Using a Cell Phone to Investigate the Skin Depth Effect in Salt Water

John Rayner, The Australian National University, Canberra, Australia

This paper describes an experimental investigation of the skin depth effect for electromagnetic waves in salt water using a cell phone that is immersed to a critical depth where it no longer responds when called. We show that this critical depth is directly proportional to the theoretical skin depth for a range of salt concentrations.

Introduction

These days ownership of a cell phone by students is virtually universal. As a way of increasing student engagement with science, this symbiotic relationship can be exploited by devising student experiments and projects that require a cell phone at the heart of the investigation. For example, a cell phone's camera might be used for recording images in the field for ecology or geology, or for data acquisition in the laboratory. Similarly a smartphone's sensor can be used for experiments in kinematics[1] or its ringtone used in the classic experiment of a source of sound in an evacuated bell jar.[2] Cell phones have also been used as the motivation governing a course in telecommunications.[3] A cell phone is, of course, a transmitter and receiver of high-frequency electromagnetic waves with frequencies ranging from 700 to 2100 MHz, i.e., wavelengths from 0.42 m to 0.14 m. It is this property that we employ in the present investigation of electromagnetic wave propagation in a conducting medium.

In introductory physics classes when discussing radio waves and light, we introduce students to the idea that they are part of a family of "electromagnetic waves" and that these waves consist of oscillating electric and magnetic fields. Acceptance of this proposition often requires an enormous leap of faith on the part of our students, where the fields are not readily apparent, so that it becomes one of those things that you simply have to learn.

As a way to make the concept more "real," experimental investigations can be employed that demonstrate how the behavior of these waves can be affected by electrical phenomena such as the permeability, permittivity, or conductivity of a medium though which the waves propagate. Thus, for example, one could use the Faraday rotation of the plane of polarization of an EM wave in a medium of a given permeability subjected to an axial magnetic field to demonstrate the magnetic nature of the waves.[4,5] Similarly one could use the speed of an EM pulse along a coaxial cable to show how it is affected by the permittivity of the dielectric between the inner and outer conductors to demonstrate the electrical nature of the wave. Both experiments, however, require relatively sophisticated equipment and deep conceptual understanding. Electrical conductivity, though, and its effect on EM wave propagation may be somewhat easier to grasp.

Many introductory physics students will have some experience of electrical circuits, the concept of electrical resistance, the idea that resistance dissipates electrical energy in the form of heat and light, and that its value depends on the electrical conductivity of the medium. Based on this prior experience, an investigation of the dissipation of an EM wave in a conducting medium by exploring the skin depth effect may well be useful in demonstrating the electrical nature of these waves.

Many schools offer enrichment programs for their more talented students designed to stimulate interest, provide challenges, and inspire a love of learning. Melrose High School[6] in Canberra, Australia, offers such a program known as its Academic Curriculum Extension (ACE) Program, which includes a strand for aspiring young scientists. The science program includes guest lectures by practicing scientists, visits to scientific institutions, work experience placements in scientific institutions, and independent research investigations mentored by professional scientists. The work described in this paper to investigate the skin depth of EM waves in a conducting solution using a cell phone was undertaken by two year-10 students enrolled in the ACE Science Program to test the viability of the idea.

Skin depth

When an electromagnetic wave enters a conducting medium, its electric field E decays exponentially with the penetration distance z according to the relationship[7]

$$E(z) = E_0 \exp(-z/\delta), \tag{1}$$

where E_0 is the field at the surface and δ is a characteristic distance known as the skin depth. The skin depth depends on a number of factors including the permeability μ, permittivity ε, and electrical conductivity σ of the medium, as well as the frequency f of the EM wave. For any medium the skin depth in SI units is given by[8]

$$\delta = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}-1\right]}}, \tag{2}$$

where ω is the angular frequency $\omega = 2\pi f$.

For a very good conductor where $\frac{\sigma}{\omega\varepsilon} >> 1$, the expression reduces to

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}. \tag{3}$$

Equation (3) shows that as the frequency or the conductivity

increases, the skin depth decreases.

The aim of the present experiment is to investigate the relationship between the electrical conductivity of salt solutions of different concentrations and the depth at which a cell phone immersed in a large tank of salt solution no longer responds when it is called. We call the depth at which the phone no longer rings the "critical depth" z_c, where it must be emphasized that the critical depth is not the same as the skin depth.

