Network Coding Noise Reduction via Relay Power Allocation in a Two-unicast Wireless System

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***Abstract*—Network coding (NC) is known as a promising approach to improve the cooperative communication network throughput. However, in certain situations, it can introduce additional noise terms which is recently referred to as *NC noise*. We consider such a problem in a two-unicast wireless system and seek to answer the following question: "Can we reduce or remove network coding noise by proper power allocation at the relay?" To this end, we provide a mathematical framework for the output signal-to-noise (SNR) ratio and instantaneous sum-rate of the network-coded cooperative communication (NC-CC) system with the notion of power assignment at the relay. Based on this framework, we provide two novel closed-form power allocation techniques that are suitable for slow and fast fading conditions. Numerical analysis is used to confirm the accuracy of the derived theory and to show the effectiveness of proposed solutions in terms of average sum-rate and outage probability. It is shown that such techniques offer a significant advantage in overcoming the adverse effects of NC noise, especially in slow fading, without introducing significant extra costs or system complexity.**

I. INTRODUCTION

A. Motivation and Background

It is well-known that cooperative communication (CC) is an effective approach to provide spatial diversity and improve system capacity by exploiting signals transmitted through direct and relay paths [1], [2]. However, such relay transmission could cost bandwidth resources, which results in a system throughput reduction. One efficient technique to increase the throughput of CC is network coding (NC) which was first proposed in [3] as a promising solution to improve network throughput and robustness. It is therefore of great interest to find out how much the performance of CC is affected due to the use of digital or physical network coding (PNC) [4]–[6]. A large body of work has been focused on exhibiting the *performance gains* that can be obtained by incorporating network coding in CC, referred to as network-coded cooperative communication (NC-CC), via information theoretic metrics such as outage probability [7]–[9], network throughput [10]–[12], and ergodic capacity [8], [13].

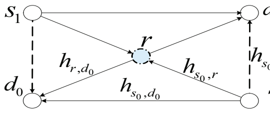

Fig. 1. Multi-unicast system with two source-destination pairs.

On the other hand, in one recent work [14], Sharma *et al.* investigated the adverse effects of network coding on the achievable rate of CC and claimed that using NC is not always beneficial in CC. The two-unicast NC-CC scenario considered in this work is shown in Fig. 1. Under this scheme, during the first two time slots, each source broadcasts its data in its respective time slot, which is overheard by the relay and both destination nodes. In the third time slot, the relay performs PNC and employs the amplify and forward (AF) strategy to transmit the sum of the received signals to the destinations. Therefore, each destination can extract a copy of its data from the combined received signal in the third time slot using the overheard signal in the previous time slots. However, as discussed in [14] and as will be shown in Section II-A, such signal cannot be completely extracted, and hence results in a non-negligible noise term at the destination node which was called *NC noise*. It was shown that the data rates of source-destination pairs in NC-CC are highly dependent on the NC noise and can even be lower than the case where only the direct link is used. Therefore, the advantages of employing NC can notably decrease or disappear if the effect of NC noise is not taken into account in the system analysis and design.

The consequent new challenge is how to effectively evaluate and reduce the adverse effect of NC noise on NC-CC system performance. To the best of our knowledge this problem is still not well addressed. The NC noise analysis in [14] was developed with the premise that the relay used equal power to mix the signals of two sources while performing PNC, although PNC has been performed in three time slots. Therefore, an interesting question would be whether power assignment at the relay can overcome the detrimental effects of NC noise in the three-time-slot multi-unicast NC-CC system, without introducing significant extra costs or system complexity.

Interestingly, in a recent work [15], Louie *et al.* have investigated the performance of two and three-time-slot PNC schemes in terms of sum-bit error rate (BER) and sum-rate for two-way relay channels, where two sources communicate with each other through the aid of a relay node using an AF protocol. This work showed that the three-time-slot PNC with power allocation at the relay has a performance which either lies between or exceeds the two-time-slot PNC and four-time-slot transmission (*i.e.*, relaying scheme without exploiting NC) schemes and thus offers a good compromise. This leads us to the idea of using relay power allocation in a multi-unicast scenario and revisit PNC scheme in [14] with the main purpose of reducing the effect of the NC noise.

