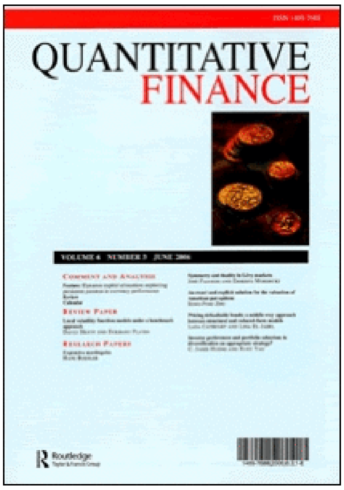

Quantitative Finance
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713665537

Does beta react to market conditions? Estimates of 'bull' and 'bear' betas using a nonlinear market model with an endogenous threshold parameter

George Woodward [a]; Heather M. Anderson [b]
[a] University of Colorado, Colorado Springs, CO 80918, USA [b] Australian National University, Canberra, ACT, 0200 Australia

First published on: 25 March 2009

To cite this Article Woodward, George and Anderson, Heather M.(2009) 'Does beta react to market conditions? Estimates of 'bull' and 'bear' betas using a nonlinear market model with an endogenous threshold parameter', Quantitative Finance, 9: 8, 913 — 924, First published on: 25 March 2009 (iFirst)
To link to this Article: DOI: 10.1080/14697680802595643
URL: http://dx.doi.org/10.1080/14697680802595643

Quantitative Finance, Vol. 9, No. 8, December 2009, 913–924

Does beta react to market conditions? Estimates of 'bull' and 'bear' betas using a nonlinear market model with an endogenous threshold parameter

GEORGE WOODWARD† and HEATHER M. ANDERSON*‡

†University of Colorado, Colorado Springs, CO 80918, USA
‡Australian National University, Canberra, ACT, 0200 Australia

(*Received 21 January 2007; in final form 17 October 2008*)

The authors use a logistic smooth transition market (LSTM) model to investigate whether 'bull' and 'bear' market betas for Australian industry portfolios returns differ. The LSTM model allows the data to determine a threshold parameter that differentiates between 'bull' and 'bear' states, and it also allows for smooth transition between these two states. Their results indicate that 'bull' and 'bear' betas are significantly different for most industries, and that up-market risk is not always lower than down-market risk. LSTM models indicate that the transition between 'bull' and 'bear' states is abrupt, supporting a dual-beta market modelling framework.

Keywords: Bull and bear betas; Dual-beta market (DBM); Models; Linearity tests; Logistic smooth transition market (LSTM) models; Sequential conditional least squares (SCLS)

1. Introduction

The simple linear market model has long been used in tests of the Capital Asset Pricing Model (CAPM), for the measurement of abnormal returns in event studies, and as a benchmark for the performance of mutual funds. See Sharpe (1966), Fama *et al*. (1969) and Fama and French (1992) for some examples. In such studies, the stability of the beta coefficient in the market model over 'bull' and 'bear' market conditions is assumed, but if beta does in fact vary with market conditions then inferences based on single beta models are likely to be misleading.† Theory that allows for beta to vary over market conditions is well advanced, with work by Bawa and Lindenberg (1977) and Harlow and Rao (1989) providing some important early developments. Direct evidence of the importance of the beta/market condition relationship issue is given by the fact that investment houses regularly publish separate alphas and betas over 'bull' and 'bear' markets, for a range of securities, to offer differing levels of upside potential and downside risk.

Many studies have investigated the relationship between beta risk and stock market conditions. These include studies of individual securities (Fabozzi and Francis 1977, Kim and Zumwalt 1979, and Clinebell *et al*. 1993), mutual funds (Fabozzi and Francis 1979, and Kao *et al*. 1998), size based portfolios (Wiggins 1992, Bhardwaj and Brooks 1993, and Howton and Peterson 1998), risk based portfolios (Spiceland and Trapnell 1983, and Wiggins 1992) and past performance based portfolios (DeBondt and Thaler 1987, and Wiggins 1992). While most of these studies have found evidence that beta varies with market conditions, this evidence is mixed and quite weak. Furthermore, most of these studies have used the dual-beta market (DBM) model and simple *t*- and *F*-testing methods in conjunction with crude 'up'- and 'down'-market definitions of 'bull' and 'bear' markets to investigate this phenomenon,

*Corresponding author. Email: heather.anderson@anu.edu.au.

†In particular, Kim and Zumwalt (1979), Pettengill *et al*. (1995) and Jagannathan and Wang (1996) each show that, when beta is allowed to vary with market conditions, the importance of beta for explaining the cross-section of realized stock returns increases.

Quantitative Finance
ISSN 1469–7688 print/ISSN 1469–7696 online © 2009 Taylor & Francis
http://www.informaworld.com
DOI: 10.1080/14697680802595643

and they have not considered more general forms of nonlinear behaviour.

There has been substantial divergence in the definition of 'bull' and 'bear' markets in the literature, but only a few studies of beta non-stationarity over 'bull' and 'bear' markets have used a continuously changing time-varying parameter model. An example of a continuously changing beta model is provided by Chen (1982). However, even with considerable refinements in the definition of 'bull' and 'bear' markets, especially in work that uses trend-based indicators of market states, almost all of the existing definitions model the transition from down-market to up-market states (or vice versa) as a discrete jump. Even the latest Markov-switching model by Maheu and McCurdy (2000) assumes an abrupt switch between regimes.

Complementing the existing evidence of two-regime market models is evidence of other sorts of nonlinearities in stock prices. These nonlinearities have been related to various behavioural dynamics of investors, with prominent work along these lines including (i) papers by Peterson (1994) and Guillaume *et al.* (1995) on investor heterogeneity arising from different risk profiles and different investment horizons; (ii) work on herd behaviour by Lux (1995); and (iii) papers on heterogeneous beliefs regarding market conditions by Brock and Hommes (1998) and Brock and LeBaron (1998). While homogeneous beliefs among investors will imply that they share the same information and that they collectively switch from assuming one market condition (say a 'bull' market) to another (say a 'bear' market), this is quite hard to accept unless we believe in a strong form of efficient market theory. Heterogeneous beliefs among investors are likely to blur the distinction between 'bull' and 'bear' markets, because different investors will interpret their information in different ways.

In this paper we investigate these phenomena with three main aims in mind. First, like many other studies, we wish to determine whether 'bull' and 'bear' market betas differ. Second, in contrast to other studies, we want to investigate the possibility that the transition between market regimes might be gradual, and thereby address the heterogeneous beliefs theory. Third, we want to let the data determine a benchmark criterion for differentiating between market states, allowing for the possibility raised in Harlow and Rao (1989) that such a benchmark might depend on 'investor targets', and might not be the same for all stocks. With these aims in mind, we apply a logistic smooth transition market model (LSTM) to a sample of returns on Australian industry portfolios over the period 1979–2002.† While the dual-beta market (DBM) model used in other studies implies a discrete jump between market states, the LSTM model replaces the indicator function with a logistic smooth function that allows for smooth and continuous transition between these two states. LSTM models are an adaptation of LSTAR (logistic smooth transition autoregressive) models popularized by Teräsvirta (1994), and in this context smooth transition between states seems particularly appropriate for modelling stock markets with many participants, each switching at different times due to heterogeneous beliefs, different investment targets and differing investment horizons. The LSTM formulation allows the transition between market states to depend on a 'transition variable' that can be chosen by the researcher, and it parametrizes the threshold between states so that this threshold can be estimated. The LSTM model also allows for both the DBM and constant risk models as special cases.

The choice of an appropriate transition or indicator variable is an important consideration when modelling the transition between different market states, because ideally this variable will reflect broad swings in the market, rather than temporary market aberrations. Previous work on 'bull' and 'bear' betas has often failed to recognize this consideration, and has simply used the return on the market index as the market indicator. As shown below, the return on the market index is highly erratic, and does not reflect typical notions of 'bull' and 'bear' states. To overcome this problem, our study uses a rolling twelve-month moving average of market returns to model movement between 'bull' and 'bear' markets. The rolling average series is much smoother than the return on the market portfolio series itself, and it abstracts from noisy and unsystematic movements in the stock market to better capture long-run dependencies and trends in the data.

Our estimates of LSTM models exhibit persuasive evidence that Australian industry portfolio betas vary between 'bull' and 'bear' phases. This result seems to be partly attributable to our use of a trend-based market indicator, but it is also due to the fact that we allow the data to define 'bull' and 'bear' states. We find that although industries tend to spend more time in 'bull' than 'bear' states, the risk associated with 'bull' states is not always smaller than the risk in 'bear' market states. Our LSTM estimates indicate that the transition between 'bull' and 'bear' states is abrupt for most industries, which can be viewed as support for homogeneous investors.

