

Radial earth models revisited

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SUMMARY

The current set of reference models for the radial variation of Earth structure have been in use for several decades, and provide a good representation of many aspects of the seismic wavefield. Nevertheless, strong constraints from the differential times between pairs of *SmKS* phases indicate the need to modify the *P* wave speed profile in the upper part of the outer core. In order to incorporate such a change and maintain the representation of the full suite of seismic phases compensatory adjustments have to be made, dominantly in the mantle. Using multi-objective optimization, a new preferred radial model *ek137* has been generated that provides a good representation of the traveltimes of all core phases. An adiabatic profile can be maintained through most of the outer core, but departures are needed at the base, as in the *ak135* model. The latest estimates for inner core shear wave speed are included in *ek137*.

Key words: Core; Structure of the Earth; Body waves.

1 INTRODUCTION

It is now nearly 40 years since the development of the Preliminary Reference Earth Model (PREM) by Dziewoński & Anderson (1981), and 30 years since the publication of the *iasp91* travel-time model for earthquake location (Kennett & Engdahl 1991). The *ak135* model with an improved rendering of later seismic phases followed in 1995 (Kennett *et al.* 1995). This group of radial earth models have taken dominant roles as references for a wide range of studies of the Earth. Yet, they have significant limitations that are often not recognized by their users.

Since these models were developed, a number of studies have indicated the need for slight modification of the radial profile in the core. However, one cannot make piecemeal adjustments to radial structure, because even a small change in seismic wave speed affects a wide range of seismic phases. By using multi-objective optimization it is possible to find a radial representation that preserves the desirable features of existing earth models, but also is compatible with the additional constraints.

We first review the character of current models, and look at the way that these relate to their mode of construction, and then consider the way that new information can be incorporated to introduce mild changes in the radial wave speed profiles, notably in the upper part of the outer core. The resulting model *ek137* should provide a suitable reference for body-wave studies of core phases. An attempt is also to provide an assessment of the likely uncertainty of seismic wave speeds as a function of depth.

1.1 Exploiting normal modes

The radial earth model PREM model of Dziewoński & Anderson (1981) was a contribution to the Standard Earth Model project sponsored by the International Association of Seismology and Physics of the Earth's interior (IASPEI). PREM includes transverse isotropy for seismic wave speeds in the upper part of the mantle. The model is parametric with seismic wave speeds and density represented as piecewise polynomials in radius, following the approach initiated by Dziewoński *et al.* (1975) with the PEM model. This parametric representation provides only a modest number of independent variables, and so enabled a wide range of data to be used in the inversion without excessive computational effort.

For PREM the data sets comprised the values of the mass and moment of inertia of the Earth, and an extensive set of centre frequencies for free oscillation, both spheroidal and toroidal, and travel-time information. PREM consists of both a radial velocity model and a set of time offsets and slope corrections for traveltimes (Table 1). These additional parameters were introduced to compensate for uncertainties in epicentral locations and frequency effects from attenuation.

The primary data set was a suite of more than 1000 normal mode frequencies, with attenuation factors where measured. The mode data provided the main control on absolute seismic wave speed, and the body wave data was used mainly for control on finer-scale variations in wave speed (Dziewoński & Anderson 1981).

Table 1. Traveltime information used in construction of PREM.

Differential times			
Phases	Source		
<i>PcP</i> – <i>P</i>	Engdahl & Johnson (1974)		
<i>PKiKP</i> – <i>PcP</i>	Engdahl <i>et al.</i> (1974)		
<i>ScS</i> – <i>S</i> deep focus (600 km) <i>SH</i> & <i>SV</i>	Jordan & Anderson (1974)		
Absolute times			
Phase	Source	Baseline correction	slope
		[s]	[s deg ^{−1}]
<i>P</i>	Global average from ISC data	− 1.88	− 0.0085
<i>P</i>	Hales <i>et al.</i> (1968)	− 1.18	0.0085
<i>P</i>	Herrin <i>et al.</i> (1968)	− 0.94	− 0.0141
<i>P</i> deep focus (550 km)	Sengupta & Julian (1976)	0.67	− 0.0173
<i>PKP</i> _{AB}	Gee & Dziewonski	− 4.22	
<i>PKP</i> _{BC}	Gee & Dziewonski	− 2.18	
<i>PKiKP</i> (<i>PKP</i> _{DF})	Engdahl <i>et al.</i> (1974)	− 2.59	
<i>S</i> (<i>SH</i>)	Gogna <i>et al.</i> (1980)	− 1.87	0.0329
<i>S</i> (<i>SV</i>)	Gogna <i>et al.</i> (1980)	− 0.29	0.0188
<i>S</i> (<i>SH</i>)	Hales & Roberts (1970)	− 1.14	− 0.0068
<i>S</i> (<i>SV</i>)	Hales & Roberts (1970)	0.44	− 0.0210
<i>S</i> (<i>SH</i>) deep focus (550 km)	Sengupta (1975)	2.28	− 0.0433
<i>S</i> (<i>SV</i>) deep focus (550 km)	Sengupta (1975)	3.00	− 0.0495
<i>ScS</i> (<i>SH</i>) deep focus (550 km)	Sengupta (1975)	− 0.57	
<i>SKS</i> (<i>SV</i>)	Hales & Roberts (1970)	− 3.13	

PREM has been a very effective reference model, and remains the dominant model used for longer-period studies. However, traveltime offsets from the predictions of the radial model have persisted for many phases, particularly for core phases, even with improved event locations. As a result PREM is rarely used for short periods.

The parametric representation in PREM certainly provides an economical description of the earth model, but imposes specific functional forms on the behaviour of seismic wave speeds and density in the different sectors of the Earth. For example, seismic wave speed in the lower mantle is represented by a single cubic polynomial in radius. In consequence, the wave speed gradients are forced to a quadratic polynomial and have no independent information. In hindsight, an equally economical polynomial representation in terms of elastic moduli and density would have provided much greater flexibility, and would be more closely aligned to the configurations of equations of state (Kennett 2006).

1.2 Traveltime models

During the 1930s Jeffreys & Bullen developed a set of empirical traveltime tables for major seismic phases, with careful analysis to overcome the limitations of time keeping and limited control on the location of earthquakes. Their set of traveltime tables (Jeffreys & Bullen 1940) became the standard for international event location. As well controlled explosion information became available, it became clear that the tables for *P* were too slow by around 1.8 s at teleseismic distances. In part, this offset arose from piecing together the best constraints from around the globe, with some consequent regional biases. A further limitation was that depth estimates from direct location were discordant with those extracted from the analysis of depth phases, for example, *pP*–*P*.

In 1988 IASPEI established a Working Group on Traveltime Tables, and a preparatory workshop established the concept that a radial earth model would be used to match the behaviour of empirical traveltimes. Calculations for the model could then be

used directly rather than relying on interpolation in tables, and so aid inversion.

Initial attention was focused on *P* and *S*, with a parametric model of a similar style to PREM with isotropic seismic wave speeds. The *iasp91* model (Kennett & Engdahl 1991) specified *P* and *S* wave speeds through the mantle, and added core structure from PEM (Dziewoński *et al.* 1975) to provide traveltimes for a broad range of seismic phases. The presence of substantial heterogeneity in the upper mantle and transition zone means that the definition of a single structure imposes choices on wave speed gradients and jumps at major discontinuities (Kennett 1991). Fortunately, the *iasp91* model provided a reconciliation of direct estimates of event depth and those derived from depth phases, even though reliable control from deep events was rather limited.

With the aid of the *iasp91* model, it was possible to relocate events with wide geographic coverage to provide a self-consistent set of empirical traveltimes for *P* and *S*, as well as new representations for later seismic phases. This suite of empirical traveltimes for 18 seismic phases was used by Kennett *et al.* (1995) to produce the *ak135* model with a slightly modified mantle and core structure. The good fit to *P* and *S* times was retained, whilst the representation of core phases was significantly improved. The *ak135* model is specified by control points with linear interpolation in radius, rather than by any higher order polynomial. But since the starting point was polynomial, *ak135* carries over aspects of this representation.

The distribution of seismic events and stations means that the empirical traveltimes have a northern hemisphere bias. Nevertheless, the *ak135* model has proved very effective for global earthquake location and tomographic studies using body waves. The model is now used by major international agencies as the standard for location. The computational convenience of the direct use of a radial earth model means that nonlinear inversion techniques can aid the inversion for hypocentral parameters (Bondár & Storchak 2011).

An alternative approach to developing an earth model from traveltimes was adopted by Morelli & Dziewoński (1993). Rather than

fit empirical traveltimes directly, they sought a representation of the spherical average of wave speed structure as a function of depth. The process of construction required the development of complex station corrections to compensate for the lateral heterogeneity of the Earth. The resulting model *sp6* retains the PREM parametrization and differs modestly from *ak135* in the mantle. There are differences in the transition zone and in the lower part of the mantle, mostly likely due to geographic bias in the empirical times. The *sp6* profile in the core shows significant differences from PREM with lowered *P* wave speed at the top of the core and reduced contrast at the inner core boundary. The general trends are similar to *ak135*, with the largest differences in the inner core.

1.3 Equation of state models

Stacey & Davis (2004) have developed models of elastic moduli and density based on equations of state for the lower mantle and core starting from PREM. They point out that the parametrization used in PREM is incompatible with the expected behaviour of derivatives such as $\partial K/\partial p$, the pressure derivative of the bulk modulus.

The relatively rapid fluid motion in the outer core can be expected to maintain it in an adiabatic state, which should be well represented by an equation of state. This has led Irving *et al.* (2018) to use normal modes sensitive to core structure to invert for *P* wave speed and density profiles in the outer core as a function of depth, whilst maintaining the rest of the Earth at PREM values. The set of 319 normal mode centre frequencies used includes 79 modes not available for use in PREM. Modes that are inner core sensitive were not used, and this might affect structure at the base of the outer core. The inversion was carried out using a Markov-chain Monte Carlo technique with a Bayesian interpretation of the ensemble of results. The preferred equation of state was the Rydberg-Vinet form (Irving *et al.* 2018), though this representation does not satisfy the conditions that Stacey & Davis (2004) insist must apply at infinite compression.

