CONVERGENCE OF AGE-SEX DISTRIBUTIONS AND POPULATION CHANGE IN THE PRESENCE OF MIGRATION

by

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This thesis is based entirely on my own research, except where I have acknowledged, and was done whilst I was a Research Scholar in the Department of Demography, The Australian National University, Canberra, during 1967-70.

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THE MAIN SYMBOLS USED IN THE STUDY

Symbol Used	Definition
x	Age (in completed years).
w	The highest age to which persons can live.
P(x,t), V(x,t), W(x,t)	Population at age x years.
p(x)	Proportion of the population at age x years.
(P_t), (V_t), (W_t)	Population vector: A column vector giving population by age.
S(b,t)	Survival rate from birth to age 0 years.
S(x,t)	Survival rate from age x years to (x+1) years.
f(x,t)	Age-specific fertility rate at age x years.
α and β	The youngest and the oldest ages at which reproduction occurs among the females. (α' and β' are used when fertility rates for both males and females are used.)
n(t)	Net migration rate taking the population at the beginning of the year as base.
n(x,t)	Proportion of net number of migrants at age x years at the time of migration. $\left[\sum_{x=0}^{w} n(x,t) = 1.0\right]$.
u(t)	Net migration coefficient for the cohort born during the year [derived from n(t), n(0,t), f(x,t), S(b,t), and a(x,t)].
a(x,t)	Net migration coefficient at age x years [derived from S(x,t), n(t), and n(x,t)].
u'(t)	Age-specific net migration rate for the cohort born during the year.

Symbol Used	Definition
$a'(x,t)$	Age-specific net migration rate at age x years.
$GI(x,t)$	Growth index at age x years.
MaxGI, MinGI	Maximum and minimum of the growth indexes at individual ages.
$GI(t)$	Growth index for the total population.
s_t and s'_t	Sex ratio at birth and among net migrants, respectively.
$SR(x,t)$ and $SR(t)$	Sex ratio at age x years and in the total population.
$L(t)$, $L'(t)$, $L''(t)$ and $M(t)$	Population projection matrices in the absence of migration.
$L_M(t)$, $L'_M(t)$	Population projection matrices when migration is included using a net migration rate and an age-sex composition of net migrants.
$P(T,i,x)$	Population at age x years at the end of the i-th period in the T-th cycle.
$\phi(T,i,x)$	Ratio of the population at age x years to the population aged 0 years at the end of the i-th period in the T-th cycle.
$R(i)$	Growth index at age 0 years in the i-th period. [$R(i)$, $i = 1, 2, \ldots, k$ are the parameters of the cyclical model of population change.]

Note: The presence of t in the symbols denotes that the value refers to the year t. The absence of t indicates that the values are constant over time. Suffix f and m are used with these symbols to distinguish the corresponding values for females and males.

PRECIS

(Summary of Argument)

In recent years considerable interest has been shown for analytical research in demography. With the increased application of matrix methods, it is becoming possible to investigate problems more thoroughly than before.

In the earlier studies, the effects of fertility and mortality on the growth and age-sex composition of human populations were examined extensively both through theoretical and empirical investigations. But it appeared that the effects of introducing migration into the process of population change had not received the same attention. When migration was included, two procedures had been used: one in which a set of age-sex-specific net migration rates was assumed, and another in which an overall net migration rate and an age-sex composition of net migrants (i.e. of net number of migrants) were assumed. In almost all theoretical investigations the first procedure had been followed. This reduced the mathematical difficulties because the age-sex-specific net migration rates, suitably defined, could be incorporated into the survival rates. But the procedure is not suited to examine the effects of either a given overall net migration rate or of a specified age-sex composition of net migrants on the growth and the changes in the age-sex distribution of a population. These can be studied only when the second procedure is adopted.

Hence, an attempt is made in this study to examine, analytically, the effects of migration on the growth and the changes in the age-sex structure of a population when migration is specified by an overall net migration rate and an age-sex composition of net migrants at the time of migration. The results in the absence of migration are used as the standard of reference to compare the effects of migration.

The investigations are carried through the use of deterministic models of one sex and two sexes. The one-sex case is used only for analytical convenience, and the results are always extended to the two-sex case. The outcomes of numerical illustrations using the two-sex model are presented. The effects of migration when it is specified by age-sex-specific net migration rates, are also given for comparison.

After presenting, in Chapter 2, the results of an analysis of the numerical data used in the illustrations, the problem of the convergence of age-sex distributions is taken up for investigation. In Chapter 3, the following questions are studied: Whether, as in the case of a closed population, an unchanging age-sex distribution and a constant growth rate are evolved if a constant set of fertility, mortality and migration schedules operates on an arbitrary age-sex distribution over a long period of time?, and How would the time required for this convergence (the duration of convergence) be changed due to the inclusion of migration into the population process?

Then Chapter 4 deals with a natural generalization and examines the convergence of two arbitrary age-sex distributions when they are subjected to identical schedules of fertility, mortality and migration that are varying over time. The changes in the duration of convergence due to the presence of migration are studied in this case also.

Next, the relationship between the growth and the changes in the age-sex structure of a population on the one hand, and the operating schedules of fertility, mortality and migration on the other, is examined both when the operating conditions remain constant over time and vary over time.

Under the assumption of constant schedules, two situations are considered: one in which a set of single schedule of each of the components operates constantly over time, and another in which a set of k schedules of each operates repeatedly over time. In the first case, a constant growth rate (i.e., the intrinsic growth rate) and a constant age-sex distribution (i.e., the equilibrium state age-sex distribution) are evolved, while in the second a stable set of k growth rates and k age-sex distributions is evolved. Hence, Chapter 5 concentrates on the derivation of expressions which show explicitly the relationship between these characteristics of the ultimate populations and the given schedules of fertility, mortality and migration.

On the other hand, when the operating schedules are changing over time, no fixed growth rate or age-sex distribution is obtained.

But both are changed over time due to the operation of the components of change. Hence, in the final chapter, an attempt is made to assess the contribution of the changes in the components during a certain period of time towards the changes in the characteristics of the population during that period. A method called the factorial projections method, is suggested for this purpose and is applied to study the changes in the population of Australia during 1911-66.

PART I

INTRODUCTION AND ANALYSIS OF THE NUMERICAL DATA USED IN THE STUDY

CHAPTER 1

USE OF MODELS IN STUDYING POPULATION CHANGE

1.1 INTRODUCTION

To analyse the mechanisms that produce an observed real situation in any field of research in social or natural sciences is not an easy task because of the multiplicity of variables involved in it, their interdependence, and the difficulties of measuring the effects of some of them. Further, in social sciences, no controlled experiments can be used to study the real situations. Demography is no exception in this regard. The use of models helps to delimit the number of variables and to produce a replica of a real situation under the operation of precisely known values of these variables. Thus, models deliberately simplify the given situations for the convenience of analysis.

Though hypothetical and to an extent limited in scope, the value of models lies in the fact that they enable a clearer understanding of the mechanisms at work in producing a real situation. Using this knowledge, they may also be employed to estimate the missing values of some of the variables involved in producing an observed situation. Models can be made more sophisticated depending on the demands made by real situations and on the mathematical and/or numerical manoeuvrability. With the increasing availability of computer facilities, models are gaining greater importance and are finding wider applications in Demography.

1.2 WHAT IS A MODEL?

A model is a 'design, style of structure'.

Whilhelm Winkler[1] specifies the characteristics of a model as follows:

> There must be an a priori assumption about the necessary building stones of the population in its whole or its parts or facts happening on it, which then, worked out in an appropriate way, lead to the model. In most cases the necessary assumptions will be taken from observed populations, but that is not a constitutive character of a model but enhances its practical value.

According to this definition, the projections of a population under assumed conditions are population models. Such models have been used in many demographic studies. We give below a brief account of their use.

Models can be constructed either by defining the processes in mathematical terms or by using certain empirical assumptions. The models of the first kind are called mathematical models, while those of the second the numerical models. There is now a vast amount of literature on mathematical models and active research is still in

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[1] H.C. Whilhelm Winkler, 'Types and Models in Demography', Proceedings of the International Population Conference, New York, 1961, Tome 1, pp.358-367. (p.358)

progress. The works by Keyfitz,[2] Joshi,[3] Sheps,[4] Tabah,[5] among others, provide an adequate account of the developments in the field, and only a brief summary is presented in Section 1.3.

On the numerical models too there exists quite an extensive literature. But it appears that very little attention has been given to collate them. Some relevant material is referred to in Section 1.4.

1.3 MATHEMATICAL MODELS

Fitting of simple mathematical functions such as the linear, the geometric or the exponential were, perhaps, the earliest of the mathematical models used in the analysis of population growth. The study of the long term trends in population growth, then, led to the use of the logistic model. However, the limitations of such methods of curve fitting were soon realised and more elaborate models based on the principle of the component method of population projections were built and the results analysed.

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2 N. Keyfitz, Introduction to the Mathematics of Population, Addison-Wesley Publishing Company, Reading, Massachusetts, 1968.

N. Keyfitz, 'Mathematical Demography', paper presented at the International Union for the Scientific Study of Population Conference, London, 1969.

3 D.D. Joshi, 'Stochastic Models Utilized in Demography', United Nations World Population Conference, Belgrade, 1965.

4 M.C. Sheps, J.A. Menken and A.P. Radick, 'Probability Models for Family Building: An Analytical Review', Demography, Vol. 6, No. 2, 1969, pp.161-183.

5 L. Tabah, 'Relationships between Age Structure, Fertility, Mortality and Migration: Population Replacement and Renewal', Background paper No. B.7/15/E/476, United Nations World Population Conference, Belgrade, 1965.

A study of the mathematical models currently in use may be divided under two main headings: (1) the deterministic, and (2) the stochastic. These can be further classified as one-sex or two-sex models depending on whether they consider only the female (or the male) population or whether they consider the male and the female populations together.

Among the deterministic models of one-sex, we may consider the life table as the first. But it takes into account only the deaths. Also, its interpretation as a stationary population model gained wider recognition with the work of Lotka who, in a way, extended it to formulate the stable population model of one sex using an integral equation which related births of one generation to the births of the preceding one. Feller continued the work of Lotka and employed the Laplace transform to solve the integral equation. Later, Coale gave a convenient method for finding the numerical solution of the dominant root of the integral equation.

The treatment of age and time as discrete variables instead of continuous as considered by Lotka, was examined by Bernardelli, Lewis and Leslie using matrix representation of the population projection model. Lopez and Keyfitz made extensive use of matrix algebra in this direction. Rogers employed it to analyse the population growth over spatial units. Keyfitz and Goodman studied both the continuous and the discrete approaches and showed that they can be reconciled. Goodman later gave an elementary approach to the

population projection model which avoided the use of the matrix algebra and brought out clearly the relationships between the population growth and the demographic variables - fertility and survival rates. Murphy generalized the stable population model by taking into account age and parity.

The inconsistencies that arose when the one-sex model was used separately for males and females, gave rise to the development of models which considered both the sexes simultaneously, viz. the two-sex models. Early work which could be classified under the two-sex model was done by Karmel and A.H. Pollard, and this was followed by Kendall, Yntema and Goodman. In studying the two-sex model, we are faced with the problem as to which sex the births should be related. If the births are related to the female population only, it is called female dominance; while if they are related to the male population only, it is called male dominance. On the other hand, if the births are related to both sexes in some way - for example, by taking a linear or a geometric or a harmonic average of the male and the female populations - it is called intermediate dominance. Though both the one-sex dominant methods and the intermediate dominance methods have been used by different authors, there appears to be no definite answer as yet to this problem. Here also, as in the one-sex models, age and time have been studied as continuous and discrete variables.

The models are called deterministic if no allowance is made for the chance variations in the components which are inherently

dependent on chance. Kendall, perhaps inspired by Bartlett, constructed stochastic versions of the one-sex and two-sex population models with time as a continuous variable. Goodman and Leslie followed him up. However, it was J.H. Pollard who studied general stochastic models with age and time as discrete variables. He employed the technique of matrix multiplications for this purpose. Recently, Thomas[6] has used probability generating functions to study these models; while Sykes[7] has examined, among others, a model which treats the projection matrix itself as a matrix random variable. These models, except some by Kendall, are one-sex models.

