

Two Kinds of Logical Impossibility

ALEXANDER SANDGREN

Umeå University

KOJI TANAKA

Australian National University

Abstract

In this paper, we argue that a distinction ought to be drawn between two ways in which a given world might be logically impossible. First, a world w might be impossible because the laws that hold at w are *different* from those that hold at some other world (say the actual world). Second, a world w might be impossible because the laws of logic that hold in some world (say the actual world) are *violated* at w . We develop a novel way of modelling logical possibility that makes room for both kinds of logical impossibility. Doing so has interesting implications for the relationship between logical possibility and other kinds of possibility (for example, metaphysical possibility) and implications for the necessity or contingency of the laws of logic.

1. What Would Be Logically Impossible?

Consider the following two conditionals:

- A. If Classical Logic were the correct logic, it would be impossible to validly infer every proposition from a contradiction.
- B. If LP (Logic of Paradox) were the correct logic, it would be impossible to validly infer every proposition from a contradiction.

These conditionals are examples of what we call *countermetalogicals*.¹ Countermetalogicals are a special kind of counterlogicals which invoke logics that may or may not hold at the actual world in the antecedent.² According to Classical Logic, every proposition can be validly inferred from a contradiction ($p \wedge \neg p \models q$ for every p and q (*ex contradictione quodlibet* (ECQ))). So A should be false. But, since this is exactly the inference that LP (and other paraconsistent logics) disallows, B should be true.

We can get the same kind of effect with pairs of countermetalogicals that both invoke non-classical logics in their antecedent.

- C. If LP were the correct logic, it would be impossible to validly infer p from $\neg\neg p$.
- D. If Intuitionist Logic were the correct logic, it would be impossible to validly infer p from $\neg\neg p$.

Double negation elimination is invalid according to Intuitionist Logic but valid according to LP. So C is false and D is true.

The logics invoked in the antecedents of conditionals do not have to be familiar. Consider Nihilist Logic which invalidates all inferences and Trivial Logic which validates all inferences. We can see the same kind of effect from the following pairs of countermetalogicals:

- E. If Trivial Logic were the correct logic, it would be impossible to validly infer p from p .
- F. If Nihilist Logic were the correct logic, it would be impossible to validly infer p from p .

E is false since Trivial Logic validates all inferences³ and F is true since Nihilist Logic invalidates all inferences.⁴

Countermetalogicals of this kind are commonplace in the contemporary study of logic, though they are not known by that name. Given the proliferation of non-classical logics, it is often crucial to determine which inferences would be valid if this or that logic were correct. Even the most avid classical logician should be able to say which inferences would be valid or invalid if some non-classical logic were correct, even if they say in the very next breath that the non-classical logic is incorrect. Given that we can legitimately ask questions about what inferences are valid or invalid according to this or that logical system—and this is the case even if the notion of ‘the correct logic’ is shown to be misguided⁵—it is important to make sense of countermetalogicals of this sort.

What licenses us to judge that A, C and E are false but B, D and F are true? In making judgments about the truth-values of the countermetalogicals, we have to go through (at least) two steps. First, we make an supposition about the actual laws of logic as specified in the antecedent. In making this supposition, we may have to suspend our judgment about what the correct logic is. Second, we consider what is logically impossible (and possible) *given that supposition*. So when we judge that D is true, for instance, we judge that *under the supposition that Intuitionist Logic is correct*, validly inferring p from $\neg\neg p$ is logically impossible.⁶

We will argue that if we are to make sense of the countermetalogicals like A–F, we need to make room for two kinds of logical impossibility. First, we will present two ways in which a situation can be logically impossible. Second, we will develop a picture of logical possibility/impossibility that makes room for both kinds of impossibility. Third, we will compare our account to an extant proposal in the literature and explore some of the features of the picture we propose. Finally, we will consider some implications of adopting our account and respond to an objection. We will show that our account of logical possibility/impossibility does not only help

us understand countermetalogicals and logical disputes but also has important implications for the relationship between logical possibility and other kinds of possibility such as metaphysical possibility, and for claims about the necessity or contingency of the laws of logic.