B. Problem Statement and Key Contributions

The question that we will address in this work is as follows. "Can we reduce or remove network coding noise by proper

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signal combination at the relay?" To answer this question we provide two novel closed-form power allocation techniques that are suitable for slow and fast fading conditions. It is worthwhile to mention that these techniques have a remarkable effect on the NC-CC performance without introducing significant overhead or computational complexity to the system and hence, are suitable for practical implementations. Our three contributions in this work are summarized as follows:

- We first introduce the notion of power assignment in NC-CC system with NC noise. Then, we study the output SNR at the destination nodes for two-unicast NC-CC system with relay power allocation.
- We use the instantaneous sum-rate to provide a novel closed-form relay power allocation solution for the NC-CC system in slow fading. Then using this methodology, we propose a power allocation technique which is suitable for fast fading conditions.
- Finally, we explore the adverse impact of NC noise on the NC-CC performance and investigate the effectiveness of proposed solutions in alleviating the effect of this noise in slow and fast fading conditions. We show that our simple power allocation schemes can notably reduce the NC noise effect compared to [14], especially in slow fading environments.

II. SYSTEM MODEL

The two-unicast system considered in this paper is depicted in Fig. 1 which consists of five nodes, where two mobile source nodes s_0 and s_1 communicate respectively with two mobile destination nodes d_0 and d_1 through one fixed relay r using an AF protocol and PNC. We assume the channels between all the nodes are independent and denoted by $h_{i,j}$, $i \neq j$ where $i \in \{s_0, s_1, r\}$ and $j \in \{d_0, d_1, r\}$. All channels are modeled as zero-mean, circularly symmetric complex Gaussian random variables with variance $\sigma^2_{h_{i,j}} = (1/d^a_{i,j})$, where $d_{i,j}$ is the distance between node i and node j and a is the path loss exponent [16]. This channel model includes both long-term path loss and short-term fading. The long-term path loss $\sigma^2_{h_{i,j}}$ determines the strength of the short-term fading i.e., the variance of the fading channel $h_{i,j}$ or the mean of $|h_{i,j}|^2$. In addition, we assume that the noise at destination node d_0 (d_1) in time slot T_k, denoted by $z_{d_0,T_k}(z_{d_1,T_k})$, is zero-mean additive white Gaussian noise (AWGN) with variance $\sigma^2_{d_0}(\sigma^2_{d_1})$, while the noise at the relay in the first and second time slots, presented respectively by z_{r,T_1} and z_{r,T_2}, is AWGN with variance σ^2_r. We use Maximum Ratio Combining (MRC) receiver at the destination nodes for better performance.

A. Three-Time-Slot PNC

In this subsection, in order to introduce the NC noise concept we review [14] briefly and then reformulate the NC noise for our proposed scheme. In the first time slot s_0 transmits a data symbol x_0 and then the received signals at other nodes can be written as

$$y_{s_0,r} = h_{s_0,r}x_0 + z_{r,T_1}, \tag{1}$$

$$y_{s_0,d_0} = h_{s_0,d_0}x_0 + z_{d_0,T_1}, \tag{2}$$

$$y_{s_0,d_1} = h_{s_0,d_1}x_0 + z_{d_1,T_1}. \tag{3}$$

As source node s_1 transmits a data symbol x_1 in the second time slot, the signals received by other nodes are

$$y_{s_1,r} = h_{s_1,r}x_1 + z_{r,T_2}, \tag{4}$$

$$y_{s_1,d_0} = h_{s_1,d_0}x_1 + z_{d_0,T_2}, \tag{5}$$

$$y_{s_1,d_1} = h_{s_1,d_1}x_1 + z_{d_1,T_2}. \tag{6}$$

In the third time slot, the relay node combines the signals which it received in the first two time slots (*i.e.*, performing PNC), and then amplifies and forwards them to the two destination nodes. Since signal analysis at two destinations are similar, in the following we write signal equations for destination d_0. With appropriate changes of indices, equations can be derived for d_1. The combined received signal at d_0 in the third time slot, y_{r,d_0}, can be expressed as

$$y_{r,d_0} = Gh_{r,d_0}[y_{s_0,r} + y_{s_1,r}] + z_{d_0,T_3}, \tag{7}$$

where G is the AF amplification factor to maintain a constant average power at the relay output [2]. Note here that based on this scheme three time slots are required for the transmission of signals from two sources to the intended destinations. In contrast, a conventional cooperative scheme (*e.g.*, [2]) requires four time slots. As it can be seen from the above equations, destination node d_0 receives one copy of signal x_0 in the first time slot. Further, it obtains another copy of x_0 in the third time slot as follows. Using (4), the combined signal in (7) can be expanded as

$$y_{r,d_0} = Gh_{r,d_0}[y_{s_0,r} + h_{s_1,r}x_1 + z_{r,T_2}] + z_{d_0,T_3}. \tag{8}$$

Using (5), (8) can be rewritten as

$$\begin{aligned} y_{r,d_0} = {} & Gh_{r,d_0}y_{s_0,r} + \frac{Gh_{r,d_0}h_{s_1,r}}{h_{s_1,d_0}}[y_{s_1,d_0} - z_{d_0,T_2}] \\ & + Gh_{r,d_0}z_{r,T_2} + z_{d_0,T_3}. \end{aligned} \tag{9}$$

