International Congress Series 1269 (2004) 129–132

www.ics-elsevier.com

Nearest neighbour coupled systems of four 1-D oscillators

Y. Nagai[a,*], T. Maddess[b]

[a] *Center for Information Science, Kokushikan University, 4-28-1 Setagaya, Setagaya, Tokyo 154-8515, Japan*
[b] *Center for Visual Sciences, RSBS, Australian National University, Australia*

Abstract. A one-dimensional (1-D) map oscillator is used to investigate the network properties of connected oscillators. The oscillators are connected as nearest neighbours, like Purkinje cells. There are two kinds of oscillators, p-type and n-type. The two types correspond to excitatory and inhibitory neurons. Thus, the 1-D oscillator network becomes an artificial system based on knowledge from the brain.

Keywords: 1-D oscillator; Network; Mapping; Basin; Bifurcation diagram; Nearest neighbour coupling

1. Introduction

We considered a system consisting of four one-dimensional oscillators to investigate the nature of network systems interacting via averaged values. The one-dimensional (1-D) oscillator is a mapping from the interval $[-1,1]$ to itself [1]. The oscillators are cubic functions of a one-parameter family. Hence, we are limited to three inputs for each element when we apply these oscillators to a network. We also considered the minimal sized network. Thus, a network of four 1-D oscillators satisfies our minimal requirements.

We examined two types of 1-D oscillators for the positive and negative signs of cubic functions; hence, we refer to them as the p-type and n-type, respectively [1]. Mixed p-and n-type oscillator networks are examined to understand their network features. Unmixed networks are also investigated. Basic oscillations, like sine waves, are found in the network of one n-type and three p-type oscillators. We examined several types of interactions. In particular, interactions using inputs corresponding to averaging over all outputs of the connected oscillators are rather stable compared with other interactions. The idea of nearest neighbour coupling comes from neural connections in the brain. The coupled-oscillator system is applicable to artificial neural systems.

In the next section, the main features of 1-D oscillators are illustrated. The mapping functions describing the 1-D oscillators have a *basin* structure [2], where the final behaviour of the oscillators depends upon the initial values. The network behaviour is

* Corresponding author. Tel.: +81-3-5481-3220; fax: +81-3-5481-3227.
E-mail address: nagai@kokushikan.ac.jp (Y. Nagai).

0531-5131/
doi:10.1016/j.ics.2004.05.135

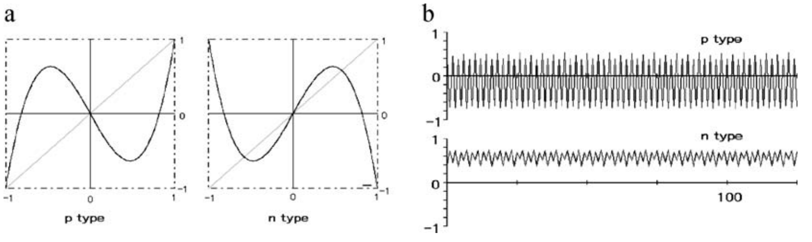

Fig. 1. (a) Maps for both types of oscillators. (b) Period 4 oscillations.

discussed in Section 3. Section 4 is devoted to a discussion of 1-D oscillator networks.

2. One-dimensional oscillator

We designed a one-dimensional mapping f: $I \rightarrow I$ where I signifies the interval $[-1,1]$. The mapping has oscillatory features and, hence, is called a 1-D oscillator. The mapping is described by cubic functions with a parameter, A, corresponding to the amplitude of an ordinary oscillator. The cubic function can adopt positive or negative signs. Thus, p-type and n-type oscillators are available. These oscillators are described by the following equations:

$$\mathrm{p-type}: \; f(x) = Ax(x^2 - 1) + x; \;\; \mathrm{n-type}: \; f(x) = -Ax(x^2 - 1) - x.$$

The parameter range of A is between 0 and 4. The effective range is [1,4]. The above oscillator behaves like the van der Pol oscillator [3]. Fig. 1a illustrates the p- and n-type oscillators. Fig. 1b shows oscillations obtained for $A = 3.3$ and initial value $x(0) = 0.4$. The origin of the n-type map is an unstable marginal point. This causes the difference of oscillation feature. The left and right areas of map are separated if the fixed point of each area is stable and attractive.

