Economic and environmental impacts of demographic changes in a life-cycle model

Tsendsuren Batsuuri
Crawford School of Public Policy
The Australian National University

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Declaration

I declare that this thesis is my own work and has not been submitted for another degree or diploma at any university or other higher education institution. It does not contain any material previously published and or written by another person unless it is duly noted with appropriate referencing.

Tsendsuren Batsuuri

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Abstract

This thesis develops a tractable life-cycle model with rich demographic details to investigate the impact of demographic changes on the macroeconomy, external imbalances, and countries' carbon emissions. In doing so, it makes two contributions to the literature. First, it offers a tractable life-cycle model that can capture demographic dividends in both the short and long runs, making it a suitable framework to incorporate into large-scale economic models used for climate policy. Second, it contributes to the climate economics literature by emphasizing the importance of using life-cycle models for long-run projections when countries are under significant demographic changes.

The thesis is comprised of three interrelated chapters. The second chapter develops a tractable life-cycle model that captures demographic dividends through three distinct channels: changing productivity with age, the time cost of children, and life-expectancy changes. It then combines this framework with an energy-dependent production function used widely within the climate economics literature to investigate the impact of demographic changes on economic growth, factor prices, and carbon emissions in the short and long run. The results show that the effects of demographic changes on emissions can significantly depend on projected fertility and mortality patterns. The annual change in economic growth due to demographic dividends may be small. Still, demographic dividends can accumulate over many years, causing significant differences in the long run, with implications for models that inform climate policy. Models that can't consider the demographic dividends due to changes in fertility and mortality may underestimate a country's baseline emissions when fertility goes down and overestimate them when fertility goes up.

The third chapter further develops the model by incorporating the consumption cost of children along with pay-as-you-go pensions and government expenditure. Using this framework, the third chapter investigates the importance of child dependency (that is, consumption and time cost of children) on the economy and external imbalances between two large countries, both during demographic changes and at steady-state. It finds that some key parameters determining factor prices in the model are endogenous to model structure. Specifically, it shows that the parameters must be chosen differently across models with and without child dependency in order to start from the same interest rate in the steady-state. Different calibration affects the magnitude of the transition dynamics of all models. These results show how important it is to use realistic OLG models to explain the past and predict the future.

The fourth chapter combines the frameworks developed in the second and third chapters by merging a two-country life-cycle model with an energy-dependent production function to investigate the sensitivity of carbon emissions baselines to alternative demographic outlooks across countries. Comparing carbon emissions mitigation efforts can be

misleading if countries' baseline emissions trajectories do not account for demographic dividends and spillovers due to unsynchronized demographic changes and asymmetric institutions. Through capital flows, differences in the timing, speed, and magnitude of demographic changes can reduce the emissions baseline in one country while increasing it in another relative to the baseline with no spillovers. Furthermore, differences in institutions such as pension and social security systems amplify spillover impacts. Models that do not consider the effect of demographic changes and institutions on the economy run the risk of underestimating or overestimating countries' carbon emission reduction efforts. So, this paper suggests using life-cycle models in studies that intend to compare countries' carbon emissions mitigation efforts. It also offers a suitable modeling framework.

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Chapter 1

Introduction

The world has been undergoing rapid demographic changes since the 1950s, driven by shifts in fertility and mortality patterns. While research on the economic impact of these demographic changes continues to expand, methodological constraints still limit the incorporation of demographic considerations into structural economic models used for climate policy. This presents severe shortcomings. Because understanding changes shaping the future global economy and the world is crucial to planning policies to mitigate climate change. Given that countries are undergoing profound demographic changes, neglecting them in economic models used for climate policy limits its usefulness and reduces the opportunity to look at shared solutions for multiple problems, namely population aging and climate change.

One main challenge for climate policy analysis is the uncertainty around future economic growth and emissions (Gillingham et al., 2018; Christensen et al., 2018; Morgan, 2018; McKibbin et al., 2009). Economists try to capture as many scenarios as possible by using multiple models that are fundamentally different (i.e., based on varying assumptions) to make robust analyses (Barron et al., 2018; Aldy et al., 2017). Despite this awareness and the effort to use multiple models, life-cycle OLG models, which is the main framework to study the impact of demographic changes, are rarely used in climate policy models due to methodological issues.[1] Specifically, two main analytical tools used to model households' intertemporal consumption and savings decisions are the infinitely lived representative agent (ILA) framework and the finitely lived overlapping generations (OLG) framework.[2] The latter is mainly used to examine how demographic changes affect the economy, while the former is more often used for climate policy models. Moreover, OLG models provide different implications than the ILA models on many fronts, including the effect of government debt and deficit on savings and the persistence of trade and capital flows (Blanchard, 1985; Buiter, 1981). These differences together with demographic changes that countries are undergoing make it an appealing framework to be considered together with the ILA framework.

[1]For example, Bryant and McKibbin (1998) argues that numerical Overlapping Generations (OLG) models which track individuals by age are complex and not suitable for incorporation into models used for climate policy, which tend to include many sectors and countries. In addition, they may have issues of overfitting (i.e., the fallacy of composition), where models that can fit microeconomic data well can give misleading results at the macro level.

[2]See figure E.1 for modeling households' consumption and savings in the general equilibrium, different types of OLG models, and their strengths and weaknesses.

Thus, this thesis develops a tractable life-cycle OLG model that can be incorporated into large-scale economic models used for climate policy.[3] In doing so, this thesis contributes to the literature that aims to use a tractable OLG model to study the impact of demographic changes on the global economy. For instance, one tractable OLG model based on the perpetual youth model of Blanchard (1985) and Weil (1989) has been successfully incorporated into large-scale policy models used for climate policy.[4] However, the Blanchard and Weil OLG framework lacks life-cycle features by assuming everyone has the same life expectancy and hence savings rate regardless of when they were born, therefore potentially underestimating the impact of demographics on aggregate consumption and savings (Gertler, 1999). Instead, this thesis is based on a tractable OLG model proposed by Gertler (1999), which has life-cycle features by assuming different savings rates across workers and retirees. It, however, extends the Gertler (1999) framework along several dimensions to make it better suited to analyze the impact of demographic changes on economic growth, energy use, and carbon emissions.

This thesis contains three interrelated chapters. Each chapter further develops the model to investigate the effect of demographic changes[5] on the macro-economy, external imbalances, and carbon emissions. Because the model is solved in annual frequency, it can show short-run and long-run transition dynamics. It is also highly tractable compared to its numerical counterparts, making it a suitable framework to incorporate into large-scale structural economic models used for climate policy. Therefore, this thesis makes two contributions to the literature. First, it contributes to the literature that aims to develop a suitable framework to study the implications of global demographic transition by offering a tractable life-cycle model that can capture demographic dividends[6] in the short and long run. Second, it contributes to the literature that emphasizes the importance of using life-cycle models in explaining the past (e.g., historic external imbalances) and projecting future economic and carbon emissions trends when countries are under significant demographic changes.

Specifically, chapter 2 extends the two life stage model developed by Gertler (1999) into three life stages to capture effects due to productivity differences between young and mature workers (empirically known as the age-earnings profile). It also adds children's

[3]Models used for climate policy tend to have many sectors and countries and are, therefore, untractable in the production structure. To simplify the problem, these models tend to assume a simpler structure regarding the household's intertemporal consumption and savings decision. Numerical OLG models, on the other hand, try to simplify the production side to solve the more complicated intertemporal consumption savings decision.

[4]For example, Bryant and McKibbin (2001) incorporated Blanchard (1985) and Weil (1989)'s OLG framework into a multi-sector multi-country dynamic intertemporal general equilibrium model of the global economy to study the impact of global demographic transitions. The resulting framework has been applied to study the impact of global demographic transition (Batini et al., 2006; Liu & McKibbin, 2022) and impact of demographic transitions in Japan (McKibbin, Nguyen, et al., 2004).

[5]Demographic change refers to changes in a country's fertility and mortality patterns. One specific demographic change known as demographic transition refers to a long-term decline in a country's fertility and mortality rates.

[6]Demographic dividends pertains to change in economic growth that comes from the changing age structure of the population (D. Bloom et al., 2003). In a country with a falling fertility rate, additional economic growth comes from higher workforce growth and higher savings following the reduction in total dependency rates. D. E. Bloom and Williamson (1998) found that almost one-third of the high economic growth experienced by East Asia between 1965 and 1990 was due to demographic dividends.

time costs to capture the impact of changing labor force participation following a change in child dependency ratio during demographic changes. It then combines the framework with an energy-dependent production function used widely within the climate economy literature to investigate the impact of demographic changes on economic growth, factor prices, energy use, and carbon emissions in the short and long run. This chapter finds that the effect of demographic changes on emissions can be large depending on projected fertility and mortality patterns. Models which cannot capture demographic dividends associated with a changing population age-structure following fertility and mortality shifts can underestimate a country's emissions baseline for fertility reductions and overestimate a country's emissions baseline for increased fertility. Moreover, while annual change in economic growth due to demographic dividends can be small, demographic dividends can accumulate over many years, causing significant differences in the long run with implications for climate economy models that deal with long-run projections. In addition, through changes in factor pricing, demographic changes also affect sectoral growth differently (i.e., capital-intensive sectors grow faster when the interest rates decline). This demonstrates the importance of applying this framework in climate policy models with many sectors of varying factor intensities.

Chapter 3 further extends the model in Chapter 2 to a two-country model and adds the consumption cost of children along with governmental institutions such as pay as you go pension and government expenditure financed by government borrowing and taxes. However, unlike the model in chapter 2, it assumes an aggregate production function without energy to focus the analysis on trade and current account imbalances across countries. Adding children's time and consumption cost is important given mounting empirical evidence that points to the significance of child dependency rates on aggregate savings and capital flow across countries. The model then investigates the difference these modifications make on the impact of demographic changes on external imbalances between two-large countries. In doing so, it addresses shortcomings of previous research, which neglected the impact of child dependency on external imbalances between large economies. This chapter shows two main results. First, including the consumption and time cost of children changes the model's steady-state aggregate consumption and savings, which has implications for model calibration. Second, child dependency amplifies the effect of asymmetric fertility changes on aggregate savings and capital flows across countries. These two findings show the importance of incorporating child dependency when investigating the impact of demographic changes on external imbalances.

Chapter 4 is closely related to chapters 2 and 3. It combines the frameworks developed in those chapters by merging the two-country life-cycle model with an energy-dependent production function to investigate the sensitivity of carbon emissions baselines to alternative demographic outlooks across countries. In one sense, the results in this chapter directly follow the results from the previous two chapters. However, it's main significance centers on two important topics in the climate economy literature. First, it contributes to the literature on uncertainty around future global emissions by showing the importance of demographic differences in determining the territorial emissions of countries through capital flows. Because different countries have different energy and carbon intensities, capital flows can change global emissions by relocating economic activities across different countries.

Second, chapter 4 makes an important contribution to the literature on the politi-

cal economy of climate change mitigation. Specifically, it shows that comparing carbon emissions mitigation efforts can be misleading if countries' baseline emissions trajectories do not account for demographic dividends and spillovers due to unsynchronized demographic changes and asymmetric institutions. Through capital flows, differences in the timing, speed, and magnitude of demographic changes can reduce the territorial emissions baseline in one country while increasing it in another relative to the baseline with no spillovers. Furthermore, differences in institutions such as pension and social security systems amplify spillover impacts. Models that do not consider the effect of demographic changes and institutions on the economy may underestimate one country's carbon emissions reduction effort while overestimating another's. Therefore, this chapter highlights the importance of using life-cycle models in studies that intend to compare countries' carbon mitigation efforts to increase their robustness and proposes a suitable modeling framework for such purposes.

Chapter 2

Economic and environmental implications of a demographic transition in a tractable life-cycle model

Abstract: This chapter explores the demographic transition's macroeconomic and environmental effects by using a tractable life-cycle model with an energy-dependent production function widely used for climate policy. Simulations indicate that demographic shifts can have significant short- and long-term impacts on a country's economic growth and carbon emissions, even under a modest and slow transition scenario. Specifically, during a demographic transition, carbon emissions may increase due to rising capital accumulation, increasing energy intensity of the economy, increasing labor force participation, and increasing prime-age workers in the workforce. Furthermore, the effect of the demographic transition is non-linear. It will eventually disappear as fertility and mortality patterns converge to low levels. Together, these findings have several policy implications. First, they suggest that comparing countries' efforts to cut their carbon emissions under the Paris Agreement will be challenging when the countries are in different stages of their demographic transition. Second, unlike what is frequently claimed in popular narratives, there may be no free lunch in reducing carbon emissions, as the decline in the population growth rate and population aging may not produce emissions reductions. This study demonstrates how crucial it is to include demographic factors in climate-economy models and suggests a suitable modeling approach to do so.

Keywords: Demographic transition, Carbon emissions, Life-cycle model, Energy dependent production function
JEL Codes: E2, J11, J13, J14, O13, Q43, C6

2.1 Introduction

Rapid demographic change is occurring worldwide, driven by changes in fertility and mortality patterns. Due to a long-term decline in birth and death rates known as a demographic transition, a population's age structure can significantly change to an older age structure (Tulchinsky & Varavikova, 2014). In practically all countries, fertility rates have decreased to replacement levels (below 2.1), except for a small number of nations, especially in Africa, where fertility rates are declining but are still significantly higher than replacement levels (UNDESA, 2019). Simultaneously, life expectancy is rising around the globe. By the middle of the century, life expectancy is predicted to increase in Africa by 11 years and in other regions by about 6-7 years (UNDESA, 2019). Moreover, the pace of change is much faster than in the past, leading to rapid population aging. It took about 100 years for the proportion of the population over 60 to rise from 10% to 20% in countries that started their demographic transition first, such as France, Sweden, and the United Kingdom. While Japan and China only needed two and three decades, respectively, to reach the same level of aging population (UNDESA, 2020).

While research on the economic impact of the global demographic transition is expanding, methodological challenges make it difficult to adequately include demographic effects into structural economic models that inform climate policy.[1] This is a severe shortcoming, given that the effectiveness of climate change policies will largely depend on their ability to foresee and respond to changes in the global economy and the world more broadly. The usefulness of economic models and the opportunity to consider common solutions to many problems including population aging and climate change, are constrained by the neglect of demographic effects in climate policy models. The numerical Overlapping Generations (OLG) models developed by Auerbach and Kotlikoff (1987), which are widely used for studying the impact of demographic changes and related policies, are computationally challenging (Laitner, 1990) and unsuitable for incorporating into models used for climate policy, which cover many sectors and countries (Bryant & McKibbin, 1998). Additionally, numerical OLG models may suffer from overfitting concerns (often referred to as "the fallacy of composition"), where models that perfectly match microeconomic data do not fit macroeconomic data well (Bryant & McKibbin, 1998).

Thus, this chapter develops a tractable[2] life-cycle overlapping generations (OLG) model suitable both to study the impact of demographic changes and to incorporate into large-scale economic models that are used to inform climate policy. As a result, this chapter adds to the body of research that aims to develop tractable OLG models that can be used to investigate how demographic changes impact the global economy and the environment. Another tractable OLG model based on the perpetual youth model of Blanchard (1985) and Weil (1989) has been successfully incorporated into large-scale policy models used for climate policy. For example, Bryant and McKibbin (2001) integrated

[1] For example, 11 different models have been used in a study by the Stanford Energy Modeling forum to assess how an economy-wide carbon price policy in the US will affect its economy, energy consumption, and emissions (Barron et al., 2018). The models used in this study differed mainly on production structure, with the household sector modeled either as a representative agent or at most dis-aggregated by income level rather than by age.

[2] The resulting framework has only a few state variables compared to the AK model's 108 state variables as shown by Laitner (1990). A key determinant of models' computational complexity is the number of their state variables (Reiter, 2015).

the Blanchard and Weil framework into a multi-sector, multi-country, dynamic, intertemporal general equilibrium model of the world economy in order to examine the economic consequences of global demographic changes. The resulting framework has been used to analyze the effects of the global demographic transition on Japan (McKibbin et al., 2004) and the world as a whole (Batini et al., 2006; Liu & McKibbin, 2022). However, the Blanchard and Weil framework has the drawback of not accounting for life-cycle savings because it assumes that everyone has the same life expectancy regardless of when they were born and, consequently, the same savings rate as well. As a result, this framework may understate the effect of demographics on aggregate consumption and savings, according to Gertler (1999). This study extends the tractable OLG model with life-cycle savings proposed by Gertler (1999) along several dimensions in order to improve its ability to analyze how demographic changes affect economic growth, energy consumption, and carbon emissions.

This chapter connect with few strands of existing literature. The first of these is the growing number of empirical studies that investigate the effect of demographics on carbon emissions. Econometric studies find that population age structure has a non-negligible impact on aggregate carbon emissions outcomes at the national and regional levels. However, these studies find conflicting results. Using panel data from 25 OECD countries between 1980 and 2009, Hassan and Salim (2015) finds that a 1% increase in the proportion of the population over 65 reduces per capita carbon emissions by 1.55 percent. In contrast, (Menz & Welsch, 2012) uses panel data from 26 OECD countries between 1960 and 2005 to show an increase in carbon emissions when the share of the population over 65 increases. On the other hand, Lugauer et al. (2014) uses panel data for 46 countries from 1990 to 2006 to find that carbon emissions go up by 6.1% for every percentage point increase in the number of people aged 35 to 49. Cole (2004) examines data on 86 countries between 1975 and 1998 to find a marginally significant positive relationship between the share of the population aged 14-64 and national level CO_2 emissions. Other country-specific studies include a study by Zhu and Peng (2012) who, after examining regional data for China between 1978-2008, finds that emissions are higher when the share of the working-age population is higher.[3] The results in this chapter can reconcile these conflicting empirical findings by showing that demographic transitions can have a non-linear impact on economic growth and emissions, with the effect declining over time.

In contrast, only a handful of modeling studies investigate the impact of demographic changes on carbon emissions. Using a dynamic general equilibrium model with overlapping dynasties of households, Dalton et al. (2008) finds that US carbon emissions can be lower by up to 40% by 2100 compared to a representative agent modeling case due to the reduction in the working-age population and population aging. Similarly, a study by Garau et al. (2013) finds that population aging reduces emissions by reducing the working-age population and economic growth in Italy, even as consumption becomes more energy-intensive. Studies that looked at global emissions rather than country-level emissions include one by Neill et al. (2010), who used the same model type as Dalton et al. (2008) but with the interlinked global economy, to analyze how demographic changes affect global carbon emissions. Neill et al. (2010) finds that emissions can be lowered by

[3]Liddle (2014) provides a comprehensive literature review on the impact of demographic variables on carbon emissions.

up to 20% by 2050 due to lower population growth and aging.

This research makes several contributions to the existing literature. First, this chapter investigates the impact of demographic changes on carbon emissions using a tractable life-cycle OLG that can capture demographic dividends. Specifically, this chapter extends a life cycle framework developed by Gertler (1999) along several dimensions to make it better suited to capture demographic dividends associated with a changing age structure of the population. First, the working life stage has been separated into the young and prime-age working life stages in order to capture the effects from the empirical age-earnings profile.[4] Similar research includes a study by Grafenhofer et al. (2007) who has divided the working life stage into eight distinct life stages to reflect better changing labor income by age. Second, the model in this chapter adds child dependency by incorporating the effect of an increased labor supply by workers following the decline in fertility rate, which is found to be important empirically.[5] Specifically, young workers' (between the ages of 20 and 39) labor supply is modeled as dependent on the child dependency ratio, similar to Gerlagh et al. (2017). Last but not least, to address the main research questions on how population shifts affect carbon emissions, the model used in this chapter explicitly includes an energy-dependent production function commonly used in climate-economy models.

This chapter shows that changes in the labor supply, shifts in the age distribution of the workforce, and adjustments in aggregate savings and capital per effective labor are three separate mechanisms through which demographic transitions affect economic growth. Specifically, the simulations demonstrate that changes in fertility rates have a time-varying, non-linear impact on short- to medium-run capital per effective labor. For instance, the declining fertility rate reduces the capital per effective labor in the short-run, as young workers' labor supply increases since they bear the time burden of raising children. However, the positive growth effect of a growing labor supply from young employees more than offsets the negative growth effect of declining capital per effective labor, resulting in a positive net impact of reduced fertility on overall economic growth. Additionally, approximately 20 years after the initial reduction in the fertility rate, the growth rate of workers starts to slow down, which results in an increase in capital per effective labor, which is beneficial for overall economic growth. On the other hand, life expectancy gains have a linear effect on capital per effective labor, with the impact being greatest in the early years when households are updating their life expectancy. Due to the rise in the old-age dependence ratio, the effect of the longer life expectancy on savings gradually decreases over time.

Explicitly modeling energy production yields an additional finding. The impact of the demographic transition on aggregate energy production and emissions differs from its impact on output. A reduction in the interest rate due to an increase in savings relative to workers causes the energy intensity of output to increase. This increase in aggregate emissions is offset to some extent by the decline in the energy sector's carbon intensity. The results show the importance of explicitly modeling the effect of fertility declines and life expectancy increases on aggregate economic growth and emissions.

[4]Data on age-earnings profiles for different countries can be accessed from the National Transfers Account (NTA) website at https://www.ntaccounts.org/.

[5]For example, D. E. Bloom et al. (2009) finds that each additional child reduces the female labor supply by an average of two years, using data from 97 countries between 1960 and 2000.

The chapter is organized as follows. Section 2 describes the model used to answer the research question on how population aging affects economic growth and emissions. Section 3 explains the calibration and hypothetical scenarios, with section 4 reporting the results and section 5 summarizing the conclusion.

2.2 The Model

This chapter extends a tractable life-cycle model proposed by Gertler (1999) along several dimensions to investigate the macroeconomic and environmental impact of demographic changes. First, to capture the effect of changes in the age structure of the workforce, the chapter divides the working life stage into two distinct phases, the young and the mature, with different labor productivity. Second, young workers' labor supply depends on the time cost of raising children. Third, an energy-dependent production function[6] widely used within climate-economy literature has been incorporated into the model to capture the impact of demographic changes on energy use and emissions. The model is solved in annual frequency, allowing one to see both short and long-run effects of demographic changes.

2.2.1 Population dynamics

Population dynamics are exogenous in the model. A set of probabilities that govern transition from one life stage to another, given in table 3.1, generates population dynamics in the model. These probabilities are independent of the age of an individual to facilitate aggregation. Time is discrete and goes forever, i.e., $t = 0, 1, 2, ...$ There are four types of individuals in the economy: children, young workers, mature workers, and retirees. Let

Table 2.1: Transition probabilities from period t-1 to t

	Children (t)	Young worker (t)	Mature worker (t)	Retiree (t)	Deceased (t)
Children (t-1)	$1-z$	z	0	0	0
Young worker (t-1)	0	1-θ	θ	0	0
Mature worker (t-1)	0	0	1-ω	ω	0
Retiree (t-1)	0	0	0	1-$\gamma_{t-1,t}$	$\gamma_{t-1,t}$
Deceased (t-1)	0	0	0	0	1

N_t^c denote the stock of children at time t. At time t, $(z + n_t - 1)N_{t-1}^c$ identical children are born which implies that the stock of children grow by $(n_t - 1)$[7] at time t.

$$N_t^c = (z + n_t - 1)N_{t-1}^c + (1 - z)N_{t-1}^c = n_t N_{t-1}^c \tag{2.1}$$

Let N_t^y denote the stock of young workers at time t. The stock of young workers at time t consists of children who became young workers at the beginning of time t (end of t-1) and young workers who didn't become a mature worker at the beginning of time t (end of t-1).

[6] see Van der Werf (2008))'s study for the empirical basis of the production functions used within climate, energy and economy literature.

[7] n_t includes 1. For example if growth rate is 1 percent, n_t is 1.01

$$N_t^y = zN_{t-1}^c + (1-\theta)N_{t-1}^y \tag{2.2}$$

Let's also denote children to younger worker ratio at time t by ψ_t^c:

$$\psi_t^c = \frac{N_t^y}{N_t^c} \tag{2.3}$$

The ratio of younger worker to children ratio evolves according to:

$$\psi_t^c = \frac{z}{n_t} + \frac{(1-\theta)(\psi_{c,t-1})}{n_t} \tag{2.4}$$

We can also define the growth rate of young workers as a function of the growth rate of children at time t as:

$$n_t^y = \frac{N_t^y - N_{t-1}^y}{N_{t-1}^y} = \frac{\psi_t^c(1+n_t)}{\psi_{t-1}^c} \tag{2.5}$$

Let N_t^o denote stock of mature or older workers at time t. The stock of older workers at time t consist of young workers who became mature at the beginning of time t (end of t-1) and mature workers who didn't retire at the beginning of time t (end of t-1).

$$N_t^o = \theta N_{t-1}^y + (1-\omega)N_{t-1}^o \tag{2.6}$$

Let's denote ratio of older to younger workers at time t by ψ_t^1:

$$\psi_t^1 = \frac{N_t^o}{N_t^y} \tag{2.7}$$

The ratio of older to younger workers evolves according to:

$$\psi_t^1 = \frac{\theta}{n_t^y} + \frac{(1-\omega)\psi_{t-1}^1}{n_t^y} \tag{2.8}$$

Let N_t^r denote the stock of retirees at time t. The stock of retirees at time t consists of older workers who retired at the beginning of time t (end of time t-1) and retirees who survived at the beginning of time t (end of time t-1).

$$N_t^r = \omega N_{t-1}^o + (1-\gamma_t)N_{t-1}^r \tag{2.9}$$

Let's denote the ratio of retirees to total number of mature and young workers at time t by ψ_t^2:

$$\psi_t^2 = \frac{N_t^r}{N_t^y + N_t^o} \tag{2.10}$$

The ratio of retirees to total number of older and young workers evolves according to:

$$\psi_t^2 = \frac{\omega\psi_{t-1}^1}{(\psi_t^1+1)n_t^y} + \frac{(1-\gamma_t)(\psi_{t-1}^1+1)\psi_{t-1}^2}{(\psi_t^1+1)n_t^y} \tag{2.11}$$

The growth rate of the workforce (total number of young and mature workers) can be estimated as:

$$n_t^{wf} = \frac{(N_t^o + N_t^y)}{(N_{t-1}^o + N_{t-1}^y)} - 1 = \frac{(1+\psi_t^1)n_t^y}{1+\psi_{t-1}^1} - 1 \tag{2.12}$$

Labor force in the economy as equal to number of mature workers and number of young workers in same effective units and it is given as:

$$N_t = \xi_t(1 - \frac{\lambda_c}{\psi_t^c})N_t^y + N_t^o = (\xi_t(1 - \frac{\lambda_c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{2.13}$$

Growth rate of the economy's labor force in efficiency unit:

$$n_t^{wf\xi} = \frac{\xi_t(1 - \frac{\lambda_c}{\psi_t^c})N_t^y + N_t^o}{\xi_{t-1}(1 - \frac{\lambda_c}{\psi_{t-1}^c})N_{t-1}^y + N_{t-1}^o} - 1 = \frac{(\xi_t(1 - \frac{\lambda_c}{\psi_t^c}) + \psi_t^1)n_t^y}{\xi_{t-1}(1 - \frac{\lambda_c}{\psi_{t-1}^c}) + \psi_{t-1}^1} \tag{2.14}$$

Because labor supply of young workers depend on child dependency ratio, it is given as:

$$N_t^{y\xi} = \xi_t(1 - \frac{\lambda_c}{\psi_t^c})N_t^y \tag{2.15}$$

Growth rate of effective labor supply of young workers are equal to:

$$n_t^{y\xi} = \frac{N_t^{y\xi} - N_{t-1}^{y\xi}}{N_{t-1}^{y\xi}} = \frac{\xi_t(1 - \frac{\lambda_c}{\psi_t^c})n_t^y}{\xi_{t-1}(1 - \frac{\lambda_c}{\psi_{t-1}^c})} \tag{2.16}$$

where λ_c reflects time cost of children as discussed in section 2.2.2.

Demographic shock process

The growth rate of the number of children and the mortality rate is exogenously determined following a demographic shock process used by Carvalho et al. (2016).

Growth rate of child dependants is specified as:

$$n_t = n_{ss}e^{u_{n,t} - v_{n,t}} \tag{2.17}$$

where n_{ss} growth rate of population in the long run when population age structure is stable.

$$u_{n,t} = \rho_{un}u_{n,t-1} + e_{n,t} \tag{2.18}$$

$$v_{n,t} = \rho_{vn}v_{n,t-1} + e_{n,t} \tag{2.19}$$

Death probability is specified as:

$$\gamma_t = \gamma_{ss}e^{u_{\gamma,t} - v_{\gamma,t}} \tag{2.20}$$

where γ_{ss} is the steady state mortality rate that correspond to the long-run stable population structure.

$$u_{\gamma,t} = \rho_{ug}u_{\gamma,t-1} + e_{\gamma,t} \tag{2.21}$$

$$v_{\gamma,t} = \rho_{vg}v_{\gamma,t-1} + e_{\gamma,t} \tag{2.22}$$

while $u_{i,t}$ and $v_{i,t}$ for $(i = \gamma, n)$ are a stationary autoregressive process with common shock $e_{i,t}$. Persistence parameters (ρ_j) for $(j = un, vn, ug, vg)$ and common shocks are chosen to mimic population growth rate and dependency ratio in the data.

2.2.2 Households

There are four types of individuals in the economy that are retirees, mature workers, young workers, and dependent children. All types of individuals, except children, maximize one's respective lifetime utility subject to one's intertemporal budget constraint. The utility of an individual is represented by the non-expected utility function proposed by Farmer (1990), which provide certainty-equivalent decision rule in the face of income risk that arises due to stochastic transition probability from one life stage to another. Different types of individuals differ by their respective intertemporal budget constraints where the superscript "i" denotes the individual type: r for retirees, o for mature workers, and y for young workers.

$$V_t^i = \left[(C_t^i)^\rho + \beta_i \mathbb{E}(V_{t+1}|i)^\rho\right]^{\frac{1}{\rho}} \tag{2.23}$$

where

$$\beta_i = \begin{cases} (1-\gamma_{t,t+1})\beta & \text{if i = r} \\ \beta & \text{if i = o,y} \end{cases}$$

$$\mathbb{E}(V_{t+1}|i) = \begin{cases} V_{t+1}^r & \text{if i = r} \\ (1-\omega)V_{t+1}^o + \omega V_{t+1}^r & \text{if i = o} \\ (1-\theta)V_{t+1}^y + \theta V_{t+1}^o & \text{if i = y} \end{cases}$$

Retirees

In addition to notation for whether a person is a retiree, a mature worker or a young worker, we can also differentiate individuals by their date of birth and the period when he/she transitioned to his/her next life stage. For example, j denotes the period when an individual became a young worker, k denotes the period when he or she became a mature worker, and s denotes the period when he or she became a retiree. Individuals differ by their accumulated assets, which depend on how much time an individual spent in each life stage. Retirees do not work and consume out of their asset income. A retiree's expected lifetime utility is given as a recursive value function:

$$V(A_t|rjks) = \max_{c_t} \left\{ [C_t^{rjks}]^\rho + \beta(1-\gamma_{t,t+1})\mathbb{E}_t V(A_{t+1}^{rjks})^\rho \right\}^{\frac{1}{\rho}} \tag{2.24}$$

subject to one's intertemporal budget constraint:

$$A_{t+1}^{rjks} = \frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} - C_t^{rjks} \tag{2.25}$$

with expected lifetime utility next period being:

$$\mathbb{E}_t V(A_{t+1}^{rjks}) = V(A_{t+1}^{rjks}) \tag{2.26}$$

where ρ is associated with intertemporal elasticity of substitution σ with $\sigma = \frac{1}{1-\rho}$. β is the subjective discount rate or time preference rate. C_t^{rjks} is the consumption at t of the retiree who was born in period j, become a mature worker at period k, and retired at

period s. $V(A_t|r)$ is the lifetime value function of a retiree with an asset of A_t at time t that is expressed in recursive form.

Since retirees can die at the end of the current period, the discount rate for the expected lifetime utility from the next period is augmented by retirees' survival probability $1-\gamma_t$. The assumed value function and associated intertemporal budget constraint imply, retirees do not value leaving bequests and are constrained to have 0 or positive assets at the end of their life. Furthermore, to eliminate unintentional bequest either positive or negative due to uncertain time of death, it is assumed there exist a fair-insurance company as in Yaari (1965) and Blanchard (1985). Under this setup, retirees who face death probability at the end of the current period turn their assets to a mutual fund that invests the proceeds at the beginning of the current period. The fund and the interest rate is distributed to the surviving retirees at the end of the current period in proportion to his initial contribution to the mutual fund. The existence of an insurance fund makes the return on assets of a retiree who turns his wealth to this mutual fund $\frac{R_t}{1-\gamma_{t-1,t}}$ where R being market interest rate.

Assets are measured at the beginning of each period while interest is paid at the end of each period. Therefore, intertemporal budget constraint of a retiree who faces probability of death at the end of current period becomes:

$$A_{t+1}^{rjks} = \frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} - C_t^{rjks} \tag{2.27}$$

First order necessary condition for the decision problem of a retiree gives the consumption euler equation of the form:

$$C_{t+1}^{rjks} = C_t^{rjks}(\beta R_{t+1})^{\sigma} \tag{2.28}$$

Furthermore, the consumption function for a retiree is conjectured as:

$$C_t^{rjks} = \epsilon_t \pi_t \Big(\frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} \Big) \tag{2.29}$$

Combining the euler equation with the conjectured consumption function yields solution to a retiree's decision problem in the form of the evolution of marginal propensity to consume out of wealth for a retiree as:

$$\epsilon_t \pi_t = 1 - \beta^{\sigma} R_{t+1}^{\sigma-1} (1-\gamma_{t,t+1}) \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}} \tag{2.30}$$

Mature workers

Mature workers supply one unit of labor inelastically and receive wage income W_t at the end of each period. Workers save and consume out of assets and labor income. Workers face uncertain time of retirement with $1-\omega$ is the probability to stay in the workforce in period t+1 of a mature worker at t while ω is the probability to retire in the beginning of next period. Risk of retirement will not complicate the decision rule of mature workers since they are risk neutral by the assumption in the form of value function being used here.

$$V(A_t|ojk) = \max_{c_t} \Big\{ [C_t^{ojk}]^{\rho} + \beta \mathbb{E}_t V(A_{t+1}^{ojk})^{\rho} \Big\}^{\frac{1}{\rho}} \tag{2.31}$$

subject to one's intertemporal budget constraint:

$$A_{t+1}^{ojk} = R_t A_t^{ojk} + W_t - C_t^{ojk} \tag{2.32}$$

where expected next period life-time value function is the weighted sum of life-time value function of being a worker at t+1 and being a retiree at t+1.

$$\mathbb{E}_t V(A_{t+1}^{ojk}) = (1-\omega) V(A_{t+1}^{ojk}) + \omega V(A_{t+1}^{rjks}) \tag{2.33}$$

As in the case of retirees, workers' assets are measured at the beginning of each period while the wage and the interest is paid at the end of each period. A worker's consumption expenditure is also incurred at the end of each period. Therefore, assets of a mature worker evolve according to:

$$A_{t+1}^{ojk} = R_t A_t^{ojk} + W_t - C_t^{ojk} \tag{2.34}$$

First order necessary condition for the decision problem of a mature worker gives the consumption euler equation of the form:

$$(1-\omega) C_{t+1}^{ojk} + \omega \Lambda_{t+1} C_{t+1}^{rjks} = \Big(R_{t+1} \Omega_{t+1} \beta \Big)^{\sigma} C_t^{ojk} \tag{2.35}$$

where $\Lambda_{t+1} = \epsilon_{t+1}^{\frac{\sigma}{1-\sigma}}$

Furthermore, the consumption function for a mature worker is conjectured as:

$$C_t^{ojk} = \pi_t \Big(R_t A_t^{ojk} + H_t^{ojk} \Big) \tag{2.36}$$

Combining the euler equation with the conjectured consumption function yields solution to a mature worker's decision problem in the form of the evolution of marginal propensity to consume out of wealth for a mature worker as:

$$\pi_t = 1 - \beta^{\sigma} (R_{t+1} \Omega_{t+1})^{\sigma - 1} \frac{\pi_t}{\pi_{t+1}} \tag{2.37}$$

where

$$\Omega_{t+1} = (1-\omega) + \omega \epsilon_{t+1}^{1/(1-\sigma)} \tag{2.38}$$

Human wealth is defined as the present value of expected life-time labor income which is the discounted sum of wages. Human wealth of a mature worker evolves according to:

$$H_t^{ojk} = W_t + (1-\omega) \frac{H_{t+1}^{ojk}}{R_{t+1} \Omega_{t+1}} \tag{2.39}$$

Young workers

Up till now, aside from changing the population dynamics described in section 3.1 and changing the individual identification notations to reflect added life stage, derivation of a retiree's and a mature worker's decision rules closely followed those in Gertler (1999). This section provides the solution and the derivation of a young worker's decision rule who not only has lower productivity compared to a mature worker but also bears the cost

of raising children. Child dependency is modeled as time spent out of work similar to the way it is modeled in Gerlagh et al. (2017). This extension makes the model described in Gertler (1999) a model better suited to study the impact of demographic transition with declining fertility rate and changing population age structure affecting female labor supply, aggregate productivity, and savings.

When transitioning from being a dependent child to being a young worker, a young worker starts without any assets. This means $A_t^{yj} = 0$ when j is current period. A young worker supplies 1 unit of labor inelastically and consumes out of assets and labor income. However, young workers' productivity level is lower compared to the productivity of mature workers due to inexperience. Besides, young workers bear the cost of raising children modeled as time spent out of work raising children determined by parameter λ_c. Therefore, the actual labor supply of a young worker is equal to their labor endowment minus time spent raising children. Because young workers' have lower productivity compared to mature workers, they receive a fraction of the wage of mature workers corresponding to their productivity level represented, here, by ξ .

With the above constraints reflected, a young worker's problem is to maximize one's life-time utility:

$$V(A_t|yj) = \max_{c_t} \left\{ [C_t^{yj}]^\rho + \beta \mathbb{E}_t V(A_{t+1}^{yj})^\rho \right\}^{\frac{1}{\rho}} \tag{2.40}$$

subject to one's intertemporal budget constraint:

$$A_{t+1}^{yj} = R_t A_t^{yj} + \xi_t (1 - \frac{\lambda_c}{\psi_t^c}) W_t - C_t^{yj} \tag{2.41}$$

where expected next period life-time value function is

$$\mathbb{E}_t V(A_{t+1}^{yj}) = (1-\theta) V(A_{t+1}^{yj}) + \theta V(A_{t+1}^{ojk)}) \tag{2.42}$$

First order necessary condition for the decision problem of a young worker gives the consumption euler equation of the form:

$$(1-\theta) C_{t+1}^{yj} + \theta \Lambda_{2,t+1} C_{t+1}^{ojk} = \Big(R_{t+1} \Omega_{2,t+1} \beta \Big)^\sigma C_t^{yj} \tag{2.43}$$

where $\Lambda_{2,t+1} = \epsilon_{2,t+1}^{\frac{\sigma}{\sigma-1}}$ Furthermore, the consumption function for a young worker is conjectured as:

$$C_t^{yj} = \epsilon_{2,t} \pi_t \Big(R_t A_t^{yj} + H_t^{yj} \Big) \tag{2.44}$$

Solution to a young worker's decision problem gives the evolution of marginal propensity to consume out of wealth for a young worker:

$$\epsilon_{2,t} \pi_t = 1 - \beta^\sigma (R_{t+1} \Omega_{2,t+1})^{\sigma-1} \frac{\epsilon_{2,t} \pi_t}{\epsilon_{2,t+1} \pi_{t+1}} \tag{2.45}$$

where

$$\Omega_{2,t+1} = (1-\theta) + \theta \epsilon_{2,t+1}^{1/(\sigma-1)} \tag{2.46}$$

$$H_t^{yj} = \xi_t (1 - \frac{\lambda_c}{\psi_t^c}) W_t + (1-\theta) \frac{H_{t+1}^{yj}}{R_{t+1} \Omega_{2,t+1}} + \theta \frac{H_{t+1}^{ojk} \epsilon_{2,t+1}^{\frac{1-\rho}{\rho}}}{R_{t+1} \Omega_{2,t+1}} \tag{2.47}$$

2.2.3 Aggregation

Because every person in retirement has the same life expectancy regardless of when he/she is retired or being born, they all have the same planning horizon implying everyone has the same marginal propensity to consume out of wealth. Therefore, it is straightforward to aggregate consumption of retirees as:

$$C_t^r = \epsilon_t \pi_t R_t A_t^r \tag{2.48}$$

with A_t^r equal to total assets held by retirees at time t. Similarly, everyone who is a mature worker also faces the same probability of retiring the next period making their planning horizon the same regardless of their age. Therefore, the aggregate consumption of mature workers are:

$$C_t^o = \pi_t (R_t A_t^o + H_t^o) \tag{2.49}$$

where A_t^o is total assets held by mature workers at time t and H_t^o is the present value of total lifetime labor income of those who are a mature worker at time t. Aggregate consumption of young workers is also:

$$C_t^y = \epsilon_{2,t} \pi_t (R_t A_t^y + H_t^y) \tag{2.50}$$

Where A_t^y is total assets held by young workers at time t and H_t^y is the present value of total lifetime labor income of those who are a young worker at time t.