The study of the two-sex stochastic models appears to present serious difficulties. So far, only simple models which did not take age into account have been investigated. J.H. Pollard[8] has now constructed a two-sex age-specific stochastic population program incorporating marriage, and claims that it enables the demographer to investigate

> ... the effect on a population of a change in marriage rates, or divorce rates, or due to changes in economic conditions,

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6 V.J. Thomas, 'A Stochastic Population Model Related to Human Populations', Journal of the Royal Statistical Society, Series A, Vol. 132, No. 1, 1969, pp.89-104.

7 Z.M. Sykes, 'Some Stochastic Versions of the Matrix Model for Population Dynamics', Journal of the American Statistical Association, Vol. 64, No. 325, 1969, pp.111-130.

8 J.H. Pollard, 'A Discrete-time Two-sex Age-specific Stochastic Population Program Incorporating Marriage', Demography, Vol. 6, No. 2, 1969, pp.185-221. [p.186]

> or due to changes in government immigration policy, etc. It is possible with this model to carry out objective numerical investigations of such problems on digital computers.

However, there is need for more work in this direction.

The account of the deterministic and the stochastic models employed to study population change, given in this section, is admittedly very brief. The literature has grown very vast and has been well documented in the references cited earlier.

Both the deterministic and the stochastic models of varying complexity, have also been used in the study of the variations in fertility, mortality and migration themselves. Since the present study is concerned with the models representing population change, we have restricted our discussion to these models only.

1.4 NUMERICAL MODELS

Numerical model building is similar in nature to conducting scientific experiments. It enables us to evaluate the mathematical models and also allows for the introduction of considerable complexity into the models to make them more realistic. The numerical models are, generally, more restricted in nature and are applicable only to the situations represented in the set of data used in their construction. But,

> ... since calculations are strictly arithmetical, the results

> are necessarily dictated by the hypotheses adopted, and it is thus possible to reproduce a given situation or even, by comparing two models which differ in only one variable, to estimate the "weight" of that variable.[9]

It is this that has made the numerical models popular in demographic analyses. In fact, every population projection is a numerical model though it has not been the practice to name it so.

This technique has been employed by different researchers to investigate several problems in demography. In this section we shall give a brief account of some of these attempts.

As in Section 1.3 we may distinguish the one-sex and the two-sex models; the deterministic and the stochastic ones. The elementary form of the one-sex model is the stationary population model or the life table. This shows the resulting population structure if the survival probabilities observed in a certain period of time, or for a generation, remain unchanged and a constant number of births - equal to the radix of the life table, ℓ_0 - occurs every year. Though very unrealistic, the model has proved very useful in comparing the mortality conditions in different populations or at different time points and also in constructing more realistic models. These models are now constructed for all populations for which the basic data needed for their construction are available. Also, attempts have been made to

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[9] L. Tabah, 'Relationships between Age Structure, ...', 1965. [p.62]

produce hypothetical sets of these models by utilizing the relations observed in the populations for which reliable data were available.[10] Such models have become extremely handy in the analysis of the characteristics of the populations with deficient vital statistics.

The highly unrealistic nature of the stationary model was partly relaxed by the one-sex stable population model developed by Lotka. This model represents the population structure which results from the continuous operation of a constant set of survival and fertility rates over a long period of time. Hence the demographers turned their attention to the construction of the stable population models[11] and utilized them in analysing certain consequences of population growth,[12] in estimating the characteristics of the

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10 A.J. Coale and P. Demeny, Regional Model Life Tables and Stable Populations, Princeton University Press, Princeton, 1966.

L. Tabah, Poblaciones Modelos Estables, Cuasi-estables y en Trancision Demográphica, [Population Models: Stable, Quasi-stable and in Demographic Transition], Centro Latinoamericano de Demografia, Santiago de Chile, 1960.

United Nations, Age and Sex Patterns of Mortality, ST/SOA/Series A, Population Studies No. 22, United Nations, New York, 1955.

11 A.J. Coale and P. Demeny, Regional Model Life Tables ...., 1966.

L. Tabah, Poblaciones Modelos Estables ...., 1960.

United Nations, The Future Growth of World Population, ST/SOA/Series A, Population Studies No. 28, United Nations, New York, 1958.

12 J.J. Spengler, 'Aging of Populations: Mechanics, Historical Emergence, Impact', Law and Contemporary Problems (Problems of Aging), Vol. 27, No. 1, 1962, pp.2-21.

United Nations, The Aging of Populations and its Economic and Social Implications, ST/SOA/Series A, Population Studies No. 26, United Nations, New York, 1956.

populations for which data were either lacking or defective.[13] Keyfitz and Murphy,[14] and Keyfitz and Flieger[15] have constructed stationary and stable population models and have analysed the projection matrices from simple data on births and deaths for several countries.

Though many of the studies which used the stable population models considered only one sex, there were some studies which used the female dominant method to construct the two-sex stable population models.[16]

Other one-sex and two-sex numerical models have been employed by research workers to examine the consequences of the changes in

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13 A.J. Coale and E.M. Hoover, Population Growth and Economic Development in Low Income Countries, Princeton University Press, Princeton, N.J., 1958. (Appendix A)

M.A. El-Badry, 'Some Demographic Measurements for Egypt Based on the Stability of Census Age Distributions', Milbank Memorial Fund Quarterly, Vol. 33, No. 3, 1955, pp.268-305.

United Nations, Manual IV: Methods of Estimating Basic Demographic Measures from Incomplete Data, ST/SOA/Series A, Population Studies No. 42, United Nations, New York, 1967.

14 N. Keyfitz and E.M. Murphy, Comparative Demographic Computations, Population Research and Training Centre, University of Chicago, Chicago, 1964.

15 N. Keyfitz and W. Flieger, World Population - An Analysis of Vital Data, The University of Chicago Press, Chicago, 1968.

16 A.J. Coale and P. Demeny, Regional Model Life Tables ..., 1966.

D.V. Glass, Population Policies and Movement in Europe, Frank Cass & Co. Ltd., London, 1940. Reprinted in 1967. (Appendix)

mortality,[17] or fertility, or both,[18] and also to study the effects of the change in the age pattern of mortality[19] and fertility.[20]

Stochastic versions of the numerical models require the use of giant computers. Hence, there does not appear to be many studies which explored population change through empirical stochastic models. J.H. Pollard[21] conducted such an experiment using the data for the Australian female population of 1960, and obtained the expected size and its variance for the year 2210 and 2211. He has also performed

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17 K.G. Basavarajappa, 'Effect of Declines in Mortality on the Birth Rate and Related Measures', Population Studies, Vol. 16, No. 3, 1963, pp.237-256.

G.J. Stolnitz, 'Mortality Declines and Age Distribution', Milbank Memorial Fund Quarterly, Vol. 34, No. 2, 1956, pp.178-215.

18 Karl Schwarz,'Influence de la Natalité et de la Mortalité sur la Composition par Age de la Population et sur l'evolution Démographique', [Influence of Natality and Mortality on Age Composition of the Population and on the Demographic Evolution], Population, Vol. 23, No.1, 1968, pp.61-92.

F. Lorimer, 'Dynamics of Age Structure in a Population with Initially High Fertility and Mortality', United Nations Population Bulletin, No. 1, 1951, pp.31-41.

United Nations, The Determinants and Consequences of Population Trends, ST/SOA/Series A/17, United Nations, New York, 1953. (Chapter VII)

19 K.G. Basavarajappa, 'The Significance of Differences in Patterns of Mortality for Projections of Population', Demography, Vol. 5, No. 1, 1968, pp.185-191.

20 A.J. Coale and C.Y. Tye, 'The Significance of Age Patterns of Fertility in High Fertility Populations', Milbank Memorial Fund Quarterly, Vol. 39, No. 4, 1961, pp.631-646.

21 J.H. Pollard, 'On the Use of the Direct Matrix Product in Analysing Certain Population Models', Biometrika, Vol. 53, No. 3 & 4, 1966, pp.397-415.

such experiments including both males and females.[22]

Apart from these, the numerical stochastic models have been applied to study the variations in fertility[23] and mortality[24] themselves.

1.5 POPULATION MODELS INCLUDING MIGRATION

While considering mathematical models, migration has often been dispensed with by saying that emigration is similar to death and could be incorporated into the survival rates and immigration could well be dealt with by constructing 'presence ratios'.[25] Following the application of matrix algebra to study population growth, it has become possible to incorporate migration into the growth process without much difficulty.[26] However, the field is still relatively

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22 J.H. Pollard, 'A Discrete-time Two-sex Age-specific ...', Demography, 1969.

23 M.C. Sheps and E.B. Perrin, 'Changes in Birth Rates as a Function of Contraceptive Effectiveness: Some Applications of a Stochastic Model', American Journal of Public Health, Vol. 53, No. 7, 1963, pp.1031-1046.

24 N. Keyfitz, 'Une Table de Survie Européenne et sa Version Stochastique', [A European Life Table and its Stochastic Version], Population, Vol. 23, No. 1, 1968, pp.29-34

25 H. Hyrenius, 'Population Growth and Replacement', in P.M. Hauser and O.D. Duncan (ed.), The Study of Population (An Inventory and Appraisal), The University of Chicago Press, Chicago, 1959. Fifth Impression 1966, pp.472-485.

L. Tabah, 'Relationships between Age Structure ...', 1965, pp.57-58.

26 A. Lopez, Problems in Stable Population Theory, Office of Population Research, Princeton University, Princeton, N.J., 1961.

J.H. Pollard, 'On the Use of Direct Matrix Product ...', Biometrika, 1966.

A. Rogers, Matrix Analysis of Inter-regional Population Growth and Distribution, University of California Press, Berkeley, 1968.

unexplored. While commenting on this aspect, Tabah[27] remarked that

> ... many other models remain to be developed, introducing new variables and new hypotheses such as migration differentials according to sex, differences in fertility and mortality between the migrants and the receiving populations, migrant age-structure curves of different shapes, various possible fertility and mortality trends, etc.

The effect of migration on the population growth and on the age-sex structure in particular populations, has been studied. These could be counted as studies in numerical models. For example, Notestein[28] analysed the growth of the female population in the United States during 1930-55 with the help of numerical models, and tried to estimate the effect of migration on the growth and age structure of the female population, while Stone[29] examined 'in what respects and to what extent would its age distribution have differed from that actually observed' if Canada were closed to migration from 1851 to 1961. In Australia, where migration has played an important role in the

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27 L. Tabah, 'Relationships Between Age Structure ...', 1965, p.63.

28 F.W. Notestein, 'Mortality, Fertility, the Size-age Distribution, and the Growth Rate', in Demographic and Economic Change in Developed Countries, A Report of the National Bureau of Economic Research, Princeton University Press, Princeton, N.J., 1960, pp.261-284.

29 L.O. Stone, 'External Migration and the Age Structure of the Canadian Population, 1851-1961', Contributed Papers, International Union for the Scientific Study of Population, Sydney Conference, 1967, pp.775-785.

population growth, there appears to have been no detailed investigation of the long term effects of migration, though there are several studies which examined the contribution of migration to population growth during certain periods.[30]

Among the hypothetical experiments conducted using numerical models for investigating the effects of migration, two may be mentioned here. The first of these was a United Nations Study[31] in which a stationary population constructed on the basis of the French life table for 1933-38, was projected to study the effects of a continuous stream

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30 W.D. Borrie, Population Trends and Policies, Australian Publishing Company, Sydney, N.S.W., 1948. (Chapters 3 to 5)

W.D. Borrie, Immigration, Angus and Robertson, Sydney-London, 1949.

W.D. Borrie, 'The Growth of the Australian Population with Particular Reference to the Period since 1947', Population Studies, Vol. 13, No. 1, 1959, pp.4-18.

W.F. Geyl, Sr., 'A Brief History of Australian Immigration', International Migration, Vol. 1, No. 3, 1963, pp.157-166.

K. Jupp, 'Factors Affecting the Structure of the Australian Population, with Special Reference to the Period 1921-33', Thesis submitted for the Degree of Master of Arts in the Australian National University.

C.A. Price, 'Effects of Post-war Immigration on the Growth of Population, Ethnic Composition and Religious Structure of Australia, 1945-56', R.E.M.P. Bulletin, (Research Group for European Migration Problems), Vol. 7, No. 2, 1959, pp.48-56.

C.A. Price, 'Overseas Migration to and from Australia, 1947-1961', Australian Outlook, Vol. 16, No. 2, 1962, pp.160-174.

