Public Good Mix in a Federation with Incomplete Information

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Abstract

We analyze a model of resource allocation in a federal system in which the center transfers real resources between member states. The center is assumed to be unable to observe the precise value of the cost differences across jurisdictions that motivate the transfers. Moreover, the center cannot observe the output levels of the individual local public goods provided by the jurisdictions, but must condition its transfers on a coarse aggregate of expenditures on public goods. We find that when the jurisdiction with private information realizes a high unit cost, it is generally worthwhile for the center to allow it a level of expenditure on public goods that differs from the "first best" level. However, whether that level is higher or lower than its first best level depends on the magnitudes of demand parameters for the local public good.

The existence and persistence of differences across local jurisdictions with respect to both the costs of providing local public services and also the access to local resources required to fund those services are both widely accepted features of the fiscal landscape. In established federal systems, notably the U.S., the level of real transfers ostensibly motivated by the desire to achieve fiscal equalization across jurisdictions is substantial, and such transfers are made at many levels—between school districts, cities, townships, counties, and member states of a federation. Their ubiquity, quantitative importance,

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We wish to thank David Wildasin and an anonymous referee for comments and suggestions that led to a significant improvement in the paper. In addition, Cornes would like to thank the Leverhulme Foundation for a Research Fellowship which helped to support this research.

Received February 2001; Accepted August 2002.

Journal of Public Economic Theory, 5 (2), 2003, pp. 381–397.

and the equity and efficiency problems that their present designs are widely perceived as generating, continue to spark lively controversy—see, for example, Oakland (1994) and Ladd (1994). In the European Community, too, rapidly changing relationships between member states have lent urgency to debates on the design of explicit regional transfer mechanisms.

Until very recently, formal analyses of transfers between member states of a federation have been predicated on the assumption that such transfers are determined in full knowledge of those preference or cost differences that motivate them in the first place. Wildasin (1986) provides a good discussion of this literature. By contrast, applied and policy-oriented discussions of these matters reflect a concern with the informational problems that create difficulties for the design of transfer policies. In Europe, such concern is relatively recent—see Costello (1993) for an explicit recognition of the difficulties. The U.S. has a longer history of interest in such issues. In order to compensate for cost differences it is necessary to measure costs, and in order to do this, it is necessary to be able to measure output. As Bradford, Malt, and Oates (1969) note, this raises particularly acute difficulties in the case of a wide variety of publicly provided goods and services. More recently, Grosskopf and Yaisawarng (1990, p. 65) find that "the most difficult problem encountered in identifying economies of scope for local/municipal governments is the specification of the vector of outputs provided by those governments." In her review of recent empirical work that attempts to measure differences in fiscal condition across jurisdictions, Ladd (1994) shows that these works are unable to rely on direct measures of public outputs and overall managerial costs. Instead, they rely on proxies—for example, density, poverty rate, and age of housing—to estimate a jurisdiction's cost of providing a bundle of public services. Because of its importance as a local expenditure item, education has been a particular focus for attention. Downes and Pogue (1994) observe that existing formulae for school aid schemes use data on observed spending on particular types of student. They point out that this fails to take into account the likelihood that districts with relatively high (low) costs may respond in part by providing lower (higher) service levels. Variations in the true underlying costs will, to this extent, be understated by the observed expenditures. This fact may provide a rationale for the following observation made by Bergstrom, Rubinfeld, and Shapiro (1982, p. 1188) in their study of micro-based estimates of demand functions for local school expenditures: "Even with data such as ours, where we observe expenditure levels in several places, the precision of our estimates of the parameters is seriously limited by the relatively small amount of variation in expenditure levels." Downes and Pogue (1994, p. 65) echo the earlier writers in observing that "[t]he primary barrier to obtaining reliable cost indices has been the definition and measurement of educational output." Even when information on costs is in principle available, it is not always used. Ireland, for example, collects data on a regional basis, yet the information that is used to determine transfers under some of the EU structural funds programs is aggregated across these regions, and treats Ireland as a single region. Wilson (1989) discusses various reasons why bureaucratic institutions may provide little incentive to use potentially available data on cost levels.

Recently, theorists have become interested in the information problems attending attempts at fiscal equalization. However, the resulting papers—see, for example, Boadway et al. (1994), Bordignon et al. (2001), Bucovetsky et al. (1996), Cornes and Silva (2000, 2001), Lockwood (1999), and Persson and Tabellini (1996)—have generally assumed outputs to be observable, and any hidden action or information has been assumed to involve other variables, such as cost levels or preferences. By contrast, the present paper takes seriously the implications of the observation by Bradford, Malt, and Oates that output levels of publicly provided goods and services may not be readily verifiable. We suppose that the individual jurisdiction has a vector of public goods that it can produce, and that the unit cost levels of a subset of these goods—we formally model the situation in which there is one such good—are unobservable by the center. Moreover, we suppose that the center cannot observe the level of output of that good. It can, however, observe the total expenditure on a vector of public goods, which includes the good whose unit cost is unobservable.

In such a situation, the center has to condition its transfer on observed expenditure on a bundle of goods and services, many of whose unit cost levels may be known to it. However, being unable to monitor their individual output levels, the center may have to design a transfer mechanism that will allow informational rents to the jurisdiction with private information in the event of its realizing a particularly low unit cost, and will have to take account of the possibility that the jurisdiction, though realizing a low unit cost, may benefit by increasing output, not only of the good whose unit price is unknown, but also of other goods in the bundle whose total cost is observed, in order to increase its observed expenditure and thereby the transfer that it will receive from the center. In light of the informational problems faced by state policymakers when designing public education equalizing formulas, this finding may provide an explanation of the fact that "increases in school aid have been offset by even greater increases in school district spending" (Reschovsky and Wiseman 1994, p. 88).