Aggregate, present value of human wealth of mature workers who are alive at time t evolves according to:

$$H_t^o = W_t N_t^o + (1-\omega) \frac{H_{t+1}^o}{n_{t+1}^o R_{t+1} \Omega_{t+1}} \tag{2.51}$$

whereas, aggregate, present value of human wealth of young workers who are alive at time t evolves according to:

$$H_t^y = W_t \xi (1 - \frac{\lambda_c}{\psi_t^c}) N_t^y + (1-\theta) \frac{H_{t+1}^y}{R_{t+1} \Omega_{2,t+1} n_{t+1}^{y\xi}} + \theta \frac{H_{t+1}^o \epsilon_{2,t+1}^{\frac{1-\rho}{\rho}}}{R_{t+1} \Omega_{2,t+1} n_{t+1}^o} \tag{2.52}$$

with the economy-wide consumption function being

$$C_t = \pi_t \Big[A_t R_t \Big(1 + (\epsilon_t - 1)\lambda_t^1 + (\epsilon_{2,t} - 1)\lambda_t^2 \Big) + \epsilon_{2,t} H_t^y + H_t^o \Big] \tag{2.53}$$

where $\lambda_t = A_t^r / A_t$ is the fraction of total assets in the economy held by retirees, $\lambda_{2,t} = A_t^y / A_t$ is the fraction of total assets in the economy held by young workers.

Evolution of capital distribution between different generations follows:

$$\lambda_{t+1}^1 + \frac{\omega}{1-\theta} \lambda_{t+1}^2 = (1-\omega) R_t \lambda_t^1 \frac{A_t}{A_{t+1}} (1 - \epsilon_t \pi_t) + \omega \tag{2.54}$$

$$\lambda_{t+1}^2 = \frac{(1-\theta)((1-\epsilon_{2,t}\pi_t)\lambda_t^2 R_t A_t + W_t \xi_t (1 - \frac{\lambda_c}{\psi_t^c}) N_t^y - \epsilon_{2,t} \pi_t H_t^y)}{A_{t+1}} \tag{2.55}$$

2.2.4 Firms

There are three production sectors in the economy, a final output sector and two intermediate goods sector for clean and dirty energy inputs. The final output sector of the model resembles those in Gerlagh and Van der Zwaan (2003) who modeled final output production as CES aggregate of energy and the Cobb-Douglas composite of capital-labor services. The main appeal of this specification is its nesting of the Solow growth model as a special case that has been applied widely for both empirical and theoretical work to explain long-run economic growth. CES allows the elasticity of substitution between capital, labor composite, and energy to be different than one which is consistent with empirical evidence such as those in Kander et al. (2015), Van der Werf (2008) and Stern and Kander (2012). Energy service is assumed to be provided by both dirty and clean energy sources following specification in Golosov et al. (2014). Further, it is assumed that both dirty and clean energy is reproducible by combining capital and labor as in Balke and Brown (2018). Only dirty energy emits CO_2. The exhaustible nature of dirty energy is not explicitly modeled, however, it can be reflected as a negative technology shock in the dirty energy sector.

Final output producers

It is assumed that there is a representative firm producing output with energy and capital-labor composite with the technology represented by a production function[8].:

$$Y_t = \left(\upsilon_1((X_{t,L}N_{t,y})^{\alpha}K_{t,y}^{1-\alpha})^{\Psi} + (1-\upsilon_1)(X_{t,e}E_t)^{\Psi}\right)^{\frac{1}{\Psi}} \tag{2.56}$$

with labor augmenting technology growing at an exogenous rate of x_l.

$$X_{t+1,L} = (1+x_l)X_{t,L} \tag{2.57}$$

And energy augmenting technology is assumed to be constant along the balanced growth path, but can be changing due to a deterministic technology shock that follows an AR(1) process.

Let W_t denote the wage rate and r_t, the interest rate from period $t-1$ to t and $p_{t,e}$ the energy price. Firms problem is setup in a way that makes firms to maximize their profits for just one period. Firms' problem is to maximize its profit under perfectly competitive market taking $r_t, W_t, p_{t,e}$ as given.

$$\max_{K_{t,y},N_{t,y},E_t} \pi_{t,y} = Y_t - W_tN_{t,y} - r_tK_{t,y} - p_{t,e}E_t \tag{2.58}$$

with:

$$Y_t = \left(\upsilon_1((X_{t,L}N_{t,y})^{\alpha}K_{t,y}^{1-\alpha})^{\Psi} + (1-\upsilon_1)(X_{t,e}E_t)^{\Psi}\right)^{\frac{1}{\Psi}} \tag{4.3}$$

[8] Ψ is related to elasticity of substitution between Capital-Labor composite and Energy input via $\sigma_{KL,E} = \frac{1}{1-\Psi}$. $\Psi \in (-\infty, 0) \cup (0, 1]$. CES function includes 3 special cases under different Ψ values. When Ψ approaches -∞, $\sigma_{KL,E}$ becomes 0 and the production function becomes a Leontief production function with perfect complementarity of inputs, when Ψ approaches 1, $\sigma_{KL,E}$ approaches ∞ and inputs become perfect substitute in production. Lastly, when Ψ approaches 0, $\sigma_{KL,E}$ becomes 1 and the production function becomes Cobb-Douglas production function.

Solving firm's problem in the final output sector gives demand for labor, capital stock and energy in the final output sector:

$$\frac{\partial \pi_{t,y}}{\partial N_{t,y}} = 0 \Leftrightarrow \frac{\partial Y_t}{\partial N_{t,y}} - W_t = 0 \tag{2.59}$$

$$\frac{\partial \pi_{t,y}}{\partial K_{t,y}} = 0 \Leftrightarrow \frac{\partial Y_t}{\partial K_{t,y}} - r_t = 0 \tag{2.60}$$

$$\frac{\partial \pi_{t,y}}{\partial E_t} = 0 \Leftrightarrow \frac{\partial Y_t}{\partial E_t} - p_t = 0 \tag{2.61}$$

Energy production

Energy is assumed to be a CES aggregate of clean and dirty energy as in Golosov et al. (2014).

$$E_t = ((1-\eta_1)S_t^{\Phi} + \eta_1 O_t^{\Phi})^{\frac{1}{\Phi}} \tag{2.62}$$

This gives prices of clean and dirty energy in terms of aggregate energy price as:

$$E_t^{(1-\Phi)}((1-\eta_1)S_t^{\Phi-1})p_t = p_{t,s} \tag{2.63}$$

$$E_t^{(1-\Phi)}(\eta_1 O_t^{\Phi-1})p_t = p_{t,o} \tag{2.64}$$

Clean energy firms

Clean energy is assumed to be produced via

$$S_t = X_{t,s}(X_{t,L}N_{t,s})^{\alpha_2}K_{t,s}^{1-\alpha_2} \tag{2.65}$$

Clean energy production technology is stable along the BGP but can deviate from the steady state with a deterministic technology shock that follows an AR(1) process. Firms in the clean energy sector are maximizing profits in a competitive market taking $W_t, r_t, p_{t,s}$ as given.

$$\max_{K_{t,s},N_{t,s}} \pi_{t,s} = p_{t,s}S_t - W_t N_{t,s} - r_t K_{t,s} \tag{2.66}$$

with:

$$S_t = X_{t,s}(X_{t,L}N_{t,s})^{\alpha_2}K_{t,s}^{1-\alpha_2} \tag{2.67}$$

Solving firm's problem in the clean energy sector gives demand for labor and capital in the clean energy sector:

$$\frac{\partial \pi_{t,s}}{\partial N_{t,s}} = 0 \Leftrightarrow p_{t,s}\frac{\partial S_t}{\partial N_{t,s}} - W_t = 0 \tag{2.68}$$

$$\frac{\partial \pi_{t,s}}{\partial K_{t,s}} = 0 \Leftrightarrow p_{t,s}\frac{\partial S_t}{\partial K_{t,s}} - r_t = 0 \tag{2.69}$$

Dirty energy firms

Dirty energy is assumed to be produced via

$$O_t = X_{t,o}(X_{t,L}N_{t,o})^{\alpha_3}K_{t,o}^{1-\alpha_3} \tag{2.70}$$

Dirty energy production technology is stable along the BGP but can deviate from the steady state with a deterministic technology shock that follows an AR(1) process. Firms in the dirty energy sector are also assumed to be maximizing its profit in a competitive market taking prices $W_t, r_t, p_{t,o}$ as given. First order necessary condition for dirty energy firms gives demand for capital and labor in the dirty energy sector:

$$\max_{K_{t,o},N_{t,o}} \pi_{t,o} = p_{t,o}O_t - W_tN_{t,o} - r_tK_{t,o} \tag{2.71}$$

with:

$$O_t = X_{t,o}(X_{t,L}N_{t,o})^{\alpha_3}K_{t,o}^{1-\alpha_3} \tag{2.72}$$

$$\frac{\partial \pi_{t,o}}{\partial N_{t,o}} = 0 \Leftrightarrow p_{t,o}\frac{\partial O_t}{\partial N_{t,o}} - W_t = 0 \tag{2.73}$$

$$\frac{\partial \pi_{t,o}}{\partial K_{t,o}} = 0 \Leftrightarrow p_{t,o}\frac{\partial O_t}{\partial K_{t,o}} - r_t = 0 \tag{2.74}$$

2.2.5 Market clearing

Since there is no government and no other assets in the economy, total assets are equal to aggregate capital stock in the economy:

$$A_t = K_t \tag{2.75}$$

Aggregate capital is sum of capital stock in three sectors:

$$K_t = K_{t,y} + K_{t,s} + K_{t,o} \tag{2.76}$$

which evolves according to:

$$K_{t+1} = Y_t - C_t + (1-\delta)K_t \tag{2.77}$$

Equation (4.9) also ensures aggregate resource constraint and provides goods market equilibrium.

Total labor force in the economy is equal to the number of mature workers plus number of young workers multiplied by their labor productivity

$$N_t = \xi_t(1 - \frac{\lambda_c}{\psi_t^c})N_t^y + N_t^o = (\xi_t(1 - \frac{\lambda_c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{2.78}$$

Labor market equilibrium is when demand for labor equals supply of labor:

$$N_t = N_{t,y} + N_{t,s} + N_{t,o} \tag{2.79}$$

Definition of equilibrium and its characterization

A competitive equilibrium is a sequence of endogenous predetermined varibales

$$K_{t+1}, \lambda_{t+1}, \lambda_{2,t+1}$$

and a sequence of endogenous variables

$$\epsilon_t, \pi_t, \epsilon_t^2, \Omega_t, \Omega_t^2, H_t^o, H_t^y, C_t, W_t, r_t, A_t, p_{t,y}, p_{t,o}, p_{t,s}$$

that satisfy equations B.7, B.9, B.8, B.16, B.17, 3.50, 4.2, 3.55, 3.56, 2.59, 2.60, 2.63, 2.64, 2.68, 2.69, 2.73, 2.74 given the sequence of exogenous predetermined variables and the initial values of all the predetermined variables. Because there is growth in the steady-state due to exogenous growth in labor productivity and the population growth rate, all quantity variables that are in level are normalized by the growth rate of the effective labor force. The system of equations characterizing the model is in the appendix B.

2.3 Calibration and quantitative investigation

Parameters and exogenous variables in the model are calibrated with values in Table 2.2. Then two demographic transition scenarios are analyzed. The first scenario assumes a gradual decline in the fertility rate and a gradual increase in life expectancy, which mimics the demographic transition being experienced by developed countries. Scenario 1 is characterized by a gradual decline of the growth rate of children from 1% in period 1 to 0% by 2100, together with life expectancy at birth increasing from 75.3 in 1980 to 92 by 2100.

The second scenario assumes a faster decline in the fertility rate and a more rapid increase in life expectancy compared to scenario 1. Scenario 2 mimics the demographic transition being experienced by developing countries such as China. The second scenario is characterized by a rapid fall in the growth rate of children from 2 percent in period 1, which turns negative by period 20 and reaches -3 percent by end of the period while life expectancy at birth increasing from 69.37 in 1980 to 82.5 by end of the period. Data on past and projected changes in the growth rate of children and the old-age dependency ratio for the US is compared with scenario 1 in Figures 2.2 and 2.1, and scenario 1 and 2 is compared in Figures 2.4 and 2.3.

The parameter ξ has been calibrated to match the ratio between mean incomes of workers aged 19-39 and 40-65 in the US Economy. Age-earnings profile in the US and model assumptions are compared in Figure 2.5. The parameter λ_c, which determines the impact of fertility changes on the labor supply of young workers, is calibrated to match an estimate provided in the study by Lee and Mason (2010).

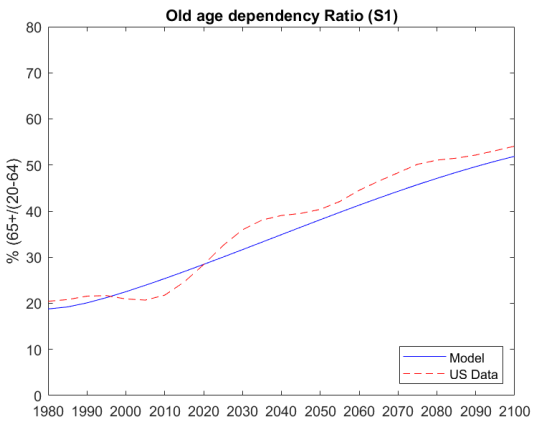


Figure 2.1: Life expectancy in S1

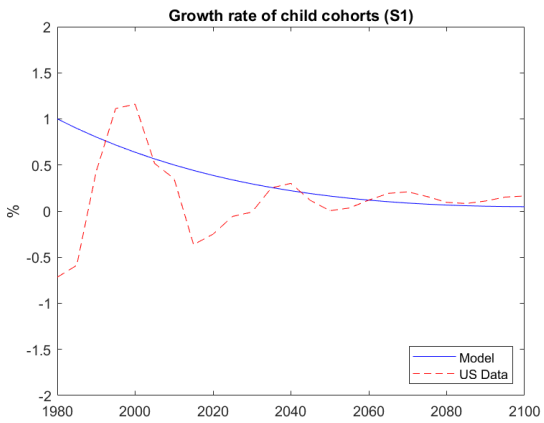


Figure 2.2: Fertility in S1

Source:Data from UN World Population prospects 2019

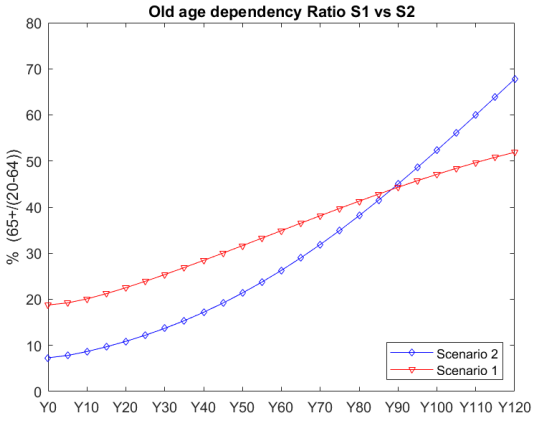


Figure 2.3: Old age dependency ratio in Scenario 1 and 2

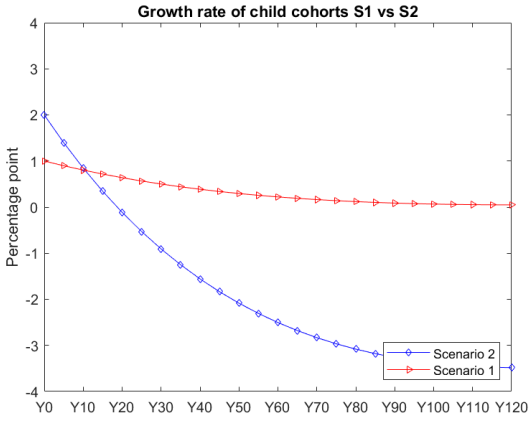


Figure 2.4: Growth rate of children in S1 and S2

Table 2.2: Parameter Values

Parameter	Value	Description
α	0.650	Labor share in the final output
$\alpha 2$	0.380	Labor share in the clean energy sector
$\alpha 3$	0.500	Labor share in the dirty energy sector
$\frac{1}{1-\Psi}$	0.69	Elasticity of substitution between capital labor composite and energy
$\frac{1}{1-\varphi}$	0.945	Elasticity of substitution between clean and dirty energy
δ	0.100	Capital stock depreciation rate
$v1$	0.960	Share of capital labor composite in the final output
$\eta 1$	0.800	Share of dirty energy in the energy production
β	0.975	Time preference rate
σ	0.5	Household intertemporal elasticity of substitution
$1/\omega$	25	Duration of mature life stage

Table 2.2 – Continued

Parameter	Value	Description
$1/\theta$	20	Duration of young life stage
$1/z$	19.6	Age at which child matures
λ_c	0.278	Time cost of children
$(x_l - 1) * 100$	2.3%	Growth rate of labor productivity
ξ	0.745	Productivity of young workers compared to mature workers
$1/\gamma_{ss}$	10.3	Life expectancy after retirement at age 65 (S1)
$(n_{ss} - 1) * 100$	1%	Initial growth rate of children (S1)

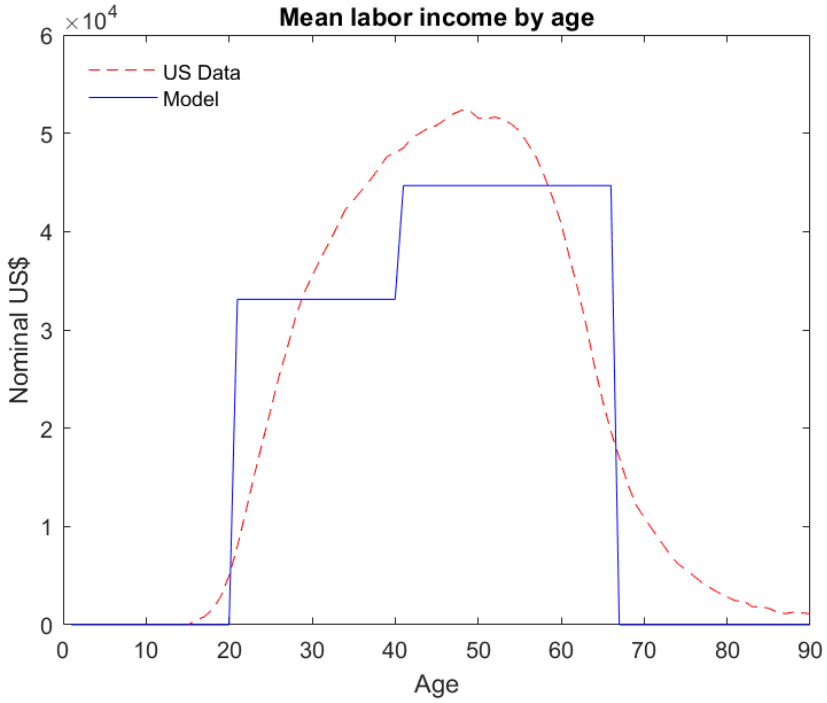


Figure 2.5: Age-earnings profile

2.3.1 Results

A demographic transition and its characteristics

Demographic change causes the growth rates of different age populations to diverge from each other, thereby producing a change in the population age structure. A demographic transition, characterized by a declining fertility rate and increasing life expectancy, will cause population aging by increasing the old-age dependency ratio in the long run. As shown in the graph in the left side of Figure 2.6, a decline in the fertility rate today will affect the growth rate of young workers with a delay of around 20 years. On the other hand, an increase in life expectancy at old age, as modeled here, causes

the growth of this segment of the population to increase for many years. The graph in the right side of Figure 2.6 shows that an increase in life expectancy will offset a decline in the fertility rate, causing total population growth to be higher than if there was no change in life expectancy.

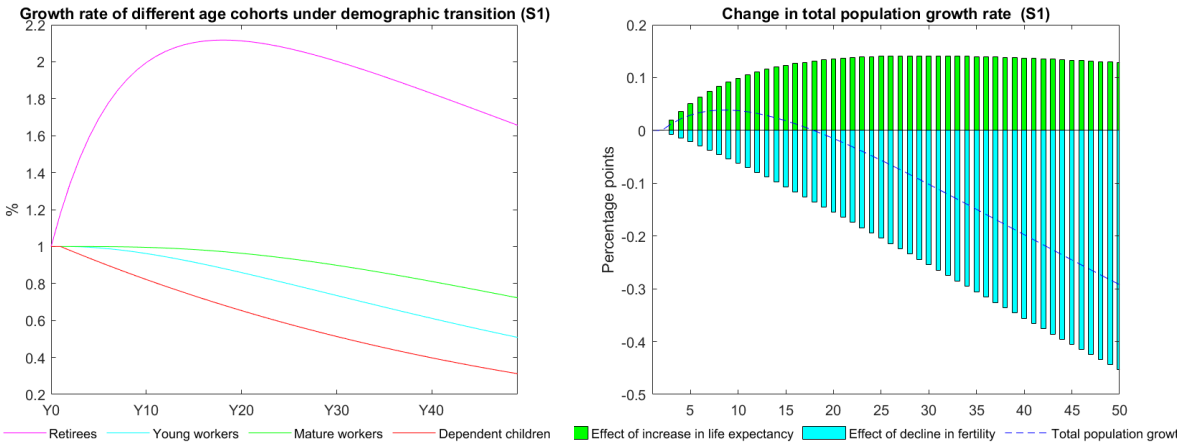


Figure 2.6: Scenario 1: Population growth rate under demographic transition

Impact of a demographic transition on the macroeconomy

Changes in the population age structure driven by the diverging growth rates of different age-cohorts have implications on the economy and on emissions because labor supply, productivity, and savings behavior differ across age groups. Figure 2.7 shows the impact of a demographic transition, in scenario 1, on the real interest rate. It further breaks down the impact of the demographic transition on the interest rate to its two components: the decline in fertility rate and increase in life expectancy. As can be observed from Figure 2.7, the increase in life expectancy explains the majority of the impact of the demographic transition on interest rates in scenario 1.

Another important observation is that the impact of fertility reduction on the interest rate does not come instantly, but is delayed almost 20 years, when the reduction in fertility rate starts affecting the growth rate of the working-age population. Capital per effective labor rises for two reasons under the demographic transition. The increase in life expectancy increases aggregate savings per effective labor, while the decline in fertility rate reduces aggregate savings by increasing the old-age dependency ratio in the long-run. However, the reduction in the growth rate of the working-age population more than offsets the negative effect of declining aggregate savings causing capital per effective labor to rise.

Figure 2.8 shows the impact of the demographic transition on individual behavior as well as on aggregate savings. When the demographic shock happens in period 1, workers and retirees all increase their savings rate, as shown in Figure 2.8 by the sudden decline in the marginal propensity to consume out of wealth for young and mature workers as well as retirees. However, young workers reduce their consumption the least (less than 30% in the long run) because they expect an increase in their human wealth due to an increase in the wage rate under the demographic transition. Conversely, the decline in the marginal propensity to consume out of wealth is highest among retirees (more than 40% in the

long run) because they also expect that income on their savings will be lower due to a reduction in the interest rate. Overall, despite the decline in the marginal propensity to consume by all cohorts, because of the change in age structure, the number of retirees who have a higher marginal propensity to consume out of wealth increase, causing aggregate savings in the GDP to decline over time under demographic transition.

Even though aggregate savings as a share of GDP declines over time, capital per effective labor increases under demographic transition because the reduction in the growth of workers more than offsets the decline in the savings rate. As shown in Figure 2.10, capital per effective labor rises by almost 20% compared to the initial steady-state in scenario 1 and by more than 40% in scenario 2 in the long run. In turn, the increase in capital per effective labor causes almost a 7 percent increase in output per effective labor over the same horizon in scenario 1 and a 14 percent increase in output per effective labor in scenario 2.

Another interesting observation that cannot be seen from a two-life-stage model as in Gertler (1999) is the reduction in savings by young workers under demographic transition, as reflected by the decline in the share of assets they hold Figure 2.9. This is due to both the declining share of young workers in the population as well as increase in their consumption following increase in their human wealth.

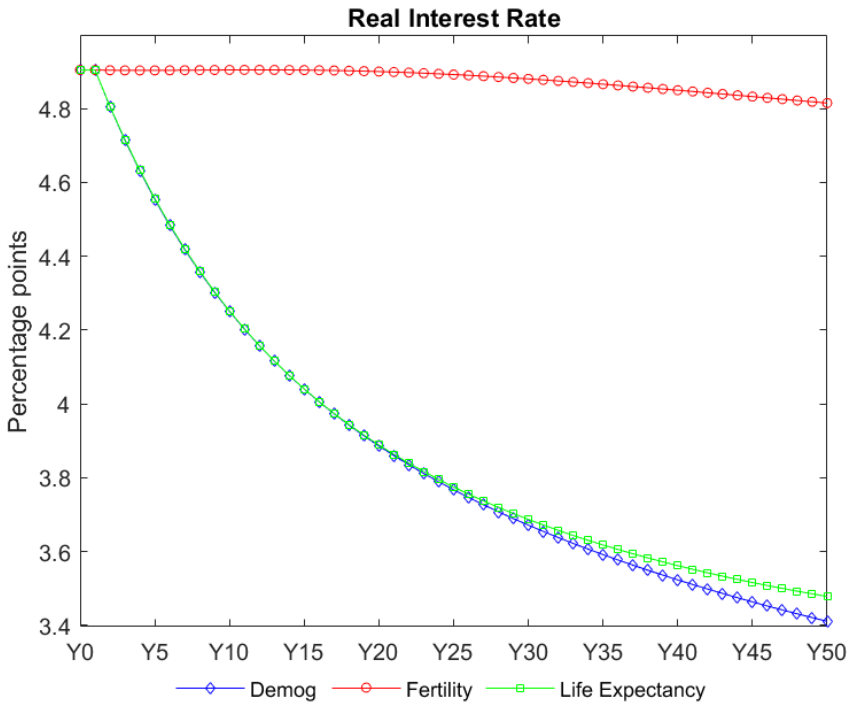


Figure 2.7: Impact of demographic transition on interest rate (S1)

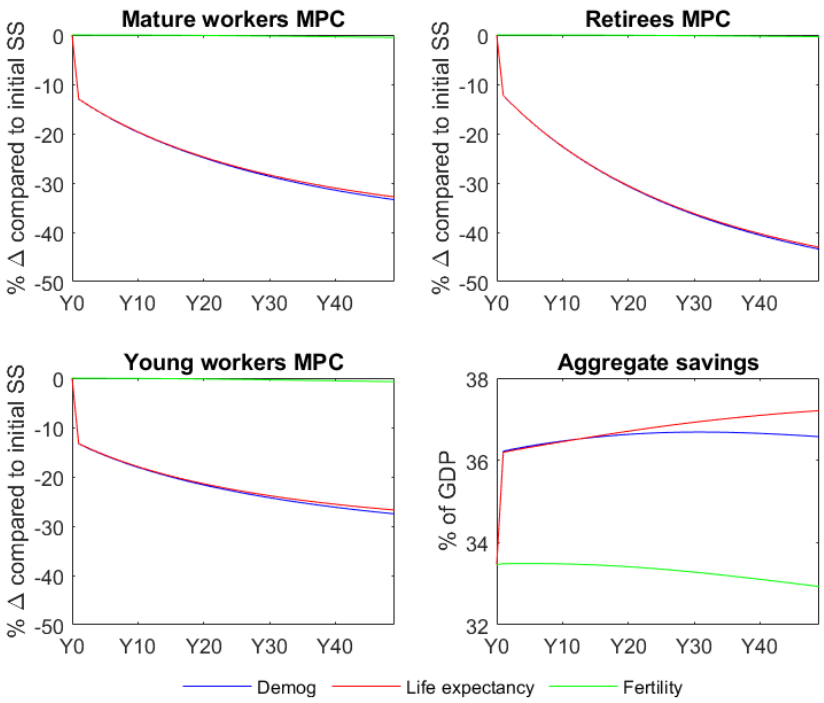


Figure 2.8: Impact of demographic transition on aggregate savings (S1)

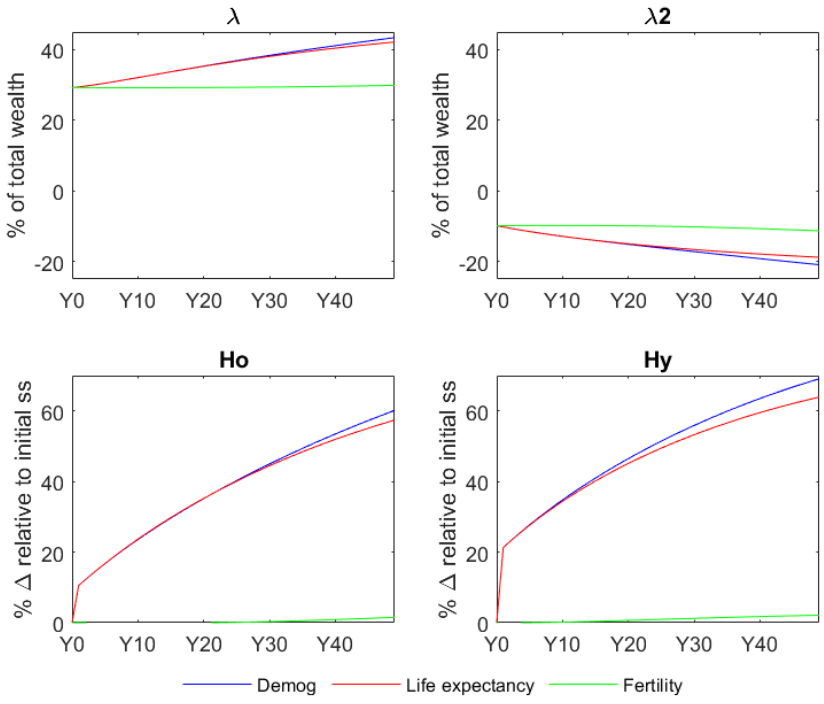


Figure 2.9: Impact of demographic transition on distribution of wealth (S1)

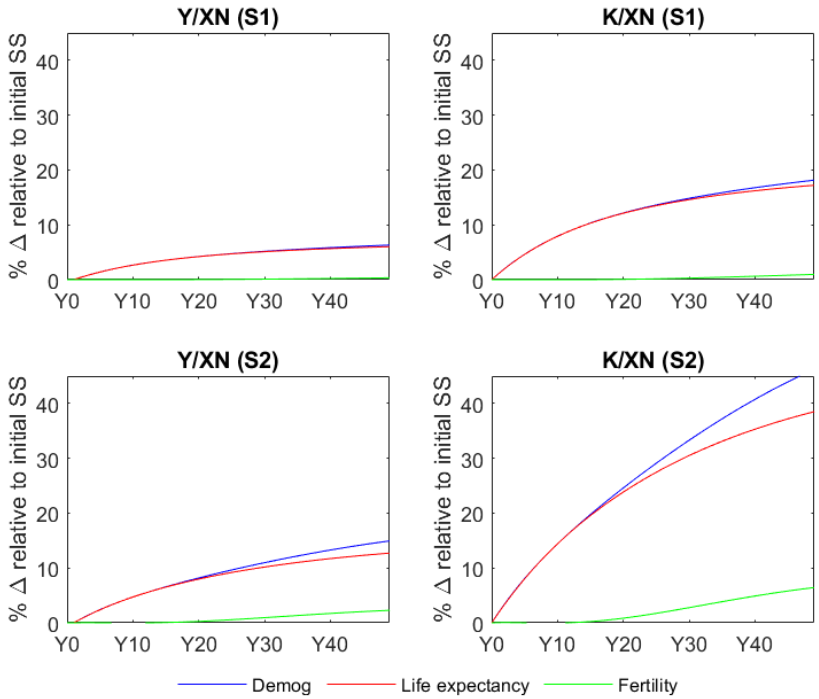


Figure 2.10: Impact of demographic transition (S1 & S2) on K/XN and Y/XN

Impact of demographic transition on energy and CO_2 emissions

An interesting finding that would not have been possible to see within a one-sector growth model is that the impact of the demographic transition on aggregate emissions differs from its impact on aggregate output through two channels. Empirically, the energy sector is a relatively capital intensive sector. Moreover, the clean energy sector is more capital intensive compared to the fossil energy sector (Best, 2017). By reducing the interest rate in the economy, the demographic transition causes capital intensive industries to grow faster. Figure 2.11 shows the impact of demographic change on the output, energy, fossil fuel, and clean energy production per effective labor compared to initial steady state.

In figure 2.12 $\Delta(E/Y)$ denotes percentage change in emissions coming from change in energy intensity of output, $\Delta(Y/XN)$ denotes percentage change in emissions due to change in output per effective labor, ξ denotes percentage point change in emissions due to changes in age structure of the workforce, λ_c denotes change in emissions due to change in child dependency ratio, $\Delta(O/E)$ denotes percentage change in emissions due to change in carbon intensity of energy. A demographic transition leads to an increase in the energy intensity of the economy, causing CO_2 emissions to grow faster than output. On the other hand, clean energy production grows faster than overall energy production in the model, offsetting the increase in emissions by reducing the carbon intensity of energy production. The net effect of the demographic transition on aggregate CO_2 emissions growth is shown in Figure 2.12 for both scenario 1 and 2.

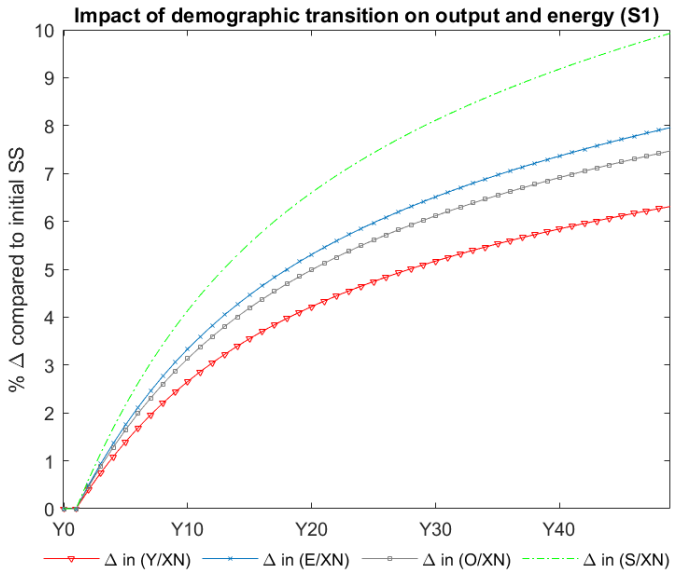
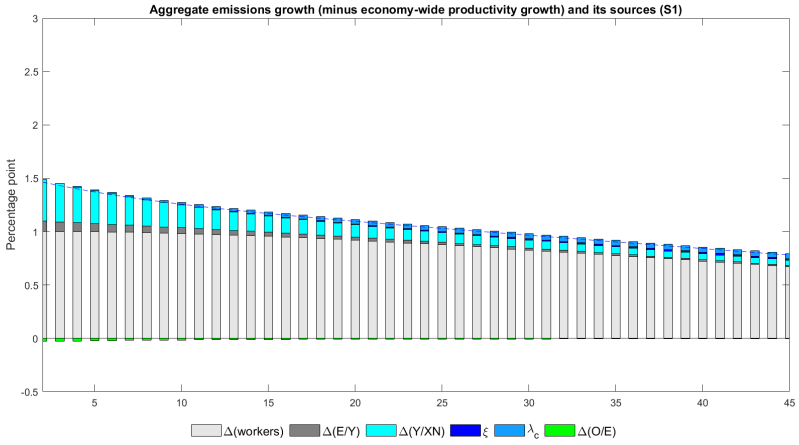
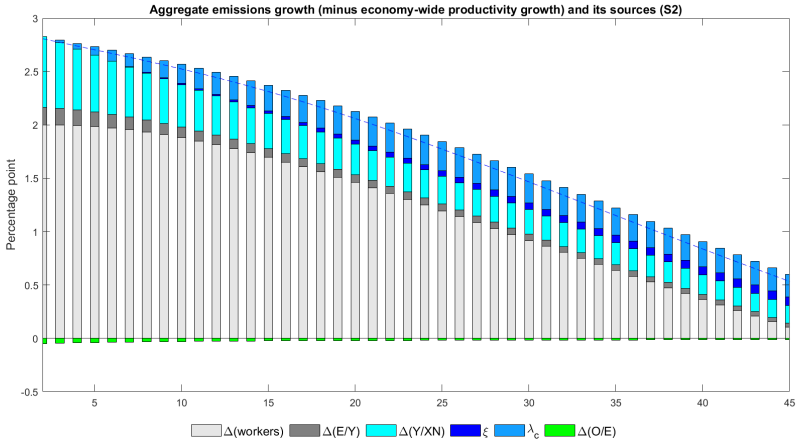


Figure 2.11: Impact of demographic transition on Energy and Output



(a) Scenario 1



(b) Scenario 2

Figure 2.12: Decomposition of impact of the demographic transition on aggregate emissions growth

Emissions growth is higher in both scenarios due to growth in energy intensity, growth in output per effective labor, labor productivity growth due to the changing age composition of the labor force, and the increase in labor force participation due to the reduction in the child dependency ratio. On the other hand, the decline in the carbon intensity of energy production offsets emissions growth. However, the contribution of renewable energy growth to emissions reduction is almost negligible due to its small share.

2.3.2 Sensitivity analysis

Intertemporal elasticity of substitution (IES) is the key parameter that determines the direction and magnitude of the response of savings to changes in the interest rate. There is great uncertainty around the value of IES (σ) for countries and periods due to mixed empirical evidence. For example, for a mean value of 0.5, Havranek et al. (2015) reports a standard deviation of 1.4 for the estimate of σ reported in 33 studies published in top journals, excluding outliers. Furthermore, meta analysis of estimated IES shows significant differences across countries, which is related to a country's income level, its financial market development, and credit constraint of households (Havranek et al., 2015). Therefore, some sensitivity analysis has been done by changing the value of IES. Figure 2.13 shows the response of the gross interest rate (big R) to the demographic transition shock in scenario 1 under different σ values.

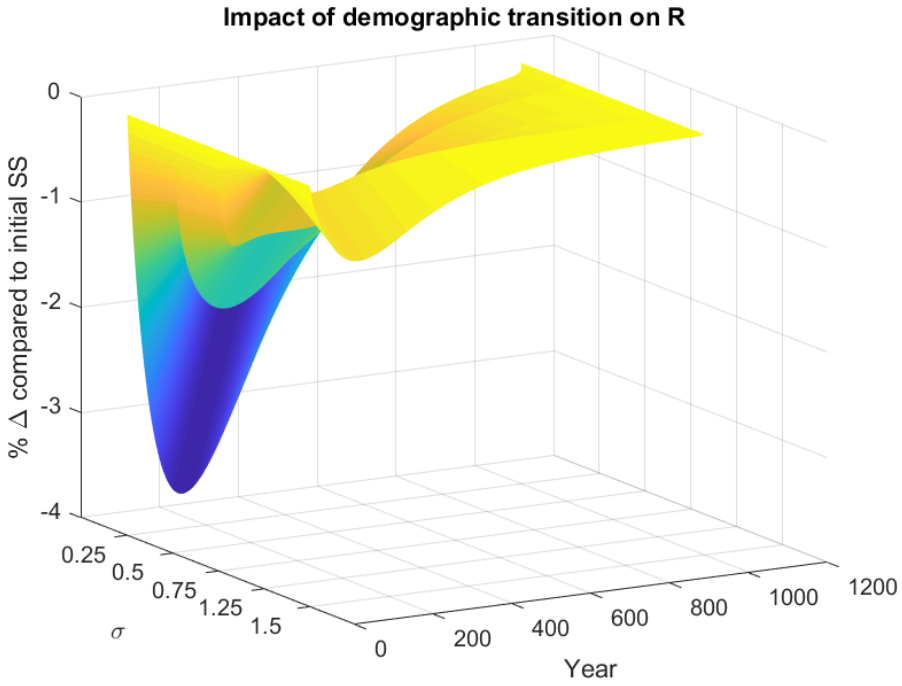


Figure 2.13: Intertemporal elasticity of substitution and sensitivity of interest rate to demographic transition

As can be seen from Figure 2.13, the magnitude of the change in interest rate is sensitive to σ. The impact is lowest as σ approaches 1. For a given value of time preference (β) parameter, R declines as σ approaches 1, making the economy dynamically inefficient ($r < g$). Similar results have been reported by Gertler (1999). The σ values

most often used in the OLG literature, which range from 0.25 to 0.55, correspond to a higher response of interest rate to the demographic transition.

Furthermore, it is interesting to look at how different σ and β parameters interact, because β is also an important determinant of interest rate and researchers calibrate parameter β to generate a certain interest rate for a given value of σ. Panel A of Figure 2.14 shows that when σ equals 0.75 and β is less than or equal to 0.94, then the interest rate rises in response to the demographic transition. However, the β value of 0.94 or less implies a very high discount rate, which is way beyond the value most commonly used in economic models. Therefore, we can say it is highly unlikely that the interest rate will rise in response to the demographic transition. Nevertheless, panel B of Figure 2.14 shows the impact of demographic transition on different combinations of β and σ values. Specifically, value of β is calibrated to generate same steady state value of interest rate for three different values of σ.