J. Zubrzycki, Immigration in Australia, a Demographic Survey Based on the 1954 Census, Melbourne University Press, 1960.

31 United Nations, 'Some Quantitative Aspects of the Aging of Western Populations', Population Bulletin of the United Nations, No. 1, 1951, pp.42-57.

of emigration and immigration on the population characteristics. It was assumed that migration occurred for a period of thirty years and then suddenly discontinued. In the case of emigration, the 'age-specific propensities to emigrate' (age-specific emigration rates) were computed on the assumption that emigration took place at an initial rate of 1 per cent of the total population per annum and the age distribution of the initial emigrants was that of the emigrants from Norway in the year 1900. These age-specific emigration rates were then combined with the life table survival rates. Using these modified survival rates, the populations alive at the end of the five year periods were obtained. On the other hand, when immigration was assumed, the annual number of immigrants amounting to 1 per cent of the total population was distributed by age according to the age distribution of emigrants from Norway in 1900, and was added to the surviving population every year. The immigrants were then projected on the assumption of the same constant mortality and fertility rates as in the receiving population. It was observed that in the case of emigration, the old-age dependency ratio increased greatly, and it took another thirty years after the emigration stopped for the ratio to return to its initial level. In the case of immigration, the child dependency started increasing after ten years and continued to increase until immigration came to an end. But the old-age dependency decreased during the period of immigration. A peak of aging was observed some thirty to forty years after the cessation of immigration. The study

also showed that, if a country had no population and was built up over a period of thirty years by immigration, the child dependency rises as the young immigrants start reproducing, and the old-age dependency remains small during the period of immigration. As immigration discontinues the child dependency falls off and the old-age dependency rises very rapidly.

The second study was conducted by Tabah and Cataldi.[32] They projected, over a period of 150 years, a female stable population corresponding to the mortality level approximately equal to that in the United Nations model life table with a life expectation at birth, ${}^{o}e_{0}$, of 30 years,[33] and a gross reproduction rate of 3. For one set of projections they assumed no migration. For another set, they assumed that immigration occurred every year according to the age-specific immigration rates calculated on the basis of the results of the Santiago Fertility Survey. The immigrants were incorporated into the population at the middle of each five year period and were then subjected to the same fertility and mortality rates as in the receiving population. This accounted for an average rate of immigration of 1 per cent per annum over the period. The whole period of 150 years was divided into three 50 year periods. In the first of these sub-periods, the fertility and survival rates were kept constant, while in the second only mortality

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32 L. Tabah and A. Cataldi, 'Effets d'une Immigration dans Quelques Populations Modèles' [The Effects of Immigration in a Few Model Populations], Population, Vol. 18, 1963, pp.683-696.

33 They have reported that the model life tables actually used were taken from L. Tabah, Poblaciones Modelos Estables ..., 1960.

was assumed to decline, and in the third both fertility and mortality were assumed to decline. Their study indicated that though the part played by immigration in increasing the size of the population was very significant, it did not produce marked changes in the age structure.

It may be noticed that in the two hypothetical examples presented here, two different procedures have been used to incorporate migration into the population. In the first procedure, the total number of migrants is obtained by multiplying the total population by an assumed overall migration rate. Then distribution of the migrants by age is obtained on the basis of an assumed proportionate age distribution. Finally, the migrants are added to the survivors of the population in the respective age groups and are treated as members of the population. This is how the immigrants were added into the population in the United Nations study.

In the second procedure, the numbers of migrants in the age groups, are calculated by multiplying the populations in the corresponding age groups at the beginning of a year, by assumed age-specific migration rates. The migrants are then added to the survivors of the population in the relevant age groups. If the age-specific migration rates are computed using the numbers of migrants alive in the respective age groups at the end of the year, they can be incorporated into the survival rates, as the emigration rates were incorporated in the United Nations study. Whereas, if the age-specific migration rates are obtained by using the migrants during the year, the migrants in the

respective age groups calculated as above, must then be survived to the end of the year on the assumption that they all came into the population at the mid-year, as was done by Tabah and Cataldi. We may also survive the migrants on the assumption that they are evenly distributed over the year in which they migrate, and over the year of their age.

1.6 THE PRESENT STUDY: ITS OBJECTIVES AND SCOPE

From the brief survey of literature given in Section 1.5, it is clear that research on streams of migration does not seem to have received the same attention as that on the other components of population change.[34] This may partly be due to the fact that often migration may not be a continuing force. But for theoretical completeness, it is necessary that this component be given the same attention as the others. Such theoretical expositions may reveal interesting information such as the implications of intended policy measures on migration, etc., which may be of considerable practical value.

When migration is included into the process of population growth, it is the age-specific net migration rates, calculated from the net numbers of migrants alive at the end of the year, that have been utilized in almost all theoretical investigations. So, they have been incorporated as additions to, or subtractions from, the survival rates.

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[34] United Nations, The Determinants and Consequences ..., 1953, p.138.

No new mathematical procedures have therefore been found necessary. This appears to be another reason why migration has not been given adequate attention in theoretical investigations.

But it may be noted that, if we use the age-sex-specific migration rates, the numbers of migrants in the respective age-sex groups are determined by the sizes of the populations in the corresponding age-sex groups. Hence, the total number of migrants as well as the proportionate age-sex distribution of migrants are determined not only by the age-sex-specific migration rates but also by the age-sex structure of the particular population.

For example, consider a female population which has the following sizes at specified ages at the beginning of a year: Age 1, P_1; Age 2, 0; and Age 3, P_3. Suppose the net immigration rates at these ages are a_1', a_2' and a_3', and are strictly positive. Then the number of immigrants alive at the end of the first year in which these rates operate, are: Age 2, $a_1' P_1$; Age 3, 0; and Age 4, $a_3' P_3$. The corresponding population figures are: Age 2, $(S_1 + a_1') P_1$; Age 3, $(S_2 + a_2') 0 = 0$; and Age 4, $(S_3 + a_3') P_3$, where S_x denotes the survival rates. Now if we continue the process for one more year using the same mortality and migration rates, we obtain the immigrants as: Age 3, $a_2' (S_1 + a_1') P_1$; Age 4, 0; and Age 5, $a_4' (S_3 + a_3') P_3$. The values in the population can be written down as before. It is easy now to see that the age composition of the surviving migrants is not the same for the first year and the second year. This is not because

we have assumed a different migration situation for the two years, but simply because the age distribution of the population has changed over the years. Thus, in this case, the overall net migration rate and the age composition of the net migrants are affected by the characteristics of the population and hence cannot be maintained as desired.

Therefore, by using the age-sex-specific net migration rates in the process of population growth, we cannot investigate questions such as: What would be the effect of a given rate of net migration on the growth and the age-sex distribution of the population? or, How would a given age-sex composition of net migrants affect the population growth and its age-sex structure?, etc. From a practical point of view such questions are important. For instance, government policies are often aimed at obtaining a specified number of migrants and/or to maintain a certain age-sex composition of net migrants, and it would be of interest to know their effects on the population characteristics.

As against this, if we use an overall migration rate and an age-sex composition of migrants to introduce migration into the process of population change, the numbers of migrants in the age-sex groups are determined not by the sizes of the populations in the corresponding age-sex groups, but by the total size of the population and the assumed age-sex composition of migrants. Hence it is possible, in this case, to investigate the consequences of a migration stream occurring at a given overall rate and having a certain age-sex composition of migrants. Thus, this procedure would be quite suitable

to investigate the type of questions raised above.

The present study, therefore, attempts a theoretical investigation into the effects of migration when it is specified by an overall net migration rate and an age-sex composition of net migrants at the time of migration. For convenience, throughout this study, the term net migrants is used for the net number of migrants. The case with no migration is used as the standard of reference to examine the effects of migration on the population characteristics. Though the results obtained by using the age-sex-specific net migration rates are used for comparison, no detailed discussion of this case is presented because the procedures that are applicable to the case with no migration, are directly applicable in this case also. Hence, unless it is specifically mentioned otherwise, the effect of migration in this study means the effect of migration when it is specified by an overall net migration rate and an age-sex composition of net migrants at the time of migration. But in the analysis of the population change in Australia, the actual net numbers of migrants by age and sex are utilized, since they are known from the observed statistics and need not be estimated.

It must be mentioned, at this stage, that a practical difficulty may arise when a net migration rate and an age-sex composition of net migrants are used to represent a migration situation, if one sex shows net emigration and the other net immigration. Similar difficulties may arise in interpreting the

percentage age distributions of net migrants when there is net emigration at some ages and net immigration at others. However, these do not affect the theoretical considerations or the numerical analyses but call for care in the interpretation of the age-sex composition of net migrants. If the net migration rate is zero, the sex composition and the age structures of net migrants have no meaning. But, there may be situations where a zero net migration is the result of a net immigration for one sex and a net emigration for another. This kind of situation may also arise in the case where the net immigration at some ages exactly balances the net emigration at others. Such situations are of a special nature and are rarely found in the actual populations. For instance, except in periods of war or economic stringency, or such other disturbances, it is observed that an immigration or emigration of males is accompanied by a similar movement of females. However, these difficulties could easily be overcome by considering immigration and emigration separately instead of taking net migration. This would not alter the procedures developed in this study, except for the fact that the number of variables would be increased because, wherever net migration rates and the age-sex compositions of net migrants are used, we must substitute the gross immigration rates and the age-sex compositions of the immigrants along with the corresponding details regarding the emigrants. Moreover, the effect of migration would, in any case, depend on the net effect of immigration and emigration on the population characteristics. Hence, we have retained the use of the net

migration rate and the age-sex composition of net migrants.

The study concentrates on the following problems:

(1) To examine the convergence of an arbitrary age-sex distribution to an equilibrium state age-sex distribution, defined below, under the operation of a constant set of fertility, mortality and migration schedules; and to study how the duration required for such a convergence changes due to the inclusion of migration.

(2) To investigate whether the weak ergodicity theorem which states that any two age-sex distributions, in course of time, become arbitrarily close to each other when subjected to identicial schedules of fertility and mortality rates, even if these rates vary over time, holds true in the presence of migration; and whether the presence of migration has any effect on the duration of this process of convergence.

(3) To study the characteristics - growth rate and age-sex distribution - of the equilibrium state population that results from a given set of fertility, mortality and migration schedules, and to examine the growth rates and the age-sex distributions resulting from the repeated operation of a sequence of k schedules of fertility, mortality and migration.

(4) To decompose the observed changes in the growth and age-sex distribution of a population during a certain period into the effects of the changes in the components during that period and those of the interactions of these changes. The actual population changes analysed

are those of Australia during 1911-66. The population of Australia was affected by considerable migration during this period, and there were large variations in fertility as well.

An equilibrium state population is defined here as a population which has an unchanging age-sex structure and grows at a constant rate under the operation of a specified set of fertility, mortality and migration schedules that remain constant over time. It has constant birth, death and net migration rates. Thus a stable population which is well known in the literature, is an equilibrium state population under the operation of a given set of fertility and mortality schedules but with zero net migration. So, all stable populations are equilibrium state populations, whereas all equilibrium state populations are not stable populations. The unchanging age-sex distribution of the equilibrium state population is called the equilibrium state age-sex distribution, and the constant growth rate as the intrinsic growth rate.

The analysis, in this study, is carried through the use of the deterministic models of population change, of one sex and two sexes. In the case of the two-sex model, though both the female dominance case and the equal dominance case are considered, the former is given greater emphasis because it is the one often used in practice. Hence, unless specifically mentioned, it is assumed that the female dominant method is used.

In order to make the analysis more realistic, we keep the

study of the two-sex model as our ultimate objective. But we use the one-sex model for the sake of clarity and convenience of argument. Thus, we begin our exposition with a one-sex model with no migration, and then introduce migration by assuming an overall net migration rate and an age-sex composition of net migrants at the time of migration. We conclude by extending the results to the two-sex case with female dominance and with equal dominance, and presenting the results of some numerical analyses using the two-sex model.

For the numerical illustrations, we have made use of the demographic data observed in Australia during the period 1911-66 and some hypothetical data prepared on the basis of certain assumptions. The population of Australia was used because Australia is one of the few countries where migration is an important factor of population growth and for which the required data on the components, including those on migration, are readily available and are reliable for all practical purposes. The following were the reasons for choosing the period 1911-66. The period had witnessed two wars - the First and the Second World Wars; an epidemic - the influenza epidemic of 1919; and a severe economic depression - that of the early nineteen thirties. Consequently, large variations in the components had occurred during this period, except in mortality which was already comparatively low by 1911. Further, except for the first decade, i.e. 1911-20, the data required were available in fair details for almost the whole of the period.

