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A single-probe-beam double-heterodyne polarimeter-interferometer for plasma Faraday rotation measurements

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ABSTRACT: This article describes a novel millimetre-wave polarimeter/interferometer for measuring the Faraday rotation of the plane of polarization of a probing coherent electro-magnetic beam that traverses a magnetized plasma. Independent frequency offset heterodyne carriers for the interferometric and polarimetric angles are generated using a chirped microwave source. The system has the advantage of requiring just a single frequency-modulatable source and a single detector for each spatial probing beam. The paper describes the system operating principles and reports measurements using a benchtop system that demonstrate the measurement principles and expected performance. A system based on this principle is now being constructed for measurements on the HIT-SI tokamak.

KEYWORDS: Plasma diagnostics - interferometry, spectroscopy and imaging; Nuclear instruments and methods for hot plasma diagnostics

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Contents

1 Introduction

The rotation of the polarization of a linearly-polarized probing laser beam traversing a magnetized plasma is a useful electron density or internal magnetic field diagnostic [1, 2]. The Faraday rotation angle of the electric vector is given by [3]:

$$\psi \approx 2.6 \times 10^{-13} \lambda_0^2 \int_0^L n_e B_{\|} \mathrm{d}l. \tag{1.1}$$

where λ_0 is the laser vacuum wavelength and $B_{\|} = B.\hat{l}$, where B is the magnetic field, $\hat{l}$ is the unit vector in the laser propagation direction, n_e is the electron density, dl is a line element and SI units are used. For toroidal magnetic confinement systems, the choice of probe wavelength ranges from the infrared for large high-power devices (e.g. ITER) down to millimeter-wave systems for smaller devices as determined by the ratio of the probing frequency to the plasma and electron cyclotron frequencies. Usually, n_e is determined simultaneously by measuring interferometrically the accompanying refractive phase shift [3]

$$\varphi = r_e \lambda_0 \int_0^L n_e \mathrm{d}l \tag{1.2}$$

where $r_e = e^2/(4\pi\varepsilon_0 m_e c^2) = 2.82 \times 10^{-15}$m is the classical electron radius.

The Cotton-Mouton effect, which is sensitive to the magnetic field components perpendicular to the viewing line, also causes elliptization of the probing beam. For propagation across inhomogeneous plasmas, the Faraday and Cotton-Mouton effects couple the polarization ellipse parameters so that equation (1.1) is an approximation whose validity depends on the probing geometry and the initial polarization orientation [2]. While the instrument described here can be generalised so as to measure all components of the Stokes vector, for our purposes we make the usual assumption that the coupling from the Cotton-Mouton effect is small and the approximation given by equation (1.1) is valid. Under these conditions, and subject also to appropriate symmetry assumptions (e.g. toroidal or flux-surface symmetry), the parallel magnetic field $B_{\|}$ can be inferred from simultaneous measurements of ψ and φ at a sufficient number of positions across the plasma.

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The simplest scheme to estimate the polarization orientation is to measure the intensity of the rotated electric field component through an analyzer oriented to reject the original probe beam polarization [4]. In this case, the transmitted component is usually mixed with an independent, frequency offset local oscillator and the amplitude of the resulting beat signal is approximately proportional to ψ. Various phase-encoded heterodyne systems have also been developed [5–8].

Most heterodyne schemes require either multiple independent far-infrared laser sources or mechanical modulators such as rotating gratings or waveplates to generate the frequency offset local oscillator. In the former case, the probing sources must be carefully co-aligned to remove differential refraction effects. Homodyne schemes (no frequency offset required) require two detectors per spatial channel in order to extract the interferometric and polarimetric phase angles φ and ψ. These systems are also difficult to balance in order that beam refraction effects are negligible.

Our approach measures both interferometric and polarimetric information using a chirped microwave source and a single detector per probe beam. Because the system uses only a single radiation source and a single detector per spatial channel, it should be robust against errors caused by refraction within the plasma [7]. The usefulness of the technique relies on the ability to frequency modulate the radiation source, and so is particularly well-suited to lower-density devices where voltage-tunable microwave sources are applicable and readily available.