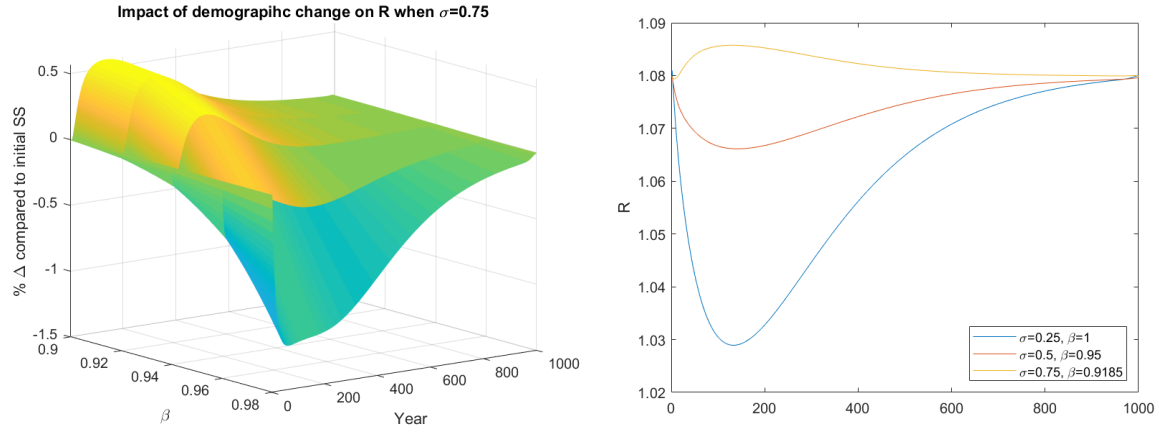


Figure 2.14: Scenario 1: Impact of demographic transition on R under different β and σ values

Two other sensitivity analyses involved the values of ξ and λ_c, as the value of these two parameters can be different across countries and periods. As can be seen in Figure 2.15, the larger the difference is between the productivity of young and mature workers (as measured by ξ), the larger the impact of the changing age structure of the workforce on economic growth during the demographic transition. Similarly, Figure 2.16 shows that the larger the time cost of raising children measured by λ_c, the larger the effect of a reduced fertility rate will be on economic growth during the demographic transition. Moreover, Figure 2.15 and 2.16 both show an inverted u-shaped relationship between contribution of these two channels to economic growth during the transition. Specifically, in scenario 1, the impact of the changing age structure of the labor force and the increase in female labor force participation peaks around 50 years after the transition begins. Due to the small size of the fertility shock in scenario 1, the effect of the changing age structure of the workforce as well as female labor force participation, both of which are driven by fertility shock, are almost negligible, with less than 0.05 percentage points at their peak impacts. However, the effect from these two channels will be sizable in countries where the fertility rate is rapidly falling from a high level.

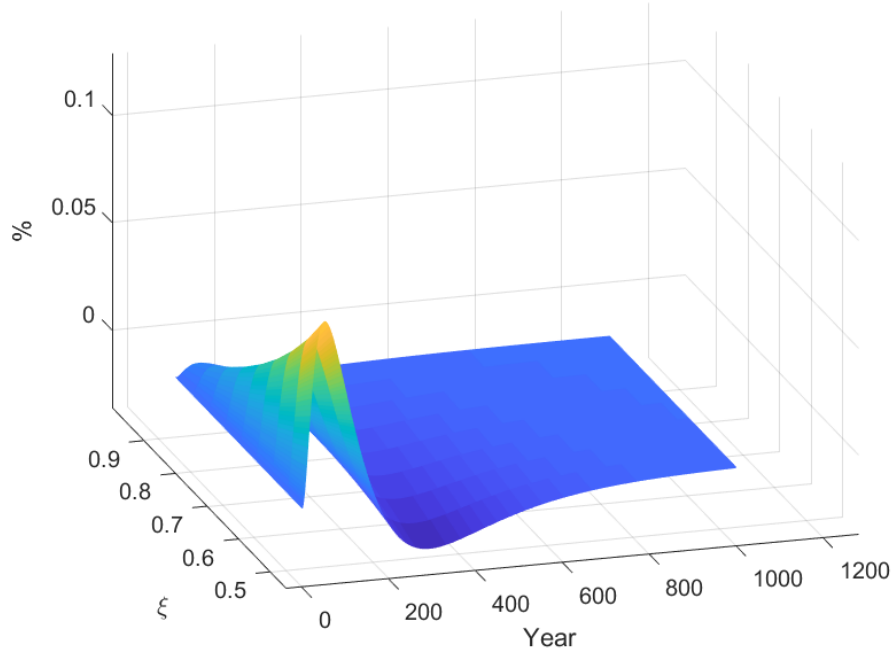


Figure 2.15: Contribution of changing age structure of workers to aggregate economic growth (S1)

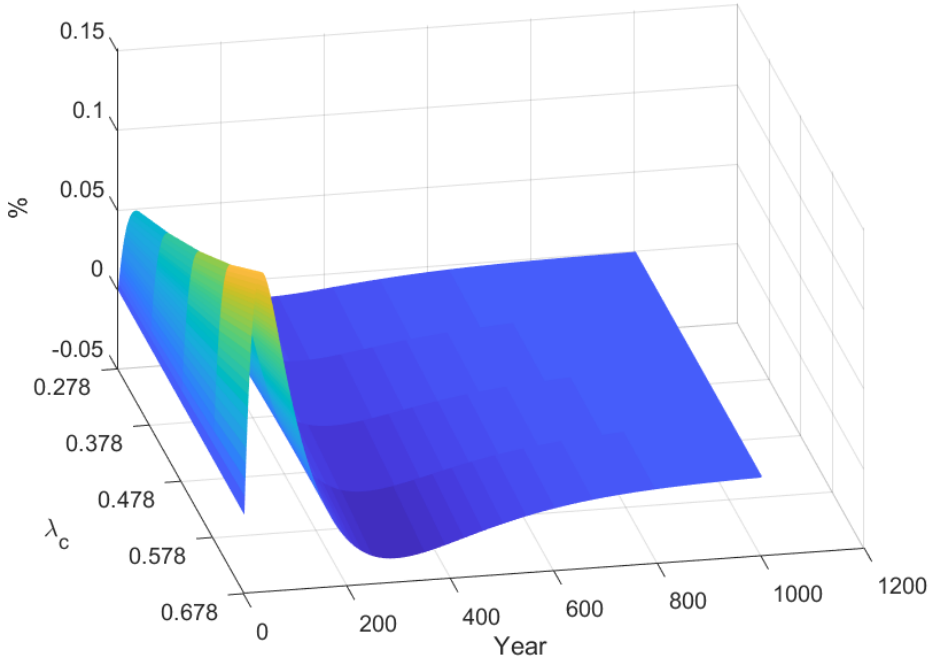


Figure 2.16: Contribution of female labor supply to aggregate economic growth (S1)

2.4 Conclusion

This chapter investigated the macroeconomic implications of demographic transitions using a life-cycle model with an energy-dependent production function. A model of a

closed economy without government is considered to simplify the analysis. Further, technological change is assumed to be exogenous and labor augmenting, which is a standard assumption in most applied economic models used for climate policy. The results show that there is no simple answer to the question of how the demographic transition impacts economic growth and carbon emissions. The answer depends on what is driving the demographic change, how big the difference is between the productivity of young and prime-age workers, how significant the impact of fertility is on the labor supply of workers, as well as how and when people update their expectation about life expectancy. In contrast to findings in previous studies, the results show that population aging brought on by demographic transition has contributed between 0.5 to 1 percentage points to aggregate GDP growth independent from its effect on the growth rate of workers.

The results in this chapter show that demographic transitions, characterized by declining fertility and increasing longevity, will increase emissions through the three distinct channels built into the model. First, it will increase emissions by increasing the number of prime-age workers in the workforce, who have higher labor productivity. Second, it will increase emissions by increasing labor force participation by reducing number of dependent children. Third, it will increase emissions by increasing capital per effective labor. The first channel is based on the model's assumption that older workers are more productive. This is a natural assumption when working life is divided into only two distinct stages. Overall, findings in this chapter may look different from the findings by Dalton et al. (2008), who concludes that population aging will reduce emissions. The different findings are due to the different baseline against which the impact of population aging is compared. In Dalton et al. (2008) study, they compared the population aging scenario with scenarios from a representative agent model where changes in the age structure of the model are not taken into account.

Another important finding is that the effect of the demographic transition is not constant: Its magnitude will decline over time before disappearing as fertility and mortality patterns stabilize around some level. It's effect will also depend on what is driving the transition. For example, the effect of an increase in life expectancy at older ages will depend on when and how people change their beliefs on life expectancy as well as their ability to respond. Fertility reduction, meanwhile, will have a non-linear effect, as it has consequence both for labor force participation and age composition, and the magnitude of the impact depends on the magnitude of fertility changes from one year to the other. Overall, despite being a simple model, it can provide a coherent explanation to seemingly conflicting results found in empirical studies by highlighting different demographic factors that can explain those results.

The demographic transition causes the population age structure of a country to change due to the varying growth rates of different age populations during the transition. Because savings rates, labor productivity and labor force participation differ by age, the demographic transition will cause changes in aggregate savings and labor supply, both of which are important in determining a country's aggregate production potential. Despite being called a transition, its effects will continue for over a hundred years.

Eventually, the effect of the demographic transition will decline and vanish as the transition completes. Another important observation is that an increase in life expectancy offsets a decline in the fertility rate to some extent, causing the total population growth to fall less than if there were no life expectancy changes. This observation has an important

implication for energy-economy models that use aggregate population growth projections to project economic growth and emissions. By doing so, they may capture the effect of an increase in life expectancy as an increase in the growth rate of workers. It is essential to differentiate between changes in life expectancy and changes in the growth rate of workers because one puts downward pressure on savings and interest rates. In contrast, an increase in the fertility rate will put positive pressure on interest rates. This indicates that energy-economy models may incorrectly capture the effect of an increase in life expectancy.

Although simplified, the framework developed in this chapter can capture the effect of the demographic transition on the macroeconomy through three distinct mechanisms. First, by having two different working life stages, it can capture the change in effective labor supply due to a change in the age structure of a working-age population. Second, it can capture the change in the effective labor force caused by a decline in fertility rate, which causes female labor force participation to increase. Third, with its forward-looking savings behavior, it can show changes in the savings rate due to changes in life expectancy and changes to economic growth under demographic transition.

The exact magnitude of the impact of the demographic transition on aggregate economic growth and emissions depends on the speed and magnitude of the demographic transition as well as on several parameters that summarize demographic components of the effective labor force and intertemporal savings behaviors of households. The results show the importance of a model's ability to distinguish the effect of a declining fertility rate from the effects of increasing life expectancy at older ages, as well as the importance of explicitly modeling energy and emissions.

Finally, the impact of population aging on the economy and emissions is sensitive to the reference scenario with which we are comparing the effect of demographic transition. Counter to the popular narrative in the literature, the results in this chapter have shown that longer life and population aging have contributed positively to GDP growth in the past.

Chapter 3

Children matter: Global imbalances and the economics of demographic transition

Abstract: This paper investigates the effect of child dependency on the economy and external imbalances under an asymmetric demographic and productivity transition within a life-cycle model. It embeds dependent children within a two-country model with life-cycle features to examine child dependency's effect on the economy and external imbalances. Specifically, the paper compares the effects of the same fertility and mortality shocks across models with and without children. Simulations show that child dependency changes both the steady-state and the transition dynamics under a demographic shock. The paper finds that while child dependency changes the direction of the impact of the fertility transition on external imbalances in the short run, it changes the magnitude of the effects in the long run. Furthermore, the model comparison shows that parameters must be chosen differently across models with and without child dependency to start from the same interest rate in the steady-state. Different calibration affects the magnitude of the transition dynamics of different models. These findings illustrate the importance of considering child dependency in studies that seek to explain the historical contribution of demographic changes to external imbalances, and suggest to approach studies that use models without child dependency for this purpose with caution.
Keywords: Global imbalances,Trade imbalances, Demographic transition, Life-cycle model
JEL Codes:

3.1 Introduction

Trade and current account imbalances across major economies have become a persistent feature of the global economy. Figure 3.1 shows the increasing trade imbalances of the US, China, and G6 (UK, France, Germany, Italy, Canada, and Japan) since the 1980s. A growing literature has attempted to understand the underlying causes. As emphasized by Blanchard and Milesi-Ferretti (2012), such research must be interpreted with caution due to its potential impact on the nature and scope of policy responses. This paper highlights some of the issues that arise in quantifying the effects of demographics on historical external imbalances using calibrated life-cycle models that do not consider the impact of changes in child dependency.

Traditional theories of trade á la David Ricardo and Heckscher-Ohlin may explain why we trade but fail to answer why there is a persistent imbalance across countries. To explain imbalances, economists rely on intertemporal theories of trade which see imbalances as a means of smoothing consumption across time in the face of non-smooth income. However, the neoclassical theory, based on an infinite horizon representative agent, falls short in explaining China's recent experience, as the standard model would predict China to have a large trade deficit driven by high investment and consumption given high productivity growth. Despite the high investment rate, China was in fact able to have a trade surplus thanks to its even higher savings rate (see the second figure in Figure 3.1). Hence, understanding diverging savings rates across countries is thought to be the key to understanding trade imbalances (Gourinchas & Rey, 2014). The failure of the standard model to account for changes in savings, and hence in predicting the trade imbalances, opens room for alternative theories.

Furthermore, the timing of the growing imbalances in these economies has led economists to attribute some of the external imbalances to unprecedented yet asymmetric demographic transitions, namely different speed and timing in the falling fertility rates and increasing life expectancy in these countries. From around the 1970s, the child dependency ratio has fallen in these economies while the old age dependency ratio has increased (see Figure 3.2). Notably, China is experiencing a rapid and profound demographic change, with China's child dependency ratio falling by almost three quarters from the early 1980s to 2020. Meanwhile, with the dramatic fertility decline and the increasing life expectancy at old age, the old age dependency ratio in China is expected to surpass the US level by 2040 (UNDESA, 2019).

Prompted by these facts, a growing literature aims to investigate the factors behind the persistent trade deficits of the US using calibrated life-cycle models. For example, by extending a tractable life-cycle model proposed by Gertler (1999) to two-country settings, Ferrero (2010) analyzed the contribution of productivity, demographics, and fiscal policy differentials on the trade imbalance between US and G6. He attributes the persistent US trade deficit against G6 between the 1980s to 2005 to its lower life expectancy and higher productivity. Other papers found institutions such as social security (Eugeni, 2015; Niemeläinen, 2021) and financial systems (Caballero et al., 2008; Coeurdacier et al., 2015) to be important in determining changes in aggregate savings and trade imbalance during the demographic transition. For example, using the model similar to Ferrero (2010), Niemeläinen et al. (2020, Chapter 3 & 4) examined the trade imbalances between the US and China, allowing different pension coverage, variable labour supply, and exchange

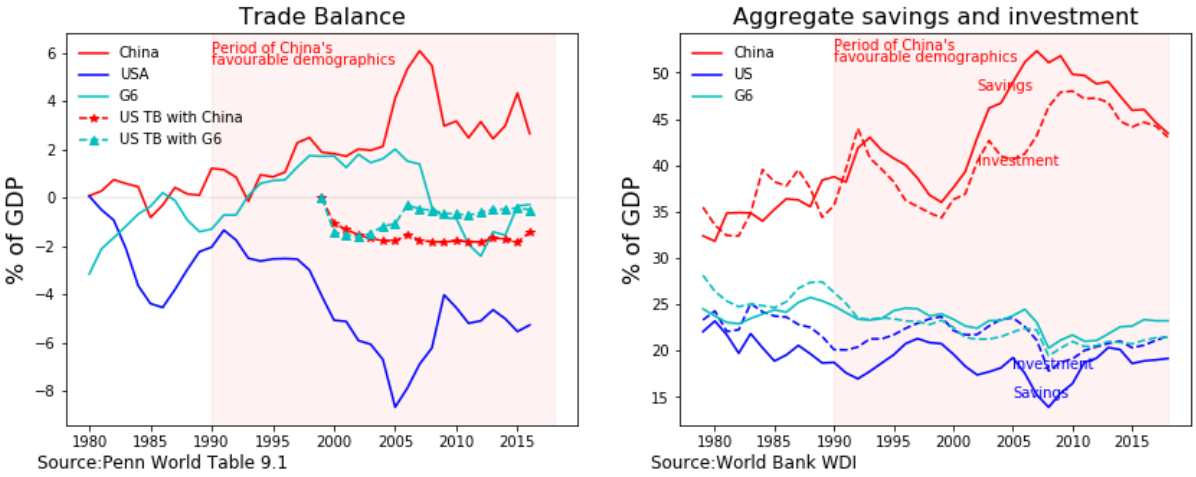


Figure 3.1: External imbalances

rates. She found that the high savings in China, which drive Chinese trade surpluses, can be attributed to its low pension coverage in the face of increasing life expectancy. On the other hand, Coeurdacier et al. (2015) emphasizes credit constraints along with demographic change and productivity growth differentials as other mechanisms whereby diverging rates of savings occur between the US and China, which underlies its trade imbalances.

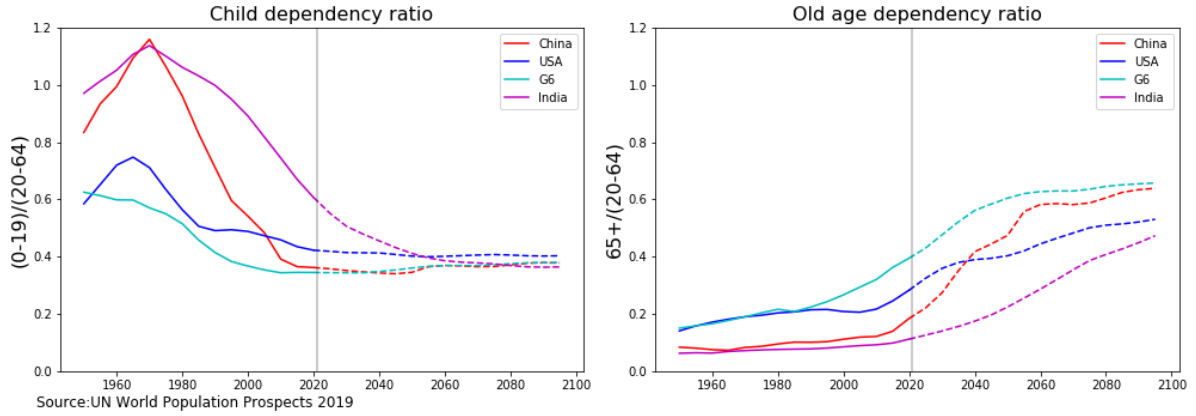


Figure 3.2: Child and old age dependency

Despite being the most prominent feature of the global demographic transition, as shown in Figure 3.2, the effect of changes in child dependency on aggregate savings has been neglected altogether by the research mentioned above. In the frameworks used by Ferrero (2010); Niemeläinen (2021); Coeurdacier et al. (2015), the population dynamics start with the working-age population. Neglecting child dependency is a serious shortcoming if one aims to understand the effect of demographic changes (Bommier & Lee, 2003). At the micro and macro levels, empirical evidence points to dependent children's significance in affecting savings. Micro-level studies estimate the consumption cost of children to fall in the range between 10 and 40 percent of household's income or budget (Donni, 2015; Letablier et al., 2009). However, these estimates do not include the time cost of children. Econometric studies find that the number of children can affect

labour force participation, particularly women's (D. E. Bloom et al., 2009; Lundborg et al., 2017). D. E. Bloom et al. (2009) finds that each additional child reduces the female labour supply on average by two years. On the other hand, macro-level empirical studies find child dependency to be significant in determining aggregate savings (Leff, 1969; Masson et al., 1998; Park & Shin, 2009; Curtis et al., 2015, 2017; Lugauer et al., 2019; Higgins & Williamson, 1997) and capital flow (Lührmann, 2003; Chinn & Prasad, 2003).

The main research question addressed in this paper is the consequence of neglecting consumption and time cost of children when examining the effect of demographic changes on the economy and external imbalances within a life-cycle model. This paper incorporates the consumption and time cost of children in a way similar to Barro and Becker (1989) [1] in a two-country tractable life-cycle model á la Gertler (1999) and compares the effects of same fertility and mortality transitions between models with and without child dependency. Previous modeling studies have shown that child dependency depresses the savings of young workers (Brooks, 2003), and it changes both the steady-state and the transition dynamics associated with demographic change (Bryant et al., 2004). The contribution of the current paper is to embed consumption and time cost of children within a tractable life-cycle model with realistic demography that is solved on an annual basis to investigate the consequence of neglecting child dependency.

The paper finds that child dependency matters for at least two reasons. First, within a non-Ricardian environment (as in life-cycle settings) where the timing of taxes and transfers do matter for aggregate savings and investment, the existence of consumption and time cost of children matters for the aggregate savings and capital per effective labour during the fertility and longevity transitions. Simulations show that child dependency changes the direction of the impact of the fertility transition on external imbalances in the short run. In contrast,in the long-run, it changes the magnitude of the effect. Second, even when there is no fertility transition and no change in child dependency, the existence of child dependency changes the steady-state capital per effective labour, the interest rate, and the distribution of assets between workers and retirees. Workers who bear the cost of raising children both in terms of time and money end up with lower savings and a lower share of aggregate assets in the economy. These differences in the steady-state and the distribution of assets matter for models that intend to investigate the contribution of demographic transitions on the economy and, particularly, external imbalances.

Table D shows the parameter values and calibration strategy of three different studies that use a life-cycle framework to explain historical trade and current account imbalance of the US. The calibration strategy used in these papers includes setting the key parameters that determine savings to target savings or interest rates in the data for a specific year. For example, Ferrero (2010) calibrated his subjective time preference rate to match the interest rate in the 1970s. By using the same parameter values, Niemeläinen (2021) gets a much lower interest rate in the steady-state for the fixed labour supply case due to the change in the model structure where she considers pension. However, it is

[1] However, the current paper abstracts from endogenous fertility and intergenerational transfers by assuming that the parents do not choose the number of children nor care about their children's utility but pay a fixed amount per child for consumption and allocate time out of work to raise children. Introducing children's consumption into parent's utility reduces the life-cycle motive of savings and makes the savings behavior of the individual closer to the behavior of a representative agent with infinite horizon (Weil, 1989).

unclear whether Niemeläinen (2021) targets the interest rate even though she is doing historical analysis. On the other hand, Coeurdacier et al. (2015) used a calibration strategy where key parameters that determine individual savings are chosen to target a close match between model predicted savings rate and the household savings rate in the data. Even though these papers conduct sensitivity analysis, sensitivity analysis may not be adequate for at least two reasons. First, model misspecification (e.g. omission of child dependency) may result in differences in parameter values across models much wider than the range considered in the sensitivity analysis. Second, sensitivity analysis for each parameter separately may not capture the interaction between parameters. In the case of Coeurdacier et al. (2015), both subjective time preference rate and the elasticity of intertemporal substitution are chosen at low levels, which increases the credit constraint mechanism artificially.

This paper shows that incorporating children's consumption and time cost will reduce steady-state aggregate savings and increase the steady-state interest rate. Using the same calibration strategy as in Ferrero (2010); Niemeläinen (2021); Coeurdacier et al. (2015), the different steady-states will result in the need to calibrate the subjective time preference to be higher in the model that considers the cost of children to target the same interest rate and savings in the data. The differences in the value of the calibrated parameters lead to differences in the dynamic impact of fertility and longevity transitions even though the latter does not affect the child dependency rates. Furthermore, the existence of child dependency increases the importance of credit constraints, thereby reducing the need to choose intertemporal elasticity of substitution and subjective time preference rate to be set at unconventional levels, as in Coeurdacier et al. (2015), which itself affects the dynamics. In other words, the models with and without child dependency represent different economies at the micro-level, which cause differences in terms of their response to demographic shocks even if they correspond to the same aggregate outcome at a certain point in time.

The paper contains six sections, including the introduction section. Section 4.2 presents the model used for simulation, with section 3.3 providing some analytical explanation of the external imbalance determinants. Section 3.4 outlines the calibration strategy and the description of the quantitative experiment; section 3.5 and 3.6 reports the steady-state and the dynamic analysis respectively, and section 4.5 concludes. In addition, a separate appendix section A provides steady-state equations of the model together with the description of the solution method and mathematical proofs of the relationship between the agent's marginal propensity to consume, the life expectancy, and the interest rate.

3.2 Model

The analysis in section 3.6 is based on an extension of the closed economy life-cycle model proposed by Gertler (1999) to a two-country model with four different cohorts, the children, young and mature workers, and retirees. The production side of the model is kept simple as in Ferrero (2010); Niemeläinen (2021); Coeurdacier et al. (2015) with the assumption of a representative firm producing a single consumption good using capital and labour. The model also considers government debt (as in Ferrero (2010)) and pension

policy (as in Niemeläinen (2021)). The pension is modeled as a pay-as-you-go system where taxes on current workers are distributed among current retirees. The countries, referred to as home and foreign, are initially identical but later differ due to their different demographic transition shocks.

Furthermore, it is a deterministic model with no aggregate uncertainty. Agents correctly foresee the future of prices and quantities in the economy except for an exogenous shock. Both economies have the same production technology and produce a single good traded internationally with no transaction costs. The structure of the home country is described in detail in sections 4.2.1 to 4.2.4. All equations apply to the foreign country except when upper index f refers to variables for foreign countries.

3.2.1 Population dynamics

Population dynamics are exogenous in the model. A set of probabilities that govern the transition from one life stage to another, given in table 3.1, generates population dynamics in the model. These probabilities are independent of the age of an individual to facilitate aggregation. Time is discrete and goes forever, i.e., $t = 0, 1, 2, ...$ There are four age groups of individuals: children, young workers, mature workers, and retirees.

Table 3.1: Transition probabilities from period t-1 to t

	Children (t)	Young worker (t)	Mature worker (t)	Retiree (t)	Deceased (t)
Children (t-1)	$1-z$	z	0	0	0
Young worker (t-1)	0	1-θ	θ	0	0
Mature worker (t-1)	0	0	1-ω	ω	0
Retiree (t-1)	0	0	0	1-$\gamma_{t-1,t}$	$\gamma_{t-1,t}$
Deceased (t-1)	0	0	0	0	1

Let N_t^c denote the stock of children at time t. At time t, $(z + n_t - 1)N_{t-1}^c$ identical children are born which implies that the stock of children grow by n_t at time t.

$$N_t^c = (z + n_t - 1)N_{t-1}^c + (1 - z)N_{t-1}^c = n_t N_{t-1}^c \tag{3.1}$$

Let N_t^y denote the stock of young workers at time t. The stock of young workers at time t consists of children who became young workers at the beginning of time t (end of t-1) and young workers who didn't become a mature worker at the beginning of time t (end of t-1).

$$N_t^y = zN_{t-1}^c + (1 - \theta)N_{t-1}^y \tag{3.2}$$

Let's also denote children to younger worker ratio at time t by ψ_t^c:

$$\psi_t^c = \frac{N_t^y}{N_t^c} \tag{3.3}$$

The ratio of younger worker to children ratio evolves according to:

$$\psi_t^c = \frac{z}{n_t} + \frac{(1 - \theta)(\psi_{c,t-1})}{n_t} \tag{3.4}$$

We can also define the growth rate of young workers as a function of the growth rate of children at time t as:

$$n_t^y = \frac{N_t^y - N_{t-1}^y}{N_{t-1}^y} = \frac{\psi_t^c(1+n_t)}{\psi_{t-1}^c} \tag{3.5}$$

Let N_t^m denote stock of mature or older workers at time t. The stock of older workers at time t consist of young workers who became mature at the beginning of time t (end of t-1) and mature workers who didn't retire at the beginning of time t (end of t-1).

$$N_t^m = \theta N_{t-1}^y + (1-\omega)N_{t-1}^m \tag{3.6}$$

Let's denote ratio of older to younger workers at time t by ψ_t^1:

$$\psi_t^1 = \frac{N_t^m}{N_t^y} \tag{3.7}$$

The ratio of older to younger workers evolves according to:

$$\psi_t^1 = \frac{\theta}{n_t^y} + \frac{(1-\omega)\psi_{t-1}^1}{n_t^y} \tag{3.8}$$

Let N_t^r denote the stock of retirees at time t. The stock of retirees at time t consists of older workers who retired at the beginning of time t (end of time t-1) and retirees who survived at the beginning of time t (end of time t-1).

$$N_t^r = \omega N_{t-1}^m + (1-\gamma_t)N_{t-1}^r \tag{3.9}$$

Let's denote the ratio of retirees to total number of mature and young workers at time t by ψ_t^2:

$$\psi_t^2 = \frac{N_t^r}{N_t^y + N_t^m} \tag{3.10}$$

The ratio of retirees to total number of older and young workers evolves according to:

$$\psi_t^2 = \frac{\omega\psi_{t-1}^1}{(\psi_t^1+1)n_t^y} + \frac{(1-\gamma_t)(\psi_{t-1}^1+1)\psi_{t-1}^2}{(\psi_t^1+1)n_t^y} \tag{3.11}$$

The growth rate of the workforce (total number of young and mature workers) can be estimated as:

$$n_t^{wf} = \frac{(N_t^m + N_t^y)}{(N_{t-1}^m + N_{t-1}^y)} - 1 = \frac{(1+\psi_t^1)n_t^y}{1+\psi_{t-1}^1} - 1 \tag{3.12}$$

labour force in the economy as equal to number of mature workers and number of young workers in same effective units and it is given as:

$$N_t = \xi_t(1 - \frac{\lambda_c}{\psi_t^c})N_t^y + N_t^m = (\xi_t(1 - \frac{\lambda_c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{3.13}$$

where λ_c is time cost of children as in previous chapter. Growth rate of the economy's labour force in efficiency unit:

$$n_t^{wf\xi} = \frac{\xi_t(1-\frac{\lambda_c}{\psi_t^c})N_t^y + N_t^m}{\xi_{t-1}(1-\frac{\lambda_c}{\psi_{t-1}^c})N_{t-1}^y + N_{t-1}^m} - 1 = \frac{(\xi_t(1-\frac{\lambda_c}{\psi_t^c}) + \psi_t^1)n_t^y}{\xi_{t-1}(1-\frac{\lambda_c}{\psi_{t-1}^c}) + \psi_{t-1}^1} \tag{3.14}$$

Because labour supply of young workers depend on child dependency ratio, it is given as:

$$N_t^{y\xi} = \xi_t(1 - \frac{\lambda_c}{\psi_t^c})N_t^y \tag{3.15}$$

Growth rate of effective labour supply of young workers are equal to:

$$n_t^{y\xi} = \frac{N_t^{y\xi} - N_{t-1}^{y\xi}}{N_{t-1}^{y\xi}} = \frac{\xi_t(1 - \frac{\lambda_c}{\psi_t^c})n_t^y}{\xi_{t-1}(1 - \frac{\lambda_c}{\psi_{t-1}^c})} \tag{3.16}$$

3.2.2 Households

There are four different age groups of individuals: retirees, mature workers, young workers, and dependent children. All age groups, except children, maximize their respective lifetime utility subject to their intertemporal budget constraints. Individuals maximize a non-expected utility function proposed by Farmer (1990), which provides a certainty-equivalent decision rule in the face of income risk that arises due to stochastic transition probability from one life stage to another. Different age individuals have different intertemporal budget constraints. The superscript 'i' denotes the individual type: 'r' for retirees, 'm' for mature workers, 'y' for young workers, and 'c' for children.

$$V_t^i = \left[(C_t^i)^\rho + \beta_i \mathbb{E}(V_{t+1}|i)^\rho\right]^{\frac{1}{\rho}} \tag{3.17}$$

where

$$\beta_i = \begin{cases} (1 - \gamma_{t,t+1})\beta & \text{if i = r} \\ \beta & \text{if i = m,y} \end{cases}$$

$$\mathbb{E}(V_{t+1}|i) = \begin{cases} V_{t+1}^r & \text{if i = r} \\ (1-\omega)V_{t+1}^m + \omega V_{t+1}^r & \text{if i = m} \\ (1-\theta)V_{t+1}^y + \theta V_{t+1}^m & \text{if i = y} \end{cases}$$

Children

Children do not make any decisions in the economy, and each child consumes a fixed portion of labour income. Workers pay for aggregate child consumption proportional to their labour productivity. Workers give away a part of their labour income for child consumption every period. Because the workers in this model supply labour inelastically, the specification of children's consumption in this way works as a lump-sum tax. Furthermore, because the wage rate is increasing at the rate of labour productivity growth, children's resources are also growing at the same rate as the wages increase. Every period, a worker gives away $t^c\psi_t^3$ portion of his/her labour income for child consumption. How much a worker gives away for child consumption depends on child dependency ratio (children to effective labour force ratio) given by ψ_t^3.

Young workers

This section provides the solution and the derivation of a young worker's decision rule. In addition to notation for whether a person is a retiree, a mature worker, or a young worker, we can differentiate individuals by their date of birth and the period when he/she transitioned to his/her next life stage. For example, 'j' indicates when people become young workers, 'k' denotes when they become mature workers, and 's' denotes when they become retirees. Individuals differ by their accumulated assets, depending on how much time an individual spent in each life stage.

Young workers have lower productivity than mature workers as well as lower labour supply due to the need to take care of children as in Lau (2014). Like mature workers, young workers also have to transfer a portion of their labour income for child consumption and pay taxes for pension and government expenditure.

When transitioning from a dependent child to a young worker, a young worker starts without any assets. This means $A_t^{yj} = 0$ when j is current period. A young worker supplies 1 unit of labour inelastically and consumes out of assets and labour income. However, young workers' productivity level is lower than the productivity of mature workers due to inexperience. Besides, young workers have to take some time out raising children. Therefore, a young worker's actual labour supply equals their labour endowment minus time spent raising children. Because young workers' have lower productivity than mature workers, they receive a fraction of mature workers' wage corresponding to their labour productivity level represented, here, by ξ . Young workers also transfer a portion of their labour income given by $\xi W_t t^c$ for child consumption and pay taxes.

With the above constraints reflected, a young worker's problem is to maximize lifetime utility:

$$V(A_t|yj) = \max_{\{C_t, A_{t+1}\}} \left\{ [C_t^{yj}]^\rho + \beta \mathbb{E}_t V(A_{t+1}^{yj})^\rho \right\}^{\frac{1}{\rho}} \tag{3.18}$$

subject to an intertemporal budget constraint:

$$A_{t+1}^{yj} = R_t A_t^{yj} + \xi(1 - \frac{\lambda_c}{\psi_{c,t}})(W_t(1 - \psi_t^3 t^c) - T_t^w) - C_t^{yj} \tag{3.19}$$

where expected next period lifetime value function is

$$\mathbb{E}_t V(A_{t+1}^{yj}) = (1-\theta)V(A_{t+1}^{yj}) + \theta V(A_{t+1}^{mjk}) \tag{3.20}$$

First order necessary condition for the decision problem of a young worker gives the consumption euler equation of the form:

$$(1-\theta)C_{t+1}^{yj} + \theta \Lambda_{2,t+1} C_{t+1}^{mjk} = \Big(R_{t+1}\Omega_{2,t+1}\beta\Big)^\sigma C_t^{yj} \tag{3.21}$$

where $\Lambda_{2,t+1} = \epsilon_{2,t+1}^{\frac{\sigma}{\sigma-1}}$ Furthermore, the consumption function for a young worker is conjectured as:

$$C_t^{yj} = \epsilon_{2,t}\pi_t\Big(R_t A_t^{yj} + H_t^{yj} + S_t^{yj}\Big) \tag{3.22}$$

Solution to a young worker's decision problem gives the evolution of marginal propensity to consume out of wealth for a young worker:

$$\epsilon_{2,t}\pi_t = 1 - \beta^{\sigma}(R_{t+1}\Omega_{2,t+1})^{\sigma-1}\frac{\epsilon_{2,t}\pi_t}{\epsilon_{2,t+1}\pi_{t+1}} \tag{3.23}$$

where

$$\Omega_{2,t+1} = (1-\theta) + \theta\epsilon_{2,t+1}^{1/(\sigma-1)} \tag{3.24}$$

$$H_t^{yj} = \xi(1 - \frac{\lambda_c}{\psi_{c,t}})(W_t(1-\psi_t^3 t^c) - T_t^w) + (1-\theta)\frac{H_{t+1}^{yj}}{R_{t+1}\Omega_{2,t+1}} + \theta\frac{H_{t+1}^{mjk}\epsilon_{2,t+1}^{\frac{1-\rho}{\rho}}}{R_{t+1}\Omega_{2,t+1}} \tag{3.25}$$

Social security wealth for a young worker is defined as the present value of expected pension income by a young worker:

$$S_t^{yj} = (1-\theta)\frac{S_{t+1}^{yj}}{R_{t+1}\Omega_{2,t+1}} + \theta\frac{S_{t+1}^{mjk}\epsilon_{2,t+1}^{1/(\sigma-1)}}{R_{t+1}\Omega_{2,t+1}} \tag{3.26}$$

Mature workers

Mature workers supply one unit of labour inelastically and receive wage income of W_t at the end of each period. However, workers need to transfer a portion of their labour income ($\psi_t^3 t^c W_t$) for child consumption which depends on child dependency ratio (children to total labour force ratio given by ψ_t^3) as well as has to pay taxes. Workers save and consume from assets and remaining labour income after transfer to children and taxes. Workers face an uncertain time of retirement with $1-\omega$ is the probability to stay in the workforce in period t+1 of a mature worker at t. At the same time, ω is the probability of retiring at the beginning of the next period. The retirement risk will not complicate the mature workers' decision rule since they are risk-neutral by assumption.

$$V(A_t|mjk) = \max_{\{C_t, A_{t+1}\}}\left\{[C_t^{mjk}]^{\rho} + \beta\mathbb{E}_t V(A_{t+1}^{mjk})^{\rho}\right\}^{\frac{1}{\rho}} \tag{3.27}$$

subject to one's intertemporal budget constraint:

$$A_{t+1}^{mjk} = R_t A_t^{mjk} + (W_t(1-\psi_t^3 t^c) - T_t^w) - C_t^{mjk} \tag{3.28}$$

where expected next period lifetime value function is the weighted sum of lifetime value function of being a worker at t+1 and being a retiree at t+1.

$$\mathbb{E}_t V(A_{t+1}^{mjk}) = (1-\omega)V(A_{t+1}^{mjk}) + \omega V(A_{t+1}^{rjks}) \tag{3.29}$$

As in the retirees' case, workers' assets are measured at the beginning of each period while the wage and the interest are paid at the end of each period. A worker's consumption expenditure is also undertaken at the end of each period. Therefore, the assets of a mature worker evolve according to:

$$A_{t+1}^{mjk} = R_t A_t^{mjk} + (W_t(1-\psi_t^3 t^c - T_t^w) - C_t^{mjk} \tag{3.30}$$

First order necessary condition for the decision problem of a mature worker gives the consumption euler equation of the form:

$$(1-\omega)C_{t+1}^{mjk} + \omega\Lambda_{t+1}C_{t+1}^{rjks} = \Big(R_{t+1}\Omega_{t+1}\beta\Big)^{\sigma}C_t^{mjk} \tag{3.31}$$

where $\Lambda_{t+1} = \epsilon_{t+1}^{\frac{\sigma}{1-\sigma}}$

Furthermore, the consumption function for a mature worker is conjectured as:

$$C_t^{mjk} = \pi_t\Big(R_tA_t^{mjk} + H_t^{mjk} + S_t^{mjk}\Big) \tag{3.32}$$

Combining the Euler equation with the conjectured consumption function yields the solution to a mature worker's decision problem in the form of the evolution of marginal propensity to consume out of wealth for a mature worker as:

$$\pi_t = 1 - \beta^{\sigma}(R_{t+1}\Omega_{t+1})^{\sigma-1}\frac{\pi_t}{\pi_{t+1}} \tag{3.33}$$

where

$$\Omega_{t+1} = (1-\omega) + \omega\epsilon_{t+1}^{1/(1-\sigma)} \tag{3.34}$$

Human wealth is defined as the present value of expected lifetime labour income which is the discounted sum of wages. The human wealth of a mature worker evolves according to:

$$H_t^{mjk} = (W_t(1-\psi_t^3t^c) - T_t^w) + (1-\omega)\frac{H_{t+1}^{mjk}}{R_{t+1}\Omega_{t+1}} \tag{3.35}$$

Social security wealth for a mature worker is defined as the present value of expected pension income a mature worker gets if retired next period:

$$S_t^{mjk} = (1-\omega)\frac{S_{t+1}^{mjk}}{R_{t+1}\Omega_{t+1}} + \omega\frac{S_{t+1}^{rjks}\epsilon_{t+1}^{\frac{\rho-1}{\rho}}}{R_{t+1}\Omega_{t+1}} \tag{3.36}$$

Retirees

Retirees do not work and consume out of their asset income. A retiree's decision problem is to choose consumption in each period and assets for the next period to maximize his/her lifetime utility given by a recursive value function:

$$V(A_t|rjks) = \max_{\{C_t,A_{t+1}\}}\Big\{[C_t^{rjks}]^{\rho} + \beta(1-\gamma_{t,t+1})\mathbb{E}_tV(A_{t+1}^{rjks})^{\rho}\Big\}^{\frac{1}{\rho}} \tag{3.37}$$

subject to an intertemporal budget constraint:

$$A_{t+1}^{rjks} = \frac{R_t}{1-\gamma_{t-1,t}}A_t^{rjks} - C_t^{rjks} + \frac{P_t}{N_t^r} \tag{3.38}$$

with expected lifetime utility next period being:

$$\mathbb{E}_tV(A_{t+1}^{rjks}) = V(A_{t+1}^{rjks}) \tag{3.39}$$

where ρ is associated with intertemporal elasticity of substitution σ with $\sigma = \frac{1}{1-\rho}$. β is the subjective discount rate or time preference rate. C_t^{rjks} is the consumption at t of the retiree born in period j, become a mature worker at period k, and retired at period s. $V(A_t|r)$ is the recursive form of a retiree's lifetime value function with an asset of A_t at time t. P_t is the aggregate pension expenditure that is divided among retirees equally.

Since retirees can die at the end of the current period, the discount rate for the expected lifetime utility from the next period includes retirees' survival probability $1-\gamma_t$. The assumed value function and associated intertemporal budget constraint imply that the retirees do not value leaving bequests. Retirees are constrained to have 0 or positive assets at the end of their life. Furthermore, to eliminate unintentional bequest, either positive or negative, due to uncertain time of death, it is assumed there exists a fair-insurance company as in Yaari (1965) and Blanchard (1985). Retirees turn their assets to a mutual fund that invests the proceeds at the beginning of each period. The surviving retirees receive the fund and interest earnings at the end of each period in proportion to their initial contribution. The existence of an insurance fund makes the return on assets of a retiree who turns his wealth to this mutual fund $\frac{R_t}{1-\gamma_{t-1,t}}$ where $R_t = (1-\delta) + r$ and r being market interest rate.