With the assumption of full channel state information (CSI), destination node d_0 can cancel the unwanted term y_{s_1,d_0} in (9) as follows. Since d_0 overhears a copy of s_1 transmission in the second time slot, (*i.e.*, y_{s_1,d_0} which is given by (5)), it can multiply (5) by a factor $\frac{Gh_{r,d_0}h_{s_1,r}}{h_{s_1,d_0}}$ and subtract it from (9). The resulting signal at d_0, indicated by $\hat{y}_{r,d_o}$, is

$$\begin{aligned} \hat{y}_{r,d_0} = {} & y_{r,d_0} - \frac{Gh_{r,d_0}h_{s_1,r}}{h_{s_1,d_0}}y_{s_1,d_0} = Gh_{r,d_0}y_{s_0,r} \\ & - \frac{Gh_{r,d_0}h_{s_1,r}}{h_{s_1,d_0}}z_{d_0,T_2} + Gh_{r,d_0}z_{r,T_2} + z_{d_0,T_3}. \end{aligned} \tag{10}$$

From (10), it is observed that instead of z_{d_0,T_3}, we now have a new noise term in this constructed signal as $\hat{z}^{NC}_{d_0} + z_{d_0,T_3}$, where

$$\hat{z}^{NC}_{d_0} = -\frac{Gh_{r,d_0}h_{s_1,r}}{h_{s_1,d_0}}z_{d_0,T_2} + Gh_{r,d_0}z_{r,T_2} \tag{11}$$

known as NC noise [14]. We observe that even with perfect knowledge of h_{r,d_0}, $h_{s_1,r}$ and h_{s_1,d_0}, the NC noise is present at destination node.

In contrast with [14] in which the relay transmits the sum of the received signals without optimization of power allocation, we use the power assignment at the relay where each received signal from s_0 and s_1 is weighted by a power allocation coefficient such that a particular metric performance is optimized. In the sequel, we will derive the new NC noise expression for the NC-CC employing power assignment where in the third time slot the relay node processes the received signals, and amplifies and forwards the processed signal to the two destination nodes. The combined received signal at d_0, $\acute{y}_{r,d_0}$, can be expressed as

$$\acute{y}_{r,d_0} = G h_{r,d_0}[\alpha_0 y_{s_0,r} + \alpha_1 y_{s_1,r}] + z_{d_0,T_3}, \qquad (12)$$

where α_0 and α_1 are the power allocation coefficients such that ${\alpha_0}^2 + {\alpha_1}^2 = 1$. The amplifying gain G of the relay is

$$G = \sqrt{\frac{P_r}{P_{s_0}\alpha_0^2|h_{s_0,r}|^2 + P_{s_1}\alpha_1^2|h_{s_1,r}|^2 + \sigma_r^2}}, \qquad (13)$$

where P_{s_0}, P_{s_1} and P_r are the transmission powers of nodes s_0, s_1 and r, respectively. Using (4), (5), we can rewrite $\acute{y}_{r,d_0}$ in (12) as

$$\begin{aligned}\acute{y}_{r,d_0} &= G\alpha_0 h_{r,d_0} y_{s_0,r} + \frac{G\alpha_1 h_{r,d_0} h_{s_1,r}}{h_{s_1,d_0}}[y_{s_1,d_0} - z_{d_0,T_2}] \\ &\quad + G\alpha_1 h_{r,d_0} z_{r,T_2} + z_{d_0,T_3}. \qquad (14)\end{aligned}$$

Again with the assumption of full CSI and the knowledge of α_0 and α_1 at destination nodes, d_0 can cancel the unwanted term y_{s_1,d_0}. In order to do so, it multiplies signal y_{s_1,d_0} in (5), overheard from the s_1 transmission in the second time slot, by a factor $\frac{G\alpha_1 h_{r,d_0} h_{s_1,r}}{h_{s_1,d_0}}$ and subtracts it from (14). The resulting signal at destination node d_0, denoted by $y^*_{r,d0}$, is obtained as

$$y^*_{r,d0} = G\alpha_0 h_{r,d_0} y_{s_0,r} + z^{new}_{d_0}, \qquad (15)$$

where $z^{new}_{d_0}$ indicates the noise term in $y^*_{r,d0}$ as $z^{new}_{d_0} = z^{NC}_{d_0} + z_{d_0,T_3}$ and

$$z^{NC}_{d_0} = -\frac{G\alpha_1 h_{r,d_0} h_{s_1,r}}{h_{s_1,d_0}} z_{d_0,T_2} + G\alpha_1 h_{r,d_0} z_{r,T_2} \qquad (16)$$

is the new NC noise at destination node d_0 for NC-CC scheme with power allocation. It can be shown that $z^{new}_{d_0}$ has a zero mean and variance

$$\sigma^2_{z^{new}_{d_0}} = G^2\alpha_1^2\left(\frac{|h_{r,d_0}|^2|h_{s_1,r}|^2}{|h_{s_1,d_0}|^2}\sigma^2_{d_0} + |h_{r,d_0}|^2\sigma_r^2\right) + \sigma^2_{d_0},$$

which is larger than the original noise variance $\sigma^2_{d_0}$ and a function of α_1. Using a similar method, we can derive the new NC noise at destination d_1.