The models in the ensemble created by Irving *et al.* (2018) have a self-consistent pressure and sensitivity distribution, but the primary constraints on the mass and moment of inertia are only weakly applied. A consequence is that the outer core density in the *EPOC-V* model is higher than in PREM, even though the exterior zones are unchanged. The mean model from the ensemble provides an excellent comparator for *P* wave speed because it satisfies the Adams–Williamson condition for adiabaticity to very high precision, so that the Bullen parameter cannot be distinguished from unity (Irving *et al.* 2018).

2 NATURE OF CURRENT MODELS

The PREM model was constructed as a spherical average of the whole globe. As a result it has a 3-km-thick surficial water layer and a hybrid crust extending to 24.4 km depth that is a result of averaging oceanic and continental properties. The upper mantle down to the prominent ‘210 km’ discontinuity is transversely isotropic. This anisotropy was introduced by Dziewoński & Anderson (1981) to fit both Rayleigh and Love wave dispersion represented by the highest frequency modes.

In the *iasp91* and *ak135* models a choice was made to suppress the ‘210 km’ discontinuity, since by 1991 it had become clear that it was not a global phenomenon. Further, the upper mantle model was constructed to avoid any low velocity zones so that traveltimes could be calculated for any distance. The crustal model is continental since

this reflects the location of most seismic stations, with the Moho set at 35 km depth.

As stressed by Morelli & Dziewoński (1993), a full reference model needs to include density and attenuation structure so that waveform calculations can be made in addition to traveltimes. Montagner & Kennett (1996) investigated the extent to which the wave speed model *ak135* can be made compatible with normal mode results. They undertook an inversion of free oscillation centre frequencies for density and attenuation with isotropic wave speeds held fixed, except for an averaged crust. The main influence of attenuation was to allow for the modest velocity dispersion with frequency, associated with intrinsic attenuation, when fitting the normal mode frequencies. The fit achieved to the normal modes was quite good, but transverse isotropy is definitely needed to reconcile Rayleigh and Love wave properties at shorter periods. Montagner & Kennett (1996) suggest that such radial anisotropy may extend to the upper mantle transition zones, weakening with depth.

Both PREM and *ak135* were designed as reference models, and in consequence adopt simplified structures. For example the upper mantle discontinuities are introduced as first-order discontinuities at fixed depth, without any considerations of the details of phase transitions.

The models are compared in Fig. 1, where in addition the major zones of heterogeneity are indicated. The most obvious differences in the wave speeds and density distributions occur down to around 800 km depth, where much of Earth’s heterogeneity is concentrated. Other differences occur in the *D''* zone just above the core mantle boundary. At the scale of Fig. 1 there is little obvious difference in the core, but as we shall see these minor differences do have significant effects on some seismic phases.

The differences in *Q* profiles are rather larger than for wave speeds, with *ak135* having somewhat lower attenuation (higher *Q*) than PREM in most of the mantle. Durek & Ekström (1996) have developed an improved *Q* profile as a function of depth for long periods ($T > 100$ s) using the same parametrization as PREM. This *Q* profile *QL6* lies intermediate between *ak135* and PREM and provides a better fit to normal mode attenuation than either.

It is commonly assumed that a traveltime model such as *ak135* is defined for a reference frequency of 1 Hz, based on the visual character of seismograms. However, Oki *et al.* (2004) have investigated the characteristics of *S–P* differential traveltimes from hand-picked records by comparison with waveform fitting. They conclude that the different sets of times can be reconciled by adopting a reference frequency of 2 Hz. The change in the reference frequency used for velocity dispersion requires the frequency-independent *Q* values for PREM to be raised by 15 per cent, which again brings them closer to the estimates for *ak135*.

In Fig. 1 the bulk-sound speed $\phi = [K/\rho]^{1/2} = [\alpha^2 - \frac{4}{3}\beta^2]^{1/2}$, associated with just the bulk modulus *K*, is plotted alongside the *P* wave speed α and *S* wave speed β . The gradient of ϕ just below the ‘660 km’ discontinuity is slightly discordant with the rest of the mantle, which suggests that there is a mild discrepancy in the gradients in *P* and *S* wave speed. Such auxiliary quantities can play a useful role in providing checks on model behaviour.

PREM and *ak135* are reference models derived for specific purposes, and are not accompanied by any estimates of uncertainty, or resolution. A significant problem is distinguishing between the natural variation of the Earth expressed via lateral heterogeneity, and the limitations of the radial earth model itself. Kennett *et al.* (1995) used a suite of randomly generated models within restricted bounds to assess the extent that different traveltime constraints on

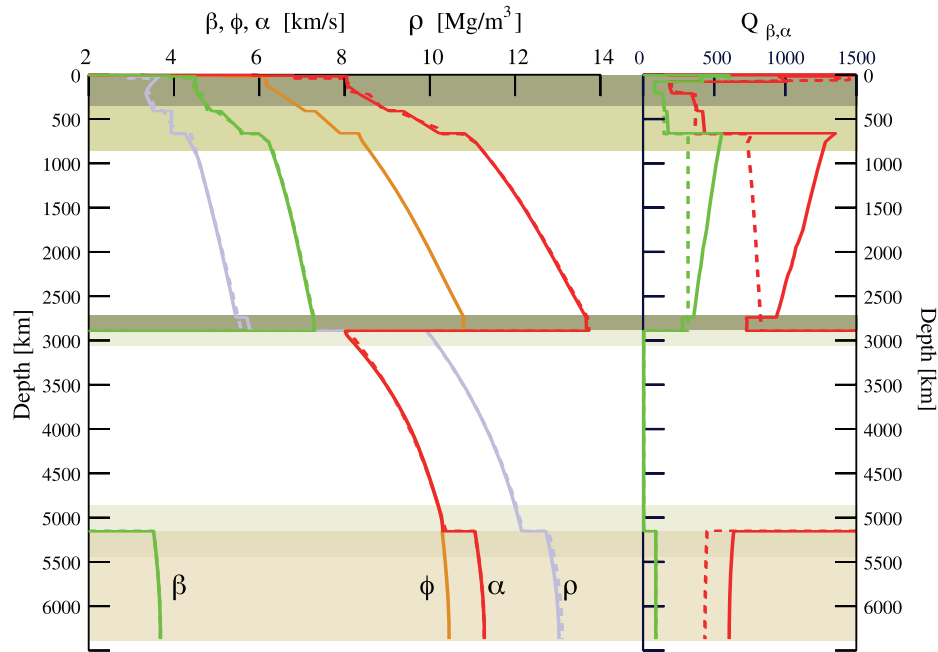


Figure 1. Wave speed and density as a function of depth for PREM (dashed line) and *ak135* (solid line). In addition to *P* wave speed (α) and *S* wave speed (β), the bulk-sound speed (ϕ) is illustrated. The brown shading in the mantle indicates the level of lateral heterogeneity, which is most pronounced in the top 300 km and near the core–mantle boundary. The light shading in the outer core highlights the boundary layers at the top and bottom. The inner core is both heterogeneous and anisotropic with strong variability near its surface.

Earth structure conformed to the *ak135* model. Masters & Gubbins (2003) have made a major contribution to the issues of resolution by conducting a linearized analysis around the PREM model. They have used a suite of 850 normal modes with very well constrained centre frequencies of which 50 sample the inner core. With the aid of these observations they construct resolving kernels for the various classes of physical properties for various target error levels. With a target of 0.5 per cent accuracy, the spread of the kernels is around 150 km for *S* wave speed and density in the mantle, and around 250 km for *P* wave speed through most of the Earth. For a target of 0.5 per cent, the spread for density is around 250 km in the outer core. However, if the target error is reduced below 0.5 per cent, acceptable resolving kernels cannot be found for density, since the spread becomes too extensive to be meaningful.

3 ZONES OF EARTH STRUCTURE

We now survey the various zones of the Earth, and look at the ways in which recent studies can be incorporated to improve the representation of Earth structure. When discussing the properties of radial models, it is important to remember the potential influence of lateral heterogeneity, already noted in Fig. 1, but this 1-D plot does not enable a judgement of the volumetric behaviour. The significance of heterogeneity is better appreciated in the context of the globe, so we show in Fig. 2(a) sector of a cross-section through the sphere with an indication of the zones of enhanced smaller-scale heterogeneity.

3.1 Lithosphere and upper mantle

The upper portion of the Earth displays rather strong horizontal contrasts superimposed on the dominant trend of wave speed increase

with depth. As a result, any single 1-D model will not provide an adequate representation of all aspects of structure. A partial solution is to develop a suite of regional models (e.g. Gudmundsson & Sambridge 1998; Kennett 2006), but global seismic phases may then traverse multiple domains. Much also depends on the style of parametrization. For a reference model it is attractive to keep the parametric representation as simple as possible, but this may mean that aspects of Earth structure are missed. Thus, even allowing for regional effects, the single linear gradient between 210 and 410 km used in *ak135* appears to be too simple (Kennett 2006). The average traveltime behaviour near 15° epicentral distance suggests that two linear gradients would be preferable. In a similar way there is no allowance for the presence of a 520 km discontinuity. Such a feature is evident in longer period studies (e.g. Deuss 2009), yet has little presence at short periods (e.g. Cummins *et al.* 1992).

A prominent feature of the PREM model is a large jump in physical properties at 210 km. Although such a feature is compatible with the normal mode constraints, it now appears that there is considerable variability in the behaviour near 200 km across the globe (e.g. Gu *et al.* 2001; Deuss 2009). The presence of the sharp 210 km boundary can produce residual effects in tomographic inversions starting from PREM (Ekström & Dziewoński 1998). This has led many authors to produce their own variant reference models with the discontinuity suppressed by smoothing.

