

THE GENERATION MODEL OF PARTICLE PHYSICS AND GALACTIC DARK MATTER

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Galactic dark matter is matter hypothesized to account for the discrepancy of the mass of a galaxy determined from its gravitational effects, assuming the validity of Newton's law of universal gravitation, and the mass calculated from the "luminous matter", stars, gas, dust, etc. observed to be contained within the galaxy. The conclusive observation from the rotation curves of spiral galaxies that the mass discrepancy is greater, the larger the distance scales involved implies that either Newton's law of universal gravitation requires modification or considerably more mass (dark matter) is required to be present in each galaxy. Both the modification of Newton's law of gravitation and the hypothesis of the existence of considerable dark matter in a galaxy are discussed. It is shown that the Generation Model (GM) of particle physics, which leads to a modification of Newton's law of gravitation, is found to be essentially equivalent to that of Milgrom's modified Newtonian dynamics (MOND) theory, with the GM providing a physical understanding of the MOND theory. The continuing success of MOND theory in describing the extragalactic mass discrepancy problems constitutes a strong argument against the existence of undetected dark matter haloes, consisting of unknown nonbaryonic matter, surrounding spiral galaxies.

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1. Introduction

Galactic dark matter is matter hypothesized to account for the discrepancy of the mass of a galaxy determined from its gravitational effects, assuming the validity of Newton's law of universal gravitation, and the mass calculated from the "luminous matter", stars, gas, dust, etc. observed to be contained within the galaxy.

Early preliminary evidence for this mass discrepancy was found by Oort¹ in 1932 and Zwicky² in 1933. Oort, during a detailed study of the amount of matter in the disk of the Milky Way, hypothesized the existence of dark matter to account for the

orbital velocities of stars in the Milky Way. Although later work indicated³ that the disk of the Milky Way probably contains little dark matter, Oort's work introduced the idea of dark matter on a galactic scale. Zwicky, during an investigation of the motions of galaxies in clusters of galaxies, hypothesized the existence of very large amounts of dark matter to provide an explanation for the orbital velocities of the galaxies. Although, Zwicky's observations were essentially valid they had less impact on understanding the mass discrepancy problem than Oort's work, partly because of the difference in their personalities and partly because, Zwicky's observations on galactic clusters required much larger amounts of dark matter than Oort's observations on the disk of the Milky Way.³ Indeed, the mass discrepancy problem lay dormant for nearly three decades, in spite of the fact that Babcock,⁴ by measuring the "rotation curve" for the Andromeda galaxy, found that the mass-to-luminosity ratio increases radially but this was not attributed to any missing matter.

The rotation curve of a galaxy is the dependence of the orbital velocity of the visible matter in the galaxy on its radial distance from the center of the galaxy. Further measurements of the rotation curve of the Andromeda galaxy were undertaken by Rubin and Ford⁵ in the late 1960s. They found that the observed rotation curve implied that the total mass of the galaxy increased linearly from 4 kpc to about 14 kpc from the galactic center and more slowly thereafter. This was in agreement with Babcock but again they did not attribute the increase in mass to any missing matter.

While studying the surface photometry of spiral galaxies, Freeman^{3,6} found rotation curves for a small number of galaxies which did not agree with expectation based upon the assumption that the galaxies consisted of stars, gas and nothing else. He suggested that these galaxies contained considerably more invisible (dark) matter than visible matter but not in the disks and its distribution must be quite different from the approximate exponential distribution which holds for the luminous galaxy.

In 1974, Ostriker, Peebles and Yahil⁷ stated that the current observations strongly indicated that the mass of a spiral galaxy increases almost linearly with radius to nearly 1 Mpc so that the total mass was of the order of 10^{12} solar masses and that the ratio of this mass to the observed luminous mass was large. Their evidence included neutral hydrogen rotation curves of three nearby spiral galaxies measured by Roberts and Rots⁸ using 21 cm line data. They concluded that the very large mass-to-light ratio and the very great extent of spiral galaxies could most plausibly be understood as due to a giant halo of dark matter. This conclusion agreed with earlier work by Ostriker and Peebles⁹ that spiral galaxies needed to be surrounded by a large halo of dark matter in order to be stable, although this argument was later shown to be untenable.¹⁰

Further evidence for the mass discrepancy in spiral galaxies was found in the 1970s by Rubin, Ford and Thonnard¹¹ who obtained high quality optical rotation

curves for many spiral galaxies. These rotation curves were essentially “flat” at the extremities of the visible matter. However, this evidence was not considered conclusive for dark matter since each rotation curve did not reach out far enough from the center of the galaxy.³ In 1978, the results of Rubin, Ford and Thonnard were supplemented by the work of Bosma¹² who compiled 21 cm rotation curves for a number of spiral galaxies and showed that these rotation curves were essentially flat out to the edge of the 21 cm data, which extended far beyond the visible matter of each galaxy.

