

Rotating Horizontal Convection in a Rectangular Box

C. A. Vreugdenhil¹, R. W. Griffiths¹, and B. Gayen¹

¹Research School of Earth Sciences
Australian National University, Australian Capital Territory 0200, Australia

Abstract

We examine the effect of rotation on horizontal convection using laboratory experiments in a rectangular box domain. Horizontal convection is a basic model for the meridional overturning circulation (MOC) in the ocean; we also expect the MOC is influenced by the Earth's rotation. The Rayleigh number is two orders of magnitude larger than any previous study ($Ra \sim 10^{12}$) to ensure the flow lies in a regime with a turbulent boundary layer and endwall plume (to better match realistic ocean conditions). Other governing parameters are the Prandtl number $Pr \sim 5$, aspect ratios $A_H = 0.16$ and $A_W = 0.24$, and Rossby number $Ro \sim 0.001 - 0.1$. Particle tracking velocimetry is used to measure horizontal velocity fields at three interior depths, away from the boundary layer that forms adjacent to the thermal forcing. With increasing rotation, the steady state time-averaged flow dynamics changes from a full length cyclonic gyre, to a series of five counter-rotating baroclinic eddies (at non-dimensional Rossby deformation scale of $O(1)$) and then to a large anticyclonic gyre. The large scale horizontal flow dynamics are largely independent of depth. The divergence and vorticity of the horizontal velocity fields are used to estimate the overturning, which consistently decreases with increases in rotation. Direct Numerical Simulations are ongoing, and will allow access to the energetics of this complicated system.

Introduction

A buoyancy gradient applied at a horizontal surface has been found to generate large-scale overturning in both laboratory experiments and numerical simulations [2, 10, 12]. Although surface wind stress is also likely to provide significant forcing, this surface buoyancy forced flow, known as horizontal convection, could substantially influence the rate and strength of the oceanic meridional overturning circulation (MOC) [6, 8, 9, 13]. As the MOC occurs at geostrophic scales of motion, the Earth's rotation will control the patterns of circulation and, potentially, influence the heat transport. Here we study horizontal convection (without applied stress) in order to better understand the roles of buoyancy and rotation in the MOC.

A thermal gradient is applied along the base of a rectangular box, where one half of the boundary (at one end of the box) is held at a uniform (lower) temperature and the other half is held at either a uniform (higher) temperature or a uniform heating flux. Cold water parcels are drawn across the heated region, where they undergo an increase in buoyancy via thermal convection, and form a destabilising boundary layer. In the non-rotating case, the thermal boundary layer feeds an end wall plume that penetrates into the domain interior. In long-term thermal steady state (no net buoyancy through any horizontal level) the water must travel back through the interior of the domain to the cooled region, closing the circulation. The governing parameters are the Rayleigh number, Prandtl number, and vertical aspect ratio,

$$Ra = \frac{g\alpha\Delta TL^3}{\nu\kappa}, \quad Pr = \frac{\nu}{\kappa}, \quad A_H = \frac{H}{L}, \quad (1)$$

where g is gravity, α is the thermal expansion coefficient, ΔT is

the applied temperature difference, L is the typical length scale of the forcing boundary, ν is the kinematic viscosity, κ is the thermal diffusivity, and H is the height of the domain. For the onset of turbulence in the boundary layer and plume (required to more closely model ocean overturning), Ra must be above the transition regime $Ra \sim 10^{10} - 10^{11}$ [3].

The presence of rotation introduces an Ekman layer, which may influence the uptake of buoyancy of the thermal boundary layer. The Q parameter, defined in terms of the Ekman number E ,

$$Q = \frac{\delta_T}{\delta_E} \sim \left(\frac{1}{Ra^{1/5} E^{1/2}} \frac{L}{H} \right)^2, \quad (2)$$

compares the Ekman (δ_E) and thermal (δ_T) layer length scales [5]. In an annulus laboratory experiment [5], the circulation was similar to the non-rotating case up to a critical Q value ($Q = 3.4$ with the geometry used) above which baroclinic eddies were present and the thermal boundary layer thickened. Similar results were noted in numerical simulations with a re-entrant channel domain having no meridional boundaries, hence no mean flow in the direction of the applied temperature gradient, where both the lateral and vertical buoyancy flux were found to be governed by baroclinic eddies [1].

