it is a necessary condition). An increase in complementarity through a fall in the surplus generated by a single object, leads to a rise in the measure of cases where holdout occurs, whether it be through pure or mixed strategy equilibria. Welfare also falls for either type of equilibrium in this case. Regardless of whether holdout occurs through pure or mixed strategies, a rise in complementarity, whatever its source, increases deadweight loss.

Our analysis shows that holdout may be important for a multitude of situations where objects held by two separate sellers need to be combined. Applications include the problem of combining patents, the land assembly problem, and merger decisions, among many others.

4 Appendix

4.0.3 Pure Strategy Equilibria in the Generic Game

Proof of Lemma 1 (pure strategies) parts (i)(a) and (ii)(a). Proof. For (i), first let $p_2 = 1$. Substitution into (2), yields

$$EV_1 = p_1 u^t + (1 - p_1) u^d, \tag{9}$$

which is maximized for $p_1 = 1$ as $u^t > u^d$ (ruling out $(1, \gamma)$, $0 \leq \gamma < 1$ as an equilibrium). The same argument holds for player 2. Therefore $(1, 1)$ is an equilibrium.

To rule out other pure strategy and hybrid equilibria, suppose instead that $p_2 = 0$. Then

$$EV_1 = p_1 u^e + (1 - p_1) \delta EV_1 \tag{10}$$

from (2). A choice of $p_1 = 1$ maximizes EV_1 since from (10), $EV_1 = \frac{p_1 u^e}{1-\delta+\delta p_1}$, and $\frac{dEV_1}{dp_1} \propto u^e (1 - \delta) > 0$ for $u^e > 0$ and $\delta < 1$. This also rules out any $(\gamma, 0)$, $0 \leq \gamma < 1$ as an equilibrium. However, we have already shown that $(1, 0)$ cannot be an equilibrium. Therefore, $(1, 1)$ is unique in the set of hybrid or pure strategy equilibria.

For (ii), consider 2's choice for $p_1 = 1$. $EV_2 = p_2 u^t + (1 - p_2) u^d$, which is maximized at $p_2 = 0$ since $u^d > u^t$. This also rules out equilibria of the form $(1, \gamma)$, $0 < \gamma \leq 1$. Thus, $(1, 0)$ is an equilibrium, and so is $(0, 1)$ by a symmetric argument.

To rule out $(0, 0)$, suppose $p_1 = 0$, so that

$$EV_2 = p_2 u^e + (1 - p_2) \delta EV_2, \tag{11}$$

and consider 2's choice. Similarly to above, EV_2 is maximized at $p_2 = 1$: $EV_2 = \frac{p_2 u^e}{1-\delta+\delta p_2}$, and $\frac{dEV_2}{dp_2} \propto u^e (1 - \delta) > 0$ for $u^e > 0$ and $\delta < 1$. This also rules out equilibria of the form $(0, \gamma)$, $0 \leq \gamma < 1$. ■

4.0.4 Mixed Strategy Equilibria in the Generic Game

Below we derive all the (non-trivial, i.e. positive measure) conditions for mixed strategy equilibria in the generic game. The equation defining mixed strategy equilibria was derived in the text as

$$f(p) \equiv \delta \left(u^e - u^t\right) \cdot p^2 + \left[(1 - \delta) \left(u^e - u^t\right) + u^d - \delta u^e\right] \cdot p - (1 - \delta) u^e = 0. \tag{12}$$

To evaluate the roots of this quadratic, use the following facts:

(a) $f(0) = -(1 - \delta) u^e$

(b) $f(1) = u^d - u^t$

(c) $f'(p) = 2\delta \left(u^e - u^d\right) p + (1 - \delta)(u^e - u^t) + u^d - \delta u^e$; $f'(\hat{p}) = 0$ at $\hat{p} = -\frac{(1-\delta)\left(u^e - u^t\right) + u^d - \delta u^e}{2\delta(u^e - u^d)}$, and $f(\hat{p}) = -\frac{\left[(1-\delta)\left(u^e - u^t\right) + u^d - \delta u^e\right]^2}{4\delta(u^e - u^d)} - (1 - \delta) u^e$

(d) $f''(p) = \delta (u^e - u^t)$

We consider the following four regions, which exhaust the set of possibilities for non-trivial cases:

(I) $u^e - u^t > 0$ and $u^d - u^t > 0$

(II) $u^e - u^t > 0$ and $u^d - u^t < 0$

(III) $u^e - u^t < 0$ and $u^d - u^t > 0$

(IV) $u^e - u^t < 0$ and $u^d - u^t < 0$

Case I: $u^e - u^t > 0$ **and** $u^d - u^t > 0$ The quadratic is convex by substitution of $u^e - u^t > 0$ in (4.0.4), and has one positive root because it is convex and $f(0) = -(1-\delta) u^e < 0$. This root is less than unity because $f(1) = u^d - u^t > 0$. Moreover, $f'(p) > 0$ locally at this root.

Case II: $u^e - u^t > 0$ **and** $u^d - u^t < 0$ There is no mixed strategy equilibrium in this case, since $f(p)$ is convex, $f(0) = -(1-\delta) u^e < 0$, and $f(1) = u^d - u^t < 0$, so that the only positive root is greater than unity.