This paper is organized as follows. The operating principle of the polarimeter-interferometer and a prototype benchtop system are described in section 2 with a Jones matrix analysis of the system given in the appendix. Results using the prototype system are presented in section 3. The system performs as expected with little crosstalk between the interferometric and polarimetric signals. A multi-channel interferometer/polarimeter based on the principles of the prototype system is now being constructed for Faraday rotation measurements on the helicity injected tokamak HIT-SI [9].

2 Operating principle and experimental layout

The layout of the polarimeter-interferometer is shown in figure 1. In this scheme, the interferometer is configured as a single-pass system. By replacing two of the interferometer mirrors by reflecting waveplates of adjustable path delay (components PR1 and PR2), and using a final polarizer to mix the various wave components, the signal at the detector is sensitive to both the interferometric phase shift and the polarization state of the probing radiation emerging from the plasma. The combination of two waveplates of adjustable delay and/or orientation and a final analyzer is a standard approach to recover the Stokes vector describing the state of polarization of light (see for example [10]).

When the source frequency f can be swept linearly (or "chirped"), the time delay $\tau = L/c$ between probe and local oscillator beams gives rise to a frequency difference between the mixed waves at the detector given by $\omega_i = 2\pi(\partial f/\partial t)\tau$ (see figure 1). In practice, a continuous linear chirp is not possible and the frequency must be modulated by a sawtooth wave. By applying a sawtooth control voltage (frequency $\Omega/2\pi$ =1kHz), it is possible to adjust the amplitude of the voltage sweep and the path difference L so that, over one sawtooth period $2\pi/\Omega$, the phase difference at the detector is an integer number of waves $N = (2\pi\tau/\Omega)\partial f/\partial t$. Ignoring the sawtooth reset discontinuity, this results in an effective heterodyne carrier frequency $\omega_i = N\Omega$.

The birefringent polarizing reflecting elements can be constructed from parallel orthogonal polarizers (or polarizer and backing mirror) whose separation is also adjustable. By introducing

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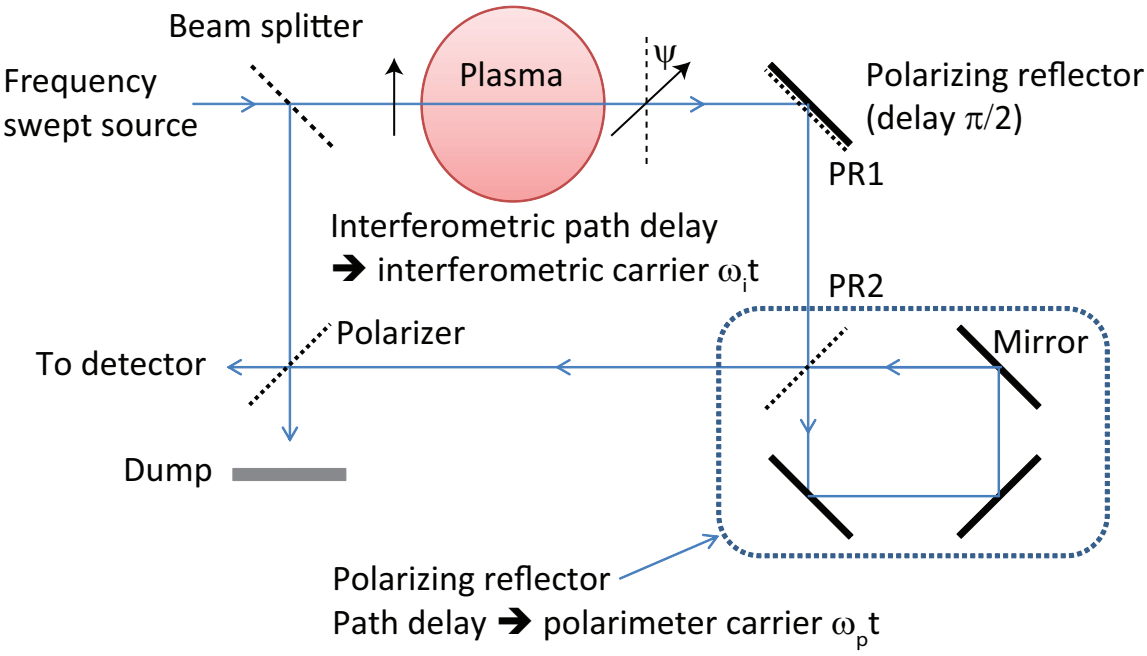