B. Output SNR

We first derive the output SNR at the destination node d_0 considering NC noise and employing power allocation at the relay. By substituting (1) into (15) we have

$$y^*_{r,d0} = G\alpha_0 h_{r,d_0}(h_{s_0,r}x_0 + z_{r,T_1}) + z^{new}_{d_0}. \qquad (17)$$

Now (2) and (17) present the appropriate channel model for NC-CC scheme with AF relaying and direct path from source node s_0. We can rewrite these equations in vector form as

$$\boldsymbol{Y} = \boldsymbol{H}x_0 + \boldsymbol{B}\boldsymbol{Z}, \qquad (18)$$

where

$$\boldsymbol{Y} = \begin{bmatrix} y_{s_0,d_0} \\ y^*_{r,d0} \end{bmatrix}, \qquad \boldsymbol{H} = \begin{bmatrix} h_{s_0,d_0} \\ G\alpha_0 h_{s_0,r} h_{r,d_0} \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 0 \\ \alpha_0 G h_{r,d_0} & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{Z} = \begin{bmatrix} z_{r,T_1} \\ z_{d_0,T_1} \\ z^{new}_{d_0} \end{bmatrix}.$$

As it was discussed in [2], the AF cooperative protocol with direct path produces an equivalent one-input, two-output complex Gaussian noise channel with different noise levels in the outputs. Therefore, it can be easily shown that for the above channel, the output SNR at d_0 is given by

$$\gamma_{d_0} = \det\left(\boldsymbol{I} + (P_{s_0}\boldsymbol{H}\boldsymbol{H}^\dagger)(\boldsymbol{B}\mathrm{E}[\boldsymbol{Z}\boldsymbol{Z}^\dagger]\boldsymbol{B}^\dagger)^{-1}\right) - 1, \qquad (19)$$

where $\det(\cdot)$ is the determinant function, $\boldsymbol{I}$ is the identity matrix, † symbolizes the complex conjugate transposition, and $\mathrm{E}[\boldsymbol{Z}\boldsymbol{Z}^\dagger] = \mathrm{diag}(\sigma_r^2, \sigma^2_{d_0}, \sigma^2_{z^{new}_{d_0}})$ is the covariance matrix of noise. After algebraic manipulations on equation (19), we have

$$\gamma_{d_0} = \frac{|h_{s_0,d_0}|^2 P_{s_0}}{\sigma^2_{d_0}} + \frac{\alpha_0^2\frac{P_{s_0}}{\sigma_r^2}|h_{s_0,r}|^2}{1 + \frac{\sigma^2_{d_0}}{P_r|h_{r,d_0}|^2}\left(\frac{\alpha_0^2|h_{s_0,r}|^2}{\sigma_r^2} + \frac{\alpha_1^2|h_{s_1,r}|^2}{\sigma_r^2} + 1\right) + \frac{\alpha_1^2\sigma^2_{d_0}|h_{s_1,r}|^2}{\sigma_r^2|h_{s_1,d_0}|^2}}. \qquad (20)$$

The output SNR at destination node d_1, γ_{d_1}, can be obtained in a similar fashion.

III. PERFORMANCE ANALYSIS AND POWER ALLOCATION

In this section we aim to study the performance of NC-CC scheme employing power allocation for slow and fast fading conditions first and provide two closed-form relay power assignment solutions.

A. Performance Criteria and Power Allocation in Slow Fading

1) Instantaneous Sum-Rate : As an important performance criterion in slow fading, we consider the instantaneous sum-rate which is denoted by R_{inst}. Note that for considered three-time-slot NC-CC transmission scheme every source node, as well as the relay node, is allocated one time slot and hence, we have the following expression for the instantaneous sum-rate

$$R_{\text{inst}}(\alpha_0, \alpha_1) = \frac{W}{3}\left[\mathcal{I}_{s_0,d_0} + \mathcal{I}_{s_1,d_1}\right], \qquad (21)$$

