WEAK DECAYS OF STRANGE PARTICLES.

by

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STATEMENT.

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any University, and that, to the best of my knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text.

Maxwell Kac.

Maxwell Kac.

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Layout of Thesis.

The following thesis is divided into three chapters, the first of which is a brief outline of basic concepts, which will be employed in the following chapters.

In chapter two, after an outline of K13 decay theory, work done on the experimental and theoretical aspects of K13 decay is presented in sections 2.2 and 2.3.

Chapter three consists of an introductory section on the non-leptonic decays which is followed by a calculation of the S and P wave amplitudes for non-leptonic hyperon decays. This chapter is concluded with a discussion of the results of the calculation and a fairly detailed comparison of this calculation with those of other authors.

CONTENTS.

CHAPTER 1.

REVIEW OF BASIC THEORY.

In the first section the advent of strange particles is briefly reviewed with the concept of strangeness assigned to them. The generalization of SU(2) symmetry to SU(3), to include the concept of strangeness is dicussed in sect., 1.2, and the relation between the matrix and tensor notation used in the description of these symmetries is outlined in preparation for chapter 3. In sect., 1.3. the concept of currents in the framework of unitary symmetry is discussed, which leads naturally to a review, in 1.4, of the Vector-Axial Vector (V-A) current-current (c-c) form of the weak interaction Hamiltonian. Sect., 1.5 deals with the development of the V-A c-c Hamiltonian to the present day phenomenological form.

The Gell-Mann Current Algebra (CA) hypthesis is introduced in sect., 1.6 and it's relevence to the concept of the universality of the weak interactions is reviewed. The ideas of Partial Conservation of the Vector and Axial-Vector Current (PCVC and PCAC resp.,) are also introduced, and will be used extensively along with the Algebra of Currents in Chapters 2 and 3. The final section outlines briefly the idea of symmetry breaking. A specific model is introduced and will be employed in chapter 2.

Section 1.1.

When 'Strange Baryon'particles were first discovered by the characteristic 'V' shaped tracks of their decay products, the absence of leptons in their decay lead theorists to believe their production and decay were similar processes. It was envisaged that the decay of the first discovered 'Strange Baryon', the Λ baryon, was induced by the strong production process $\pi^- + P \longrightarrow \Lambda + \pi_1^{\circ}$.
in the following chain of events, $\Lambda \rightarrow \pi^- + (P' + \pi^{\circ}) \rightarrow \pi^- + P$.
As both of these processes were considered to be strong interaction processes, compatible rates for both were expected.

The paradox of the different rates, of the order of 10^{-22}sec for production, and 10^{-10}sec for the decay, was resolved by Pais(1) in 1952, with the hypothesis of Associated Production. It was suggested that 'Strange Particles' are produced in pairs, and not singly. This necessitated the introduction of 'Strange Mesons', or Kaon Mesons, as they are now called, to give rise, for example, to the associated production reaction. $\pi^- + p \rightarrow \Lambda + K^0$.

In 1958 Gell-Mann and Nishijima (2,3) extended the concept of isotopic spin to 'Strange Particles', to account for the occurance of associated prduction, by introducing the new quantum number, Strangeness(S).

Strangeness was required to be conserved in all strong interactions, with the consequence that the K^0 and Λ were assigned equal and opposite strangeness. The presence of the K^0 now prevented the Λ decay from being a strong inverse process under the requirement of conservation of strangeness. This led to the conclusion of the existence of a new type of weak interaction, in which leptons play no role, which is the Non-Leptonic decay, forming the basis of chapter 3.

Section 1.2.

The requirement that S be conserved in all strong interactions, and that strongly interacting baryons and mesons, of definite spin and parity are now specified by two independent quantum numbers, S and T_3, led Gell-Mann and Nishijima to the generalization of charge independence of nuclear forces(2,3). The strangeness quantum number was introduced by generalizing the relation $Q = I_3 + B/2$, relating electric charge to the third component of isospin, I_3, and the baryon number, B, to the relation $Q = I_3 + \frac{S+B}{2}$. Defining the hypercharge quantum number by $Y = S+B$, the Gell-Mann-Nishijima (GNN) relation can be written $Q = I_3 + Y/2$. 1.1

SU(2), the isotopic spin group, was subsequently generalized to SU(3), whose multiplets are now described by the two independent commuting quantum numbers, S and T_3 . Baryons and mesons are now assigned to the irreducible representations of SU(3), according to strangeness and the third compone nt of isospin associated with the particle. The identification of the $J=\frac{1}{2}^+$ baryons and the $J=0^-$ mesons with the octet or eight-dimensional representation of SU(3) by Gell-Mann and Ne'eman (4), opened the way for higher symmetries to play a definitive role in particle physics.

SU(3), in complete analogy with SU(2) (5 , 6), is the group of all continuous 3×3 unitary unimodualr transformations(5 , 6), which can be represented by the form $U = e^{1/2 \sum_{i=1}^{8} \alpha_i \lambda_i}$, with the 3×3 traceless and hermitian matrices λ_i (7). These matrices can be constructed from nine independent fundamental matrices of the form $a_1 \begin{pmatrix} 1 & 0 & 0 \\ & 0 & \end{pmatrix} + a_2 \begin{pmatrix} 0 & 1 & 0 \\ & 0 & \end{pmatrix} + \cdots$ (6,7). The trace condition allows one of these matrices to be expressed in terms of the remaining eight, and hence there are in all 8 independent λ_i, which we take in the form chosen by Gell-Mann(4 , 8 , 9).

λ_i play the same role as the 2×2 Pauli σ matrices of SU(2)(6), and obey the following commutation relations (CR's) of the SU(3) group.

$$[\lambda_i/2 , \lambda_j/2] = i f_{ijk} \lambda_k/2 \qquad (i,j,k = 1 \dots 8) \qquad 1\cdot 2$$
$$\{\lambda_i/2 , \lambda_j/2\} = 2/3\, \delta_{ij} \underset{\sim}{1} + 2 d_{ijk} \lambda_k$$

Relinquishing the unimodular condition, the SU(3) group becomes the U(3) group. Following Gell-Mann, the trace can be introduced by defining $\lambda_0 = \sqrt{\tfrac{2}{3}}\, \underset{\sim}{1}$, which preserves the CR's, for 'i' now (i=0.....8).Ie, eg

$$\{\lambda_i/2 , \lambda_j/2\} = 2 d_{ijk} ,$$

where the factor $\sqrt{\tfrac{2}{3}}$ arises from the normalization condition

$$\mathrm{Tr}(\lambda_i \lambda_j) = 2\delta_{ij},$$

and f_{ijk}, d_{ijk} are respectively the completely antisymmetric and symmetric structure constants of the SU(3) Lie algebra above, in direct analogy with the completely antisymmetric structure constants e_{ijk} of SU(2).

The smallest non-trivial representation of SU(3) is the 3-D or triplet representation ($\underset{\sim}{3}$), corresponding to the 2-D representation of SU(2). Denoting the basis functions of this representation by the three component spinor χ, invariance under SU(3) symmetry requires

$$\chi \rightarrow \chi' = U\chi \equiv u \chi u^{-1} \qquad 1\cdot 3$$

where the abstract generators F_i of the group are defined by $U = e^{i\alpha_i F_i}$, and correspond to the infinitesimal generators of rotation in F-spin space. The corresponding case for SU(2) are the three abstract generators T_i ($i = 1 \dots 3$) (6),corresponding to the infinitesimal generators of rotations of the isospinors for the 2-D representation, in isotopic spin space, or T-spin space.

In the same way that the σ matrices provide a realization of the

T_i for the 2-D representation of SU(2), the λ_i provide a realization of the F_i in 3-D F-Spin space. From equation 1·3 we find $F_i = \lambda_i/2$. Consequently the CR's obeyed by the F_i are,

$$[F_i, F_j] = i f_{ijk} F_k . \qquad 1\cdot 4$$

Unlike the case of SU(2), the conjugate representation $\underset{\sim}{3}^*$, with basis functions χ^*, is not equivalent to $\underset{\sim}{3}$, as there is no 3×3 matrix S which can effect the similarity transformation $S\lambda S^{-1} = -\lambda^*$, due to the following properties of the λ matrices under conjugation.

$$\left(\lambda^*_{2,5,7} = \lambda_{2,5,7} \ , \ \lambda^*_{1,3,4,5,6,8} = -\lambda_{1,3,4,5,6,8} . \right) \qquad 1\cdot 5$$

SU(3), being a rank 2 group, admits two independent quantum numbers, S and T_3, or equivalently, Y and T_3, whose conservation is implied when the theory is invariant under the transformations of the group. In terms of λ_i this allows two of these to be similtaneously diaganalized. Following Gell-Mann (8), the conventional choice is $\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1&&\\&1&\\&&-2\end{pmatrix}$ & $\lambda_3 = \begin{pmatrix}1&&\\&-1&\\&&0\end{pmatrix}$, whose eigenvalues correspond physically

to the two independent commuting charges or quantum numbers, Y and T_3.

Although no physically known particles can be assigned to the triplet representation (c/f the 2-D representation of SU(2)), it is nevertheless useful to consider the field operators forming the basis of the $\underset{\sim}{3}$ representation as comprising a triplet χ of three equal mass spin $\frac{1}{2}$ Dirac fields, denoted in the liturature as Quarks(10). The assignment of quantum numbers to the components of the triplet χ have been made with the octet representation assignments in mind, and are summarized below,

TABLE. 1.

Q(charge)	B(baryon no.,)	Y(hypercharge)	(third comp of isospin)	χ_i
$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	χ_1
$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	χ_2
$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	χ_3

with opposite signs for χ^*, as a consequence of the properties of λ_i, eq., 1·5, under conjugation, and the GNN. relation, which is required to hold. We will often have cause to refer to the idea of a triplet of

Dirac particles or Quarks, as they are called, at a later stage.

We return now to the CR's $[F_i, F_j] = i f_{ijk} F_k$.

Although true for the 3-D representation of SU(3), these CR's are generally valid for any higher N dimensional representation of SU(3)(11), where the F_i are now N×N matrices, providing a realization of the representation. In particular, the most important representation for our purposes is the 8-D or octet representation (4), in analogy with the $\underset{\sim}{3}$ representation of SU(2), These representations are called the regular representations of the groups SU(3) and SU(2) respectively. The matrices F_i of these representations, are constructed from the structure constants f_{ijk} and e_{ijk} in the following way.

$$F_i \equiv (F_i)_{jk} = -i f_{ijk} \; ; \quad (ijk = 1 \cdots 8) \qquad T_i \equiv (T_i)_{jk} = -i e_{ijk}. \quad (ijk = 1, \cdots 3) \qquad 1\cdot 6$$

In SU(2), the triplet φ_i $(i = 1 \cdots 3)$, of eg., the pion fields, form the basis of the 3-D representation, and transform as components of a 3-vector under rotations in isospin space. In an analogous way, the basis of $\underset{\sim}{8}$ is regarded as an octet of operator fields, φ_i $(i = 1 \cdots 8)$, and is said to transform according to the regular representation, as a vector in the abstract 8-D F-spin space, if $\varphi_i \longrightarrow \varphi_i' = U \varphi_i U^{-1}$,

where $U = e^{i\alpha_i F_i}$ $(i = 1 \cdots 8)$. Under infinitesimal transformations this implies $[F_i, \varphi_j] = i f_{ijk} \varphi_k$ $(ijk = 1 \cdots 8)$,which henceforth defines 1·7

the properties of φ_i given above, for $F_i \equiv (F_i)_{jk}$.

The 8 generators, F_i, correspond to the 8 components of F - spin. With Gell-Mann's choice of the λ matrices (4), $\lambda_i = \left(\begin{array}{c|c} \tau_i & 0 \\ \hline 0 & 0 \end{array}\right)$ $(i = 1 \cdots 3)$,

and hence F_i $(i = 1 \cdots 3)$ can be put in correspondence with the three components of isospin T_i $(i = 1 \cdots 3)$, of the three-D representation of SU(2). Further, in complete amalogy with SU(2), the raising and lowering operators $T_\pm = F_1 \pm i F_2$ can be introduced. From the SU(2) CR's $[T_i, T_j] = i \varepsilon_{ijk} T_k$, the following relations are obeyed,

$$[F_3, T_\pm] = \pm T_\pm \quad , \quad [T_+, T_-] = 2 F_3 . \qquad 1\cdot 8a$$

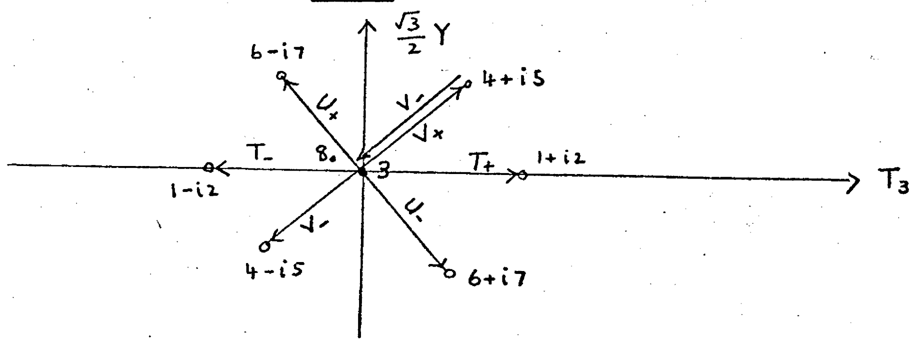
Generalizing these relations to the case of the 8 generators of SU(3), it is possible to construct the operators

$U_\pm = F_6 \pm i F_7$, $V_\pm = F_4 \pm i F_5$ and again $T_\pm = F_1 \pm i F_2$, 1·8b

with $T_3 = F_3$, $Y = \frac{2}{\sqrt{3}} F_8$, $V_3 = \frac{1}{2}(T_3 + \frac{3}{2} Y)$ and $U_3 = \frac{1}{2}(T_3 + \frac{3}{2} Y)$.

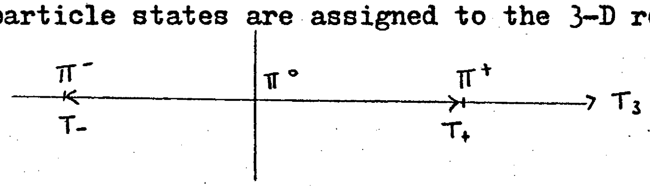
The pairs $U_\pm$, U_3 , $V_\pm$, V_3 , and $T_\pm$, T_3 . form sets of SU(2) algebras, and obey the CR's, eq., 1·8a (6, 12), and can be used effectively to analise the properties of SU(3) by appealing to the known results of SU(2)(12). The action of $T_\pm$, $U_\pm$,and $V_\pm$ as raising and lowering operators, on any one of the states belonging to an SU(3) multiplet, generates all states of an irreducible representation by repeated application (11,12). In this way the octet representation can be diagramatically displayed(Fig 1), where the action of the operators are as indicated. $U_\pm$ and $V_\pm$ act on states, corresponding to the eigenvalues of V_3 and U_3 respectively.

FIG 1.



Returning now to the basis functions of the 3-D representation of SU(2), the φ_i $(i = 1..3)$ are related to the $\pi^\pm$ and π° meson field operators by the following relations. $\pi^\pm = \frac{1}{\sqrt{2}}(\varphi_1 \mp i\varphi_2)$, $\pi^\circ = \varphi_3$.

Employing fig 1, these particle states are assigned to the 3-D representation of SU(2) thus,



The assignment corresponds to the action of the isospin raising and lowering operators $T_\pm$ on the isospin state $|\pi^\circ\rangle$. Eg., $T_+|\pi^\circ\rangle = |\pi^+\rangle$, for SU(2) symmetry holding. π^+ is defined as the complex meson field operator that creates π^- and destroys π^+ , with the quantum number assignment of the 1+i2th state of the 3-D SU(2) representation.

Employing fig.,1, Levinson et al(12) have shown the SU(2) algebras formed by $V_\pm$, V_3, $U_\pm$, and U_3 can be used directly to generalise the above results to SU(3). For the $J = 0^-$ mesons, the results are summarised below.

FIG 2.

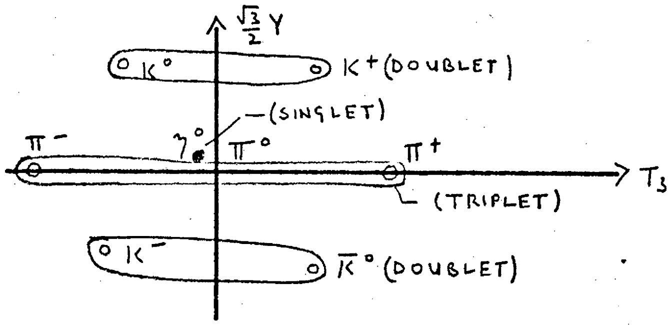


TABLE 2.

	S	T	T_3	Mass.(MEV). (13).
K^+	1	$\frac{1}{2}$	$\frac{1}{2}$	493.82.
K°	1	$\frac{1}{2}$	$-\frac{1}{2}$	497.76.
K^-	-1	$\frac{1}{2}$	$-\frac{1}{2}$	493.82.
$\bar{K}^\circ$	-1	$\frac{1}{2}$	$\frac{1}{2}$	497.76.
π^+	0	1	1	139.578.
π^-	0	1	-1	139.578.
π°	0	1	0	134.974.
η	0	0	0	548.8.

The connection between the octet representation, $\underset{\sim}{8}$, and the $\underset{\sim}{3}$ and $\underset{\sim}{3}^*$ representations can be seen in analogy with the 2x2 σ matrices of SU(2), where the 3x3 λ matrices can be employed to construct 8 Pseudo-scalar (PS) objects $\varphi_K = i\chi^\dagger\gamma_5\lambda_K\chi \equiv i(\lambda_K)_{ij}\chi_i^*\gamma_5\chi_j$ $\begin{smallmatrix}(i,j=1..3)\\(K=1..8)\end{smallmatrix}$, 1.9

from the triplets χ_i , χ_i^* , which form a basis for the regular representation. This corresponds to the group theoretic result of decomposing the direct product of triplet representations into irreducible representations, in the following way, $\underset{\sim}{3}\otimes\underset{\sim}{3} = \underset{\sim}{1} + \underset{\sim}{8}$, (6,14). $\underset{\sim}{1}$ is the trivial 1-D representation, corresponding to the trace $\lambda_0=\sqrt{\frac{2}{3}}\begin{pmatrix}1&&\\&1&\\&&1\end{pmatrix}$.

An important notation that arises naturally from the considerations of $\underset{\sim}{3}$ and $\underset{\sim}{3}^*$, and which will be employed later in chapter 3, is the tensor notation(15), used extensively by Okubo(15). The relationship between Gell-Mann's 8 3x3 hermitian operators F_i ($F_i = \lambda_i/2$, $i = 1..8$), and the 8 tensor generators F_a^b($a, b = 1..3$), is given by the relation(11,16),

$$F_a^{\;b} = \sum_{i=1}^{8}(\lambda_i)_{ab}\,F_i . \qquad 1.10$$

Denoting now χ^* and χ as the contravariant and covariant vectors χ^a and χ_a respectively, the (eg.,P-scalar meson octet) is given by the traceless tensor $P_a{}^b = \chi_a \gamma_5 \chi^b - \frac{1}{3}\delta_a{}^b \chi^c \gamma_5 \chi_c$, corresponding 1·11
to equation 1·9 , by employing eq.,1·10 ,and the tracelessness of the λ matrices. Hence we also can write $P_a{}^b = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} (\lambda_i)_{ab} \varphi_i$. 1·12
The group theoretic result $\underset{\sim}{3} \otimes \underset{\sim}{3}^* = \underset{\sim}{1} + \underset{\sim}{8}$ (14) is explicit in the tensor notation, by considering all possible ways of contracting the tensor indices, and forming traceless tensors. The trace term corresponds to the $\underset{\sim}{1}$ representation.

As the quantum numbers are additive under the direct product, in order to form an octet of spin $\frac{1}{2}$ baryons, it is necessary to consider the direct product of three triplets(all contravariant or all covariant), to give B =-1 and + 1 respectively, from the quantum number assignment of the triplets(Table 1). In the tensor notation the baryon octet is reached from $T_{abc} = \chi_{abc}$ (or $T^{abc} = \chi^{abc} \not\equiv T_{abc}$, due to opposite baryon number) through reduction by contraction of indices.

$$B_a{}^b = \epsilon^{b\lambda\mu} T_{a\lambda\mu} - \frac{1}{3}\delta_a{}^b \epsilon^{\mu\nu\lambda} T_{\mu\nu\lambda}\text{, where } T_a{}^a = \frac{1}{3}\delta_a{}^b \epsilon^{\mu\nu\lambda} T_{\mu\nu\lambda}.$$

corresponds to the baryon singlet. The contraction of indices is equivalent in this case to the reduction of the direct product $\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3}$ to yield $\underset{\sim}{10} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{1}$ (11) . Unlike the meson octet, the baryon octet is not self conjugate. ie $\underset{\sim}{3} \otimes \underset{\sim}{3}^* \equiv \underset{\sim}{3}^* \otimes \underset{\sim}{3}$, but $\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} \not\equiv \underset{\sim}{3}^* \otimes \underset{\sim}{3}^* \otimes \underset{\sim}{3}^*$. (5, 11) . Similarly, as for the meson octet, the relation between the baryon octet $B_a{}^b$ and the baryon fields is

$B_a{}^b = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} (\lambda_i)_{ab} \psi_i$, whose relation to the physical particle field operators is given by $\Sigma^{\pm} = \frac{1}{\sqrt{2}}(\psi_1 \pm i\psi_2)$, $\Sigma^0 = \psi_3$,
$p = \frac{1}{\sqrt{2}}(\psi_4 - i\psi_5)$, $n = \frac{1}{\sqrt{2}}(\psi_6 - i\psi_7)$, $\Xi^- = \frac{1}{\sqrt{2}}(\psi_4 + i\psi_5)$
$\Xi^0 = \frac{1}{\sqrt{2}}(\psi_6 + i\psi_7)$, $\Lambda = \psi_8$,
and similarly for the antibaryon octet $\bar{B}_a{}^b$. Explicitly, in tensor notation the PS and baryon octets are,

$$P_a{}^b = \begin{pmatrix} \frac{1}{\sqrt{6}}\eta^0 + \frac{1}{\sqrt{2}}\pi^0 & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{6}}\eta^0 - \frac{1}{\sqrt{2}}\pi^0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix} \qquad 1\cdot 13a$$

$$B_a{}^b = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} \qquad 1\cdot 13b$$

Although the triplet representation is the most basic representation of the SU(3) group, the philosophy of the Eight-fold-way (4, 8) is the hypothesis that the $\underset{\sim}{8}$ representation forms the basis of all physically realizable particle multiplets. Ie, apart from the assignment of zero strangeness and zero isotopic spin particles to the singlet representation, the smallest irreducible representation into which physically known particles can be assigned, is the octet. The higher spin particles are then assigned to the multiplets of higher dimensional representations, formed from the direct product of octets, in an exactly analogous way to the methods used above with the 3-D representations.

SECTION 1.3.

After the discovery of SU(3) symmetry(4)in particle physics, it became apparent that SU(3)furnished a unified basis for describing both electromagnetic and weak interactions through the advent of 'Current Algebra'(17, 18, 19, 5).

In the SU(3) symmetry scheme, the weak and electromagnetic interactions are described by the same octets of currents, $J^j_\lambda(x)$ $(j = 1 \,..\, 8)$ $(\lambda = 0, \cdots 3)$, where j is the unitary spin index, and λ is the space-time index. The postulate that J^j_λ form an octet of currents in the framework of SU(3) symmetry requires $[F_i, J^j_\lambda(x)] = i f_{ijk} J^k_\lambda(x)$. 1·14

(c/f eq., 1·7 for Q_j), with $F_i = (F_i)_{jk} = -if_{ijk}$.

The idea of an octet of currents is made clearer by returning to the octets of baryon fields χ_i. Under SU(3), the χ_i transform as vectors in the 8-D F-spin space. Using the structure constants f_{ijk} and d_{ijk}; antisymmetric and symmetric linear combinations of the octets χ_j can be formed, to give(11, 7) $O^A_i = -if_{ijk}\bar{\chi}_j \chi_k$, $O^S_i = d_{ijk}\bar{\chi}_j \chi_k$, 1·15

which have the same transformation properties as χ_i under SU(3), and likewise form a basis for the regular representation. f_{ijk} and d_{ijk} act as Clebsch-Gordon coefficients(11) for the antisymmetric and symmetric $\underset{\sim}{8}$ representations in the direct product decomposition

$$\underset{\sim}{8} \otimes \underset{\sim}{8} = \underset{\sim}{1} + \underset{\sim}{8}_a + \underset{\sim}{8}_s + \underset{\sim}{10} + \underset{\sim}{10}^* + \underset{\sim}{27}, \quad (11, 14), \qquad 1\cdot16$$

to which O^A and O^S belong respectively.

As SU(3) symmetric transformations are only enacted upon the unitary symmetry indices $i\ j\ k$, these 'bilinear operators' can be generalized with the aid of the 16 independent elements of the Dirac algebra, Γ_A ($\Gamma_A \equiv 1, \gamma_\mu, \sigma_{\mu\nu}, i\gamma_\mu\gamma_5, \gamma_5$) for the scalar(S) , vector (V) , antisymmetric tensor(T), axial vector(A) and pseudoscalar(PS) elements respectively. As the bilinears (eq., 1.15) transform as scalars under the inhomogeneous proper orthochronous Lorentz group $L_+^\uparrow$, and the operations of space reversal(P) and time reversal(T), the corresponding quantities

$O_i^A = -i f_{ijk} \bar{\chi}_j \Gamma_A \chi_k$, $O_i^S = d_{ijk} \bar{\chi}_j \Gamma_A \chi_K$ transform as bosons 1.17

under $L_+^\uparrow$,P and T with the tensor character of Γ_A (5,11). These objects are also octets, and hence it is possible to construct with these, octets of vector and axial-vector currents, in the following way(16,20)

$$V_i^\lambda(x) = F'\left(-i f_{ijk} \bar{\chi}_j i\gamma_\lambda \chi_K(x)\right) + D'\left(d_{ijk} \bar{\chi}_j(x) \gamma_\lambda \chi_K(x)\right) \quad 1.18a$$

$$A_i^\lambda(x) = F\left(-i f_{ijk} \bar{\chi}_j i\gamma_\lambda\gamma_5 \chi_k(x)\right) + D\left(d_{ijk} \bar{\chi}_j(x) \gamma_\lambda\gamma_5 \chi_k(x)\right), \quad 1.18b$$

where F',D',F and D give the relative amounts of antisymmetric and symmetric coupling of the baryon octets to form the current. There are in all 5 independent currents J_i^λ , where the term current will henceforth refer to the Dirac bilinears, unless specified otherwise.

Another model which we will have occaision to use later, is the quark model. Generalizing $\varphi_i = i(\lambda_i)_{jk} \chi_j^* \chi_k$ by introducing the Γ_A matrices as above, an octet of objects having the same space-time properties as eq's 1.17 can be constructed. In this model the octet of currents belong to the octet representation of the direct product decomposition $\underset{\sim}{3} \otimes \underset{\sim}{3}^*$. This model is used extensively by Sakurai(21).

Section 1.4. Weak interaction Hamiltonian.

Currents were first introduced into weak interactions by Fermi(22) in 1934, who proposed that the most general weak interaction Hamiltonian governing the weak β decay $n \to p + e^- + \bar{\nu}_e$, which is also invariant under the space-time symmetry groups $L_+^\uparrow$,P and T, is given by

$$H_\beta = \sum_{A=1}^{5} g_A \left\{ (\bar{\chi}_p \Gamma_A \chi_n)(\bar{\chi}_e \Gamma_A \chi_{\nu_e}) + h.c. \right\}, \quad 1.19$$

where G_A are the weak coupling- constants, and in the present day terminology of SU(3) symmetry, $\chi_n \equiv \chi_6 + i\chi_7$ etc., and the Γ_A are as defined in eq., 1.17 . Fermi originally conjectured that only the vector currents contributed to H_β, although the possibility of S,T,A, and P currents was not excluded.

By 1956, studies of the shapes of the beta spectra and the angular correlation measurements in the Gamov-Teller transition $He^6 \rightarrow Li^6 + e^- + \bar{\nu}_e$ (23) favoured the combination of S and T currents for the weak interaction Hamiltonian. On the other hand, measurements of the e^- spectrum in muon decay, $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$, confirmed the choice of V and A currents for H_μ.

With the leptonic decay of the pion requiring an A or P current to explain the interaction, the possibliity of a universal current-current weak interaction did not become clear until the suggestion of non-conservation of parity in weak interactions, by Lee and Yang (24) in 1956. The non-conservation of parity was promptly verified by a series of experiments suggested by Lee and Yang(24), including the e^-assymmetry from a polarized nucleus in β decay(25), and the e^-assymmetry in the pion decay(26). The emmission of the neutrino in the parity break-down experiments led to the observation that the neutrino may play an important role in parity violation. It was suggested that the neutrino be described by the two component Weyl equation (27), rather than the 4-component Dirac equation. This formulation automatically explained the zero mass of the neutrino and built in parity and charge conjugation non-conservation, while maintaining CP invariance, and consequently time-reversal invariance.

As the neutrino has zero mass it's spin direction is either parallel or antiparallel to it's direction of momentum. The two helicity states, as they are called in the two component neutrino theory, are projected out by the positive and negative chiral projections $\chi_\pm = \left(\frac{1 \pm \gamma_5}{2}\right)\chi$, where χ is the Dirac 4-component spin-$\frac{1}{2}$ field for zero mass particles, and the chiral projection operators $1 + \gamma_5$ and $1 - \gamma_5$ each select out two components of ψ to give ψ_+ and χ_- respectively(16), with negative and positive helicity respectively, in the notation of Källen (28).

Both the validity of parity non-conservation and the introduction of the two-component neutrino theory led to the explanation of the backward e^- assymmetry from the μ decay, with either positive or negative helicity of the neutrino(29) for a V-A current-current interaction. On the other hand the parity break-down experiments in nuclear β decay could be explained by either an S-T combination of currents with positive helicity (23), or a V-A combination of currents with negative helicity (30).

The principle of 'chirality invariance' put forward by Sudarshan and Marshak (31) in 1958, led to a theoretical principle which favoured the V-A combination, and hence, negative helicity, as per the nuclear β decay experiments. The proposition was that under the chirality transformation, γ_5 , $\psi(x) = \gamma_5 \psi(x)$, Ie invariance under chirality, and hence, as parity is not conserved, the current-current interaction is indifferent to the choice of ψ or $\gamma_5 \psi$, and ψ could now be an arbitary Dirac spinor describing both massless and massive spin-$\frac{1}{2}$ particles. This led finally to a unique V-A current-current weak Hamiltonian

$$H_w = -\frac{G}{\sqrt{2}} \left\{ \left[\bar{\psi}_a \gamma_\lambda (1+\gamma_5) \psi_b \right] \left[\bar{\psi}_c \gamma_\lambda (1+\gamma_5) \psi_d \right] + h.c. \right\}, \quad 1.20$$

where the covariant bilinears $i\bar{\psi}_a \gamma_\lambda \psi_b$ and $i\bar{\psi}_a \gamma_\lambda \gamma_5 \psi_b$ are the vector and axial-vector currents respectively, and the $1/\sqrt{2}$ factor is conventional, with G, the weak interaction constant given by $G \approx 1.026 \times 10^{-5}/M^2_{PROTON}$. The universality of weak interactions (32) means simply that G is defined as the universal strength of all weak interactions described by H_w.

The advent of SU(3) symmetry, along with the experimental absence of neutral leptonic and hadronic currents(16) in the weak Hamiltonian, led to the form $H_w = -\frac{G}{\sqrt{2}} \{ J_\lambda, \bar{J}^\lambda \}$ as the universal weak interaction, with $J_\lambda = A_\lambda + V_\lambda + L_\lambda$, and similarly for $\bar{J}_\lambda$. ($\bar{J}_\lambda \stackrel{def^n}{=} J_\lambda^+ (1 - 2\delta_{\lambda 4})$ as $J_\lambda = (\underset{\sim}{J}, iJ_o)$). A_λ and V_λ are the charged hadronic currents, $L_\lambda = \psi_{\bar{\nu}_\mu} \gamma^\lambda (1+\gamma_5) \psi_\mu + \psi_{\bar{\nu}_e} \gamma^\lambda (1+\gamma_5) \psi_e$ is the leptonic current, and $J^h_\lambda = V_\lambda + A_\lambda$ is the total hadronic current. 1.21 1.22

Writing H_w out in full, the various components of H_w are;

$H_{leptonic} = -\frac{G}{\sqrt{2}} \{ L_\lambda, \bar{L}_\lambda \}$ -Purely leptonic weak interactions. 1.23a

$H_{semi-leptonic} = -\frac{G}{\sqrt{2}} \{ L_\lambda, \bar{J}^h_\lambda \} + h.c$ -Semileptonic weak interactions(Chpt.2.) 1.23b

$H_{non-leptonic} = -\frac{G}{\sqrt{2}} \{ A_\lambda \bar{A}_\lambda + V_\lambda \bar{V}_\lambda + A_\lambda \bar{V}_\lambda + V_\lambda \bar{A}_\lambda \}$, (Chpt.3.) 1.23c

where $\{ - \}$ denotes the symmetrized product of currents.

In 1958 the conserved vector current (CVC) hypothesis was introduced by Gell-Mann and Feynman (32) to explain the near equality of the observed values of the muon decay constant G_μ and the vector coupling constant g_V, occuring in the muon and neutron β decays $\mu \rightarrow e + \nu$, $n \rightarrow p + e + \nu$, respectively (5, 32).

It was originally expected that the vector coupling constant would be renormalized by the strong interactions, resulting from the presence of the strongly interacting hadrons in the semi-leptonic decay in which g_V occurs, whereas G_μ, being the decay constant of a purely leptonic decay, is only subject to electromagnetic corrections. This situation is analogous to the equality of the renormalized (observed) charges of the proton and electron, provided the bare charges are equal, and which is a consequence of the divergence free nature of the electromagnetic current J_λ^{em}. Defining the hypercharge current by $V_\lambda^8 = \frac{\sqrt{3}}{2} Y_\lambda$, and the third component of isospin current by $V_\lambda^3 = I_\lambda^3$, where V_λ^3 is a member of the isotopic spin currents $V_\lambda^i = I_\lambda^i$ $(i = 1..3)$ of SU(2), J_λ^{em} can be constructed from these currents in the form $J_\lambda^{em} = V_\lambda^3 + V_\lambda^8/\sqrt{3}$.

The requirement of the equality of the renormalized charges implies $\partial_\lambda J^\lambda_{em} = 0$, and hence $\partial_\lambda V_3^\lambda = 0$, where $\partial_\lambda V_i^\lambda (i = 1, 2, 3) = 0$ for SU(2) symmetry holding.

Gell-Mann and Feynman proposed that the vector currents responsible for β decay ($V_\lambda^{1 \pm i2}$) be identified with the 1±i2th components of the divergenceless isospin currents of SU(2), hence automatically ensuring the equality of G_μ and g_V up to corrections of the order of the electromagnetic interaction.

From eq., 1·18a, that $\partial_\lambda V_i^\lambda (x) = 0$ $(i = 1, 2, 3)$, implies, with the use of the Dirac equation,

$$- F' f_{ijk} [m_k - m_j] \bar{\chi}_j \chi_k - D' d_{ijk} [m_k + m_j] \bar{\chi}_j \chi_k = 0 \qquad 1\cdot24$$

Hence for $m_k = m_j$, $D' = 0$. The neglect of mass differences for the hadrons m_k and m_j in the same isotopic spin multiplet is equivalent to neglecting electromagnetic mass-splitting effects. To this order then, V_λ^i $(i = 1, 2, 3)$ are purely F-type currents, and are divergenceless. For pion β decays the divergence is proportional to the $\pi^\pm - \pi^\circ$ mass difference, and the P-N mass difference for the neutron β decays.

Section 1.5. Cabibbo Phenomenological V-A Weak Hamiltonian.

The bare V-A universal weak interaction Hamiltonian H_W underwent a substantial further interpretation in 1963 by Cabibbo(33), in the framework of SU(3) unitary symmetry.

Cabibbo proposed that the charged hadronic currents occuring in eq.1.21 belong respectively to an octet of vector and axial-vector currents. Thus

$$A_\lambda = (A_1 + iA_2)_\lambda + (A_4 + iA_5)_\lambda \qquad 1\cdot25$$

and $$V_\lambda = (V_1 + iV_2)_\lambda + (V_4 + iV_5)_\lambda,$$

where these are formed from the 1+i2 and 4+i5 components of an octet, with the corresponding transformation properties under SU(3) holding, as discussed in sect., 1.2. The electromagnetic interactions, as alluded to earlier, are described by J_λ^{em}, belonging to the same octet of vector currents as occuring in the semi-leptonic and non-leptonic weak interactions. The explicit connection between J_λ^{em} and the octet of vector currents is through the CVC hypothesis.

Up to this time it had been observed that the strangeness changing semi-leptonic (sl) decays induced by J_λ^{4+i5}, were an order of magnitude slower than the strangeness conserving sl. decays induced by J_λ^{1+i2}, contrary to the predictions of the bare V-A C-C weak interaction Hamiltonian.

To preserve the universality of strength of the weak interactions (ie the sl. decays having the same overall strength as the leptonic and non-leptonic decays), Cabibbo proposed to introduce the supression of the strength of the strangeness changing sl. decay by writting the total hadron current $J_\lambda^h = A_\lambda + V_\lambda$ in the following phenomenological form. 1·26

$$J_\lambda^h = \cos\Theta_c\,(J_1 + iJ_2)_\lambda + \sin\Theta_c\,(J_4 + iJ_5)_\lambda.$$

The requirement that Θ_c be the same for both $\Delta S = 0$ and $\Delta S \neq 0$ components, automatically ensures the universality of strength of the weak interactions by maintaining the equality of strength of the total sl. weak interactions with the leptonic and non-leptonic, through the relation $g^2\sin^2\Theta_c + g^2\cos^2\Theta_c = g^2$, for the strength of the sl. interactions.

Section 1.6. C.A. Hypothesis.