Case III: $u^e - u^t < 0$ **and** $u^d - u^t > 0$ $f(p)$ is concave by substitution of $u^e - u^t < 0$ in (4.0.4). It has two positive roots because it is concave and $f(0) = -(1-\delta) u^e < 0$. Since $f(1) = u^d - u^t > 0$, the larger root is greater than unity and is therefore not feasible, and the smaller root is less than unity. Thus, there is one feasible root. Moreover, $f'(p) > 0$ locally at the feasible root.

Case IV: $u^e - u^t < 0$ **and** $u^d - u^t < 0$ Subcase (a): $u^e - u^d > 0$. $f(p)$ has no real roots if $u^e - u^d > 0$, since the it is concave by substitution of $u^e - u^t < 0$ in (4.0.4), and the maximum is $f(\hat{p}) < 0$ by substitution of $u^e - u^d > 0$ in (4.0.4).

Subcase (b): $u^e - u^d < 0$. $f(p)$ is concave and $\hat{p} > 0$ by (4.0.4). There are two possibilities. Either there are two roots or none, depending on the parameters. In this case $u^e < u^d < u^t$, so that any mixed strategy equilibria are Pareto dominated by the pure strategy equilibrium $(1,1)$ which exists by lemma 1.

Proof of Lemma 1 (mixed strategies) part (i)(b).

Proof. To prove that $u^d < \min[u^t, u^e]$ implies non-existence of mixed strategy equilibrium. First note that it is a combination of cases II and IV(a), in which non-existence was demonstrated. For II, it was demonstrated for $u^d < u^t < u^e$, and for IV(a), for $u^d < u^e < u^t$ The union of these cases is $u^d < \min[u^t, u^e]$.

Alternatively, a direct proof is more informative: Player 1's payoff from certain delay $p_1 = 0$, given that player 2 chooses p_2 is $EV_{1|_{p_1=0}} = p_2 u^d + (1-p_2)\delta EV_{1|_{p_1=0}}$, which yields

$$EV_{1|_{p_1=0}} = \frac{p_2 u^d}{1-(1-p_2)\delta}.$$

This payoff is maximized[10] when $p_2 = 1$, and attains the value u^d. However, the payoff from certain entry $p_1 = 1$, is $EV_{1|_{p_1=1}} = p_2 u^t + (1-p_2) u^e > u^d$ as $u^d < \min[u^t, u^e]$. This rules out interior solutions. ■

Proof of Lemma 1 (mixed strategies) part (i)(c).

Proof. The case $u^t > u^d > u^e$ is covered above by case IV(b). ■

Proof of Lemma 1 (mixed strategies) part (ii)(b)

Proof. The lemma combines cases I and III, in which mixed strategy equilibria are shown to exist with $f'(p) > 0$ locally. Case I has $u^e - u^t > 0$ and $u^d - u^t > 0$, and case III has $u^e - u^t < 0$ and $u^d - u^t > 0$. The union of these conditions is the condition of the lemma $u^d - u^t > 0$. ■

Welfare in the Generic Game with Mixed Strategy Equilibria Expected welfare is given by

$$EW(p;\theta) = p^2 S + (1-p)p \cdot 2(s + \delta(S-s)) + (1-p)^2 \delta EW(p;\theta).$$

Differentiation with respect to s yields

$$\frac{dEW}{ds} \propto \frac{dp}{ds}(1-p)\delta(S-EW) + p\frac{dp}{ds}(1-\delta)(S-2s) + (1-\delta)p(1-p),$$

[10] Since $\frac{d}{dp_2}[\frac{p_2}{1-\delta+\delta p_2}] = \frac{1-\delta}{(1-\delta+\delta p_2)^2} > 0$.

which, if $\frac{dp}{ds} > 0$, is clearly positive under complementarity, i.e. if $S - 2s > 0$, since $S \geq EW$. A fall in s (and hence a rise in complementarity $(S - 2s)$) leads to and a fall in EW and a rise in deadweight loss $S - EW$.

Differentiation of EW with respect to S gives

$$\frac{dEW}{dS} \propto \frac{dp}{dS}(1-p)\,\delta\,(S - EW) + p\frac{dp}{dS}(1-\delta)(S-2s) + (1-\delta)\frac{dp}{dS}s + \left[\frac{1}{2}p^2 + \delta p(1-p)\right].$$

If $\frac{dp}{dS} < 0$, the first three terms are negative. However, the last term is positive and independent of S and s, so the sign of $\frac{dEW}{dS}$ is indeterminate. The derivative of the deadweight loss with respect to S is

$$1 - \frac{dEW}{dS} = -2p\frac{dp}{dS}(1-\delta)[S-2s] - \delta(1-p)\frac{dp}{dS}S - 2(1-\delta)\frac{dp}{dS}s$$
$$- \delta(1-p)\frac{dp}{dS}[S - EW] + (1-\delta)\left(1-p^2\right).$$

If $\frac{dp}{dS} < 0$, then this derivative is positive.