1. A Model with Two Local Public Goods

Resource Allocation Within a Jurisdiction

The preferences of the typical individual, of whom there are n in each jurisdiction, can be represented by the following utility function:

$$u(x, Q, R) = u(x + f(Q, R)) \tag{1}$$

where x is a private good, Q and R are two local public goods, and $u(\cdot)$ and $f(\cdot)$ are strictly increasing and concave functions of their arguments. Preferences are quasi-linear, there by removing income effects from the determination of public good levels. However, we apply a concave transformation $u(\cdot)$ to capture declining marginal utility of income, $x + f(Q, R)$, in each jurisdiction. The center maximizes a utilitarian expected social welfare function $W(\cdot)$, where

$$W(u_a, u_b) = Eu[x^a + f(Q^a, R^a)] + Eu[x^b + f(Q^b, R^b)]. \tag{2}$$

The jurisdiction j's resource constraint is

$$nx^j + c^j Q^j + R^j + n\tau^j = nI, \quad j = a, b, \tag{3}$$

where c^j is the unit cost of producing Q^j, I is the exogenous per capita income, and τ^j is the transfer taken from jurisdiction j by the center. A negative value of τ^j therefore indicates a transfer of resources by the center to jurisdiction j. The unit cost of public good R^j is exogenously given, and the units of measurement are chosen so that it is unity in both jurisdictions.

Two Jurisdictions and Transfers with Complete Information

For the moment, we suppose the center observes all parameters in both jurisdictions, including the unit costs c^a and c^b. The center and the individual jurisdictions have coincident interests in the choice of public good provision. The resulting allocation is the solution to the problem

$$\underset{Q^a, R^a, Q^b, R^b, \tau^a, \tau^b}{\text{Maximize}} \sum_{j=a}^{b} nu\left\{I + f(Q^j, R^j) - \frac{c^j Q^j + R^j}{n} - \tau^j\right\} \tag{4}$$

subject to: $\tau^b = -\tau^a$.

The first-order conditions imply that

$$nf_Q(Q^j, R^j) = c^j, \tag{5a}$$

$$nf_R(Q^j, R^j) = 1, \quad j = a, b, \tag{5b}$$

$$u'_a = u'_b. \tag{5c}$$

Equations (5a) and (5b) are simply the set of Samuelson optimality conditions for public good provision. Equation (5c) requires the value of τ^a $[= -\tau^b]$ to equate the realized marginal utilities across the two jurisdictions. Since the utility functions are identical, this in turn implies equality of the incomes that are their sole arguments in the two jurisdictions:

$$\begin{aligned} &nI + nf(Q^{a*}, R^{a*}) - c^a Q^{a*} - R^{a*} - n\tau^{a*} \\ &= nI + nf(Q^{b*}, R^{b*}) - c^b Q^{b*} - R^{b*} + n\tau^{a*}, \end{aligned} \tag{6}$$

where Q^{j*}, R^{j*}, and τ^{a*} denote the optimal values for the choice variables. It follows that

$$\tau^{a*} = \frac{[nf(Q^{a*}, R^{a*}) - c^a Q^{a*} - R^{a*}] - [nf(Q^{b*}, R^{b*}) - c^b Q^{b*} - R^{b*}]}{2n}. \tag{7}$$

If $c^a < c^b$, then the center will transfer resources from a to b. This transfer is larger, the greater the difference $c^b - c^a$. The same reasoning applies, *mutatis mutandis*, if $c^a > c^b$.

This section is summarized in the following proposition:

PROPOSITION 1: *In a regime of complete information,*

(i) *Contingent transfers equate marginal, and hence total, utility levels across the jurisdictions.*

(ii) *Samuelson's optimality condition is fulfilled for both of the public goods.*

We henceforth refer to the values of variables at this allocation as their "first best" values.

2. Transfers with Incomplete Information

Suppose now that the center cannot observe the unit cost of producing Q in jurisdiction a. It does, however, know that with probability π, $c^a = \underline{c} < c^b$ and, with probability $(1-\pi)$, $c^a = \overline{c} > c^b$. Moreover, we assume that the center does not observe the quantities Q^a and R^a separately. For the time being, we suppress the superscript that identifies the jurisdiction under discussion. The center, unable to observe the unit cost c in one of the jurisdictions, has to make its equalizing transfer dependent upon the observed total expenditure E in that jurisdiction, where $E \equiv cQ + R$.

It is convenient to think of the jurisdiction's allocation process in two stages. For any given level of total expenditure E, the jurisdiction will allocate its resources efficiently between Q and R. In addition, it chooses its total expenditure optimally taking into account the transfer, as well as the level of public goods, associated with each level of total expenditure E.