Dynamics was originally introduced into the group theoretic structure by Gell-Mann (8,9) through the 'Current Algebra' hypothesis (9,17). The idea proposed was that the underlying algebraic structure of the group is more fundamental than the group symmetry itself, and that though the symmetry may be broken(through the mass differences in the SU(3) multiplets), the commutation relations between the generators(eq., 1·4) continue to hold as equal-time commutators (ETC's). When the symmetry is broken, equation 1·4 is generalized to $[F_i(t), F_j(t)] = i f_{ijk} F_k(t)$. 1·27 a

The above hypothesis is made more powerful physically by the introduction of charges. From the definition of a charge

$$Q^i = \int J_0^i(x)\, d^3\underset{\sim}{x} = -i\int J_4^i(x)\, d^3\underset{\sim}{x}$$ 1·28

we can define, from the electromagnetic current, the electric charge Q in the following way. $Q = \int J^0_{em}(x)\, d^3\underset{\sim}{x} = \int V_3^0(x)\, d^3\underset{\sim}{x}$ 1·29

$$+ \int \frac{1}{\sqrt{3}} V_8^0(x)\, d^3\underset{\sim}{x} = I_3 + Y/2$$

(from the GNN relation). Now $F_3 \leftrightarrow T_3$ and $F_8 \leftrightarrow \frac{\sqrt{3}}{2} Y$, and hence quite generally we can relate the generators F_i of SU(3) to the physical charges by defining $F_i = \int V_i^0(x)\, d^3\underset{\sim}{x}$, for SU(3) 1·30
symmetry holding, with $\partial_\mu V_i^\mu = 0$ and $dF_i/dt = 0$. For broken SU(3) we have $F_i(t) = \int V_0^i(x)\, d^3\underset{\sim}{x}$ where $\partial_\mu V_i^\mu \neq 0$, and the corresponding charges now acquire a time dependence, governed by the dynamics of the system. $dF_i/dt \neq 0$, and charge is not conserved.

With the hypothesis of an octet of axial-vector currents A_λ and their corresponding charges, $F_i^5(t)$, the hypothesis was extended by Gell-Mann to include the commutation relations

$$[F_i(t), F_j^5(t)] = i f_{ijk} F_k^5(t)$$ 1·27 b

$$[F_i^5(t), F_j^5(t)] = i f_{ijk} F_k(t)$$ 1·27 c

Although 1·27a,b are true in any reasonable field theory(34), 1·27c is more restrictive. Theories which do obey 1·27c are the quark current model and the octet baryon current model , discussed above.

The octet transformation properties of $A_\lambda^i(x)$ and $V_\lambda^i(x)$ ($J_\lambda^i(x)$), require the relation $[F_i , J_j^\lambda(x)] = i f_{ijk} J_k^\lambda(x)$ to hold,

which in the presence of symmetry breaking, is correspondingly generalized to $[F_i(t), J_j^\lambda(x)] = i f_{ijk} J_k^\lambda(x)$. To fix the scale 1·32 a
of the axial vector current it was postulated in addition(8, 9)that the following CR's also hold.

$$[F_i^5(t), V_j^\lambda(x)] = i f_{ijk} A_k^\lambda(x)$$ 1·32 b

$$[F_i^5(t), A_j^\lambda(x)] = i f_{ijk} V_k^\lambda(x)$$ 1·32 c

These CR's allow a precise statement of the hypothesis of the universality of the strength of the weak interaction. Gell-Mann's version of universality, from the once integrated CR's (eq. 1·32) requires that the hadronic charges defined by $F_h^+ = \frac{1}{2}\int d^3\underset{\sim}{x}\, J_h^0(x)$, with $F_h^- = F_{h^+}^\dagger$ (for both V_λ and A_λ charged currents occuring in H_w) and $F_h^3 = \frac{1}{2}\int d^3\underset{\sim}{x}\, J_{h0}^3(x)$, satisfy the same SU(2) CR's as do the leptonic charges $F_{l^+} = \frac{1}{2}\int d^3\underset{\sim}{x}\, L^0(x)$ with $F_{l^-} = F_{l^+}^\dagger$.

ie $[F_{h^+}, F_{h^-}] = 2F_h^3$, $[F_h^3, F_{h^\pm}] = \pm F_h^\pm$ imply

$[F_{l^+}, F_{l^-}] = 2F_l^3$, $[F_l^3, F_l^\pm] = \pm F_l^\pm$. These CR's fix the relative scale of the hadronic and leptonic charged currents, by restricting F_h^3 to have the same spectrum of eigenvalues as F_l^3, with $F_h^\pm$ and $F_l^\pm$ both having well defined normalisation with the same strength (35).

Further useful CR's are obtained by defining the chiral currents

$$J_\lambda^{Li}(x) = \frac{1}{2}\left(V_\lambda^i(x) + A_\lambda^i(x)\right) \text{ and } J_\lambda^{Ri}(x) = \frac{1}{2}\left(V_\lambda^i - A_\lambda^i(x)\right), \qquad 1\cdot33$$

so called because of the implicit presence of the chirality projection operators $1 \pm \gamma_5$ respectively. Under equal-time-commutation, the time components of these currents form two separate SU(3) algebras(eq. 1·4)

$$[J_i^{L0}(x), J_j^{L0}(y)]\big|_{x^0=y^0} = i\delta(\underset{\sim}{x}-\underset{\sim}{y})f_{ijk}J_k^{L0}(x) \qquad 1\cdot34a$$

$$[J_i^{R0}(x), J_j^{R0}(y)]\big|_{x^0=y^0} = i\delta(\underset{\sim}{x}-\underset{\sim}{y})f_{ijk}J_k^{R0}(x) \qquad 1\cdot34b$$

with $$[J_i^{L0}(x), J_j^{R0}(y)] = 0. \qquad 1\cdot34c$$

This is the Lie algebra of the direct product group $SU(3)_L \otimes SU(3)_R$, which readily follows by taking the appropriate linear combinations of the CR's 1·32 a, b and c . $SU(3)_L$ and $SU(3)_R$ are the groupsof tranformations defined by (c/f eq. 1·3) $U_L = e^{\frac{1}{2}i\sum_{j=1}^{8}(\lambda_j(1+\gamma_5))\alpha_j}$ and $U_R = e^{\frac{1}{2}i\sum_{j=1}^{8}(\lambda_j(1-\gamma_5))\alpha_j}$ respectively. In the baryon octet current model, J_L transforms as a member of $\underset{\sim}{8}$, and J_R as $\underset{\sim}{1}$ under $SU(3)_L$, and vica-versa for $SU(3)_R$.

It will be important ,when discussing the symmetry breaking model (sect., 1·7) below to consider the quark model, for which J_L and J_R are defined as

$$(J_L)_i^\lambda = i\bar{\chi}\gamma^\lambda\left(\frac{1+\gamma_5}{2}\right)\frac{\lambda_i}{2}\chi \qquad 1\cdot35$$

$$(J_R)_i^\lambda = i\bar{\chi}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)\frac{\lambda_i}{2}\chi$$

$$\doteq (V_R)^\lambda - (A_R)^\lambda$$

Hence J_R+J_L and J_R-J_L transform as members of an SU(3) multiplet of V and A currents respectively. Making the identifications

$$\chi_L = \tfrac{1}{2}(1+\gamma_5)\chi \quad , \quad \bar{\chi}_L = \tfrac{1}{2}\bar{\chi}(1-\gamma_5)$$
$$\chi_R = \tfrac{1}{2}(1-\gamma_5)\chi \quad , \quad \bar{\chi}_R = \tfrac{1}{2}\bar{\chi}(1+\gamma_5) \qquad 1\cdot36$$

with $\chi = \chi_L + \chi_R$ etc., J_L can be written in the form $i\bar{\chi}_L\gamma^\lambda \frac{\lambda_i}{2}\chi_L$, and similarly for J_R.

In this model the free quark Lagrangian(8,17,21) $L = -\bar{\chi}(\gamma_\mu\partial_\mu + m)\chi$ has definite transformation properties under SU(3)⊗SU(3). The kinetic term

$$-\bar{\chi}\partial_\mu\gamma_\mu\chi = -\bar{\chi}_L\gamma_\mu\partial_\mu\chi_L - \bar{\chi}_R\gamma_\mu\partial_\mu\chi_R \qquad 1\cdot37$$

transforms like $(\underset{\sim}{1},\underset{\sim}{1})$, which in the notaton of Gell-Mann denotes (Right-hand SU(3) repr., , Left-hand SU(3) repr.,). In this case the right and left hand components of $-\bar{\chi}\partial_\mu\gamma_\mu\chi$ transform as scalar singlets under SU(3), and are thus invariant under SU(3). On the other hand the mass term $-m\bar{\chi}\chi = -m(\bar{\chi}_R\chi_L + \bar{\chi}_L\chi_R)$ transforms according to the $(\underset{\sim}{3}^*,\underset{\sim}{3}) + (\underset{\sim}{3},\underset{\sim}{3}^*)$ representation of SU(3)⊗SU(3), but is not invariant under the transformations of SU(3)⊗SU(3), due to the presence of the mass. Hence this term breaks the chiral symmetry.

This led Gell-Mann(8) to investigate the behaviour of the nonets of S and PS quark densities $u_i = \bar{\chi}\lambda_i\chi$ and $v_i = -i\bar{\chi}\lambda_i\gamma_5\chi$, which transform as the mass term under chiral symmetry, and whose consideration was motivated by the search for a symmetry breaking mechanism in the frame-work of unitary symmetry(Sect., 1.7).

Assuming the correctness of the SU(3)⊗SU(3) ETC's for the V_λ and A_λ charges, F_i and F_i^5 respectively(eq. 1·27), defined by $F_i^5 = \int \bar{\chi}\frac{\lambda_i}{2}\gamma_4\gamma_5\chi \, d^3\underset{\sim}{x}$ etc., in the quark current model, the CR's between the charge operators and the quark densities can be written directly as

$$[F_i, u_j] = i f_{ijk} u_k \quad , \quad [F_i, v_j] = i f_{ijk} v_k \qquad 1\cdot38.$$
$$[F_i^5, u_j] = -i d_{ijk} v_k \quad , \quad [F_i^5, v_j] = i d_{ijk} u_k$$
$$(i, j, k = 1 \ldots 8).$$

An important subalgebra of the SU(3)⊗SU(3) Lie algebra above, is the algebra formed from the 1,2,and 3 components of the vector and axial-vector currents and charges.This is the Lie algebra of the group SU(2)⊗ SU(2). In the limit of SU(2)⊗SU(2) symmetry, $\partial_\lambda A_i^\lambda = 0$ (i= 1,2,3), and $\partial_\lambda V_i^\lambda = 0$(i 1,2,3),which also is a consequence of the CVC hypothesis. In the real world SU(2)⊗SU(2) is only an approximate symmetry, and the axial-vector currents are only partially conserved.

The Partial Conservation of the Axial-Vector Current (PCAC) was first introduced by Nambu(36), and further employed by Gell-Mann and Levy (37), to clarify and justify the derivation of the Goldberger-Treiman (GT) relation(38), relating the pion decay constant f_π, to the nucleon axial-vector coupling constant g_A, ie., $f_\pi \simeq 2M_N g_A / G_{NN\pi}$. Explicitly it was postulated (37) that $\partial_\mu A_\mu^i(x) = f^i (m^i)^2 \varphi^i(x)$, where $\varphi^i(x)$ is the octet of $J = 0^-$ meson field operators, and f_i is the strength of the ith PS meson hadron decay matrix, defined by $\langle 0 | \partial_\mu A_\mu^j(0) | \varphi^i \rangle = \delta_{ij} f_i m_i^2 / (2\pi)^{3/2} \sqrt{2} \sqrt{2q^0}$. 1.39

The PCAC condition can be interpreted in the sense of the Haag-Nishijima-Zimmermann theorem(39), which states that the divergence of the A_λ-current can be used to define a local pion field operator, in correspondence with the discussion of the Dirac bilinears in sect., 1.2.

This relation will be used to relate the matrix elements of the axial-charges to those of the PS meson fields (chapt., 2 and 3), which are known from the analysis of the strong interaction processes involving these mesons.

When $\partial^\mu A_\mu^i = 0$, $m_i^2 = 0$, rather than f_i , which is zero only for mesons stable against decay. In general, $\partial^\mu A_\mu \neq 0$ (37, 38), as can be seen from equation 1·18b .

From equation 1·18a , the strangeness changing components V_i ($i = 4, 5, 6, 7$) of the vector currents are only conserved in the SU(3) symmetric limit of equal baryon masses, due to the dependence of $\partial_\mu V_\mu^i$ on the mass differences between particles belonging to different isotopic spin multiplets in the $\underset{\sim}{8}$ representation. The concept of Partial Conservation of the Vector Current (PCVC) was introduced to describe the non-vanishing of $\partial_\mu V_\mu^i$ ($i = 4, 5, 6, 7$) . $\partial_\mu V_\mu^i$ transforms as a scalar boson (c/f eq. 1·18a) under $L_+^\uparrow$, P and T. The principle of PCVC implies $\partial_\mu V_\mu^i$ can be used to define a scalar field operator, belonging to an octet of scalar mesons by the identity $\partial_\mu V_\mu^i = C_i \varphi_i^S$ (39) ($\varphi_i^S \leftrightarrow$ scalar octet). This relation is trivially fulfilled in the field theoretic model considered by Haag-Nishijima-and Zimmerman(39), with C_i defined by $C_i = i f_i m_i^2$. The factor of 'i' is seen to arise from eq. 1·18a, which is often not made clear in the liturature using PCVC. Unlike PCAC, $\partial_\mu V_\mu^i = 0$ implies $f_i = 0$ and $m_i^2 \neq 0$ (40) That is, the strange scalar mesons behave as Goldstone bosons in the symmetry limit(40, 41).

Section 1.7. Symmetry Breaking.

In the Gell-Mann-Ne'eman symmetry scheme(4) the strong interactions are assumed to be invariant under the tramsformation groups SU(3)⊗SU(3) and SU(3), which are broken by a weaker,symmetry breaking mechanism (8, 17), in such a way that hypercharge and isospin are still conserved.

The electromagnetic interaction further breaks down the remaining symmetry(SU(2)), such that only hypercharge and the third component of isospin are conserved, which is the requirement $\partial_\mu J_\mu^{em} = 0 = \partial_\mu (V_\mu^3 + V_\mu^8/\sqrt{3})$, and ensures the equality of the renormalized charges for the proton and positron.

Finally, for the weak interaction, hypercharge and the third component of isospin are no longer generally conserved. That the weak interaction relys on the phenomenon of symmetry breaking can be readily seen by noting that the decay of a particle into another particle or particles of mass equal to or less than itself is energetically impossible.

The chain of symmetry breaking processes can be illustrated by writing the most general Hamiltonian

$$H^{TOTAL} = H^\circ + \epsilon H' + H_{EM} + H_{WEAK} , \qquad 1.40$$

in order of magnitude of strength. $H^\circ + \epsilon H'$ is the Hamiltonian of the strong interactions, of order 10°, where H° is the SU(3)⊗SU(3) invariant part, and H' breaks SU(3)⊗SU(3) and SU(3) with strength ϵ . H_{EM} is the electromagnetic interaction term, with strength of the order of the fine structure constant, $1/_{137}$, and H_{WEAK} is proportional to the weak Fermi decay constant $G \sim 1\cdot0 \times 10^{-5}/M_p^2$.

It was originally proposed by Gell-Mann(8) that the Hamiltonian density $\mathcal{H}'$ ($\int \mathcal{H}' d^3\underset{\sim}{x} = H'$) responsible for the symmetry breaking is of the form $\mathcal{H}' = \mathcal{H}^\circ - u_0 - c u_8$. $\mathcal{H}^\circ$ is defined to be invariant under SU(3)⊗SU(3) and u_0 and u_8 are assumed to belong to a nonet of scalar densities(sect., 1.6), transforming according to the $(\underset{\sim}{3}^*,\underset{\sim}{3})+(\underset{\sim}{3},\underset{\sim}{3}^*)$ representation of this group. u_0 breaks SU(3)⊗SU(3) but conserves SU(3), while u_8 violates SU(3).

Employing this model and the Heisenberg equations of motion, H' can be related to the current-divergences by writing

$$d/dt \int J_0(\underset{\sim}{x},t)\, d^3\underset{\sim}{x} \equiv - \int \partial_\mu J_\mu \, d^3\underset{\sim}{x} = i\left[H', \int J_0(\underset{\sim}{x},t)\, d^3\underset{\sim}{x} \right].$$

For scalar symmetry breaking terms(8), this can be written in the local form

$$\partial_\mu J_\mu(\underset{\sim}{x},t) = i\left[\mathcal{H}'(\underset{\sim}{x},t), \int J_0(\underset{\sim}{y},t)\, d^3\underset{\sim}{y} \right]. \qquad 1\cdot41$$

From the CR's (eq. 1·38), for u_0 and u_8 transforming according to

$(\underset{\sim}{3}^*,\underset{\sim}{3})\oplus(\underset{\sim}{3},\underset{\sim}{3}^*)$, the current-divergences can be uniquely determined(8) by the quark densities in the following way.

$$\partial_\mu V_\mu^i = c\, f_{i8k}\, u_k \;, \quad \partial_\mu A_\mu^i = -d_{iok} v_k - c\, d_{i8k} v_k \qquad 1\cdot 42$$

Hence from these relations and the CR's (eq., 1·38), the charge-current divergence CR's are uniquely determined in this model of symmetry breaking (8,42).

At the time this model was first proposed(8) it was believed that in the limit $\mathcal{H}' \to 0$,SU(3)⊗SU(3) was a degenerate group of zero mass mesons and baryons, where u_o broke SU(3)⊗SU(3) down to SU(3), which was considered to be a good symmetry of the strong interactions.(This case follows directly by considering the baryon octet current model, sect., 1.3). As a consequence, it was argued that 'c', the measure of the relative strength of the breaking of SU(3)⊗SU(3) and SU(3),was close to zero, due to the expected smallness of the ratio of mass splitting induced by u_8 in SU(3) degenerate multiplets, to the baryon masses induced by u_o .ie 'c' is of the order $\Delta M/M$. This scheme corresponded to the symmetry breaking chain

$$SU(3)\otimes SU(3) \to SU(3) \to SU(2).$$

In a later paper, Gell-Mann,Oakes and Renner demonstrated(43), with the use of PCAC and the CR's(eq. 1·38) that 'c' was not small and close to zero, but rather $\sim -1\cdot 25$. For 'c' $= -\sqrt{2}$, $\mathcal{H}'$ commutes with F_i and F_i^5 ($i = 1, 2, 3$), and hence is invariant under SU(2)⊗SU(2), which is now considered to be a better symmetry of the strong interactions than SU(3). $\mathcal{H}' \to 0$ now implies an SU(3)⊗SU(3) symmetry, with only the F_i generating a degererate SU(3) symmetry of non-zero equal mass baryons, while the F_i^5 are the charge operators corresponding to 8 massless PS mesons(from the requirement that $\partial_\mu A_\mu^i = 0$ in this limit, and PCAC).

In the SU(2)⊗SU(2) symmetric limit, with 'c' $= \sqrt{2}$, only $\partial_\mu A_\mu$ and $\partial_\mu V_\mu$ ($i = 1, 2, 3$) are conserved, corresponding as above,to an SU(2) degenerate group, generated by F_i ($i = 1,2,3$), and zero mass pions.This scheme corresponds to the symmetry breaking chain

$$SU(3)\otimes SU(3) \to SU(2)\otimes SU(2) \to SU(2),$$

and will be employed in section 2.3 in calculations involving the K_{l3} form-factors.

CHAPTER 2.

K_{l3} DECAYS.

Introduction.

K_{l3} decay is the study of the semileptonic decay modes of the non-zero-strangeness K meson

$$K \rightarrow \pi + e + \bar{\nu} \qquad (K_{e3})$$

$$K \rightarrow \pi + \mu + \bar{\nu} \qquad (K_{\mu 3}).$$

The strongly interacting mesons participating in these weak decays belong to an octet of PS mesons, with the octet properties and quantum numbers as illustrated in fig 1. and table 1. of chapter 1.

These decays occur for both the neutral and charged K mesons, and form a branch of weak interactions known as semi-leptonic decays, in which both hadrons and leptons participate.

The theoretical study of K_{l3} decays, and in general weak decays involving strongly interacting particles, serves two purposes. The first is the study of the weak interaction itself, while on the other hand

the weak interaction is used as a probe for exploring the strong interactions of the baryons and mesons involved, through the occurance of the strong interaction matrix elements entering into the description of the decays.

Considerable interest has centred around the experimental and theoretical problem of the K_{l3} form-factor ratio $\xi = f_-(0) / f_+(0)$ arising from the hadronic matrix element occuring in the decay amplitude (sect., 2.1). Many theoretical attempts have been made to calculate this ratio due to it's importance in testing models of symmetry breaking and it's relevance to the mode by which higher symmetries are broken. The wide spectrum of experimental results, particularly over the years 1966 to the present time, and the correspondingly large spectrum of theoretical predictions subsequently obtained, indicate the fluid situation of the determination of ξ.

The experimental difficulties in extracting the value of ξ are easing, particularly with the higher statistics available to recent experiments, with the result that a clearer idea of the value of ξ is emerging.

The experimental aspects of K_{l3} and recent results, are discussed in sections 2.2 and 2.3 . The two categories discussed are (1) the branching ratio measurments ($\Gamma_{e3}/\Gamma_{\mu 3}$) and (2) the μ-polarizaton measurments. Computer computations were carried out to analyse the experimental situation, and compare the methods of obtaining ξ ,to see whether or not these two approaches are compatible with one another.

Both approaches have led in the past to strongly differing results for ξ, with recent experimental results favouring the original polarization measurments. Branching ratio measurments indicated ξ small and positive, while on the other hand the polarization measurments have favoured $\xi \simeq -1$. In terms of the theoretical calculations of ξ these differing values indicate entirely different behaviour of the K_{l3} amplitudes (sect., 2.3).

The theoretical importance of the K_{l3} form-factor ratio is centred around the long standing question of the mode by which higher symmetries are broken(c/f chapter 1.), and it's calculation as a testing ground, for the Current Algebra hypothesis of Gell-Mann(8), and the limit of validity of the assumptions made when employing this hypothesis to practical calculations. ξ is also a measure of the effect of the symmetry breaking upon the hadronic matrix elements appearing in the weak decays. These aspects are discussed in section 2.3 in relation to a theoretical investigation carried out, of the ratio ξ ,and a comparison with the calculations of other authors is made.

Section 2.1 Review of K_{l3} decays.

That no higher order theory than first order for weak interactions is available, allows only an effective Hamiltonian to be written down, which describes directly the lowest order weak interaction. This can be seen from the matrix element for the specific K_{l3} decay, $K^+(p) \rightarrow \pi^0(k) + l^+(p') + \nu(k')$. 2.1

$$S = \langle \pi^0 \, l \, \nu \mid T(\exp(-i\int H_w^{SL}(x)\, d^4x))\mid K^+ \rangle, \qquad 2.2$$

for which the corresponding T matrix element is defined by,

$$T \doteq -i \langle \pi^0 \, l \, \nu \mid \int H_w^{SL}(x)\, d^4x \mid K^+ \rangle. \qquad 2.3$$

From chapter 1. eq. 1.23b , $H_w^{SL} = -\frac{G}{\sqrt{2}} \left(J_h^{\lambda} L_{\lambda} + \bar{J}_h^{\lambda} L_{\lambda} \right)$.

Substituting this into equation 2.3, we obtain

$$T = +\frac{iG}{\sqrt{2}} \langle \pi^0 \, l \, \nu \mid \int d^4x \left(J_h^{\lambda} L_{\lambda} + \bar{J}_h^{\lambda} L^{\lambda} \right) \mid K^+ \rangle \qquad 2.5$$

with $\bar{J}_\lambda$ defined by $J^{+}(1-2\delta_{\lambda 4})$, where $\dagger$ denotes hermitian conjugate, and $J_\lambda = (\underset{\sim}{J}, iJ^{\circ})$. Using the fact that leptons do not interact strongly, and neglecting the electromagnetic interactions of the leptons with the K and π mesons, the weak decay can be factored into hadronic and leptonic parts, thus(16,28),

$$T = -g/\sqrt{2} \int d^4x <\pi^{\circ} \mid \bar{J}_h^{\lambda}(x) \mid K^+> \cdot \sqrt{\frac{m_\nu m_l}{p_o' k_o'}} \frac{1}{(2\pi)^3} \cdot$$
$$\cdot\ \bar{u}(p')\gamma_\lambda(1+\gamma_5)\nu(k')e^{-ik'x}e^{-ip'x}, \qquad 2\cdot 6$$

using the definition of $\bar{L}_\lambda$(eq.,1·22)

From fig 1 and table 1, we see that only $\bar{J}_h = \sin\theta_c J^{4-i5}$ is eligible to effect the transition 2·6, due to the direction of change of strangeness, and the properties of the octet current operators. Hence $J_\lambda^h L^\lambda$ is the only term contributing, with J_λ^h defined by eq.,1·26.

As the hadronic matrix element $<\pi|\bar{J}_\lambda|K>$ must be even under parity, $<\pi|A_\lambda|K>=0$, and hence only V_λ can mediate the transition. With the property of translational invariance, the T matrix becomes

$$T = -g\frac{\sin\theta_c}{\sqrt{2}}(2\pi)^4\delta^4(p-k-p'-k')<\pi^{\circ}(k)|V_\lambda^4(0) - iV_\lambda^5(0)|K^+(p)>$$
$$\times\ 1/(2\pi)^3\sqrt{m_\nu m_l/p_o' k_o'}\ [\bar{u}(p')\gamma_\lambda(1+\gamma_5)\nu(k')].$$

Defining the reduced T matrix by $T_{ab} = \tilde{T}_{ab}\, i(2\pi)^4\delta(p_a - p_b)$, $\qquad 2\cdot 7a$

we can write $\tilde{T} = ig/\sqrt{2} \cdot <\pi^{\circ}(k)|V_\lambda^4(0) - iV_\lambda^5(0)|K^+(p)>$

$$\cdot\ 1/(2\pi)^3\sqrt{\frac{m_\nu m_l}{p_o' k_o'}} \cdot (\bar{u}(p')\gamma_\lambda(1+\gamma_5)\nu(k')) \qquad 2\cdot 7b$$

As p_μ and k_μ are the only independent 4-vectors of the hadronic matrix element, the requirement of lorentz invariance leads to the following conventional decomposition in terms of the f-f's $f_+(q^2)$ and $f_-(q^2)$.

$$<\pi^{\circ}(k)|V_4^\lambda(0) + iV_5^\lambda(0)|K^+(p)>$$
$$= \frac{1}{(2\pi)^3\sqrt{4p^{\circ}k^{\circ}}}[f_+(q^2)(p+k)_u + f_-(q^2)(p-k)_u]; \quad (q_u = p_\mu - k_\mu). \qquad 2\cdot 8$$

The μ - e universality form of the leptonic current (ie μ and e appear symmetrically with the same strength in the total leptonic current (eq.,1·22)), ensures the same form-factors (f-f's) apply for both K_{e3} and $K_{\mu 3}$ decays. Further, time reversal invariance requires f_+ and f_- to be relatively real.

Taking the divergence of both sides of equation 2·8, we have,

$$<\pi^{\circ}(k)|\partial_\lambda V_\lambda^{4-i5}(0)|K^+(p)>$$
$$= \frac{-i}{(2\pi)^3\sqrt{4p^{\circ}k^{\circ}}}[(m_K^2 - m_\pi^2)f_+(q^2) - q^2 f_-(q^2)] \qquad 2\cdot 9$$

In the SU(3) symmetric limit, $m_K^2 = m_\pi^2$. In this limit $f_-(q^2) = 0$, and hence $\xi = 0$. On the other hand, for broken SU(3), $\xi \neq 0$ and $\partial_\lambda V_\lambda^{4-i5}(0) \neq 0$. That $f_+(0) = \frac{1}{\sqrt{2}}$ in the SU(3) limit can be seen by considering the integrated form of eq., 2·8 .

$$<\pi^{\circ}(k)|F_{4-i5}(t)|K^+(p)> = \frac{\delta^3(\underset{\sim}{p}-\underset{\sim}{k})}{\sqrt{4p^{\circ}k^{\circ}}}[f_+(-q_o^2)(k_o+p_o) + f_-(q_o^2)$$
$$\times\ (k_o - p_o)]e^{-i(k_o-p_o)t}, \qquad 2\cdot 10$$

where $F_{4-is}(t) = \int V^{0}_{4-is}(x)\, d^3\underset{\sim}{x}$ .In the limit of SU(3) , the momentum δ function implies $k_0 = p_0$, and thus,

$$< \pi^0(k) \mid F_{4-is}(0) \mid K^+(p) > \; = \; \delta^3(\underset{\sim}{p} - \underset{\sim}{k})\, f_+(0) \qquad 2\cdot 11$$

Next employing the fact that $[\, F_{4+is} ,\; F_{4-is}] = I_3 + {}^3\!/\!_2 Y$ (eq., 1.27), 2.11a

and sandwiching the commutator between $<\pi^0|$ states, and using the constraint that in the SU(3) symmetric limit, only octet states contribute, we find,

$$| < \eta^0 \mid F_{4-is} \mid K^+ > |^2 \; = \; | < \pi^0 \mid F_{4-is} \mid K^+ > |^2, \qquad 2\cdot 12$$

where we have used $< \pi^0 \mid I_3 + Y\cdot{}^3\!/\!_2 \mid \pi^0 > = 0$. Then sandwiching the commutator between $|K^+>$ states and summing over intermediate states, we obtain

$$\int \frac{d^3 k}{(2\pi)^3} \left\{ |<\eta^0(\underset{\sim}{k}) \mid F_{4-is}(0) \mid K^+(\underset{\sim}{p}) >|^2 + |<\pi^0(\underset{\sim}{k}) \mid F_{4-is}(0) \mid K^+(\underset{\sim}{p}) >|^2 \right\} \qquad 2\cdot 13$$

$$= \; < K^+(\underset{\sim}{p}) \mid T_3 + {}^3\!/\!_2\, Y \mid K^+(\underset{\sim}{k}) > \; = \; \delta^3(\underset{\sim}{p} - \underset{\sim}{k})$$

From equations 2.12 and 2.13, we require $<\pi^0(\underset{\sim}{k}) \mid F_{4-is}(0) \mid K^+(\underset{\sim}{p}) > = \frac{1}{\sqrt{2}}\, \delta^3(\underset{\sim}{p} - \underset{\sim}{k})$, with onlythe positive sign possible,due to the sign of the eigenvalues of the charge operator F_{4-is} acting on the K^+ state vector. Thus $f_+(0) = \frac{1}{\sqrt{2}}$.

$f_+(0)$ will come under discussion in the calculation of section 2.4, where the value $f_+(0) = \frac{1}{\sqrt{2}}$ is a natural consequence of the model chosen, and the assumptions used in it's theoretical evaluation. Any deviation of $f_+(0)$ from $1/\sqrt{2}$ is due to renormalization by the strong symmetry breaking forces.

An important theorem,concerning the renormalization of $f_+(0)$, is the Ademollo-Gatto theorem(44), which shows that to first order in SU(3) symmetry breaking, matrix elements of the vector current between single particle states of the same SU(3) multiplet are not renormalized in the limit of zero momentum transfer. Ie $f_+(0) = \frac{1}{\sqrt{2}}$ up to at least second order the symmetry breaking. For the first three components of the vector current, the theorem holds trivially, from the CVC hypothesis(32).

In the case ofthe 4,5,6, and 7 components of the vector current, the theorem can be demonstrated by appealing to the CR 2.11a , which from the C.A. hypothesis of Gell-Mann(8) is valid for broken symmetry, with time dependent charges.For simplicity we will sandwich eq., 2.11a between $|\pi^+>$states, and sum over intermediate states. In this case only one octet state can contribute, with the other contributions arising from outside the octet multiplet. Hence we may write

$$\int d^3k/(2\pi)^3 < \pi^+(\underset{\sim}{k}) \mid F_{4+is}(t) \mid \bar{K}^0(\underset{\sim}{p}) >< \bar{K}^0(\underset{\sim}{p}) \mid F_{4-is}(t) \mid \pi^+(\underset{\sim}{k}) >$$

$$+ \sum_{n \neq \bar{K}^0} \Big\{ < \pi^+(\underset{\sim}{k}) \mid F_{4+is}(t) \mid n >< n \mid F_{4-is}(t) \mid \pi^+(\underset{\sim}{k}) >$$

$$- < \pi^+(\underset{\sim}{k}) \mid F_{4-is}(t) \mid n >< n \mid F_{4+is}(t) \mid \pi^+(\underset{\sim}{k}) > \Big\},$$

$$= \langle \pi^+(\underline{k}) | I_3 + \tfrac{3}{2}Y | \pi^+(\underline{p}) \rangle = \delta^3(\underline{k} - \underline{p}).$$

From equation 1.40, the relevant part of H^{TOTAL}, to the order to which we we are working(ie neglecting electromagnetic interactions in the hadron matrix elements) is $H^0 + \epsilon H'$. Employing the Heisenberg equations of motion and time translational invariance, we have

$$\langle \pi^+(k) | F_{4+i5}(t) | n \rangle = \langle \pi^+(k) | [H', F_{4+i5}(t)] | n \rangle / (E_\pi - E_n) \qquad 2.14$$

For $|n\rangle = |\bar{K}^0\rangle$, in the SU(3) limit as $\epsilon \to 0$, $E_{\pi^+} \to E_{\bar{K}^0}$. Thus $E_{\pi^+} - E_{\bar{K}^0}$ is proportional to ϵ, and the first two terms are independent of ϵ. For $E_\pi - E_n$ (n not a member of the same octet as π^+) $E_\pi - E_n \not\to 0$ as $\epsilon \to 0$, and thus terms under the summation are of order ϵ^2 in the symmetry breaking. Whence $f_+(0)$ is only renormalized in the second order of the symmetry breaking interaction. ie $f_+(0) = 1/\sqrt{2} + O(\epsilon^2)$ 2.15
The theorem is generally valid for other octets of particles and their corresponding f-f's, besides the PS octet example above (44). In summary,

$$f_+(0) = \frac{1}{\sqrt{2}}(1 + O(\epsilon^2)), \quad \text{and} \quad \xi(0) = O(\epsilon). \qquad 2.15$$

It is often useful, when analysing the energy dependence of the f-f's to decompose the matrix element (eq., 2.8) in the following way,

$$\left(1/(2\pi)^3\sqrt{4p^0k^0}\right)\left[f_+(q^2)(p+k)_\mu + \left((f_0(q^2) - f_+(q^2))\frac{(m_K^2 - m_\pi^2)}{q^2}\right)(p-k)_\mu\right] \qquad 2.16$$

and to consider the f-f's (45)

$$f_+(q^2) \quad \& \quad f_0(q^2) = \left(f_+(q^2) - \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)\right), \qquad 2.17$$

which receive contributions from only $J = 1^-$ and $J = 0^+$ states respectively. The f-f's f_+ and f_0 are usually assumed to satisfy at most once subtracted dispersion relations (Sect., 2.3) with the absorptive parts of f_+ and f_0 determined by spin 1 and spin 0 intermediate states respectively. If f_+ and f_0 are assumed to be dominated by single particle states ,in this aprroximation f_+ and f_0 can be written as

$$f_+(q^2) \cong \frac{f_+(0)\, m_{K^*}^2}{m_{K^*}^2 + q^2} \; ; \quad f_0(q^2) = \frac{f_+(0)\, m_\kappa^2}{m_\kappa^2 + q^2}, \qquad 2.18$$

where K^* and κ are assumed to belong to an octet of $J = 1^-$ vector and $J = 0^+$ scalar mesons respectivley, with the quantum number assignment of the 4-i5th state in the above case.

Although the existence of an octet of vector mesons is well established by now(5, 16), the existence of the scalar kappa mesons, and even more so, an octet of scalar mesons, is much less certain, although recent evidence of a K- π enhancement of mass $\sim$ 1100 Mev has been obtained by Trippe et al., and Crennell et al.,(46).

In the simple pole model, the energy dependence of the f-f's f_+ and f_0 in the physical q^2 range of the decay, can be written to first order as

$$f_+(q^2) \simeq f_+(0)(1+\lambda_+ q^2/m_\pi^2)$$
$$f_0(q^2) \simeq f_0(0)(1+\lambda_0 q^2/m_\pi^2) \equiv f_+(0)(1+\lambda_0 q^2/m_\pi^2), \qquad 2.19$$

for $\lambda_+ \doteq m_\pi^2/m_{K^*}^2 \sim 0.023$ and $\lambda_0 = m_\pi^2/m_\kappa^2 \sim 0.014$ (sect., 2.3).

An important relation relating f_+ and f_- at the unphysical point $q^2=-m_K^2$, and which will be discussed later in section 2.3, is the Callan-Treiman (CT) relation (47)

$$f_+(-m_K^2)+f_-(-m_K^2) = 1/\sqrt{2}\, f_K/f_\pi \qquad 2.20$$

This was derived by Callan and Treiman from the matrix element 2.8, by employing PCAC in the soft pion limit ($k_\mu^\pi = 0$). The use of PCAC to obtain information on the matrix elements of processes involving soft pions ($m_\pi^2 = 0$) was first introduced by Nambu (36) and later employed in conjunction with current algegra by Callan and Treiman (see also sect., 3.3)

The derivation of equation 2.20 proceeds from the matrix element 2.8 by contracting the pion , using pion PCAC, $\partial_\lambda A_i^\lambda = f_i m_i^2 \varphi_i/\sqrt{2}$, and then partially integrating with respect to k_μ.

$$\lim_{k_\mu\to 0} \langle \pi^0 | \partial_\mu V_\mu^{4-i5}(0) | K^+\rangle$$
$$= \lim_{k_\mu\to 0} \frac{-i}{(2\pi)^3\sqrt{4p^0k^0}} \left[(m_K^2-m_\pi^2)f_+(q^2) - q^2 f_-(q^2)\right] \qquad 2.21$$
$$= \lim_{k_\mu\to 0} i\int e^{-ikx}\frac{d^4x}{\sqrt{2q_0(2\pi)^3}} \langle 0|T(\varphi_{\pi^0}(x)\,\partial_\mu V_\mu^{4-i5}(0))|K^+\rangle\cdot(k^2+m_\pi^2)$$

Using PCAC, and then partially integrating w.r.t. k_μ, we obtain,

$$\lim_{k_\mu\to 0} \frac{-\sqrt{2}\,i}{m_\pi^2 f_\pi}\int \frac{d^4x(k^2+m_\pi^2)}{\sqrt{2q_0}\,(2\pi)^{3/2}}\langle 0|\delta(x_0)[A_0^{\pi^0}, \partial_\mu V_\mu^{4-i5}(0)]|K^+\rangle. \qquad 2.22$$

Evaluating the ETC with the use of the relations 1.42 and the CR's 1.38, 2.22 becomes

$$+\frac{\sqrt{2}\,i}{f_\pi}\int\frac{d^4x\,\delta^4(x)}{\sqrt{2q^0}\,(2\pi)^{3/2}}\langle 0|\partial_\mu A_\mu^{4-i5}(0)|K^+\rangle\left(\frac{3c}{(\sqrt{2}-c/2).4}\right).$$

In the SU(2)⊗SU(2) symmetric limit ($m_\pi^2 = 0$) 'c'$=-\sqrt{2}$, and hence from equation 1.39 we find,

$$\frac{m_K^2 f_K}{\sqrt{2}f_\pi} = (\text{eq., } 2.21) = \left(f_+(-m_K^2) + f_-(-m_K^2)\right)m_K^2,$$

which is the CT relation. We will refer to the above method as the soft-pion method.