Thus, by 1980 it was clear that there existed a serious mass discrepancy problem for spiral galaxies. An important conclusion, from all the evidence involving extragalactic distance scales, is that the mass discrepancy is greater the larger the distance scales involved: e.g., cluster of galaxy distance scales indicate greater mass discrepancies than single galaxy distance scales, and the outer regions of a spiral galaxy exhibit a greater mass discrepancy than the inner regions of the same galaxy.

The conclusive observation from the rotation curves of spiral galaxies that “the mass discrepancy is greater the larger the distance scales involved” implies that either Newton’s law of universal gravitation requires a significant modification in order to provide a stronger gravitational field than expected at large (galactic) distance scales or considerably more mass is required to be present in each galaxy. In the conventional cosmological model¹³ of spiral galaxies, each spiral galaxy is considered to be surrounded by a giant halo of invisible (dark) matter, which provides a large contribution to the gravitational field at large distances from the center of the galaxy. Since spiral galaxies provide the most extensive and clearest evidence for this dark matter hypothesis, the discussion in this paper concerning the galactic mass discrepancy will be restricted to such galaxies.

Both the modification of Newton’s law of gravitation and the hypothesis of dark matter scenarios have been considered, although until recently the existence of dark matter surrounding spiral galaxies has been favored.³ In 1983, Milgrom¹⁴⁻¹⁶ developed a modification of Newtonian dynamics known as the modified Newtonian dynamics (MOND) theory as a possible alternative to dark matter. Recently, Milgrom’s approach has been applied to extragalactic systems with considerable success.¹⁷ This is discussed in Sec. 2.

The main purpose of this paper is to show that the Generation Model (GM) of particle physics,¹⁸⁻²⁰ which also leads to a modification of Newton’s law of universal gravitation especially at large (galactic) distance scales, may describe the observed cosmological data for spiral galaxies. This is discussed in Sec. 3. Section 4 states the conclusion.

2. Modified Newtonian Dynamics

Bekenstein²¹ has provided a useful discussion of the two approaches: (i) dark matter and (ii) MOND, in attempts to provide an understanding of the “galactic rotational problem” for spiral galaxies. He states that there are two overarching empirical facts

in the phenomenology of spiral disk galaxies, which need to be described by any successful theoretical model.

First, the work of Bosma¹² as well as Rubin and coworkers^{5,11} indicated that stars and gas clouds in the disks of spiral galaxies, which serve as tracers of the gravitational field, circle around each galaxy's center with a linear velocity which first rises as one moves out from the center of the galaxy but then remains fairly constant as the distance (r) increases to well beyond the visible disk's edge. The observed "flat" rotational curves at large distances from the centers of the galaxies are completely at variance with Newtonian gravitation, which predicts that the rotation curves should fall off as $1/\sqrt{r}$ outside the luminous parts of these galaxies.

Second, as pointed out by Tully and Fisher,²² for spiral galaxies there is a good correlation between the global neutral hydrogen line profile width, a distance-independent observable, and the absolute magnitude of the galaxy. It is also well known that the intrinsic luminosity of a galaxy is correlated with the total visible mass. The Tully-Fisher correlation is an empirical relationship between the intrinsic luminosity L (proportional to the total visible mass) of a spiral galaxy and the velocity, v_f , of the matter circulating at the extremities of the disks of the galaxies (i.e., at large distances from the centers of the galaxies corresponding to the outer flat parts of the rotation curves). In fact

$$L \propto v_f^\alpha, \tag{1}$$

where α is approximately four.

The flat extended rotation curves have provided the most convincing evidence for dark matter on galactic scales: in the flat part of a rotation curve, the rotation velocity r is independent, so that the centripetal acceleration goes as $1/r$ and the gravitational field must be decreasing as $1/r$. According to Poisson's equation of Newtonian theory, such a gravitational field (assuming spherical symmetry) must be sourced by a mass distribution with a r^{-2} profile. Since the visible mass density in the inner disk drops much faster (exponentially) than this, it is consistent to assume that the total mass distribution in the outer parts of the galaxy is approximately spherical. The conclusion is reached that each spiral galaxy must be immersed in a spherical dark matter halo with a mass density profile tending at large r to r^{-2} . In Newtonian dynamics the circular velocity is expected to be

$$v(r) = \sqrt{GM(r)/r}, \tag{2}$$

where G is Newton's gravitational constant, $M(r) = 4\pi \int \rho(r)r^2 dr$ and $\rho(r)$ is the mass density profile, which should be falling $\propto 1/\sqrt{r}$ beyond the visible disk. The empirical fact that $v(r)$ is roughly constant implies the existence of a dark matter halo with $M(r) \propto r$ and $\rho \propto r^{-2}$.