Fig. 2a and b presents bifurcation diagrams demonstrating the basin structure. The bifurcation diagrams above demonstrate that different oscillations exist depending on the initial values. The n-type oscillator resembles the z-value time development of the Lorenz model [4]. The 1-D oscillator is simple, but it has a clear, nonlinear nature.

3. Four-oscillator coupled system

To consider the network properties of connected oscillators, a nearest neighbour coupled system of four 1-D oscillators is investigated. Several kinds of interactions among

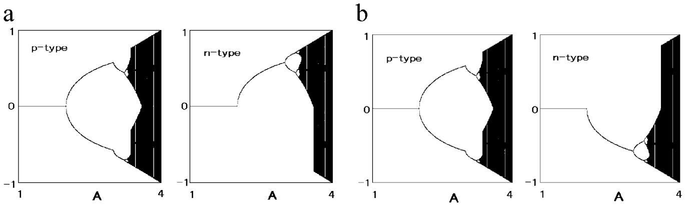

Fig. 2. (a) Bifurcation diagrams for $x(0) = 0.4$. (b) Bifurcation diagrams for $x(0) = 0.05$.

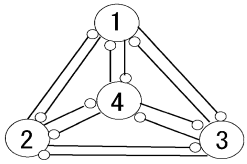

Fig. 3. Four-element network scheme.

oscillators were examined. We found that the input averaged over other oscillator outputs gives stable oscillator behaviour over almost the entire parameter range. Thus, an average quantity interaction is used. The network is described by the following equation,

$$x_i(t+1) = \eta_i \left\{ A_i \left(\frac{1}{3} \left(\sum_{j=1}^{4} x_j(t) - x_i(t) \right) \right)^3 - \frac{1}{3} \left(\sum_{j=1}^{4} x_j(t) - x_i(t) \right) + \frac{1}{3} \left(\sum_{j=1}^{4} x_j(t) - x_i(t) \right) \right\},$$

where η_i denotes the sign corresponding to the oscillator type, namely, $\eta_i =$ " + " for p-type and $\eta_i =$ " − " for n-type. The 1-D oscillator network considered is illustrated in Fig. 3. Each oscillator receives all of the other three oscillator outputs, but not itself. The oscillator is therefore driven by the other three, i.e., implying a delayed feedback.

3.1. Same-type oscillator network

The behaviour of same-type oscillator networks is investigated by changing the combination of the four parameters (A_1 to A_4). The oscillation feature of four oscillators is shown from Fig. 4a to Fig. 5b. In Fig. 4a and b, the same parameter values (A_1 to A_3) are used for oscillators 1–3, and A_4 is set to 2.2 or 3.6. In the setting of parameter values of A_1 to A_3, the 1-D oscillator adopts chaotic oscillations. It is immediately known from Fig. 4a and b that oscillator 4 governs the feature of all the oscillators. Similar results occur in Fig. 5a and b. Our speculation for the fact is as follows. Each oscillator input is the average of the other three so that the inputs of oscillators 1–3 are smaller than that of oscillator 4 at the initial stage. This causes a synchronous change of oscillators 1–3.

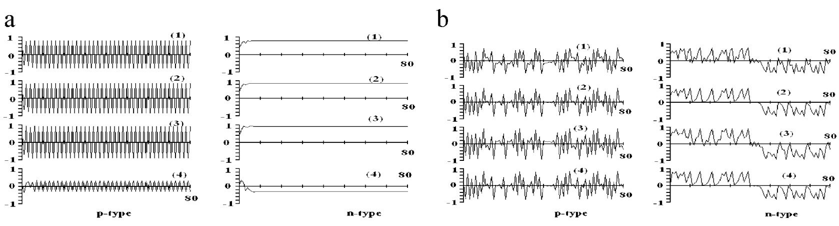

Fig. 4. (a) $A_1=3.4$, $A_2=3.6$, $A_3=3.8$, $A_4=2.2$, $x_1(0)=x_2(0)=x_3(0)=0.4$, $x_4(0)=0.05$. (b) $A_4=3.6$, A_1–A_3, and initial values as in Fig. 5.