The plan of the paper is as follows. In Section 2 we review the literature on definitions of 'bull' and 'bear' markets and describe a market indicator that will be used in this study to define 'bull' and 'bear' states. In Section 3 we develop our model and describe the methodologies employed in the study. Section 4 discusses the data and the results of our analysis, and Section 5 finishes with some concluding remarks.

2. Phases of the market

The studies reviewed in Section 1 either compare the market index to a critical threshold value to separate 'up'-from 'down'-market periods, or they use a trend-based

†We choose to analyse industry portfolios because the existence of industry-specific risk is recognized, and because one can be more confident of the response of a portfolio beta to changes in market conditions than in the case of a single security beta.

scheme to classify markets as 'bull' or 'bear'. The 'up'- and 'down'-market scheme dichotomizes the market by comparing the market index to a critical threshold value. Wiggins (1992), for example, defines up (down) months as months when the (excess) market return is greater (less) than zero, while Bhardwaj and Brooks (1993) use the median return on the market portfolio as the demarcating value. Fabozzi and Francis (1977, 1979) define substantial up (down) months as months in which the return on the market portfolio is greater (less) than 1.5 times its standard deviation, and thereby separate the market into periods when the market is substantially up, substantially down, or neither. Another way of classifying the market is offered by Granger and Silvapulle (2001), who separate the market into 'bullish', 'bearish' and 'usual' states using quantiles of return distributions. Taleb (2004, pp. 95–97) notes that 'bullish' and 'bearish' characterizations of the market typically incorporate expectations of returns and actions such as whether investors choose to go long or short in a stock, but he also points out that the terms 'bullish' and 'bearish' are commonly used without taking into account all of the relevant features of return distributions.

Several economists (e.g. Neftci 1984 and Skalin and Teräsvirta 2002) have suggested that monthly observations on changes in economic time series are noisy and therefore do not reveal cyclical characteristics. Recognizing this, some authors have used a trend-based approach in their analysis of stock market conditions. Fabozzi and Francis (1977, 1979), for example, use 'bull' and 'bear' market dates published in Cohen *et al.* (1987) to classify their data into 'bull' and 'bear' categories, and, in a similar vein, Gooding and O'Malley (1977) define non-overlapping 'bull' and 'bear' phases based on major peaks and troughs found in the S&P425 Industrial Index. Dukes *et al.* (1987) used the S&P500 Index to define 'bull' ('bear') markets as periods in which the index increased (decreased) by at least 20% from a trough (peak) to a peak (trough). More noteworthy are the recent studies by Pagan and Sossounov (2003) and Lunde and Timmermann (2004), who each developed sophisticated trend-based definitions of 'bull' and 'bear' markets that focus on systematic movements in the market while ignoring the short-term noise effects. Both papers define 'bull' and 'bear' markets in terms of movements between peaks and troughs, and use pattern recognition dating algorithms to classify 'bull' and 'bear' markets. Both papers find that 'bull' markets tend to last longer than 'bear' markets.

We also use a trend-based definition of 'bull' and 'bear' markets in our analysis, and capture the underlying cyclical movement in the stock market by using the 12-month moving average of the (logarithmic) returns associated with the All Ordinaries Accumulation index as a stock market indicator. Like Pagan and Sossounov (2003) and Lunde and Timmermann (2004), we intend to capture sustained periods of growth or contraction that are normally associated with the concepts of 'bull' and 'bear' markets. Figures 1 and 2 illustrate the rationale behind our approach. The plot of returns for the market

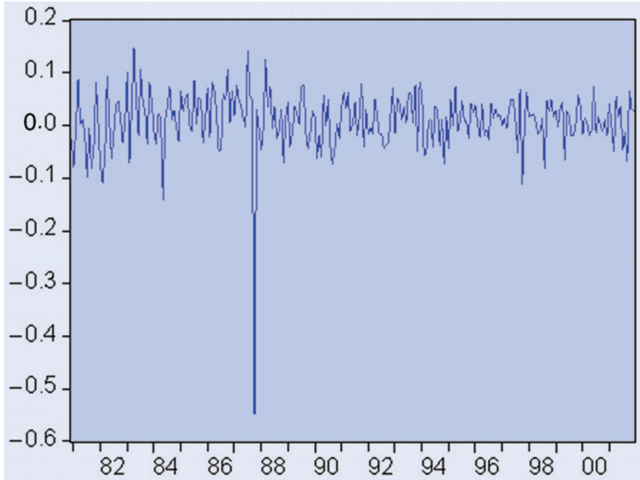

Figure 1. Return on the market index R_{mt} (XAO).

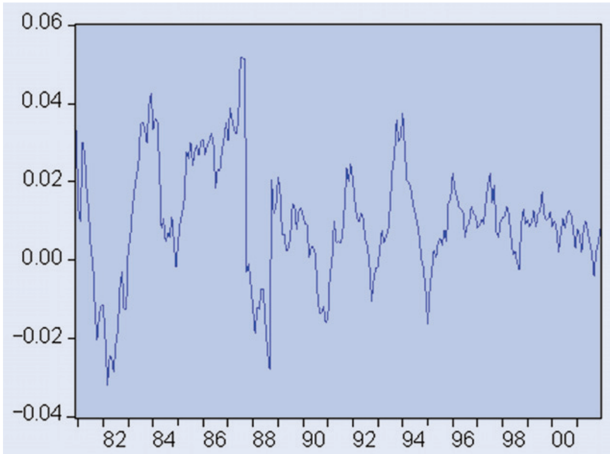

Figure 2. The transition variable (R^*).

index (R_{mt}) in figure 1 reveals that, by using this noisy series as an indicator of 'bull' and 'bear' states, most researchers have implicitly assumed that the market jumps in and out of market phases very rapidly, and very frequently. On the other hand, our use of R_t^*, the smoother 12-month moving average of this variable illustrated in figure 2 ensures that our market indicator follows a much smoother path.

2.1. *The Logistic Smooth Transition Market Model (LSTM)*

An unconditional beta for any asset or portfolio can be estimated using the constant risk market model (CRM) regression given by

$$R_{it} = \alpha_i + \beta_i R_{\mathrm{mt}} + \varepsilon_{it}, \tag{1}$$

where R_{it} is the return on portfolio i for period t, R_{mt} is the return on the market index for period t, $\beta_i = cov(R_{it}, R_{\mathrm{mt}})/\sigma^2_{\mathrm{mt}}$ and ε_{it} is the disturbance term which has zero mean and is assumed to be serially independent and homoskedastic. Under this specification α_i and β_i are constant with respect to time.

A dual-beta market model (DBM) can be specified as

$$R_{it} = \alpha_i + \alpha_i^{\mathrm{U}} \times D_t + \beta_i R_{\mathrm{mt}} + \beta_{ii}^{\mathrm{U}} \times D_t \times R_{\mathrm{mt}} + \varepsilon_{it}, \quad (2)$$

where D_t is a dummy variable defining up and down markets (i.e. 'bull' and 'bear' markets) by taking the value one if a market state indicator denoted by M_t exceeds some critical value c, and zero otherwise. The market return R_{mt} is often used as the market indicator M_t, and the parameter c is often set equal to zero, $\overline{R}_{\mathrm{mt}}$ or the median of R_{mt}. It is also common to omit the $\alpha_i^{\mathrm{U}} \times D_t$ term in (2), thereby assuming that the intercept does not vary with the market state.

In this paper we set $M_t = R_t^*$, for reasons discussed above. Since there is no theory with which to specify c and no reason to believe that c will be the same for all industries, we estimate a separate value (c_i) for each industry. Thus, for our DBM models, the market for stocks in industry i will be classified as being in a 'bear' state when $R_t^* < c_i$ and in a 'bull' state when $R_t^* > c_i$. Notice that in this specification the value of β_i^{U} measures the difference between the up- and down-market values of the slope coefficient, so that the up-market value of beta is given by $\beta_i + \beta_i^{\mathrm{U}}$.