4.0.5 Derivation of Payoffs in the Specific Bargaining Game

Here we consider each subgame of our specific bargaining game in turn. The most straightforward is the trilateral subgame. We adopt the variant of Rubinstein bargaining analyzed by Shaked and Sutton, where one player makes an offer, and the other players accept or reject the offer in turn. If both accept, then the proposal is implemented. If not then another round occurs with a different player making an offer. A pie of size one is assumed. Shaked proves that there exists a stationary subgame perfect equilibrium with payoffs

$$\frac{\delta^{k-1}}{1+\delta+\delta^2},\ k = 1, 2, 3,$$

if the order of offer is 1, 2 and then 3. To avoid unnecessary complexity in the model, we assume that the order of offers is random: Specifically, the probability of every combination of orders of the three players (there are six possibilities) is $\frac{1}{6}$. This leads to an expected payoff of

$$\frac{2}{6}\cdot\frac{1}{1+\delta+\delta^2} + \frac{2}{6}\cdot\frac{\delta}{1+\delta+\delta^2} + \frac{2}{6}\cdot\frac{\delta^2}{1+\delta+\delta^2} = \frac{1}{3}.$$

The total surplus in our game is S, leading to $u^t = \frac{S}{3}$.

Consider diagram 3, boxes B and C. Box B represents entry by seller 1, and subsequent bargaining with the buyer. Box C represents the bargain between seller 2 and the buyer, which occurs after the seller 1—buyer bargain. Note that we make the assumption that seller 2 can *only* enter after the previous bargain has concluded. Alternatively, we could assume (for example) that seller 2 is able to enter into a bargain directly after only one round of bargaining has been completed between seller 1 and the buyer. We found that the model with this alternative assumption was complicated, without adding significantly to the analysis: delay is still possible, and it is related to complementarity: The alternative assumption of one round entry admits an outcome where seller 2 will never wish to enter if bargaining between the others has not been concluded, because the trilateral payoff that seller 2's entry induces falls below the delay payoff seller 2 otherwise receives.

The delay subgame is the labeled oval in diagram 5. We assume that there is a probability of $\frac{1}{2}$ of seller 2 or the buyer being the first offerer. However, the diagram depicts the case where seller 2 makes the first offer, and we note that if the buyer is first-offerer, the game is symmetric. After nature has made its move, seller 2 always decides to enter, because by not entering the best it can do is obtain the discounted payoff from entering. The total available surplus to be split between the players is $S - s$, since the buyer already holds one of the objects. We assume that players engage in a standard Rubinstein alternating offers bargain. There is immediate agreement and a unique subgame perfect equilibrium. This yields date 0 present value payoffs of $\delta\left[\frac{1}{1+\delta}(S-s)\right]$ to seller 2, and $\delta\left[\frac{\delta}{1+\delta}(S-s)\right]$ to the buyer. With probability $\frac{1}{2}$ of being first offerer, the payoff to each the buyer and seller 1 in expectation is

$$u^d = \frac{1}{2}\delta(S-s).$$

Now consider the entry subgame depicted in diagram 6. We have replaced the delay subgame with its payoffs. Suppose, as in the diagram, that nature selects 1 to be first offerer. The total pie over which 1 and the buyer bargain is the sum of the surplus obtained from 1's object alone, i.e. s, and the payoff that the buyer subsequently receives from its bargain with 2, i.e. $\frac{1}{2}\delta(S-s)$. With a total pie of size $s+\frac{1}{2}\delta(S-s)$, the players engage in a standard Rubinstein bargaining game. Player 1, being the first offerer, receives $\frac{1}{1+\delta}[s+\frac{1}{2}\delta(S-s)]$. The buyer receives $\frac{1}{1+\delta}[s+\frac{1}{2}\delta(S-s)]$. Since nature selects first offerer at the beginning of the game with probability $\frac{1}{2}$, the expected payoff obtained by buyer and seller 2 is

$$\frac{1}{2}\left[s+\frac{1}{2}\delta(S-s)\right].$$

4.0.6 Mixed Strategy Equilibria in the Specific Bargaining Game

Derivation of diagram 4. The bargaining game delivers the following payoffs

$$u^t = \frac{S}{3}, \tag{13}$$

$$u^e = \frac{1}{4}(\delta S + (2-\delta)s), \tag{14}$$

and

$$u^d = \frac{\delta}{2}(S-s). \tag{15}$$

This yields

$$u^d - u^t \propto S\cdot\frac{3\delta-2}{3\delta} - s, \tag{16}$$

$$u^e - u^t \propto S\cdot\frac{3\delta-4}{6-3\delta} + s, \tag{17}$$

and

$$u^e - u^d = -S\frac{\delta}{2} + s. \tag{18}$$

First consider case I. Here, $u^d - u^t > 0$ and $u^e - u^t > 0$ are required. From (16), the former inequality implies that $\delta > \frac{2}{3}$ and admits $S > 3s$ as a lower bound on the smallest possible S consistent with the inequality. From (17), the latter inequality implies for $\delta > \frac{2}{3}$ that $S < 2s$, which is inconsistent with (16). Thus, the specific bargaining game does not yield a solution of the type in case I.