2. Logical Impossibility

In evaluating counterlogicals like A–F, we are forced to consider worlds (or situations or points of evaluation) where the laws of logic may be different from what we take them to be. A classical logician, for instance, would have to consider a world where the laws of logic are what LP specifies them to be in evaluating B. If the actual laws of logic are classical, those worlds may be described as logically impossible given that the ‘correct’ logic does not hold at those worlds. However, it is crucial to understand how exactly they can be thought of as logically impossible. As we will see, it is important that we distinguish two kinds of logical impossibility: logical *difference* between worlds and the presence of a *violation* of some set of logical laws at a world. In this section, we will present two ways in which a world can be logically impossible.

2.1 Logically different worlds

One kind of logical impossibility involves difference in which logical laws hold at different worlds. If Classical Logic is correct (i.e., the actual laws of logic are classical), worlds in which non-classical logical laws hold are *logically different* from the actual world. To illustrate, consider a world w_1 where the laws of logic are classical and there are no true contradictions. At such a world, it is valid to infer every proposition from a contradiction (ECQ) even though there are no propositions such that both p and $\neg p$ are true at w_1 . Now consider another world, w_2 , at which the laws of LP hold. At w_2 , one cannot validly infer an arbitrary proposition from a contradiction. The laws of logic that hold at w_1 are different from those that hold at w_2 . They are what we will call *logically different* worlds.

Logically different worlds can be characterised as logically impossible with respect to each other. For instance, in the case under consideration, w_2 is a logically impossible world with respect to w_1 , since ECQ is not a valid inference at w_2 but it is at w_1 . In fact, logically impossible worlds are often described in terms of logical difference.⁷

However, since LP is a sub-logic of classical logic, if w_2 ’s laws are those of LP and w_1 ’s are classical, w_1 *need not* contain a violation of w_2 ’s logical laws even though w_1 and w_2 are logically different. In this way, logical difference comes apart from what we call *logical violation*.

In the next sub-section, we will make the notion of logical violation more precise. In so doing, we will distinguish logical violation from logical difference in order to better understand logical possibility and impossibility.

Before doing so, however, let’s make explicit what we mean by saying that laws of logic *hold* at a world. There are two ways in which we might understand what it is for laws of logic to hold at a world. First, laws of logic may be *operative*. If laws

of logic are operative, they immanently guide our inferences. In proving a theorem, we may remind ourselves of some of the laws of logic in order to direct our thought to a certain conclusion. If logical laws are operative, they are often *necessary* for making inferences and the laws of logic help to explain why we make the inferences we make.

Second, laws of logic may be *corrective* in that they need only specify what the valid inferences are. If we make an inference that deviates from any of the logical laws, we can say that the inference is not valid according to those laws. In the corrective sense, laws of logic serve as the standard to which our inferences should conform and our judgments about validity of inferences can be made in reference to the laws of logic. However, they are not always necessary for making inferences and the holding of this or that set of logical laws does not necessarily help to explain why we make the inferences we do.

In this paper, we will understand logical laws in the corrective sense. We will say that when a set of logical laws holds at a world, there is a list of entailment-statements that are true at that world. These statements specify which inferences are valid according to the logical laws that hold at that world.

Call the list of entailment statements that are true at a world w , Θ_w . Entries in these lists will often take the form: $p \models q$.⁸ The corrective sense of logical laws is more general than the operative sense. According to the operative sense, logical laws are a necessary part of the psychological episodes of inference making. If laws of logic are operative in this way, we can specify a list of entailment-statements that express the way that valid inferences are immanently guided by the laws of logic. In this way the operative sense of laws can be subsumed under the corrective sense of laws. However, the subsumption does not go the other way. The fact that there is a list of entailment statements that are true at a world does not mean that these laws directly guide reasoning at that world. We may make inferential steps that are invalid according to the laws of logic or we might reason in accordance with those laws. But our inferences themselves may not be explained by our ‘grasp’ of some logical laws and so the laws themselves may not explain why we reason as we do. So the corrective sense of logical laws is more general than the operative sense.

2.2 Worlds at which logical laws are violated

We can now characterise logical impossibility in terms of a violation of logical laws. Consider two worlds w_1 and w_2 . Suppose that $\emptyset \models p$ is true at w_1 and p does not hold at w_2 . In this case, we say that the laws of logic that hold in w_1 are *violated* at w_2 . Given that the laws of logic that hold at w_1 are violated at w_2 , w_2 is logically impossible from the point of view of w_1 .