In equation 2.22 we have dropped the term

$$\lim_{k_\mu \to 0} \frac{-\sqrt{2}k_\mu}{m_\pi^2 f_\pi} \int d^4x / (\sqrt{2q^0}(2\pi)^{3/2}) \langle 0 | T \left(A_\mu^{\pi^0}(x), \partial_\mu V_\mu^{4-i5}(0) \right) | K^+ \rangle (k^2 + m_\pi^2),$$

which only contributes if there is a single intermediate particle state which is degenerate in mass with either the initial or final states, $\langle K^+|$ and $|0\rangle$ respectively. In this case no eligible single particle state exists, so this term is vanishing in the soft pion limit.

We also note that as SU(2)⊗SU(2) symmetry is turned off, we obtain corrections to the Callan-Treiman relation, of the order (m_π^2/m_K^2) which indicates the approximate validity of the SU(2)⊗SU(2) symmetric limit.

Returning to the matrix element $\langle \pi^0 | V_\lambda^{4-i5} | K^+ \rangle$, as V_λ^{4-i5} is hypothesised to belong to an octet of vector currents (33), transforming as the 4 - i5 th components of this octet, an immediate consequence of this (fig., 1 and table 1) are the following constraints on the decay.

(1). $|\Delta S| = 1$

(2). $|\Delta Q| = 1$, and hence only $|\Delta I_3| = \frac{1}{2}$ transitions are possible, from the GNN relation

(3). $\Delta Y = \Delta Q$, which is a consequence of the properties of the $V_\pm$ raising and lowering operators (Fig., 1)

(4). As V_λ^{4-i5} belongs to an isotopic doublet of currents (V_λ^{4-i5} , V_λ^{6+i7}) within the octet, and taking into account the properties of the octet operators (Fig., 1), only $|\Delta I| = \frac{1}{2}$ transitions can be mediated by V_λ^{4-i5} . The $|\Delta I| = \frac{1}{2}$ rule is a direct consequence, in this case, of the octet properties of V_λ^{4-i5} .

Evidence for support of the V - A current - current phenomenological weak Hamiltonian for the strangeness changing semi-leptonic decays comes from the measurements of the $e^\pm$- energy spectrum in K_{e3} decay (58) and the π and μ spectra in $K_{\mu 3}$ decay (59). On covariance grounds alone, both the scalar and the tensor currents could also contribute to the K decays. Nevertheless, the fit to the energy spectra experiments is wholly consistent with the vector current alone(58, 59).

Further evidence for the Cabibbo phenomenological c-c weak interaction arises from considering the 4 possible neutral Kaon decay modes.

$$\left.\begin{array}{l} K^0 \to \pi^- + l^+ + \nu_l \\ K^0 \to \pi^+ + l^- + \bar{\nu}_l \end{array}\right\} \Delta Y = \Delta Q = \Delta S$$
$$\left.\begin{array}{l} K^0 \to \pi^+ + l^- + \bar{\nu}_l \\ K^0 \to \pi^- + l^+ + \nu_l \end{array}\right\} \Delta Y = -\Delta Q = \Delta S \qquad 2\cdot23$$

The last two decay modes are ruled out by the above weak Hamiltonian, as both are $|\Delta I_3| = 3/2$ transitions, which implies the possibility of $|\Delta I| = 3/2$ transitions.

A test of this is obtained by considering the consequences of the $|\Delta I| = \frac{1}{2}$ rule in these decays. From the matrix elements of the CR's

$$[F_{1-i2}, V^{\lambda}_{4-i5}] = 0 = [F_{1+i2}, V^{\lambda}_{4+i5}] = 0 \qquad 2\cdot24$$

between the states $\langle\pi^+|$ and $|K^-\rangle$, the following $|\Delta I| = \frac{1}{2}$ rule relations follow. $\sqrt{2}\langle\pi^0|V^{\lambda}_{4+i5}|K^-\rangle = \langle\pi^+|V_{\lambda}^{4+i5}|\bar{K}^0\rangle$,

and similarly $\sqrt{2}\langle\pi^0|V^{\lambda}_{4-i5}|K^+\rangle = \langle K^-|V^{\lambda}_{4-i5}|K^0\rangle$. $\qquad 2\cdot25$

The relations between the corresponding decey rates are thus,

$$2\Gamma(K^+) = \Gamma_{l^-}(\bar{K}^0) \quad ; \quad 2\Gamma(K^-) = \Gamma_{l^+}(K^0). \qquad 2\cdot26$$

As hypercharge is not conserved in weak interactions, K^0 and $\bar{K}^0$ can transform into one another through the hypercharge non-conserving interactions of the form(48) $K^0 \xrightarrow{\text{WEAK}} \pi^+ + \pi^- \xrightarrow{\text{WEAK}} \bar{K}^0$.

Thus the appropriate states when discussing weak interactions are the even and odd CP eigenstates (16,48) $(\bar{K}^0 + K^0)/\sqrt{2} = K^0_L$ & $\frac{(K^0 - \bar{K}^0)}{\sqrt{2}} = K^0_S$

Hence the $|\Delta I| = \frac{1}{2}$ relations imply

$\Gamma(K^0_L) = 2\Gamma(K^-)(\equiv 2\Gamma(K^+)$ from the CPT theorem$)$ and $\Gamma(K_S^0) = 0$.

Experimentally (49) only K_L is observed to undergo semi-leptonic decay, and hence this lends support to both the $\Delta Y = \Delta Q$ and $\Delta I = \frac{1}{2}$ rules, as required by the Cabibbo phenomenological weak interaction.

With the Cabibbo strangness changing current of the form,

$$J^{\lambda}_{h(4+i5)} = \sin\theta_c A_{\lambda}^{4+i5} + \sin\theta_c V_{\lambda}^{4+i5},$$

consideration of the amplitudes

$\langle 0|A_{\lambda}^{4+i5}|K^-(p)\rangle = \frac{iP_{\mu} f_K}{(2\pi)^{3/2}\sqrt{2p^0}}$ and $\langle 0|A_{\lambda}^{1+i2}|\pi^-(k)\rangle = ik_{\lambda} f_{\pi}/\sqrt{(2\pi)^3 2k^0}$,

and thence the ratio $\Gamma(K\to e+\nu)/\Gamma(\pi\to\mu+\nu)$ determines the quantity $f_K/f_{\pi} \tan\theta_c = \tan\theta_A$(50), where θ_A is simply a parametrization of the LHS, indicating the Cabibbo angle occurs with the axial-vector current. A determination of θ_A by Brene et al gives $\theta_A = 0.2655 \pm 0.0006$.

In the limit of SU(3), $f_K = f_\pi$ ($m_K^2 = m_\pi^2$), and hence $\Theta_c = \Theta_A$. In this sense Θ_A is renormalized by the strong symmetry breaking interactions.

Similarly, a measurement of Γ_{Ke3} ($K \to \pi + e + \nu$) (51) determines the quantity $\sqrt{2} f_+(0) \sin\theta_c = \sin\Theta_V = 0.21 \pm 0.01$, where Θ_V again is a parametrization of the LHS involving Θ_c occuring with the vector current. Renormalization of $f_+(0)$ by the strong interactions is expected to be small, by the Ademollo-Gatto theorem, and hence $\sin\Theta_V \approx \sin\Theta_c$. Eliminating Θ_c between these two experimental determinations, for $\Theta_V = \Theta_c$, Brene et al find $f_K / (f_\pi f_+(0)\sqrt{2}) = 1.28 \pm 0.06$, indicating that Θ_A is appreciably different from Θ_V for broken SU(3) symmetry when $f_K \neq f_\pi$. For $f_K = f_\pi$, in the limit of SU(3), $\Theta_A = \Theta_V = \Theta_c$.

The experimental value for the ratio $f_K / f_\pi f_+(0)$ will be important in the next section, when comparing the predictions of the Callan-Treiman relation with the experimental status of the form-factors f_+ and f_- .

Section 2.2 Experimental determination of ξ .

Introduction.

By 1967, the experimental status of ξ , from both the neutral and charged K_{l3} decay had become very uncertain (52), with the large discrepancy between the value of ξ as obtained from μ-polarization ($\underset{\sim}{P}_\mu$) and the branching ratio (R) and Dalitz plot experiments. In particular, up to the Heidelberg conference (53) ξ_{AVERAGE} (branching ratio) $= 0.64 \pm 0.3$, while $\xi^{\text{POLAR.}}_{\text{AV}} = -1.25 \pm 0.35$ for $K^{\pm}_{l3}$ decays. We will henceforth specifically consider the decays of the charged Kaons, although exactly analogous considerations apply for the neutral Kaon decays.

Up to this time the energy dependence of the f-f's (54)had not been taken into account in analysing the experimental data. It was suggested by Cabibbo in 1967 (55) that the widely differing results arose from the neglect of the energy dependence of the f-f's, where q^2 is an energy variable in the rest frame of Kaon decay. The energy depenence has since been introduced for $f_\pm(q^2)$ by linearly parametrising w.r.t. q^2, and retaining first order terms. This corresponds from the dispersion theoretic point of view, to a single particle pole approximation, expanded to first order in the energy. In this simple model the standard parametric form for $f_{\pm}(q^2)$ is (54) $f_{\pm}(q^2) = f_{\pm}(0)\,(1 - \lambda_{\pm}\, q^2/m_\pi^2)$, 2.27

where $\lambda_{\pm}$ are related to the single pole expressions $f_{\pm}(q^2) = \dfrac{f_{\pm}(0)\, m_{\pm}^2}{m_{\pm}^2 + q^2}$ for $\lambda_{\pm} = m_\pi^2/m_{\pm}^2$.

By introducing the energy dependence, the expressions for $\underset{\sim}{P}_\mu$ and R are now dependent on three parameters, namely λ_+, λ_- and ξ, with the consequence that ξ is coupled to the energy dependence of the f-f's for it's determination. In this way, for particular values of λ_+ and λ_-, the values of ξ obtained from $\underset{\sim}{P}$ and R measurements can be made compatible with each other. Very recent experiments show evidence of the compatibility of ξ from the $\underset{\sim}{P}_\mu$ and R experiments, in favour of the results originally obtained from the $\underset{\sim}{P}_\mu$ measurements, which yields a direct determination of $\xi(q^2)$ (56), and is less sensitive to the energy dependence of the form factors, as shown below.

We outline below the derivation of the theoretical expressions (16, 28, 57) for the branching ratio and μ-polarization, which we subsequently computed, to study theoretically, the degree of compatibility of the experimental determination of ξ from these two approaches.

Section 2.2

K_{l3} branching ratio, and μ - polarization.

The branching ratio R is defined by R = $\Gamma_{\mu3}/\Gamma_{e3}$, where $\Gamma_{\mu3}$ and Γ_{e3} denote the decay rates for the processes $K \to \pi + \mu + \nu$ and $K \to \pi + e + \nu$ respectively. As the $|\Delta I| = \frac{1}{2}$ rule requires $\xi(q^2)$ is the same for both the charged and neutral Kaon decays (57) we will henceforth consider the decay $K^+(p) \to \pi^0(k) + l^+(p') + \nu_l(k')$.

From equations 2.6, 2.7, the reduced matrix element, $\tilde{T}$, for the process $K^+ \to \pi^0 + l^+ + \nu_l$ is given by

$$\tilde{T} = i g \frac{\sin\theta_c}{\sqrt{2}} \cdot \frac{[f_+(q^2)(p+k)_\mu + f_-(q^2)(p-k)_\mu]}{\sqrt{4p^0k^0}\,(2\pi)^3} \frac{(\bar{u}_\nu(k')\gamma_\mu(1+\gamma_5)v_l(p'))}{(2\pi)^3} \sqrt{\frac{M_\nu M_l}{p_0' k_0'}}. \qquad 2.28$$

The differential rate for the decay can be wwitten in terms of $\tilde{T}$ as

$$d\Gamma = (2\pi)^4 \delta^4(p-k-p'-k') \sum_{\text{spin}} |\tilde{T}|^2\, d^3k\, d^3k'\, d^3p'. \qquad 2.29$$

With the definition

$$M = \frac{\sin\theta_c}{\sqrt{2}} g \left(f_+(q^2)(p+k)_\mu + f_-(q^2)(p-k)_\mu (\bar{u}_\nu(k')\gamma_\mu(1+\gamma_5)v_l(p')) \right),$$

we can write equation 2.29 as

$$d\Gamma = \delta^4(p-k-p'-k') \frac{|M|^2}{(2\pi)^5} \left(\frac{M_\nu M_l}{4p^0}\right) \frac{d^3k\, d^3k'\, d^3p'}{k_0\, k_0'\, p_0'}. \qquad 2.30$$

Using the momentum constraint condition $(p-k)_\mu = (p'+k')_\mu$ we have

$$M = g\frac{\sin\theta_c}{\sqrt{2}} [\, f_+(q^2)p_\mu - \tfrac{1}{2}(f_+(q^2) - f_-(q^2)(p'+k')_\mu) \qquad 2.31$$

$$\cdot\, (\bar{u}_\nu(k')\gamma_\mu(1+\gamma_5)v_l(p')).$$

Next, employing the Dirac equation, and the property $\gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu$ M becomes

$$M = (-i)\, G \frac{\sin\theta_c}{2\sqrt{2}}\, \bar{u}_\nu(k')(1-\gamma_5)(-m_l f_+(q^2) + m_l f_-(q^2) + i\gamma\cdot p\, f_+(q^2))\, v_l(p').$$

Time reversal invariance ensures the relative reality of the form- factors, and hence

$$M^* = i G \frac{\sin\theta_c}{2\sqrt{2}}\, v_l^+(p')(-m_l f_+ + m_l f_- - i\gamma\cdot p^* f_+)(1-\gamma_5)\gamma_4 u_\nu(k'),$$

where the property $\bar{u} = u^+\gamma_4$ has been used, and we have dropped the q^2 dependence for future convenience. With the Dirac - Pauli represenration of the γ matrices, and the properties

$$\gamma_4^2 = 1, \quad \gamma_4\gamma_5 = -\gamma_5\gamma_4, \quad \bar{v}_l(p') = v_l^+\gamma_4, \quad \gamma\cdot p^* = -\gamma\cdot p,$$

M^* can finally be written in the form

$$M^* = i G \frac{\sin\theta_c}{2\sqrt{2}}\, \bar{v}_l(p')(-m_l f_+ + m_l f_- + i\gamma\cdot p\, f_+)(1+\gamma_5).$$

Summing over the spins, by means of the spin projection operators $\sum_{spin} v_l(p')\bar{v}_l(p') = \frac{-i\gamma\cdot p + m}{2m}$,

we find $\sum_{spin} |M|^2 = G^2 \frac{\sin^2\theta_c f_+^2}{4 m_l m_\nu}\, i\,Tr\{((-\frac{m_l}{2}(1-\xi) + i\gamma_p))(\gamma\cdot k')(1-\gamma_5)((-\frac{m_l}{2}\cdot(1-\xi) + i\gamma\cdot p)(m_l + i\gamma\cdot p)\}$.

Evaluation of the trace then yields

$$\sum_{spin} |M|^2 = G^2 \sin^2\theta_c f_+^2 \{[2p\cdot p' - 2m_l \operatorname{Re}(-\frac{m_l}{2}(1-\xi))]\, p\cdot p' + (m_K^2 - m_l^2/4\,(1-\xi)^2\, p'\cdot k'\}/m_\nu m_l,$$

where f_+ has been factored out in the above to obtain the expressions in terms of $\xi = f_+ / f_-$.

Working in the Kaon rest frame, we have the following conditons holding,

$$p\cdot p' = -m_K p^{o\prime}, \quad p\cdot k' = -m_K k_o', \quad (p-k)^2 = (p'+k')^2 \Rightarrow$$
$$-m_K^2 - m_\pi^2 + 2k^o m_K = -m_l^2 - m_\nu^2 - 2p'\cdot k',$$

and hence $f_+(q^2) = f_+'(k^o)$. In this three body decay, the maximum energy of the π is given by $W_o = (m_K^2 + m_\pi^2 - m_l^2)/2m_K$,

for decay of the Kaon at rest. Expressing $\sum|M|^2$ in the rest frame of the Kaon, and integrating with respect to $\underset{\sim}{p'}$ and the angular variables, we obtain for the differential decay rate

$$d\Gamma(k^o, p^o) = \frac{1}{(2\pi)^5}\frac{1}{m_K}(m_l\cdot m_\nu)\, 8\pi^2\, dk^o\, dp_o'$$
$$\times G^2\sin^2\theta_c\, f_+^2\, \frac{m_K}{m_l m_\nu}\Big[\Big\{m_K[2p_o'k_o' - m_K(W_o - k^o)]$$
$$+ \frac{1}{4}m_l^2(W_o - k_o) - m_l^2 k_o'\Big\} + \Big\{m_l^2[k_o' - \frac{1}{2}(W_o - k^o)]\Big\}\xi$$
$$+ \Big\{\frac{1}{4}m_l^2(W_o - k_o)\Big\}|\xi|^2\Big]. \qquad 2\text{-}32$$

The $8\pi^2 dk^\circ dp_0'$ arises from the phase space factor

$$\frac{d^3\underset{\sim}{k}\, d^3\underset{\sim}{p}'\, d^3\underset{\sim}{k}'}{k^\circ p^\circ k_0'}\,\delta^3(\underset{\sim}{k}+\underset{\sim}{p}'+\underset{\sim}{k}')\,\delta(m_K - k_0 - k_0' - k_0') \qquad 2.33$$

$$= (d^3k\, d^3p' / k_0 p_0' k_0)\,\delta(M_K - k^\circ - p_0' - k_0')$$

Regarding the direction of the π as arbitrary, only the angular variation of the direction of l relative to the π's direction, $d\Omega_{\pi l}$, need be considered, and hence equation 2.33 can be written as

$$8\pi^2 \frac{k^2 dk}{k_0}\,\frac{p'^2 dp'}{p_0' k_0'}\, d\cos\theta_{\pi l}\,\delta(m_K - k^\circ - p_0' - k_0'),$$

where

$$k_0^{2\prime} = \underset{\sim}{k}^2 + \underset{\sim}{l}'^2 + 2|\underset{\sim}{l}'||\underset{\sim}{k}|\cos\theta_{\pi l} \qquad (m_\nu = 0,\ \underset{\sim}{l} = 0).$$

Integrating over $\cos\theta_{\pi l}$ we obtain $8\pi^2 dk_0\, dp_0'$.

From equation 2.32 the pion energy spectrum is defined by

$$d\Gamma/dk^\circ = g^2 \sin^2\theta_c f_+^2 \cdot \int_{p_0'\,min}^{p_0'\,max} [-m_l^2(1-\xi)(M_K - k^\circ)$$

$$- (M_K^2 - M_l^2/4\,(1-\xi)^2)(W_0 - k_0)) + (2M_K(M_K - k_0)$$

$$+ m_l^2(1-\xi))\,p_0' - 2M_K p_0'^2\,]\,dp_0'. \qquad 2.34.$$

To calculate the maximum and minimum values of the lepton energy p_0', for a particular value of k_0, we use the constraint,

$$\left.\begin{matrix}max\\min\end{matrix}\right\}\ |\underset{\sim}{l}'|^2 = (p_0'^2 - M_l^2) = (|\underset{\sim}{k}'| \pm |\underset{\sim}{k}|^2), \qquad = (k_0' \pm |\underset{\sim}{k}|)^2;$$

which is a consequence of $\underset{\sim}{l}' = (\underset{\sim}{k} + \underset{\sim}{k}')$ & $\underset{\sim}{l}^2 = \underset{\sim}{k}^2 + \underset{\sim}{k}'^2 + 2|\underset{\sim}{k}||\underset{\sim}{k}'|\cos\theta,$

and hence

$$\left.\begin{matrix}max\\min\end{matrix}\right\}\ p_0 = [(M_K - k_0 \pm |\underset{\sim}{k}|^2) - M_l^2\,] / (2M_K - k_0 \pm |\underset{\sim}{k}|) \qquad 2.35$$

Integrating over p_0 we then obtain,

$$d\Gamma_{l3}/dk_0 = g^2 \frac{\sin^2\theta_c}{16\pi^3} f_+^2 |\underset{\sim}{k}| \left(\frac{W_0 - k_0}{W_0 - k_0 + M_l^2/2M_K}\right)^2 \Big\{ \tfrac{1}{3} M_K^2 |\underset{\sim}{k}|^2$$

$$+ \tfrac{1}{3} m_l^2 |\underset{\sim}{k}|^2 / (W_0 - k_0 + M_l^2/2M_K) + (M_l^2/8M_K)(M_K^2 + M_\pi^2 + 2M_K k_0)$$

$$+ \tfrac{1}{4} M_l^2 \frac{(M_K^2 - M_\pi^2)}{M_K}\,\xi + \tfrac{1}{4} m_l^2 (W_0 - k_0 + m_l^2/2M_K)\,|\xi|^2 \Big\}. \qquad 2.36$$

For $m_l = m_e$, contributions of $O(m_e^2)$ can be neglected in relation to the other terms, and equation 2.36 simplifies to

$$d\Gamma_{e3}/dk_0 = g^2 \frac{\sin^2\theta_c}{48\pi^3} M_K (f_+(q^2))^2 |\underset{\sim}{k}|^3. \qquad 2.37.$$

For $l = \mu$ in equation 2.36, $d\Gamma_{\mu 3}/dk_0$ is the energy spectrum of the π^0 meson for $K^+_{\mu 3}$ decay, and similarly $d\Gamma_{e3}/dk_0$ for K^+_{e3} decay. The lepton energy spectra for the respective decays is obtained by integrating over k_0 first.

The integration over k_0 (max -min), for the expressions 2.36 and 2.37 was computed, where as for p_0, the maximum and minimum values of k_0 are given by $\left.\begin{matrix} max \\ min \end{matrix}\right\} k_0 = [(m_K - P_0' \pm |\underset{\sim}{p}'|)^2 + m_\pi^2]/2(m_K - P_0' \pm |\underset{\sim}{p}'|)$

In the computation we employed the standard parametrization (16, 60)

$$f_\pm(q^2) = f_\pm(0)(1 - \lambda_\pm q^2/m_\pi^2)$$

$$= f_\pm(0)[1 + \bar{\lambda}_\pm 2(w_0 - k_0 + m_l^2/2m_K)/m_K] \qquad 2.38$$

where $\bar{\lambda} = \lambda . m_K^2/m_\pi^2$. This parametrization simply corresponds to a Taylor expansion of $f_\pm(q^2)$ about $q^2 = 0$, and can be interpreted in terms of the simple pole model as discussed in section 2.1.

So far all experiments are consistent with pure vector coupling (61), which we have assumed above. In particular Bellotti et al have found, from the π and e spectra measurements, that the upper limit for the contribution from the scalar and tensor form-factors (f_S and f_T) is less than 9 %(62).

The expressions computed for the branching ratio are recorded below, for which we define the quantity $B = W_0 - k_0 + m_l^2/2m_K$.

$$\Gamma_{\mu 3} = \int_{k_0 min}^{k_0 max} dk_0 \left\{ g^2 \frac{\sin^2\theta_c}{16\pi^3} f_+^2(0) \right\} \left(1 + \frac{4B}{m_K}\bar{\lambda}_+\right)$$

$$\times \left\{ \left[1/3\, m_K |\underset{\sim}{k}|^2 + 1/3\, m_l^2 |\underset{\sim}{k}|^2/B + \frac{m_l^2}{8m_K}(m_K^2 + m_\pi^2 + 2m_K k_0) \right.\right.$$

$$\left. + \frac{1}{4} m_l^2 . \frac{m_K^2 - m_\pi^2}{m_K} \xi(0)\left(1 + \frac{2B\bar{\lambda}_-}{m_K} - \frac{2B\bar{\lambda}_+}{m_K}\right) + \frac{1}{4} m_l^2 B \right] (\xi(0))^2$$

$$\left. \cdot \left(1 + \frac{4B\bar{\lambda}_-}{m_K} - \frac{4B\bar{\lambda}_+}{m_K}\right) \right\}. \qquad 2.39$$

$$\Gamma_{e3} = \int_{k_0 max}^{k_0 min} dk_0 \left\{ g^2 \frac{\sin^2\theta_c}{16\pi^3} f_+^2(0) \right\} \left[|\underset{\sim}{k}| \frac{(k_0^2 - m_\pi^2)}{3m_K^2} \right.$$

$$\left. + \frac{4B\bar{\lambda}_+}{3m_K^4} |\underset{\sim}{k}| (k_0^2 - m_\pi^2) \right]. \qquad 2.40$$

Taking the computed ratio of $\Gamma_{\mu 3}/\Gamma_{e3}$, the final result obtained was

$$\Gamma_{\mu 3}/\Gamma_{e3} = 0.6456 + 0.1055\,\bar{\lambda}_+ + 0.1264\,\xi + 0.0006\,\bar{\lambda}_+\xi$$
$$+ 0.0355\,\xi\,\bar{\lambda}_- + 0.0192(\xi)^2 + 0.0126\,\bar{\lambda}_-(\xi)^2 - 0.0531\,\bar{\lambda}_+(\xi)^2 \qquad 2\cdot 41$$

A similar relation has been obtained by Cabibbo (55). We find the first term of his expression is somewhat larger than ours, which is particularly significant for experimental values of R close to the value of this term. In fact Cabibbo obtains 0.6487, which is significant for R $\approx$ 0.64 when determining ξ in terms of $\bar{\lambda}_+$ and $\bar{\lambda}_-$.

From equation 2·41 it is apparent that $\Gamma_{\mu 3}/\Gamma_{e3}$ is dependent on the three parameters ξ, λ_+ and λ_-, and hence the extraction of the value of ξ from measurements of R is dependent on a knowledge of the energy dependence of the form-factors. Therefore experiment can only determine a relation between the three parameters λ_+, λ_- and ξ. In fact, the branching ratio is fairly insensitve to λ_-, and we essentially determine a relation between ξ and λ_+ for different values of the branching ratio. Early experiments simply ignored λ_+ and λ_-, which is equivalent to settting these parameters equal to zero in the above.

Both the sensitivity of ξ to the values of λ_+ and λ_-, and the difficulty in determining ξ from the experimental values of R, due largely to the lack of conclusive knowledge about λ_+ and λ_-, motivated Cabibbo and Maksymowicz (56) to point out the desirability of μ - polarization measurements. They showed that due to the zero spin of π and K, and the definite helicity state of the neutrino, once the kinematics of the decay are fixed, the meson is totally polarized along some direction (56, 63), and that the direction of the polarization is very sensitive to the value of $\xi(q^2)$.

This can be seen below from the expressions for the longitudinal and transverse components of the μ - polarization (56, 64, 65) These are obtained by projecting the μ spin onto the z-axis (quantization axis z $\perp$ to $\underset{\sim}{p}'$) from the expressions for the decay rates (eq.'s 2·39, 40), to give the rates $\Gamma(z, +\tfrac{1}{2})$ and $\Gamma(z, -\tfrac{1}{2})$. $\Gamma(z, \pm\tfrac{1}{2})$ are defined to be the decay rates for the projection of the μ spin onto the z-axis, having the values $\pm\frac{1}{2}$ respectively. This is effected by multiplying the Dirac μ - spin projection operator $\dfrac{-i\gamma\cdot p + m_\mu}{2m_\mu}$ by the covariant spin projection operator $\frac{1}{2}\left(1 \pm 2W\cdot z / m_\mu\right)$, to give $\Gamma(z, \pm\tfrac{1}{2})$ respectively (65). 2·42

W_λ is the Pauli-Lubanski covariant spin vector(65), which reduces in the rest frame of the μ to the spin of the μ, and $\frac{2W_\lambda}{m_\mu}$ is defined as the ' covariant polarization vector' S (65), which coincides in the rest of

μ , to the polarization vector, $\underset{\sim}{S}_R$ of the muon. $\underset{\sim}{S}_R$ is the experimentally observable polarization, in contrast to S which is not in general, except in the muon rest frame. In this frame $2w_\lambda/m_\mu$ is orthogonal to p_λ' . Ie $w_\lambda \cdot p_\lambda' = 0$, as in this frame $\frac{2w_\lambda}{m_\mu} \to \underset{\sim}{S}_R$ and $p_\lambda' = (0, i m_\mu)$. Since $w_\lambda \cdot p_\lambda'$ is a lorentz scalar, it must be zero in all frames, and in particular the muon rest frame.

From equations 2.39,40,41, $\Gamma(\mathfrak{z}, \pm 1/2)$ can be written in the general form ($A \pm B \cdot \mathfrak{z}$). Hence we can define the 'covariant polarization vector' S in analogy with the non-relativistic case (64) by,

$$\frac{\Gamma(\mathfrak{z}, +1/2) - \Gamma(\mathfrak{z}, -1/2)}{\Gamma(\mathfrak{z}, +1/2) + \Gamma(\mathfrak{z}, -1/2)} = \frac{B \cdot \mathfrak{z}}{A} = S \cdot \mathfrak{z}$$

From the properties

$$S \cdot p' = 0 \quad , \quad p_\lambda'^2 = -m_\mu^2 \quad , \quad p' \cdot \mathfrak{z} = 0 ,$$

S can be written in the general form

$$S_\lambda = B_\lambda / A + \frac{B \cdot p'}{A} \cdot \frac{p_\lambda'}{m_\mu^2} .$$

Introducing the covariant spin projection operator in the previous trace calculation for $\sum_{spin} |M|^2$, the expressions $\Gamma(\mathfrak{z}, \pm 1/2)$ are obtained. By a change of basis, the $\Gamma(\mathfrak{z}, \pm 1/2)$ are then transformed to the rest frame of the muon (65) to obtain the longitudinal and transverse components ofthe polarizatinn vector $\underset{\sim}{S}_R$. The final covariant expression for $\underset{\sim}{S}_R$ is then evaluated for the Kaon decaying at rest to yield,

$$\underset{\sim}{S}_R = \underset{\sim}{B} / |\underset{\sim}{A}| = \left\{ -b \left(\underset{\sim}{k} - \frac{(\underset{\sim}{p}' \cdot \underset{\sim}{k})}{|\underset{\sim}{p}'|^2} \underset{\sim}{p}' \right) + \left[-\frac{m_K a}{m_\mu} - \frac{b}{m_\mu} \left(\frac{\underset{\sim}{p}' \cdot \underset{\sim}{k}}{|\underset{\sim}{p}'|^2} p_0' + (m_K - k_0) \right) \right] \underset{\sim}{p}' \right\} / |\underset{\sim}{A}| , \qquad 2\cdot 43.$$

using the linear parametrization

$$\xi(q^2) = \xi(0) \left[(1 - \lambda - (q^2/m_\pi^2)) / (1 - \lambda + (q^2/m_\pi^2)) \right] = f_+(q^2)/f_-(q^2).$$

The various quantities occuring in equation 2.43 are defined by,

$$a/m_\mu = m_K \left[(1 - \xi(0))(1 - \lambda - q^2/m_\pi^2 + \lambda + q^2/m_\pi^2)(W_0 - k_0) - 2(m_K - k_0 - p_0') \right].$$

$$b/m_\mu = \left[m_K^2 + \frac{m_\mu^2}{4} (1 - \xi^2(0)) . (1 - 2\lambda - q^2/m_\pi^2 + 2\lambda + q^2/m_\pi^2) - p_0' m_K (1 - \xi(0)) (1 - \lambda - q^2/m_\pi^2 + \lambda + q^2/m_\pi^2) \right].$$

$$W_0 = (m_K^2 + m_\pi^2 - m_\mu^2)/2m_K ; \quad q^2 = 2m_K(k_0 - W_0 - m_\mu^2/2m_K) ; \quad k_0' = (m_K - p_0' - k_0).$$

As the muon is totally polarized (56,63),we require $|\underset{\sim}{A}| = |\underset{\sim}{B}|$. The first term corresponds to the transverse polarization, and the second to the logitudinal polarization. As time reversal invariance requires the relative reality of the f-f's, both of these components lie in the decay plane defined by $\underset{\sim}{k} \times \underset{\sim}{p}$. This follows due to the fact that components transverse to $\underset{\sim}{k} \times \underset{\sim}{p}$ transform as pseudo-vectors, and would be required to be multiplied by imaginary components from ξ, in order to transform overall as a vector. Hence this component dos'nt arise.

From equation 2·43, $\underset{\sim}{S}_k = \underset{\sim}{S}_k(k^c, p_c')$ is seen to be directly dependent upon $\xi(q^2)$, or in the parametrization we are using, upon $\xi(0)$ and $\xi(0)((\lambda_+ - \lambda_-) q^2/M_\pi^2)$. Hence a measurement of $\underset{\sim}{S}_k$, for fixed values of k_c and p_c offers a direct determination of $\xi(q^2)$. A measure of the energy dependence of $\xi(q^2)$ can be determined by evaluating $d\xi(0)/d[(\lambda_+ - \lambda_-)\xi(0)]$ which is obtainable from a plot of $\xi(0)$ verses $(\lambda_+ - \lambda_-)$, and which is studied below in comparison with the information obtainable from the branching ratio measurements.

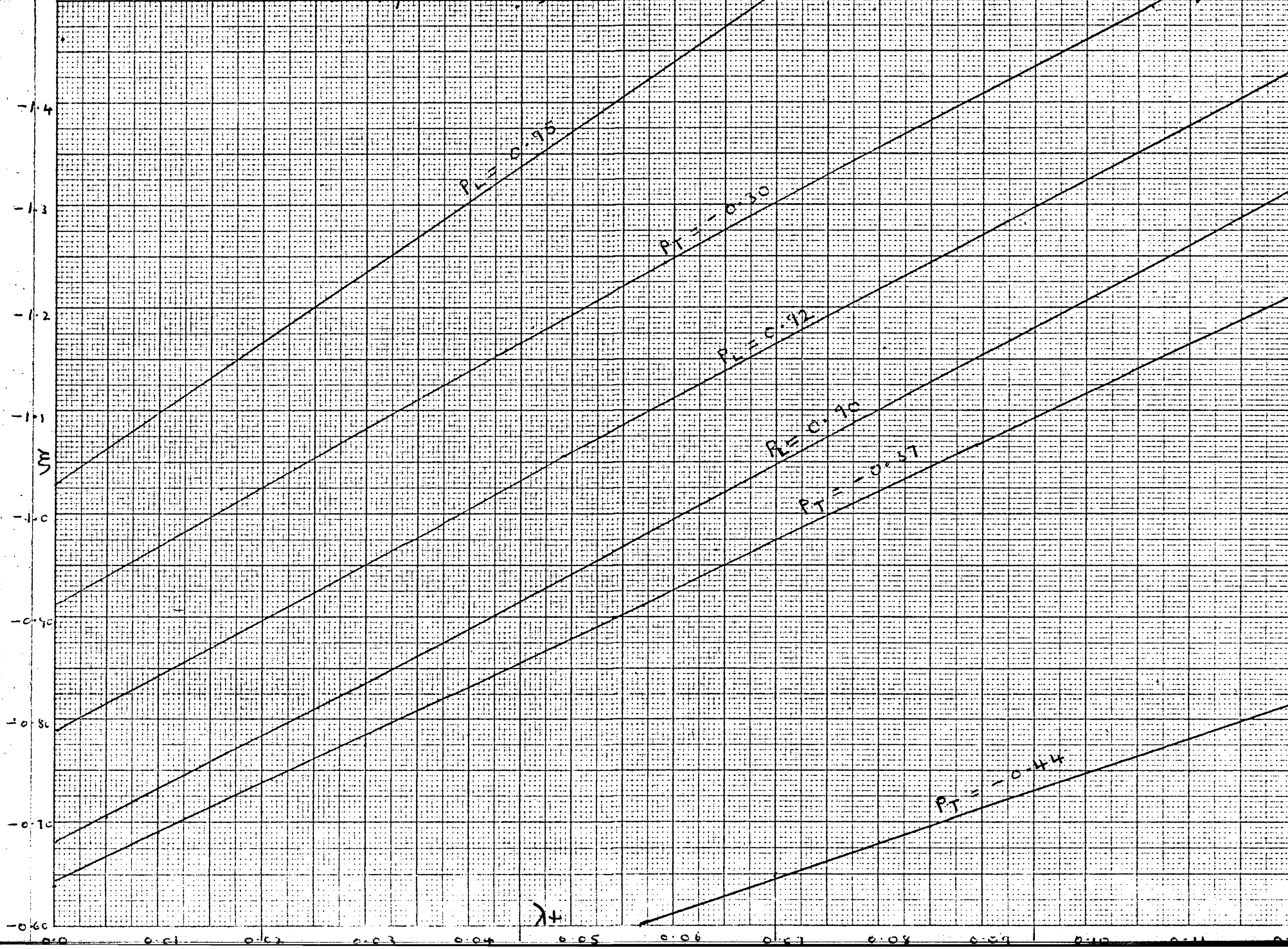
Experimental determination of ξ.

Recent experiments studying the three parameter fit of ξ, λ_+ and λ_- to the data of the R and Dalitz plot experiments, show that, while $\xi(0)$ is strongly correlated with λ_+, both λ_+ and ξ are much less correlated with λ_-. In this regard λ_- is difficult to determine experimentally, particularly in the case of the R experiments for which $d\xi/d\lambda_+ \gg d\xi/d\lambda_-$, as is apparent from the expression for R (eq., 2·41). As λ_- is only weakly coupled theoretically for R, to ξ and λ_+, measurments of R are not very sensitive to λ_-.