Assets are measured at the beginning of each period, while interest is paid at the end of each period. Retirees also receive pension benefits equal to total pension expenditure divided by the number of retirees $\frac{P_t}{N_t^r}$ at the end of each period. The intertemporal budget constraint of a retiree who faces the probability of death at the end of the current period becomes:

$$A_{t+1}^{rjks} = \frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} - C_t^{rjks} + \frac{P_t}{N_t^r} \tag{3.40}$$

First order necessary condition for the decision problem of a retiree gives the consumption euler equation of the form:

$$C_{t+1}^{rjks} = C_t^{rjks}(\beta R_{t+1})^{\sigma} \tag{3.41}$$

Furthermore, the consumption function for a retiree is conjectured as:

$$C_t^{rjks} = \epsilon_t \pi_t \Big(\frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} + S_t^{rjks} \Big) \tag{3.42}$$

Combining the Euler equation with the conjectured consumption function yields the solution to a retiree's decision problem in the form of the evolution of marginal propensity to consume out of wealth for a retiree as:

$$\epsilon_t \pi_t = 1 - \beta^{\sigma} R_{t+1}^{\sigma-1} (1-\gamma_{t,t+1}) \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}} \tag{3.43}$$

Social security wealth for a retiree who are alive at t is defined as the present value of expected pension income over the retirement:

$$S_t^{rjks} = \frac{P_t}{N_t^r} + (1-\gamma) \frac{S_{t+1}^{rjks}}{R_{t+1}} \tag{3.44}$$

Aggregation

Because every person in retirement has the same life expectancy regardless of when he/she is retired or being born, they all have the same planning horizon implying everyone has the same marginal propensity to consume out of wealth. Therefore, it is straightforward to aggregate consumption of retirees as:

$$C_t^r = \epsilon_t \pi_t (R_t A_t^r + S_t^r) \tag{3.45}$$

with A_t^r equal to total assets held by retirees at time t. Similarly, everyone who is a mature worker also faces the same probability of retiring the next period making their planning horizon the same regardless of their age. Therefore, the aggregate consumption of mature workers are:

$$C_t^m = \pi_t (R_t A_t^m + H_t^m + S_t^m) \tag{3.46}$$

where A_t^m is total assets held by mature workers at time t and H_t^m is the present value of total lifetime labour income of those who are a mature worker at time t. Aggregate consumption of young workers is also:

$$C_t^y = \epsilon_{2,t} \pi_t (R_t A_t^y + H_t^y + S_t^y) \tag{3.47}$$

Where A_t^y is total assets held by young workers at time t and H_t^y is the present value of total lifetime labour income of those who are a young worker at time t.

As for consumption by children, since each child consumes $t^c W_t$ amount in every period, aggregate consumption by children becomes

$$C_t^c = t^c W_t N_t^c \tag{3.48}$$

Because, as shown in equation 3.49 and 3.50, workers discount the future at a higher rate due to finite work and lifetime, they do not capitalize the stream of future taxes fully, whether it is for child consumption or government spending. In other words, the increase in taxes reduces the present value of lifetime human wealth less in this framework than in the infinitely lived representative agent case causing the workers to internalize the future taxes partially. Thus, the introduction of child consumption does not crowd-out workers' consumption one to one, and the presence of child consumption increases aggregate consumption.

The aggregate, present value of the human wealth of mature workers who are alive at time t evolves according to:

$$H_t^m = (W_t(1 - \psi_t^3 t^c) - T_t^w) N_t^m + (1 - \omega) \frac{H_{t+1}^m}{n_{t+1}^m R_{t+1} \Omega_{t+1}} \tag{3.49}$$

whereas, aggregate, present value of human wealth of young workers who are alive at time t evolves according to:

$$H_t^y = \xi (1 - \frac{\lambda_c}{\psi_t^c})(W_t(1 - \psi_t^3 t^c) - T_t^w) N_t^y + (1 - \theta) \frac{H_{t+1}^y}{R_{t+1} \Omega_{2,t+1} n_{t+1}^{y\xi}} + \theta \frac{H_{t+1}^m \epsilon_{2,t+1}^{\frac{1-\rho}{\rho}}}{R_{t+1} \Omega_{2,t+1} n_{t+1}^m} \tag{3.50}$$

Aggregate lifetime social security wealth evolve according to:

$$S_t^y = (1 - \theta) \frac{S_{t+1}^y}{R_{t+1} \Omega_{2,t+1} n_{t+1}^y} + \theta \frac{S_{t+1}^m \epsilon_{2,t+1}^{1/(\sigma-1)}}{R_{t+1} \Omega_{2,t+1} n_{t+1}^m} \tag{3.51}$$

$$S_t^m = (1-\omega)\frac{S_{t+1}^m}{R_{t+1}\Omega_{t+1}n_{t+1}^m} + \omega\frac{S_{t+1}^r \epsilon_{t+1}^{\frac{\rho-1}{\rho}}}{R_{t+1}\Omega_{t+1}n_{t+1}^r} \tag{3.52}$$

$$S_t^r = P_t + (1-\gamma)\frac{S_{t+1}^r}{R_{t+1}n_{t+1}^r} \tag{3.53}$$

with the economy-wide consumption function being

$$C_t = \pi_t\Big[A_t R_t\Big(1+(\epsilon_t-1)\lambda_t^1+(\epsilon_{2,t}-1)\lambda_t^2\Big)+\epsilon_{2,t}(H_t^y+S_t^y)+\epsilon_t S_t^r+H_t^m+S_t^m\Big]+t^c W_t N_t^c \tag{3.54}$$

where $\lambda_t = A_t^r/A_t$ is the fraction of total assets in the economy held by retirees, $\lambda_{2,t} = A_t^y/A_t$ is the fraction of total assets in the economy held by young workers.

Evolution of capital distribution between different generations follows:

$$\lambda_{t+1}^1 + \frac{\omega}{1-\theta}\lambda_{t+1}^2 = \frac{P_t - \epsilon_t\pi_t S_t^r + (1-\omega)R_t\lambda_t^1 A_t(1-\epsilon_t\pi_t)}{A_{t+1}} + \omega \tag{3.55}$$

$$\lambda_{t+1}^2 = \frac{(1-\theta)((1-\epsilon_{2,t}\pi_t)\lambda_t^2 R_t A_t + \xi(1-\frac{\lambda_c}{\psi_t^c})(W_t(1-\psi_t^3 t^c)-T_t^w)N_t^y - \epsilon_{2,t}\pi_t(H_t^y+S_t^y))}{A_{t+1}} \tag{3.56}$$

3.2.3 Government

In each period, government levies a lump-sum tax and incurs debt to finance its wasteful spending and pays retirees . The period budget constraint of the government is:

$$B_{t+1} = R_t B_t + G_t + P_t - T_t \tag{3.57}$$

where

$$T_t = T_t^w N_t \tag{3.58}$$

The intertemporal budget constraint of the government is:

$$\sum_{v=0}^{\infty}\frac{T_{t+v}}{\Pi_{s=1}^v R_{t+z}} = R_t B_t + \sum_{v=0}^{\infty}\frac{G_{t+v}+P_{t+v}}{\Pi_{s=1}^v R_{t+z}} \tag{3.59}$$

In the model, it is assumed that debt to GDP ratio ($\overline{b_y}Y_t$), government spending to GDP ratio ($\overline{g_y}Y_t$) and the pension expenditure to GDP ratio ($\overline{p_y}Y_t$) is fixed and taxes adjust endogenously to satisfy the government's intertemporal budget constraint.

Comparing equation 3.59 with equations 3.49 and 3.50 shows that the discount rate used by the workers is different from the discount rate the government uses. Workers discount rate is higher, including parameters that reflect finite work (presence of θ and ω) and lifetime (presence of Ω). In other words, the government expenditure does not crowd-out private consumption to the extent it does in the infinite horizon representative agent economy because workers only partially internalize the future tax burden. Therefore, taxes and debts matter in this life-cycle economy than in the economy populated by representative agents with infinite horizons. In addition, because of different marginal

propensities to consume out of the wealth of different age cohorts, redistribution of wealth across cohorts also have an additional effect on aggregate consumption and savings separate from its effect due to change in timing of taxes and transfers as in the model of Gertler (1999).

3.2.4 Firms

The production structure of the economy is kept simple as in Ferrero (2010); Niemeläinen (2017); Coeurdacier et al. (2015) to make it comparable and contrast how child dependency and age-earnings profile changes the results found in previous papers. It is assumed that there is a representative firm producing output with capital and labour with a technology represented by a production function:

$$Y_t = (X_t N_t)^{\alpha} K_t^{1-\alpha} \tag{3.60}$$

with labour augmenting technology growing at an exogenous rate of x

$$X_t = (1 + x_{l,t}) X_t \tag{3.61}$$

I considered capital adjustment cost with same specification as in Ferrero (2010); Niemeläinen (2021). However, the simulations showed that the adjustment costs do not change the results much within a perfect foresight model. Within a perfect foresight model, agents know the demographic and productivity transition paths except only in the first period. Therefore, the model's results with capital adjustment cost differ from the model without adjustment cost only in the first period. In the absence of capital adjustment costs, the capital stock in the economy evolves according to:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{3.62}$$

First-order conditions of the firm's problem give the demand for labour and capital:

$$W_t = \alpha \frac{Y_t}{N_t} \tag{3.63}$$

$$r_t = (1 - \alpha) \frac{Y_t}{K_t} \tag{3.64}$$

3.2.5 Characterization of equilibrium in the world economy

The competitive world equilibrium is defined as a sequence of endogenous quantity and price variables given the sequence of exogenous predetermined variables, and the initial values of all the predetermined variables such that in each country i) households maximize their utility subject to their budget constraints, ii) firms maximize their profits subject to their technology constraints, and iii) all markets clear.

Total assets in the home country is equal to the value of total capital stock in the home country, government debt and net foreign asset holdings of the home country.

$$A_t = K_t + B_t + F_t \tag{3.65}$$

Foreign assets evolve according to

$$F_{t+1} = R_t F_t + NX_t \tag{3.66}$$

which links the goods and asset markets. Trade deficits and surpluses change net foreign asset holdings.

The aggregate capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{3.67}$$

The trade balance is determined by the difference between aggregate output and aggregate expenditures which include both private and government consumption and investment expenditure.

$$NX_t = Y_t - (C_t + I_t + G_t) \tag{3.68}$$

Free flow of assets between economies ensures equalization of gross interest rate R_t between countries. In equilibrium, net asset holdings of the two countries cancel each other out.

$$F_t + F_t^f = 0 \tag{3.69}$$

Current account is the trade balance plus net interest payment on foreign assets.

$$CA_t = NX_t + (R_t - 1)F_t \tag{3.70}$$

The total labour force in the economy equals the number of mature workers plus the number of young workers multiplied by their labour productivity. labour market equilibrium is when the total supply of labour equals the total demand for labour.

$$N_t = \xi(1 - \frac{\lambda_c}{\psi_t^c})N_t^y + N_t^m = (\xi(1 - \frac{\lambda_c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{3.71}$$

All quantity variables are normalized by population and productivity growth to estimate the model's steady state. The steady-state and the transition dynamics are solved using the non-linear Newton method. Government debt is assumed to be exogenous and a fixed share of GDP to close the model. The appendix section B provides a description of the solution method and the algorithm used to solve the model. Julliard (2005) also offers further explanation of the solution method.

3.3 Determinants of external imbalances

A perfectly integrated capital market implies that capital per effective labour is equalized across countries in each period in the absence of capital adjustment cost.

$$\kappa = \kappa^f \tag{3.72}$$

Since output per effective labour is only a function of capital per effective labour, output per effective labour is also equalized across countries in each period.

$$y = y^f \tag{3.73}$$

When the government expenditure per effective labour is equal across countries, the differences in consumption and investment completely characterize the trade balance between countries 3.74.

$$nx_t = \frac{-1}{(1+\Psi_t^r)}\Big[(c_t - c_t^f) + (i_t - i_t^f)\Big] \tag{3.74}$$

where Ψ_t^r reflects relative country sizes as expressed by:

$$\Psi_t^r = \frac{X_t N_t}{X_t^f N_t^f} \tag{3.75}$$

which evolves according to:

$$\Psi_{t+1}^r = \Psi_t^r \frac{n_{t+1}^{f,wf\xi} x_{l,t+1}^f}{n_{t+1}^{wf\xi} x_{l,t+1}} \tag{3.76}$$

Small alphabets denote variables expressed in effective labour units. As in Ferrero (2010) we can denote a variable that is showing the difference between variable in the home country relative to the foreign country as $z_{R,t} = z_t - z_t^f$. Then equation 3.74 becomes:

$$nx_t = \frac{-1}{(1+\Psi_t^r)}\Big[c_{R,t} + i_{R,t}\Big] \tag{3.77}$$

Further, consumption differentials can be written as:

$$c_{R,t} = c_{R,t}^c + c_{R,t}^y + c_{R,t}^m + c_{R,t}^r \tag{3.78}$$

Different from the model in Ferrero (2010), consumption differences stem from not only the differences in consumption of the workers and the retirees but also from the differences in consumption by the children, which depend on child dependency ratio and cost per child. The importance of this mechanism grows when one considers China. Despite examining the trade imbalances of China, which experienced a sharp drop in its child dependency ratio, Niemeläinen (2021) has also neglected the role of this mechanism. Child consumption differences can be written as:

$$c_{R,t}^c = \left(t^c w_t \frac{\psi_t^4}{\psi_t^c} - t^{c,f} w_t^f \frac{\psi_t^{4,f}}{\psi_t^{c,f}}\right) \tag{3.79}$$

With the same children to labour force ratio, consumption differences between children across countries depend only on the cost per child:

$$c_{R,t}^c = \left(t^c - t^{c,f}\right) w_t \frac{\psi_t^4}{\psi_t^c} \tag{3.80}$$

However, the effect of child dependency on trade balance works through the consumption of children and its impact on workers' consumption. Higher consumption costs of children will reduce consumption by the workers. However, it doesn't reduce workers' consumption one to one because workers discount the future more. Therefore, even though the relative consumption of workers in a country with high child consumption may be less, it is not enough to offset the higher consumption of children, resulting in higher aggregate consumption, which causes a country to run a trade deficit.

In addition to the consumption cost of children, dividing the work-life stage into two stages allow us to introduce credit constraints as in Coeurdacier et al. (2015). Credit constraint limits the effect of productivity growth on consumption growth by lowering access to future human wealth. Coeurdacier et al. (2015) argues that the credit constraint is one of the critical factors underlying the divergence of savings between the US and China. Consumption differentials can be written for young workers as:

$$c^{y}_{R,t} = R_t\Big(\epsilon_{2,t}\pi_t\lambda_t^2 a_t - \epsilon^f_{2,t}\pi^f_t\lambda_t^{2,f}a^f_t\Big) + (\epsilon_{2,t}\pi_t(h^y_t + s^y_t) - \epsilon^f_{2,t}\pi^f_t\zeta(h^{y,f}_t + s^{y,f}_t)) \tag{3.81}$$

for mature workers as:

$$c^{m}_{R,t} = R_t\Big(\pi_t(1-\lambda_t-\lambda_t^2)a_t - \pi^f_t(1-\lambda^f_t-\lambda_t^{2,f})a^f_t\Big) + (\pi_t(h^m_t+s^m_t) - \pi^f_t(h^{m,f}_t+s^{m,f}_t)) \tag{3.82}$$

for retirees as:

$$c^{r}_{R,t} = R_t\Big(\epsilon_t\pi_t\lambda_t a_t - \epsilon^f_t\pi^f_t\lambda^f_t a^f_t\Big) + (\epsilon_t\pi_t s^r_t - \epsilon^f_t\pi^f_t s^f_t) \tag{3.83}$$

As can be seen from equations 3.81 to 3.83, not only differences in the MPCs but also in age-earnings profile (ξ) that affects the human wealth of young workers and the credit constraint (ζ) is important in determining consumption differentials.

Another determinant of the trade imbalances across countries is the differences in investment growth. Capital accumulation equation in 3.67 provides clues to the investment differentials. When there are no capital adjustment costs in the symmetric steady-state, capital per effective labour force is equalized across countries. Therefore investment differentials reflect the differences in the growth of the effective labour force.

$$i_{R,t} = (n^{wf\xi}_t - n^{wf\xi,f}_t + x_{l,t} - x^f_{l,t})\kappa_t \tag{3.84}$$

If economy-wide labour productivity growth is the same across countries, then investment differentials depend only on the differences in labour force growth. However, when you take into account age-dependent labour productivity and labour cost of children, relative differences in labour force growth during the demographic transition becomes more complex than just the difference between the growth of the working-age population as assumed in Ferrero (2010).

Specifically, in a country with a declining fertility rate and an age-earnings profile higher at mature age on average, the labour force growth will be higher than the growth rate of the young workers during the medium term. At the same time, when children require time out of work, a declining fertility rate will also have a short-run positive impact on labour force participation. However, both of these channels' effects will be transitory and depend on country-specific circumstances that determine the age-earnings profile and children's time cost.

3.4 Calibration

The experiment in this paper is purely hypothetical, and it uses parameter values commonly used in the literature, mainly for the US economy. A summary is given below

for each parameter. The current paper highlights issues with calibration using historical values. Section 3.5 reports results of sensitivity analysis to illustrate the importance of different parameters on the steady-state of the model. The section below discusses possible strategies for calibrating critical parameters that determine the effect of demographic transition, particularly changes in the child dependency ratio, on external imbalances.

Consumption cost of children: In the baseline simulation, the paper assumes the consumption cost of children t^c to be equal to an average estimated value commonly found by empirical papers (see Letablier et al. (2009) for a review of the literature) as the share of annual income spent for children. Estimating the consumption cost of children is challenging, and the estimates differ quite a bit depending on the assumption and the methodology (Letablier et al., 2009). Therefore, a sensitivity analysis is conducted in section 3.5 to understand the implication of different values for the steady-state of the model.

labour cost of children: Similarly, to calibrate λ_c, findings from econometric studies are taken as guidance. Specifically, a study by D. E. Bloom et al. (2009) finds that each additional child causes two years out of work for the mother. Therefore, one can calibrate the parameter λ_c to generate an outcome where young workers must spend 1-2 years out of work per child. Section 3.5 presents the result from a sensitivity analysis for this parameter for the steady-state of the model.

Shape of the age-earnings profile: The age-earnings profile works as a weight given to different age cohorts for the growth of the workforce. The national transfer accounts database provides age-earnings profiles by country. In addition, Coeurdacier et al. (2015) provides more detailed data on the empirical age-earnings profile for both China and the US. Baseline simulation assumes the shape of the profile to be increasing at an older age and the same across countries. As highlighted by Coeurdacier et al. (2015), the shape of the age-earnings profile matters more when the young workers face credit constraints. Specifically, a profile with an increasing income at a later age will induce a greater consumption motive at a young age. In addition, in the presence of an age-earnings profile, the growth of the effective labour force diverges from the growth of the workforce multiplied by economy-wide productivity growth. Then the question of how this discrepancy is taken into account when explaining the past arises. In studies that use Solow residual as a proxy for productivity growth as in Ferrero (2010), productivity growth measurements may incorrectly capture the changes in effective labour force growth due to age-earnings profile as economy-wide productivity growth.

Table 3.2: Demographic and household parameters

Parameter	Values	Description
ω	0.039	Probability to retire
θ	0.048	Probability to become mature worker
z	0.05	Probability to become young worker
ξ	0.75	Productivity of young workers relative to mature workers
γ_{ss}	0.08	Initial mortality rate

Continued on next page

Table 3.2 – continued from previous page

Parameter	Values	Description
n_{ss}	1.028	Initial population growth rate
β	0.99 0.946	Time preference rate (The rest, NC)
σ	0.55 0.34	Household intertemporal elasticity of substitution (The rest, NC3)

Table 3.3: Child dependency parameters

Parameter	Values	Description
t^c	0.25	Consumption cost of children as a share of wage
λ_c	0.23	Time cost of children

Table 3.4: Production parameters

Parameter	Values	Description
α	0.667	labour share in the final output
δ	0.1	Capital stock depreciation rate
x_{lss}	1.028	Initial productivity growth rate

Table 3.5: Other parameters

Parameter	Values	Description
Ψ^r_{ss}	1	Relative size of countries at the steady state
p_y	0.06	Share of pension expenditure in GDP
b_y	0.2	Government debt to GDP ratio
g_y	0.2	Government expenditure to GDP ratio
ζ	1	Share of capitalized value of labour income of the young workers that can be borrowed

3.5 Steady-state analysis

This section reports the results from sensitivity analysis concerning the steady-state of the model. Figure 3.3 shows the steady-state impact of the consumption cost of children (t^c) on the output per effective labour, interest rate and the distribution of assets at the steady-state. It shows that the higher consumption cost of children corresponds to lower output per effective labour and higher interest rate reflecting lower capital per effective labour. Not only do aggregate savings and assets in the economy decline in absolute value in the presence of the consumption cost of children, but also its distribution among the adult population changes. The bottom half of figure 3.3 shows that the asset holdings of workers differ markedly between models with and without child consumption. Higher the consumption cost of children, lower the share of assets held by workers and higher the share of assets by retirees'. Specifically, the share of assets held by retirees can increase by

almost 16 percentage points. In comparison, young and mature workers can be reduced by around seven percentage points each when the consumption cost of children increases from 0 to 50% of workers' income.

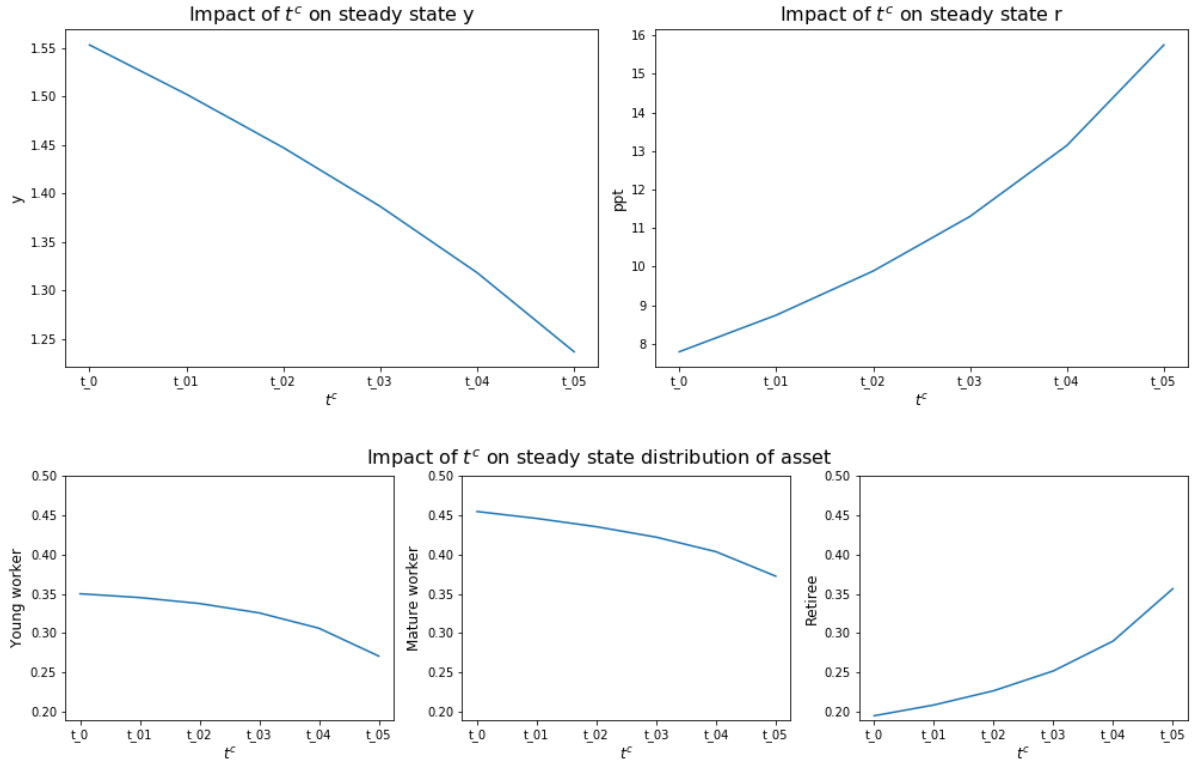


Figure 3.3: Steady state impact of consumption cost of children

Figure 3.4 shows the steady-state impact of the time cost of children. Unlike the consumption cost of children, the paper assumes that only young workers (20-39 age) spend time out of work to care for children. Like the consumption cost of children, the time cost of children reduces output per effective labour. It increases the interest rate reflecting lower capital per effective labour at the steady state. In contrast to the consumption cost of children, the time cost of children only reduces the share of assets held by young workers as they are the ones who bear the time cost of children. Specifically, the share of assets held by retirees and mature workers can increase by more than 6 and 5 percentage points each. In comparison, young workers' share can decline by more than 12 percentage points when the time cost of children increases from 0 to 30 % of young workers' available labour supply.

Two other critical parameters determining the interest rate at the steady-state are subjective time preference rate and the elasticity of intertemporal substitution. Figure 3.5 reports the steady-state impact of sigma (elasticity of intertemporal substitution). Higher sigma implies a greater motive for consumption smoothing, and it is associated with higher savings (lower MPCs) and capital per effective labour. In turn, this translates to higher output per effective labour and a lower interest rate. Higher sigma is also associated with a higher share of assets held by young workers who have the longest horizon and greater consumption smoothing motive. However, compared to children's time and consumption cost, the effect of sigma on the distribution of assets is modest.

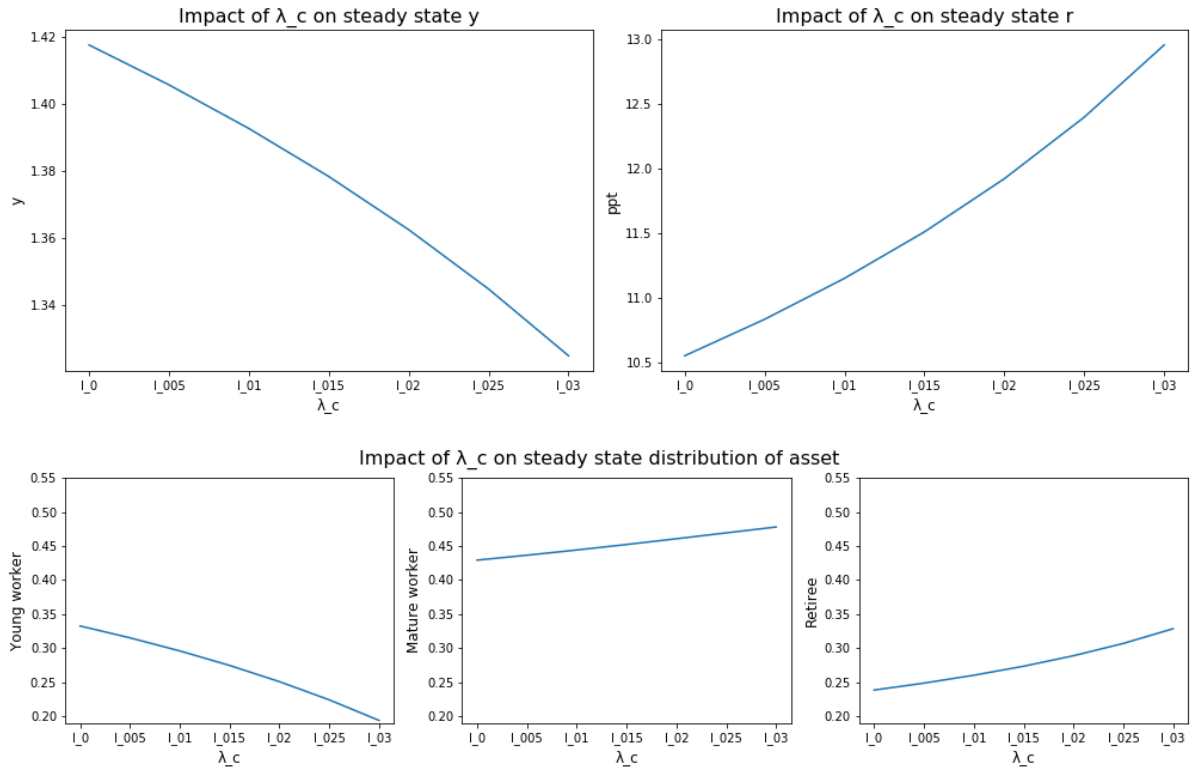


Figure 3.4: Steady state impact of time cost of children

It increases the share of assets held by young workers while reducing the share of assets held by mature workers by around two percentage points each when sigma rises from 0.3 to 0.6. In contrast, the share of assets held by retirees decline when sigma increases, but the magnitude of impact is almost negligible.

The effect of subjective time preference rate (beta) on output per effective labour and interest rate is comparable to sigma. Figure 3.7 reports the steady-state impact of different values for parameter beta. Higher beta, implying lower discounting of the future, is associated with higher savings (lower MPCs) and higher capital per effective labour. It translates to higher output per effective labour and a lower interest rate. Like lower sigma, lower beta is also associated with a lower share of assets held by young workers and a higher share of assets held by retirees and mature workers. The magnitude of the effect is also comparable to the impact of sigma. It ranges from less than 1 to 3 percentage points.

To summarize, the effect of child consumption and the time cost of children on aggregate savings and interest rate is similar to the impact of taxes and transfers within the life-cycle model. In a life-cycle model, discount rates used by cohorts differ from the interest rate. Specifically, all cohorts discount the future more because of finite life. In addition, workers discount their future income even more to take into account retirement. These additional discounts mean that the present value of the reduction in child consumption stream is more than the present value of the increase in human wealth due to reduced child consumption or time cost.

Second, the time and consumption cost of children and the subjective time preference rate and the elasticity of intertemporal substitution change the consumption of differ-

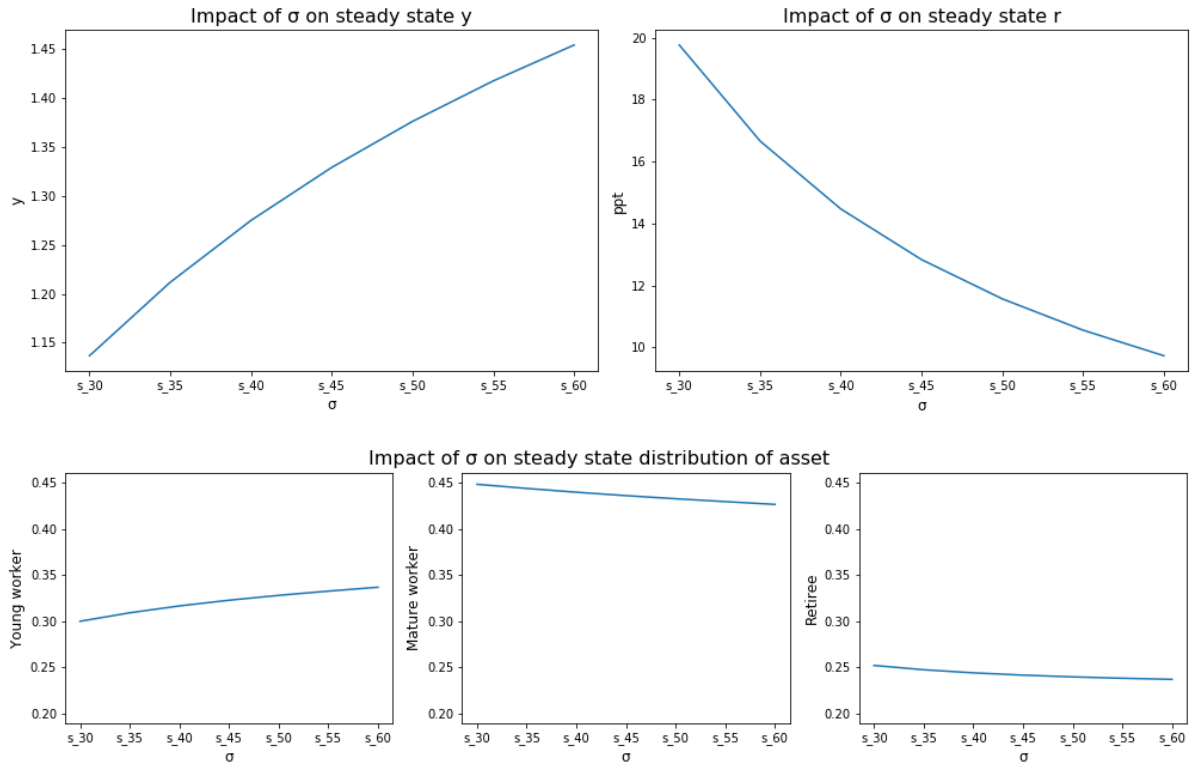


Figure 3.5: Steady state impact of elasticity of intertemporal substitution

ent age groups and affect the distribution of assets. By choosing lower values for beta and sigma than conventional, the models such as Coeurdacier et al. (2015) may end up selecting a micro-situation where younger workers, who have a longer planning horizon, consume more, thereby unjustly increasing the importance of their credit constraint mechanism. The steady-state analysis here shows that incorporating the cost of children may not only eliminate the need for setting beta and sigma to unconventionally low values, but it may increase the importance of the credit constraint channel.

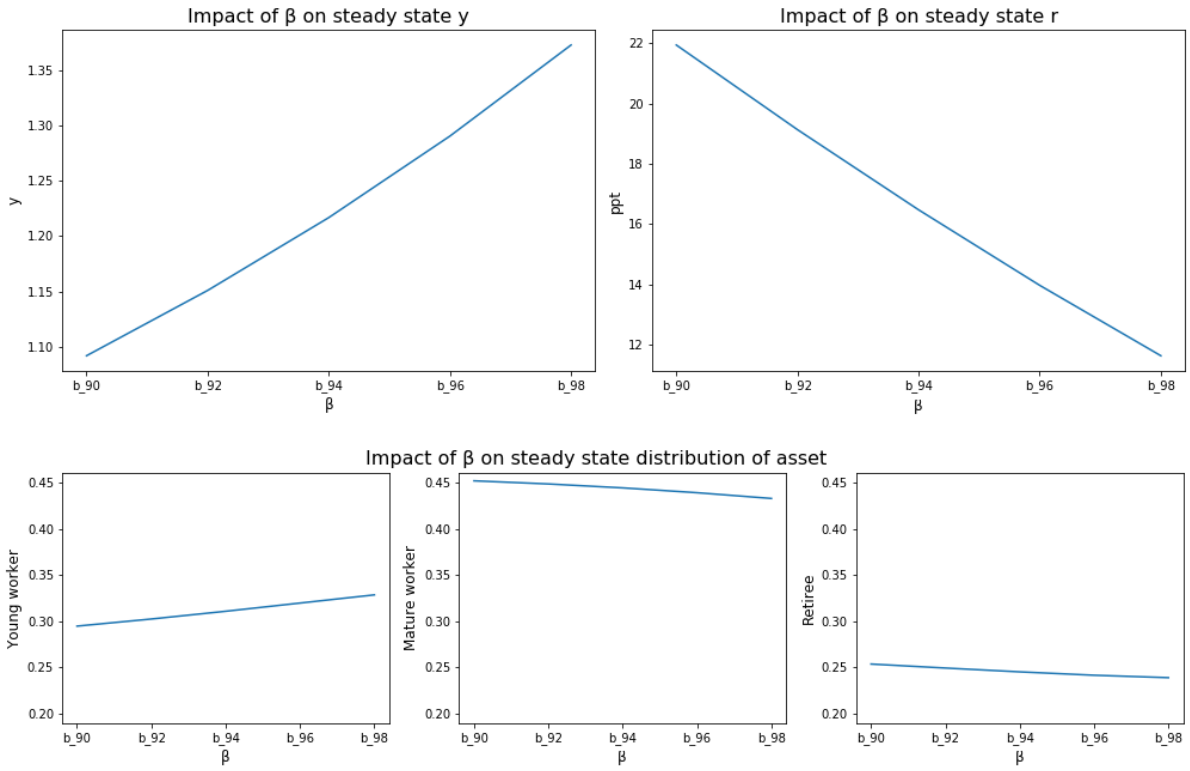


Figure 3.6: Steady state impact of subjective time preference rate

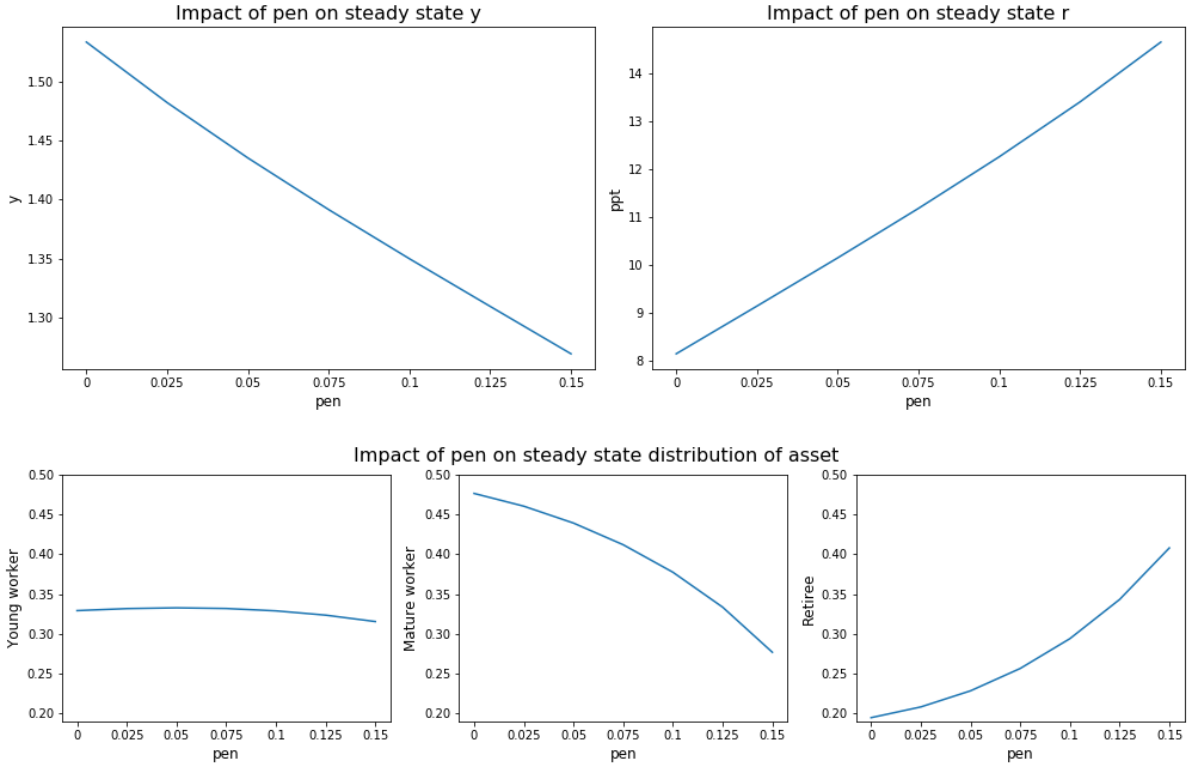


Figure 3.7: Steady state impact of PAYG pension

3.6 Simulation results

To understand how consumption cost of children affect the sign and magnitude of the impact of a demographic shock on external balances, an asymmetric fertility (section 3.6.1) and mortality (section 3.6.2) shock is introduced into the model described in section 4.2 in sequence. Then, to contrast the effect of a fertility shock with a productivity shock on external imbalances, an asymmetric productivity shock is introduced in section 3.6.3.

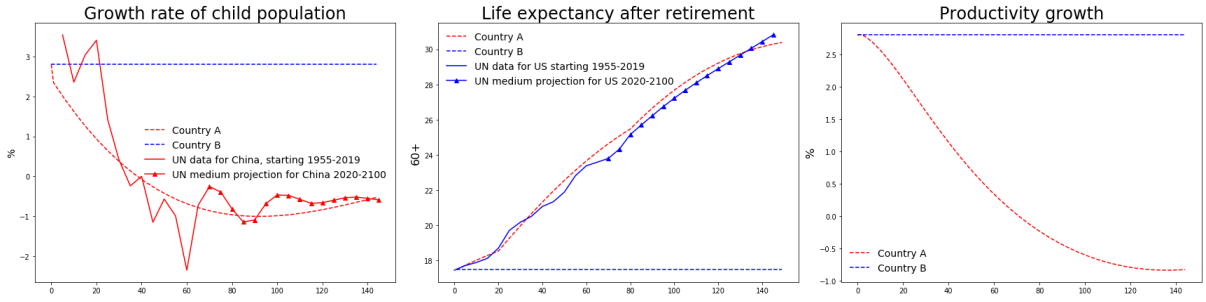


Figure 3.8: Model shocks and data

Initially, the two countries in the simulation are identical, and the trade balance is zero. The paper compares the results from a model with and without child dependency to isolate the children's impact on aggregate savings and external imbalances. Real-world examples are taken as guidance when choosing the magnitude of the demographic shocks. Figure 3.8 compares the fertility and mortality shocks introduced into the model with actual data for China and the US. In the fertility scenario, country A experiences a relative fertility decline, as shown in the first panel of figure 3.8. In the longevity transition, country A experiences a relative increase in life expectancy at old age, as shown in the second panel of figure 3.8. In the third scenario, country A experiences a relative decline in its economy-wide productivity growth as shown in the last panel of figure 3.8. The sections 3.6.1, 3.6.2 and 3.6.3 analyze the impact of these three transition scenarios on the external imbalances of both countries in sequence.

