Fano resonance in layered structures with disorder

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Abstract

Transport properties of the layered structures with disorder was studied numerically and analytically both. We find extinction spectra of disordered structure demonstrate the Fano-like line shape on the Bragg frequencies. Besides disorder can transforms conventional stop-band into pass-band. This transformation is governed by disorder-induced Fano resonance. Differen regimes of light localizations were considered.

1. Introduction

The fluctuation of the structure parameters is the most fundamental physical properties of real samples. The simplest model to study fluctuation effects is a disordered one-dimensional (1D) layered structures. Although disordered 1D structures were studied numerically [1,2] and analytically, [3,4] here we reveal that the physics of light localization in such systems is much richer than it was suggested earlier. In particular, we study an interplay between the Bragg scattering and disorder and demonstrate an (completely unexpected) enhancement of the light transmission when the Bragg stop-bands are converted into the Bragg pass-bands. The origin of the unique transmutability of the Bragg bands is the interference between multiple Bragg scattering and disorder-induced single-layer Fabry-Perot scattering, and we demonstrate that this interference can be described within the Fano approach [5]. For the first time to our knowledge, we introduce the Fano resonance where one of the scattering channels is random.

2. Numerical calculations

We calculate by means of the transfer matrix method the extinction χ of 1D layered structure consisting of the alternating slabs A and B. Extinction is defined as $\chi = -\ln(T)/L$, where T is the transmission coefficient, and L is the total length of the structure. The slabs A have constant permittivity ε_A and thickness w_A. The slabs B are allowed to possess some degree of disorder both in thickness (w-disorder, with dispersion σ_{wB}) and in permittivity (ε-disorder, with dispersion $\sigma_{\varepsilon B}$). Fig. 1 demonstrates results of the calculation for 4 typical line shapes. This figure affirms the ergodic assumption for the structures under the study. In particular extinction spectra of long enough structure have well-defined limit, which coincide with ensemble-averaged extinction of structure with moderate layer number.