Now consider the logistic smooth transition regression model, henceforth called the LSTM model, which is given by

$$R_{it} = \alpha_i + \beta_i R_{\mathrm{mt}} + (\alpha_i^{\mathrm{U}} + \beta_i^{\mathrm{U}} \times R_{\mathrm{mt}}) \times F(M_t) + \varepsilon_{it} \quad (3)$$

with

$$F(M_t) = (1 + \exp[-\gamma_i(M_t - c_i)])^{-1}, \quad \gamma_i > 0, \quad (4)$$

and which has (1) and (2) as special limiting cases. The superscript U signifies up-market differential values of the parameters α and β, F is the logistic smooth transition function with transition variable M_t and threshold value c_i, and $\varepsilon_{it} \sim i.i.d.\ \mathrm{N}(0, \sigma_i^2)$. In our case $M_t = R_t^*$, the 12-month moving average of the return on the market index, because this variable seems to capture the cyclical upturns and downturns that are typically associated with concepts of 'bull' and 'bear' markets. As for our DBM models, we allow for (and estimate) a separate value of c for each industry.

Equation (3) with (4) and $M_t = R_t^*$ show that beta changes monotonically with the transition variable R_t^*, because $F(R_t^*)$ in (4) is a smooth and continuously increasing function of R_t^*. The transition function $F(R_t^*)$ takes a value between 0 and 1, depending on the magnitude of $(R_t^* - c_i)$. When $(R_t^* - c_i)$ is large and negative, $F(R_t^*) \approx 0$ and R_{it} is effectively generated by the linear model $R_{it} = \alpha_i + \beta_i R_{\mathrm{mt}} + \varepsilon_{it}$. In such cases the market for stocks in industry i is very 'bearish'. On the other hand, when $(R_t^* - c_i)$ is large and positive, R_{it} is effectively generated by $R_{it} = (\alpha_i + \alpha_i^{\mathrm{U}}) + (\beta_i + \beta_i^{\mathrm{U}})R_{\mathrm{mt}} + \varepsilon_{it}$, and the market for stocks in industry i is very 'bullish'. Intermediate values of $(R_t^* - c_i)$ give a convex combination of the two extreme regimes, and as $F(R_t^* - c_i)$ increases from 0 to 1, the market for industry i moves through a series of market states that range from very 'bearish' to very 'bullish'. The γ_i parameters determine the smoothness of transition between 'bearish' and 'bullish' market states. Note that the DBM is a special case of the LSTM, because $F(R_t^*)$ becomes an indicator function when γ_i approaches infinity in (4), with $F(R_t^*) = 0$ for all values of R_t^* smaller than c_i and $F(R_t^*) = 0$ otherwise. Also notice that the constant risk market (CRM) model is a special case of the LSTM, because (3) approaches (1) as the smoothness parameter γ_i in (4) approaches zero.

Our LSTM model is a simple adaptation of the logistic smooth transition autoregressive (LSTAR) models promoted by Teräsvirta (1994), and tests for smooth transition in our framework are very closely related to tests for LSTAR behaviour. To our knowledge, LSTM models have not been used to study 'bull' and 'bear' markets before, although Coutts *et al.* (1997) used an adaptation of an LSTAR model with a polynomial trend as a transition variable, in an attempt to capture the timing of changes in beta in response to major events.

2.2. Tests of linearity against LSTM

As mentioned in Section 3.1, (3) becomes the CRM model when γ_i approaches zero, implying that the constant risk market model is nested within the LSTM model. Thus a natural first step in specifying the model is to test for linearity against the LSTM form. If the null of linearity cannot be rejected then we can conclude that the constant risk market model adequately represents the data generating process. On the other hand, if linearity is rejected we can go on to estimate the nonlinear LSTM model using nonlinear least squares (NLS).

Tests of $H_0 : \gamma_i = 0$ are non-standard, since the parameters of (3) are only identified under the alternative $H_A : \gamma_i \neq 0$. Following Luukkonen *et al.* (1988) we replace $F(M_t)$ in (3) by either a third- or first-order Taylor series linear approximation to $F(R_t^*)$, and expand this to form an auxiliary model in which the alternative $H_A : \gamma_i \neq 0$ is equivalent to $H_A : \beta_i^{\mathrm{U}} \neq 0$ in equation (3). When a third-order Taylor series approximation is used, the expanded and reparametrized equation is

$$\begin{aligned} R_{it} = {} & \phi_1 + \phi_2 R_{\mathrm{mt}} + \phi_3 R_t^* + \phi_4 (R_t^*)^2 + \phi_5 (R_t^*)^3 \\ & + \phi_6 R_{\mathrm{mt}} R_t^* + \phi_7 R_{\mathrm{mt}} (R_t^*)^2 + \phi_8 R_{\mathrm{mt}} (R_t^*)^3 + u_{it}, \end{aligned} \quad (5)$$

with the last six variables in this equation acting as proxies for the nonlinearity. Within this reparametrized setting, the null hypothesis of linearity is $H_0 : \phi_j = 0$ $(j = 3, \ldots, 8)$. A standard Wald test can be used to test this hypothesis (for each i), and the test statistic (called S_3 in what follows) has an $F_{6, T-8}$ distribution under the null. When a first-order Taylor series is used the expanded and reparametrized equation is

$$R_{it} = \phi_1 + \phi_2 R_{\mathrm{mt}} + \phi_3 R_t^* + \phi_4 R_{\mathrm{mt}} R_t^* + u_{it}. \quad (6)$$

The null hypothesis is then $H_0 : \phi_j = 0$ $(j = 3, 4)$, and the Wald test statistic (denoted by S_1) has an $F_{2, T-4}$ under the null. When the intercept is not time varying, an augmented first-order test statistic, S_1^* can be used. This test includes a fifth regressor, $\phi_5 R_{\mathrm{mt}} (R_t^*)^3$ in (6), and tests $H_0 : \phi_j = 0$ $(j = 3, 4, 5)$. Luukkonen *et al.* (1988) derive the three tests S_3, S_1^* and S_1 as Lagrange Multiplier statistics that have asymptotic χ^2 distributions, but they point out that the F versions (presented here) perform better in small samples. All of these tests can be adjusted to account for potential heteroskedasticity, and we use White's (1980) adjustments in all that follows.

Simulations undertaken by Luukkonen *et al.* (1988) and Petruccelli (1990) show that the S_3, S_1^* and S_1 tests are quite powerful in small samples when the true alternative is either a smooth transition or an abrupt regime switching regression. They also show that these tests outperform Tsay's (1989) test for threshold behaviour, even when the data are generated by a threshold process. Thus we expect that the S_3, S_1^* and S_1 tests will have power against our DBM and LSTM models.† Generally, the tests with more regressors (i.e. S_3 tests) are more powerful because they search for non-linearity in more directions, but S_1 tests can become more powerful, when fitting in directions where the nonlinearity is weak interferes with efficiency. The S_1^* test is more efficient than the S_3 test when the intercept does not vary, because the latter includes $(R_t^*)^2$ and $(R_t^*)^3$ terms, which only appear in the Taylor's series expansion (4) when $\alpha_i^{\mathrm{U}} \neq 0$. We perform the S_3 and S_1^* tests in the analysis that follows, omitting the S_1 tests because they are similar to the S_1^* tests, but not as powerful. Finding evidence of nonlinearity before embarking on the estimation of DBM and LSTM models is important, not only because such evidence justifies the nonlinear form, but also because a failure to reject $H_0: \gamma_i = 0$ provides a warning that the separate identification of parameters for two regimes might not be possible.

Nonlinear least squares (NLS) will provide consistent estimates of the parameters in LSTM and DBM models when the errors follow a martingale difference process, and our assumptions on ε_{it} satisfy this requirement. Given normality, NLS is equivalent to MLE in the LSTM case, but this does not hold in the special case of the DBM model, because an abrupt threshold violates the continuity and differentiability conditions needed for MLE. Estimation of the LSTM models is difficult, because the error sum of squares (or likelihood) is often quite flat with respect to γ_i and c_i (see Teräsvirta 1994 for a discussion), but all parameter estimates will have normal distributions, provided that $\gamma_i \neq 0$. The standard NLS technique for estimating a threshold model is to use sequential conditional least squares (SCLS). This involves estimating α_i, α_i^{U}, β_i and β_i^{U} conditionally for each value of c_i as OLS estimates given by

$$(\hat{\alpha}_i, \widehat{\alpha}_i^{\mathrm{U}}, \hat{\beta}_i, \hat{\beta}_i^{\mathrm{U}})' = \left[\sum_{t=1}^{n} x_t(c_i)x_t(c_i)'\right]^{-1} \times \left[\sum_{t=1}^{n} x_t(c_i)R_{it}\right], \tag{7}$$

where $x_t(c_i) = (1,\ I[R_t^* > c_i],\ R_{\mathrm{mt}},\ R_{\mathrm{mt}}I[R_t^* > c_i])$, with $I[R_t^* > c_i] = 1$ if $R_t^* > c_i$ and zero otherwise. A grid search over the set of potential values for c_i (i.e. C_i) is conducted to determine the value of $\hat{c}_i$ which minimizes the residual variance $\widehat{\sigma}^2(c_i)$, and the final estimates of the parameters are then given by $\hat{\alpha}_i = \alpha_i(\hat{c}_i)$, $\hat{\alpha}_i = \alpha_i(\hat{c}_i)$, $\hat{\beta}_i(\hat{c}_i)$ and $\hat{\beta}_i^{\mathrm{U}}(\hat{c}_i)$. Chan (1993) has demonstrated that the estimator $\hat{c}_i = \min_{c_i \epsilon C_i}\left[\widehat{\sigma}^2(\hat{c}_i)\right]$ is (super) consistent at the rate T, but that the actual distribution of $\hat{c}_i$ depends on the parameters $\alpha_i, \alpha_i^{\mathrm{U}}, \beta_i$ and β_i^{U}.‡ The parameter estimates $\hat{\alpha}_i$, $\widehat{\alpha}_i^{\mathrm{U}}$, $\hat{\beta}_i$ and $\hat{\beta}_i^{\mathrm{U}}$ have normal distributions.