3.6.1 Fertility decline

Figure 3.9 shows the population dynamics under the fertility transition. Fertility changes affect the population's age structure,first by changing the child dependency ratio and the age structure of the workforce, and then by changing the old-age dependency ratio. Specifically, a decline in fertility reduces the child dependency ratio, increases the mature to young worker ratio, and ultimately increases the old-age dependency ratio.

As pointed out earlier, the model's steady-state with and without child dependency is different under the same parameter values. The paper calibrated the model without children in three different ways. Model NC2 has the same elasticity of intertemporal substitution (σ) and subjective time preference rate (β) as the model Base, which considers child dependency. Consequently, model NC2 has a different steady-state interest rate. Models NC and NC3 have different β (NC) and σ (NC3) respectively to target the same interest rate at the steady-state as in the model Base. Specifically, model NC has a lower β than the rest, while model NC3 has a lower sigma. In other words, the inclusion of the

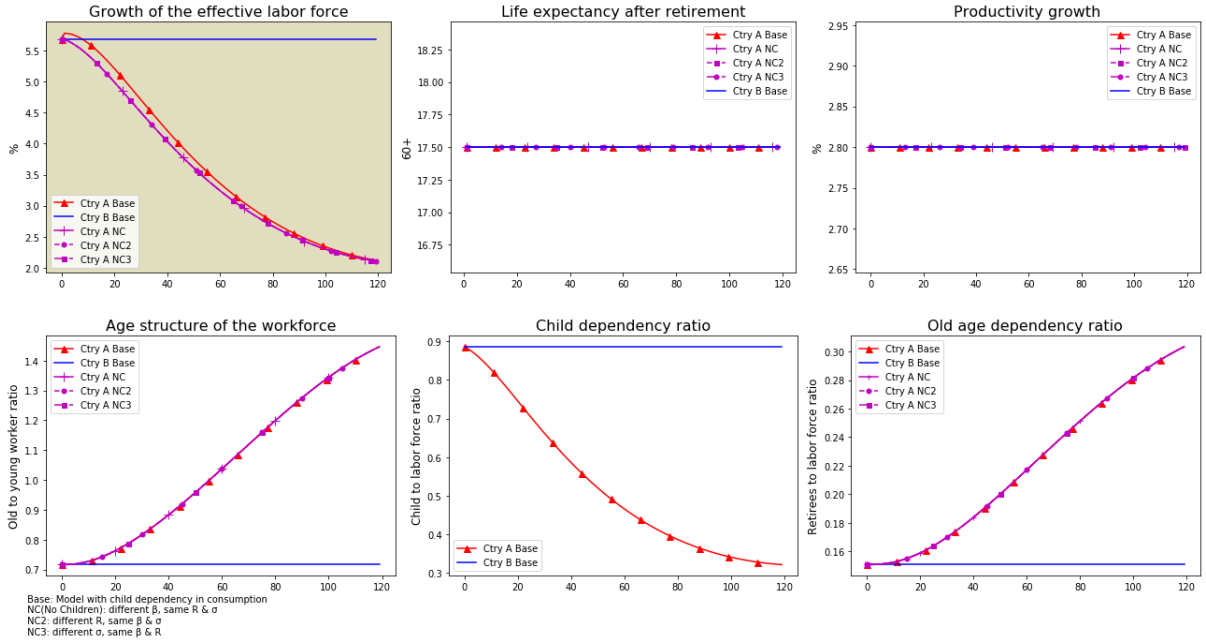


Figure 3.9: Population dynamics after the fertility shock

consumption cost of children reduces the steady-state savings and increases the steady-state interest rate. Therefore, the same model without children needs to be calibrated with a lower beta (higher discounting of the future) and a lower sigma (lower elasticity of intertemporal substitution) to generate the same interest rate as the model Base.

The declining fertility rate reduces the growth of the effective labour force for any given path of economy-wide productivity growth. However, as shown in the first plot of figure 3.9, the growth of the effective labour force diverges between models with and without the time cost of children. Specifically, the effective labour force can grow under a fertility decline in the medium run due to declining child dependency in models with the time cost of children whereas a model without the time cost of children will show declining labor force. Moreover, in the long run under a fertility transition, the growth of the effective labour force falls less in the model with children's time cost than the model without one.

Figure 3.10 compares the effect of fertility decline across models with and without child dependency on aggregate savings, investment, and trade balance in the medium-run (top panel) and the long-run (bottom panel). Comparing results from different models shows that children's time and consumption cost may change the sign of the impact of the fertility decline on aggregate savings, investment, and trade balance in the medium run. In contrast, it changes the magnitude of the long-run impact, particularly for a country undergoing a fertility change.

One can explain the short to medium-run dynamics by looking at the change in the marginal propensity to consume out of wealth for different age cohorts and the change in capitalized value of different types of wealth, reflecting changing factor prices and changing transfers between different generations. Under a fertility decline, the interest rate declines, reflecting lower growth in the effective labour force while the wage rate increases. The decrease in the interest rate is the largest in the model that considers children because the declining child dependency also affects aggregate savings under a

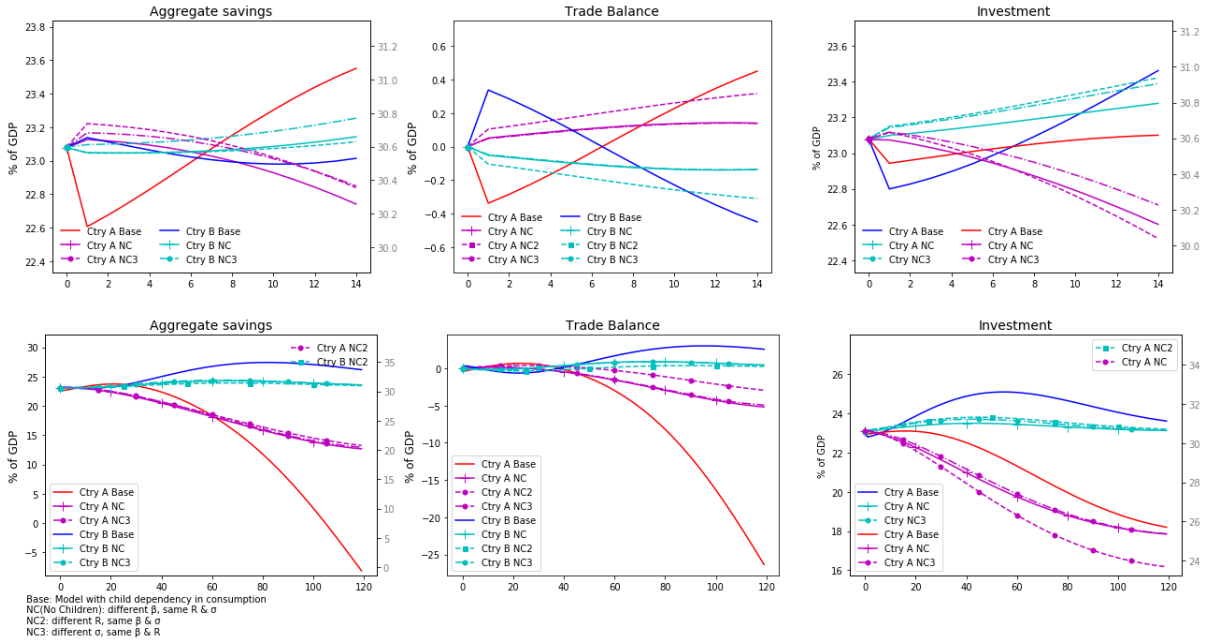


Figure 3.10: Aggregate savings, investment and trade balance after the fertility shock

fertility shock. Consequently, when σ is less than 1, a decline in the interest rate reduces the marginal propensity to consume (MPC) out of wealth for all cohorts. Across models, the magnitude of the fall in the MPCs is the largest for the model with children, while among generations, the fall is the largest for young workers who have the longest planning horizon.

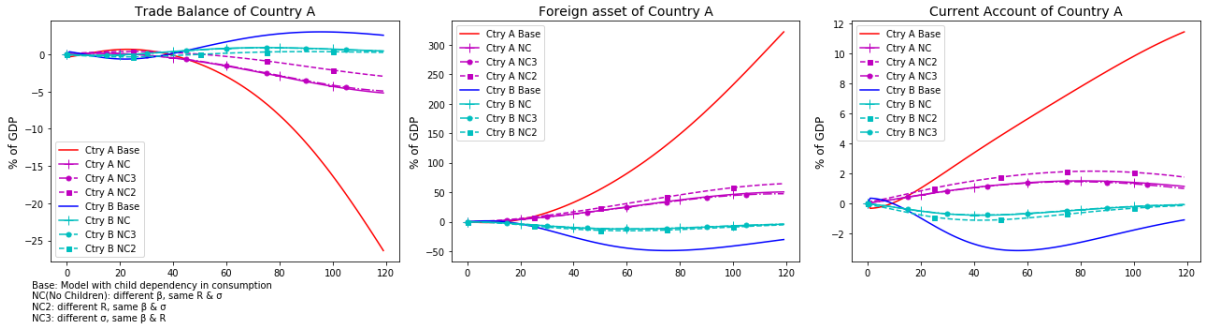


Figure 3.11: External balances after the fertility shock

The lower half of figure 3.10 shows that trade imbalances are driven to a larger extent by savings change in the long run. Country A experiences a larger decline in its savings than its investments which causes its trade deficit. Figure 3.11 shows that neglecting the consumption cost of children will result in significant differences in the estimated magnitude of the fertility shock on external imbalances in the long run, particularly for the country under fertility shock. The model with child dependency is associated with a higher increase in foreign assets and current account position, allowing higher trade deficits in the long run — in fact, they are twice as high in the model with children than without children.[2]

[2]Equation 3.66 shows that, next period foreign asset is equal to the current period foreign asset with

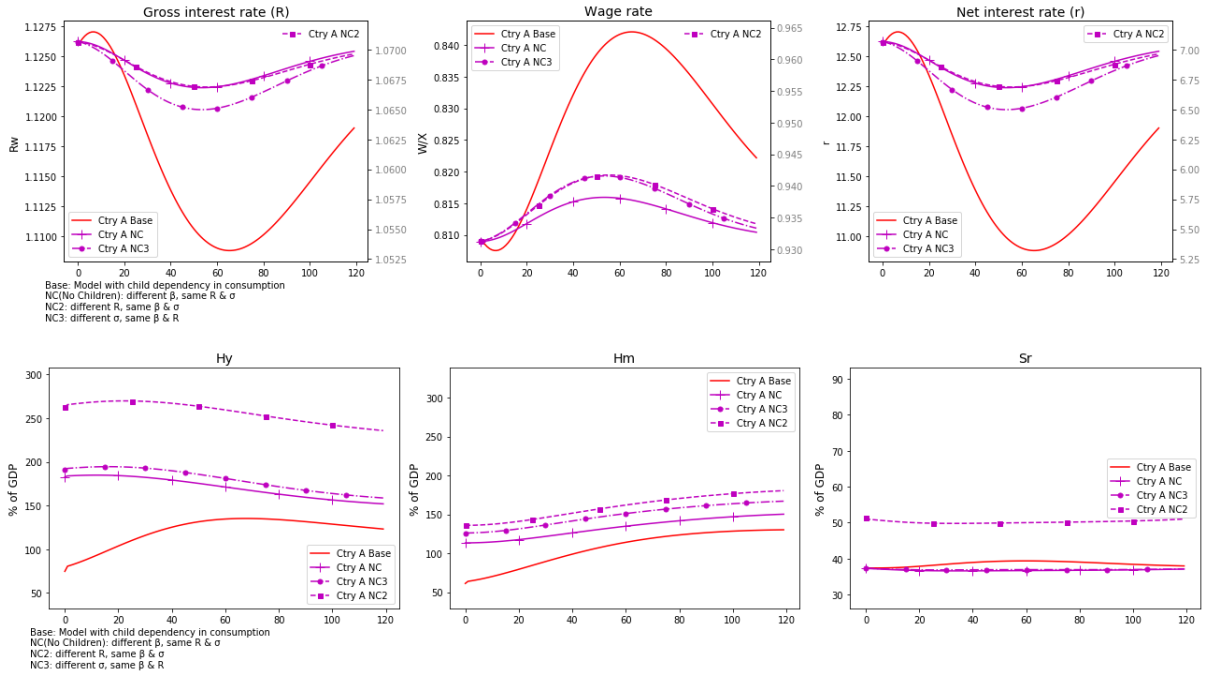


Figure 3.12: Factor prices and wealth dynamics after the fertility shock

The different short to medium-run dynamics across models with and without child dependency can be explained by looking at the differences in the dynamic response of the different types of wealth to the fertility shock. The bottom half of figure 3.12 shows the human wealth for young and mature workers and the social security wealth of retirees as a percent of GDP. The decline in the interest rate increases the capitalized value of the forward-looking wealth by reducing the discount rate. In addition to the effect of the interest rate on human wealth, child dependency will further increase the human wealth (which is calculated after all taxes and transfers to children) of workers in country A by reducing transfers to children. The impact of the fertility reduction on aggregate consumption can be different in the short run and the long run because it simultaneously changes the current and future dependency rates. In the long run, a country with lower child dependency will have higher aggregate savings following a demographic transition. However, at the beginning of the transition, when the workers expect the fertility rate to fall further, the increase in the human wealth of young and mature workers could raise consumption before child consumption declines. The size and path of the fertility shock and the consumption cost of children simultaneously determine the magnitude of the impact of fertility decline on aggregate consumption in the short to medium run.

In contrast, the long-run dynamics are affected more by the aggregation effect caused by the changing relative size of different age cohorts. The bottom half of figure 3.12 shows

its interest earnings plus trade balance. While equation 3.70 defines the current account as equal to the sum of trade balance and interest income on foreign assets. Therefore, it is possible to have a positive current account and negative trade balance when a country has a positive foreign asset position where the interest earnings on the foreign asset more than offsets the trade deficit. In other words, current account surplus is not only driven by the difference in domestic production and consumption (trade balance) but also by interest earnings on foreign assets. The savings in figure 3.10 doesn't include earnings on foreign assets.

that in the long run, the human wealth of young workers as a percentage of GDP declines while the human wealth of mature workers increases, reflecting the increase in the size of the mature working population relative to young workers. Consequently, in the long run, population aging pushes consumption up in country A by increasing the share of retirees with higher consumption propensities. Thus, country A's savings decline in the long run while increasing in country B. Across all models, investment falls in country A, driven by the decline in its workforce's growth, but increases in country B, reflecting an increase in savings and capital inflow.

Figure 3.12 compares factor prices across different model setups under the same fertility shock. As expected, the change in factor prices is largest in the model with child dependency because it introduces an additional mechanism that affects aggregate savings. Figure 3.12 also shows that the interest rate can increase in the model with children in the short run, reflecting an increase in the growth of the effective labour force. On the other hand, simulations show that the different calibrations also affect the dynamics of the factor prices under the fertility shock. Comparing the dynamics of the factor prices under the fertility shock in three different NC models is not straightforward. Specifically, because the steady-state interest rate is low in the NC2 model, the percentage change in the interest rate relative to its steady-state or initial value is higher in the NC2 model than in the NC and NC3 models. However, if we compare the change in factor prices in absolute terms, then the change in factor prices among the models without children is highest under lower σ values as shown in figure 3.12. Overall, declining fertility results in decline in the interest rate and an increase in the wage rate in the long run across all models for a given path of productivity growth. The exact magnitude of the effect depends on parameter values and the model specification.

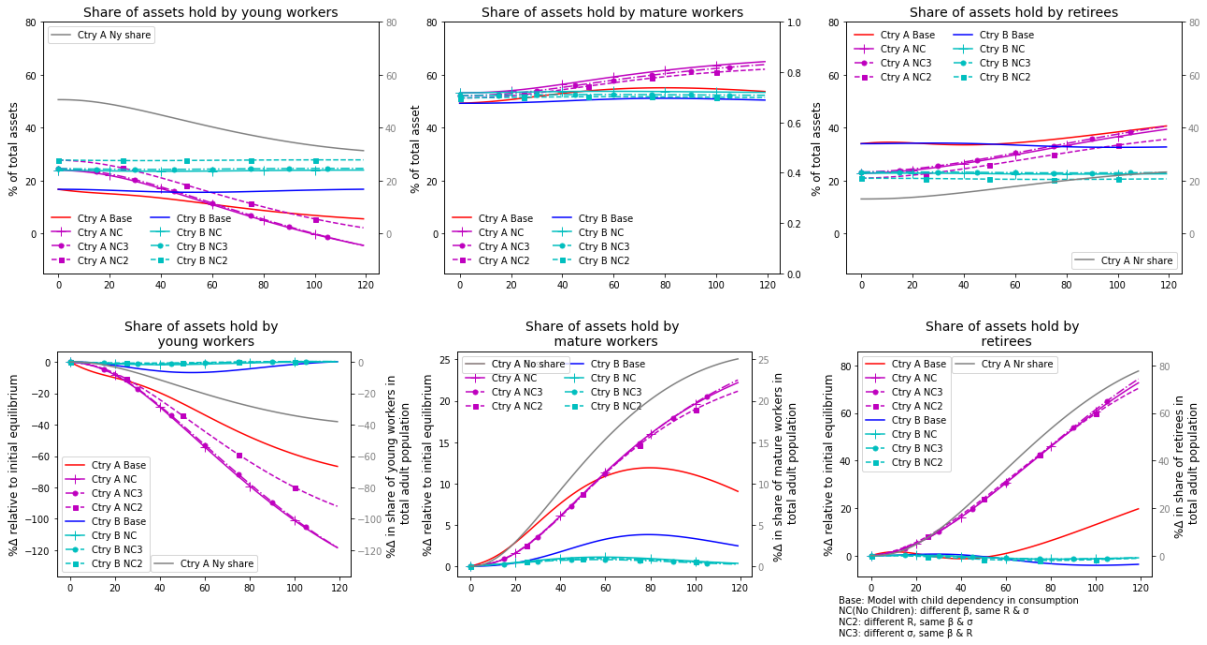


Figure 3.13: Distribution of assets after the fertility shock

Figure 3.13 and 3.14 shows the effect of fertility transition on the distribution of assets and marginal propensity to consume of different age cohorts. The first observation is that

the consumption cost of children changes the distribution of assets across age groups. Specifically, the share of assets held by retirees is larger in the model with children. In addition, the marginal propensity to consume (the opposite of savings rate) is different across all models. As shown in section C, MPCs have a negative relationship with β and σ, while they have a positive relationship with the interest rate.

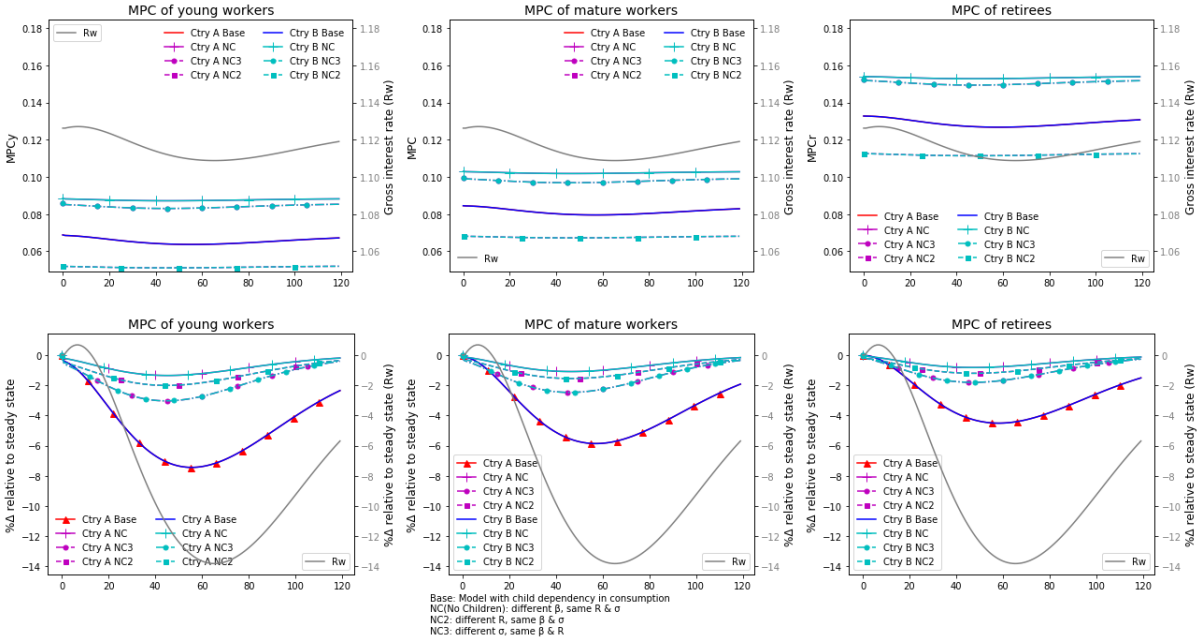


Figure 3.14: MPCs after the fertility shock

Figure 3.13 makes it clear that the change in the distribution of assets is driven mainly by the aggregation effect, which is the result of the changing relative size of different age cohorts. Blue lines show that the share of assets held by different age groups is relatively constant in country B, whose population age structure is stable. Any changes are due to change in factor prices. The share of assets held by young workers in country A declines more than the change in their population share, reflecting greater consumption by these groups under the fertility transition. On the other hand, the share of assets held by retirees increases less than the increase in their share of the adult population, reflecting the decline in interest rate. In other words, retirees reduce their consumption less relative to the decrease in interest rate. Figure 3.14 confirms this by showing that despite falling in line with the reduction in the interest rate, retirees' MPCs react the least to changes in the interest rate compared to workers reflecting their shorter planning horizon.

To summarize, the presence of time and the consumption cost of children changes the sign of the effect of fertility decline on aggregate savings and trade balances in the short run. In contrast, it changes the magnitude of the long-run impact. By adding additional mechanisms, the presence of children's time and consumption cost reduces the factor prices the most compared to models without children across different parameter specifications. This affects the magnitude of the impact of fertility shock on external imbalances across countries. This section also shows that different calibrations and model specifications correspond to different economies at the micro-level, despite representing the same economy at the aggregate level. Furthermore, simulation results indicate that

while the differences could be minor in the short run, they become quite large in the long run.

3.6.2 Longevity

Unlike fertility shocks, longevity shocks do not affect the child dependency ratio and workforce age structure, as shown in figure 3.21. However, they do change the old-age dependency ratio. Because a longevity shock doesn't affect the growth of the effective labour force in the models in this paper, investment dynamics across the two countries follow the same path, as shown in figure 3.16. The absence of blue lines in figure 3.16 indicates that the blue and red lines overlap.

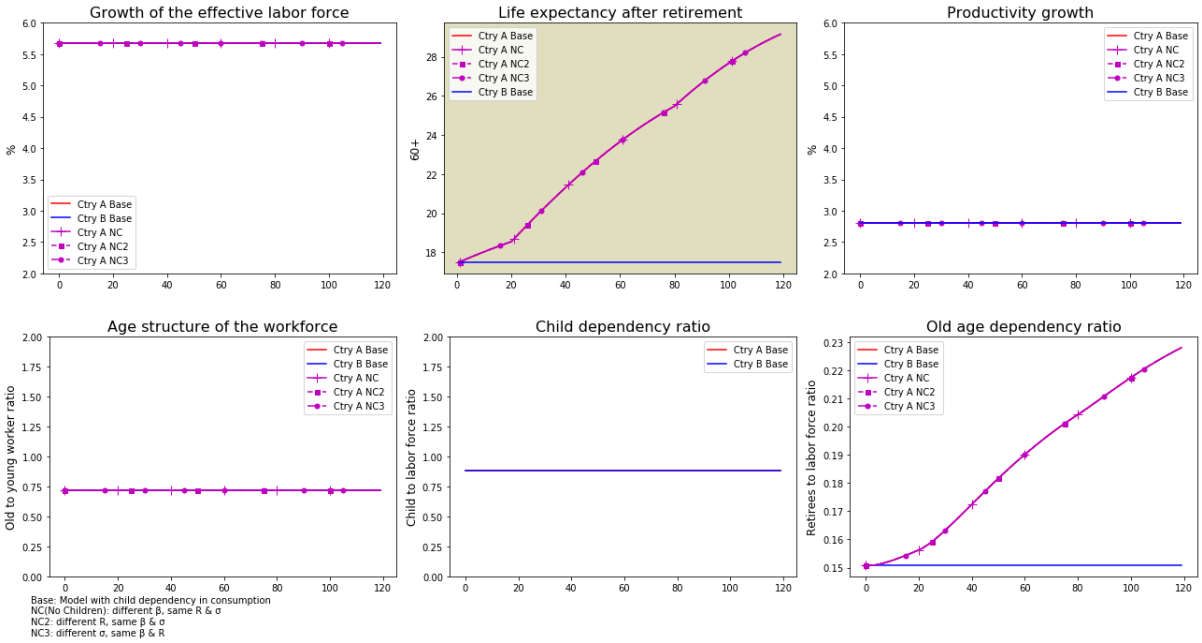


Figure 3.15: Population dynamics after the longevity shock

Figure 3.16 shows the effect of a longevity shock on aggregate savings, investment, and trade balance of two countries in the short and long run. Because investment dynamics are the same across the two countries, differences in savings alone drive trade imbalances between them. In country A, aggregate savings increase in response to the longevity shock in the short to medium run as households anticipate a more extended retirement period. Thus, country A runs a trade surplus in the short to medium run when it invests parts of its increased savings in foreign assets.

Across all models except NC2, trade imbalances display similar patterns in response to longevity shocks both in the short and long run, not counting minor differences in magnitude. The model with child dependency predicts country A to have a larger trade surplus in the short run and a larger trade deficit in the long run. However, the observed differences are minimal compared to the effect of the fertility shock. Interestingly, in model NC2, the longevity shock increases the magnitude and lengthens the horizon over which country A accumulates foreign assets through a trade surplus. Because, the steady-state world interest rate is low in the NC2 model, country A needs to increase its savings

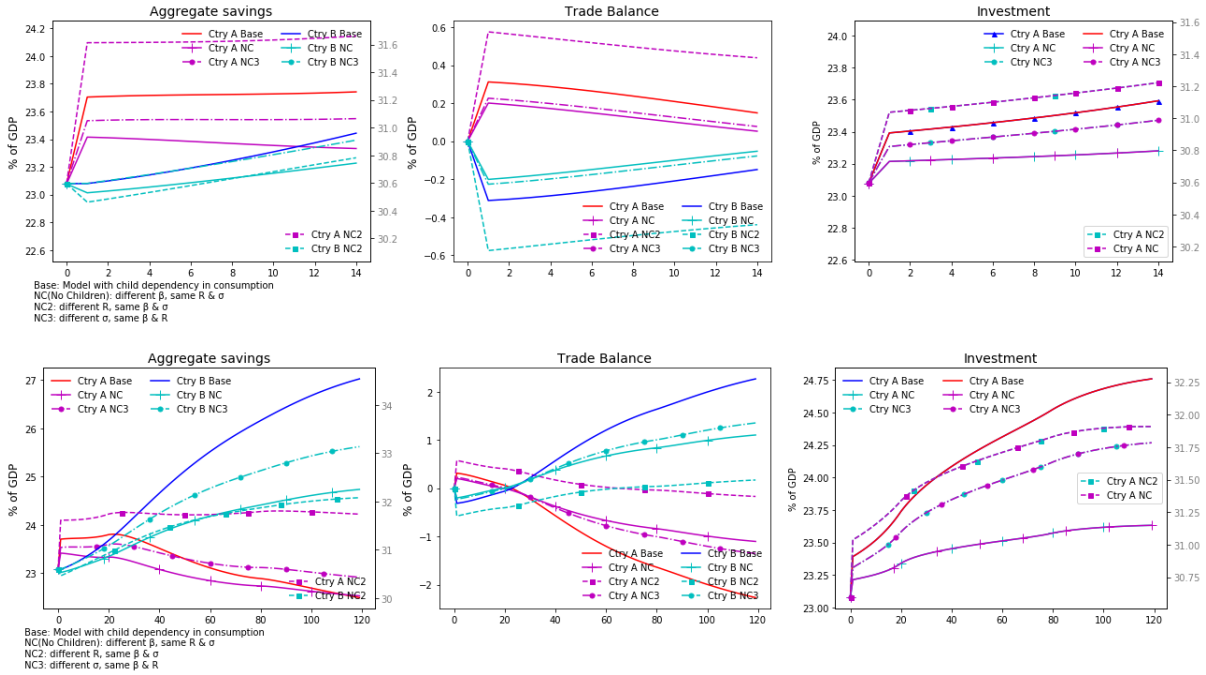


Figure 3.16: Aggregate savings, investment and trade balance after the longevity shock

more, and for far longer, to accumulate a certain target level of wealth to finance its consumption for a more extended retirement period. Figure 3.17 demonstrates this, as country A, despite having a higher trade surplus for far longer in model NC2, ends up with foreign assets and current accounts similar to those in other models.

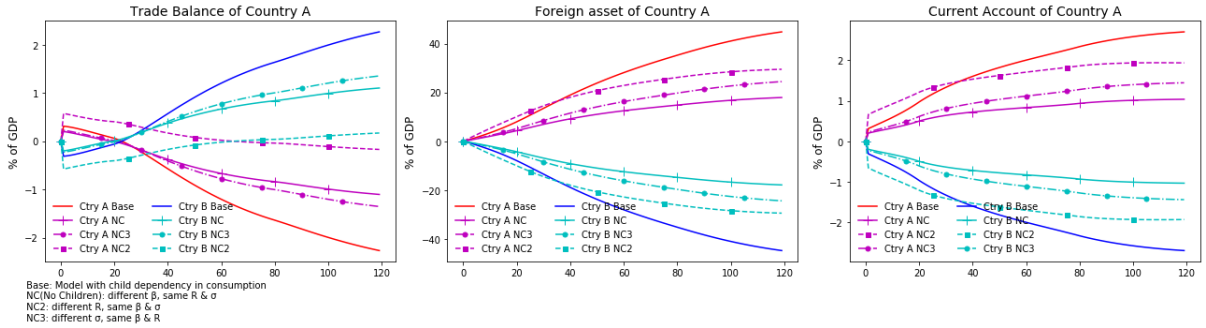


Figure 3.17: External balances after the longevity shock

In contrast to fertility shock, the effect of longevity shock on interest rates is modest and comparable across models with and without child dependency. Figure 3.18 shows that a lower beta is associated with the smallest decline in interest rate under a longevity transition. It also indicates that longevity shocks impact factor prices the most in the model with children due to a higher share of assets being held by retirees.

The dynamics of the share of assets held by different age groups under a longevity shock display a similar pattern to those seen under the fertility shock. Longevity shock also changes the share of different age groups in the adult population, thereby affecting the share of assets held by these groups through the aggregation effect. Specifically, the share of assets held by retirees increases much more than the increase in the share of

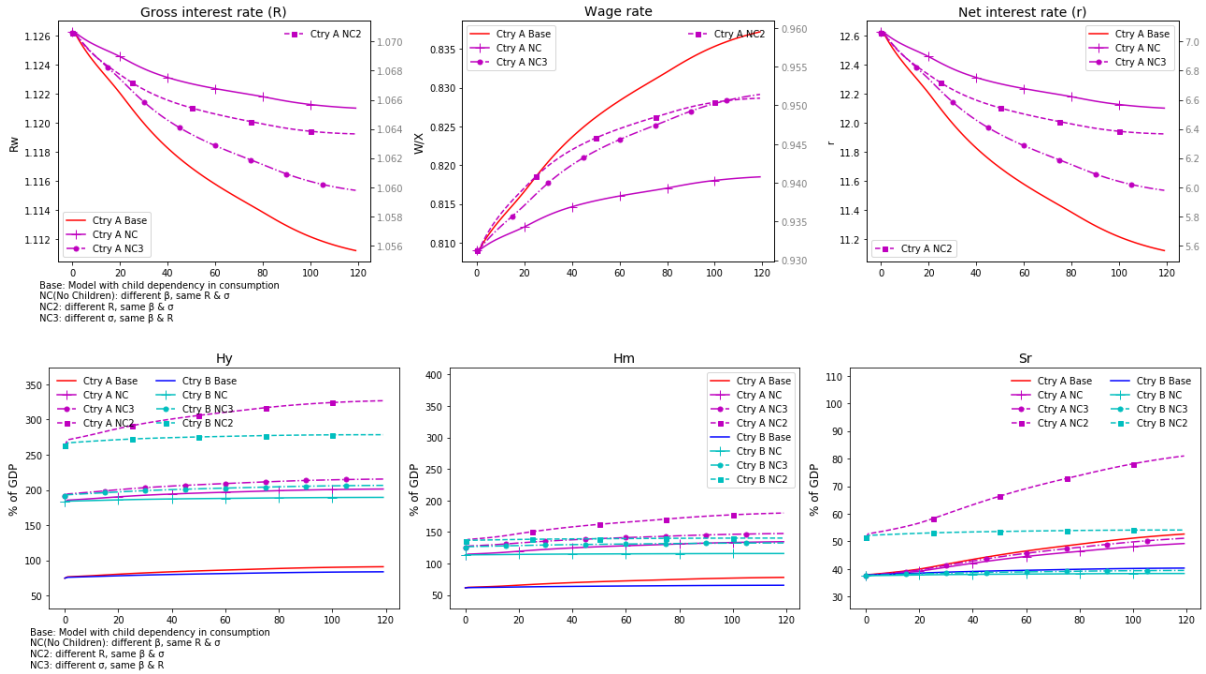


Figure 3.18: Factor prices after the longevity shock

retirees in the adult population, as shown in figure 3.19. In contrast, the share of assets held by young workers declines much more than their share in the adult population. A more pronounced reduction in retirees' consumption reflects the fact that the retirees' MPCs decline the most in response to longevity shock and young workers' the least, as shown in Figure 3.20.

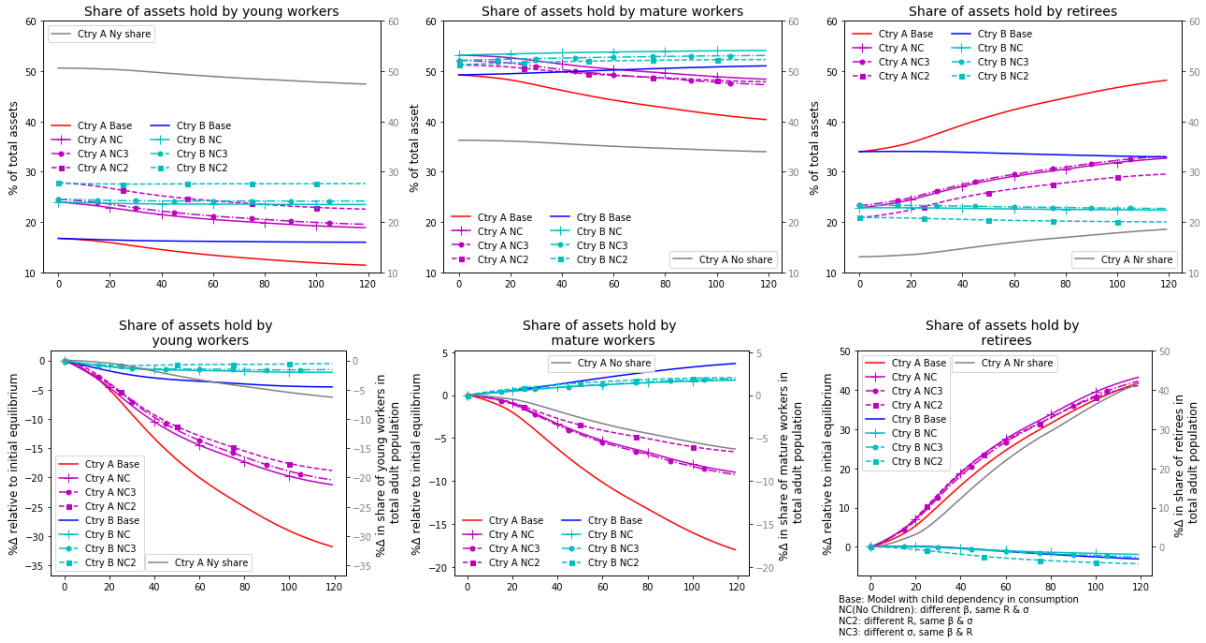


Figure 3.19: Distribution of assets after the longevity shock

First, the effect of longevity shock on aggregate savings, investment, and trade bal-

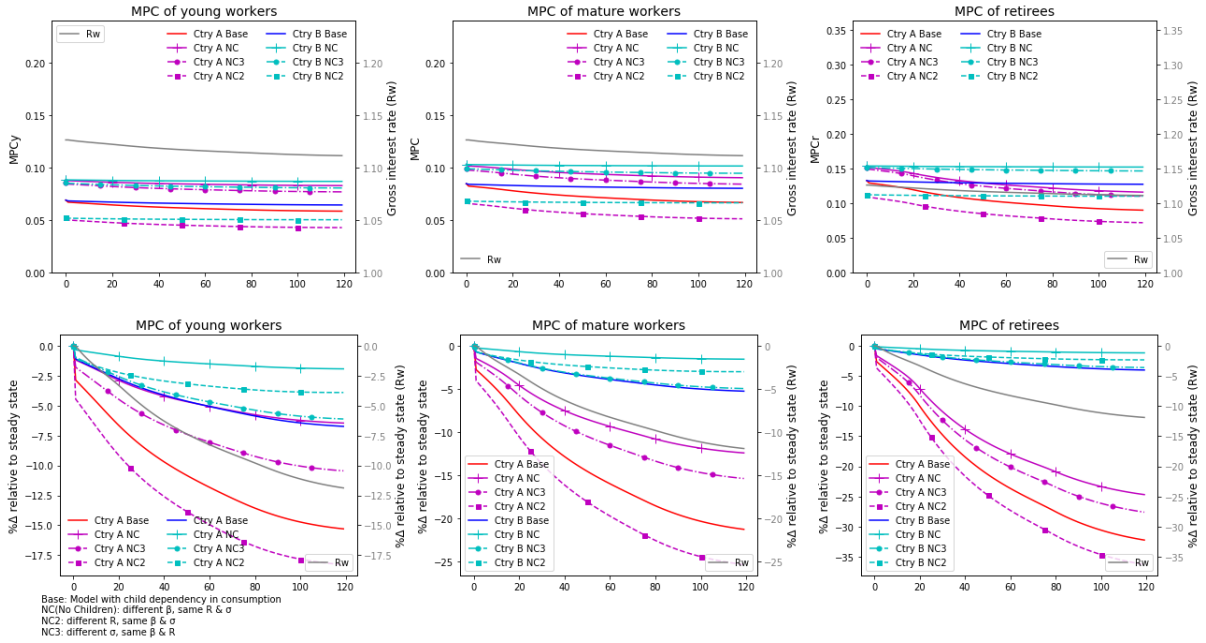


Figure 3.20: MPCs after the longevity shock

ances across models with and without child dependency with the same steady-state interest rate is quite comparable. The model with child dependency predicts a larger impact on the trade imbalance across countries for a given steady-state interest rate. Savings differences alone determine the trade balances, as longevity shock is assumed to not affect the growth of the effective labour force. In reality, longer life expectancy may extend the retirement age or increase labour supply by retirees. Then longevity shock will also affect the investment dynamics to the extent that the increased labour force by the retirees requires new capital investments. The model is capable of simulating this scenario, but the current paper omits it to simplify the analysis. Second, in a low-interest-rate environment, as in NC2, a country under longevity shock has to incur a trade surplus for longer at a much higher rate than if the interest rates were higher. It implies that, as the interest rate declines globally, the demographic transition may lead to larger trade imbalances across countries.

3.6.3 Decline in productivity

This section analyzes the effect of productivity shock on external imbalances for two reasons. First, it is difficult to differentiate the impact of demographic dividends from economy-wide productivity growth in some models. Economy-wide productivity decline will have the same effect on the growth of the effective labour force as the same magnitude decline in the fertility rate. Therefore, its impact on investment differentials will be the same as the decline in fertility for a given interest rate path. However, unlike fertility declines, productivity changes do not affect the population's age structure. They will therefore have a different impact on savings. Because both the savings and investment differentials determine trade imbalances across countries, productivity shock will have a different effect on trade imbalance than a fertility shock. Second, globally, productivity growth is declining, and it is suspected to be one of the main reasons behind

the declining interest rate. Productivity differentials are also one of the main reasons behind capital flows between countries. Therefore, it is interesting to see the impact of productivity changes on external imbalances within this framework to compare its effect to demographic changes.