where dependence of R_{inst} on α_0 and α_1 is explicity shown, W is the available bandwidth, $\mathcal{I}_{s_0,d_0} = \log_2(1 + \gamma_{d_0})$ and $\mathcal{I}_{s_1,d_1} = \log_2(1 + \gamma_{d_1})$ are the mutual information between the pair (s_0, d_0) and the pair (s_1, d_1), respectively. Also, γ_{d_0} and γ_{d_1} are the output SNR given in the previous section. In the following, we consider the R_{inst} criterion and derive the optimum power allocation coefficients to maximize it. Without loss of generality, we assume $\sigma^2_{d_0} = \sigma^2_{d_1} = \sigma_r^2 = \sigma^2$. Using γ_{d_0}, γ_{d_1}, and $\alpha_0^2 + \alpha_1^2 = 1$ expressions we rewrite R_{inst} in (21) in terms of α_0 (or α_1) as

$$\begin{aligned}R_{\text{inst}}(\alpha_0) = \frac{W}{3}&\left[\log_2\left(C_7 + \frac{C_1\alpha_0^2}{C_2\alpha_0^2 + C_3}\right)\right. \\ &\left.+ \log_2\left(C_8 + \frac{C_4(1-\alpha_0^2)}{C_5\alpha_0^2 + C_6}\right)\right], \quad 0 \le \alpha_0 \le 1, \qquad (22)\end{aligned}$$

where C_i, $i = 1, \ldots, 8$ are defined in the Appendix. Now the maximum point of (22) can be calculated by finding the zeros of its gradient. If we differentiate (22) with respect to α_0 and let it be equal to zero, then we have the following biquadratic equation to solve

$$A\alpha_0^4 + B\alpha_0^2 + C = 0, \tag{23}$$

where A, B, and C are defined in the Appendix. Fortunately, by solving equation (23) we can derive the closed-form solution for power allocation. By performing simple algebraic manipulation and using the condition $\alpha_0^2 + \alpha_1^2 = 1$, it is straightforward to show that the optimum value of α_0 can be obtained as

$$\alpha_{0,opt} = \begin{cases} \sqrt{\frac{-B-\sqrt{B^2-4AC}}{2A}}, & 0 < \frac{-B-\sqrt{B^2-4AC}}{2A} < 1; \\ \underset{\alpha_0 \in \{0,1\}}{\arg\max}(R_{\text{inst}}(\alpha_0)), & \text{otherwise}, \end{cases} \tag{24}$$

and then $\alpha_{1,opt} = \sqrt{1 - \alpha_{0,opt}^2}$. Note that in the second case in (24), $\mathrm{d}R_{\text{inst}}(\alpha_0)/\mathrm{d}\alpha_0 \neq 0$ and is strictly positive or negative in the range $\alpha_0 \in (0, 1)$. Hence, the end points of the range, *i.e.*, $\alpha_0 = 0$ and $\alpha_0 = 1$, should be tested. Therefore, under this condition, the relay does not perform network coding and only forwards data for one of the users. It can be easily verified that the provided solution results in the global maximum of $R_{\text{inst}}(\alpha_0, \alpha_1)$. Note that, we will apply the derived power allocation coefficients at the relay for every channel realization while the channels remain constant for many transmissions in slow fading conditions.

2) Outage Probability: Outage probability is another important performance metric defined as the probability that the instantaneous rate falls bellow a predetermined threshold necessary to support the desired data rate. To the best of our knowledge finding exact expressions for the outage probability of the considered NC-CC scheme is difficult, if not impossible. Therefore in the next section, we will numerically evaluate this probability using Monte Carlo simulation and investigate the efficiency of power allocation developed above for optimizing sum-rate as a suboptimal choice for reducing outage.

B. Performance Criterion and Power Allocation in Fast Fading

We consider average sum-rate, denoted by R_{avg}, as a performance criterion to measure the performance in fast fading environments. R_{avg} is defined as the average sum of the data rate at d_0 and d_1, and is given by

$$R_{\text{avg}} = \frac{W}{3}\left(\mathbf{E}_{\gamma_{d_0}}\left[\log_2(1+\gamma_{d_0})\right] + \mathbf{E}_{\gamma_{d_1}}\left[\log_2(1+\gamma_{d_1})\right]\right), \tag{25}$$

where $\mathrm{E}(\cdot)$ is the statistical expectation. Unfortunately, finding an exact expression for the average sum-rate, *i.e.*, the sum-rate averaged over the fading channels, using the given output SNR (20) is difficult. As a result, we numerically evaluate the above average sum rate using Monte Carlo simulation in the next section. Note that finding a closed-form expression for power assignment at the relay using (25) is involved. Therefore, we have to use some approximations to find an acceptable power allocation. Our approach is as follows. In (25), we substitute all channel gains ($|h_{i,j}|^2$) in γ_{d_0} and γ_{d_1} with their means ($\sigma^2_{h_{i,j}}$). Therefore, no expectation is required. Hence, we can use the same power allocation methodology which was proposed for instantaneous sum-rate in slow fading. That is, (24) can be used for fast fading as well, by replacing ($|h_{i,j}|^2$) with their means ($\sigma^2_{h_{i,j}}$) in the definitions in the Appendix.