Consider case II, $u^e - u^t > 0$ and $u^d - u^t < 0$ where no symmetric mixed strategy equilibria exist. Using lemma 1, $(1,1)$ is the unique equilibrium. From (17), the first inequality defines a region $\left\{(S,\delta): 0 \le S < \frac{6-3\delta}{4-3\delta}s\right\}$. From 16, the second inequality is satisfied for $S < \frac{3\delta}{3\delta-2}s$ for $\delta > \frac{2}{3}$, and for all S for $\delta < \frac{2}{3}$, i.e. the region $\left\{(S,\delta): 0 \le S < \frac{3\delta}{3\delta-2}s\right\}$. It is straightforward to show that that $\frac{3\delta}{3\delta-2} > \frac{6-3\delta}{4-3\delta}$ for $\frac{2}{3} < \delta < 1$. Therefore the intersection of the regions given by the inequality is the former region

$$\left\{(S,\delta): 0 \le S < \frac{6-3\delta}{4-3\delta}s\right\}. \tag{19}$$

We will combine this with case IV (a) below to find the entire region in which $(1, 1)$ is the pertinent equilibrium.

In case III, $u^d - u^t > 0$ and $u^e - u^t < 0$. From (16), the former inequality implies $\delta > \frac{2}{3}$ and $S > \frac{3\delta}{3\delta-2}s$. From (17), the latter inequality implies $S > \frac{6-3\delta}{4-3\delta}s$. Since $\frac{3\delta}{3\delta-2} > \frac{6-3\delta}{4-3\delta}$ for $\frac{2}{3} < \delta < 1$, satisfaction of the former inequality implies satisfaction of the latter for $\delta > \frac{2}{3}$. Therefore there exists a symmetric mixed strategy equilibrium for the region $\left\{(S,\delta) : S > \frac{3\delta}{3\delta-2}, \delta > \frac{2}{3}\right\}$. This corresponds to region A in diagram 4.

Consider case IV(a) where $u^e - u^t < 0$ and $u^d - u^t < 0$, and $u^e - u^d > 0$ imply that there are no symmetric mixed strategy equilibria. From (17), $u^e - u^t < 0$ defines the region $\left\{(S,\delta) : \frac{6-3\delta}{4-3\delta}s < S\right\}$, and we argued above for case II that $u^d - u^t < 0$ defines the region $\left\{(S,\delta) : 0 \le S < \frac{3\delta}{3\delta-2}s\right\}$. From (17), $u^e - u^d > 0$ defines the region $\left\{(S,\delta) : 0 \le S < \frac{2}{\delta}s\right\}$. Since $\frac{2}{\delta} < \frac{3\delta}{3\delta-2}$, the case defines the region

$$\left\{(S,\delta) : \frac{6-3\delta}{4-3\delta}s < S\right\} \cap \left\{(S,\delta) : 0 \le S < \frac{2}{\delta}s\right\}. \tag{20}$$

The intersection of regions (19) and (20) is represented by region C in diagram 4, where $(1, 1)$ is the unique mixed strategy equilibrium.

For case IV(b), $u^e - u^t < 0$, $u^d - u^t < 0$, and $u^e - u^d < 0$, which implies that if any symmetric mixed strategy equilibria exist they are Pareto dominated. This region is given by the intersection

$$\left\{(S,\delta) : \frac{6-3\delta}{4-3\delta}s < S\right\} \cap \left\{(S,\delta) : 0 \le S < \frac{3\delta}{3\delta-2}s\right\} \cap \left\{(S,\delta) : \frac{2}{\delta}s < S\right\}, \tag{21}$$

and is denoted region B in diagram 4.

Proof of Proposition 3

Proof. Existence of symmetric mixed strategy equilibria with holdout in region $\left\{(S,\delta) : S > \frac{3\delta}{3\delta-2}, \delta > \frac{2}{3}\right\}$ is demonstrated above. Application of Lemma 1 demonstrates that asymmetric pure strategy equilibria also exist in this region, and that $(1, 1)$ is the only undominated equilibrium in the complementarity region. ■

4.0.7 Comparative Statics in the Specific Bargaining Game

Recall from lemma 2 equation (6) that

$$\frac{df(p;\theta)}{d\theta} = -(1-p)(1-\delta+\delta p)\frac{\partial u^e}{\partial \theta} - p(1-\delta+\delta p)\frac{\partial u^t}{\partial \theta} + p\frac{\partial u^d}{\partial \theta}.$$

Differentiation of (13), (14) and (15) with respect to s yields $\frac{du^t}{ds} = 0$, $\frac{du^e}{ds} = \frac{\delta}{4}$, and $\frac{du^d}{ds} = -\frac{\delta}{2}$. Substitution into (6) implies $\frac{df(p;s)}{ds} < 0$, so that a fall in s leads to a rise in f, hence a fall in p since$\frac{df(p;s)}{dp} > 0$ locally when $u^d > u^t$ according to lemma 1. Differentiation of (13), (14) and (15) with respect to S yields $\frac{du^t}{ds} = \frac{1}{3}$, $\frac{du^e}{ds} = \frac{\delta}{4}$, and $\frac{du^d}{ds} = \frac{\delta}{2}$. Substitution into (6) gives

$$\frac{df(p;S)}{dS} = \left(\delta p^2 + (1-\delta)p\right)\left(\frac{\delta}{2} - \frac{1}{3}\right),$$

which is positive when $\delta > \frac{2}{3}$. Again by lemma 1, $\frac{df(p;s)}{dp} > 0$ locally when $u^d > u^t$, so that p decreases when S rises.