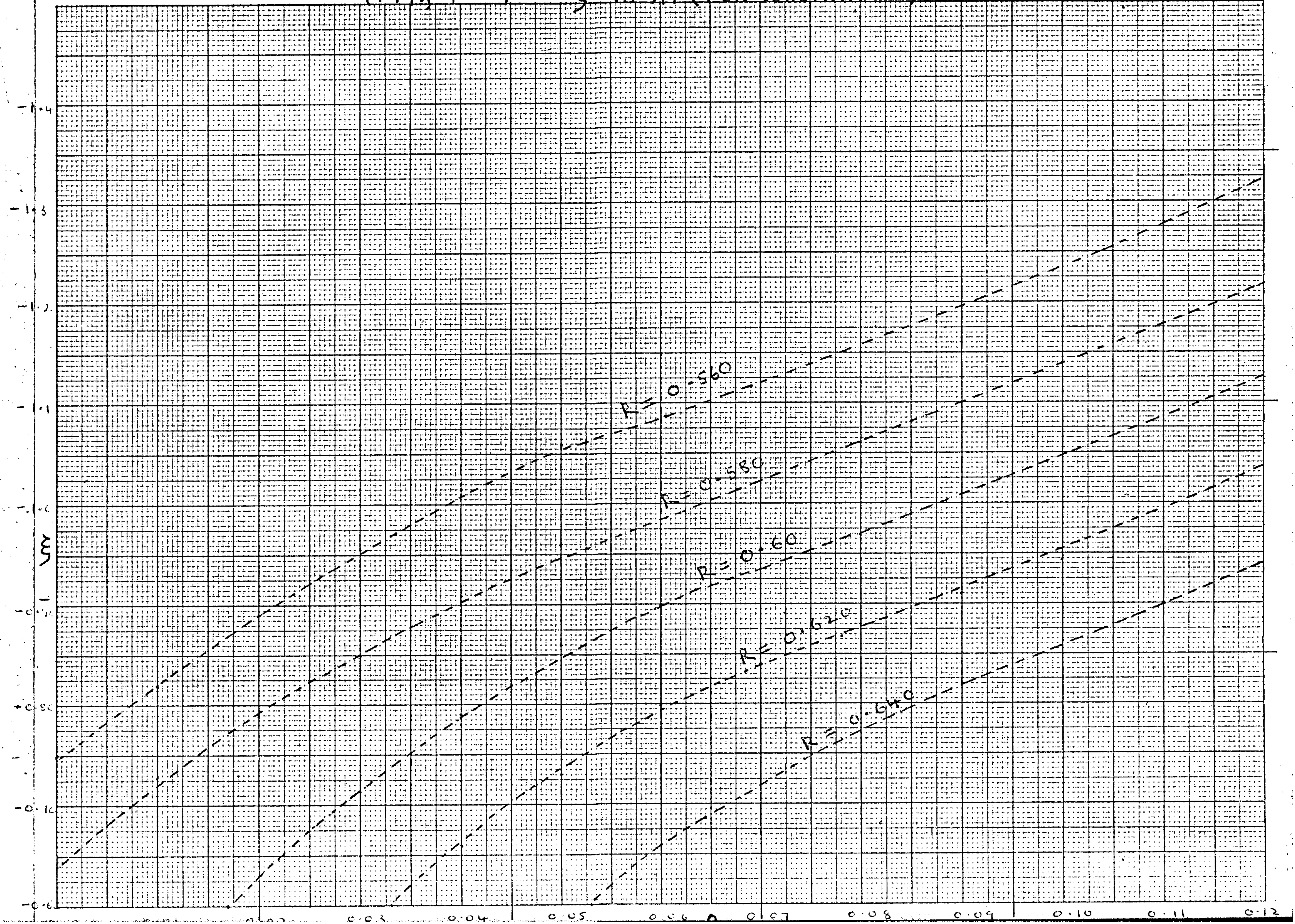
μ-Polarization, however, depends on $\xi(q^2) = \xi(0)\left[1 - \frac{\lambda_- - \lambda_+}{M_\pi^2} q^2\right]$, so these experiments are equally sensitive to λ_+ and λ_-, and can't distinguish between them. However, Chounet (66), in a very recent compilation of the experimental determination of ξ, concludes that at the present time λ_- is con sistent with zero. We will follow a number of authors (SEE 66) in setting $\lambda_- = 0$ in the analysis below, and in particular in the analysis of the polarization experiments, to determine the variation of ξ with λ_+. Theoretically there is no basis for setting $\lambda_- = 0$, although it is a useful device in comparing the results of the R and P measurements, by reducing the problem down from a three parameter fit to a two parameter fit.

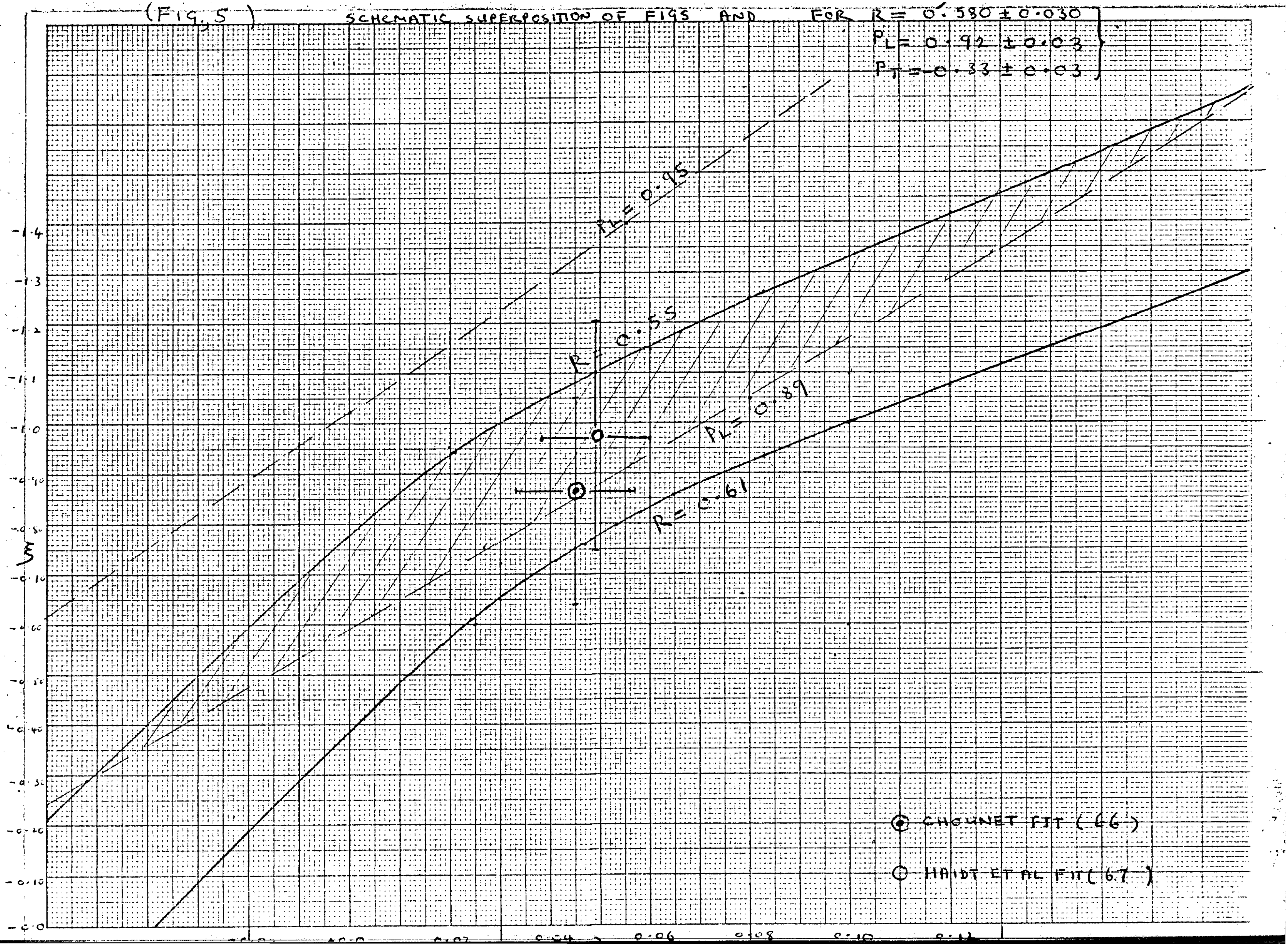
From the expressions for both longitudinal and transverse polarization (P_L and P_T respectively), and the relation for the branching ratio, we have computed plots for ξ verses λ_+, on setting $\lambda_- = 0$ (Figs 3 and 4 respectively).

(Fig., 3) ξ vs λ+ (FOR CONSTANT PL AND PT′ IN 60-100 MEV)



(FIG., 4) ξ vs λ_+ (FOR CONSTANT R)

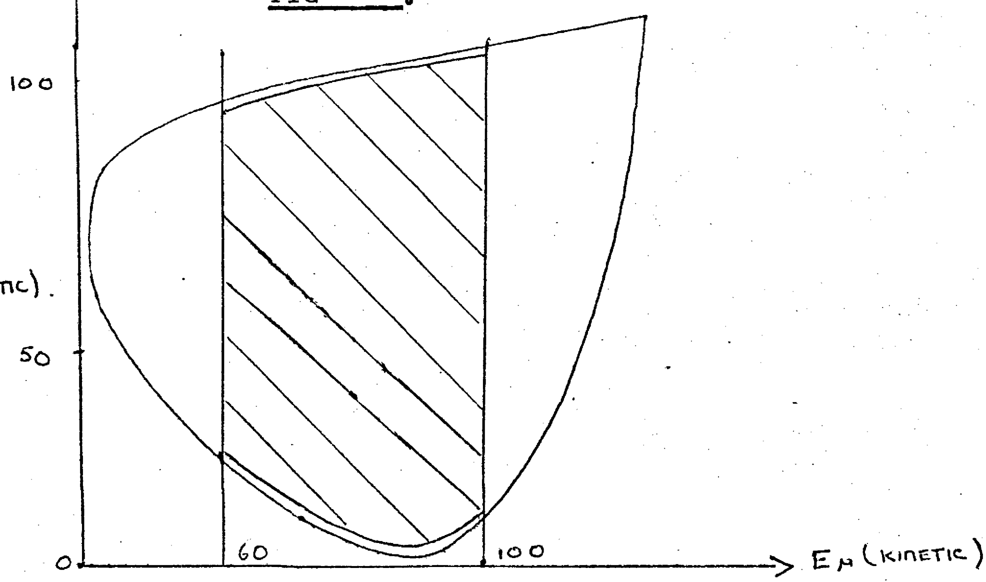




(FIG. 5) SCHEMATIC SUPERPOSITION OF FIGS AND FOR $R = 0.580 \pm 0.030$, $P_L = 0.92 \pm 0.03$, $P_T = -0.33 \pm 0.03$

In the case of P_L and P_T, a double integration was first computed to obtain a plot of ξ verses the weighted average of P_L(P_T) (Ie the average polarization over the region of the Dalitz plot under consideration), for different values of λ_+. We chose the 60-100 Mev muon energy band accross the Dalitz plot, to integrate over (Fig 6), as this lies on the largest and most important band for the reconstruction of the K^+_{l3} decays for determining the polarization of the muon (67). The regions surrounding this band are excluded either because of the resolution of the polarization, or because these fall in the physical regions of the τ and $K_{\pi 2}$ decay modes which would impair the reconstruction of the K^+_{l3} mode(67).

FIG 6.



From the computed graphs mentioned above, plots of ξ vs λ_+ were readily obtained for different constant weighted average values of P_L and P_T (Fig 3) . Figure 4 is similarly a plot of ξ vs λ_+ for varying values of R. The two plots were then superimposed and are presented in fig ,5

Using the graphs as a guide, we will first briefly outline the experimental situatuon regarding λ_+ and ξ , R and P_L(P_T), for $K^+_{\mu 3}$ and K^+_{e3} , for which there exists more abundant data (66) at present than for the K^c_{l3} decay mode. From this discussion and the graphs ,we then draw some conclusions about the compatibility of the R and P_μ determinations of ξ , and finally, we compare a number of simple theoretical model predictions with the most recent experimental results.

The most direct way of determining λ_+ is a study of the e or π spectra, given by the expressions $d\Gamma_{e3}/dp_e'$ and $d\Gamma_{e3}/dk_c'$ respectively (eq ., 2.40), which amounts to a study of the K^+_{e3} Dalitz plot. A recent determination of the average value of λ_+ , obtained by Chounet (66), fromthe latest experiments up to 1970 studying the e and π energy spectra, yields $\lambda_+ = 0.030 \mp 0.007$. Nevertheless, considerable variation exists in the values correlated, with the more recent values indicating a trend to higher values of λ_+. Bellotti et al (62) obtained $\lambda_+ = 0.045 \pm 0.018$ from a Dalitz plot analysis. Similatly Botterill et al (60) obtained λ_+ as large as 0.08 ± 0.04 from a study of the e spectrum, although a more recent determination by this group, studying the π^0 energy spectrum in K^+_{e3} decay, finds $\lambda_+ = 0.045 \pm 0.015$, which is more significant by virtue of it's better accuracy.

A less direct determination of λ_+ is obtained from a measurement of the K^+_{e3} decay rate Γ^+_{e3} ,which is related to λ_+ by

$$\Gamma(K^+_{e3}) = \sin^2\theta_c f_+^2(0) (1 + 3.707 \lambda_+) \times 7.42 \times 10^7 sec^{-1}. \quad (55).$$

This method requires a priori the value of $f_+(0)$ $\sin\theta_c = \sin\theta_V$ for the determination of λ_+. In a recent determination of $\Gamma(K^+_{e3})$, Haidt et al (67) find $\lambda_+ = 0.029 \pm 0.010$ for $f_+(0) \sin\theta_c = 0.218 \pm 0.004$. For a small change in $f_+(0) \sin\theta_c$ to 0.206 ± 0.006 they find λ_+ increases sharply to 0.060 ± 0.019. In the light of the analysis of Brene et al (sect., 2.1.) we expect, from the above, that λ_+ lies in the vicinity of 0.04.

The above values of λ_+ indicate a considerable increase in the values over the last two years, from the value 0.020 ± 0.008, estimated to be the world average up to the 1968 Heidelberg conference (53).

The increasingly large values are supported further by Chounet (66) who finds the overall fit to the $K^+_{\mu 3}$data, obtained from μ- polarization and branching ratio experiments, requires $\lambda_+ = 0.045 \pm 0.012$ for $\lambda_- = 0$ and $\xi = -0.85 \pm 0.20$. Chien et al (68) , in a recent high statistics experiment, have investigated the $K^0_{\mu 3}$ Dalitz plot, and find $\lambda_+ = 0.09 \pm 0.002$ for $\lambda_- = 0$ and $\xi = -0.68 \pm 0.12$. The most recent and highest statistics experiments tend to support $\lambda_+ > 0.04$ for $\lambda_- = 0$.

For the determination of ξ from the polarization measurements, the most recent , highest statistics experiments, of in particular, Cutts et al (69),Haidt et al (67) and Bettels et al (70) are all consistent with $\xi \sim -1$ for $K^+_{\mu 3}$ decay. With $\lambda_- = 0$, Haidt et al find the most likely value of $\xi = -1.0 \pm 0.3$ for P_L 0.95 ± 0.09 and $P_T = -0.29 \pm 0.09$, in the energy range 47 Mev $\leqslant E_\mu \leqslant$ 94 Mev. This can be seen from the curves of fig 3 for the values of P_L and P_T above. As the μ is totally polarized, we plot graphs in pairs of (P_L, P_T) for $P_L^2 + P_T^2 \simeq 1$ ($\neq 1$, as we have taken a weighted average over a region of the Dalitz plot, rather than values of P_L and P_T at particular points in the Dalitz plot).The pairs plotted are (0.95 , -0.30) , (0.92 , - 0.37) , (0.90 , -0.44), all of which are well within the experimental limits of the latest highest statistics experiments (67,66). Large values of ξ of ~ -1 are seen to require a large component of longitudinal polarization and a small component of of transverse polarization. Cutts et al also find $\xi = -0.95 \pm 0.09$ in the μ energy range $55\,MeV \leq E_\mu \leq 90\,MeV$ for $\lambda_- = 0$, but give no values for P_L and P_T.

A number of earlier experiments (71) with statistics of an order of 10^1 less than the above, found P_L consistently smaller than 0.80, with large positive values of ξ obtained as a consequence. This is readily seen from the graphs (fig., 3, 5).From the approximate linearity of the plot of ξ vs P_L from which the graphs were obtained, we expect positive values for ξ . Chounet (66) finds the best overall fit to the $K^+_{\mu 3}$ polarization data requires $\xi = -1.45 \pm 0.70$ for $\lambda_+ = 0.07 \pm 0.15$ and $\lambda_- = 0$, by fitting a linear variation of λ_+ with the experimental data on the values of ξ obtained for bands of q^2 accross the Dalitz plot. We find this variation of ξ with λ_+ compatible with the experiment of Haidt et al from the graph (fig.,3) above, for their values of P_L and P_T. Hence the polarization data requires $\xi \approx -1$.

From equation 2.41 for R , setting $\lambda_- = 0$, we obtain

$$R = \Gamma_{\mu 3}/\Gamma_{e3} = 0.6456 + 0.1055\bar{\lambda}_+ + 0.1264\xi + 0.0006\bar{\lambda}_+\xi + 0.0192\xi^2, \quad (\bar{\lambda}_+ = \lambda_+ m_K^2/m_\pi^2) \qquad 2.44.$$

for K^+ decay. In the case of the R measurements, the first term is close to the experimenatal value of R, and hence ξ and λ_+ are very sensitive to the value of R. Small changes in R mean significant changes in ξ or λ_+. For $R \lesssim 0.6456$, as $\bar{\lambda}_+ > 0$, this implies ξ is negative. On the other hand for $R \gtrsim 0.6456 + 0.1055\,\bar{\lambda}_+$, ξ is positive.

The world average obtained by Chounet is $R = 0.626 \pm 0.019$, which was calculated by averaging separately the data on the decay rates of $K^r_{\mu 3}$ and K^r_{e3}, and turns out to be somewhat lower than the individual measurements of R. For example, Botterill et al (72) obtained $R = 0.667 \pm 0.17$, while Auerbach et al (73) obtained R as large as 0.753 ± 0.076. The particle data group (74), in contrast to Chounet (66), found an average value of $R = 0.656 \pm 0.023$, by averaging values of R.

Many of the earlier experiments (53,66,67) obtained values of R significantly larger than 0.7, with the consequence of large positive results for ξ (eq., 2.44), particularly as those experiments , up to 1967, neglected any energy dependence of the form factors in extracting ξ from the expression for R. This is especially important for the energy dependence of the $J = 1^-$ form factor $f_+(q^2)$, which if included can significantly lower the value of ξ . The error in setting $\lambda_+ = 0$ is also compounded wihh the difficulties in the earlier experiments of the detection of the $K_{\mu 3}$ mode, due to the difficulty in separating the $K_{\mu 3}$ mode from the background (66)

Contrary to the values above , the recent X2 experiment (67), which is the most comprehensive and accurate experiment to date, obtained $R = 0.596 \pm 0.025$ for $\lambda_- = 0$ (as in all the cases quoted above), which is significantly smaller than any of the previous values of R, and lends support to a large value of λ_+ , for ξ in agreement with the results of the polarization experiments. The essential feature of the recent higher statistics experiments, is that they indicate a smaller value of R, and subsequently with the larger values of λ_+ , as indicated by the most recent Dalitz plot analyses discussed above, values of ξ compatible with the polarization measurments are possible.

Haidt et al obtain an overall fit to their polarization, branching ratio and Dalitz plot analyses with $\lambda_- = 0$, for $\xi = -0.98 \pm 0.23$ and $\lambda_+ = 0.049 \pm 0.011$. This is to be compared with the overall fit obtained by Chounet(66) in the compilation of the up to date above experiments, who finds for $\lambda_- = 0$ $\xi = -0.85 \pm 0.20$ and $\lambda_+ = 0.045 \pm 0.012$. We have marked both these fits on the graph (fig. 5). Both fall well within the region of overlap of the μ-polarization and R experiments, as shown more diagramatically in fig 5 , where the heavy lines lie well within experimental limits of the most recent experimental results. Both points fall well within the the experimental errors of the values for P_L and P_T and the experimental values quoted for the latest value of R.

It is interesting to compare the apparent consistency between the R and μ- polarization determination of ξ and λ_+ from the graph above (fig 5), in the region $0.08 \geq \lambda_+ \geq 0.02$, with a similar analysis carried out by Auerbach et al (75). They took $P_T = -0.32 \pm 0.08$ and $R = 0.73 \pm 0.03$ for the average values of these quantities at the time, and concluded that unless $\lambda_+ > 0.24$ for $\lambda_- = 0$, the μ- polarization determination of λ_+ and ξ is not compatible with the R determination of these quantities.

This conclusion is apparent from the graph above (fig 4), and arises chiefly through the large R value. We argue then that while the μ-polarization measurements of ξ have remained reasonably stationary, the recent measurements of R have shown a marked tendency to smaller values of R, and that compatibility within experimental error for these two methods of determination of $\xi(0)$ is possible, especially since newer experiments indicate a larger value of λ_+ than before. If we regard the values of P_L and P_T, as obtained by Haidt et al (67), as being very close to the true values of these quantities, within $\pm$ 0.07, then for small values of R persisting, inthe region of R ~ 0.59, we conclude from figs 4, 5 above, that somewhat larger values of $\lambda_+ (\lambda_- = 0)$, will become possible.

We will now briefly compare the predictions of a number of simple models relating to the energy dependence of the form- factors, with the predictions we would expect from the trend of the most recent experiments.

The Callan-Treiman relation, discussed in section 2.1. relates the form-factors $f_\pm$ at the unphysical point $q^2 = -m_K^2$ in the following way,

$$f_+(-m_K^2) + f_-(-m_K^2) = f_K / f_\pi \sqrt{2} + O(m_\pi^2),$$

and requires for it's validity that the off-mass shell form-factors, defined by $f_+(p^2, k^2, q^2)$ and $f_-(p^2, k^2, q^2)$, are smooth functions of k^2, such that the extrapolation of k^2 from zero to the pion mass shell is well defined. Substituting the standard linear parametrization for $f_\pm(q^2)$, we can rewrite the Callan-Treiman relation in the form

$$f_-(0)/f_+(0) = \xi(0) = \left[f_K/(f_\pi f_+(0)\sqrt{2} - \frac{f_+(-m_K^2)}{f_+(0)} \right] \frac{f_-(0)}{f_-(-m_K^2)} + O(m_\pi^2).$$

Using the recent experimental determination of $f_K/(f_\pi \sqrt{2} f_+(0))$ by Haidt et al (67), we find for $\lambda_- = 0$, $\xi(0) \simeq -0.48 + O(m_\pi^2)$, $\lambda_+ = 0.060$.

$$\xi(0) \simeq -0.20 + O(m_\pi^2), \quad \lambda_+ = 0.029.$$

Hence the C-T relation yields values of $\xi(0)$ which are substantially smaller than recent experiments indicate, unless values of $\lambda_+ > 0.06$ are taken.

From the definition of the spin zero form-factor f_0 (eq., 2-17), if we set $m_\pi^2 = 0$ and $q^2 = -m_K^2$, corresponding to the unphysical soft pion point, then

$$f_0(-m_K^2) = f_+(-m_K^2) + f_-(-m_K^2) = \frac{1}{\sqrt{2}} f_K/f_\pi, \quad \text{(From the C-T relation)},$$

so $f_0(-m_K^2)/f_+(0) = f_0(-m_K^2)/f_0(0) = \frac{1}{\sqrt{2}} \frac{f_K}{f_\pi f_+(0)} = 1.25$ approx (67).

In the kappa- dominance model (Kappa pole model), we have

$$f_0(-m_K^2)/f_0(0) = \frac{m_k^2}{m_k^2 - m_K^2}, \quad \text{for } \lambda_+ = m_\pi^2/m_k^2$$

If $m_k \sim 1100$ Mev (46), then for $m_K = 500$ Mev we find

$m_k^2/(m_k^2 - m_K^2) = 1.26 \simeq$ LHS, to an excellent approximation, so the

C-T relation is not in conflict with the dominance of the S-waves by a kappa meson pole.

Possible conflict comes when demands are made on λ_+. In the pole model approximation, for f_0 dominated by kappa, and f_+ by the K^*- meson, we have from the definition of $f_0(q^2) = f_+(q^2) - \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$,

the following pole model result

$$f_-(q^2)/f_+(0) = (m_K^2 - m_\pi^2)\left[\frac{1}{m_\kappa^2+q^2} - \frac{1}{m_{K^*}^2+q^2}\right],$$

for which λ_- is fixed and not equal to zero. This treatment requires

$$\xi(0) = (m_K^2 - m_\pi^2)\left(\frac{1}{m_\kappa^2} - \frac{1}{m_{K^*}^2}\right).$$

For $\xi(0) \cong -0.85$ we require

$$-0.85 \cong 0.25 - (m_K^2 - m_\pi^2)/m_{K^*}^2 \qquad \text{whence} \quad m_{K^*}^2 \simeq (m_K^2 - m_\pi^2)/1.10,$$

which implies a value $m_{K^*}^2 \approx 460$ Mev, which is about half the mass of the usual K^* meson, predicting λ_+ to be $\lambda_+ = m_\pi^2/m_{K^*}^2 = \left(\frac{140}{460}\right)^2 = 0.09$ aprrox., in agreeement with upper estimates of λ_+.

This result follows similarly from the C-T relation which we can write in the form

$$(1 + \lambda_+ m_K^2/m_\pi^2) + \xi(0)(1 + \lambda_- m_K^2/m_\pi^2) \cong 1.25,$$

where we have used the value of Haidt et al for $f_K/f_\pi\sqrt{2}$.

Setting $\lambda_- = 0$ we find $\lambda_+ \cong 0.09$ for $\xi(0) = -0.85$, which is not incompatible with the latest data. Hence the C-T relation is compatible with recent experiments, whether one uses the kappa-K^*- pole approximation to take account of the possibility of $\lambda_- \neq 0$, or one uses a strict linear approximation for the form-factors with λ_- set equal to zero, provided a large value for λ_+ is taken.

The above models do not give any indication of why λ_+ is large, and in particular the K^* pole model of the spin 1^- form-factor, for the experimentally observed K^*-vector meson at $\sim$ 890 Mev, predicts $\lambda_+ =$ 0.023 , which was the old world averge(53), but is now in serious disagreement with the changing experimental scene. Hence if large λ_+ persists, the simple K^* pole model may have to be substantially modified. Two possible directions which this may take are by either considering the possibility of inelastic contributions to the form-factor , or by considering a more complicated parametric form to replace the pole structure, as for example, the dipole form-factor.

An indication that λ_- may be approximately equal to zero can be obtained by considering kappa dominance of the spin zero form-factor, and equating this to the strict linear parametrizations (eqs., 2.19) thus

$$\frac{f_+(0)m_K^2}{m_K^2+q^2} = f_+(0)\left[1 - q^2/m_\pi^2\left(\lambda_+ + \frac{\xi(0)m_\pi^2}{m_K^2-m_\pi^2}\right) + \frac{\xi(0)\lambda_- q^4}{m_\pi^2(m_K^2-m_\pi^2)}\right].$$

equating powers of q^2, we obtain,

$$\xi(0) = -\lambda_+\frac{(m_K^2-m_\pi^2)}{m_\pi^2} + \frac{(m_K^2-m_\pi^2)}{m_K^2}. \qquad 2\cdot45$$

and

$$\lambda_-\xi(0) = m_\pi^2(m_K^2-m_\pi^2)/m_K^4 \sim 0\cdot003$$

For $\xi \sim -0.9$, we find $\lambda_- \simeq -0.003$ and $\lambda_+ \approx 0.09$.
For $\xi \sim -0.8$ we find $\lambda_- \approx -0.003$ and $\lambda_+ \approx 0.07$. On the other hand the strict pole model yields $\xi(0)\lambda_- = m_\pi^2(m_K^2-m_\pi^2)\left(\frac{1}{m_K^4} - \frac{1}{m_{K^*}^4}\right)$ which for the values of $\xi(0)$ above, again suggests that λ_- is small.

The expression for $\xi(0)$ from the assumption of kappa dominance alone (eq., 2·45), is consistent with the experiments for large λ_+. In particular we find for $\xi(0) = -0.6$, $\lambda_+ = 0.07$; $\xi = -0.7$, $\lambda_+ = 0.08$. In the next section, we show among other things, that a theorem for $\xi(0)$, recently derived by Dashen and Weinstein, reduces to the simple kappa model above, with the assumption of PCVC, and hence we can remark now, that this theorem is also consistent with the recent experimental results for large λ_+.

In conclusion the main points are,

1). $\lambda_+ > 0.04$, in conflict with the K^*- dominance model, and that still higher values appear feasible, particularly from future values of R.

2). λ_- is essentially unknown, and very difficult to determine experimentally, due to its very weak coupling to λ_+ and ξ in measurements of R, and the inability of polarization measurements to distinguish between λ_+ and λ_-.

3). The increasing compatibility of the R and polarization measurements in the determination of ξ ,due mainly to the smaller values of R obtained.

4). That despite the very large errors associated with the measurements of P_L and P_T(and hence the apparent discrepancy of the predictions for ξ from P_L and P_T, from fig., 3), the polarization experiments, together with the small values of R, and the larger values obtained for λ_+, obtained, in particular, from the recent e and π spectra measurements, indicate that ξ is large and negative, and of the order of -1.0.

5). While the simple kappa pole model and the C-T relation are compatible within experimental error with the recent data, the K^*-dominance of the $J = 1^-$ form-factor is in serious disagreement with experiment, and may have to be modified substantially.

Section 2.3. Symmetry breaking and the kappa-meson dominance model.

This section is an account of work carried out in applying the Gell-Mann-Oakes-Renner (G.O.R.) symmetry breaking scheme (43) and kappa meson dominance of the $J=0^+$ form-factor to the analysis of the $K_{\ell 3}$ form-factors, following the method employed by Dashen and Weinstein (76) in the derivation of their theorem for the $K_{\ell 3}$ form-factor ratio ξ . Assuming a specific form for the symmetry breaking Hamiltonian, and including the hypothesis of kappa PCVC, a relationship for $\xi(0)$ is obtained up to second order in ϵ ,the SU(3)$\otimes$SU(3) symmetry breaking parameter (eq., 1.40). We are thus able to obtain second order corrections to the Dashen-Weinstein (D-W) theorem using this model, and also that the D-W theorem reduces to the simple kappa pole model, expanded to second order in the symmetry breaking, as a consequence of the assumption of PCVC.

In this regard the kappa meson model has been employed to obtain an explicit estimate of the effects of symmetry breaking upon $\xi(0)$. We show further, that as a consequence of this model, and the assumptions used in applying it, $f_+(0)$ is unrenormalized to all orders in the symmetry breaking by the kappa meson. A further consequence of this model is a specific measure of the renormalization of the ratio f_K/ f_π from it's SU(3) symmetric value of 1 by the symmetry breaking interaction. We find $f_K - f_\pi = f_\kappa$ where f_κ is the decay constant of the kappa meson, which vanishes in the SU(3) symmetric limit (40).

The work of other authors, related to the above calculations, is reviewed, and it is shown that seemingly different approaches, using the same basic assumptions, reach the same conclusions.

Recently Dashen and Weinstein advanced a theorem for $\xi(0)$, independently of any assumption concerning the structure of the symmetry breaking Hamiltonian. Writing $H = H_0 + \epsilon H'$, where H_0 is the part invariant under SU(3)$\otimes$SU(3), and H' simultaneously breaks SU(3)$\otimes$SU(3) and SU(3) with the strength ϵ , they argued that ϵ is small enough for a perturbation expansion in ϵ to be meaningful, and obtained for $\xi(0)$, up to second order in ϵ ,the following expression.

$$\xi(0) = \frac{1}{2}\left(\frac{f_K}{f_\pi} - \frac{f_\pi}{f_K}\right) - \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2}\right)\lambda_+ + O(\epsilon^2) \qquad 2.46$$

Under the requirement that ϵ is small, this theorem is expected to be generally valid for different models of H' and in particular, for the G.O.R. symmetry breaking model, which we have briefly reviewed in chapter 1 (sect., 1.7.), and which we assume below.

We begin the calculation by introducing the on mass shell matrix element $\langle \pi^{c}_{(k)} | \partial_u V_u^{4-is}(o) | K^{+}(p) \rangle \sqrt{4 p^o k^o} (2\pi)^3$

$$= -i[f_+(q^2)(m_K^2 - m_\pi^2) - q^2 f_-(q^2)] \qquad \text{(Sect., 2.1)}$$

$$= -i(m_K^2 - m_\pi^2) f_o(q^2), \tag{2.47}$$

where f_+ and f_- are the usual form-factors, and f_o is the spin zero f-f. $f_+(q^2)$ and $f_o(q^2)$ are expanded in the standard form

$$f_+(q^2) = f_+(o)(1 - \lambda_+ q^2/m_\pi^2)$$
$$f_o(q^2) = f_o(o)(1 - \lambda_o q^2/m_\pi^2), \tag{2.48}$$

where

$$\xi(o) = f_-(o)/f_+(o) = \frac{(m_K^2 - m_\pi^2)}{m_\pi^2}(\lambda_o - \lambda_+). \tag{2.49}$$

Using PCVC in the form

$$\partial_u V_u^{4-is}(x) = i f_k m_k^2 \varphi_k^-(k), \tag{2.50}$$

equation 2.47 can be written as

$$i f_k m_k^2 \langle \pi^c | \varphi_k^-(o) | K^+ \rangle \sqrt{4 p^o k^o} (2\pi)^3 \tag{2.51}$$

$$= i f_k m_k^2 \Delta_k(q^2)(g \Gamma(m_K^2, m_\pi^2, q^2)),$$

where Γ is the K-π-k vertex function, normalized such that $\Gamma(o,o,o) = 1$, and $g \equiv g_{kK\pi}$, is defined as the strong interaction vertex coupling constant (c-c). $\Delta_k(q^2)$ is the k meson propagator, which will be approximated below by a pole.

Following the method of D-W, on contracting the π^o and K^+, and using the PCAC conditions

$$\partial_u A_u^{4+is} = f_K m_K^2 \varphi_{K^+} \equiv D_A^{4+is}, \quad \partial_u A_u^3 = f_\pi m_\pi^2 \varphi_{\pi^o}/\sqrt{2} \equiv D_A^3, \tag{2.52}$$

we obtain for equation 2.51

$$i f_k m_k^2 \Delta_k(q^2) g \Gamma(m_K^2, m_\pi^2, q^2) = \lim_{p^2 \to m_K^2} \lim_{k^2 \to m_\pi^2} (i)^2 \sqrt{2} \frac{(p^2 + m_K^2)(k^2 + m_\pi^2)}{f_K^2 m_K^2 f_\pi^2 m_\pi^2}$$
$$\times \iint d^4x \, d^4y \, e^{+ipy} e^{-ikx} \langle o | T(D_A^3(x) \, D_A^{4+is}(y) \, D_V^{4-is}(o)) | o \rangle. \tag{2.53}$$

On removing the limits on the RHS of 2.53, we obtain the off mass shell expression for the vertex function.

$$i f_k m_k^2 \Delta_k(q^2) g \Gamma(p^2, k^2, q^2) = (i)^2 \sqrt{2} \frac{(p^2 + m_K^2)(k^2 + m_\pi^2)}{f_K m_K^2 f_\pi m_\pi^2} .$$
$$\iint d^4x \, d^4y \, e^{+ipy} e^{-ikx} \langle o | T(D_A^3(x) \, D_A^{4+is}(y) \, D_V^{4-is}(o)) | o \rangle. \tag{2.54}$$

In a similar manner, the off mass shell f-f's f_- and f_+ (eq. 2.8) are defined by

$$f_+(p^2, k^2, q^2)(p+k)_u + f_-(p^2, k^2, q^2)(p-k)_u$$

$$= (i)^2\sqrt{2}\,(p^2+m_K^2)(k^2+m_\pi^2)\,/(f_K m_K^2 f_\pi m_\pi^2)$$

$$\cdot \iint d^4x\, d^4y\, e^{+ipy} e^{-ikx} \langle 0|T(D_A^3(x)\, D_A^{4+i5}(y)\, V_\mu^{4-i5}(0))|0\rangle. \qquad 2\cdot 55$$

Using the relation $iq_\mu \longleftrightarrow \partial_\mu$,where $q_\mu = p_\mu - k_\mu$ is the momentum transfer, and multiplying both sides of equation 2·54 by iq_μ we obtain ,

$$i[\,f_+(p^2,k^2,q^2)(p^2-k^2) + f_-(p^2,k^2,q^2)q^2\,]$$

$$= i(p_\mu - k_\mu)\Big[(i)^2\sqrt{2}\,\frac{(p^2+m_K^2)(k^2+m_\pi^2)}{f_K m_K^2 f_\pi m_\pi^2}$$

$$\cdot \iint d^4x\, d^4y\, e^{+ipy} e^{-ikx} \langle 0|T(D_A^3(x)\, D_A^{4+i5}(y)\, V_\mu^{4-i5}(0))|0\rangle. \qquad 2\cdot 56$$

By partial integration and evaluation of the resultant E.T.C. (equal-time-commutator) terms, using the G.O.R. model (eq's 1·38, 42), the above becomes (Appendix B)

$$i f_k m_k^2\, \Delta_k(q^2)\, g\, \Gamma(p^2,k^2,q^2) + \frac{i}{\sqrt{2}}(m_K^2 - m_\pi^2) + \frac{i}{\sqrt{2}}(p^2-k^2). \qquad 2\cdot 57$$

Then setting $p^2 = k^2 = q^2 = 0$ and $\Delta_k(q^2) = 1/(q^2+m_k^2)$, we see that

$$0 = i g f_k + \frac{i}{\sqrt{2}}(m_K^2 - m_\pi^2), \qquad 2\cdot 58$$

and hence we may write 2·57 as

$$-\frac{i}{\sqrt{2}}(m_K^2-m_\pi^2)\frac{m_k^2}{m_k^2+q^2}\Gamma(p^2,k^2,q^2) + \frac{i}{\sqrt{2}}(m_K^2-m_\pi^2) + \frac{i}{\sqrt{2}}(p^2-k^2)$$

$$= i[\,f_+(p^2,k^2,q^2)(p^2-k^2) + f_-(p^2,k^2,q^2)q^2\,]. \qquad 2\cdot 59$$

To proceed further it is necessary to evaluate

$$\frac{i}{\sqrt{2}}(m_K^2-m_\pi^2)\frac{m_k^2}{m_k^2+q^2}\Gamma(p^2,k^2,q^2)$$

$$= \sqrt{2}\,\frac{(k^2+m_\pi^2)(p^2+m_K^2)}{f_\pi m_\pi^2 f_K m_K^2}\iint d^4x\, d^4y\, e^{+ipy} e^{-ikx} \langle 0|T(D_A^3(x)\, D_A^{4+i5}(y)\, D_V^{4-i5}(0))|0\rangle. \qquad 2\cdot 60$$

By integrating the RHS by parts sucessively, we obtain six terms involving E.T.C. terms, which may be evaluated within the G.O.R. model, in the same manner as illustrated in appendix B for obtaining equation 2·57 , together with a seventh term of the form

$$k_\alpha p_\beta q_\mu \iint d^4x\, d^4y\, e^{-ikx} e^{+ipy} \langle 0|T(A_\alpha^3(y)\, A_\beta^{4+i5}(x)\, V_\mu^{4-i5}(0))|0\rangle, \qquad 2\cdot 61$$

which is at least fourth power in the momentum variables.