3. Results

The data used in this study is adjusted price data (prices adjusted for dividends) on the 24 industry groupings within the Australian Stock Exchange, provided by the Securities Industry Research Centre of the Asia/Pacific (SIRCA). Prices are measured in $AUD. Observations are monthly, and for 19 of the industries they date from December 1979 to December 2001, giving rise to a sample of 265. For three of the industries (solid fuels, oil and gas, and entrepreneurial investors) the observations end in October 1996, giving 203 returns. The 'miscellaneous services' industry series ends in August 1997 giving 213 returns, and the 'tourism and leisure' industry series begins in December 1990, giving 144 returns. Return series for each industry and the market index are calculated as the difference of the logarithms of prices.

Figure 1 provides a graph of returns for the market index, and table 1 provides descriptive statistics for the market returns and the returns for each of the 24 industries' indices. The most noticeable feature of the illustrated market index is the sharp dip in October 1987, which was due to the crash in the United States stock market at that time. Separate graphs of individual industry returns show the same feature, although it is more pronounced in some industries (for instance 'banks' and 'insurance' and 'investment & financial services') than it is in others (such as 'gold index'). Turning to the summary statistics, the 'media' industry offered the highest return over this period and 'miscellaneous industrials' the lowest. The standard deviation was highest for 'diversified industrials' and lowest for the 'property trust' industry. In keeping with other studies of financial time series, all return series are leptokurtotic and exhibit negative skewness. Jarque–Bera tests indicate that none of the return series is normally distributed. The estimate of the constant risk market model beta is highest for the 'gold' industry and lowest for the 'property trust' industry, and all betas are statistically significant at the 1% significance level. Fifteen of the betas are statistically different from unity at the 5% level of

†Our own simulation experiments, based on 'realistic' parameter values that had been calibrated to reflect our data set, support the published results.

‡Note that since we have only a finite number of observations on R_t^*, we will only be able to estimate *interval* in which c_i lies with respect to the set of R_t^* (ordered by size). Superconsistency will, however, ensure that the probability of choosing the correct interval approaches 1 very quickly as the sample becomes larger, so that we can expect an accurate choice, given our sample of 253.

Table 1. Summary statistics for monthly returns.

Code	Industry	N_i	μ_i	σ_i	S_i	κ_i	$\widehat{\beta}_i$
XAO	All ordinaries index (R_{mt})	253	0.0095	0.0576	−3.58	35.97	n/a
XGO	Gold	253	0.0013	0.1203	−0.41	7.61	1.337*
XOM	Other metals	253	0.0031	0.0918	−2.00	18.87	1.301*
XDR	Diversified resources	253	0.0092	0.0754	−1.09	10.61	1.091
XDC	Developers & contractors	253	0.0135	0.0700	−3.60	36.74	1.039
XBM	Building materials	253	0.0108	0.0585	−1.51	12.50	0.846*
XAT	Alcohol and tobacco	253	0.0169	0.0563	−2.22	20.95	0.708*
XFH	Food & household goods	253	0.0118	0.0595	−1.39	10.24	0.718*
XCE	Chemicals	253	0.0098	0.0649	−0.59	8.37	0.805*
XEG	Engineering	253	0.0066	0.0615	−0.82	6.89	0.827*
XPP	Paper & packaging	253	0.0084	0.0565	−1.04	8.02	0.726*
XRE	Retail	253	0.0131	0.0593	−2.07	21.36	0.779*
XTP	Transport	253	0.0117	0.0733	−2.28	20.96	1.017
XME	Media	253	0.0173	0.0937	−1.12	8.01	1.052
XBF	Banks	253	0.0162	0.0593	−0.92	8.73	0.774*
XIN	Insurance	253	0.0137	0.0692	−1.68	15.47	0.840*
XIF	Investment & financial services	253	0.0096	0.0544	−3.70	38.40	0.773*
XPT	Property trusts	253	0.0111	0.0359	−1.58	15.67	0.423*
XMI	Miscellaneous industrials	253	0.0004	0.1137	−7.43	84.81	1.106
XDI	Diversified industrials	253	0.0130	0.0666	−2.59	23.75	0.976
XMS	Miscellaneous services	201	0.0093	0.0523	−2.13	17.38	0.661*
XSF	Solid fuels	191	0.0028	0.0707	−1.18	9.34	0.824*
XOG	Oil and gas	191	0.0035	0.0857	−1.33	10.46	1.062
XEI	Entrepreneurial investors	191	0.0085	0.0942	−3.96	35.57	1.138
XTU	Tourism and leisure	132	0.0130	0.0483	−0.99	6.67	0.893
R_t^*	12-month moving average of XAO	253	0.0100	0.0150	−0.02	3.29	n/a

The symbols N_i, μ_i, σ_i, s_i, κ_i and $\widehat{\beta}_i$ represent the sample size, mean, standard deviation, skewness, kurtosis and estimated beta for industry *i*. The first eleven observations of each index were trimmed to allow for the construction of R_t^* that was used as the transition variable in the subsequent analysis. The (heteroskedasicity consistent) *p*-value associated with each beta is less than 0.0000, and a star superscript indicates that $\beta_i \neq 1$ at the 5% level of significance.

significance. Summary statistics for our proposed transition variable are provided on the last line of table 1. The most notable feature here is that the standard deviation of R_t^* is much lower than that for any of the individual indices, and in particular, it is only about one fifth of that for the market index. The small variation in R_t^* relative to that in the market index R_t becomes particularly clear, once one compares figures 1 and 2 and notes that these two graphs are on different scales. The cyclical properties of R_t^* have been noted above.

We provide evidence of nonlinearity in table 2, using R_t^* as the transition variable in the tests. Since R_t^* exhibits cyclical properties, evidence of nonlinearity due to movements in R_t^* provides support for a type of time variation in beta that is consistent with 'bear' and 'bull' betas. Given that the 1987 observation associated with the stock market crash can potentially influence the test results, we conduct two sets of tests. The first set consists of (heteroskedasticity adjusted) S_3, S_1^* and S_1 tests as described in Section 2 above, while the second set of tests (also heteroskedasticity corrected) includes a 'stock market crash' dummy variable (SMC_t) in all of the test regressions, so as to remove the effect of this outlying observation. The dummy is statistically significant in 13 out of the 24 test regressions. While the two sets of *p*-values for the tests show some differences, the qualitative conclusions are quite similar. Nevertheless, the results that explicitly account for the outlier are likely to be more reliable, so we place more weight on these. For both sets of tests, the null of linearity is rejected by the S_3 (or S_3^S) tests for 11 industries at the 5% level and for 12 industries at the 10% level. These tests suggest time-varying betas for approximately half of the series. Joint tests based on systems of index returns (see the last two lines of table 2) provide overall support for differences between 'bull' and 'bear' betas,† and justify our subsequent estimation of nonlinear models. Analogous tests that use R_t rather than R_t^* as the transition find stronger evidence of nonlinearity when SMC_t is excluded from the test regressions, but much less evidence of nonlinearity once SMC_t has accounted for the 1987 stock market crash. These test results are not reported here, but are available on request.

Given the evidence of nonlinearity in table 2, we started to model it by assuming an LSTM specification and a gradual transition between the 'bear' and 'bull' regimes. We imposed the simplifying restriction that alpha was constant over the two market phases, but allowed for the possible effects of the stock market crash by including the SMC_t dummy in the lower ('bear') regime. As suggested by Teräsvirta (1994) we standardized the transition variable to simplify the joint estimation of γ and c, but

†Our joint tests are based on a single regression of the (vertically stacked $T \times k$) vector of returns for k industries on $I_k \otimes Z$, where I_k is an identity matrix of dimension k, and Z contains the set of test regressors.