Any receiving antenna, including that of a cell phone, responds not only to the electric field but also to the magnetic field of the EM waves. The sensitivity of a receiver is usually expressed in terms of the minimum power P_{min} required to give a specified signal-to-noise ratio at the output of the receiver in units of watts or dBm.[9] The power depends on the geometry of the antenna and the received power per unit area, i.e., its intensity as defined by the average Poynting vector at the receiver.[10]

Given that the received power depends on the square of the electric field and the intrinsic impedance of the medium, we posit that the sensitivity of the receiver P_{min} for which the cell phone just rings is given by

$$P_{min} = P_0 e^{-2z_c/\delta}, \tag{4}$$

where P_0 is a reference power that is a function of the field at the surface E_0, the intrinsic impedance of the medium, and the effective area A of the phone's antenna. Apart from a small variation (<12%) in the intrinsic impedance, the other factors should remain constant as we change the conductivity of the solution, and so we assume that P_0 should remain essentially constant for these experiments.

If we rearrange Eq. (4), then

$$z_c = k\delta, \tag{5}$$

where

$$k = \frac{1}{2}\ln\left(\frac{P_0}{P_{min}}\right). \tag{6}$$

Because of the large number of unknown factors (A, E_0, and P_{min}), it is not possible to arrive at a theoretical value for k and hence actual values for the skin depth based on our measurements of z_c. However, for an experiment in which the conductivity, and thus the skin depth, is the only significant variable that we will be changing, we assume that k should be constant and hence predict that the critical depth should increase linearly with the calculated depth.

In deriving Eq. (4) we have assumed that the incident wave is vertically downwards and that the only dimension that matters is the distance from the surface of the water to the phone. Although the angle of incidence of the wave is not known, we have allowed for possible oblique incidence by ensuring that at all times the horizontal distances to the walls of the tank and also the distance from the phone to the tank's bottom surface are at least a factor of two larger than the vertical distance from the phone to the water's surface. Thus the horizontal components of the incident wave should be significantly attenuated compared with the vertical component, given that the attenuation depends exponentially on distance.

As part of the analysis of the results, we need to know the relationship between the concentration of the salt solution and its electrical conductivity. Although such data are readily available for seawater and NaCl solutions, we decided to carry out our own preliminary experiment to establish the relationship for the solutions we would use and compared our results with the published data. Our somewhat scattered results with uncertainties ~12% meant that we reverted to the published data for our calculations.

Skin depth experiments

We conducted the experiments in a 15-L rectangular plastic container. We wrapped the cell phone in a Ziploc® bag and then put it in a small waterproof plastic box. We made sure that the distances through the salt solution from the box to the sides of the container and also to the bottom of the container were all larger than the distance from the top surface to the box by at least a factor of two. Thus the only distance that mattered for radio waves reaching the phone was the one from the top surface to the box. For the cell phone we used a 2G phone operating in the Australian 900-MHz band.

We held the phone in its box at different depths in the solution and tried calling it at each depth. If successful we could hear the phone ringing and could also feel its vibration. For each depth as we lowered the box into the solution, we measured the distance of the box from the top surface and noted whether or not the phone rang. Once the phone no longer rang, we slightly withdrew the box until the phone again started to ring. Doing this several times, we found that we could find the critical depth with a precision of ±2 mm for critical depths between 1 and 5 cm. We repeated this procedure for four different concentrations.

We calculated the theoretical skin depth as a function of concentration using the exact theory [Eq. (2). and also the simple theory [Eq. (3). that applies for high conductivities. For the calculations we assumed a dielectric constant of 72, which is typical of seawater of 35 g/kg concentration,[11] a frequency of 900 MHz, and published conductivities.[12] Figure 1 compares the measured cut-off depth with the theoretical

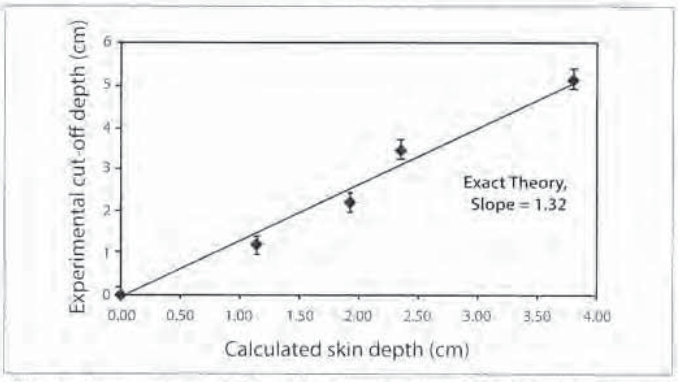


Fig. 1. Comparison of the critical cut-off depth with the theoretical skin depth based on Eq. (2).