We are thus able to expand the integral up to quadratic terms in k^2 ,q^2 , and p^2 in the form

$$i(a + bq^2 + cp^2 + dk^2). \qquad 2\cdot 62$$

Depending on which of the partial integrations are performed first (ie wrt. ∂_α or ∂_β) we find,

$$a = \frac{\sqrt{2} - c/_2}{3c} f_k^{\,2} m_k^{\,2} - \frac{3c}{4\sqrt{2} - 2c} f_K^{\,2} m_K^{\,2}$$

$$= \frac{\sqrt{2}+c}{3c} f_k^{\,2} m_k^{\,2} - \frac{3c}{4(\sqrt{2}+c)} f_\pi^{\,2} m_\pi^{\,2}. \qquad 2\cdot 63.$$

$$= {}^1\!/_2\, f_\pi f_K (m_k^{\,2} - m_\pi^{\,2}).$$

The PCAC and PCVC conditions, together with the G.O.R. (43) prediction for the current divergencies hence tell us

$$c = -\sqrt{2}\,(f_K m_K^{\,2} - f_\pi m_\pi^{\,2})/(f_K m_K^{\,2} + {}^1\!/_2 f_\pi m_\pi^{\,2}), \qquad 2\cdot 64$$

together with

$$f_k m_k^{\,2} = f_K m_K^{\,2} - f_\pi m_\pi^{\,2}. \qquad 2\cdot 65$$

These relations, together with the consistency requirement for the expr- essions involving 'a' lead to a further constraint,

$$f_k m_k^{\,2} = (f_K - f_\pi)(f_K m_K^{\,2} - f_\pi m_\pi^{\,2}), \qquad 2\cdot 66$$

which implies

$$f_k = f_K - f_\pi. \qquad 2\cdot 67.$$

The assumption of PCVC, together with the G.O.R. breaking scheme leads to the relative fixing of scale of the quantities f_K, f_π, and f_k in terms of m_K, m_π, and m_k. Employing the charge- divergence CR's (eq., 1·38) (8) and the conditions (sect., 1.7.)

$$\partial_u V_u^{\,i}(x) = c\, f_{ijk}\, u_k(x) \qquad i = 1 \cdots 8$$

$$\partial_u A_u^{\,i}(x) = {}^1\!/_{\sqrt{3}}\,(\sqrt{2}+c)\, v^i(x) \quad i = 1 \cdots 3$$

$$\partial_u A_u^{\,i}(x) = {}^1\!/_{\sqrt{3}}\,(\sqrt{2} - c/_2)\, v^i(x) \quad i = 4 \cdots 7, \qquad 2\cdot 68$$

we obtain

$$f_k m_k^{\,2} / f_\pi m_\pi^{\,2} = -3c/2(\sqrt{2}+c)$$

$$f_k m_k^{\,2} / f_K m_K^{\,2} = -3c/_2\,(\sqrt{2} - c/_2), \qquad 2\cdot 69.$$

and hence from equation 2·69 we find,

$$f_k / f_\pi = (m_K^{\,2} - m_\pi^{\,2})/(m_k^{\,2} - m_K^{\,2}) \qquad (a$$

$$f_K / f_\pi = (m_k^{\,2} - m_\pi^{\,2})/(m_k^{\,2} - m_K^{\,2}) \qquad (b$$

$$f_k / f_K = (m_K^{\,2} - m_\pi^{\,2})/(m_k^{\,2} - m_\pi^{\,2}). \qquad (c \qquad 2\cdot 70$$

These consequences automatically follow from the assumptions above, although they are not usually stated in papers dealing with this subject.

Insertion of these relations often leads to equivalence of results of many authors (below).

One point of difference from the G.O.R. scheme is in the determination of ' c '. Basic to their derivation is the assumption that $f_K \approx f_\pi$. However, the assumption of the existence of the k- meson, and PCVC (sect., 1.6) leads to a measure of this approximation through $f_k = f_K - f_\pi$. They further assume $\langle 0 | u_8 | 0 \rangle$, where u_8 is the eigth component of the scalar densities u_i (chap., 1 sect., 1.6). In the above model, using equations 2.64 – 2.69, and the charge-divergence CR's (eq's 1.38), we obtain

$$\langle 0 | u_8 | 0 \rangle = 2/3 \, f_k{}^2 m_k{}^2 / c .$$

In this way the k- meson is being used to parametrise the effects of SU(3) symmetry breaking.

Returning to the integral, we find we may expand the integral as follows. (The factor $(k^2 + m_\pi{}^2)(p^2 + m_K{}^2)/f_\pi m_\pi{}^2 f_K m_K^2$ is understood). 2.71a.

$$\iint d^4x\, d^4y\, e^{-ikx} e^{ipy} \langle 0 | T (D_A{}^3(x) D_A{}^{4+is}(y) D_V{}^{4-is}(0)) | 0 \rangle$$

$=$

A). $$- \iint d^4x\, d^4y\, e^{-ikx} e^{ipy} \, \delta(x_0 - y_0) \langle 0 | T ([A_0{}^3(x), D_A{}^{4-is}(y)] D_V{}^{4+is}(0)) | 0 \rangle$$

B). $$- \iint d^4x\, d^4y\, e^{-ikx} e^{ipy} \, \delta(x_0) \langle 0 | T ([A_0{}^3(x), D_V{}^{4+is}(0)] D_A{}^{4-is}(0)) | 0 \rangle$$

C). $$- i k_\alpha \iint d^4x\, d^4y\, e^{-ikx} e^{ipy} \, \delta(y_0 - x_0) \langle 0 | T ([A_0{}^{4-is}(y) A_\alpha{}^3(x)] D_V{}^{4+is}(0)) | 0 \rangle$$

D). $$- i k_\alpha \iint d^4x\, d^4y\, e^{-ikx} e^{ipy} \, \delta(y_0) \langle 0 | T ([A_0{}^{4-is}(y), D_V{}^{4+is}(0)] A_\alpha{}^3(x)) | 0 \rangle$$

E). $$- k_\alpha p_\beta \iint d^4x\, d^4y\, e^{-ikx} e^{ipy} \, \delta(y_0 - x_0) \langle 0 | T ([V_0{}^{4+is}(0), A_\alpha^3(x)] A_\beta{}^{4-is}(0)) | 0 \rangle$$

F). $$- k_\alpha p_\beta \iint d^4x\, d^4y\, e^{-iqy} e^{-ikx} \, \delta(y_0) \langle 0 | T ([V_0{}^{4+is}(y), A_\beta{}^{4-is}(0)] A_\alpha{}^3(x)) | 0 \rangle$$

G). $$+ i k_\alpha p_\beta q_\mu \iint d^4x\, d^4y\, e^{-iqy} e^{-ikx} \langle 0 | T (A_\alpha{}^3(x) A_\beta{}^{4-is}(0) V_\mu{}^{4+is}(y)) | 0 \rangle .$$

2.71b

Terms A and C are of the so called σ- type terms (43 , 76), disregarded by D-W (76), as they are of higher than first order in SU(3)⊗SU(3)

breaking, with $\partial_\mu V_\mu \sim O(\epsilon)$, and $[A, \partial_\mu A_\mu] \sim \partial_\mu V_\mu \sim O(\epsilon)$
The principal contribution comes from terms E and F. As the term G is of higher order in the momentum, we drop this term henceforth.

We may integrate and evaluate the various terms A–F exactly (see appendix B for example), yielding,

A). $i\left(\dfrac{\sqrt{2} - c/2}{3c}\right)(f_k m_k^2)^2 / (q^2 + m_k^2)$

B). $-ic3(f_K m_K^2)^2 / 4(\sqrt{2} - c/2)(m_K^2 + p^2)$

C). $\dfrac{-if_k m_k^2}{}(q^2 + k^2 - p^2)/(q^2 + m_k^2)$

D). $i3c f_\pi^2 m_\pi^2 k^2 / 4(\sqrt{2} + c).(k^2 + m_\pi^2)$

E). $if_\pi^2 m_\pi^2 (q^2 - k^2 - p^2) / 4(m_\pi^2 + k^2)$

F). $-i(q^2 - k^2 - p^2) f_K^2 m_K^2 / 4(m_K^2 + p^2)$. 2·72.

Counting powers of symmetry breaking ϵ, using the fact that m_k^2, f_K, $f_\pi \sim O(1)$ and f_k, m_K^2, $m_\pi^2 \sim O(\epsilon)$, we find that A and C are of order ϵ^2 and D, E and F are of order ϵ.

At this stage the C–T relation (47) may be easily deduced for either soft pions or kaons from the above formalism. For example, for soft pions, $k_\mu = 0$, and term G does not contribute. Multiplying the above (eq's., 2·72) by the factor $\dfrac{(k^2 + m_\pi^2)(p^2 + m_K^2)}{f_\pi m_\pi^2 f_K m_K^2}$, which we omitted previously for convenience, and taking the limits $k^2 = 0$, $k_\mu = 0$, and hence, $q^2 = p^2 = -m_K^2$, the only remaining term is B, for which we find,

$$B \to -i3c f_K^2 m_K^4 m_\pi^2 / 4(\sqrt{2} - c/2).f_\pi m_\pi^2 f_K m_K^2 = -i\,\frac{f_K m_K^2 - f_\pi m_\pi^2}{2 f_\pi}. \qquad 2·73$$

Whence from equation 2·59, we have

$$-im_K^2 \{ f_+(-m_K^2, 0, -m_K^2) + f_-(-m_K^2, 0, -m_K^2) \} = \frac{-i}{\sqrt{2}}\,\frac{f_K m_K^2 - f_\pi m_\pi^2}{f_\pi}$$

$$+ \lim_{k^2 \to 0,\, p^2 \to -m_K^2} \frac{i}{\sqrt{2}}(m_K^2 - m_\pi^2) + i/\sqrt{2}\cdot(p^2 - k^2); \quad \text{R.H.S.} = -i/\sqrt{2}\left(f_K\,\frac{m_K^2 - f_\pi m_\pi^2 + f_\pi m_\pi^2}{f_\pi}\right) = \frac{-i}{\sqrt{2}}\frac{f_K}{f_\pi} m_K^2$$

Whence

$$\sqrt{2}\,[f_+(-m_K^2, 0, -m_K^2) + f_-(-m_K^2, 0, -m_K^2)] = f_K / f_\pi. \qquad 2·74$$

The equivalent soft kaon theorem is

$$\sqrt{2}\,[f_+(0, -m_\pi^2, -m_\pi^2) - f_+(0, -m_\pi^2, -m_\pi^2)] = f_\pi / f_K, \qquad 2·75$$

which may be obtained by a similar procedure.

Given the terms A–F, we may expand the integral up to quadratic powers in k^2, p^2 and q^2. As G was neglected, no higher order terms than quadratic are taken. The expansion obtained from 2.72 is,

$$\tfrac{i}{2}\,(m_K^2 - m_\pi^2)\, f_K f_\pi \left\{ 1 - q^2/m_k^2 - p^2/m_K^2 - k^2/m_\pi^2 \right\}, \qquad 2.76$$

which when multiplied by the factor 2.74 becomes,

$$\tfrac{1}{\sqrt{2}}\, i\,(k^2 + m_\pi^2)(1 - q^2/m_k^2). \qquad 2.78$$

Retaining only quadratic powers for consistency, in terms of the vertex function, we obtain

$$\frac{-i}{\sqrt{2}}\,(m_K^2 - m_\pi^2)\left(1 - q^2/m_k^2\right) = \frac{-i}{\sqrt{2}}\,(m_K^2 - m_\pi^2)\,\frac{m_k^2}{m_k^2 + q^2}\,\Gamma(p^2, k^2, q^2), \qquad 2.79$$

whence $\Gamma(p^2, k^2, q^2) = 1$, correct up to quadratic terms in the momentum. The simple form of the expansion of the integral above reveals the expected pole structure, which must exist from the assumptions of the kappa pole dominance and PCAC and PCVC employed above.

Using the fact that $\Gamma(p^2, k^2, q^2) = 1$, correct to quadratic terms in the momenta, we are only able to deduce $f_+(0,0,0)$ and $f_-(0,0,0)$. From equation 2.59, due to the constancy of Γ we have,

$$f_+(0,0,0) = 1/\sqrt{2}, \qquad f_-(0,0,0) = -\frac{1}{\sqrt{2}}\,\frac{m_K^2 - m_\pi^2}{m_k^2}. \qquad 2.80$$

We need to know the dependence of Γ up to quartic powers in the momentum in order to deduce anything about the momentum dependence of f_+ and f_-, and hence to extrapolate to the physical momentum values.

To the extent that the neglect of higher order terms in Γ is acceptable, we have

$$-i(m_K^2 - m_\pi^2)\, f_0(q^2) = i f_k m_k^2\, \Delta_k(q^2)\, g\, \Gamma(-m_K^2, -m_\pi^2, q^2)$$

$$\approx \frac{i f_k m_k^2}{q^2 + m_k^2}\, g\, \Gamma(0,0,0) = \frac{i}{\sqrt{2}}\,\frac{m_k^2}{q^2 + m_k^2}\,(m_K^2 - m_\pi^2), \qquad 2.81$$

or $\qquad f_0(q^2) = 1/\sqrt{2}\,(m_k^2)/(q^2 + m_k^2)$,

or (eq., 2.17) $\quad f_+(q^2)(m_K^2 - m_\pi^2) - q^2 f_-(q^2) = \frac{1}{\sqrt{2}}\, m_k^2/(q^2 + m_k^2)$. 2.82

Hence $\lambda_0 = m_\pi^2/m_k^2$, and from equation 2.49, the simple k - pole model yields

$$\xi(0) = -\frac{m_K^2 - m_\pi^2}{m_\pi^2}\,\lambda_+ + \frac{m_K^2 - m_\pi^2}{m_k^2} \qquad 2.83$$

On the other hand the D-W theorem is

$$\xi(0) = -\frac{m_K^2 - m_\pi^2}{m_\pi^2}\,\lambda_+ + \frac{1}{2}\left(f_K/f_\pi + f_\pi/f_K \right).$$

Thus in the model we have employed the correction to the D-W theorem is in the second term and arises completely from the inclusion of the terms A and C. Had we taken account of only the terms E and F, we would have recovered the D-W result.

$$(m_K^2 - m_\pi^2)/m_\pi^2 \cdot \lambda_0 = \frac{1}{2}\left(f_K/f_\pi - f_\pi/f_K \right). \qquad (\text{Eq}^{ns}\ 2\cdot 72\text{E} \ \&\ \text{F})$$

Inclusion of the terms which are of higher order in ϵ yields simply

$$\frac{m_K^2 - m_\pi^2}{m_\pi^2}\lambda_0 = \frac{m_K^2 - m_\pi^2}{m_K^2},$$

which is the kappa pole result, as above.

Using the explicit form for f_K/f_π (eq., 2·70b), it can be readily shown that

$$\frac{1}{2}\left(f_K/f_\pi - f_\pi/f_K \right) - \left(\frac{m_K^2 - m_\pi^2}{m_K^2} \right) = \frac{m_K^4 - m_\pi^2}{2 m_K^2} + O(\epsilon^3), \qquad 2\cdot 84$$

so that in this model, the correction to the D-W theorem is small for a kappa meson around 1 Bev (46). (approx., $\simeq$ - 0.03)

ie $$\xi(0) = \frac{1}{2}\left(f_K/f_\pi - f_\pi/f_K \right) - \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2} \right)\lambda_+ - \frac{1}{2}\left(\frac{m_K^4 - m_\pi^4}{m_K^4} \right) + o(\epsilon^3) \qquad 2\cdot 85.$$

From the above it is also clear that the κ -meson and PCVC reduces the current algebra result to the standard pole model result of dispersion relations. This is what we expected, but it does not seem to be generally recognized in the liturature, where many basically equivalent results are presented.

Although a large number of different approaches have been applied to the K-π-κ system to determine theoretically the quantities $\xi(0)$, $f_+(0)$ $f_-(0)$ and f_κ, no definite predictions have yet been made of these without eventually assuming κ- pole dominance of the spin zero form-factor. This arises due to the introduction of unknown parameters and constants when extending the simple pole dominance model to include finite width corrections, about which very little is known, let alone the existence of the κ- meson. On the other hand , in the kappa pole approximation, all quantities are well determined (c/f eq., 2·70 and 2·80) through the experimentally known quantities m_π, m_K, f_K, f_π, and hence, in so far as as this model is a good approximation to the truth, definite predictions are possible. Results obtained up to the present time for $\xi(0)$ all reduce to the simple κ- pole model prediction (eq., 2·83) on employing the relations 2·67 and 2·70 .

Glashow and Weinberg (77) assume only that the SU(3)⊗SU(3) breaking term of the Hamiltonian transforms as the $(\underset{\sim}{3}^*,\underset{\sim}{3}) \oplus (\underset{\sim}{3},\underset{\sim}{3}^*)$ representation of SU(3)⊗SU(3). Making a pole ansatz for their two point functions or propagators, and assuming that the three point function (c/f eq., 2·54) corresponding to the K-π-κ vertex, is a smooth function of the momentum variables, in order that an expansion to low orders in the momentum variables is justified, they obtain quite generally for $f_+(0)$,

$$f_+(0) = \left(f_\pi^2 + f_K^2 - f_\kappa^2 \right) / 2\sqrt{2}\, f_K f_\pi \qquad 2\cdot 86$$

If we impose a further constraint on the form of $\epsilon . H'$, by assuming the G.O.R. model (43), and use PCVC, on substituting for f_κ(2·67) we obtain $f_+(0) = 1/\sqrt{2}$ as previously (eq., 2·80). We expect this, as the smoothness assumption for the three point function is identitical in consequence to assuming $\epsilon << 1$, and that only the lowest order terms in the expansion of the integral (eq., 2·71b) need be retained. In the particular case of the κ- meson dominance, we found this to be justified by the smallness of the expansion coefficents (eq's 2·62, 72, 60). The result $f_+(0) = 1/\sqrt{2}$ implies that in the model approximation we have made, $f_+(0)$ is unrenormalized to all orders of the symmetry breaking, in contrast to the result of the Ademollo-Gatto theorem (44), but not in contradiction with it. In other words the κ -meson pole model produces no second and higher order symmetry breaking effects on $f_+(0)$.

It suffices henceforth, when comparing results for $\xi(0)$, obtained by employing the G.O.R. symmetry breaking model with PCVC and κ- pole dominance, to set $f_+(0)$ equal to it's SU(3) symmetric value of $1/\sqrt{2}$.

A similar smoothness assumption for the three point function (eq 2·54) is made by Deshpande (78), who also assumes H' transforms as the $(\underset{\sim}{3}^*,\underset{\sim}{3})\oplus(\underset{\sim}{3},\underset{\sim}{3}^*)$ representation of SU(3)$\otimes$SU(3). Starting with equation 2·51, and assuming

$$T(k^2, p^2, q^2) \approx \left(1 + p^2\, \partial/\partial p^2 + k^2\, \partial/\partial k^2 + q^2\, \partial/\partial q^2 \right) T(0,0,0),$$

Deshpande similarly obtains 2·86, (on making pole ansatzes in the spectral representation he employed for the two point functions), and

$$\xi(0) = \frac{f_K^2 - f_\pi^2 + f_\kappa^2}{2 f_K f_\pi} - (M_K^2 - M_\pi^2)\lambda_+ \frac{f_+(0)}{M_\pi^2} + \frac{1}{M_\kappa^2}\left[(M_K^2 - M_\pi^2) f_+(0) + \frac{f_K M_K^2\, 3c}{f_\pi \cdot 2 \cdot (\sqrt{2} - c/2)} \right) \qquad 2\cdot 87$$

Employing equations 2·64, 67, we obtain equation 2·83, which is the kappa pole dominance result.

The above result for $\xi(0)$ has also been obtained by Gerstein and Schnitzer (79) using the Hard-pion technique (80, 81). This method has also been employed extensively by a number of other authors (82), including initially Schnitzer and Weinberg (81), to obtain expressions for $\xi(0)$. The essential assumptions of the Hard- pion technique are : 1). The validity of the chiral SU(3)$\otimes$SU(3) and SU(2)$\otimes$SU(2) current algebra (8, 17). 2). PCAC and CVC (chapt., 1 and refr's therein) with single meson pole ansatzes for the two point functions, and 3). that the meson vertices or three point functions can be approximated by low order polynomials in the momentum. With the assumption that H' transforms as the $(\bar{3},3)\oplus(3,\bar{3})$ representation, the Glashow – Weinberg result follows, plus relations of the form 2·81 . By further imposing the results of the G.O.R. model and PCVC, the results of the hard pion calculations reduce to the simple κ – pole model, which can be written down directly from equation 2·49 for $\lambda_0 = \frac{m_\pi^2}{m_\kappa^2}$.

The results of the Hard – pion technique are reproduced by the somewhat different but equivalent procedure of using phenomenological lagrangians (83). In this approach lagrangians are constructed to satisfy chiral SU(3)⊗SU(3) and SU(2)⊗SU(2) current algebra up to terms proportional to m_π^2, with a chiral symmetry breaking piece added. This corresponds to assumption 1 of the hard – pion technique. The lagrangian formalism is then treated to lowest order in perturbation theory, or the so called tree – graph approximation, in which only diagrams with no internal loops are retained, and which is equivalent to the retention of only single particle resonant states (c/f assumption 2 above). With the inclusion of only the K^* and κ - resonance states (inclusion of others leads to a host of undetermined parameters and constants), this method gives rise to the same results as the dispersion theoretic pole model. In this case $\xi(0)$ reduces to

$$\xi(0) = -\frac{m_K^2 - m_\pi^2}{m_{K^*}^2} + \frac{m_K^2 - m_\pi^2}{m_\kappa^2}, \qquad 2\cdot 88$$

where in this approach, K^* vector meson dominance of the $J=1^-$ form-factor $f_+(q^2)$ implies $\lambda_+ = \frac{m_\pi^2}{m_{K^*}^2}$ in equation 2·49 .

The dispersion relation approach has been employed by a number of authors to investigate the $J=1^-$ and $J=0^+$ form – factors f_+ and f_0 (84), the absorptive parts of which are determined by spin 1 and spin 0 intermediate states respectively. Assuming once subtracted dispersion relations and taking into account only the $K-\pi$ intermediate states, the dispersion relations reduce to the Omnès equations (85), with the solution

$$f_0(q^2) = f_+(0)\exp\left[\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} q^2/q'^2 \; \frac{\delta_0(q')}{(q'^2 - q^2 - i\varepsilon)}\right], \qquad 2\cdot 89$$

and similarly for $f_+(q^2)$. These integrals have been evaluated using resonance or scattering length approximations (84, 86), although the solutions obtained are not unique (85), as they may be multiplied by a polynomial of degree $\leq n+1$ (or $\leq n$ if n integer) , where n is defined by $\lim_{q^2\to\infty} \delta(q^2) = n\pi$ (85). Unless it is known that $n = 0$, nothing can be said about the behaviour of the f-f's in the decay region. On the other hand, assuming once subtracted dispersion relations and saturating the integrals with poles for each form – factor, there is only one parameter to be determined, as for example, for the $J = 0^+$ form-factor, $f_0(q^2)$, which takes the form

$$f_0(q^2) = f_+(0) - q^2/m_k^2 \frac{C_k}{m_k^2 + q^2}), \qquad 2\cdot 90$$

with similar expressions for the other f-f's

The above procedure is commonly replaced by the assumption of unsubtracted dispersion relations for f_+ and f_0 with approximation of the resultant integrals by poles or equivalently zero width resonances (87). In this approximation the $J=1^-$ f-f, $f_+(q^2)$, is approximated by the K^* vector meson pole (88) and $f_0(q^2)$, the $J=0^+$ f-f, by the k pole. In this model $\xi(0)$ can be written down directly as

$$\xi(0) = \frac{m_K^2 - m_\pi^2}{m_k^2} - \frac{m_K^2 - m_\pi^2}{m_{K^*}^2},$$

which corresponds to the results obtained above.

Results very similar to ours have been obtained by Dahmen, Rothe and Schulke (89), using the algebra of charges. In this approach the CR's of the charges (eq's 1·27) eg., $[F_5^{\pi^0}(t), F(\overset{k^+}{t})] = \frac{F_5^{k^+}(t)}{2}$ etc. 2·91
(and similarly for the combinations k, K and π, k) are sandwiched between the vacuum and single particle meson states, and saturated by the lowest lying eligible intermediate single particle meson states. Using the relation

$$\langle n' | F_{(5)}(t) | n \rangle = \langle n' | [H', F_{(5)}(t)] | n \rangle / (E_{n'} - E_n), \qquad 2\cdot 92$$

and the G.O.R. model to relate the CR occuring to the divergence of a current (43) (eq's 1·38, 1·42) , and then employing equation 2·9 , relating the current - divergence matrix element to the $J=0^+$ form-factor, Dahmen et al obtain from the above charge- charge CR,

$$m_K f_K = f_\pi [(m_K + m_\pi) f_0^{\pi K}((m_K - m_\pi)^2) + (m_K - m_\pi) f_0^{\pi K}((m_K + m_\pi)^2)]$$
$$+ f_k [(m_K + m_k) f_0^{kK}((m_K - m_k)^2) + (m_K - m_k) f_0^{kK}((m_K + m_k)^2)], \qquad 2\cdot 93$$

evaluated in the rest frame of the intermediate particle state. In the above, f_0^{kK} is defined by

$$(2\pi)^3 \sqrt{4k^0 p^0} \langle k(k) | \partial_\mu V_\mu^{\pi^0}(0) | K^-(p) \rangle = -i(m_K^2 - m_k^2) f_0^{kK}, \qquad 2\cdot 94$$

and similarly for the other f-f's. To connect the f-f's at the different values of q^2, Dahmen et al assumed once subtracted dispersion relations for the f-f's f_0 of the form

$$f_0(q^2) = f_0(0) + q^2/\pi \int \frac{\mathrm{Im}\, f_0(q'^2)}{q'^2(q'^2 - q^2)} dq'^2.$$

Making a pole approximation for the absorptive part of f_0 , and repeating the process for the other CR's, they obtain, with our normalization,

$$f_0^{\pi K}(0) \equiv f_+^{\pi K}(0) = (f_\pi^2 + f_K^2 - f_k^2)/2\sqrt{2} f_\pi f_K, \qquad 2\cdot 95$$

and similarly for $f_+^{KK}(0)$ and $f_+^{K\pi}(0)$. 2.95 is just the Glashow Weinberg result (77). With the assumption of unsubtracted dispersion relations they obtain directly equation 2.83 and the relations 2.64, 67, 7c .The algebra of charges approach has also been stressed by Stech (90), who points out that the full content of the current algebra assumptions is obtainable from this approach. We see again that the simple kappa pole model is returned by assuming unsubtracted dispersion relations for the f-f's. The essential feature of this approach is that the $J = 0^+$ f-f is obtained at the two unphysical points $q^2 = -(M_K - m_\pi)^2$ and $-(M_K + m_\pi)^2$, necessitating the use of dispersion relations to relate $f_0(q^2)$ at these two points. Kraemmer and Vilhyalmsson (91) also use the charge- charge algebra, in the same manner to Dahmen et al, and reach similar conclusions to ourselves.

Identical results to the above approach of Dahmen et al have also been obtained by McKay et al (92) who use the dispersive sum rule method of Fubini and Furlan (93), which was originally applied by Ademollo et al (94) to vertex functions of the form $\langle 0 | [A_\mu^\alpha(x), V_\mu^\beta(0)] | K^\gamma \rangle$, relevant to the study of K_{l3}. With the assumptions of current algebra, PCAC and PCVC with pole dominance, McKay et al applied this approach to the above vertex function, and the vertex functions

$$\langle 0 | [A_\mu^\alpha(x), V_\mu^\beta(0)] | \pi^\gamma \rangle \;,\; \langle 0 | [A_\mu^\alpha(x), A_\mu^\beta(0)] | k \rangle .$$

The method, which is outlined in detail by Carbone et al (95), essentially begins by defining

$$W^{\alpha\beta\gamma} = i \int d^4x\, e^{-ipx}\, \theta(x_0) \langle 0 | [\partial_\mu A_\mu^\alpha(x), \partial_\mu V_\mu^\beta(0)] | \pi^\gamma(p) \rangle, \qquad 2.96$$

and similarly by replacing alternately π^γ with k^γ and k^γ with K^γ . Partially integrating, this becones

$$q_\nu p_\mu T_{\mu\nu}^{\alpha\beta\gamma} = W^{\alpha\beta\gamma} - i \int d^4x\, e^{-ipx}\, \delta(x_0) \langle 0 | [\partial_\mu A_\mu^\alpha(x), V_0(0)] | \pi^\gamma(p) \rangle + f^{\alpha\beta\gamma}(k-q)_\mu f_\pi / (2\pi)^{3/2} (2k_0)^{1/2}.$$

This expression is then evaluated in the collinear frame, defined à la Fubini and Furlan (93) by $p = kx$, $q = k(1-x)$, to give,

$$x(1-x) k_\mu k_\nu T_{\mu\nu}^{\alpha\beta\gamma} = W^{\alpha\beta\gamma} + S^{\alpha\beta\gamma} + f^{\alpha\beta\gamma} f_\pi k^2 (1-x) / (2\pi)^{3/2} (2k_0)^{1/2} \qquad 2.97$$

In this frame there is one independent variable 'x' and hence $p_\mu T_{\mu\nu}^{\alpha\beta\gamma}(x)$, can be written as $p_\nu T^{\alpha\beta\gamma}(x)$ etc., Choosing the rest frame of the π^γ , equation 2.97 becomes

$$m_\pi^2 x(1-x) T^{\alpha\beta\gamma}(x) = W^{\alpha\beta\gamma} + S^{\alpha\beta\gamma}(x) - f^{\alpha\beta\gamma} f_\pi m_\pi^2 (1-x) / (2\pi)^{3/2} (2k_0)^{1/2}. \qquad 2.98$$

As discussed by Fubini and Furlan, and further by Ademollo et al (94), this choice of reference frame permits only scalar and pseudo-scalar contributions to $\mathrm{Im}\, W^{\alpha\beta\gamma}$. $W^{\alpha\beta\gamma}$ was then assumed to obey an unsubtracted dispersion relation (92, 95), and whence, for x = 1, the following sum rule for W was obtained.

$$\frac{1}{\pi}\int dx \, \frac{\mathrm{Im}\, W^{\alpha\beta\gamma}(x)}{x(x-1)} = -f^{\alpha\beta\gamma} f_{\pi} m_{\pi}^{2} / (2\pi)^{3/2} (2m_{\pi})^{1/2}$$

The same proceedure is carried out for the other two vertices by McKay et al (92), and the absorptive parts of W are saturated with single particle meson states. This leads directly to equations 2.93 , which in turn lead to the results obtained by Dahmen et al, and the conclusions we obtained using the framework of the D-W theorem derivation along with PCVC and the G.O.R. breaking scheme.

Further to the above, we obtained definite predictions about $\xi(0)$, and using a specific model, obtained corrections to the D-W theorem . We find that for the kappa dominance model the second order symmetry breaking corrections are small, and the order ≈ -0.03 . We finally draw attention to the equivalence of the approaches discussed above, and point out that where expressions for $\xi(0)$ have been calculated, these reduce to the kappa dominance model on imposing PCVC.

CHAPTER 3.

S AND P WAVE NON LEPTONIC HYPERON DECAYS.

Introduction.

Although many papers have appeared on the topic of non-leptonic decays of the hyperons (NLDH), with many different approaches and models having been applied (96), there still exists in the liturature large uncertainties as to which approach is the correct one, and also the largely unsolved problem of a consistent calculation of the S and P wave amplitudes, corresponding to the Parity Violating (PV) and the Parity Conserving (PC) amplitudes of the decay.

Despite the large number of models proposed, most of these can be shown to be equivalent to a few standard models, or shown to give rise to similar results, under the assumptions employed in applying these models.

The first section is an outline of the theory and predictions of NLDH's. In the second section an account of a current algebra calculation carried out for the S and P wave amplitudes is given, and the problems and features of this calculation are discussed in the following section, in the light of present day calculations in this area, and the present status of the theory of the NLDH's.

Section 3.1. Basic theory of NLDH's.

NLDH's is the name given to decays of baryons of non-zero strangeness, decaying to baryons of zero strangeness and π mesons. Leptons play no part in these decays, which distinguishes them from the purely leptonic and semi-leptonic decay processes.

The decays which will be considered in this chapter are,

TABLE 3.

Decay mode.	(Shorthand)	Decay rate (97) $\Gamma \times 10^{10}\,\text{sec}^{-1}$
$\Lambda^{\circ} \rightarrow p + \pi^{-}$	(Λ°_{-})	0.397 - 0.005
$\Lambda^{\circ} \rightarrow N + \pi^{\circ}$	$(\Lambda^{\circ}_{\circ})$	0.225 - 0.005
$\Sigma^{-} \rightarrow N + \pi^{-}$	(Σ^{-}_{-})	0.610 - 0.047
$\Sigma^{+} \rightarrow N + \pi^{+}$	(Σ^{+}_{+})	1.235 - 0.020
$\Sigma^{+} \rightarrow p + \pi^{\circ}$	$(\Sigma^{+}_{\circ})$	1.235 - 0.020
$\Xi^{\circ} \rightarrow \Lambda + \pi^{\circ}$	$(\Xi^{\circ}_{\circ})$	0.330 - 0.020
$\Xi^{-} \rightarrow \Lambda + \pi^{-}$	(Ξ^{-}_{-})	0.602 - 0.013

and we denote the above precesses quite generally as

$$B_1(P_1) \longrightarrow B_2(P_2) + \pi(q) \; . \qquad 3.1$$

For later reference the quantum number assignment of the participating particles, in summary, is

TABLE 4.

Y	T	B = +1 ($J^P = \frac{1}{2}^+$)	B = 0 ($J^P = 0^-$).
1	$\frac{1}{2}$	P, N	$K^+ \; K^0$
0	1	$\Sigma^+ \; \Sigma^0 \; \Sigma^-$	$\pi^+ \; \pi^0 \; \pi^-$
0	0	Λ^0	η^0
-1	$\frac{1}{2}$	$\Xi^0, \; \Xi^-$	$\bar{K}^0 \; K^-$

As the above decays are neither charge symmetric nor charge independent they need not conserve strangeness. In fact, from the q.no., assignment, each decay is seen to proceed via a $|\Delta Y| = 1$ change in hypercharge (we henceforth use Y in preference to S, where $Y = S + B$).

Parity conservation was first brought into question by Lee and Yang (98), in relation to the non-leptonic $\tau - \theta$ paradox phenomenon. Two mesons, namely τ and θ, were observed to be similar in all respects, except that τ decayed into 3π and θ to 2π, with final states of opposite parity, deduced from the final state pion configuration (99). Among the experiments suggested by Lee and Yang to test for parity non-conservation, as a consequence of the above paradox, was the measurement of the up-down assymmetry of the pion emmitted in Λ- decay, which subsequently helped confirm the postulate of the non-conservation of parity, and in particular, for the NLDH's(96). τ and θ were subsequently identified with the kaon meson.

In transitions between spin $\frac{1}{2}$ baryons, parity non-conservation implies both the S and P wave channels are open to the decay. An interference between these two allowed channels permits the measurement of three effects (96).
1). An assymmetry in the angular distribution of the pion with respect to the spin of $B_1(P_1)$ 2).and 3). A longitudinal and transverse polarization of B_2.

This is made more explicit by considering the decay in the rest frame of $B_1(P_1)$. If H_ω is identified as the weak Hamiltonian responsible for the decay, then to first order in H_ω, the reduced matrix element for the decay process (eq., 2.7a) is given by

$$\tilde{T} = - \langle B_2(P_2)\, \pi(q) \mid H_\omega(0) \mid B_1(P_1) \rangle . \qquad 3.2$$

As the final state can be a linear combination of S and P states, which we denote by S and P respectively, then by Lorentz covariance, the above equation can be written in the form

$$\frac{-i}{(2\pi)^{9/2}} \sqrt{\left(\frac{B_1 B_2}{P_1^0 P_2^0 \, 2q^0} \right)} \; \bar{u}(P_2) \left[S + \gamma_5 P \right] u(P_1) \quad \ldots \qquad 3.3$$

(where capitol letters will denote the masses of the corresponding particles), and S and P denote the S and P wave amplitudes respectively. In general S and P are invariant functions of the invariant variables $s = -(p_2+q)^2$, $t = -(p_1-p_2)^2$, $u = -(p_1-q)^2$, which for on mass shell decay take the values $s = -p_1^2 = B_1^2$, $t = -q^2 = \pi^2$ and $u = -p_2^2 = B_2^2$. The decay rate Γ is related to S and P by (100)

$$\Gamma = c_1 \{ |S|^2 + c_2 |P|^2 \},$$

for which

$$c_1 = \frac{|q|}{8\pi} \frac{(B_1+B_2)^2 - \pi^2}{B_1^2} \quad \text{and} \quad c_2 = \frac{((B_1-B_2)^2 - \pi^2)}{((B_1+B_2)^2 + \pi^2)}. \qquad (100).$$

At centre-of-mass energies of the decay, $\tilde{T}$ can be written in the non-relativistic form (100)

$$\tilde{T} = \frac{-i}{(2\pi)^{9/2}} \chi^+ (a_S + a_P \underset{\sim}{\sigma} \cdot \hat{p}_2) \chi$$

where

$$u(p_1) = \frac{(B_1 + p_1^0) - i \underset{\sim}{\gamma} \cdot \underset{\sim}{p}_1}{\sqrt{2 B_1 (p_1^0 + B_1)}} \chi \quad \text{and} \quad a_S = \left(\frac{p_1^0 + B_1}{2p_1^0}\right) S \quad : \quad a_P = \frac{|q|\, P}{\sqrt{2p_1^0 (p_1^0 + B_1)}}.$$

In isotopic spin space a_S and a_P will be either in an $I = 1$ state (Ξ - decay) or a linear combination of $I = \frac{1}{2}$ and $3/2$ (Λ and Σ decays), which is taken into account by writting $a_S = a_S(I)$ and $a_P = a_P(I)$. Experimentally, these amplitudes are determined through the measurement of the 'assymmetry parameters' α, β, γ, which are in turn determined fron the three effects enumerated above (100).

α is related to the angular distribution of B_2 relative to the polarization of B_1 (96, 100) by

$$D_{B_2}(\theta) = \text{const} (1 + \alpha P_{B_1} \cos\theta), \qquad 3\cdot4$$

where P_{B_1} is the polarization of B_1, and θ is defined as the angle between $\underset{\sim}{p}_1$ and $\underset{\sim}{p}_2$, and $\alpha = 2 \operatorname{Re} (a_S^* a_P) / (|a_S|^2 + |a_P|^2)$. (101) 3·5

For an isotropic angular distribution, $\alpha = 0$. This implies either $a_S = 0$ or $a_P = 0$, which implies the conservation of parity. The experimental verification of anisotropy in the angular distribution (100) confirms the non-conservation of parity, and leads to a determination of the product αP_{B_1}.

β and γ are determined from the polarization of B_2 (101)

$$\langle \sigma \rangle_{B_2} = \frac{1}{(1 + \alpha \underset{\sim}{P}_{B_1} \cdot \hat{\underset{\sim}{p}}_2)} \Big\{ (\alpha + \underset{\sim}{P}_{B_1} \cdot \hat{\underset{\sim}{p}}_2) \hat{\underset{\sim}{p}}_2 + \beta (\underset{\sim}{P}_{B_1} \times \hat{\underset{\sim}{p}}_2) + \gamma (\hat{\underset{\sim}{p}}_2 \times (\underset{\sim}{P}_{B_1} \times \hat{\underset{\sim}{p}}_2)) \Big\}, \qquad 3\cdot6$$

where β and γ are defined by

$$\beta = 2\,\mathrm{Im}\,(a_s^* a_p)/(|a_s|^2 + |a_p|^2), \quad \gamma = (|a_s|^2 - |a_p|^2)/(|a_s|^2 + |a_p|^2). \qquad 3\cdot5$$

For B_1 unpolarized (ie $P_{B_1} = 0$), $\langle \sigma \rangle_{B_2} = \alpha \hat{p}_2$, which allows a direct determination of α. A measurement of the B_1 and B_2 angular distributions, and the three components of the polarization of B_2 determine the values of α, β, and γ.