Introducing meridional sidewalls and creating a box domain produces a horizontal aspect ratio $A_W = W/L$ (W is domain width) and significantly changes the flow dynamics, as sidewall boundary currents can form. In a previous laboratory study, temperature profiles indicated that when rotation increased, the thermal boundary layer thickness and top-to-bottom temperature difference increased, while the overturning strength decreased [11]. The accompanying scaling analysis concluded that geostrophic flows (with small effects from bottom friction) dominated the meridional heat transport. Upwelling and downwelling primarily occurred at boundaries, where pressure gradients could be sustained.

Although the Ra transition to the turbulent regime for rotating horizontal convection has not been studied, we expect the transition to occur at or below that for the non-rotating case, as rotation confines the viscous stress to the Ekman layer which (at the high Q values considered) is significantly thinner than the thermal boundary layer. However, the maximum Rayleigh number for previous laboratory experiments (10^8 in an annulus [5], 10^{10} in a box [11]) and numerical simulations ($Ra < 10^{10}$ in a channel [1]) are too small to ensure the boundary layer and plume are turbulent.

We use a similar laboratory set-up as previously used for non-rotating horizontal convection [2, 4, 10], with $Ra \sim O(10^{12})$ considerably higher than used previously [11]. The influence of rotation rate is also described in terms of a Rossby number (and Rossby deformation length scale)

$$Ro = \frac{u_{bl}}{fL} \sim \frac{\kappa Ra^{2/5}}{fL^2} \left(L_d = \frac{\sqrt{g\delta_T}}{f} \right), \quad (3)$$

where u_{bl} is the velocity in the boundary layer (scaling from non-rotating case [7]), and f is the Coriolis parameter. Particle

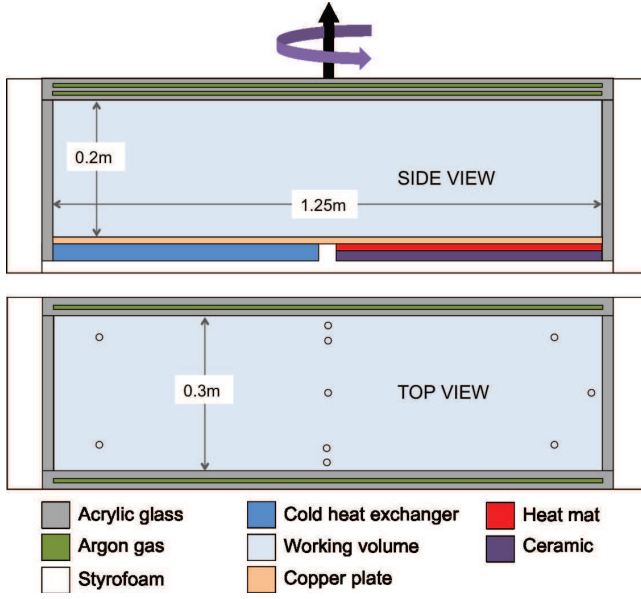


Figure 1. Rectangular box tank (not to scale). Circles in top view indicate the position of thermistor access tubes.

tracking velocimetry (PTV) is used to find velocity fields from which we estimate the transport in the overturning circulation. While the results are not included here, a thermistor array is used to measure boundary layer thickness and applied temperature difference at the base, and preliminary Direct Numerical Simulations (DNS) have been obtained.