Preferences in (E, τ) Space

It is helpful to define a jurisdiction's implied preferences over E and τ, conditional upon the realized value of the unit cost c. Let the conditional utility function $V(\cdot)$ be defined as follows:

$$V(E, \tau \mid c) \equiv \underset{x,Q,R}{Max}\{nu(x + f(Q, R)) \mid nx + cQ + R + n\tau = nI,\ cQ + R = E\}. \tag{8}$$

The Lagrangian can be written as $L(\cdot) \equiv nu(I + f(Q, R) - \tau - (cQ + R)/n) - \lambda(cQ + R - E)$. The first-order necessary conditions associated with this problem are

$$[wrtQ]\, nu'[f_Q(Q,R) - c/n] - \lambda c = 0,$$

$$\Rightarrow nf_Q(Q,R) = (1+\lambda/u')c = (1+\alpha)c[\equiv \rho_Q] \tag{9}$$

$$[wrtR]\, nu'[f_R(Q,R) - 1/n] - \lambda = 0,$$

$$\Rightarrow nf_R(Q,R) = 1+\lambda/u' = 1+\alpha[\equiv \rho_R] \tag{10}$$

where $\alpha \equiv \lambda/u'$. Our statement of these conditions introduces the variables ρ_Q and ρ_R. These are the magnitudes to which the sum of demand prices, or marginal valuations, over all residents of the jurisdiction must be equated in order for the allocation to be Pareto-optimal subject to the restriction placed on the level of total expenditure E. They can be thought of as the "restricted" supply prices of the local public goods Q and R, respectively. Equations (9) and (10) implicitly define demand functions for the public goods Q and R which we write, respectively, as $q(\rho_Q, \rho_R)$ and $r(\rho_Q, \rho_R)$.

For a given value of c, (8) implicitly defines the transfer τ as a function of E and the utility level V. By the envelope theorem, $\partial V/\partial E = \lambda$ and $\partial V/\partial \tau = -nu'$. The implicit function theorem thus implies that $\partial \tau/\partial E = \lambda/nu' = \alpha/n$. Since n is held fixed throughout, the behavior of this partial derivative—which is the slope of an indifference curve at a point in (E, τ) space—depends on that of the parameter α. Differentiating (9) and (10) and rearranging, we finally obtain

$$\frac{d\alpha}{dE} = \frac{n\begin{vmatrix} f_{QQ} & f_{QR} \\ f_{RQ} & f_{RR} \end{vmatrix}}{(1 \quad c)\begin{pmatrix} f_{QQ} & f_{QR} \\ f_{RQ} & f_{RR} \end{pmatrix}\begin{pmatrix} 1 \\ c \end{pmatrix}} = \frac{n|F|}{\Omega} < 0$$

where, due to the strict concavity of $f(\cdot)$, the quadratic form Ω is negative definite and the determinant of the partial derivatives, $|F|$, is positive. Moreover, if the level of expenditure is kept constant and the size of the transfer changed, a similar exercise reveals that the parameter α remains constant. Our assumption of quasi-linearity implies that an increase in τ alone leads to a decrease in private good consumption, but does not affect the implied values of Q, R, or α. Figure 1 shows a jurisdiction's preference map in (E, τ) space for a given value of c.

Equations (9) and (10) make it clear that, for a given value of α, an increase in c implies an increase in the supply price of Q relative to both R and the numeraire private good x. It will therefore tend to encourage substitution out of Q and into one, or both, of the other two goods. Denote total expenditure as a function of c and α by $E(c, \alpha)$. Whether $E(\bar{c}, \alpha)$ exceeds, equals, or falls short of $E(\underline{c}, \alpha)$ depends partly on the magnitude of the own substitution effect associated with Q, and partly on the cross-substitution effect that summarizes the response of demand for R.

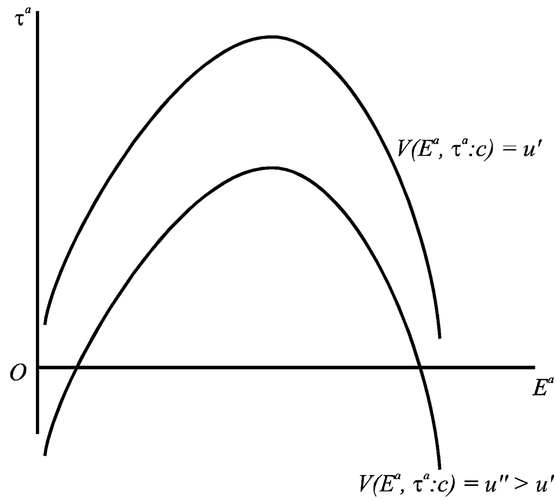

Figure 1: A Jurisdiction's Indifference Map

Consider the consequences of an infinitesimal change in c. Differentiating (9) and (10) and the resource constraint with respect to c, holding E and τ constant, we obtain

$$\frac{\partial \alpha(E, \tau \mid c)}{\partial c} = \frac{-n|F|Q}{\Omega}\left(1 + \frac{(1+\alpha)(cf_{RR} - f_{RQ})}{n|F|Q}\right) \tag{11}$$

Given our assumptions concerning $f(\cdot)$, the sign of this expression depends on that of the expression in large brackets on the right-hand side of (11). Specifically,

$$\frac{\partial \alpha(E, \tau \mid c)}{\partial c} >, =, < 0 \quad \text{according to whether}$$
$$\frac{(1+\alpha)(cf_{RR} - f_{RQ})}{n|F|Q} >, =, \text{or} < -1. \tag{12}$$

This condition can be related to demand elasticities, and thereby expressed in a more enlightening form. Recall our observation that (9) and (10) implicitly define the restricted demand, or marginal valuation, functions, $q(\cdot)$ and $r(\cdot)$, for the goods Q and R. Now consider the elasticities associated with the first of these functions, $\eta_{QQ} \equiv \frac{\partial q}{\partial \rho_Q}\frac{\rho_Q}{q}$ and $\eta_{QR} \equiv \frac{\partial q}{\partial \rho_R}\frac{\rho_R}{q}$. Differentiating (9) and (10), first with respect to a change in ρ_Q alone and then with respect to ρ_R alone, yields

$$\eta_{QQ} \equiv \frac{\partial q}{\partial \rho_Q}\frac{\rho_Q}{q} = \frac{(1+\alpha)\, cf_{RR}}{nQ|F|} \quad \text{and} \quad \eta_{QR} \equiv \frac{\partial q}{\partial \rho_R}\frac{\rho_R}{q} = -\frac{(1+\alpha)\, f_{RQ}}{nQ|F|}.$$