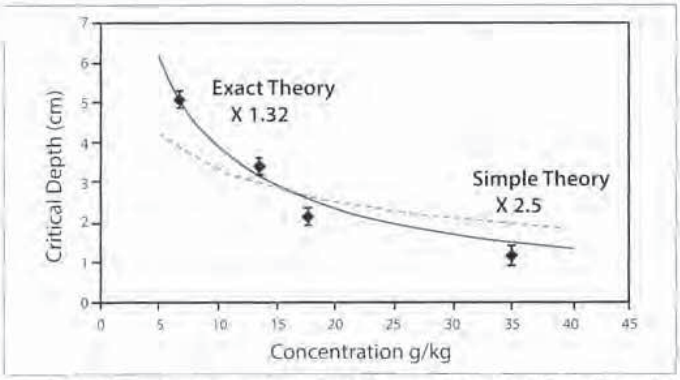


Fig. 2. Critical depth as a function of concentration. Experimental results compared with theoretical results based on Eqs. (2) and (3) multiplied by a numerical constant.

skin depth based on Eq. (2) and shows that, as predicted by Eq. (5), the cut-off depth increases linearly with skin depth with a slope $k = 1.32 \pm 0.1$. The linear relationship confirms our assumption that we can treat P_0 as a constant for these experiments.

If we put $k = 1.32$ into Eq. (6) and solve for the power ratio, then we find that $P_{min}/P_0 = 0.071$ or 7%, meaning that for our conditions the signal is attenuated by about 11.5 dB to produce cut-off. The fact that we obtained a good experimental straight line confirms that our cut-off depths are well accounted for by the skin-depth formula multiplied by the numerical constant k.

To test whether the high-conductivity approximation applies to these experiments, we have plotted in Fig. 2 the measured critical depths compared with the theoretical skin depths from Eq. (2) multiplied by the scaling factor $k = 1.32$. We have also plotted the results calculated from Eq. (3) multiplied by a scaling factor of 2.5, chosen so as to obtain as close a match as possible with the results. From this plot we can conclude that the simple expression for the skin depth is inadequate for our conditions as the curve is the wrong "shape."

We checked this conclusion by calculating the value of $\sigma/\omega\varepsilon$, which at a concentration of 20 g/kg has a value of 0.75, certainly not >>1 as is required if Eq. (3) is to be valid.

Discussion

In these experiments we have shown that it is possible to investigate quantitatively the skin depth effect in an electrically conducting medium using nothing more than a tank of salt water and a cell phone. We have also shown that at the frequency of 900 MHz used in these experiments, salt water can not be regarded as a good conductor, and that the simple expression for the skin depth of a good conductor is not valid under these conditions. If, however, the complete expression for the skin depth is employed, then good agreement is achieved between experiment and theory multiplied by an empirical constant k.

In the future the effect of different frequencies could be tested by using phones that operate in the 1800- or 2100-MHz bands. For the 2100-MHz band, however, at a concentration of 40 g/kg the theoretical skin depth drops to 0.85 cm compared with 1.3 cm at 900 MHz, so changes in skin depth would be difficult to observe. Alternatively other consumer electronics could be considered, such as a small FM radio receiver operating at around 100 MHz, leading to a skin depth of 2.3 cm at 40 g/kg or 2.2 cm based on the simple theory. At a lower concentration of 5 g/kg, critical depths of about 8 cm would be encountered, meaning that physically larger tanks of salt solution would be required.

For the tank, an alternative to plastic would be to use one constructed from metal with the top left open. In this way the metal would prevent EM waves from entering the tank through the sides or bottom so that the distance between the top surface and the phone becomes unambiguously the only dimension of importance.

From a learning perspective, the students reported that their interest and motivation were strongly enhanced by participation in the program. In particular they found that the use of a cell phone as the heart of a physics experiment was something that caught their imagination and maintained their enthusiasm for the project. They also reported that the experiment helped them to appreciate that EM waves are genuinely electrical in nature. From an educational perspective it would be interesting to repeat the experiment with a larger group of students to test whether or not their perceptions with respect to the nature of EM waves altered as a result of the experience.