Some worlds that are required to capture the semantics for some of the Lewis’ systems of modal logic such as $S2$ and $S3$ as well as the semantics for relevant logics contain a violation of logical laws in this way. What distinguishes $S2$ and $S3$ from stronger Lewis systems such as $S4$ and $S5$ is the failure of necessitation: $\models p$ does not entail $\models \Box p$. The semantics that captures this failure makes use of the worlds where $\Box p$ fails to be true even if p is a necessary truth according to

some set of logical laws.⁹ Given that the necessity of a necessary truth fails at these worlds, they are worlds that *violate* some logical laws.¹⁰ In relevant logics, in order to undermine the irrelevant conditionals such as $p \rightarrow (q \rightarrow q)$, the semantics for relevant logics require us to consider worlds in which $q \rightarrow q$ fails to be true. But $q \rightarrow q$ is a logical truth of (most) relevant logics. So it and other logical laws are *violated* at these worlds. These examples need not involve logical difference; some are examples of *mere logical violation*.¹¹

There are two ways in which a given set of logical laws may be violated at a world. First, non-logical truths at a world may not cooperate with the logical laws. Suppose that according to the logical laws of w_1 , *disjunctive syllogism* ($p, \neg p \vee q \models q$) is valid. A world at which, for some propositions p and q , p , $\neg p \vee q$, and $\neg q$ are all true contains a violation of w_1 's logical laws. To take another example, if q is not true everywhere where p is false, then it is not necessarily the case that q even if p and $\neg p \vee q$ are true.¹² This may happen if the evaluation of particular inferences (in this case $p, \neg p \vee q \not\models q$ where p and q are propositions) comes apart from the evaluation of inferential forms ($A, \neg A \vee B \models B$ where A and B are meta-variables).¹³

Second, there might be a violation at the level of the logical laws; that is, the laws of logic themselves might violate a given set of logical laws. For example, if the meta-logic of a paraconsistent logic (relevant logics are generally paraconsistent) is also paraconsistent, it may be that an inference is classified by that logic as both valid and invalid.¹⁴ In this way, a set of logical laws Θ_{w_2} might violate another set of logical laws Θ_{w_1} . For instance, this will happen if Θ_{w_2} classifies some inferences as both valid and invalid and Θ_{w_1} are the laws of Classical Logic.

3. The Picture

Now that we have characterised two ways in which a world can be described as logically impossible, we can develop a picture of logical impossibility that can capture both logical difference and logical violation. We will use this picture to shed some light on the *modal structure* of logical possibility and impossibility.

Consider the class of all worlds. Following Nolan (1997) and Priest (2016), we are extremely generous about which worlds feature in this class.¹⁵ We will say that for *any* set of statements Γ (including entailment statements), Γ is true at at least one world and false at at least one world.¹⁶ These worlds sometimes 'access' each other. This accessibility captures logical possibility, in the sense of possibility tied to lack of logical violation. To say that w_1 can access w_2 is to say that w_2 contains no violations of w_1 's logical laws. Conversely, if there is no accessibility from w_1 to w_2 , then w_2 is logically impossible from the point of view of w_1 , again in the sense that w_2 contains a violation of w_1 's laws. A statement p is logically possible from the point of view of some world w , just in case, p is true at at least one world that is accessible from w .

In developing the picture illustrating the modal structure of logical possibility and impossibility, we have made use of the notion of logical possibility and impossibility that is tied to logical violation. However, logical difference can also be

captured in this picture. We can say that a world w_1 is logically different from w_2 if, and only if, either there is some world that is accessible from w_1 that is not accessible from w_2 or that is accessible from w_2 but not from w_1 . In other words, two worlds are logically different if different things count as logical violations according to their respective logical laws.

This picture is not meant to be a complete theory of logical possibility and impossibility. However, as we will see, it provides us with a schema for a theory and it is enough to capture the basic modal structure of logically possible and impossible worlds and distinguish it from other accounts of the modal structure proposed in the literature. Crucially for our purposes, our picture only classifies worlds as logically possible or impossible *with respect to a particular set of logical laws*. What this means is that what is logically possible and impossible varies depending on which world is supposed to be actual.

Countermetalogicals can now be evaluated using the above structure. We can say that the conditional A (If Classical Logic were the correct logic, it would be impossible to validly infer every proposition from a contradiction), for instance, is true just in case the following holds: *if* the actual laws of logic were classical *then* there would be no worlds, accessible from the actual world, in which we could validly infer every proposition from a contradiction. We first suppose that a world at which Classical Logic holds (which may or may not be logically different from the actual world) is actual and then ask which worlds contain a logical violation from the point of view of the actual laws of logic under that supposition. The same goes, making the appropriate substitutions, for other countermetalogicals.