Kustowski *et al.* (2008) have combined the processes of the estimation of 3-D structure and construction of a 1-D reference model. For their reference model *STW105* they have constructed a spherical average of shear wave structure linked to inversions for the radial variations of *P* wave speed and density down to 1320 km, exploiting surface wave dispersion and traveltime information. Below 1320 km depth the PREM structure is used. The model *STW105* has once again a hybrid crust, with a rather smooth variation through the upper mantle down to the 410 km

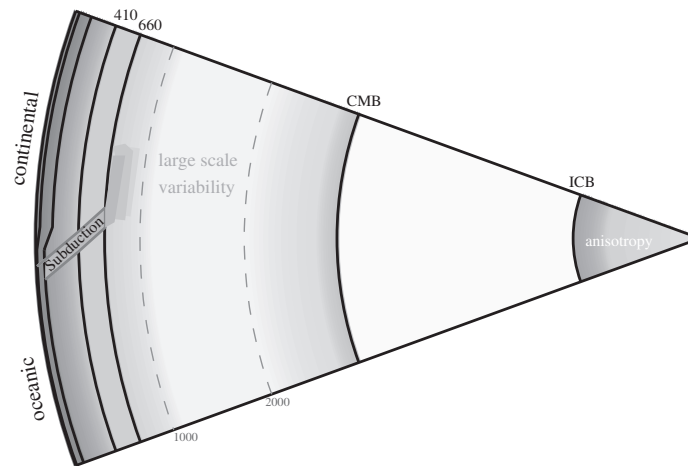


Figure 2. Schematic representation of levels of smaller-scale heterogeneity in the different zones of Earth structure.

discontinuity, the second discontinuity is placed at 650 km. The model is again radially anisotropic, but the anisotropy is weak below 220 km.

The specific choices made for the variation of physical properties with depth models such as PREM and *ak135*, and the absence of uncertainty estimates means that comparisons with mineral physics models are not readily made for the upper mantle. Cobden *et al.* (2008) suggest that there may be systematic bias in seismological reference models in this zone due to the effect of lateral heterogeneity. The problem is rather that the levels of simplification that are suitable for fitting seismological data are not the same as would be expected for a 1-D mineral physics model. Somewhat arbitrary choices have to be made to counter the effect of lateral heterogeneity in the outer part of the Earth, and would be present even if an inversion were based directly on a mineral physics model.

3.2 Lower mantle

The PREM model has a linear gradient in radius from 670–760 km, and then uses a single cubic in radius for both seismic wave speeds and density between 760 and 2740 km depth. The base of the mantle is again represented by a linear gradient to the core-mantle boundary. As noted above, Stacey & Davis (2004) have criticised this form of parametrization as too restrictive and leading to unreasonable derivatives for physically important quantities.

The *ak135* model for seismic wave speeds was developed from the parametric *iasp91* model, and so still carries an imprint of this parametrization. Although specified by linear gradient segments with values specified at around 50 km intervals, *ak135* wave speeds can be well approximated by two cubics with a junction point around 1700 km. The bulk-sound speed was not explicitly considered in the construction of the *P* and *S* wave speed models, but provides a useful check on behaviour. As can be seen from Fig. 1, the bulk-sound gradient just below the 660 km discontinuity does not match well to the behaviour at greater depth. This discrepancy can be readily removed by a slight decrease in the *S* wave speed gradient between 660 and 760 km, without any deleterious effect on *S* times.

Under lower mantle conditions there is expected to be a transition from high-spin to low spin in iron bearing minerals as pressure increases. During the spin crossover the bulk modulus is expected to reduce significantly, but there will be little effect on the shear modulus. Such a transition would therefore be expected to have its main effect on the *P* wave speed profile as a function of depth.

The spin transition has a significant effect on the bulk modulus of ferropericlasite (Wentzcovitch *et al.* 2009, Marquardt *et al.* 2018), but has more muted effects on bridgmanite (Shukla *et al.* 2016). The transition is expected to occur over a broader pressure range, and hence depth interval, as the temperature increases (Tsuchiya *et al.* 2006).

The polynomial representation used in PREM would not accommodate localised deviations due to such a transition, and the fit to the empirical traveltimes in *ak135* does not require any feature that might be associated with the spin transition. Engdahl *et al.* (1998) introduced a processing scheme to produce refined locations for well observed events, including dynamic reassignment of phase identifications, using the *ak135* model. The resulting EHB catalogue starting in 1964 provides a very large data set of well controlled traveltimes for *P* and *S*, and the results show very little deviation of the mean response from the predictions for *ak135* (Fig. 3). There is scatter in the observations associated with the 3-D variations in Earth structure and these variations can be exploited in traveltime tomography for both *P* and *S* phases (e.g. Bijwaard *et al.* 1998; Kennett *et al.* 1998).

Cammarano *et al.* (2010, fig. 2) have made a similar analysis of the EHB catalogue with a different selection of data, and obtain comparable results. Deviations of the mean of the observations from the predictions of *ak135* is small, with an excursion of around 0.1 s early near 60° for *P*, the width of one pixel in Fig. 3. Cammarano *et al.* (2010) show that the traveltime predictions for the models PREM and *STW105* do not conform to the pattern of observations. Both models are early compared with the observations at shorter distances and slightly late at 90°. There is need for a tilt correction of around $-0.017 \text{ s deg}^{-1}$ for *P* and -0.03 s deg^{-1} for *S*, which are comparable to those found in the construction of PREM using traveltime tables with less well controlled events.

Thus, the available traveltime data shows that *ak135* is self-consistent and there is no need to introduce any deviations in the mid-mantle that might correspond to the global presence of spin transition effects. Perhaps the 3-D variations in structure and temperature are sufficient to mask the presence of the transition in the global average?

3.3 Core

Because the *P* wave speeds in the core are distinctly lower than in the mantle above, the *P* waves refracted back from the core

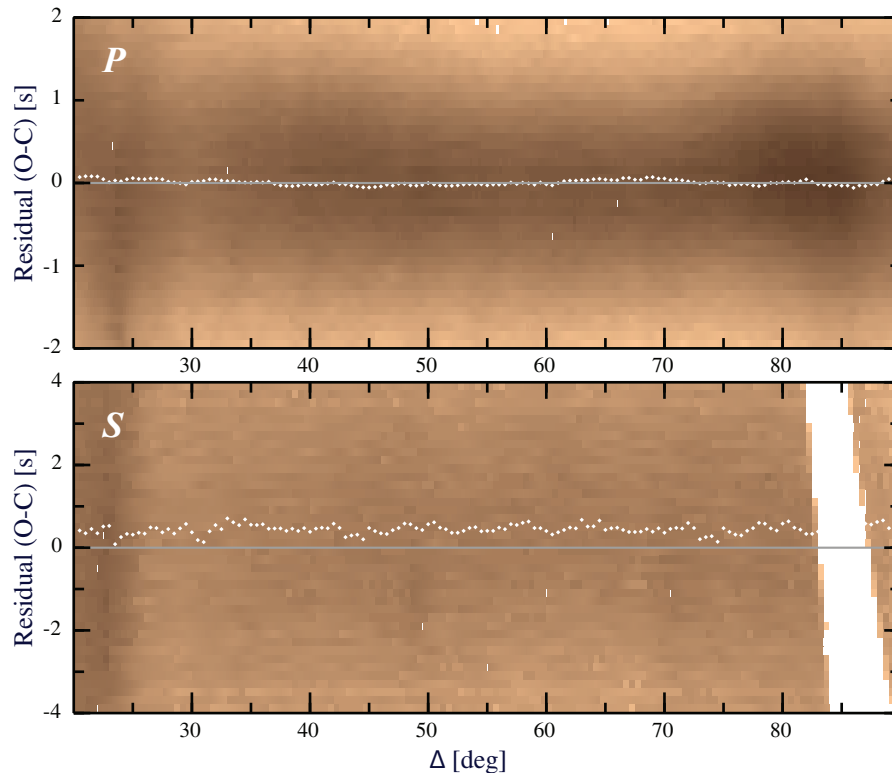


Figure 3. Binned traveltime data from 30 yr of the EHB catalogue (Engdahl *et al.* 1998) for around 1 million *P* times and 100 000 *S* times. The tonal density in each plot is logarithmic and the mean of the entire data set at each epicentral distance is indicated by white symbols. The blank area around 85° for *S* comes from the crossover with the *SKS* phase.

(*PKPAB*) do not have turning points shallower than 10 000 km below the core–mantle boundary. Fortunately, the *S* wave speed at the base of the mantle lies below the *P* wave speed at the top of the core by a small margin, and so the refracted phase *SKSAC* samples the entire outer core. The doubly refracted waves *SKKS* and the higher multiples *SmKS* for underside reflection at the core–mantle boundary sample closer to the boundary when observed at the same distance.

The construction of a *P* wave speed profile from traveltimes thus requires the reconciliation of observations from *SKS*, *SKKS*, *PKPAB* and *PKIKP*. This process immediately introduces potential trade-offs between mantle *S* structure and *P* wave speeds in the outer part of the core. The sampling of the heterogeneous lowermost mantle by reported *PKP* and *SKS* phases may well not be the same, so some compromises are needed to establish a single radial *P* profile.

In order to match the empirical times for *SKS* and *SKKS*, a significant reduction of *P* wave speed relative to PREM was introduced in *ak135* in the top 300 km of the core, as in model *sp6*. It now appears that an overcompensation has been made, and this was indeed suggested by the retrospective assessment of *ak135*, using a suite of random models (Kennett *et al.* 1995). The *S* phases favour slightly higher wave speeds, while the push to lower wave speeds comes from the *P* core phases. A number of studies (e.g. Tanaka 2007; Kaneshima & Helffrich 2013; Tang *et al.* 2015; Kaneshima 2018; Wu & Irving 2020) have investigated the differential times between *SmKS* phases and demonstrated the need for a structure intermediate between PREM and *ak135* in the top 300 km of the core. The use of differential times means that the dominant sensitivity is to the wave speed gradient in the outermost

core, rather than absolute wave speed. Because much of the paths for the different *SmKS* phase pairs in the mantle are similar, the effects of mantle heterogeneity are reduced relative to measurements on a single phase. Nevertheless, the effects on the differential traveltimes are still notable, and somewhat scattered (Wu & Irving 2020).

The *P* wave speeds for *ak135* are slightly faster than for PREM in the middle of the outer core. The main difference arises near the inner-core boundary, where *ak135* has reduced velocity gradients introduced to satisfy differential time constraints between the different branches of *PKP*. The structure in this region is similar to that proposed by Song & Helmberger (1995).

An additional source of information on deep structure in the Earth comes from the correlation of coda waves from great earthquakes (Pham *et al.* 2018). The correlation wavefield, constructed from the stacked cross-correlations between stations for the time window from 3 to 10 hr after event initiation, shows a rich harvest of core related phases. These phases arise from constructive interference between arrivals with comparable slowness at the different stations. As a necessary consequence of the cross-correlation process such correlation phases have timing which represents the difference between the physical phases being correlated. Complex phases with steep paths are enhanced and many multiples of *PKP* phases are observed. The observed correlation wavefield and simulations using *ak135* show very strong similarity in both timing and amplitude distribution with epicentral distance, indicating that control on general core and mantle structure is good.

Tkalčić & Pham (2018) have exploited the sensitivity of the correlation wavefield to changes in structure to demonstrate the effect of changing the shear wave speed structure in the inner core. They

have identified three correlation phases *PKIKPPKIKP-PKJKP* (*I2-PKJKP*), *PKIKP-PKJKP* and *PKiKP-PKJKP* that can be recognized in synthetic correlation fields and show such sensitivity. The combination *I2-PKJKP* appears at small angular distance with contributions from all azimuths that enhance the visibility of the correlation phase. The difference phases *PKIKP-PKJKP* and *PKiKP-PKJKP* are weaker because there is no focusing in the 120° to 150° angular distance in which they emerge. Tkalčić & Pham (2018) have made systematic searches for the properties of the inner core that best match the observed correlation field, and find the best fit for a shear wave speed reduction of 2.5 ± 0.5 per cent relative to PREM.

Robson & Romanowicz (2019) have undertaken an inversion of a compilation of the centre frequencies of normal modes sensitive to the inner core using a Monte Carlo parameter-space search. They seek average values for the shear wave speed, *P* wave speed and density without taking into account depth variations. Their results are compatible with a *P* wave speed close to that of PREM, but require reductions in average shear wave speed and density. The reduction in density is not well constrained, but is of the order of 1.5 per cent. The shear wave speed reduction is around 1.25 per cent, depending on the base model used. Robson & Romanowicz indicate that the 2.5 per cent reduction favoured by Tkalčić & Pham (2018) is not compatible with the behaviour of some modes, and gives a poor fit to the normal mode observations. Nevertheless, this 2.5 per cent value provides a good fit to the time behaviour in the correlation wavefield.

The inner core has complex properties (Tkalčić 2015) with apparent hemispherical differences and complex anisotropy. The variations in the traveltime for *PKIKP* (*PKP_{DF}*), with corrections for ellipticity, can amount to more 2 s between equatorial and near polar paths. These variations complicate the definition of summary traveltimes, and hence the description of the inner core. The observations of high-frequency scattered coda to *PKiKP* (Poupinet & Kennett 2004) require the presence of complex fine-scale structure and suggest the existence of a waveguide in the outermost part of the inner core. Such an effect could be created with a stronger wave speed gradient in the top 100–200 km of the inner core.

4 MODIFIED MODELS

Aside from minor adjustments in the upper part of the mantle, the main reason for seeking change in the radial earth models comes from the studies of the differential times between the *SmKS* phases with different numbers of underside reflections at the core–mantle boundary. The initial results were dominantly from deep events near Fiji to European stations (e.g. Kaneshima & Helffrich 2013) for the frequency band from 0.02 to 0.2 Hz. Much wider geographic coverage is now available (e.g. Tang *et al.* 2015; Kaneshima 2018) with reasonable consistency between results for varied paths and different measurement techniques. Waveform matching techniques need to include the effect of cumulative caustics on underside reflection that introduce successive $\pi/2$ phase shifts.

The patterns of *SmKS* differential times require the *P* wave speed at the top of the core to be slower than in PREM and faster than in *ak135* or *sp6*. Fig. 4 displays tabulated results from Kaneshima & Helffrich (2013), Kaneshima (2018) and Wu & Irving (2020) compared to a number of different radial models. The binned measurements for the differential time between *S3KS* and *SKKS* (dt: 3,2) plotted by Tang *et al.* (2015) are largely consistent with the point measurements shown in Fig. 4. They show mean times around

+0.5 s residual from PREM, with typical error bars of more than 1 s, over the epicentral distance range from 120° to 140° .

The model predictions in Fig. 4 are made for a source depth of 500 km. The patterns of differential time residuals are rather slowly varying functions of source depth, so the differences in depth for the various events used by the different authors are not significant. The patterns of differential time residuals are dominantly controlled by the gradients in wave speed at the top of the core, and are only shifted slightly in time by small changes in absolute wave speed.

Fig. 4 makes clear the need for modifications to the commonly used reference models: *sp6* and *ak135* predict too large residuals, whilst those for PREM are too small. Only *iasp91*, which uses the PEM core, lies in the general vicinity of the observations. However, we cannot simply replace a part of the model that is traversed by many different classes of seismic phase without affecting the properties of these phases. For example, *ak135* was developed to provide a fit to 18 different seismic phases. All the 10 core phases traverse the upper part of the core where change is required. If we increase the *P* wave speed in this zone from the value for *ak135*, we have to make compensating reductions in wave speed elsewhere to preserve effective traveltime representations. The only place where such wave speed reductions can readily be made is in the lower part of the mantle, but this then impinges on the properties of the mantle phases.

Already we have competing objectives, we wish to maintain or improve the fit to the traveltimes of the full suite of seismic phases whilst incorporating the constraints from the *SmKS* differential times. We are faced with a multi-objective optimization problem (see, e.g. Marler & Arora 2004) for which it is well known that improvements in fitting one class of information tend to come at the expense of degrading others (Pareto 1906). The approach taken by Kennett *et al.* (1995) was to use multiple measures of data fit combining different groups of seismic phases and seek to optimize, as far as possible, the full set of measures by suitable adjustments to models. We have followed such an interactive approach here, since it is hard to formulate any single measure of misfit that would encompass all aspects of the desired outcome.

The strategy that has been adopted is to look for classes of radial models with different characteristics, and seek improvements in model performance in each class, with comparisons between classes. We have used three different classes of radial models, in addition to prior models:

- (1) ‘ak’ class – these models are developed from *ak135* which provides a good representation of *P* core phases;
- (2) ‘kh’ class – these models start from the *KHOMC* model of Kaneshima & Helffrich (2013) for the outermost core that was constructed to fit *SmKS* differential time results, together with the *ak135* mantle.
- (3) ‘ek’ class – these models start from the combination of the *ak135* mantle and the *EPOC-V* core model of Irving *et al.* (2018).

In each case derived models are sought that match as broad a set of observations as possible. Below 4250 km depth we have adopted the *ak135* *P* wave speed structure, since this provides a good representation of the differential times between the different branches of *PKP*. The changes in wave speed in the upper core introduce slight shifts, but the general character is maintained. For the inner core *S* wave speed, the profile from Tkalčić & Pham (2018) has been adopted with a 2.5 per cent reduction from the PREM values, since this fits the traveltime patterns for the correlation phases.

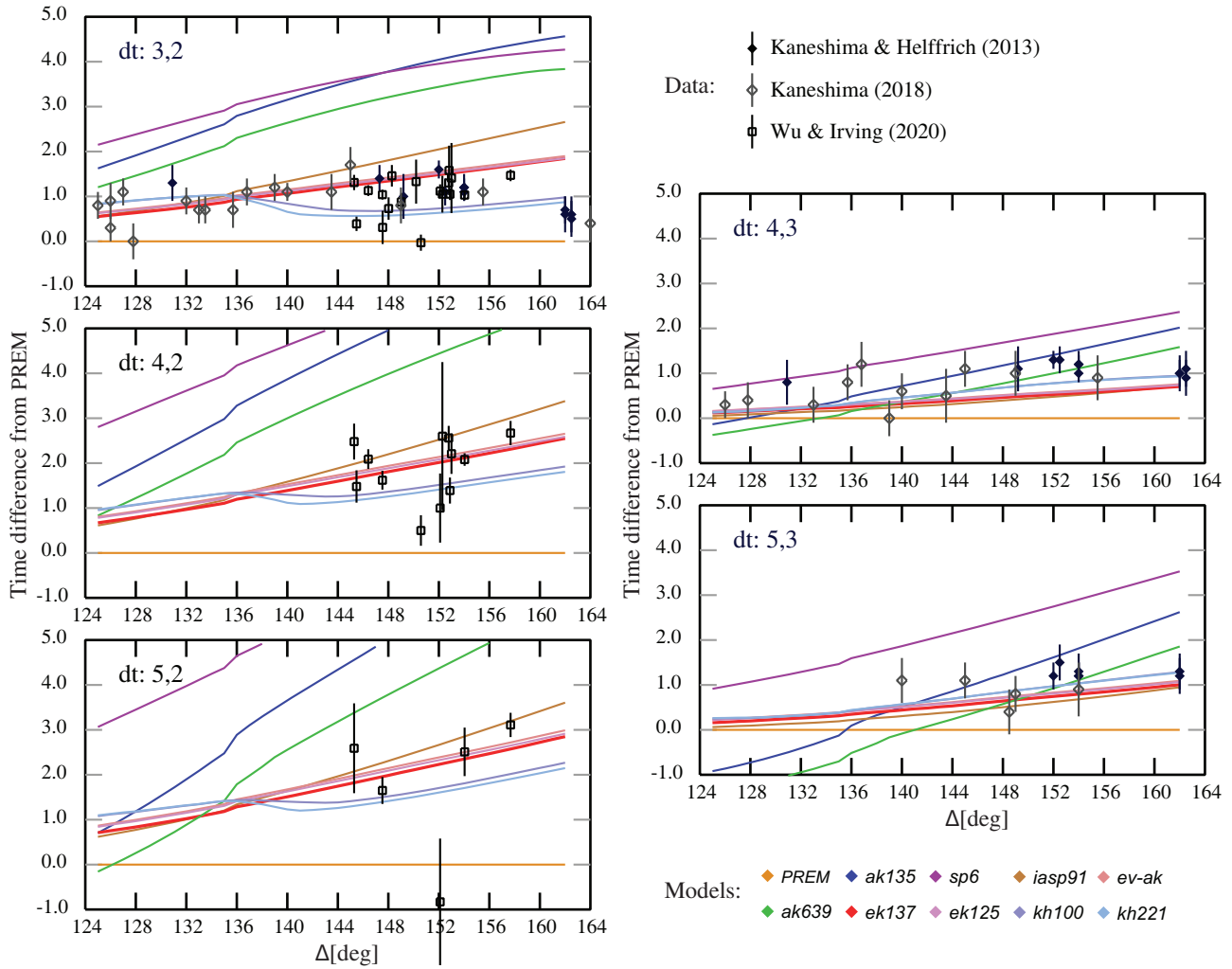


Figure 4. Differential times for *SmKS* phases compared with predictions for a variety of radial models. The new models in the lower row of the key are discussed later. The data are taken from the tabulations of Kaneshima & Helffrich (2013) and Kaneshima (2018)—class A estimates and Wu & Irving 2020. The differential times are displayed relative to the values for the PREM model, using the convention *dt: m,n* for the time difference between *SmKS* and *SnKS*. Times are calculated for a source depth of 500 km.

The new models were derived by an iterative process, with a several different classes of modification of the base models initially considered, marked with a distinctive initial number. Subsequent model designators were incremented by 10, and then as refinement occurred by 5 or fewer. The changes to the models were made by hand by modifying the wave speed values at fixed depths, with testing of all models against empirical traveltimes, monitoring a suite of more than 20 misfit measures for individual phases and composite phase collections. Poor models were immediately discarded and only promising models retained. The best were then tested against the differential time constraints displayed in Fig. 4. Over 250 models were assessed, including models taken from the set used in the construction of *ak135* and the best fitting from the subsequent random model testing (Kennett *et al.* 1995). 50 new models of different styles warranted retention, and of these 15 were checked against the differential times for *SmKS*.

When testing the new models, we have used the same set of empirical traveltimes as were used in the construction of *ak135* (Kennett *et al.* 1995). Caustic corrections of 0.5 s for *P* phases and 1.0 s for *S* phases have been introduced, based on comparisons of manual picking and waveform matching

techniques. These changes allow greater discrimination between models.

We have used the full set of weighted combinations of summary traveltimes residuals used by Kennett *et al.* (1995), using an L_2 measure of fit between the smoothed empirical times and the predictions for a particular model. For example, for the *P* phase we construct

$$\psi_P = \frac{1}{N_P} \left[\sum_{j=1}^{N_P} \left(\frac{(t_{obs})_j - (t_{calc})_j}{\sigma_j} \right)^2 \right]^{1/2}, \quad (1)$$

where N_P is the total number of 1° samples for the *P* phase. Linear combinations of these individual phase measures are then to produce composite measures of fit encompassing a number of phases. The measures for the individual phases are combined using weights w_{branch} (Table 2) based on an assessment of the likely quality of phase picks, taking into account the expected amplitude of the arrivals and the nature of the coda of the preceding phases. The same weights are used as in the construction of *ak135*, recognizing that these are somewhat subjective.

Particular useful combinations of phase measures are those associated with phases with passage through the core: for phases with

Table 2. Seismic phases used and the corresponding phase weights, for example w_{PKP} , used in the construction of the misfit measures in (2)–(4).

Phase branch	Distance range	Weight	Phase branch	Distance range	Weight
P	25–99°	5.0	$PKPdf$	118–180°	4.0
S	25–80°	3.0	$PKPbc$	151–153°	4.0
PP	53–180°	1.5	$PKPab$	156–178°	4.0
SS	56–150°	1.0	$PKKPab$	111–122°	1.5
PcP	26–70°	2.0	$PKKPbc$	83–122°	1.5
ScS	19–65°	1.0	$SKSac$	91–123°	3.0
ScP	18–62°	2.0	$SKKSac$	65–178°	1.5
SP	95–128°	1.0	$SKPdf$	113–160°	1.0
$P'P'$	56–67, 79–103°	1.0	$SKPbc$	141–148°	1.0

solely mantle P propagation

$$CPw = \sum w_{PKP} \psi_{PKP} + \sum w_{PKKP} \psi_{PKKP} + w_{P'P'} \psi_{P'P'}, \quad (2)$$

and for phases involving S waves in the mantle

$$CSw = w_{SKS} \psi_{SKS} + w_{SKKS} \psi_{SKKS} + \sum w_{SKP} \psi_{SKP}. \quad (3)$$

We also use the measures

$$PK = \sum w_{PKP} \psi_{PKP} + \sum w_{PKKP} \psi_{PKKP}, \quad (4)$$

$$SK = \psi_{SKS} + \psi_{SKKS},$$

to concentrate on the major core phases.

The ‘ak’ series of models tend to retain the good fit to phases such as the branches of PKP and $PKKP$. It proves possible with a combination of modifications in the lowermost mantle and the top of the core, to improve the fit to SKS and $SKKS$, with only a minor degradation of the fit to the P core phases and mantle phases. However, even the most successful model of this class, *ak639*, does not have the appropriate behaviour for the $SmKS$ differential times. The residuals are reduced from those of *ak135* but the trends are wrong (Fig. 4).

The ‘kh’ set of models start with a good fit to the differential time behaviour. However, even with strong adjustments to lower mantle structure, the P wave speeds in the outer core for the model *KHOMC* of Kaneshima & Helffrich (2013) are too fast to secure to good fit to the empirical traveltimes for either the PKP phases or SKS – model *kh100*. Below 3300 km depth the model *kh100* is linked to *ak135*. Retaining the gradient structure but reducing the P wave speed by 0.01 km s^{-1} down to 4250 km depth (model *kh221*) with modification of lower mantle structure has the effect of giving a good representation of all the $SmKS$ behaviour, with a tolerable representation of the P core phases.

The model *EPOC-V* of Irving *et al.* (2018) was constructed by a Bayesian inversion of normal mode frequencies. When combined with the mantle structure of *ak135* the resulting model *ev-ak* is largely consistent with the differential $SmKS$ data (Fig. 4), but is not a good fit to PKP or $PKKP$. An improved set of models can be found by shifting the P wave speeds at the top of the core faster by 0.01 km s^{-1} and linking to *ak135* below 4250 km. The densities in *EPOC-V* are somewhat higher than in PREM and our preferred model *ek137* was constructed by a shift equivalent to reducing the *EPOC-V* densities by 0.25 per cent down to 4000 km depth whilst retaining the same bulk modulus.

The reason that any single inversion for structure will run into difficulties in balancing the trends imposed by different classes of data can be well illustrated in Fig. 5, where we have plotted two classes of weighted residual measures sensitive to the core phases

with P or S mantle legs. In the left hand panel of Fig. 5 we show the measures CPw , CSw from (2), (3) that include all core phases and in the right hand panel PK , SK that concentrate on the main phases. In each case we can see the development of Pareto optimal bounds favouring fit to the P or S phases. However, the relative performance of the reference and derived models is very different in the two panels. The preferred model *ek137* is a slightly suboptimal for fit for every criterion, but lies at the ‘corner’ in each case, and so represents a very satisfactory compromise to represent the dominant trends in Earth structure.

Fig. 6 displays the character of the P wave speed in the upper part of the outer core as deviations from *ev-ak* for representative models from the set investigated. All of these models are compared with the $SmKS$ differential times in Fig. 4. The use of the outer core structure from *EPOC-V* means that the reference used in Fig. 6 is derived from an adiabatic model. Models of the ‘ak’ class are slower than the reference at the underside of the core mantle boundary; while those in the ‘kh’ group are faster than the reference. The preferred model *ek137* is intermediate with an increase of 0.01 km s^{-1} from *ev-ak*. The shifts in wave speed across the whole suite of models examined are quite small, less than 0.1 per cent, but the differences in wave speed gradients make significant changes to the properties of the body wave phases.

In Table 3 we provide a direct comparison of the misfit measures for each individual phase between the models *ak135* and *ek137*, accompanied by the composite measures introduced above. We also include the weighted sums of fit for all mantle and core phases, and the weighted sum for all phases. The changes to the mantle have improved the fit to S and SP , but there is slightly less good fit to P and the multiple phases PP , SS . The bulletin reports for the core reflections are of mixed quality, and the empirical times prove difficult to fit. Model *ek137* is a little worse than *ak135* for ScS , but better for PcP and ScP . As hoped the fits to the core phases with S mantle legs are improved, as is the fit to the differential $SmKS$ times (Fig. 4), but this has been achieved at the cost of a slight reduction in fit quality for the core phases with purely P mantle legs. It should be noted the empirical times do not account for the effects of inner core anisotropy.

5 DISCUSSION AND CONCLUSIONS

In this study, we have been able to construct a traveltime model that provides a good representation of the full range of constraints. The preferred model *ek137* is displayed in Fig. 7 along with a range of other models, including all those for which differential times are displayed in Fig. 4. An addition marked *SDcor* is the wave speed profile derived from the equation of state presented by Stacey &

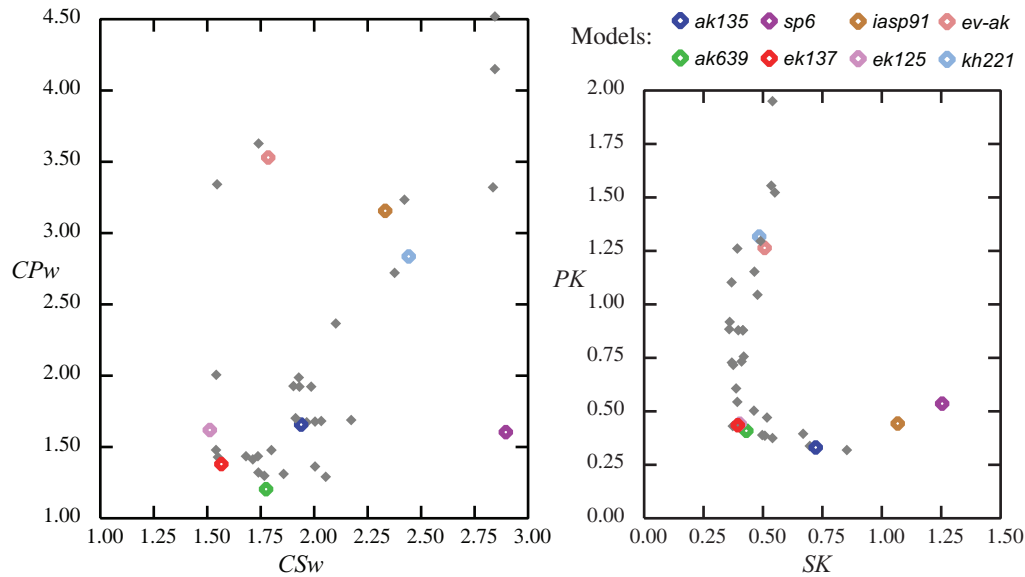


Figure 5. Trade-offs in weighted residual measures CP_w , CS_w and PK , SK introduced in (2)–(4), balancing core phases with P or S legs in the mantle. The positions of specific models are identified with coloured open symbols.

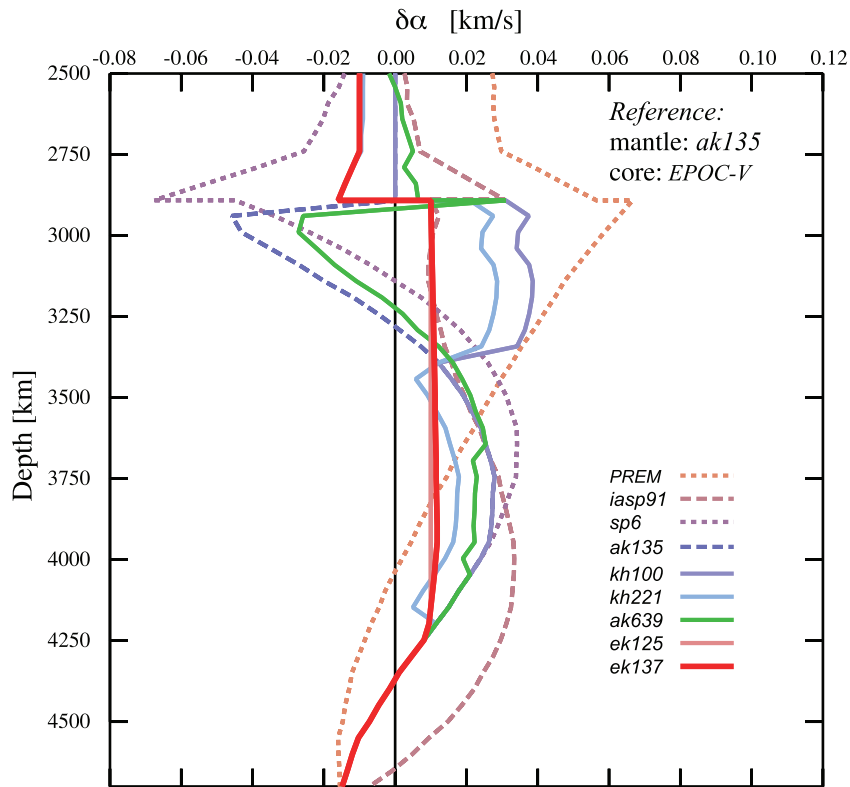


Figure 6. Structure at the top of the outer core for radial models used in Fig. 4.

Davis (2004) based on the PREM profile. The bulk-sound speed ϕ is shown in Fig. 7 along with the P wave speed α and S wave speed β . Except in the upper mantle and the lowermost mantle the differences between models are hard to discern. The subtle differences in core structure produce a thickened band at the top and bottom of the outer core.

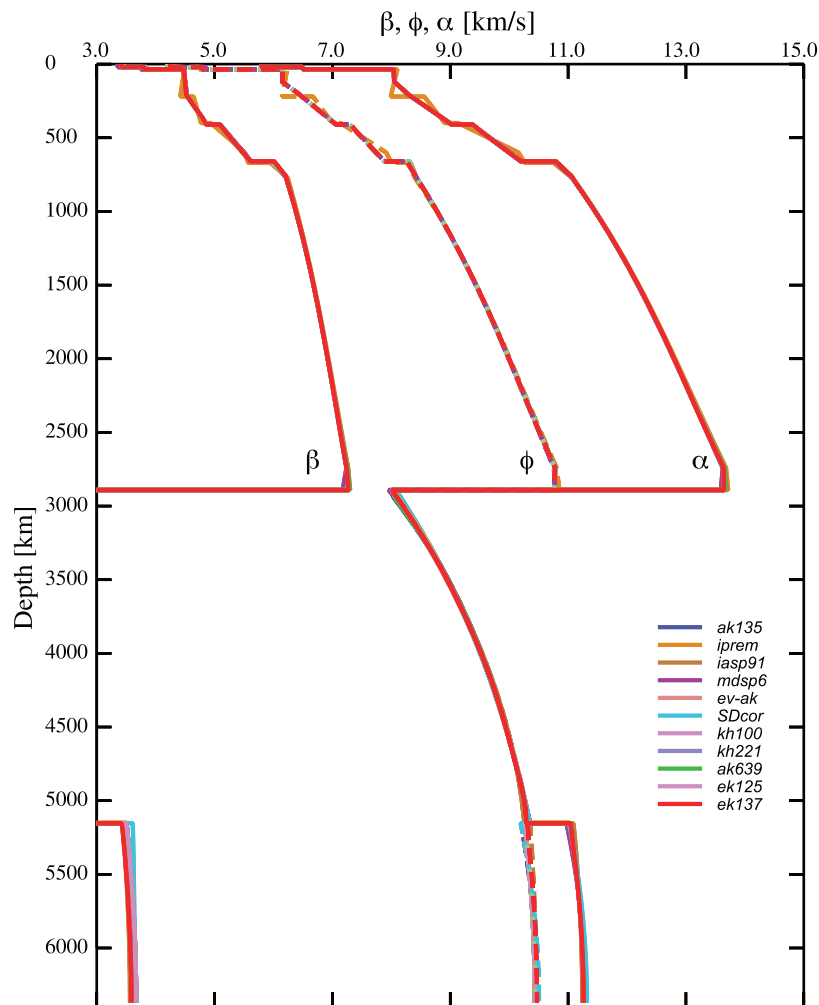
In the inner core model *ek137* includes an S wave speed profile with a reduction of 2.5 per cent from the PREM values. The wave speed gradients for S wave speed are not well controlled in the inner

core. Stacey & Davis (2004) have argued that the depth (pressure) derivatives of the shear modulus are too strong for PREM, whilst those for bulk modulus are rather weak. The P wave speed profile can be maintained by a modest change in both moduli. The inner core for model *SDcor* in Fig. 7 illustrates the difference made by such changes in gradients of moduli.

For all the models there is very little contrast in bulk-sound speed ϕ between the inner and outer core and indeed for *ek137* it is almost continuous. Nearly all of the increase in P wave speed on entering

Table 3. Comparison of phase fit measures for *ak135* and *ek137*, with inclusion of caustic adjustments.

Phase	<i>ak135</i>	<i>ek137</i>	Phase	<i>ak135</i>	<i>ek137</i>	Measure	<i>ak135</i>	<i>ek137</i>
<i>P</i>	0.008	0.013	<i>PKPdf</i>	0.023	0.043	<i>PK</i>	0.331	0.435
<i>S</i>	0.122	0.106	<i>PKPbc</i>	0.026	0.007	<i>SK</i>	0.722	0.394
<i>PP</i>	0.055	0.063	<i>PKPab</i>	0.029	0.073	<i>CPw</i>	1.291	1.380
<i>SS</i>	0.162	0.191	<i>PKKPbc</i>	0.190	0.267	<i>CSw</i>	2.054	1.566
<i>PcP</i>	0.104	0.084	<i>PKKPab</i>	0.063	0.045	<i>Mantle</i>	1.586	1.628
<i>ScS</i>	0.217	0.354	<i>SKSac</i>	0.056	0.021	<i>Core</i>	3.585	3.169
<i>ScP</i>	0.038	0.029	<i>SKKSac</i>	0.666	0.373	<i>All</i>	5.171	4.797
<i>SP</i>	0.434	0.378	<i>SKPdf</i>	0.523	0.506			
<i>P'P'</i>	0.599	0.418	<i>SKPbc</i>	0.603	0.661			

**Figure 7.** Comparison of wave speed profiles for the entire Earth for a range of radial earth models. The *P* wave speed α , *S* wave speed β are accompanied by the bulk-sound speed ϕ that depends solely on the bulk modulus.

the inner core can therefore be associated with the influence of the weak shear modulus in this solid material.

The differences between the the model *ek137* and the reference model *ak135* are displayed in Fig. 8. Some adjustments have been made to the upper mantle discontinuity jumps and the shear wave speed gradient just below 660 km, which removes the previous slight discordance in bulk-sound speed gradient. The main change is in the upper part of the outer core with higher *P* wave speeds in the outermost core. This increase in wave speed has to be compensated by modifications of both *P* and *S* wave speeds in the lower section

of the mantle, and a slight reduction in *P* wave speed in the middle of the core.

The model *ek137* provides an improved fit to most *S* phases compared with *ak135*, in particular those core phases with *S* legs in the mantle such as the *SmKS* suite. However, such improvements come at the expense of a very small degradation in the fit to purely *P* wave phases, for example, *P* beyond 90°. Intriguingly, *P'P'* is somewhat better fit. Weighted measures of fit to the empirical traveltimes that include converted phases and core multiples are better fit than for *ak135*.