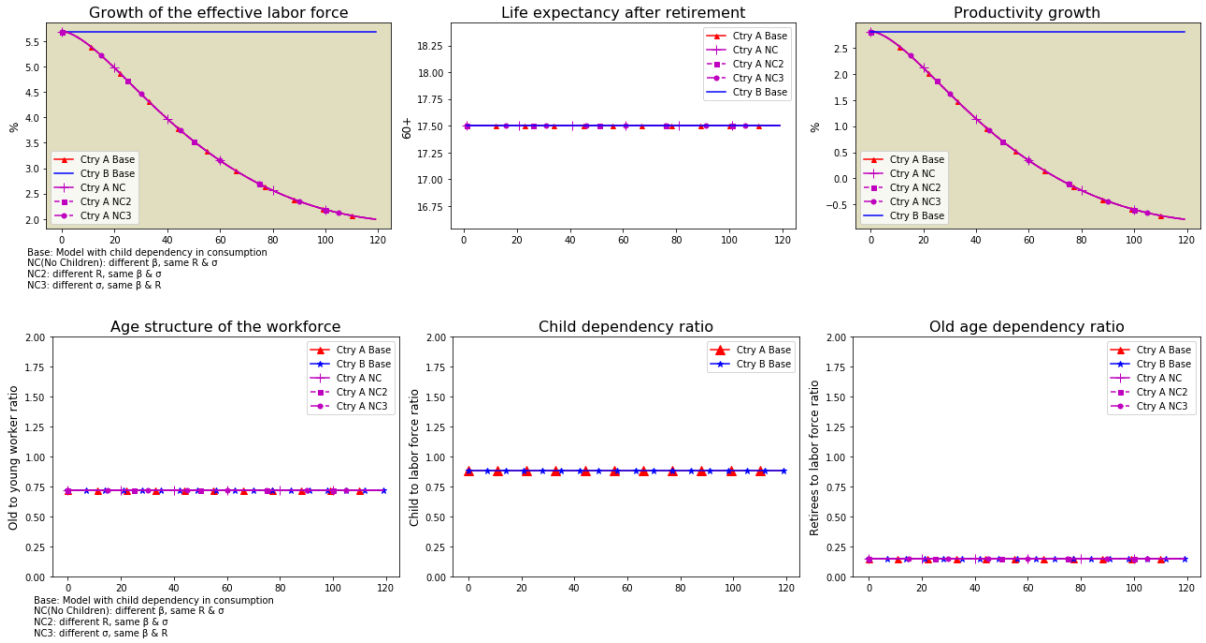


Figure 3.21: Population dynamics after the productivity shock

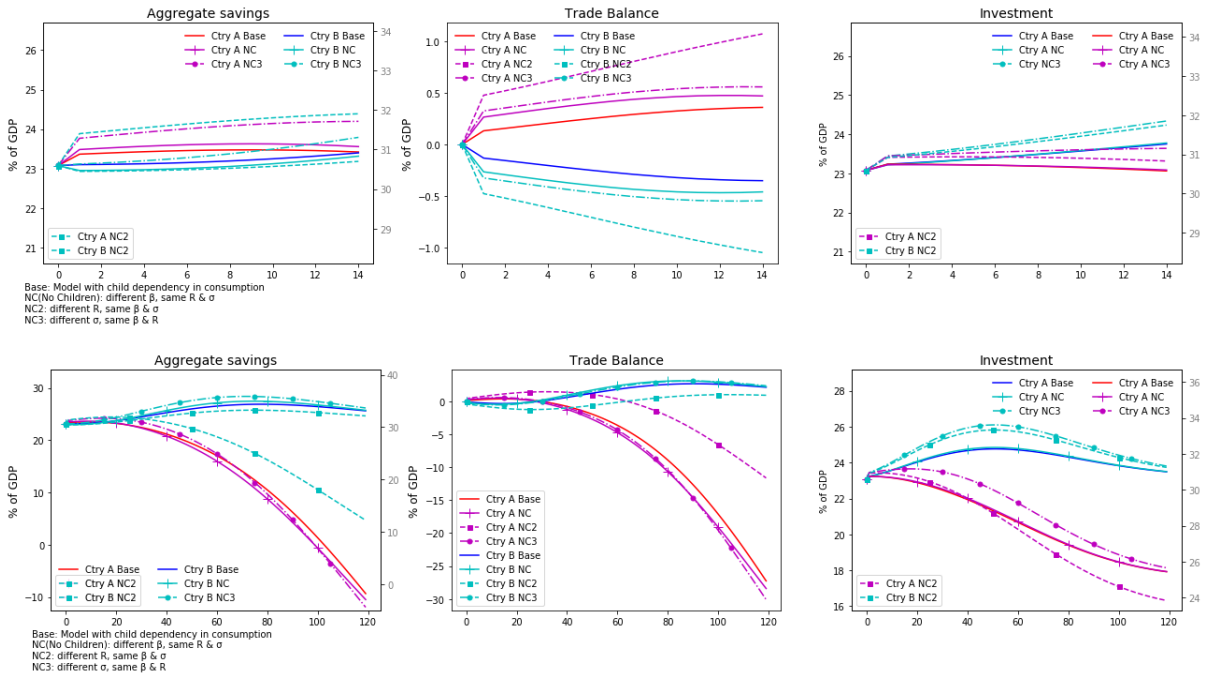


Figure 3.22: Aggregate savings, investment and trade balance after the productivity shock

Different from fertility and longevity shocks,productivity shocks do not alter the population age structure. However, productivity shocks do affect the growth of the effective

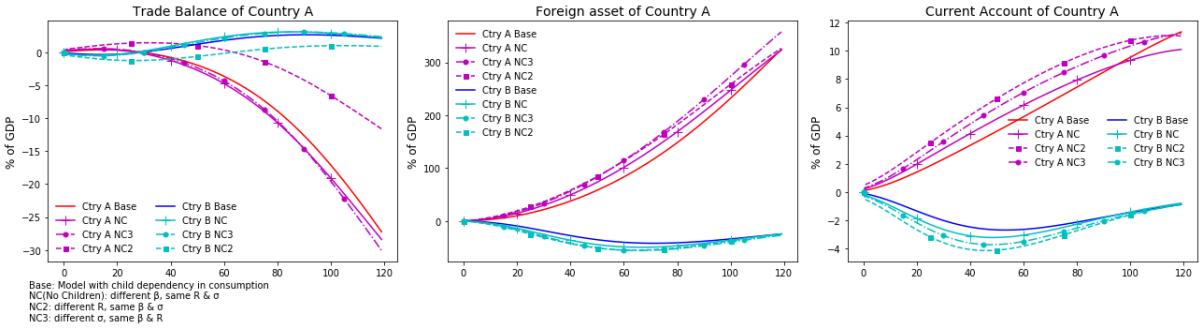


Figure 3.23: External balances after the productivity shock

labour force (calculated as growth of workforce x growth of productivity), which affects the investment rate. Factor prices determine the change in aggregate savings under the productivity shock. Because there is no change in the population's age structure, there is no effect from aggregation. Lower productivity growth in country A and associated lower interest rates increase savings in both countries (see figure 3.23) by reducing MPCs of all age groups, which has a positive association with the interest rate (when σ is less than 1). Investment in country B increases more than the increase in its savings,resulting in trade deficits in the short to medium run. Again, the world interest rate affects the magnitude and duration of the trade imbalance between countries. Figure 3.23 shows that in model NC2, country A has to run a trade surplus and country B has to run a trade deficit much longer compared to other models because of the low-interest rate.

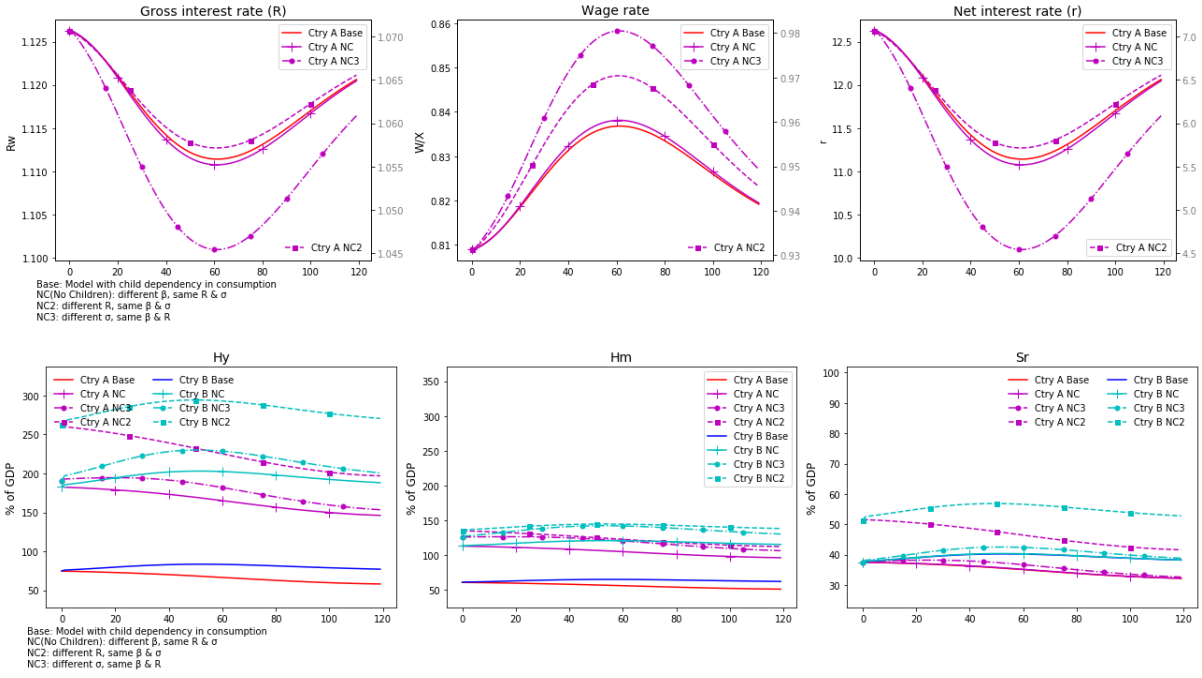


Figure 3.24: Factor prices after the productivity shock

At the micro-level, in contrast to fertility shock, productivity decline does not significantly alter asset distribution (see figure 3.25). Any minor changes are associated with changes in factor prices, with retirees' share of assets increasing slightly in country A.

An opposite pattern can be observed in country B, where higher investment increases the wage rate and thus increases the share of assets held by workers. However, any impact on the distribution of assets of the productivity shock is quite negligible. The same can be said of the effect of productivity decline on the MPCs (see figure 3.26), where the declining interest rate only slightly reduces MPCs.

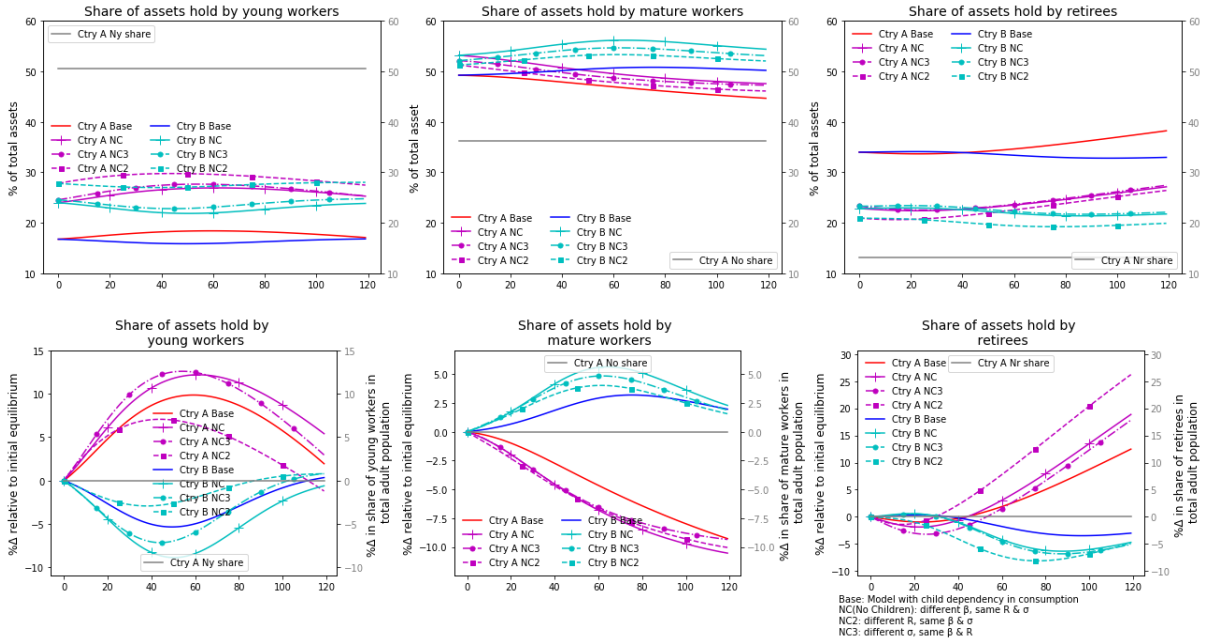


Figure 3.25: Distribution of assets after the productivity shock

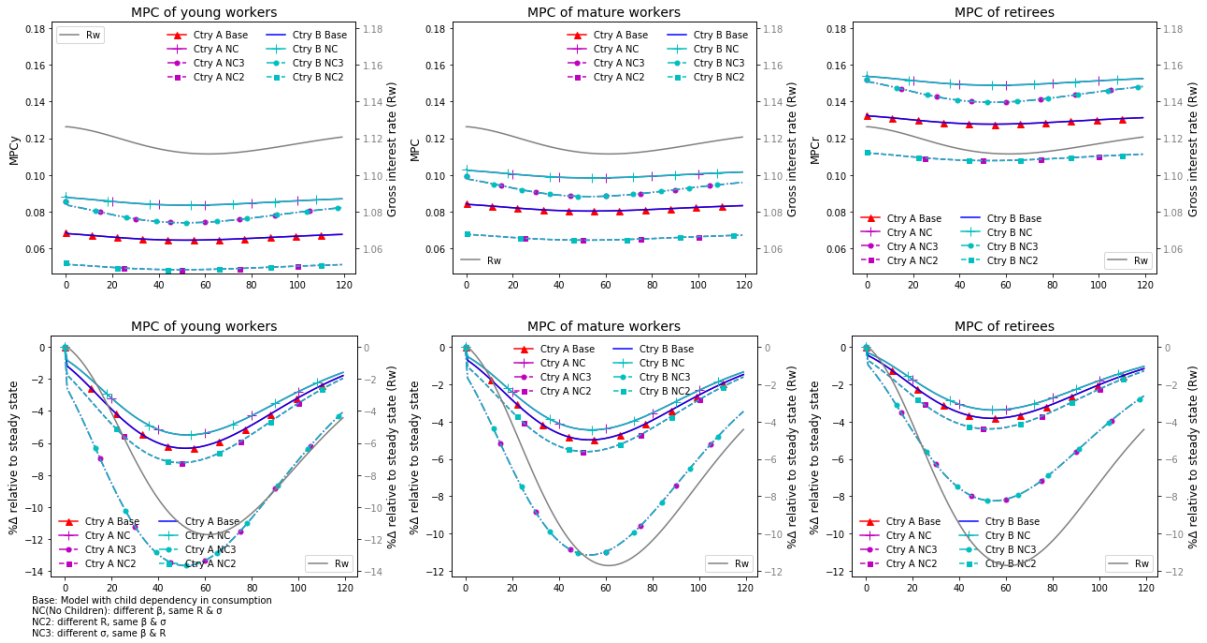


Figure 3.26: MPCs after the productivity shock

Overall, smaller values of R, sigma, and beta increase trade imbalances across countries, as shown in figure 3.23. However, the observed dynamics of the trade imbalance

across models with and without children with the same steady-state interest rate under a productivity shock are almost negligible. In contrast, when the interest rate is low, the trade surplus of country A and the trade deficit of country B have to grow for longer. In other words, in a low-interest-rate environment, it takes time for a country to build its foreign asset in anticipation of an aging population or lower productivity.

3.7 Conclusion

Understanding the causes behind the growing and persistent external imbalances across major economies has become a primary endeavor of policymakers and economists. Despite being the most prominent manifestation of the current global demographic transition, most papers neglect the effect of changing child dependency rates on external imbalances. Given the policy importance of correctly interpreting the causes behind global imbalances, this paper investigates the consequence of ignoring children when examining the impact of the demographic transition on external imbalances. To this end, it adds children's consumption and time cost to a tractable life-cycle model of two symmetric countries. To contrast, it then compares the demographic transition's impact on external imbalance across models with and without children.

Two main insights are drawn from this exercise. First, the presence of time and the consumption costs of children amplifies the effect of fertility transitions on factor prices and the external imbalances by adding additional mechanisms where changes in child dependency affect the aggregate savings. The results show that child dependency changes the sign and magnitude of a fertility transition on aggregate savings and external imbalances in the short and medium run. Though the difference is minor in the short to medium run, it is quite large in the long run.

Second, because the model with children corresponds to lower savings and higher interest rates at the steady-state, models with and without children need to be calibrated with different parameter values determining the steady-state interest rate. This implies that studies that use a calibrated life-cycle model without children may indirectly choose lower values for subjective time preference rate (β) and lower values for elasticity of intertemporal substitution (σ). The paper further shows that the different parameter values correspond to a different economy at the micro-level. Specifically, lower β and σ reduce the share of assets held by young workers who have the longest horizon while increasing retirees' share. Differences in composition and parameter values change the model's dynamics even under longevity and productivity shocks that do not change child dependency.

Overall, the simulations in this paper demonstrate that a country with a declining fertility rate, increasing longevity, and declining productivity, all relative to its trading partner, will have a trade surplus in the medium run and a trade deficit in the very long run. However, the prevailing world interest rate determines the duration over which a country runs a trade surplus before running a trade deficit during its demographic transition. Specifically, when the world interest rate is low, the trade surplus of a country with a declining fertility rate, increasing life expectancy, and declining productivity may continue for far longer than if the world interest rate was higher. Within a life-cycle framework, it is even possible for a country with positive foreign assets to run a trade

surplus in the very long run when the interest rates fall below the growth rate of the effective labour force.

To sum up, this paper highlights some of the issues in quantifying the effect of the demographic transition on the external imbalance in the past using calibrated life-cycle models that do not take child dependency into consideration. The absence of critical features underlying the demographic transition (e.g., child dependency) raises concerns that the effect of demographic transition hasn't been captured fully in previous studies. It is therefore important to be cautious in interpreting results from such studies. Further work is needed to develop a unified framework encompassing all the crucial features concerning the demographic transition is needed to quantify the contribution of demographic transition to external imbalances that can better inform policymakers.

Chapter 4

Globalized economy and national policies: Issues in comparing carbon emissions mitigation efforts under demographic and institutional asymmetry

Abstract: The success of the 2015 Paris Agreement in achieving its main temperature goal depends on its ability to increase the ambitions of individual countries to reduce their carbon emissions through effort comparison and peer pressure. Despite the empirical relevance of demographic changes in affecting factor prices, economic growth, and capital flows across countries, most comparisons of countries' carbon emissions reduction efforts are based on models that cannot capture demographic effects. Overlooking future demographic change is problematic given the profound yet asymmetric demographic changes that countries are undergoing. This paper uses a two-country life-cycle model to show that comparing carbon emissions mitigation efforts can be misleading if countries' baseline emissions trajectories do not account for demographic dividends and spillovers from one country to another from unsynchronized demographic changes and asymmetric institutions. Through capital flows, differences in the timing, speed, and magnitude of demographic changes can reduce the emissions baseline in one country while increasing it in another country relative to the baseline with no spillovers — an effect which is amplified by differences in institutions such as pension and social security systems. Models that do not consider the effect of demographic changes and the institutions on the economy and emissions may underestimate one country's carbon emissions reduction effort while overestimating that of another. Consequently, neglecting demographic changes when comparing countries' carbon emissions mitigation efforts can undermine the successful implementation of the Paris Agreement.
Keywords: Global imbalances, Demographic transition, Carbon emissions, Life-cycle model, Energy dependent production function
JEL Codes: E2, F32, F41, J11, J13, J14, O13, Q43, C6

4.1 Introduction

Since the establishment of a global policy framework to tackle climate change, economic models have become increasingly important in evaluating and successfully implementing climate change agreements. The Paris Agreement, the current global policy framework on climate change, was almost universally accepted[1] because it allowed countries to determine their own emissions mitigation targets (UNFCCC, 2015, Article 4). However, due to this flexibility, economic models are crucial to the Agreement's successful implementation for three key reasons. First, they are needed to project global emissions to assess whether we are on track to achieving the 2-degree Celsius target set out in the Paris Agreement. Second, economic models allow us to translate various emissions reduction pledges put forth by countries to make apples-to-apples emissions reduction comparisons. Third, economic models are necessary to translate those emissions reductions into economically comparable mitigation efforts.[2] Effort comparison is critical within the Paris Agreement because of the lack of externally imposed mitigation targets. Given the opportunity to free-ride due to the climate being a public good (Edenhofer et al., 2015), individual countries are eager to mitigate their emissions only if they believe they can reap the benefits in the form of reduced global climate change.

Past experiences show us the complexity and importance of setting realistic emissions mitigation targets. Ironically, since the establishment of the UN framework on climate change and the Kyoto Protocol (KP) in 1994, global emissions have grown faster (Raupach et al., 2007) despite many developed countries setting emissions reduction targets. The apparent failure of the KP was due to a combination of factors. First, large emitters such as the United States (US) didn't join the KP and therefore had no emissions restraints. Second, for some countries with emissions restraints in the KP, the targets turned out to be non-binding[3]. Third, developing countries had no emissions restraint under the KP. China, which had developing country status along with India, has experienced spectacular economic and carbon emissions growth since the start of the KP, surpassing the US as the largest emitter in the world in 2006 (NEAA, 2007; Gregg et al., 2008).[4] Though

[1] As of March 2022, 192 countries and the European Union has ratified the Paris Agreement (https://treaties.un.org).

[2] Regarding the costs and efforts of mitigation, there are two distinct ideas. The first concept relates to a direct cost associated with switching from carbon-emitting sources to carbon-neutral sources. Total mitigation costs for this are dependent on the scale of economic activity (Stern et al., 2012), population and productivity growth (Gillingham et al., 2018), the structure of the economy, and how easy it is to switch between carbon-emitting and carbon-neutral sources (McKibbin et al., 2011). Furthermore, because replacing carbon-emitting sources requires new investments, countries' real interest rates also determine mitigation costs (Lambrecht et al., 2006). The second concept regarding the effort or cost of mitigation is broader. Independent of the actual investments made to replace fossil fuels, some countries benefit more than others due to countries' transitioning to carbon-free economies. For instance, McKibbin et al. (2011) demonstrates that the economic costs associated with climate change mitigation strategies for countries that export fossil fuels are mostly caused by international rather than local policy. As a result, it is thought that the change in economic production and consumption caused by mitigation policy is the most comprehensive indicator of mitigation effort (See McKibbin et al. (2011); Aldy et al. (2017)).

[3] For example, former Soviet block countries set their mitigation targets relative to their emissions in 1990, which turned out to be non-binding given the economic downturn they experienced after the collapse of the Soviet Union (Brizga et al., 2013).

[4] Importantly, studies find that over half of China's high household savings, which accompanied its

successful in reducing emissions to some extent (Kuriyama & Abe, 2018), the failure of the KP in restraining global emissions shows the importance of universal, realistic and ambitious mitigation goals.

As an acknowledgment of the complexity of comparing countries' mitigation efforts, studies often propose the use of multiple distinct models to make a robust comparison of emissions projections and mitigation efforts[5]. However, so far, effort comparison studies have only used models which cannot capture demographic effects[6] from a changing age structure of a population following the change in a country's fertility and mortality patterns. Furthermore, Gillingham et al. (2018) argue that multiple model comparison may not be adequate to represent uncertainty around future emissions which can be smaller compared to the uncertainty in emissions caused by exogenous factors such as future population growth. However, the study by Gillingham et al. (2018) also neglected the uncertainty around global emissions that can result from uncertainty around differences in demographic outlooks across countries. In a life-cycle model, demographic differences can affect a country's economic growth and emissions through capital flows. This paper explores this aspect by applying a life-cycle model and demonstrating that the more asymmetric the demographic changes are across countries, the more spillovers[7] there will be. Different demographic outlooks, therefore, have implications for both global emissions[8] and the baseline emissions projections of individual countries, and hence for effort comparison as well.

This paper examines the effects of nine relative demographic scenarios (based on three demographic transition scenarios for each country) on the emissions trajectories of two large countries inhabited by households with life-cycle savings, to investigate the sensitivity of emissions baselines to alternative demographic scenarios. There are two main contributions of this paper, the first methodological and the second thematic. First, this paper incorporates an energy-dependent production function widely used within climate and energy modeling literature[9] into a two-country tractable life-cycle model that is solved in annual frequency that can capture demographic dividends. These model features allow us to study the implications of demographic transition on economic growth and emissions both in the short and the long run while allowing cross-country spillovers.

high economic growth causing capital outflow, in the last two decades can be attributed to its changing demographics (Curtis et al., 2015).

[5]Barron et al. (2018) and Aldy et al. (2017) have presented results from four different models evaluating mitigation efforts implied by the proposed climate change targets of China, The EU, the US, and India. Other studies such as Fragkos et al. (2018) also presented multiple modelling approaches.

[6]Demographic effect pertains to change in economic growth that comes from the changing age structure of the population (D. Bloom et al., 2003). In a country with a falling fertility rate, additional economic growth comes from higher workforce growth and higher savings following the reduction in total dependency rates. D. E. Bloom and Williamson (1998) found that almost one-third of the high economic growth experienced by East Asia between 1965 and 1990 was due to demographic changes.

[7]Here, spillovers refer to changes in a country's economic growth in the short to medium term due to capital flows from another country, resulting in differences in a country's long-run output level.

[8]Cross-country capital flows can change the growth of countries. As long as different countries' energy and carbon intensity are not the same, the international flows of capital will change global emissions relative to if there were no capital flows. Empirical evidence on the convergence of energy and carbon intensity of the economies is limited (Emir et al., 2019).

[9]See Van der Werf (2008)'s study for empirical validity of production functions used within the energy-economy models for climate policy.

Second, to my knowledge, this is the first study that applies a life-cycle framework to study carbon emissions mitigation efforts under demographic changes. This paper aims to highlight the inter-linkages of countries' emissions trajectories through capital flows under asymmetric demographics and institutions.

Previous studies have demonstrated the sensitivity of emissions baselines to demographic changes. For example, using a two-period Overlapping generations (OLG) model with logarithmic utility and an Infinitely lived representative agent (ILA) model, Gerlagh et al. (2017) has shown how the two models can generate very different outcomes when used for projections under a demographic transition. One caveat of this study is that the two-period OLG models impose demographic structures that are extreme and not suitable for analyzing the effect of demographic changes (Gertler, 1999; Ferrero, 2010). Furthermore, logarithmic utility assumes the elasticity of intertemporal substitution to be 1, which nullifies the effect of interest rates on consumption and savings. Other studies by Dalton et al. (2008) and Neill et al. (2010) have looked at the emissions implications of population aging in the US and the world (represented by six regions) using a model with overlapping dynasties of households. They find that demographic transitions will impact a country's economic growth and emissions. However, dynastic models lack life-cycle savings (Niemeläinen, 2017) and therefore underestimate the change in aggregate savings during demographic transitions, which is the main feature that explains persistent capital flows across countries in the last three decades (Gourinchas & Rey, 2014; Bernanke, 2015). Another study by Garau et al. (2013) has used a large-scale numerical OLG model to study the impact of population aging in Italy on economic growth and emissions. By confining his focus only to Italy, however, Garau et al. (2013) misses the effect of demographic transitions in other large countries on Italy[10].

The analysis in this paper demonstrates that demographic changes can bring about substantial demographic dividends. This, in turn, can change a country's emission path, complicating our ability to compare countries' mitigation efforts if we use models that cannot capture demographic effects. In addition, unlike ILA models[11] which tend to interpret persistent trade imbalances as difference in subjective time preference rates across countries,[12] capital flows can also depend on demographic and institutional differences within life-cycle models.[13] Diverging demographics and asymmetric institutions across countries can, therefore, cause larger cross-border capital flows. In other words, demographic and institutional differences can result in spillovers, making emissions higher in one country and lower in the other relative to the absence of the capital flow — thereby further complicating effort comparison. Because demographic effects are time-varying, they are difficult to account for in models without demographics through exogenous parameters unless one can explicitly forecast the impact of the demographic changes on the relevant parameters.[14]

[10]Study by Grafenhofer et al. (2006) provides useful comparison of different OLG models

[11]In the climate economics literature, the infinitely lived representative agent (ILA) framework is the one that is most frequently used to simulate households' intertemporal decisions.

[12]For example, a study by Leimbach and Bauer (2021) have assumed different time preference rates for countries to match cross country current account imbalances.

[13]Liu (2021) provides a comprehensive review of empirical and theoretical literature on demographic change's impact on cross country capital flows within life-cycle models.

[14]Liu et al. (2020) provides an analysis of the economic and environmental implications of the Paris agreement using a dynamic general equilibrium model of the global economy (G-cubed) that is well

This paper consists of 5 sections. Section 4.2 summarizes key features of the model used to answer the research question. Section 4.3 provides analytical results while section 4.4 offers a quantitative investigation to supplement analytical results. Finally, section 4.5 provides the main conclusions of the paper together with a discussion of policy implications and potential future research.

4.2 Model

This section describes the main features of the model that is used for the quantitative analysis in section 4.4. The household side of the model is based on a two-country life-cycle model developed in Batsuuri (2022), which has four types of individuals who are at different life stages. Population dynamics, which is used for aggregation, is exogenous in the model, which dictates how many people are at each life stage in each period. For the production structure, this paper uses an energy-dependent production function that is widely used within applied climate economics literature[9] which is also used in chapter 2. Only a brief explanation on the key equations describing the model is provided in this paper. Please refer to (Batsuuri, 2022) for the household equations, and 2 for the production side for a detailed description of the model. The equations describing the two countries are identical, and therefore, only equations for one country are described here. When necessary, upper index f refers to variables for the other country.

4.2.1 Population dynamics

As in Batsuuri (2022), population dynamics are exogenous in the model, with individuals going through four life stages being children, young workers, mature workers, and being a retiree. A set of probabilities that govern the transition from one life stage to another, as illustrated in Figure 4.1, generates population dynamics in the model. These probabilities are independent of the age of an individual to facilitate aggregation.

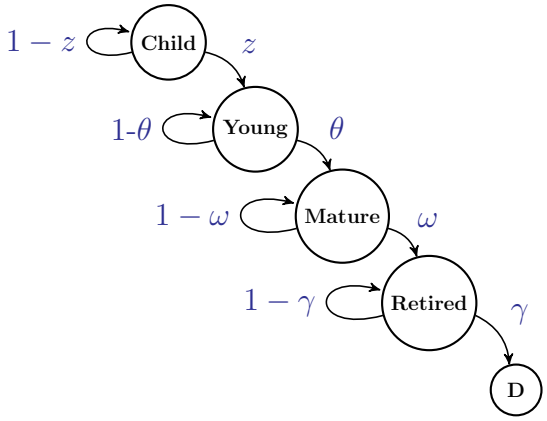


Figure 4.1: Transition probabilities from one life-stage to another

suited to take into account international capital flows. It could be possible to consider international capital flows due to future demographic differences in the G-cubed through calibration.

4.2.2 Households

There are four types of individuals in the economy: retirees, mature workers, young workers, and dependent children. The model assumes that the children are born to young workers, who are between 20-39 years old, who bear the time cost of raising children. In contrast, all workers, young and old, equally pay the children's consumption cost based on their respective labour productivity.

All types of individuals, except children, maximize their respective lifetime utility shown in equation 4.1 subject to own intertemporal budget constraints. The utility of an individual is represented by the non-expected utility function proposed by Farmer (1990), which provide certainty-equivalent decision rule in the face of income risk that arises due to stochastic transition probability from one life stage to another. Different types of individuals differ by their respective intertemporal budget constraints. The superscript 'i' denotes the individual type: 'r' for retirees, 'm' for mature workers, 'y' for young workers, and 'c' for children.

$$V_t^i = \left[(C_t^i)^\rho + \beta_i \mathbb{E}(V_{t+1}|i)^\rho\right]^{\frac{1}{\rho}} \tag{4.1}$$

where

$$\beta_i = \begin{cases} (1-\gamma_{t,t+1})\beta & \text{if i = r} \\ \beta & \text{if i = m,y} \end{cases}$$

$$\mathbb{E}(V_{t+1}|i) = \begin{cases} V_{t+1}^r & \text{if i = r} \\ (1-\omega)V_{t+1}^m + \omega V_{t+1}^r & \text{if i = m} \\ (1-\theta)V_{t+1}^y + \theta V_{t+1}^m & \text{if i = y} \end{cases}$$

The solution to the individual problems gives decision rules that govern how the marginal propensity to consume (MPC) out of wealth evolves in response to interest rate and life expectancy. The equations for the MPCs are the same for individuals at same life stages and differs across individuals at different life stages, which is the main feature that gives rise to life-cycle consumption and savings in the model. Aggregate consumption function is given by equation 4.2:

$$C_t = \pi_t \left[A_t R_t \left(1+(\epsilon_t-1)\lambda_t^1+(\epsilon_{2,t}-1)\lambda_t^2\right)+\epsilon_{2,t}(H_t^y+S_t^y)+\epsilon_t S_t^r+H_t^m+S_t^m\right]+t^c W_t N_t^c \tag{4.2}$$

where $\lambda_t = A_t^r/A_t$ is the fraction of total assets in the economy held by retirees, $\lambda_{2,t} = A_t^y/A_t$ is the fraction of total assets in the economy held by young workers. The notations π, ϵ, and ϵ_2 represent MPCs of individuals at different life-stages which is influenced by life-expectancy and the interest rate.[15] Aggregate consumption is affected by demographic changes through changes in life expectancy and the changes in the size of people who have different life expectancies and different MPCs. This is the key feature that differentiates the model from perpetual youth model proposed by Blanchard (1985) where individuals have same MPCs regardless of their age.[16] More detailed description of the population dynamics and the household problem can be found in Batsuuri (2022).

[15] The proof on the relationship between the MPCs, the interest rate and the life expectancy is provided in the appendix section of Batsuuri (2022).

[16] Blanchard (1985) and Weil (1989) proposed a tractable overlapping generations model that is easy

4.2.3 Firms

It is assumed that there is a representative firm producing final output combining energy and capital-labor composite while energy is produced combining clean and dirty energy inputs. In turn, clean and dirty energy inputs are produced combining capital, labor and resource specific factors ($X_{t.o}$ and $X_{t,s}$) that reflect technology and finite resources. The production structure of the economy is depicted in figure 4.2.

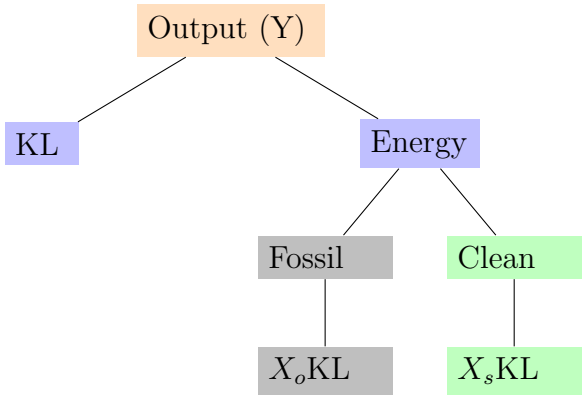


Figure 4.2: Production structure

The technology of the firm producing final output is represented by the function in equation 4.3.[17]

$$Y_t = \left(\upsilon_1((X_{t,L}N_{t,y})^{\alpha}K_{t,y}^{1-\alpha})^{\Psi} + (1-\upsilon_1)(X_{t,e}E_t)^{\Psi}\right)^{\frac{1}{\Psi}} \tag{4.3}$$

with labor augmenting technology growing at an exogenous rate of x_l.

$$X_{t+1,L} = (1+x_l)X_{t,L} \tag{4.4}$$

Energy augmenting technology ($X_{t,e}$) is assumed to be constant along the balanced growth path, but can change due to a deterministic technology shock.

Energy is produced combining clean and dirty energy inputs with technology represented by:

$$E_t = ((1-\eta_1)S_t^{\Phi} + \eta_1 O_t^{\Phi})^{\frac{1}{\Phi}} \tag{4.5}$$

where O_t and S_t represent dirty and clean energy inputs respectively while η_1 is the share of dirty energy inputs in the total energy production. Solution to final output and energy firms' problem provides quantity of final goods and energy production for given factor prices. Chapter 2 provides more detailed description of firms' problem and their solution.

to solve and widely used for policy analysis. However, it lacks a life-cycle structure by assuming that each individual has the same life expectancy hence MPCs.

[17] Ψ is related to elasticity of substitution between Capital-Labor composite and Energy input via $\sigma_{KL,E} = \frac{1}{1-\Psi}$. $\Psi \in (-\infty, 0) \cup (0, 1]$. CES function includes 3 special cases under different Ψ values. When Ψ approaches -∞, $\sigma_{KL,E}$ becomes 0 and the production function becomes a Leontief production function with perfect complementarity of inputs, when Ψ approaches 1, $\sigma_{KL,E}$ approaches ∞ and inputs become perfect substitute in production. Lastly, when Ψ approaches 0, $\sigma_{KL,E}$ becomes 1 and the production function becomes Cobb-Douglas production function.

4.2.4 Characterization of equilibrium in the world economy

The competitive world equilibrium is defined as a sequence of endogenous quantity and price variables given the sequence of exogenous predetermined variables, and the initial values of all the predetermined variables such that in each country i) households maximize their utility subject to their budget constraints, ii) firms maximize their profits subject to their technology constraints, and iii) all markets clear.

Total assets in the home country is equal to total capital stock in the home country, government debt and net foreign asset holdings of the home country.

$$A_t = K_t + B_t + F_t \tag{4.6}$$

Foreign assets evolve according to

$$F_{t+1} = R_t F_t + NX_t \tag{4.7}$$

which links the goods and asset markets. Trade deficits and surpluses change net foreign asset holdings. The aggregate capital stock in the economy is equal to sum of the capital stocks in the final output, dirty and clean energy sectors.

$$K_t = K_{t,y} + K_{t,o} + K_{t,s} \tag{4.8}$$

The aggregate capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{4.9}$$

The trade balance is determined by the difference between aggregate output and aggregate expenditures which include both private and government consumption and investment expenditure.

$$NX_t = Y_t - (C_t + I_t + G_t) \tag{4.10}$$

Free flow of assets between economies ensures equalization of gross interest rate R_t between countries. In equilibrium, net asset holdings of the two countries cancel each other out.

$$F_t + F_t^f = 0 \tag{4.11}$$

Current account is the trade balance plus net interest payment on foreign assets.

$$CA_t = NX_t + (R_t - 1)F_t \tag{4.12}$$

Total labor force in the economy is equal to the number of mature workers plus labor supply by the young workers multiplied by their labor productivity. Labor market equilibrium is when total supply of labor equal to total demand for labor.

$$N_t = (\xi(1 - \frac{\lambda_c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{4.13}$$

All quantity variables are normalized by population and productivity growth to estimate the steady-state of the model. The solution of the steady-state and the transition dynamics are solved using a non-linear Newton method. To close the model, government debt is assumed exogenous as fixed share of GDP. Detailed explanation of the algorithm used to solve the model can be found in Batsuuri (2022) and Julliard (2005).

4.3 Analysis

This section discusses how fertility and longevity transitions affect the economy. The impact of a fertility shock can be decomposed into its impact on labour force growth and aggregate savings and capital per effective labour, influencing output growth. In contrast, an increase in life expectancy at old age is assumed not to affect the growth of the labor force in effective units. Therefore, the impact of change in life expectancy at old age comes only through changes in aggregate savings and capital per effective labour. Specifically, for each transition, we can differentiate the main mechanisms of effect on capital per effective labour as direct through individual decision rules and indirect due to aggregation and general equilibrium effect.

4.3.1 Fertility shock

Growth of the effective labor force

Fertility shocks change the population dynamics, with the shock slowly propagating through different age cohorts' growth in sequence. More specifically, a fertility shock first changes the child dependency ratio and then the workforce's age structure before eventually changing the old-age dependency ratio. These ratios are important in determining labour supply, aggregate child consumption, income tax to finance child consumption, distribution of assets between different age cohorts and tax for social security which is modelled as pay-as-you-go pension. These effects will work to change aggregate savings and capital per effective labour. In addition, fertility shocks change the growth of effective labour force which is given by

$$n^{wf_\xi}_{t-1,t} = \frac{(\xi_t(1-\frac{\lambda_c}{\psi^c_t})+\psi^1_t)n^y_{t-1,t}}{\xi_{t-1}(1-\frac{\lambda_c}{\psi^c_{t-1}})+\psi^1_{t-1}} \tag{4.14}$$

Where ψ^1_t is the mature to young worker ratio, ψ^c_t is the children to young worker ratio, λ_c is the time cost of children, ξ is the productivity of young workers relative to mature workers. Linearizing equation (4.14) around its steady state provides following linear equation:

$$\hat{n}^{wf\xi}_t = \hat{n}^y_t + z_2(\hat{\psi}^1_t - \hat{\psi}^1_{t-1}) + z_3(\hat{\psi}^c_t - \hat{\psi}^c_{t-1}) \tag{4.15}$$

where z_2 and z_3 are positive constants that characterize the contribution and the effect of age structure of the workforce and the fertility rate on the effective labor supply at the steady state. The variables with hats denote deviation of a variable from its steady state in percentages. Specifically, $\hat{x}_t = Log(\frac{x_t}{x_{ss}})$. Given that both z_2 and z_3 are positive constants, it is clear from the above equation that the change in effective labour force depends on the change in young workers' growth and the change in the workforce's age structure and the change in fertility rate. In an economy where fertility is declining, both $(\hat{\psi}^1_t - \hat{\psi}^1_{t-1})$ and $(\hat{\psi}^c_t - \hat{\psi}^c_{t-1})$ will be positive. Therefore, changes in the workforce's age structure and increased labour force participation due to a reduction in fertility rate will offset the negative impact on the growth of the fewer young workers entering the workforce following a fertility reduction.

Capital per effective labor

Even though fertility shocks do not directly affect individual decision rules, they can indirectly affect them by changing expected prices and the capitalized value of human and social security wealth. Fertility shocks change population dynamics. By changing the population's relative size at different life stages, fertility shocks will alter aggregate consumption and the distribution of assets among workers and retirees with different savings rates. Also, a reduction in the child dependency ratio immediately reduces the tax rate on labour income for child consumption, assumed as fixed per child in this model. However, because the workers have a higher discounting of the future due to finite work and lifetimes, reducing the tax for child consumption will increase workers' consumption less than the aggregate child consumption decrease.

