Ultra-High Energy Pulse Generation: Dissipative Soliton Approach

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Abstract: We present an equation that allows one to approximately locate the dissipative soliton resonance in the parameter space of the complex Ginzburg-Landau equation. This equation may provide a systematic approach to ultra-high energy pulse generation.

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1. Introduction

The generation of ultra-high energy pulses in the femto-second range is very important for further progress in modern science and technology. In the past, it required a cascade of delicate amplifiers at the output of a mode-locked oscillator, and as a result, the equipment increased dramatically both in size and cost. In recent years, femto-second laser oscillators with pulse energies in excess of 100 nJ have been demonstrated. However, these results are obtained mainly by using "trial-and-error" techniques for each laser design. So far, there is no well-defined theory behind these achievements.

There exists a general approach that allows one to approximate the main properties of pulses generated by passively mode-locked lasers [1]. It is based on a single master equation derived using certain approximations. The resulting equation is the complex Ginzburg-Landau equation. The master equation approach avoids the problem of having separate models for each laser design, and is thus a powerful basis for developing a universal theory in the generation of ultra-high energy pulses from passively mode-locked lasers.

2. Models

The complex cubic-quintic Ginzburg-Landau equation (CCQGLE), in the context of optics, is given by [2]

$$i\psi_z + \frac{D}{2}\psi_{tt} + |\psi|^2\psi = i\beta\psi_{tt} + i\delta\psi + i\varepsilon|\psi|^2\psi + i\mu|\psi|^4\psi - \upsilon|\psi|^4\psi, \qquad (1)$$

where ψ is the envelope of the optical field; t is the retarded time; z is the cavity round-trip number; D determines the cavity dispersion, with a positive value for the anomalous dispersion and a negative value for the normal dispersion; υ accounts for the quintic nonlinear effects; β is responsible for the parabolic gain-dispersion; δ is the linear gain-loss coefficient; and ε and μ describe the cubic and quintic gain-loss in the system.

By solving Eq. (1) numerically, Akhmediev *et al.* [3] have found that the energy of the dissipative solitons increases dramatically when their parameters are located on a special surface in the multi-dimensional parameter space. This phenomenon has been called "dissipative soliton resonance" as this process resembles the resonance phenomenon in the theory of oscillators.

At present, there is no analytic technique that would allow one to find the resonances. However, their approximate locations can be predicted by using a system reduction. In order to obtain the simplified model, a technique called the method of moments [4] is applied. This requires a trial function detailed enough to show the main properties of the high energy soliton solutions of Eq. (1), but simple enough to achieve the desired degree of system reduction. A suitable function for this was found to be a generalized Gaussian function, as given below.

$$\psi(t,z) = A\exp\left(-\frac{t^2}{w^2} - \frac{t^4}{w^4}\right)\exp\left(ict^2\right), \qquad (2)$$

where A is the amplitude, w is the width and c is the quadratic phase parameter. These are variables that can evolve along the propagation direction z. By applying the method of moments using Eq. (2) as a trial function, a set of ordinary differential equations that describe the evolution of the function parameters is obtained as shown below.

$$Q_z = Q\left(2\delta - 3.738\frac{\beta}{w^2} - 1.158c^2w^2\beta + 1.433\frac{Q\varepsilon}{w} + 1.146\frac{Q^2\mu}{w^2}\right),$$

$$w_z = w\left(2cD + 2.142\frac{\beta}{w^2} - 0.874c^2w^2\beta - 0.290\frac{Q\varepsilon}{w} - 0.325\frac{Q^2\mu}{w^2}\right),$$

$$c_z = \frac{1}{w^2}\left(-2c^2w^2D + 6.453\frac{D}{w^2} - 1.237\frac{Q}{w} - 1.319\frac{Q^2\upsilon}{w^2} - 19.624c\beta\right). \qquad (3)$$

The dynamical system Eq. (3) has relatively simple structure. Using standard techniques, one can study the stable fixed points of the reduced system and compare them with the soliton solutions of the CCQGLE.

3. Dissipative soliton resonances

Fig. 1 shows the comparison between the CCQGLE and the reduced system around the region where the ultra-high energy solutions are found. The curves of the energy Q against the dispersion D clearly show that the reduced system well describes the main qualitative features of the CCQGLE. In fact, the indefinite growth of the curves is more pronounced in the reduced model. However, the curves to the left of the resonance do not appear in the CCQGLE, and may represent unstable solutions. The curves on the left may be artifacts of the reduction.

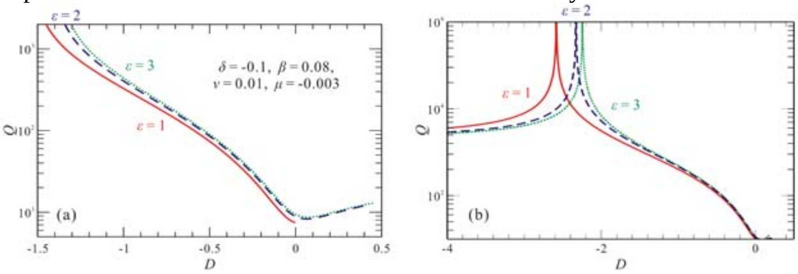

Fig. 1. Soliton energy *Q* against dispersion coefficient *D* for (a) the CCQGLE [3] and (b) the reduced model.

Fig. 2 presents the region of stable solitons in the (ε, D)-plane for the CCQGLE as well as the reduced model. The left-hand-side boundary of the region in Fig. 2(a) corresponds to the resonance in the CCQGLE, while the solid-line in the middle of the region in Fig. 2(b) is the resonance in the reduced model. Again, the grey region to the left of this curve does not appear in the CCQGLE, and hence would not be observed in experiments. Nevertheless, the two models show excellent qualitative agreement.

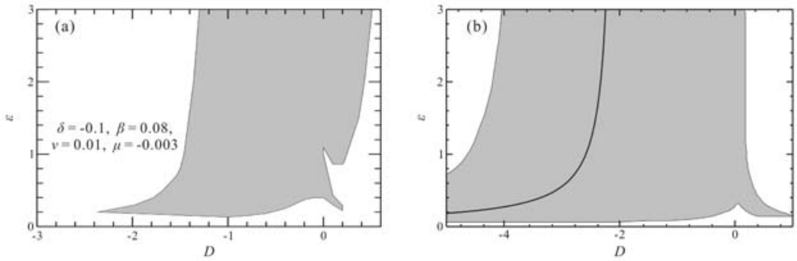

Fig. 2. Region of existence of solitons in the (ε, *D*)-plane for (a) the CCQGLE [3] and (b) the reduced model.

One of the main advantages of constructing the reduced model is that one can obtain an approximate analytic expression for the location of the resonance in terms of the master equation parameters. Using the asymptotic behaviors of Q, w and c when approaching the resonance, the expression can be derived as below.

$$D = \left(\frac{7.778\upsilon}{\mu} - \frac{6.333}{\varepsilon}\right)\beta, \qquad (4)$$

Eq. (4) can be a powerful tool for designing a passively mode-locked laser system generating ultra-high energy pulses, enabling the selection of the cavity design parameters from a broad range of values.

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