However, the hypothesis of a dark matter halo surrounding a spiral galaxy to account for the observed "flat" rotation curve of a galaxy has yet to be verified and also there are several outstanding puzzles.^{3,21} First, the nature of the proposed dark matter is unknown, although it is now considered to be nonbaryonic matter. Second,

a dark matter halo has yet to be detected directly, although many searches have been carried out. Third, the density profile of a typical dark matter halo, within Newtonian physics, is required to be fine-tuned in order to produce the observed flat rotation curve of a spiral galaxy. Fourth, the lack of dark matter in globular clusters is still a mystery: large globular clusters have about the same mass as the smallest dwarf galaxies, although their diameter is only about a tenth the diameter of a typical dwarf galaxy, which are considered to have considerable amounts of dark matter.

Consequently, in view of these considerable uncertainties concerning the existence and nature of the proposed dark matter, there have been several attempts to modify Newton's law of gravitation, instead of introducing dark matter. The most successful of these modifications is that proposed by Milgrom,^{14–16} who suggests that gravity varies from the prediction of Newtonian dynamics for low accelerations: in order to explain both the flat rotation curves of spiral galaxies and the Tully–Fisher relation, the transition to $1/r$ gravity should occur below a critical “acceleration” a_0 , rather than beyond a critical length scale r_0 .²³ While r_0 is not fixed from one galaxy to the next, a_0 is, and the modified law of gravity in terms of a_0 leads directly to the Tully–Fisher relation, whereas the modified law in terms of r_0 does not. The modified law of gravity in terms of a_0 is

$$g = GM/r^2 + (GMa_0)^{1/2}/r, \tag{3}$$

where g is the gravitational acceleration. The asymptotic flat rotational velocity of a point mass in the limit of very small accelerations is then

$$v_f = (GMa_0)^{1/4}, \tag{4}$$

for which, if the mass to luminosity ratio, M/L , is roughly constant for galaxies, leads to the Tully–Fisher relation:

$$L \propto v_f^4. \tag{5}$$

On the other hand, the corresponding modified law of gravity in terms of r_0 may be written as

$$g = GM/r^2 + GM\epsilon/(rr_0), \tag{6}$$

where ϵ is a factor representing the relative strengths of the modified and Newtonian gravitational fields. In this case, the asymptotic flat orbital velocity of a point mass for $r \gg r_0$ is

$$v_f = (GM\epsilon/r_0)^{1/2}, \tag{7}$$

which leads to

$$L \propto v_f^2, \tag{8}$$

in gross disagreement with the Tully–Fisher relation.

Thus Milgrom's MOND theory, based upon a critical acceleration, describes the two overarching facts stated by Bekenstein²¹ and discussed above to be a necessary

requirement for any successful theoretical model, while a modified law of gravity, based upon a critical length scale, does not. Indeed, Milgrom was the first to point out that any deviation from Newton's law in galactic systems had to appear below a critical acceleration in order to be consistent with observations.

Sanders²³ has made some interesting comments concerning both the MOND theory and the alternative hypothesis of dark matter.

First, in a conceptual sense, Sanders considers that it is better to view Milgrom's proposal as a modification of "gravity" rather than as MOND suggests "Newtonian dynamics", since the latter leads to problems such as the nonconservation of linear momentum of an isolated system.²⁴

Second, whether or not MOND is correct in its intended implication that gravity is non-Newtonian in the limit of low accelerations, it is certainly correct in the sense that mass discrepancies in galaxies tend to appear below a critical acceleration.

Third, even if a modification like MOND is in some sense correct, it is "incomplete". Such an idea must have its basis in a general theory of gravity, which makes some connection to more familiar physics. Viewed in this way, the central issue becomes the theory of gravity and not the explanation of flat rotation curves and the Tully–Fisher relation. These observations are simply signals that something is missing from the current theory of gravity. In particular, something is missing in General Relativity, even in the classical limit, just as anomalous planetary precession indicated an incompleteness of Newtonian theory. If one takes this point of view, then the problem with General Relativity is that it possesses the wrong weak field limit.