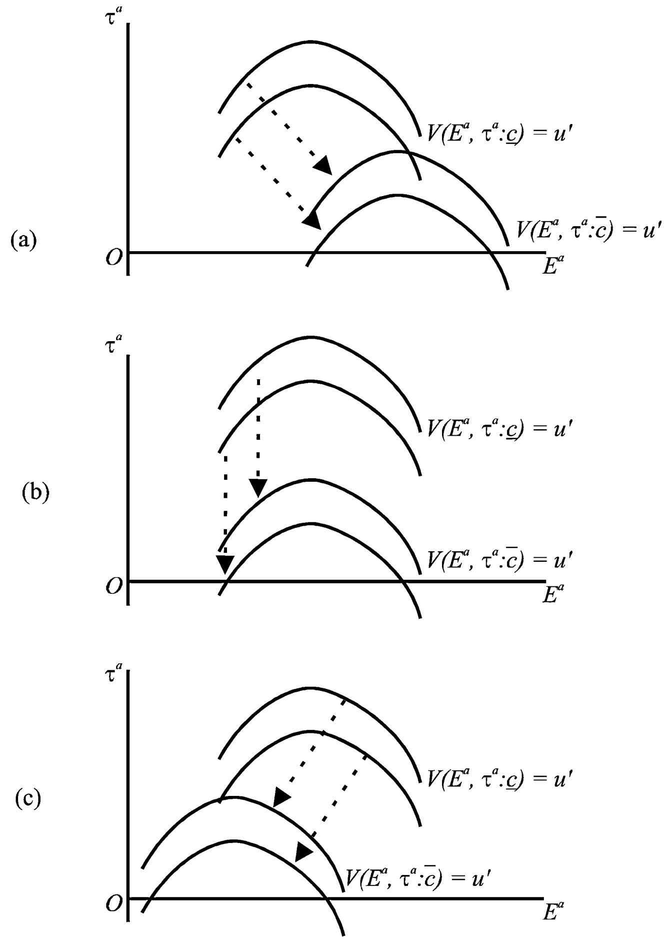

Figure 2: Shifts in the Indifference Map

Substituting these expressions into (12), we obtain

$$\frac{\partial \alpha(E, \tau \mid c)}{\partial c} >, =, < 0 \quad \text{according to whether} \quad \eta_{QQ} + \eta_{QR} >, =, \text{or}, < -1. \tag{13}$$

We can infer from this that an increase in the unit cost shifts the indifference map as portrayed by Panels (a)–(c) of Figure 2 according to whether the sum of the elasticities, $\eta_{QQ} + \eta_{QR}$, exceeds or equals -1. We will consider the following three cases:

Case I ["Low" elasticities]: $\eta_{QQ} + \eta_{QR} > -1$ everywhere. For any given value of E, an increase in c shifts the preference map down and to the right. Satisfaction of this local condition everywhere is sufficient for $E(\overline{c}, \alpha) > E(\underline{c}, \alpha)$.

Case II [Unit elasticities]: $\eta_{QQ} + \eta_{QR} = -1$ everywhere. An increase in c shifts the preference map vertically downwards. If this condition is satisfied, $E(\overline{c}, \alpha) = E(\underline{c}, \alpha)$.

Case III ["High" elasticities]: $\eta_{QQ} + \eta_{QR} < -1$ everywhere. For any given value of E, an increase in c shifts the preference map down and to the left. Hence, $E(\overline{c}, \alpha) < E(\underline{c}, \alpha)$.

The Center's Problem

Appealing to the revelation principle, we can think of the incompletely informed center as offering jurisdiction a the menu $[(\underline{E}^a, \underline{\tau}^a), (\overline{E}^a, \overline{\tau}^a)]$, which must satisfy the incentive compatibility (IC) requirements for both the low cost and the high cost type. The formal problem facing the center is

$$\begin{aligned}\underset{\underline{E}^a, \overline{E}^a, E^b, \underline{\tau}^a, \overline{\tau}^a}{\text{Maximize}} \ & \pi V(\underline{E}^a, \underline{\tau}^a \mid \underline{c}) + (1-\pi) V(\overline{E}^a, \overline{\tau}^a \mid \overline{c}) \\ & + \pi V(E^b, \underline{\tau}^a) + (1-\pi) V(E^b, \overline{\tau}^a)\end{aligned} \tag{14}$$

subject to:

$$V(\underline{E}^a, \underline{\tau}^a \mid \underline{c}) \geq V(\overline{E}^a, \overline{\tau}^a \mid \underline{c}) \quad [\underline{\mu} \geq 0] \tag{15}$$

$$V(\overline{E}^a, \overline{\tau}^a \mid \overline{c}) \geq V(\underline{E}^a, \underline{\tau}^a \mid \overline{c}) \quad [\overline{\mu} \geq 0]. \tag{16}$$

The terms in square brackets are the Lagrangian multipliers associated with the IC constraints.