Laboratory Experiment

The rectangular acrylic box has dimensions $1.25 \times 0.3 \times 0.2$ m ($L \times W \times H$) (figure 1). To insulate the working volume from the laboratory temperature ($21 \pm 1^\circ\text{C}$) the sidewalls are double glazed with argon gas in the gap, and the endwalls are solid 20 mm acrylic further insulated with 100 mm polystyrene foam. The top lid is triple glazed and filled with argon, with access tube holes for thermistors. The base of the tank is a 10 mm solid copper plate, beneath which is an electric heat mat under one half of the base and a cold water heat exchanger (connected to an external constant temperature bath) under the other half. A 50 mm polystyrene foam spacer is placed between these, under the mid-line of the box length and the whole apparatus sits on 100 mm of polystyrene foam insulation. In long-term thermal steady state (and for an appropriate time-average) the heat flux input from the hot plate must exactly balance the heat exiting through the cold plate. The cold plate is set at 10°C and the heat flux input at 530 W. The entire tank set-up is placed on a rotating table and the Coriolis parameter is set in the range $f = 2\Omega = 0 - 1.6 \text{ rads}^{-1}$ anticlockwise.

For a typical experiment $Ra \sim O(10^{12})$, which is in the turbulent boundary layer and plume regime (for the non-rotating case). Other governing parameters are $Pr = 5$, $A_H = 0.16$, $A_W = 0.24$, and $Ro = 0.001 - 0.1$. For the largest rotation rate, geopotential curvature is on the same order as the thermal boundary layer thickness and may influence the flow dynamics. However, we need to consider small Ro because this corresponds to the important case $L_d \ll W$. The Q value is $100 - 1000$, which is in the same strong rotation regime that is expected for the ocean. The Ekman layer thickness is $\delta_E = 0.7 - 4$ mm, and the viscous sidewall Stewartson layer thickness is $\delta_S = 4 - 29$ mm.

After the heating and cooling are turned on, the experiment is

left for 40 hrs to reach thermal equilibrium. Four thermistors inserted into the copper plate record the temperature at the hot and cold ends. In the tank interior, temperature is measured with an array of ten thermistors that traverse vertically through the full depth of the tank (at 0.1 mm precision and to within 3 – 4 mm of the base) by SmartMotor control.

Particle tracking velocimetry (PTV) is used to measure horizontal velocities at different depths $U = z/D = 0.15$ (upper; z increasing with depth), $M = 0.5$ (mid), and $L = 0.85$ (lower, just above the thermal boundary layer). A LED light (1×0.1 m) is placed 1 m away from the tank to create a horizontal beam of light across the entire working volume; a light slit narrows the beam to a thickness of ~ 1 cm. Two cameras (one for each half of the working volume) are placed 2 m above the tank. Pliolite resin particles ($125 - 250 \mu\text{m}$) are added to track the flow velocity in the light sheet. As the suspended pliolite particles for PIV are of density 1022 kgm^{-3} , the water is also increased to this density via the addition of sodium chloride, and the water is de-aerated to prevent bubbles forming. The HD video record (3 hrs) is processed in the computer program Streams 2.02.

Results and Discussion

Horizontal Velocity Fields

In figure 2, the velocity field for mid-range rotation, $f = 0.4 \text{ rads}^{-1}$, is displayed for three depths; each velocity field is statistically averaged over 3 hrs. The velocity fields do not quite extend to the tank edges (0.5 cm for sidewalls and 1.0 cm for endwalls) because of light refraction in these regions, and hence some of the Stewartson layer dynamics may be missed.

The flow structures in figure 2 are similar across all depths, a trend recorded for all rotating results. The two-dimensionality of this interior flow is consistent with the Taylor-Proudman (TP) theorem, that a vertical column of fluid in a geostrophic and unstratified system cannot be tilted or stretched, and therefore horizontal velocities are independent of depth. In rotating horizontal convection the TP theorem may not apply to flow at the boundaries where there are frictional effects, or in highly stratified regions such as the boundary layer. Although the flow structures are common across depths, regions in the velocity fields often differ in magnitude depending on the depth. In figure 2, the maximum horizontal velocities are in the lower depth, but this was not the case for all the rotation speeds (for 0.16 rads^{-1} and 1.6 rads^{-1} the maximum velocities are in the upper depth). The five full-width gyres may be baroclinic eddies set by the horizontal aspect ratio, as the Rossby length scale of deformation is of the same order as the tank geometry ($L_d/W = 3$, using δ_T from thermistor measurements).