It is notable that two proposed power assignments are performed at the relay node only and change how the relay mixes received signals. The destination nodes need to know α_0 (or α_1). We expect the additional costs of implementing these power allocations on NC-CC scheme in terms of computational complexity and overhead to be small.

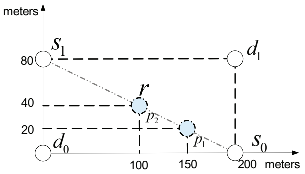

Fig. 2. Two-unicast network topology. p_1 and p_2 present two relay positions.

IV. NUMERICAL RESULTS

In this section, we study the adverse impact of NC noise on the two-unicast NC-CC performance and then the efficiency of two novel proposed power allocation schemes in alleviating the effect of this noise in terms of instantaneous sum-rate and outage probability in slow fading and average sum-rate performance in fast fading. Moreover, the AF cooperative communication (AF-CC) and direct transmission schemes are presented for comparison. We investigate two scenarios based on various relay positions and source powers for the topology shown in Fig. 2. We assume the path-loss exponent is $a = 4$ and noise variance is $\sigma^2 = 10^{-10}$ W. We consider a carrier frequency of 2.5 GHz and a bandwidth of $W = 10$ kHz, which is suitable for mobile WiMAX, *i.e.*, IEEE 802.16e [17]. In our numerical results, mobile-to-fixed channels are generated according to Clarke's model [18] and mobile-to-mobile channels are generated according to the method of exact Doppler spread (MEDS) [19]. We use normalized Doppler frequency (*i.e.*, $f_D T_S$ where f_D is the Doppler frequency shift and T_S is the symbol duration) of 0.001 and 0.03, corresponding to mobile speeds of 4.3 km/h and 129.6 km/h, to represent slow and fast fading, respectively. Note that in simulation results for sum-rate in slow fading mode we use (24) to find the optimum values of power allocation coefficients α_0 and α_1 for each transmission and then average the obtained instantaneous sum-rates over many block transmissions.

Fig. 3 demonstrates the average sum-rate of the two-unicast system for different transmission schemes and varying relay positions (from s_0 to s_1 on the dotted line in Fig. 2) in slow fading environments. Five main observations that follow from this simulation are: *1)* the NC-CC scheme without considering the NC noise, represented by 'ideal NC-CC', has the best sum-rate performance for all relay positions. However, when NC noise is present (NC-CC scheme in [14] which we refer to as 'NC-CC without power allocation') its performance is severely degraded; *2)* NC-CC with proposed power allocation significantly mitigates the effect of NC noise and performs better than NC-CC without power allocation; *3)* the sum-rate optimized using analytical power allocation in (24) is exactly equal to that optimized through global numerical search; *4)* when the relay is not positioned midway between two sources,

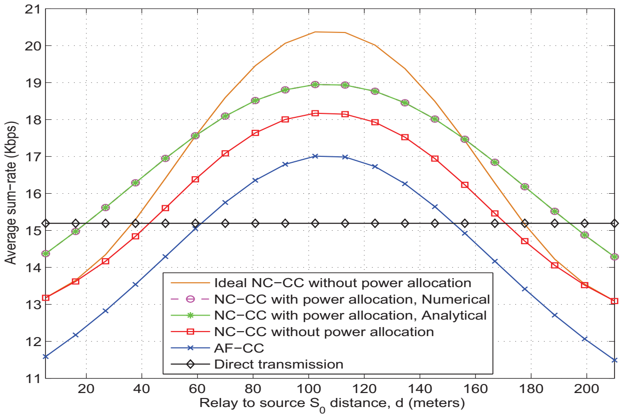

Fig. 3. The average sum-rate of two-unicast scenario vs. relay position in slow fading environments. Different transmission schemes with $P_{s_0} = P_{s_1} = P_r = 0.4$ W are compared.