Note that when we combine these two results with the welfare results in section 4.0.4, the proof of proposition 4 is complete. The condition $u^d > u^t$ means that $\frac{S}{3} - \frac{\delta(S-s)}{2} < 0$, which is only possible when $\delta > \frac{2}{3}$. Therefore, $\frac{S}{3} - \frac{\delta(S-s)}{2} < 0$ is sufficient for $\frac{dp}{ds} > 0$ and $\frac{dp}{dS} < 0$, and hence that the welfare loss is increasing in the degree of complementarity regardless of the source.

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Diagram 1: The Entry Game

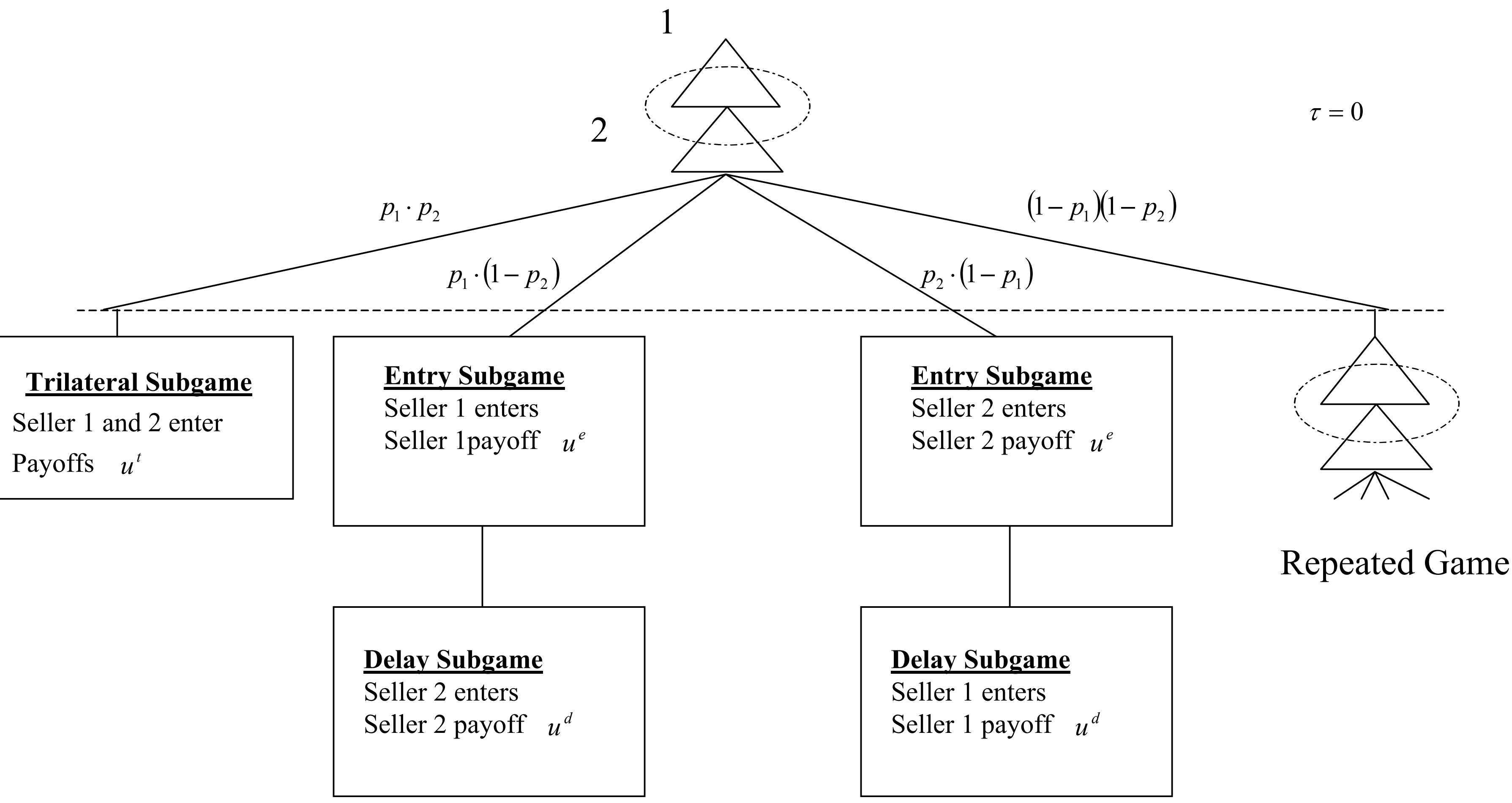

Diagram 2
Comparative Statics in the Generic Game

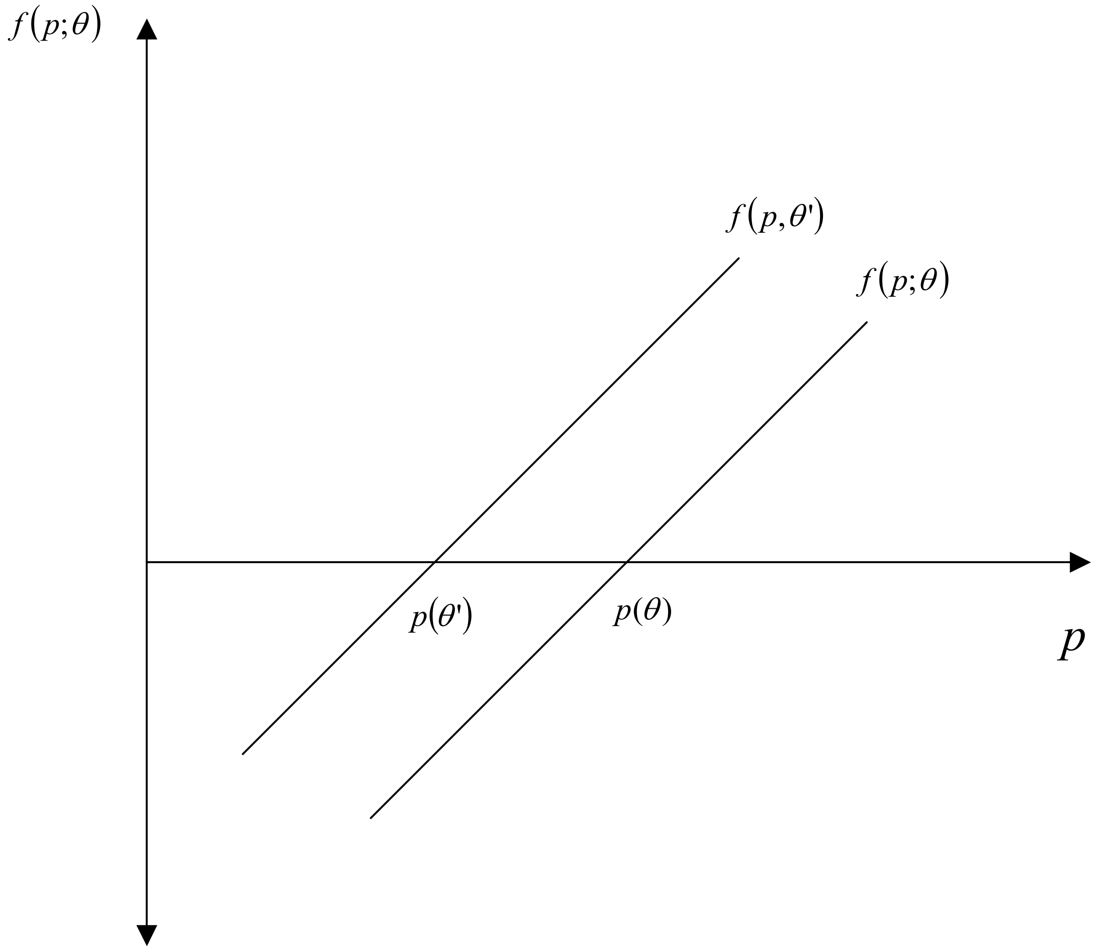

Diagram 3: A Schematic of the Extensive Form