From the CPT theorem, non-conservation of parity implies either C or T not conserved, and if Time-reversal invariance (T) holds, then CP must be conserved. The requirement of T is $\beta = 0$. ie the relative phase of a_s and a_p is zero. The condition on β is obtained from a consideration of the final state interactions of $B_2 + \pi$. As the energy of the B_1 decaying at rest is below the inelastic threshold of $B_2 + \pi$, the $B_2 + \pi$ elastic scattering phase can be defined by

$$|B_2, \pi, l, I\ \mathrm{in}\rangle = |f_{lI}, \mathrm{in}\rangle = S_{lI}\,|f_{lI}\ \mathrm{out}\rangle = e^{2i\delta_l(I)}|f_{lI}\ \mathrm{out}\rangle, \ (l = s, p),$$

where S_{lI} is the s-matrix for $B_2 - \pi$ scattering. With $a_l(I) \propto \langle f_{lI}\ \mathrm{out}|H_w|B_1\rangle$, the condition for T invariance, $T H_w(0) T^{-1} = H_w(0)$, requires

$$\langle f_{lI}\ \mathrm{out}|T H_w T^{-1}|B_1\rangle = \langle f_{lI}\ \mathrm{out}|H_w|B_1\rangle \propto a_l(I).$$

As T is an antiunitary operator, we have

$$\langle f_{lI}\ \mathrm{out}|T H_w T^{-1}|B_1\rangle = \langle \tilde{B}_1|H_w|\tilde{f}_{lI}\ \mathrm{in}\rangle,$$

where the tilde indicates the momenta and spin of the states $\langle B_1|, |f\rangle$ have been reversed by the operation of T. However for B_1 decaying at rest, only the magnetic quantum number is reversed, which does not effect the state due to rotational invariance. Hence we may write

$$\langle \tilde{B}|H_w|\tilde{f}_{lI}\ \mathrm{in}\rangle = e^{2i\delta_l(I)}\langle B|H_w|f_{lI}\ \mathrm{out}\rangle \propto e^{2i\delta_{lI}} a_l^*(I).$$

This requires $a_{lI} = e^{2i\delta_l(I)} a_l^*(I)$, and thus we find

$$\varphi(I) = \delta_s(I) - \delta_p(I) \quad ;= \delta_s(I) - \delta_p(I) - \pi.$$

Neglecting the final state phase shifts at the small energies concerned, $\varphi(I) = 0$, and hence $a_s(I)/a_p(I)$ is real, which implies $\beta = 0$. A measurement of the final state phase shifts by Overseth et al(102) place an upper limit of 10° for δ_s and δ_p of the π – N final state of Σ and Λ decays, which is consistent with time reversal invariance.

Although final state interactions can introduce small phase differences between a_s and a_p, experimentally (100) β - is very small and consistent with zero. However the accuracy of the measurements is not sufficient to decide between the final state interactions and the possibility of very small CP violation in the NLDH's, as was discovered in the non-leptonic decay of the neutral K mesons(103). Nevertheless, CP will henceforth be considered conserved in the NLDH's for the purposes of this chapter.

Invariance under Charge-conjugation (C), in the same way as the above, requires $\alpha = 0$, $\beta \neq 0$. Both the anisotropy of the angular distribution of B_2 relative to the polarization of B_1 $(\alpha \neq 0)$ and the time-reversal invariance condition, $\beta \simeq 0$, imply the violation of C invariance in these decays. This is expected from the CPT theorem, which for T invariance requires CP invariance. As P is violated, so must C be.

Of equal importance to the space-time symmetries when discussing the NLDH's are the internal symmetry properties obeyed by these decays. The first observation is that all the NLDH's listed proceed via a $|\Delta Y| = 1$ change in hypercharge. Strong evidence exits against $|\Delta Y| = 2$ transitions, as for example the ratio of the $\Delta Y = 2$ decay, $\Xi^- \to n + \pi^-$ to (Ξ^-_-) (104) $\frac{\Gamma(\Xi^- \to n + \pi^-)}{\Gamma(\Xi^- \to \Lambda^0 + \pi^-)} \leqslant 0.5\%$, which lends support to $|\Delta Y| \leqslant 1$. Further evidence arises from the small $K^0_S - K^0_L$ mass difference(105). This mass difference is proportional to the matrix element of the transition $K^0 \to \bar{K}^0$, involving a $|\Delta Y| = 2$ change in hypercharge. If the $|\Delta Y| = 2$ transition were of comparable strength to the $|\Delta Y| = 1$ transition, δM would be of first order in the weak interaction. On the other hand, for $|\Delta Y| = 2$ forbidden, δM could only be attributable to a second order process (ie second order in the weak interaction goes as the square of the weak coupling-constant $G \sim 1 \cdot 0 \times 10^{-5}/M_p^2$). Experimentally the magnitude of δM supports the latter, in favour of the $|\Delta Y| \leqslant 1$ rule.

From the GNN relation, $Q = I_3 + Y/_2$, conservation of charge implies $|\Delta I_3| = |\frac{\Delta Y}{2}| = \frac{1}{2}$. Inspection of the decay modes listed, assures that this rule follows from the $|\Delta Y| = 1$ rule for the NLDH's.

Although $|\Delta I_3| = \frac{1}{2}$ does not imply the corresponding rule $|\Delta I| = \frac{1}{2}$., that this is so, is seen by refering to tables 3 and 4 . The decay processes can proceed either by $|\Delta I| = \frac{1}{2}$ or $3/_2$ change in isospin. However, using the Wigner-Eckart theorem to relate the various amplitudes through the reduced matrix elements, and assuming only the $|\Delta I| = \frac{1}{2}$ channel is open (25,101), the following relations follow,

$$-\sqrt{2}\, S(\Lambda^0_0) = S(\Lambda^0_-) \cdot , \qquad -\sqrt{2}\, P(\Lambda^0_0) = P(\Lambda^0_-) .$$

These imply the branching ratio $\Gamma(\Lambda^0_-) / (\Gamma(\Lambda^0_-) + \Gamma(\Lambda^0_0)) = 2/_3$;

which compares favourably with the experimental value of Filthuth et al of 0.640 ± 0.14 (97), and that of Rosenfeld et al of 0.663 ± 014(104) Similarly, the relations $-\sqrt{2}\, S(\Xi^0_0) = S(\Xi^-_-)$ and $-\sqrt{2}\, P(\Xi^0_0) = P(\Xi^-_-)$

imply $\Gamma(\Xi^-_-)/\Gamma(\Xi^-_0) = 2$; to be compared with the experimental values

of 1.68 ± 0.21(104) and $\Gamma(\Xi^0_0)/\Gamma(\Xi^-_-) = 0.58 \pm 0.036$.(97).
For Σ decay, the $|\Delta I| = \frac{1}{2}$ rule leads to the relation $\sqrt{2}\,\Sigma^+_0 = \Sigma^+_+ - \Sigma^-_-$ 3.6
for both the S and P wave amplitudes. Substituting the experimental results (100) for the corresponding amplitudes, this relation is satisfied to within experimental error.

From the above discussion we will not be concerned with the (Λ^0_0) and (Ξ^0_0) decay modes, accepting the $|\Delta I| = \frac{1}{2}$ rule to be valid, although all three modes of Σ will be considered in the calculation below, as a test that the computed amplitudes do in fact obey the $|\Delta I| = \frac{1}{2}$ rule.

In summary, the most important features to arise from the experimental analysis of the NLDH's, and which are crucial in determining the form which H_ω can take are,

A. Space-time properties.
i). Parity violation.
ii). Charge -conjugation violation.
iii). CP invariance.

B. Internal symmetry properties.
i). $|\Delta Y| = 1$ rule.
ii). $|\Delta I_3| = \frac{1}{2}$ rule.
iii). $|\Delta I| = \frac{1}{2}$ rule.

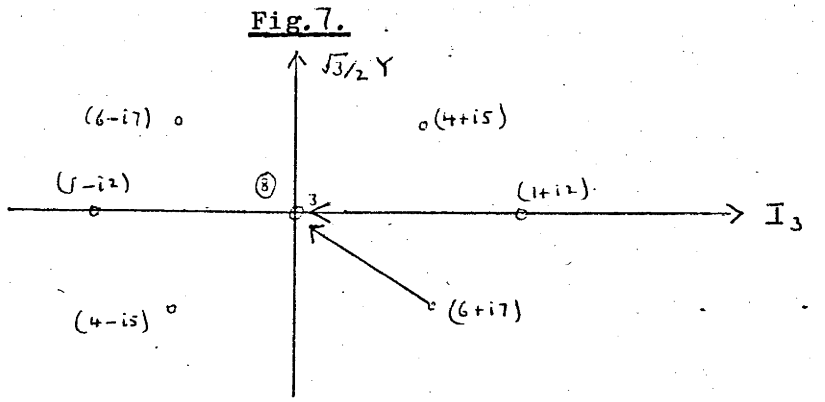
With the above properties, definite bounds on the form H_ω can take, can now be imposed. From A(i) , and appealing to equation 3.3 , parity violation can be imposed by splitting H_ω into Parity-Violating(PV) and Parity-Conserving (PC) components, thus $H_\omega = H_\omega^{PC} + H_\omega^{PV}$. 3.7

B(i) indicates that both $\Delta Y = +1$ and $\Delta Y = -1$ are possible, and hence H_ω can be further broken into hypercharge raising and hypercharge lowering parts.

$$H_\omega = H_\omega^{PC}(\Delta Y = 1) + H_\omega^{PC}(\Delta Y = -1) + H_\omega^{PV}(\Delta Y = 1) + H_\omega^{PV}(\Delta Y = -1) \quad 3.8$$

A method of imposing B(ii) and B(iii) properties upon H_ω within the framework of SU(3) unitary symmetry, is to require H_ω to transform as a member of an octet. We will demonstrate how this arises by considering the octet diagram below.

Fig.7.



The labelling of the diagram is in keeping with the discussion of chapter 1, where the action of the operators transforming as members of an octet are indicated by the example of an operator transforming as the 6 + i7 member of the octet, and where the arrow indicates the direction of action of this operator on the 6 + i7 state function of the octet in F-spin space. In this particular example, the operator raises the hypercharge by one unit and lowers the third component of isospin by ½ a unit. It follows immediately, that for all components of H_ω satisfying B(iii), they must transform as either an $I = \frac{1}{2}$ or $I = -\frac{1}{2}$ member of an isospin doublet within the octet in order to have the correct hypercharge raising and lowering properties. This ensures the $|\Delta I| = \frac{1}{2}$ rule, and employing B(ii), limits $(H_\omega^{PC} + H_\omega^{PV})^{\Delta Y = +1}$ to transform an the 6 + i7 component of an octet, and $(H_\omega^{PC} + H_\omega^{PV})^{\Delta Y = -1}$ to transform as the 6 - i7 component of an octet.

We now discuss A(ii) and A(iii). On the basis of the octet character of H_ω, H_ω^{PC} can be considered to transform as a member of a scalar octet, and H_ω^{PV} as a member of a pseudo-scalar octet, denoted generally by S_i and P_i respectively. (c/f chapter 1, where φ_i is an octet of, eg PS mesons). From equation 1.12, P_i is related to the tensor octet through $P_i = \frac{1}{\sqrt{2}} \sum_{a,b}^{3} (\lambda_i)_{ba} \mathcal{P}_a^{\ b}$ 3.9

To aid the discussion below, the relation between the tensor notation of Okubo (15) and the matrix notation of Gell-Mann(8) is summarized.

TABLE 5 .

$T_a^{\ b}$	$\mathcal{B}_a^{\ b}$	$\mathcal{P}_a^{\ b}$	O_i
$T_1^{\ 3}$	p	K^+	$1/\sqrt{2}\,(O_4 - iO_5)$
$T_2^{\ 3}$	n	K^0	$1/\sqrt{2}\,(O_6 - iO_7)$
$T_3^{\ 2}$	Ξ^0	$\bar{K}^0$	$1/\sqrt{2}\,(O_6 + iO_7)$
$T_3^{\ 1}$	Ξ^-	K^-	$1/\sqrt{2}\,(O_4 + iO_5)$
$T_1^{\ 2}$	Σ^+	π^+	$1/\sqrt{2}\,(O_1 - iO_2)$
$\frac{1}{\sqrt{2}}(T_2^{\ 2} - T_1^{\ 1})$	Σ^0	π^0	O_3
$T_2^{\ 1}$	Σ^-	π^-	$1/\sqrt{2}\,(O_1 + iO_2)$
$-\sqrt{\frac{3}{2}}\,T_3^{\ 3}$	Λ^0	η^0	O_8

The generalization of the concept of charge-conjugation to the framework of SU(3) unitary symmetry was introduced by Gell-Mann (106) through the concept of $\mathcal{C}$ parity. For a self-conjugate octet (eg $\rho_a^{\ b}$ and $S_a^{\ b}$, but not $\beta_a^{\ b}$ as $\beta_a^{\ b} \neq \bar{\beta}_a^{\ b}$) , Gell-Mann (106) defined the $\mathcal{C}$ parity as the phase factor arising through charge-conjugation thus, $C\, T_a^{\ b}\, C^{-1} = \mathcal{C}\, T_b^{\ a}$. $\mathcal{C}$ is fixed for the octet by the behaviour of the completely neutral member (Q = 0, S = 0) of the octet under charge-conjugation C . This convention is taken as there is no ambiguity in defining the charge-conjugation properties of the completely neutral members decaying into two gammas. For the self-conjugate octets $\rho_a^{\ b}$ and $S_a^{\ b}$, $\mathcal{C} = +1$. The $\mathcal{C}$ parity of the octet is refered to as abnormal if it is opposite in sign to that defined above. For the A and V self-conjugate octets, $\mathcal{C} = \pm 1$ respectively.

From equation 3·9, $\mathcal{C}$ parity can be correspondingly defined for P_i etc. Under charge-conjugation C , $P_i \xrightarrow{C} \mathcal{C}\varepsilon_i P_i$, where ε_i is defined by the property of the Gell-Mann λ_i matrices under transposition; $\tilde{\lambda}_i = \varepsilon_i \lambda_i$ (i not summed). This is due to the fact that under C, $\rho_a^{\ b}$ is transposed, which from equation 3·9 requires $\lambda_i \rightarrow \tilde{\lambda}_i$. From the definition of the λ_i matrices (4,8), $\varepsilon_i = -1$ for i = 2,5,7 and + 1 for i otherwise.

With the parity of P_i and $S_i = \mp 1$ respectively, and the requirement A(iii)(ie H_ω even under CP, or equivalently CP = + 1 for H_ω), there are four possibilities for H_ω which can be deduced fron the table below.

TABLE 6.

Parity.	Charge-conjugation	Octet transformation character. Matrix	Tensor
$P = -1$ (PV)	$\mathcal{C} = +1$ (Normal)	λ_7 (O_7)	$T_3^{\ 2} - T_2^{\ 3}$
$P = -1$ (PV)	$\mathcal{C} = -1$ (Abnormal)	λ_6 (O_6)	$T_3^{\ 2} - T_2^{\ 3}$
$P = +1$ (PC)	$\mathcal{C} = +1$ (Normal)	λ_6 (O_6)	$T_3^{\ 2} - T_2^{\ 3}$
$P = +1$ (PC)	$\mathcal{C} = -1$ (Abnormal)	λ_7 (O_7)	$T_3^{\ 2} - T_2^{\ 3}$

Hence the possibilities for H_ω are,

$(H_\omega = H_\omega^{PC} + H_\omega^{PV})$

1). $H_\omega = \lambda_s S_6 + \lambda_p P_7$

2). $H_\omega = \lambda_s S_6 \quad \lambda_p P_6'$

3). $H_\omega = \lambda_s S_7' \quad \lambda_p P_7$

4). $H_\omega = \lambda_s S_7' \quad \lambda_p P_6'$

TABLE 7.

S and P normal octets.
S$'$ and P$'$ abnormal octets.

The above choices of transformation properties of H_ω are important when discussing and comparing different models below. The final demand on the transformation property of H_ω will be made in relation to both the charge-conjugation properties ($\mathcal{C} = \pm 1$) and the ability of the model to satisfy the Lee-Sugawara relation (L-S) (107)

$$2X(\Xi^-_-) + X(\Lambda^\circ_-) = \sqrt{3}X(\Sigma^+_\circ) \qquad (X = S,P),$$

which is obeyed very closely by the experimental S and P wave amplitudes (96,100).

This relation was first derived by Lee and Sugawara (107), using the now superseded RP invariance arguments (106) for the S wave amplitudes, and later by Coleman et al (108) for RP invariance, and the requirement that H_ω^{PV} transform as ($\sim$) λ_6 . The derivation has since been strengthened by a number of authors (106,109), and obtained for both the S and P wave amplitudes, in the framework of SU(3) unitary symmetry. Gell-Mann and Okubo (106, 109) show that the L-S relation follows for the S wave amplitudes if H_ω^{PV} is even under CP and transforms as the 6th component of an octet, abnormal under charge-conjugation. Marshak et al (16) have extended Okubo's approach to show that for $H_\omega^{PC} \sim \lambda_7$, the P wave L-S relation obtains.

For particular models of H_ω, and in particular the quark density model and the current divergence models discussed in sections 3·2 & 3·3, although H_ω^{PC} and H_ω^{PV} do not $\sim \lambda_7$ and λ_6 respectively, when the methods of CA and PCAC are applied, (see calculation below) the L-S relations do obtain for both the S and P wave amplitudes. B.W.Lee (110) has pointed out that this arises for the P waves when the SU(3) mass-splitting (symmetry breaking) terms $\sim \lambda_8$ of the same octet as H_ω^{PC} act in conjunction with H_ω^{PC}, to give H_ω^{PC} transforming effectively as λ_7 .

The most natural model to be considered to describe the NLDH'S was the c-c model (32,33), which is the logical extension of the weak Hamiltonian (eq., 1·23c), and which has been so successful in describing both purely leptonic and semi-leptonic decays. For NLDH's only products of hadron currents contribute to H_ω where (eq., 1·23c)

$$H_\omega^{(\text{NON-LEPTONIC})} = -\frac{G}{\sqrt{2}} \left\{ J_{h\lambda}, \bar{J}_h^{\lambda} \right\} \qquad 3{\cdot}10$$

(G is the weak interaction constant, with $G \cong 1\times10^{-5}/M_p^2$)

with

$$J_{1\lambda}^{2} = (V_1 + iV_2)_\lambda + (A_1 + iA_2)_\lambda$$
$$J_{1\lambda}^{3} = (V_4 + iV_5)_\lambda + (A_4 + iA_5)_\lambda. \qquad 3{\cdot}11$$

From equation 3·11, in order for the $|\Delta Y| = 1$ rule to obtain, only the cross terms of the hadronic currents $J_1^3, J_1^{\,2}$ can contribute. Thus the piece of H_ω responsible for the NLDH's is

$$H_\omega(|\Delta Y| = 1) = -G/\sqrt{2}\, \sin\theta_c \cos\theta_c\, \tfrac{1}{2}\left(\{J_1^2, J_3^1\} + \{2 \leftrightarrow 3\}\right), \qquad 3{\cdot}12$$

The hypothesis that the V and A currents belong each to an octet of currents implies H_ω has components belonging to the representations formed from the direct product decomposition of octets (11)

$$\underset{\sim}{8} \otimes \underset{\sim}{8} = \underset{\sim}{1} + \underset{\sim}{8}_a + \underset{\sim}{8}_s + \underset{\sim}{10} + \underset{\sim}{10}^* + \underset{\sim}{27},$$

obtained (in the tensor notation) by the contraction of indices in the product $J_a^b J_c^d$, in all possible ways. As H_ω is symmetric under the interchange $2 \leftrightarrow 3$, only the symmetric representations will contribute. $\underset{\sim}{8}_s$ and $\underset{\sim}{8}_a$ are the symmetric and antisymmetric octets formed from the direct products of octets thus

$$S_a^b = J_c^a J_b^c + J_b^c J_c^a - \tfrac{2}{3}\delta_a^b J_e^d J_d^e \quad \& \quad A_a^b = J_c^a J_b^c - J_b^c J_c^a, \qquad 3{\cdot}13$$

and hence the symmetric product of currents in H_ω excludes $\underset{\sim}{8}_a$. Similarly $\underset{\sim}{10}$ and $\underset{\sim}{10}^*$ are antisymmetric (11), while $\underset{\sim}{27}$ is symmetric. The trivial 1-D representation $\underset{\sim}{1}$ does not contribute to H_ω as it corresponds to the trace of the direct product and has only a $\Delta Y = 0$ contribution. This leaves $\underset{\sim}{8}_s$ and $\underset{\sim}{27}$.

It was originally shown by Gell-Mann (106) that the assumption of CP invariance of the weak interaction and the c-c nature of H_ω, with the currents belonging to the octet representation of SU(3), together implied that $H \sim \lambda_6$ of an octet. We can see this from the following considerations.

From S_a^b (eq., 3·13) and the definition of $\mathcal{C}$ (106) for self-conjugate octets of currents,

$$C J_a^b C^{-1} = \mathcal{C}(J) J_b^a. \qquad 3{\cdot}14$$

(J_a^b are self-conjugate octets for the currents considered as bilinears of baryron octets, for example, Eq., 1·18) $\mathcal{C}$ parity applies separately for the current octets in the product S_a^b. IE

$$C S_a^b C^{-1} = \mathcal{C}(J)\mathcal{C}(J) S_b^a. \qquad 3{\cdot}15$$

Since H_ω^{PV} is made up of the products VA and AV, the $\mathcal{C}$ parity of H_ω is

$$\mathcal{C}(A)\mathcal{C}(V) = \mathcal{C}(V)\mathcal{C}(A) = -1,$$

and hence H_{ω}^{PV} is even under CP due to the opposite parity of the V and A octets, Similarly H_{ω}^{PC} is even under CP, and hence , considering the octet representation alone, H_{ω}^{PV} and H_{ω}^{PC} transform effectively as the 6th components of an octet, as seen from table 6 and the properties of H_{ω}^{PC} and H_{ω}^{PV} above.

Despite the desirability of this form for H_{ω} , which if it obtained, would allow the phenomenological description of all weak interactions by the c–c weak interaction, it is faced with the following difficulties with respect to the experimentally observed properties of the NLDH's (96, 100).

1). As the $\underset{\sim}{27}$ representation can enter H_{ω} , the simple properties for H_{ω} transforming as a member of an octet must now by modified to include the contribution of the $\underline{27}$ representation.

2). From the angular momentum rules for combining $I = 1$ and $I = \frac{1}{2}$ currents in equation 3·12 , both $|\Delta I| = \frac{1}{2}$ and $3/2$ contributions are possible, with equal strength, contrary to the $|\Delta I| = \frac{1}{2}$ rule, which is found to be valid within experimental error (100).

The possible ways out of this are either to generalize equation to include neutral hadron currents (96) to enforce the $|\Delta I| = \frac{1}{2}$ rule by demanding that the $|\Delta I| = 3/2$ contribution of the neutral currents cancels with the $|\Delta I| = 3/2$ contribution from the c–c Hamiltonian above, or to consider a mechanism whereby the $|\Delta I| = 3/2$ component is suppressed by strong interaction dynamics (106, 111). Gell-Mann suggested the predominance of scalar and PS meson intermediate states, selectively enhancing the $|\Delta I| = \frac{1}{2}$ contribution to the matrix elements of H_{ω} . Alternatively, if the $\underset{\sim}{27}$ contribution were suppressed relative to the $\underset{\sim}{8}$ contribution, the $|\Delta I| = \frac{1}{2}$ contribution would be enhanced relative to the $|\Delta I| = 3/2$ contribution coming from $\underset{\sim}{27}$, and an effective $|\Delta I| = \frac{1}{2}$ rule would hold. For the case of adding neutral currents, the weak Hamiltonian can be written in the form

$H_{\omega} = -\, g/\sqrt{2}\; d_{6ij} \cdot J_{\mu}^{i}\, \bar{J}_{\mu}^{j}$, where the contributions $J_{\mu}^{6}\, \bar{J}_{\mu}^{3} + J_{\mu}^{3}\, \bar{J}_{6}$ are now possible, with d_{6ij} , the symmetric SU(3) structure constant, acting as a Clebsch-Gordon coeficient for the symmetric coupling of two octets (11). As neutral currents have not been observed, the inclusion of these currents would exclude the possibility of a universal interaction for all types of weak interactions.

3). The factor $\sin\theta_c \cos\theta_c$, involving the Cabibbo angle θ_c , unlike the case for the semi-leptonic decays, where it was first introduced to account for the suppression of the strangeness changing decays relative to the strangeness conserving decays, does not appear to be warranted experimentally in NLDH's(53), and in particular for the P wave amplitudes, whose theoretical values are consistently smaller than the experiment without this factor.

4). Quite generally it has been shown that the L-S relation is a consequence of CP invariance and $H_{\omega}^{PC} \sim \lambda_6$ and $H_{\omega}^{PV} \sim \lambda_7$, for the P and S waves respectively, and hence in the c-c model the L-S relation does not follow directly for the PC amplitudes (P wave amplitudes).

Another model which alongside the c-c model forms the basis of many calculations in NLDH's is the so called quark-density model (112) in which H_{ω} is assumed to belong to an octet, transforming as the 6th and 7th components of scalar and PS quark-densities ($\eta = +1$) for the PC and PV parts respectively, from the requirement of CP invariance (table 6). Following the notation of Gell-Mann (8), H_{ω} can be written as

$$H_{\omega} = H_{\omega}^{PV} + H_{\omega}^{PC} = \lambda_S v_7 + \lambda_P u_6 , \qquad 3\text{-}16$$

where v_7 transforms like the 7th component of the quark density $i\bar{\chi}(x)\gamma_5\frac{\lambda_i}{2}\chi(x)$, and u_6 as the 6th component of the quark-density $\bar{\chi}(x)\frac{\lambda_i}{2}\chi(x)$ ($i = 1....8$).

This model is entirely similar to $H_{\omega} = \lambda_S S_6 + \lambda_P P_7$ (Table 7.), where the scalar and PS octets, S_i and P_i are normal under charge conjugation (table 6). In the quark model the u_i and v_i are postulated to transform as members of the octet representation formed from the direct product of the triplets $\underset{\sim}{3}$ and $\underset{\sim}{3}^*$ (eq., 1·4), whereas the actual form of S_i and P_i need not be specified (although we chose a specific model for these above, for which S_i and P_i belong to the octet representation formed from the direct product of octets), as long as they are understood to transform as members of octets of scalar and PS bosons respectively.

The above two models both satisfy the conditions (A) and (B) (pg.,62), although the L-S relation is only apparent when these models are employed using current algebraic methods and PCAC(110).

Variations of these models, and other related models to those already discussed will be refered to in more depth in section 3.3, in comparison with the model calculation below.

Section 3.2. Calculation of the S and P wave amplitudes

In this section we employ a Current-Divergence (C-D) model, arrived at indepepdently by the author, to calculate the S and P wave amplitudes, and we attempt to show;

1). Even though the Hamiltonian used implies a null result for the amplitudes; due to the standard approximations involved in employing the CA hypothesis, the use of CA gives rise to non-vanishing amplitudes.

2). That the resultant amplitudes obtained from the CD model are identical in form to the amplitudes obtained from the simple dispersion theoretic pole dominance model, with all first order pole terms retained.

3). That other models, including the dispersion theoretic pole dominance model obtain non-vanishing results due to the approximations employed. This is seen by comparing the terms obtained in the CD calculation with those of other calculations. As we know that the terms obtained in our model should cancel identically if we were able to calculate exactly, the results of other models retaining all the terms we do must similarly be potentially vanishing.

4). There are conditions for which some of the resultant amplitudes vanish identically- but contrary to this we demonstrate that an excellent fit to the data for the 10 S and P wave amplitudes can be obtained from the calculation with 4 free parameters. We show that a large number of authors obtaining essentially the same amplitudes as ourselves, but using different assumptions and methods to obtain them, attempt fits to the experimental data as ourselves, the merits of which we have discussed.

5). Finally we point out that the CA calculations refered to in (4) are in fact equivalent to assuming the CD model, which the authors have explicity attempted to avoid by introducing alternative or added assumptions, with particular reference to the P wave amplitudes; and that the neglect of certain terms in some of the cases, (which can be traced to the assumptions made) are responsible, as well as the inability to calculate exactly, to the non-vanishing results obtained.

CALCULATION.

The current-divergence model approach arose from considering the S and P wave NLDH's as proceeding through the two-step processes,

$$N(p_2) + \pi^-(q) \xleftarrow{\text{strong}} K^0(k) + \Sigma^-(p_1) \qquad \text{S wave.} \qquad 3\text{-}17$$

$$K^0(k) \xrightarrow{\text{weak}} \text{Vacuum.}$$

$$N(p_2) + \pi^-(q) \xleftarrow{\text{STRONG}} k^c(k) + \Sigma^-(p_1)$$
$$k^c_{(k)} \xrightarrow{\text{WEAK}} \text{vacuum}, \qquad \text{P wave.} \qquad 3\cdot 18$$

for the S and P waves respectively, in the limit as $k \longrightarrow 0$. k^c and K^c are taken to be the scalar kappa meson and the PS kaon meson, each belonging respectively to an octet of scalar and PS mesons, and transforming as the 6 + 7 th component of the octet(sect's., 2·1, 3·1). For the purposes of demonstrating the calculation we will consider as a specific example the P wave decay of Σ^-.

Writing the covariant amplitude for this process, we have

$$T = (2\pi)^3 \lambda_p \sqrt{4k^o q^o} < \pi^-(q)\, N(p_2) \,|\, \Sigma^-(p_1)\, k^o(k) >, \qquad 3\cdot 19$$

where λ_p is the strength of the process $k^c \longrightarrow$ vacuum. Contracting the k^c meson (See appendix A for conventions and notation) and using PCVC (eq., 2·5c) for the k^c-meson, we obtain

$$T = \lambda_p (2\pi)^3 \sqrt{2q^o} \left\{ i\int d^4x\, e^{ikx} \frac{(k^2 + m_k^2)}{i f_k m_k^2} < \pi^-(q^2)\, N(p_2) \,|\, \partial_u V_u^{6+i7}(x) \,|\, \Sigma^-(p_1) > . \right. \qquad 3\cdot 20$$

f_k is defined as the decay constant of the kappa meson by the relation

$$< k^o \,|\, \partial_u V_u^{6+i7} \,|\, o > \overset{\text{def}^n}{=} i f_k m_k^2 / (2\pi)^{3/2} \sqrt{2k^o}, \qquad 3\cdot 21$$

and hence $\lambda_p \propto f_k$. We use the PCVC assumption to mean here that the matrix element of $\partial_u V_u^{6+i7}$ is dominated by the kappa meson pole, which implies T is a smooth function of k, for k_μ varying in the vicinity of m_k. This is the PDDVC (pole domination of the divergence of the vector current) form of PCVC (113). Although not as justified as for the pion or kaon PDDAC (113), it will be asssumed that T is a sufficiently smooth function of k for the extrapolation $k_\mu \longrightarrow 0$ to be meaningful(the $k_\mu = 0$ limit will be taken at a later stage of the calculation.) Thus we find

$$T = (2\pi)^{3/2} \left\{ \int d^4x\, e^{ikx} < \pi(q)\, N(p_2) \,|\, \partial_u V_u^{6+i7}(x) \,|\, \Sigma^-(p_1) > \lambda_p / f_k \right.$$

$$= i(2\pi)^4 \delta^4(p_1 + k - p_2 - q)\,(2\pi)^{3/2} (-i) \sqrt{2q^o} < \pi(q)\, N(p_2) \,|\, \partial_u V_u^{6+i7}(o) \,|\, \Sigma^-(p_1) \rangle\, \lambda_p / f_k . \qquad 3\cdot 22$$

Hence from equations 3·2, 3·3 , T is related to the P wave amplitudes by

$$T = i(2\pi)^4 \delta(p_1 + k - p_2 - q)\, i \sqrt{\frac{N\cdot\Sigma}{p_1^o\, p_2^o}}\, \frac{1}{(2\pi)^6}\, \bar{u}(p_2)\, (\, P\gamma_5\,)\, u(p_1), \qquad 3\cdot 23$$

where 'P' are invariant functions of the invariant variables $s = -(p_1 + q)$, $t = -(p_1 - p_2)$ and $u = -(\, p_1 - q)$, which approach the physical decay amplitudes in the limit as $k_\mu \longrightarrow 0$, for $s = -p_1^2 = \Sigma^2$, $t = -q^2 = \pi^2$, $u = -p_2^2 = N^2$(with the particle symbols denoting the particle masses).

Equating equations 3·22 and 3·23 we find,

$$-i(2\pi)^{3/2}\sqrt{2q^o} < \pi(q)\, N(p_2) \,|\, \partial_u V_u^{6+i7}(o) \,|\, \Sigma^-(p_1) > \lambda_p / f_k$$

$$= \frac{+i}{(2\pi)^6} \sqrt{\left(\frac{N\Sigma}{p_1^o p_2^o}\right)}\, \bar{u}(p_2)\, P\gamma_5\, u(p_1) \qquad 3\cdot 24$$

Defining the reduced T matrix by $T = i(2\pi)^4 \delta^4(p_1 + k - p_2 - q)\tilde{T}$, and denoting the LHS of 3·24 by $\tilde{T}$, we now contract the π^- meson (Appendix A). Using PCAC (eq., 2·52) in the PDDAC sense (113) and taking the $q^2 \to$ limit, as for k^2 above, it follows from equation 3·24, with the definition $\langle 0 | \partial_\mu A_\mu^{1+i2}(y) | \pi^- \rangle \overset{\mathrm{def}^n}{=} f_\pi m_\pi^2 / \sqrt{2q^0}\,(2\pi)^{3/2}$, that

$$\tilde{T} = \frac{\lambda_P}{f_k f_\pi} \int d^4y \langle N(p_2) | T\left(\partial_\mu A_\mu^{4+i2}(y)\, \partial_\mu V_\mu^{6+i7}(0)\right) | \Sigma^-(p_1) \rangle e^{-iqy}$$

$$= \frac{\lambda_P}{f_k f_\pi} i q_\mu \int e^{-iqy} d^4y \langle N(p_2) | T\left(A_\mu^{1+i2}(y)\, \partial_\mu V_\mu^{6+i7}(0)\right) | \Sigma^-(p_1) \rangle \qquad 3\cdot25$$

$$- \frac{\lambda_P}{f_k f_\pi} \int e^{-iqy} d^4y \langle N(p_2) | \delta(y_0) \left[A_0^{1+i2}(y), \partial_\mu V_\mu^{6+i7}(0)\right] | \Sigma^-(p_1) \rangle$$

The first term is the so called Surface-Term (ST), and the second term will be refered to as the Equal - Time - Commutator term (ETC).

In order to be able to compute the contribution fron the ST, the Hard-Pion-Approximation (HPA)(80,115,114) is employed in favour of using the usual Soft-Pion-Approximation (SPA) (116), where we take only the limit $q^2 = 0$, rather than the $q_\mu = 0$ limit, which is more severe, in that $q_\mu = 0$ implies $\underset{\sim}{q} = q^0 = 0$, but $q^2 = 0$ does not imply $\underset{\sim}{q} = q^0 = 0$. In the $q_\mu = 0$ limit the ST only contributes in the limit when the intermediate states are degenerate in mass with the initial or final states (114) through the energy-momentum constraints on the matrix elements occuring in the ST. By taking the HPA in preference, we avoid the ambiguity of whether to take the mass-degeneracy or $q_\mu = 0$ limit first. A further important adavntage is that the mass differences between the initial and final states are taken into account (114). The SPA is in this sense equivalent to taking the SU(3) symmetric limit for those matrix elements that involve mass differences. For this reason the SPA has been found to give poor results, particularly for the P waves which rely on broken SU(3) for their evaluation (Sect., 3.3). This has been emphasized in particular by Okubo (114), who has discussed the importance of not neglecting the baryon mass differences in ST, and who similarly invoked the HPA to avoid mass degeneracy.

To evaluate the ETC, the validity of the Gell-Mann -Oakes-Renner SU(3)⊗SU(3) charge-divergence algebra (eq's., 1·38, 1·42) (43) is assumed, along with PCAC and PCVC (eq's 2·50, 2·52). It is important to stress here the assumption of an octet of scalar mesons in order to justify the use of the GOR symmetry breaking scheme (43), and the use of PCVC. Thus for the ETC term we obtain from equations 1·38 & 1·42

$$\delta(y_0)\left[A_0^{1+i2}(y), \partial_\mu V_\mu^{6+i7}(0)\right] = -\delta^4(y) \frac{3c}{2(\sqrt{2} - c/2)} \partial_\mu A_\mu^{4+i5}, \qquad 3\cdot26$$

where ' c' is the GOR symmetry breaking parameter (43). Hence we have

$$\text{ETC.} = \frac{\lambda_P}{f_K f_\pi} \int e^{-iqy} d^4y < N(P_2) | \delta^4(y) A_\mu^{4+i5}(y) | \Sigma^-(P_1) > \frac{3c}{2(\sqrt{2} - c/2)}$$

$$= \frac{\lambda_P}{f_K f_\pi} < N(P_2) | \partial_\mu A_\mu^{4+i5}(0) | \Sigma^-(P_1) > \frac{3c}{2(\sqrt{2} - c/2)} \qquad 3\cdot 27$$

We now condider the ST, which with $\lambda_P' \overset{def^n}{=} \lambda_i/f_\pi f_K$, we write as

$$ST = -\lambda_P' i q_\mu \int e^{-iqy} d^4y < N(P_2) | T(A_\mu^{1+i2}(y), \partial_\mu V_\mu^{6+i7}(0)) | \Sigma^-(P_1) > \qquad 3\cdot 28$$

Expanding out the T-produnt and summing over intermediate states we obtian,

$$= -\lambda_P' i q_\mu \Big\{ \int d^4y \, dp_0' e^{ip_0' y_0} e^{-i\underset{\sim}{q}\cdot\underset{\sim}{y}} \frac{(-1)}{2\pi i} \sum_n \sum_{spin} \frac{< N(P_2) | A_\mu^{1+i2}(y) | n >}{(P_0' - q_0 + i\varepsilon)}$$

$$\times < n | \partial_\mu V_\mu^{6+i7}(0) | \Sigma^-(P_1) >$$

$$+ \int d^4y \, dp_0' e^{ip_0' y_0} e^{-i\underset{\sim}{q}\cdot\underset{\sim}{y}} \frac{1}{2\pi i} \sum_n \sum_{spin} \frac{< N(P_2) | \partial_\mu V_\mu^{6+i7}(0) | n' >}{(P_0' - q^0 - i\varepsilon)}$$

$$\times < n' | A_\mu^{1+i2}(y) | \Sigma^-(P) > \Big\}. \qquad 3\cdot 29$$

(See appendix A for notation and conventions used).
Using translational invariance, the above becomes,

$$= -\lambda_P' i q_\mu \Big\{ - \int \frac{d^4y \, dP_0' \, d^3\underset{\sim}{P}_n}{(2\pi i)(2\pi)^3} \frac{e^{iP_0' y_0} e^{-i\underset{\sim}{q}\underset{\sim}{y}}}{(P_0' - q^0 + i\varepsilon)} \sum < N(P_2) | A_\mu^{1+i2}(0) | n >$$

$$\times < n | \partial_\mu V_\mu^{6+i7}(0) | \Sigma^-(P_1) > e^{i(P_n - P_2)y}$$

$$+ \int \frac{d^4y \, dP_0' \, d^3\underset{\sim}{P}_n}{(2\pi i)(2\pi)^3} \frac{e^{iP_0' y_0} e^{-i\underset{\sim}{q}\underset{\sim}{y}}}{(P_0' - q_0 - i\varepsilon)} \sum < N(P_2) | \partial_\mu V_\mu^{6+i7}(0) | n >$$

$$\times < n | A_\mu^{1+i2}(0) | \Sigma^-(P_1) > \Big\} \qquad 3\cdot 30$$

It is at this stage of the calculation that we take the $k_\mu = 0$ limit.
After integrating over Y and p_0', we have,

$$ST = \lim_{k_\mu \to 0} \Big\{ \lambda_P' q_\mu \sum \int d^3\underset{\sim}{P}_n \, \delta^3(\underset{\sim}{P}_n - \underset{\sim}{P}_2 - \underset{\sim}{q}) \frac{< N | A_\mu^{1+i2}(0) | n >}{(-P_2^0 + P_n^0 - q_0)}$$

$$< n | \partial_\mu V_\mu^{6+i7}(0) | \Sigma^-(P_1) >$$

$$- \lambda_P' q_\mu \sum \int d^3\underset{\sim}{P}_n \, \delta^3(\underset{\sim}{P}_1 - \underset{\sim}{P}_n - \underset{\sim}{q}) \frac{< N | \partial_\mu V_\mu^{6+i7}(0) | n > < n | A_\mu^{1+i2}(0) | \Sigma^-(P_1) >}{(-P_n^0 + P_1^0 - q^0)}$$

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Employing the four-momentum delta function $\delta^4(p_1 + k - p_2 - q)$ (which is understood in the above), in the limit as $k_\mu \longrightarrow 0$

$(p_2^0 + q^0) = p_1^0$, $p_2^0 = (p_1^0 - q^0)$, $\underset{\sim}{p}_1 = \underset{\sim}{p}_2 + \underset{\sim}{q}$, $\underset{\sim}{p}_2 = \underset{\sim}{p}_1 - \underset{\sim}{q}$.