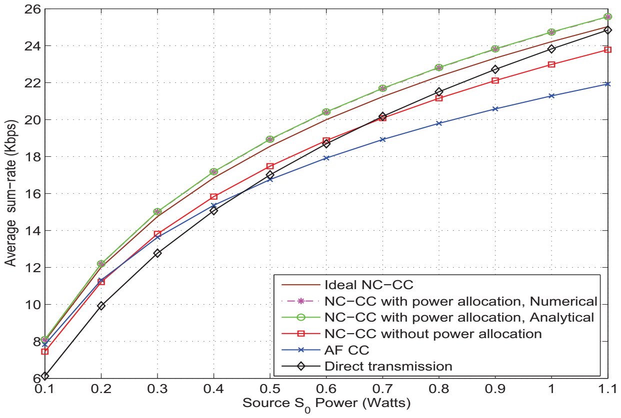

Fig. 4. The average sum-rate of two-unicast scenario vs. source s_0 power in slow fading environments. Different transmission schemes with $P_{s_0} = P_{s_1} = P_r$ are compared.

the performance gain of power allocation in NC-CC scheme is higher. Thus, in the rest of this section we focus on the position p_1 for the relay (which is presented in Fig. 2); *5)* sum-rate performance of NC-CC scheme similar to AF-CC is highly dependent on relay position (based on (22)). When the relay node is close to one of the source nodes, sum-rate is noticeably lower than other points.

Fig. 4 shows the average sum-rate for different source s_0 powers in slow fading environments. One can see NC-CC with power allocation outperforms other schemes, *e.g.*, when the node powers are high, it can provide respectively 7% and 14.8% sum-rate gain compared with NC-CC without power allocation and AF-CC schemes. It is notable that in this setup NC-CC with optimal power allocation outperforms ideal NC-CC. It is also important to see that the performance of the direct transmission scheme outperforms other relaying schemes (as in [20]) for high source power ranges.

Fig. 5 and 6 indicate the outage probability of (s_0, d_0) pair, and (s_1, d_1) pair for different transmission schemes, respectively. We have used Monte Carlo simulation for evaluating the outage probability and the analytical power allocation introduced in Section III-A for optimizing the instantaneous sum rate. First observation is that the outage probability performance is improved with NC-CC schemes compared to AF-CC or direct transmission over a wide range of required rates. Second, NC-CC with power allocation significantly improves the performance of the (s_1, d_1) pair with respect to NC-CC without power allocation; However for the (s_0, d_0) pair this power assignment causes higher values for the outage probability. Nevertheless, the percentage of outage probability performance improvement of proposed power assignment for (s_1, d_1) pair is more than the performance reduction for (s_0, d_0) pair.

Finally, we investigate the average sum-rate performance of two-unicast system in fast fading in Fig. 7 by evaluating the expression in (25) using Monte Carlo method. In this figure, the average sum-rate performance curve of NC-CC using power allocation coefficients derived from global search is compered with that obtained using the analytical power assignment in Section III-B. One can see the analytic approach closely matches numerical optimization. This also shows that replacing $|h_{i,j}|^2$ with their means $\sigma^2_{h_{i,j}}$, dose not have significant impact on the power optimization performance. In addition, in Fig. 7 the average sum-rates achieved by the four schemes are compared as a function of source s_0 power. The average sum-rate is superior with NC-CC over small to medium source s_0 powers. Moreover, the positive effect of proposed power allocation on alleviating the impact of NC noise on performance is obvious, especially for high powers.

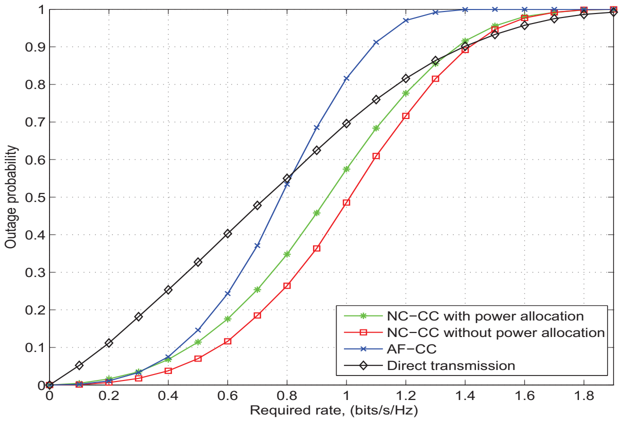

Fig. 5. The outage probability of the (s_0, d_0) pair. Different transmission schemes with $P_{s_1} = P_{s_0} = P_r = 0.4$ W are compared.

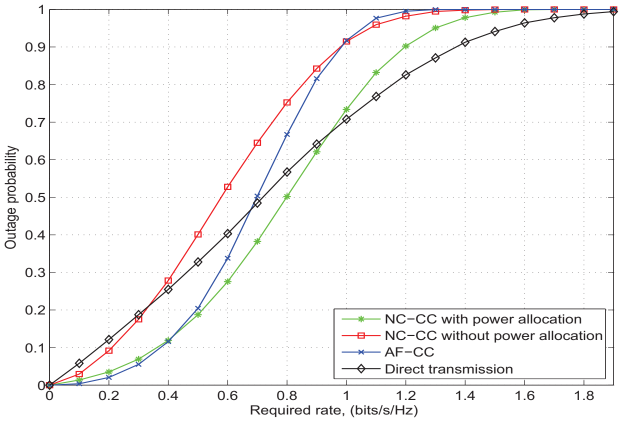

Fig. 6. The outage probability of the (s_1, d_1) pair. Different transmission schemes with $P_{s_1} = P_{s_0} = P_r = 0.4$ W are compared.