It is convenient to begin by analyzing the solution to this problem for Case I, the "low elasticities" case. In Case I a higher value of c implies a preference map that is displaced downwards and to the right. In the absence of any transfer scheme, the chosen level of expenditure on public goods in jurisdiction a will be $\underline{E}^{a*}$ in the event of realizing a low cost level, and $\overline{E}^{a*}$ in the event of realizing a high cost level. These are the pairs $(\underline{E}^{a*}, 0)$ and $(\overline{E}^{a*}, 0)$ in both panels of Figure 3. The allocation attained in the presence of a transfer scheme under complete information—which we shall call the "first best allocation"—are the pairs $(\underline{E}^{a*}, \underline{\tau}^{a*})$ and $(\overline{E}^{a*}, \overline{\tau}^{a*})$, where the values of the transfers are determined by equation (7) in the light of the realized value of c^a.

It is straightforward to show that the first best allocation may be incentive compatible. The first best allocation always satisfy IC constraint (16) because: (i) $V(\overline{E}^{a*}, \overline{\tau}^a \mid \overline{c}) \geq V(\underline{E}^{a*}, \overline{\tau}^a \mid \overline{c})$ from the definition of $\overline{E}^{a*}$; and (ii) $V(\underline{E}^{a*}, \overline{\tau}^{a*} \mid \overline{c}) \geq V(\underline{E}^{a*}, \underline{\tau}^{a*} \mid \overline{c})$ from the fact that $\overline{\tau}^{a*} < \underline{\tau}^{a*}$. The first best allocation may also satisfy IC constraint (15). Figure 3(a), for example, shows a situation in which the first best allocation satisfies (15) and is therefore incentive compatible.

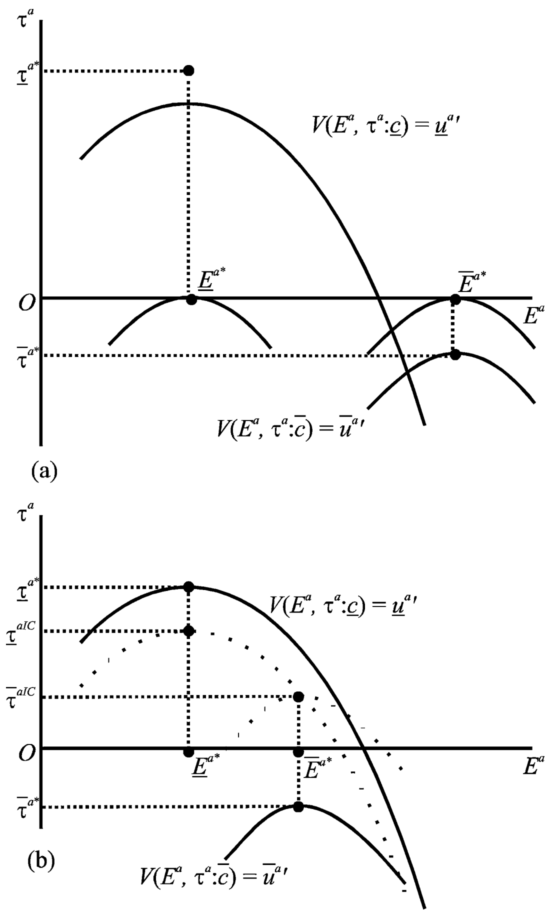

Figure 3: First Best Allocation: Incentive Compatible (a); Incentive Incompatible (b)

Figure 3(b), however, shows a situation in which (15) is violated by the first best allocation, since a low cost type would prefer $(\overline{E}^{a*}, \overline{\tau}^{a*})$ over $(\underline{E}^{a*}, \underline{\tau}^{a*})$. To summarize:

PROPOSITION 2: *In the presence of incomplete information, the first best allocation satisfies the incentive compatibility constraint for the high cost type. It may, or may not, satisfy the incentive compatibility constraint for the low cost type.*

We claim, and later prove, that constraint (16) can be ignored in the computation of the incentive compatible optimum, since it is always satisfied. Ignoring (16), consider the first-order conditions for the center's problem when the incentive compatibility condition (15) binds:

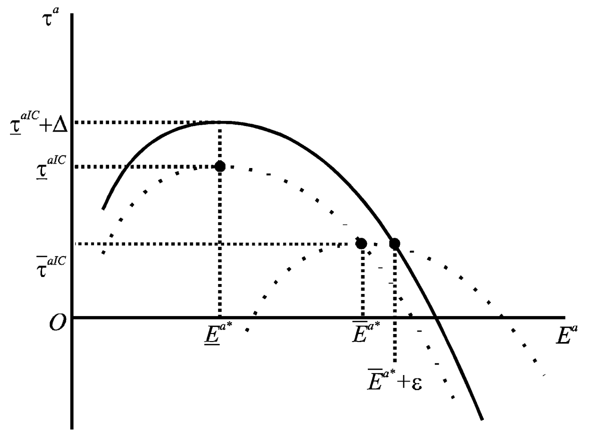

Figure 4: Optimal Incentive Compatible Allocation in the Low Elasticity Case

$$[\text{wrt}\,\underline{E}^a]\quad \pi\frac{\partial V(\underline{E}^a,\underline{\tau}^a\,|\,\underline{c})}{\partial \underline{E}^a}+\underline{\mu}\frac{\partial V(\underline{E}^a,\underline{\tau}^a\,|\,\underline{c})}{\partial \underline{E}^a}=0\Rightarrow\frac{\partial V(\underline{E}^a,\underline{\tau}^a\,|\,\underline{c})}{\partial \underline{E}^a}=0,$$