Horizontal velocity fields at mid-depth are shown in figure 3 for the smallest (non-zero) and largest rotation rates, respectively $f = 0.04 \text{ rads}^{-1}$ and $f = 1.6 \text{ rads}^{-1}$. For 0.04 rads^{-1} , the single large gyre is anticlockwise (cyclonic) and, as $L_d/W = 30$, is limited in size by the tank geometry. When 1.6 rads^{-1} , $L_d/W \sim 0.1$ and a large scale (anticyclonic) gyre has formed from the small scale motions. The result at rotation rate 0.16 rads^{-1} (not included here) is also primarily cyclonic, while for 1.0 rads^{-1} a strong anticyclonic gyre is again formed.

Overturning

To find a value for the overturning, consider the continuity equation for an incompressible closed system,

$$\nabla \cdot (u, v, w) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (4)$$

where (u, v) are the horizontal and w is the vertical direction,

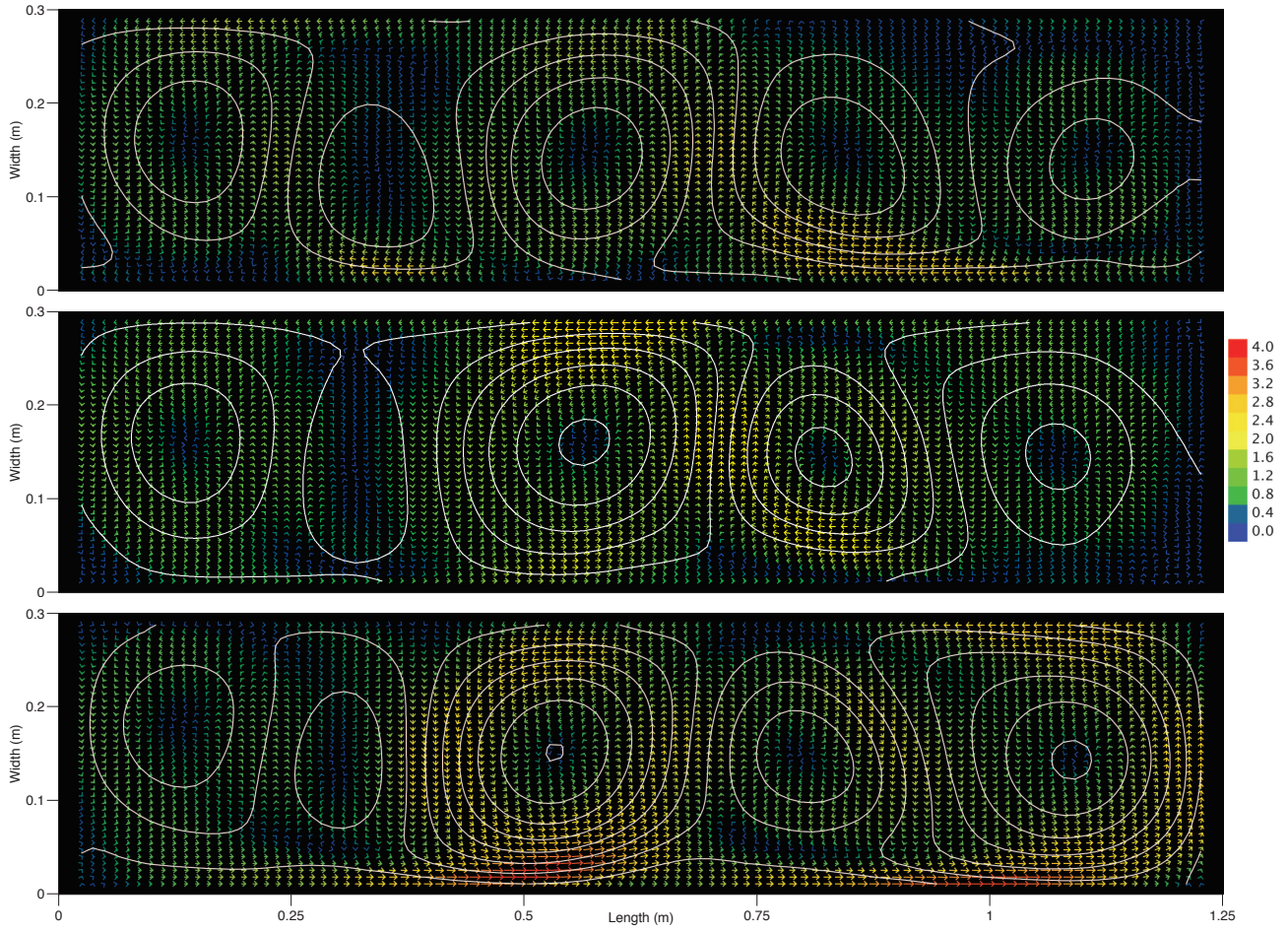


Figure 2. Horizontal velocity fields at three depths for $f = 0.4 \text{ rad s}^{-1}$. Depths are $U = z/D = 0.15$ (upper), $M = 0.5$ (mid), and $L = 0.85$ (lower). Colourbar refers to speed (mm/s), arrows indicate velocity direction, and streamlines are incremented equally.

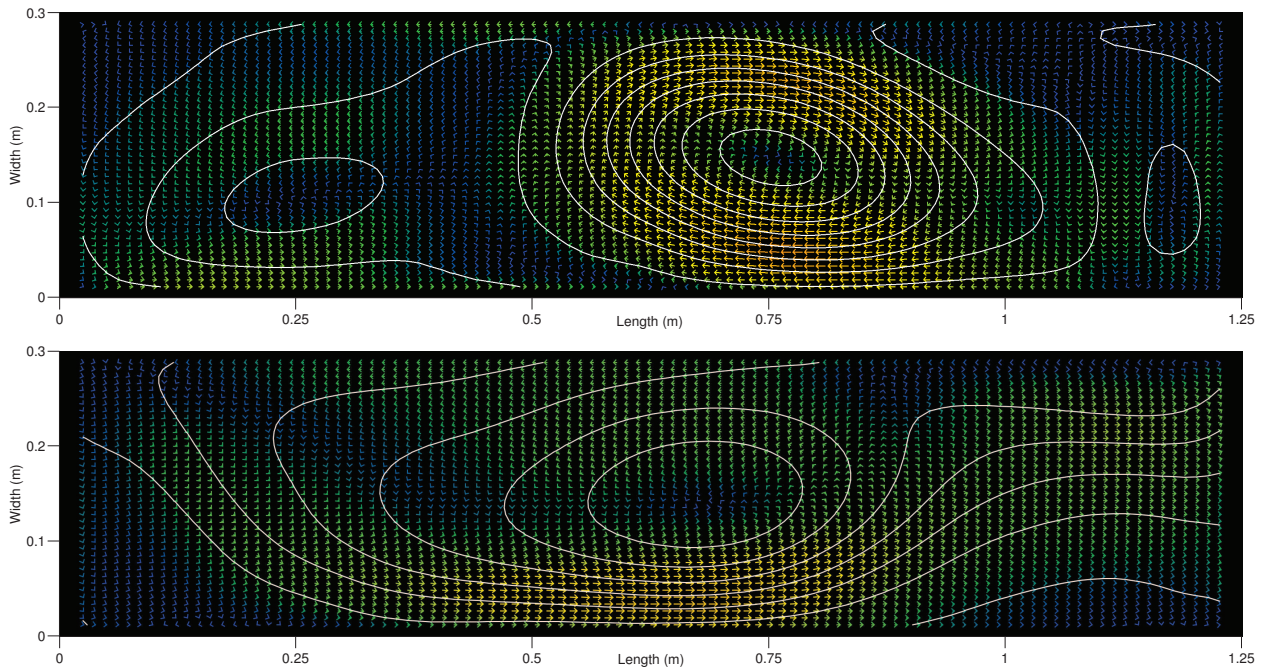


Figure 3. Mid-depth velocity field at (top) $f = 1.6 \text{ rad s}^{-1}$, (bottom) $f = 0.04 \text{ rad s}^{-1}$. Colourbar is the same as figure 2.