Table 2. *p*-Values for linearity tests.

Code	Industry	S_3	S_1^*	S_3^{SMC}	S_1^{*SMC}
XGO	Gold	0.013	0.351	0.024	0.299
XOM	Other metals	0.085	0.138	0.033	0.133
XDR^{SMC}	Diversified resources	0.003	0.015	0.002	0.006
XDC^{SMC}	Developers & contractors	0.398	0.441	0.863	0.797
XBM^{SMC}	Building materials	0.283	0.487	0.408	0.930
XAT^{SMC}	Alcohol and tobacco	0.046	0.348	0.048	0.237
XFH	Food & household goods	0.368	0.345	0.387	0.516
XCE^{SMC}	Chemicals	0.010	0.001	0.003	0.004
XEG^{SMC}	Engineering	0.214	0.070	0.447	0.482
XPP^{SMC}	Paper & packaging	0.192	0.116	0.150	0.235
XRE^{SMC}	Retail	0.661	0.692	0.895	0.833
XTP^{SMC}	Transport	0.001	0.000	0.000	0.000
XME	Media	0.448	0.119	0.543	0.339
XBF^{SMC}	Banks	0.810	0.933	0.817	0.789
XIN^{SMC}	Insurance	0.000	0.016	0.000	0.002
XIF^{SMC}	Investment & financial services	0.119	0.429	0.345	0.551
XPT^{SMC}	Property trusts	0.580	0.979	0.299	0.900
XMI	Miscellaneous industrials	0.005	0.082	0.113	0.132
XDI	Diversified industrials	0.012	0.854	0.013	0.925
XMS	Miscellaneous services	0.001	0.380	0.003	0.372
XSF^{SMC}	Solid fuels	0.326	0.463	0.173	0.254
XOG	Oil and gas	0.000	0.044	0.000	0.009
XEI	Entrepreneurial investors	0.124	0.140	0.405	0.161
XTU	Tourism and leisure	0.007	0.812	0.007	0.812
$X19^{SMC}$	Industries XGO to XDI	0.000	0.000	0.000	0.000
$X3^{SMC}$	Industries XSF to XEI	0.000	0.062	0.000	0.013

All test statistics have been corrected for heteroskedasticity. The tests labelled S_3^{SMC} and $S_1^{*SMC}$are versions of the S_3 and S_1^*tests that have been corrected for possible effects of the stock market crash in October 1987 by including a stock market crash dummy (SMC_t) in the test regressions. An SMC superscript on the industry code indicates those industries for which this dummy is statistically significant (at the 5% level). The last two lines contain (joint) tests of the null of linearity in the 19 (3) industries for which 253 (191) observations are available.

despite this standardization the error sum of squares (ESS) was very flat with respect to γ, leading to very large and imprecise estimates of γ. We found that $\gamma_i < 10$ for only two industries ('miscellaneous industrials' and 'oil and gas'), which suggested that most industries experience rather abrupt changes between 'bear' and 'bull' states. In the two cases in which $\gamma_i < 10$, we needed to restrict the c_i to be no smaller than -0.015, to ensure enough observations in the lower regime to identify the lower regime parameters. For the other industries, our NLS algorithm found γ_i to take values as high as 12 000, implying that these LSTM models were effectively DBM models. Given that $\{1+\exp[-\gamma(R_t^* - c)]\}^{-1}$ is effectively constant with respect to γ once γ becomes large, it is probably more accurate to view a large $\widehat{\gamma}$ as one that has satisfied a particular set of convergence criteria rather than an estimate of γ itself. This situation is discussed by Teräsvirta, who notes that for large γ, the estimation of the other parameters is unaffected by conditioning on γ. In his work, he simply conditions on $\gamma = 50$ or $\gamma = 100$. Here, to provide readers with an idea of how soon the ESS minimization algorithm starts to treat the model as if it were a DBM, we successively estimate our LSTM models by conditioning on a grid of integer values for each γ_i, and then we report the lowest γ_i that minimizes the error sum of squares measured to four decimal places.

The estimated LSTM models are reported in table 3. The transition parameters are high, providing support for a homogeneous beliefs story, in which investors share the same information and collectively differentiate between 'bull' and 'bear' market states. Accompanying these high values of γ_i are huge *t*-statistics that are associated with the estimation of the centrality parameter c_i. These high *t*-statistics reflect the 'super-consistency' property associated with estimating the threshold in DBM models, because the reported LSTM models are effectively DBM models and therefore have the same properties. The reported *t*-statistics for c_i are not useful for inference, but they reflect accurate estimation of this parameter. Consistent with Harlow and Rao's (1989) suggestion that investor targets do not just depend on parameters associated with the distribution of market or risk-free returns, we observe that there is no apparent common value for this parameter across different industries. Our empirical evidence shows that 'blanket' decisions to set c_i equal to zero or c_i equal to the mean of the transition variable for all industries are not necessarily appropriate, and perhaps it explains why most previous research on DBMs has failed to find systematic support for such models. It is noteworthy that more than a third of the estimated centrality parameters are negative, so that for these industries R_t^* needs to be very low before the relevant stock shows evidence of 'bearish' behaviour. Such industries include 'gold', 'building' and 'oil and gas', while other industries such as 'alcohol and tobacco' or 'chemicals' only become 'bullish' when R_t^* has become relatively high.

The up-market differentials in industry-specific betas are recorded in the β^U column, and 15 of these are statistically significant at the 5% level and another six are

Table 3. LSTM models.

Code	α	δ	β	β^{U}	c	γ	ESS
XGO	−0.009 (−1.541)	0.003 (0.041)	2.985 (5.942)	−1.723 (−3.267)	−0.015 (−85.047)	83	2.0581
XOM	−0.008 (−2.290)	0.051 (1.103)	1.425 (17.116)	−0.390 (−2.297)	0.015 (1132.2)	800	0.6889
XDR	−0.002 (−0.684)	0.197 (6.087)	1.272 (21.280)	−0.255 (−2.117)	0.019 (230.14)	226	0.4122
XDC	0.006 (2.362)	−0.179 (−5.454)	1.061 (9.220)	−0.161 (−1.247)	−0.004 (−56.932)	479	0.3121
XBM	0.002 (0.710)	0.096 (3.426)	0.780 (10.833)	0.144 (1.627)	−0.012 (−155.74)	126	0.2570
XAT	0.010 (4.269)	−0.148 (−4.578)	0.562 (9.757)	0.343 (2.472)	0.028 (502.34)	202	0.3599
XFH	0.004 (1.295)	−0.072 (−1.176)	0.551 (4.923)	0.245 (1.746)	−0.001 (−49.78)	763	0.4557
XCE	0.000 (0.052)	0.064 (1.917)	0.809 (13.389)	0.439 (2.945)	0.031 (1073.8)	381	0.4999
XEG	−0.004 (−1.553)	0.202 (6.336)	0.646 (3.381)	0.351 (1.762)	−0.011 (−81.658)	99	0.3542
XPP	0.001 (0.330)	0.136 (3.705)	0.836 (12.852)	−0.222 (−1.863)	0.031 (1208.9)	581	0.3509
XRE	0.008 (3.045)	0.025 (0.338)	0.945 (7.074)	−0.304 (−2.042)	−0.001 (−26.972)	177	0.3634
XTP	0.002 (0.642)	−0.117 (−3.110)	0.887 (130.137)	0.318 (2.765)	0.023 (623.2)	304	0.4745
XME	0.005 (1.036)	0.015 (0.217)	0.985 (8.002)	0.395 (1.997)	0.019 (333.7)	184	1.2636
XBF	0.009 (3.516)	0.181 (3.250)	1.006 (9.860)	−0.244 (−2.005)	0.003 (145.5)	604	0.3725
XIN	0.004 (1.203)	−0.231 (−5.075)	0.525 (6.305)	0.496 (3.706)	0.008 (312.2)	113	0.5793
XIF	0.003 (1.585)	−0.149 (−4.634)	0.438 (3.800)	0.255 (1.958)	−0.012 (−2198)	2640	0.2256
XPT	0.007 (4.260)	−0.087 (−3.861)	0.338 (8.397)	0.105 (1.453)	0.014 (3298)	2929	0.1705
XMI	−0.012 (−1.388)	−0.0581 (−0.793)	0.0289 (0.073)	1.1785 (1.998)	−0.015 (*B*)	2.47	2.1979
XDI	0.005 (2.295)	0.000 (0.000)	1.037 (15.248)	−0.203 (−2.281)	0.010 (178.7)	282	0.3135
XMS	0.004 (1.636)	−0.010 (−0.272)	0.682 (10.816)	−0.231 (−1.834)	0.030 (2613)	996	0.2096
XSF	−0.004 (−1.117)	0.128 (2.967)	1.008 (12.887)	−0.417 (−3.065)	0.018 (1018)	698	0.4191
XOG	−0.006 (−1.625)	0.081 (1.352)	2.009 (6.957)	−0.945 (−3.032)	−0.015 (*B*)	9.06	0.5033
XEI	0.000 (0.059)	−0.427 (−6.648)	0.739 (6.325)	0.318 (1.978)	0.014 (1094)	818	0.6220
XTU	0.004 (1.316)	n/a	1.011 (7.627)	−0.323 (−1.909)	0.011 (1882)	1856	0.1532