$$[\text{wrt}\,\overline{E}^a]\quad (1-\pi)\frac{\partial V(\overline{E}^a,\overline{\tau}^a\,|\,\overline{c})}{\partial \overline{E}^a}-\underline{\mu}\frac{\partial V(\overline{E}^a,\overline{\tau}^a\,|\,\underline{c})}{\partial \overline{E}^a}=0,$$

$$[\text{wrt}\,E^b]\quad \pi\frac{\partial V(E^b,\underline{\tau}^a)}{\partial E^b}+(1-\pi)\frac{\partial V(E^b,\overline{\tau}^a)}{\partial E^b}=0$$

$$\Rightarrow\frac{\partial V(E^b,\underline{\tau}^a)}{\partial E^b}=\frac{\partial V(E^b,\overline{\tau}^a)}{\partial E^b}=0,$$

$$[\text{wrt}\,\underline{\tau}^a]\quad \pi\left(\frac{\partial V(\underline{E}^a,\underline{\tau}^a\,|\,\underline{c})}{\partial \underline{\tau}^a}+\frac{\partial V(E^b,\underline{\tau}^a)}{\partial \underline{\tau}^a}\right)+\underline{\mu}\frac{\partial V(\underline{E}^a,\underline{\tau}^a\,|\,\underline{c})}{\partial \underline{\tau}^a}=0,$$

$$[\text{wrt}\,\overline{\tau}^a]\quad (1-\pi)\left(\frac{\partial V(\overline{E}^a,\overline{\tau}^a\,|\,\overline{c})}{\partial \overline{\tau}^a}+\frac{\partial V(E^b,\overline{\tau}^a)}{\partial \overline{\tau}^a}\right)-\underline{\mu}\frac{\partial V(\overline{E}^a,\overline{\tau}^a\,|\,\underline{c})}{\partial \overline{\tau}^a}=0.$$

Applying the envelope theorem, we can rewrite these equations in a more intuitive way:

$$\alpha(\underline{E}^a,\underline{\tau}^a\,|\,\underline{c})=0\quad\Rightarrow \underline{Q}^a=\underline{Q}^{a*},\underline{R}^a=\underline{R}^{a*},\tag{17}$$

$$(1-\pi)\,u'_{a\overline{c}}\,\alpha(\overline{E}^a,\overline{\tau}^a\,|\,\overline{c})-\underline{\mu}\,u'_{a\underline{c}}\,\alpha(\overline{E}^a,\overline{\tau}^a\,|\,\underline{c})=0,\tag{18}$$

$$\alpha(E^b,\underline{\tau}^a)=\alpha(E^b,\overline{\tau}^a)=0\quad\Rightarrow Q^b=Q^{b*},R^b=R^{b*},\tag{19}$$

$$\pi(-u'_{a\underline{c}}+u'_{b\underline{c}})-\underline{\mu}\,u'_{a\underline{c}}=0,\tag{20}$$

$$(1-\pi)(-u'_{a\overline{c}}+u'_{b\overline{c}})+\underline{\mu}\,u'_{a\underline{c}}=0.\tag{21}$$

where $u'_{j\bar{c}}$ is simply the value of the derivative of $u(\cdot)$ with respect to its argument, income, evaluated for jurisdiction j at the allocation that it enjoys when jurisdiction a realizes the unit cost level $\bar{c}$, and so on.

Equation (17) tells us that the optimal incentive compatible mechanism does not distort the Samuelson conditions for the low cost type. This represents the standard "no distortion at the top" result of contract theory in our framework. Equation (19) states that the Samuelson conditions for jurisdiction b should not be distorted. Equations (20) and (21) are the conditions that determine the optimal incentive compatible transfers. It is important to note that the first best transfers are not incentive compatible. The optimal incentive compatible transfers fall short from equating marginal utilities of income across jurisdictions in both states of nature. To see this, note that equations (20) and (21) imply that

$$\frac{\pi\,(u'_{b\underline{c}} - u'_{a\underline{c}})}{u'_{a\underline{c}}} = \frac{(1-\pi)\,(u'_{a\underline{c}} - u'_{b\underline{c}})}{u'_{a\underline{c}}} = \underline{\mu} > 0.$$

Figure (3b) depicts incentive compatible transfers $\underline{\tau}^{aIC}$ and $\bar{\tau}^{aIC}$. The pairs $(\underline{E}^{a*}, \underline{\tau}^{aIC})$ and $(\overline{E}^{a*}, \bar{\tau}^{aIC})$ are incentive compatible, but not optimal. As we explain below, the incentive compatible optimum distorts the expenditure level experienced by the high cost type in order to reduce the informational rent accrued by the low cost type.

We now turn to equation (18). Since the IC constraint (15) is binding, in Figure 3(b) the expenditure/transfer pairs are on the same indifference curve for the low cost type—see the dotted indifference curve for the low cost type. The shapes of the indifference curves, together with the fact that the incentive compatibility is binding for the low cost type but not for the high cost type, imply that $\alpha(\overline{E}^a, \bar{\tau}^a \mid \underline{c})$ is negative at the optimal allocation. Equation (18) implies that $\alpha(\overline{E}^a, \bar{\tau}^a \mid \bar{c})$ and $\alpha(\overline{E}^a, \bar{\tau}^a \mid \underline{c})$ are of the same sign. Thus $\alpha(\overline{E}^a, \bar{\tau}^a \mid \bar{c})$ is also negative. It follows that the optimal level of expenditure, $\overline{E}^{a**}$, faced by the high cost type exceeds its first best level.