Hence in this limit ST becomes

$$ST = \lambda_p' q_\mu \sum \int \frac{\langle N(p_2) | A_\mu^{1+i2}(0) | n \rangle \langle n | \partial_\mu V_\mu^{6+i7}(0) | \Sigma^-(p_1) \rangle}{(p_n^0 - p_1^0)} d^3\underset{\sim}{p}_n \, \delta^3(\underset{\sim}{p}_n - \underset{\sim}{p}_1)$$

$$- \lambda_p' q_\mu \sum \int \frac{\langle N(p_2) | \partial_\mu V_\mu^{6+i7}(0) | n \rangle \langle n | A_\mu^{1+i2}(0) | \Sigma^-(p_1) \rangle}{(p_2^0 - p_n^0)} d^3\underset{\sim}{p}_n \, \delta^3(\underset{\sim}{p}_n - \underset{\sim}{p}_2) \qquad 3\cdot32$$

In order to evaluate the P wave amplitudes, it will be assumed that only the single particle intermediate states contribute significantly. In this case the single particle states are single baryon octet states. This constitutes the single baryon pole dominance approximation(SBPD) (117). With this approximation, we obtain finally, after integration over momentum,

ETC + ST = $\tilde{T}$

$$= \lambda_p' \langle N(p_2) | \partial_\mu A_\mu^{4+i5}(0) | \Sigma^-(p_1) \rangle \frac{3c}{2(\sqrt{2} - 1/2)}$$

$$+ \lambda_p' q_\mu \sum \frac{\langle N(p_2) | \partial_\mu V_\mu^{6+i7}(0) | \Lambda(p_2) \rangle \langle \Lambda(p_2) | A_\mu^{1+i2}(0) | \Sigma^-(p_1) \rangle}{(\Lambda^0 - N^0)}$$

$$+ \lambda_p' q_\mu \sum \frac{\langle N(p_2) | \partial_\mu V_\mu^{6+i7}(0) | \Sigma^0(p_2) \rangle \langle \Sigma^0(p_2) | A_\mu^{1+i2}(0) | \Sigma^-(p_1) \rangle}{(\Sigma^0 - N^0)}$$

$$= \frac{i}{(2\pi)^6} \sqrt{\left(\frac{N \Sigma}{N^0 \Sigma^0} \right)} \, \bar{u}(p_2) \, \gamma_5 \, P \, u(p_1), \qquad 3\cdot33$$

where the four-momentum δ function has been canceled from both sides of equation $3\cdot24$, and is now $\delta^4(p_1 - p_2 - q)$.We take this to be understood in the above equation. ' P ' are now the physical decay amplitudes, at the physical values of s,t,u, for the decay process.

If we return to equation $3\cdot24$, it can be readily seen that the LHS is equivalent to

$$-i(2\pi)^{3/2} \frac{\sqrt{2q^0}}{f_k} \langle \pi(q) N(p_2) | \partial_\mu V_\mu^{6+i7}(0) \pm \partial_\mu V^{6-i7}(0) | \Sigma^-(p_1) \rangle \lambda_p ,$$

where $\partial_u V_u^{6-i7}$ does not contribute to the matrix element due to the direction of change of Y . Choosing the minus sign to maintain the reality of P, we obtain

$$(2\pi)^{3/2}\,\sqrt{2q^0}\,.\,\frac{2}{f_k}\,.\,<\pi(q)\,N(p_2)\,|\,\partial_u V_u^7(0)\,|\,\Sigma^-(p_1)>\lambda_p$$

and hence,

$$-(2\pi)^{3/2}\sqrt{2q^0}<\pi(q)\,N(p_2)\,|\left(\frac{2\lambda_p}{f_k}\right)\partial_u V_u^7(0)\,|\,\Sigma^-(p_1)>$$

$$= -i/(2\pi)^3\sqrt{\frac{N\Sigma}{N^0\Sigma^0}}\;\bar{u}(p_2)\,(\gamma_5\,P\,)\,u(p_1)\,, \qquad 3\cdot34$$

where δ^4 ($p_1 - p_2 - q$) for on mass shell decay is understood on both sides of 3·34 . From equations 3·2 and 3·3 we identify $H_\omega^{PC}(0)$ with $\lambda_p''\partial_u V_u^7(0)$, and similarly for the S wave, to give

$$H_\omega(0) = \lambda_p''\,\partial_u V_u^7(0) + \lambda_s''\,\partial_u A_u^7(0)\,, \qquad 3\cdot35a$$

where $\lambda_s'' = 2\lambda_s/f_k$.

This is the current-divergence model of H_ω, implicit in this calculation, and which from the discussion of section 3.1 obeys the properties A(i,ii,iii) and the rules B(i,ii,iii). This model can be readily related to the quark density model using equations 1·42 , relating the current divergencies to the corresponding scalar and PS charge densities. In particular, if we interpret the currents in the quark model (eq 1·18), we obtain

$$H_\omega(0) = \gamma_p\,u_6 + \gamma_p v_7\;; \qquad 3\cdot36$$

which is identical to the quark-density model under the assumption that the Gell-Mann charge-divergence CR's (eq's., 1·38) hold. As we assume these CR's to hold, in order to apply the GOR model of symmetry breaking for quark currents, we are effectively working with the the quark-model, although from a different point of view. With reference to table 7 , the CD model is also directly proportional to

$$H_\omega(0) = \lambda_p\,P^7 + \lambda_s\,S^6, \qquad 3\cdot35b$$

with the octet baryon interpretation for the currents, although this is not necessary.

At this stage we point out that 3·34 is vanishing for on mass shell decay, to which we refer later, and which implies that if we had been able to calculate exactly to this point, we would expect the ETC term to cancel identically with ST. Also for the CR's (eq's 1·38) holding exactly, and the validity of the qark model interpretation of the currents, we should expect this model to give vanishing results if calculated exactly. Nevertheless we pursue this line of approach as it turns out to reproduce the amplitudes of a large number of calculations, thus enabling us to highlight the difficulties facing NLDH's.

Before proceeding to calculate out the final amplitudes, a number of important remarks have to be made. In evaluating the matrix elements of the current divergencies, the validity of the generalized Goldberger-Treiman relations (118) are implicitly assumed to be a good approximation in the light of the $q^2 = 0$ and $k^2 = 0$ limits taken, which are necessary in the derivation of these relations. The relevant Goldberger-Treiman relations are (5, 16, 118)

$$g_{B_1 B_2 K(\pi)} = 1/f_{K(\pi)} \cdot (M_{B_1} + M_{B_2}) g^A (B_1 \to B_2)/\sqrt{2}$$

$$g_{B_1 B_2 k} = 1/f_k \cdot (m_{B_1} - m_{B_2}) g^V (B_1 - B_2)/\sqrt{2}, \qquad 3\cdot 37$$

which arise implicitly (appendix A) in the evaluation of the matrix elements below. In this way the effects of SU(3) breaking are introduced by retaining the physical masses, and it is possible to avoid taking SU(3) symmetric values for the strong coupling-constants (due to lack of experimental knowledge of these) in favour of using SU(3) symmetric values for the weak vector and axial-vector coupling-constants.

As both the g^A 's and the g_V's , to a very good approximation, take their SU(3) symmetric values in the $q^2 = 0$ and $k^2 = 0$ limits (119), this will henceforth be assumed. The validity of this approach is founded on the Ademollo-Gatto theorem (44), where for the $\Delta Y = 0, \pm 1$ matrix elements, to first order in SU(3) symmetry breaking, the mass splitting occurs, but the weak coupling-constants remain invariant to this order. The above is also consistent with the avoidance of the SPA , in which the baryon masses are degenerate for on mass shell decay, from the four-momentum conservation constraint $\delta^4(p_1 - p_2 - q)$.

Finally, a small amount of CVC (32) breaking is introduced for the $\Delta Y = \pm 1$ currents, where the effect is found to be small,as expected, This amounts to adding a small amount of D-type coupling (eq., 1.18a chapt, 1), in the evaluation of the weak vector coupling constants. This is equivalent to adding some D-type vector current (eq., 1.18a). As we are taking symmetry breaking effects into account, this effect will be of second order in SU(3) breaking. We feel further justified in including D-type coupling or CVC breaking, in as much as we are employing the GOR breaking scheme which requires that SU(2)$\otimes$SU(2) is a better symmetry of the strong interactions than SU(3). In the limit of SU(2)$\otimes$SU(2) $\partial_\mu V_\mu^i = 0$ (i = 1,2,3) while $\partial_\mu V_\mu^i \neq 0$ (i = 4,5,6,7), except in the limit of SU(3)$\otimes$SU(3), and hence for the $|\Delta Y| = 1$ currents we may expect some D-type coupling for SU(2)$\otimes$SU(2) holding approximately. On the other hand, if we argue that SU(3) is a good symmetry, this implies $\partial_\mu V_\mu^i = 0$ (i = 1...8) in the limit of SU(3), and there can be no D-type coupling or D-type component to the vector current, as is apparent from the baryon octet model for the vector currents (eq., 1·18).

Summing over the spins of the intermediate states, and with the help of the Dirac equation and the definitions of the current-divergence matrix elements (appendix A), the final form for the P wave amplitude for Σ^- decay is

$$P(\Sigma^-_-) = -\frac{\lambda_P}{f_\pi f_K}\left\{\frac{3c}{2(\sqrt{2}-c/_2)}(N+\Sigma)\left(\frac{-F+D}{2}\right) - \frac{(\Lambda+N)(\Lambda+\Sigma^-)}{2\Lambda}\right.$$

$$\left.\times\,(-3F'-D')\frac{D}{6} - \frac{(N+\Sigma^\circ)(\Sigma^\circ+\Sigma^-)}{2\Sigma^\circ}(F'-D')\frac{F}{2}\right\}, \qquad 3\cdot38$$

where we have evaluated the amplitude in the $\underset{\sim}{p}_1 = 0$ frame, in which $\Sigma = p_1^\circ$ (Appendix A). F , D and F', D' are defined in the appendix A and equations 1·18 , where in the limit of CVC (32), $F' = 1$ and $D' = 0$. (Capital letters will henceforth represent the corresponding masses).

By exactly analogous methods of calculation, the remaining amplitudes can be evaluated in the same manner and are recorded below.

$$S(\Lambda^\circ_-) = \frac{\lambda_S}{f_K f_\pi}\left(\frac{2(\sqrt{2}-c/_2)}{3c}(\Lambda-P)(-3F'-D')(6)^{-1/2} - \frac{(P+N)(\Lambda+N)}{2N}(F+D)\right.$$

$$\left.\times\,(-3F-D)\frac{(6)^{-1/2}}{2} - \frac{(P+\Sigma^+)(\Sigma^++\Lambda)}{2\Sigma^+}(D)(-F+D)(6)^{-1/2}\right)$$

$$P(\Lambda^\circ_-) = -\frac{\lambda_P}{f_K f_\pi}\left(\frac{3c}{2(\sqrt{2}-c/_2)}(N+\Lambda)(-3F-D)\frac{(6)^{-1/2}}{2} + \frac{(P+N)(N+\Lambda)}{2N}(F+D)\right.$$

$$\left.\times\,(-3F'-D')\frac{(6)^{-1/2}}{2} - \frac{(P+N)(\Sigma^++\Lambda)}{2\Sigma^+}(-F'+D')(D)(6)^{-1/2}\right)$$

$$S(\Xi^-_-) = \frac{\lambda_S}{f_K f_\pi}\left(\frac{2(\sqrt{2}-c/_2)}{3c}(\Xi-\Lambda)(3F'-D')(6)^{-1/2} - \frac{(\Lambda+\Sigma^-)(\Sigma^-+\Xi^-)}{2\Sigma^-}\right.$$

$$\left.\times\,(D)(F+D)(6)^{-1/2} - \frac{(\Lambda+\Xi^\circ)(\Xi^\circ+\Xi^-)}{2\Xi^\circ}(3F-D)(-F+D)\frac{(6)^{-1/2}}{2}\right)$$

$$P(\Xi^-_-) = -\frac{\lambda_P}{f_K f_\pi}\left(\frac{3c}{2(\sqrt{2}-c/2)}(\Lambda+\Xi^-)(3F-D)\left(\frac{6}{2}\right)^{-1/2} + \frac{(\Lambda+\Sigma^-)(\Sigma^-+\Xi^-)}{2\Sigma^-}\right.$$

$$\left.\times (D)(F'+D')(6)^{-1/2} - \frac{(\Xi^0+\Lambda)(\Xi^0+\Xi^-)}{2\Xi^0}(3F'-D')(-F+D)\left(\frac{6}{2}\right)^{-1/2}\right)$$

$$S(\Sigma^-_-) = -\frac{\lambda_s}{f_K f_\pi}\left(\frac{2(\sqrt{2}-c/2)}{3c}(\Sigma^- - N)(-F'+D') + \frac{(N+\Lambda)(\Lambda+\Sigma^-)}{2\Lambda}\right.$$

$$\left.\times(-3F-D)\left(\frac{D}{6}\right) + (N+\Sigma^0)(\Sigma^0+\Sigma^-)(F-D)\left(\frac{F}{2}\right)\right).$$

$P(\Sigma^-_-)$ as above.

$$S(\Sigma^+_+) = -\frac{\lambda_s}{f_K f_\pi}\left(\frac{(N+P)(P+\Sigma^+)}{2P}(F+D)(-F+D)\frac{1}{2} + \frac{(N+\Lambda)(\Lambda+\Sigma^+)}{2\Lambda}\right.$$

$$\left.\times(-3F-D)\left(\frac{D}{6}\right) + \frac{(N+\Sigma^0)(\Sigma^0+\Sigma^+)}{2\Sigma^0}(F-D)\left(-\frac{F}{2}\right)\right).$$

$$P(\Sigma^+_+) = -\frac{\lambda_P}{f_K f_\pi}\left(\frac{(N+P)(P+\Sigma^+)}{2P}(F+D)\left(-\frac{F'+D'}{2}\right) - \frac{(N+\Lambda)(\Lambda+\Sigma^+)}{2\Lambda}\right.$$

$$\left.\times(-3F'-D')\left(\frac{D}{6}\right) - \frac{(N+\Sigma^0)(\Sigma^0+\Sigma^+)}{2\Sigma^0}(-F)\left(\frac{F'-D'}{2}\right)\right)$$

$$S(\Sigma^+_0) = -\frac{\lambda_s}{f_K f_\pi}\left(\frac{2(\sqrt{2}-c/2)}{3c}(\Sigma^+ - P)(-F'+D')(2)^{-1/2} + \frac{(P+P)(P+\Sigma^+)}{2P}\right.$$

$$\left.\times(F+D)(-F+D)(2)^{-3/2} + \frac{(P+\Sigma^+)(\Sigma^+ + \Sigma^+)}{2\Sigma^+}(-F+D)(F)(2)^{-1/2}\right)$$

$$P(\Sigma^+_0) = +\frac{\lambda_P}{f_K f_\pi}\left(\frac{3c}{2(\sqrt{2}-c/2)}(P+\Sigma^+)(-F+D)\left(\frac{2}{2}\right)^{-1/2} - \frac{(P+P)(P+\Sigma^+)}{2P}\right.$$

$$\left.\times(F+D)(-F'+D')(2)^{-3/2} + \frac{(P+\Sigma^+)(\Sigma^+ + \Sigma^+)}{2\Sigma^+}(-F'+D')(F)(2)^{-1/2}\right). \qquad 3.39$$

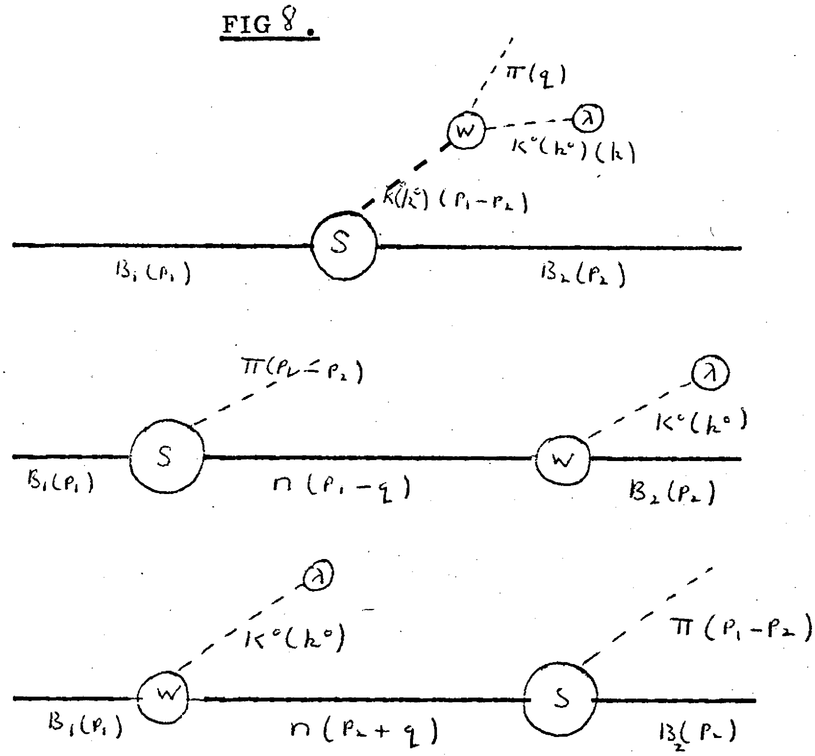
From equation 3.33 we can identify the terms in the amplitudes with the following simple pole model diagrams, corresponding to the single particle dominance assumption of the matrix elements through the use of PDDAC and PDDVC and the retention of only single particle intermediate states in the sum over intermediate states.

FIG 8.

(A) ETC term.

(B) ST.

(C) ST.



S and W denote the strong and weak interaction vertices respectively, and λ denotes the strength of the weak amplitudes $K^{\circ}, \bar{k}^{\circ} \longrightarrow$ vacuum, and is refered to in the liturature as the 'Tadpole contribution ' (111).

To obtain a comparison of the amplitudes above with experiment we took the experimentally determined sum $F + D = 1.20 = g_A^{NP}(0)$(120), and varied F and D within the ranges $0.48 \geqslant F \geqslant 0.40$ and $0.82 \geqslant D \geqslant 0.74$, with a small variation of F' and D' about their CVC limit values of 1.0 and 0.0 respectively. λ_P/f_K and λ_S/f_K have been absorbed into λ_P' and λ_S', and the rounded experimental value of $f_\pi = 125$ Mev was taken. For ' c ' we used the value of -1.25, as determined by GOR (43), as this accounts reasonably well for the symmetry breaking interaction.

On choosing the values of F and D : $F = 0.43$ and $D = 0.77$, in agreement with the experimetal values of these quantities (121), we obtained a fit of the amplitudes for the following values of the parameters; $F' = 1.194$, $d = -0.120$, $\lambda_S' = 0.275 \times 10^{-5}$, $\lambda_P' = 0.254 \times 10^{-5}$. We compare the fitted amplitudes below with two sets of compiled experimental results for the S and P wave amplitudes, from Berge (100) and Filthuth (97) respectively.

$S \times 10^{-5}$ $\mathrm{sec}^{-1/2}$.

Theory.		Experiment.	
		Berge.	Filthuth.
$S(\Lambda^0_-)$	= 1.46	1.551 - 0.024	1.52 - 0.02
$S(\Xi^-_-)$	= - 2.46	- 2.022 - 0.029	- 2.07 - 0.03
$S(\Sigma^-_-)$	= 1.40	1.861 - 0.017	1.87 - 0.03
$S(\Sigma^+_+)$	= - 0.04	0.008 - 0.034	0.02 - 0.04
$S(\Sigma^+_0)$	= - 1.00	- 1.168 - 0.187	- 1.15 - 0.18
		$P \times 10^{-5}$ $\mathrm{sec}^{-1/2}$.	
$P(\Lambda^0_-)$	= 11.00	11.045 - 0.475	10.44 - 0.33
$P(\Xi^-_-)$	= 0.20	- 0.152 - 0.386	- 0.55 - 0.43
$P(\Sigma^-_-)$	= 6.61	6.628 - 0.574	4.54 - 1.02
$P(\Sigma^+_+)$	= 19.11	19.081 - 0.347	19.08 - 0.35
$P(\Sigma^+_0)$	= 13.59	15.61 - 1.42	15.36 - 1.40

A brief summary of the assumptions taken in the above calculation is given below before the results are discussed and related to the other calculations.

C(i) Validity of the $k_\mu = 0$ limit to relate the strong interaction scattering process to the weak interaction on mass shell amplitudes.

C(ii) Hard pion approximation

C(iii) PCAC and PCVC for the kaons and the scalar kappa mesons respectively, used in the PDDAC and PDDVC interpretation.

C(iv) Assumption of an octet of scalar mesons.

C(v) Validity of the Gell-Mann charge-divergence CR's and the GOR summetry breaking scheme.

C(vi) Implicit use of the generalized G.T. relations and the validity of the Ademollo- Gatto theorem for taking the SU(3) symmetric values of the weak coupling constants.

C(vii) Single octet baryon pole saturation of the surface term.

C(viii) A small amount of CVC breaking.

From C(i) and C(iii) it is to be expected that the dominant contributions to the amplitudes arise from the lowest lying intermediate states, C(vii), from low energy dispersion relation arguments (114, 117). C(vi) and C(iv) rest for their validity upon one another, and both in turn upon C(i), C(ii) and C(iv). C(v) and C(viii) are extra assumptions, although to use C(v) we require the first four assumptions.

Section 3.3. Discussion of the results.

We will first demomstrate that the S and P wave amplitudes very closely satisfy the L–S relation, by showing this independently for the ETC and ST, as this will be important when comparing the results with other methods of calculation. For the S waves the L–S relation is (107)

$$2\,S(\Xi^-_-) + S(\Lambda^0_-) = \sqrt{3}\,S(\Sigma^+_0).$$

From equation 3·39, for this relation to hold we require,

$$3(P+\Sigma)\left((F+D)(-F+D)(2)^{-3/2} + (F)(-F+D)(2)^{-1/2}\right)$$

$$= \frac{(P+N)}{2N}(\Lambda+N)(F+D)(-3F-D)(F+D)(2)^{-3/2} + \frac{(P+\Sigma^+)}{2\Sigma^+}(\Sigma^+ +\Lambda)(D)(-F+D)(2)^{-1/2}$$

$$+\ 2\frac{(\Lambda+\Sigma^-)}{2\Sigma^-}(\Sigma^- +\Xi^-)(D)(F+D)(2)^{-1/2}$$

$$+\ 2\frac{(\Lambda+\Xi^0)}{2\Xi^0}(\Xi^0+\Xi^-)(3F-D)(-F+D)(2)^{-3/2} \qquad 3\cdot40$$

If we regard it as safe to neglect electro-magnetic mass differences, then we can regard $P = N$, $\Sigma^+ = \Sigma^- = \Sigma^0$, and dividing through by $(\Lambda + \Xi^0)$, we find, apart from very small deviations proportional to the mass splitting $\frac{\Delta M}{2M}$, that the L–S relation holds. A similar analysis for the P waves yields the same conclusions as above. Making the approximation $\Lambda = \Sigma$ the L–S relation is obeyed exactly for the P waves. This has also been noted by B.W. Lee (110) for the dispersion relation pole model, in which he showed that for $H^{PC} \sim \lambda_6$ and $\Lambda = \Sigma$, the L–S relation follows for the P waves.

It is interesting to note that if we demand SU(3) symmetry (ie all the baryon masses degenerate, with 'c' and D = 0 in this limit), then the L–S relation holds exactly for both the S and P wave amplitude's ST's, with the P wave ETC terms vanishing in this limit. When the SU(3) breaking is turned on again, there arise small corrections to the L–S relations, of the order of the symmetry breaking mass splittings. These corrections indicate the fact that we have chosen to be very careful about retaining a broken symmetry situation when calculating the amplitudes.

Considering now the ETC terms for the S wave amplitudes, we require for the L-S relation to hold, that

$$(\Lambda - P)(-3F' - D') + 2(\Xi - \Lambda)(3F' - D') = -3(\Sigma - P)(-F' + D').$$

$$\text{ie}\quad (\Lambda - N)(-3F' - D') + (\Lambda + \Sigma - 2N)(3F' - D') = -3(\Sigma - N)(-F' + D'),$$

using the Gell-Mann- Okubo mass formula, $2(M_N + M_\Xi) - 3M_\Lambda - M_\Sigma = 0$. (4,122), we find the correction to the above equality holdng exactly, is $(2\Lambda D' - 2\Sigma D')$, which is third order in SU(3) breaking. On the other hand for SU(2)$\times$SU(2) a good symmetry of the strong interactions, the correction only vanishes if we take $\Lambda = \Sigma$. For the P wave ETC terms, the correction is of the order of the mass splitting, and vanishes in the SU(3) symmetric limit. In this limit though, the ETC terms are seen to vanish. ('c' $= 0$).

For the S and P wave amplitudes, both the ETC terms and the surface terms obey the $|\Delta I| = \frac{1}{2}$ rule $\sqrt{2}\,\Sigma^+_0 = \Sigma^+_+ - \Sigma^-_-$, apart from correctoons of the order of the electromagnetic mass splittings which we are disregarding in comparison to SU(3) breaking corrections. Hence we regard the above relation as holding exactly, which is to be expected, as the $|\Delta I| = \frac{1}{2}$ rule is explicitly built into the form of the Hamiltonian (eq., 3·359), up to first order of SU(3) breaking.

If we turn to S(Σ^+_+), the condition for this vanishing identically, which is consistent with experiment, is that D / F$=\pm\sqrt{3}$. Taking the positive solution, this value is in agreement with experiment (121), and the D / F ratio found for the best fit to the amplitudes.

At this stage it is worth pointing out that if we had had a criteria for retaining only the ETC terms or the ST's of the S and P wave amplitudes, the amplitudes would have been found to obey the relations discussed above. The second point is that from the table below,

TABLE 8 .

Terms.	Order of SU(3) symmetry breaking.
SU(3) degenerate masses ($M_B = M_{SU(3)} \pm \delta M$) and numerical constants.	Zeroth order.
' c ' , mass differences ($m_B - m_{B'}$), and f_K , λ_P.	First order.
D'	Second order.
$D'(m_B - m_{B'})$ etc.,	Third order.

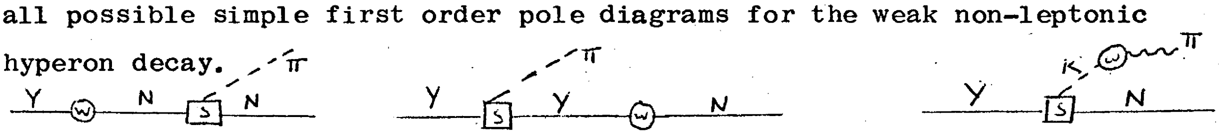
we see that there is a mixture of orders of SU(3) symmetry breaking, namely 0th 1st, 2nd and 3rd orders in the amplitudes. Hence it is not surprising that the amplitudes obtained are non-vanishing, as not all these orders have a complete complement of terms due to the approximate nature of the calculation. If we were able to calculate to all orders exactly, we would have canceling between terms of each order of symmetry breaking to produce a vanishing result for both the S and P wave amplitudes, as discussed in section 3.2. This fact has often been overlooked by authors using similar models to ourelves and will be discussed further in section 3.4. As an example, if we remove the CVC breaking terms, of second order in SU(3) breaking, we see $P(\Sigma^+_+) = 0$ for equal baryon masses. In effect we have separated out the mass differences to include them with the 1st order breaking terms,and find that we have a full complement of zeroth order terms,which cancel among themselves to give $P(\Sigma^+_+) = 0$ in the SU(3) symmetric limit.

Section 3.4. Discussion of models

Dispersion relation approach and pole models.

As the CD calculation above generates the lowest order pole diagrams of the dispersion theoretic approach, we compare here the resultant amplitudes we obtained with those of the dispersion relation approach.

The simple pole model approach was first advanced by Feldman et al (117), prior to the advent of SU(3) unitary symmetry, by writing down all possible simple first order pole diagrams for the weak non-leptonic hyperon decay.

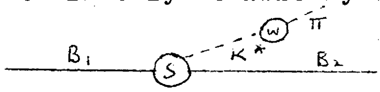


These were first analysed using a generalization of the method employed by Goldberger and Treiman in their derivation of the GT relation (118), obtained from the pole term in the dispersion relation of the matrix element of the axial-vector current divergence. Feldman et al argued that the NLDH's are dominated by pole terms in the same manner.

These pole diagrams were subsequently employed by Lee and Swift (123) in the framework of SU(3) unitary symmetry for both the S and P wave amplitudes, using an effective Hamiltonian, transforming as λ_6 . For $H_w^{PV} \sim \lambda_6$, the requirement that H_w^{PV} belong to an octet abnormal under charge conjugation (table 6) implies that H_w^{PV} can not be coupled to a baryon-antibaryon pair, B-$\bar{B}$, or to a pair of PS mesons in the SU(3) symmetric limit, when the members of the pair belong to the same octet. Equivalently, the single particle marrix elements $\langle B_1 | H_w^{PV} | B_2 \rangle = \langle P_1 | H_w^{PV} | P_2 \rangle = 0$, 3.41

as these are not CP invariant in the SU(3) symmetric limit. This can be seen by writing $H_w^{PV} \sim \lambda_6$, in the general form

$$f' \mathrm{Tr}\, \bar{B} \gamma_5 [B, \lambda_6] + d' \mathrm{Tr}\, \bar{B} \gamma_5 \{B, \lambda_6\} + g \,\mathrm{Tr}\, \lambda_6 P P, \qquad 3.42$$

where B and P are baryon and PS meson tensor octets (sect., 3.1). Under CP, H_w^{PV} changes sign (sect., 3.1 eq., 3.15 and 123), and hence the above matrix elements of H are vanishing under CP. Hence for the PV or S wave decays , Lee and Swift argued that the FMS (Feldman- Mathew-Salam) model is only tenable by using K^*-vector meson dominance, or the diagram  ,where the above matrix elements do not enter. This corresponds to the decay proceeding via the two step process $B_1 \rightarrow B_2 + K^*$ ($K^* \rightarrow \pi$).

This model has also been employed by Sakurai [124] to calculate the S wave amplitudes. Assuming an effective Hamiltonian of the form $2\lambda d_{6ij} \partial_\lambda P_i V_j^\lambda$, corresponding to the symmetric derivative coupling of an octet of PS mesons P_i to an octet of vector mesons V_i, We can readily show by direct evaluation of the diagram abovethat this model leads to results identical to the ETC terms of the S waves for pure F - type coupling of the K^*- mesons to the baryons, apart from a proportionality constant, dependent on the models used to calculate the K^* contribution. In this particular model the octet transformation property, and the λ_6 transformation character is enforced by the SU(3) structure constant d_{6ij} , which acts here as a Clebsch-Gordon coefficiect for the coupling (11), and which only obtains for P_i and V_i belonging to octets.

As with the discussion of the results (eq., 3-39), for pure F-type coupling (ie $D' = 0$), the L-S rule holds identically. Apart from the ability of the model to predict the L-S relation for the S waves and the ratios $S(\Sigma^-_-)/ S(\Sigma^+_0)$, $S(\Xi^-_-)/ S(\Lambda^0_-)$ within reasonable agreement with experiment, it has been argued by a number of authors (125) that the K^* dominance model for S waves gives amplitudes which are consistently too small. We will see that this is attributable in part to the neglect of the surface terms, which are expected to contribute when SU(3) breaking obtains, and $\langle B' | (H_\omega^{PV})_6 | B \rangle \neq 0$.

On comparing this calculation with the ETC terms of the CD calculation, we can at least conclude that, for conserved vector currents, the derivative coupling of the vector meson octet to the PS octet gives the same results as the non-derivative coupling of the scalar octet to the PS octet, where in the CD we have scalar meson pole dominance giving rise to the S wave ETC term.

For the P waves H_ω^{PC} has well defined matrix elements when $H_\omega^{PC} \sim \lambda_6$, in contrast to H_ω^{PV} , and the FMS model is tenable in this case. The transformation properties of the PC and PV parts of H_ω correspond to the c-c model (eq., 3-12) for the octet parts only cotributing. Lee amd Swift assumed however that H_ω^{PC} transforms as the 6th component of an octet of scalar spurions.(In the two step process $B_1 + S \longrightarrow B_2 + \pi$, ($S \rightarrow vac$) ,S is called a scalar spurion if it is a scalar quantity with the quantum no., assignment of H_ω, c/f the kappa meson in eq., 3-18). Contrary to the CD calculation, they assumed SU(3) symmetric parametrization of the strong coupling constants, which as we have pointed out, leads to the neglect of baryon octet mass differences. We refer again to this in the context of the dispersion theoretic approach below.

H. Sugawara (126) similarly employed the FMS pole model for both the PC and PV amplitudes with SU(3) symmetric strong interaction vertices, and $H_\omega \sim \lambda_6$,and assumed to obey the $|\Delta I| = \frac{1}{2}$ rule, which in the Lee-Swift paper automatically follows from the octet property they imposed on H_ω . It was assumed by Sugawara that the K meson pole contribution is smaller than that of the baryon pole terms by arguing that the couplings of the kaon are smaller than those of the pion. With this criteria he neglected the K pole diagram for the P waves.

The more recent dispersion theoretic approach, and in particular that of M.Sugawara (127), forms a convenient summary of the pole model results in a more rigorous context, and is a convenient basis for comparison between the above approach and the CD calculation.

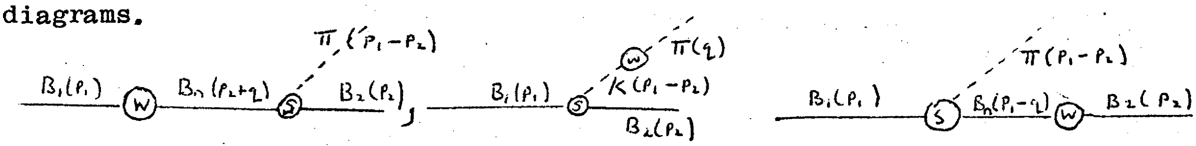
We begin by employing the reduced matrix formalism for the 2-body decay amplitides (eq.., 3·24)

$$-\langle B_1(p_1)\,\pi(q)\,|\,H_\omega(0)\,|\,B_2(p_2)\rangle$$
$$= i/(2\pi)^{9/2}\sqrt{\left(\frac{B_1\,B_2}{2\,p_1^0\,p_2^0\,q_0}\right)}\;\bar{u}(p_2)\,(\,S + P\gamma_5\,)\,u(p_1), \qquad 3\cdot43$$

defined for off mass shell S and P wave amplitudes, considered as analytic functions in the variables $s = -(\,p_2 + q\,)$, $t = -(\,p_1 - p_2)$ and $u = -(\,p_1 - q\,)$. Following Sugawara, we assume the Mandelstam representation is valid for S(s,t,u) and P(s,t,u), which allows us to write,

$$S(s,t,u) = \frac{R_s}{s-s_0} + \frac{R_t}{t-t_0} + \frac{R_u}{u-u_0} + \frac{1}{\pi^2}\left[\iint \cdots\cdots \right], \qquad 3\cdot44$$

and similarly for P. In this way the analyticity of S and P are exploited, where S and P are defined to be the on mass shell decay amplitudes as s,t and u approach the values $s = m^2_{B_1}$, $t = m^2_\pi$, $u = m_{B_2}$. The first three terms correspond to the simple pole model approximation, with the corresponding pole diagrams.



S-CHANNEL. t-CHANNEL. u-CHANNEL.