V. CONCLUSIONS

We addressed the problem of network coding noise reduction in a two-unicast NC-CC system and proposed relay power assignment as a potential solution. We studied the output SNR with the notion of power assignment and NC noise. Based on instantaneous sum-rate we provided two closed-form power allocation techniques that are suitable for slow and fast fading. Our numerical results showed that such proposed schemes reduce effectively the impact of NC noise without introducing significant extra costs or system complexity. Note, however, that the obtained performance gains in terms of average sum-rate in fast fading are smaller than those in slow fading environments. The main reason is that in slow fading the power allocation coefficients are updated at the relay for every channel realization. However, in fast fading these coefficients are calculated only once based on average channel powers and do not change fast in every channel realization. Thus, we expect a tradeoff between increasing the performance gain and decreasing the amount of required computation and overhead costs for power allocation.

APPENDIX

In this appendix we define the constants which were used in Section III as

$$
\begin{aligned}
&C_1 \triangleq P_{s_0} P_r |h_{r,d_0}|^2 |h_{s_0,r}|^2 |h_{s_1,d_0}|^2,\\
&C_2 \triangleq \sigma^2 |h_{s_1,d_0}|^2 (P_{s_0}|h_{s_0,r}|^2 - P_{s_1}|h_{s_1,r}|^2)\\
&\qquad - \sigma^2 P_r |h_{s_1,r}|^2 |h_{r,d_0}|^2,\\
&C_3 \triangleq \sigma^2 |h_{s_1,d_0}|^2 (P_{s_1}|h_{s_1,r}|^2 + \sigma^2 + P_r |h_{r,d_0}|^2)\\
&\qquad + \sigma^2 P_r |h_{r,d_0}|^2 |h_{s_1,r}|^2,\\
&C_4 \triangleq P_{s_1} P_r |h_{r,d_1}|^2 |h_{s_1,r}|^2 |h_{s_0,d_1}|^2,\\
&C_5 \triangleq \sigma^2 |h_{s_0,d_1}|^2 \left(P_{s_0}|h_{s_0,r}|^2 - P_{s_1}|h_{s_1,r}|^2\right)\\
&\qquad + \sigma^2 P_r |h_{s_0,r}|^2 |h_{r,d_1}|^2,\\
&C_6 \triangleq \sigma^2 |h_{s_0,d_1}|^2 \left(P_{s_1}|h_{s_1,r}|^2 + \sigma^2 + P_r |h_{r,d_1}|^2\right),\\
&C_7 \triangleq 1 + \frac{|h_{s_0,d_0}|^2 P_{s_0}}{\sigma^2},\\
&C_8 \triangleq 1 + \frac{|h_{s_1,d_1}|^2 P_{s_1}}{\sigma^2},\\
&\;A \triangleq C_1 C_3 C_5 (C_8 C_5 - C_4) - C_2 C_4 (C_6 + C_5)(C_7 C_2 + C_1),\\
&\;B \triangleq C_1 C_3 \left[2 C_8 C_5 C_6 + C_4 (C_5 - C_6)\right]\\
&\qquad - C_3 C_4 (C_5 + C_6)(2 C_7 C_2 + C_1),\\
&\;C \triangleq C_3 \left[C_1 C_6 \left(C_8 C_6 + C_4\right) - C_4 C_7 C_3 (C_5 + C_6)\right].
\end{aligned}
$$

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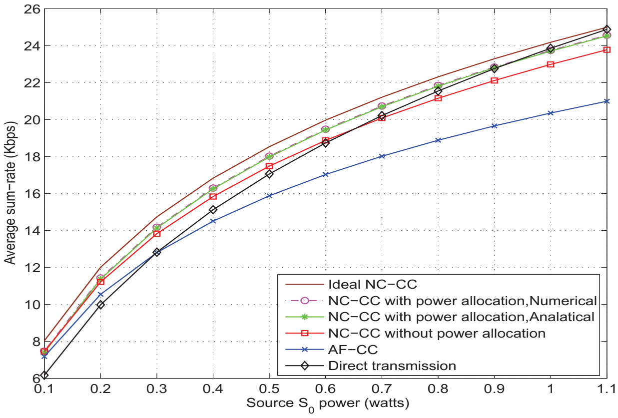

Fig. 7. The average sum-rate of two-unicast NC-CC scenario vs. source s_0 power in fast fading environments. Different transmission schemes with $P_{s_0} = P_{s_1} = P_r$ are compared.

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