The table reports coefficient estimates (and heteroskedasticity corrected *t*-statistics) for $R_{it} = \alpha_i + \delta_i \mathrm{SMC}_t + \beta_i R_{\mathrm{mt}} + \beta_i^{\mathrm{U}}\{1 + \exp[-(\frac{\gamma}{\sigma_{R^*}})(R_t^* - c)]\}^{-1} \times R_{\mathrm{mt}} + \varepsilon_{it}$. We do not report *t*-statistics for γ, since this parameter was determined via a grid-search. ESS is the minimized error sum of squares. (*B*) under an estimate for c indicates that c was restricted to be no less than −0.015.

statistically significant at the 10% level. Our LSTAR models therefore provide widespread support for time-varying betas, with variation linked to movements in the business cycle. Compared with Faff's (2001) work on Australian industry stock portfolios, we provide much stronger evidence of differences between 'bull' and 'bear' betas, and we believe that this is largely attributable to our use of a different regime switching mechanism. Of the statistically significant $\widehat{\beta}^{\mathrm{U}}$s, 11 are negative, consistent with literature that claims that risk in up markets is lower than risk in down markets. However, we also observe ten industries in which up-market betas are higher, so that overall we cannot conclude that down-market betas are systematically larger than up-market betas (or vice versa).† Industries for which down-market betas are bigger than up-market betas include all five in the 'resource & mining' sectors (i.e. industries in table 1 with codes XGO, XOM, XDR, XSF and XOG). Faff (2001) found that out of all 24 sectors, these five sectors provided the weakest support for a dual-beta CAPM, but here the support is quite clear. Industries in which up-market betas are bigger than down-market betas include 'transport', 'insurance' and 'investment & financial services'. We note that such industries can offer upside potential with minimal downside risk.

Nearly all of the LSTM models in table 3 are essentially DBM models, so we next consider the direct estimation of DBM specifications. This is relatively straightforward and simply involves using a grid of potential thresholds. For each of these potential thresholds we apply OLS to the two subsamples to obtain a composite ESS, and then we choose the threshold and associated model by minimizing over these composite ESSs. We determine our grid of potential thresholds by considering the ordered values of R_t^* in our sample, using successively larger values of R_t^* to subdivide the sample into two.‡ Figure 3 shows how the recursive ESS changes with the choice of different thresholds. We illustrate the case for the

†Over all industries, the average down-market beta is approximately 1 (as might be expected), and the average up-market differential is about zero (as might also be expected).
‡We considered only those R_t^* that implied at least 15 observations in each regime.

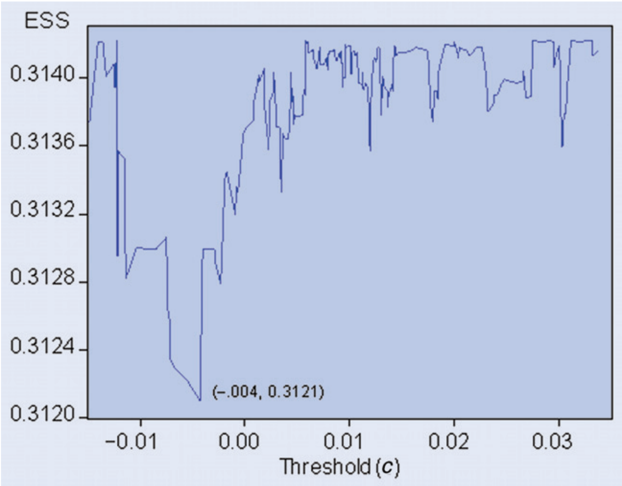

Figure 3. DBM error sum of squares plotted against possible thresholds for models of the returns for the 'developers & contractors' industry.

'developers & contractors' industry, which had not shown evidence of nonlinearity in table 2. The plot shows that despite the lack of evidence provided by the nonlinearity tests, the point where ESS is minimized is nevertheless well defined. Analogous plots for other industries that were characterized by stronger evidence of nonlinearity in table 2 typically showed sharper and more dramatic downward spikes at the point where ESS was minimized.

We present our estimated DBM models in table 4, and not surprisingly they are very similar to the LSTM models in table 3. The parameter estimates and their standard errors are almost identical, and we observe only slight increases in the ESS for DBM models in the few cases where industry models had experienced smoother transition (i.e. smaller γ) in the LSTM framework. As for the LSTM models, the estimation procedure is affected by numerical accuracy limitations, but in this case the limitations arise because our grid of possible values for c can be no finer than the set of observed R_t^* in the sample. We show these limitations by reporting the interval (defined by two values of the ordered R_t^*) in which $\widehat{c}$ lies. The most noticeable difference between the $\widehat{c_i}$ reported in the LSTM and DBM tables occur for the 'engineering', 'miscellaneous industrials', 'oil and gas' and 'tourism' industries, and it is in these industries where we see changes in the other estimated parameters. Such changes do not affect our overall conclusion that the betas for most industries are time varying. Further, of those industries in which changes in beta are statistically significant, about half have up-market betas that are lower than down-market betas, and about half have up-market betas that are higher than down-market betas. It is interesting to note that more than half of the industries have less than half of their observations in the down-market regime ($T^{\mathrm{L}} < 126$). This result concurs with those of Pagan and Sossounov (2003) and Lunde and Timmermann (2004), both of whom used trend-based definitions of 'bull' and 'bear' markets to analyse market phase durations and amplitudes.

The R^2 measures reported in table 4 indicate that the DBM models fit the data quite well, and the reported p-values for Ramsey (1969) reset tests find evidence of mispecification in only one of the 24 industries ('retail'). Thus, it appears that the DBM models have captured the time variation in the beta quite well. There is a little evidence of serial correlation in the model residuals for six of the industries (see the LM_1 and LM_{12} columns in table 4), but since this was not widespread over all industries and it did not imply any bias in the parameter estimates, we did not explicitly correct for this. However, as noted above, we used White's heteroskedasticity corrections for all of our t-statistics, because heteroskedasticity characterized each of the industry series that we were modelling.

Our estimated DBM models provide consistent evidence that beta varies according to values of R_t^*, and it is interesting to see whether our use of data-determined thresholds is driving this result, whether our use of R_t^* is responsible, or whether each of these factors plays a part. We believe that the use of data-determined thresholds is important, because if we re-estimate table 4 using $R_t^* \gtrless 0$ to define up and down-market betas, then we find statistically significant differences between up- and down-market betas for only one industry. Defining R^*_{median} to be the median of R_t^* (0.0096) and repeating the experiment using $R_t^* \gtrless R^*_{\mathrm{median}}$ leads to only four industries in which up- and down-market betas are statistically different. The decision to use R_t^* is also influential because if we re-estimate table 4 using R_t as the transition variable, we find statistically significant differences between up- and down-market betas in only eight cases at the 5% level of significance and in only 12 cases at the 10% level. This last finding confirms our earlier intuition regarding our choice of transition variable, because R_t^* reflects broad swings in the market rather than temporary market movements, and this allows the resulting beta to take on 'bull' and 'bear' market interpretations.

We re-estimated table 4 using 6-month and 18-month moving averages of R_t to explore the effects of varying the extent of lagged information contained in our market indicator. Our new estimates were very similar to those in table 4 (especially when using the 18-month moving average), with the industry-specific signs of β^{U} remaining the same in almost all cases. However, the number of statistically significant β^{U} was smaller in both cases, down to 15 (at the 5% level of significance) when the moving average was taken over 18 months, and 14 when the moving average was taken over six months. Further, the R^2 for the regressions involving the six-month moving average were consistently lower. This evidence suggests that a classification of the current state of the market can usefully incorporate returns that go back for at least six months, but not much further than a year. A justification of why lagged information might be relevant when classifying a current market state is that it helps investors to distinguish between temporary and more persistent stock market movements.