In contrast to countermetalogicals, *counterlogicals* do not always require this two-step process to evaluate. For instance, consider the following counterlogical:

G. If I was human and not human, pigs would fly.

When evaluating G, we do not need to invoke the notion of logical possibility and impossibility tied to logical difference. G's truth-value depends partly on what the actual laws are like. It does not require supposing some world that may be logically different from the actual world is actual. In counterlogicals like G, the antecedent merely concerns a matter of fact which may or may not include a violation of the actual logical laws. Counterlogicals, including countermetalogicals, all have antecedents that may be logically impossible *in some sense*. Countermetalogicals are a special kind of counterlogicals whose antecedents concern the laws of logic directly and may be logically impossible in the sense tied to logical difference. On the other hand, the more familiar kind of counterlogical like G have antecedents that may be impossible only in the sense tied to logical violation. In this way, the distinction between countermetalogicals and other counterlogicals is, for us, tied to the distinction between logical violation and logical difference.

4. Contrasting Our Picture From Priest's

In our picture capturing the modal structure of logical possibility and impossibility, a world cannot be classified as possible or impossible *full stop* but only *relative*

to some set of logical laws. This feature of our account can be brought out by comparing it with that recently presented by Priest (2016). Priest claims that there is a subclass of the class of all worlds that is carved out at *the* class of possible worlds (notice the definite article) (p. 2652). Priest takes the actual world to be a member of this class. The idea is that the actual laws of logic characterise a set of worlds as logically possible in what Priest calls ‘the veridical sense’ (p. 2657). Priest also insists that all the worlds in this class access all and only the worlds in this class (p. 2657). This means that if possibility ‘in the veridical sense’ behaved as Priest suggests, we would be able to make the inferences associated with modal logics as strong as *S4* such as the inference from $\diamond\diamond p$ to $\diamond p$ since any world accessible from a world accessible from the actual world is also accessible from the actual world.

It is with respect to claims about possibility ‘in the veridical sense’ that our picture differs from Priest’s. To begin with, given the motivation to make room for logical difference between worlds, it seems odd to talk about *the* set of logically possible worlds. What is actually logically possible seems to crucially depend on which logical laws actually hold. If this is right, the claim that all the worlds that are accessible from the actual world access all and only those accessed from the actual world should be understood as a claim about the laws that hold at the actual world, rather than a fact about logical space itself. Once we allow for both kinds of logical impossibility, we are driven to say that we cannot classify a given world as possible or impossible *full stop*; we can only say that it is possible or impossible *with respect to some particular set of logical laws*. Unlike Priest’s account, our picture allows the boundary between logical possibility and impossibility to shift, depending on which world (with which logical laws) is considered as actual.

Because of this feature, our picture leaves room for *mere logical difference*, logical difference *without* logical violation.¹⁷ A world w_1 is merely logically different from w_2 just in case w_2 is logically different from w_1 and w_1 does not contain a violation of w_2 ’s logical laws. Since mere logical difference is a relative notion, if one is to capture it, one will have to give up on the claim that there is something about logical possibility *itself* that forces one into validating the inferences associated with *S4* and *S5*. Based on this observation, we can formulate an argument against some of Priest’s contentions.

Consider three worlds w_1 , w_2 , and w_3 . At w_1 , $\neg a$ is true, Θ_{w_1} contains $\emptyset \models \neg a$, and w_1 does not contain a violation of the laws that hold at w_1 . All the same propositions are true at w_2 and w_1 except that the list of laws that hold at w_2 does not contain $\emptyset \models \neg a$. This means that w_2 does not contain a violation of the laws of w_1 and that w_2 is merely logically different from w_1 . At w_3 a is true, and w_1 accesses w_2 and w_2 accesses w_3 but w_1 *does not* access w_3 (since w_3 contains a violation of w_1 ’s logical laws). Priest’s ‘primary directive’ ensures that there will be three worlds with these features (Priest 2016: 2653).

Now, $\diamond\diamond a$ is true at w_1 since w_1 accesses a world, namely w_2 , that accesses a world, namely w_3 , in which a is true. But $\diamond a$ is *false* at w_1 since every world in which a is true contains a violation of w_1 ’s logical laws. So we *cannot* always infer from $\diamond\diamond a$ to $\diamond a$. Put informally, if we allow for mere logical difference, something can be impossible but possibly possible. But this constellation of circumstances is

difficult to fit with Priest's claims because Priest requires that possible worlds access only worlds that access all and only the possible worlds. Given this claim, anything that is possibly possible at a world should also be possible, thus crowding out the kind of mere logical difference involved in the case described.