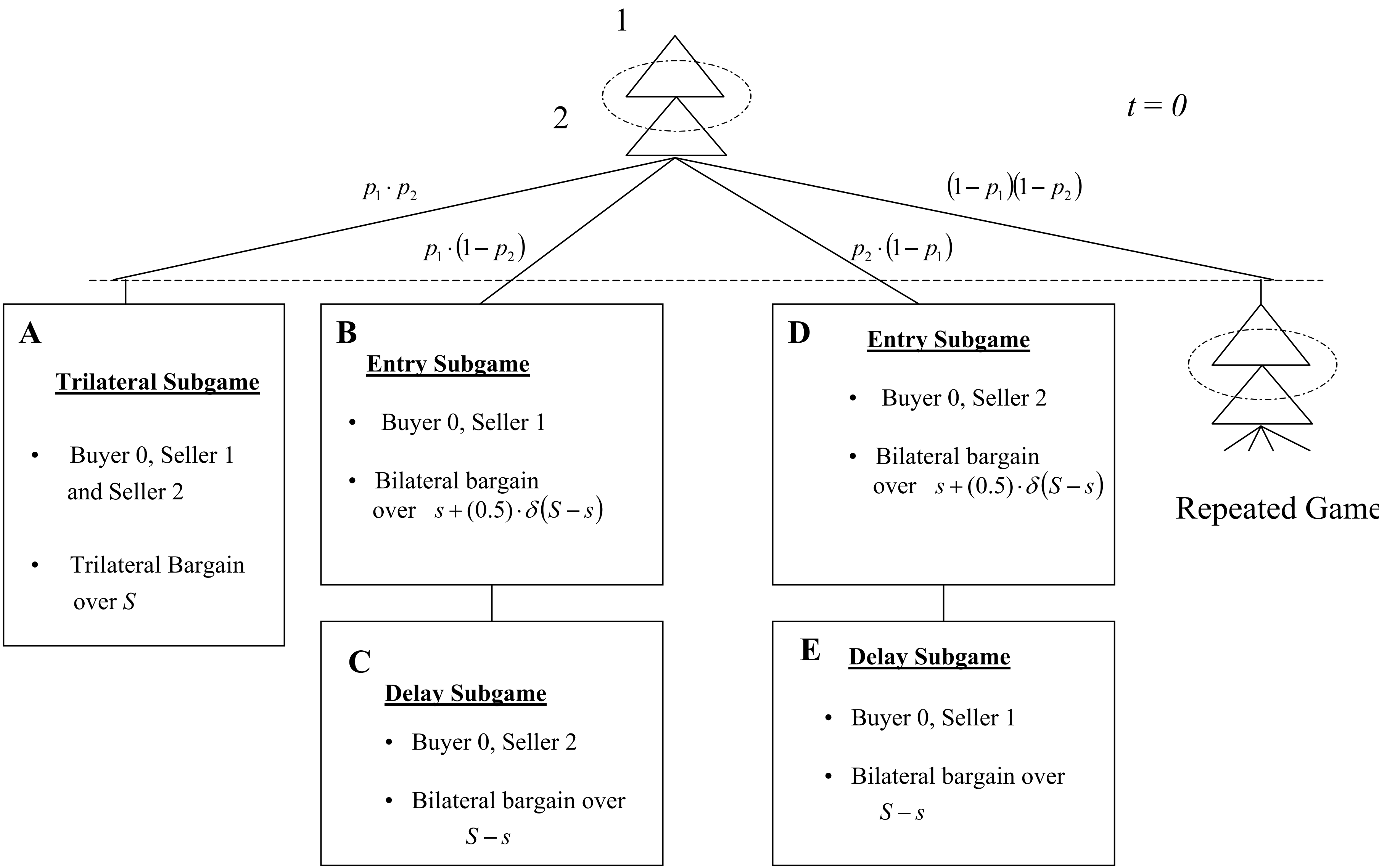

Diagram 4: Symmetric Equilibria

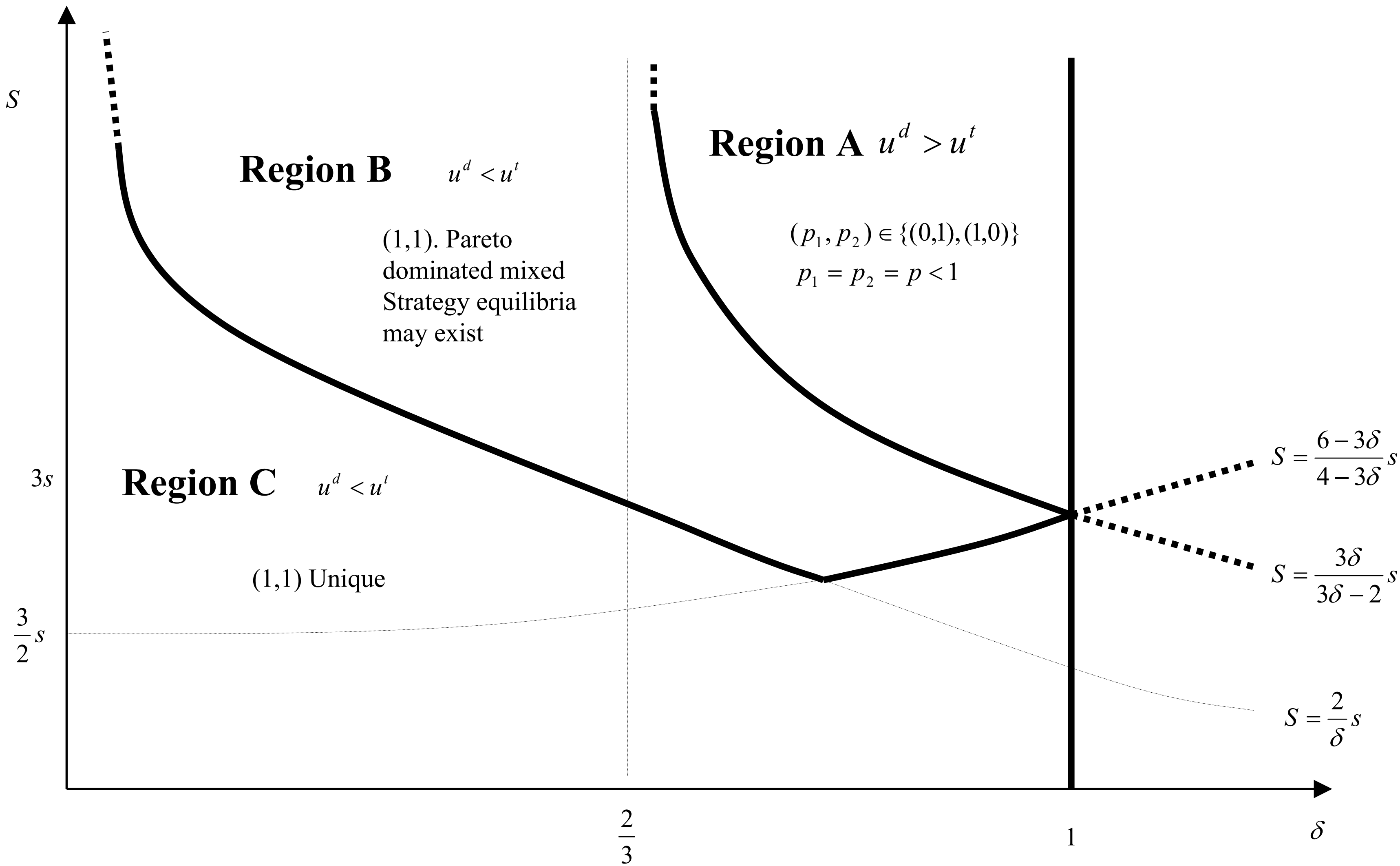

Diagram 5: The Entry and Delay Subgames

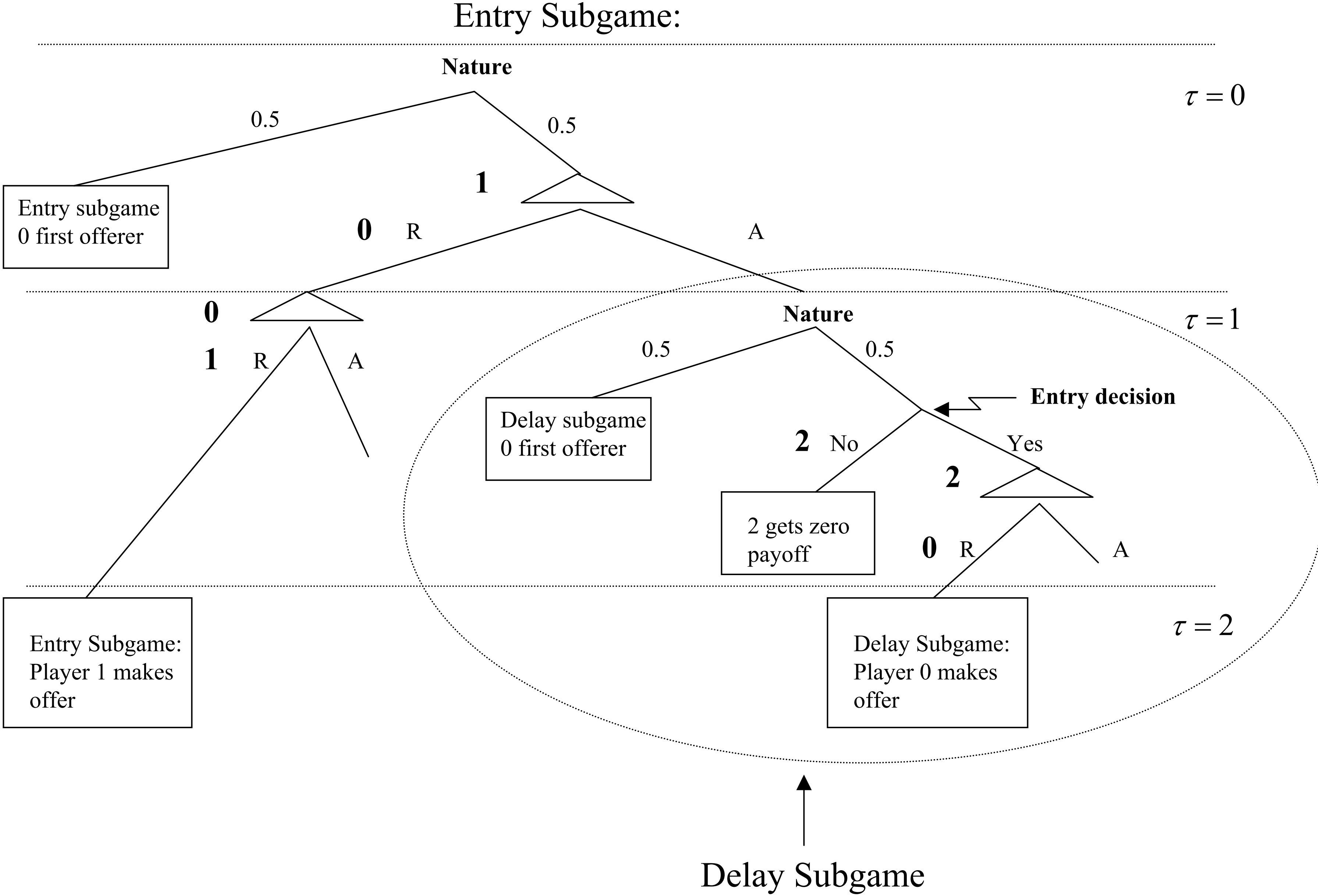

Diagram 6: Solving The Entry and Delay Subgames

Entry Subgame: