

The Effects of Sills and Mixing on the Meridional Overturning Circulation

Kial Douglas Stewart

A Thesis submitted for the degree of Doctor of Philosophy

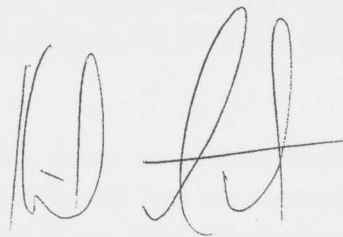
Research School of Earth Sciences

Australian National University

15 June 2012

Declaration

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

A handwritten signature in black ink, appearing to read 'Kial D. Stewart', with a stylized, cursive script.

Kial D. Stewart
15 June 2012

Acknowledgements

I remember my introduction to the GFD group; the first lecture of PHYS3034 “Physics of Fluid Flows” in Semester 2, 2006. I had been keen to take this class for as long as I had known about it, and the only reason I managed to do it was literally by dumb luck. What I learned in PHYS3034 goes beyond what you’ll find in any textbook. The combined teachings of Andy Hogg, Graham Hughes and Ross Griffiths, with Ross Kerr in the lab, made for the most inspiring and rewarding course I had ever taken; and I was hooked. 5 years later, I’m leaving the GFD group with a PhD, and I have many people to thank.

First and foremost, my supervisors Ross Griffiths and Graham Hughes. You granted me the opportunity to further not only my studies but myself, for which I am forever indebted. Your guidance over the years and encouragement of independent work have allowed me to find my feet in the academic world.

The other academic stalwarts of the GFD group, Andy Hogg and Ross Kerr, for all the advice and assistance. I have gained much from the examples you set.

None of this would have been possible without the technical expertise of Tony Beasley and Ben Tranter. You are both assets to the group and invaluable to the science.

My parents, for always being there, no matter what. Lucy and Adam, for not only giving me something to strive for but also demonstrating that it is possible to achieve. And for sheltering me in my time of homelessness. Joel, for reinforcing my belief that work ethic speaks volumes.

My long-term and long-suffering housemate, Tom, thank you for helping me stay sane. It has been so much fun.

Kelsey, thank you for your patience, support and love. I promise you it will all be worth it.

I want to dedicate this thesis to my grandfather, Bill Fripp OAM. He was very interested in my work and progress and was looking forward to seeing this thesis, but sadly passed away shortly before it was complete. Papa, you are dearly missed.

Abstract

The meridional overturning circulation of the global oceans is thought to be a result of an interplay, not yet well understood, of surface buoyancy fluxes, wind-driven upwelling and turbulent mixing. One factor believed to be important to this interplay is the latitudinal distribution of the surface buoyancy fluxes. The resulting convective circulation can be modeled in idealized laboratory experiments and numerical solutions. The convection has a number of features in common with the meridional overturning circulation and is used in this thesis to examine the effects of topography and turbulent mixing. This work is motivated by the need to better understand the roles that topographic sills and mixing play in the oceans.

A set of laboratory experiments is used to explore the effect of a partial barrier (modeling a sill in an ocean basin) on the global density structure of an overturning circulation. The experiments with a steady-state circulation forced by a gradient in either surface temperatures or heat fluxes, show that the sill will influence the density field when the sill depth is less than twice the boundary layer (or thermocline) depth. The overflow over shallow sills interacts directly with the surrounding thermocline stratification, decreasing the abyssal density relative to an ocean with no sill and increasing the density difference across the sill. The experiments show that the effect of the sill depends on the surface boundary conditions, with a larger response for conditions approaching a prescribed distribution of temperature rather than of heat flux. Application of the results to the North Atlantic circulation suggests that the Greenland-Scotland Ridge is shallow enough to lead to a significant reduction of the density of North Atlantic Deep Water, and this is consistent with the conclusions from an analysis of water mass properties.

A set of numerical solutions is used to investigate the effect of a sill on an overturning circulation from an energetics viewpoint. Specifically, the numerical solutions provide a means to test three methods of defining a reference potential energy state of no motion, a necessary exercise that is complicated by the presence of topographic barriers. The first method finds the true unique reference state as a local minimum in potential energy to which the water will relax adiabatically under the influence of gravity alone; this is achieved by an iterative process and can be computationally expensive. In the second method the domain is simplified by ignoring minor basins and calculating the reference state of the largest basin only. These two methods are applied, along with a third method that neglects the influence of topography, to the numerical solutions for an overturning circulation driven by surface buoyancy fluxes in an idealized multi-basin domain. The method that ignores topography underestimates the potential energy of the reference state, resulting in an overestimate of available potential energy. However, it is found that the method used to determine the reference state does not significantly affect the calculated rate of energy input from surface buoyancy fluxes, which is the quantity

of dynamic importance to the maintenance of the circulation. For a simple case of a ridge that defines a relatively small high-latitude basin within a much larger basin, the presence of the ridge increases the available potential energy by a large amount if the sill is shallow. (The same condition found experimentally to give mixing of the dense overflow with the overlying less dense thermocline water.) The surface buoyancy forcing, on the other hand, is insensitive to the presence of the ridge.

A second set of laboratory experiments is used to examine the effects of externally imposed rates of small-scale mixing on the overturning circulation forced by differential surface buoyancy fluxes. The destabilizing component of buoyancy flux was imposed in a turbulent plume of saline water that sank full-depth against one endwall of the experiment. The stabilizing component of buoyancy flux was supplied by a continuously replenished layer of fresh water that maintained a fixed density at the surface. A known rate of mechanical stirring was imposed by horizontal rods continuously traversed through the full depth of the experiment. The rate at which tracer and buoyancy is transported vertically in the interior can be parameterized by a vertical diffusivity, which is estimated using: (a) transport across the boundary layer, and (b) the Munk advective-diffusive balance. These estimates for the diffusivity are found to be consistent and they collapse all the measured data to a single line. For weak or no external stirring, internal processes maintain a level of vertical transport approximately 200 times larger than that expected by molecular diffusion and it is argued that this includes a component that is not irreversible mixing but which is instead advection due to turbulent entrainment into the sinking plume. Thus the parameterizations of vertical diffusivity over-estimate the mixing rate. When the added stirring (and consequent mixing) provides the dominant contribution to the total vertical transport, measurements of the equilibrated circulation show a dependence of boundary layer thickness and overturning transport on the mixing rate in accord with a theoretical model. With reference to the oceans, it is shown that the primary effect of mixing (with energy sourced from winds, tides and convection) is to deepen the thermocline, thereby influencing the buoyancy entrained by the plume. Thus, it is concluded that the advective transport of buoyancy by the plume, and not vertical mixing, is crucial for coupling the surface to the abyss.

Contents

Acknowledgements	v
Abstract	vii
1 Introduction	1
1.1 The Meridional Overturning Circulation of the Oceans	1
1.2 Horizontal Convection	3
1.3 Layout of this Thesis	4
2 Background	5
2.1 The Oceanic MOC	5
2.2 Horizontal Convection	12
2.2.1 Fundamental Experiments and Principles	12
2.2.2 The Effect of Topography	17
2.2.3 The Effect of an External Input of Mechanical Energy	20
2.2.4 The Role of Available Potential Energy	23
3 Horizontal Convection with a Sill	27
3.1 Experiments	27
3.1.1 Apparatus	27
3.1.2 Temperature Measurement Calibration	31
3.1.3 Dimensionless Parameters	33
3.1.4 Experimental Procedure	34
3.2 Results	36
3.2.1 The Effect of Sill Depth	37
3.2.2 Non-Uniform Sills	44
3.2.3 Effects of Sill Position	46
3.3 Discussion	47
3.4 Application	50
3.5 Conclusions	54
4 Numerical Solutions for Overturning Circulations with Topography	55
4.1 Methods to Determine the Reference State of a Multi-Basin Domain	55
4.1.1 Multi-Basin Relaxation (MBR)	58
4.1.2 Capped-Basin Resort (CBR)	60
4.2 Numerical Solutions	61
4.2.1 Testing and Performance of the Model	61
4.2.2 Model Setup	63

4.3	Model Results	67
4.3.1	Equilibrated Circulations	67
4.3.2	Comparison to Laboratory Experiments with Topography . .	72
4.3.3	The Dependence of E_a and Φ_{b2} on the Reference State	74
4.4	Discussion	77
4.5	Conclusions	80
5	Horizontal Convection with an External Input of Mechanical En- ergy	81
5.1	Experiments	81
5.1.1	Apparatus	81
5.1.2	Measurement Calibration	90
5.1.3	Dimensionless Numbers and Scaling	92
5.1.4	Procedure	94
5.2	Results	96
5.2.1	Qualitative Description	96
5.2.2	Quantitative Measurements	98
5.3	Discussion	102
5.4	Conclusions	110
6	Conclusions	115
6.1	Overview and Implications	115
6.2	Further Work	116
	Bibliography	119

List of Figures

1.1	Schematic of the MOC (Kuhlbrodt <i>et al.</i> , 2007)	2
1.2	Photo of Horizontal Convection	3
2.1	Global map of net radiative flux (Barkstrom <i>et al.</i> , 1989).	5
2.2	Mean latitudinal heat transport (Fasullo & Trenberth, 2007).	6
2.3	Inferred diapycnal diffusivity in the Brazil Basin Polzin <i>et al.</i> (1997) .	9
2.4	The experimental setup of Pierce & Rhines (1996)	16
2.5	‘Recycling Box’ model of Hughes <i>et al.</i> (2007)	17
2.6	Two-basin Filling-Box model of Wong & Griffiths (2001)	18
2.7	The convective circulation of Coman <i>et al.</i> (2006 <i>b</i>)	19
2.8	Phillip’s Red Sea model (Phillips, 1966)	19
2.9	Semi-enclosed buoyancy-driven exchange	20
2.10	The experimental setup of Whitehead & Wang (2008)	21
2.11	The available potential energy of simple domains (Hughes <i>et al.</i> , 2009)	23
2.12	Calculating E_p , E_b and E_a of simple cases	24
3.1	Experimental setup	28
3.2	Photo of the laboratory setup	29
3.3	Thermistor positions	31
3.4	Calibration curve for a thermistor	32
3.5	Calibration curve for a bridge	33
3.6	Experiments with an imposed temperature difference and a flat sill .	38
3.7	Experiments with an imposed flux and a flat sill	39
3.8	Vertical profiles of normalized temperature	41
3.9	Mid-depth temperatures	42
3.10	Thermal boundary layer thickness	43
3.11	Nusselt numbers	44
3.12	Experiments with an imposed flux and a V-shaped sill	45
3.13	Nusselt number for sill position	47
3.14	Mid-Depth Temperatures	48
3.15	Thermal boundary layer thickness	49
3.16	Map of the Greenland-Scotland Ridge	52
3.17	Θ - S of the Greenland-Scotland Ridge area	53
4.1	The effect of topography on E_a	56
4.2	Domain used to demonstrate the resorting methods	57
4.3	Iterative process of the MBR	59
4.4	The reference state returned by the CBR	60
4.5	The reference states returned by three methods.	61

4.6	Grid used for the numerical solutions	62
4.7	Profiles of temperature for the resolutions tested	63
4.8	Timestep testing	64
4.9	Computational scaling performance	64
4.10	Domains used for the numerical solutions	65
4.11	Mean temperature fields for $K_v = 10^{-2} \text{ m}^2 \text{ s}^{-1}$	68
4.12	Time log of the potential energy	69
4.13	Profiles of temperature for $K_v = 10^{-3} \text{ m}^2 \text{ s}^{-1}$	70
4.14	Profiles of temperature for $K_v = 10^{-2} \text{ m}^2 \text{ s}^{-1}$	71
4.15	Normalized temperatures at mid-depth	73
4.16	Time logs of E_a and Φ_{b2} for Run #7.	74
4.17	Time logs of E_a and Φ_{b2} for Run #8.	75
4.18	Time logs of E_a and Φ_{b2} for Run #9.	75
4.19	Values of normalized E_a and Φ_{b2}	76
4.20	Fractional errors for E_a and Φ_{b2} for the CBR and SBR	77
4.21	Profiles of inter-basinal temperature difference	78
5.1	Experimental Setup	82
5.2	Spool design	85
5.3	Setup of the calibration experiments	86
5.4	Evolution of the Calibration Experiment	87
5.5	Evolution of the Density Field	89
5.6	Calibrated value of κ_E and $\bar{V}$	91
5.7	Vertical profiles of temperature	96
5.8	Sequence of Photos for Experiment #1	97
5.9	Measurements of the density fields.	99
5.10	Ultrasonic Doppler Velocimeter measurements	100
5.11	Maximum of the streamfunction	101
5.12	Parameterized diffusivities	103
5.13	Data with parameterized diffusivities	105
5.14	The effect of the plume on the circulation	107
5.15	Response to a positive perturbation of κ_E	111
5.16	Comparison of numerical solutions to the laboratory experiments	112

List of Tables

3.1	Table of Experiments	35
4.1	Grid Testing	62
4.2	Table of Numerical Solutions	66
5.1	Table of Calibration Experiments	88
5.2	Table of Experiments	95
5.3	Perturbation Observations	109

Introduction

1.1 The Meridional Overturning Circulation of the Oceans

The difference between insolation and outgoing radiation at Earth's surface leads to a net heating at low latitudes and a net cooling towards the poles, requiring a net latitudinal heat transport for Earth to maintain an approximate thermal equilibrium. This heat is transported by the mean large-scale circulation of the closely coupled atmosphere-ocean system. The present-day setting is one in which the majority of the total heat is stored in the oceans but transported in the atmosphere (e.g., Bryden & Imakawi, 2001; Trenberth & Caron, 2001; Wunsch, 2005). While most of our daily experiences are more directly affected by the state of the local atmosphere, we require a good knowledge of ocean dynamics to understand and predict our ever-changing climate.

The length scales of ocean motions range from turbulent eddies to large-scale currents and gyres. To first-order, the zonally-integrated ocean circulation is comprised of a large-scale meridional overturning (Fig. 1.1; Kuhlbrodt *et al.*, 2007). The governing physics of this oceanic meridional overturning circulation (MOC) are not well understood and remain an active area of oceanographic research, to which the current study will contribute.

The MOC is a result of two primary forces: surface buoyancy fluxes (e.g., Winton, 1995; Marotzke & Scott, 1999; Hughes & Griffiths, 2006) and surface wind stress (e.g., Toggweiler & Samuels, 1998; Webb & Sugimotohara, 2001; Gnanadesikan *et al.*, 2005). For the first, the latitudinal distribution of surface buoyancy fluxes (e.g., the sum of heating, cooling, evaporation, precipitation, river discharge, etc.) leads to dense waters forming at high-latitudes and sinking as localized gravity currents. The dense sinking waters are heavily constrained by topography as they fill the ocean basins, and are balanced by a broad upwelling and entrainment of ambient water into the dense gravity currents. The range of densities observed throughout the oceans are maintained by surface buoyancy fluxes alone. In the second, the prevailing atmospheric winds at certain locations (particularly in the Southern Ocean and at continental boundaries) establish a divergent velocity field in the surface waters, leading to an oceanic upwelling to maintain continuity. It is these two forces, surface buoyancy fluxes and surface wind stress, that result in the MOC.

There are many other processes of profound importance to the MOC, primarily

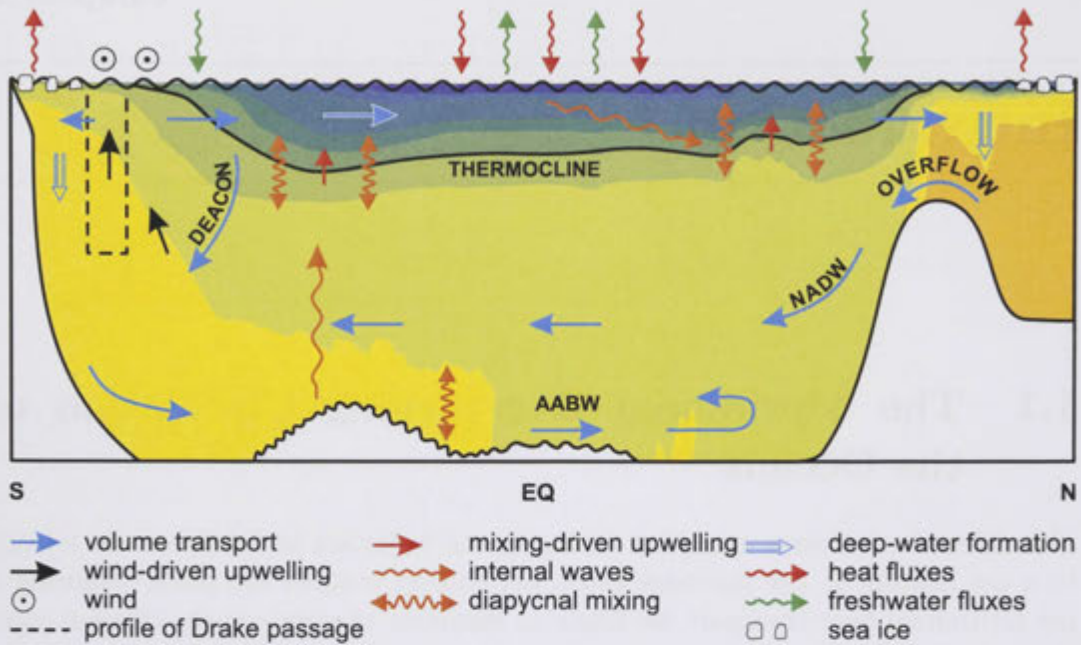


Figure 1.1: A simplified schematic of the zonally-integrated meridional overturning circulation in the Atlantic Ocean. From Kuhlbrodt et al. (2007)

through their ability to mix the oceans (e.g., Munk, 1966; Munk & Wunsch, 1998; Wunsch & Ferrari, 2004). Atmospheric winds impart a large amount of energy into the upper ocean, some of which goes into surface and internal gravity waves, leading to an increase in turbulent mixing. Tides act on the full-depth of the ocean and, through interaction with bottom topography, mix the deep ocean. Both winds and tides are able to excite and sustain a broadband internal wave field, enabling their respective influences to propagate throughout the global oceans and maintain an increased level of mixing. The biosphere also contributes to the turbulent mixing of the oceans (e.g., Dewar *et al.*, 2006; Kunze *et al.*, 2006; Visser, 2007). However, no combination of these processes will result in a mean oceanic MOC in the absence of surface buoyancy fluxes and wind-driven upwelling.

The observed oceanic stratification and MOC can coexist only if there is a continual down-gradient transport of density. This transport is enhanced by mixing, which acts to broaden the vertical density profile and accommodate a more rapid MOC (e.g., Munk, 1966; Bryan, 1987; Park & Whitehead, 1999; Hughes & Griffiths, 2006). It is the interplay between the surface buoyancy fluxes, wind-driven upwelling and turbulent mixing that is of primary importance to the MOC. However, a detailed physical understanding of the role of mixing is yet to emerge.

In addition to heat, the oceans store and globally redistribute large amounts of carbon dioxide. While the oceans are currently a net sink of atmospheric CO_2 (e.g., Sabine *et al.*, 2004), this may not have always been the case (e.g., Broecker, 1982; Toggweiler, 1999). There is current concern into how the oceans, in particular the



Figure 1.2: The ‘polar’ end of a laboratory experiment of horizontal convection driven by thermal boundary conditions of a piecewise uniform temperature difference at the base of the tank. This laboratory experiment is inverted relative to the oceans.

Southern Ocean, will respond to anthropogenic global warming, especially in terms of their ability to absorb CO_2 and potential climatic feedbacks (Rintoul, 2011). Shifts in the state of the MOC, and thus atmosphere-ocean CO_2 fluxes, have been suggested as a possible trigger mechanism for palaeoclimate change (e.g., Toggweiler *et al.*, 2006). Hence greater knowledge of the MOC is required to better understand Earth’s changing climate.

1.2 Horizontal Convection

Differential buoyancy fluxes along a single horizontal boundary of a fluid establishes an overturning circulation known as ‘horizontal convection’ (after Stern, 1975). Horizontal convection is fundamentally different to the more commonly studied forms of convection, such as Rayleigh-Bénard and Grashof convection, in which buoyancy is transported between the horizontal and vertical boundaries, respectively. Idealized laboratory experiments and numerical solutions for horizontal convection have a number of features in common with the MOC, and have been used to help understand the role that surface buoyancy fluxes might play in the latter (e.g., Rossby, 1965; Mullaney *et al.*, 2004). For instance, the horizontal (meridional) variation of buoyancy fluxes in horizontal convection (and the global oceans) leads to a (poleward) transport of heat in a (surface) boundary layer and highly localized sinking plumes situated in (high latitude) regions where the buoyancy flux is destabilising (Fig. 1.2). There is a (deep) horizontal return flow and a broadly distributed (upwelling) flow towards the forcing boundary surface.

Only simple rectangular box geometries have been used in experiments with horizontal convection, thus excluding some physical processes fundamental to the MOC. For instance, the majority of deep water sinking occurs as localized outflows from marginal seas or semi-enclosed basins (Dickson & Brown, 1994; Orsi *et al.*, 1999; Schott *et al.*, 2004). The submarine topography in these regions strongly influence

the sinking, mixing and lateral spreading of these waters. Bottom topography also tends to dam dense water in abyssal basins and restrict exchange flows between basins (Whitehead, 1998; Hansen & Østerhus, 2000). Climate models suggest that the overturning circulation in the Atlantic Ocean is particularly sensitive to perturbations, and this is attributed to the complex topography of the North Atlantic and Nordic Seas (Ganopolski & Rahmstorf, 2001, 2002). The role that topography plays in the global MOC, and implications for the dynamics and transport of heat by the circulation, are largely unknown.

Most experimental work on horizontal convection has considered only buoyancy forcing, where the internal transport of heat is by advection, molecular diffusion and any mixing generated by the convection itself. Turbulent mixing powered by external sources of mechanical energy (e.g., winds, tides, biosphere, etc.) is widely thought to play an important role in the ocean, and has been shown to strongly influence the circulation in experiments with horizontal convection (Whitehead & Wang, 2008). However, a complete understanding of the effect of turbulent mixing in horizontal convection is yet to emerge.

1.3 Layout of this Thesis

The current study is motivated by the need to better understand the potential roles that topography and turbulent mixing play in the MOC. These effects will be introduced into idealized and controllable laboratory experiments and numerical solutions for horizontal convection. The first section of this thesis reports the results of laboratory experiments with topography, which is introduced in the form of a bottom ridge that defines a sub-basin and allows an exchange flow over the top of the ridge (or sill). The presence of the topography influences the overturning circulation if the topography is large enough. The experiments investigate the effect of the topography size and geometry, and position relative to the buoyancy sources. The second section of this thesis reports numerical solutions for horizontal convection with topography. These solutions extend the laboratory experiments with topography to investigate the influence of the topography on the energy budget of the circulation. The final section of this thesis reports laboratory experiments with horizontal convection in which mechanical mixing is imposed on the circulation (using stirring rods to mimic turbulent mixing generated by the winds, tides and biosphere in the oceans). We examine how the circulation and density field are modified by the mixing.

An overview of previous work is given in §2. The laboratory experiments and numerical solutions for horizontal convection with topography are presented in §3 and §4, respectively. Laboratory experiments with horizontal convection and an external input of mechanical energy are presented in §5.

Background

2.1 The Oceanic MOC

The latitudinal distribution of net radiation at the top of the atmosphere (Fig. 2.1; Barkstrom *et al.*, 1989) requires a latitudinal transport of heat in order for Earth to maintain a thermal equilibrium. This heat transport is performed by the mean large-scale circulations in the tightly coupled atmosphere-ocean system. The coupling of the atmosphere and ocean plays an important role in the latitudinal heat transport. However, there is merit in studying the heat transport processes of the atmosphere and ocean individually, so long as we are mindful of the coupled nature of the system

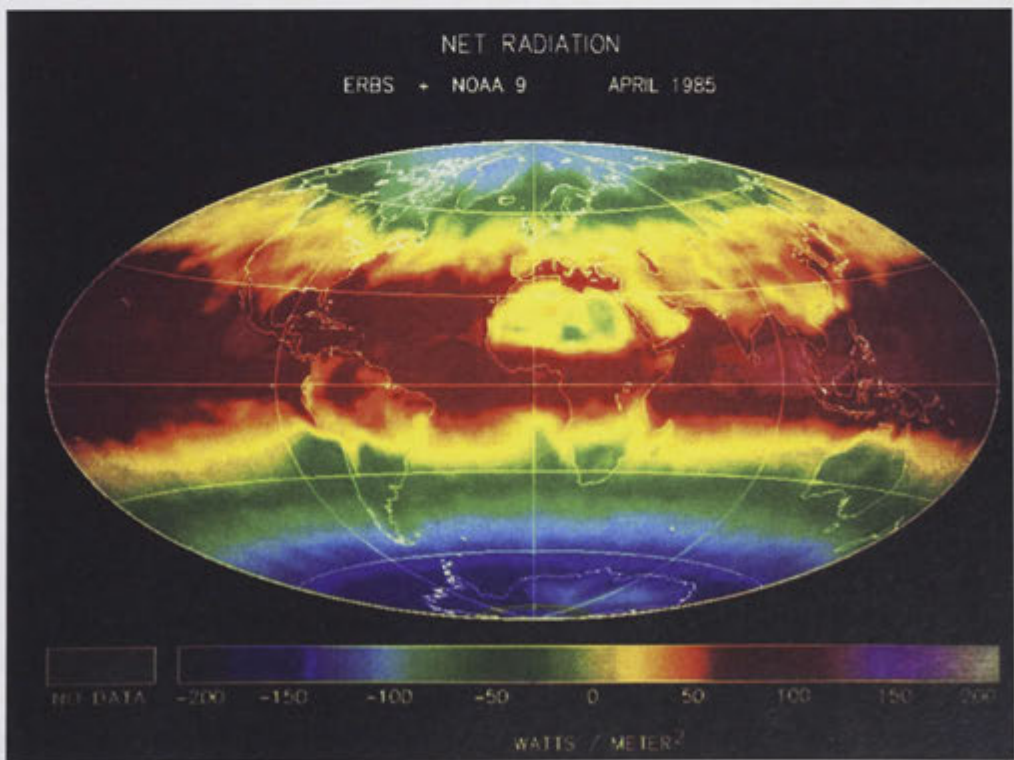


Figure 2.1: Global map of net radiative flux for April 1985. *From Barkstrom et al. (1989)*

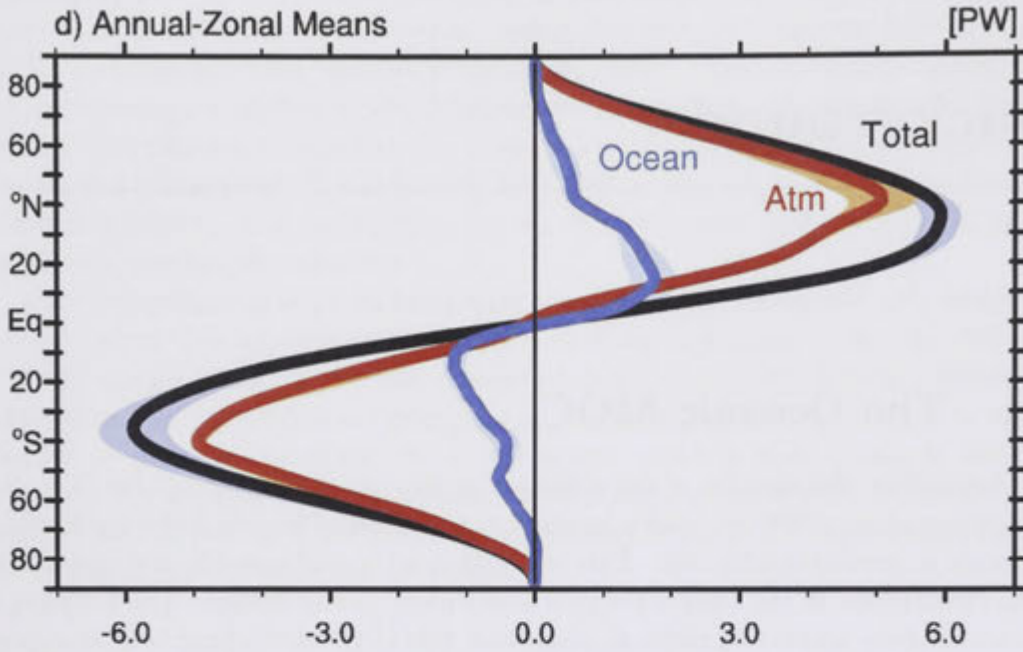


Figure 2.2: Zonally-integrated annual mean heat transport (in PW north) for latitude. The blue, red and black lines represent the ocean, atmosphere and total, respectively, with the shading showing the 2σ range. From Fasullo & Trenberth (2007)

as a whole, and interpret the results as such.

Surface waters of the oceans at latitudes equatorward of approximately 20° in both hemispheres experience a net heating (in terms of the zonally-integrated annual mean; Fasullo & Trenberth, 2007). The complicated global network of surface currents, heavily influenced and steered by atmospheric wind stress, carry a net poleward transport of heat to balance the latitudinal oceanic heat budget (Fig. 2.2). This oceanic heat is released to the atmosphere at higher latitudes, cooling and densifying the surface waters. The formation of sea-ice and subsequent brine rejection further increases the density of these surface waters, which eventually become unstable and downwell in very localized regions, occupying less than 1.5% of the total ocean surface area (Whitehead, 2000). The downwelling primarily occurs as dense gravity currents which follow topography as they sink (Killworth, 2001), bringing water from the surface to the abyss (Stommel, 1962; Munk, 1966; Winton, 1995). Entrainment of ambient water increases the transport of the sinking gravity currents and modifies their properties from those of the high-latitude surface waters (Hughes & Griffiths, 2006). The localized sinking is balanced by an upwelling of the abyssal and intermediate waters, closing the MOC. Most of this upwelling is thought to be broadly distributed throughout the ocean area (e.g., Munk & Wunsch, 1998), a configuration that gives a maximum overturning for a minimum upwelling velocity and minimum potential energy (Winton, 1995). Some of the upwelling occurs as advection along isopycnal surfaces that outcrop to the ocean surface as a re-

sult of prevailing atmospheric wind stress, particularly in the Southern Ocean (e.g., Toggweiler & Samuels, 1998; Webb & Sugimotohara, 2001; Gnanadesikan *et al.*, 2005).

The Effect of Topography

The sinking limbs of the MOC occur in regions of complex topography. The observation that Antarctic Bottom Water (AABW) originates from the likes of the Ross and Weddell Seas (Orsi *et al.*, 1999), and that North Atlantic Deep Water (NADW) forms from surface waters transformed in the Nordic and Labrador Seas (Dickson & Brown, 1994; Schott *et al.*, 2004), might be interpreted as evidence that high-latitude marginal seas are necessary for the formation of Deep and Bottom waters. However, the densification of water in these seas, which is assisted by the restriction of circulation by topographic sills, does not necessarily lead to a greater density of abyssal ocean waters. Previous studies (e.g., Stommel, 1962; Winton, 1995) have shown that localized dense water formation at high latitudes does not require such enclosing topography. This brings into question the extent to which deep and bottom water formation at high latitudes (which is a consequence of the large-scale meridional gradient of surface buoyancy fluxes; Stommel, 1962) is modified by the presence of shallow or semi-enclosed seas. The existence of these seas may simply make them the preferred sites for the production of densest water, or for the greatest rate of production of density anomaly (*i.e.* largest buoyancy flux). The answer to the above question and to what abyssal densities would exist in the absence of marginal seas cannot readily be determined from observations in the present oceans, and is the motivation for the experiments detailed in §3.

For the purpose of this thesis, a marginal sea is either a relatively shallow semi-enclosed sea connected to the main ocean basin, or a second deep but smaller basin separated from the main ocean by a relatively high (*i.e.* shallow) topographic sill. Dense overflows from such marginal seas are the major contributors to the sinking legs of the MOC and help maintain the abyssal density stratification (Saunders, 1996, 2001). Thus any factors influencing the density or mass flux of these dense overflows are likely to affect the abyssal densities and the overall stratification (Bryden & Nurser, 2003).

Topographic sills can give rise to hydraulic control of the mass flux in dense overflows (Killworth, 1995; Whitehead, 1998; Hansen & Østerhus, 2007). Control of the mass exchange by sills also implies control of the density difference between the semi-enclosed sea and the ocean. In a long-term steady state, the lateral density difference must be large enough to force an exchange that facilitates a net export of heat from the semi-enclosed sea to balance the surface cooling of the sea. Shallower sills will tend to give rise to larger density differences. Thus sills could potentially govern the abyssal stratification. Simple experiments and numerical solutions have demonstrated the effects of sills and relative basin sizes on the density fields in two basins connected by a sill overflow when there is a destabilizing buoyancy flux into one basin in the form of a turbulent plume (Wong & Griffiths, 2001). Other experiments provide a relationship between the surface buoyancy flux in a marginal sea and the resulting baroclinic mass exchange flux in the sill overflow into an

infinitely large basin (Finnigan & Ivey, 1999, 2000; Finnigan *et al.*, 2001), but these studies did not address the role of forcing, circulation and density stratification in the large basin. The experiments were not intended to achieve a global equilibrium, with zero net surface heat input, as might be supposed for the oceans in a fixed climate regime.

Observations of the Greenland-Scotland Ridge (GSR) overflows (e.g., Voet & Quadfasel, 2010; Fer *et al.*, 2010) highlight the important role that mixing and entrainment of ambient waters at the ridge play in setting the properties of NADW, ventilating the deep North Atlantic and setting the global ocean density stratification. The Strait of Gibraltar is another example of an overflow modified by mixing (in the vicinity of Carmarinal Sill) with the North Atlantic lower thermocline waters, leading to a density difference between the “Mediterranean Waters” in the Mediterranean and in the North Atlantic (Bryden *et al.*, 1994).

The overflows, and particularly the associated entrainment, are a complicated convective process usually unresolved in numerical solutions and GCMs, thus requiring parameterization. Typically, convective parameterizations act to effectively invert or homogenize the unstable water column, or perform bulk mixing via large vertical diffusion coefficients prescribed spatially (*a priori*) or adaptively whenever a water column becomes locally unstable (e.g., Marotzke & Scott, 1999; Marshall & Schott, 1999). The parameterization of overflows has been identified as a weakness of GCMs and is the focus of much current work (e.g., Legg *et al.*, 2009; Cenedese & Adduce, 2010; Danabasoglu *et al.*, 2010; Wells *et al.*, 2010).

The Effect of Mixing

Like overflows, mixing is another process requiring parameterization in GCMs. In the oceanographic sense, mixing refers to an enhanced rate of diapycnal diffusive transport arising from (predominantly) turbulent processes, such as stirring or instabilities in the circulation (shear, convective or otherwise). Observations of velocity microstructure infer a background diapycnal diffusivity of $O(10^{-5}) \text{ m}^2 \text{ s}^{-1}$, 100 times larger than the molecular diffusion of heat, with substantially larger values in regions of rough topography (e.g., Fig. 2.3; Polzin *et al.*, 1997). Although mixing operates on the smallest of length scales, it is thought by many to have a leading order influence on ocean density and the MOC. Parameterizations of mixing generally involve manipulating the diffusion coefficients, either globally, spatially or adaptively depending on local density and velocity fields or Richardson number (e.g., Large *et al.*, 1994; Marotzke & Scott, 1999). These parameterizations attempt to describe the roles of unresolved scales and can inadvertently have a leading-order influence on the solution.

An enhanced rate of diapycnal diffusive transport is required for the observed stratification and rate of deep water formation to coexist. This relationship between mixing and the MOC was described by Munk (1966) as a one-dimensional advective-diffusive balance;

$$w \frac{\partial C}{\partial z} = \kappa \frac{\partial^2 C}{\partial z^2}, \quad (2.1)$$

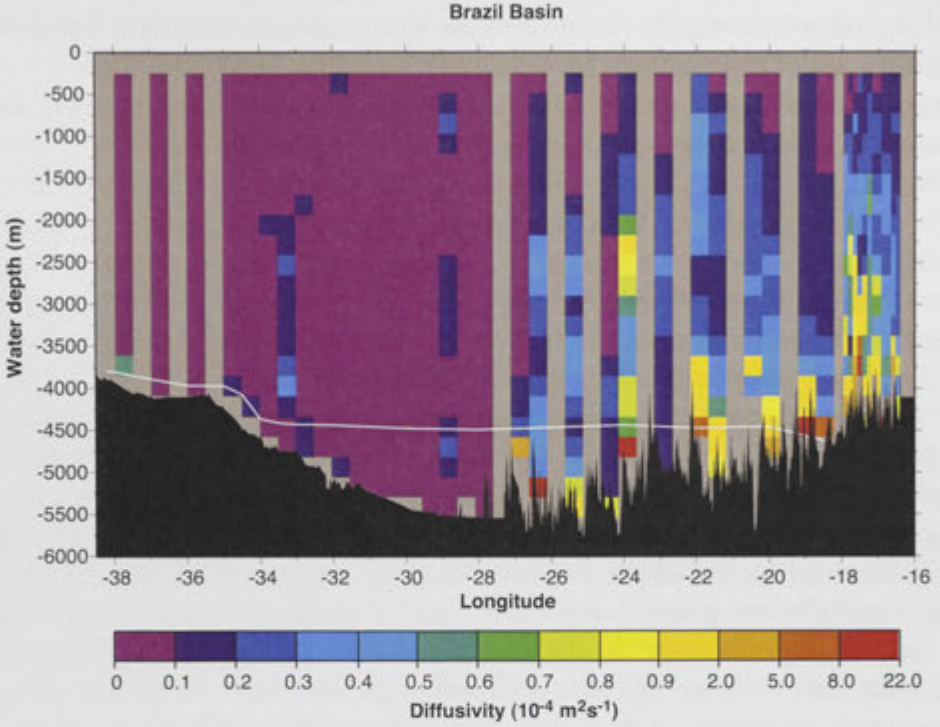


Figure 2.3: Longitudinal section through the Brazil Basin of diapycnal diffusivity inferred from microstructure observations. *From Polzin et al. (1997)*

where w (m s^{-1}) and κ ($\text{m}^2 \text{s}^{-1}$) are the upwelling velocity and diapycnal diffusion coefficient of the stratifying species, respectively, and C is the concentration of a given tracer. If we consider heat as the tracer of interest, the left-hand term of (2.1) reflects the advective heat flux associated with the upwelling of cold deep water displaced by the flux of new dense water and required to close the MOC. The right-hand term refers to the diffusive heat flux from the surface downward through the water column. For a given stratification to be steady these advective and diffusive fluxes must be in balance, such that there is a zero net heat flux at all depths. To maintain the observed abyssal stratification against the estimated upwelling of the global oceans, the Munk (1966) balance calls for a diapycnal diffusivity of $\kappa = 10^{-4} \text{ m}^2 \text{s}^{-1}$, which requires an input of 2.1 TW of mechanical energy to the oceans to power the mixing (Munk & Wunsch, 1998; Wunsch & Ferrari, 2004).

We note here a simple but widespread misconception that by increasing the diapycnal diffusivity, or mixing rate, one increases the total potential energy (E_p) of the ocean by raising the centre of mass of the water column (e.g., Munk & Wunsch, 1998; Huang, 1999; Wunsch & Ferrari, 2004; Kuhlbrodt *et al.*, 2007). This fallacy can arise by considering the case of an initial stratification at rest that is subsequently exposed to an increased diapycnal diffusion (mixing). The final state will indeed have a higher centre of mass and an increased E_p relative to the initial state. However, when the initial stratification is the result of the Munk (1966) advection-diffusion

balance, the equilibrated final state with enhanced mixing will be one where heat is mixed downwards to a greater depth in the water column, and therefore decreasing the density and E_p of the water column relative to the initial state.

Diapycnal diffusivity is not a directly measurable entity; rather, it is inferred or parameterized from observations of the density or velocity fields, or by tracer release experiments (e.g., Gregg, 1989; Ledwell *et al.*, 1993; Kunze & Sanford, 1996; Polzin *et al.*, 1997). Such studies indicate that the background diapycnal diffusivity is an order of magnitude smaller than the value required by Munk (1966) and Munk & Wunsch (1998). Ocean observations also reveal large spatial variability to the diapycnal diffusivity field that appears correlated with the local complexity, or roughness, of the bottom topography (e.g., Fig. 2.3; Toole *et al.*, 1994; Polzin *et al.*, 1996, 1997; Ledwell *et al.*, 2000). Regions have been identified where the mixing rates are up to 1000 times larger than the background value (Naveira Garabato *et al.*, 2004; Lauderdale *et al.*, 2008; Fer *et al.*, 2010; Voet & Quadfasel, 2010). However, it seems unlikely that these isolated regions can account for the globally averaged diffusivity required by the Munk balance (Webb & Sugimotohara, 2001; Rudnick *et al.*, 2003). Nevertheless, these localized regions of intense mixing are postulated to have far reaching effects on the MOC (Jayne & St. Laurent, 2001; Friedrich *et al.*, 2011), particularly those regions coincident with deepwater formation sites, where the upstream properties of deep and abyssal waters can be significantly influenced by enhanced mixing (e.g., Mauritzen *et al.*, 2005).

The discrepancy between observations and canonical theoretical estimates of diapycnal diffusivity remains an outstanding question. The quest to find this apparent ‘missing mixing’ has motivated work from an energetics viewpoint (e.g., Munk & Wunsch, 1998; Huang, 1999; Wunsch & Ferrari, 2004; Hughes *et al.*, 2009; Tailleux, 2009). Central to this approach is identifying the sources of mechanical energy that sustain the mixing (observed or otherwise). The wider energy budget for the overturning is also under debate, specifically the energy sources and sinks. In one view mixing is put forward as a source of mechanical energy (*i.e.* the driver) for the MOC, with the energy for turbulent mixing ultimately coming from winds and tides (e.g., Wunsch & Ferrari, 2004). This case has been referred to as “pulling” of the circulation by (diabatic) mixing (Visbeck, 2007); however, the term “pulling” has also been used to refer to the (adiabatic) upwelling of deep water as a result of the wind-driven Ekman transport of the upper Southern Ocean (e.g., Wolfe & Cessi, 2011) or even to the hypotheses that the sinking of dense water “pulls” the less dense thermocline water poleward. The dynamical alternative (again diabatic) is sometimes referred to as “pushing” of the circulation, in which the MOC is forced by the sinking of dense water (e.g., Marotzke & Scott, 1999; Marshall & Schott, 1999). Hughes & Griffiths (2006, 2008) argued that both “pushing” and “pulling” hypotheses, in which either sinking or interior mixing is dominant, are misleading and that the circulation requires a balance of the two mechanisms.

The potential sources of mechanical energy to the oceans are currently believed to be the winds, tides, surface buoyancy fluxes, bio-locomotion, geothermal heating and atmospheric pressure loading. However, an understanding of exactly how these physical processes couple and their relative importance in sustaining the MOC is

lacking and a topic of much debate. Munk & Wunsch (1998) estimate that 2.1 TW of mechanical energy is required to generate the mixing necessary to sustain the observed stratification, and assert that winds and tides are the only contributors capable of providing this amount of energy. They reason that surface buoyancy fluxes can be neglected because they act to expand and contract the oceans at the same geopotential, such that there is no significant change to the total potential energy (E_p), and therefore they are unable to provide any significant contribution of mechanical energy to the circulation. This argument was taken further with the hypothesis that surface buoyancy fluxes cannot maintain an overturning circulation in the absence of an additional source of mechanical mixing; a view that has been adopted more widely (e.g., Huang, 1999; Wunsch & Ferrari, 2004; Wang & Huang, 2005; Kuhlbrodt *et al.*, 2007) and often erroneously associated with the theory of Paparella & Young (2002) and postulate of Sandström (1908, 1916), both of which will be reviewed in more detail in §2.2.3. However, as pointed out by Hughes *et al.* (2009), this reasoning is flawed because it fails to distinguish between E_p and available potential energy (E_a , as defined by Oort *et al.*, 1994; Winters *et al.*, 1995); this will be covered in detail in §2.2.4. Indeed, it has been demonstrated experimentally that an overturning circulation can be established and sustained without an external source of mechanical mixing (e.g., Rossby, 1965; Miller, 1968; Mullaney *et al.*, 2004; Coman *et al.*, 2006a, 2010).

Neglecting surface buoyancy fluxes as a significant contributor to the MOC has far reaching consequences. First and foremost, the absence of surface buoyancy fluxes will lead to a homogenous ocean, which is obviously not consistent with observations. The sinking of dense water at high-latitude (a direct result of surface buoyancy fluxes) supplies buoyancy to all depths and ventilates the deep ocean, and in doing so sets and maintains the stratification. Convection arising from surface buoyancy fluxes ostensibly provide a means of contributing to the turbulent mixing of the ocean. Entrainment into the dense sinking currents is one form of such buoyancy-driven mixing and has a substantial influence on the abyssal density (as revealed by measured overflow dilution, Mauritzen *et al.*, 2005) and overturning strength. Additionally, the sinking currents provide an efficient advective coupling of surface conditions with the abyss (Winton, 1995; Hughes & Griffiths, 2006). Neglecting surface buoyancy fluxes requires this coupling to be achieved by turbulent mixing in a diffusive manner, at great expense energetically.

As mentioned above, atmospheric winds at certain locations are capable of sustaining a divergent velocity field in the surface waters, leading to an oceanic upwelling to maintain continuity (e.g., Toggweiler & Samuels, 1998; Webb & Suginohara, 2001; Gnanadesikan *et al.*, 2005). The consequences of such an upwelling for the energy budget are not included in previous accounting (Munk & Wunsch, 1998), and would reduce the energy required to maintain the observed stratification. Another potential source of mechanical energy often overlooked is the biosphere, which consumes an estimated ~ 63 TW of chemical power (Behrenfeld & Falkowski, 1997). As a conservative estimate, if 1% of this power is converted into mechanical energy, the contribution from the biosphere is comparable to that from the winds or tides (e.g., Dewar *et al.*, 2006; Kunze *et al.*, 2006; Visser, 2007). The contributions of

mechanical energy arising from geothermal heating and atmospheric pressure loading are believed to be $O(10^{-2})$ TW and therefore not significant (Wunsch & Ferrari, 2004).

There is strong evidence that mixing plays an integral role in the MOC (Bryan & Cox, 1967; Bryan, 1987; Park & Whitehead, 1999; Park & Bryan, 2000, 2001; Hughes & Griffiths, 2006), but a clear understanding of this role is yet to emerge. Approaching the problem from an energetics viewpoint has raised additional questions and revealed much about the complex nature of oceanic mixing global circulation. The aim of this thesis is to improve the understanding of the relationship between mixing and the MOC.

2.2 Horizontal Convection

2.2.1 Fundamental Experiments and Principles

Given that we cannot test the dependence of the actual MOC on variables, we must use models to understand the processes involved. One process believed to play a role in the MOC is the latitudinal distribution of surface buoyancy fluxes. A simple laboratory model of this process is one which prescribes heating and cooling at one horizontal boundary of a fluid. Such boundary conditions give rise to an overturning circulation known as ‘horizontal convection’ (after Stern, 1975), a particular class of thermal convection recently reviewed by Hughes & Griffiths (2008).

A thought experiment proposed by Stommel (1962) suggested that differential heating at one horizontal boundary would lead to an asymmetric overturning circulation where the downwelling is highly localized and the upwelling is broadly distributed. This was put forward to explain the observed smallness of sinking regions in the global oceans. The thought experiment motivated Rossby (1965) to perform laboratory experiments, in which a linear gradient of temperature was prescribed to the base of an enclosed and insulated rectangular tank of water. For practical reasons this forcing arrangement is inverted relative to the oceans, however the convective circulation can be compared to that of the MOC providing the flow is Boussinesq. The asymmetry between the horizontal scales of upwelling and downwelling legs predicted by Stommel (1962) was observed (Fig. 1.2). Rossby (1965) described the circulation as largely two-dimensional and independent across the width of the tank, with subsequent laboratory studies reporting additional small-scale three-dimensional turbulence (e.g., Mullarney *et al.*, 2004). A boundary layer is established at the base of the tank; the differential heating maintains a horizontal density gradient in this layer, resulting in a flow towards the heated end of the tank. The boundary layer directly feeds a plume which rises against the endwall at the heated end. The plume is turbulent and entrains ambient fluid as it rises and it forms an outflow along the top of the tank, supplying buoyancy to the uppermost regions of the tank. Fluid in the plume outflow slowly downwells through the interior of the tank and is either entrained horizontally into the plume or recycled back into the lower boundary layer, closing the convective circulation.

As suggested by Rossby (1965), the asymmetric nature of the overturning circulation in experiments with horizontal convection has been shown to be qualitatively insensitive to the temperature distribution along the forcing boundary (Pierce & Rhines, 1996; Mullarney *et al.*, 2004; Wang & Huang, 2005; Wåhlin *et al.*, 2010), including in the rotating case (Hignett *et al.*, 1981; Speer & Whitehead, 1988; Park & Whitehead, 1999). The asymmetric circulation was initially thought to arise from the relative efficiencies of the heat transfer processes (*i.e.* advection of heat in the plume compared to diffusion of heat in the interior, Rossby, 1965). It was later shown on energetic grounds that an asymmetric configuration achieves a given overturning for a minimum upwelling velocity and minimum potential energy (Winton, 1995). The asymmetry of convection has been shown to be retained when the buoyancy fluxes vary non-monotonically with distance or becomes two-dimensional. For example, Coman *et al.* (2010) report on the convection arising from a forcing boundary with two regions of destabilizing buoyancy flux separated by a central region that provided stabilizing buoyancy. Their circulation involved two plumes, of which the dominant plume (that having maximum buoyancy) traversed the full-depth of the tank, with the weaker plume outflowing at an intermediate level where it reached neutral buoyancy. Incorporating a second plume weakened the interior stratification relative to that arising from a single plume of equal (total) buoyancy flux. An example of the converse distribution of buoyancy was used by Miller (1968), who performed experiments with a temperature distribution at the forcing boundary that varied smoothly from a destabilizing flux in the centre to stabilizing fluxes at the two ends of the box. This forcing arrangement removed the plume from either endwall, allowing it with an extra degree of lateral freedom and leading to increased unsteadiness in the circulation.

The potential oceanographic application for horizontal convection was first raised by Sandström (1908, 1916), who reported on the convective circulation arising from heating and cooling at opposite ends of a long tank of water, in which the relative vertical positions of the heat source and sink were varied. Defant (1961) stated as a postulate drawn from empirical conclusions of Sandström (1908, 1916) that “a closed steady circulation can only be maintained in the ocean if the heat source is situated at a lower level than the cold source”. Jeffreys (1925) argued theoretically that this postulate is incorrect because it discounts diffusion. This has been interpreted by some to imply that surface buoyancy fluxes can only give rise to a ‘slow, diffusively-driven’ circulation (Huang, 1999), an interpretation is at odds with numerous observations of laboratory experiments with horizontal convection (*e.g.*, Rossby, 1965; Mullarney *et al.*, 2004; Wang & Huang, 2005). The use of Sandström’s reports as outlined above are confused as his experiments certainly included the diffusion of heat, and yet he reported no sustained circulation. Indeed, Coman *et al.* (2006a) recreated the experiments of Sandström (1908, 1916), finding a significant steady circulation is maintained for all experiments with an imposed horizontal gradient of buoyancy and concluding that the original observations were in error. Thus, there is no physical basis, experimental or otherwise, that supports this conclusion of Sandström (1908, 1916).

It is worth elaborating on the depth of the circulation. The overturning cir-

ulation of equilibrated horizontal convection is confined to the depths to which buoyancy is able to be supplied by the forcing boundary (Hughes & Griffiths, 2008). For example, Wunsch & Ferrari (2004) and Kuhlbrodt *et al.* (2007) argue that solar radiation only penetrates into the top 100 m of the ocean water column, thus any circulation established by buoyancy forcing alone will be confined to above this depth. Additionally, in the original publication Sandström (1908) himself, showing great insight, concluded that the oceanic circulation will penetrate to the depth of mixing, which at that early time he suggested might be around 1000 m. Both of these cases rely on diffusive processes (albeit turbulent in the latter) to supply the buoyancy to depth and set the vertical extent of the circulation. A more efficient means of supplying buoyancy to depth is via advection in the plume. In this process the plume entrains buoyant ambient water (including from within the surface boundary layer) and advects it to depth. It makes good physical sense that the depth of the circulation is inherently related to the depth at which the plume outflows. The question is now, what determines the vertical extent of the plume?

When the boundary conditions are steady and the circulation is in equilibrium, there is a zero net buoyancy flux through any level (Paparella & Young, 2002; Hughes & Griffiths, 2006; Hughes *et al.*, 2007), such that the buoyancy supplied by the plume to any level is balanced by a buoyancy flux in the ambient. The only input and output of buoyancy for the volume is the forcing boundary, and the most buoyant (in the case of forcing at the base) is located in the plume where it leaves the forcing boundary. The maximum horizontal difference in buoyancy is at the forcing boundary, and this is always larger than the maximum vertical difference in buoyancy (Rossby, 1998; Hughes & Griffiths, 2008). That is, as soon as the plume water separates from the forcing boundary its buoyancy decreases via interaction with ambient fluid. For the entire plume to become neutrally buoyant at an intermediate depth requires the region beyond that depth to have a buoyancy excess. This implies a finite stratification at the level of the plume outflow, across which the buoyancy excess will diffuse (violating the zero net buoyancy flux condition for equilibrium), eroding the anomaly and eventually reestablishing a full-depth plume. The only ways for a partial-depth plume to be in a steady equilibrium state is when there is an additional source of buoyancy distant to the forcing boundary to maintain the buoyancy excess, or if there exists a non-diffusive interface that prevents the erosion of the buoyancy excess and maintain the zero net buoyancy flux condition. It remains possible that the long-time equilibrium state of horizontal convection supports a fluctuating circulation in which the plume outflow depth oscillates as would be the case for a well-mixed interior.

Rossby (1965) identifies three dimensionless parameters important to the equilibrated circulation; these were generalized by Hughes & Griffiths (2008) to be the Rayleigh or flux Rayleigh number,

$$Ra_{\Delta b} = \frac{\Delta b L^3}{\nu_m \kappa_m} \quad \text{or} \quad Ra_B = \frac{B L^3}{\nu_m \kappa_m^2}, \quad (2.2)$$

respectively, the aspect ratio of the tank,

$$A = \frac{D}{L}, \quad (2.3)$$

and the Prandtl number of the fluid,

$$Pr = \frac{\nu_m}{\kappa_m}, \quad (2.4)$$

where Δb (m s^{-2}) is the imposed buoyancy difference or B ($\text{m}^3 \text{s}^{-3}$) is the imposed buoyancy flux (per unit width spanwise), ν_m is the kinematic viscosity of the fluid, κ_m is the diffusivity of the stratifying species, and D and L are the tank depth and length, respectively. The style of the imposed boundary condition determines which Rayleigh number is appropriate.

Using a scaling analysis to balance viscous and buoyancy forces in the boundary layer, Rossby (1965), Hignett *et al.* (1981) and Mullarney *et al.* (2004) derive and experimentally verify relationships between the appropriate Rayleigh number and boundary layer thickness, volume transport and heat transport as,

$$\frac{\delta}{L} \sim Ra_{\Delta b}^{-1/5} \sim Ra_B^{-1/6}, \quad \frac{\Psi_{bl}}{\kappa_m} \sim Ra_{\Delta b}^{1/5} \sim Ra_B^{1/6}, \quad Nu \sim Ra_{\Delta b}^{-1/5} \sim Ra_B^{-1/6}, \quad (2.5)$$

where δ (m) is the boundary layer thickness, Ψ_{bl} ($m^2 s^{-1}$) is the volume transport (per unit width spanwise) in the boundary layer, and Nu is the Nusselt number, which characterizes the rate of convective buoyancy transport relative to the conductive transport, calculated as,

$$Nu = \frac{B}{\kappa_m \Delta b}. \quad (2.6)$$

These scaling relationships were further supported by the results of numerical solutions by Beardsley & Festa (1972), and, following computational advances, by Rossby (1998), Paparella & Young (2002), Siggers *et al.* (2004), Mullarney *et al.* (2004), Ilıcak & Vallis (2011) and Scotti & White (2011). Such numerical solutions are able to overcome some of the practical constraints faced by the laboratory experiments, particularly to explore a wider range of parameter space. However, the numerical solutions have their own limitations, especially in relation to resolution.

Laboratory experiments with horizontal convection have predominantly employed heat as the stratifying species by prescribing either a distribution of constant temperature or heat flux at a horizontal boundary (Rossby, 1965; Mullarney *et al.*, 2004). The Prandtl number of these experiments is therefore of order one, as it is in the oceans. However, using heat to provide the buoyancy does have some practical limitations. For example, thermally-driven experiments are prone to insulation problems, have a small range of usable temperatures (and thus densities), and heat has a relatively large diffusion coefficient. Hence the Rayleigh number is limited, and heat transfer through the sidewalls is difficult to eliminate. This motivated Pierce & Rhines (1996) to turn to salt to provide solutal buoyancy fluxes;

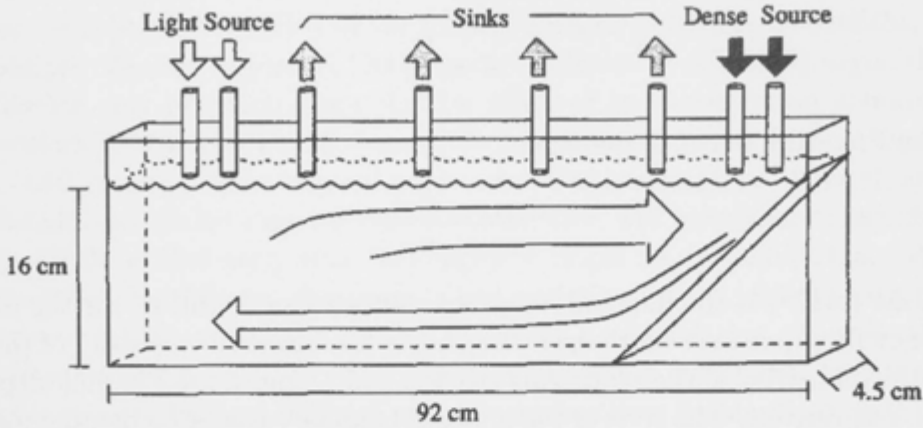


Figure 2.4: The experimental setup used by Pierce & Rhines (1996). The sloping endwall forces the sinking fluid to act as a gravity current as opposed to a vertical plume, altering the entrainment dynamics. *From Pierce & Rhines (1996).*

they report on the convective flow that arises from surface fluxes of fresh and saline water located at opposite ends of a rectangular tank (Fig. 2.4). A series of sinks, situated at the surface between the sources, actively pumped water out of the tank to maintain approximately constant volume. Although their experiments did not achieve equilibrium, they observed the full-depth asymmetric circulation characteristic of horizontal convection. Whitehead & Wang (2008) extended the experiment of Pierce & Rhines (1996) to impose turbulent mixing, as described below in §2.2.3.

The forcing arrangements of Pierce & Rhines (1996) and Whitehead & Wang (2008) both impose constraints on the density field at the location(s) of the sink/spillway. The density of the water exiting the tank must approach the average of the source waters, and thus the density field is sensitive to the depth of the sink/spillway. In §5 we describe laboratory experiments with solute-driven horizontal convection where this constraint is removed by using boundary conditions that maintain a layer of fresh water at the surface and impose a flux of saline water into the tank.

The generalized theory of Hughes *et al.* (2007) builds upon those of Peterson (1979) and Hughes & Griffiths (2006) to combine models for a turbulent entraining plume and an ambient return flow of the interior, coupled at all depths, such that there are no undetermined parameters (Fig. 2.5). This model is referred to as a ‘recycling box’ model, in contrast to the ‘filling box’ model of Baines & Turner (1969), for which, like horizontal convection, entrainment into the plume plays a significant role. For instance, altering the entrainment characteristics of the plume by simply changing the geometry or location/configuration of a plume of given buoyancy flux will lead to a significant global effect on the flow and density fields (e.g., Miller, 1968; Pierce & Rhines, 1996; Paparella & Young, 2002; Hughes *et al.*, 2007; Coman *et al.*, 2010). It is interesting to note that the recycling box theory is strictly inviscid (and in the solution the Prandtl and Rayleigh numbers always

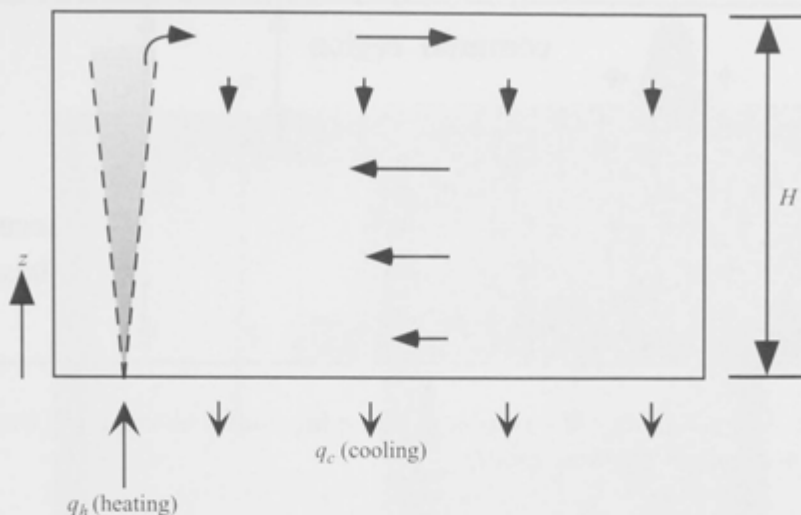


Figure 2.5: A schematic of the ‘recycling box’ model of Hughes *et al.* (2007). A point source (or line source) of heat maintains a localized region of upwelling (represented theoretically as a turbulent plume). Cooling is distributed uniformly over the base such that there is a zero net heat flux into the volume. From Hughes *et al.* (2007).

appear as a product), whilst viscosity was thought to play an important role in setting the dynamic balance in the boundary layer scaling of (2.5). Despite this, the model of Hughes *et al.* (2007) gives solutions consistent with the high-Rayleigh number laboratory experiments and numerical solutions of Rossby (1965, 1998); Mullarney *et al.* (2004). It warrants further verification.

2.2.2 The Effect of Topography

Regions of deep-water formation have been referred to as the Achilles heel of the climate system (e.g., Broecker, 1997), where the large-scale overturning circulation is sensitive to local factors. One such factor is submarine topography, which is prevalent in marginal seas, semi-enclosed basins and continental shelves, where much of the deep-water formation occurs. The extent to which the MOC is influenced by topography is unclear, and is the motivation for the experiments detailed in §3. To date, all experiments with equilibrated horizontal convection have been performed in simple rectangular boxes.

Wong & Griffiths (2001) were motivated by the significant topography of deep-water formation sites to introduce a vertical partition to the enclosed volume of a filling-box. The barrier restricts the horizontal spreading of fluid and leads to a circulation characterized by different stratifications in each basin (Fig. 2.6). The basin directly ventilated by the turbulent plume ‘fills’ as per a filling-box and overflows to the second basin. The overflow is denser than the fluid of the second basin and sinks against the barrier as an overflow plume which entrains the ambient fluid. Fluid in the ventilated basin is denser and has a stronger stratification than that in

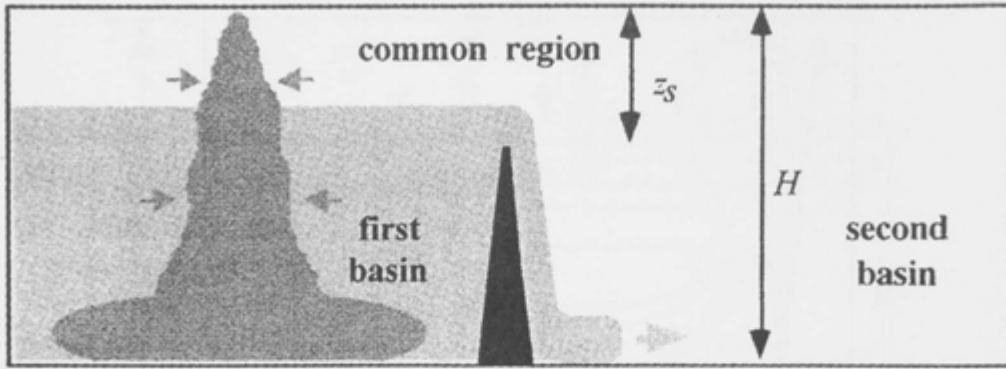


Figure 2.6: A single plume in a two-basin filling-box model showing the overflow across the sill. *From Wong & Griffiths (2001).*

the second basin. The stratification of the fluid above the barrier becomes virtually identical to that expected for a single-basin filling-box. The height of the barrier and relative basin size are of first order importance to the circulation.

It has long been established that two endwalls of an enclosed volume held at different constant temperatures will drive a convective circulation dominated by narrow boundary layers against the vertical endwalls, with the interior evolving to become approximately linearly stratified (e.g., Simpkins & Chen, 1986). Coman *et al.* (2006b) introduce a vertical sill to such a convective configuration (Fig. 2.7). The convective flow was in a laminar regime and the vertical partition significantly altered the thermally-equilibrated steady-state circulation. The temperature fields in the two chambers are very different with the development of a near-stagnant “very cold chamber” developing between the barrier and cool endwall. Atop the cold chamber is a fast overflow that is formed predominantly of fluid detrained from the narrow boundary layer adjacent to the cold endwall. The region above the partition develops a number of horizontal shear layers. No change in heat transport occurred unless the height of the partition was greater than 80% of the total tank depth, under which conditions the heat transport rapidly decreased to zero as the partition height was increased to the full height of the box.

Experiments investigating the influence of topography on convection arising from surface buoyancy fluxes have been limited to semi-enclosed volumes. For instance, an analytical model proposed by Phillips (1966) considers the buoyancy driven exchange across a sill (Fig. 2.8), motivated by the observed circulation within the Red Sea. The Phillips Red Sea model exhibits a circulation limited to the depth above the barrier. This model was built upon by, among others, Tragou & Garrett (1997) and Maxworthy (1997) to include a turbulent vertical diffusivity and allow for a hydraulically critical sill exchange, respectively, which leads to a circulation that occupies the entire basin region. Such studies focus on the circulation within the semi-enclosed basin or marginal sea and treat the large-scale ocean circulation as an infinite source of water with no feedback from the semi-enclosed basin.

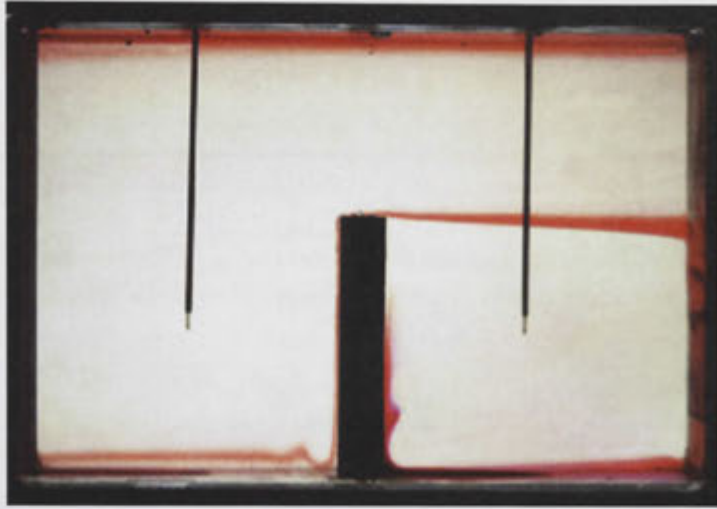


Figure 2.7: The convective circulation of Coman *et al.* (2006b). The left- and right-hand endwalls are heated and cooled, respectively. *From Coman et al. (2006b).*

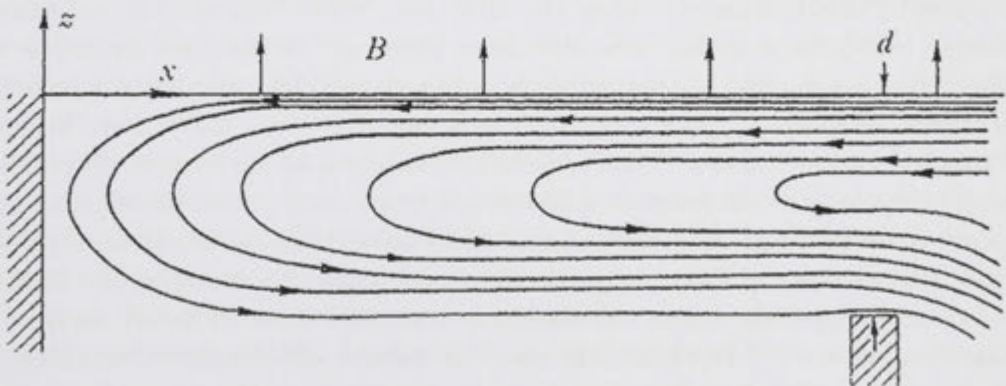


Figure 2.8: Sketch of the streamlines in a convective motion driven by a uniform surface buoyancy flux, with a barrier of depth D far from a vertical boundary at $x = 0$. *From Phillips (1966)*

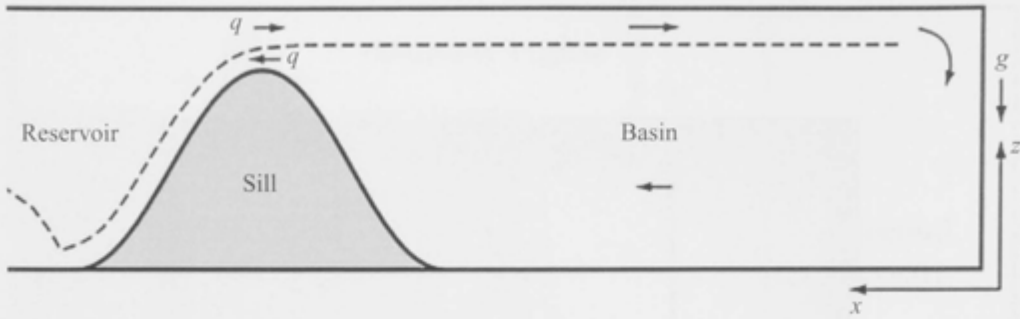


Figure 2.9: Sketch of the flow pattern for a semi-enclosed basin where the surface above the small basin is maintained at a constant density. *From Finnigan & Ivey (1999).*

Finnigan & Ivey (1999, 2000) and Finnigan *et al.* (2001) reported laboratory and numerical experiments with buoyancy driven exchange flows. Figure 2.9 depicts their arrangement, in which they found the circulation maintains a well-mixed basin and an exchange across the sill into the (effectively) infinite reservoir. Although they achieved a steady-state circulation in the small basin under an imposed density boundary condition, they did not attempt to attain a global equilibrium having a zero net buoyancy flux. They reported that the exchange across the sill in their laboratory experiments was hydraulically critical.

2.2.3 The Effect of an External Input of Mechanical Energy

Paparella & Young (2002) revisited the work of Sandström (1908, 1916) and presented a special case when the postulate is rigorous; that is, in the limit of $\kappa \rightarrow 0$ for a fixed Prandtl number. They use this case, which removes the objection of Jeffreys (1925), to pose the ‘anti-turbulence theorem’: if the only forcing is non-uniform heating applied at the surface of a Boussinesq fluid and if the viscosity, ν , and thermal diffusivity, κ , are lowered to zero, with $Pr = \nu/\kappa$ fixed, then the energy dissipation ϵ also vanishes. With this strict definition of turbulence, Paparella & Young (2002) argue that horizontal convection is not truly turbulent, thus an ocean exposed to surface buoyancy fluxes alone could not exhibit the observed small-scale marine turbulence; therefore, it is the winds and tides that energise the turbulent motions and enable the ocean circulation to transport heat. However, in this vanishing limit of $\kappa \rightarrow 0$ the heat flux into the volume also vanishes, resulting in a null circulation. This limit is known to be inappropriate to the oceans (or any fluid with a finite diffusivity), and thus conclusions drawn from it are limited. The claim that surface buoyancy fluxes alone cannot sustain turbulent motions is at odds with many laboratory experiments (e.g., Mullarney *et al.*, 2004; Coman *et al.*, 2010) and numerical solutions (e.g., Paparella & Young, 2002; Mullarney *et al.*, 2004; Scotti & White, 2011). Despite this, authors continue to use the theorem of Paparella & Young (2002) to vindicate the postulates of Sandström (1908, 1916) and justify discounting the role of surface buoyancy fluxes in maintaining the MOC.

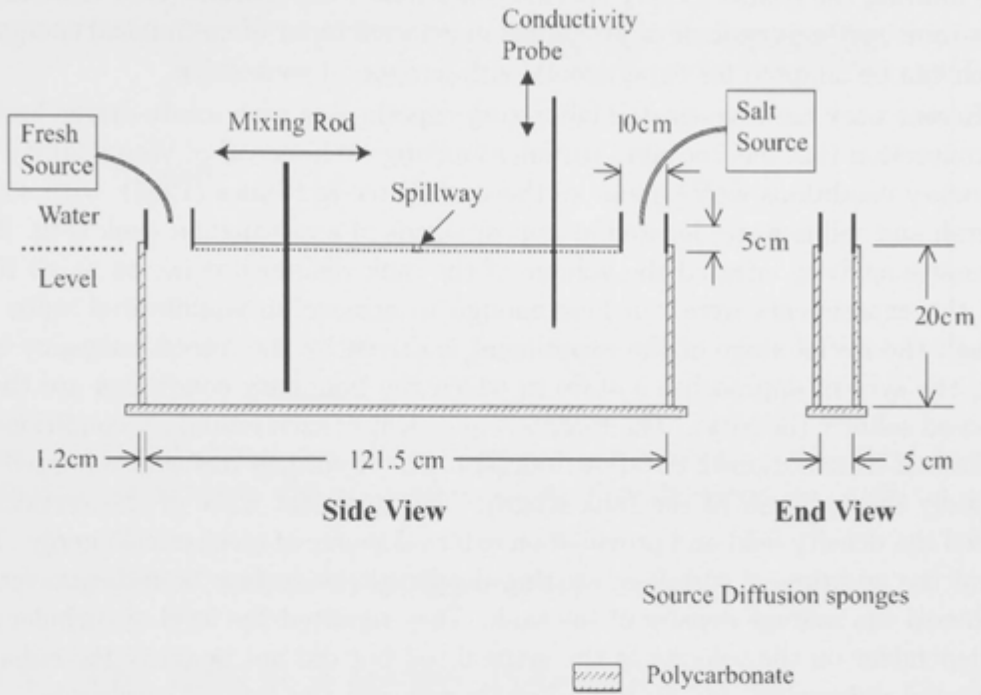


Figure 2.10: The experimental setup of Whitehead & Wang (2008). *From Whitehead & Wang (2008).*

The experimental observations of Sandström (1908, 1916) and theorem of Paparella & Young (2002) imply that a distribution of surface buoyancy alone cannot maintain an overturning circulation in the absence of diffusion ($\kappa \rightarrow 0$). This is often interpreted to mean that surface buoyancy forcing cannot maintain an overturning circulation in the absence of *mixing*, and that this mixing requires an additional source of energy (e.g., Kuhlbrodt *et al.*, 2007; Ilıcak & Vallis, 2011). Diffusion is required to transfer buoyancy to and from the surface, and this transfer of buoyancy is enhanced by mixing. In fact, for an overturning circulation arising from surface buoyancy forcing to be in equilibrium, the surface buoyancy fluxes are necessarily balanced by the (irreversible) mixing (Hughes *et al.*, 2009). In this sense both diffusion and mixing are required for surface buoyancy forcing to maintain an overturning circulation. However, the mixing can be energized by sources other than an external input of mechanical energy, for instance, instabilities in the convective circulation itself.

Experimental studies of turbulent mixing generally involve stirring an initial stratification and tracking the evolution of the density field in time (e.g., Linden, 1980; Fernando, 1991; Holford & Linden, 1999; Whitehead & Stevenson, 2007). The mixing or diapycnal diffusion cannot be measured directly and is inferred from measurements of the density and velocity fields. Such experiments find the diapycnal diffusion for a given stirring (dependent on the geometry of the tank) to be sensitive to the initial stratification and motion of the agitator (e.g., the Richardson number),

thus limiting the results to very specific conditions. Nevertheless, they demonstrate repeatable methods capable of providing an external input of mechanical energy and which can be adapted for experiments with horizontal convection.

Recent work has investigated laboratory experiments with solute-driven horizontal convection that incorporated turbulent mixing (Whitehead & Wang, 2008). The boundary conditions were similar to those of Pierce & Rhines (1996), with sources of fresh and saline water located at opposite ends of a rectangular tank (Fig. 2.10). A passive spillway ensured the volume of the tank remained constant at all times, and the experiments were run long enough to achieve an equilibrated state. Although the initial stage of the experiment is driven by the forced buoyancy input flux, the system approaches a state in which the boundary conditions are that of imposed salinity (density). The thermal equivalent of such boundary conditions is a prescribed distribution of constant temperature. A vertical rod was oscillated horizontally through half of the tank length. The turbulent wake of the vertical rod stirred the density field and provided an external source of mechanical energy. They found the addition of turbulent stirring deepened the surface boundary layer and freshened the average density of the tank. They reported the level of turbulence to be dependent on the velocity of the vertical rod but did not quantify the enhanced diapycnal diffusivity; nor do they directly measure the rate of overturning. The mechanical energy input by the rod was estimated to scale as the velocity of the rod cubed, but it is not known how much of this energy was dissipated and how much went into raising the background potential energy.

It is well established that the rate of overturning increases with parameterized diapycnal diffusion in numerical solutions and GCMs of convectively-driven circulations (e.g., Bryan, 1987; Park & Bryan, 2000, 2001; Hughes & Griffiths, 2006; Ilıcak & Vallis, 2011). It is often the case that large diapycnal diffusivities are required to ensure the solution is numerically stable for given spatial and temporal resolutions, or for computational efficiency, leading to spurious estimates of overturning. This sensitivity is often taken advantage of to tune the numerical models to better reproduce observations, without considering the physical implications such as the additional energy sources required to perform the extra mixing (as explained by Rahmstorf, 2002; Wunsch & Ferrari, 2004). Counter-intuitively, a coarse-resolution model will occasionally out-perform (in the sense of reproducing observations) a fine-resolution model, calling into question the scale-independence of the parameterizations.

An early geostrophic theory derived by Bryan & Cox (1967) incorporates the vertical advective-diffusive heat balance and balances the meridional density gradient with the zonal velocity. The theory predicts the overturning to scale with diapycnal diffusivity as $\Phi \sim \kappa_*^{2/3}$ for the case of an imposed temperature difference at the surface. This theory was extended to the case of an imposed heat flux and predict the slightly weaker dependence of $\Phi \sim \kappa_*^{1/2}$ (Park & Whitehead, 1999). The non-rotating case is addressed by the recycling box model of Hughes *et al.* (2007), who show for a flux condition, the overturning to scale as $\Phi \sim \kappa_*^{1/4}$. These theories consider the total diapycnal diffusivity (κ_*), which is irreversible and includes molecular diffusion and turbulent contributions from both external sources of mechanical

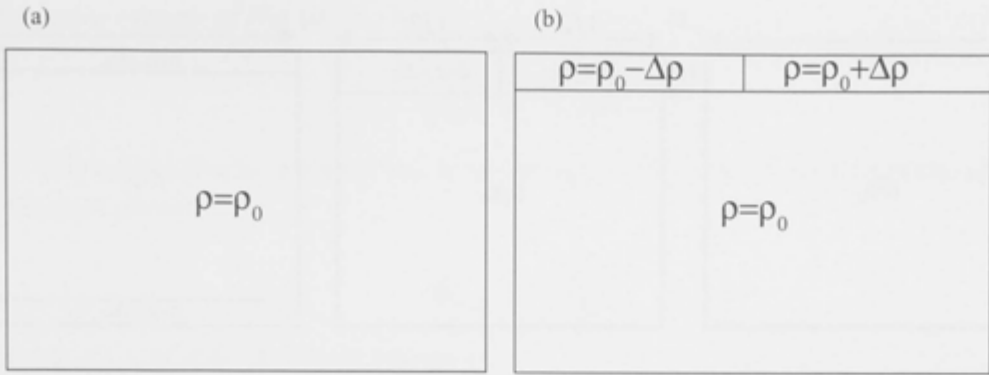


Figure 2.11: Both boxes have the same total potential energy, but only (b) possesses available potential energy and will result in a flow. *From Hughes et al. (2009)*

energy and internal instabilities arising from the circulation itself.

2.2.4 The Role of Available Potential Energy

Heating and cooling at the same geopotential (e.g., approximately the case for the ocean surface to within a couple of meters) results in a small total change in potential energy (E_p ; Huang, 1999; Paparella & Young, 2002). This observation been used to support the theory that surface buoyancy fluxes are unimportant in (and perhaps even unable to contribute to) the energy budget of the MOC (e.g., Huang, 1999; Munk & Wunsch, 1998; Wunsch & Ferrari, 2004). Thus, it has been proposed that turbulent mixing in the oceans, and hence the MOC as a whole, is driven by other sources of energy (*i.e.* winds and tides). However, this reasoning fails to recognize the important distinction between the total potential energy and the available potential energy (E_a , e.g., Oort *et al.*, 1989, 1994; Hughes *et al.*, 2009; Tailleux, 2009). For instance, consider the simple example in Figure 2.11. The total potential energy in boxes is identical, but a flow will only occur in (b). This is because it is not in gravitational equilibrium and possesses available potential energy.

Recent work (Tailleux, 2009; Hughes *et al.*, 2009) has built on the framework proposed by Lorenz (1955) and Winters *et al.* (1995) to decompose the total potential energy into that which is ‘available’ to promote fluid motions and that which is not; namely the available gravitational potential energy, E_a , and the reference (or background) potential energy, E_b , respectively. Surface buoyancy fluxes produce available potential energy at the expense of the background potential energy. For a circulation to be in steady state, this rate of generation of E_a is necessarily balanced by the rate of irreversible mixing, which returns energy to E_b . The rates of transfer of energy between the various reservoirs is important to the maintenance of the MOC, and not the total amount of any given reservoir (Hughes *et al.*, 2009).

Following Winters *et al.* (1995), the total potential energy of a Boussinesq fluid with a linear equation of state is given by,

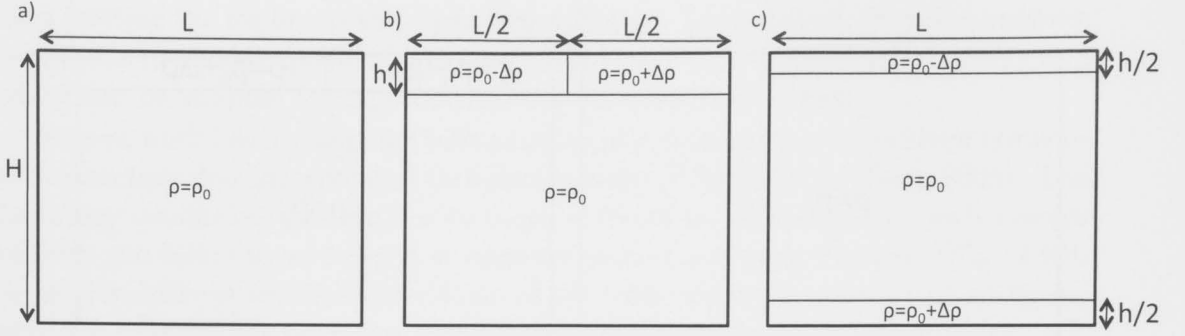


Figure 2.12: Calculating E_p , E_b and E_a for the two boxes in Figure 2.11. Adiabatically resorting Fig. 2.11a does not change the density field, thus $E_p = E_b$ and $E_a = 0$. Adiabatically resorting Fig. 2.11b returns the density field pictured in (c); in this case $E_p > E_b$ and $E_a > 0$. Adapted from Hughes et al. (2009)

$$E_p = g \int_V \rho z dV, \quad (2.7)$$

where ρ is the density of the fluid and z is the vertical coordinate. The reference potential energy defined as,

$$E_b = g \int_V \rho z_* dV, \quad (2.8)$$

where z_* is the height to which a fluid element will move if the entire domain is adiabatically resorted to be stably stratified and in a state of no motion.

The available gravitational potential energy is defined as the difference between the total potential energy and reference potential energy,

$$E_a = E_p - E_b = g \int_V \rho (z - z_*) dV. \quad (2.9)$$

This is the energy available to promote motion (*i.e.* for conversion to kinetic energy). In addition to determining the reference and available potential energy, z_* is also required in order to calculate the rate of supply of energy by surface buoyancy fluxes into available potential energy. Adopting the notation of Hughes *et al.* (2009), this conversion rate, referred to as the surface buoyancy forcing Φ_{b2} , is given by

$$\Phi_{b2} = -\rho_0 \oint_A z_* B dA, \quad (2.10)$$

where ρ_0 is a reference density of the fluid and B is the buoyancy flux per unit area ($\text{m}^2 \text{s}^{-3}$) imposed at the surface of area A .

Equations 2.7–2.9 can be understood for the illustrated example in Figure 2.11. We assume both boxes are of equal height and length H and L , respectively, and that the height of the density anomalies in Fig. 2.11b is h ; this is depicted in Figure 2.12 alongside the adiabatically resorted box for the case of Fig. 2.11b. The total

potential energy of Fig. 2.11a is

$$E_{p,(a)} = \frac{g\rho LH^2}{2}. \quad (2.11)$$

The adiabatically resorted box is unchanged for Fig. 2.11a such that the reference potential energy is

$$E_{b,(a)} = \frac{g\rho LH^2}{2}, \quad (2.12)$$

giving an available potential energy of

$$E_{a,(a)} = E_{p,(a)} - E_{b,(a)} = 0. \quad (2.13)$$

For the case shown in Figure 2.11b, calculating the total potential energy requires the sum of the contributions of each of the individual water masses,

$$E_{p,(b)} = \frac{g\rho L(H-h)^2}{2} + \frac{g(\rho + \Delta\rho)Lh(H - \frac{h}{2})}{2} + \frac{g(\rho - \Delta\rho)Lh(H - \frac{h}{2})}{2}, \quad (2.14)$$

which reduces to,

$$E_{p,(b)} = \frac{g\rho LH^2}{2}, \quad (2.15)$$

the same value calculated for the case in Fig. 2.11a. Adiabatically resorting Fig. 2.11b gives the density field shown in Fig. 2.12c, for which the potential energy is

$$E_{b,(b)} = \frac{g\rho LH(H-h)}{2} + \frac{g(\rho + \Delta\rho)Lh^2}{8} + \frac{g(\rho - \Delta\rho)Lh(H - \frac{h}{4})}{2}, \quad (2.16)$$

which reduces to

$$E_{b,(b)} = \frac{g\rho LH^2}{2} - \frac{g\Delta\rho Lh(H - \frac{h}{2})}{2}. \quad (2.17)$$

This gives the available potential energy for the case in Fig. 2.11b as

$$E_{a,(b)} = E_{p,(b)} - E_{b,(b)} = \frac{g\Delta\rho Lh(H - \frac{h}{2})}{2}, \quad (2.18)$$

which is positive definite. The density field in Figure 2.11b is produced from that of Fig. 2.11a by introducing no net buoyancy, and this can be achieved by applying surface buoyancy fluxes with a zero net flux. Any process that leads to an increase in the depth (h) and magnitude ($\Delta\rho$) of the near-surface density anomalies produced from the surface buoyancy fluxes will increase the available potential energy of the system; one such process is mixing.

Horizontal Convection with a Sill

In this chapter we explore the effect of a sill in laboratory experiments with thermally-driven horizontal convection. The experiments are conducted in a rectangular tank that is differentially heated from below. A partial-depth vertical barrier, modelling a topographic ridge, divides the tank into two coupled basins. We investigate how the equilibrated circulation and density field depends on the height, cross-sectional shape and lateral position of the barrier. We find the barrier influences the interior (abyssal) density when the deepest point of the barrier (from the surface) is less than twice the thickness of the boundary layer, such that water overflowing the barrier interacts directly with the boundary layer water. An application of the results to the North Atlantic circulation suggests that the Greenland-Scotland Ridge is shallow enough to lead to a significant reduction of the density of North Atlantic Deep Water owing to interaction of the southward flowing dense overflow with the northward flowing thermocline water, consistent with the conclusions from an analysis of water mass properties.

3.1 Experiments

3.1.1 Apparatus

The experiments with horizontal convection and a topographic sill were carried out in a tank with internal dimensions of length $L = 1.25$ m, width $W = 0.305$ m, and depth $D = 0.2$ m (Fig. 3.1). The tank was identical to that used for the imposed temperature experiments of Mullarney *et al.* (2004), which was twice the width of that used for their imposed flux experiments. The tank incorporated a 10 mm thick copper base and was filled to capacity with de-aired water. The acrylic sidewalls were double-glazed, the lid was triple glazed, and the gaps filled with argon; the endwalls were 20 mm thick. The entire tank was thermally insulated with 0.1 m expanded polystyrene foam board encased in aluminium foil. The experiments were conducted in a special constant temperature laboratory (Fig. 3.2) held at 30 ± 0.3 °C, where the temperature control operated on a high frequency cycle of period ~ 15 minutes, giving little time for the fluctuations to conduct into the tank. The copper base was in direct contact with heating and cooling systems; either two water heat exchangers (each 0.6×0.305 m), or one water heat exchanger and an electric heating mat. The heat source and sink were insulated from each other by a 50 mm spacer

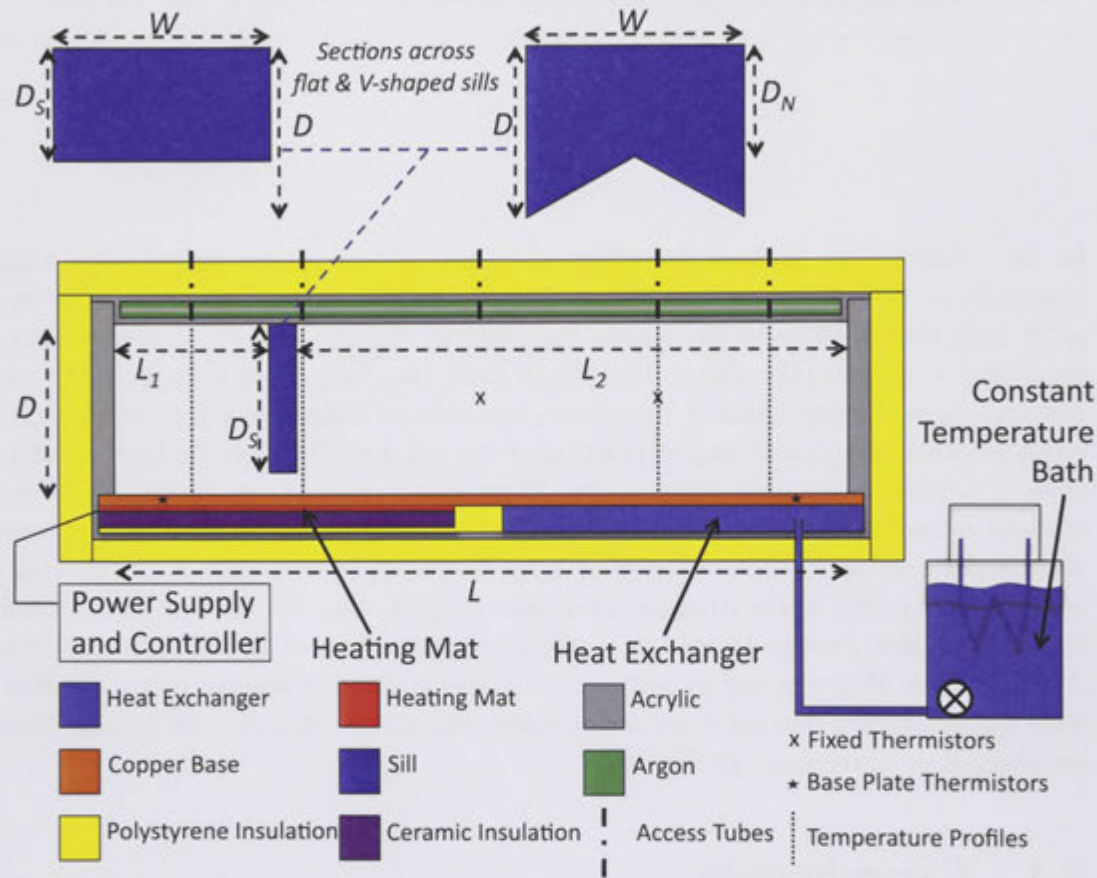


Figure 3.1: Schematic diagram of the apparatus for the experiments with an imposed heat flux. Vertical dotted lines show the location of the six profiling thermistors; one thermistor each for the profiles closest to the endwalls, and two thermistors each for the other two profiles (one on, and the other 0.1 m off, the centerline). The normalized temperature profiles (Fig. 3.8) are from the two centerline thermistors above the cold plate, and the mid-depth temperatures (Fig. 3.9) are from the thermistors closest to the endwalls. The sill is shown for a basin length ratio $L_1/L_2 = 1/4$, but ratios of 1 and 4 were also used. Cross-stream sections of the flat and V-shaped sill geometries (rotated from the vertical plane across the tank width) are also shown. A uniform heat flux is delivered to the base on the left half. For runs with imposed temperature difference, the heating element and underlying ceramic insulation were replaced with a heat exchanger identical to that shown under the right half of the base and connected to a second water bath. In both systems at thermal equilibrium there was a zero net heat input to the tank.

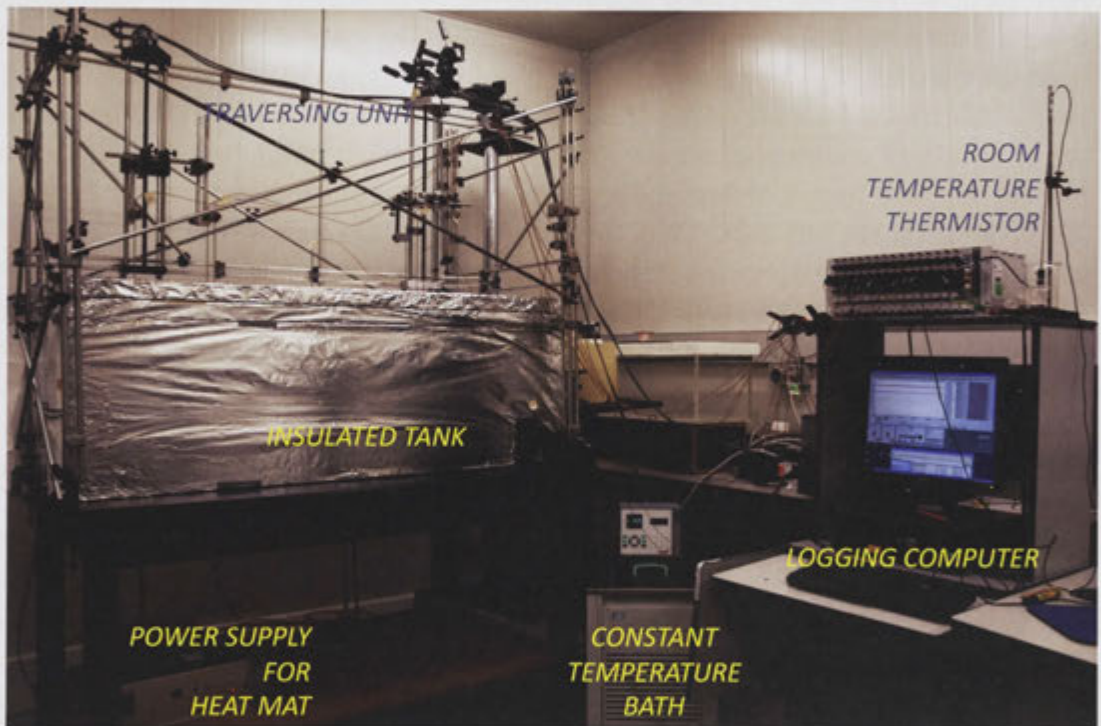


Figure 3.2: A photo of the laboratory used for the experiments of horizontal convection impeded by a sill.

placed between them. The direct transport of heat between the source and sink by conduction along the copper base was estimated from a numerical solution of the heat equation and found to be less than 0.3% of the total heat input. The insulating side and endwalls extended downwards to completely surround the heating/cooling system and the whole arrangement was insulated beneath by 0.1 m of foam board.

One set of boundary conditions consisted of a uniform, constant (input) heat flux, F_T through one half of the base, and a uniform, constant temperature on the other half of the base. The total heat input from the heating power controller was set between 210 and 420 W ($F_T = 1148$ and 2295 W m^{-2} , respectively) and most runs were with 280 W ($F_T = 1530 \text{ W m}^{-2}$). The microprocessor based controller held the input power constant to within 0.1 W. The applied temperature at the right-hand half of the base was maintained by recirculating water at $16 \pm 0.02^\circ\text{C}$ from a Julabo constant temperature bath. In steady state the applied heat input must be matched by the time average rate of heat removal at the cooled region. Hence this case is referred to below as “imposed heat flux” (Table 3.1). The second set of boundary conditions consisted of the same applied temperature on the right half of the base and a higher applied temperature, from a second water bath at $40 \pm 0.02^\circ\text{C}$, on the left half. Thus the temperature difference in the base was held at $\Delta T = 24^\circ\text{C}$, and this case is hereinafter referred to as “imposed temperature difference”. This temperature difference was chosen to match that measured in a reference experiment with an imposed heat flux of 280 W (and no sill) so that conditions were almost identical apart from the nature of the boundary conditions.

Topographic sills were simple vertical barriers of 50 mm thick impermeable polystyrene foam that sealed against the tank walls and lid. The barriers were at the top of the tank rather than against the base, as the whole system was inverted relative to the oceans for practical reasons. For the majority of the experiments the sill was positioned to give a ratio of basin lengths $L_1/L_2 \approx 1/4$, with the smaller basin on the left over the heating region. This choice creates one relatively small basin at the end having destabilizing buoyancy flux, and leaves some of that buoyancy flux applied directly to the larger basin. The basin length ratios of $L_1/L_2 \approx 1$ and 4 were also examined (Table 3.1). The barrier height D_S was such that normalized gap heights $R_D = (D - D_S)/D$, (recall D is the depth of the box) varied over the range $0 \leq R_D \leq 1$ for imposed temperature difference and $0.05 \leq R_D \leq 1$ for imposed heat flux (note that $R_D \rightarrow 0$ was not a useful limit in the latter case because the net heat flux into the small basin would lead to an unlimited temperature increase). A small number of experiments were carried out to examine the effect of a non-uniform sill height: in these runs the sill had a full-width V-shape removed from its base and of height $D - D_N$ at the apex, as a crude representation of an ocean channel (Fig. 3.1). In these cases the normalized gap height, defined as $R_D = (D - D_N)/D$, was varied over the range $0.1 \leq R_D \leq 0.95$, where D_N is the height of the triangular opening at its vertex. For these V-shaped sills, only imposed heat flux was used and the length ratio of the two basins was fixed at $L_1/L_2 = 1/4$.

Eight Thermometrics Fastip thermistor probes (FP07) were inserted into the tank through access tubes in the lid (Fig. 3.3); when used along with associated active bridge circuits these were able to detect temperature changes of $O(0.01)^\circ\text{C}$

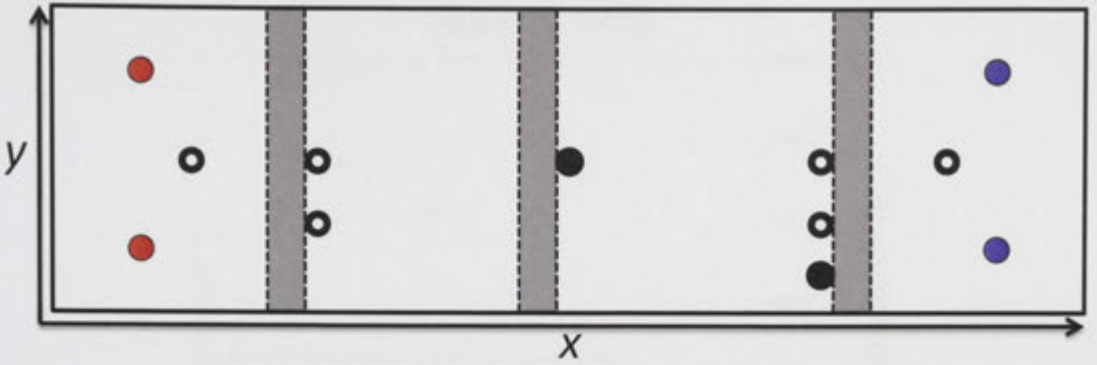


Figure 3.3: A plan view of the box showing thermistor and sill positions. The black dots represent the positions of the thermistors suspended in the water column; the hollow dots show those able to be vertically profiled through the tank. The red and blue dots represent the locations of the thermistors in the hot and cold ends of the base plate, respectively. The three lateral positions of the sill are also depicted; $L_1/L_2 \approx 1/4, 1$ and 4 , from left to right.

with a response frequency of 142 Hz and an absolute accuracy of $O(0.1)^\circ\text{C}$. Temperatures were monitored to indicate when the flow had reached thermal equilibrium (a period of approximately 3-4 days, depending on the initial water temperatures and boundary conditions), and the profiling thermistors were subsequently used to measure the temperature field. The six profiling thermistors were traversed through the depth at a constant velocity of 5 mm s^{-1} . A further four thermistors were positioned in the copper base plate; located at $(x, y) = (100, 75), (100, 230), (1150, 75) \text{ \& } (1150, 230)\text{ mm}$. A thirteenth thermistor was suspended in an open glass beaker and dedicated to logging the room air temperature.

Signals from the thermistors were sampled at 100 Hz by a dedicated computer using the National Instruments (NI) LabVIEW program. For long-time logging the 100 Hz samples were stored until 1000 samples had been collected (10 seconds) and the 10 s average was saved. For vertical temperature profiling, samples were collected at 100 Hz and a windowed-average based on 10 successive samples (0.1 seconds) was saved, giving a vertical resolution of 0.5 mm at the 5 mm s^{-1} profiling speed.

3.1.2 Temperature Measurement Calibration

Each thermistor had a resistance that is temperature dependent and requires routine calibration. Calibration of the thermistors involved submerging them in the Julabo constant temperature bath alongside a Hart Scientific[®] thermistor probe. The Hart probe was coupled to a digital readout and provided temperature readings rated to an accuracy of 0.005°C between 0 and 60°C . A 30 second average of the resistance of each thermistor was recorded using a Fluke[®] digital ohm meter (89IVRMS) for 6 bath temperature settings spanning $5\text{--}65^\circ\text{C}$. As a check for hysteresis, the bath temperature was first increased in steps through the range and then decreased in

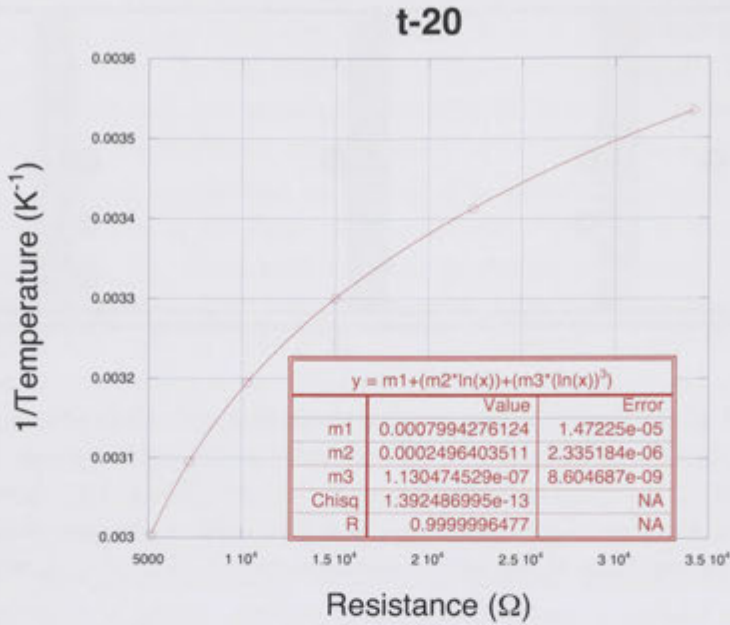


Figure 3.4: A typical calibration curve for a thermistor. The line is the best fit of Equation (3.1).

steps, with two resistances recorded at each of the temperature settings for each thermistor.

The resistance R of each thermistor was related to temperature by the Steinhart-Hart equation (Steinhart & Hart, 1968):

$$\frac{1}{T} = a + b (\ln R) + c (\ln R)^3, \tag{3.1}$$

where T is the temperature in Kelvin and R is the resistance in ohms, and a , b and c are constant coefficients. The coefficients were found by fitting the form (3.1) to the set of calibration measurements (of R and T from the digital ohm meter and Hart probe) for each thermistor; an example of a fit is shown in Figure 3.4.

During experiments the resistances of the thermistors were monitored by a set of bridge circuits, which output voltages proportional to R and logged by the LabVIEW program. The bridges require routine calibration, whereby a resistance box provided a known resistance to each active bridge in turn and the output voltage was recorded. The fixed resistance provided by the resistance box was measured by the Fluke® digital ohm meter. The output voltages of between 15 and 21 resistance settings (selected to ensure the output voltage spanned the range 1 to 10 V) were recorded. The process was repeated twice for all 13 bridge channels. The recorded resistances and voltages were fitted by a polynomial of the form,

$$\frac{1}{R} = d + eV + fV^2 + gV^3 + hV^4, \tag{3.2}$$

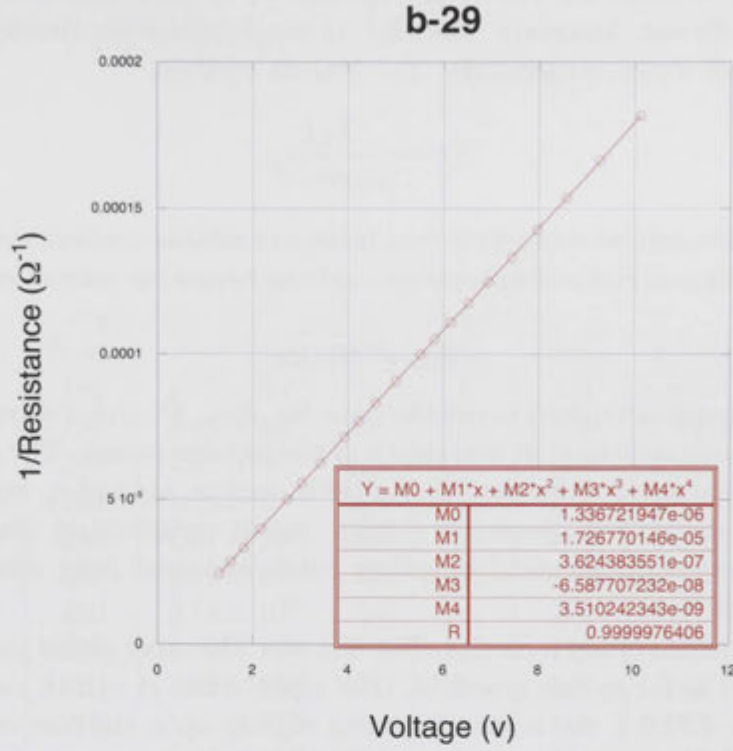


Figure 3.5: A typical calibration curve for a bridge. The line is the best fit of Equation (3.2).

where R is the resistance in ohms and V is the voltage in volts, and d , e , f , g , and h are constant coefficients. The coefficients were found using the KaleidaGraph program and recorded for each bridge; an example of a fit is shown in Figure 3.5. The calibrations of the thermistors and bridges were routinely tested and found to remain stable for the experimental period.

3.1.3 Dimensionless Parameters

In addition to R_D and L_1/L_2 , dimensionless parameters governing the experiments were a Rayleigh number (either Ra based on the imposed temperature difference or Ra_F based on the imposed flux),

$$Ra = \frac{g\alpha\Delta TL^3}{\nu\kappa_T} \quad \text{or} \quad Ra_F = \frac{g\alpha F_T L^4}{\rho c_p \nu \kappa_T^2}, \quad (3.3)$$

the aspect ratio,

$$A = \frac{D}{L}, \quad (3.4)$$

and the Prandtl number,

$$Pr = \frac{\nu}{\kappa_T}, \quad (3.5)$$

where g is the acceleration due to gravity, and α , ν , κ_T , ρ and c_p are the thermal expansion coefficient, kinematic viscosity, thermal diffusivity, density and specific heat capacity of water, respectively. The Nusselt number,

$$Nu = \frac{F_T L}{\rho c_p \kappa_T \Delta T}, \quad (3.6)$$

characterizes the rate of convective heat transport relative to that due to molecular conduction in the case of no fluid motion, and can be used to relate the two Rayleigh numbers as,

$$Ra_F = Nu Ra. \quad (3.7)$$

The water properties required to calculate the Ra , Ra_F , Pr and Nu are derived using the temperatures measured at mid-depth of the interior region. The density model for water is that of McCutcheon *et al.* (1993), and ν , κ_T and c_p are interpolated from values tabulated in Batchelor (1967). Small variations in Ra and Ra_F for experiments driven by identical boundary conditions arise from minor changes in the water properties.

The focus in this study is on the effect of a sill. The other global parameters were therefore fixed as far as was practical. The aspect ratio $A = 0.16$ and the Prandtl number $Pr = 4.7 \pm 0.2$, the latter depending slightly upon the temperature in each experiment. Unless stated otherwise (Table 3.1), the results are for runs with a basin length ratio $L_1/L_2 \approx 1/4$ and either an imposed heat flux of 280 W ($Ra_F = 1.85 \times 10^{14}$) or an imposed temperature difference of $\Delta T = 24^\circ\text{C}$ ($Ra = 1.55 \times 10^{12}$). The Rayleigh number achieved was large enough for the plume to be turbulent and the circulation to be in the entrainment regime of horizontal convection (Hughes & Griffiths, 2008).

3.1.4 Experimental Procedure

The tank was completely filled with de-aired water of room temperature¹ and the thermal boundary conditions were applied. A 3–4 day period of adjustment was allowed for each experiment to reach thermal equilibrium. Once this state was achieved, 13 vertical profiles were taken of each experiment over a period of 1–2 days; a vertical profile consists of traversing six thermistors simultaneously through the entire depth of the tank. The 13 vertical temperature profiles were averaged to give a single vertical profile from each thermistor. Differences between the 13 profiles were discernible only in the bottom 20 mm over the heated base, where the thermistors traversed through small-scale convective eddies (Mullarney *et al.*, 2004).

Temperature data is generally presented here in normalized form, $T_n(z) = (T(z) - T_c)/\Delta T$, where $T(z)$ is the temperature measured at depth z , T_c is the fixed temperature of the cooled boundary and ΔT is the temperature difference between the heated and cooled regions of the boundary in the same experiment (measured in the copper base). In the cases of an imposed heat flux, ΔT was free to

¹ $\sim 5^\circ\text{C}$ de-aired water was used for the experiments in which L_S was varied (exp. #18–25)

Table 3.1: Experimental conditions and calculated values for key dimensionless parameters defined in the text. Boundary conditions of imposed heat flux and imposed temperature difference are denoted by “*flux*” and “ ΔT ”, respectively.

Exp.	B.C.s	Watts	Ra_F (Ra)	Sill	R_D	L_1/L_2	δ/D	Nu	R_0
1	<i>flux</i>	280	1.85×10^{14}	flat	1	1/4	0.055	145	17.79
2	<i>flux</i>	280	2.06×10^{14}	flat	0.05	1/4	0.203	103	0.8005
3	<i>flux</i>	280	1.97×10^{14}	flat	0.1	1/4	0.103	133	1.601
4	<i>flux</i>	280	1.89×10^{14}	flat	0.15	1/4	0.064	142	2.668
5	<i>flux</i>	280	1.87×10^{14}	flat	0.2	1/4	0.059	144	3.558
6	<i>flux</i>	280	1.87×10^{14}	flat	0.25	1/4	0.060	144	4.447
7	<i>flux</i>	420	3.74×10^{14}	flat	0.1	1/4	0.077	144	1.797
8	<i>flux</i>	210	1.30×10^{14}	flat	0.1	1/4	0.134	125	1.420
9	<i>flux</i>	420	3.53×10^{14}	flat	1	1/4	0.049	152	20.41
10	<i>flux</i>	210	1.22×10^{14}	flat	1	1/4	0.062	135	16.13
11	ΔT	–	(6.26×10^{11})	flat	0	1/4	0.055	–	0
12	ΔT	–	(1.48×10^{12})	flat	0.05	1/4	0.083	–	0.8224
13	ΔT	–	(1.57×10^{12})	flat	0.1	1/4	0.091	–	1.645
14	ΔT	–	(1.56×10^{12})	flat	0.125	1/4	0.074	–	2.284
15	ΔT	–	(1.56×10^{12})	flat	0.15	1/4	0.062	–	2.741
16	ΔT	–	(1.55×10^{12})	flat	0.25	1/4	0.056	–	4.569
17	ΔT	–	(1.55×10^{12})	flat	1	1/4	0.055	–	18.28
18	<i>flux</i>	280	2.10×10^{14}	V	0.1	1/4	0.153	125	1.779
19	<i>flux</i>	280	1.99×10^{14}	V	0.15	1/4	0.088	135	2.668
20	<i>flux</i>	280	1.96×10^{14}	V	0.2	1/4	0.074	137	3.558
21	<i>flux</i>	280	1.93×10^{14}	V	0.25	1/4	0.066	139	4.447
22	<i>flux</i>	280	1.89×10^{14}	V	0.95	1/4	0.057	142	16.90
23	ΔT	–	(1.38×10^{12})	flat	0.05	1	0.695	–	0.8224
24	ΔT	–	(1.61×10^{12})	flat	0.05	4	0.880	–	0.8224
25	ΔT	–	(1.55×10^{12})	flat	0.15	1	0.065	–	2.741
26	ΔT	–	(1.55×10^{12})	flat	0.15	4	0.065	–	2.741
27	<i>flux</i>	280	2.21×10^{14}	flat	0.05	1	0.890	88	0.8005
28	<i>flux</i>	280	2.24×10^{14}	flat	0.05	4	0.880	113	0.8005
29	<i>flux</i>	280	1.90×10^{14}	flat	0.15	1	0.069	141	2.668
30	<i>flux</i>	280	1.90×10^{14}	flat	0.15	4	0.055	141	2.668

adjust to ensure the imposed heat flux was accommodated. The thermal boundary layer thickness δ is defined here as the region accommodating 98% of the top-to-bottom temperature difference, based on the two profiles above the cooled region in the large basin. The heat transport for the experiments with imposed flux boundary conditions is characterized by the Nusselt number (3.6).

As the aim was to obtain accurate temperature measurements, removal of the sidewall insulation was undesirable and only brief intervals (≤ 30 min) of flow visualization were carried out during which no measurable temperature perturbations were detected. Potassium permanganate crystals were dropped through access tubes in the tank lid.

In a separate calibration experiment the cooling was turned off and the water in the tank was stirred and heated above the temperature of the room, before the heating was turned off, and the subsequent cooling rate of the water in the insulated tank measured. This gave the bulk heat loss from the water in the tank to the room as $1.013 \times (T_{\text{tank}} - T_{\text{lab}})$ (in Watts). Applying this heat transfer coefficient to the flow experiments implies that, in general, 0.5% of the total heat input to the tank was lost to the room. For the cases with either very large sills and imposed heat flux, or a larger imposed heat flux (exp. #2, 7, 9, 27 & 28), sections of the tank reached up to $\sim 50^\circ\text{C}$, for which we estimate the total heat lost to the room approached 3.5% of the total heat input. The conductive heat transfer through the barrier forming the sill was calculated for the worst case to be 0.2% of the total heat input.

3.2 Results

The circulation (e.g., Fig. 3.6) was similar to that in previous observations of horizontal convection with no sill (e.g., Fig. 1.2 and the descriptions of Rossby, 1965; Mullarney *et al.*, 2004). There was a stable thermal boundary layer above the cooled portion of the base. Water in the boundary layer flowed continually from the cooled region of the base to the heated portion, where a convective mixed layer (CML) was maintained. The CML was the result of local convective instabilities on the heated boundary and was capped by the overlying stable boundary layer stratification. For small and moderate sill heights (*i.e.* for $R_D > 0.15$), the depth of the CML increased towards the endwall of the tank, at which point all the overlying stratification had been eroded and the CML carried most of the boundary layer transport into an unsteady, buoyant endwall plume which rose into the small basin. The small basin was kept well-mixed above the level of the sill by the turbulent convection that was free to rise from the base while being swept in a basin-scale overturning. For the highest sills (smallest gaps, $R_D \leq 0.15$), the vertical extent of the CML was instead limited by the warm well-mixed region above the level of the sill. A small fraction of the boundary layer transport did not acquire sufficient buoyancy to rise in the plume and was returned above the boundary layer in a weak horizontal intrusion. The horizontal extent of this weak intrusion was steady and finite, and determined by balancing the rate at which intrusion water was being supplied by the boundary layer with the rate at which intrusion water was being recycled back into the lower

boundary layer. Water was continually displaced from the small basin in a sill overflow that supplied a turbulent buoyant plume in the large basin. (In contrast, recall that for cases without a sill and hence no small basin, there was a single plume against the tank endwall; Mullarney *et al.*, 2004). Entrainment of ambient water from the large basin into the overflow plume maintained a weak stable density stratification throughout the depth of the large basin. The large-scale circulation was dependent in detail upon the sill geometry, but included the boundary layer flow towards the small heated basin, entrainment into the endwall plume and sill overflow, an overflow plume outflow along the top of the tank, and a broad downward flow in the large basin towards the forcing boundary.

3.2.1 The Effect of Sill Depth

The effect of a sill on the water properties (temperature) in the main basin is the key question of oceanographic interest. The effect is likely to depend of sill depth. In the experiments, sill depth is represented by the normalized gap height R_D and its effect on the circulation is illustrated by Figures 3.6 and 3.7. When the gap was large, no direct interaction is apparent between the sill overflow and the relatively cooler boundary layer flow at the tank base. However, when $R_D \lesssim 0.15$, the overflow and the (cooler) boundary layer were forced to interact in the vicinity of the sill. This interaction increased the range of temperatures in the overflow, leading to an increased proportion of overflow water not sufficiently buoyant to be fully incorporated into the plume and rise through the depth of the large basin. It instead formed a weak horizontal intrusion of steady and finite length at the level of the sill; intrusion water was recycled into the lower boundary layer. Thus, the intrusion also introduced an alternate circulation pathway for boundary layer fluid that bypassed the small basin. This pathway is dominant for very small gaps ($R_D \lesssim 0.05$), for which exchange across the sill was strongly restricted. In this extreme, a larger proportion of the water from the boundary layer was present in the plume that rose through the depth of the large basin (Figures 3.6c,d and 3.7c). In the limit where the gap was closed, circulation in the large basin resembled that expected for horizontal convection in a box of reduced length L_2 and reduced heat input.

The two types of boundary condition gave qualitatively different circulations when the gap was small. For imposed heat flux the sill exchange flow must accommodate a fixed proportion of the heat input (which depends on the lateral position of the sill), whereas there is no constraint on the heat transport for imposed temperature difference. Consequently, the stable boundary layer and sill overflow were confined to more rapidly flowing (and thinner) layers for smaller gap heights with imposed flux conditions (Fig. 3.7). With imposed temperature difference, on the other hand, the smaller gap tended to shut off the exchange flow (Fig. 3.6c). Convective instabilities at the heated base then mixed throughout the small basin the small proportion of water that passed through the gap. Note that for imposed temperature difference and $R_D = 0$ (Fig. 3.6d), there was a weak horizontal intrusion from the base of the plume: a possible explanation for this particular case is that the tank length is effectively shortened and the Rayleigh number reduced sufficiently to

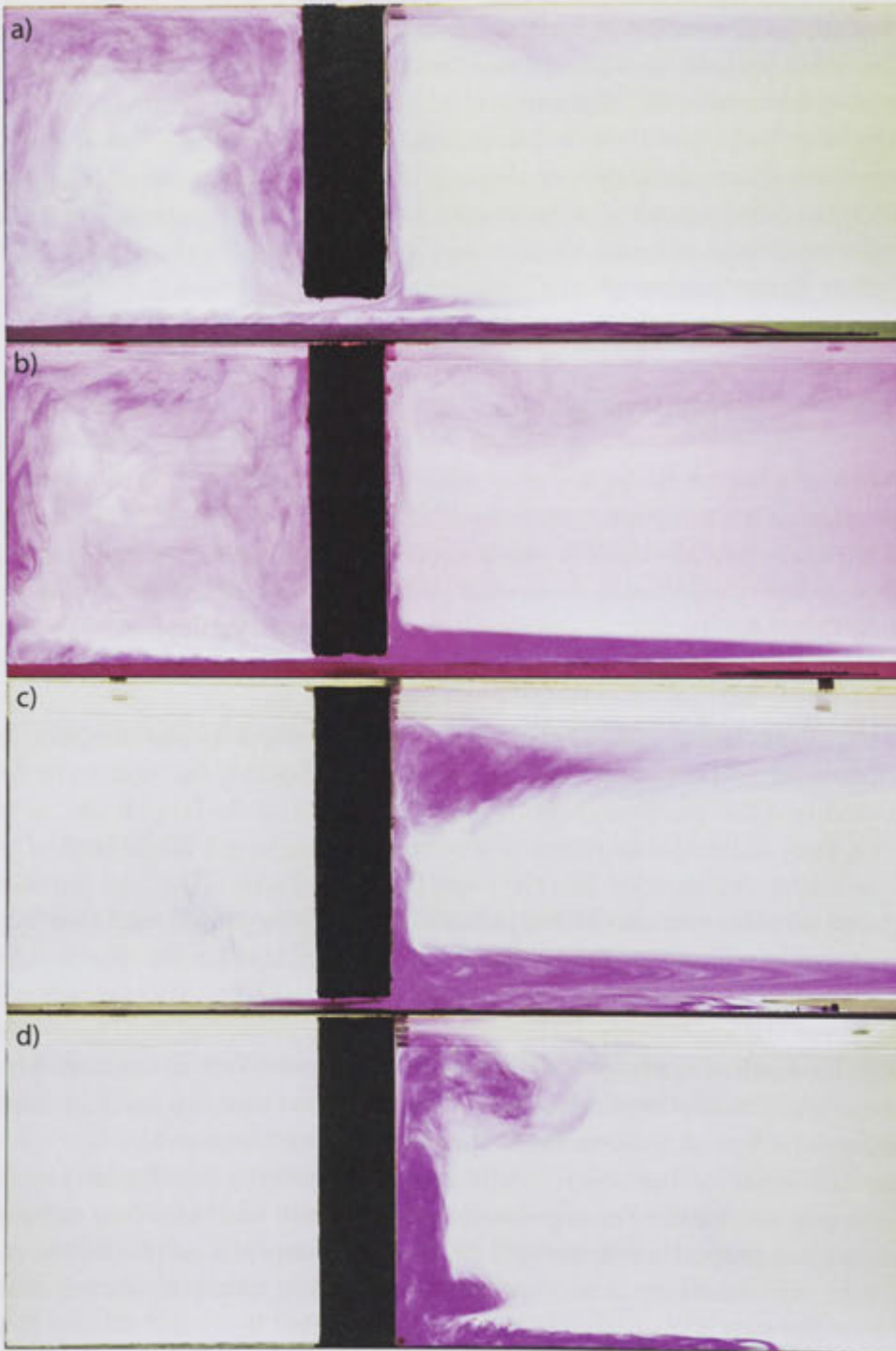


Figure 3.6: Photos of experiments with an imposed temperature difference and a flat sill for R_D values of (a) 0.15, (b) 0.1, (c) 0.05 and (d) 0. Just over half of the tank is visible: the field of view extends 0.75 m from the left-hand end and incorporates the heated region. Potassium permanganate crystals were dropped into the bottom boundary layer at the mid-point of the box, where they dissolved. The tracer was advected with the flow, beginning with mixing through the convective mixing layer over the heated plate. The weak horizontal intrusions at the sills are visible; in photos they appear prominent due to their steady nature and minimal dilution of the tracer dye, but in reality they do not significantly modify the circulation.



Figure 3.7: Photos of experiments with an imposed flux and a flat sill with $R_D =$ (a) 0.15, (b) 0.1 and (c) 0.05. Only the left-hand (heated) half of the tank is shown. Dye tracer was added as in figure 3.6.

support the intrusive regime of horizontal convection (Hughes & Griffiths, 2008).

Examples of vertical profiles of normalized temperature $T_n(z)$ in the large basin are shown in Figure 3.8. The profiles show that the temperature field was almost unchanged by the presence of a sill, for a given set of boundary conditions, if the normalized gap height was not small ($R_D \gtrsim 0.15$). Slight thickening of the thermal boundary layer and warming (towards the temperature of the heated base, $T_n = 1$) in the basin interior was measured as the gap was decreased from $R_D = 1$ to $R_D = 0.15$. The effects of the sill depth on interior temperatures and on the difference between the two basins are summarized in Figure 3.9, where normalized temperatures at mid-depth are shown as a function of gap height. Hughes *et al.* (2007) predict $T_n(D/2) = 0.87$ for the imposed heat flux experiments with no sill, consistent with our results. The temperature difference between the two basins (Fig. 3.9c) was very small for $R_D \gtrsim 0.15$, a further sign that the sill did not significantly impede the circulation. The thermal boundary layer thickness, plotted in Figure 3.10, is normalized by that measured for identical boundary conditions in the absence of a sill and is unchanged for $R_D \gtrsim 0.2$, or only slightly increased for $0.1 \lesssim R_D \lesssim 0.2$.

For $R_D \lesssim 0.15$ the presence of a sill resulted in a global modification of the temperature field (Figures 3.8a,b, 3.9, 3.10). Although the details show some dependence upon boundary conditions, a number of consistent features emerge. As the gap height was decreased below $R_D = 0.15$, the temperature difference between the two basins increased (Fig. 3.9c). This increase follows a warming of the small basin and allows buoyancy forcing to maintain an exchange flow across the sill. For imposed heat flux the exchange must carry (at least) all the heat input from the base of the small basin, and the lateral temperature difference is expected to increase without bound as $R_D \rightarrow 0$. (Although not apparent in Fig. 3.9b the range of temperatures measured in the large basin increased monotonically as R_D decreased for all R_D .) In contrast, for the case of imposed temperature difference there is no requirement for the exchange to carry a particular amount of heat or mass, and the small basin approached the boundary temperature. The large basin too was relatively warmer for $0.1 \leq R_D \leq 0.2$, but by a lesser amount, whereas for $R_D \lesssim 0.1$ there was a sharp drop in the normalized temperature of the large basin (Figures 3.8b and 3.9b; the range of temperatures in the large basin also begins to decrease below very small values of R_D). For the case of an imposed heat flux this sharp decrease in normalized temperature can be partially attributed to an increase in ΔT resulting from the restriction of mass and heat transport across the sill.

The modification of the temperature field for small gaps is also evident in the boundary layer thickness δ , which we compare to the thickness δ_0 under identical surface boundary conditions but without the sill (Fig. 3.10). For $R_D \leq 0.2$, the boundary layer was thicker. We attribute this to the formation of the strong horizontal intrusion near the level of the sill (Figures 3.6 and 3.7), which coincides with a reduction of heat removed from the box by conduction to the cooled boundary for imposed temperature difference and with an increased temperature difference across the boundary layer for imposed heat flux. However, for dimensionless gap heights $R_D \lesssim 0.1$, the boundary layer thickness was more strongly dependent upon the boundary conditions. For imposed temperature difference it thinned from a

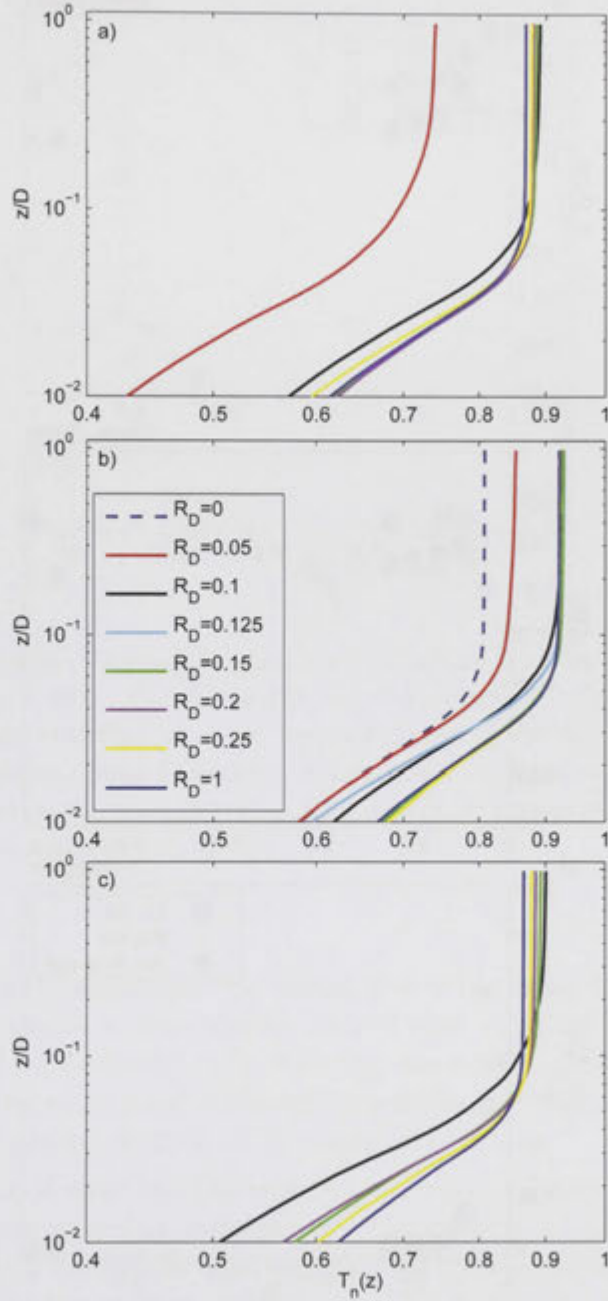


Figure 3.8: Vertical profiles of normalized temperature, $T_n(z) = (T(z) - T_c)/(\Delta T)$, where T_c is the temperature measured in the cooled base. Each plotted profile represents an average from two thermistors positioned in the large basin above the cooled plate, where horizontal temperature variations are small. Note that the vertical axis is logarithmic and height is normalized by the total depth: (a) imposed heat flux experiments with a flat sill (#1-6), (b) imposed temperature difference experiments with a flat sill (#11-17), and (c) experiments with a V-shaped sill (#18-22).

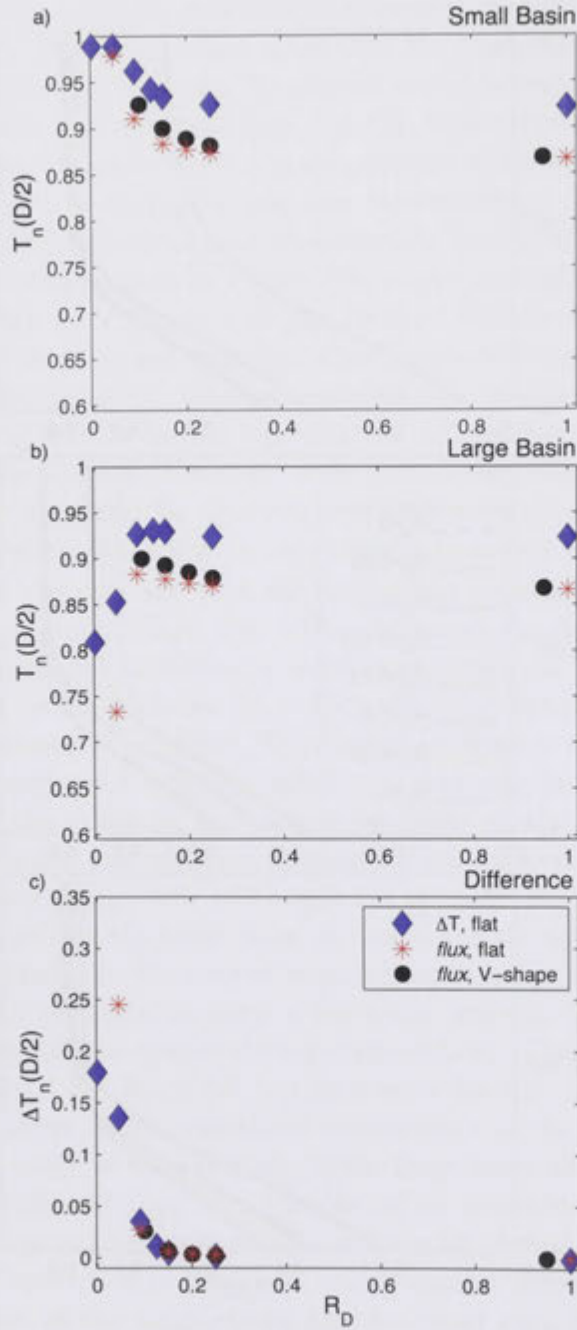


Figure 3.9: Normalized temperatures T_n at mid-depth in the (a) small basin and (b) large basin, and (c) the normalized temperature difference ΔT_n between the two basins, where $T_n(D/2) = (T(D/2) - T_c)/(\Delta T)$: (diamond) imposed temperature difference and flat sill; (star) imposed heat flux and flat sill; (solid circle) imposed heat flux and V-shaped sill. For imposed heat flux with $R_D \rightarrow 0$, $T_n(D/2)$ in the small and large basins will approach the limits of 1 and 0, respectively (recall that ΔT will however continue to increase), and thus $\Delta T_n(D/2)$ will approach 1.

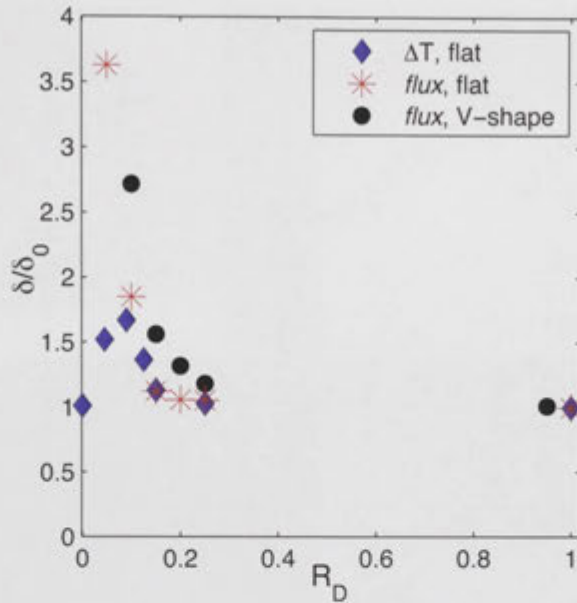


Figure 3.10: Normalized thermal boundary layer thickness δ/δ_0 as a function of normalized gap height $R_D = (D - D_S)/D$ for flat and V-shaped sills. Data are for boundary conditions of imposed heat flux (star) and imposed temperature difference (diamond) with a flat sill, and for imposed heat flux with a V-shaped sill (solid circle). The normalization is in each case based on the boundary layer thickness δ_0 measured in control cases having identical boundary conditions but no sill.

maximum at $R_D \approx 0.1$ as the gap was closed, due to the reduction of heat throughput (δ decreases towards a value slightly greater than δ_0 as R_D approaches 0). For imposed heat flux no boundary layer thinning was measured at $R_D \geq 0.05$ as the heat throughput remained fixed (although it is again expected that δ will decrease to a value slightly greater than δ_0 as R_D approaches 0).

The variation of Nusselt number with dimensionless gap size is shown in Figure 3.11 for the imposed heat flux experiments. The increase in ΔT as R_D decreased below 0.2 (with heat input held constant) led to a monotonic reduction in Nusselt number with decreasing gap size (Fig. 3.11). The Nusselt number is expected to decrease to 0 as $R_D \rightarrow 0$, based on the unbounded increase in base plate temperature difference for $R_D \rightarrow 0$.

Four additional experiments (#7–10 in Table 3.1) were conducted at two other imposed heat fluxes, 210 and 420 W, for $R_D = 0.1$ and 1. The results were similar to those with an imposed heat flux of 280 W, although measurements indicated that the sill had a relatively greater effect on the temperature field for smaller imposed heat fluxes. This behavior can be attributed to a thicker boundary layer for smaller Rayleigh numbers (Rossby, 1965; Mullarney *et al.*, 2004), and we return to these results in §3.3.

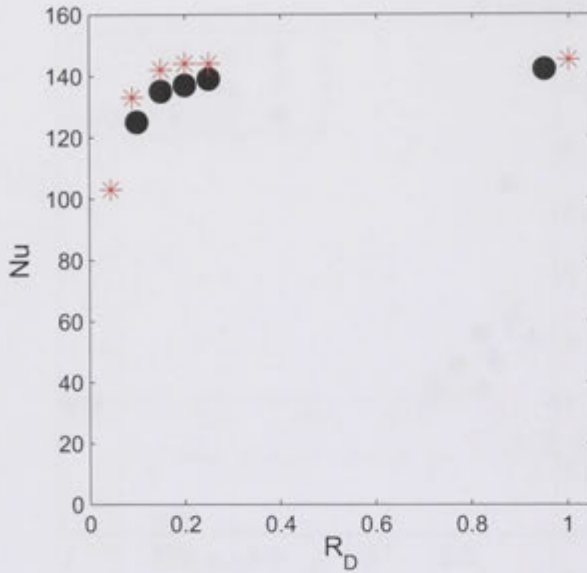


Figure 3.11: Nusselt numbers for the experiments with an imposed heat flux of 280 W and flat (star) and V-shaped sills (solid circles). The Nusselt number is expected to go to zero for $R_D \rightarrow 0$.

3.2.2 Non-Uniform Sills

Cross-stream variation in the sill height caused the overflow to be concentrated in the vicinity of the maximum sill gap, at the centre-line of the tank. The V-shaped sills (#18–22 in Table 3.1) presented only a small obstruction to the flow close to the tank base, where the boundary layer flows into the small basin, but a much greater obstruction (in terms of the cross-sectional area) at the heights where the sill overflow is exiting the small basin. For most gap heights ($R_D > 0.1$) the overflow carried sufficient horizontal momentum to cause the buoyant plume to separate from the vertical side of the sill, subsequently rising through the large basin at an angle of $\sim 30^\circ$ to the vertical (Fig. 3.12). The separated plume was initially laminar but unsteadiness and turbulence developed as it rose. This separation behaviour in the laboratory is attributed to greater momentum of the relatively narrow overflow through the apex of the V-shaped gap compared with the uniform overflows for flat sills, and is therefore also a result of the vertical sides of the sill and is unrealistic in the ocean context. However, the experiments offer useful insight into relevant flow dynamics associated with entrainment into the plume. For the largest gap examined ($R_D = 0.95$) the overflow plume reached the top prior to the onset of unsteadiness. For the smallest gap ($R_D = 0.1$) the overflow remained attached to the side of the sill (Fig. 3.12d). Otherwise, the circulation details were almost identical to those observed with a flat sill and the same gap.

The shape of the sill led to only minor changes in the temperature field. Normalized temperature profiles in the large basin (Fig. 3.8 a,c) and for a given R_D

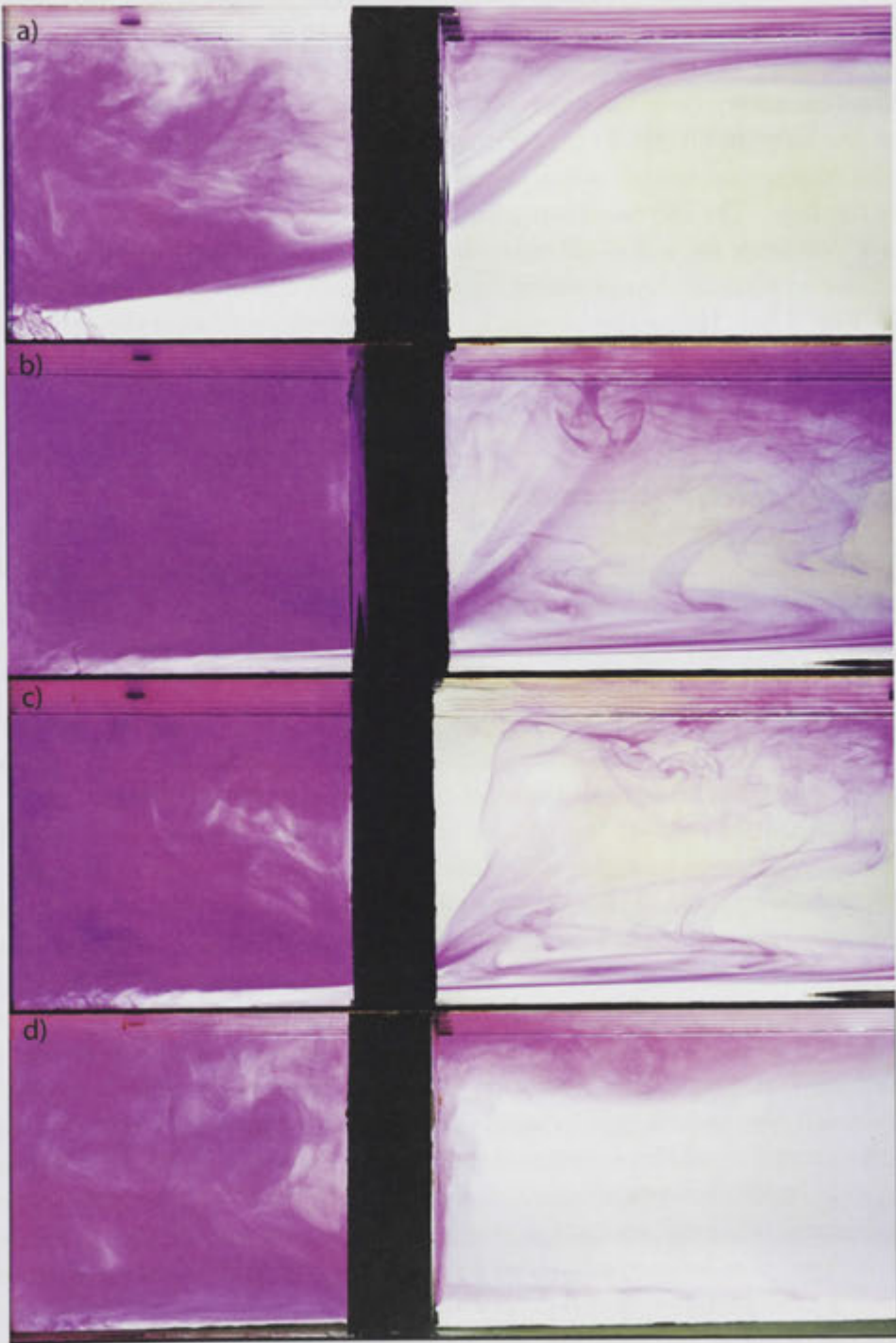


Figure 3.12: Photographs of experiments with an imposed heat flux and a V-shaped sill with $R_D =$ (a) 0.95, (b) 0.2, (c) 0.15 and (d) 0.1. Only the left-hand (heated) half of the tank is shown. Potassium permanganate crystals were dropped into the bottom boundary layer in the small basin on the left.

show that the temperature throughout the flow is slightly warmer with a V-shaped sill (see also Fig. 3.9a,b). Cross-stream variation of the sill height also resulted in a larger boundary layer thickness for a given R_D (Fig. 3.10), which we suggest is due to blocking of the boundary layer and return flow by the sill geometry. The increased boundary layer thickness is consistent with the slight warming in the interior of the large basin, thus maintaining the temperature gradient in the boundary layer for conduction to the cooled plate to accommodate the imposed heat throughput in the flow. The increased temperature difference also implies a smaller Nusselt number Nu than for a flat sill with the same maximum gap height (Fig. 3.11). Note however that the temperature difference across the sill was independent of sill shape (Fig. 3.9c). Hence the interior buoyancy difference driving the exchange flow is determined primarily by the maximum gap height.

3.2.3 Effects of Sill Position

An additional eight experiments (#23–30 in Table 3.1) were carried out to investigate the effect of lateral sill position; the basin length ratio was changed from $L_1/L_2 = 1/4$ to 1 and 4 for both types of boundary conditions, with flat sills having $R_D = 0.05$ and 0.15. The position of the sill played only a minor role in all experiments with $R_D = 0.15$ and no significant differences were measured. However, the position of the sill had a significant influence on the circulation for the smaller gap height ($R_D = 0.05$) with both types of boundary conditions. For $L_1/L_2 = 1$ and 4 the basin to the right of the sill (no longer the larger basin, but cooled by the base) contained an overturning circulation that was largely confined to the lower $1/5$ to $1/3$ of the tank. This shallow overturning was attributed to the large temperature range of water feeding the sill overflow from the left-hand basin, which for these sill positions had a stratified thermocline overlapping the depth of the sill. The exchange flow across the sill in these cases was minimal, except for the case of $L_1/L_2 = 1$ and an imposed heat flux in which the exchange must accommodate all the heat transport. The circulation in the left-hand basin (which was bounded below by all the heated base) resembled that of standard horizontal convection, with the exception of the experiment with an imposed temperature difference for $L_1/L_2 = 1$. In this exceptional case, the absence of any differential heating in either basin, coupled with the restriction presented by the sill, means that there is little reinforcement of a large-scale overturning. Instead of the boundary layer feeding into an endwall plume, convection was observed above the thermal boundary layer throughout much of the left-hand basin.

The Nusselt numbers for the experiments with imposed heat flux investigating the effect of sill position (Exps. #2, 4 & 27–30) are shown in Figure 3.13; they are plotted for $L_S = L_1/L$. The Nusselt number of the control case is shown by the blue line. For $R_D = 0.15$ there is no significant difference in Nu observed for any sill position. For $R_D = 0.05$ there is a minima in Nu at $L_S = 0.5$, with Nu approaching the value of the control case as L_S goes to 0 or 1.

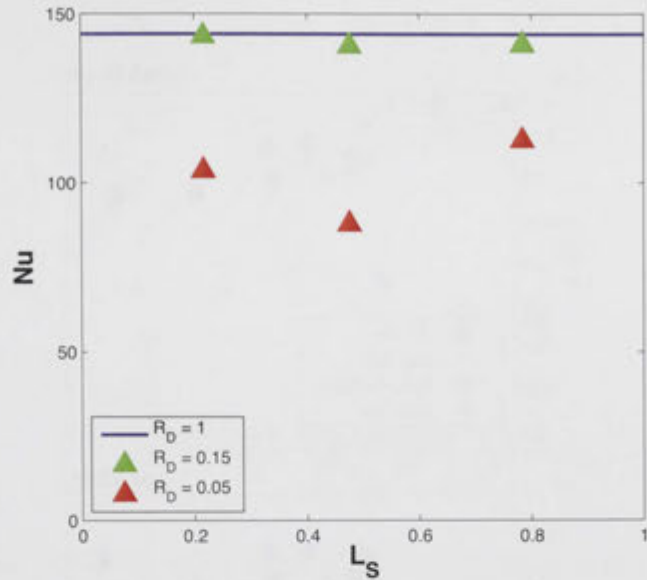


Figure 3.13: Nusselt number of the experiments with imposed heat flux at the different sill positions (Exps. #2, 4 & 27–30). The green and red triangles represent the experiments with $R_D = 0.15$ and 0.05 , respectively. The blue line is the Nusselt number of the control experiment with $R_D = 1$.

3.3 Discussion

The results show that the influence of a sill on the basin-scale circulation becomes especially strong when the stable boundary layer flow interacts directly with the sill overflow, tending to reduce the buoyancy of the sill plume and modify the range of densities in the large basin. We therefore define $R_0 = (D - D_S)/\delta_0 = R_D(D/\delta_0)$, where δ_0 is the boundary layer thickness in corresponding control cases having the same boundary conditions but no sill. Thus the ratio of the gap height and boundary layer thickness is the parameter of interest. Given that the boundary layer and the counter-flowing overflow at the crest of the sill are of similar thickness, we anticipate that $R_0 \lesssim 2$ implies an interaction that impedes the sill exchange flow, modifies the boundary layer thickness from the control case and leads to mixing that modifies the overall circulation. Note that as the boundary layer thickness δ_0 is a function of Rayleigh number ($\delta_0 \sim Ra_F^{-1/6}$; Mullarney *et al.*, 2004), we might expect a given sill to pose less of an obstacle to flow for larger Rayleigh numbers. This is indeed apparent in Figures 3.14c and 3.15 (triangle symbols, imposed flux increasing to the right), where an increased imposed flux returns a decreased inter-basin normalized temperature difference and normalized boundary layer thickness.

The data from Figures 3.9 and 3.10 are replotted as a function of R_0 in Figures 3.14 and 3.15, respectively, together with the additional data from experiments in which Ra_F was varied. The normalization largely collapses all the temperature data in Figure 3.14 for $R_0 \gtrsim 1$, irrespective of sill geometry, though a small difference

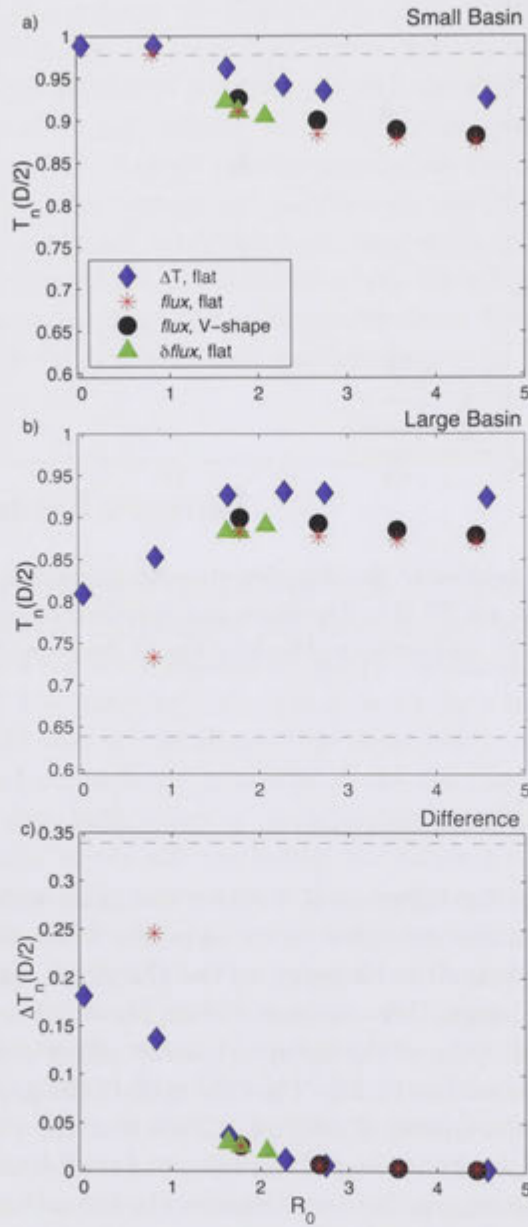


Figure 3.14: Normalized temperature and temperature difference ΔT_n between the two basins at mid-depth as a function of the ratio of gap height to reference boundary layer thickness in the absence of a sill: Experiments with a flat sill and an imposed flux of 280 W (stars); a range of imposed fluxes (triangles, inputs of 210, 280 and 420 W, increasing to the right); an imposed temperature difference (diamonds); and with a V-shaped sill and imposed heat flux (circles). The North Atlantic observations of Figure 3.17 are plotted as magenta dashed lines; the method for calculating the position of the lines is described in §5.

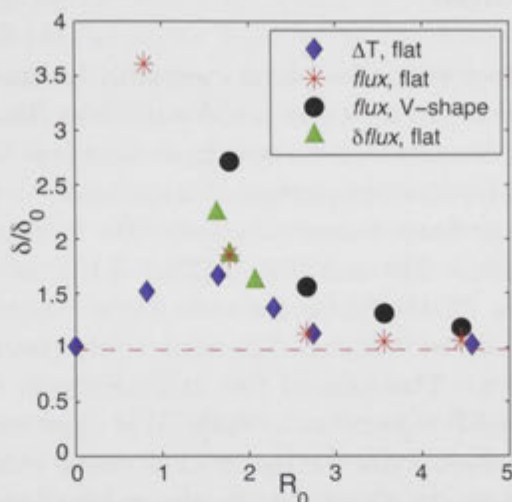


Figure 3.15: Normalized thermal boundary layer thickness δ/δ_0 as a function of R_0 for flat and V-shaped sills. Symbols as defined in Fig. 3.14. The red dashed line represents the theoretical boundary layer thickness for $R_D=0$ and an imposed heat flux (allowing a 50% smaller heat flux and a 25% shorter box length available to the circulation owing to the closure of the sill passage); thus a maximum of δ/δ_0 exists in the range $0 < R_D < 0.05$.

between imposed temperature and imposed flux boundary conditions (Fig. 3.14a,b) is apparent. The normalized boundary layer thickness (Fig. 3.15) similarly collapses for $R_0 \gtrsim 1$, again with a small dependence on the boundary conditions. Importantly, Figures 3.14 and 3.15 show that the influence of a sill on the circulation is apparent for $2 \lesssim R_0 \lesssim 4$, and strong for $R_0 < 2$. A significant dependence on the boundary conditions develops for $R_0 < 2$.

Partial blocking of the boundary layer flow by the sill alters the temperature stratification in the large basin (see Fig. 3.8) and shifts the normalized temperature at mid-depth in the large basin closer to that of the cooled boundary (Fig. 3.14b; note that this corresponds to an increased range of temperatures, except for imposed temperature difference at very small R_0). The normalized temperature at mid-depth in the small basin approaches that of the heated boundary for large sills ($R_0 \rightarrow 0$, Fig. 3.14a). Larger temperature differences are required to drive the exchange flow across larger sills (smaller R_0 ; Fig. 3.14c), causing the water entering the overflow plume in the large basin to be more buoyant. As the overflow is the source of the most buoyant water in the large basin, the range of temperatures (and therefore the deep stratification) in that basin is increased. The restriction of the sill exchange also results in an intrusion into the large basin at levels comparable with the sill, and the intrusion results in a thicker thermal boundary layer (as measured on the basis of a vertical temperature profile).

3.4 Application

Transformation of surface waters into what eventually becomes North Atlantic Deep Water (NADW) occurs in the Labrador and Nordic Seas (Hansen *et al.*, 2004). The outflow of dense water from the Nordic Seas must overcome the Greenland-Scotland Ridge (GSR), but no equivalent topographic feature exists in the Labrador Sea. The two main paths that the dense waters take across the GSR are the Denmark Strait (DS) and the Faroe Bank Channel (FBC) (Fig. 3.16), at 620 and 840 m depth, respectively (Saunders, 2001). These channels might be crudely approximated as V-shaped sills having $R_D \approx 0.15$ and 0.2 , respectively (taking the North Atlantic Ocean as 4000 m deep). The base of the thermocline in the North Atlantic at latitudes south of the GSR is found at a depth (δ) of approximately 1000 m. We are unable to predict the exact depth the thermocline would take in the absence of the GSR (δ_0) as we cannot accurately account for the effects of winds, Coriolis and other processes involved in setting the thermocline depth of the North Atlantic. However, it is unlikely that the value of δ_0 would be less than 400 m (a value required to give $R_0 > 2$), and therefore we expect the GSR to be characterized by dimensionless gap heights $R_0 < 2$. Thus the experimental results imply that these sills will have an ocean-wide effect on the density field and circulation. A verifiable prediction from the experimental results is that the restriction of the circulation results in significant modification of the dense overflow plume waters by interaction with thermocline waters in the vicinity of the GSR. As outlined below, this prediction is consistent with observations.

The Labrador and Nordic Seas each account for approximately 1/3 of the total transport of NADW flowing south in the North Atlantic (Hansen *et al.*, 2004). Entrainment of ambient water into the GSR overflow plumes contributes the final third of the total production rate of NADW. This entrainment doubles the GSR overflow transport (Dickson & Brown, 1994) and significantly modifies the properties of the sinking plumes. Our experiments demonstrate a means by which the modification may depend on whether the overflow plume at the sill is shallow enough for it to interact with the thermocline. In the case of the Denmark Strait Overflow (DSO) the plume has a maximum warming rate of 0.4 to 0.5°C per 100 km during the first 200 km downstream of the sill (where the bulk of the slope plume descends through the thermocline), and a decreased warming rate of 0.05 – 0.1°C per 100 km for the remainder of the descent before reaching Cape Farewell (Voet & Quadfasel, 2010). The Faroe Bank Channel Overflow (FBCO) undergoes intense mixing with overlying thermocline waters within the FBC, diluting the entire FBCO (Fer *et al.*, 2010). Once the FBCO plume has exited the FBC and sunk below a depth of 1200 m, the rate of change, with downstream distance, of the outflow plume properties is much reduced in potential temperature–salinity (θ – S) space; Mauritzen *et al.* (2005) have already attributed this to the overflow no longer encountering and entraining thermocline waters. In addition, mixing rates for the GSR overflows have been found from hydrographic data to be 10 times higher than further downstream (and 100 times greater than the background mixing rate; Lauderdale *et al.*, 2008).

Fig. 3.17 shows the range of θ – S properties that characterize the dense southward

flowing waters entering the DS and FBC (green and blue thick bars, respectively), where mixing proceeds (in the region bounded by the green and blue broken lines) with the overlying, much warmer and saltier thermocline waters (large green and blue circles). The properties of these mixed waters, which then sink beyond the thermocline depths, are shown by the small green and blue circles. The temperature and salinity of the overflows are increased by approximately 1°C and 0.1 psu during this stage due to the interaction with thermocline waters. At greater depths the overflow plumes entrain North Atlantic intermediate water (red circle), which causes a relatively small modification of the overflow properties (in the region bounded by magenta broken lines), before forming the deep western boundary current that rounds Cape Farewell (magenta circle) at depths of $1900 - 3000$ m and enters the Labrador Sea (yellow circle). The waters from the GSR overflow plumes then interact with waters in the Labrador Sea before exiting as NADW (black diamond). Thus the increase in temperature and salinity during the transformation from GSR source waters to NADW is seen to occur almost exclusively in the GSR overflows, with most of this change occurring as a result of interaction with the thermocline waters.

The properties of Antarctic Bottom Water and Arctic Ocean Water are included in Figure 3.17 (black circle and square, respectively); while Arctic Ocean Water (the ultimate source water for the GSR overflows) is significantly denser than AABW, the NADW is less dense and overlies AABW. This raises the question of what role the GSR might play in influencing the relative densities of AABW and NADW. The experimental results show that the presence of a sill increases the density difference between basins (regardless of surface boundary conditions) and we expect to see the greatest response in the density of abyssal waters in the main (lower-latitude) basin: NADW will be warmer (assuming temperature is the main player in density) than with a small (or no) sill. The experiments also show that the range of temperatures (*i.e.* strength of the stratification) in the main basin is increased by a significant sill, implying that the GSR causes North Atlantic surface waters to be warmer. This adds to the global influence of the GSR.

Other oceanographic observations lend support to the hypothesis that the GSR has a global influence. In Figure 3.14 key potential densities (shown by dashed horizontal lines as we lack a robust measure of R_0 for the GSR) are normalized in the same manner as the experimental data: these are the potential densities of the Arctic Ocean (> 2000 m deep) and upper North Atlantic water immediately south of the GSR, which are representative of the range of densities that interact in the vicinity of the sill, as well as the potential densities of the southward flowing waters entering the GSR and the NADW. If the GSR were to have little influence on the circulation, we would expect the normalized mid-depth density (Figure 3.14a,b) to be approximately 87% of the normalized density range (see Hughes *et al.*, 2007) and to have negligible density difference across the sill (Figure 3.14c). This is clearly not the case, supporting the case that the GSR leads to a decrease in the potential density of the NADW (and an accompanying small increase in the potential density of the high-latitude waters). This represents a major influence on the density structure of the ocean.

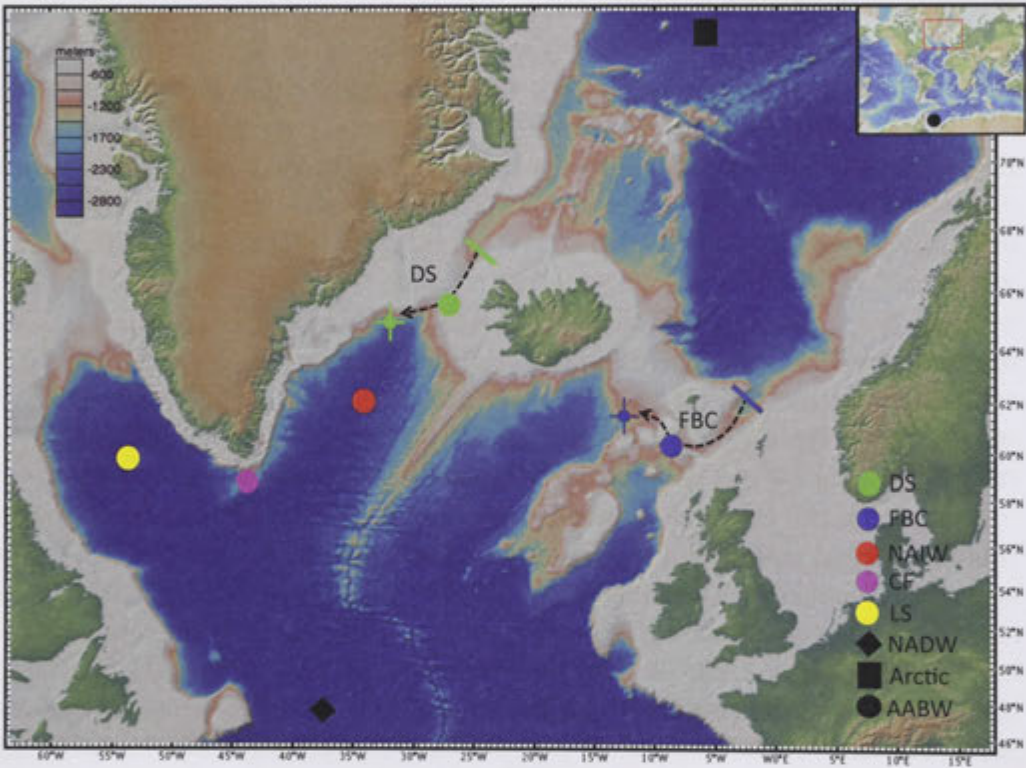


Figure 3.16: Schematic of Denmark Strait Overflow (green) and Faroe Bank Channel Overflow (blue) pathways across the Greenland-Scotland Ridge. The locations of the water properties used to generate Figure 3.17 are shown; the green and blue bars and cross-haired circles show the overflow source and outflow locations, respectively. Figure generated in GeoMapApp utilizing the Global Multi-Resolution Topography synthesis of Ryan *et al.* (2009).

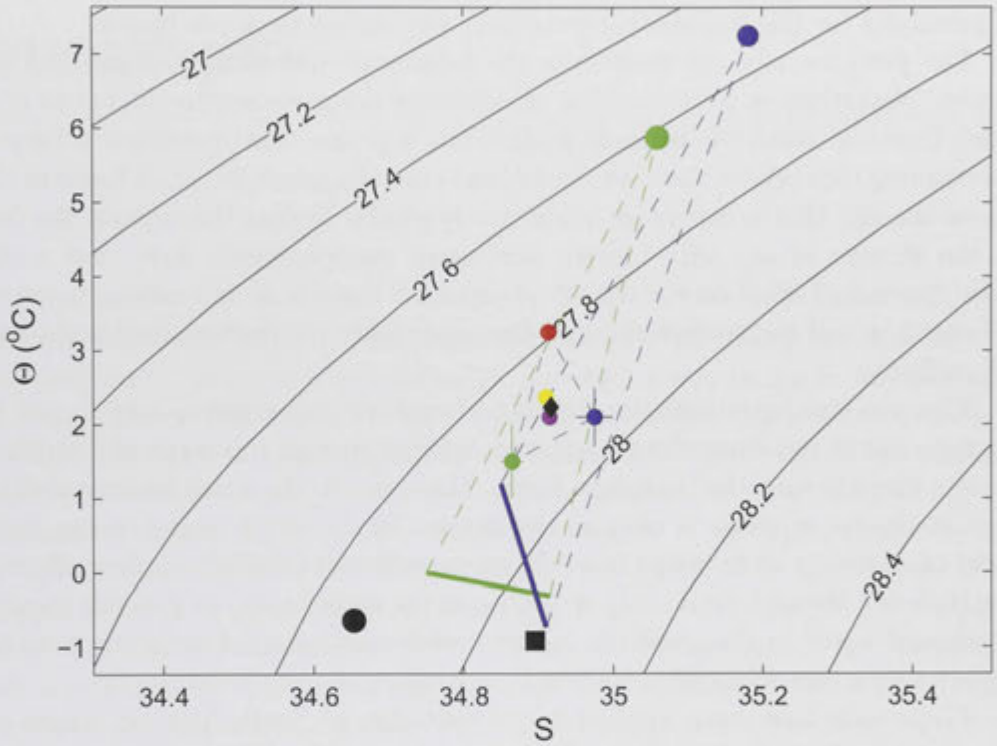


Figure 3.17: Characteristic potential temperature–salinity (θ - S) diagram for the water masses involved in the Greenland-Scotland Ridge exchange. The water mass properties (taken from Saunders, 2001; Locarnini *et al.*, 2010; Antonov *et al.*, 2010) are the dense southward flowing waters entering the Denmark Strait and Faroe Bank Channel (green and blue bars, respectively), the relatively warmer and saltier overlying thermocline waters (large green and blue circles, respectively) and the overflow plumes sinking beyond the thermocline depths (cross-haired green and blue circles, respectively). These overflow plumes entrain North Atlantic intermediate waters (red circle) to give the deep western boundary current rounding Cape Farewell (magenta circle) and entering the Labrador Sea (yellow circle), subsequently exiting as North Atlantic Deep Water (black diamond). The dashed lines indicate mixing triangles coloured as per their respective product waters. Antarctic Bottom Water and Arctic Water are shown by the black circle and square, respectively.

3.5 Conclusions

The effect of a topographic sill on the convective circulation in a model rectangular basin forced by a meridional gradient of surface heating has been investigated. The sill is assumed to divide the model ocean into a small (high-latitude) basin, in which a sinking plume is situated, and a main (lower latitude) basin. Various sill sizes, shapes and surface boundary conditions have been examined, yielding a robust set of principles for the large-scale overturning circulation in ocean basins.

The presence of a sill interrupts the large-scale overturning circulation in the model. Nevertheless, a sill overflow of relatively dense water (in the ocean orientation) from the small to the large basin feeds a plume that maintains a large-scale overturning throughout the main basin (and extending into the small basin at depths above the sill) that is otherwise qualitatively similar to that throughout the domain in the absence of any sill. Indeed, laboratory measurements show that a sill has little dynamical effect on the density structure if there is no interaction between the sill overflow and the surface thermal boundary layer (or thermocline) waters at the sill exchange.

The overturning circulation in the laboratory experiments was found to be strongly restricted when the topography is large enough to ensure the overflow interacts directly with the boundary layer. The water in the small basin is well-mixed and, in the ocean sense, is close to the densest in the entire model ocean. Entrainment of relatively warm water from the thermocline into the sill overflow affects both the bulk density and the density structure in the main basin. In general, the density of abyssal water in that basin is reduced while the range of densities is increased (relative to a case of no sill), leading to a stronger density stratification.

The results have been applied to the exchange of North Atlantic waters across the Greenland-Scotland Ridge and the formation of NADW associated with these overflows. We find clear evidence of entrained thermocline waters in the NADW that is consistent with a strong role for these topographic sills in the basin-scale circulation. The topographic constriction of these sills leads to NADW that is warmer than it would be if the sills were absent. Further reasoning has suggested that the Greenland-Scotland Ridge could lead to the modification of water properties on a global scale because of the interaction of NADW and AABW. It seems likely that feedbacks involving a modification of the surface circulation and buoyancy fluxes are an important part of this process.

Numerical Solutions for Overturning Circulations with Topography

We report numerical solutions for thermally-driven overturning circulations with topography and a parameterized turbulent mixing. These solutions are primarily used to investigate the convective flow from an energetics viewpoint. Analysis of the energy budget of the overturning circulation requires knowledge of the available gravitational potential energy in the flow and the mechanical power input from surface buoyancy fluxes. Both in turn require calculation of an appropriate reference state of no motion. Defining this reference state is complicated by the presence of topographic barriers. Two methods of addressing this issue are proposed and applied, along with a method that neglects the influence of topography, to an overturning circulation driven by surface buoyancy fluxes in an idealized multi-basin domain. The method that ignores topography underestimates the potential energy of the reference state, resulting in an overestimate of available potential energy. However, it is found that the method used to determine the reference state does not significantly affect the calculated rate of energy input from surface buoyancy fluxes, which is the quantity of dynamical importance to the maintenance of the overturning circulation. For a simple case of a ridge that defines a relatively small high-latitude basin within a much larger basin, the presence of the ridge increases the available potential energy by a large amount only if the sill is shallow enough for the dense overflow to mix with the overlying less dense thermocline water. The surface buoyancy forcing, on the other hand, is insensitive to the presence of the ridge. Additionally, we compare the results of the numerical solutions with those from the laboratory experiments of §3.

4.1 Methods to Determine the Reference State of a Multi-Basin Domain

Current frameworks for calculating z_* (see §2.2.4 for the definition of z_*) seek the global minimum potential energy (Winters *et al.*, 1995; Huang, 2005; Hughes *et al.*, 2009) by adiabatically resorting the entire domain into a single stable stratification,

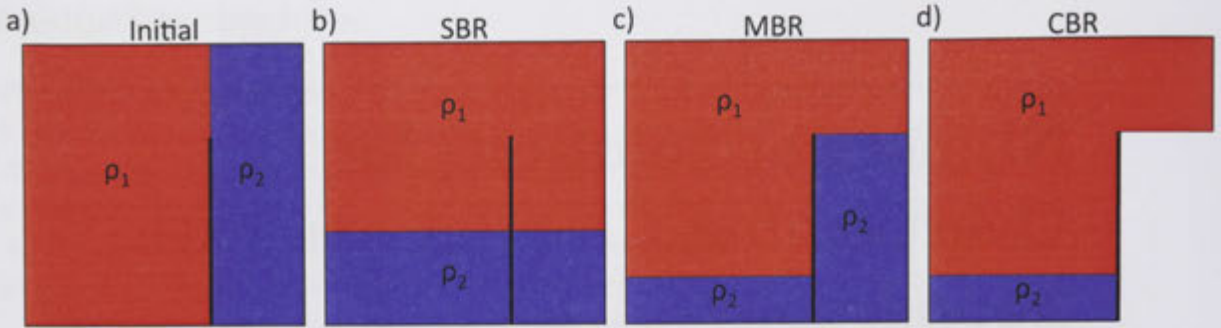


Figure 4.1: Diagram of an initial two-fluid density field (a) and the three reference states of no motion achieved by three different methods of resorting (defined in the text).

thus returning density ρ as a single function of z_* . This is sufficient for an ocean without topography. However, to physically achieve the global minimum state would require that all basins are able to communicate and exchange water masses at all depths. The oceans have topographic barriers that divide the oceans into basins and tend to act as weirs, damming dense water that lies nears the bottom into one or more of the basins while water at shallower depths is free to flow across the whole domain. There is no consensus as to the correct reference state to use for calculating z_* (and thus E_b) in a complex domain.

To illustrate the problem, consider the simple case depicted in Figure 4.1a. The domain contains water of two density classes, with $\rho_1 < \rho_2$, and is divided into two basins by a vertical barrier. A ‘Single-Basin Resort’ (SBR) of the water having different densities will return the global minimum potential energy (Fig. 4.1b). To physically achieve this state, a portion of the water of density ρ_2 that is initially at a level lower than the top of the barrier (the sill) must be raised to flow over the sill. Raising this water even temporarily requires an energy input. Therefore this is not the state of no motion to which the domain will adiabatically relax in the absence of forces other than gravity.

A physically plausible method would find the state of minimum potential energy that the fluid can reach whilst respecting the impermeable topography. We refer to this method as ‘Multi-Basin Relaxation’ (MBR, Fig. 4.1c) and will be described in detail in §3. The final state achieved by the MBR method recognizes that a topographic ridge is able to support the dense water in the right-hand basin at a level higher than the depth of dense water in Figure 4.1b, thereby returning a larger value of E_b and a smaller value of E_a . The MBR method returns ρ as a function of z_* for each basin and accounts for the fact that, although topography can support dense water higher in the water column, the potential energy associated with this water is not ‘available’ to the system. The MBR method will return a true, unique reference state for a multi-basin domain. The potential energy of this reference state is a local minimum in potential energy with respect to perturbations of the resorted density field, rather than the global minimum in potential energy given by the SBR method. However, the resorting must be an iterative process and

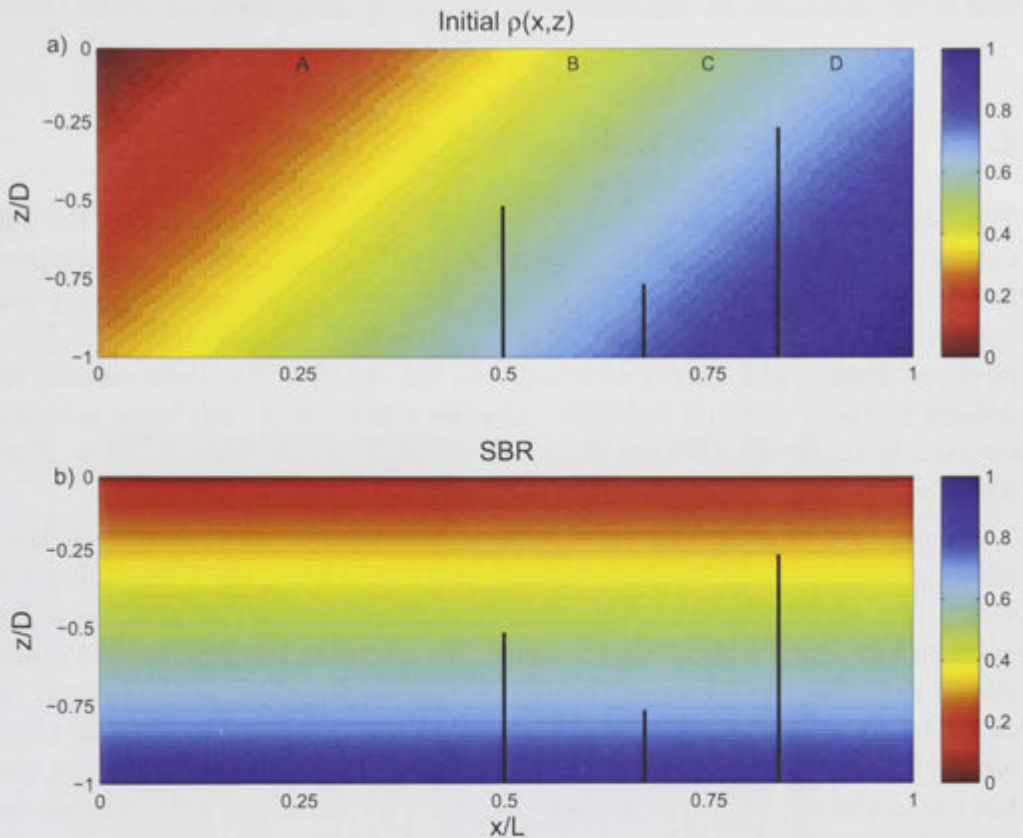


Figure 4.2: (a) An idealized domain with an arbitrary density field and basin configuration used to demonstrate the proposed methods. The reference state calculated by the SBR method is shown in (b).

can be computationally expensive to calculate for the complex three-dimensional topography and density fields of the oceans.

An alternative approach is to ignore the volume below the level of the sill in all but the largest basin; effectively ‘capping’ the minor basin(s) at the level of the sill and resorting the remaining volume into a single stratification. We refer to this method as the ‘Capped-Basin Resort’ (CBR, Fig. 4.1d). The resorting methods used to achieve the resorted domains of Figures 4.1c and d will return no difference in the available potential energy of the simple case shown in Fig. 4.1a, and we propose that it may represent an adequate approximation to the true reference state for the complex density distribution and topography of the oceans.

In order to demonstrate the various methods we use the arbitrary density field and idealized two-dimensional topography depicted in Figure 4.2a. The methods are equally applicable to three-dimensional basins, topography and density distributions. The reference state calculated by the SBR method is shown in Figure 4.2b. This is the global minimum potential energy and allows for the densest waters (ini-

tially located in basin D) to occupy all basins irrespective of topography. It is a state that is not attainable by adjustment under gravity alone from the initial conditions.

4.1.1 Multi-Basin Relaxation (MBR)

The ‘Multi-Basin Relaxation’ method seeks the state of no motion that the fluid will approach by adiabatic gravitational adjustment. The solution is found by iteration, first identifying the basins of the domain, then adiabatically rearranging the fluid such that each individual basin is stably stratified. The 4 basins (A–D) of the idealized domain in Figure 4.2a occupy the full depth of the water column and are defined by the 3 sills and endwalls. The lower portion of each basin is completely enclosed by topography, where contact forces balance gravitational forces. Exchange with the adjacent basin(s) is able to occur at depths above the sills.

In the resorting method each basin is individually rearranged such that it is stably stratified and in it’s state of minimum potential energy (iteration #1 in Fig. 4.3). This rearranged state results in unbalanced horizontal pressure differences between adjacent basins at depths above the enclosing topography and would give rise to inter-basin exchange at these depths. However, there is one basin (C in this example) for which this first iteration state below the depth of the sill cannot be altered by inter-basin exchanges; this is the basin containing the densest water at the top of the sill, such that any water able to be exchanged from another basin is not sufficiently dense to enter the enclosed lower region. Thus, the topographically enclosed region of this basin is isolated, by way of stratification, from the rest of the domain and plays no part in subsequent rearrangements; we refer to this isolated region as being ‘locked’ from the domain. Water above the sill and overlying the locked region is included into the adjacent basin (B), completing the first MBR iteration. It is important to note that the locked region is not necessarily associated with the highest sill nor the densest basin (*i.e.* not basin D in our example).

The second iteration begins by identifying the remaining basins (A, B & D) of the domain, less the locked region, then rearranging these basins into their respective stable states (iteration #2 in Fig. 4.3); basins A & D will be unaltered from the previous iteration. The densities at the top of the topographically enclosed regions of each stably stratified basin are again compared and a second locked region is identified (D in this example). The fluid overlying the second locked region becomes part of the adjacent basin (B), and the second iteration is complete.

This iterative process continues until the entire volume of the domain has reached a stable state; n iterations are required for a domain of n basins. The MBR method returns the unique density distribution from which the reference potential energy is calculated for the multi-basin domain. The order in which regions are locked is not predetermined by topography and strongly depends on the density field at each iteration, corresponding to an evolving field as the water adjusts adiabatically.

The final stable density profiles of the four regions are shown in Figure 4.5a (dashed blue); it is from these density profiles that E_b is calculated, namely,

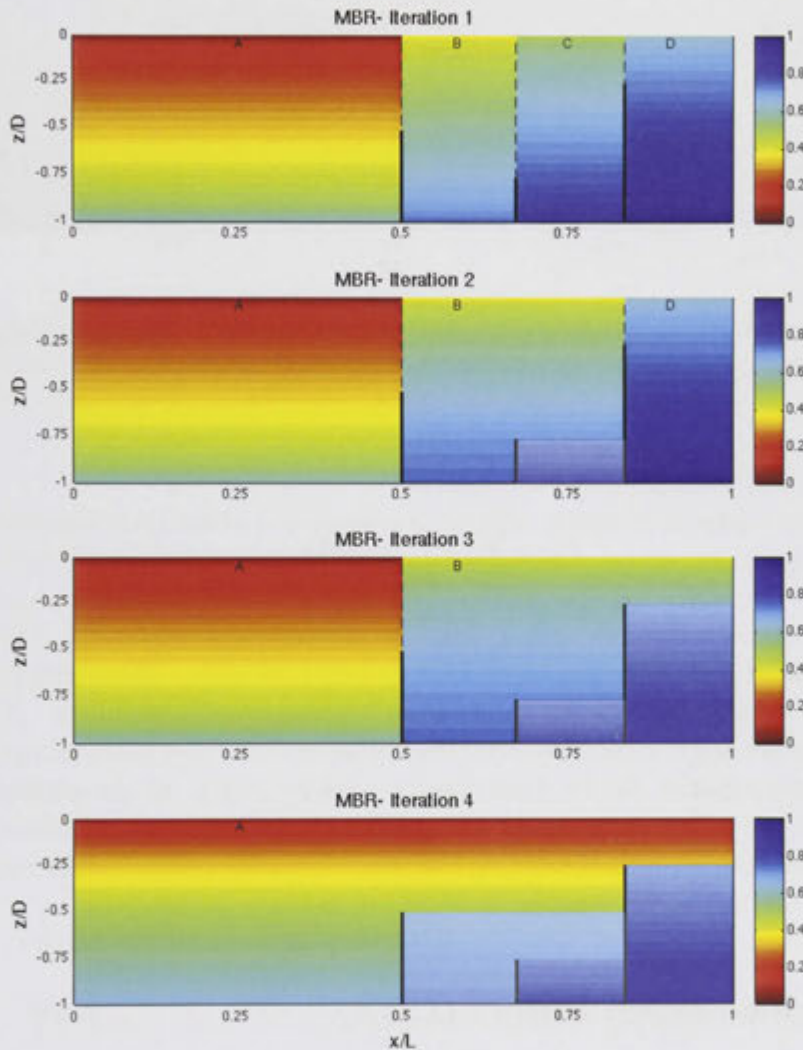


Figure 4.3: The evolution of the density field in the domain from that in Figure 4.2a through the four-stage iterative process of the MBR method. The first iteration identifies the basins of the domain (A–D, indicated by the dashed lines) and adiabatically resorts each basin individually. The lower enclosed region of basin C (indicated by the transparent region) is ‘locked’ for the second iteration, with the remainder of basin C incorporated into basin B. Basin D is the next basin to be locked, with the overlying region becoming part of basin B. For the final iteration the enclosed region of basin B is locked.

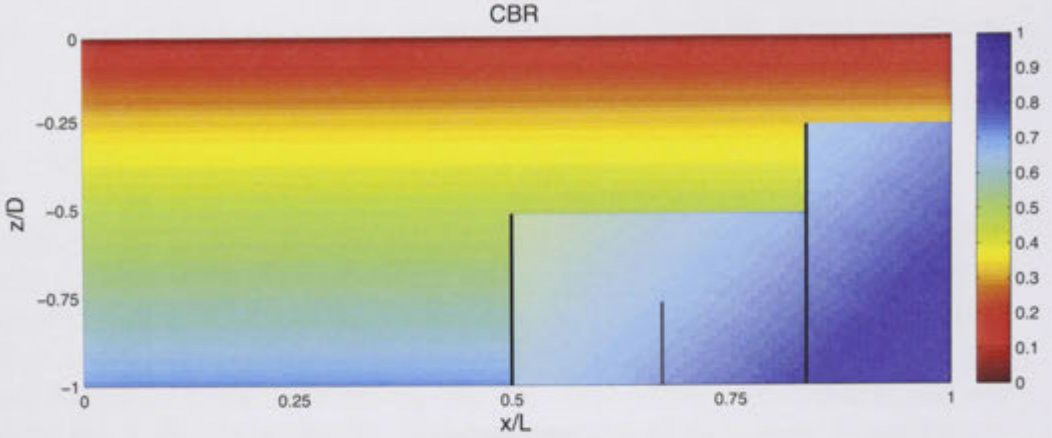


Figure 4.4: The reference state calculated by the CBR; the transparent regions are ‘capped’ and treated as solid.

$$E_b = \sum_{i=1}^n g \int_{V_i} \rho z_{*i} dV_i, \quad (4.1)$$

for n basins, each associated with a different density profile and z_* . Figure 4.5b shows the value of z_* at the upper surface for the initial state. This quantity is used (with Q) to calculate the energy input from surface buoyancy fluxes. Note that the surface waters are not confined to their initial basin by the bottom topography. The obvious discrepancy in the final density profiles and z_* of the surface waters determined by the different methods will result in a difference in calculated E_b , E_a and Φ_{b2} . We test the significance of this difference for a buoyancy forced overturning circulation in a multi-basin domain.

4.1.2 Capped-Basin Resort (CBR)

A multi-basin domain can be simplified by ‘capping’ the enclosed regions of minor basins; the water within the enclosed region is ignored and the remaining single basin is resorted to find the z_* field of the initial state. This ‘Capped-Basin Resort’ approximation is expected to be valid if the capped regions occupy a small fraction of the total volume of the domain, and if the density fields of the ‘capped’ regions at any stage of the gravitational relaxation do not significantly differ from those generated by the MBR method (e.g., the dense water formation occurring in the capped basins simply fills them with dense water). The CBR method is motivated by the configuration of the global oceans: large basins (Atlantic, Pacific, Southern and Indian Oceans) are topographically separated from relatively small polar basins, where much of the surface cooling and dense water formation occurs. As no part of the surface of the domain will ever be included in the capped region, z_* will be defined over the entire surface and Φ_{b2} can be calculated. For an overturning

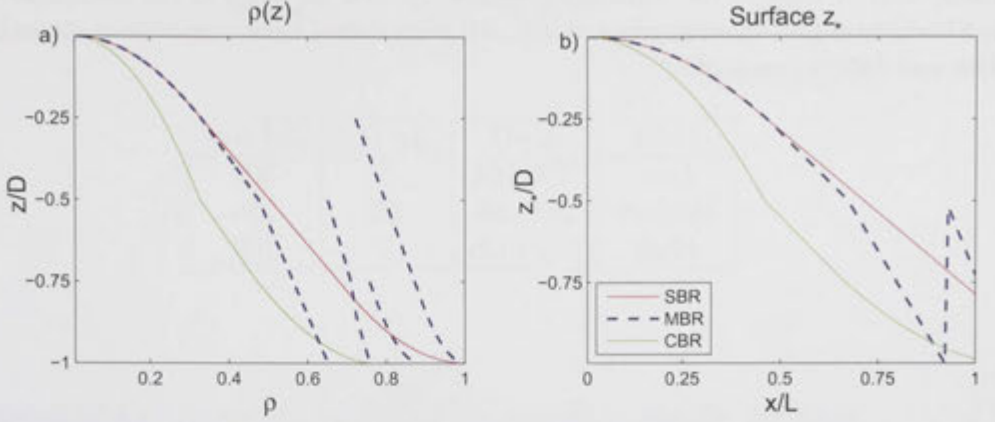


Figure 4.5: The final density profiles (a) and corresponding z_* (b) of the surface waters in the initial state (Fig. 4.2a), for the reference states of no motion calculated by the SBR (red), MBR (dashed blue) and CBR (green) methods.

circulation driven by surface buoyancy fluxes the range of densities at the surface is much wider than in the interior, hence z_* will range up to the maximum depth of the uncapped region.

For the example in Figure 4.2a, the CBR method caps the volumes enclosed by the sills in basins B, C and D, ignoring the water in these regions and treating it as solid (Fig. 4.4). The water initially located in the remaining region is adiabatically resorted to return a single stable stratification (shown by the green line in Fig. 4.5a), from which we can find an approximate field of z_* (Fig. 4.5b) and values for E_b , E_a and Φ_{b2} .

4.2 Numerical Solutions

4.2.1 Testing and Performance of the Model

Numerical solutions were used to test the proposed methods of determining the reference state of a multi-basin domain, and expand upon the parameter space covered by the laboratory experiments with horizontal convection. The MITgcm (Marshall *et al.*, 1997) was used, selecting non-hydrostatic mode to model the Boussinesq flow in an idealized non-rotating domain, which is 1600 m deep, 300 km long and 500 m wide. The domain was two grid cells wide with a periodic condition at the lateral boundary to ensure the circulation remains two-dimensional. The convective circulation was forced by an imposed surface heat flux which had a half-sinusoidal distribution, antisymmetric about the centre of the domain with an amplitude $Q_0 = 250 \text{ W m}^{-2}$, such that there was a zero net heat flux into the domain. The model was run on National Computational Infrastructure (NCI) National Facility.

Three grids were tested to determine the dependence of the numerical solutions on model resolution. The grids were of length $\times$ depth of 256×64 , 384×96 (Fig. 4.6)

Table 4.1: A table of the three grids tested. Δz and Δx refer to the minimum vertical and horizontal grid spacing, respectively. All grids tested have a maximum Δz and Δx of 90 m and 7500 m, respectively.

Grid	L×D	Δz (m)	Δx (m)
Low	256×64	5	500
Medium	384×96	2.5	250
High	512×128	2	100

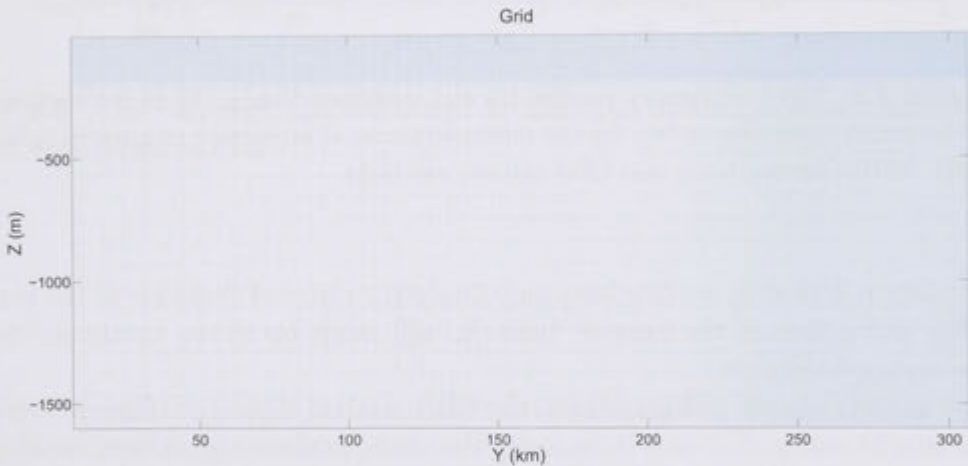


Figure 4.6: The grid of medium resolution used for the numerical solutions (384×96). The figure is vertical axis is exaggerated 85 times the horizontal.

and 512×128; these are referred to as ‘low’, ‘medium’ and ‘high’ resolution. All three grids increased in vertical and horizontal resolution towards the surface and sinking regions, respectively (Tab. 4.1, Fig. 4.6). The models were run for 50 model days with identical parameters and at the lowest value of vertical diffusivity ($K_v = 10^{-5} \text{ m}^2 \text{ s}^{-1}$) that will be investigated. It has been shown previously that the boundary layer at the forcing boundary is of prime importance to horizontal convection. Thus, any grid used to model an overturning circulation driven by surface heat fluxes must be able to adequately resolve this feature.

Figure 4.7a,b show profiles of the final 5 day mean temperature through the upper 20 m in the regions of surface heating and cooling, respectively, for each of the grids tested. The surface temperatures for the medium and high resolution grids are in good agreement. Both of the higher resolution grids capture the temperature inversion in the cooling region. The temperature profiles for the low resolution grid bears little resemblance to those of the other test cases. We determine that the grid of medium resolution is adequate for the numerical solutions in the test domain.

The independence of the solution on the timestep is tested for $\delta t = 1, 2 \text{ \& } 5$ seconds. The model was run for 50 model days with the same configuration as above

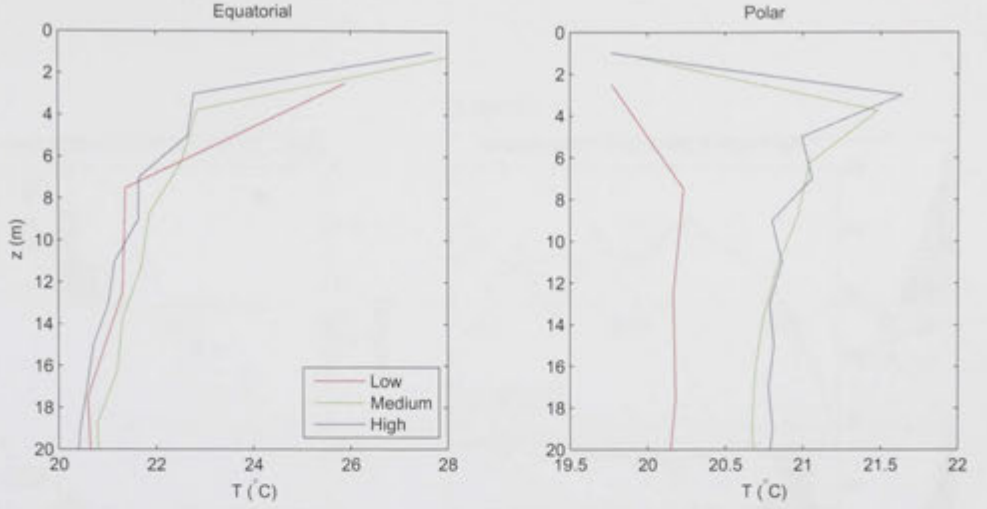


Figure 4.7: Profiles of temperature through the upper 20m of the heated and cooled regions. The three resolutions are shown by the red (low), green (medium) and blue (high) lines.

for each δt . Figure 4.8a shows a time log of minimum and maximum temperatures in the domain. There is good agreement between all the timesteps tested; thus, we turn to the computational performance. Figure 4.8b shows the total processor time per 10 model day period for the three timesteps tested. The timestep $\delta t = 2$ s was selected for use in the numerical solutions.

The resolution and timestep testing were performed on 32 CPUs. We tested the parallel performance of the model by running it on 8, 16, 32, and 64 CPUs. The model was run at the medium resolution with $\delta t = 2$ over 3 model days for each of the cluster sizes. Figure 4.9a,b show the NCI service units and wall time, respectively, per model day for each of the cluster sizes. It is concluded that 32 CPUs provides the optimal ratio between the two metrics.

4.2.2 Model Setup

The MITgcm was run using the grid shown in Figure 4.6. The domain has sloping endwalls (8% gradient) to allow the deep sinking to occur as a gravity current. We set no-slip conditions at the bottom and endwall boundaries, and an implicit free surface. Density is given by a linear equation of state as a function of heat alone, with the thermal expansion coefficient, specific heat capacity, and reference density prescribed at $\alpha = 2 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$, $c_p = 3900 \text{ J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$, and $\rho_0 = 1000 \text{ kg m}^{-3}$, respectively. The coefficients of diffusion of heat and momentum were chosen to be equal (*i.e.* turbulent Prandtl number $Pr_* = 1$) but anisotropic; the horizontal coefficients were fixed at $K_h = 15 \text{ m}^2 \text{ s}^{-1}$ for all runs, and the vertical coefficients are varied between $K_v = 10^{-5}$ to $10^{-2} \text{ m}^2 \text{ s}^{-1}$. The flux Rayleigh number (2.2) covered by the numerical solutions is $Ra_F = 2.2 \times 10^{23}$ to 2.2×10^{32} .

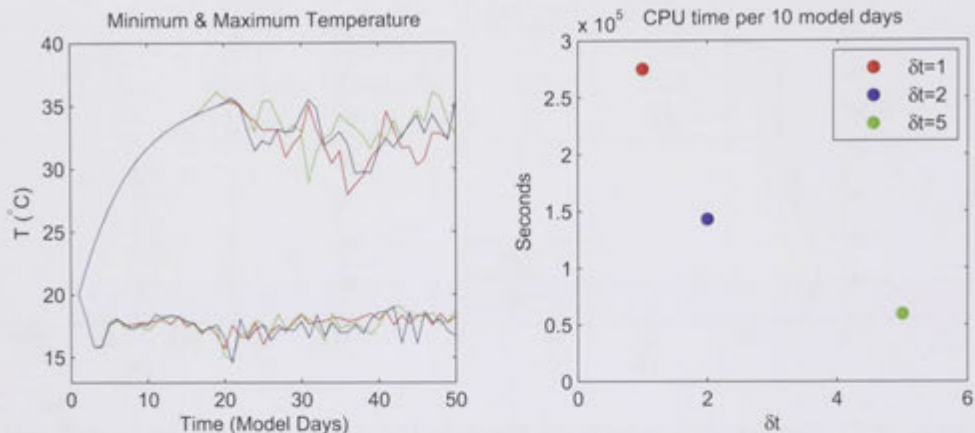


Figure 4.8: Figure (a) shows a log of minimum and maximum temperatures for the three different timesteps tested. There was no significant difference for the range tested, so we turned to the computational performance. Figure (b) plots the average CPU time per 10 model day period for each of the δt tested.

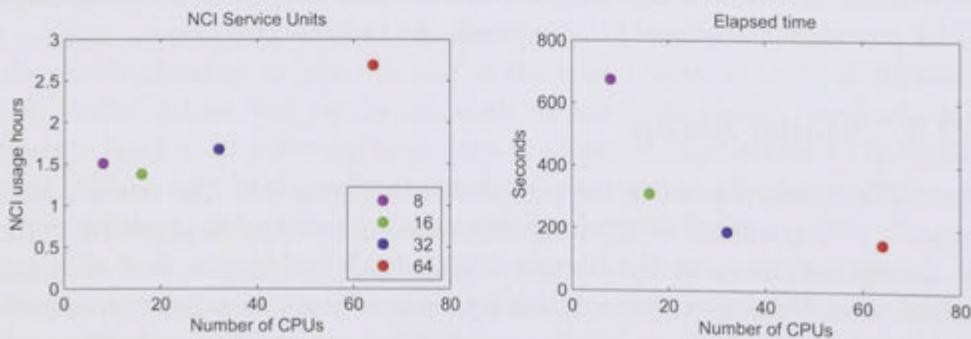


Figure 4.9: Scaling performance per model day; 384x96 grid, $\delta t = 2$.

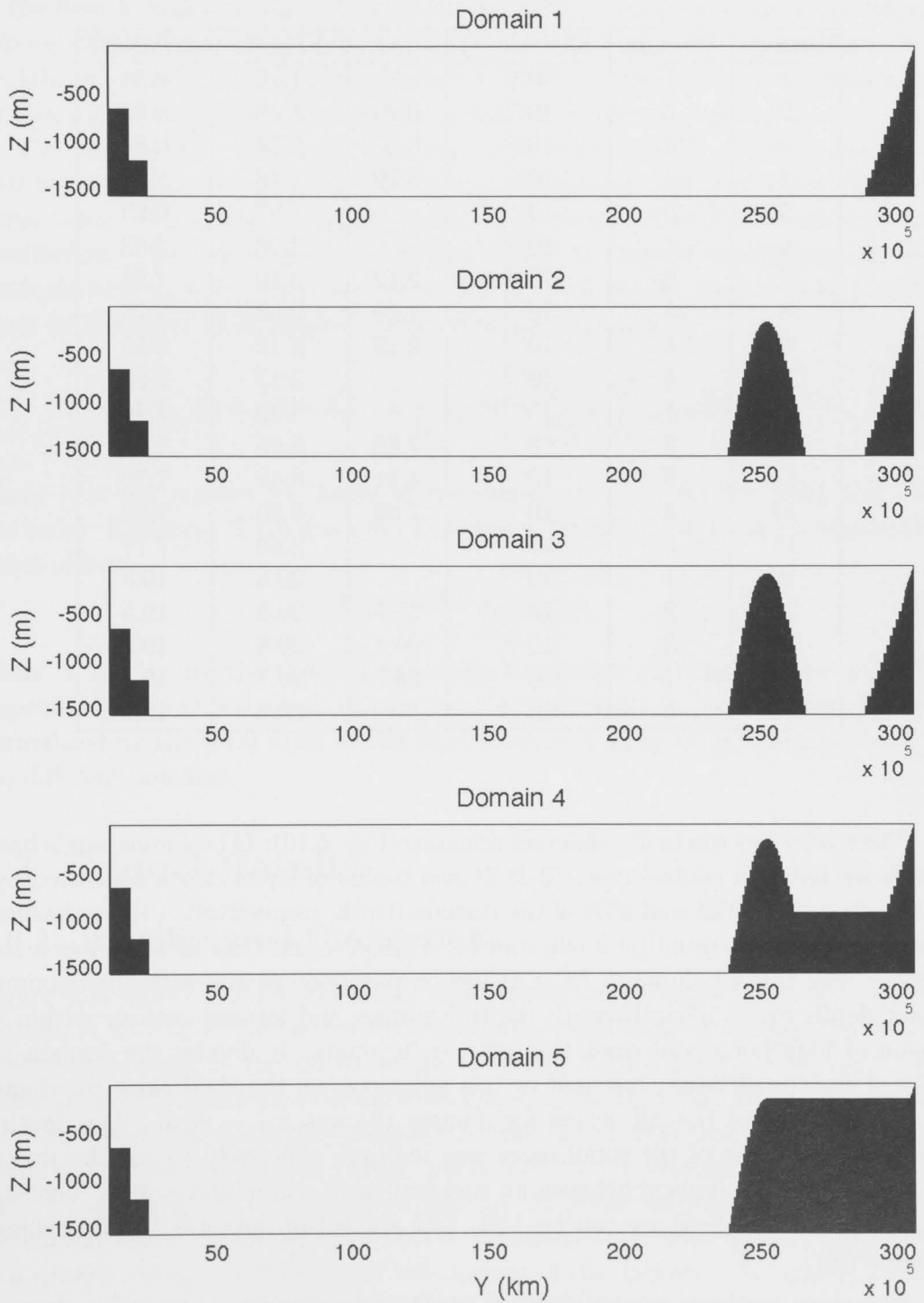


Figure 4.10: The 5 domains used for the numerical solutions.

Table 4.2: Table of the numerical solutions; the columns list the domain, vertical diffusivity, S_D/δ_0 , the mean temperature differences at the surface (ΔT_h) and top-to-bottom in the large basin (ΔT_v).

Run #	Domain	K_v (m^2s^{-1})	S_D/δ_0	ΔT_h ($^{\circ}\text{C}$)	ΔT_v ($^{\circ}\text{C}$)
1	1	10^{-2}	–	1.16	0.81
2	2	10^{-2}	0.79	1.48	0.83
3	3	10^{-2}	0.45	2.24	0.87
4	4	10^{-2}	0.79	1.48	0.83
5	5	10^{-2}	–	1.67	0.83
6	1	10^{-3}	–	3.28	2.63
7	2	10^{-3}	2.12	3.16	2.56
8	3	10^{-3}	1.19	3.60	2.75
9	4	10^{-3}	2.12	3.12	2.55
10	5	10^{-3}	–	3.17	2.68
11	1	10^{-4}	–	8.49	7.46
12	2	10^{-4}	7.66	8.65	7.84
13	3	10^{-4}	4.31	8.49	7.79
14	4	10^{-4}	7.66	8.60	7.82
15	5	10^{-4}	–	8.60	7.77
16	1	10^{-5}	–	20.5	19.3
17	2	10^{-5}	35.7	20.8	19.5
18	3	10^{-5}	20.1	20.6	19.3
19	4	10^{-5}	35.7	20.7	19.4
20	5	10^{-5}	–	20.3	18.9

The model was run in five different domains (Fig. 4.10): (1) a simple single basin, which we use as a control case; (2 & 3) two basins of equal depth separated by a sill that occupies 90% and 95% of the domain depth, respectively; (4) two basins of unequal depth separated by a sill; and (5) a single basin that incorporates a shelf region. The sills for domains (2 & 4) have a parabolic profile with the maximum water depth $S_D = 160$ m beneath the free surface and located entirely within the region of high horizontal resolution (the ‘polar’ end). It divides the domain into a large and small basin; one half of this sill serves as the shelf slope for domain (5). The depth of the sill vertex for domain (3) was $S_D = 90$ m. The depth of the enclosed region of the small basin was reduced, effectively to zero for domain (5), to make a comparison between an enclosed basin region and a shelf. The small basins of domains (2–4) account for 13%, 13% and 9% of the total domain volumes, respectively.

The three methods return identical results for domains (1) and (5); these are included in the study to illustrate the influence of a sill on the circulation. §3 and Stewart *et al.* (2011) show that a sill will significantly alter the global density field (relative to the case of no sill) only if the shallowest point of the sill (S_D) is less than twice the depth of the thermocline (δ_0) in the control case. This implies that

the energetics of the circulation will be similar for all domains having $S_D/\delta_0 > 2$, so long as the influence of the sill is adequately accounted for when determining the reference potential energy state. Twenty cases are examined (Tab. 4.2); each of the five domains for four different values of K_v , which was varied in order to achieve different thermocline depths. If the numerical solutions behave similarly to the laboratory experiments, the influence of the sill is expected to be significant only for Runs #2–4 & 8, where the diffusivity and sill geometry give $S_D/\delta_0 < 2$.

The models were initialized with the entire domain at $T = 20^\circ\text{C}$; due to the zero net heat flux the mean temperature of the domain remained constant at this initial value. The forcing was applied and the circulation allowed to come to thermal equilibrium (a period of up to 1600 model days). At thermal equilibrium the three methods were used to determine values for E_a and Φ_{b2} for each time step. E_a is given by Equation (2.9) adapted for a multi-basin domain,

$$E_a = E_p - E_b = g \int_V \rho z dV - \sum_{i=1}^n g \int_{V_i} \rho z_{*i} dV_i, \quad (4.2)$$

where n is the number of basins in the domain ($n = 1$ for the SBR and CBR methods). Equation (2.10) gives Φ_{b2} in terms of buoyancy, and can be expressed in terms of heat,

$$\Phi_{b2} = -\frac{g\alpha}{c_p} \oint_A z_* Q dA, \quad (4.3)$$

where α and c_p are the thermal expansion coefficient and specific heat capacity, respectively, and Q is the heat flux imposed at the surface A . Both E_a and Φ_{b2} were normalized by the total mass of the fluid volume to allow for comparison between the different domains.

4.3 Model Results

4.3.1 Equilibrated Circulations

Figure 4.11 shows the 100-day mean equilibrated temperature fields for solutions with $K_v = 10^{-2} \text{ m}^2 \text{ s}^{-1}$ in each of the five domains; this value of K_v is shown because it allows the effect of the sill to be demonstrated with a linear colour scale. The mean circulation overturned the full-depth in all of the domains. The surface heating over the ‘equatorial’ half of the domain established a thermocline in which relatively warm water was transported to the ‘polar’ end to the domain. The thermocline water cooled along the way and upon reaching the polar end the surface had become the coldest water in the domain. This cold surface water sank against the endwall as a gravity current, outflowing at the bottom of the domain. The gravity current warmed, owing to entrainment of ambient water as it sank, resulting in the density of the outflow at depth being less than that at the surface. The return upwelling was broadly distributed along the length of the basin.

For the numerical solutions with a sill (domains 2–4), the sinking gravity current mixed and ventilated the small basin below the level of the sill, as shown in previous

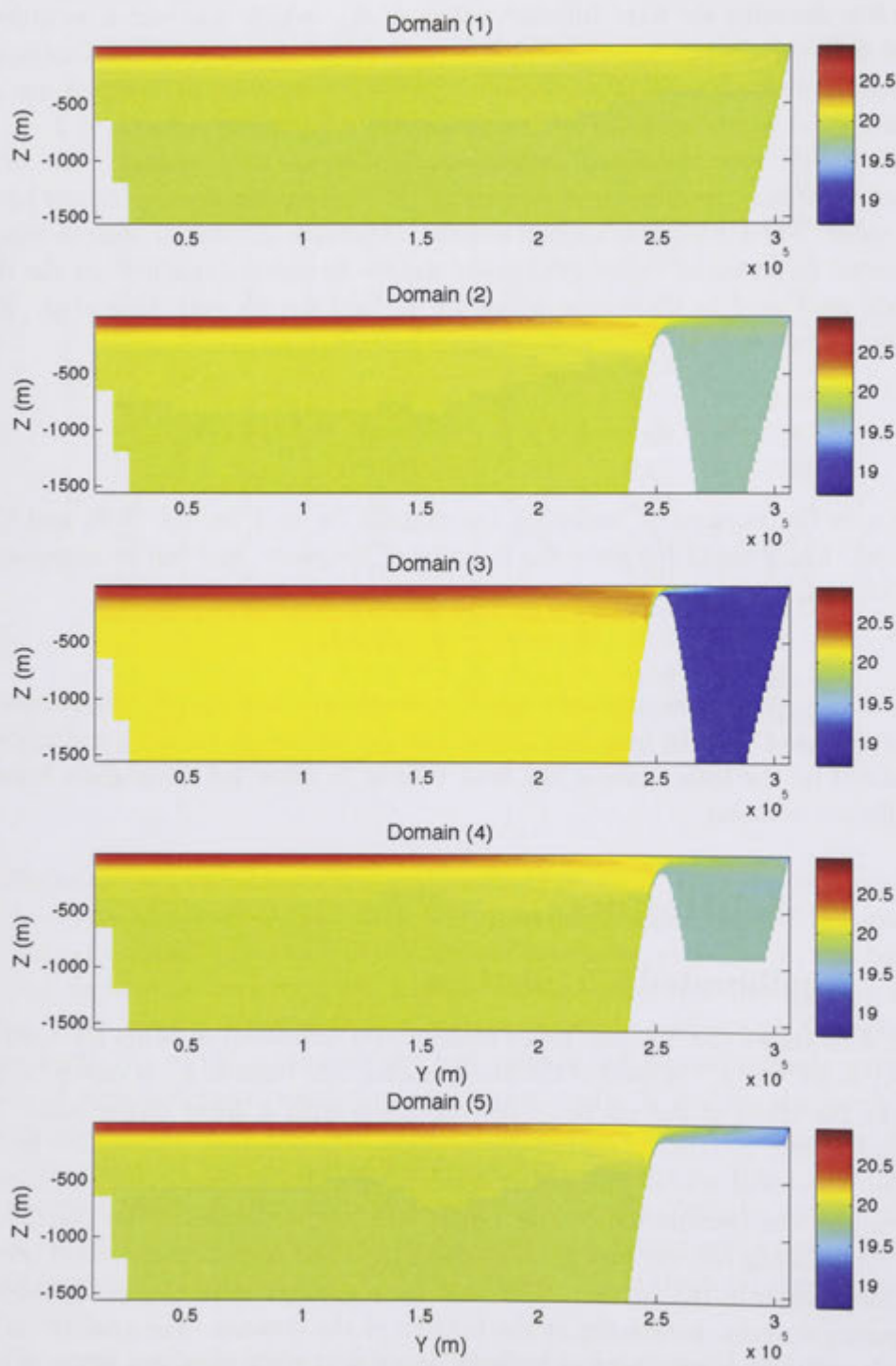


Figure 4.11: The mean temperature fields of the equilibrated circulations for solutions with $K_v = 10^{-2} \text{ m}^2 \text{ s}^{-1}$.

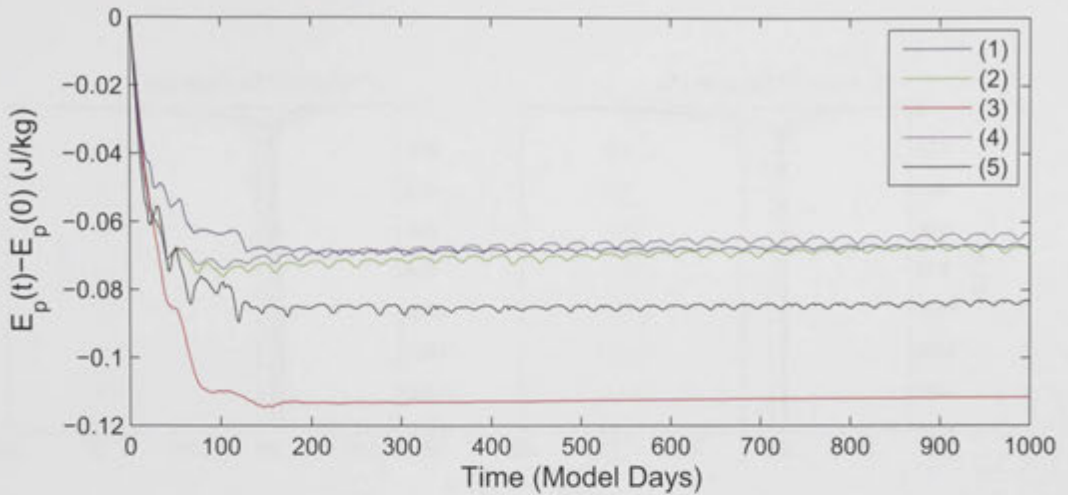


Figure 4.12: Time log of the total potential energy less the initial potential energy, normalized by the total mass of fluid in each domain, for the solutions with $K_v = 10^{-3} \text{ m}^2 \text{ s}^{-1}$.

laboratory experiments (§3 and Stewart *et al.*, 2011). For $K_v < 10^{-2} \text{ m}^2 \text{ s}^{-1}$, the density of the well-mixed region is that of the gravity current at the level of the sill, beneath which the gravity current no longer accesses ambient waters of different densities and ceases to evolve in density. Water was continually displaced from the small basin in a sill overflow, representing a baroclinic exchange with the overlying less dense thermocline water. The shelf regions in the solutions with domain (5) were well-flushed by entering thermocline water and exiting dense water. The sill and shelf overflows sank to the bottom of the large basin against the sill and shelf slope, respectively, and again were balanced by a broad upwelling throughout the large basin.

There existed a periodic time-dependence to the circulations similar to that described in Hughes *et al.* (2009). Surface water which was not dense enough to penetrate the underlying stratification appeared to collect at the polar end and continue to cool, deepening the thermocline in this region. Eventually this water became dense enough to sink as the aforementioned gravity current, giving a periodic intermittency to the deep sinking. The thermocline quickly readjusted as the overlying surface waters sink, initiating an internal gravity wave which propagated into the interior along the thermocline. Upon reaching the equatorial endwall, the internal gravity wave was reflected and returned to the polar end as a surface wave. The propagation speed of the surface wave was significantly faster than that of the internal gravity wave. The time that the surface wave reached the polar endwall appeared to be coincident with the subsequent sinking ‘pulse’ of surface water. Interestingly, there was no time-dependence to the circulations for $S_D/\delta_0 < 2$; we attribute this to the two-layer critical exchange at the sill decoupling the sinking pulse from the larger basin. As in the case of Hughes *et al.* (2009) the oscillation amplitude (relative to the mean) was larger for smaller K_v . The calculated values

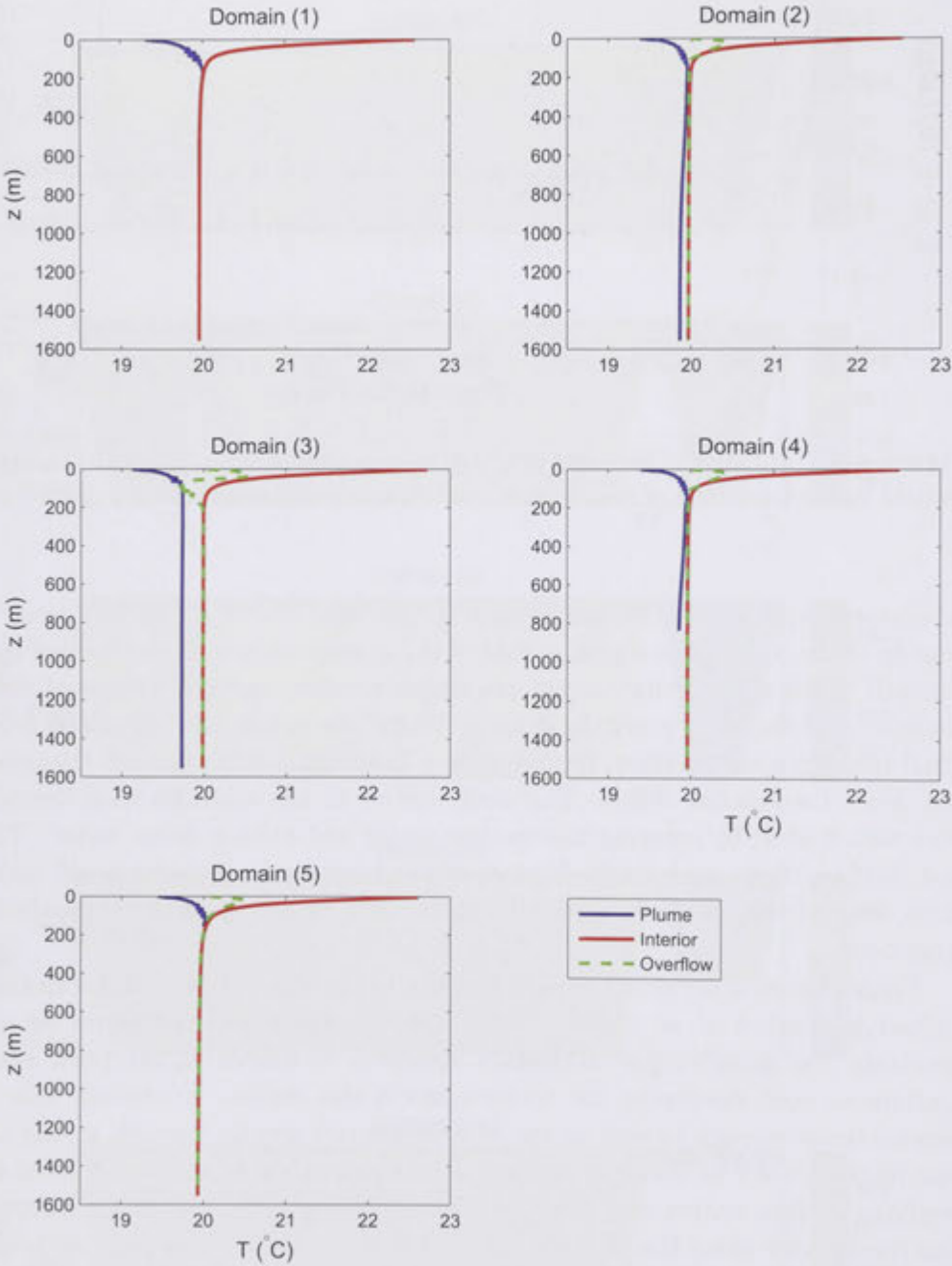


Figure 4.13: Profiles of temperature with depth for solutions with $K_v = 10^{-3} \text{ m}^2 \text{ s}^{-1}$. The profiles are taken through the endwall gravity current (blue), sill exchange and overflow current (dashed green), and the interior (red).

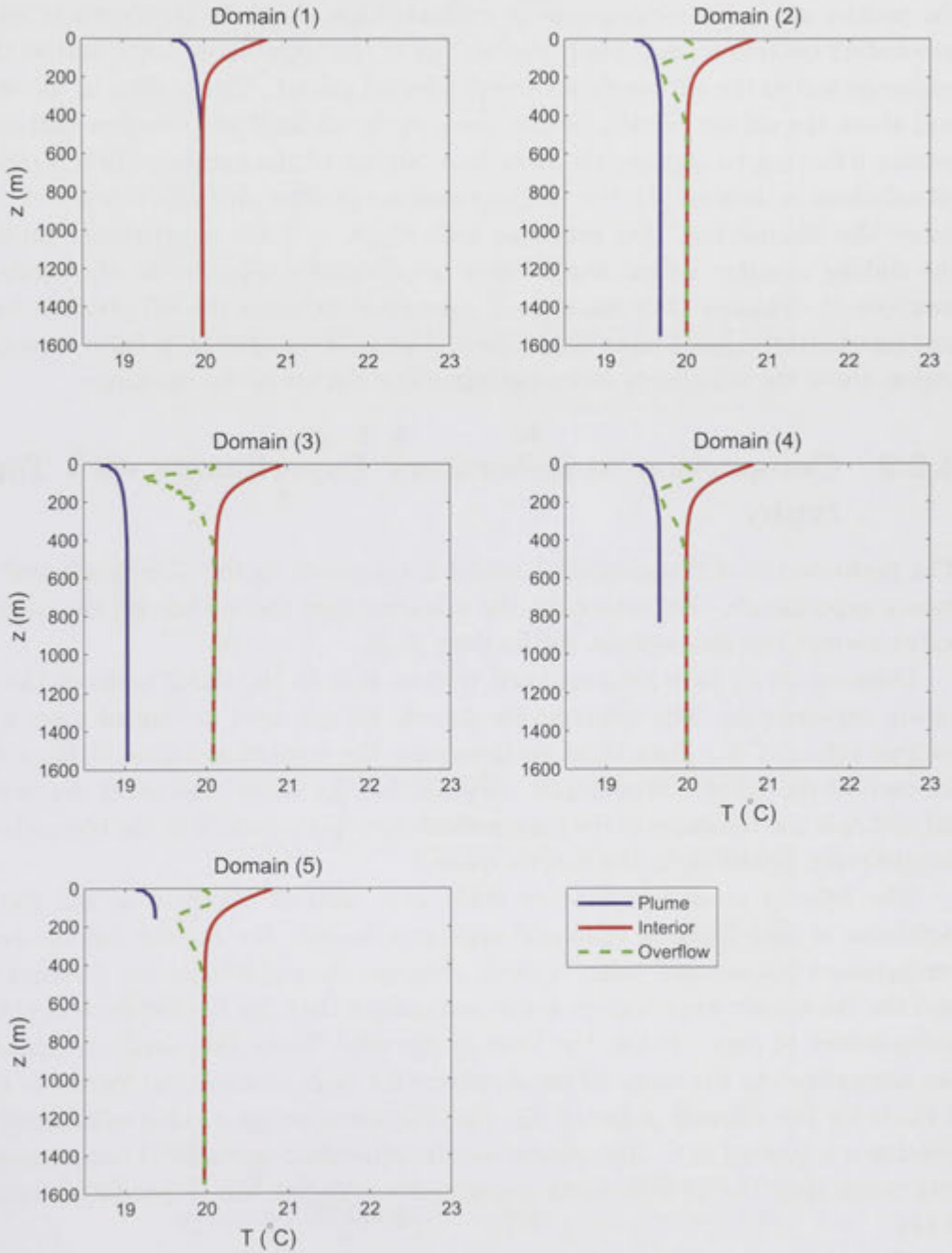


Figure 4.14: Profiles of temperature with depth for solutions with $K_v = 10^{-2} \text{ m}^2 \text{ s}^{-1}$. The profiles are taken through the endwall gravity current (blue), sill exchange and overflow current (dashed green), and the interior (red).

of E_a and Φ_{b2} were therefore averaged over a period of 500 model days.

Figures 4.13 and 4.14 plot the temperature with depth at three different locations in each of the domains for $K_v = 10^{-3}$ and $10^{-2} \text{ m}^2 \text{ s}^{-1}$, respectively. The locations of the profiles are in the sinking gravity current (blue, shown to the depth at which it encounters bottom topography), the interior of the large basin (red), and at the sill exchange and in the sill overflow current (dashed green). The profiles in the interior and above the sill are vertical, whilst those in the endwall and overflow currents are terrain following to capture the density evolution of the currents with depth. For all solutions in domain (1), the two temperature profiles virtually overlap at depths below the thermocline. For solutions with $S_D/\delta_0 < 2$ the temperature profiles in the sinking gravity current and interior are distinctly separate at all depths. All solutions in domains (2–5) show good agreement between the sill overflow current and interior throughout most of the lower depths. Temperature profiles through the region above the sill clearly show the baroclinic nature of the exchange.

4.3.2 Comparison to Laboratory Experiments with Topography

The performance of the numerical model is compared to the aforementioned laboratory experiments. We determine the influence that the sill has on the numerical solutions and test this against the findings of §3.

Domains (2–4) have semi-enclosed regions akin to the small basin of the laboratory experiments. The solutions for domain (1) are used as control cases for the various values of K_v ; from these we determine the respective values of $R_0 = S_D/\delta_0$ for each of the numerical solutions. (Recall that S_D is the shallowest depth to the sill and δ_0 is the thickness of the region which incorporates 95% of the top-to-bottom temperature difference in the control cases.)

The 100-day mean temperature fields were used to calculate δ_0 and the temperatures at mid-depth in the small and large basins. For domain (4) the bottom temperature in the small basin is used, although the small basins of domains (2–4) and the laboratory experiments are so well-mixed that the temperature is virtually independent of depth below the level of the sill. These mid-depth temperatures are normalized by the range of temperatures for each solution and shown in Figure 4.15a,b for the relevant value of R_0 ; the difference between these mid-depth temperatures is plotted in c. Also shown are the equivalent normalized temperatures at mid-depth from the §3 laboratory experiments with flat sills (reproduced from Fig. 3.14).

There is good agreement between the numerical models and laboratory experiments. One key result from §3 is that a sill will significantly influence the global density structure if $R_0 < 2$; we find this result is robust. Interestingly, there is good agreement between the bulk properties of the laboratory experiments and numerical solutions, despite the latter not resolving either mixing at the sill or entrainment into the plume. This suggests the parameterization of the sub-grid processes by the enhanced horizontal and vertical diffusivities in the numerical solutions is adequate for modeling the dynamics of the large-scale circulation.

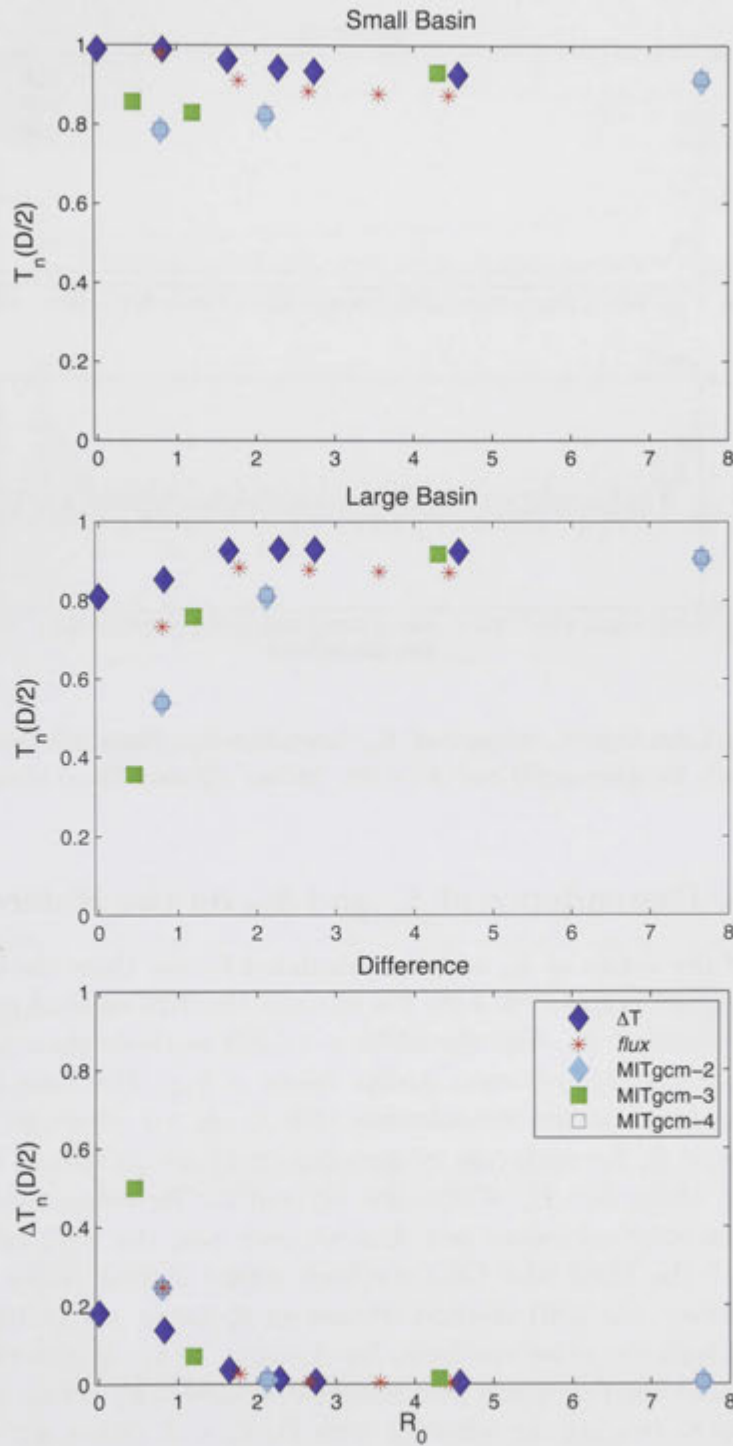


Figure 4.15: Normalized temperatures at mid-depth in the small (top) and large (middle) basins, and the difference between them (bottom). The results from §3 for laboratory experiments with flat sills and an imposed temperature difference (blue diamonds) or heat flux (red stars) are compared with these from the numerical solutions in domains (2–4) represented by the cyan diamonds, green squares and hollow magenta squares, respectively. There is good agreement between the laboratory experiments and the numerical solutions.

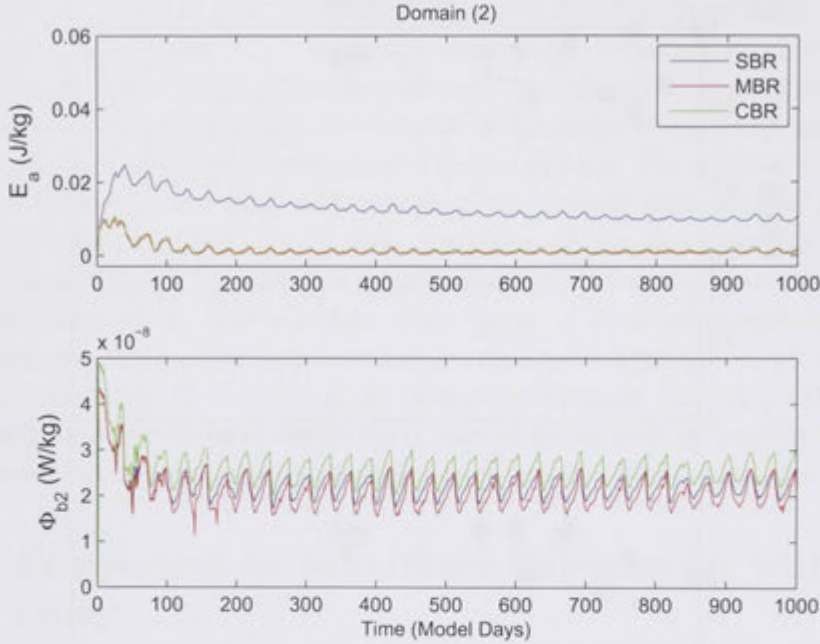


Figure 4.16: Time logs of normalized E_a (top) and Φ_{b2} (bottom) calculated by the different methods for domain (2) and $K_v = 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (Run #7).

4.3.3 The Dependence of E_a and Φ_{b2} on the Reference State

Time series of the values of E_a and Φ_{b2} calculated by the three methods for Runs #7–9 are shown in Figures 4.16–4.18. For all cases the SBR method returned significantly larger values of E_a , with the MBR and CBR methods showing good agreement. All three methods returned similar values of Φ_{b2} . The time dependence of the circulations is evident for the solutions with $S_D/\delta_0 > 2$ (Runs #7 & 9).

The values of E_a for each case in domains (2–4) are plotted in Figure 4.19a–c, respectively, along side E_a of domains (1) and (5) for comparison. The three methods return identical values and thus we only plot the SBR method results. For $D_s/\delta_0 > 2$ the MBR and CBR methods return similar values of E_a for all domains. However, the SBR method returns an apparent 10- to 100-fold greater E_a (compared with the other methods) for domains (2–4). In general, for a given K_v the MBR and CBR methods return similar values of E_a across all 5 domains. The exceptions to this are the solutions with $D_s/\delta_0 < 2$, (Runs #2–4 & 8, shown as enlarged symbols), for which the influence of the sill is expected to be global (Stewart *et al.*, 2011); in these cases the MBR and CBR methods return values of E_a for domains (2–4) that are much larger than that of the respective domain (1). This is a real physical difference, with the sill causing a large increase in horizontal density difference and available potential energy. As with the smaller values of K_v , there is good agreement between the MBR and CBR methods for domains (2–4), while the SBR method again overestimates E_a .

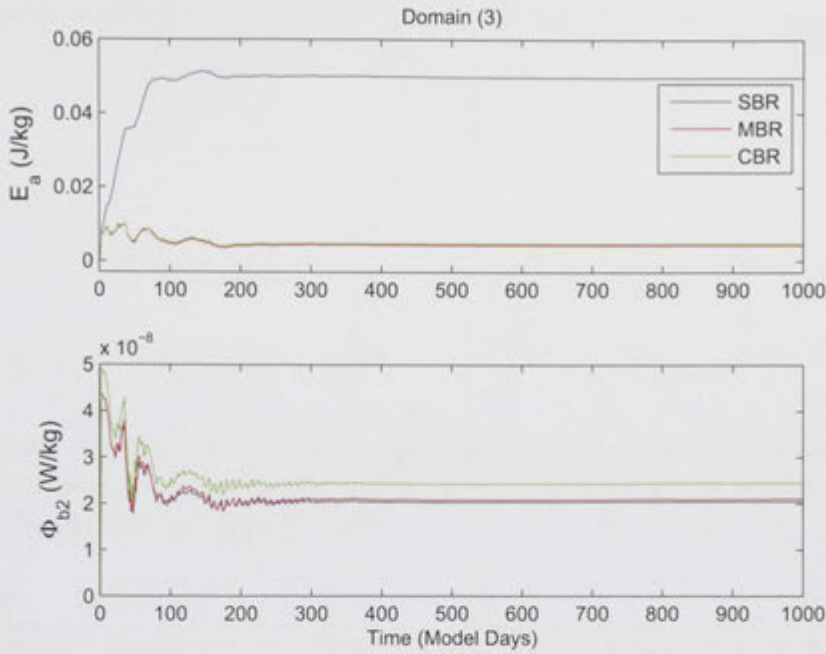


Figure 4.17: Time logs of normalized E_a (top) and Φ_{b2} (bottom) calculated by the different methods for domain (3) and $K_v = 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (Run #8).

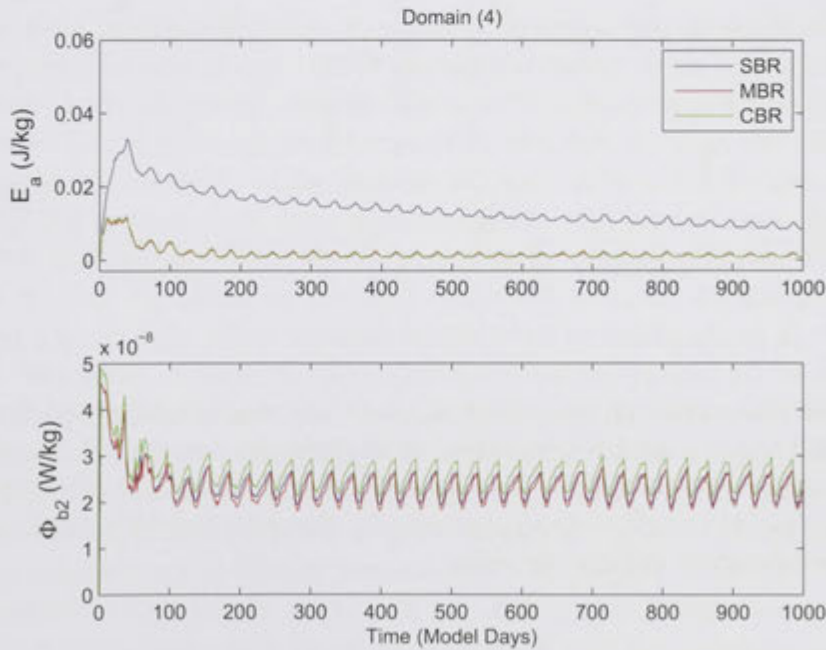


Figure 4.18: Time logs of normalized E_a (top) and Φ_{b2} (bottom) calculated by the different methods for domain (4) and $K_v = 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (Run #9).

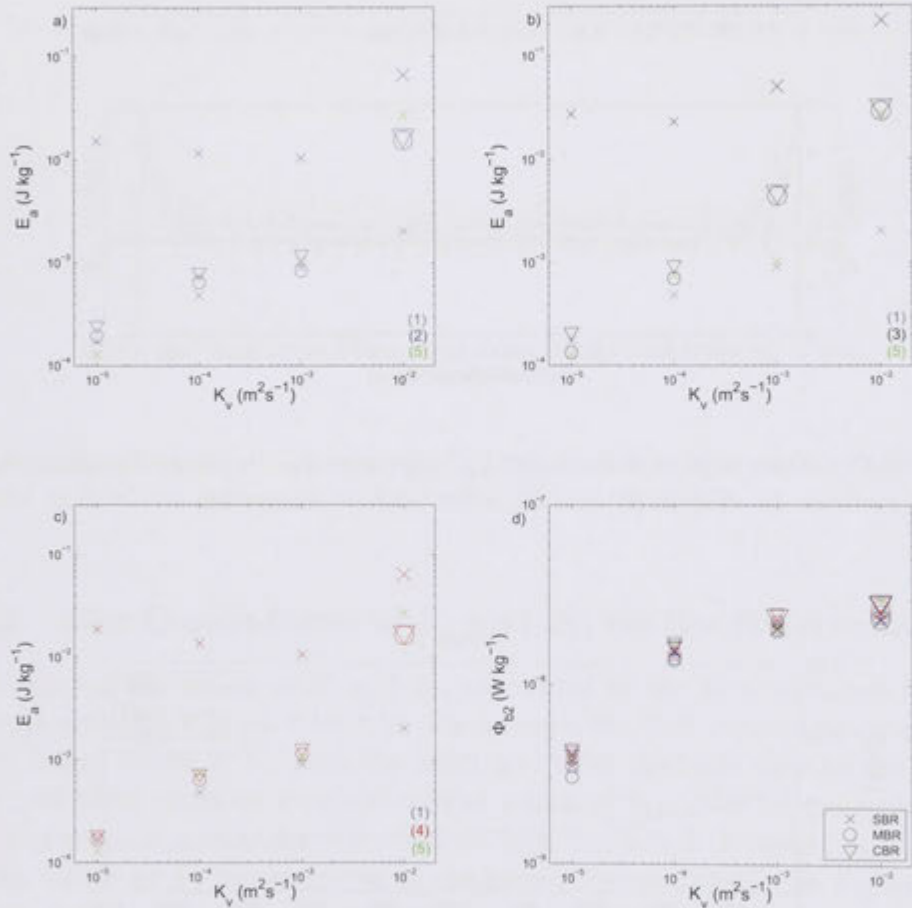


Figure 4.19: The values of normalized E_a (a–c) and Φ_{b2} (d) calculated by the SBR (crosses), MBR (circles) and CBR (triangles) methods for the domains 1–5 (magenta, blue, black, red and green, respectively). The values of E_a for domains (2–4) are compared with those of domains (1) and (5) in (a–c), respectively. The solutions with $S_D/\delta_0 < 2$ (Runs #2–4 & 8) are shown by enlarged symbols.

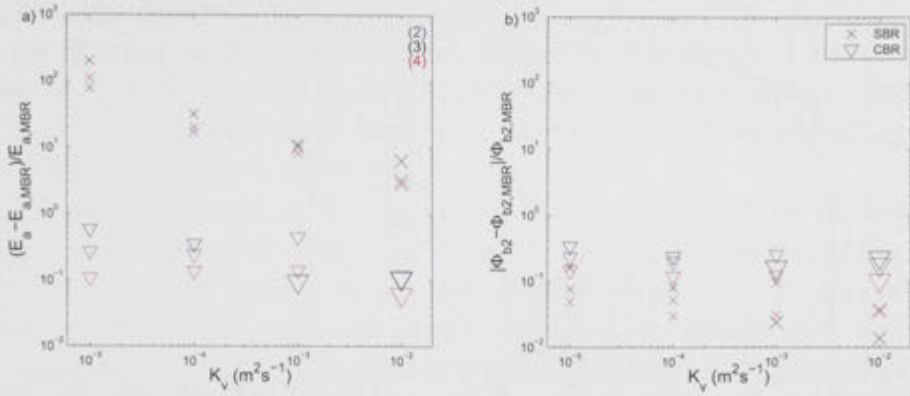


Figure 4.20: The fractional errors for E_a (a) and Φ_{b2} (b) of the SBR (crosses) and CBR (triangles) methods relative to the MBR. The blue, black and red points represent the domains (2), (3) and (4), respectively.

The surface buoyancy forcing Φ_{b2} calculated using the three methods is comparable for each of the domains (Fig. 4.19d). This suggests that any of the three methods are suitable to use to calculate the rate of energy input from buoyancy.

4.4 Discussion

The numerical solutions provide a means to evaluate the methods of estimating available potential energy and rate of mechanical energy input by surface buoyancy fluxes in an idealized oceanic context. The results indicate that for deep sills or thin thermoclines ($D_s/\delta > 2$), the true values of E_a and Φ_{b2} (given by the MBR method) are comparable irrespective of the presence of topography in the domain. We assume that the MBR method gives the unique physically relevant reference state, and use this state to test the veracity of the computationally simpler methods. The fractional errors in E_a and Φ_{b2} given by the SBR and CBR methods for domains (2–4) are plotted in Figure 4.20. Most obvious is the gross overestimate of E_a by the SBR method. The CBR method, on the other hand, overestimated E_a by only 10–30% for both domains; the error in E_a is smaller for domain (4), which has a smaller capped basin. This error will generally be smaller for smaller capped basin volumes.

The SBR method appears to increasingly overestimate E_a for small K_v , despite these circulations existing further into the regime of $S_D/\delta_0 > 2$ and exhibiting no significant dependence of density structure on the domain. This can be explained by the profiles in Figure 4.21; these plot the difference in temperature between the interior and small basin as a function of depth for all the solutions performed in domain (2). The differences are largest in the upper 1% of the domain depth, with the size of the differences decreasing towards the level of the sill. Below the level of the sill the temperature differences generally increase with depth; the exception to this is Run #2, for which $S_D/\delta_0 < 2$ and the temperature difference continues

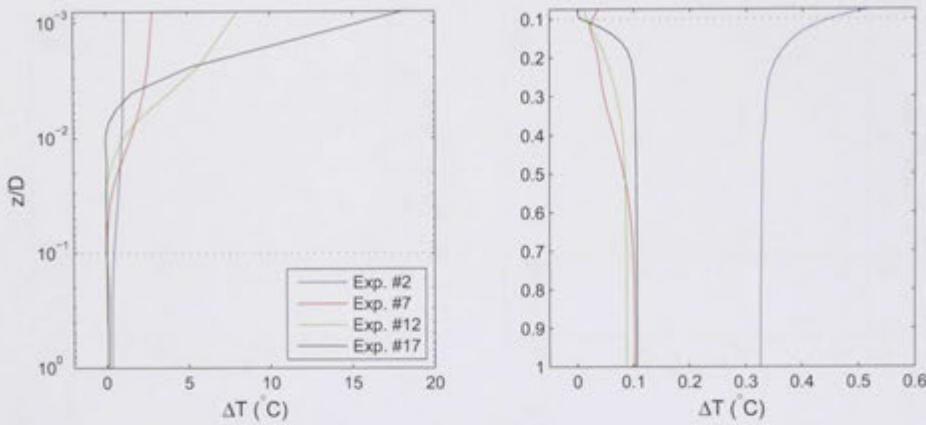


Figure 4.21: The temperature difference between the large and small basin as a function of normalized depth for the numerical solutions in domain (2); Runs #2 (blue), 7 (red), 12 (green) & 17 (black). The left plot shows the full depth of the domain (note the logarithmic scale of the ordinate); the right plot focusses on depth below the level of the sill, which is indicated by the dotted line.

to decrease through the whole depth. For Runs #7, 12 & 17 the temperature difference increases towards, and reaches, a maximum value below the level of the sill. The depth at which this maximum temperature difference is reached is higher in the water column for the lower values of K_v . The small basin is intermittently ventilated by the dense gravity current, which brings dense waters to all depths and maintains the stratification. For small values of K_v , this dense water is unable to diffuse (vertically) out of the enclosed region and must be displaced by relatively younger sinking water. This leads to the enclosed region of the small basin “filling” with the densest water, maintaining the temperature difference with the interior up to the level of the sill. As K_v increases, the effect of vertical diffusion reduces the temperature difference through the upper part of the enclosed region. The ability to diffuse out the dense water in the small basin leads to a reduction in the across-sill temperature difference, thus reducing the relative error introduced by the SBR method.

In terms of Φ_{b2} , both simple methods returned values close to the true value. The details depend on size of basin and results from the simple methods are closer to that from the MBR method in the case of domain (4) than for (2) and (3); this makes good physical sense given the smaller fractional volume of the small basin in domain (4). For $K_v = 10^{-2} \text{ m}^2 \text{ s}^{-1}$ in the present configurations the SBR method returns a value of Φ_{b2} that is approximately 5% smaller than that from the MBR method.

For all three methods the majority of the surface waters tended to end up in the largest basin. Therefore, the values of z_* for the surface water are most sensitive to the stratification in the largest basin, which is predominantly set by the end-wall gravity current or the sill/shelf overflows. For thin thermoclines or deep sills

($S_D/\delta_0 < 2$ in the present solutions) the small basin is well-mixed below the level of the sill: the gravity current falling into this region and the displaced overflow water are of equal density. The profiles of density with depth in the overflows of domains (2–5) were identical to that of the endwall gravity current in domain (1). As such, the stratification in the large basin, and hence z_* at the surface, are largely insensitive to the presence of the sill.

The weak dependence of Φ_{b2} on the resorting method can be understood via Equation (2.10); essentially, Φ_{b2} is the surface integral of the product of the imposed surface buoyancy flux (which in this case is given by $B = g\alpha Q/c_p\rho_0$) and z_* for the surface water. The regions of maximum B can potentially give rise to large differences in Φ_{b2} , provided z_* in these regions are affected by the choice of resorting method. However, the warmest and coldest waters of the domain are found in these same regions. As any of the resorting methods will move these waters to the top and bottom of the water column, respectively, the methods will yield similar z_* for the surface water and thus Φ_{b2} . Differences in z_* at locations where B is relatively small are not of leading order when calculating Φ_{b2} .

This raises the issue of the sensitivity of terms in the available potential energy loop (Hughes *et al.*, 2009) to the kind of surface boundary conditions. Specifically, would Φ_{b2} remain insensitive to the resorting method for a prescribed surface temperature distribution or relaxation boundary condition? In these cases we can expect that the maximum amplitude of surface heat fluxes will not necessarily be collocated with the warmest and coldest waters of the domain, possibly giving rise to significant differences in Φ_{b2} as calculated by the various methods.

Another issue concerns parameterizations and resolution of convection in GCMs, where statically unstable density gradients are not permitted. While these have already been shown to render the available potential energy loop irrelevant to the model output (Hughes *et al.*, 2009), the same issue arises in regards to the hydrostatic criterion often incorporated into the analysis of ocean data sets. In such cases z_* for the surface waters will not be the full depth of the domain. Thus it is not clear whether Φ_{b2} remains insensitive to the resorting method for a hydrostatic ocean.

The role of the Arctic Ocean in the overall global circulation provides an ideal case to apply the findings of this study. The Arctic Ocean is completely enclosed below depths of 840 m (Saunders, 2001), the maximum depth of the Greenland-Scotland Ridge which separates the relatively dense Arctic Ocean from the Atlantic Ocean. The Arctic basin contains 1.4% of the global ocean volume with an average depth of 1205 m (Eakins & Sharman, 2010) and we estimate the total enclosed volume of the Arctic Ocean is less than 1% of the global ocean. Our results indicate that the topography physically increases E_a by $O(10)$, as $D_s/\delta < 2$ (Stewart *et al.*, 2011). In addition, neglecting the topography when calculating the available potential energy (the SBR method) leads to a very substantial overestimation of E_a , whereas the CBR method will adequately account for the global influence of the Greenland-Scotland Ridge when calculating E_a . In terms of the dynamically important quantity, the power input from surface buoyancy fluxes Φ_{b2} , the relatively small Arctic Ocean contains both the densest surface waters and regions of maximum surface buoyancy fluxes, and thus any of the three methods tested will

suffice.

4.5 Conclusions

We have proposed a method (Multi-Basin Relaxation) for calculating the reference state of potential energy and hence the available potential energy for a multi-basin domain. The method returns the unique physically relevant state of no motion to which the fluid would relax by adiabatic adjustment under the influence of gravity alone. For complex basin topography this calculation is complicated and time-consuming.

A computationally simpler method (Capped-Basin Resort) is also examined that estimates the reference state by ignoring the influence of minor basins. The results of both methods proposed here are compared with an existing method (Single-Basin Resort) that neglects all pooling of dense water behind topography, using a simple model of an overturning circulation driven by surface buoyancy fluxes in an idealized multi-basin domain. For the two-dimensional flow solutions the rectangular model basin includes a topographic ridge that defines a relatively small sub-basin at the polar end. If the available potential energy is estimated without taking into account the pooling of dense water behind the topographic barrier, the estimate is much greater than the true available potential energy.

The numerical solutions indicate that topography increases the available potential energy when the ridge is high enough to cause the dense overflow to interact and mix with the overlying thermocline, but causes little difference in the rate of mechanical energy input by surface buoyancy fluxes. Considering the relative size and density of the Arctic Ocean, we conclude the computationally efficient Capped-Basin Resort method will adequately account for the global influence of the Greenland-Scotland Ridge. On the other hand, all of the three methods tested are adequate for calculating the surface buoyancy forcing of the global oceans, and this is the quantity of dynamical importance.

Horizontal Convection with an External Input of Mechanical Energy

We turn our attention to laboratory experiments that investigate the effect of externally imposed rates of turbulent mixing on a buoyancy-driven overturning circulation in a rectangular tank. Buoyancy was supplied at the surface by an imposed flux of saline water, with the rest of the surface maintained at a fixed salinity by a continuously replenished layer of fresh water. Mechanical mixing was provided at a controlled rate by a pair of horizontal rods continuously traversed through the full-depth of the experiment, enhancing the total diffusive transport to a known amount that could be calibrated for each rod velocity. The dependence of the density field structure and overturning transport on the mixing rate is found to be in accord with a theoretical model. With reference to the oceans, it is shown that the primary effect of mixing (with energy sourced from winds, tides and convection) is to deepen the thermocline, thereby influencing the properties of the abyssal ocean by modifying the environment through which the turbulent plume sinks and entrains. The advective transport of buoyancy by the plume, and not vertical mixing, is crucial for coupling the surface to the abyss.

5.1 Experiments

5.1.1 Apparatus

The convection experiments were performed in an acrylic tank with internal dimensions of length, depth and width $1.2\text{ m} \times 0.6\text{ m} \times 0.08\text{ m}$ (Fig. 5.1); a vertical partition wall was inserted such that the model basin in which the overturning circulation was to be measured (and which will be referred to as the ‘working volume’) was $1.16\text{ m} \times 0.53\text{ m} \times 0.08\text{ m}$ ($L \times D \times W$, respectively). Thus the aspect ratio was fixed at $A = 0.457$. Fresh and saline water were gravity-fed from two constant-head supplies to the upper right- and left-hand ends of the working volume, respectively. The vertical partition at the left hand end allowed an overflow at the top, thus maintaining a constant working volume. The experiments were carried out in a laboratory held at $20 \pm 1^\circ\text{C}$ and all water entering the tank was equilibrated to the

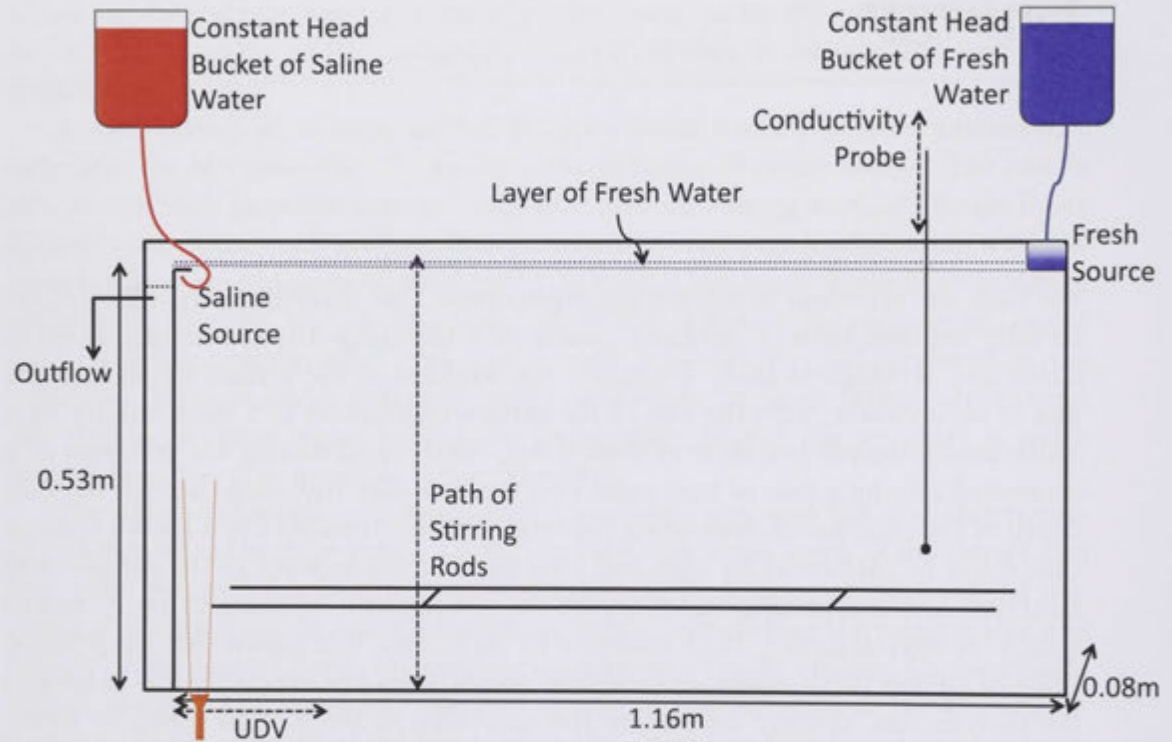


Figure 5.1: Schematic drawing of the experimental setup for experiments #1–9 (for Exp #10 & 11 the saline source is relocated to the bottom left-hand corner of the experiment). The fresh water source maintained a layer of fresh water at the top of the experiment, above the working volume; the fresh layer leaves in the overflow of the partition. The saline water source at upper left of the tank produced a turbulent plume that sank to the bottom of the tank against the partition; note the lip at the upper left-hand corner. Ultrasonic Doppler Velocimeter measurements using a vertical beam were taken at the discrete locations along the dashed horizontal arrow. The stirring rods repeatedly traversed the full-depth from the top of the partition wall to the base. The conductivity-temperature probe was profiled vertically through the full-depth at a fixed horizontal position.

laboratory temperature. Nevertheless, the tank was insulated on all sides by 30 mm of expanded polystyrene foam board; this was removed only for short periods for flow visualization.

In a separate, well-mixed and insulated reservoir 400 L of saline water were prepared. This saline water was continuously recycled through one of the constant-head supplies, from which the same saline water was gravity-fed into the tank via a small tube at a constant volume flux. The outlet of the small tube directed the saline water upward, as a jet, into the upper left-hand corner of the working volume, where a small ‘lip’ separated the saline water from the overflow. This ‘lip’ prevented the immediate escape of saline water from the working volume into the overflow, and also allowed the momentum of the jet to reshape the saline source into a full-width (line) endwall plume that was turbulent at all levels below the corner. The volume flux and the density of the saline source were held constant for all experiments at $Q_s = 35.42 \text{ mm}^2 \text{ s}^{-1}$ and $\rho_s \approx 1022 \text{ kg m}^{-3}$ (salinity $S_s \approx 0.034 \text{ kg/kg}$), respectively, thus imposing the buoyancy flux ($\text{m}^3 \text{ s}^{-3}$, per unit tank width),

$$B = g\beta(S_s - S_f)Q_s, \quad (5.1)$$

supplied to the tank. The coefficient of expansion β for salt has the value 0.69 and, with the salinity of fresh water $S_f = 0 \text{ kg/kg}$, substituting (5.1) into (2.2) gives a flux Rayleigh number $Ra_B = 5.7 \times 10^{18}$ for the experiments. The Prandtl (Schmidt) number (2.4) was held fixed at $Pr = 671$.

A 200 L reservoir of fresh water ($\rho_f \approx 998.3 \text{ kg m}^{-3}$) was maintained at constant volume by a ballcock set plumbed to the filtered mains water supply. This water was continuously recycled through the second constant-head supply, from which a portion was gravity-fed to the tank via a small tube at a constant rate. The fresh water entered at the surface through a small chamber that had a porous base designed to remove excess momentum, create a smooth outflow, and maintain an overlying layer of fresh water there; the volume flux per unit width was $Q_f = 83.33 \text{ mm}^2 \text{ s}^{-1}$. The fresh water flowed along the length of the tank and exited in the overflow above the partition wall. It is the interface between this layer of fresh water and the underlying denser waters that defined the upper surface of the working volume. The boundary condition at the upper surface was therefore a constant and uniform density, corresponding to fresh water. Unlike the injection of saline water into the working volume, note that the transport of fresh water across the upper surface was free to adjust to that required by the internal dynamics of the volume.

The boundary conditions closely approximate a surface maintained at constant buoyancy, except for a line source of destabilizing buoyancy at one end, as used in the theoretical models of Hughes & Griffiths (2006); Hughes *et al.* (2007) and which is argued therein to be similar to the thermal boundary conditions applied in the applied heat flux experiments of Mullarney *et al.* (2004). In a steady-state, both the amount of salt and volume in the working section must be constant. The volume of the working section was maintained by the overflow and, although a steady volume input was associated with the saline source, this was small relative to the measured overturning circulation. The rate that salt entered the volume (kg s^{-1}) from the

saline source was balanced by a diffusive flux into the layer of fresh water at the upper surface and a small advective transport associated with the finite volume flux of the saline source. In the hypothetical non-diffusive limit no salt could exit the experiment for a time and the volume would initially evolve as a filling box (Baines & Turner, 1969) before completely filling with homogeneous saline water and eventually balancing the salt content through a very small volume flux of large density in the overflow (having salinity and volume flux equal to the saline source).

Profiles of salinity/density were measured through the depth of an experiment using a PME MicroScale Conductivity and Temperature probe (Model 125) mounted to a computer-controlled traversing mechanism. The conductivity probe was calibrated before each profile and a few small-volume water samples were withdrawn during each profile to enable the calibration accuracy to be monitored (see §5.1.2). The probe was immersed in the fresh water layer when not in use.

Vertical velocities in the sinking plume were measured using a *Signal Processing* 8 MHz Ultrasonic Doppler Velocimeter (UDV) transducer (Model DOP2000) directed upwards from underneath the left-hand end of the tank. The transducer could be translated horizontally along one quarter of the tank length, and was acoustically coupled with the acrylic base through a thin gel layer. Good-quality velocity data were obtained by pre-seeding the circulation in the tank with near neutrally-buoyant pliolite particles (70–125 μm diameter, which were pre-wetted in a salt solution of density ρ_s). The vertical component of velocity was obtained at 240 intervals along the vertical beam line to a distance of 180 mm above the base, at 0.5 s intervals.

Mechanical Stirring

Mechanical agitation was generated in the flow (in addition to the stirring associated with the convection) by a pair of coplanar horizontal rods that were oscillated and continuously traversed through the full depth of the tank. The rods were stainless steel, each 1.12 m in length and $l = 8$ mm in diameter, with their axes separated by two 32 mm struts and extending along the length of the tank. Small spacers ensured that each of the rods remained parallel to and equidistant from the closest respective sidewall. The rods were suspended by stainless steel wires (diameter 0.02 mm) from a frame above the tank. This frame was suspended by a nylon line that was repeatedly wound on and off a spool, which was mounted off-axis on a computer-controlled *Animatics* “SmartMotor”. This complicated arrangement was designed to drive the rods at a vertical velocity consisting of a constant mean component ($\bar{V}$, alternately upward or downward between reversal points at the top and bottom) and a relatively high frequency sinusoidal fluctuation (V'),

$$V(\omega, t) = \bar{V}(\omega) + V'(\omega, t) = \frac{3l\omega}{4\pi} + \frac{3l\omega}{2}\sin(\omega t), \quad (5.2)$$

where the velocity is expressed in terms of the diameter l of the stirring rods, and ω the angular velocity of the spool rotation (Fig. 5.2). The motions generated by the stirring rods have length scale l and, within each period of sinusoidal oscillation, the

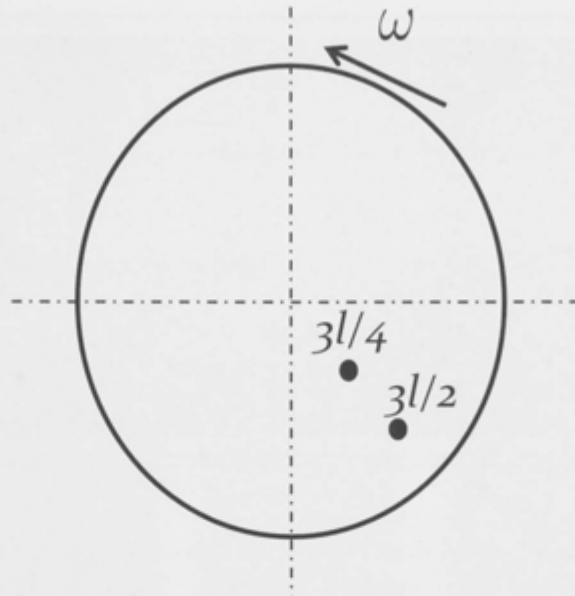


Figure 5.2: The design of the spool used for the experiments. The black dots represent horizontal pins, located at radius $3l/4$ and $3l/2$ from the rotational axis, upon which the nylon supporting line is wound. For a constant ω , the velocity of the stirring rods is that given by Equation (5.2).

rods are first displaced in the direction of $\bar{V}$ by $3l$ and then by $1.5l$ in the opposite direction. Advection of tracer in the rod wake over large vertical distances was thus prevented, and the consequent mixing can be represented as a Fickian diffusion process. The time taken for the stirring rods to pass through the depth ranged from 14 to 189 s (infinite for $\omega = 0$), with the associated periods of oscillation ranging from 0.48 to 6.71 s (also infinite for $\omega = 0$). Uniform stirring was imposed throughout the working volume and the time between stirring events was much less than the overturning timescale of the circulation ($\sim 2 \times 10^3$ s). Active stirring at any given location occurred for the fraction $3l/D \approx 0.045$ of the total time (an intermittency coincidentally similar to that in the ocean thermocline, Gregg, 1987).

Calibrating the Mixing Input

The overall effect of the rods was to introduce small-scale three-dimensional motions that stirred the density field, causing irreversible diapycnal mixing. The time-averaged rate of mixing was measured for each level of stirring in calibration experiments starting with a two-layer stratification (fresh water overlying saline water) and conducted in the same apparatus as the actual experiments (Fig. 5.3). A known quantity of passive dye tracer ($KMnO_4$) was added initially to the saline water and the subsequent evolution of the concentration field was measured optically. Light passing through the flow was attenuated by an amount dependent upon the local dye concentration, and a high resolution digital camera (Nikon D300 DSLR) was

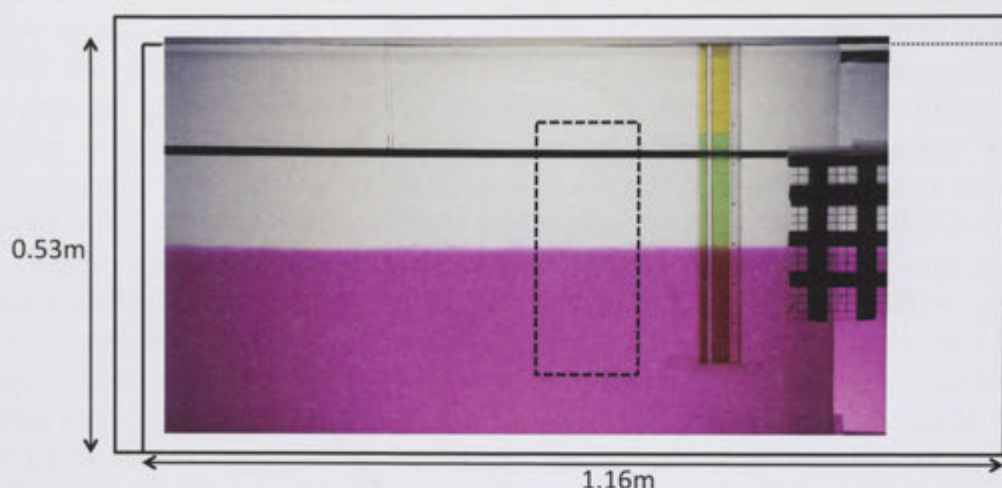


Figure 5.3: An example of the initial state of a calibration experiment. The approximate location in the experimental apparatus is indicated. The dashed rectangle outlines the region of interest, the evolution of which is shown in Figure 5.4.

used to measure the relative attenuation field at many times during each calibration experiment (which lasted of $O(100)$ traverses of the stirring rods).

A uniform background light field was achieved by using PALIGHT foamed PVC sheeting and a bank of fluorescent lights. The Nikon D300 DSLR digital camera was set to capture 8-bit RGB Tagged Image File Format (TIFF) images with the maximum possible resolution (4288×2848 pixels). The focus and white balance settings were manually preset to concentrate on a region of interest of height and length approximately 360×90 mm (2000×500 pixels), respectively, and centered at the initial density interface. For a tank full of un-dyed fresh water, an image of this region was found to be uniformly white to ± 1 bit in 256 over the three RGB channels; such an image served as a reference to which all other images were compared.

The initial density of the lower layer ranged between 1011 – 1028 kg m^{-3} in order for the calibration experiments to cover the expected range of bulk densities of the convection experiments. The concentration of the potassium permanganate dye in the lower layer was selected to avoid the regime of non-linear optical responses¹ for the addition of fresh water to the dyed solution. This concentration was 10 – 20 μg of KMnO_4 per litre of saline solution.

The tank was filled to a depth of 265 mm with a premixed solution of salt, KMnO_4 and water at room temperature. A polystyrene boat with a porous base (sponge and tissue paper) was floated atop this dyed layer. Fresh water at room temperature was carefully added to the tank through the porous base of the boat,

¹An optically saturated solution has no detectable change in light attenuation for the addition of fresh water, and a non-linear regime exists between this saturation concentration and that which has a linear response in light attenuation for the addition of fresh water.

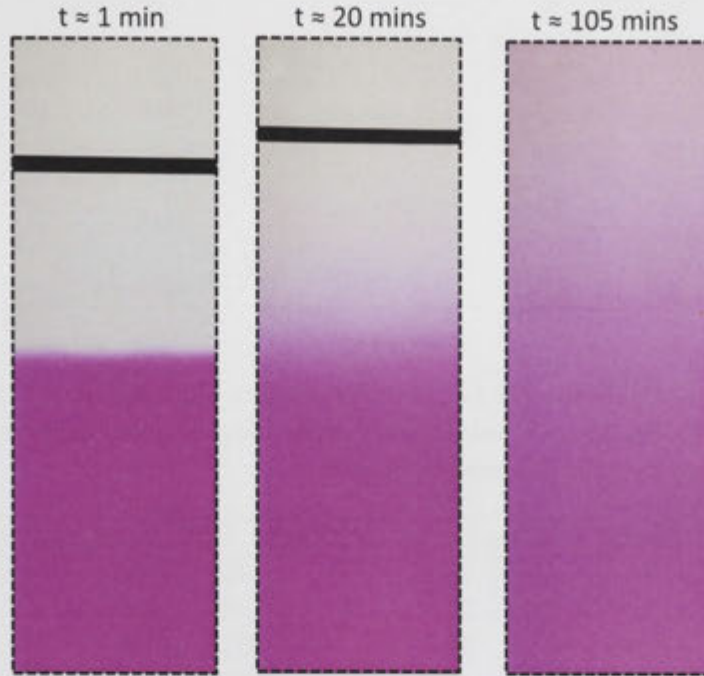


Figure 5.4: Progression of the region of interest for $\bar{V} = 23.1 \text{ mm s}^{-1}$. The shadow of the stirring rods is visible in the two early photos.

establishing a sharp interface between the two layers. The working volume of the tank was completely filled, the boat removed and the stirring rods activated at a predetermined $\bar{V}$, performing the identical stroke it would for the convection experiments. The camera captured images of the tank at regular time intervals for the duration of the calibration experiment. This procedure was repeated twice for each $\bar{V}$ setting, each time with a different density difference between the initial layers.

Ten different $\bar{V}$ settings were investigated from 0 to 40.5 mm s^{-1} (Table 5.1). The duration of the calibration experiments ranged between 10^3 and 10^5 s ; for large stirring rod velocities the tank became well-mixed relatively quickly, such that further evolution of the density field was no longer visually detectable. The time interval between photos varied for the different experiments between 90 and 1560 seconds depending on $\bar{V}$; each repeat experiment used a slightly different time interval to avoid aliasing with the position of the rods. 10–20 images of each experiment were analyzed using the Image Processing Toolbox of MATLAB.

The green channel of the images was found to be most sensitive to the presence of KMnO_4 dye. The green channel of the region of interest was loaded into MATLAB and the 500-pixel horizontal average taken to produce a single vertical profile of green channel intensity. A filter was applied to each image to remove the shadow cast by the stirring rods should they be in the region of interest when the image was taken (see Fig. 5.4); this filter linearly interpolated across either side of the rods. Each profile was normalized by that of the reference image and the minimum value

Table 5.1: List of Calibration Experiment parameters. Quantities are $\bar{V}$ the mean velocity of the stirring rods, $\Delta\rho_c$ the initial density difference, the photo interval and experiment duration times, and κ_E the calibrated diffusivity.

Exp. #	$\bar{V}$ (mm s ⁻¹)	$\Delta\rho_c$ (kg m ⁻³)	Photo, Exp. time (s)	κ_E (m ² s ⁻¹)
1	0	16.8	1200, 24000	2.89×10^{-9}
2	0	27.0	1560, 31200	2.44×10^{-9}
3	2.9	30.7	1250, 25000	1.22×10^{-8}
4	2.9	18.0	1500, 30000	6.93×10^{-9}
5	5.8	21.7	1000, 18000	4.91×10^{-8}
6	5.8	14.2	950, 19000	1.10×10^{-7}
7	8.7	19.5	1000, 18000	1.48×10^{-7}
8	8.7	27.4	970, 14550	2.34×10^{-7}
9	11.6	17.3	620, 12400	4.58×10^{-7}
10	11.6	29.1	640, 12800	4.94×10^{-7}
11	17.4	24.2	540, 10800	1.16×10^{-6}
12	17.4	15.7	560, 11200	1.26×10^{-6}
13	23.1	20.4	576, 10944	2.59×10^{-6}
14	23.1	28.6	550, 10450	1.45×10^{-6}
15	28.9	19.4	471, 6123	3.16×10^{-6}
16	28.9	28.5	393, 4716	3.53×10^{-6}
17	34.7	19.3	210, 3150	6.16×10^{-6}
18	34.7	24.9	250, 3000	5.02×10^{-6}
19	40.5	19.2	90, 1440	1.36×10^{-5}
20	40.5	27.8	100, 1200	1.91×10^{-5}

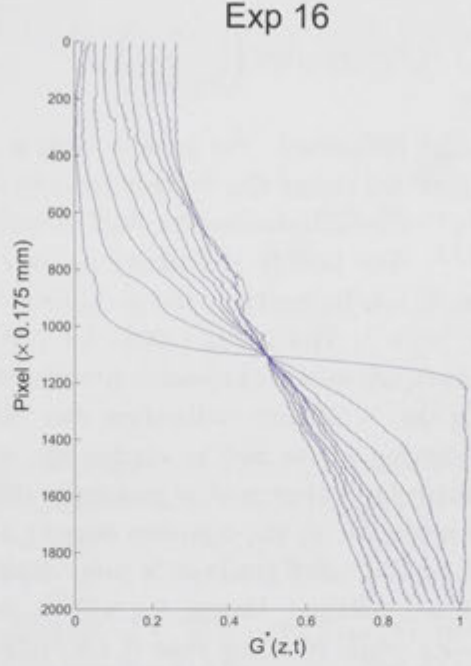


Figure 5.5: An example of the typical evolution of G^* for a calibration experiment. The initial profile is the widest, with the profiles narrowing in time. The evolution depicted here takes place over a period ~ 70 minutes, with each profile separated by ~ 7 minutes, and is in response to a $\bar{V} = 28.9 \text{ mm s}^{-1}$.

of the green channel for the initial image,

$$G^*(z, t) = \frac{G_B(z) - G(z, t)}{G_B(z) - G_{i, \min}}, \quad (5.3)$$

where $G^*(z, t)$ is the profile of normalized intensity at time t and depth z (relative to the initial interface position), $G_B(z)$ is the profile of intensity for the reference image, and $G_{i, \min}$ is the minimum intensity value of the green channel in the region of interest for the initial image. The camera automatically applied a filter to the image to smooth highlights prior to saving each TIFF file, resulting in an asymmetry of G^* about the interface such that $G^*(0, t) \neq 0.5$. Thus, a quadratic was fitted to G^* that removed the effect of this filter. The number of images analyzed for each experiment was limited to those which had a range of $G^* \geq 0.5$; this criteria excluded any images of a well-mixed tank. The progression of G^* through a typical experiment is shown in Figure 5.5.

Care was taken to ensure the initial stratification was as close as possible to a step function. However, practical limitations required that a virtual $t = 0$ be calculated for each experiment and the correction made to t for each image.

The profiles of normalized intensity $G^*(z, t)$ for each time t were then fitted to

the solution for Fickian diffusion:

$$G^*(z, t) = \operatorname{erf}\left(\frac{z}{2\sqrt{\kappa_E t}}\right), \quad (5.4)$$

where κ_E and is the (best-fit) diffusivity. The profiles were accurately described by the error function and Figure 5.6 shows the diffusivities obtained. Over the range $\kappa_E \approx O(10^{-9} - 10^{-5}) \text{ m}^2 \text{ s}^{-1}$ the diffusivities are well described by the empirical relationship $\kappa_E = 0.027 \bar{V}^{2.5}$. The best-fit diffusivity $\kappa_E = 2.67 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ measured for $\bar{V} = 0$ is a factor of two larger than the accepted coefficient of molecular diffusion ($\kappa_m = 1.39 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$, Washburn (2003) for NaCl in water at 20°C); the difference is likely a consequence of additional convective transport arising from imperfect insulation during the ~ 24 hour calibration run. Although the case with no additional mechanical stirring serves well to anchor the calibration results, the contribution to the total vertical salt transport of molecular diffusion down the mean gradient is expected to be negligible at the non-zero stirring rod speeds used. Thus the product of κ_E and the local density gradient is proportional to the density flux due to the externally imposed stirring. Hence, κ_E will be referred to here as the externally imposed diffusivity while recalling that it also reflects the component of transport due to molecular diffusion down the mean gradient and approaches κ_m when there is no mechanically-induced mixing. No physical argument is offered for the particular functional relationship between κ_E and $\bar{V}$ found here, which may be valid only for the present apparatus and stirring style. The motion of the stirring rods was particularly violent for $\bar{V} > 34.7 \text{ mm s}^{-1}$ and a good fit to (5.4) was not possible; we take this velocity as the upper limit for which the apparatus and method are useful.

5.1.2 Measurement Calibration

The PME MicroScale Conductivity and Temperature probes had a specific bridge circuit which output voltages logged by the National Instruments (NI) LabVIEW program. The temperature reading was calibrated by fitting the recorded voltages for known temperatures to the equation,

$$\ln(V(T) - V_o) = a + \frac{b}{T}, \quad (5.5)$$

where $V(T)$ is the output voltage for temperature T (in Kelvin), V_o is the reference voltage output by the bridge without the probe, and a and b are constant coefficients. A Hart Scientific[®] thermistor probe is used as a reference for T . The output voltages for 5 different temperatures spanning 10 to 30°C are used to calculate the calibration coefficients. The temperature calibration was routinely tested throughout the experimental period.

The conductivity was calibrated by fitting the output voltages measured for standard solutions of known conductivities to the equation,

$$V(C) - V_o = h * C, \quad (5.6)$$

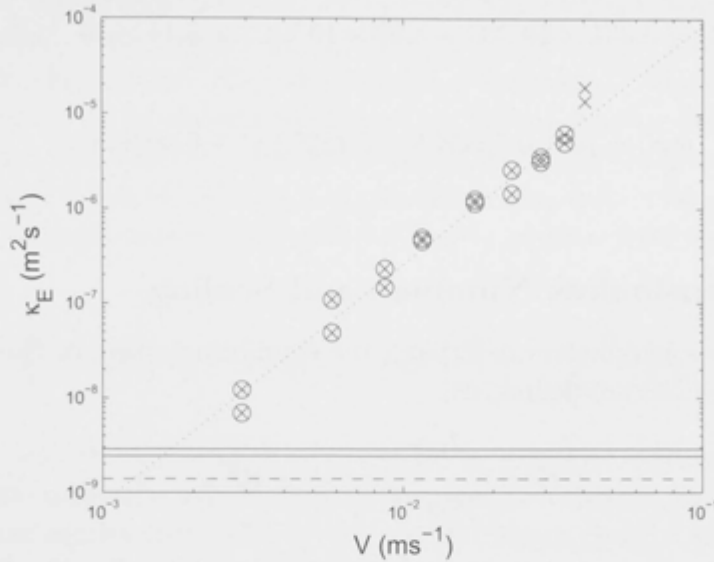


Figure 5.6: κ_E as a function of $\bar{V}$. The solid lines are the calibrated values of κ_E for $\bar{V} = 0$ (*i.e.* no rods), with the dashed line showing the accepted value for the molecular diffusion of salt in water ($\kappa_m = 1.39 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$, Washburn, 2003). The circles are the data used to develop the $\kappa_E = 0.027 \bar{V}^{2.5}$ relationship, shown by the dotted line. The largest value of $\bar{V}$ tested exceeded the practical limits of the apparatus.

where $V(C)$ is the output voltage for conductivity C (siemens per meter), V_o is the reference voltage output by the bridge without the probe, and h is a constant coefficient. The conductivities of the standard solutions were calculated from the salinities, which were in turn calculated from the standard solutions' densities. The densities of the standard solutions were measured by an Anton-Paar precision densimeter (Model DMA5000) to an accuracy of 5 parts in 10^7 . The salinities, S , in weight percent, of the standard solutions are given from their densities by Equation 2.3(d) of Ruddick & Shirtcliffe (1979),

$$S(\rho) = 1.41397(\rho - \rho_0) - 0.42868(\rho - \rho_0)^2 - 0.5536(\rho - \rho_0)^3, \quad (5.7)$$

where the reference density is $\rho_0 = 0.99823 \text{ g cm}^{-3}$ (the equations of Ruddick & Shirtcliffe, 1979, are formulated in cgs units). The salinities of the standards are then used to calculate the conductivities by Equation 2.6(a) of Ruddick & Shirtcliffe (1979),

$$C(S) = 1581 S - 3776 S^2 + 5617 S^3. \quad (5.8)$$

Three standard solutions are used to calibrate the probe immediately following each profile.

The profiles of conductivity are converted to salinity and then density through

Equations 2.6(b) and 2.3(c) of Ruddick & Shirtcliffe (1979), namely,

$$S(C) = 6.331 \times 10^{-4} C + 8.92 \times 10^{-7} C^2 + 3.11 \times 10^{-9} C^3, \quad (5.9)$$

and

$$\rho(S) = \rho_0 + 0.70864 S + 0.12252 S^2 + 0.3649 S^3, \quad (5.10)$$

respectively.

5.1.3 Dimensionless Numbers and Scaling

The dimensionless parameters describing the equilibrium state of the overturning circulation (2.2)–(2.4) are defined as,

$$Ra_B = \frac{B L^3}{\nu_m \kappa_m^2} = 5.7 \times 10^{18}, \quad (5.11)$$

$$Pr = \frac{\nu_m}{\kappa_m} = 671, \quad (5.12)$$

$$A = \frac{D}{L} = 0.457. \quad (5.13)$$

The flux Rayleigh number divided by the appropriate Nusselt number ($Nu = B/\kappa_m \Delta b$) gives the usual Rayleigh number ($Ra_{\Delta b} = \Delta b L^3/\nu_m \kappa_m$), where Δb is the reduced gravity based on the density contrast between sources that are separated horizontally by a length scale of order L . It is important to note that these governing parameters are defined in terms of the molecular diffusivities κ_m and ν_m , fluid properties that are known *a priori*. However, when additional stirring is included in the problem, two further governing parameters are required to describe the effective (turbulent) diffusivity and viscosity ratios,

$$K = \frac{\kappa_*}{\kappa_m}, N = \frac{\nu_*}{\nu_m}, \quad (5.14)$$

respectively.

A boundary layer scaling analysis for the two-dimensional flow in a basin of rectangular surface area (Rossby, 1965; Mullarney *et al.*, 2004) gives the boundary layer thickness, δ , normalized by L ,

$$\frac{\delta}{L} \sim Ra_B^{-1/6} \left[\left(\frac{B L^3}{\nu_m \kappa_m^2} \right)^{-1/6} \right]. \quad (5.15)$$

This predicts that for fixed buoyancy flux and basin length the boundary layer thickness depends on viscosity and diffusivity as

$$\delta \sim \nu_m^{1/6} \kappa_m^{1/3}. \quad (5.16)$$

In a very different approach, Hughes *et al.* (2007) obtained a closed solution for flow at large Rayleigh numbers by coupling the equations describing a turbulent plume

to those for the transport of density by advection and diffusion in the interior of the basin. In this inviscid solution the Prandtl and Rayleigh numbers only appear as their product, removing all dependence on the viscosity. Furthermore, it was assumed that the relevant diffusivity in such a formulation would be the total effective diffusivity κ_* for density resulting from both molecular diffusion (down the mean gradient) and that promoted by turbulent stirring of the interior. For the case of a rectangular basin (which has a plume descending from a two-dimensional line source with an entrainment coefficient of 0.1) this solution gives the boundary layer thickness as

$$\frac{\delta}{L} = 3.95 Pr_*^{-1/6} Ra_{B*}^{-1/6} \left[= 3.95 \left(\frac{\nu_*}{\kappa_*} \right)^{-1/6} \left(\frac{B L^3}{\nu_* \kappa_*^2} \right)^{-1/6} \right], \quad (5.17)$$

where Ra_{B*} and Pr_* are defined in terms of the turbulent diffusivity and viscosity in place of the molecular values, and the constant is evaluated for a boundary layer thickness that incorporates 95% of the top-to-bottom density difference. In this case, again holding the buoyancy flux fixed,

$$\delta \sim \kappa_*^{1/2}. \quad (5.18)$$

The total diffusivity (and viscosity) would reduce to the molecular value at sufficiently small Rayleigh numbers and no externally imposed mechanical mixing. The results (5.16) and (5.18) are then reconciled for a constant (turbulent) Prandtl number, such that $\nu_* \sim \kappa_*$, in which case (5.16) reduces to (5.18). Hughes *et al.* (2007) argued that viscosity is ineffective on the large length scales of the interior and hypothesized that at high-Rayleigh-number the flow in the boundary layer, even if it is viscous, is set by the interior flow. (We note that when holding Δb fixed in both the Rossby scaling and the inviscid model the boundary layer scales as $\delta \sim \kappa_*^{2/5}$.)

The model solution of Hughes *et al.* (2007) also gives the top-to-bottom density difference, $\Delta\rho_V$, and streamfunction maximum, Ψ_{max} . These predictions scale with the diffusivity (holding B fixed) as

$$\Delta\rho_V \sim \kappa_*^{-1/2} \quad \text{and} \quad \Psi_{max} \sim \kappa_*^{1/4}, \quad (5.19)$$

respectively. This solution was tested using experiments and numerical solutions of thermal convection in which the heat input was varied (and assuming $\kappa_* = \kappa_m$), with good agreement found for the Nusselt number – Rayleigh number relation and boundary layer thickness (5.17). However, the dependence on diffusivity has not been tested explicitly.

In the experiments reported here the aim is to investigate the dependence of the rate of overturning for a range of known vertical diffusivities. While this approach does not allow examination of the energetics of convective overturning with added mechanical mixing, it does provide a clear picture of the dynamics of the flow. An additional aim is to remove the constraint imposed on the density field by the sink or spillway in previous work, leaving the densities free to adjust to the surface boundary conditions and flow.

5.1.4 Procedure

The working volume and reservoirs were filled with fresh water. Approximately 12.5 kg of salt were added to the saline reservoir, which was then allowed to thermally equilibrate to the laboratory (~ 1 day). The fresh source, stirring rods and saline source were then switched on, and the experiment left to come to equilibrium: a state in which there was zero net buoyancy/salt flux into the experiment. The time taken to reach equilibrium was largely determined by the ‘filling timescale’ (*i.e.* the time required for the saline source to replace the tank volume $\sim 2 \times 10^4$ s); each experiment was run for 5–6 filling timescales (25–30 hours) before detailed measurements were carried out. Monitoring of the overflow salinity was used to further verify equilibration of the experiment.

Conservation of volume and salt requires the overflow to have the volume transport,

$$Q_o = Q_s + Q_f, \quad (5.20)$$

and hence the equilibrium salinity,

$$S_o^* = \frac{Q_s S_s + Q_f S_f}{Q_o}. \quad (5.21)$$

The overflow salinity was observed to asymptotically approach the value given by (5.21) on the filling timescale.

It is worth noting that the overflow in these experiments does not have the effect of the sink or spillway used in previous studies (Pierce & Rhines, 1996; Whitehead & Wang, 2008), where water was withdrawn directly from the working volume, therefore participating in the circulation. Here the overflowing layer accommodates the salt transported out of the working volume by vertical diffusion and the large throughput of fresh water that keeps the overflow layer constantly flushed.

After the equilibrium state was reached a profile of conductivity and temperature was measured through the depth of the tank 0.17 m from the right-hand endwall, which was chosen as a location representative of the interior. The conductivity-temperature probe was traversed with a velocity and sampling rate that gave a vertical resolution of 0.625 mm. Immediately following each profile three standard solutions were used to check the conductivity probe calibration. In addition, 3 ml water samples were taken at depths of 0, 20 and 200 mm below the fresh–saline water interface and in the same location as the profile. The densities of the standards, samples, sources and outflow were measured to an accuracy of 5 parts in 10^7 in an Anton-Paar precision densimeter (Model DMA5000) and used in conjunction with the conductivity data (Ruddick & Shirtcliffe, 1979) to produce each vertical profile of density. The temperature was measured to vary less than 1°C through the depth (and between all experiments, Fig. 5.7). In order to check the stability of the equilibrium state, the profiling and sampling procedure was carried out twice for each experiment, separated by at least one filling timescale (~ 6 hours).

Between conductivity profiles the saline water source was seeded with pliolite particles (70–125 μm diameter) for UDV measurements. Profiles of vertical velocity were constructed above seven locations 20 mm apart, between 12 mm and 152 mm

Table 5.2: Table of experiments; (a,b) denote the two profiles performed for each experiment. Quantities are $\bar{V}$ the mean velocity of the stirring rods, κ_E the diffusivity measured in the calibration runs, $\Delta\rho_H$ the density difference between the source waters, $\Delta\rho_n$ the normalized ‘top-to-bottom’ density difference, δ the boundary layer thickness, and Ψ_{max} the maximum streamfunction defining the total overturning transport.

Exp.#	$\bar{V}$ (mm s ⁻¹)	κ_E (m ² s ⁻¹)	$\Delta\rho_H$ (kg m ⁻³)	$\Delta\rho_n$	δ (mm)	Ψ_{max} (mm ² s ⁻¹)
1(a)	0	2.67×10^{-9}	24.31	0.99	13.3	216
1(b)	0	2.67×10^{-9}	24.31	0.99	11.7	216
2(a)	2.9	9.57×10^{-9}	23.53	0.99	12.3	247
2(b)	2.9	9.57×10^{-9}	23.53	0.99	10.1	247
3(a)	5.8	7.95×10^{-8}	22.97	0.98	12.1	238
3(b)	5.8	7.95×10^{-8}	22.97	0.98	12.3	238
4(a)	8.7	1.91×10^{-7}	23.90	0.92	15.1	264
4(b)	8.7	1.91×10^{-7}	23.90	0.92	17.0	264
5(a)	11.6	4.76×10^{-7}	23.79	0.86	31.2	318
5(b)	11.6	4.76×10^{-7}	23.79	0.86	46.3	318
6(a)	17.4	1.21×10^{-6}	23.98	0.68	39.6	410
6(b)	17.4	1.21×10^{-6}	23.98	0.69	44.0	410
7(a)	23.1	2.02×10^{-6}	24.11	0.55	50.7	476
7(b)	23.1	2.02×10^{-6}	24.11	0.54	62.3	476
8(a)	28.9	3.35×10^{-6}	24.18	0.42	70.0	538
8(b)	28.9	3.35×10^{-6}	24.18	0.43	98.7	538
9(a)	34.7	5.59×10^{-6}	23.31	0.37	109.4	648
9(b)	34.7	5.59×10^{-6}	23.31	0.35	106.6	648
10	0	2.67×10^{-9}	25.18	1	–	35
11	17.4	1.21×10^{-6}	25.52	1	–	35

from the left-hand end of the working volume. At each location more than 500 individual profiles were recorded over a duration of approximately four minutes. Both spikes and null readings were first removed from these profiles, which were then averaged to give a single profile for each location that was relatively insensitive to perturbations introduced by passage of the stirring rods. The mean vertical velocity was integrated horizontally, from the endwall to the position where the vertical velocity changed sign (thus defining the outer edge of the plume), giving the streamfunction in the plume as a function of position. The maximum value of the streamfunction, Ψ_{max} (mm² s⁻¹), occurred near the top of the plume outflow along the base of the tank, and this is taken as the maximum streamfunction value for the circulation.

Eleven experiments were conducted (Tab. 5.2); in nine of these only the mechanical stirring was varied. In two further runs the turbulent plume was eliminated. The results are discussed below in terms of the vertical density structure, boundary layer thickness, top-to-bottom density difference and maximum streamfunction.

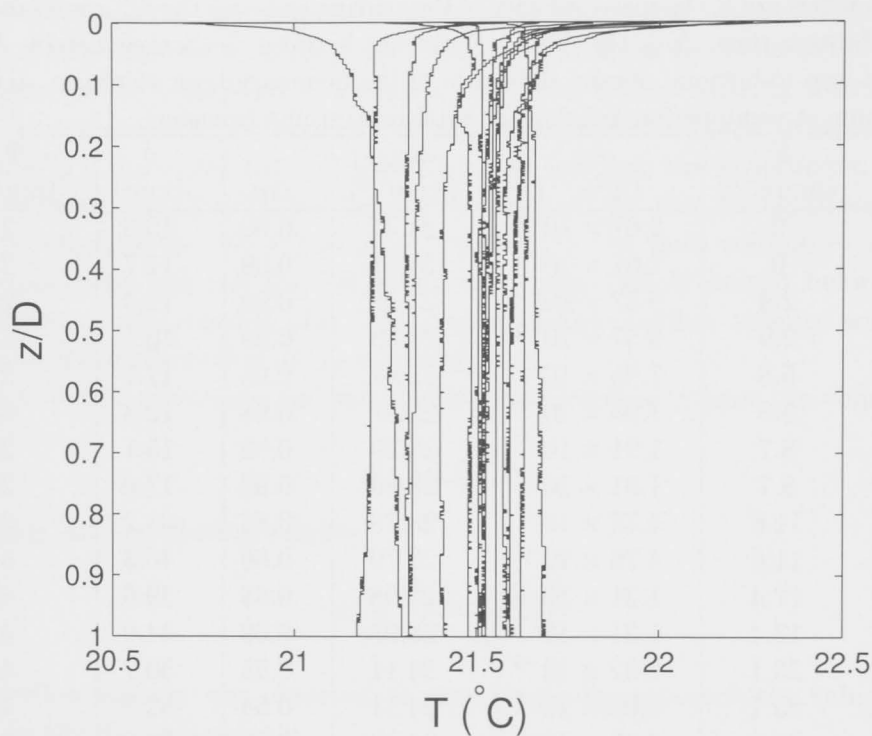


Figure 5.7: Vertical profiles of temperature for all of the experiments, showing the variation over each and every experiment to be less than 1°C .

5.2 Results

5.2.1 Qualitative Description

The mechanical stirring in these experiments did not qualitatively alter the large-scale circulation observed previously in horizontal convection experiments (e.g., Rossby, 1965; Mullarney *et al.*, 2004, Fig. 5.8). The intermittent passage of the stirring rods excited local three-dimensional turbulence that quickly decayed. In the equilibrated convective overturning circulation, the saline source sustained a turbulent line plume that descended against the left-hand endwall (Fig. 5.8a). The horizontal entrainment velocity into the plume (as indicated by the distortion of initially vertical dye lines generated by dropping $KMnO_4$ crystals through the flow) varied with depth and was largest in the upper one-fifth of the depth. The plume penetrated to the tank bottom only intermittently, as a consequence of large fluctuations induced by the stirring rods throughout the tank. An outflow from the plume was established along the base of the tank (Fig. 5.8b), with a depth that was approximately one-third that of the working volume. The mean shear in the outflow always supported three-dimensional eddies and mixing, but this was less vigorous than in the plume. Water in the outflow tended to continually displace

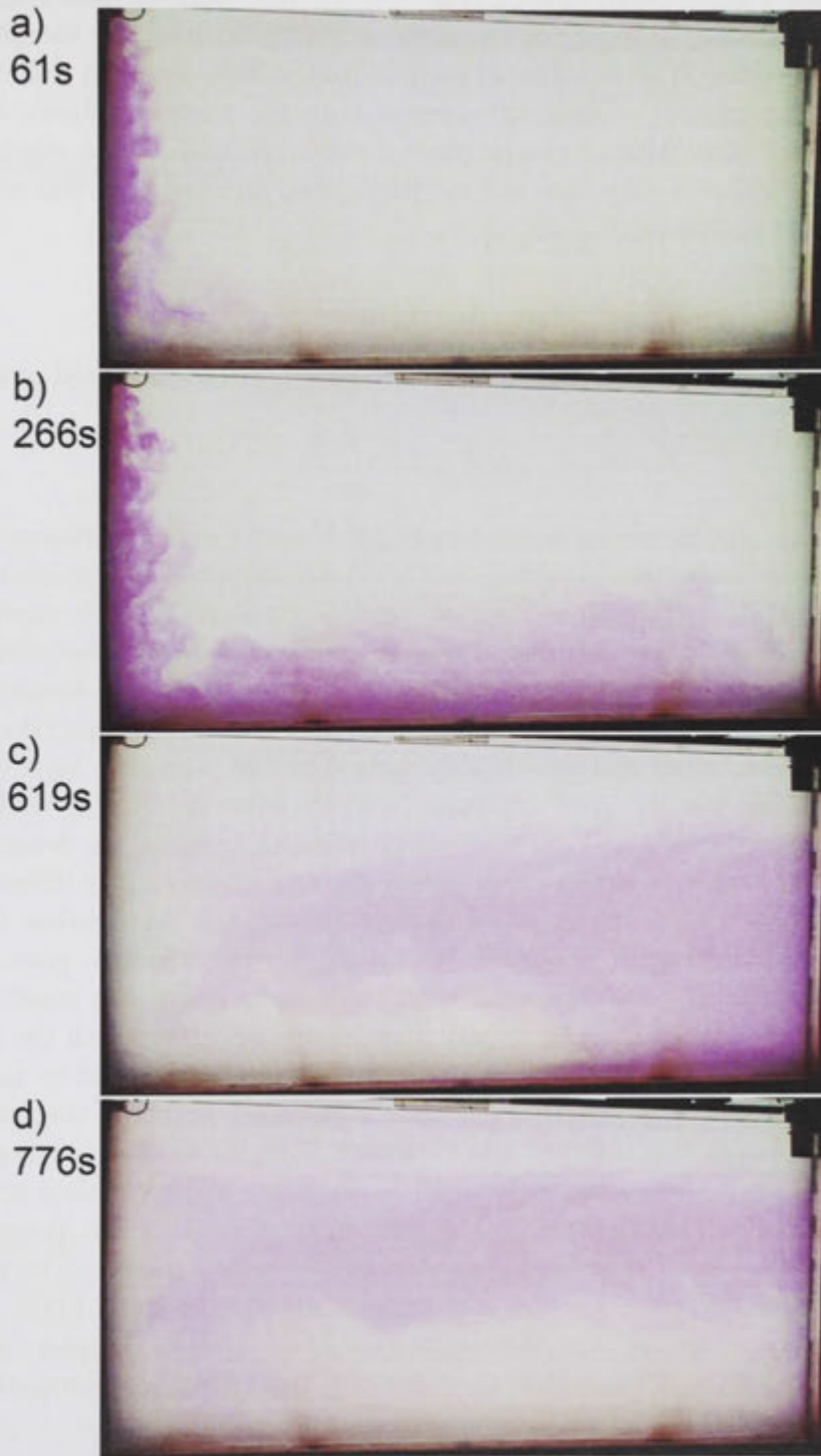


Figure 5.8: A sequence of photos of the equilibrated circulation of Experiment #1. The time since the initial introduction of the dye is shown; the dye was removed from the saline source at $t = 459$ s. Each photo indicates the relative strength of the flow in different parts of the circulation: (a) the turbulent plume descending the left endwall; (b) the outflow from the plume along the base of the tank; (c) and (d) a relatively dispersed cloud of dye indicates the broad upwelling over time of water in the circulation interior.

the overlying water column upward (Fig. 5.8c,d). However, the upwelling decreased towards the surface, as expected for substantial entrainment into the plume at intermediate depths. The overflow of predominantly fresh water in this experiment resulted in a negligible transfer of momentum to the working volume; other than the temporary disruption of the interface with the passage of the stirring rods no sustained instabilities were seen and the Richardson number there was estimated to be larger than $O(10)$ in all cases.

5.2.2 Quantitative Measurements

The measured profiles of density shown in Figure 5.9a are presented in normalized form,

$$\rho_n(z) = \frac{\rho(z) - \rho_f}{\Delta\rho}, \quad (5.22)$$

where $\Delta\rho = \rho_s - \rho_f$ is the maximum density difference for each experiment (measured between the source water reservoirs), and $\rho(z)$ is the density calculated from the conductivity measured at depth z . The two profiles measured for each experiment are almost coincident (hence only one of these is plotted in Figure 5.9a), verifying that the circulation had reached a well-defined equilibrium state. The density structure is similar to that measured in thermally-forced horizontal convection (Rossby, 1965; Mullarney *et al.*, 2004) and reveals a strongly stratified boundary layer overlying a weakly stratified interior. As $\bar{V}$ increases (with the other parameters held fixed) the boundary layer thickens, the water at any given depth freshens, the density gradient is everywhere smaller and there is a smaller top-to-bottom density difference.

The boundary layer thickness δ is defined here as the depth below the surface that accommodates 92.5% of the top-to-bottom density difference; given the shape of the density profiles, this threshold was chosen to be as large as possible without δ being unduly sensitive to the density fluctuations associated with the turbulence (some of which are just visible in Figure 5.9a). Thus δ , normalized by the depth of the working volume D , is plotted in Figure 5.9b as a function of the ratio κ_E/κ_m . The data indicates two regimes: the boundary layer thickness δ is independent of the mixing rate κ_E for $\kappa_E/\kappa_m < 10^2$, and for $\kappa_E/\kappa_m > 10^2$ the scaling predicted by Hughes *et al.* (2007) is recovered. The constant predicted by this theory is about a factor of two greater than that suggested by the measurements. The most likely reasons for this difference are that a marginally greater threshold (95%, which was impractical to use for the experiments) was used to define the boundary layer in the theory and that the saline source does not match an idealized plume close to the upper surface.

Measurements of the top-to-bottom density difference

$$\Delta\rho_V = \rho(D) - \rho_f, \quad (5.23)$$

are presented in normalized form in Fig. 5.9c as an adjusted salinity difference,

$$\beta\Delta S_n = \frac{\Delta\rho_V}{\bar{\rho}_f} \times \frac{\bar{\Delta\rho}}{\Delta\rho}, \quad (5.24)$$

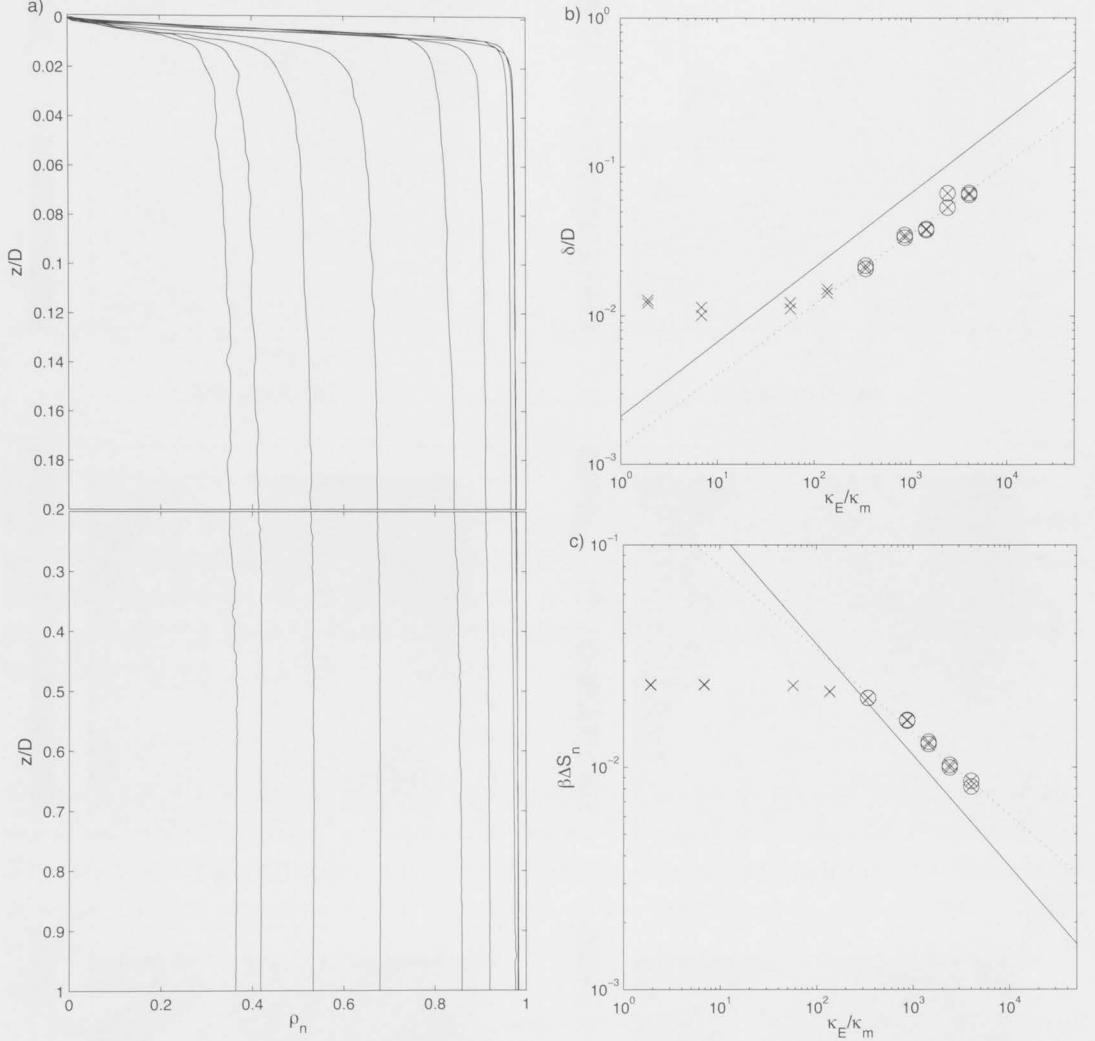


Figure 5.9: (a) Vertical profiles of normalized density for experiments #1–9; two (virtually coincident) profiles were recorded for each experiment – only one is shown. The right-most profile is for $\bar{V} = 0$, with increasing $\bar{V}$ trending to the left. Panels (b) and (c) show the normalized boundary layer thickness and normalized top-to-bottom salinity difference, respectively, as functions of the externally-imposed diffusivity κ_E , normalized by κ_m . The dotted lines depict the least-squares fit of the (circled) data in the ‘strong mixing’ regime: $\delta/D = 1.3 \times 10^{-3} (\kappa_E/\kappa_m)^{0.475}$ and $\beta\Delta S_n = 0.19 (\kappa_E/\kappa_m)^{-0.373}$, respectively. The solid lines are those predicted by the theory of Hughes *et al.* (2007): $\delta/D = 2.1 \times 10^{-3} (\kappa_E/\kappa_m)^{0.5}$ and $\beta\Delta S_n = 0.36 (\kappa_E/\kappa_m)^{-0.5}$, respectively.

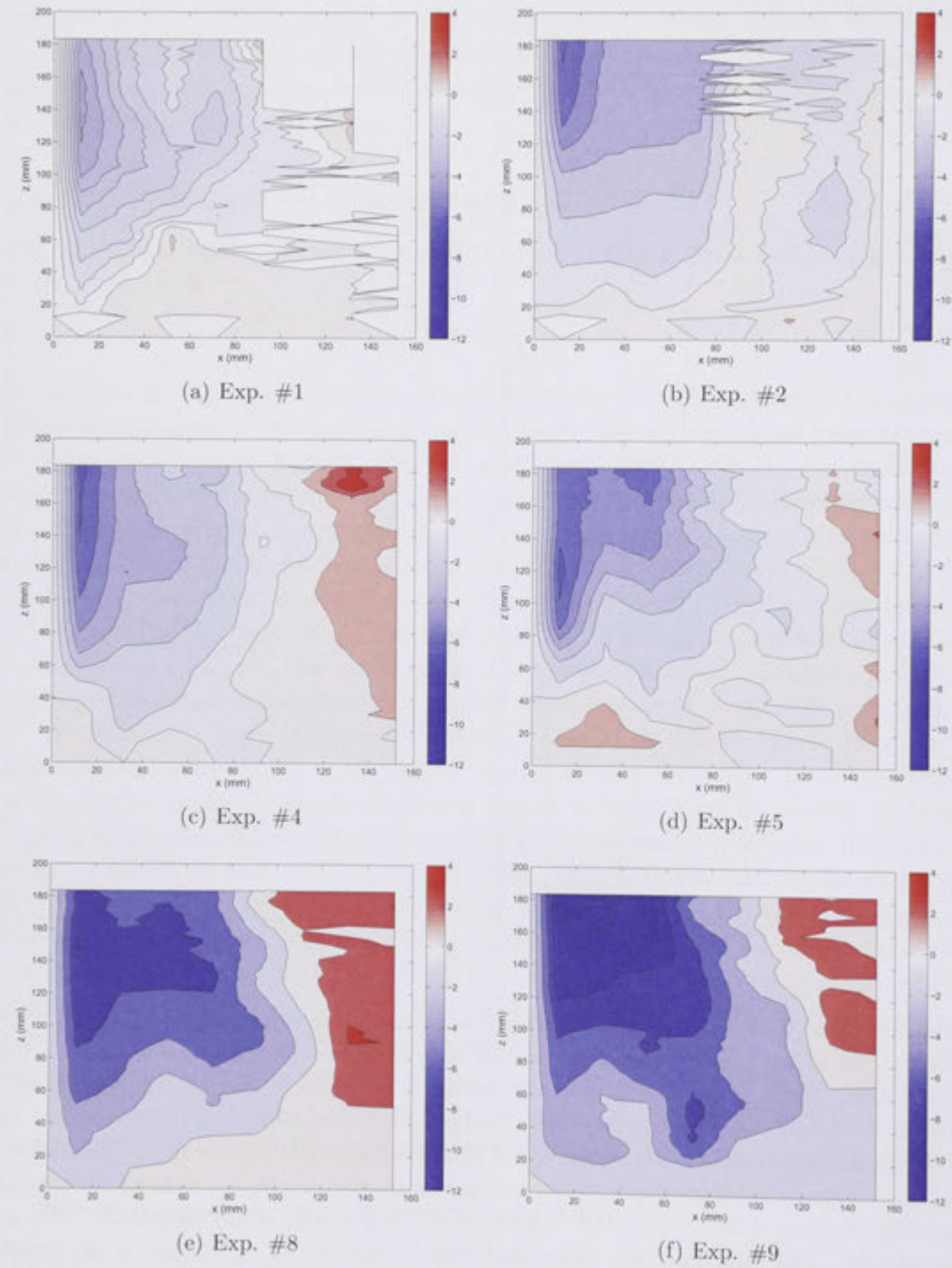


Figure 5.10: Contour plots of the UDV measurements of vertical velocity near the base of the plume.

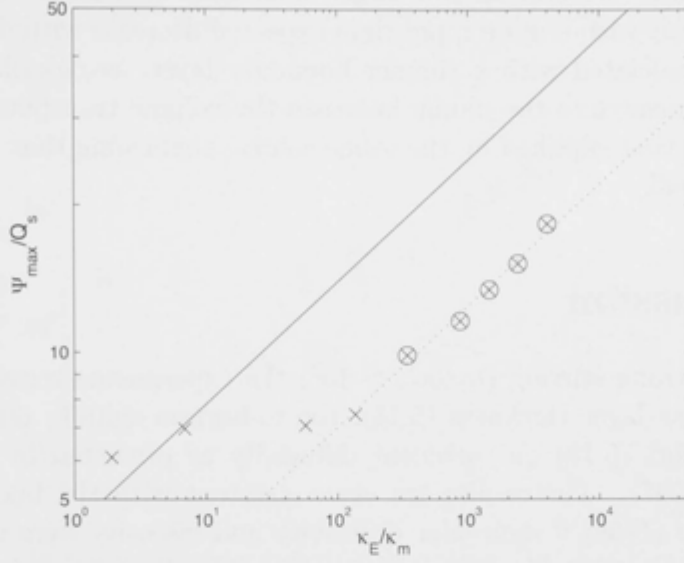


Figure 5.11: The maximum volume transport in the plume, normalized by the volume transport of the saline source, as a function of κ_E/κ_m . The dotted line indicates the least-squares fit to the (circled) data in the ‘strong mixing’ regime: $\Psi_{max}/Q_s = 2.2 (\kappa_E/\kappa_m)^{0.25}$. The solid line is that predicted by the theory of Hughes *et al.* (2007): $\Psi_{max}/Q_s = 4.4 (\kappa_E/\kappa_m)^{0.25}$.

where $\rho(D)$ is the density measured at the base and an overbar denotes a reference value calculated here as a mean over the set of experiments under consideration. The adjustment factor $\overline{\Delta\rho}/\Delta\rho$ therefore allows experiments with minor differences in source water densities to be compared. Two regimes are again evident: $\beta\Delta S_n$ is independent of κ_E for $\kappa_E/\kappa_m < 10^2$ and, for $\kappa_E/\kappa_m > 10^2$, decreases with κ_E in agreement with the theory of Hughes *et al.* (2007).

Note that $\rho_n(D) \rightarrow 1$ and $\beta\Delta S_n$ approaches $\overline{\Delta\rho}/\overline{\rho_f}$ when the mixing is weak (Fig. 5.9a,c). This occurs primarily because the boundary layer becomes thinner as κ_E decreases, reducing the amount of fresh water entrained into the plume and thus carried to depth. The diffusive transport of fresh water to depths below the boundary layer is negligible. Furthermore, the theoretical model of Hughes *et al.* (2007) was formulated for ideal point and line plume sources that have finite buoyancy flux but infinite density. The theory does not describe the weak mixing regime observed in experiments because the range of densities in the flow (excluding the plume) approaches that imposed by the sources.

Contours of the vertical velocities measured by the UDV are shown in Figure 5.10; these are used to calculate the maximum volume transport (Ψ_{max}) in the overturning circulation. Ψ_{max} is shown in Fig. 5.11, normalized by the volume flux of the saline source. The two regimes are apparent: volume transport was independent of κ_E for $\kappa_E/\kappa_m < 10^2$ (*i.e.* weak mixing), and for $\kappa_E/\kappa_m > 10^2$ the transport increases by a factor of approximately three for a 30-fold increase in diffusivity. The latter

sensitivity accords with the scaling of Hughes *et al.* (2007), however, the predicted value is consistently a factor of two larger than that measured. This difference would be consistent with a more rapid than expected decrease with depth of plume buoyancy flux associated with a thinner boundary layer, as described above. In all cases, entrainment into the plume increases the volume transport by at least a factor of six over that supplied by the saline source, confirming that this input can be considered small.

5.3 Discussion

For sufficiently strong stirring ($\kappa_E/\kappa_m > 10^2$) the experiments recover the dependence of boundary layer thickness (5.18), top-to-bottom salinity difference (5.19) and total transport (5.19) on turbulent diffusivity as predicted by the model of Hughes *et al.* (2007). The results are also consistent with the buoyancy-viscous scaling of Rossby (1965) if molecular diffusivity and viscosity were to be replaced in that scaling by turbulent values and the turbulent Prandtl (Schmidt) number assumed fixed.

For experiments with weak stirring ($\kappa_E/\kappa_m < 10^2$) the flow and density structure are independent of the externally imposed mixing rate. A plausible reason for this is an apparent background level of mixing that is present even when no mechanical energy is added to the system (aside from available potential energy generated by the imposed surface buoyancy fluxes).

In order to test this hypothesis we first examine more thoroughly the diffusive transport of salt. The transport per unit width of salt, M_s (in $\text{kg s}^{-1} \text{m}^{-1}$), into the working volume is an externally imposed quantity and can be calculated from the applied volume flux and salinity difference between the sources, $M_s = \rho_s(S_s - S_f)Q_s$. In an equilibrium state M_s must be exactly balanced by a transport out of the working volume. Neglecting the advective transport of salt out of the working volume associated with the small volume throughput from the saline source, the salt balance is accommodated by diffusion across the upper fresh water interface. Hence $\beta M_s = \kappa_\delta L \Delta \rho_\delta / \delta$, where $\Delta \rho_\delta = \rho(\delta) - \rho_f$ is the density difference across the boundary layer and κ_δ is the relevant ('total') diffusivity in the boundary layer. Thus the 'total diffusivity', obtained independently of the calibration for the external mixing, is given by

$$\kappa_\delta = \frac{\beta \rho_s (S_s - S_f) Q_s \delta}{\Delta \rho_\delta L}. \quad (5.25)$$

The total diffusivity κ_δ is plotted as a function of the external mixing κ_E (given by the calibration runs) in Figure 5.12, where both quantities are normalized by κ_m . There is close agreement between κ_E and κ_δ for $\kappa_E/\kappa_m > 10^2$, conditions under which the stirring rods provide the dominant contribution to the total diffusivity. For $\kappa_E/\kappa_m < 10^2$ the total diffusivity is independent of the externally-induced mixing rate, with $\kappa_\delta/\kappa_m \approx 2 \times 10^2$ – this value will be referred to as the 'apparent background mixing rate'. In particular, $\kappa_\delta/\kappa_m \approx 2 \times 10^2$ for the experiment with no externally induced mixing, which implies that the salt transport through the boundary layer is

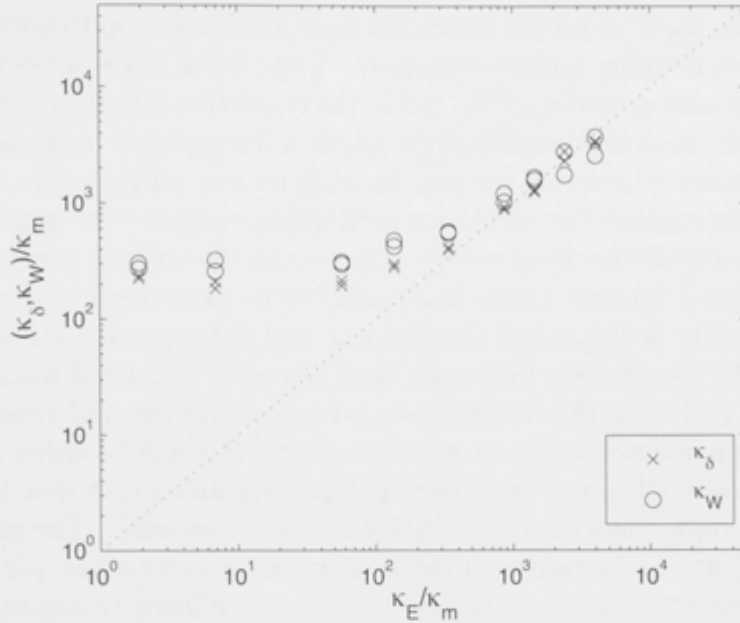


Figure 5.12: The parameterized diffusivities, κ_δ and κ_W , plotted as a function of the corresponding κ_E , all normalized by κ_m . The dotted line has unity slope and passes through the origin, indicating where the external stirring sets the total diffusivity in the experiment ($\kappa_E/\kappa_m > 10^2$). For small levels of external stirring ($\kappa_E/\kappa_m < 10^2$) the parameterized total diffusivities are much larger than κ_E .

always (for the Rayleigh and Prandtl numbers and aspect ratio used here) dominated by mechanisms other than molecular diffusion down the mean gradient.

Another independent method to evaluate the total vertical diffusivity in the experiment is that of Munk (1966). A homogeneous advection-diffusion balance is assumed of the form

$$w \frac{\partial \rho_n(z)}{\partial z} = \kappa_W \frac{\partial^2 \rho_n(z)}{\partial z^2}, \quad (5.26)$$

where κ_W is a (uniform) diffusivity and w is a uniform upwelling velocity. The ratio κ_W/w is first found by fitting an exponential function to the vertical profile of (normalized) density, and substituting into (5.26). The fit yielded an excellent approximation to the measured profile in all cases and was insensitive to the domain size (which was approximately one third of the working volume depth, below the threshold in the surface boundary layer corresponding to 66% of the top-to-bottom density difference). This ratio is then multiplied by w (calculated as the area-averaged upwelling required to balance the measured volume transport in the plume) to give the diffusivity κ_W . This diffusivity is plotted alongside κ_δ as a function of κ_E in Figure 5.12 and also exhibits behavior spanning two regimes. The fact that both the boundary layer transport parameterization (κ_δ) and the interior parameterization of the Munk form (κ_W) return approximately the same total

diffusivity lends confidence to the value as well as to the nature of this quantity.

The level of externally-induced mixing at which the regime transition occurs in Figure 5.12 is the same as for the boundary layer thickness, top-to-bottom density difference and overturning volume transport. Thus, when these quantities are re-plotted as functions of κ_δ and κ_W (Fig. 5.13), the respective data largely collapse to a single slope. This raises the question of the origin of the apparent background mixing rate. In the absence of external mixing, instabilities and stirring were observed to be widespread throughout the circulation with the exception of the upper boundary layer. Hence the circulation itself may be able to generate at least some of the total irreversible mixing. Another mechanism available to cause vertical down-gradient transport of density is horizontal entrainment and subsequent vertical advection in the plume. In the process, buoyancy (and tracer) is extracted from the upper boundary layer and from the interior over a range of depths and re-injected into the interior by the plume outflow at a greater depth; this will be either at the level of neutral buoyancy (thus not changing the interior buoyancy at that level), or at the base of the tank (thus displacing ambient water upwards). The effect of this advective transport is a freshening of the ambient water everywhere, particularly at large depths. Hence the parameterizations for the total diffusive transport, embodied in κ_δ and κ_W (Fig. 5.12), include contributions to the vertical transport from molecular diffusion down both the mean and stirring-enhanced gradients, and from the advective circulation (the latter being an apparent mixing, *i.e.* not irreversible).

The theoretical model of Hughes *et al.* (2007) was formulated in terms of a total diffusivity κ_* for irreversible vertical mixing in the interior, which cannot be determined in these experiments. This total diffusivity does not include advective transport associated with the plume because it is explicitly allowed for in the model. Thus κ_* will be smaller than the parameterized total diffusivities, κ_δ and κ_W , and larger than the diffusivity characterizing externally-induced mixing κ_E . In the strong mixing regime κ_* is expected to be comparable with κ_δ , κ_W and κ_E . The comparison of experimental results with the theoretical predictions in Figs. 5.9b, 5.9c, 5.11 and 5.13 can be viewed consistently in this light. The data and the predictions should only be expected to approach the same scaling where the mixing is strong in these plots (*i.e.* for $\kappa_E/\kappa_m > 10^2$) and $\kappa_* \sim \kappa_E \sim \kappa_{\delta,W}$, as is indeed the case. Although κ_E was varied over four orders of magnitude in these experiments, the corresponding variation in $\kappa_{\delta,W}$ was less than two orders of magnitude; κ_* therefore varied by a factor intermediate to that of $\kappa_{\delta,W}$ and κ_E . Thus, over the range of experimental conditions examined, we expect the theoretical scaling based on $\kappa_{\delta,W}$ (because κ_* is unknown) to yield a weaker than observed increase in boundary layer thickness and maximum streamfunction, and a stronger than observed decrease in top-to-bottom density difference, as is the case in Fig. 5.13.

In order to further emphasize the plume's role in vertical transport in the convective circulation along with the concept of an apparent background mixing rate, two auxiliary experiments are performed in which the saline source was simply relocated to the bottom left-hand corner of the tank. The source density and volume flux was the same as in the previous experiments, but the turbulent plume was eliminated from the circulation. This configuration removed all convection and any small-scale

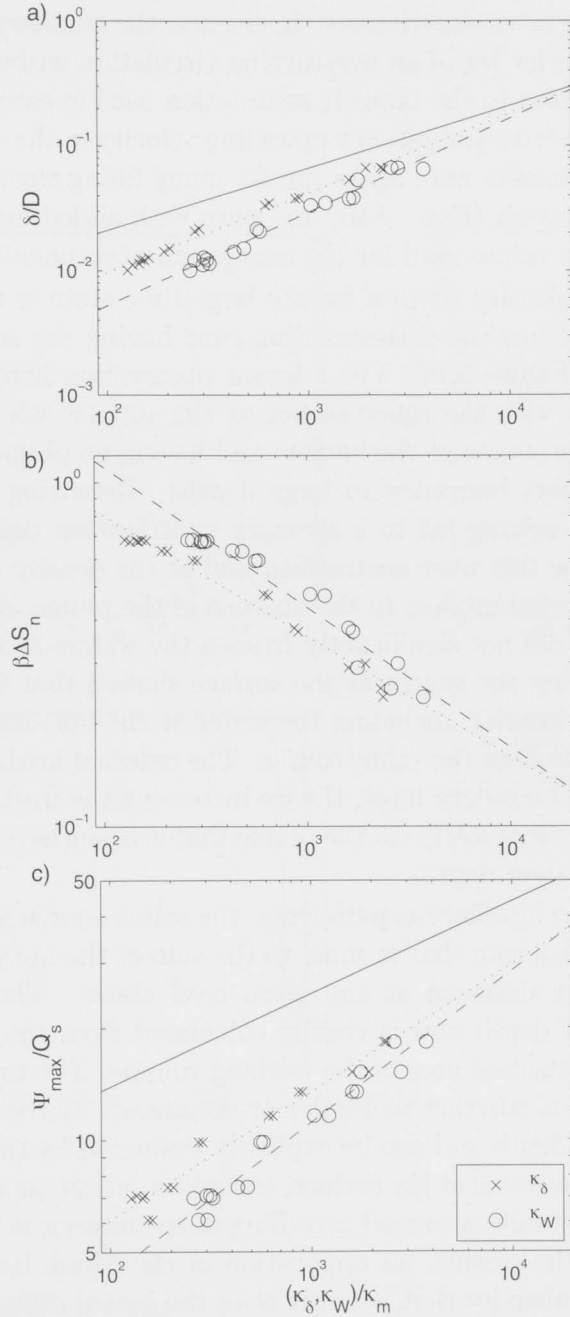


Figure 5.13: Panels showing the (a) normalized boundary layer thickness, (b) top-to-bottom salinity difference and (c) plume outflow as a function of the parameterized total diffusivities κ_δ and κ_W , both normalized by κ_m . All the data has collapsed and the two regimes in Figures 5.9b, 5.9c & 5.11 are no longer evident. The dotted lines represent the least-squares fits in terms of the parameterized total diffusivity κ_δ (based on the boundary layer transport): $\delta/D = 5 \times 10^{-4} (\kappa_\delta/\kappa_m)^{0.639}$, $\beta\Delta S_n = 4.0 (\kappa_\delta/\kappa_m)^{-0.35}$ and $\Psi_{\max}/Q_s = 1.24 (\kappa_\delta/\kappa_m)^{0.345}$. The dashed lines represent the least-squares fits for the Munk parameterization: these are $\delta/D = 2 \times 10^{-4} (\kappa_W/\kappa_m)^{0.751}$, $\beta\Delta S_n = 7.2 (\kappa_W/\kappa_m)^{-0.41}$ and $\Psi_{\max}/Q_s = 0.66 (\kappa_W/\kappa_m)^{0.41}$. The solid lines are the same as those in Figures 5.9b, 5.9c & 5.11, predicted by the theory of Hughes *et al.* (2007).

instabilities and irreversible mixing that may have been caused by the overturning circulation in the previous experiments. In essence, the auxiliary experiments model the upwelling or interior leg of an overturning circulation without requiring closure by a sinking leg, similar to the implicit assumption used in early attempts to evaluate the vertical diffusivity and interior upwelling velocity in the ocean (Munk, 1966). The auxiliary experiments were again run for many filling timescales, once with no added mechanical mixing (Exp. #10) and once with added mixing (Exp. #11) in the mid-range of the values used for the convection experiments.

The normalized density profiles for the large-time state in the auxiliary experiments are contrasted to the corresponding runs having the source at the surface (Exp. #1 & 6) in Figure 5.14. The relevant observation here is that $\rho_n(D) < 1$ for the experiments with the saline source at the surface, whereas $\rho_n(D) = 1$ for experiments with the source at the bottom and having no plume; a plume is needed to effectively transport buoyancy to large depths. Removing both the turbulent plume and external mixing led to a stronger stratification tightly confined at the surface; waters below this were unstratified and at the density of the saline source. The addition of external mixing, in the absence of the plume, deepened the surface boundary layer but did not significantly freshen the waters at large depth. In contrast, the cases having the source at the surface showed that the turbulent plume freshened all of the interior, including the water at the bottom; the plume outflow was always less dense than the saline source. The external mixing in this case again acted to thicken the boundary layer, thence increasing the fresh water entrained by the turbulent plume. It is solely via the plume that near-surface water and buoyancy are transported to larger depths.

In the case of the auxiliary experiments, the saline source at the bottom introduces dense water at a rate that is equal to the sum of the upward mass transports by advection and by diffusion at any given level above. The upwelling velocity w is independent of depth and is readily calculated from the volume flux of the saline source and horizontal area of the working volume. The total diffusivity in the auxiliary experiments, whether molecular or dominated by the external stirring, is also independent of depth and can be explicitly evaluated by the Munk balance. In contrast, when the source is at the surface, turbulent entrainment into the plume ensures that the horizontally-averaged upwelling in the interior is a function of depth. In this case, as in the oceans, an application of the Munk balance returns a parameterized total diffusivity that is larger than the actual diffusivity characterizing mixing in the interior and surface boundary layer.

The values of κ_W obtained by applying the Munk solution for experiments #1, 6, 10 & 11 are plotted against the calibrated external mixing rate κ_E in Figure 5.14b, again normalized by κ_m . In the regime $\kappa_E/\kappa_m > 10^2$, mixing induced by the stirring rods provided the dominant contribution to the vertical transport, whether or not there was a plume, and therefore the parameterized total diffusivities obtained with the saline source at the bottom and at the surface are similar; the value for the circulation including a plume is only twice as large as for that without. However, for no external stirring ($\kappa_E/\kappa_m \sim O(1)$) there is a factor of approximately 30 between the two values of κ_W : hence the plume entrainment very substantially enhanced the

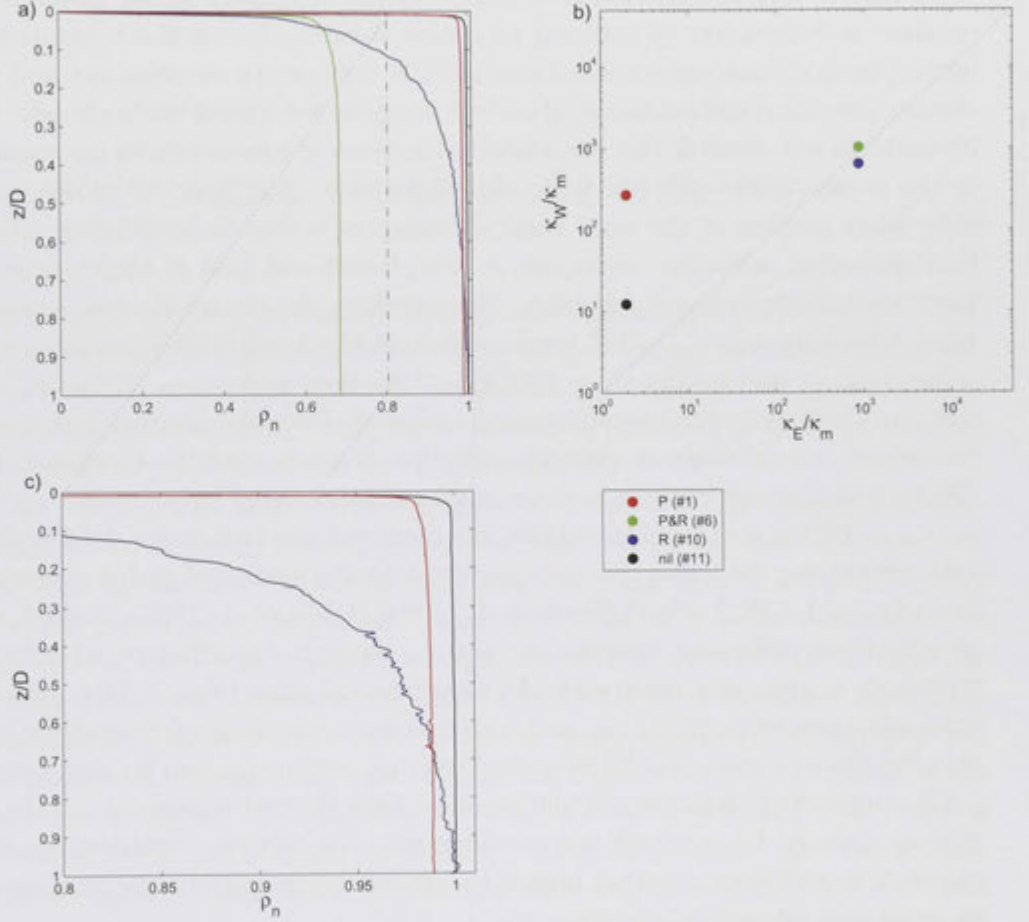


Figure 5.14: (a) Vertical profiles of normalized density for experiments #1 (red), 6 (green), 10 (blue) & 11 (black). (b) The parameterized total diffusivity for the same experiments as in (a). For large κ_E the presence of a plume has little influence on κ_W , indicating that the external-stirring sets the total vertical transport; this is not the case for small κ_E . (c) shows a close-up of the region outlined in (a); at this scale it is clear that even in the case of no external mixing, the plume is able to advect fresh water to depth.

parameterized measure of total mixing (note that κ_W in the auxiliary experiment without a plume is still significantly greater than κ_E in the case having no external stirring because of the stress applied to the working volume by the overflow and the spreading of the injected salt). We conclude that depth-dependent vertical transport in the convective overturning circulation (recall that Ψ_{max} is at least six times larger than Q_s) appears in the Munk analysis as a larger parameterized diffusivity.

A closely related question is whether vertical transport in the global ocean circulation is dominated by internal processes associated with the overturning or by mixing induced through external means. The laboratory experiments and previous studies provide some guidance. However, we have not varied the molecular diffusivity and it is not possible to conclude that the rates of irreversible or apparent mixing in the oceans scale with the molecular diffusivity. Nor have we shown systematically what portion of the total vertical transport is due to irreversible mixing (*i.e.* that excluding advective transport in the plume) and how it might depend upon parameters such as basin geometry. Nevertheless, the oceans have a parameterized total diffusivity $\kappa_W/\kappa_m \sim 10^3$ (evaluated from the Munk (1966) analysis at depths below 1000 m, and taking $\kappa_m = 10^{-7} \text{ m}^2 \text{ s}^{-1}$ for heat and $\kappa_W \sim 10^{-4} \text{ m}^2 \text{ s}^{-1}$). When the contribution to the overall vertical transport from the internal processes of entrainment and subsequent vertical advection is made explicit, Hughes & Griffiths (2006) find that irreversible interior mixing characterized by $\kappa_* \sim 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (*i.e.* $\kappa_*/\kappa_m \sim 10^2$) is sufficient to explain the observed top-to-bottom density difference and overturning rate, and this is consistent with the measured global average mixing rate of $\kappa_* \approx 1 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (Toole *et al.*, 1994; Polzin *et al.*, 1997). Indeed, an order of magnitude difference between κ_W and κ_* owing to the effect of advective plume transport is also consistent with the experimental data (Fig. 5.14b). As κ_E is in turn less than or equal to κ_* , and hence at least an order of magnitude less than κ_W , the global oceans will lie in a ‘weak mixing’ regime (as can be seen from Figure 5.12), suggesting that the circulation may have limited sensitivity to the level of mixing induced by external means. However, the relative contributions from the circulation and from external means to irreversible mixing in the interior remains an open and interesting question.

Insights from Perturbed Circulations

A topical subject is the sensitivity of deep-water formation to perturbations of surface buoyancy fluxes. Palaeo-reconstructions and numerical solutions suggest a sudden freshening of high-latitude surface waters, arising from physical processes such as glacial melt or lake outflows, can lead to a “shut-down” of deep-water formation with significant follow-on effects to the climate.

Qualitative observations of the response of the laboratory overturning circulation to six different types of perturbation are presented. In each case the equilibrium circulation, with a full-depth plume, was temporarily (in the sense that the circulation is not able to equilibrate to the perturbed conditions) exposed to a perturbation. The temporary perturbations took the form of either increases or decreases of Q_s , ρ_s and κ_E (Tab. 5.3). The magnitude of each perturbation was not quantified, but

Table 5.3: The types of perturbations temporarily imposed on the equilibrated circulations. Listed in each column is the type of perturbation, the depth of the plume penetration (and thus, circulation) during and immediately after the perturbation, and the type of anomaly produced by the perturbation (relative to the equilibrated state).

Perturbation	During	Post	Anomaly
Increase Q_s	Partial	Partial	Overlying fresh
Decrease Q_s	Full	Partial	Underlying dense
Increase ρ_s	Full	Partial	Underlying dense
Decrease ρ_s	Partial	Partial	Overlying fresh
Increase κ_E	Partial	Partial	Overlying fresh
Decrease κ_E	Full	Partial	Underlying dense

was large enough to elicit an observable response. The response of the circulation is described in terms of the depth of the plume outflow during and immediately following each of the perturbations. The circulation was insensitive to perturbations of Q_f , supporting the argument that the surface layer provided by Q_f is an imposed buoyancy boundary condition and not a flux condition.

The key result is that all perturbations produced a partial-depth plume and shallow circulation immediately following the return to steady-state forcing conditions. The reason for the shallow circulation state was dependent upon the type of density anomaly each perturbation produced. These anomalies fall into two categories; an overlying layer of fresh water, or an underlying layer of dense water (relative to the equilibrated circulation). An overlying layer of relatively fresh water resulted in the plume entraining relatively fresh water from the upper depths of the volume, and thus not possessing enough buoyancy to sink full-depth; this is an example of the ambient water modifying the buoyancy of the plume. An underlying layer of dense water resulted in the plume being insufficiently dense to penetrate into the anomalously dense region. Both styles of anomaly were stably stratified and removed through erosion by the turbulent plume. A full-depth circulation is re-established in a timescale ranging from minutes to hours; much shorter than that of molecular diffusion through the depth of the tank, $D^2/\kappa_m \approx 2 \times 10^8 \text{ s}$ (~ 6 years).

The observed responses arising from perturbations of Q_s were surprising. The initial response to an increase of Q_s was a partial-depth plume which outflowed at an intermediate depth, despite the perturbation imposing a larger buoyancy flux. The response is attributed to an increased level of turbulence associated with the saline source leading to an increased amount of entrainment into the plume, particularly from within the upper boundary layer. The relatively fresh plume led to the development of a stronger stable density gradient beneath the outflow. A decrease in Q_s , on the other hand, initially resulted in a less turbulent plume, which entrained less ambient fluid and remained full-depth. This perturbation led to a lower layer of relatively dense water.

Perturbations of ρ_s gave rise to more intuitive responses. Increasing ρ_s resulted in a full-depth plume and led to an underlying dense anomaly. Decreasing ρ_s gave

rise to a partial-depth plume and an overlying fresh anomaly.

The four types of perturbations described above are of the form of a sudden change in the destabilizing buoyancy flux. Perturbations of κ_E , on the other hand, amount to a perturbation of stabilizing buoyancy flux given the fixed buoyancy surface condition. Increasing κ_E enabled more fresh water to enter the upper region of the working volume. This fresh water was subsequently entrained into the plume, leading to a partial-depth circulation and the formation of an upper layer fresh anomaly (Fig. 5.15). Decreasing κ_E reduced the rate of fresh water mixing into the upper region of the working volume and led to a relatively denser plume, which continued to sink full-depth and establish a denser deep layer.

Comparison with Numerical Solutions

We turn our attention to the numerical solutions detailed in §4. The four numerical solutions obtained in domain (1) span 4 orders of magnitude of K_v and can be compared with the experimental results. The laboratory experiments and numerical solutions are characterized by the product of their respective Prandtl and flux Rayleigh numbers (Equations 2.2 and 2.4, using κ_E or K_v) and can also be compared to the theory of Hughes *et al.* (2007).

Figure 5.16a shows the boundary layer thickness normalized by the depth of the domain for the laboratory experiments (red stars) and the numerical solutions (green squares). There is good agreement between the data and the respective predictions of the theoretical model. The offset between the laboratory and numerical data arises from the significant difference in domain length ($L = 1.16$ and 3×10^5 m for the laboratory and numerical domains, respectively).

Figure 5.16b shows the measured normalized top-to-bottom density difference against that predicted by Hughes *et al.* (2007). Again, there is good agreement between model predictions and the data. It is interesting to note this agreement despite entrainment into the plume not being resolved in the numerical solutions. This suggests that the parameterization of the turbulent mixing (by the increased K_h and K_v) is having a similar effect on the density field as that which arises from entrainment.

5.4 Conclusions

Laboratory experiments have examined the overturning circulation forced by differential surface buoyancy fluxes and subjected to an imposed level of mechanical stirring. The experimental design used sources of saline and fresh water to provide the buoyancy forcing, and a pair of traversing horizontal rods to provide a controlled and calibrated rate of stirring throughout the circulation. The resulting flow resembled that observed previously in studies of horizontal convection. An important finding is the dependence of the circulation on the vertical transport in the circulation: the overturning strength and boundary layer thickness increased with the transport, but the stratification in the circulation decreased. These dependences can be successfully quantified by parameterizing the transport as a vertical diffusivity

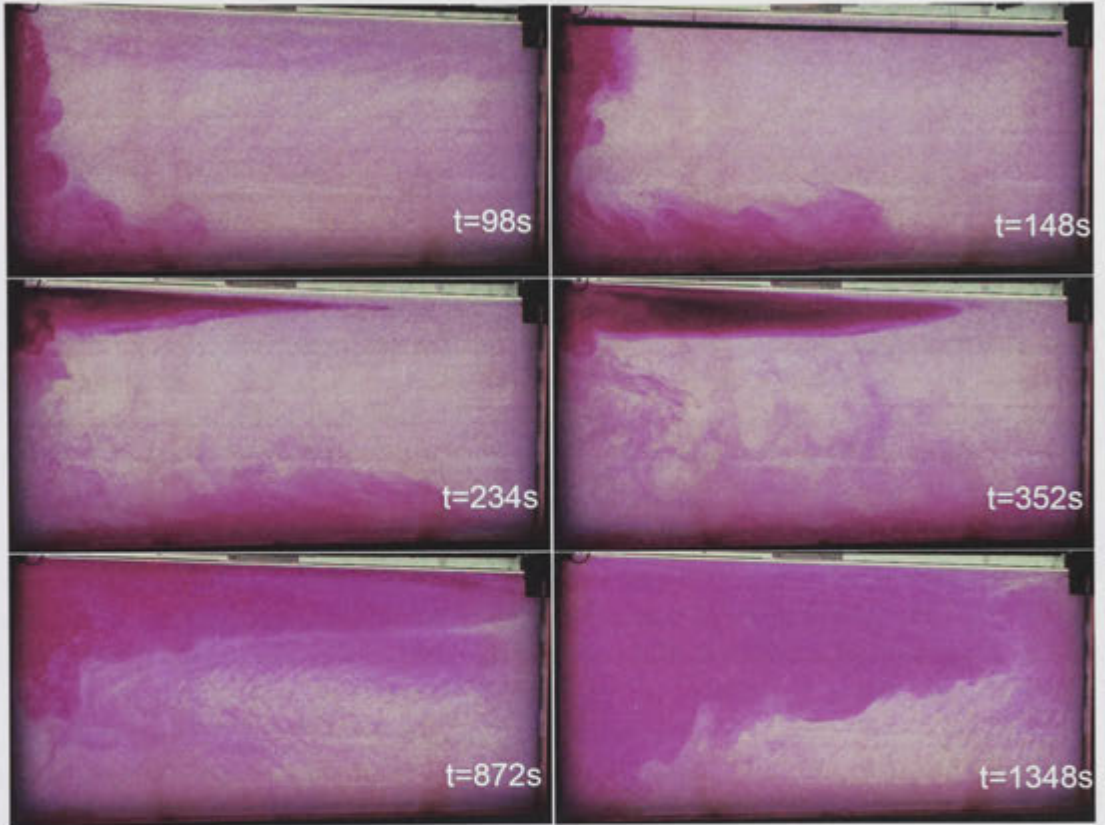


Figure 5.15: A sequence of images depicting the response of an equilibrated circulation to a temporary increase in κ_E . The time stamp refers to time since the dye was introduced; there initially exists a residual amount of dye from an earlier visualization of the equilibrated circulation (Fig. 5.8). A full-depth plume and circulation (evident at $t = 98$ s) is perturbed by traversing the stirring rods through the upper 0.2 m of the working volume at $t = 100$ s. The rods are seen leaving the tank at $t = 148$ s. The perturbation brings an excess of fresh water into the upper region of the working volume; this fresh water is entrained by the plume, which is subsequently no longer dense enough to sink full-depth and outflows at an intermediate level ($t = 234$ s). The fresh water anomaly remains after the perturbation ceases as an overlying layer of relatively fresh water. The vertical extent of the shallow overturning gradually deepens as the turbulent plume erodes away the overlying fresh anomaly. A full-depth circulation is reestablished by $t = 1348$ s.

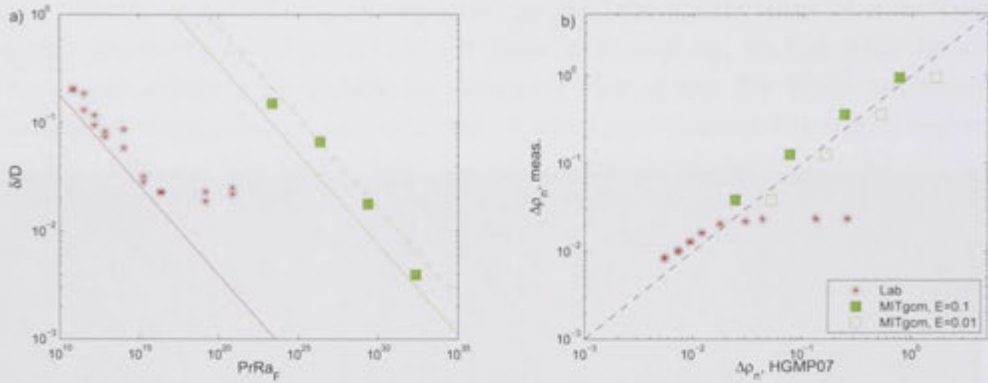


Figure 5.16: (a) Normalized boundary layer thickness as a function of $PrRa_F$ for the laboratory experiments (red) and MITgcm solutions (green); the solid and dashed lines represent the theoretical predictions of Hughes *et al.* (2007) with the entrainment coefficient of $E = 0.1$ and $E = 0.01$, respectively. (b) The top-to-bottom density difference predicted by the theoretical model against the measured values; the solid and hollow squares represent theoretical predictions made with entrainment coefficients of $E = 0.1$ and $E = 0.01$, respectively.

using both the Munk advection–diffusion balance (applied over a range of depths in the interior) and an independent estimate of transport at the surface (applied in the surface boundary layer).

Understanding the vertical transport in the circulation is not straightforward, however. Three significant contributions to the transport were found: 1) irreversible mixing throughout the circulation interior attributable to the imposed mechanical stirring, 2) irreversible mixing arising from spontaneous small-scale stirring events associated with the circulation itself, 3) and advective transport following the entrainment of interior water into the turbulent plume. The latter is a robust feature of the circulation at high Rayleigh number. Only the first two contributions correspond to genuine irreversible mixing processes (the combined effect of which is denoted by the turbulent diffusivity κ_*).

Two circulation regimes were found, depending on the strength of the externally-imposed mechanical stirring (in turn characterized by the turbulent diffusivity κ_E , which is measured from independent calibration experiments without an overturning circulation). When that stirring is sufficiently strong, κ_E is comparable with the parameterized vertical diffusivities obtained using either the Munk balance or an estimate based on the surface boundary layer. The characteristics of the circulation in this regime closely followed the theoretical predictions of Hughes *et al.* (2007); the boundary layer thickness increased as $\delta \sim \kappa_E^{1/2}$ and the maximum overturning transport increased as $\Psi_{\text{max}} \sim \kappa_E^{1/4}$. Because the theory explicitly includes vertical transport by entrainment into the turbulent plume, this agreement suggests that it is also important in this strong mixing regime. Entrainment must occur at all depths in the surface boundary layer and much of the interior, giving rapid advective

transport of buoyancy from the upper regions to the bottom. External stirring acts to deepen the boundary layer by enhancing downward mixing of buoyancy from the surface. This leads to a greater buoyancy (at a given level) driving the dense plume and larger area of entrainment, and thence to greater entrainment of fresh water buoyancy into the plume. The buoyancy is advected to the bottom and results in a smaller top-to-bottom density difference (Hughes & Griffiths, 2006).

When the externally-imposed stirring is weak, κ_E is significantly smaller than the parameterized vertical diffusivities obtained using the Munk balance and an estimate based on the surface boundary layer. The measured stratification and circulation were independent of the rate of external stirring and remained unchanged from the limiting case of no external stirring, in which it might have been expected that only molecular diffusion would operate to give irreversible mixing. The boundary layer thickness, for example, was very much greater than that expected for molecular diffusion alone. These results are explained in terms of an apparent minimum, or background, level of mixing, which in the experiments was some 200 times greater than the molecular diffusion of salt. It is concluded that this apparent background mixing was generated by internal processes associated with the convection. It has been shown that at least a part of the vertical transport is not irreversible mixing; this component is associated with the lateral transfer of water between the interior and the plume as a result of entrainment. The corresponding vertical transport can be interpreted as the downward advection of relatively fresh water entrained into the plume at any given height; an equivalent interpretation is in terms of the upward advection of relatively saline water in the circulation interior, where the upwelling decreases with height (as a result of entrainment). This advective transport is parameterized as a diffusivity by both the Munk advection–diffusion balance and the independent surface boundary layer transport estimate. The irreversible component of the background mixing is attributed to flow instabilities in the convective circulation and is qualitatively consistent with laboratory observations of weak stirring throughout much of the flow interior.

The experimental results serve to emphasize several lessons that are directly relevant and of great importance when considering the meridional overturning of the oceans. Turbulent mixing arising from energy input to the oceans by winds and tides (the external stirring) is available to deepen the boundary layer (thermocline). Turbulent diffusion is, however, an inefficient means of transporting heat to warm the abyssal depths. Instead, and contrary to much of the thinking of recent decades, the primary function of mixing is to modify the upper ocean density. The dense sinking overflows and slope currents then entrain the warmth from the thermocline on their way to forming North Atlantic Deep Water and Antarctic Bottom Water. The density of deep and bottom waters, and the overturning transport, are determined by entrainment and vertical advection by the plumes. In this picture, abyssal mixing plays a significantly lesser qualitative role than has commonly been suggested. Furthermore, the rate of irreversible mixing at abyssal depths required to explain ocean measurements is substantially less than that indicated by parameterizations based on the total vertical transport of density.

Conclusions

6.1 Overview and Implications

Two kinds of laboratory experiments and a set of numerical solutions have been used to investigate the effects of topography and mixing on an idealized overturning circulation driven by surface buoyancy fluxes. The first set of laboratory experiments (detailed in §3) showed that a sill has a global influence on the density structure if the sill depth is less than twice the boundary layer (or thermocline) depth. These large sills ensure the sill overflow interacts directly with the surrounding thermocline water, decreasing the density of the overflow plume and thus the upstream properties of the abyssal water. The experiments show that the effect of the sill depends on the surface boundary conditions and indicate a larger response for conditions of a prescribed distribution of temperature rather than of heat flux. Application of the results to the North Atlantic circulation suggest that the Greenland-Scotland Ridge is shallow enough to lead to a significant reduction of the density of North Atlantic Deep Water, and this is consistent with the conclusions from an analysis of water mass properties. During epochs of lower sea level the influence of the Greenland-Scotland Ridge on the global ocean circulation is predicted to have been even greater than present.

The second set of laboratory experiments (§5) showed that an external input of mechanical mixing leads to an increased rate of overturning, thicker thermocline and fresher abyss. When the imposed rate of mechanical mixing was the dominant contributor to the total mixing, measurements of the overturning rate and density structure are in accord with a theoretical model (Hughes *et al.*, 2007), which is based on a turbulent plume and a steady state advection–diffusion balance in the interior. For small rates of imposed mixing the circulation was insensitive to the mechanical mixing, and was greater than that expected for the case of a stratification maintained by a balance between molecular diffusion and advection. This behaviour is attributed to mixing by instabilities in the convective flow and entrainment into the turbulent plume. Application of these physical principles to the oceans suggests that turbulent mixing (due to energy sourced from winds and tides) acts primarily to deepen the thermocline. Mixing thereby influences the properties of the abyssal ocean by modifying the upper ocean environment through which the dense plumes sink and from which they entrain water and heat. The bulk transport in the plumes, and not vertical mixing throughout the interior, as was previously thought, is the

crucial mechanism for transporting heat to large depths.

The two distinctly different kinds of experiments reported here share a common result; the abyssal density is set by the sinking plume and the water it entrains. In §3 the abyss was freshened by topographically forcing the sill overflow to interact with thermocline water. In §5 the abyss was freshened by increasing the depth through which the plume accesses thermocline water. Thus in both cases the abyss is freshened by increasing the amount of buoyancy entrained into the sinking plume. The results illustrate that the abyssal density is influenced by the thermocline depth and plume entrainment, both of which are strongly influenced by micro-scale processes and require parameterizations in GCMs (e.g., bulk mixed-layers, convective adjustment, and turbulent mixing). Care must be taken when interpreting the results of GCMs, particularly relating to abyssal densities, where such parameterizations have been made.

The numerical solutions (§4) show good agreement with both sets of laboratory experiments and provide an opportunity to investigate the energetics of the overturning circulation. In particular, we use these solutions to test three different methods of determining the appropriate reference state of no motion for an overturning circulation in a multi-basin domain. The results show that although the different methods give different values of available potential energy, they all return similar values for the power input from surface buoyancy fluxes, which is the quantity of dynamic importance to the maintenance of the overturning circulation.

6.2 Further Work

The findings detailed in this thesis have motivated further questions into the dynamics of an overturning circulation driven by surface buoyancy fluxes. All quantitative results presented here relate to equilibrated circulations under steady boundary conditions. Recent work uses laboratory experiments to investigate the response of an overturning circulation to oscillatory surface buoyancy forcing conditions and identify the adjustment timescales of the deep-water formation and convective overturning (Griffiths *et al.*, in press). It is likely that incorporating topography into such experiments will introduce an additional timescale associated with the ventilation time of the small basin. Additionally, the small basin may act as a reservoir of dense water such that the influence of the oscillatory forcing on the deep-water formation is damped. If this is the case it may go some way to explaining why seasonal variability is observed in the Weddell Sea outflow (which does not have a substantial sill; Gordon *et al.*, 2010), but not Greenland-Scotland Ridge overflows (Macrandar *et al.*, 2007).

The experiments in §5 use a spatially uniform rate of externally imposed mechanical mixing. It is known that oceanic mixing is enhanced in regions of rough topography and towards the surface. This observed spatial dependence of oceanic mixing is thought to be dynamically important to the MOC. The results presented here suggest that the most significant effect of the imposed mixing is to deepen the thermocline, thus increasing the access of the plume to surface water. It is possible

to re-calibrate the apparatus used in §5 to incorporate a depth-dependent rate of externally imposed mechanical mixing, thus allowing examination of the sensitivity of the circulation to mixing at specific depths.

Using a solute to supply the buoyancy flux has many advantages over heat in laboratory experiments. However, the source of destabilising buoyancy in §5 does not provide a pure buoyancy flux condition (unlike that in §3): it also imposes a finite volume flux and sets a maximum density for the experiment. This must be taken into account when interpreting the results, particularly when comparing to the theory of Hughes *et al.* (2007) which prescribes a buoyancy flux with no limit on the density difference. Future laboratory experiments could focus on achieving a source of destabilising buoyancy that is closer to a pure buoyancy flux.

The MOC is thought to be a result of the interplay between surface buoyancy forcing, wind-driven upwelling and turbulent mixing. Previous laboratory experiments investigating overturning circulations have generally focussed on surface buoyancy forcing alone, with the more recent addition of turbulent mixing. The boundary conditions employed in §5 offer a simple means of introducing surface stress (winds) to the overturning circulation by adjusting the flow rate and location of the fresh water source. Experiments of this type are already being conducted (K. Matusik, personal communication).

The laboratory experiments detailed in this thesis were conducted in a non-rotating reference frame, and an obvious step is to introduce Coriolis effects. It has been shown that the circulations in the laboratory are dominated by small-scale mixing processes. While rotation may give rise to additional instabilities and mixing pathways, practical limitations (e.g., aspect ratio) may not allow access to the geostrophic flow conditions required to quantify any influence of planetary rotation. R. Griffiths (personal communication) has carried out thermally-forced experiments with rotating horizontal convection in the same box used in §3. Apart from the addition of horizontally-recirculating gyres, the overall flow is qualitatively similar to that in the non-rotating case. These experiments could include imposed mixing and require extensive measurements of the three-dimensional flow.

The non-hydrostatic numerical solutions driven by an imposed distribution of buoyancy flux (§4) showed that the calculated rate of energy input from surface buoyancy forcing is insensitive to the adiabatic resorting method. Many GCMs are often run in a hydrostatic mode, incorporate non-linear equations of state and prescribe (or relax to) a distribution of surface buoyancy; it is not clear if a solution employing any these conditions will display the same insensitivity to the adiabatic resorting method.

Bibliography

- ANTONOV, J. I., SEIDOV, D., BOYER, T. P., LOCARNINI, R. A., MISHONOV, A. V., GARCIA, H. E., BARANOVA, O. K., ZWENG, M. M. & JOHNSON, D. R. 2010 World Ocean Atlas 2009, Volume 2: Salinity. In *NOAA Atlas NESDIS 69* (ed. S. Levitus), p. 184. U.S. Government Printing Office, Washington, D.C.
- BAINES, W. D. & TURNER, J. S. 1969 Turbulent buoyant convection from a source in a confined region. *Journal of Fluid Mechanics* **37**, 51–80.
- BARKSTROM, B., HARRISON, E., SMITH, G., GREEN, R., KIBLER, J., CESS, R. & THE ERBE SCIENCE TEAM 1989 Earth Radiation Budget Experiment (ERBE) archival and April 1985 results. *Bull. Amer. Meteor. Soc.* **70**, 1254–1262.
- BATCHELOR, G. K. 1967 *An Introduction to Fluid Dynamics*. Cambridge University Press.
- BEARDSLEY, R. C. & FESTA, J. F. 1972 A numerical model of convection driven by a surface stress and non-uniform heating. *Journal of Physical Oceanography* **2**, 444–455.
- BEHRENFELD, M. & FALKOWSKI, P. 1997 Photosynthetic rates derived from satellite-based chlorophyll concentration. *Limnology and Oceanography* **42**, 1–20.
- BROECKER, W. S. 1982 Glacial to interglacial changes in ocean chemistry. *Progress in Oceanography* **11**, 151–197.
- BROECKER, W. S. 1997 Thermohaline circulation, the Achilles heel of our climate system: Will man-made CO₂ upset the current balance? *Science* **278**, 1582–1588.
- BRYAN, F. 1987 Parameter sensitivity of primitive equation ocean general circulation models. *Journal of Physical Oceanography* **17**, 970–985.
- BRYAN, K. & COX, M. 1967 A numerical investigation of oceanic general circulation. *Tellus* **19**, 54–80.
- BRYDEN, H. L., CANDELA, J. & KINDER, T. H. 1994 Exchange through the Strait of Gibraltar. *Progress in Oceanography* **33**, 201–248.
- BRYDEN, H. L. & IMAKAWI, S. 2001 In *Ocean Circulation and Climate* (ed. G. Siedler, J. Church & J. Gould), *International Geophysics Series*, vol. 77, chap. Ocean heat transport, pp. 455–474. San Diego: Academic Press.
- BRYDEN, H. L. & NURSER, A. J. G. 2003 Effects of strait mixing on ocean stratification. *Journal of Physical Oceanography* **33**, 1870–1872.

- CENEDESE, C. & ADDUCE, C. 2010 A new parameterization for entrainment in overflows. *Journal of Physical Oceanography* **40**, 1835–1850.
- COMAN, M. A., GRIFFITHS, R. W. & HUGHES, G. O. 2006a Sandström's experiments revisited. *Journal of Marine Research* **64**, 783–796.
- COMAN, M. A., GRIFFITHS, R. W. & HUGHES, G. O. 2010 The sensitivity of convection from a horizontal boundary to the distribution of heating. *Journal of Fluid Mechanics* **647**, 71–90.
- COMAN, M. A., HUGHES, G. O., KERR, R. C. & GRIFFITHS, R. W. 2006b The effect of a barrier on laminar convection in a box with differentially heated endwalls. *International Journal of Heat and Mass Transfer* **49**, 2903–2911.
- DANABASOGLU, G., LARGE, B. G. & BRIEGLEB, B. P. 2010 Climate impacts of parameterized Nordic Sea overflows. *Journal of Geophysical Research* **115** (C11005).
- DEFANT, A. 1961 *Physical Oceanography, Vol. 1*. Pergamon Press.
- DEWAR, W. K., BINGHAM, R. J., IVERSON, R. L., NOWACEK, D. P., LAURENT, L. C. S. & WIEBE, P. H. 2006 Does the marine biosphere mix the ocean? *Journal of Marine Research* **64**, 541–561.
- DICKSON, R. R. & BROWN, J. 1994 The production of North Atlantic Deep Water: Sources, rates and pathways. *Journal of Geophysical Research* **99** (C6), 12319–12341.
- EAKINS, B. W. & SHARMAN, G. F. 2010 In *Volumes of the World's Oceans from ETOPO1*. NOAA National Geophysical Data Center, Boulder, CO.
- FASULLO, J. T. & TRENBERTH, K. E. 2007 The annual cycle of the energy budget. part II: Meridional structures and poleward transports. *Journal of Climate* **21**, 2313–2325.
- FER, I., VOET, G., SEIM, K., RUDELS, B. & LATARIUS, K. 2010 Intense mixing of the Faroe Bank Channel overflow. *Geophysical Research Letters* **37** (L06606), doi 10.1029/2009GL041924.
- FERNANDO, H. J. S. 1991 Turbulent mixing in stratified fluids. *The Annual Review of Fluid Mechanics* **23**, 455–493.
- FINNIGAN, T. D. & IVEY, G. N. 1999 Submaximal exchange between a convectively forced basin and a large reservoir. *Journal of Fluid Mechanics* **378**, 357–378.
- FINNIGAN, T. D. & IVEY, G. N. 2000 Convectively driven exchange flow in a stratified sill-enclosed basin. *Journal of Fluid Mechanics* **418**, 313–338.
- FINNIGAN, T. D., WINTERS, K. B. & IVEY, G. N. 2001 Response characteristics of a buoyancy-driven sea. *Journal of Physical Oceanography* **31**, 2721–2736.

- FRIEDRICH, T., TIMMERMANN, A., DECLOEDT, T., LUTHER, D. S. & MOUCHET, A. 2011 The effect of topography-enhanced diapycnal mixing on ocean and atmospheric circulation and marine biogeochemistry. *Ocean Modelling* **39**, 262–274.
- GANOPOLSKI, A. & RAHMSTORF, S. 2001 Rapid changes of glacial climate simulated in a coupled climate model. *Nature* **409**, 153–158.
- GANOPOLSKI, A. & RAHMSTORF, S. 2002 Abrupt glacial climate change due to stochastic resonance. *Physical Review Letters* **88**.
- GNANADESIKAN, A., SLATER, R. D., SWATHI, P. S. & VALLIS, G. K. 2005 The energetics of ocean heat transport. *Journal of Climate* **18**, 2604–2616.
- GORDON, A., HUBER, B., MCKEE, D. & VISBECK, M. 2010 A seasonal cycle in the export of bottom water from the Weddell Sea. *Nature Geoscience* **3**, 551–556.
- GREGG, M. C. 1987 Diapycnal mixing in the thermocline: A review. *Journal of Geophysical Research* **92**, 5249–5286.
- GREGG, M. C. 1989 Scaling turbulent dissipation in the thermocline. *Journal of Geophysical Research* **94**.
- GRIFFITHS, R. W., MAHER, N. & HUGHES, G. O. in press Ocean stratification under oscillatory surface buoyancy forcing. *Journal of Marine Research*.
- HANSEN, B. & ØSTERHUS, S. 2000 North Atlantic–Nordic Seas exchanges. *Progress in Oceanography* **45**, 109–208.
- HANSEN, B. & ØSTERHUS, S. 2007 Faroe Bank Channel overflow 1995–2005. *Progress in Oceanography* **75**, 817–856.
- HANSEN, B., ØSTERHUS, S., QUADFASEL, D. & TURRELL, W. 2004 Already the Day After Tomorrow? *Science* **305**, 953–954.
- HIGNETT, P., IBBETSON, A. & KILLWORTH, P. D. 1981 On rotating thermal convection driven by non-uniform heating from below. *Journal of Fluid Mechanics* **109**, 161–187.
- HOLFORD, J. M. & LINDEN, P. F. 1999 Turbulent mixing in a stratified fluid. *Dynamics of Atmospheres and Oceans* **30**, 173–198.
- HUANG, R. X. 1999 Mixing and energetics of the oceanic thermohaline circulation. *Journal of Physical Oceanography* **29**, 727–746.
- HUANG, R. X. 2005 Available potential energy in the world's oceans. *Journal of Marine Research* **63**, 141–158.
- HUGHES, G. O. & GRIFFITHS, R. W. 2006 A simple convective model of the global overturning circulation, including effects of entrainment into sinking regions. *Ocean Modelling* **12**, 46–79.

- HUGHES, G. O. & GRIFFITHS, R. W. 2008 Horizontal Convection. *The Annual Review of Fluid Mechanics* **40**, 185–208.
- HUGHES, G. O., GRIFFITHS, R. W., MULLARNEY, J. C. & PETERSON, W. H. 2007 A theoretical model for horizontal convection at high Rayleigh number. *Journal of Fluid Mechanics* **581**, 251–276.
- HUGHES, G. O., HOGG, A. M. & GRIFFITHS, R. W. 2009 Available potential energy and irreversible mixing in the meridional overturning circulation. *Journal of Physical Oceanography* **39**, 3130–3146.
- ILICAK, M. & VALLIS, G. K. 2011 Simulations and scaling of horizontal convection. *Tellus* **submitted**.
- JAYNE, S. R. & ST. LAURENT, L. C. 2001 Parameterizing tidal dissipation over rough topography. *Geophysical Research Letters* **28**, 811–814.
- JEFFREYS, H. T. 1925 On fluid motions produced by differences of temperature and humidity. *Quarterly Journal of the Royal Meteorological Society* **51**, 347–356.
- KILLWORTH, P. 1995 Hydraulic control and maximal flow in rotating stratified hydraulics. *Deep Sea Research I* **42**, 859–871.
- KILLWORTH, P. 2001 On the rate of descent of overflows. *Journal of Geophysical Research* **106**, 22267–22275.
- KUHLBRODT, T., GRIESEL, A., MONTOYA, M., LEVERMANN, A., HOFMANN, M. & RAHMSTORF, S. 2007 On the driving processes of the Atlantic Meridional Overturning Circulation. *Reviews of Geophysics* **45** (RG2001), doi 10.1029/2004RG000166.
- KUNZE, E., DOWER, J. F., BEVERIDGE, I., DEWEY, R. & BARTLETT, K. P. 2006 Observations of biologically generated turbulence in a coastal inlet. *Science* **313**, 1768–1770.
- KUNZE, E. & SANFORD, T. B. 1996 Abyssal mixing: Where is it not? *Journal of Physical Oceanography* **26**, 2286–2296.
- LARGE, W. G., MCWILLIAMS, J. C. & DONET, S. C. 1994 Oceanic vertical mixing: a review and a model with a nonlocal boundary layer parameterization. *Reviews of Geophysics* **32**, 363–403.
- LAUDERDALE, J. M., BACON, S., NAVEIRA GARABATO, A. C. & HOLLIDAY, N. P. 2008 Intensified turbulent mixing in the boundary current system of southern Greenland. *Geophysical Research Letters* **35** (L04611).
- LEDWELL, J. R., MONTGOMERY, E. T., POLZIN, K. L., ST. LAURENT, L. C., SCHMITT, R. W. & TOOLE, J. M. 2000 Evidence for enhanced mixing over rough topography in the abyssal ocean. *Nature* **403**, 179–182.

- LEDWELL, J. R., WATSON, A. J. & LAW, C. S. 1993 Evidence for slow mixing across the pycnocline from an open-ocean tracer release experiment. *Nature* **364**, 701–703.
- LEGG, S., EZER, T., JACKSON, L., BRIEGLEB, B., DANABASOGLU, G., LARGE, W., WU, W., CHANG, Y., ÖZGÖKMEN, T. M., PETERS, H., XU, X., CHASSIGNET, E. P., GORDON, A. L., GRIFFIES, S., HALLBERG, R., PRICE, J., RIEMENSCHNEIDER, U. & YANG, J. 2009 Improving oceanic overflow representation in climate models: The Gravity Current Entrainment Climate Process Team. *Bull. Amer. Meteor. Soc.* **90**, 657–670.
- LINDEN, P. F. 1980 Mixing across a density interface produced by grid turbulence. *Journal of Fluid Mechanics* **100**, 691–703.
- LOCARNINI, R. A., MISHONOV, A. V., ANTONOV, J. I., BOYER, T. P., GARCIA, H. E., BARANOVA, O. K., ZWENG, M. M. & JOHNSON, D. R. 2010 World Ocean Atlas 2009, Volume 1: Temperature. In *NOAA Atlas NESDIS 68* (ed. S. Levitus), p. 184. U.S. Government Printing Office, Washington, D.C.
- LORENZ, E. N. 1955 Available potential energy and the maintenance of the general circulation. *Tellus* **7**, 157–167.
- MACRANDER, A., KÄSE, R., SEND, U., VALDIMARSSON, H. & JÓNSSON, S. 2007 Spatial and temporal structure of the Denmark Strait Overflow revealed by acoustic observations. *Ocean Dynamics* **57**, 75–89.
- MAROTZKE, J. & SCOTT, J. R. 1999 Convective mixing and the thermohaline circulation. *Journal of Physical Oceanography* **29**, 2962–2970.
- MARSHALL, J., ADCROFT, A., HILL, C., PERELMAN, L. & HEISEY, C. 1997 A finite-volume incompressible navier stokes model for studies of the ocean on parallel computers. *Journal of Geophysical Research* **102**, 5753–5766.
- MARSHALL, J. & SCHOTT, F. 1999 Open-ocean convection: observations, theory and models. *Reviews of Geophysics* **37**.
- MAURITZEN, C., PRICE, J., SANFORD, T. & TORRES, D. 2005 Circulation and mixing in the Faroese Channels. *Deep-Sea Research I* **52**, 883–913.
- MAXWORTHY, T. 1997 Convection into domains with open boundaries. *Annual Reviews in Fluid Mechanics* **29**, 327–371.
- MCCUTCHEON, S. C., MARTIN, J. L. & BARNWELL, T. O. J. 1993 Water density as function of temperature and concentration. In *Handbook of Hydrology* (ed. D. R. Maidment). McGraw-Hill.
- MILLER, R. C. 1968 A thermally convecting fluid heated nonuniformly from below. PhD thesis, Massachusetts Institute of Technology.

- MULLARNEY, J. C., GRIFFITHS, R. W. & HUGHES, G. O. 2004 Convection driven by differential heating at a horizontal boundary. *Journal of Fluid Mechanics* **516**, 181–209.
- MUNK, W. H. 1966 Abyssal recipes. *Deep-Sea Research* **13**, 707–730.
- MUNK, W. H. & WUNSCH, C. 1998 Abyssal recipes II: energetics of tidal and wind mixing. *Deep-Sea Research I* **45**, 1977–2010.
- NAVEIRA GARABATO, A. C., POLZIN, K. L., KING, B. A., HEYWOOD, K. J. & VISBECK, M. 2004 Widespread intense turbulent mixing in the Southern Ocean. *Science* **303**, 210–213.
- OORT, A. H., , S. C. A., LEVITUS, S. & PEIXÓTO, J. P. 1989 New estimates of the available potential energy in the world ocean. *Journal of Geophysical Research* **94**, 3187–3200.
- OORT, A. H., ANDERSON, L. A. & PEIXÓTO, J. P. 1994 Estimates of the energy cycle of the oceans. *Journal of Geophysical Research* **99**, 7665–7688.
- ORSI, A. H., JOHNSON, G. C. & BULLISTER, J. L. 1999 Circulation, mixing, and production of Antarctic Bottom Water. *Progress in Oceanography* **43**, 55–109.
- PAPARELLA, F. & YOUNG, W. R. 2002 Horizontal convection is non-turbulent. *Journal of Fluid Mechanics* **466**, 205–214.
- PARK, Y. & BRYAN, K. 2000 Comparison of thermally driven circulation from a depth-coordinate model and an isopycnal model. Part I: Scaling law sensitivity to vertical diffusion. *Journal of Physical Oceanography* **30**, 590–605.
- PARK, Y. & BRYAN, K. 2001 Comparison of thermally driven circulation from a depth-coordinate model and an isopycnal model. Part II: The difference and structure of the circulations. *Journal of Physical Oceanography* **31**, 2612–2624.
- PARK, Y. G. & WHITEHEAD, J. A. 1999 Rotating convection driven by differential bottom heating. *Journal of Physical Oceanography* **29**, 1208–1220.
- PETERSON, W. H. 1979 A steady thermohaline convection model. PhD thesis, University of Miami.
- PHILLIPS, O. M. 1966 On turbulent convection currents and the circulation of the Red Sea in early summer. *Deep-Sea Research* **13**, 1148–1160.
- PIERCE, D. W. & RHINES, P. B. 1996 Convective building of a pycnocline: Laboratory experiments. *Journal of Physical Oceanography* **26**, 176–190.
- POLZIN, K. L., SPEER, K. G., TOOLE, J. M. & SCHMITT, R. W. 1996 Intense mixing of Antarctic Bottom Water in the equatorial Atlantic Ocean. *Nature* **380**, 54–57.

- POLZIN, K. L., TOOLE, J. M., LEDWELL, J. R. & SCHMITT, R. W. 1997 Spatial variability of turbulent mixing in the abyssal ocean. *Science* **276**, 93–96.
- RAHMSTORF, S. 2002 Ocean circulation and climate during the past 120,000 years. *Nature* **419**, 207–214.
- RINTOUL, S. R. 2011 In *Science Diplomacy: Antarctica, Science and the Governance of International Spaces* (ed. P. Berkman, M. A. Lang, D. W. H. Walton & O. R. Young), chap. The Southern Ocean in the Earth System. Smithsonian Institution Scholarly Press.
- ROSSBY, H. T. 1965 On thermal convection driven by non-uniform heating from below: an experimental study. *Deep-Sea Research* **12**, 9–16.
- ROSSBY, T. 1998 Numerical experiments with a fluid non-uniformly heated from below. *Tellus* **50**, 242–257.
- RUDDICK, B. R. & SHIRTCLIFFE, T. G. L. 1979 Data for double diffusers: physical properties of aqueous salt-sugar solutions. *Deep-Sea Research* **26A**, 775–787.
- RUDNICK, D. L., BOYD, T. J., BRAINARD, R. E., CARTER, G. S., EGBERT, G. D., GREGG, M. C., HOLLOWAY, P. E., KLYMAK, J. M., KUNZE, E., LEE, C. M., LEVINE, M. D., LUTHER, D. S., MARTIN, J. P., MERRIFIELD, M. A., MOUM, J. N., NASH, J. D., PINKEL, R., RAINVILLE, L. & SANFORD, T. B. 2003 From tides to mixing along the hawaiian ridge. *Science* **301**, 355–357.
- RYAN, W. B. F., CARBOTTE, S., COPLAN, J., O'HARA, S., MELKONIAN, A., ARKO, R., WEISSEL, R., FERRINI, V., GOODWILLIE, A., NITSCHKE, F., BONCZKOWSKI, J. & ZEMSKY, R. 2009 Global Multi-Resolution Topography (GMRT) synthesis data set. *Geochemistry, Geophysics, Geosystems* **10**, Q03014, doi 10.1029/2008GC002332.
- SABINE, C. L., FEELY, R. A., GRUBER, N., KEY, R. M., LEE, K., BULLISTER, J. L., WANNINKHOF, R., WONG, C. S., WALLACE, D. W. R., TILBROOK, B., MILLERO, F. J., PENG, T.-H., KOZYS, A., ONO, T. & RIOS, A. F. 2004 The oceanic sink for anthropogenic CO₂. *Science* **305**, 367–371.
- SANDSTRÖM, J. W. 1908 Dynamische versuche mit meerwasser. *Annalen der Hydrographic und Maritimen Meteorologie* **36**, 6–23.
- SANDSTRÖM, J. W. 1916 Meteorologische studien im Schwedischen Hochgebirge. *Göteborgs Kungl. Vetenskaps- och Vitterhetssamhälles Handlingar* **17**, 1–48.
- SAUNDERS, P. M. 1996 The flux of dense cold water overflow water southeast of Iceland. *Journal of Physical Oceanography* **26**, 85–95.
- SAUNDERS, P. M. 2001 In *Ocean Circulation and Climate* (ed. G. Siedler, J. Church & J. Gould), *International Geophysics Series*, vol. 77, chap. The dense northern overflows, pp. 401–418. San Diego: Academic Press.

- SCHOTT, F. A., ZANTOPP, R., STRAMMA, L., DENGLER, M., FISCHER, J. & WIBAUX, M. 2004 Circulation and deep-water export at the western exit of the subpolar North Atlantic. *Journal of Physical Oceanography* **34**, 817–843.
- SCOTTI, A. & WHITE, B. 2011 Is horizontal convection really “non-turbulent”? *Geophysical Research Letters* **38** (L21609).
- SIGGERS, J. H., KERSWELL, R. R. & BALMFORTH, N. J. 2004 Bounds on horizontal convection. *Journal of Fluid Mechanics* **517**, 55–70.
- SIMPKINS, P. G. & CHEN, K. S. 1986 Convection in horizontal cavities. *Journal of Fluid Mechanics* **166**, 21–39.
- SPEER, K. G. & WHITEHEAD, J. A. 1988 A gyre in a non-uniformly heated rotating fluid. *Deep-Sea Research* **35**, 1069–1077.
- STEINHART, J. S. & HART, S. R. 1968 Calibration curves for thermistors. *Deep Sea Research* **15**, 497–503.
- STERN, M. 1975 *Ocean Circulation Physics*. Academic.
- STEWART, K. D., HUGHES, G. O. & GRIFFITHS, R. W. 2011 When do marginal seas and topographic sills modify the ocean density structure? *Journal of Geophysical Research* **116**, C08021, doi:10.1029/2011JC006980.
- STOMMEL, H. 1962 On the smallness of sinking regions in the ocean. *Proceedings of the National Academy of Sciences, Washington* **48**, 766–772.
- TAILLEUX, R. 2009 On the energetics of stratified turbulent mixing, irreversible thermodynamics, Boussinesq models, and the ocean heat engine controversy. *Journal of Fluid Mechanics* **638**, 339–382.
- TOGGWEILER, J. R. 1999 Variation of atmospheric CO₂ by ventilation of the ocean’s deepest water. *Paleoceanography* **14**, 571–588.
- TOGGWEILER, J. R., RUSSELL, J. L. & CARSON, S. R. 2006 Midlatitude westerlies, atmospheric CO₂, and climate change during the ice ages. *Paleoceanography* **21**, PA2005.
- TOGGWEILER, J. R. & SAMUELS, B. 1998 On the ocean’s large-scale circulation near the limit of no vertical mixing. *Journal of Physical Oceanography* **28**, 1832–1852.
- TOOLE, J. M., POLZIN, K. L. & SCHMITT, R. W. 1994 Estimates of diapycnal mixing in the abyssal ocean. *Science* **264**, 1120–1123.
- TRAGOU, E. & GARRETT, C. 1997 The shallow thermohaline circulation of the Red Sea. *Deep-Sea Research I* **44** (8), 1355–1376.
- TRENBERTH, K. E. & CARON, J. M. 2001 Estimates of meridional atmosphere and ocean heat transport. *Journal of Climate* **14**, 3433–3443.

- VISBECK, M. 2007 Power of pull. *Nature* **447**, 383.
- VISSER, A. W. 2007 Biomixing of the oceans? *Science* **316**, 838–839.
- VOET, G. & QUADFASEL, D. 2010 Entrainment in the Denmark Strait overflow plume by meso-scale eddies. *Ocean Sciences* **6**, 301–310.
- WÄHLIN, A. K., JOHANSSON, A. M., AAS, E., BROSTRÖM, G. & WEBER, J. E. H. 2010 Horizontal convection in water heated by infrared radiation and cooled by evaporation: scaling analysis and experimental results. *Tellus* **62A**, 154–169.
- WANG, W. & HUANG, R. X. 2005 An experimental study on thermal convection driven by horizontal differential heating. *Journal of Fluid Mechanics* **540**, 49–73.
- WASHBURN, E. W. 2003 In *International Critical Tables of Numerical Data, Physics, Chemistry and Technology (1st Electronic Edition)* (ed. E. W. Washburn). Knovel.
- WEBB, D. & SUGINOHARA, N. 2001 Oceanography– vertical mixing in the ocean. *Nature* **409**, 37.
- WELLS, M. C., CENEDESE, C. & CAULFIELD, C. P. 2010 The relationship between flux coefficient and entrainment ratio in density currents. *Journal of Physical Oceanography* **40**, 2713–2727.
- WHITEHEAD, J. A. 1998 Topographic control of oceanic flows in deep passages and straits. *Reviews of Geophysics* **36** (3), 423–440.
- WHITEHEAD, J. A. 2000 Stratified convection with multiple states. *Ocean Modelling* **2**, 109–121.
- WHITEHEAD, J. A. & STEVENSON, I. 2007 Turbulent mixing of two-layer stratified fluid. *Physics of Fluids* **19**, 125104.
- WHITEHEAD, J. A. & WANG, W. 2008 A laboratory model of vertical ocean circulation driven by mixing. *Journal of Physical Oceanography* **28**, 1091–1106.
- WINTERS, K. B., LOMBARD, P. N., RILEY, J. J. & D'ASARO, E. A. 1995 Available potential energy and mixing in density-stratified fluids. *Journal of Fluid Mechanics* **289**, 115–128.
- WINTON, M. 1995 Why is the deep sinking narrow? *Journal of Physical Oceanography* **25**, 997–1005.
- WOLFE, C. L. & CESSI, P. 2011 The adiabatic pole-to-pole overturning circulation. *Journal of Physical Oceanography* **41**, 1795–1810.
- WONG, A. B. D. & GRIFFITHS, R. 2001 Two-basin filling boxes. *Journal of Geophysical Research* **106** (C11), 26929–26941.

- WUNSCH, C. 2005 The total meridional heat flux and its oceanic and atmospheric partition. *Journal of Climate* **18**, 4374–4380.
- WUNSCH, C. & FERRARI, R. 2004 Vertical mixing, energy, and the general circulation of the oceans. *Annual Reviews in Fluid Mechanics* **36**, 281–314.