1.7 NUMERICAL DATA USED IN THE STUDY

The data required for the construction of the numerical examples in the study, were: (a) some populations having significantly different initial age-sex distributions - for testing the convergence of age-sex distributions; (b) survival rates by single years of age for males and females; (c) fertility rates by single years of age of females (and of males for some models); (d) net numbersof migrants by age and sex, the overall net migration rates and the age-sex compositions of net migrants at the time of migration, and the age-sex-specific net migration rates; and (e) the sex ratio at birth. We shall present in this section a brief description of the data mentioned above. Their main characteristics will be discussed in the next chapter.

(a) The Initial Populations

Three populations were used in the numerical illustrations. These were identified as the 1911 obs. population, the 1911 stb. population and the 1966 obs. population. The sizes of these populations were assumed to be the same at the initial point of time and were taken to be equal to the size of the population of Australia as at 30 June 1911. Their age-sex distributions were assumed to be different. The age-sex distribution of the 1911 obs. population was taken as the graduated age-sex distribution observed in the 1911 census of Australia, while the age-sex distribution of the 1911 stb. population was computed by the female dominant method using the

fertility and survival rates by single years of age for the year 1911, with a sex ratio at birth of 105.22 which was the average value for the three years 1910-12. On the other hand, the age-sex distribution obtained by projecting the 1911 obs. population to 1966 with the fertility, mortality and migration schedules as observed during 1911-66 and with a sex ratio of 105, was taken as the 1966 obs. population age-sex distribution. It may be seen from Appendix C that the age-sex distribution of the projected population in 1966 was not significantly different from that of the population enumerated in the 1966 census.

The size of the Australian population was used for convenience and in order to facilitate the analysis of the population change in Australia during 1911-66. The results of the study, except those presented in Chapter 6, would not have been different if we had used a different size of the initial population.

(b) The Survival Rates

In this study, the mortality situation was taken to be represented by the survival rates by single years of age of males and females. In Australia, these were available for the census years 1921-66 from the official life tables of 1920-22, 1932-34, 1946-48, 1953-55, 1960-62, and from the life table of 1965-67 constructed for the present study. The details of the procedures used for the construction of this life table are given in Appendix B. For the census year 1911, the survival rates were obtained using the L_x values

linearly interpolated from the official life tables of 1901-10 and 1920-22. The survival rates for each of the intercensal years were also computed using the L_x values linearly interpolated from the life tables at the census years. Thus the complete set of survival rates for the years 1911-66 was obtained and used in the study. No hypothetical survival rates were prepared.

(c) The Fertility Rates

Age-specific birth rates by single years of age of females and for some models by single years of age of males, were used in the study. For the period 1921-66, the rates for females were provided by the Commonwealth Bureau of Census and Statistics, while for the decade 1911-20, they were obtained from the published data on births and the estimated population by single years of age. The procedure adopted is discussed in Appendix A. Some hypothetical fertility rates were also used along with these observed ones. The fertility rates by single years of age of males were computed only for the years 1911 and 1966.

(d) The Migration Data

In Australia, the basic data on migration, viz. the numbers of arrivals and departures by sex and age, are recorded in fair detail in recent years. But, for the earlier years, only total numbers of arrivals and departures by sex were published. Hence, for the period 1911-20, the age distributions of the male and female arrivals and departures had to be estimated. Also, for the years 1921-23, the first half of 1924 and for some of the years during the Second World War, these details were not available. For many of the

years, the age distributions were published in 5 year age groups. However, a complete series of arrivals and departures by single years of age and sex for all the years from 1921 to 1966, was prepared by Brown[35] using the published and unpublished information given by the Bureau of Census and Statistics. This series was utilized in the study, and a separate estimation was done only for the period 1911-20. The estimation procedure is given in Appendix A.

These data on arrivals and departures were used to compute: the net migration rates for each of the financial years with the population at the beginning of the year as the base; the sex ratio among net migrants; the age distributions for male and female net migrants; and the age-sex-specific net migration rates. These, along with certain hypothetical data on the net migration rates and on the age-sex compositions of net migrants, were used in the numerical illustrations. However, for estimating the effects of the changes in the components - fertility, mortality and migration - on the growth and the changes in the age-sex structure of the population of Australia during 1911-66 (see Chapter 6), the actual net numbers of migrants by age and sex were used.

(e) The Sex Ratio at Birth

This was assumed to be 105 males per 100 females in all the

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35 H.P. Brown, Department of Economics, Australian National University, Canberra, has prepared a Demographic Data Bank for Australia for the years 1921 onwards.

numerical illustrations and in the analysis of population change in Australia.

1.8 ASSUMPTIONS AND LIMITATIONS

The general assumptions on which the investigations rest, are as follows:

(1) Age x and time t are discrete variables taking values 0, 1, 2, ...

(2) w is the maximum age beyond which no person, male or female, ever lives. We assume that w does not change with time.

(3) f(x,t) is the age-specific fertility rate at age x years defined as the ratio of the number of births occurring to women aged x years during a year to the mid-year female population at that age. These are assumed to be positive in the range α to β years, and are zero at other ages. For theoretical purposes, it is enough if this age range consists of two fixed consecutive ages.

(4) S(b,t) and S(x,t), (x = 0, 1, 2, ..., (w-1)) are, respectively, the proportion of births during a year t surviving to the end of the year and the survival rate for persons aged x years at time t to become aged (x+1) years at time (t+1). These are assumed to be positive and bounded above.

(5) n(t) is the net migration rate defined as the ratio of the net number of migrants during a year t, to the total population at the beginning of the year. It is finite and is not equal to zero; but may be positive or negative. If it is equal to zero, the situation is assumed to be identical with the case where there is no migration. If

it is positive, we call it immigration and if negative, emigration. Whenever n(t) is not equal to zero, it is associated with an age composition of net migrants at the time of arrival or departure. n(x,t) denotes the proportion at age x years in the age composition of net migrants at the time of migration. It is finite and may be positive, negative or zero; but the sum of n(x,t) over all values of x, is equal to 1, for all t.

When age-sex-specific net migration rates are used to specify the migration conditions, we define u'(t), the net migration rate for the cohort born during a year t, as the ratio of the net number of migrants aged 0 years at the end of a year to the number of births during that year;[36] and a'(x,t), the net migration rate at age x years, as the ratio of the net number of migrants surviving at age (x+1) years at the end of a year t to the population aged x years at the beginning of that year. These are assumed to be finite and could be positive, negative or zero.

(6) Once they are in the population, the fertility and survival rates in the general population apply to the migrants as well. Further, the fertility and survival rates are not changed by the occurrence of migration.

(7) The flow of migration, namely the arrival and departure, is even

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[36] Rogers [A. Rogers, Matrix Analysis of ..., 1968] had not explicitly taken account of this direct effect of migration. Stone [L.O. Stone, 'Stable Migration Rates ...', Demography, 1968] introduced a correction by assuming that the migrant females would give birth for the full year during which they migrate. The definition adopted here seems to be more satisfactory in the sense that it explicitly takes account of the direct effect of net migration on the age cohort born during the year.

throughout the year and the migrants are uniformly distributed over the years of their age at arrival or departure. Thus, the migrants entering or leaving the population are assumed to experience the fertility and mortality of the population only for half of the year in which they migrate, and one half of the survivors remains at the same age while the other half joins the next higher age.

(8) Whenever there is emigration, the net migration rate and the age-sex composition of the net migrants are such that no age cohort becomes negative at the end of any year. This restriction applies to the age-sex-specific net migration rates also. In practice, this is not really a restriction because there cannot be more emigrants than there are people in any age group.

(9) The changes in the components - fertility, mortality and migration - occur independently. For example, even when net immigration as large as 5 per cent occurs every year, it will not alter the prevailing conditions of fertility and mortality; or even when fertility declines significantly, mortality and migration remain unchanged; etc.

(10) The sex ratio at birth is independent of t, the time and x, the age of the person.

Among the assumptions made above, (6) and (9) need some justification. Assumption (6) implies that the fertility and mortality rates do not differ significantly between migrant and non-migrant populations. Investigations in this direction have shown that, though

differences do exist between these two groups, they are not very significant.[37] It has also been observed that, generally, migrants tend to acquire the rates of the populations to which they migrate;[38] and the rates pertaining to migrants actually remain intermediate between those of the sending and the receiving population.[39] Some numerical data required to examine this assumption were available for Australia. These have been analysed, and the results will be discussed briefly here.

Figure 1.1 presents the ratios of the survival rates of those born in Australia to the survival rates of those born outside Australia at the respective census dates. The relevant data were not available for the census years 1947 and 1954. For computing the life tables for

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37 K.G. Basavarajappa, 'Trends in Fertility in Australia, 1911-61', A Thesis submitted for the Ph.D. Degree in the Australian National University, Canberra, 1964, p.103.

W.S. Bernard, American Immigration Policy - a Reappraisal, Harper and Brothers, New York, 1950, pp.131-137.

W.D. Borrie, Population Trends and ..., 1948, Chapter 9.

L. Tabah and A. Cataldi, 'Effets d'une Immigration ...', Population, 1963.

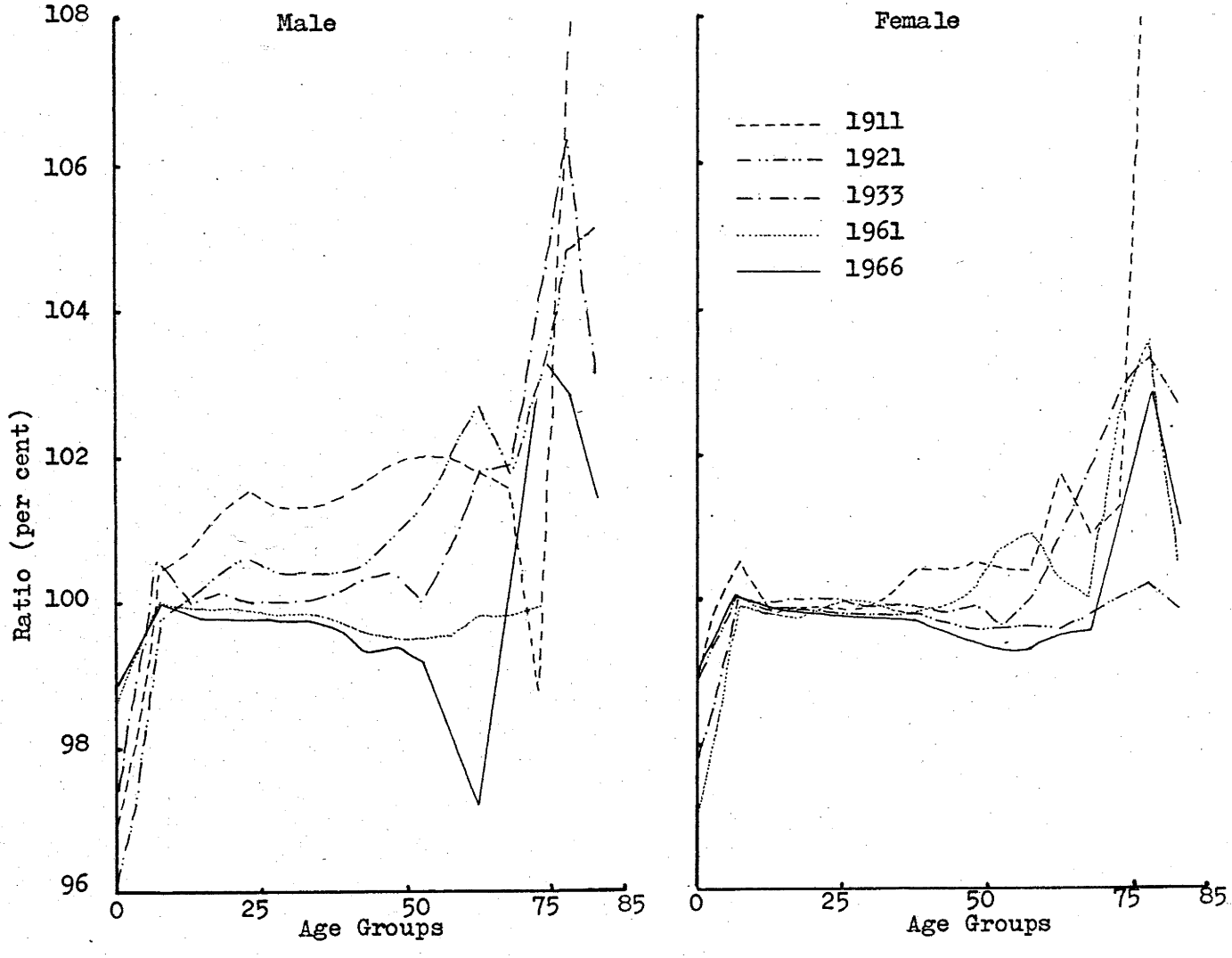
United Nations, The Determinants and ..., 1953, pp.138-140.