To begin with we will not consider a specific form for H_ω , and define quite generally the matrix elements of H_ω , by for example (c/f appendix A)

$$\langle B_1(p_1)\,|\,H_\omega(0)\,|\,B_n(p_2+q)\rangle = \frac{1}{(2\pi)^{3/2}}\sqrt{\frac{B_1 B_n}{p_1^0\,B_n^0}}\;\bar{u}(p_2+q)(\,\bar{S}_{n1}\gamma_5 + \bar{P}_{n1}\,)u(p_1);\ \text{etc} \qquad 3\cdot45$$

and

$$\langle \pi(q)\,|\,H_\omega(0)\,|\,\rho(k)\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{1}{4\pi^0\,\rho^0}}\;M_{\pi\rho}\,. \qquad 3\cdot46$$

Denoting the strength of the strong vertex by $G_{BB\pi}$, and employing the Feynman rules to write down the contribution to the pole diagrams, for the weak vertices defined above in equations 3·45 and 3·46 , we obtain for the S and P wave amplitudes, evaluated at $s = B_1^2$, $t = m_\pi^2$ and $u = B_2^2$,

$$S_{pole} = \left[\frac{g_{B_n B_2 \pi} . \bar{S}_{n_1}}{B_1 + B_2} + \frac{g_{B_n B_1 \pi} . \bar{S}_{n_2}}{B_2 + B_n} + \frac{g_{B_1 B_2 k} . m_{k\pi}}{m_k^2 - m_\pi^{2}} \right] \qquad 3{\cdot}47a$$

$$P_{pole} = - \left[\frac{g_{B_n B_2 \pi} . \bar{P}_{n_1}}{B_1 - B_n} + \frac{g_{B_n B_1 \pi} \bar{P}_{n_2}}{B_2 - B_n} - \frac{g_{B_1 B_2 K} \, m_{K\pi}}{m_K^2 - m_\pi^2} \right] \qquad 3{\cdot}47b$$

(Capitol letters denote the masses of the corresponding particles.) Unlike Sugawara we have included a scalar kappa meson pole term to correspond to the CD calculation. For the sake of comparison of the pole terms generated by the current-algebra approach and the pole terms of the dispersion theoretic approach or pole diagram approach,we consider the $P(\Sigma^-)$ amplitudes. Employing the generalised GT relations for the $G_{BB\pi}$'s (eq., 3·37), and substituting the divergence model for H_ω^{pc} (eq., 3·35a),we obtain with the aid of the expressions for the matrix elements of H_ω in appendix ,

$$P_{pole}(\Sigma^-) = -\frac{\lambda_p}{f_k f_\pi} \left[-\frac{f_\pi}{f_K}(N+\Sigma^-)(-F+D) - (\Lambda+\Sigma^-)(-3F'-D')\frac{D}{6} - (\Sigma^- + \Sigma^\circ)(F'-D')F/2 \right] \qquad 3{\cdot}48a$$

c/f

$$P_{CD}(\Sigma^-) = -\frac{\lambda_p}{f_k f_\pi} \left[\frac{3c}{2(\sqrt{3}-c/2)}(N+\Sigma^-)(-F+D) - \left(\frac{\Lambda+N}{2\Lambda}\right)(\Lambda+\Sigma^-)(-3F'-D')D/6 - \left(\frac{N+\Sigma^\circ}{2\Sigma^\circ}\right)(\Sigma^\circ+\Sigma^-)(F'-D')F/2 \right] \qquad 3{\cdot}48b$$

(NB. the matrix elments of $H_{\omega(CD)}$ are well defined quantities, even though the reduced matrix element eq., 3·34 is vanishing for on mass shell decay.) For ' c ' = - 1.25 , $3c/2(\sqrt{3}-c/2) \approx -1.08$. On the other hand the mass factors $(\Lambda+N)/2\Lambda$ and $(N+\Sigma^\circ)/2\Sigma^\circ \approx 0.9$. In the GOR model $f_\pi \approx f_K$, and hence the above are identical,apart from the small mass factor deviations which arise solely from the the use of PCAC in the current-algebra calculation. That is the matrix elements are assumed approximately constant for the $q^2 = 0$ extrapolation in the CD calculation ($m_\pi^2 = 0$), whereas in the dispersion relation approach m_π^2 is on the mass shell. Hence we not only see that the corrections to the HPA are small, but that calculating exactly to each order in symmetry breaking in the dispersion theoretic approach, similarly leads to vanishing amplitudes.

In this respect Adler and Dashen (17) have noted that quite generally,for $H_\omega^{pc} \sim \lambda_6$ (this obtains in the CD model) for the simple pole model, to leading order in SU(3) breaking, the pole diagrams cancel against each other and make no contribution to any of the P wave amplitudes. This is demonstrated by using SU(3) symmetric values for the strong coupling constants in equation 3·47 . All terms are then of the same order,($O(1)$ SU(3)-breaking $)^{-1}$, and cancel identically. Although H.Sugawara uses SU(3) symmetric values for the strong coupling constants in the P wave amplitudes, the non-vanishing of his results is attributable to the neglect of the K meson pole.

Okubo (114) similarly considers the weak decay from a dipersion theoretic point of view,for the S waves. By temporarily giving up energy-momentum conservation (114) for on mass shell decay, Okubo considers the scattering process $S(k) + B_1(p_1) \to \pi(q) + B_2(p_2)$, 3·49

which corresponds to the physical amplitude in the limit $k_\mu = 0$, as we have considered using a current algebra approach. Dropping unnecessary factors in the reduced T matrix for the decay, we define as Okubo, the matrix

$M = \bar{u}(p_2)\,[\,S + \gamma_5 P\,]\,u(p_1)$,

proportional to $\tilde{T}$ for on mass shell decay. For S and P off mass shell, M is proportional to the pion production amplitude (eq. 3·49) , defined by

$M = \bar{u}_2(p_2)\,\{\,(H_1 + \gamma_5 H_2) + [\,i\gamma.k\,,\,i\gamma.q\,](J_1 + \gamma_5 J_2)\,\}\,u(p_1)$,

where H_1 ,H_2,J_1 ,J_2 are in general functions of the invariant variables s, t and u, as defined above. In order to avoid the soft pion ambiguity (sect., 3·2) Okubo considers the HPA, $q^2 = 0$. For the physical decay amplitudes, M is related to the S and P wave amplitudes by

$S = H_1(B_1^2, 0, 0)$ and $P = H_2(B_1^2, 0, 0)$, for H (s,t,u), 3·50

where we have used the relation $s + t + u = B_1^2 + B_2^2 - q^2 - k^2$, with s,t and k^2 chosen as independent variables, and with the understanding that the hard pion limit, $q^2 = 0$, is to be taken. For t and k^2 fixed, we can write, following Okubo,an unsubtracted s channel dispersion relation for H_i ,

$$H_i(s,t,k^2) = \frac{1}{\pi}\int_{-\infty}^{\infty} ds'\, \mathrm{Im}\, H_i(s',t,k^2)\,/\,(s'-s-i\varepsilon), \quad i (= 1,2) \qquad 3·51$$

which on evaluating the RHS, gives rise to s and u channel baryon pole terms for H. In order to generate poles in the t channel, Okubo argues on the Regge-asymptotic behaviour of H_i , the simplest means is to require at least a once-subtracted dispersion relation,

$$H_i(s,t,k^2) = H_i(s_0,t,k^2) + \frac{1}{\pi}(s-s_0)\int_{-\infty}^{\infty} ds'\,\mathrm{Im}\,H_i(s',t,k^2)/(s'-s_0)(s'-s-i\varepsilon) \qquad 3·52$$

where $H_i(s_0,t,k)$ is a function of t alone. Formally setting $q_\mu = 0$, we find $s = B_2^2$, $k^2 = -t$, and hence at $s_0 = B_2^2$ in the soft pion limit, $H_i(s,t,-t) = H_i(B_2^2,t,-t) \overset{\mathrm{defn}}{=} K_i(t)$, and equation 3·52 becomes

$$H_i(s,t,-t) = K_i(t) + \frac{s-B_2^2}{\pi}\int_{-\infty}^{\infty} \frac{ds'\,\mathrm{Im}\,H_i(s',t,-t)}{(s'-B_2^2)(s'-s-i\varepsilon)}, \qquad 3·53$$

For the physical S wave decay, in the $q^2 = 0$ limit, with $k_\mu = 0$, $s = B_1^2$ and $t = -(p_1 - p_2)^2 = -(q-k)^2 = 0$,

$$S = K_1(0) + \frac{B_1^2 - B_2^2}{\pi}\int_{-\infty}^{\infty} ds' \, \mathrm{Im}\, H_1(s',0,0)\Big/(s' - {}_2^{\,2})(s' - {}_1^{\,2} - i\varepsilon) \qquad 3\cdot 54$$

The important point is that in the soft pion limit, $B_1^2 \to B_2^2$, and the s and u channel pole terms are eliminated, with only the subtraction constant or t channel pole term contributing. We can readily show, using the soft pion theorem (see sect., 2.1 eq's 2·21, 22.) to calculate S from equations 3·2 and 3·3 , that K_1 corresponds to the K^*- dominance model or the ETC term in the CD calculation.

The soft pion theorem (eq's 2·21,22) was first applied to NLDH's by Suzuki and Sugawara (128, 129) to the S wave amplitude $\langle B_2\pi \mid H_\omega^{PV}(0) \mid B_1\rangle$. By contracting the pion and taking the soft pion limit, $q_\mu = 0$, with neglect of the surface terms, it readily follows that,

$$\langle B_2\pi \mid H_\omega^{PV}(0) \mid B_1\rangle \propto \langle B_2 \mid H_\omega^{PC} \mid B_1\rangle . \qquad 3\cdot 55$$

This can be seen in the framework of the c-c model for example, for $H_\omega \sim \lambda_6$, where from the Gell-Mann (8) charge-charge-density CR's (eq's., 1·27), the proportionality $[F_5^i, (H_\omega^{PV})^6] \propto (H_\omega^{PC})^i$ holds, and the above result follows for the ETC term remaining, on the neglect of the ST's. From equation 3·55 , Suzuki and Sugawara were able to show for the c-c model, that the $|\Delta I| = \frac{1}{2}$ rule obtains for $\Lambda \to N + \pi$ and $\Xi \to \Lambda + \pi$ S waves, by noting that between single particle octet states Λ, N and Ξ, Λ, only the $|\Delta I| = \frac{1}{2}$ component of H_ω^{PV} contributes. This gives rise to an effective $|\Delta I| = \frac{1}{2}$ rule for these amplitudes in the $q_\mu = 0$ limit, although for Σ decay both $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ transitions are possible for the matrix element $\langle\Sigma \mid H_\omega^{PC} \mid N\rangle$. For Σ_+^+ decay, only the $\underset{\sim}{27}$ component of H_ω^{PC} is found to contribute to $\langle \Sigma^+ \mid H_\omega^{PC} \mid N\rangle$. This is not surprising, as for only the $\underset{\sim}{8}$ component contributing, as in the CD model, no ETC term is possible for $S(\Sigma_+^+)$, and hence $S(\Sigma_+^+)_{ETC} = 0$. By relating the amplitudes to the reduced matrix elements of the components of H_ω^{PC} belonging to the $\underset{\sim}{8}$ and $\underset{\sim}{27}$ representations, Suzuki finds the condition for the $|\Delta I| = \frac{1}{2}$ rule $\Sigma_-^- - \sqrt{2}\,\Sigma_0^+ + \Sigma_+^+ = 0$ for Σ decay holding is $S(\Sigma_+^+) = 0$ ($\equiv$ ($S(\Sigma_+^+)$)$_{ETC} = 0$). This condition automatically holds in the CD model, where we have shown that the $|\Delta I| = \frac{1}{2}$ rule holds for the S wave amplitudes, and amounts to the assumption of octet dominance. Ie $(H_\omega^{PC})^{\underset{\sim}{27}} = 0$, which for Suzuki's analysis implies $S(\Sigma_+^+) = 0$. With ($S(\Sigma_+^+))_{ETC} = 0$, Suzuki further shows that the L-S relation obtains for the S waves, which corresponds in terms of the CD calculation, to the ETC terms obeying this relation.

An important feature of the soft pion analysis above is the implied SU(3) symmetry, arising from the $B_1 - B_2$ mass degeneracy, on taking the $q_\mu = 0$ limit. This property is also apparent in the approach of Okubo, and leads to the neglect of the baryon pole terms in each case, for the S waves.

On the other hand, for the P waves, in the soft pion limit, the mass degeneracy implies the P wave amplitude $\langle \pi B_1 | (H^{PC})^c | B_2 \rangle \propto \langle B_1 | (H^{PV})^c | B_2 \rangle$ is vanishing in this limit, as Lee and Swift (123) demonstrated in their application of the c-c model to the FMS pole model, and hence for the c-c model, no predictions for the P waves can be made in the soft pion limit defined above.

The soft pion approach was later extended by a number of authors(130, 131) to the analysis of the P waves. In particular we follow Brown and Sommerfeld (130) in discussing this approach in relation to the DR and CD calculations of the P waves. The important advance was the extension of the analysis of Suzuki and Sugawara to include the ST's by establishing a well defined procedure for taking the $q_\mu^\pi = 0$ limit.

The calculation proceeds by applying the soft pion technique to the reduced T matrix (eq., 3·3) for which,

$$\lim_{q_\mu \to 0} T(q) = \lim_{q_\mu \to 0} (2\pi)^{3/2} \sqrt{2q^0} \langle B_2(p_2)\, \pi(q) | H_w(0) | B_1(p_1) \rangle$$

$$= \frac{-i}{f_\pi} \langle B_2(p_2) | [F_i^5(0), H_w(0)] | B_1(p_1) \rangle + \lim_{q_\mu \to 0} \frac{i q_\mu}{f_\pi} S_t . \qquad 3\cdot 56$$

We define the ST, for H_w unspecified, as

$$S_t = i \int d^4y \, e^{-iqy} \, \theta(y_0) \langle B_2(p_2) | [A_\mu^i, H_w(0)] | B_1(p_1) \rangle, \qquad 3\cdot 57$$

where

$F_i = -i \int d^3y \, A_4^i(\underset{\sim}{y}, 0)$ and $q_\mu S_t$ = ST of the CD calculation, to display q_μ explicitly. The ambiguity of which limit to take first (the mass degeneracy or the $q_\mu = 0$ limit) is overcome by defining the combination

$$\lim_{q_\mu \to 0} \left[-\frac{i q_\mu}{f_\pi} S_t + T_{BORN}(q) \right] = \hat{T}(q = 0), \qquad 3\cdot 58$$

which is well defined in the $q_\mu = 0$ limit. T(Born) are the baryon pole terms (eq., 3·47). Writing $T(q) = T(\mathrm{Born})(q) + T'(q)$, where T' are the remaing contributions (apart from baryon pole terms) to T(q), we find from equation 3·56,

$$T'(q) = T(q) - T_{BORN}(q, q^2 = -m_\pi^2) = \frac{-i}{f_\pi} \langle B_2(p_2) | [F_i^5, H_w(0)] | B_1(p_1) \rangle$$

$$- \lim_{q_\mu \to 0} \left[-i q_\mu \frac{S_t}{f_\pi} + T_{BORN} \right]. \qquad 3\cdot 59$$

When evaluating T_{Born} ($q^2 = -m_\pi^2$) for unspecified H_w at $q^2 = -m_\pi^2$ (eq., 3·47), a difference between the results of the current-algebra calculation(eq., 3·39) and the pole terms' direct evaluation does arise, in as much as PCAC, with $q^2 = 0$, is assumed in the CA calculation, with the amplitude assumed to be a smooth function in q^2, as $q^2 \to 0$. The difference in fact is what we found before on comparing the pole diagram terms with the CD terms (eq's 3·45a and b). Hence $T_{BORN}(q, q^2 = -m_\pi^2) \approx T_{BORN}(q, q^2 = 0)$. Substituting for $T_{BORN}(q)$ and S_t in equation 3·58, and taking $q_\mu = 0$, we find

$$\hat{T}(0) = \frac{i}{(2\pi)^3}\sqrt{\left(\frac{m_1 m_2}{p_0^{\,2} p_0^{\,1}}\right)}\;\bar{u}(p_2)\left[\sum_n g_{\pi n 2}\,\gamma_5\,\frac{(\bar{s}_{1n} + \gamma_5 \bar{p}_{1n})}{(m_n + m_2)}\right.$$

$$\left. + \sum_{n'} \frac{(\bar{s}_{n'2} + \gamma_5 \bar{p}_{n'2})}{(m_1 + m_{n'})}\,\gamma_5\, g_{\pi 1 n'}\right] u(p_1), \qquad 3·60$$

where $\bar{s}$ and $\bar{p}$ are defined in equation 3·45, and the strong interaction coupling constants can be related to the physical masses through the generalized GT relations (eq's., 3·37) as previously.

With $T_{BORN}(q, q^2 = m_\pi^2) = T^{PV}_{BORN} + T^{PC}_{BORN}$, the total amplitude for the PC and PV decays can be written from equations 3·58 and 3·59 as

$$T(q, q^2 = -m_\pi^2) \simeq \frac{-i}{f_\pi} \langle B_2(p_2) | [F_i^5, H_\omega(0)] | B_1(p_1)\rangle + T_{BORN}\quad \hat{T}(0), \qquad 3·61$$

where the assumption is made that $\hat{T}$ is a slowly varying function of q_μ as $q_\mu \to 0$.

Equation 3·61 is essentially the so called soft pion result, where the cancelation of $T_{BORN}(q^2 = -m_\pi^2)$ is implicit from the form of $\hat{T}$ (eq., 3·58), but is written in the above form in order that the $q_\mu = 0$ limit is well defined. For $q_\mu \not\to 0$, the previous CD terms are returned, as the Born terms cancel, and we are left with the surface terms as before.

Using the result from the analysis of Suzuki and Lee and Swift (123,128), that in the SU(3) symmetric limit $\langle B_2 | H^{PV} | B_1\rangle = 0$, for H^{PV} of the c–c form and $\sim \lambda_6$, and also the relation

$$\langle B_2 | [F_i^5, H^{PC}_\omega] | B_1\rangle = \langle B_2 | [F_i, H^{PV}_\omega] | B_1\rangle \propto \langle B_2 | H^{PV} | B_1\rangle, \qquad 3·62$$

obtained from both the transformation property of H_ω and the charge-current CR's (eq's., 1·32 a,b,c), we see immediately in the c–c model, for $H_\omega \sim \lambda_6$ in the soft pion approximation, only the ETC terms contribute to the S waves, and only the baryon pole terms to the P waves. We refer again to the fact that the soft pion limit effectively imposes SU(3) symmetry on the matrix elements $\langle B_2 | H^{PV}_\omega | B_1\rangle$, and hence the above terms that don't contribute have been eliminated as a consequence of the soft pion limit alone.

Relinquishing the soft pion limit, the neglect of the terms otherwise contributing to the pole diagrams (Fig 8) is no longer justified, and that taking the less severe hard pion limit is one step to calculating more exactly, although the problem of the vanishing of the amplitudes is more apparent, as it should be. In as much as the soft pion limit implies the SU(3) symmetric limit for the matrix elements $\langle B_2 | H_\omega^{PV} | B_1\rangle$ etc., we remark that on comparing the CD results with the pole model results, we appear to be much closer to an $SU(2)\times SU(2)$ symmetric world than an $SU(3)$ symmetric world, for ' c ' $\doteq$ − 1.25 rather than $\approx$ 0, in so far as the CD calculation should reproduce the pole model results up to corrections arising from the PCAC assumption. We also note that contrary to Okubo's conclusion that the S wave ETC terms vanish in the SU(3) symmetric limit, the ETC terms for the S waves in the CD calculation are of zeroth order in SU(3) breaking due to the presence of ' c ' in the denominator, and hence don't vanish in this limit, although this is not apparent for the K^{*} and soft pion models.

The comparison of the soft pion results with the L-S relation and the $|\Delta I| = \frac{1}{2}$ rules is essentially the same as the analysis of Suzuki, so we will not repeat it here.

A number of authors have obtained results for the P waves, using the soft pion techniques with the c-c $(H_{\omega}^{c-c})^{i}$ (eq., 3·12), for which the $|\Delta I| = \frac{1}{2}$ rules and the L-S relation obtain only in the absence of the $\underset{\sim}{27}$ contribution. Apart from setting this contribution equal to zero, and assuming octet dominance (H_{ω}^{c-c} as a member of an octet only), a number of attempts have been made to see if there is any justification for neglecting the $\underset{\sim}{27}$ contribution in comparison to the octet contribution of H_{ω}^{c-c}.

Chiu et al (132) have obtained a suppression of the $\underset{\sim}{27}$ component of H_{ω}^{c-c} in the framework of the soft pion calculation, by saturating the commutators arising in the S wave amplitudes, namely $\langle B_2(P_2)|[F_i^5, H_{\omega}^{PV}(0)]|B_1(P_1)\rangle$, with both baryon octet and decuplet intermediate states, in order to obtain more information from this term. The $\underset{\sim}{27}$ component of H_{ω}^{c-c} is found to be suppressed by a factor of $\approx$ 20 with respect to the octet component. This is attributed both to the smallness of the C-G coeeficients coupling the $\underset{\sim}{27}$ component to the baryon states (7, 132), and partly as a dynamical accident. However there is a great deal of uncertaintity about which are the important states to keep, and whether the cancelation of the $\underset{\sim}{27}$ contribution obtains by including all possible intermediate states. A similar analysis has been conducted for the P waves by Chan and Ram Mahon, by extending the baryon pole saturation to include baryon decuplet states (133, 134). Chan, in particular, employing the hard-pion technique, obtained reasonable agreement with experiment by including the usual baryon octet states in the surface term plus one decuplet state, $Y_0^{*}(1405)(J=\frac{3}{2}^{+})$. As the coupling constants for the $Y_0^{*}(1405)$ are barely known, let alone all of the octet baryons', almost any result can be got by suitably choosing the unknown parameters. As no clearly defined criteria exist for which states to keep, this field is still wide open to speculation.

We now briefly compare the CD calculation with similar calculations attempted by other authors, and point out the similarity between the results we obtained and their's. The overriding feature is that the results are essentially the same, apart from terms of the order of the baryon mass-splitting, and that despite the various novel assumptions employed in these calculations, they are one and the same thing.

Guralnik, Mathur and Pandit (135) avoid stating that their model is of the CD form, by assuming that H_ω^{PC} and H_ω^{PV} are only defined between single particle states, thus 3·63

$$\langle B_1 | H_\omega^{PV} | B_2 \rangle = \lambda_S \partial_\mu \langle B_1 | A_\mu^7(x) | B_2 \rangle ; \langle B_1 | H_\omega^{PC} | B_2 \rangle = \lambda_P \partial_\mu \langle B_1 | V_{\mu 7}(x) | B_2 \rangle$$

Clearly this amounts to assuming the CD model, as when applying this model for H_ω we found that we end up calculating just these matrix elements only. They insert these matrix elements into the conventional soft pion formalism, and neglecting the S wave baryon pole terms, obtain the ETC terms of the CD calculation. On the other hand for the P waves they retain both the ETC and ST's, to obtain the amplitudes of equation 3·39 , apart from terms of the order $\Delta M / M$. Rather than apply SU(3) for the weak vector and AV coupling constants, they choose to use three of these as parameters, and take experimental values for the remainder. Two of the five P wave amplitudes, $P(\Xi^-_-)$ and $P(\Xi^-_0)$ are then used as input to the fit, but unfortunately they have neglected the Σ^c baryon pole contribution to $P(\Sigma^+_+)$, which casts doubt on the fit.

The divergence model has also been employed recently by Ohya (136), using the GOR symmetry breaking scheme. The divergences have been related to the corresponding quark densities, with the consequence that the F / D ratio for $H_\omega^{PC} \sim u_6$, is taken to be equal to the F / D ratio for the medium strong mass splitting spurion (Chapt 1) u_8 , assumed to belong to the same octet as H_ω^{PC}. In contrast we related the matrix elements of $\partial_\mu V_\mu$ to the physical mass differences, and used SU(3) symmetry for the weak vector coupling constants. These two proceedures would be identical , if, instead of taking physical masses, we chose the masses obeying the Gell-Mann- Okubo mass formula (122), such that the resultant mass differences would be directly proportional to the reduced matrix elements, F and D, of the mass splitting interaction, u_8 .

To fit the resultant amplitudes, to experiment, Ohya requires the GOR symmetry breaking parameter ' c ' = 2.1 , which is inconsistent with the use of this model, which finds ' c ' $\simeq$ - 1.25. Comparing calculations, we find that taking such a large value of c is equivalent to our having to assume a very large value for the ratio λ_P / λ_S .

Fenster and Panchapakeson(137) employ the quark density model with $H_\omega^{PC} \sim u_6 \sim \frac{1}{2} \bar{\chi} \lambda_6 \chi$ and $H_\omega^{PV} \sim v_7 \sim \frac{1}{2} i \bar{\chi} \lambda_7 \gamma_5 \chi$, starting from equation 3.24. We have previously shown that this model is equivalent to the CD model for the GOR symmetry breaking scheme holding. Apart from employing SU(3) symmetric values for the strong coupling- constants, the results obtained are the same as the CD calculation, as expected. Fenster et al also retain some CVC breaking for the strangeness changing vector currents by including some D-type coupling as ourselves, and they obtain similary $S(\Sigma^+_+) = 0$ for $D / F = \sqrt{3}$. Their fit is also good, except for the $P(\Sigma^+_+)$ term, in which they have omitted the proton baryon pole term.

Rather than assume the existence of an octet of scalar mesons, Kumar et al (135) assumed the existence of a single kappa meson, first discussed by Nambu et al (139), before the idea of an octet of scalar mesons had become popular. In order to relate this meson to the four divergence of a strangeness changing vector current and determine the constant of proportionality, they drew upon current algebra results outside the realm of NLDH's and this thesis. The main point is, this was subsequently called H_ω^{pc}, and the amplitudes obtained are identical to those of Guralnik et al and the CD calculation, apart from terms of the order of $\Delta M / M$. The main reason Kumar et al endeavoured to avoid the assumption that H_ω^{pc} transforms as a member of a scalar octet, will be discussed in relation to a theorem of Coleman et al (111) below.

We compare for example the amplitude $P(\Lambda^0_-)$ with the result of Kumar et al.

$$P(\Lambda^0_-)\ (\text{Kumar}) = \frac{-\lambda_p}{f_\pi f_k}\Big(-(\Lambda+N)(-3F-D)\left(\frac{6}{2}\right)^{-1/2} + (N+N)(F+D)(-3F'-D')\left(\frac{6}{2}\right)^{-1/2} - (\Lambda+\Sigma)(-F'+D')(D)(6)^{-1/2}\Big)$$

$$P(\Lambda^0_-)\ (\text{CD}) = -\frac{\lambda_p}{f_\pi f_k}\Big(\frac{3c}{2(\sqrt{2}-c/2)}(\Lambda+N)(-3F-D)\left(\frac{6}{2}\right)^{-1/2} + \frac{(N+\Lambda)(N+N)}{2N}(F+D)(-3F'-D')(6)^{1/2}/2 - \left(\frac{N}{\Sigma}\right)(\Lambda+\Sigma)(-F'+D')(D)(6)^{-1/2}\Big), \qquad 3.64$$

where we have substituted the SU(3) parametrization of the the weak coupling-constants for the strong, and neglected electromagnetic mass-differences.

These examples suffice to show that very similar results are being obtained by essentially the CD form of H_ω^{pc}, particularly for the P waves which we discussed above, despite the varying interpretations of the authors to avoid the explicit statement of the fact.

The most obvious reason why the CD model implies vanishing results is the fact that for on mass shell decay, the reduced matrix elements of the divergence of a current is vanishing due to momentum - energy conservation (17, 140).

This is also supported for the P waves by a theorem first proposed by Coleman and Glashow (111), which states that for H_ω^{pc} transforming as the 6th component, u_6, of a scalar octet, to which also the mass-splitting interaction, u_8, belongs, then by performing a unitary transformation in SU(3) symmetric space on the total Hamiltonian

$$H = H_{inv} + u_0 + c\,u_8 + d\,u_6 + e\,v_7 + H_{em}\,.$$

it is possible to 'rotate out' identically the u_6 contribution (96, 111), to give $H = H_{inv} + u_0 + c'u_8' + H_{em}$. Hence for $H_\omega^{pc} \sim u_6$, no predictions for the P waves are possible, and we see the motive behind Kumar et al's single scalar meson hypthesis.

For the GOR symmetry breaking model holding, u_6 anf v_7 are related to the corresponding current divergencies, and the model of Fenster et al gives vanishing results for the S and P waves, if calculated to all orders of SU(3) symmetry breaking. Guralnik et al avoided denoting directly H_ω^{pc} by a current divergence for the first reason above, but nevertheless ended up with the CD amplitudes, as they must.

The applicability of the Coleman-Glashow (CG) theorem to models other than those which can be related to the divergence of a current, can be demonstrated by using the analysis of Suzuki(128) with a c-c model of $H_\omega^{c-c} \sim \lambda_6$. In the soft pion limit the S waves are proportional to $\langle B_2 | H_\omega^{pc} | B_1 \rangle \propto \langle B_2 | [F_i^5, H_\omega^{pv}] | B_1 \rangle$, where B_1 and B_2 are members of an octet. Fitting the S wave decay parameters to experiment (128), one finds that the D / F ratio for the matrix element of H_ω^{pc} is very close to that of $\langle B_2 | u_8 | B_1 \rangle$. Following Adler and Dashen (17), this result can be written as the operator statement relation $\langle B_2 | H_\omega^{pc} | B_1 \rangle \propto \langle B_2 | u_6 | B_1 \rangle$, or $H_\omega^{pc} \sim u_6$, 3·65
and hence for this model, no predictions for the P waves can be made if equation 3·65 obtains. In fact, for octet dominance of the c-c interaction, we expect this to be the case.

Conclusion.

What has been achieved by the CD calculation is a synthesis of a large number of calculations in NLDH's up to the present time, with emphasis on the underlying problems facing these models, and reference to the assumptions employed by the authors to overstep these difficulties.

The most salient feature is that NLDH's, particularly for the P waves, are not at all well understood. This is made apparent by the diversity of the assumptions employed and the recurrence of the simple pole model diagram amplitudes, which emerge in each case.

The natural consequence of assuming PCAC and PCVC together with current-algebra, is the simple pole model picture, for PCAC and PCVC implying single particle dominance of the matrix elemets in which they occur, and thus we are not suprised at the recurrence of the pole diagram amplitudes. On the other hand the simple divergence model, which leads naturally to the pole diagrams, gives vanishing amplitudes if calculated accurately to each order of symmetry breaking. As we have seen that the very basic calculation attempted with the CD model does not vanish unless certain severe symmetry limits are imposed, we can only conclude that this occurs because of (1) the approximations that had to be made in order to apply the model, and (2) the incomplete hierarchies of orders in the symmetry breaking, due to (1). This in our opinion is why a large number of authors obtain non-vanishing results, not forgetting the neglect of terms in some calculations, which would contribute for less restrictive assumptions. We note that by imposing severe symmetry limits on the amplitudes, one is in effect sending all higher order terms to zero and reducing the terms to a subset of terms of zeroth order in the symmetry breaking, which in the particular case of the CD model, must cancel identically among themselves.

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Appendix A.

Notation and conventions used in chapters 2 and 3.

$$(\gamma_\mu \partial_\mu + M)\,\psi(x) = 0$$

$$(i\gamma_\mu p_\mu + M)\,u_r(p) = 0 \qquad (r \text{ spin index}).$$

$$\bar{u}(p)(i\gamma_\mu p_\mu + M) = 0$$

$$\sum_r u_r(p)\,\bar{u}_r(p) = (-i\gamma\cdot p + M)/2M$$

$$u_r^+(p) = u_r(p) = p_0/M.$$

$$AB = A_\mu B_\mu = \underset{\sim}{A}\cdot\underset{\sim}{B} - A_0 B_0 \qquad (A_4 = -iA_0)$$

$$p^2 = -M^2$$

$$\theta(x_0) = \begin{cases} 0 & x_0 < 0 \\ 1 & x_0 \geqslant 0 \end{cases} \qquad \gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu \quad \gamma_5^2 = 1;$$

$$T(A(y)B(x)) = \theta(y_0 - x_0)\,A(y)B(x) + \theta(x_0 - y_0)\,B(x)A(y)$$

$$e^{ip_0x_0}\,\theta(\pm x_0) = \frac{\pm}{2\pi i}\int_{-\infty}^{\infty}\frac{dp_0'\,e^{ip_0'x_0}}{(p_0' - p_0 \mp i\varepsilon)}.$$

The matrix elements occuring in the calculation of section 3.2 are defined by

$$\langle B_2(p_2)\,|\,A_\mu^{|\Delta Y| = 1}(0)\,|\,B_1(p_1)\rangle$$

$$= i\sqrt{\frac{B_1B_2}{p_1^0\,p_2^0}}\;\bar{u}(p_2)\Big(g_A^{B_1B_2}(q^2)\gamma_\lambda\gamma_5 + i f_A^{B_1B_2}(q^2)\gamma_5 q_\mu - h_A^{B_1B_2}(q^2)$$

$$\times\,\gamma_5\sigma_{\mu\lambda}q_\lambda\Big)\,u(p_1).$$

$$\langle B_2(p_2)\,|\,V_\mu^{|\Delta Y| = 1}(0)\,|\,B_1(p_1)\rangle$$

$$= i\sqrt{\frac{B_1B_2}{p_1^0\,p_2^0}}\;\bar{u}(p_2)\Big(g_V^{B_1B_2}(q^2)\gamma_\mu - f_V^{B_1B_2}(q^2)\sigma_{\mu\lambda}q_\lambda - i h_V^{B_1B_2}(q^2)$$

$$\times\,q_\mu\Big)\,u(p_1).$$

The divergence of these matrix elements in the $q^2 = 0$ limit is defined by

$$\langle B_2(p_2)\,|\,\partial_\mu A_\mu^{|\Delta Y| = 1}(0)\,|\,B_1(p_1)\rangle = -i\sqrt{\frac{B_1B_2}{p_1^0\,p_2^0}}\;\bar{u}(p_2)$$

$$\times\Big(g_A^{B_1B_2}(0)\,(M_{B_1} + M_{B_2})\Big)\gamma_5\,u(p_1).$$

$$\langle B_2(p_2)\,|\,\partial_\mu V_\mu^{|\Delta Y| = 1}(0)\,|\,B_1(p_1)\rangle = i\sqrt{\frac{B_1B_2}{p_1^0\,p_2^0}}\;\bar{u}(p_2)$$

$$\times\Big(g_V^{B_1B_2}(0)\,(M_{B_1} - M_{B_2})\Big)\,u(p_1).$$

Using the SU(3) symmetrical values for the weak coupling constants, 'g' to evaluate the current-divergence matrix elements above, we define the reduced matrix elements F,D,F$'$ and D$'$ by

$$\langle B_k(p_2) \,|\, \partial_\mu A_\mu^{i\,|\Delta Y|=1}(0) \,|\, B_j(p_1)\rangle$$

$$= i\sqrt{\frac{B_k B_j}{p_0^k\, p_0^j}}\;\bar{u}(p_2)\left(i f_{ijk} F + d_{ijk}\right)\left(m_B{}^k + m_B{}^j\right)\gamma_5\, u(p_1)$$

$$\langle B_k(p_2) \,|\, \partial_\mu V_\mu^{i\,|\Delta Y|=1}(0) \,|\, B_j(p_1)\rangle$$

$$= -i\sqrt{\frac{B_k B_j}{p_0^k\, p_0^j}}\;\bar{u}(p_2)\left(i f_{ijk} F' + d_{ijk} D'\right)\left(m_B{}^j - m_B{}^k\right) u(p_1).$$

For the contractions performed on the matrix elements in sections 2.3 and 3.2 we take

$$\langle B_2(p_2)\, \pi(q)\, \mathrm{out} \,|\, J(y) \,|\, B_1(p_1)\rangle$$

$$= i\int \frac{d^4x\, e^{-iqx}}{(2\pi)^{3/2}\sqrt{2q^0}}\left(q^2 + m_\pi^2\right)\langle B_2(p_2) \,|\, T\left(\varphi_\pi(x), J(y)\right) \,|\, B_1(p_1)\rangle$$

$$\langle B_2(p_2) \,|\, J(y) \,|\, B_1(p_1)\, \pi(q)\, \mathrm{in}\rangle$$

$$= i\int \frac{d^4x\, e^{iqx}}{(2\pi)^{3/2}\sqrt{2q^0}}\left(q^2 + m_\pi^2\right)\langle B_2(p_2) \,|\, T\left(J(y), \varphi_\pi(x)\right) \,|\, B_1(p_1)\rangle$$

Appendix B.

Calculation of equation 2.57, section 2.3.

The T (time-ordered-product)for three fields is defined by

$$T(A(x)B(y)C(0)) = \theta(x_0-y_0)\theta(y_0)ABC$$
$$+\theta(x_0)\theta(-y_0)ACB + \theta(y_0-x_0)\theta(x_0)BAC$$
$$+\theta(y_0)\theta(-x_0)BCA + \theta(-x_0)\theta(x_0-y_0)CAB$$
$$+\theta(-y_0)\theta(y_0-x_0)CBA \qquad \text{B 1}$$

Employing the translation operation for the fields φ , $e^{-iqy}\varphi(0)e^{iqy}=\varphi(y)$, the integral on the RHS of equation 2.56 may be written

$$iq_u\frac{(i)^2\sqrt{2}(p^2+m_K^2)(k^2+m_\pi^2)}{f_K m_K^2 f_\pi m_\pi^2}\iint d^4x\,d^4y\,e^{-ik(x+y)}e^{-iqy}\langle 0|T(D_A^3(x+y))D_A^{4+is}(0)V_u^{4-is}(y))|0\rangle \qquad \text{B.2}$$

Partially integrating w.r.t. $\partial/\partial y_u$, and noting that $\partial/\partial y_u\theta(x_0-y_0)=-\delta(x_0-y_0)$, we obtain

$$(i)^2\sqrt{2}\frac{(p^2+m_K^2)(m_\pi^2+k^2)}{f_K m_K^2 f_\pi m_\pi^2}\Bigg\{\iint d^4x\,d^4y\ldots\langle\partial_u V_u^{4-is}\rangle\ldots$$
$$+\langle 0|-\delta(x_0-y_0)\theta(y_0)D_A^3D_A^{4+is}V_u^{4-is}+\theta(x_0-y_0)\delta(y_0)D_A^3D_A^{4+is}V_u^{4-is}$$
$$-\theta(x_0)\delta(y_0)D_A^3V_u^{4-is}D_A^{4+is}+\delta(y_0-x_0)\theta(x_0)D_A^{4+is}D_A^3V_u^{4-is}$$
$$+\delta(y_0)\theta(-x_0)D_A^{4+is}V_\mu^{4-is}D_A^3-\theta(-x_0)\delta(x_0-y_0)V_\mu^{4-is}D_A^3D_A^{4+is}$$
$$-\delta(y_0)\theta(y_0-x_0)V_u^{4-is}D_A^{4+is}D_A^3+\theta(-y_0)\delta(y_0-x_0)V_u^{4-is}D_A^{4+is}D_A^3|0\rangle \qquad \text{B.3}$$

We readily identify the first term above with the first term of equation 2.57 (eq 2.
Rearranging the remaining terms to form equal-time-commutators, and employing the trick that $\theta(x_0-y_0)\delta(y_0)=\delta(y_0)\theta(x_0)$, we may write the above as (apart from the first term)

$$-(i)^2\sqrt{2}\frac{(p^2+m_K^2)(k^2+m_\pi^2)}{f_K m_K^2 m_\pi^2 f_\pi}\iint\cdots\Big\{\langle 0|\delta(y_0)T(D_A^3(x+y)[V_0^{4-is}(y),D_A^{4+is}(0)]|0\rangle$$
$$+\langle 0|\delta(y_0-x_0)T([V_0^{4-is}(y),D_A^3(x+y)]D_A^{4+is}(0))|0\rangle\Big\} \qquad \text{B.4}$$

We now evaluate the ETC's using the GOR model(43), with the use of equations 1.38 and 1.42. Noting that through the use of the contraction proceedure and PCAC , $(2\pi)^{3/2}\sqrt{2p^0}\langle 0|D_A^{4+is}(0)|K^+(p)\rangle \overset{def^n}{=} f_K m_K^2$ B.5
$= \frac{i(p^2+m_K^2)}{f_K m_K^2}\int d^4y\,e^{ipy}\langle 0|T(D_A^{4-is}(0)D_A^{4+is}(y)|0\rangle$, and applying this result to the terms obtained directly above in B.4 , we immediately obtain equation 2.57.