Fourth, Sanders concludes: "In the context of dark matter, any observed rotation curve can be explained by adjusting the parameters of some model for a dark halo or dark disk; the dark matter hypothesis can never be falsified. In this sense, it is quite accurate to compare the dark matter hypothesis to the medieval system of crystal spheres. Any improvement in the accuracy of observations of the apparent motion of planets across the sky, could be accommodated in the system of unseen spheres simply by adding additional spheres. The system worked perfectly; it finally just became rather cumbersome. MOND, on the other hand, is quite analogous to Kepler's laws of planetary motion. Kepler's three rules worked no better than the older epicyclic hypothesis, but they were a far more efficient summary of the phenomena. Carrying the analogy further, it should be stressed that Kepler's rules were a mathematical prescription without physical content in the modern sense. It remained for Newton to unify such diverse phenomena as planetary motion and falling objects on earth in a single law of universal attraction. In a similar way, MOND, if it is correct, remains incomplete."

In Sec. 3, we shall show that the GM of particle physics,^{18–20} which also leads to a modification of Newton's law of gravitation, not only describes the two overarching empirical facts in the phenomenology of spiral galaxies, but also provides a physical basis for the modification.

3. GM of Particle Physics

The GM²⁰ is an alternative model to the Standard Model (SM) of particle physics.²⁵ This model provides agreement with the SM for all the transition probabilities arising from every interaction involving any of the six leptons or the six quarks, which form the elementary particles of the SM.

In the GM, both leptons and quarks have a substructure, consisting of spin- $\frac{1}{2}$ massless particles, rishons and/or antirishons, each of which carries a single color charge. The constituents of leptons and quarks are bound together by strong color interactions, mediated by massless vector hypergluons, acting between the colored charged rishons and/or antirishons. These strong color interactions of the GM are analogous to the strong color interactions of the SM, mediated by massless vector gluons, acting between colored charged elementary quarks and/or antiquarks.

In the GM²⁰ between any two leptons and/or quarks there exists a residual interaction arising from the color interactions acting between the constituents of one fermion and the constituents of the other fermion. Robson¹⁸ proposed that such “inter-fermion” color interactions may be identified with the usual gravitational interaction. More recently, Robson¹⁹ has presented a quantum theory of gravity, briefly described below, based upon the earlier conjecture.

The mass of a body of ordinary (baryonic) matter is essentially the total mass of its constituent electrons, neutrons and protons. In the GM, each of these three particles is considered to be colorless (a colorless particle contains one color charge of each of the possible color charges: red, green and blue). All three particles are assumed to be essentially in a three-color antisymmetric state, so that their behavior with respect to the strong color interactions is expected basically to be the same. This similar behavior suggests that the inter-fermion interactions of the GM between electrons, neutrons and protons have several properties associated with the usual gravitational interaction: universality, infinite range, very weak strength and attraction.^{19,20}

In the GM, the above inter-fermion color interactions suggest a universal law of gravitation, which closely resembles Newton’s original law that a body of mass m_1 attracts another body of mass m_2 by an interaction proportional to the product of the two masses and inversely proportional to the square of the distance (r) between the centers-of-mass of the two bodies:

$$F = H(r)m_1m_2/r^2, \tag{9}$$

where Newton’s gravitational constant is replaced by a function of r , $H(r)$.

This difference arises from the self-interactions of the hypergluons mediating the inter-fermion color interactions.^{19,20} These self-interactions cause antiscreening effects,^{26,27} which lead to an increase in the strength of the residual (inter-fermion) interaction acting between the two masses, so that H becomes an increasing function of r .

In a spiral galaxy, the gravitational interaction of a point mass at a distance r from the center of the galaxy will depend upon two factors: (i) the total mass

M distributed within the sphere of radius r and (ii) the nature of the function, $H(r)$. Thus, for small values of r , these two factors will be entwined, each making a contribution to the orbital velocity of the point mass. However, for large values of r , only the second factor, $H(r)$, will make a significant contribution to the orbital velocity.

It is known from particle physics that the strong color interactions tend to increase with the separation of color charges in order to confine quarks within baryons, so that one expects $H(r)$ to increase as a function of r . The flat rotation curves observed for spiral galaxies indicate that $H(r)$ is essentially a linear function of r . In this case, the modified law of gravity based upon gravity being identified with the very weak residual color interactions may be written as

$$g = GM/r^2 + GM\epsilon/(rr_S), \tag{10}$$

where we have used

$$H(r) = G(1 + \epsilon r/r_S). \tag{11}$$

Here, G is Newton’s gravitational constant, ϵ is again a factor representing the relative strengths of the modified and Newtonian gravitational fields and r_S is a radial length scale, dependent upon the radial mass distribution of the spiral galaxy, i.e., r_S varies from galaxy to galaxy.