38 F. Lorimer and F. Osborn, Dynamics of Population, The MacMillan Co., New York, 1934, p.54.

United Nations, The Determinants and ..., 1953, pp.138-139.

39 E.S. Lee, 'A Theory of Migration', Demography, Vol. 3, No. 1, 1966, pp.47-57.

FIGURE 1.1: RATIO OF THE SURVIVAL RATES FOR AUSTRALIAN BORN TO THOSE FOR OVERSEAS BORN IN 5 YEAR AGE GROUPS FROM ABRIDGED LIFE TABLES: CENSUS YEARS



Note: Relevant data were not available for 1947 and 1954.

those born in Australia and born outside, we have used the average number of deaths in 5 year age groups during the three calendar years around the census dates and the respective population data from the census publications. Since the 1911 and 1921 censuses were taken in April, the respective populations had to be estimated as at 30 June. The total numbers of males and females as at 30 June were taken from the estimates published by the Bureau of Census and Statistics. By multiplying these numbers by the proportions of the Australian born observed in the censuses, the numbers of the Australian born were obtained. Then the distributions of the total numbers and the numbers of Australian born into 5 year age groups, were computed using the respective age structures observed in the censuses. Finally, the populations born outside Australia were obtained by subtracting the Australian born groups from the total populations age group by age group. Thus, the outside born group included the population of unknown birth place. Similarly, among the deaths to outside born also, this group was included. This is justifiable, to some extent, because there is more likelihood of birth places of persons born outside being not known than that of persons born in Australia. In constructing the abridged life tables with 5 year age groups, the following approximate formulaswere used:

$$\text{Probability of survival} = 1 - q(x) = \frac{2 - n\,{}_nm_x}{2 + n\,{}_nm_x} \qquad (1.1)$$

where ${}_nm_x$ is the death rate for the age group [x, (x+n)],

$$L(0\text{-}4) = 2\,\ell_0 + 3\,\ell_5$$

$$L(x,x+4) = 2.5\,[\ell_x + \ell_{x+5}], \quad \text{for } x = 5, 10, \ldots, 80$$

and $$L(85+) = \ell_{85} \log \ell_{85} \qquad (1.2)$$

The survival rates were then computed in the usual manner.

The abridged life tables were also constructed for the total population using the same approximate formulas and the ${}^0e_x$ values from the abridged life tables were compared with those given in the official complete life tables. The differences were found to be negligible. The comparison of the survival rates, instead of the death rates as is often done in such studies, was resorted to because it is the differences in the former that are important for the purpose of this study.

The figure (i.e. Fig. 1.1) reveals that the differences for males were slightly larger than for females. In the case of females, the differences were almost negligible while in the case of males they were fairly small, except in the youngest age group (0-4) and in the old age groups (65-69), etc. The differences in these age groups might have arisen partly due to the fact that the proportion of the outside born population in the younger age groups was very small while that in the older age groups was very high when compared with the respective proportions of the Australian born population (see Table 1.1).

A similar procedure to the above could not be used for the

TABLE 1.1

AGE-SEX STRUCTURE OF THE POPULATION BORN IN AUSTRALIA AND BORN OVERSEAS: CENSUS YEARS

Census Year		Age Groups 0-14	15-64	65+	Total	0-14	15-64	65+	Total	Proportion Born in Australia
		Born in Australia				Born Overseas				
1911	Male	37.90	61.24	0.86	100.00	3.17	78.86	17.97	100.00	79.67
	Female	37.33	61.75	0.92	100.00	4.24	72.27	23.49	100.00	85.44
	Sex Ratio	102.48	100.12	95.09	100.95	113.21	164.92	115.63	151.14	-
1921	Male	37.30	60.88	1.82	100.00	5.83	77.31	16.86	100.00	82.14
	Female	35.73	62.31	1.96	100.00	7.43	72.88	19.69	100.00	86.16
	Sex Ratio	102.82	96.22	91.25	98.48	104.51	141.45	114.18	133.34	-
1933	Male	31.88	63.65	4.47	100.00	3.62	79.55	16.83	100.00	84.59
	Female	30.46	64.50	5.04	100.00	4.67	77.12	18.21	100.00	88.22
	Sex Ratio	103.59	97.66	87.93	98.96	104.63	139.21	124.69	134.96	-
1947	Male	28.60	65.47	5.93	100.00	0.22	80.14	19.64	100.00	89.02
	Female	26.94	65.78	7.28	100.00	0.20	76.42	23.38	100.00	91.35
	Sex Ratio	103.89	97.38	79.72	97.84	137.20	133.69	107.08	127.48	-
1954	Male	32.31	61.24	6.45	100.00	10.63	77.17	12.20	100.00	83.86
	Female	30.41	61.28	8.31	100.00	12.95	71.22	15.83	100.00	87.55
	Sex Ratio	104.20	98.01	76.04	98.07	108.87	143.82	102.25	132.72	-
1961	Male	34.89	58.38	6.73	100.00	11.87	78.69	9.44	100.00	81.42
	Female	32.74	57.97	9.29	100.00	13.73	73.51	12.76	100.00	84.77
	Sex Ratio	104.64	98.89	71.12	98.20	107.84	133.52	92.27	124.73	-
1966	Male	34.64	58.68	6.68	100.00	10.62	80.61	8.77	100.00	80.17
	Female	32.39	58.01	9.60	100.00	11.65	76.34	12.01	100.00	82.95
	Sex Ratio	104.83	99.17	48.24	98.04	107.52	124.54	76.08	117.94	-

[Source: Computed from the population data in the respective censuses of the Commonwealth of Australia]

comparison of the fertility rates because, although the total number of confinements occurring to overseas born and Australian born females were available, they were not classified by the age of females. Hence, an indirect method was used. The age-specific confinement rates in 5 year age groups for the females in the total population were computed using the average number of confinements during the three years around the census dates and the female populations in the respective censuses - the census data being adjusted as described earlier where necessary. By applying these rates to the female populations born in Australia and born outside, the expected numbers of confinements were calculated. These were compared with the observed numbers of confinements among the Australian born and outside born females. A similar procedure was applied in the case of married females when the required data were available. The results are presented in Table 1.2.

The table shows that the females born outside Australia had lower fertility than those born in Australia, except during 1920-22 and 1965-66. A similar observation was made by Borrie.[40] Since the population in the reproductive ages was dominated by the females born in Australia in all the censuses, the expected numbers of confinements among those born in Australia were very close to the observed numbers of confinements. Even in the case of those born outside, the differences did not exceed more than 6 per cent except for the period

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40 W.D. Borrie, Population Trends and ..., 1948, p.124.

TABLE 1.2

RATIO (PER CENT) OF OBSERVED TO EXPECTED CONFINEMENTS FOR FEMALES BORN IN AUSTRALIA AND BORN OVERSEAS: AROUND CENSUS YEARS

Years	All Women		Married Women	
	Aust. Born	Overseas	Aust. Born	Overseas
1910-12	101.44	89.59	NA	NA
1920-22	99.28	105.34	101.44	91.81
1932-34	99.97	100.30	NA	NA
1946-48	100.40	94.19	NT	NT
1953-55	100.83	94.12	101.47	90.21
1960-62	101.40	94.04	NT	NT
1965-67	98.93	103.73	100.12	99.64

Note:- NA - The distribution by age of the married females was not available.
NT - The distribution by marital status and birth place was not available.
[Source: Commonwealth Bureau of Census and Statistics: Demography Bulletin and respective Census volumes. For 1965-67 unpublished but made available by the Bureau.]

1910-12. The large difference for 1910-12 might have arisen due to the fact that there was heavy immigration during the period and there is likely to be some time lag between the time of arrival of immigrants and the time they start reproducing. The comparison of the nuptial confinements, given in the table, reveals that, when the age-sex composition of the outside born population was not dominated very much

TABLE 1.3

AVERAGE ISSUE OF EXISTING MARRIAGE OF WIVES AGED (45-49) YEARS AT THE CENSUS DATE: AUSTRALIA, 1911-66

Census Year	Born in Australia	Total Population	Post-World War II Migrants
1911	5.33*	5.25	-
1921	4.13	4.02	-
1933	NA	NA	-
1947	NA	2.77	-
1954	2.47	2.43	2.18
1961	2.52	2.50	2.44
1966	NA	2.66	NA

Note:- NA - Not available.
* Based on all marriages.
[Source: Commonwealth Bureau of Census and Statistics, Census of the Commonwealth of Australia:
1911 Census Vol. III p.1160;
1921 Census Vol. II p.1966-67;
1947 Census Vol. III - Statistician's Report p.322;
1954 Census Vol. VIII - Statistician's Report p.327,331;
1961 Census Vol. VIII - Statistician's Report p.396;
1966 Unpublished - made available by the Bureau.]

by males, as for instance in 1965-67, the differences became negligible. However, the available series was insufficient to draw any firm conclusions. On the other hand, the comparison of the average number of issues per existing marriage, given in Table 1.3, suggests that, though the outside born had a slightly smaller completed family

size, the difference was not very significant. On the whole, assumption (6) does not seem to be as unsatisfactory as it appears at first glance.

Now, we examine the validity of assumption (9). There is no doubt that the components of population change vary concomitantly to some extent. But, speaking less rigorously, declines in mortality are generally associated with the efforts of the institutions - for instance, public health and sanitation improvements, new discoveries in the field of medicine, etc.; while changes in fertility are dependent more on the individual behaviour - such as the use or non-use of family planning or family limitation methods, etc. Migration, on the other hand, is very much conditioned by the political and economic conditions in the country and by the policies of the ruling government.

In Australia, for example, mortality declined continuously during the period 1911-66 [see Section 2.3.1]; whereas fertility declined, increased and again showed a downward trend [see Section 2.3.2]; and net migration was highly sensitive to the measures adopted by the government such as the granting of assistance to migrants, and the political and economic situation in the country [see Section 2.3.3]. We should not, however, conclude that the factors effecting changes in one component will not cause any change in others. For instance, an economic depression such as the one during the early nineteen thirties would cause not only a reduction in migration but also a decline in

fertility rates due to the postponement of marriages and births.[41] But the degree of dependence seems to vary.

Further, when the population is affected by migration, the fertility and survival rates may change due not only to the differences in these rates between the migrant population and the rest of the population, but also to the indirect effects such as the increase (or decrease) in the number of eligible brides or grooms, or the re-union or separation of spouses, or the changes in the living conditions, or the changes in the social values, etc.

It is difficult to assess the impact of these indirect effects on the fertility and mortality rates because of the lack of relevant data and also because such an assessment would probably involve a very detailed examination of several related aspects. However, it can be inferred that some of these effects will act in opposite directions and hence, the resulting net changes in the fertility and mortality rates are not likely to be significant.[42]

1.9 PLAN OF THE STUDY

Having given the objectives and the conditions assumed, we give here an outline of the study. In Chapter 2, we present the

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41 K.G. Basavarajappa, 'Trends in Fertility ...', Ph.D. Thesis, 1964, (Chapter 11).

P.K. Whelpton, Cohort Fertility, Princeton University Press, Princeton, N.J., 1954, (Chapter 6).

42 International Labour Organization, International Migration, 1945-1957, Studies and Reports, New Series No. 54, I.L.O., Geneva, 1959, pp.332-333.

United Nations, Determinants and Consequences ..., 1953, p.139.

results of an analysis of the fertility, mortality and migration data for Australia for the period 1911-66, those derived from them, and the hypothetical data, with a view that this would aid the understanding of the outcome of the numerical illustrations in the study. Then the main results of the study are presented in Parts II and III.