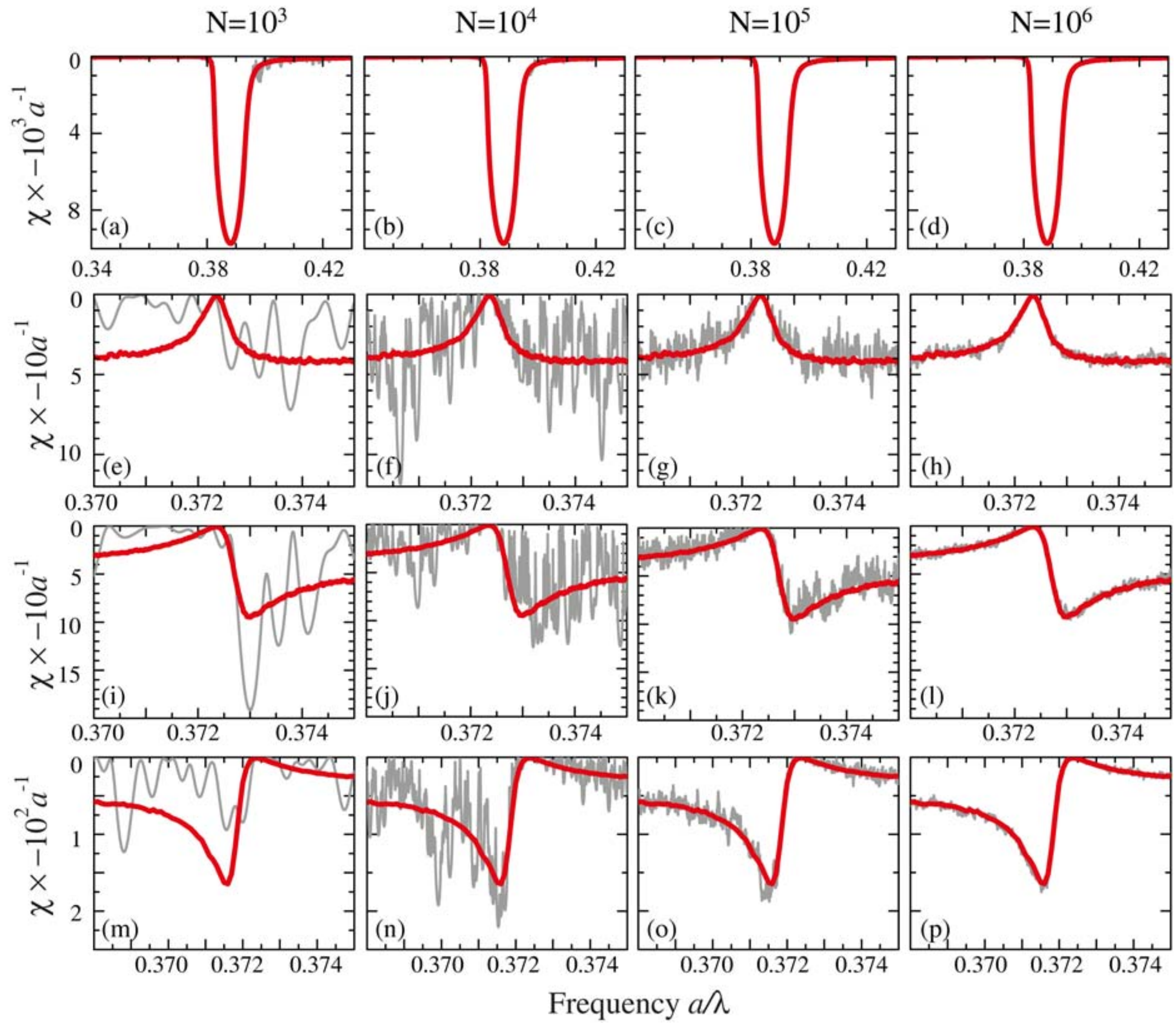

Fig. 1: Extinction spectra as a function of layer number (thin gray curves) and ensemble-averaged extinction spectra of the structure with layer number N=10^4 (thick red curve). First row - structure with stop-band, second row – structure with pass-band, third and fourth rows – structures with asymmetric fano-like line shape.

3. Analytical model

Under the ergodic assumption we have developed a theoretical model to explain the results of numerical calculation. Transfer matrix method allowed us to obtain the Langevin-type equations, describing the electric field evolution under the influence of the random dielectric contrast.[2,4] The Langevin equations were reduced to the Fokker-Planck equation for the phase of the electric field. Focusing on the spectral range close to the given Bragg frequency and assuming weak dielectric contrast we were able to express extinction as a Fano function [5]

$$\chi(\Omega) = \frac{\sin^2 \Phi_h \sigma_{\varepsilon B}^2}{4(w_A + w_B)\varepsilon_A^2} \frac{(\Omega + q)^2}{\Omega^2 + 1} \tag{1}$$

of the dimensionless frequency $\Omega = (\omega - \omega_h)/\Gamma_h$ and the Fano parameter q. Here $\Phi_h = \omega_h \sqrt{\varepsilon_A} w_B / c$ is the average phase incursion in the layers B at the Bragg resonance frequency ω_h.

Far from the Bragg resonance ($|\Omega| >> q$) second factor in Eq. (1) is unity and the extinction is determined by independent Fabry-Perot scattering on different layers B. At the Bragg frequency, corresponding to $\Omega = 0$, the multiple scattering effects become important. In particular, since the layer thicknesses do not fluctuate, interference of waves, scattered on different layers is essential. Interplay between the single-scattering events, determining the broad Fabry-Perot background, and the multiple scattering, essential only in the narrow vicinity of the Bragg frequency, results in the Fano profile (1). The shape of the spectrum is determined by the Fano parameter *q*. It depends on Φ_h and dielectric parameters of the structure. In the simplest case of $\bar{\varepsilon}_B = \varepsilon_A$ Fano parameter reads $q = -\cot \Phi_h$. For $q = 0$ one gets dip in extinction (pass-band) at the Bragg frequency. For $q \to \infty$ extinction has a maximum corresponding to the stop-band. In general case the spectrum is asymmetric.

The Fano regime holds when the thickness w_B is of the order of the wavelength λ, i.e. $1 \le \Phi_h \le 10$. Microscopic origin of the Fano-like effect is related to: (a) a stochastic analogue of the Borrmann effect known as the extraordinary transmission of X-rays through the absorbing crystal at the stop-band edge, and (b) non-vanishing disorder-averaged amplitude reflection coefficient of a single unit cell AB. The result of individual action of these two effects is the enhanced and suppressed transmission, respectively, while being combined they lead to the Fano-type resonance.

3. Conclusions

Results for arbitrary thickness of layers B can be summarized as follows: (i) for $w_B << \lambda$ ($\Phi_h << 1$, similar to the Rayleigh scattering of light on a sphere), extinction spectrum has a narrow pass-band originating from the Borrmann-like effect and, for non-zero average contrast, also an adjoining Bragg stop-band, (ii) in the Fabry-Perot regime $w_B \sim \lambda$ (similar to the Mie scattering on a sphere), the Fano-like extinction strongly and periodically depends on the ratio w_B / λ. Finally, (iii) for the large layer thicknesses ($w_B >> \lambda$, geometrical optics regime) the Fano feature is smeared out. Such transition from the Rayleigh scattering regime to the geometrical optics has been discussed for photonic crystals already in the seminal paper of S. John [1], the distinctive feature of our results is the Fano interference discovered in the intermediate case.

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