It should be noted that the MOND modified theory of gravity, based upon a critical acceleration a_0 , implies that the modified gravitational interaction is associated with a different radial parameter for each galaxy, unlike the corresponding modified law of gravity in terms of a critical length scale r_0 . This is the fundamental reason why the MOND theory satisfies the Tully–Fisher relation and the critical length scale theory does not.

As indicated above, the GM expression, Eq. (10), for a modified gravitational interaction is also associated with a radial parameter, r_S , which varies from galaxy to galaxy. Indeed, using Eqs. (3) and (10), one can relate the modified terms in the gravitational acceleration expressions to obtain

$$a_0 = GM\epsilon^2/r_S^2. \tag{12}$$

Thus, the scale factor r_S may be regarded as the radial parameter beyond which weak acceleration takes place, and the value of r_S will depend upon the radial mass distribution of the galaxy. Equation (12) implies that the physical basis of the critical weak acceleration a_0 of the MOND theory is the existence of a radial parameter r_S , which defines a region beyond which the gravitational field behaves essentially as $1/r$. This occurs in the GM as a consequence of the universal nature of the weak color residual interaction identified as the universal gravitational interaction.

Equation (10) describes both the flat velocity rotation curves of spiral galaxies and also the correlation of their asymptotic orbital velocity with their luminosity, the Tully–Fisher relation.

If one assumes that r_S is sufficiently large so that practically all the mass of a spiral galaxy is contained within a sphere of radius r_S (spherical symmetry is assumed for simplicity), then the asymptotic flat rotational velocity of a point mass for $r > r_S$ is given by

$$v_f = (GM\epsilon/r_S)^{\frac{1}{2}}. \tag{13}$$

Furthermore, the relative luminosity L may be written in terms of the average surface brightness Σ and the radial parameter r_S :

$$L = 4\pi\Sigma r_S^2, \tag{14}$$

so that

$$v_f^4 = (GM\epsilon/r_S)^2 \propto (M/L)^2(L/r_S)^2 \propto \tau^2\Sigma L, \tag{15}$$

where $\tau = M/L$. Equation (15) is the Tully–Fisher relation provided $\tau^2\Sigma$ is approximately constant. Indeed, as discovered by Freeman³ in 1970, for galaxies which fall into the high surface brightness (HSB) family of galaxies, both τ and Σ are approximately constant, so that Eq. (15) corresponds to the Tully–Fisher relation. However, in 1976 Disney²⁸ suggested that the constancy of the disk surface brightness noticed by Freeman was not a physical reality, but instead was an artifact of sample selection. More recently, another family of spiral galaxies has been observed.^{29,30} These galaxies have a much fainter surface brightness and this family is called low surface brightness (LSB) galaxies for which Σ is quite different from that for HSB galaxies. However, observation indicates that the LSB galaxies also satisfy the same Tully–Fisher relation as the HSB galaxies.³¹ This implies that $\tau^2\Sigma$ is approximately a constant for both families of galaxies and this has been verified^{29,32} for spiral LSB galaxies.

4. Conclusion

Gravity in the GM is identified with the very weak universal and attractive residual color interactions acting between the particles of ordinary matter (electrons, neutrons and protons). This gravitational interaction is mediated by vector bosons (hypergluons), which self-interact so that the gravitational interaction coefficient H (Eq. (11)) gradually increases in intrinsic strength as the separation of the interacting color charges increases. This unusual property of the GM gravitational interaction leads to a significant modification of Newton’s law of universal gravitation, especially at large distance scales.

Observation of the flat rotational curves at large distances from the centers of spiral galaxies indicates that the gravitational field for spiral galaxies changes from the $1/r^2$ dependence of Newton’s law to a $1/r$ dependence for large values of r . Such a transition in the gravitational field is simply accommodated within the GM-modified law of gravity by the addition of the term $GM\epsilon/rr_S$ as in Eq. (10).

This modification of Newton's universal law of gravitation is found to be essentially equivalent to that of Milgrom's MOND theory, with the GM modification providing a physical understanding of the MOND theory. Furthermore, the continuing success^{17,32} of MOND theory in describing the extragalactic mass discrepancy problems constitutes a strong argument against the existence of undetected dark matter haloes, consisting of unknown nonbaryonic matter, surrounding spiral galaxies.

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