Part II consists of Chapters 3 and 4, and is devoted to a discussion of the convergence of age-sex distributions under constant conditions and under changing conditions of fertility, mortality and migration. From the stable population theory, it is known that, when an arbitrary age-sex distribution of a closed population is subjected to unchanging schedules of fertility and mortality rates for a sufficiently long period of time, it converges to a stable age-sex structure which is determined entirely by the known schedules of fertility and mortality rates. An examination of whether such a convergence occurs if a constant stream of migration is introduced into the process of population growth and how the time required for the convergence, which is called the duration of convergence, is affected due to the inclusion of migration, is the chief concern of Chapter 3. On the other hand, when the schedules of fertility, mortality and migration are changing over time, no fixed age-sex structure or a constant growth rate, is evolved. But it is known from the weak ergodicity theorem that, in an arbitrary closed population, the initial shape of the age-sex distribution is 'forgotten', in course of time, and the resulting age-sex distribution depends completely on the

history of the fertility and mortality rates. In Chapter 4, it is proved that this theorem holds good even when migration is included in the process of population growth. The changes in the duration of convergence due to the presence of migration are also analysed. Throughout Part II, matrix multiplication is used as the main technique of analysis.

Part III includes Chapters 5 and 6, and examines the population change under constant and changing conditions of fertility, mortality and migration. When the conditions are constant over time, it is shown in Chapter 3 that a unique equilibrium state population is ultimately established. Though the intrinsic growth rate and the age-sex distribution of this equilibrium state population could be computed from matrix methods, the relationship between these characteristics of the equilibrium state population on the one hand and the fertility, mortality and migration schedules on the other, will not, in that case, be shown explicitly. The availability of explicit expressions to obtain these characteristics is an advantage. This is accomplished in Chapter 5 through elementary methods using purely demographic considerations. The chapter also examines the age-sex distributions and the growth rates resulting from the repeated operation of a set of k schedules of fertility, mortality and migration conditions, which are called by Namboodiri[43] as the cyclical model of

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[43] N.K. Namboodiri, 'On the dependence of Age Structures on a Sequence of Mortality and Fertility Schedules: An Exposition of a Cyclical Model of Population Change', Demography, Vol. 6, No. 3, 1969, pp.287-299.

population change in the case of the closed populations. If the fertility, mortality and migration schedules are changing over time, no definite age-sex distribution or a fixed growth rate is evolved (see Chapter 4). But both are modified during each period of time. Therefore, we can only examine how these changes in the age-sex distribution and the growth rate are caused by the variations in the fertility, mortality and migration schedules during a certain period of time. This means that we must carry out the analysis for several populations which have experienced diverse changes in these schedules and for different time periods. Obviously, it is not easy to undertake such an exhaustive investigation. Hence in Chapter 6, we propose a method of analysis, called the factorial projections method, to decompose the observed changes in the population characteristics into the effects of the changes in the components and the effects of the interactions of these changes. Using the proposed method, we analyse only the changes in the characteristics of the population of Australia during three time periods 1911-66, 1933-66 and 1947-66.

We conclude our presentation with a summary of findings in Part IV.

Throughout the study, we use the terms age-sex distribution, age-sex composition and age-sex structure to denote the proportionate age-sex distribution.

1.10 CONCLUDING REMARKS

Before closing this chapter, we may mention that it may be possible to define demographic processes with a given growth rate and given fertility and mortality schedules assuming some age pattern for migrants; or with a given crude birth rate and given mortality and migration schedules as specified in this study; etc., which would lead to equilibrium state populations, in analogy with the processes studied by the United Nations[44] to obtain Malthusian populations. In each case, however, we must examine whether the defined process is determinate or not in the sense that, given the assumptions and an arbitrary initial population, it is always possible to compute a projection of the population for any time.[45] We shall confine in this study to the cases where fertility and mortality schedules are known and migration is specified either by a known net migration rate and an age-sex composition of net migrants, or by a known set of age-sex-specific net migration rates. These give rise to equilibrium state populations which are similar in nature to the well known stable populations. The processes as defined in the present study are always determinate under the conditions assumed here.

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44 United Nations, The Concept of a Stable Population - Application to the Study of Populations of Countries with Incomplete Demographic Statistics, ST/SOA/Series A/39, United Nations, New York, 1968.

45 United Nations, The Concept of a Stable Population ..., 1968, p.45.

Our attempt in this chapter has been to give a brief account of how the population projection models have been used to study population change and to indicate the purpose for which they are used in this study. We have also given the assumptions involved and have indicated the limitations.

CHAPTER 2

SOME CHARACTERISTICS OF THE DATA USED IN THE NUMERICAL ILLUSTRATIONS

2.1 INTRODUCTION

Before embarking on the theoretical investigations, it is considered worthwhile to analyse the characteristics of the basic data used in the numerical illustrations because the results that are obtained from the numerical models depend on the nature of the changes in the basic data used in their construction. With this in view, we present in this chapter a comparison of the age-sex distributions of the initial populations and an examination of the changes in the survival rates, fertility rates and migration data observed in Australia during 1911-66 and indicate briefly the nature of the other hypothetical data derived for the study.

2.2 AGE-SEX DISTRIBUTIONS OF THE INITIAL POPULATIONS

Comparison of the age-sex distributions of any two populations could be done in two ways: (1) by computing the percentages of the populations in the age-sex groups to the total population and then comparing these percentages for the two populations; or (2) by computing the percentage age distributions for males and females separately and then comparing these age distributions in the two populations along with a comparison of the sex ratios in the same age groups in the two populations. The first

procedure may be called the joint analysis and the second the two-sex analysis. The latter is used in this study since it seems to give a better picture of the sex compositions and the age distributions in the populations.

Figure 2.1 presents the age distributions in 5 year age groups separately for males and females, and Figure 2.2 the sex ratios in those age groups. Though single year age distributions were used in the study, the percentages in 5 year age groups, are presented for convenience. In fact, in all the numerical illustrations the age distributions in 5 year age groups were used for comparison.

The curves of the age distributions for males and females in the 1911 obs. and 1966 obs. populations compared with the smooth curves of those in the 1911 stb. population, reveal the impact of the demographic history of Australia on the age distributions. Immigration into Australia, which had virtually ceased since 1891, started gaining force from 1909 and there was comparatively heavy immigration during 1910-11. The birth rate which was declining during that period, also showed an increasing trend during a few years prior to the census of 1911.[1] The effect of these events on the male and female age distributions was to make the proportions in the younger and the middle age groups higher; and consequently, those in the older ages lower than in the 1911 stb. population. The very high sex ratios in the 1911 obs.

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[1] Commonwealth Bureau of Census and Statistics, Demography, Bulletin No. 83, 1965, pp.132 & 137.

FIGURE 2.1: COMPARISON OF THE PERCENTAGE AGE DISTRIBUTIONS OF THE INITIAL POPULATIONS IN 5 YEAR AGE GROUPS

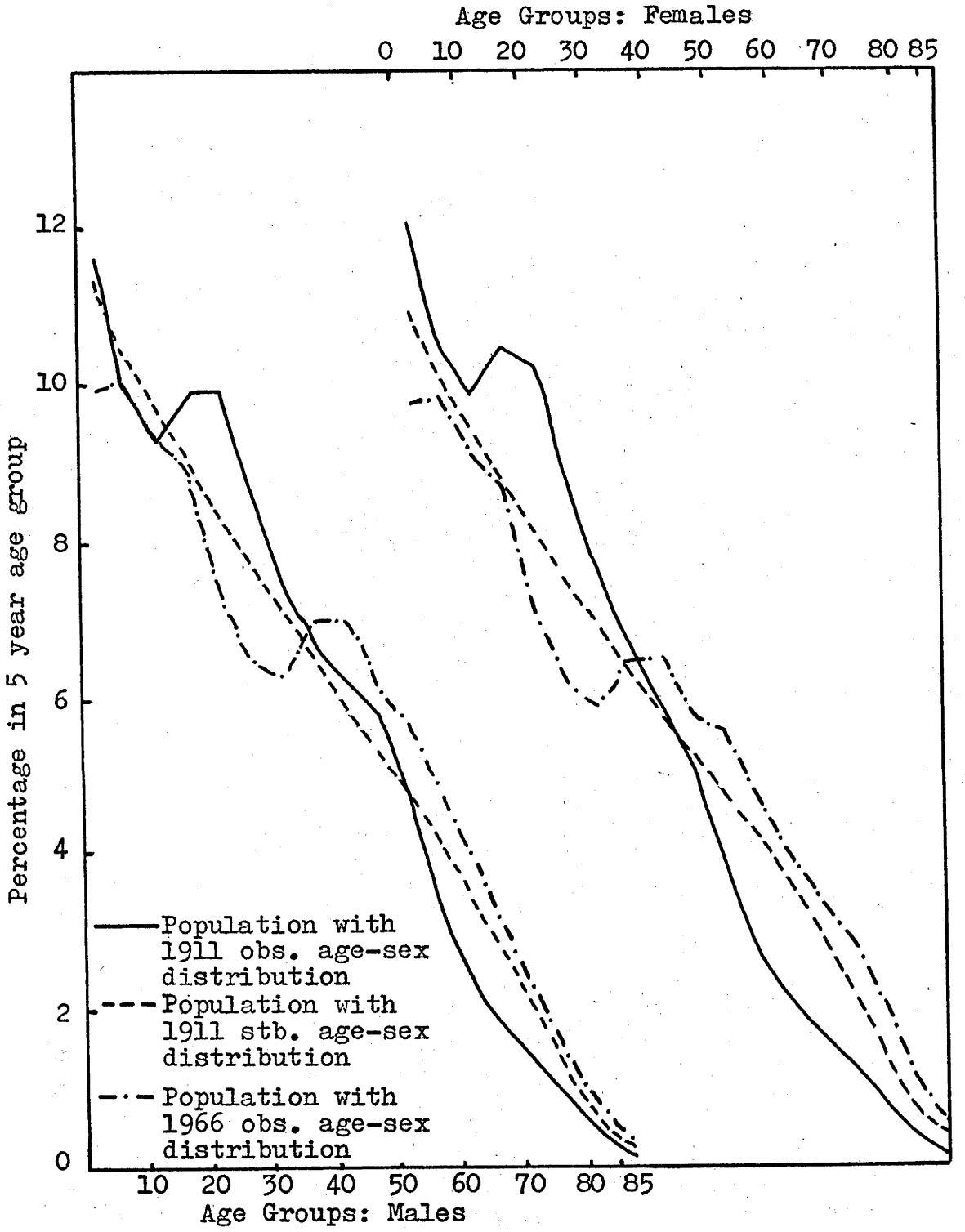
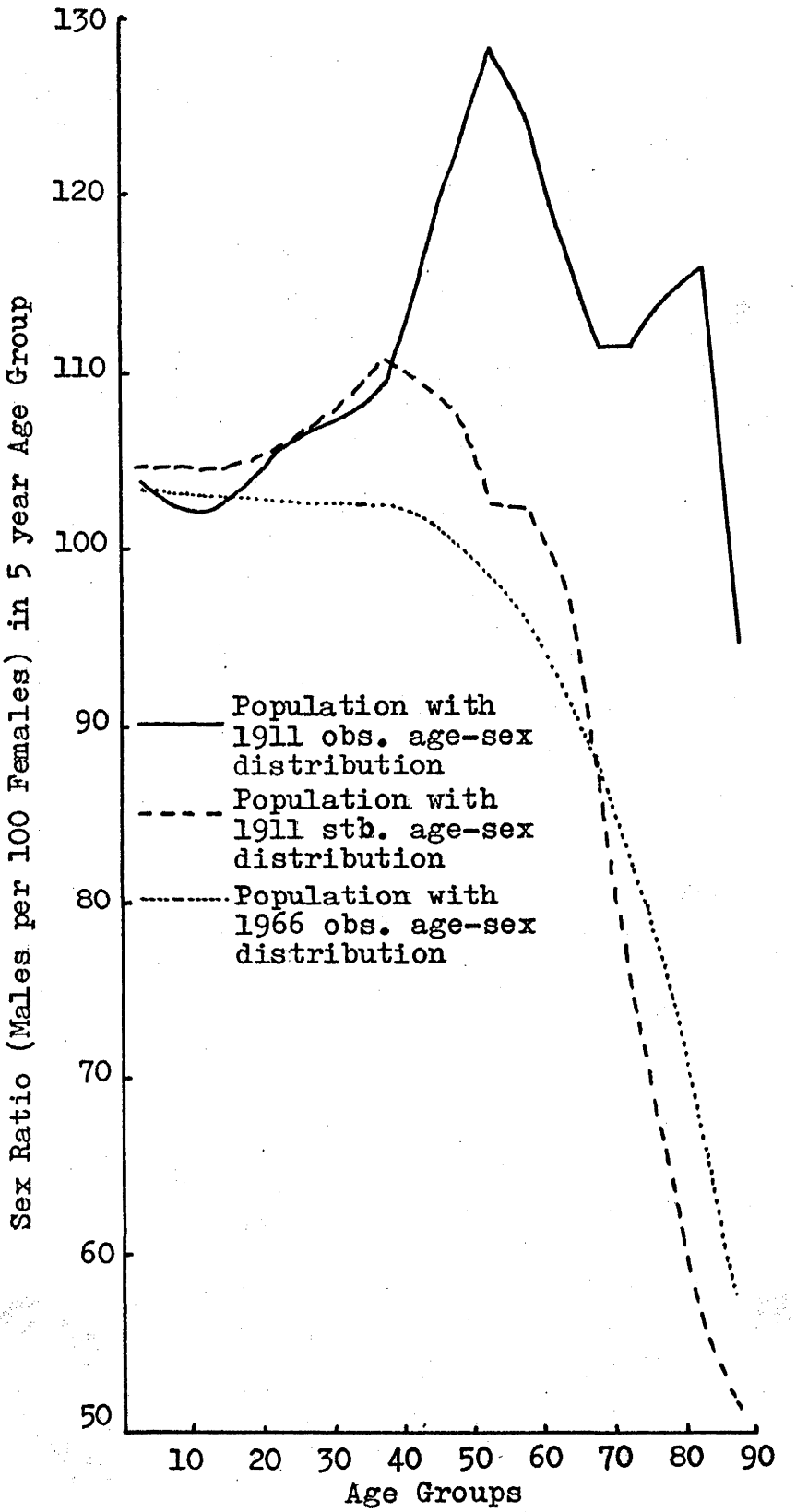