The intuition for this result is shown in Figure 4, which reproduces some of the information shown in Figure 3(b). The incentive compatible transfer $\underline{\tau}^{aIC}$ is less than the first best level. However, starting from the allocation $(\overline{E}^{a*}, \bar{\tau}^{aIC})$, consider the implications of a slight increase in the level of expenditure from $\overline{E}^{a*}$ to, say, $\overline{E}^{a*} + \varepsilon$. A sufficiently small increase ε imposes only a second-order loss of social welfare in the event of jurisdiction a realizing a high unit cost. But such an increase permits a relaxation of the incentive compatibility constraint for the low cost type, thereby allowing an increase in the transfer that can be taken from jurisdiction a in the event of its realizing a low unit cost. This has a first-order beneficial effect on the center's objective. As the level of expenditure is raised further above $\overline{E}^{a*}$, the distortion implied by this "excessive" expenditure generates inefficiency, and $\overline{E}^{a**} \equiv \overline{E}^{a*} + \varepsilon$ is the level at which the direct efficiency cost associated with a further increment of total expenditure by the high cost type just cancels out the indirect gain permitted by the resulting relaxation of the incentive compatibility constraint for the low cost type, implying fulfillment of the first-order conditions for optimality.

We can now prove that constraint (16) is satisfied in the incentive compatible optimum, which is characterized by pairs $(\underline{E}^{a*}, \underline{\tau}^{aIC} + \Delta)$ and $(\overline{E}^{a**}, \overline{\tau}^{aIC})$, with the help of Figures 1 and 4. Consider the family of indifference curves for the high cost type in Figure 4. The pair $(\underline{E}^{a*}, \underline{\tau}^{aIC} + \Delta)$ would lie on an indifference curve—not shown in the figure—located above from the indifference curve on which we find the pair $(\overline{E}^{a**}, \overline{\tau}^{aIC})$. Hence, (16) is satisfied slack in the optimum because, as Figure 1 clearly demonstrates, the latter allocation yields greater utility than the former one.[1]

The implied levels of provision of the public goods by the high cost type can be determined by looking at equations (9) and (10). These show that, by comparison with the first-best allocation, the supply prices of both Q and R have fallen relative to that of the private good, but that ρ_Q/ρ_R is unchanged. Thus, by comparison with the first-best allocation, the incentive compatible allocation will involve substitution out of the private good into the local public goods. However, there also may be substitution between Q and R. It may be shown that

$$\frac{\partial Q}{\partial \alpha} = \frac{Q}{(1+\alpha)}(\eta_{QQ} + \eta_{QR}) \quad \text{and} \quad \frac{\partial R}{\partial \alpha} = \frac{R}{(1+\alpha)}(\eta_{RR} + \eta_{RQ}).$$

We cannot be absolutely certain on *a priori* grounds whether both Q and R will exceed their first-best levels, or whether one will fall short. We can say, however, that at least one of them will exceed its first-best level, since $\overline{E}^{a**} \equiv \overline{c}\overline{Q}^{a**} + \overline{R}^{a**} > \overline{c}\overline{Q}^{a*} + \overline{R}^{a*} \equiv \overline{E}^{a*}$. Clearly the own-price elasticities will tend to make the responses negative. However, our assumptions concerning the function $f(Q, R)$ do not rule out the possibilities that either $\eta_{QR} > -\eta_{QQ}$, in which case a negative value of α implies a lower value of Q, or that $\eta_{RQ} > -\eta_{RR}$ in which case it is R whose provision level in the incomplete information setting falls short of its first-best level. If the cross-elasticities η_{RQ} and η_{QR} are not too large, then both public goods will be "over-provided" by comparison with the first-best allocation. We can summarize our discussion of the low elasticity case as follows:

PROPOSITION 3: *Suppose the incentive compatibility constraint (15) binds, and the public goods are poor substitutes* $[\eta_{QQ} + \eta_{QR} > -1]$. *Then*

(i) *The transfers between the jurisdictions fall short of their complete information levels.*

(ii) *When jurisdiction a realizes a low cost, Q and R equal their complete information levels.*

[1]A similar argument can be used to prove that constraint (16) is satisfied slack in Case III. Hence, for the sake of brevity, we will omit the proof. As for Case II, both constraints (15) and (16) bind in the optimum and no self-selection is possible.

(iii) *In the event of jurisdiction a realizing a high cost, aggregate expenditure on Q and R exceeds its complete information level.*

(iv) *Depending on the precise magnitudes of the demand elasticities, expenditure on either Q, or R, or both, exceeds its complete information level.*

Now consider Case II, the "unit elasticities" case. It implies that the indifference map associated with the high value of c is simply a vertical displacement of the preference map associated with the low value. It follows that $\alpha(\overline{E}^a, \overline{\tau}^a \mid \overline{c}) = \alpha(\overline{E}^a, \overline{\tau}^a \mid \underline{c})$. However, the first-order conditions require that

$$\alpha(\overline{E}^a, \overline{\tau}^a \mid \overline{c}) = \left(\frac{\mu u'_{a\underline{c}}}{(1-\pi)\, u'_{a\overline{c}}}\right)\alpha(\overline{E}^a, \overline{\tau}^a \mid \underline{c}).$$