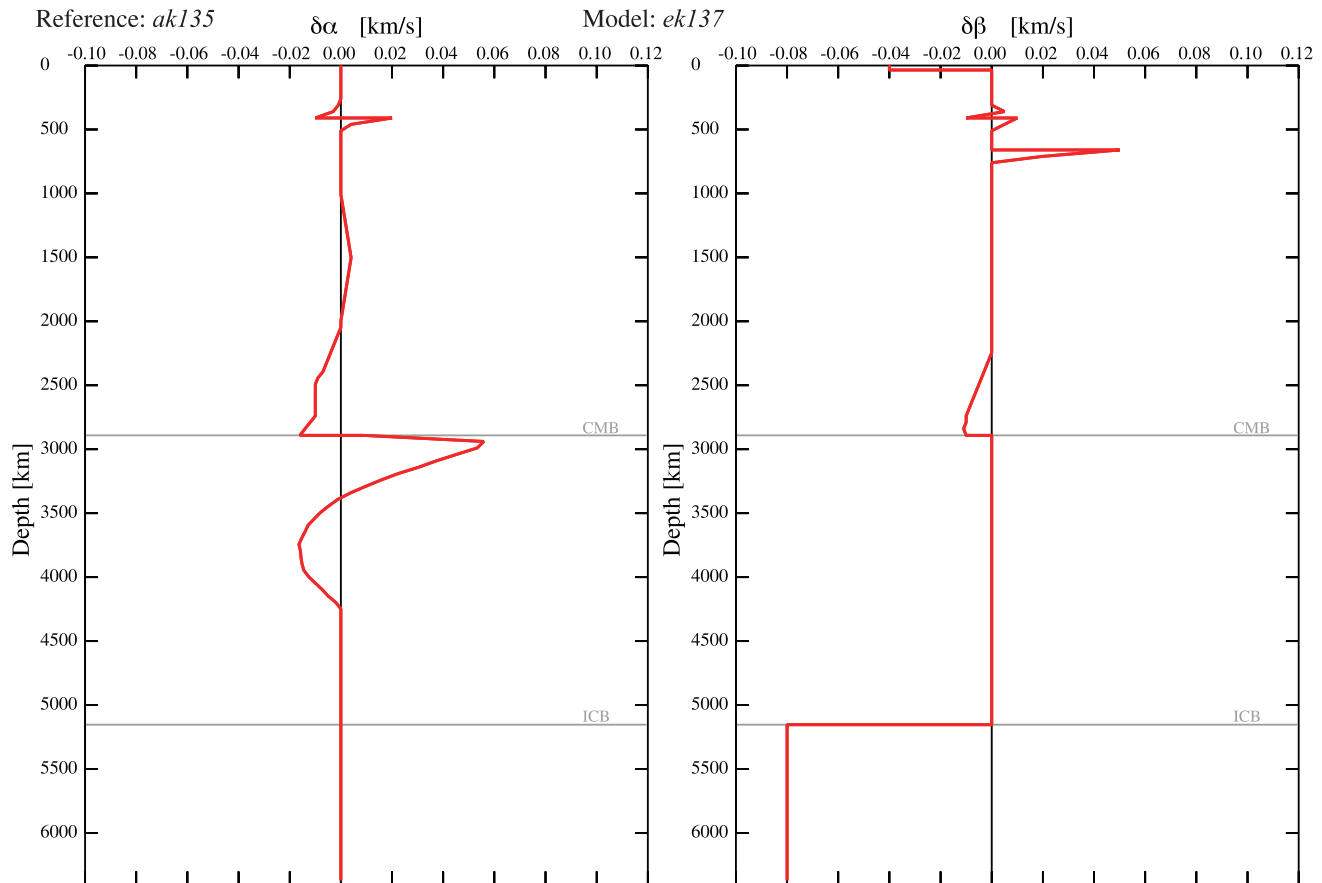


Figure 8. Deviations of the P and S wave speeds for the preferred model *ek137*, from the reference model *ak135*.

It has proved possible to provide a representation of core structure that provides a balanced fit to a wide range of phases and the constraints from the differential times of *SmKS*. The P wave speed for model *ek137* has similar wave speed gradients to the adiabatic model *EPOC-V* through the outer core from the core–mantle boundary down to 4200 km depth. There is a modest deviation in gradient at the base of the outer core that is needed to provide a good representation of the differential times between the branches of *PKP*. Perhaps the changes in the lower part of the outer core are associated with modifications of the light element distribution due to the freezing out of the inner core?

There have been suggestions that the *S3KS–SKKS* differential times require the presence of non-monotonic structure in the outermost core (Tang *et al.* 2015). Here we find that the full set of differential times are compatible with a simple monotonic profile. It is rather easy for 3-D variations, for example in the mantle, to get mapped into corrugations in radial structure. Finding suitable event–station pairs to provide full data coverage will be very difficult, and so it is preferable to look for rather smooth structure provided this gives a good fit to the data.

The model *ek137* is tabulated in the Appendix (Table A1) in a form that should be suitable for a broad range of body-wave applications to the core. Although it is not feasible to provide a formal error analysis, the wide range of structures tested in the process of developing the new model have provided considerable insight into the extent to which the model can be modified without prejudice to data fit. Fig. 9 displays a subjective assessment of

the levels of uncertainty that can be ascribed to the new reference model for core studies. Inner core properties have a high level of uncertainty, due in part to strong influences from heterogeneity and anisotropy. The inclination of the S wave speed zone in the inner core reflects the uncertainties in shear wave speed gradients, and may well be underestimated. The lighter tone represents the effect of the alternative estimate of inner core S wave speed from Robson & Romanowicz (2019).

The mode of construction of the new model may well be criticized, because it is not the product of a formal inversion. However, any fit to multiple classes of data requires trade-offs between the importance of fit, that are often introduced through additional parameters that are specified by some external procedure. With the iterative approach to multi-objective optimization it has been possible to balance the fitting of more than 20 classes of information, without only slight retreats from the best fit for any class.

The *ek137* model is a traveltime model, designed to fit as broad a range of observations of absolute and differential times as possible. Although some aspects of models derived from normal mode and traveltime studies appear to be converging, there is still no single model that provides an equally satisfactory fit to both classes of observations. We have noted above the discrepancy in recent estimates of inner core structure.

Density is not constrained by traveltimes, and remains the least well defined aspect of radial earth structure. Trade-offs between density and wave speeds can be important in normal mode studies,

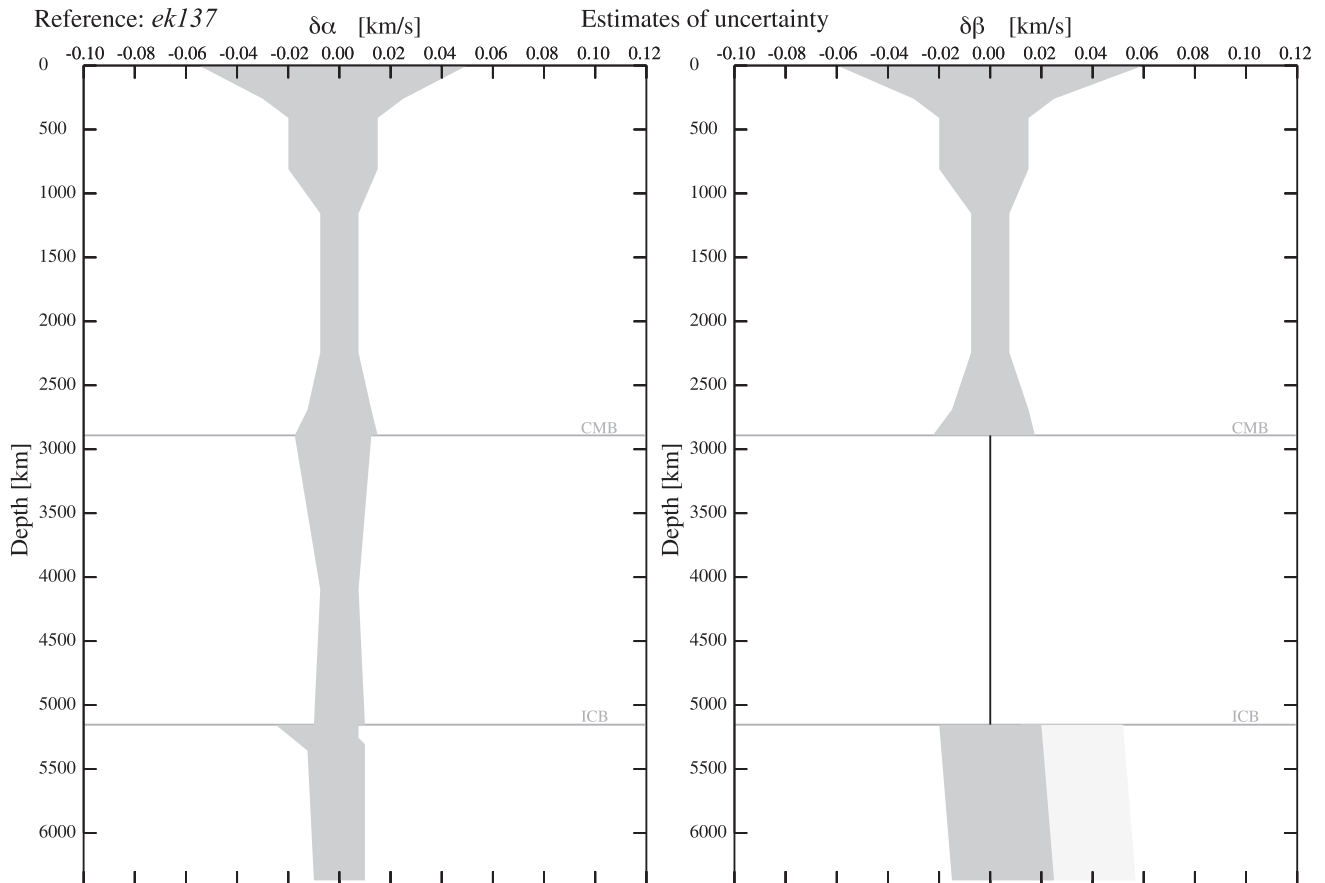


Figure 9. Assessment of levels of uncertainty in P and S wave speeds with the preferred model *ek137* as reference. The light grey area in the inner core represents the effect of the alternative estimate of inner core S wave speed from Robson & Romanowicz (2019).

as observed by Robson & Romanowicz (2019), and time constraints, where available, can help to resolve ambiguities.