Figure 1. Schematic layout for the heterodyne polarimeter-interferometer. If each of the polarizing reflectors PR1 and PR2 is replaced by simple mirrors, the system reverts to a standard chirped-source interferometer.

a large path difference, the second waveplate PR2 in figure 1 becomes effectively a temporally modulated waveplate of delay $\omega_p t = M\Omega t$ where M is an integer. The combination of independent frequency-offset carriers ω_i and ω_p at harmonics of the sawtooth modulation frequency Ω allows separation of the interferometric and polarimetric phase angles. Using a Jones matrix analysis, it is shown in the appendix, that the signal at the detector output is given by:

$$\begin{aligned} I = {} & (I_{\mathrm{LO}} + I_0/2) + I_0/2\cos(\omega_p t + 2\psi) \\ & + \sqrt{I_0 I_{\mathrm{LO}}}\cos(\omega_i t + \varphi - \psi) \\ & + \sqrt{I_0 I_{\mathrm{LO}}}\cos[(\omega_i + \omega_p)t + \varphi + \psi] \end{aligned} \tag{2.1}$$

where I_0 is the probe beam intensity and I_{LO} is the local oscillator intensity. The result indicates that the detector will see carrier signals at three distinct frequencies $\omega_p = M\Omega$, $\omega_i = N\Omega$ and $\omega_i + \omega_p = (N+M)\Omega$ with independent phase modulations 2ψ, $\varphi - \psi$ and $\varphi + \psi$ respectively. It is straightforward to demodulate the carrier signals to recover the Faraday angle ψ and the interferometer phase shift φ. The scheme proposed here is reminiscent of, but much simpler than the multiple laser-beam scheme proposed elsewhere [8].

3 Results

The radiation source is a 140 GHz, 25 mW voltage tunable IMPATT diode source (ELVA-1 VCO 140/1) [11]. The quasi-optical interferometer is constructed using polyethylene lenses to control

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the beam diameter, aluminium mirrors and various free-standing wire grids to manipulate the polarization state. The detector is a horn-coupled ELVA-1 ZBD-6 (zero biased detector), with an in-line optical isolator to suppress back reflections.

Both the source power and source frequency vary only approximately-linearly with the applied control voltage. This non-linear frequency dependence of the source parameters ($\lesssim 20\%$ variation over the control voltage sweep range of 6-8 V) generates harmonic sidebands that couple the various interferometric and polarimetric carriers. The crosstalk due to the source non-linearity can be largely reduced by separating the carriers by more than one harmonic of the sawtooth modulation frequency. In our case, the path lengths and drive voltage were adjusted to give $N = 4$ and $M = 2$ so that the carrier frequencies correspond to the 2^{nd}, 4^{th} and 6^{th} harmonics of the fundamental sawtooth frequency. In practice, this was achieved using an interferometer arm path length difference of 6 m (to generate ω_i) and a delay-plate path length difference of 3 m (ω_p). Figure 2 shows the multiple-harmonic detector waveform (solid) and the sawtooth control voltage for two periods of the sawtooth wave. The associated frequency spectrum shows the primary carrier waves at 2, 4 and 6 kHz, and the nonlinearly generated contaminating sidebands at a level 15-20 dB lower. To further suppress these sidebands, a non-linear sweep voltage could, in principle, be applied in order to deliver a linear frequency sweep. Alternatively, a wide-band voltage-tunable backward wave oscillator might deliver more ideal performance.

In order to test the polarimeter performance, a rotatable wire grid polarizer was interposed in the nominal plasma arm to set the polarization angle ψ. Because of its large aperture (100 mm) the polarizer was rotated by mounting it in a circular ring that could be driven by a rigid bar connected to a translation stage, such that 1mm of translation corresponded approximately 1.1° of polarizer rotation. Figure 3 (top) shows that the measured polarisation angle obtained by ramping up and down the polarizer angle in steps of 1° across an angle range of 10°. The test angle is accurately recovered by the system using the 4kHz and 6kHz carriers, and the rms angle noise for a 10ms integration period is 0.1°. There is some evidence of mechanical backlash in the rotation mechanism when the direction of rotation is reversed. As seen in figure 3 (bottom), the coupling between the polarimetric and interferometric angles is small.