In the short run, the reduction in fertility rate increases the aggregate savings. However, it is ambiguous how the capital per effective labour changes in the short run since the effective labour force also grows. Increase in savings will positively affect the capital per effective labour, while the workforce's growth will negatively affect it. In the long run, it is also ambiguous how the reduction in fertility rate affects the capital per effective labour. The decrease in fertility rate increases the old-age dependency ratio. Because retirees have lower savings rates, it will put negative pressure on aggregate savings. However, in the long run, the reduced fertility rate also reduces the growth of the population entering the workforce. Similar to its short-run effect, the effect of fertility reduction on the capital per effective labor will depend on the interaction between change in aggregate savings and the change in the growth of the effective labor force. With its non-linear and often conflicting effect both at the individual and aggregate level, it is impossible to analytically derive the impact of the fertility shock on capital per effective labour in this model. Therefore, a quantitative investigation is undertaken in the next section to investigate the effect of fertility reduction on capital and output per effective labour.

4.3.2 Longevity shock

The shock to life expectancy beyond working age doesn't affect the workforce's growth[18], age structure, and children to labor force ratio. Therefore, its effect mainly centers on aggregate savings and capital per effective labour through the individual savings decision and the retirees to worker ratio.

Capital per effective labor

In contrast to a fertility shock, a longevity shock, or transition will directly affect individual decision rules. Everyone, who make their consumption and savings decision based on their expected lifetime, reduces their marginal propensity to consume when life expectancy after retirement increases. Only consumption by the children is not affected because they consume what they are given and do not decide.[19] An increase in life

[18]To simplify the analysis, retirement age is assumed as fixed. It is possible to simulate change in retirement age.

[19]This paper abstracts from endogenous fertility and bequest. With endogenous fertility and bequest, transfer to children could change as well. For example, Day (2016) presents a model with endogenous fertility and transfer to children. The model shows a non-linear relationship between wage rate, fertility

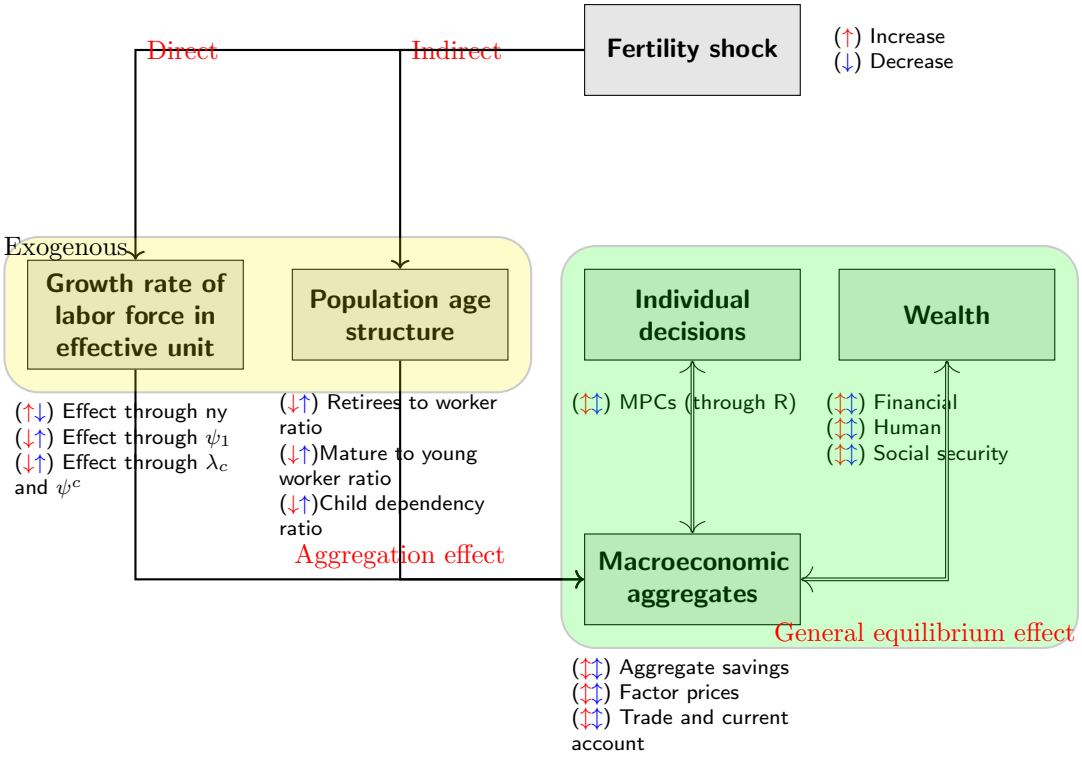


Figure 4.3: Effect of fertility shock

expectancy reduces retirees' MPC the largest when the shock hits. In addition to its direct impact on the individual's MPCs, longevity changes can indirectly impact the MPCs and the individual consumption by changing factor prices and wealth through aggregation. The reduction in interest rate will reduce retirees' consumption by further decreasing the MPCs, under the selected value used for the elasticity of intertemporal substitution (EIS) and reducing the financial wealth of the retirees who are net lenders. On the other hand, a reduction in the interest rate will increase the capitalized value of workers' human wealth by increasing wage and reducing the discount rate or the cost of borrowing against future income. Therefore, the effect of the longevity changes on workers' consumption is ambiguous.

Moreover, the aggregation effect is further complicated by the effect of a longevity transition on aggregate savings and capital per effective labour. The increase in life expectancy at old age increases retirees to worker ratio, increasing aggregate assets held by retirees who have a higher marginal propensity to consume. Again, with the rich demographic details and the effect of the longevity shock working through individual, general equilibrium, and aggregation, it is ambiguous how the longevity transition affects savings and capital per effective labour. Therefore, the analysis is supplemented by a quantitative investigation in the next section.

rate and investment in children.

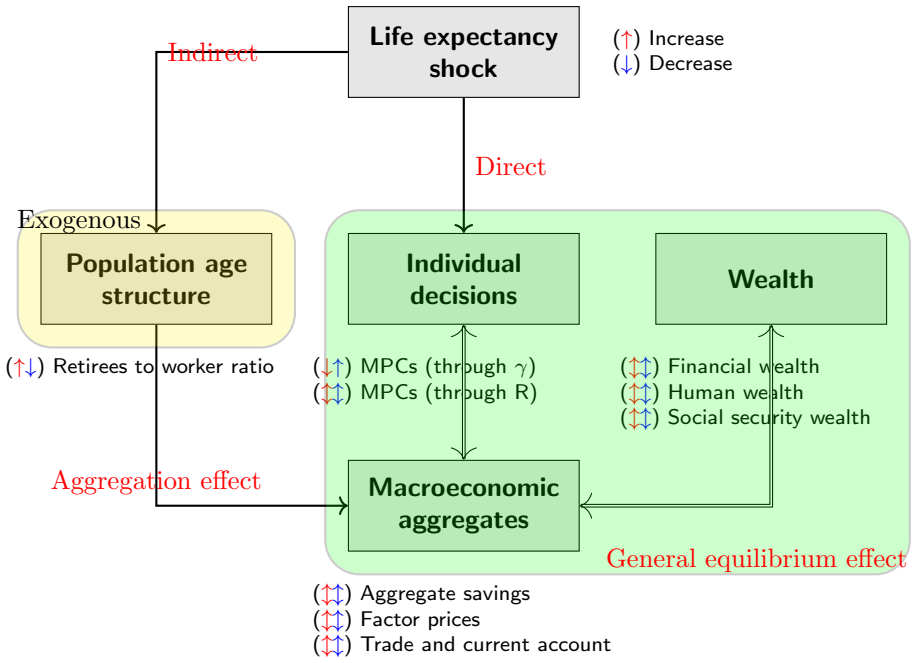


Figure 4.4: Effect of life expectancy shock

4.3.3 Determinants of emissions baseline and emissions mitigation effort

Previous studies have proposed several different indicators as a possible proxy to show a country's mitigation effort, such as energy and carbon prices, the energy intensity of economies, and the emissions reduction from a baseline (Aldy et al., 2016, 2017). By construction, energy and carbon prices and energy intensity of the economies are equal in all periods in this paper. However, baseline emissions projections, which determine mitigation efforts, are sensitive to whether one captures demographic dividends and spillovers through capital flows. Through capital flows, asymmetric demographic changes can increase emissions in one country while reducing them in the other country. Therefore, models that cannot capture spillovers can overestimate one country's emissions baseline while underestimating the emissions baseline of another country.

To understand the impact of demographic changes on emissions, let's look at the equations that determine total emissions in the economy. Only dirty (fossil) energy emits carbon. Total carbon emissions in a country are equal to carbon emissions per effective labor times its total effective labor force. For simplicity, this paper assumes only one type of fossil fuel. Therefore, the quantity of fossil energy use corresponds to total emissions in the economy. In response to a demographic shock, total carbon emissions in the economy change according to:

$$\Delta_t(O_t) = \Delta_t(o_t) + n_t^{wf\xi} \tag{4.16}$$

where big O_t stands for total fossil energy use while small o_t stands for fossil energy use per effective labor. As explained in section 4.3.1, $n^{wf\xi}$ represents growth in the effective labor force. Section 4.3.1 have already discussed demographic transitions impact on the effective labor force. With a falling fertility rate, an effective labor force can grow in the short run. The effective labor force will be larger relative to the workforce that does

not consider its age composition in the long run. Therefore, models that do not take into account demographic changes will underestimate a country's total emissions from a larger effective labor force in the case of fertility reductions. On the other hand, because of changes in aggregate savings (assuming the country is a large economy and does not take the world interest rate as given), output per effective labor and consequently emissions per effective labor will also grow under a fertility reduction scenario. A lower interest rate will result in capital-intensive sectors expanding more. Because the energy sector is relatively capital intensive, emissions per effective labor will grow more than output per effective labor, as shown in chapter 2.

However, how much fossil energy use (or emissions) per effective labor changes for a country depends on the interest rate the country faces. Within a life-cycle model, the demographic structure is one factor that affects a country's aggregate savings rate. However, the factor prices a country faces depend on whether the country is open to capital flows from other economies or not and on how big the country is relative to the rest of the world. As discussed in Ferrero (2010); Eugeni (2015); Niemeläinen (2021); Batsuuri (2022) capital flows within life-cycle models depend on demographic and institutional differences that determine each countries' aggregate savings and investment patterns. In addition, unlike ILA models, trade imbalances can be permanent in a life-cycle model as there are no transversality conditions on a country's borrowing. In life-cycle models, the age structure of the population endogenously determines the steady-state values of capital per effective labor as highlighted by Ferrero (2005). Therefore, life-cycle models can generate different outcomes from ILA models in predicting capital flows between countries for different demographic outlooks. Because the energy and carbon intensity of economies are different (Emir et al., 2019), this has implications for global emissions. In addition, because of not accounting for negative (in the case of increased total dependency) or positive (in the case of reduction in total dependency) demographic dividends, the model could overestimate or underestimate the emissions baseline of countries for given demographic patterns.

4.4 Calibration and quantitative investigation

This section undertakes a hypothetical quantitative investigation to investigate the effect of an asymmetric fertility and longevity transition on aggregate savings and capital per effective labor within the model described in section 4.2. To focus on asymmetry and spillovers, only one of the countries undergoes a demographic shock. To bring the analysis closer to the real-world applications and understand demographic uncertainty scenarios on emissions baseline, section 4.4.2 uses demographic shocks obtained from demographic outlooks for China and the US under different probabilistic projections by the UN. Quantitative analysis in sections 4.4.1 and 4.4.2 is calibrated with parameter values commonly used in the literature. The values for the parameters used in the models in section 4.4.1 and 4.4.2 are provided in Table 4.1 to 4.3. Section 4.4.3 undertakes a sensitivity of the findings to key parameters that reflect differences in institutions across countries.

Table 4.1: Production parameters

Parameter	Values	Description	Source
α	0.667	Labor share in the final output	
α_2	0.380	Labor share in the clean energy sector	
α_3	0.500	Labor share in the dirty energy sector	
Ψ	- 0.449	$\frac{1}{1-\Psi}$ Elasticity of substitution between capital labor composite and energy	
φ	- 0.058	$\frac{1}{1-\varphi}$ Elasticity of substitution between clean and dirty energy	
δ	0.060	Capital stock depreciation rate	
$v1$	0.960	Share of capital labor composite in the final output	US EIA
η_1	0.800	Share of dirty energy in the energy production	US EIA

Table 4.2: Demographic parameters

Parameter	Values	Description	Source
ω	0.039	Probability to retire	
θ	0.048	Probability to become mature worker	
z	0.045	Probability to become young worker	
λ_c	0.278	Time cost of children	Bloom (2017)
ξ	0.500	Productivity of young workers compared to mature workers	NTA database
γ_{ss}	0.08	Initial life expectancy	UN WPP 2019
n_{ss}	1.01	Initial growth rate of children	UN WPP 2019

Table 4.3: Household preferences and other parameters

Parameter	Values	Description	Source
β	0.994	Time preference rate	
σ	0.750	Household intertemporal elasticity of substitution	
	1.000	Relative size of countries	Hypothetical
x_{lss}	1.01	Productivity growth	Penn World Table 9.1
pen	0.020	Share of pension expenditure in GDP	
by	0.150	Government debt to GDP ratio	

Continued on next page

Table 4.3 – continued from previous page

Parameter	Value	Description	Source
gy	0.150	Government expenditure to GDP ratio	
tax	0.050	Share of wage for child	
ζ	1.000	Share of capitalized value of labor income of the young workers that can be borrowed	

Continued on next page

4.4.1 Hypothetical shock with two countries

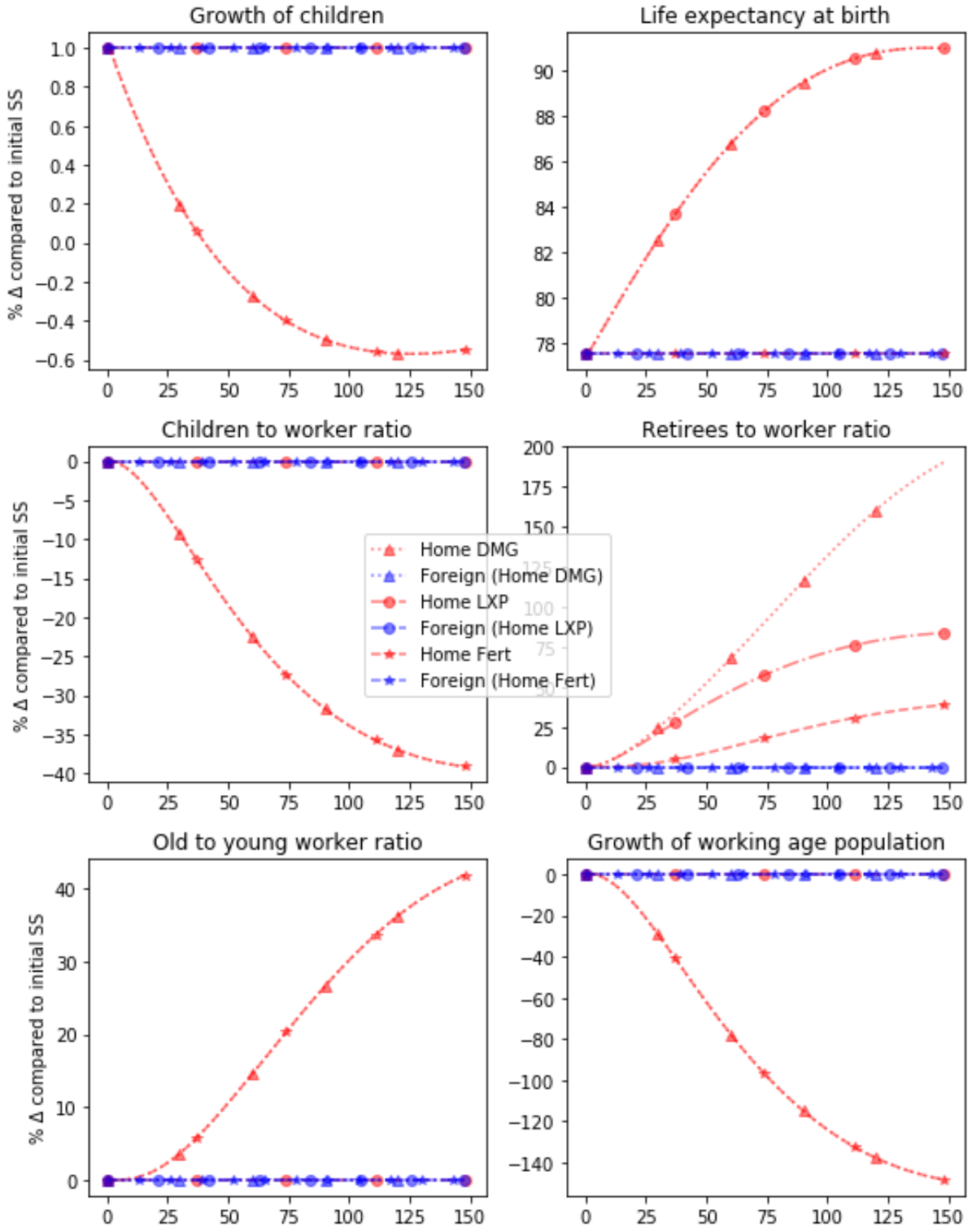
An asymmetric demographic shock is introduced into a hypothetical model of otherwise symmetric two countries, the home and the foreign. In the first scenario, called Fert, it is assumed that only the home country's fertility rate falls, causing the growth rate of children in the Home country to fall from 1% in the beginning to -0.6 % within a 100-year horizon. In the 2nd scenario, called LXP, it is assumed that only the home country experiences a reduction in its mortality rate after retirement, causing the life expectancy to rise from 78 in the beginning to over 90 years within a 100-year horizon. In a third scenario called DMG, it is assumed that the Home country experiences both the fertility and the life expectancy shock described in the Fert and LXP scenarios simultaneously.

Figure 4.5 contrast population dynamics in the home and the foreign country under the three shocks. Because there is no fertility and life expectancy shock in the foreign country, there is no change in population growth and population age structure in the foreign country. Only the fertility shock affects the children to worker ratio and the workforce's age structure for the home country. In contrast, the fertility and life expectancy shock affect the retirees to worker ratio. Furthermore, the growth of the working-age population is only driven by the fertility shock but not by increasing life expectancy after retirement. In figure 4.5, one can also see that both the reduction in fertility and increase in life expectancy after retirement drive up the retirees to worker ratio, which changes the most under both shocks introduced together.

Figure 4.6 contrasts individual savings decisions and the two countries' wealth distribution under the three scenarios. As explained in section 4.3.2, life expectancy shocks alter the MPCs of individuals directly in addition to their indirect effect due to changes in the interest rate. In contrast, fertility shocks only affect the MPCs through the interest rate change. The impact of interest rate on the MPCs moves in the same direction as life expectancy shocks on the MPCs, reinforcing its effect. In the second country (the foreign country), MPCs change only in response to interest rate changes.

Regarding the effect of the demographic factors in explaining wealth distribution, we can see interesting dynamics from the bottom two figures of figure 4.6. In the home country under the fertility scenario, young workers' share of wealth rises initially before declining. In contrast, the share of wealth held by young workers in the second country falls in a pattern similar to the interest rate changes. This contrasting result shows the importance of aggregation and the child dependency effect. There is no change in the relative size of different age populations in the foreign country. The relative decline in young workers' financial assets is driven by the change in factor prices, which start around

Figure 4.5: Asymmetric demographic transition scenario



20 years after the fertility shock hits. Whereas in the home country, a reduction in the fertility rate immediately increases the relative financial wealth of the young workers by reducing transfer to children and releasing labor for work which more than offsets the

Figure 4.6: Divergence of savings rate and wealth holdings by different generations

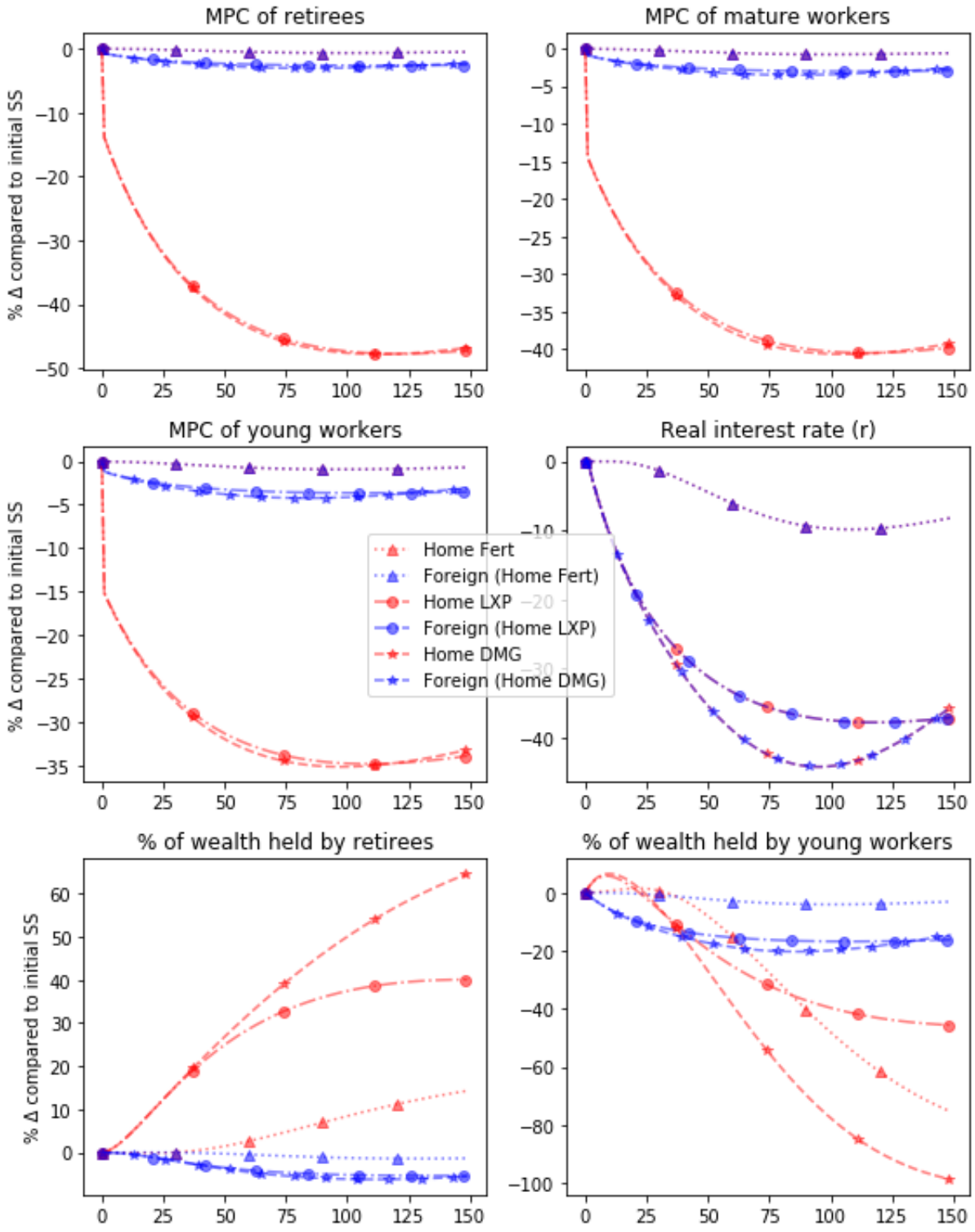
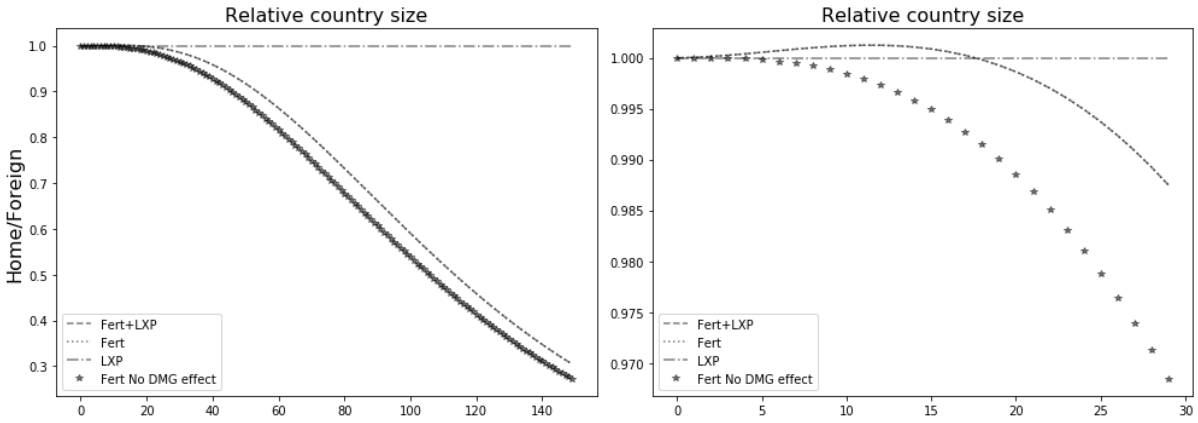


Figure 4.7: Relative country sizes



negative effect of factor prices on the young workers' relative asset holdings. However, after a few decades, the aggregation effect dominates the child dependency effect. A reduction in the fertility rate causes the relative size of the retirees and the mature workers to increase, causing the share of wealth held by young workers to decline.

Under the LXP[20] scenario, the share of wealth held by young workers declines in both countries. The magnitude of the reduction in the interest rate is such that the positive effect from the decrease in child dependency on the share of assets of the young workers is more than offset by the factor prices shown in the DMG scenario when the two shocks happen simultaneously. The negative effect of the factor prices on the share of assets held by the young workers is due to their different life horizons and wealth composition. Young workers have the longest life expectancy. In response to the reduction in the interest rate and the increase in life expectancy, young workers' MPCs decline the least compared to mature workers and retirees. Furthermore, the decline in the interest rate and increase in wages increase the capitalized value of the young workers' human wealth the most because they expect to stay in the workforce the longest.

Only fertility transitions drive changes in the relative country sizes, with the home economy's size compared to the second country falling to 30 % as shown in Figure 4.7. Furthermore, there is a marked difference between relative country sizes when one doesn't consider changes in the workforce's age structure and the workers' labor supply changes. The right panel of Figure 4.7 shows the relative country sizes with and without demographic effects for a 30-year horizon, while the left-hand side shows the same for a 150-year horizon. Models that do not consider demographic effects will underestimate the size of countries which are under a demographic transition relative to countries which are not under demographic transitions.

Demographic transition changes projected emissions for both countries linked through trade and capital flows. Figure 4.8 shows that the output and energy use is higher in both countries relative to a case where there are no demographic effects. However, once demographic effects are taken into account, the economy and emissions are lower in the open economy under the demographic transition than if the country was a closed economy.

[20]Fert: fertility change scenario, LXP: Life-expectancy change scenario, DMG: Scenario where both fertility and life-expectancy change at the same time

Figure 4.8: Projection of output and energy use per effective labor

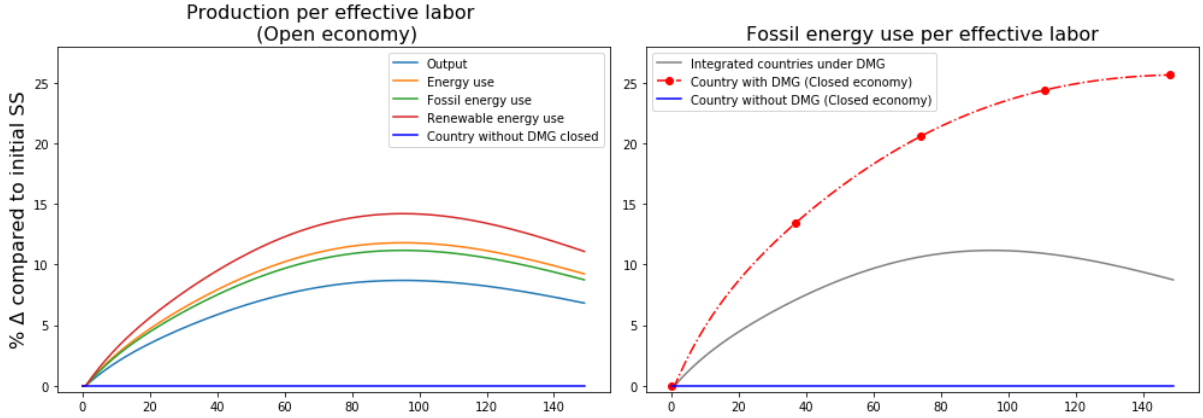
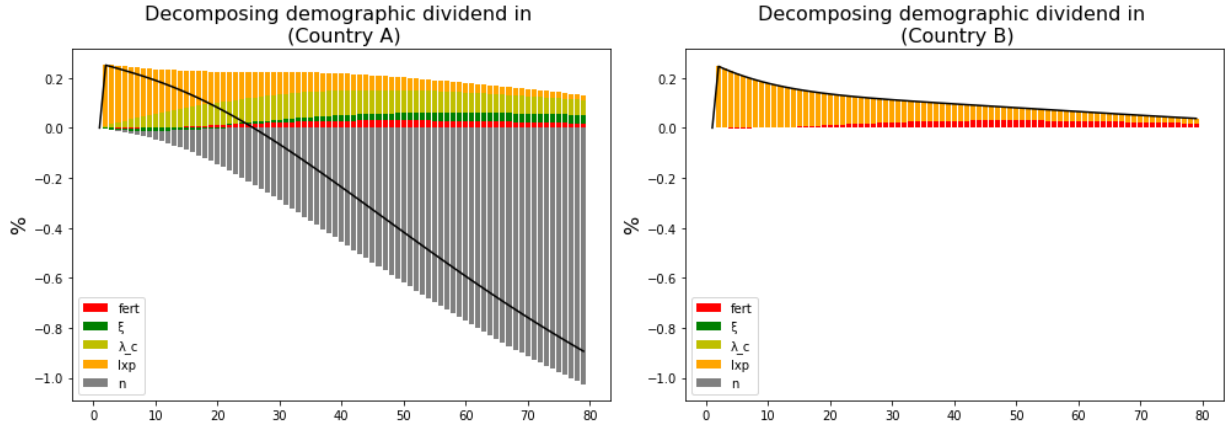


Figure 4.9: Changes in aggregate economic growth



At the same time, the economy and emissions are higher in the country, not under any demographic changes than if the country was a closed economy. As highlighted in chapter 2, energy consumption grows faster than the output reflecting its capital intensity. An additional insight from an integrated model is that global emissions can be permanently different for two reasons. When countries are integrated, the global output is higher than if the countries were closed, showing the positive effect of exchange between the two countries with different domestic interest rates. Second, the impact on global emissions of this reallocation of output across countries depends on the economies' carbon intensity. If the two economies' carbon intensities are very different, the spillovers can change global emissions. Section F show two graphs from our world in data website that show China and US's historical energy and carbon intensities. For example, the US and China's historical energy intensities were similar between 1990-2000s while China had much higher carbon intensity due to its coal intensive energy mix.

Figure 4.9 shows the change in annual growth of aggregate output of the Home and the Foreign country compared to their initial long-run values. Because the demographic shock in our model is minimal, its effect on aggregate growth in a specific year is small. However, even small differences can accumulate into a larger difference in the projected outcomes

over a longer horizon. The black line in figure 4.9 shows the net demographic effect. Models which cannot capture demographic dividends will underestimate the growth of a country under declining fertility and increasing longevity. At the same time, one can also underestimate or overestimate growth in countries that do not have demographic changes if one doesn't account for cross-country spillovers due to demographic differentials across countries.

4.4.2 Uncertain demographic outlooks

This section examines nine different relative fertility and mortality change scenarios based on the US and China's three different probabilistic demographic outlooks. This section aims to understand the uncertainty around countries' emissions paths and emissions spillovers due to uncertainty around demographic differences across countries. Using demographic outlooks for China and the US, two large countries closely linked with trade and capital flows, makes the shock magnitude closer to reality.

Fertility rate and emissions baselines

Relative fertility scenarios are constructed by combining three different probabilistic projections for each country. Scenarios reflect fertility changes with varying asymmetries of magnitude across countries where scenarios four (S4), seven (S7), and eight (S8) are the least asymmetric while scenario three[21] (S3) is the most asymmetric, as shown in figure 4.10. Figure 4.10 shows that the fertility patterns may unfold quite symmetrically (S7 shows almost no difference in the growth rates of the populations under 19),but it is more likely that the fertility changes will unfold asymmetrically (i.e., there are many more scenarios where the relative difference is not zero).

Figure 4.10: Demographic outlook for China and the US

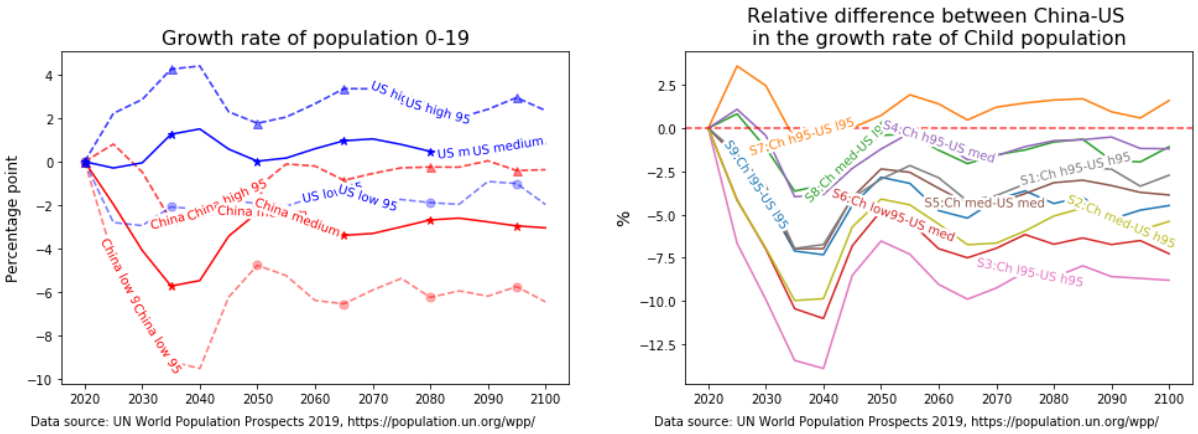
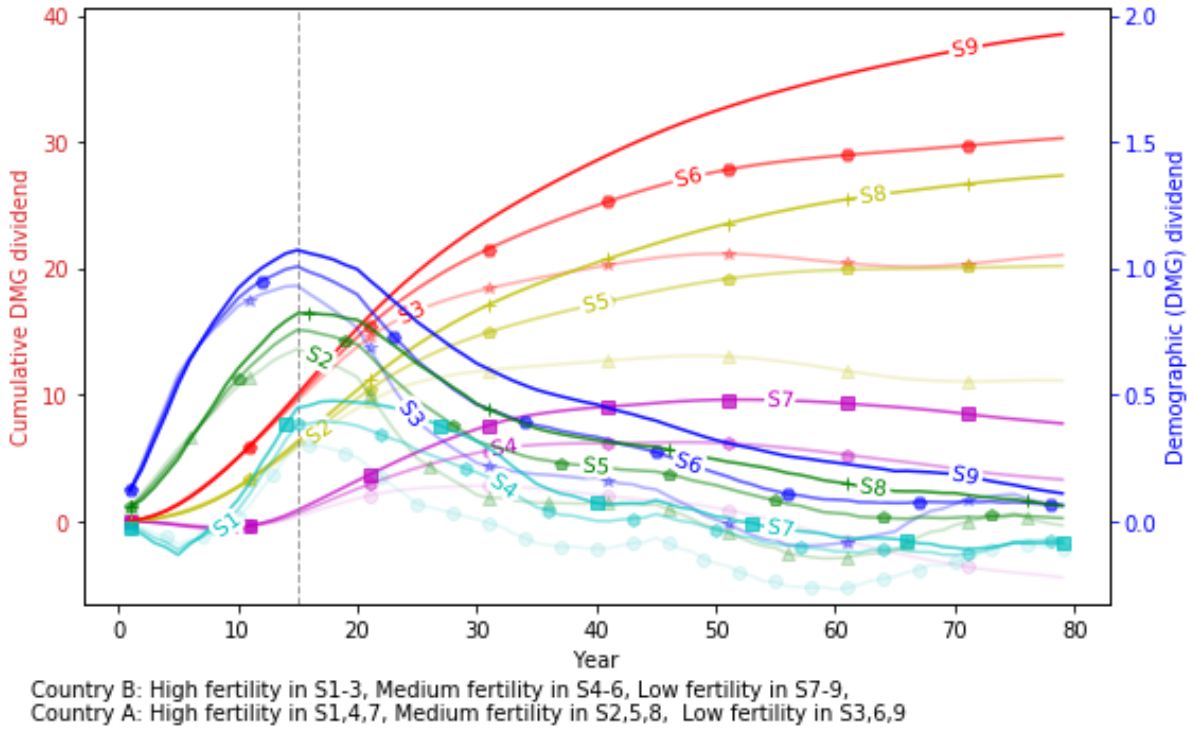


Figure 4.11 estimates demographic effects from nine different scenarios for Country A (which has a fertility shock that mimics China). Blue, green, and cyan-colored lines show

[21]Scenario three (S3) is where country A follows a low fertility scenario similar to China's 95th percentile low fertility path while Country B follows a high fertility path that is similar to US's 95th percentile high fertility path.

the effect of demographic changes on the annual growth of GDP, with the value shown on the right axis. In contrast, the red, yellow, and purple lines show the cumulative growth effect of demographic changes with the value on the left axis. The lines with the same colors in 4.11 represent the same fertility shock in Country A combined with three different possible fertility shocks in country B (which has a fertility shock that mimics the US). Figure 4.11 shows few interesting results. First, demographic changes have a non-negligible impact on economic growth and emissions both in the short and long-run, which the models without demographics cannot capture. Second, the magnitude of the demographic effect depends on the magnitude of demographic shocks (in this case, the fertility shock), as shown by the variation between lines with different color. Third, the magnitude of the demographic effect depends not only on the country's demographic changes but also on the demographic changes of its trading partners, as shown by the difference between lines with the same color. Fourth, even if the demographic effect is small in a given year, it can accumulate over many years to cause large disparities in projected outcomes, as shown by the difference between the lines with same colors showing cumulative effects.

Figure 4.11: Demographic dividends under a fertility shock (Country A)



Life expectancy and emissions baselines

Similar to the fertility scenarios, nine different relative mortality change scenarios have been constructed by combining three different mortality change scenarios for both China and the US as shown in figure 4.12. Unlike the fertility scenarios, which tilted toward China having lower fertility rates than the US, relative differences in mortality

may be positive or negative. In other words, the odds that US life expectancy increases more than Chinese life expectancy are the same as the odds that life expectancy in China increases more than the life expectancy in the US (see 2nd panel of Figure 4.12).

Figure 4.12: Demographic outlook for China and the US

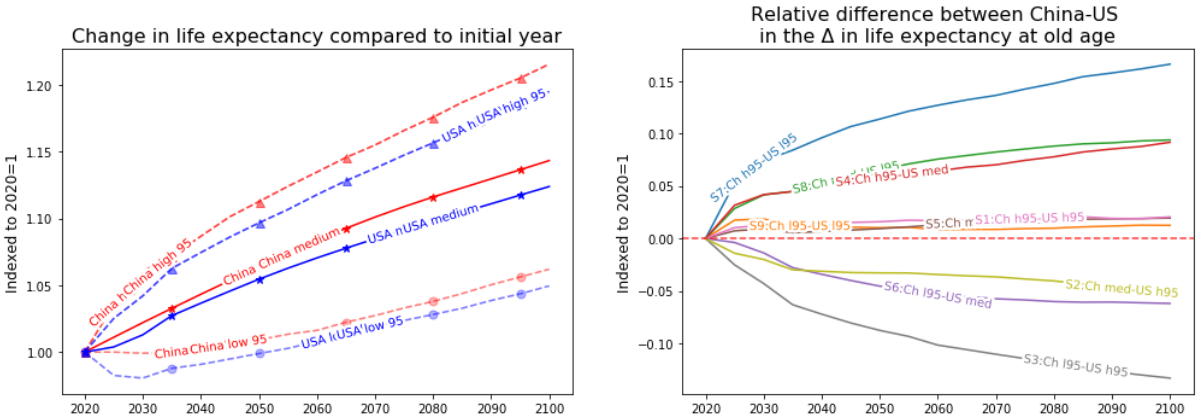
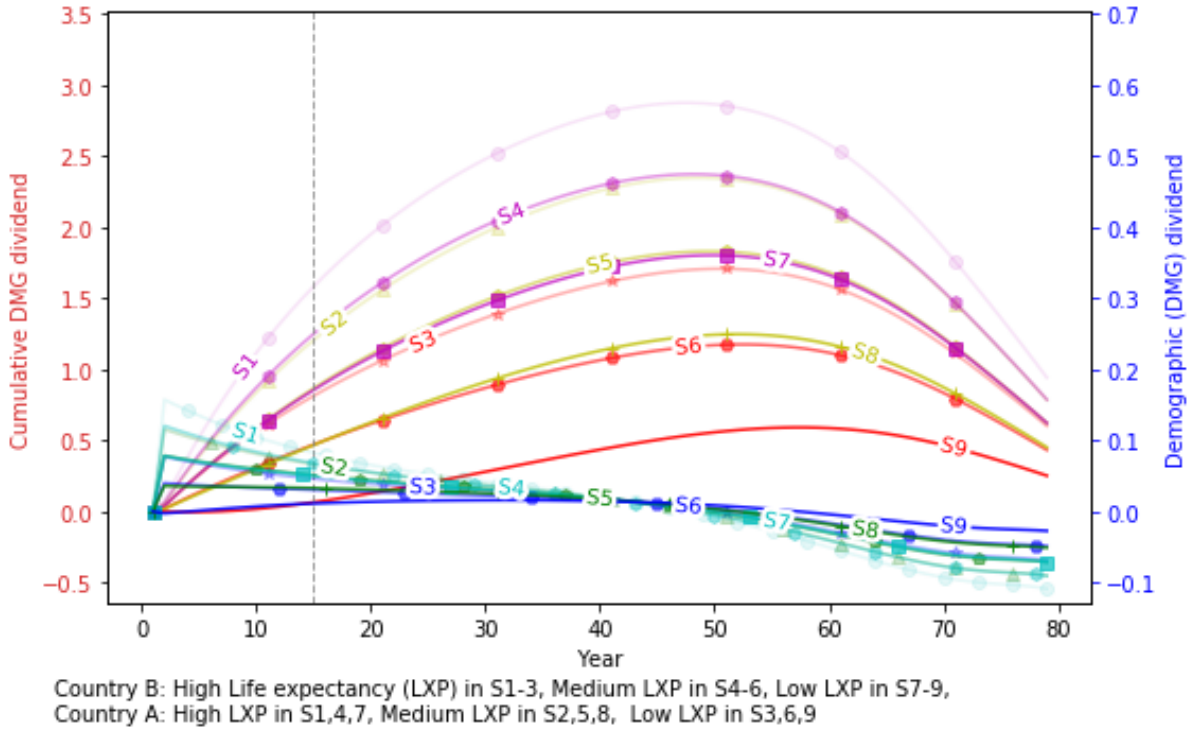


Figure 4.13 shows both the annual and cumulative growth effect in Country A based on nine different life expectancy scenarios. Similar to figure 4.11, lines with same color represent the same life expectancy change of Country A combined with three different life expectancy changes in country B. Blue,green, and cyan lines show the annual growth effect with the value shown on the right axis. Red, yellow and purple lines show cumulative growth effects with the value shown on the left axis. Compared to fertility shocks, changes in life expectancy (as projected for China and the US) have only minor impacts on a country's annual and cumulative growth rate. Again this is also because it is assumed that the life expectancy at old age affects the economy only through changes in savings. If there are employment impacts of life-expectancy changes, such as longer labor force participation by the retiree population, there will be a larger demographic dividends from the changing life expectancy at old age. Similar to fertility scenarios, the magnitude of the impact of the life-expectancy changes on annual and cumulative growth depends not only on a country's life expectancy changes but also on its relative life-expectancy changes, as shown by the variation between lines with the same color.