To relate this point to countermetalogicals, consider H:

H. If Θ_{w_1} were the correct laws of logic, a would be logically possible.

Since Θ_{w_1} contains $\emptyset \models \neg a$, H should be false. But if we insist that the actual world only accesses worlds which access all and only the same worlds as the actual world and suppose that w_1 is the actual world, anything that is possibly possible at w_1 should also be possible at w_1 . So, since a is logically possible from the point of view of w_2 and w_2 is accessible from w_1 , a is possible from the point of view of w_1 after all and H comes out true. But this is a wrong result: H should come out false.

That being said, the actual laws of logic might disallow any worlds being merely logically different from the actual world. For instance, the laws of logic at w_1 could require that any world that w_1 can access, accesses all and only the worlds that w_1 accesses. We discuss this feature of some sets of logical laws below when we come to talk about obstinacy. Nonetheless, if we are right to distinguish logical violation from logical difference and leave room for mere logical difference then one ought to think that the claim that the logically possible worlds (from the point of view of the actual world) are subject to these constraints on accessibility is a *contingent* claim about the actual laws of logic.

Perhaps Priest's talk of 'the' set of logical possibilities and of logical possibility 'in the veridical sense' obeying these constraints could be read as an expression of the conviction that the actual laws of logic are such that they impose the appropriate constraints on logical possibility and impossibility. But if this is so, it is worth emphasising that this is a substantive claim about the actual laws of logic, and not a feature of the overall framework for understanding logical possibility and impossibility in general.

5. Logical Possibility and Other Kinds of Possibility

The conception of logical possibility and impossibility that we have advanced has some consequences for the relationship between logical possibility and other flavors of possibility. It suggests that the connection between logical possibility and other kinds of possibility is not what is often assumed.

It is a common thought that logical possibility does not entail metaphysical possibility. For instance, it is a metaphysical impossibility that something be crimson but not red, but this is not logically impossible.¹⁸ Moreover, it is a common thought that metaphysical possibility entails logically possibility.

Our picture leaves room to doubt this connection between metaphysical and logical possibility. If we are right, it is not a feature of the general framework for characterising logical possibility that the worlds or accessibility relations used to capture logical possibility correspond to those one might use to describe

metaphysical possibility. If there is such connection, it depends on the logical or metaphysical laws that hold at the actual world.

Take, for instance, the claim that metaphysical possibility entails logical possibility. We are not denying this claim. All we argue is that, *if* it is actually true, it is so in virtue of the actual logical laws, the actual metaphysical laws, or both. That is to say, the claim that metaphysical possibility entails logical possibility is a substantive claim about how the logical and/or metaphysical laws actually stand, rather than a claim about logical space itself. If the actual laws of logic and metaphysics are such that, for any world that is metaphysically possible, it does not contain a violation of the actual logical laws, then the inference from metaphysical possibility to logical possibility (in the sense tied to logical violation) will be valid at the actual world. But notice that this validity does not flow from the structure of logical space itself. One might have very good reasons to think that the actual metaphysical and/or logical laws have this feature, but these reasons should be considered independent of one's overall picture of logical space.

The same is true for the connection (or lack thereof) between logical possibility and other kinds of possibility (for example, epistemic, nomic, and deontic possibility). There may be interesting connections, but there is nothing *about the way one ought to model logical possibility itself* that will give rise to those connections.

6. Necessity, Contingency and Obstnacy

At this point one may object that our picture makes logical possibility too contingent. If anything is necessary, says the objector, logical laws are. They could not possibly have been otherwise than they are. Modal logics as strong as *S4* and *S5* at least capture the sense in which logical laws are necessary. By saying that we only get to say that a world is logically possible or impossible relative to some set of logical laws, we have made logical possibility too contingent, so the objection might go.