4 Conclusion

We have demonstrated a new type of millimeter-wave interferometer/polarimeter that requires a single source and a single detector for each spatial probing beam. A modulated waveplate produced using a chirped microwave source, is used to discriminate between the interferometric and polarimetric angles. A system based on this principle is now being constructed for measurements on the HIT-SI tokamak.

A Jones matrix analysis

The modified polarization state of a linearly polarized probing beam can be represented by the complex vector

$$E = \begin{pmatrix} \cos\eta \\ \sin\eta \exp j\phi \end{pmatrix} \tag{A.1}$$

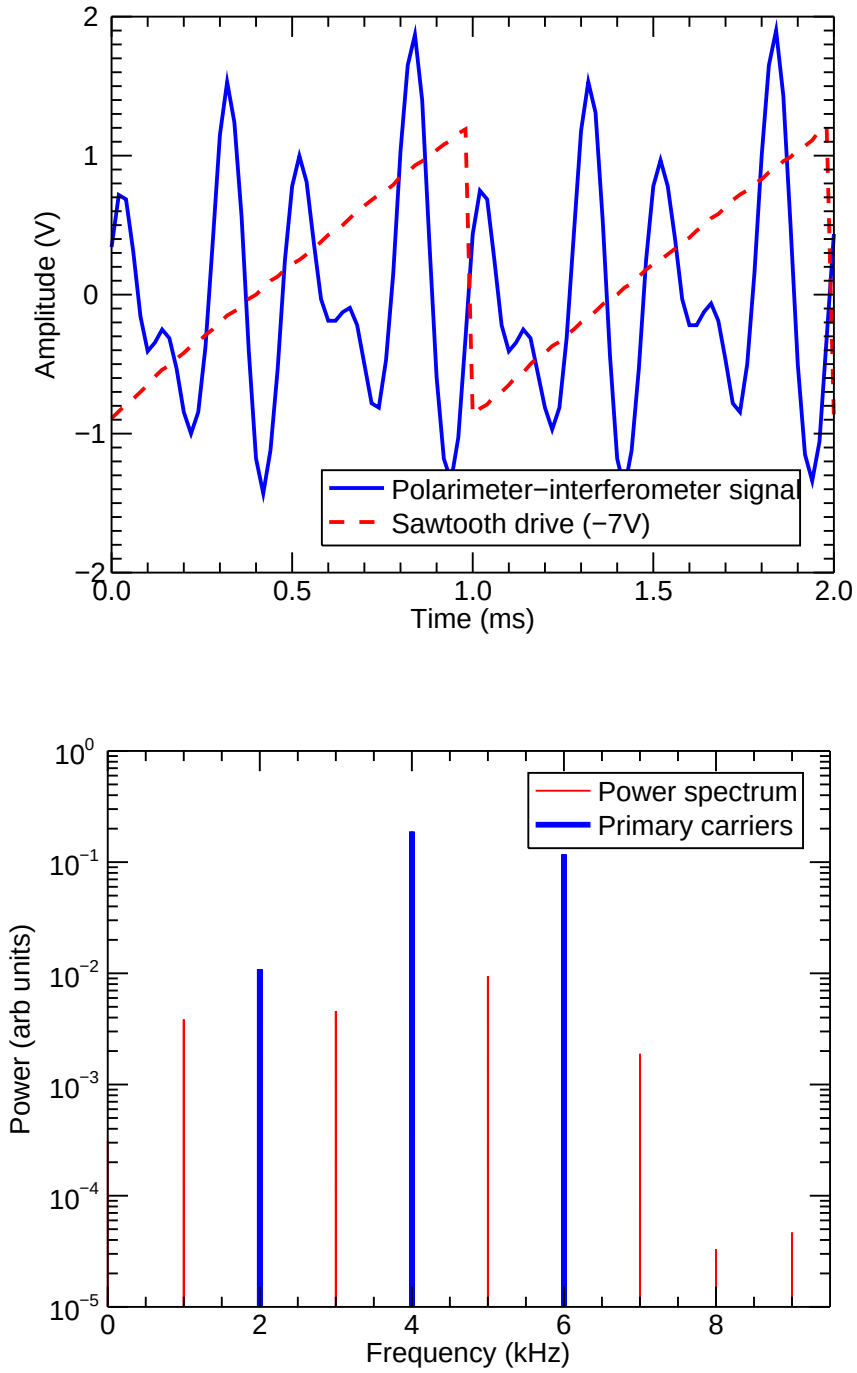

Figure 2. Top: the sawtooth control signal applied to the frequency swept source (dashed) and the resulting polar-interferometric carrier signal. Bottom: the computed carrier signal power spectrum. The primary polarimetric and interferometric carriers are indicated by the bold traces. Note that the sideband power decreases rapidly away from the principal carriers - see harmonics 7, 8 and 9.