Spillovers and emissions baselines

To further highlight spillover effects, this section focuses on two scenarios. Figure 4.14 shows the effect of demographic changes under scenarios three (S3) and nine (S9) for both countries under closed and open economy assumptions. S3 and S9 represent the same demographic change for Country A each combined with a different demographic changes for Country B. Specifically, S3 is the most asymmetric in terms of fertility changes across the two countries. This is shown in panel B of figure 4.10. In S3, Country A follows its low fertility scenario while Country B follows its high fertility scenario. When country B follows its high fertility path as in scenario 3, there is a capital dilution effect wherein high growth in the population under 19 depresses savings while more younger workforce reduces capital per effective labor. As shown in figure 4.14, due to the capital dilution

Figure 4.13: demographic dividends under a longevity scenarios (Country A)



and an increase in the interest rate, fossil energy use per effective labor also declines. In contrast, in S9, both countries follow their low fertility scenarios. In S9, low fertility in both countries bring larger demographic dividends and a bigger decline in interest rate, which explains the increase in fossil energy use per effective labor.

Figure 4.14: Emissions baseline (Open vs Closed economy)

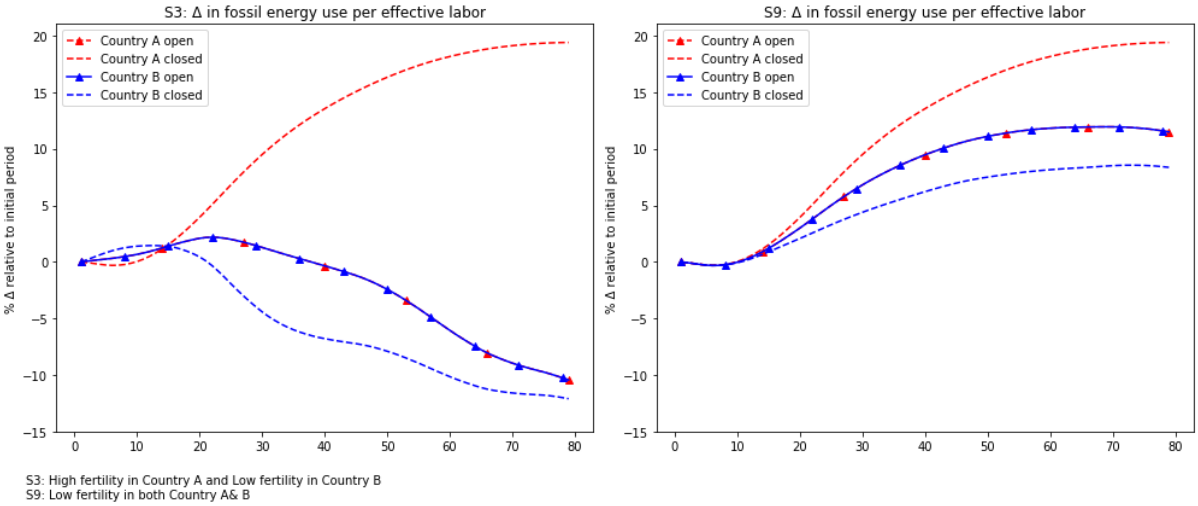


Figure 4.14 shows three results. First, a change in a country's emissions baseline depends on whether the country is an open economy or closed economy, based on the

apparent difference between emissions per effective labor across open and closed economy assumptions under both scenarios. Second, when a country is an open economy, its emission baseline depends on its demographics and its trading partners' demographics. Figure 4.14 shows a significant difference in emissions per effective labor for Country A in S3 and S9 despite Country A having the same demographics in both cases. Third, the magnitude of the demographic changes on emissions baselines depend on the magnitude of demographic changes as well as asymmetries between countries. In other words,in addition to their impact on population scales, demographic changes affect emissions baselines through demographic dividends and capital flows which is most often neglected by models which do not explicitly include demographic changes.

4.4.3 Sensitivity analysis: Institutional and other asymmetries

Next, the two-country model's steady-state is solved under different model parameters to reflect initial asymmetries between countries. Considered asymmetries included differences in life expectancy, institutions such as pension, consumption and time cost of children, government spending, production (energy share of GDP), and preference parameters (subjective time preference rate, elasticity of intertemporal substitution). The previous literature (Ferrero, 2010; Niemeläinen, 2021; Eugeni, 2015; Batsuuri, 2022; Buiter, 1981) have found these differences to be important in explaining the persistent trade and current account imbalances across countries.

Figure 4.15: Emissions under various asymmetries

Asymmetries	Difference in emissions open vs closed economy (%)	Difference in energy intensity open vs closed economy (%)
Higher LXP@birth	-4.615	-1.247
Higher pension	2.037	0.537
Higher transfer to children	0.127	0.034
Higher child labor cost	1.439	0.38
Higher gov. spending	1.969	0.52
Lower energy share of the economy	-2.471	-0.663
Lower β	1.294	0.342
Lower σ	1.398	0.37

Figure 4.15 shows the difference in total emissions and energy intensity of the output of a country relative to its value under a closed economy assumption for each case listed in the column 'Asymmetries'. Everything else equal, the country with a higher life expectancy and a lower energy share of the economy relative to its trading partners will have a positive foreign asset position and negative trade balance in the steady-state. Longer life expectancy causes the country to have higher savings in anticipation of a longer retirement period, some of which is invested in foreign assets. Similarly, the lower energy

share of the economy implies that it's labor share is higher than its trading partners, which allows for higher savings and positive foreign asset positions. In turn, positive foreign asset positions imply that, in the long run, the country can consume more than its domestic production. Therefore, emissions in the open economy with longer life expectancy and lower energy share of the economy can result in lower emissions for the country in the long-run relative to the case if the country had to invest all its savings domestically. In contrast, all else equal, countries with higher pension, higher consumption and time cost of children, higher government spending, lower subjective time preference rate[22] and lower elasticity of intertemporal substitution[23] will have lower aggregate savings compared to its trading partners and hence negative foreign asset position and positive trade balance in the long run. Everything else equal, higher government spending does not crowd out private spending one to one reducing aggregate savings as emphasized in Ferrero (2010). On the other hand, lower pension expenditure reduces tax on labor income for pensions under the pay as you go (PAYG) pension scheme. Because of the mechanisms mentioned in section 4.3, a reduction in the taxes increases aggregate savings. Eugeni (2015) has investigated the effect of an asymmetric pension scheme on aggregate savings and trade imbalance between countries using a two-period theoretical OLG model. She further provided empirical evidence that supports her theoretical predictions that a country with lower PAYG pension coverage will have a positive foreign asset position (Eugeni, 2015). A negative foreign asset position and positive trade balance imply higher national emissions in the long run than if the country was a closed economy.

In addition, relative differences in consumption and time cost of children can affect a country's trade and foreign asset position, which Batsuuri (2022) explores in detail. Reduction in transfer to children reduces the tax on labor income and reduces the time cost of raising children, thereby increasing labor supply and income. Because Ricardian equivalence does not hold in this economy due to finite work and lifetime, children's consumption has a non-negligible effect on aggregate consumption and savings. Therefore, everything else equal, higher transfer to children and higher time cost of children causes a country to accumulate foreign debt and increase its output (positive impact of lower interest rate) and long-run emissions.

4.5 Conclusion

There is a growing need for economic models to evaluate and successfully implement climate change agreements. To successfully implement the Paris Agreement, the latest global policy agreement on climate change, economic models will be required to compare countries' emissions mitigation efforts which will help push countries to have more ambitious policies. Despite using multiple distinct models to increase the robustness of the studies to alternative assumptions, modelers continue to neglect the effect of demographic transitions on economic and emissions projections by not using life-cycle models, the main framework to study the impact of demographic changes.

Life-cycle models will be fundamental given the considerable uncertainty around future population patterns across countries. This paper investigates the robustness of the

[22] implying higher discounting of the future

[23] implying lower preference to smooth consumption across time

carbon emissions baseline to alternative demographic assumptions using a two-country life-cycle model with realistic demographic scenarios. Specifically, the paper constructs nine different demographic scenarios by combining low, medium, and high probabilistic projections of fertility and mortality rates of the two largest economies (China and the US). This paper shows that a country's emissions baseline is sensitive to its domestic and trading partner's demographics.

The model used in this paper has several advantages over previous modeling studies. First, because the model is solved in annual frequency, thereby capturing the effect of changing child and old-age dependency rates and the age structure of the workforce on aggregate savings and labor force growth both in short and the long run. Second, two-country features enable the model to capture spillover impacts from asymmetric demographic changes. Third, the inclusion of an energy-dependent production function allows for the examination of the impact of changes in factor prices on final output and energy and emissions, with each having different capital intensities.

The analysis in this paper leads to two main results. First, the impact of demographic changes on emissions can be large depending on projected fertility and mortality patterns. Models which cannot capture demographic dividends associated with changing age-structure of the population following fertility and mortality changes can underestimate a country's emissions baseline for fertility reductions and overestimate a country's emissions baseline for increased fertility. Annual change in economic growth due to demographic dividends can be small. Still, demographic dividends can accumulate over many years, causing significant differences in the long run with implications for climate economy models that deal with long-run projections.

Second, relative demographics and institutions can determine capital flows between countries within life-cycle models. Therefore, differences in the demographic outlooks across countries can affect cross-country capital flows and resulting emissions. This paper shows that the size of cross-border capital flows depends on how asymmetric the demographic changes will be across countries. Models which do not take into account spillovers can overestimate (underestimate) a country's emissions baseline with relative decline (increase) in its fertility and mortality rates. Furthermore, differences in governmental and household institutions such as pensions and social security system, time and labor cost of children, and energy intensity of the economies will amplify the effect of asymmetric demographic changes. As long as the economies' carbon intensities do not converge, global emissions will significantly differ from what they would have been if the effect of the demographic shocks was contained domestically.

The interconnected emissions trajectories of different countries through capital flows, as determined by demographic differences, illustrates the difficulty in comparing carbon emissions mitigation efforts using a model that is not well suited to capture the demographic dividends and associated spillovers. Through capital flows, asymmetric demographic changes can increase emissions in one country while reducing them in the other country compared to a baseline that does not consider the spillover impact. Hence, the rankings of countries' emissions mitigation efforts' may change if one uses life-cycle models. In addition, life-cycle OLG models provide different implications from the standard models on many fronts, including the effect of government debt and deficit on savings and the persistence of trade and current account imbalances. Therefore, to increase robustness, this paper suggests using life-cycle models in studies that intend to compare

countries' mitigation efforts and proposes a suitable modeling framework.

Chapter 5

Conclusion

Understanding changes shaping the future is crucial to crafting climate change mitigation policies. Despite the rapid and profound demographic changes many countries are undergoing, most economic models used for climate policy neglect demographic effects, primarily due to methodological constraints. This thesis developed a tractable life-cycle overlapping generations (OLG) model that can be incorporated into large-scale economic models used for climate policy. Using the framework created, the three main chapters of this thesis investigated the impact of demographic changes on economic growth, external imbalances, and carbon emissions in the short and long run. Specifically, Chapters 2 and 4 focused on the importance of using life-cycle OLG models for climate policy analysis that deals with long-run emissions scenarios. Chapter 3 highlighted the importance of using OLG models with rich demographic details that can fully capture the effects of demographic changes when explaining the past and projecting the future.

Taking the three chapters together, this thesis draws three broad conclusions. First, the impact of demographic changes on economic growth, energy use, and emissions can be significant depending on projected fertility and mortality patterns. Models which cannot capture demographic dividends associated with a changing age-structure of the population following fertility and mortality changes can underestimate a country's emissions baseline for fertility reductions and overestimate its emissions baseline for increased fertility. Moreover, while annual change in economic growth due to demographic dividends can be small, these can accumulate over many years, causing significant differences in the long-run projections, as shown in Chapter 2.

Second, demographic changes affect both child and old-age dependency. OLG models that don't take changing child dependency into account can be inaccurate in explaining the past and predicting the future. Specifically, chapter 3 demonstrates that differing OLG model structures affect parameter choices, which also affect the magnitude of the effects of demographic changes. Because OLG models with and without child dependency produce different steady-state aggregate savings rates, they need to be calibrated with varying parameters which affect model dynamics also. In addition, fertility declines first reduce child dependency levels before eventually causing an aging population. Models that assume changing child dependency won't have any effects (which is furthermore not supported by empirical evidence) will underestimate the effects of falling fertility on economic growth, energy use, and emissions.

Third, life-cycle OLG models would be essential for understanding the uncertainty

around global emissions and predicting countries' baseline emissions in the face of uncertain future population patterns across countries. OLG models have endogenous external imbalances between countries that reflect differences in institutional and demographic structures. Chapter 4 shows that differences in the timing, speed, and magnitude of demographic changes can lead to changes in external imbalances between countries. Specifically, the more asymmetric these demographic changes are, the more spillovers there will be through capital flows. Capital movements will affect national and global emissions because of the countries' differences in energy and carbon intensities.

5.1 Future research needs

Future research can focus on several crucial elements through which the demographic transition may impact the economy and environment. These elements include endogenous technological growth; adjustments to aggregate consumption patterns (which can affect intratemporal trade patterns); adjustments to resource extraction rates; and adjustments to global migration patterns. One potential extension concerns how demographic changes affect technological growth. A number of theories emphasize the link between technical and demographic development. In the neoclassical growth model, which forms the theoretical foundation of most applied intertemporal general equilibrium models, exogenous technological progress is the only source of long-run per capita output growth. Since there is currently no single comprehensive theory explaining how technology develops through time, general equilibrium models primarily regard it as an exogenous variable. But several studies point to a possible link between changes in the population and technological progress.

Theories that argue that a demographic transition characterized by declining fertility and increasing life expectancy would positively affect technological advancement emphasize the possibilities of building up physical and human capital. Technology growth may accelerate with an increase in aggregate savings and capital stock investment when new technology is embedded in capital stock (Solow, 1960; Scott, 1989). By boosting investment in children, fertility decline can also accelerate the accumulation of human capital (Lee & Mason, 2010). On the other hand, theories that point to an adverse effect include those that link it to a population's size and age composition. For instance, if technology is mainly transferred by young workers with fresh education, for example, Van Imhoff (1989) hypothesizes that population aging could slow down technology's growth by lowering the influx of young people into the workforce. According to views that emphasize the size of the population for technological advancement (See Romer (1986)), the influence of demographic transition may also be detrimental. A more hybrid theory being popularized by Acemoglu (2002) emphasizes how technological growth can change direction given a change in the relative scarcity of production factors. He further argues that population aging may accelerate growth by increasing automation (Acemoglu & Restrepo, 2017). However, the potential for capital deepening to spur economic growth, according to Eggertsson et al. (2019), may be constrained by the zero lower bound on interest rates.

When theory is applied to data, things get much more complex. In truth, measurements of technical advancement include everything other than the changes in the input quantities. Total factor productivity, frequently regarded as a proxy for technological ad-

vancement, is calculated as a residual from a production function with only capital and labor inputs. As has been shown in the simulations, even after taking out its effect on the quantity of capital and labor, demographic change can affect aggregate production through changes in the age structure of workers and labor force participation. Furthermore, human capital accumulation is empirically important in contributing to GDP growth (Mankiw et al., 1992), and there are studies linking the rate of human capital accumulation to fertility reduction (Lee & Mason, 2010) which implies there could be additional growth coming from this mechanism.

Two extensions may be essential for energy and emissions. The first is to incorporate an age-dependent consumption bundle into the model. This thesis made an implicit assumption that, despite the demographic change, an economy's consumption bundle stays constant by assuming a single consumption good. Independent of the other effects of the demographic transition, emissions could increase if the consumption bundle shifts toward more energy-intensive commodities. However, Dalton (2008) concluded that the consumption channel had a minor impact on emissions after analyzing the effect of the demographic transition on US emissions using US household data on 17 consumption commodities. Another study by Kronenberg (2009) found that changes in emissions due to changes in consumption patterns in an aging population may be insignificant in Germany. Specifically, he found that the rise in residential emissions may balance the increase in transportation emissions. According to Garau et al. (2013), who studied the impact of population aging on carbon emissions in Italy, emissions may increase due to the emission intensity of consumption at old age. He concluded, however, that this increase will be more than offset by a decrease in emissions caused by decreased production potential.

Explicitly modeling finite resource extraction problems is the second extension that can be crucial for energy and emissions. The demographic transition may affect the relative attractiveness of various asset types by incentivizing savings and lowering interest rates. In particular, it may change the value of natural resources and the fossil fuel extraction rates. Some possible interactions between the return to capital and natural resources are provided in the paper by Siegmeier et al. (2018).

Finally, this thesis didn't touch upon the impact of asymmetric demographic changes on international migration patterns. Demographic factors were found to be important in empirical investigations that looked at the determinants of international migration patterns. For example, Kim and Cohen (2010) finds that demographic variations are significant in explaining cross-country migration trends using migration data from 17 countries between 1950 and 2007. Specifically, they find that people migrate from countries with a younger population to countries with an older population. In this way, demographic changes that cause countries to age differently can change magnitude and pattern of international migration.

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Appendix A

The steady state equations of the model in Chapter 3

Equations from A.1 to A.5 provides steady-state equations for age structure of the population. In the steady-state, population age structure is constant with different age groups growing at the same rate n.

Young worker to children ratio, equation A.1, depends on population growth rate and the two transition probabilities that matter for children and young workers.

$$\psi^c = \frac{z}{n + \theta - 1} \tag{A.1}$$

Similarly, the mature to young worker ratio, equation A.2, depends on the population growth rate and two transition probabilities that matter for young workers and mature workers.

$$\psi^1 = \frac{\theta}{n + \omega - 1} \tag{A.2}$$

The old-age dependency ratio, equation A.3, depends not only on the population growth rate and all the transition probabilities from one life stage to another but also on the mortality rate given by γ.

$$\psi^2 = \frac{\psi^1 \omega}{(1 + \psi^1)(n + \gamma - 1)} \tag{A.3}$$

Equation A.4 provides the children to workforce ratio.

$$\psi^3 = \frac{1}{\psi^1 \psi^c + \xi(\psi^c - \lambda_c)} \tag{A.4}$$

Equation A.5 provides the young worker to workforce ratio.

$$\psi^4 = \frac{1}{\psi^1 + \xi \left(1 - \frac{\lambda_c}{\psi^c}\right)} \tag{A.5}$$

What is important to notice here is that, since z, θ and, ω is fixed, reflecting the average length of each life stage, the age structure of the population below retirement

age only depends on population growth rate. In an integrated model where the initial population growth rate is assumed to be the same, the child dependency ratio and the workforce's age structure are constrained to be the same. Therefore, simulation models that assume the same initial population growth rate should start their simulation from a period where the child dependency ratio and workforce's age structure were not vastly different.

Equations A.6 to A.18 describes equations characterizing household side of the economy. Because of population and productivity growth, the model has a steady-state only in de-trended variables. Small alphabets denote the variable de-trended by population and productivity growth. Specifically, a small alphabet v denotes a variable $\frac{V}{XN}$.

Section **??** provides mathematical proofs of the relationship between marginal propensity to consume out of lifetime wealth for young and mature workers and the mortality rate for the steady-state. It also provides proof that the additional discount rates (Ω and Ω_2) used by young and mature workers are greater than 1.

Equation A.6 provides marginal propensity to consume out of wealth for a retiree:

$$\epsilon\pi = 1 - (1-\gamma)\beta^{\sigma} R_w^{\sigma-1} \tag{A.6}$$

Equation A.7 provides marginal propensity to consume out of wealth for a mature worker:

$$\pi = 1 - \beta^{\sigma}(R_w\Omega)^{\sigma-1} \tag{A.7}$$

where, mature worker's additional discount rate Ω that reflect his/her finite life is given in equation A.8:

$$\Omega = 1 - \omega + \omega\,\epsilon^{\frac{1}{1-\sigma}} \tag{A.8}$$

Equation A.9 provides marginal propensity to consume out of wealth for a young worker:

$$\epsilon_2\pi = 1 - \beta^{\sigma}(R_w\Omega_2)^{\sigma-1} \tag{A.9}$$

where, the young worker's additional discount rate Ω_2 that reflect his/her finite life is given in equation A.10:

$$\Omega_2 = 1 - \theta + \theta\,\epsilon_2^{\frac{1}{\sigma-1}} \tag{A.10}$$

Aggregate household consumption is given in equation A.11:

$$c = \pi\Big[aR_w(1-(1-\epsilon)\lambda-(1-\epsilon_2)\lambda^2) + s^m + h^m + \epsilon s^r + \epsilon_2\zeta(h^y + s^y)\Big] + \frac{\psi^4 w t^c}{\psi^c} \tag{A.11}$$

Human wealth of mature workers are given by equation A.12:

$$h^m = \frac{R_w\Omega\psi^1(w(1-\psi^3 t^c)-\tau)}{(R_w\Omega-(1-\omega)x_l)(\psi^1+\xi(1-\frac{\lambda_c}{\psi^c}))} \tag{A.12}$$

Human wealth of young workers are given by equation A.13:

$$h^y = \frac{R_w\Omega_2(1-\frac{\lambda_c}{\psi_c})\xi(w(1-\psi^3 t^c)-\tau)}{(\psi^1+(1-\frac{\lambda_c}{\psi_c})\xi)(R_w\Omega_2 - x_l(1-\theta))} + \frac{\theta h^m x_l{}^1 \epsilon_2^{\frac{1}{\sigma-1}}}{(R_w\Omega_2 - x_l(1-\theta))} \tag{A.13}$$

Social security:

$$s^y = \frac{\theta x_l \epsilon_2^{\frac{1}{\sigma-1}} s^m}{R_w\Omega_2 - x_l(1-\theta)} \tag{A.14}$$

$$s^m = \frac{x_l \epsilon^{\frac{1}{\sigma-1}} \omega s^r}{(R_w\Omega - x_l(1-\omega))} \tag{A.15}$$

$$s^r = \frac{R\overline{p_y}y}{(R_w - x_l(1-\gamma))} \tag{A.16}$$

Share of assets held by retirees at the steady state is given by:

$$\lambda = \frac{\omega x_l n(1-\frac{\lambda^2}{1-\theta}) + py - \epsilon\pi s^r}{(x_l n - (1-\epsilon\pi)(1-\omega)R_w)} \tag{A.17}$$

Share of assets held by young workers' at the steady state is given by:

$$\lambda^2 = \frac{(1-\theta)(\psi^4(1-\frac{\lambda_c}{\psi_c})\xi(w(1-\psi^3 t^c)-\tau) - \epsilon_2\pi(h^y+s^y))}{a(x_l n - (1-\epsilon_2\pi)(1-\theta)R_w)} \tag{A.18}$$

Output:

$$y = \kappa^{1-\alpha} \tag{A.19}$$

$$w = y\,\alpha \tag{A.20}$$

Government:

$$b = \frac{\overline{p_y}y + \overline{g_y}y - \tau}{x_l n - R_w} \tag{A.21}$$

The government debt is assumed to be equal to a constant fraction of output at the steady-state, which closes the model.

$$x_l n b = \overline{b_y}y \tag{A.22}$$

The capital accumulation equation shows that investment is a constant fraction of capital stock at the steady state to compensate for population and productivity growth rate.

$$\kappa = \frac{i}{x_l n + \delta - 1} \tag{A.23}$$

Assets are held in capital stock, government bonds and foreign assets.

$$a = b + \kappa + f \tag{A.24}$$

In an efficient steady-state, where $R_w > x_l n$, a country with a positive foreign asset position will have a negative trade balance reflecting the difference between domestic savings and investment.

$$f = \frac{nx}{x_l n - R_w} = \frac{y(1-g) - (c+i)}{x_l n - R_w} \tag{A.25}$$

and will have a positive current account balance given as:

$$ca = nx\Big(\frac{x_l n - 1}{x_l n - Rw}\Big) \tag{A.26}$$

Appendix B

Solution method

The savings rate declines as individual ages, reflecting their horizon over which consumption decisions are made. Which implies that $\epsilon_2 < 1$ and $\epsilon > 1$. It is possible to prove it, at least for the steady-state of the model.

Getting ratio of the MPCs of the retirees and mature workers at the steady state provides following ratio.

$$\frac{1-\gamma}{\Omega^{\sigma-1}} = \frac{(\epsilon\pi - 1)}{\pi - 1} \tag{B.1}$$

If retirees have lower MPC than mature workers implying $\epsilon < 1$, then.

$$1 - \gamma > \Omega^{\sigma-1} \tag{B.2}$$

and

$$1 - \gamma > \left[(1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}}\right]^{\sigma-1} \tag{B.3}$$

Since $0 < \gamma < 1$, and when $0 < \sigma < 1$, the last condition holds only if $\epsilon > 1$ which contradicts our guess.

Similarly, the ratio of the MPCs of the young worker and the mature worker at the steady-state provides the following ratio.

$$\left(\frac{\Omega_2}{\Omega}\right)^{\sigma-1} = \frac{(\epsilon_2\pi - 1)}{\pi - 1} \tag{B.4}$$

If young workers have higher consumption rate than mature workers implying $\epsilon_2 > 1$, then.

$$\left[(1-\theta) + \theta\epsilon_2^{\frac{1}{\sigma-1}}\right]^{\sigma-1} < \left[(1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}}\right]^{\sigma-1} \tag{B.5}$$

For values of $0 < \sigma < 1$, this equality implies

$$\left[(1-\theta) + \theta\epsilon_2^{\frac{1}{\sigma-1}}\right] > \left[(1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}}\right] \tag{B.6}$$

Since we know that $\epsilon > 1$ and values of $0 < \sigma < 1$, the right-hand side of the equation is greater than one while the left-hand side should be less than 1 when ϵ_2 is greater than one, which contradicts our initial assumption. Therefore, it proves that ϵ_2 is less than 1.

We can establish the relationship between the MPCs of workers and life expectancy. The life expectancy parameter enters workers MPCs through Ω for mature workers and Ω_2 for the young worker.

In the steady state, Marginal propensity to consume (MPC) of mature worker is given by:

$$\pi = 1 - \beta^{\sigma}(R\Omega)^{\sigma-1} \tag{B.7}$$

where

$$\Omega = (1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}} \tag{B.8}$$

where ϵ is the ratio of MPC of retirees to MPC of mature workers:

$$\epsilon = \frac{1 - \beta^{\sigma}R^{\sigma-1}(1-\gamma)}{\pi} \tag{B.9}$$

With a function F of γ and π given as:

$$F(\gamma,\pi) = \pi - 1 + \beta^{\sigma}[\Omega(\gamma,\pi)R]^{\sigma-1} \tag{B.10}$$

where

$$\Omega(\gamma,\pi) = (1-\omega) + \omega\Big(\frac{(1-(1-\gamma)\beta^{\sigma}R^{\sigma-1})}{\pi}\Big)^{\frac{1}{1-\sigma}} \tag{B.11}$$

the effect of mortality rate γ on MPC of mature workers can be deduced by using the implicit function theorem:

$$\frac{\partial\pi}{\partial\gamma} = -\frac{\partial F(\gamma,\pi)/\partial\gamma}{\partial F(\gamma,\pi)/\partial\pi} \tag{B.12}$$

Where

$$\frac{\partial F}{\partial\gamma} = \beta^{\sigma}(\sigma-1)\Omega^{\sigma-2}R^{\sigma-1}(\omega(\frac{1}{1-\sigma})(\epsilon)^{\frac{1}{1-\sigma}-1})(\frac{\beta^{\sigma}R^{\sigma-1}}{\pi}(1)) < 0 \tag{B.13}$$

$$\frac{\partial F}{\partial\pi} = 1 + \beta^{\sigma}(\sigma-1)\Omega^{\sigma-2}R^{\sigma-1}\omega(\frac{1}{1-\sigma}\epsilon^{\frac{1}{1-\sigma}-1})\Big(-\frac{(1-(1-\gamma)\beta^{\sigma}R^{\sigma-1})}{\pi^2}\Big) > 0 \tag{B.14}$$

Therefore,

$$\frac{\partial\pi}{\partial\gamma} = -\frac{-}{+} > 0 \tag{B.15}$$

In other words, higher mortality increases MPCs by lowering the length of the horizon over which consumption decisions are made.

Similarly, we can deduce the sign of the effect of mortality rate on MPC of young workers as follows:

Mortality rates do not enter equation of the MPC of young workers directly.

$$MPC^{y} = 1 - \beta^{\sigma}(R\Omega_2)^{\sigma-1} \tag{B.16}$$

However, through Ω_2 which links the MPC of young workers to the MPC of mature workers, life expectancy can influence the MPC of young workers.

$$\Omega_2 = (1-\theta) + \theta(\frac{MPC^y}{1-\beta^\sigma(R\Omega)^{\sigma-1}})^{\frac{1}{\sigma-1}} \tag{B.17}$$

where

$$\Omega(\gamma,\pi) = (1-\omega) + \omega\Big(\frac{(1-(1-\gamma)\beta^\sigma R^{\sigma-1})}{\pi}\Big)^{\frac{1}{1-\sigma}} \tag{B.18}$$

For a function G of γ and MPC^y given as:

$$G(\gamma, MPC^y) = MPC^y - 1 + \beta^\sigma(R\Omega_2(MPC^y,\gamma))^{\sigma-1} \tag{B.19}$$

the effect of mortality rate on the MPC of young workers can be deduced using, again, the implicit function theorem:

$$\frac{\partial MPC^y}{\partial\gamma} = -\frac{\partial G(\gamma, MPC^y)/\partial\gamma}{\partial G(\gamma, MPC^y)/\partial MPC^y} \tag{B.20}$$

And the sign of the two partial derivative is:

$$\frac{\partial G(\gamma, MPC^y)}{\partial\gamma} > 0 \tag{B.21}$$

$$\frac{\partial G(\gamma, MPC^y)}{\partial\gamma} < 0 \tag{B.22}$$

$$\frac{\partial MPC^y}{\partial\gamma} = -\frac{+}{-} > 0$$

(B.23)

Appendix C

Proofs

From a mathematical point of view, the detrended model described in section 4.2 can be thought of as system of non-linear equations that can be written in the form:

$$f(y_{t+1}, y_t, y_{t-1}, u_t) = 0 \tag{C.1}$$

where

- y is the vector of endogenous variables with dimension n
- u is the vector of perfectly anticipated exogenous shocks except the one that occurring in period 1. Shocks can occur in any period, however, only the period 1 shocks come as surprise.
- f is a real valued continuous function defined as f: $\mathbb{R}^{3n+q} \rightarrow \mathbb{R}^n$
- The model admits a steady state when $f(\bar{y}, \bar{y}, \bar{y}, \bar{u}) = 0$.
- $t = 1, ...T$, with initial condition y_0 given for the state variables and $y_T = \bar{y}$.

The steady-state of the model is solved using the Newton algorithm, with the following steps summarizing its solution steps:

1. For a given exogenous variables, provide initial guess of the endogenous variables $y^{(0)}$. Upper indice (i) denote $(i)^{th}$ guess or iteration.
2. Update the guess value by solving the equation for next guess $y^{(i+1)}$:

$$F(y^{(i)}) + JF(y^{(i)})(y^{(i+1)}y^{(i)}) = 0 \tag{C.2}$$

   where $JF(y) = \frac{F(y)}{y}$ is the jacobian matrix of F.
3. Stop the iterations if.

$$\|F(y^{(i)})\| < \epsilon \tag{C.3}$$

   where ϵ is the tolerance level close to zero.

Newton method is also used to solve the model's dynamic path but take advantage of the sparsity of the Jacobian matrix of the dynamic model. Description below summarizes the solution steps :

1. It is assumed that the model reaches its steady-state in a finite period T after a disturbance.

2. Then the dynamic solution is found when the endogenous variables satisfy all the dynamic equations given by first-order conditions and the budget constraints for each period until it reaches the steady-state at period T starting from given initial values for state variables. This problem is known as two boundary value problem in the literature.

3. The system of equations can be written in the form of a function F: f: $\mathbb{R}_+{}^{nT} \rightarrow \mathbb{R}^{nT}$ such that:

$$F(y) = 0 \tag{C.4}$$

   y includes all the endogenous variables in all periods until the terminal period T. Values for deep parameters and initial values for the state variables and the terminal conditions of the endogenous variables need to be known to solve the function.

4. The non-linear system of equations is approximated by its first derivative stored in the Jacobian matrix. The Jacobian matrix is then solved using the Newton-Rapson algorithm.

The dimension of the problem increases with the number of endogenous variables and the periods. For example, for a system with n endogenous variables and T periods, the number of unknowns and equations to be solved is equal to nT. Because first-order conditions and the budget constraints are expressed only in first lag or lead, the Jacobian matrix of the dynamic model is sparse, meaning most of its columns consist of zeros. In this case, specialized algorithms can be used to reduce the size of the problem and solve the system faster.

Appendix D

Calibration comparison

Table D.1: Calibration comparison (US)

Parameters	Paper 1[1]	Paper 2[2]	Paper 3[3]
Initial Gross interest rate (R_{ss})	1.04		1.016[4] 1.059[4]
Growth of workforce (n)	0.01	0.015	0.015
Productivity growth (x_l)	0.015	0.015	0.014
Capital depreciation rate (δ)	0.1	0.1	0.2
Capital share (α)	1/3	0.28	1/3
Time discount rate (β)	0.98	0.91	0.98
Elasticity of intertemporal substitution (σ)	0.5	0.32	0.5
Relative productivity of young workers (ξ)	N/A	0.79[5]	N/A
Relative productivity of retirees (ξ_2)	N/A	0.28[5]	0.58
Consumption elasticity (v)	N/A	N/A	0.8
Government expenditure (g)	0.2	N/A	0.15
Government debt (b)	0.26	N/A	0.3
Social security (s)	N/A	N/A	0.04
Bequest	N/A	0.19	N/A
Credit constraint	N/A	0.16	N/A

[1] Ferrero (2010) Discount factor targets a 5% real interest rate in 1970.
[2] Coeurdacier et al. (2015) β, σ and other key parameters are calibrated to match household savings data in 1988.
[3] Niemeläinen et al. (2020) In addition to following the previous literature, demographic and productivity parameters chosen to match the average between China and the US between 1980 and 2015.
[4] Fixed and variable labour supply
[5] Calculated based on given age-earnings profile.

Appendix E

Modeling households' consumption and savings in the General Equilibrium

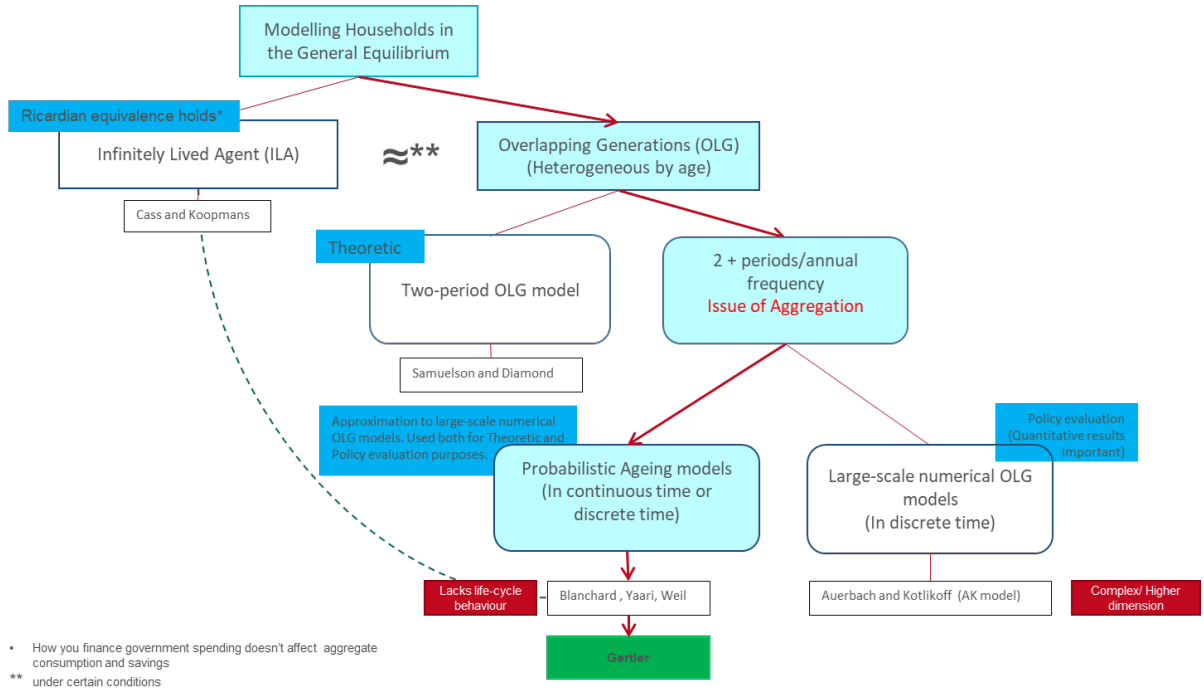

Figure E.1: Modeling households' consumption and savings decision in the General Equilibrium

Appendix F

Energy and Carbon intensity of USA and China

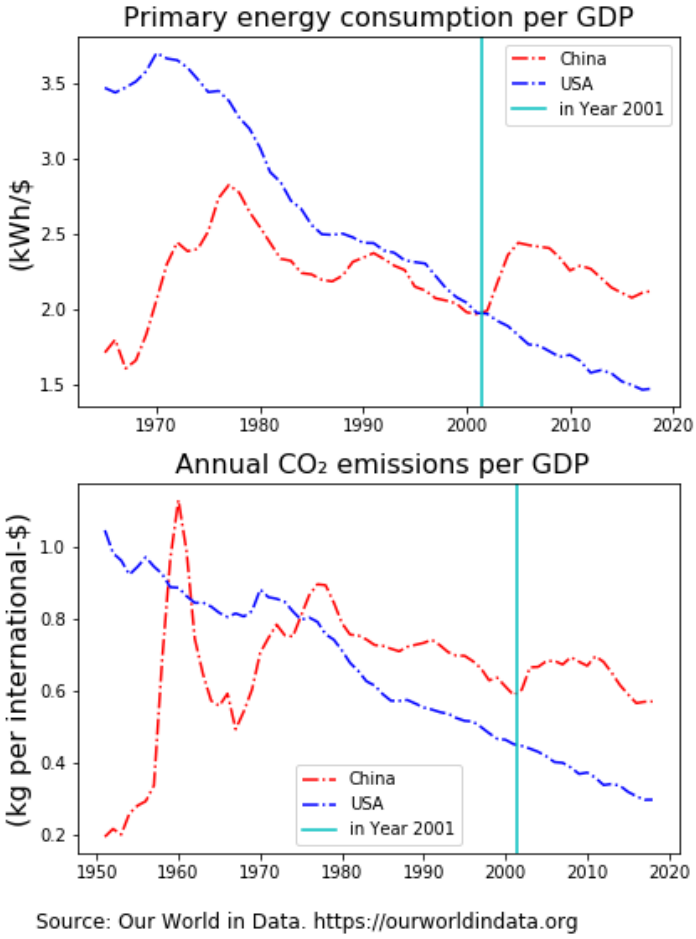

Figure F.1: Energy and carbon intensity of China and the USA