FIGURE 2.2: SEX RATIOS IN 5 YEAR AGE GROUPS IN THE INITIAL POPULATIONS



population in the age groups above 40 years, seen in Figure 2.2, show the effect of the large preponderance of males among the migrants prior to 1890.[2]

Similarly, the smaller population in the younger age groups and a trough in the middle age range seen in the curves for the 1966 obs. population as compared with those for the 1911 stb. population were, respectively, the results of the recent decline in fertility since 1961, and its decline to a very low level during and following the economic depression of the early nineteen thirties which was also accompanied by a decrease in immigration around the same years. These variations in fertility and migration during 1911-66 are discussed in greater detail in Section 2.3.

The dissimilarity in the age distributions can be quantified by computing an age distribution dissimilarity index (ADI) which is defined as the sum of the absolute differences in the percentages in the respective age groups in the two populations being compared.[3] The value of this index was 9.85 between the age distributions of the 1911 obs. and 1911 stb. populations; 10.93 between those of the 1911 stb. and 1966 obs. populations; and 17.34 between those of the 1911 obs. and 1966 obs. populations, in the case of males. The corresponding values in the case of females were 15.36, 10.73 and 25.75. A zero

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2 W.D. Borrie, Population Trends and ..., 1948. (Chapter 3)

3 N. Keyfitz and E.M. Murphy, Comparative Demographic Computations, 1968, p.8. They have used half of this sum as the index.
N. Keyfitz, Introduction to the Mathematics ..., 1968, p.47.

value of this index would indicate that the two age distributions are identical. But this index does not show whether the differences are in the younger age groups or in the older age groups. For this purpose we have to examine other indexes such as the mean age, proportion in the old age group, etc. The mean ages in the three populations - 1911 obs., 1911 stb. and 1966 obs. - were, respectively, 27.67, 29.13 and 31.08 years for males and 26.64, 30.20 and 32.43 years for females. The same order was observed when the populations were classified according to the increasing magnitudes of the proportion of the population in the old age group (65+). Thus the 1911 obs. population was young compared to both the 1911 stb. and 1966 obs. populations, whereas the 1966 obs. was the oldest of all.

The overall sex ratios (number of males per 100 females) in the three populations were 108.26, 100.18 and 102.49 respectively. The dissimilarity in the sex ratios by age between any two populations may be measured by the sex ratio dissimilarity index (SRDI) which is defined as the sum of the absolute differences of the sex ratios in 5 year age groups between the populations being compared. As in the case of the ADI, the value of SRDI would be zero if the sex ratios are the same for the two populations. The value of SRDI was 315.42 between the sex ratios in the 1911 obs. and 1911 stb. populations; 94.54 between those in the 1911 stb. and 1966 obs. populations; and 307.86 between those in the 1966 obs. and 1911 obs. populations.

From these numerical values and the graphs presented, it can

be seen that the age-sex distributions of the initial populations were considerably different from one another.

2.3 VARIATIONS IN THE COMPONENTS OF POPULATION CHANGE IN AUSTRALIA DURING 1911-66

2.3.1 SURVIVAL RATES

Since the survival rates used in the study were interpolated for each calendar year from the official life tables for Australia and from the life table constructed on the basis of the 1966 census (see Appendix B), it would be sufficient as well as convenient to examine the variations in the survival rates observed at these points of time during the period 1911-66. The values for the year 1911 were also interpolated from the life tables for 1901-10 and 1920-22. However, it was found that the survival rates for 5 year age groups obtained from the interpolated life table for 1911, did not differ much from those obtained from an abridged life table constructed on the basis of the deaths during 1910-12 and the estimated population as at 30 June 1911.

The expectation of life at birth, $^{o}e_{0}$, which was 56.59 years for males and 60.39 for females in 1911, increased, respectively, to 67.71 and 74.11 years in 1966. The increasing trend in the $^{o}e_{0}$ which continued up to 1961, experienced a slight setback during 1961-66 in the case of males and a negligible increase in the case of females. However, in this study we are not interested in the changes in the

expectation of life at birth but in the variations in the survival rates by single years of age. The importance of examing the survival rates in studies of population change, is not often stressed. But some authors have made it clear that it is the variations in the survival rates that are important in studying population change.[4] Hence, we shall examine how the survival rates by single years of age for males and females have changed in Australia during the period 1911-66.

Apart from ${}^{o}e_{0}$, there seems to be no single composite index to represent the survival rates. But the variations in ${}^{o}e_{0}$ may not be a good guide to the variations in the survival rates. For instance, ${}^{o}e_{0}$ increased from 1911 to 1966 by about 20 per cent in the case of males and by about 23 per cent in the case of females while even the survival rate from birth to age 0 years which showed the maximum improvement, increased by only 4 to 5 per cent in either case. The survival rates at most of the other ages, especially in the age range 15-64 years, did not change by more than half a per cent. The best way to examine the variations in the survival rates would, therefore, be to draw and compare the histograms of the survival rates with age as the abscissa and the value of the survival rate as the ordinate at the mid-year of age. Then the changes in the area under the survival rates

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[4] G.J. Stolnitz, 'Mortality Declines and ...', 1956.

R.H. Daw, 'The Comparison of Male and Female Mortality Rates', Journal of the Royal Statistical Society, Series A, Vol. 124, Part 1, 1961, pp.20-43. (Comment by J. Hajnal)

curve would show the changes in the survival rates, if we keep the age range the same.

For obtaining a quantitative measure, we may assume that S(b) and the survival rate for the last open end age group have the same width for their age intervals as the survival rates at other ages. The age interval in the present study is one year. Then the sum of the product of the respective survival rates and the width of the age interval, gives the area under the survival rates curve. We shall call this quantity the total lifetime survived (TLS). If we now obtain a measure of location for this survival rates curve, say the mean age ($\overline{S}$) or the median age ($\overline{S}_{md}$), then that would indicate whether the weight of the improvements in the survival rates was more on the younger ages than on the older ages, or vice versa.

The changes in these measures, viz. TLS and $\overline{S}$ or TLS and $\overline{S}_{md}$, could be used for examining the changes in the survival rates. These would not only show the absolute changes in the survival rates, but would also indicate the age pattern of the change to some extent. However, due to the averaging effect, the variations in these measures conceal the relatively large variations in the survival rates at the very young and at old ages. This may be seen from Table 2.1 which presents the areas under the survival rates curves in the infant ages and in broad age groups along with $^{o}e_{0}$, TLS, $\overline{S}$ and $\overline{S}_{md}$.

It is clear from the table that the improvements in the survival rates for both males and females were considerable only in the

TABLE 2.1

CHANGES IN THE AREA COVERED BY THE SURVIVAL RATES CURVE IN BROAD AGE GROUPS AND IN OTHER MEASURES: AUSTRALIA, 1911-66

Census Year	S(b)	Age Groups 0	1-5	6-14	15-34	35-44	45-64	65+	TLS	Mean Age $\bar{S}$	Median Age $\bar{S}_{md}$	Expectation of Life at Birth $^{o}e_{o}$
						MALE						
1911	0.93588	0.96766	4.97355	8.98301	19.91856	9.92418	19.60953	17.97930	83.29167	41.15	40.91	56.59
									(100.00)	(100.00)	(100.00)	(100.00)
1921	0.94556	0.97558	4.97576	8.98490	19.93267	9.93713	19.63895	18.07026	83.46081	41.18	40.95	59.26
									(100.20)	(100.07)	(100.10)	(104.72)
1933	0.96203	0.98840	4.98468	8.98820	19.95080	9.95248	19.67142	18.15667	83.65468	41.20	40.98	63.49
									(100.44)	(100.12)	(100.17)	(112.19)
1947	0.97238	0.99387	4.99217	8.99238	19.96546	9.96440	19.66910	18.17320	83.72296	41.20	40.96	66.09
									(100.52)	(100.12)	(100.12)	(116.79)
1954	0.97816	0.99518	4.99337	8.99386	19.96560	9.96881	19.68560	18.18618	83.76676	41.20	40.97	67.13
									(100.57)	(100.12)	(100.15)	(118.63)
1961	0.98031	0.99634	4.99548	8.99541	19.96906	9.96870	19.69055	18.22830	83.82415	41.22	40.99	67.91
									(100.64)	(100.17)	(100.20)	(120.00)
1966	0.98195	0.99667	4.99597	8.99582	19.96879	9.96742	19.67896	18.16673	83.75231	41.22	40.95	67.71
									(100.55)	(100.17)	(100.10)	(119.65)
						FEMALE						
1911	0.94827	0.97204	4.97579	8.98505	19.92280	9.93543	19.72195	18.32471	83.78604	41.31	41.13	60.39
									(100.00)	(100.00)	(100.00)	(100.00)
1921	0.95816	0.97985	4.97860	8.98751	19.93728	9.94781	19.75478	18.38475	83.92874	41.33	41.15	63.39
									(100.17)	(100.05)	(100.05)	(104.97)
1933	0.97024	0.98993	4.98673	8.99108	19.95354	9.95865	19.77032	18.51076	84.13125	41.36	41.20	67.13
									(100.41)	(100.12)	(100.17)	(111.16)
1947	0.97828	0.99498	4.99386	8.99508	19.97380	9.97055	19.79409	18.54650	84.24714	41.37	41.20	70.62
									(100.55)	(100.15)	(100.17)	(116.94)
1954	0.98312	0.99572	4.99475	8.99630	19.98370	9.97745	19.81995	18.64153	84.39252	41.41	41.25	72.70
									(100.72)	(100.24)	(100.29)	(120.38)
1961	0.98466	0.99687	4.99618	8.99716	19.98571	9.98042	19.83961	18.74534	84.52595	41.45	41.31	74.07
									(100.88)	(100.34)	(100.44)	(122.65)
1966	0.98603	0.99711	4.99680	8.99737	19.98525	9.98001	19.83290	18.75141	84.52688	41.45	41.31	74.11
									(100.88)	(100.34)	(100.44)	(122.72)

Note:- Figures in brackets show the ratio to the corresponding value in 1911.

