

# Efficient Methodologies for Real-time Image Restoration

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DEDICATED TO

MY LATE FATHER, WHO ALWAYS INSPIRED ME WITH HIS  
DETERMINATION

AND

MY MOTHER, WHOSE COMMITMENT TO MY CHILDREN MADE THIS  
THESIS A REALITY.

# Declaration

The contents of this thesis are the results of original research and have not been submitted for a higher degree to any other university or institution. The research represented in this thesis has been performed jointly with Professor Rodney A. Kennedy and Dr. Hongdong Li.

Much of the work in this thesis has been published or has been submitted for publication as journal papers or conference proceedings. In some cases, the conference papers contain material overlapping with the journal publications. The following is a list of those publications.

## Published

1. R. A. Kennedy and P. D. Samarasinghe, “Efficient blind separable kernel deconvolution for image deblurring”, in *International Conference on Signal Processing and Communication Systems, ICSPCS*, Dec. 2008, pp. 1-7.
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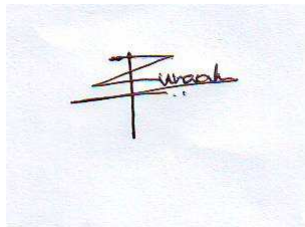
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# Abstract

In this thesis we investigate the problem of image restoration. The main focus of our research is to come up with novel algorithms and enhance existing techniques in order to deliver efficient and effective methodologies, applicable in real-time image restoration scenarios.

Our research starts with a literature review, which identifies the gaps in existing techniques and helps us to come up with a novel classification on image restoration, which integrates and discusses more recent developments in the area of image restoration. With this novel classification, we identified three major areas which need our attention.

The first developments relate to non-blind image restoration. The two mostly used techniques, namely deterministic linear algorithms and stochastic nonlinear algorithms are compared and contrasted. Under deterministic linear algorithms, we develop a class of more effective novel quadratic linear regularization models, which outperform the existing linear regularization models. In addition, by looking in a new perspective, we evaluate and compare the performance of deterministic and stochastic restoration algorithms and explore the validity of the performance claims made so far on those algorithms. Further, we critically challenge the necessity of some complex mechanisms in Maximum A Posteriori (MAP) technique under stochastic image deconvolution algorithms.

The next developments are focussed in blind image restoration, which is claimed to be more challenging. Constant Modulus Algorithm (CMA) is one of the most popular, computationally simple, tested and best performing blind equalization algorithms in the signal processing domain. In our research, we extend the use of CMA in image restoration and develop a broad class of blind image deconvolution algorithms, in particular algorithms for blurring kernels with a separable prop-



erty. These algorithms show significantly faster convergence than conventional algorithms.

Although CMA method has a proven record in signal processing applications related to data communications systems, no research has been carried out to the investigation of the applicability of CMA for image restoration in practice. In filling this gap and taking into account the differences of signal processing in image processing and data communications contexts, we extend our research on the applicability of CMA deconvolution under the assumptions on the ground truth image properties. Through analyzing the main assumptions of ground truth image properties being zero-mean, independent and uniformly distributed, which characterize the convergence of CMA deconvolution, we develop a novel technique to overcome the effects of image source correlation based on segmentation and higher order moments of the source.

Multichannel image restoration techniques recently gained much attention over the single channel image restoration due to the benefits of diversity and redundancy of the information between the channels. Exploiting these benefits in real time applications is often restricted due to the unavailability of multiple copies of the same image. In order to overcome this limitation, as the last area of our research, we develop a novel multichannel blind restoration model with a single image, which eliminates the constraint of the necessity of multiple copies of the blurred image. We consider this as a major contribution which could be extended to wider areas of research integrated with multiple disciplines such as demosaicing.



# List of Acronyms

1D	One Dimensional
2D	Two Dimensional
BCCB	Block Circulant Circulant Block
BTB	Block-Toeplitz-Block
CFA	Color Filter Array
CLS	Constrained Least Squares
CMA	Constant Modulus Algorithm
DFT	Discrete Fourier Transform
EM	Expectation Maximization
FIR	Finite Impulse Response
FSE	Fractionally Spaced Equalizer
FOPDO	First Order Partial Derivative Operators
FSOPDO	First and Second Order Partial Derivative Operators
GTI	Ground Truth Image
ICA	Independent Component Analysis
i.i.d	independently and identically distributed
IRLS	Iterative Re-weighted Least Squares
ISI	Inter Symbol Interference
MAP	Maximum A Posteriori
MCR	Multi Channel Restoration
MIMO	Multiple-Input Multiple-Output
ML	Maximum Likelihood
MRE	Mutually Referenced Equalizer
MSE	Mean Square Error
MSSIM	Mean SSIM
PAM	Pulse Amplitude Modulation
PCA	Principal Component Analysis
PDO	Partial Derivative Operators
PSF	Point Spread Function
PSR	Perfect Source Recovery
rgb	red, green and blue components of a color image
SIMO	Single-Input Multiple-Output
SOPDO	Second Order Partial Derivative Operators

# Notations and Symbols

## Mathematical Variables

In laying out this thesis, some symbols have been reserved in their meaning such as  $\sigma$  and others have been chosen to allow for easy mnemonic reference such as  $n$  for noise,  $g$  for ground truth image, as listed below. In chapters where the number of symbols proliferate uncomfortably, we have tried to reuse symbols, within a similar context as its earlier usage such as  $i$  and  $j$ , used for indexes.

The lowercase letters represent two-dimensional matrices while the bold lowercase represent a vector, where a two-dimensional matrix can be converted into a vector by lexicographically ordering the elements if necessary. Capital letters represent the Fourier transforms of their lower case counterparts with the exception of  $N$ , which is used to represent the normal distribution. Capital letters in square brackets represent special matrices such as Block-Toeplitz-Block matrix or Block-Circulant-Block Matrix.

$g$	Original or ground truth image
$b$	Degraded image
$n$	Noise corruption
$k$	point spread function
$\hat{g}$	Estimate of ground truth image
$\mathbf{g}$	Vector of $g$ : stacks the columns of $g$
$g(i, j)$	A point represented by $i$ th row and $j$ th column in matrix $g$
$\sigma_g$	Standard deviation of $g$
$\mu_g$	Mean of $g$
$G$	Fourier Transform of $g$
$[G]$	Special matrices, such as the Block-Toeplitz-Block matrix of $g$
$L_1 \times L_2$	Support of a matrix

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## Mathematical Operators

$\ \cdot\ $	Frobenius norm
$\ \cdot\ _1$	L1 norm
$E\{\cdot\}$	mathematical expectation
$\mathcal{F}(\cdot)$	discrete Fourier transform
$\partial_x$	first order derivative in $x$ direction
$\partial_y$	first order derivative in $y$ direction
$\overline{(\cdot)}$	complex conjugate
$\star$	element-wise product
$\sum$	Sum over all elements
$\prod$	Product of all elements
$H^T$	Transpose of matrix $H$
$H^{-1}$	Inverse of matrix $H$
$\otimes$	Convolution operator
$\log(\cdot)$	Natural logarithm
$\arg_x$	Argument $x$
$\max$	Maximum
$\min$	Minimum
$N(\cdot)$	Gaussian distribution
$\triangleq$	Defined as being equal to
$p(\cdot)$	Probability



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