Given the work in Siegel (1998) and Resnick and Shoesmith (2002) that looks at the relationship between

Table 4. DBM (threshold) models.

Code	α	δ	β	β^{U}	c	T^{L}	RR_1	LM_1	LM_{12}	R^2	ESS
XGO	−0.010 (−1.584)	0.002 (0.032)	2.761 (6.178)	−1.489 (−3.139)	−0.0149 −0.0141	15	0.63	0.01	0.51	0.43	2.0733
XOM	−0.008 (−2.271)	0.051 (1.093)	1.425 (16.978)	−0.390 (−2.279)	0.0151 0.0153	181	0.48	0.16	0.35	0.68	0.6889
XDR	−0.002 (−0.679)	0.198 (6.071)	1.272 (21.150)	−0.255 (−2.101)	0.0184 0.0191	191	0.68	0.06	0.73	0.71	0.4122
XDC	0.006 (2.347)	−0.179 (−5.416)	1.061 (9.322)	−0.161 (−1.260)	−0.0043 −0.0041	37	0.61	0.55	0.82	0.75	0.3121
XBM	0.002 (0.711)	0.096 (3.414)	0.780 (10.907)	0.144 (1.632)	−0.0115 −0.0014	26	0.82	0.10	0.01	0.70	0.2570
XAT	0.010 (4.239)	−0.146 (−4.506)	0.564 (9.711)	0.339 (2.427)	0.0277 0.0282	218	0.55	0.02	0.03	0.55	0.3599
XFH	0.004 (1.286)	−0.072 (−1.165)	0.551 (4.885)	0.245 (1.731)	−0.0008 −0.0007	48	0.29	0.12	0.20	0.49	0.4557
XCE	0.000 (0.054)	0.064 (1.902)	0.809 (13.306)	0.439 (2.925)	0.0309 0.0312	231	0.34	0.58	0.21	0.53	0.4999
XEG	−0.004 (−1.571)	0.204 (6.244)	0.669 (3.997)	0.331 (1.868)	−0.0074 −0.0072	32	0.83	0.06	0.08	0.63	0.3543
XPP	0.001 (0.328)	0.134 (3.677)	0.836 (12.751)	−0.222 (−1.852)	0.0309 0.0312	231	0.99	0.15	0.31	0.56	0.3509
XRE	0.008 (3.022)	0.025 (0.337)	0.945 (7.021)	−0.304 (−2.027)	−0.0017 −0.0009	46	0.03	0.38	0.74	0.59	0.3634
XTP	0.002 (0.637)	−0.117 (−3.083)	0.887 (13.034)	0.318 (2.746)	0.0232 0.0238	205	0.66	0.96	0.30	0.65	0.4745
XME	0.005 (1.027)	0.015 (0.217)	0.985 (7.940)	0.395 (1.982)	0.0184 0.0191	191	0.44	0.02	0.39	0.43	1.2636
XBF	0.009 (3.486)	0.181 (3.223)	1.006 (9.784)	−0.244 (−1.987)	0.0024 0.0027	62	0.96	0.22	0.27	0.58	0.3725
XIN	0.004 (1.197)	−0.231 (−5.027)	0.525 (6.258)	0.496 (3.681)	0.0081 0.0085	113	0.79	0.41	0.21	0.52	0.5793
XIF	0.003 (1.547)	−0.149 (−4.577)	0.438 (4.177)	0.255 (2.100)	−0.0122 −0.0122	22	0.31	0.94	0.05	0.70	0.2256
XPT	0.007 (4.172)	−0.087 (−3.751)	0.338 (8.097)	0.105 (1.459)	0.0131 0.0131	164	0.26	0.83	0.29	0.43	0.1705
XMI	−0.011 (−1.401)	−0.003 (−0.029)	0.383 (1.968)	0.778 (2.621)	−0.0122 −0.0122	22	0.91	0.93	1.00	0.32	2.1988
XDI	0.005 (2.274)	−0.000 (−0.006)	1.037 (15.148)	−0.203 (−2.262)	0.0096 0.0098	129	0.81	0.65	0.07	0.72	0.3135
XMS	0.004 (1.618)	−0.010 (−0.268)	0.682 (10.720)	−0.231 (−1.820)	0.0301 0.0301	172	0.42	0.26	0.81	0.62	0.2096
XSF	−0.004 (−1.108)	0.128 (2.934)	1.008 (12.747)	−0.417 (−3.036)	0.0180 0.0181	128	0.52	0.25	0.43	0.56	0.4191
XOG	−0.006 (−1.620)	0.080 (1.374)	1.753 (5.112)	−0.688 (−1.930)	−0.0122 −0.0122	22	0.12	0.00	0.02	0.64	0.5055
XEI	0.000 (0.057)	−0.427 (−6.586)	0.739 (6.262)	0.318 (1.961)	0.0142 0.0143	120	0.86	0.03	0.03	0.63	0.6220
XTU	0.005 (1.617)	n/a	1.068 (6.380)	−0.325 (−1.756)	0.0079 0.0080	54	0.12	0.11	0.46	0.49	0.1532

We report coefficient estimates (together with heteroskedasticity corrected t-statistics) for $R_{it} = \alpha_i + \delta_i \mathrm{SMC}_t + \beta_i R_{\mathrm{mt}} + \beta_i^{\mathrm{U}} \times D_t \times R_{\mathrm{mt}} + \varepsilon_{it}$ with $D_t = 1$ for $R_t^* > c$. The two values for c indicate the upper and lower bounds for the threshold R_t^*, and T^{L} counts the number of observations in the down-market regime. We also report p-values for one fitted term Ramsey (1969) reset tests (RR_1) and first- and twelfth-order LM serial correlation tests (LM_1 and LM_{12}) R^2 and ESS provide measures of fit.

stock market performance and business cycle indicators, we also re-estimated table 4 using (i) a coincident business cycle indicator, and (ii) a lagged interest rate spread as a transition variable.† As in the moving average experiments, the resulting estimates were quite similar to those that used R_t^*, although again the number of instances for which β^{U} differed from zero at the 5% level of significance was smaller. Thus it appears that although business cycle indicators contain information that is relevant for assessing the state of the stock market, they do not discriminate between 'bull' and 'bear' phases as well as our market-specific indicator.

4. Conclusion

Previous research on the relationship between beta and market phase offers rather weak evidence that security and portfolio betas are influenced by the alternating forces of 'bull' and 'bear' markets. However, most previous studies have arrived at their conclusions by using the simple threshold DBM model in conjunction with crude 'up'- and 'down'-market definitions that involve a comparison of the market return to an arbitrarily chosen threshold value. In this paper we reinvestigate this phenomenon, testing for differential 'bull' and 'bear' market effects using a trend-based definition of 'bull' and 'bear' markets, and allowing the data to determine an appropriate value for the threshold parameter for each separate industry. We use logistic smooth transition market models for our study, which incorporate smooth transition between the two extreme regimes, and allow an additional investigation of the extent to which investors exhibit heterogeneous beliefs.

Our results differ from previous work in several important respects. Most importantly, we find strong and consistent evidence that beta varies between 'bull' and 'bear' phases. This result seems to be partly attributable to our use of a trend-based market indicator, but it is also due to the fact that we allowed the data to define 'bull' and 'bear' states rather than working with an arbitrarily defined definition. Our LSTM estimates indicate that transition is abrupt for most industries, and we view this as support for homogeneous beliefs among investors due to information symmetry. Specifically, investors collectively differentiate between 'bull' and 'bear' market states for each industry, and once the market indicator crosses each industry-specific threshold value, the beta for that industry reflects a corresponding change in state.

We estimate DBM models because the estimated value of the smoothness parameter in the LSTM models *is* very large for most industries, and we find that up-market and down-market betas are significantly different in 21 out of 24 industries. The former is lower than the latter in only 11 cases, and is not consistent with literature that claims that risk in up markets is lower than down-market risk. A possible explanation for the lack of a clear relationship between risk and beta is offered by Ang *et al.* (2006), who use a 'disappointment utility function' and downside correlations (not downside beta) to show that the compensation for bearing downside risk is not simply compensation for market beta.

Our analysis finds very strong evidence that betas in 'bull' and 'bear' markets differ because we are particularly careful about how we define these states. We are careful to ensure that our market indicator captures persistent movements in market returns and that it abstracts from market noise. Our analysis indicates that a noisy indicator such as the market return series itself does not help to identify market states, but that smoother indicators (such as our moving average indicators) do. Our results also show that definitions of 'bull' and 'bear' markets depend on which industry is under consideration. When our market indicator R^* is declining, some industries go into the 'bear' state well before other industries. This supports Harlow and Rao's (1989) theoretical observation that investor thresholds need not just depend on parameters associated with the distributions of the market or risk-free returns.