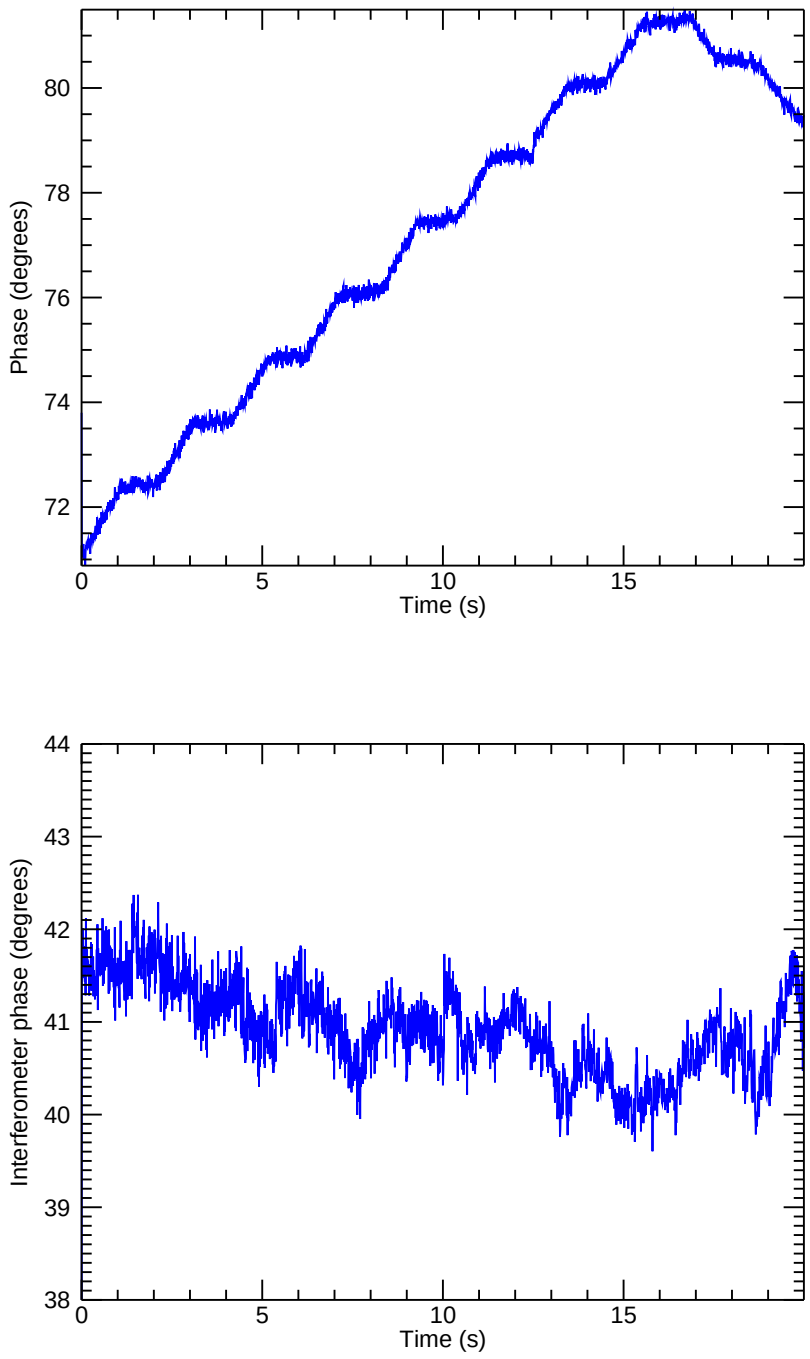

Figure 3. Top: The observed change in polarization angle as the test polarizer is rotated in approximately 1 degree steps and Bottom: the associated interferometric angle (with an arbitrary DC offset). The traces were smoothed using a boxcar filter of width 10 ms. It can be seen that the interferometric signal is only weakly influenced by the imposed step-variation in the polarization angle.