Our response is that we are able to capture a good sense in which the laws of logic might be necessary within our picture. We call it logical *obstnacy*. The logical laws at a world w_1 are obstinate if, and only if, for any world w_2 that does not contain a violation of w_1 's logical laws, all and only those worlds accessible from w_1 are accessible from world w_2 . In other words, the laws of logic at a world are obstinate if, by the lights of those logical laws, any logical difference counts as a logical violation.¹⁹

This allows inferences analogous to those licensed by *S4* and *S5* to be captured within our picture. For example, under the supposition that the actual logical laws are obstinate, we will be able to infer from the logical possibility of some proposition, p , from the point of view of the actual world, to the logical possibility of p at *all* the worlds that do not contain a violation of the actual laws of logic. Given the supposition that the actual laws of logic are obstinate, we can make the inference from the fact that p is logically possible from the point of view of the actual world to the claim that p is necessarily possible from the point of view of the actual world. Under the supposition that the actual laws of logic are obstinate,

we can infer from $@\diamond p$ to $@\Box\diamond p$, from $@\Box p$ to $@\Box\Box p$, from $@\Box\diamond p$ to $@\Box p$, and so on. In this way, we are able to recapture an interesting sense in which the laws of logic may be necessary; they are necessary if they are obstinate. If they are obstinate, the actual logical laws could not possibly have been different than they are.

We will not attempt to answer the question of whether or not the actual laws of logic are obstinate. All we claim is that nothing in the way one ought to characterise logical possibility in general forces one to believe that the actual laws of logic are obstinate or necessary in any other sense. There may be independent reasons to think that they are but the claim that the laws of logic are necessary in this or any other sense should be understood as a substantive claim about how the laws of logic actually stand.

7. Conclusion

We have argued that we need to make a distinction between two kinds of logical impossibility: one tied to logical difference and another tied to logical violation. This distinction allows us to correctly assess countermetalogicals that are commonplace in the philosophy of logic. The resulting conception of logical possibility and impossibility suggests that the boundary between logical possibility and impossibility is, in a sense, not absolute; what is logically possible and impossible shifts depending on which logical laws are considered as actual. It is this feature of our account that allows us to make proper assessments of countermetalogicals. Moreover, one implication of the shifting nature of the boundary between logical possibility and impossibility is that claims about the connections between logical possibility and other kinds of possibility that are often taken for granted are substantive claims about how the actual world is. For instance, the claim that logical possibility entails other flavours of possibility is a substantive claim about how the laws actually stand rather than a claim about logical space itself. The laws of logic may be necessary in some important sense. However, if they are necessary, there is a kind of contingency about their necessity.

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Notes

- ¹ This terminology is borrowed from Alexander Kocurek and Ethan Jerzak.
- ² In using the term ‘counterlogicals’, we follow Goodman (2009).
- ³ See Russell (forthcoming), especially section 3.1.
- ⁴ See Russell (2018).
- ⁵ See Beall and Restall (2006).
- ⁶ Note that what are called *counterlogicals* do not always require this two step process. In section 3 we discuss the difference between countermetalogicals like A–F and other counterlogicals.
- ⁷ See Cresswell (1973), Mares (1997), and Priest (1992).
- ⁸ We do not insist that *all* logical laws take this form, however.
- ⁹ See Kripke (1963, 1965).
- ¹⁰ For an alternative interpretation of these worlds, see Hughes and Cresswell (1996). They characterise these worlds as worlds where every proposition is possible (p. 201).
- ¹¹ See Tanaka (2013, 2018).
- ¹² This is a possibility in the semantics for relevant logics.
- ¹³ See Tanaka (forthcoming).
- ¹⁴ Strictly speaking, this has not shown to be a possibility (though it has not been shown to be impossible either) as a paraconsistent meta-theory to derive all the properties of a paraconsistent logic has yet to be fully developed. Nevertheless, it is plausible to think that it is possible. See Weber (2013).
- ¹⁵ We also side with Nolan (1997) against Restall (1997) and Mares (1997) in denying that all worlds, possible and impossible, are closed under some logic or other. For instance, we countenance worlds that are not closed under any logic whatever.
- ¹⁶ This claim is analogous to Priest’s ‘Primary Directive’ (Priest (2016: 2653)).
- ¹⁷ Our picture also allows for mere logical violation, the violation of a set of logical laws without a *difference* in which laws govern the relevant worlds. Mere logical violation is interesting for reasons that are not the focus of this paper; see Tanaka (2018).
- ¹⁸ More controversially one might think that ‘water is not H₂O’ is logically possible but metaphysically impossible.
- ¹⁹ In section 4 we discussed an interpretation of Priest according to which his talk of *the* class of possible worlds being able to access all and only the logically possible worlds as a claim about the actual laws of logic. We can now characterise this as the claim that the actual laws of logic are obstinate.

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