For both conditions to hold, we must have $\alpha(\overline{E}^a, \overline{\tau}^a \mid \overline{c}) = \alpha(\overline{E}^a, \overline{\tau}^a \mid \underline{c}) = 0$. We already know that $\alpha(\underline{E}^a, \underline{\tau}^a \mid \overline{c}) = \alpha(\underline{E}^a, \underline{\tau}^a \mid \underline{c}) = 0$. In short, there should be no distortion of the Samuelson conditions. However, this does not imply that the allocation is first best. Rather, it reflects the fact that the center has no scope for exploiting self-selection in order to identify cost types. The transfer cannot profitably be conditioned on the expenditure level, since the latter contains no useful information. It must instead be a non-contingent transfer: $\overline{\tau}^a = \underline{\tau}^a$. Maximizing (14) with respect to the transfers subject to $\overline{\tau}^a = \underline{\tau}^a$ yields the first-order condition $u'_b = \pi u'_{a\underline{c}} + (1-\pi)\, u'_{a\overline{c}}$, which deviates from the first best conditions (5c). To summarize this case:

PROPOSITION 4: *Suppose the incentive compatibility constraint (15) binds and there are unit elasticities $[\eta_{QQ} + \eta_{QR} = -1]$. Then, at the information-constrained optimum,*

(i) *Aggregate expenditure on public goods provides no information to allow the center to exploit self-selection. There is a noncontingent transfer, the magnitude of which equates the realized marginal utility in jurisdiction b to the expected marginal utility in jurisdiction a.*

(ii) *Whichever cost level is realized in jurisdiction a, Samuelson's optimality conditions are satisfied for both public goods.*

The closer is the sum of demand elasticities to the value minus one, the less scope there is for the center to exploit self-selection in order to mitigate the information problem. Empirically, this suggests that we would not expect transfers to be contingent on expenditure levels in situations where the total expenditure level E is relatively uninformative. The observation by Bergstrom, Rubinfeld, and Shapiro (1982) to which we referred in our introduction suggests that this situation may be empirically relevant in some settings.

Finally, consider Case III, the "high elasticities" case, when $\eta_{QQ} + \eta_{QR} < -1$. The reasoning for this case is similar to that used for the case involving "low elasticities," with some significant reversals of sign. As in that case, the first best allocation may or may not be incentive compatible. As before, the allocation that emerges when a low cost is realized involves no distortion of

Q or R away from their first best levels. Now recall that, when $\eta_{QQ} + \eta_{QR} < -1$, the situation is the one shown in Figure 2(c) and $\overline{E}^I \leq \underline{E}^I$. When condition (15) binds, the optimum requires that both $\alpha(\overline{E}^a, \overline{\tau}^a \mid \overline{c})$ and $\alpha(\overline{E}^a, \overline{\tau}^a \mid \underline{c})$ be positive. Hence, total expenditure $\overline{E}^a$ is lower than in the first best. The implications for the levels of Q and R are easily inferred, with appropriate sign changes.

PROPOSITION 5: *Suppose the incentive compatibility constraint (15) binds, and the public goods are close substitutes* $[\eta_{QQ} + \eta_{QR} < -1]$. *Then*

(i) *The transfers between the jurisdictions fall short of their complete information levels.*

(ii) *When jurisdiction a realizes a low cost, Q and R equal their complete information levels.*

(iii) *In the event of jurisdiction a realizing a high cost, aggregate expenditure on Q and R falls short of its complete information level.*

(iv) *Depending on the precise magnitudes of the demand elasticities for the public goods, either Q, or R, or both, falls short of their complete information levels.*

In sum, when the jurisdiction with private information is a low cost type, there is no distortion of expenditure level away from the complete information allocation, the transfer paid by a low cost type is less than under complete information, and the low cost type earns an informational rent. Whether, in its attempt to reduce this rent, the center should allow the high cost jurisdiction to spend more or less on public goods than in the complete information setting, however, depends on the numerical values of the relevant demand responses.

3. Concluding Remarks

Higher levels of government of federal systems often undertake substantial redistribution of real resources between those at lower levels. Even without factor mobility and explicit inter-jurisdiction spillovers, such transfers can be interpreted, not simply as redistribution, but as providing a certain level of insurance, with jurisdictions receiving a transfer when, according to some criterion, they experience a bad state of nature and paying a transfer when they experience a good state. Whatever their interpretation or intention, however, transfers will generally have to be determined without as much knowledge of the differences between jurisdictions that give rise to them as has typically been assumed. Transfer policies therefore have to use information about observable magnitudes. Existing literature persuasively demonstrates that the levels of many publicly provided services—such as education, health, police protection—are not readily verifiable by the higher level government. We have investigated the implications of conditioning transfers on the total observed expenditure incurred in providing a service, or an aggregate of such services. To the extent to which jurisdictions can influence this cost by their decisions concerning their mix of publicly provided goods, a moral hazard problem arises, and the center may face binding information constraints that imply that, by comparison with the complete information allocation, jurisdictions' public good levels may be distorted. Whether such goods are likely to be over- or under-provided by comparison with the Samuelson optimality condition depends on demand elasticities for the public goods in question. Indeed, the values of these elasticities will determine whether aggregate cost provides information that can be used to condition transfers. Our analysis, therefore, illustrates the potential for future empirical research to shed light on whether local public goods are under- or over-provided when fiscal equalization programs are adopted to reduce fiscal disparities across jurisdictions. Perhaps, the method introduced by Bergstrom, Rubinfeld, and Shapiro (1982) of estimating demand functions for local school expenditures from survey data may prove very useful in this regard. Although their findings for the price elasticity of demand seem to support our "low elasticity case," it would be very interesting to extend their analysis to account for the impact of public school finance equalization on such price elasticity.

This is very much a first step in modeling these issues. We intend to introduce labor mobility and also explicit spillovers across jurisdiction borders. Both of these considerations may also provide efficiency motives for undertaking transfers between jurisdictions.

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