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where $\tan\eta = |E_y| \,/\, |E_x|$ and ϕ is the phase angle between the x and y components of the wave and the time dependence is ignored. The parameters η and ϕ are related to the Faraday rotation ψ of the vibrational ellipse and the ellipticity χ by

$$\begin{aligned} \tan 2\psi &= \tan 2\eta \cos\phi \\ \sin 2\chi &= \sin 2\eta \sin\phi . \end{aligned}$$

The plasma birefringence (Cotton-Mouton effect) is often negligible. It is thus later convenient to ignore this effect, allowing us to set $\phi = 0$ and identify $\eta = \psi$.

The first element of the polarimeter is a retarding optic (reflecting or transmitting) aligned so that the initial electric vector is parallel to one of the characteristic axes. We write its Jones matrix as

$$R_0(\delta_1) = \begin{pmatrix} 1 & 0 \\ 0 & \exp -2j\delta_1 \end{pmatrix}. \tag{A.2}$$

with $\delta_1 = \pi/4$ in the case of a quarter wave delay. This is followed by a second delay component oriented at 45° to the first and having Jones matrix

$$R_{45}(\delta_2) = \begin{pmatrix} \cos\delta_2 & j\sin\delta_2 \\ j\sin\delta_2 & \cos\delta_2 \end{pmatrix}. \tag{A.3}$$

The final component is an ideal analyzer (in our case, a free-standing wire grid polarizer) whose matrix in transmission is

$$P^T_\alpha = \begin{pmatrix} \sin^2\alpha & -\sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha & \cos^2\alpha \end{pmatrix} \tag{A.4}$$

and in reflection $P^R_\alpha = -P^T_{\alpha-\pi/2}$ where α is the polarizer wire orientation. For this polarimeter, the analyzer is oriented at $\alpha = 90^\circ$.

For an initial quarter wave delay followed by a general delay δ, the electric field of the wave transmitted by the final analyzer is given by

$$E_{\mathrm{T}} = P^T_{90} R_{45}(\delta) R_0(\pi/4) E \tag{A.5}$$

This can be easily evaluated, leading to complex scalar electric field

$$\begin{aligned} u_{\mathrm{T}} = \frac{\sqrt{I_0}}{2} \big[&(1+\exp 2j\delta)\cos\eta \\ &-j(1-\exp 2j\delta)\sin\eta \exp j\phi \big] \end{aligned} \tag{A.6}$$

where I_0 is the probe beam intensity.

In order to encode the polarimetric information it is convenient to inject a frequency shifted local oscillator beam which we write as

$$u_{\mathrm{LO}} = \sqrt{I_{\mathrm{LO}}} \exp\left[-j(\omega_i t + \varphi)\right] \tag{A.7}$$

where ω_i is the interferometric carrier frequency difference between the LO beam and the probe, φ is the plasma produced interferometric phase shift and I_{LO} is the local oscillator intensity. The

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combined intensity on a square law detector is given by $I = \mid u_{\mathrm{T}} + u_{\mathrm{LO}} \mid^2$. In the interests of simplicity we assume the plasma linear birefringence is negligible and take $\phi = 0$ and $\psi \equiv \eta$. The intensity at the detector can then be written

$$\begin{aligned} I \;=\; & I_{\mathrm{LO}} + I_0/2\big[1+\cos(2\psi+2\delta)\big] \\ & +\sqrt{I_0 I_{\mathrm{LO}}}\cos(\omega_i t+\varphi-\psi) \\ & +\sqrt{I_0 I_{\mathrm{LO}}}\cos(\omega_i t+\varphi+\psi+2\delta) \end{aligned} \tag{A.8}$$

For a frequency swept source, it is possible to construct the final waveplate delay such that $2\delta = \omega_p t$ where $\omega_p t$ is the polarimetric carrier frequency. In practice, the optical path lengths giving rise to both ω_i and ω_p are chosen such that the carrier frequencies are integer multiples of the frequency $\Omega/2\pi$ of the control sawtooth wave applied to the microwave source.

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