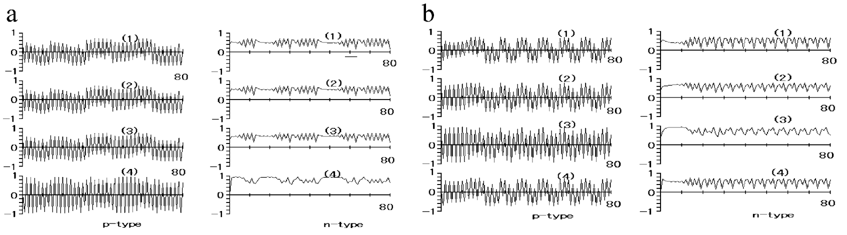

Fig. 5. (a) $A_1 = 3.2$, $A_2 = 3.25$, $A_3 = 3.3$, $A_4 = 3.85$, $x_1(0) = x_2(0) = x_3(0) = 0.4$, $x_4(0) = 0.05$. (b) $A_1 = 3.2$, $A_2 = 3.43$, $A_3 = 3.83$, $A_4 = 3.3$, $x_1(0) = x_2(0) = x_3(0) = 0.4$, $x_4(0) = 0.05$.

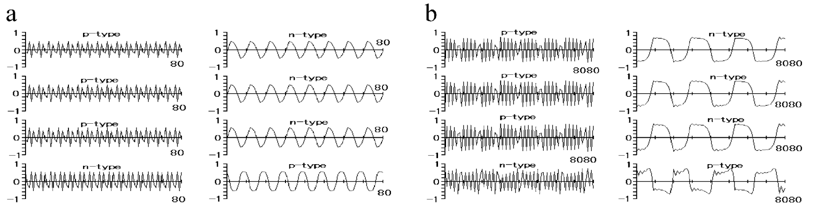

Fig. 6. (a) $A_1 = 3.2$, $A_2 = 3.43$, $A_3 = 383$, $A_4 = 3.9$, $x_1(0) = x_2(0) = x_3(0) = 0.4$, $x_4(0) = 0.05$. (b) $A_1 = 3.4$, $A_2 = 3.6$, $A_3 = 3.8$, $A_4 = 3.27$, $x_1(0) = x_2(0) = x_3(0) = 0.4$, $x_4(0) = 0.05$.

3.2. Different-type oscillator mixed networks

Here, one opposite-type oscillator is mixed with three same-type oscillators. A network, consisting of three n-type oscillators and a p-type oscillator, gives an oscillation like a sine wave (Fig. 6a), or a somewhat chaotic rectangular oscillation (Fig. 6b).

4. Discussion

The 1-D oscillator provides wide modes of oscillation, such as damping, regular, period doubling, and chaotic oscillations by varying the parameter A. The oscillator network with average quantity interaction is usually governed by the lowest parameter value oscillator. In other cases, the network follows a majority decision. The network responds well when the majority of oscillators are in the weakly chaotic parameter region. The complete features of 1-D oscillator networks are not yet clear. Future work will provide more detailed analysis of 1-D oscillator networks.

References

[1] Y. Nagai, T. Maddess, Mechanics of control dynamics, Mem. Kokushikan U Cent. Inform. Sci. 25 (2004) 21–45.
[2] W. de Melo, S. van Strein, One-Dimensional Dynamics, Springer-Verlag, Berlin Heidelberg, 1993.
[3] P. Bergé, Y. Pomeau, C.H. Vidal, L'ordere Dans Le Chaos, Hermann, Paris, 1984.
[4] E.N. Lorenz, Deterministic nonperiodic flow, J. Atmos. Sci. 20 (1963) 130–141.