Acknowledgments

This work formed the basis of a student research project undertaken under the science strand of the ACE Program operated by Melrose High School, Canberra, Australia, created and run by Mr. Geoff McNamara. The experimental results were generated by two grade-10 students, Jack Whiter and Prateek Rai, whose major contributions to this project are gratefully acknowledged.

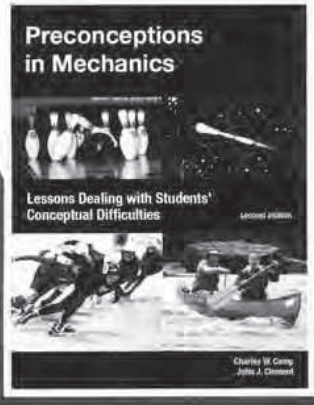
References

1. Patrik Vogt and Jochen Kuhn, "Analyzing free fall with a smartphone acceleration sensor," *Phys. Teach.* **50**, 182 (March 2012).
2. Christian Villa, "Bell-jar demonstration using cell phones," *Phys. Teach.* **47**, 59 (Jan. 2009).
3. Mark Davids, Rick Forrest, and Don Pata, "Teaching the fundamentals of cell phones and wireless communications," *Phys. Teach.* **48**, 217 (March 2010).
4. Frank J. Loeffler, "A Faraday rotation experiment for the undergraduate physics laboratory," *Am. J. Phys.* **51**, 661–663 (July 1983).
5. Michael Faraday firmly believed that light was electromagnetic in nature. His experiment on what became known as Faraday rotation was the first one to demonstrate that light and electromagnetism were related.
6. The school covers years 7 to 10 in the Australian K-12 system.
7. The concept of the skin effect is discussed in many textbooks on EM Waves. See, for example, Edward C. Jordan and Keith

Look What's in The Physics Store!

Preconceptions in Mechanics

This second edition of Charles Camp and John J. Clement's book contains a set of 24 innovative lessons and laboratories in mechanics for high school physics classrooms that was developed by a team of teachers and science education researchers. Research has shown that certain student preconceptions conflict with current physical theories and seem to resist change when using traditional instructional techniques. This book provides a set of lessons that are aimed specifically at these particularly troublesome areas: Normal Forces, Friction, Newton's Third Law, Relative Motion, Gravity, Inertia, and Tension. The lessons can be used to supplement any course that includes mechanics. Each unit contains detailed step-by-step lesson plans, homework and test problems, as well as background information on common student misconceptions, an overall integrated teaching strategy, and key aspects of the targeted core concepts. A CD of all duplication materials is included.



Members: $28
Non-Members: $30

AAPT PHYSICS EDUCATION

Order yours now at www.aapt.org/store

G. Balmain, *Electromagnetic Waves and Radiating Systems*, 2nd ed. (Prentice Hall, 1968), p. 126, or http://en.wikipedia.org/wiki/Skin_effect#CITEREFVanderVorst2006, accessed Sept. 18, 2012.

8. Op. cit., p. 130.
9. Bruce A. Carson, *Communication Systems: An Introduction to Signals and Noise in Electrical Communication* (McGraw-Hill, 1975), p. 445, or https://en.wikipedia.org/wiki/Sensitivity_(electronics), accessed May 27, 2016.
10. Although there is no agreed standard, the "bars" on a cell phone that indicate the received signal strength are roughly related to the received signal power in dBm.
11. D. K. Cheng, *Field and Wave Electromagnetics*, 2nd ed. (Addison-Wesley, 1989), p. 415.
12. G. W. C. Kaye and T. H. Laby, *Tables of Physical and Chemical Constants*, 14th ed. (Longman, London, 1973), p. 141.

John Rayner *(AM) holds a PhD in hypersonics. For many years he taught physics and telecommunications engineering at the University of Canberra. Prior to his retirement he was dean of faculty of Information Sciences and Engineering and chair of the Academic Board at UC. These days he is a visiting fellow in the National Centre for the Public Awareness of Science (CPAS) at the Australian National University. His interests include innovative teaching in physics and engineering and the creation and delivery of professional development programs for science teachers. His research work relates to the study of wave-heated plasmas and their applications, and the history of physics and wireless in Australia, with a particular interest in geophysics. In 2007 he was made a member of the Order of Australia (AM) for his services to science education.*
john.rayner@anu.edu.au