Consistent with Maheu and McCurdy (2000), Pagan and Sossounov (2003), and Lunde and Timmermann (2004), we find that, for many industries, stocks spend more time in 'bull'-market than 'bear'-market states. We also find that the threshold between states is typically lower than the mean and/or median of market returns, suggesting that many of the standard 'symmetrical' definitions of 'bull' and 'bear' markets are inappropriate. Taleb (2004, Chapter 6) emphasizes the ways in which asymmetries in the distributions of stock market returns can confound seemingly straightforward notions of 'bull' and 'bear' market classifications, and the alternative 'bull'/'bear' classification mechanism outlined in this paper offers a practical way of accounting for and studying stock market asymmetries.

Acknowledgements

This research was supported by a Monash Graduate Scholarship and the Financial Derivatives Unit at Monash University, Australia. Heather Anderson acknowledges the financial assistance of Australian Research Council Discovery Grant # DP0449995. The authors are grateful to Robert Brooks and Timo Teräsvirta for helpful suggestions.

†We used the WESTPAC Coincident Index series, obtained from the Melbourne Institute of Applied Economic and Social Research (University of Melbourne), and the interest rate spread between Australian 10-year and 3-month government bonds. The spread was lagged by four months, following the market timing work undertaken by Resnick and Shoesmith (2002).

References

Ang, A., Chen, J. and Xing, Y., Downside risk. *Rev. Finan. Studies*, 2006, **19**, 1191–1239.

Bawa, V.S. and Lindenberg, E.B., Capital market equilibrium in a mean-lower partial moment framework. *J. Finan. Econ.*, 1977, **5**, 189–200.

Bhardwaj, R. and Brooks, L., Dual betas from bull and bear markets: reversal of the size effect. *J. Finan. Res.*, 1993, **16**, 269–283.

Brock, W. and Hommes, C., Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *J. Econ. Dynam. & Control*, 1998, **22**, 1235–1274.

Brock, W. and LeBaron, B., Mutual fund performance: evidence from the UK. *Eur. Finan. Rev.*, 1998, **2**, 57–77.

Chan, K.S., Consistency and limiting distributions of the least squares estimator of a threshold autoregressive model. *Ann. Statist.*, 1993, **21**, 520–533.

Chen, S., An examination of risk return relationship in bull and bear markets using time varying betas. *J. Finan. & Quant. Anal.*, 1982, **17**, 265–285.

Clinebell, J.M., Squires, J.R. and Stevens, J.L., Investment performance over bull and bear markets: Fabozzi and Francis revisited. *Quart. J. Bus. & Econ.*, 1993, **32**, 14–25.

Cohen, J.B., Zinbarg, E.D. and Zeikel, A., *Investment Analysis and Portfolio Management*, 5th ed., 1987 (R.D. Irwin: Homewood, IL).

Coutts, A.J., Roberts, J. and Mills, T., Parameter stability in the market model: tests and time varying parameter estimation with UK data. *Statistician*, 1997, **46**, 57–70.

DeBondt, W. and Thaler, R., Further evidence on investor overreaction and stock market seasonality. *J. Finan.*, 1987, **42**, 557–582.

Dukes, W.P., Bowlin, O.D. and MacDonald, S.S., The performance of beta in forecasting portfolio returns in bull and bear markets using alternative market proxies. *Quart. J. Bus. & Econ.*, 1987, **26**, 89–103.

Fabozzi, F.J. and Francis, F.C., Stability tests for alphas and betas over bull and bear market conditions. *J. Finan.*, 1977, **32**, 1093–1099.

Fabozzi, F.J. and Francis, F.C., Mutual fund systematic risk for bull and bear markets: an empirical examination. *J. Finan.*, 1979, **34**, 1243–1250.

Faff, R., A multivariate test of a dual-beta CAPM: Australian evidence. *The Finan. Rev.*, 2001, **36**, 157–174.

Fama, E., Fisher, L., Jensen, M. and Roll, R., The adjustment of stock prices to new information. *Int. Econ. Rev.*, 1969, **10**(1), 1–21.

Fama, E. and French, K., The cross-section of expected stock returns. *J. Finan.*, 1992, **47**, 427–465.

Gooding, A.E. and O'Malley, T.P., Market phase and the stationarity of beta. *J. Finan. & Quant. Anal.*, 1977, **12**, 833–857.

Granger, C.W. and Silvapulle, P., Large returns, conditional correlation and portfolio diversification: a value-at-risk approach. *Quant. Finan.*, 2001, **1**, 542–551.

Guillaume, J., Robson, G. and Edelen, R., Fundamental economic variables, expected returns, and bond fund performance. *J. Finan.*, 1995, **50**, 195–220.

Harlow, V.W. and Rao, R.K.S., Asset pricing in a generalized mean-lower partial moment framework: theory and evidence. *J. Finan. & Quant. Anal.*, 1989, **24**, 285–311.

Howton, S.W. and Peterson, D.R., An examination of cross–sectional realized stock returns using a varying risk beta model. *The Finan. Rev.*, 1998, **33**, 199–212.

Jagannathan, R. and Wang, Z., The conditional CAPM and the cross-section of expected returns. *J. Finan.*, 1996, **51**, 3–53.

Kao, W.G., Cheng, L.T.W. and Chan, K.C., International mutual fund selectivity and market timing during up and down market conditions. *The Finan. Rev.*, 1998, **33**, 127–144.

Kim, M.K. and Zumwalt, K.J., An analysis of risk in bull and bear markets. *J. Finan. & Quant. Anal.*, 1979, **14**, 1015–1025.

Lunde, A. and Timmermann, A., Duration dependence in stock prices: an analysis of bull and bear markets. *J. Bus. & Econom. Statist.*, 2004, **22**, 253–321.

Luukkonen, R., Saikkonen, P. and Terasvirta, T., Testing linearity against smooth transition autoregressive models. *Biometrika*, 1988, **75**, 491–499.

Lux, T., Herd behavior, bubbles and crashes. *Econ. J.*, 1995, **105**, 881–896.

Maheu, J. and McCurdy, T., Identifying bull and bear markets in stock returns. *J. Bus. & Econom. Statist.*, 2000, **18**, 100–112.

Neftci, S.N., Are economic time series asymmetric over the business cycle? *J. Polit. Econ.*, 1984, **92**, 307–328.

Pagan, A. and Sossounov, K., A simple framework for analyzing bull and bear markets. *J. Appl. Econom.*, 2003, **18**, 23–46.

Peterson, D.R.Mutual fund performance: a nonparametric empirical investigation, Working Paper, 1994 (University of Wisconsin).

Petruccelli, J., On tests for SETAR-type nonlinearity in time-series. *J. Forecasting*, 1990, **9**, 25–36.

Pettengill, G., Sundaram, S. and Mathur, I., The conditional relation between beta and returns. *J. Finan. & Quant. Anal.*, 1995, **30**, 101–116.

Ramsey, J.B., Tests for specification errors in classical linear least squares regression analysis. *J. Roy. Statist. Soc. B*, 1969, **31**, 350–371.

Resnick, B.R. and Shoesmith, G.L., Using the yield curve to time the market. *Finan. Analysts J.*, 2002, **58**, 82–90.

Sharpe, W.F., Mutual fund performance. *J. Bus.*, 1966, **39**, 119–138.

Siegel, J.J., *Stocks for the Long Run*, 2nd ed., 1998 (McGraw–Hill: New York).

Skalin, J. and Teräsvirta, T., Modelling asymmetries and moving equilibria in unemployment rates. *Macroecon. Dynam.*, 2002, **6**, 202–241.

Spiceland, D.J., and Träpnell, J.E., The effect of market conditions and risk classifications on market model parameters. *J. Finan. Res.*, 1983, **6**, 217–222.

Taleb, N.N., *Fooled by Randomness: the Hidden Role of Chance in Life and the Markets*, 2004 (Thompson/Texere: New York).

Teräsvirta, T., Specification, estimation and evaluation of smooth transition autoregressive models. *J. Amer. Statist. Assoc.*, 1994, **89**, 208–218.

Tsay, R.S., Testing and modeling threshold autoregressive processes. *J. Amer. Statist. Assoc.*, 1989, **84**, 231–240.

White, H., A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity. *Econometrica*, 1980, **48**, 817–838.

Wiggins, J.B., Betas in up and down markets. *The Finan. Rev.*, 1992, **27**, 107–123.