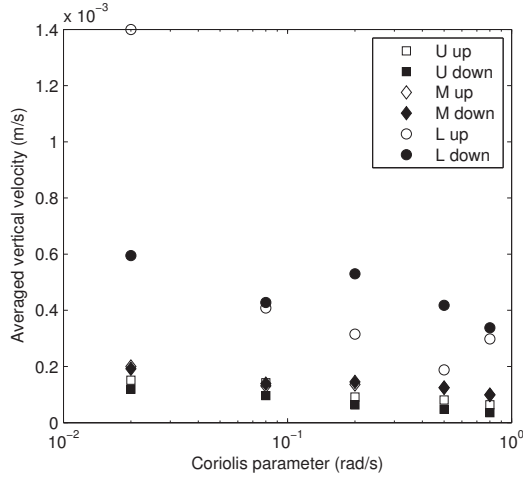


Figure 4. The averaged vertical velocities going upwards and downwards through each horizontal level ($U = 0.15$, $M = 0.5$, $L = 0.85$).

increasing with depth. Integrating with respect to z

$$\int \frac{\partial w}{\partial z} dz = - \int \nabla \cdot (u, v) dz. \quad (5)$$

Taking the z -derivative of the original continuity equation, $\partial w / \partial z$ is independent of z when (u, v) is independent of z , which is the case where the TP theorem holds. As the velocity fields are generally consistent over the interior depths considered, let us assume that the TP theorem applies to our system. Using the depth independence of (u, v) ,

$$w(z) = -z \nabla \cdot (u, v) + C_1. \quad (6)$$

To find the constant C_1 , define $z = 0$ at the top boundary, increasing in depth. At the Ekman layer depth $z = z_E = \sqrt{\nu / \Omega}$ ($f = 2\Omega$) assume the vertical velocity is entirely set by Ekman pumping,

$$w(z_E) = \frac{1}{2} \sqrt{\frac{\nu}{\Omega}} \nabla \times (u, v). \quad (7)$$

The vertical velocity is then given by

$$w(z) = -z \nabla \cdot (u, v) + \sqrt{\frac{\nu}{\Omega}} \left(\frac{1}{2} \nabla \times (u, v) + \nabla \cdot (u, v) \right). \quad (8)$$

Vertical velocities are calculated using the divergence and vorticity from horizontal velocity fields. By integrating in terms of upwards and downwards transport, and dividing by the area over which the transport occurred, a final value for averaged w upwards and downwards is calculated — this is a measure of total overturning. In figure 4 the averaged vertical velocities are generally smaller than the horizontal velocities we see in figures 2 and 3. The larger values for the lower depth could be convective motions occurring in the top of the boundary layer region, however there is an overall trend for the vertical velocity in all three planes to decrease as rotation increases.

Conclusions

Rotating horizontal convection in a rectangular box is considered in a laboratory experiment for a range of rotation rates ($0.04 - 1.6 \text{ rad s}^{-1}$). With increases in rotation (decreases in L_d) the time-averaged equilibrium flow evolved into a large cyclonic gyre at small rotation rates, into five full-width baroclinic eddies at intermediate rotation rate ($L_d/W \sim O(1)$), and a strong anticyclonic gyre at large rotation rate. The space- and

time-averaged vertical velocity (an estimate for overturning) decreases with increased rotation. As the PTV has only been applied to three horizontal levels, does not include the boundary layer, and is not designed to measure dissipation, it is unlikely that we can fully access the energy budget using only experimental results. Hence DNS solutions are ongoing, and will complement the laboratory experiments while allowing full access to the energy budget and boundary layer dynamics.

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