6 RESOURCES

The *ek137* velocity model and the set of empirical traveltimes used in the paper are available at the Research pages on the Australian Passive Seismic server: <http://auspass.edu.au/research/traveltimes.html>. The *ek137* model is provided as a .tvel file suitable for traveltime calculations. The empirical times developed for Kennett *et al.* (1995) are provided at 1° intervals. *SKKSAC* identified at short distances may well actually correspond to the *SKSAC* branch before the cross-over with S for which the traveltimes are very similar.

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APPENDIX: TABULATED MODEL *ek137*

The *ek137* model is presented with isotropic *P* and *S* wave speeds as a function of depth, together with a density profile suitable for synthetic seismogram calculations, based on Montagner & Kennett (1996) with the removal of the low density layer in the upper mantle and consequent adjustment to match the mass of the Earth and moment of inertia.

Table A1. Model *ek137*.

Depth (km)	Radius (km)	α (km s ⁻¹)	β (km s ⁻¹)	ρ (Mg m ⁻³)
0.00	6371.00	5.8000	3.4200	2.6510
20.00	6351.00	5.8000	3.4200	2.6510
20.00	6351.00	6.5000	3.8100	2.8630
35.00	6336.00	6.5000	3.8100	2.8630
35.00	6336.00	8.0400	4.4800	3.2900
77.50	6293.50	8.0450	4.4900	3.2940
120.00	6251.00	8.0500	4.5000	3.2960
165.00	6206.00	8.1750	4.5090	3.3425
210.00	6161.00	8.3000	4.5180	3.3889
210.00	6161.00	8.3000	4.5230	3.3889
260.00	6111.00	8.4825	4.6090	3.4512
310.00	6061.00	8.6640	4.6960	3.5203
360.00	6011.00	8.8445	4.7880	3.5894
410.00	5961.00	9.0200	4.8600	3.6586
410.00	5961.00	9.3800	5.0900	3.7836
460.00	5911.00	9.5320	5.1910	3.8482
510.00	5861.00	9.6960	5.2920	3.9108
560.00	5811.00	9.8640	5.3980	3.9735
610.00	5761.00	10.0320	5.5040	4.0371
660.00	5711.00	10.2000	5.6100	4.1008
660.00	5711.00	10.7900	6.0100	4.3253
710.00	5661.00	10.9229	6.1097	4.3756
760.00	5611.00	11.0558	6.2095	4.4259
809.50	5561.50	11.1353	6.2426	4.4561
859.00	5512.00	11.2221	6.2798	4.4899
908.50	5462.50	11.3068	6.3160	4.5220
958.00	5413.00	11.3896	6.3512	4.5654
1007.50	5363.50	11.4705	6.3854	4.5926
1057.00	5314.00	11.5499	6.4187	4.6198
1106.50	5264.50	11.6277	6.4510	4.6467
1156.00	5215.00	11.7038	6.4828	4.6736
1205.50	5165.50	11.7782	6.5138	4.7001
1255.00	5116.00	11.8511	6.5439	4.7266
1304.50	5066.50	11.9224	6.5727	4.7528
1354.00	5017.00	11.9923	6.6008	4.7790
1403.50	4967.50	12.0609	6.6285	4.8050
1453.00	4918.00	12.1281	6.6555	4.8307
1502.50	4868.50	12.1952	6.6815	4.8562
1552.00	4819.00	12.2586	6.7073	4.8817
1601.50	4769.50	12.3217	6.7326	4.9069
1651.00	4720.00	12.3847	6.7573	4.9321
1700.50	4670.50	12.4450	6.7815	4.9570
1750.00	4621.00	12.5051	6.8052	4.9817
1799.50	4571.50	12.5647	6.8286	5.0062
1849.00	4522.00	12.6233	6.8515	5.0306
1898.50	4472.50	12.6812	6.8742	5.0548
1948.00	4423.00	12.7386	6.8972	5.0789
2047.00	4324.00	12.8526	6.9418	5.1264
2096.50	4274.50	12.9086	6.9627	5.1499
2146.00	4225.00	12.9648	6.9855	5.1732
2195.50	4175.50	13.0192	7.0063	5.1963
2245.00	4126.00	13.0743	7.0281	5.2192
2294.50	4076.50	13.1286	7.0490	5.2420
2344.00	4027.00	13.1834	7.0700	5.2647
2393.50	3977.50	13.2395	7.0901	5.2870
2443.00	3928.00	13.2928	7.1104	5.3092
2492.50	3878.50	13.3485	7.1319	5.3313
2542.00	3829.00	13.4056	7.1526	5.3532
2591.50	3779.50	13.4641	7.1737	5.3748
2640.00	3731.00	13.5212	7.1951	5.4012
2690.00	3681.00	13.5800	7.2168	5.4276
2740.00	3631.00	13.6394	7.2390	5.4537
2740.00	3631.00	13.6394	7.2390	5.6114
2789.67	3581.33	13.6410	7.2497	5.6476

Table A1. Continued

Depth (km)	Radius (km)	α (km s ⁻¹)	β (km s ⁻¹)	ρ (Mg m ⁻³)
2839.33	3531.67	13.6426	7.2594	5.6828
2891.50	3479.50	13.6442	7.2711	5.7591
2891.50	3479.50	8.0090	0.0000	9.9145
2939.33	3431.67	8.0941	0.0000	9.9942
2989.66	3381.34	8.1816	0.0000	10.0722
3039.99	3331.01	8.2666	0.0000	10.1485
3090.32	3280.68	8.3496	0.0000	10.2233
3140.66	3230.34	8.4305	0.0000	10.2964
3190.99	3180.01	8.5084	0.0000	10.3679
3241.32	3129.68	8.5848	0.0000	10.4378
3291.65	3079.35	8.6591	0.0000	10.5062
3341.98	3029.02	8.7320	0.0000	10.5731
3392.31	2978.69	8.8024	0.0000	10.6385
3442.64	2928.36	8.8714	0.0000	10.7023
3492.97	2878.03	8.9382	0.0000	10.7647
3543.30	2827.70	9.0034	0.0000	10.8257
3593.64	2777.36	9.0664	0.0000	10.8852
3643.97	2727.03	9.1286	0.0000	10.9434
3694.30	2676.70	9.1889	0.0000	11.0001
3744.63	2626.37	9.2470	0.0000	11.0555
3794.96	2576.04	9.3047	0.0000	11.1095
3845.29	2525.71	9.3604	0.0000	11.1623
3895.62	2475.38	9.4145	0.0000	11.2137
3945.95	2425.05	9.4669	0.0000	11.2639
3996.28	2374.72	9.5180	0.0000	11.3127
4046.62	2324.38	9.5677	0.0000	11.3604
4096.95	2274.05	9.6159	0.0000	11.4069
4147.28	2223.72	9.6623	0.0000	11.4521
4197.61	2173.39	9.7080	0.0000	11.4962
4247.94	2123.06	9.7513	0.0000	11.5391
4298.27	2072.73	9.7914	0.0000	11.5809
4348.60	2022.40	9.8304	0.0000	11.6216
4398.93	1972.07	9.8682	0.0000	11.6612
4449.36	1921.74	9.9051	0.0000	11.6998
4499.60	1871.40	9.9410	0.0000	11.7373
4549.93	1821.07	9.9761	0.0000	11.7737
4600.26	1770.74	10.0103	0.0000	11.8092
4650.59	1720.41	10.0439	0.0000	11.8437
4700.92	1670.08	10.0768	0.0000	11.8772
4751.25	1619.75	10.1092	0.0000	11.9098
4801.58	1569.42	10.1415	0.0000	11.9414
4851.91	1519.09	10.1739	0.0000	11.9722
4902.24	1468.76	10.2049	0.0000	12.0001
4952.58	1418.42	10.2329	0.0000	12.0311
5002.91	1368.09	10.2565	0.0000	12.0593
5053.24	1317.76	10.2745	0.0000	12.0867
5103.57	1267.43	10.2854	0.0000	12.1133
5153.50	1217.50	10.2890	0.0000	12.1391
5153.50	1217.50	11.0427	3.4243	12.7037
5204.61	1166.39	11.0585	3.4387	12.7289
5255.32	1115.68	11.0718	3.4514	12.7530
5306.04	1064.96	11.0850	3.4635	12.7760
5356.75	1014.25	11.0983	3.4751	12.7980
5407.46	963.54	11.1166	3.4861	12.8188
5458.17	912.83	11.1316	3.4965	12.8387
5508.89	862.11	11.1457	3.5064	12.8574
5559.60	811.40	11.1590	3.5157	12.8751
5610.31	760.69	11.1715	3.5244	12.8917
5661.02	709.98	11.1832	3.5326	12.9072
5711.74	659.26	11.1941	3.5402	12.9217
5762.45	608.55	11.2041	3.5472	12.9351
5813.16	557.84	11.2134	3.5537	12.9474
5863.87	507.13	11.2219	3.5596	12.9586
5914.59	456.41	11.2295	3.5650	12.9688

Table A1. Continued

Depth (km)	Radius (km)	α (km s ⁻¹)	β (km s ⁻¹)	ρ (Mg m ⁻³)
5965.30	405.70	11.2364	3.5698	12.9779
6016.01	354.99	11.2424	3.5740	12.9859
6066.72	304.28	11.2477	3.5777	12.9929
6117.44	253.56	11.2521	3.5808	12.9988
6168.15	202.85	11.2557	3.5833	13.0036
6218.86	152.14	11.2586	3.5867	13.0074
6269.57	101.43	11.2606	3.5875	13.0100
6320.29	50.71	11.2618	3.5878	13.0117
6371.00	0.00	11.2622	3.5878	13.0122