[Source: 1911 values from the interpolated L_x values; 1921-61 values from the Official life tables for Australia; and 1966 values from the life table given in Appendix B]

infant ages and in the old ages - 65 years and over. Since 1961 there was a retardation in the improvements in the survival rates for males in the middle and in the old age range and for females in the old age range. This appears to have happened due to the fact that, in recent years, there were significant increases in the death rates from motor vehicle and other accidents and from suicide which affected the age range 15-34 years; from the diseases of the heart and lungs which accounted for the maximum tolls in the age range 45 years and over (for example, these accounted for more than 50 per cent of deaths in this age range during 1967 both for males and females); and from malignant and lymphatic neoplasms which, along with the two mentioned earlier, were the major causes of deaths in the age range 35-44 years.[5] Further, the survival rates for females, which were already at a higher level than those for males in 1911, recorded a faster improvement since 1933. Much of this improvement occurred in the survival rates at the older ages as can be observed from Figure 2.3. The slower improvement in the survival rates for males was probably due to the greater impact on males of the deaths from the causes mentioned above. Similar trends in mortality have been observed in the populations of England and Wales, and the United States.[6]

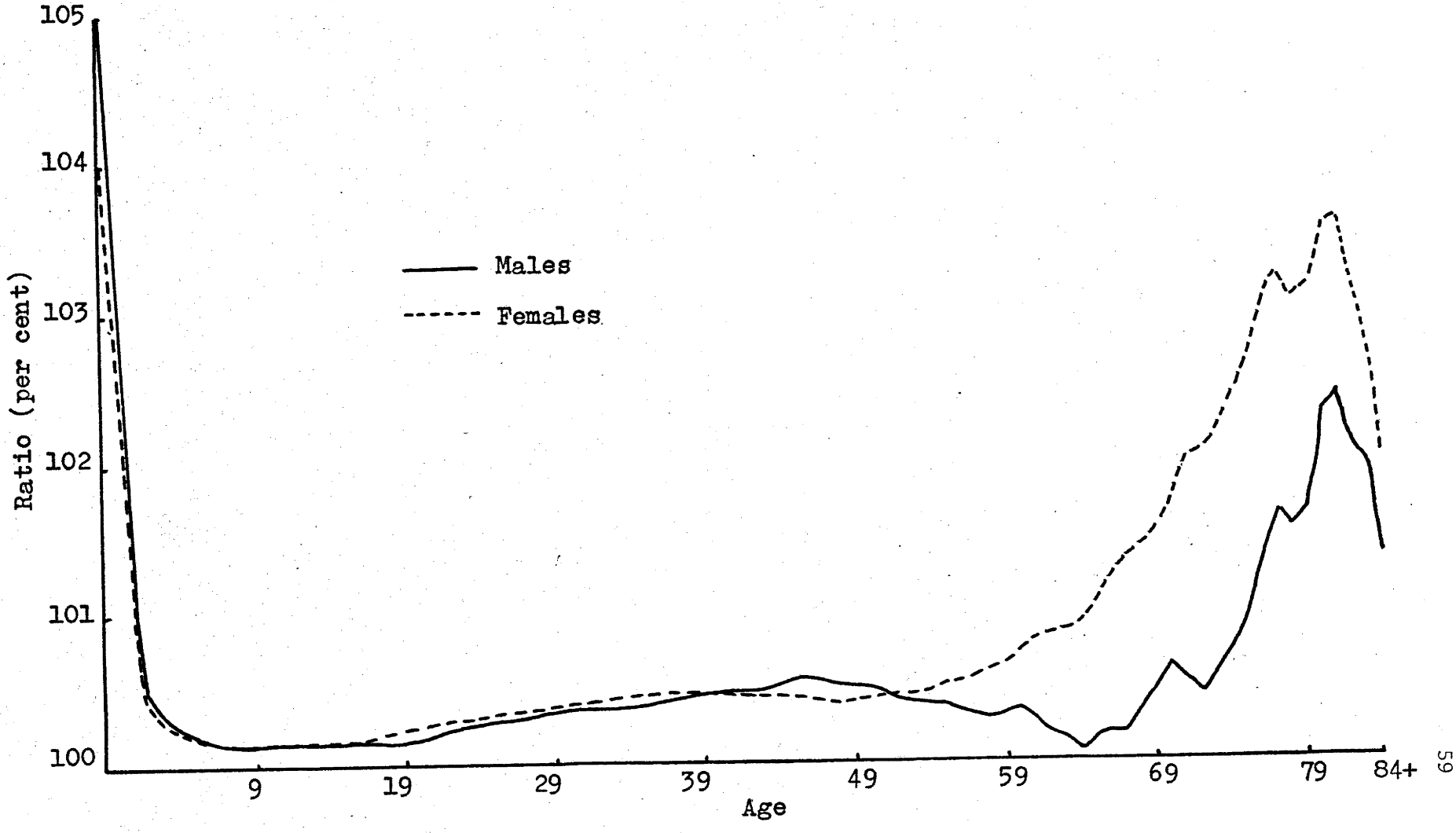
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5 P.F. McDonald, 'Trends in Major Causes of Death in Australia, 1950-67', a paper presented at a Departmental Seminar, Department of Demography, The Australian National University, Canberra, 1969. (Unpublished manuscript.)

6 L.V. Martin, 'The Recent Trend of Mortality in Great Britain', Journal of the Institute of Actuaries, Vol. 93, 1967, pp.439-443

M. Spiegelman, 'Mortality in the United States: A Review and Evaluation of Special Reports of the National Center for Health Statistics', Demography, Vol. 5, No. 1, 1968, pp.525-533.

FIGURE 2.3: RATIOS OF SURVIVAL RATES IN 1966 TO THOSE IN 1911 AT SINGLE YEARS OF AGE, AUSTRALIA



Before closing this discussion, it must be mentioned that no allowance was made for the effect of the epidemic on the survival rates for 1919. Therefore, the deaths during the years 1918-20, especially for 1919, were underestimated and consequently the survived populations at the younger ages were slightly overestimated when compared with the populations enumerated at the 1921 census. However, the differences between the projected and the enumerated populations in the different age groups were not so large to invalidate the use of the interpolated survival rates (see Appendix A).

2.3.2 FERTILITY RATES

The age-specific fertility rates could change in two ways - (a) in their total intensity which is called the level, and (b) in their age pattern. The variation in the level of fertility is measured by the variation in the total fertility rate (TFR) which is the sum of all the fertility rates at single years of age. This is the area under the fertility rates curve, as explained in the case of the survival rates curve. The variation in the age pattern is measured by the variation in the mean age of the fertility schedule ($\bar{m}$) which is a measure of location for the fertility rates curve. Recently, it has also become customary to compute the variance of the fertility rates curve, which indicates the concentration of births around the mean age.

Using these measures, the age-specific fertility rates observed in Australia during 1911-66 were analysed and the results are presented here. The analysis concentrated more on the changes in the

fertility rates by age of females than on those by age of males because the female dominant method was used in most of the numerical illustrations. The fertility rates by age of males which were required in a few cases where equal dominance was used, were analysed only for the two years 1911 and 1966.

(i) Fertility Rates by Age of Females

TFR and $\overline{m}$ for Australia during 1911-66 are displayed in Figure 2.4. The graph shows that the TFR declined from a relatively high value of 3.51 in 1911 till 1920 when it registered a temporary recovery, only to decline again. After reaching a minimum of 2.10 in 1935, it started increasing and recorded a very rapid growth from about 1947 - the period described as the period of the 'baby boom'. But from 1951 onwards the rate of increase in TFR slowed down and since 1961 a downward trend has set in.

During all these years, from 1911 to 1966, $\overline{m}$ declined from 30.49 years in 1911 to 27.34 years in 1966 except at brief intervals during or immediately following the two World Wars, viz. during 1916-19 and 1944-47.

Figure 2.5 brings out some of the features of how these changes were brought about. In general, the observed variations in TFR and $\overline{m}$ were caused by the increase in the fertility rates at the younger ages and by the decrease in those at the older ages. If we divide the female reproductive life of 35 years from age 15 to 49 years into three broad age groups - (15-19), (20-34) and (35-49) - we can obtain the

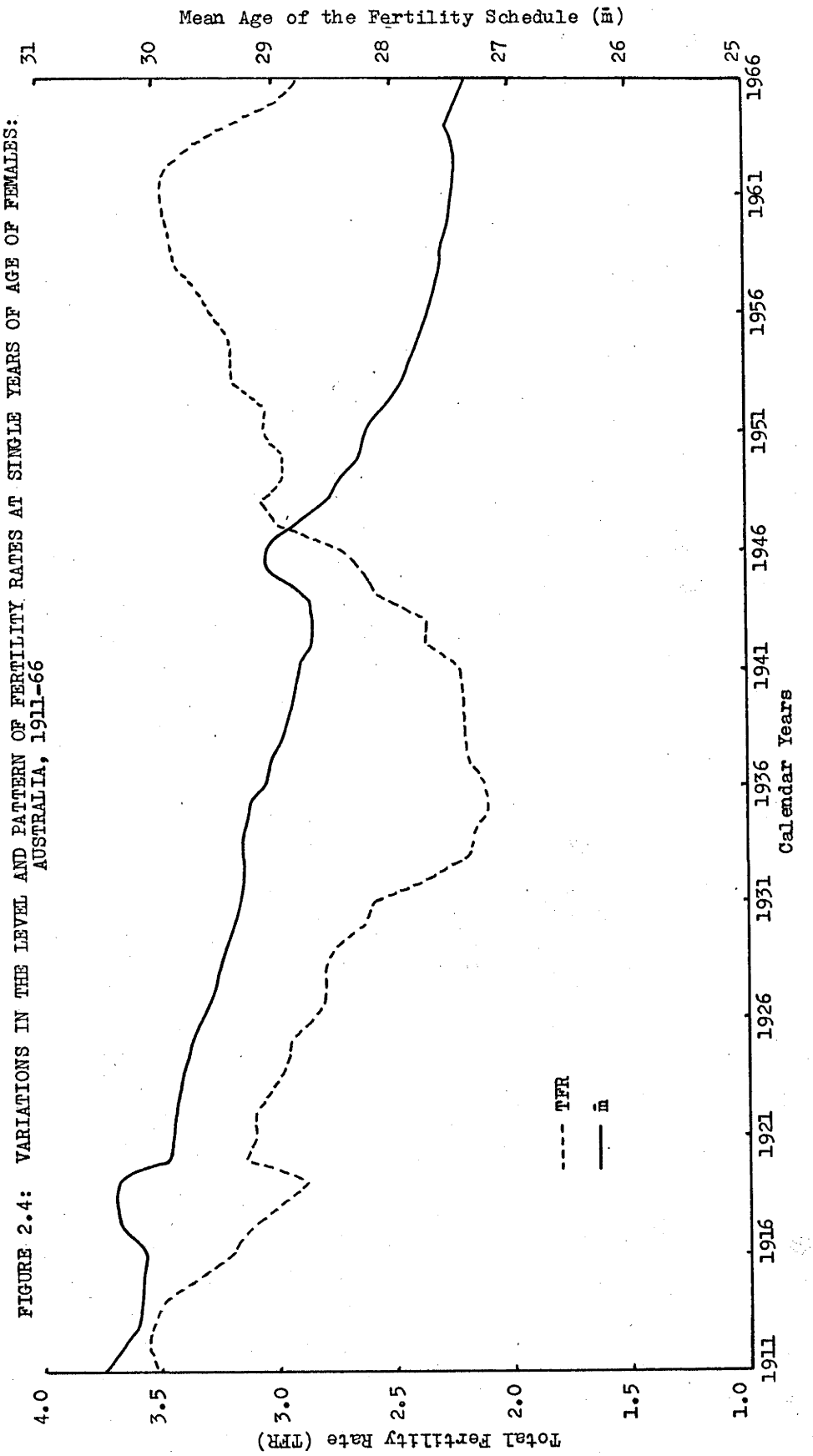


FIGURE 2.4: VARIATIONS IN THE LEVEL AND PATTERN OF FERTILITY RATES AT SINGLE YEARS OF AGE OF FEMALES: AUSTRALIA, 1911-66

FIGURE 2.5: VARIATIONS IN THE AGE-SPECIFIC FERTILITY RATES AT SINGLE YEARS OF AGE OF FEMALES: SELECTED CALENDAR YEARS, AUSTRALIA

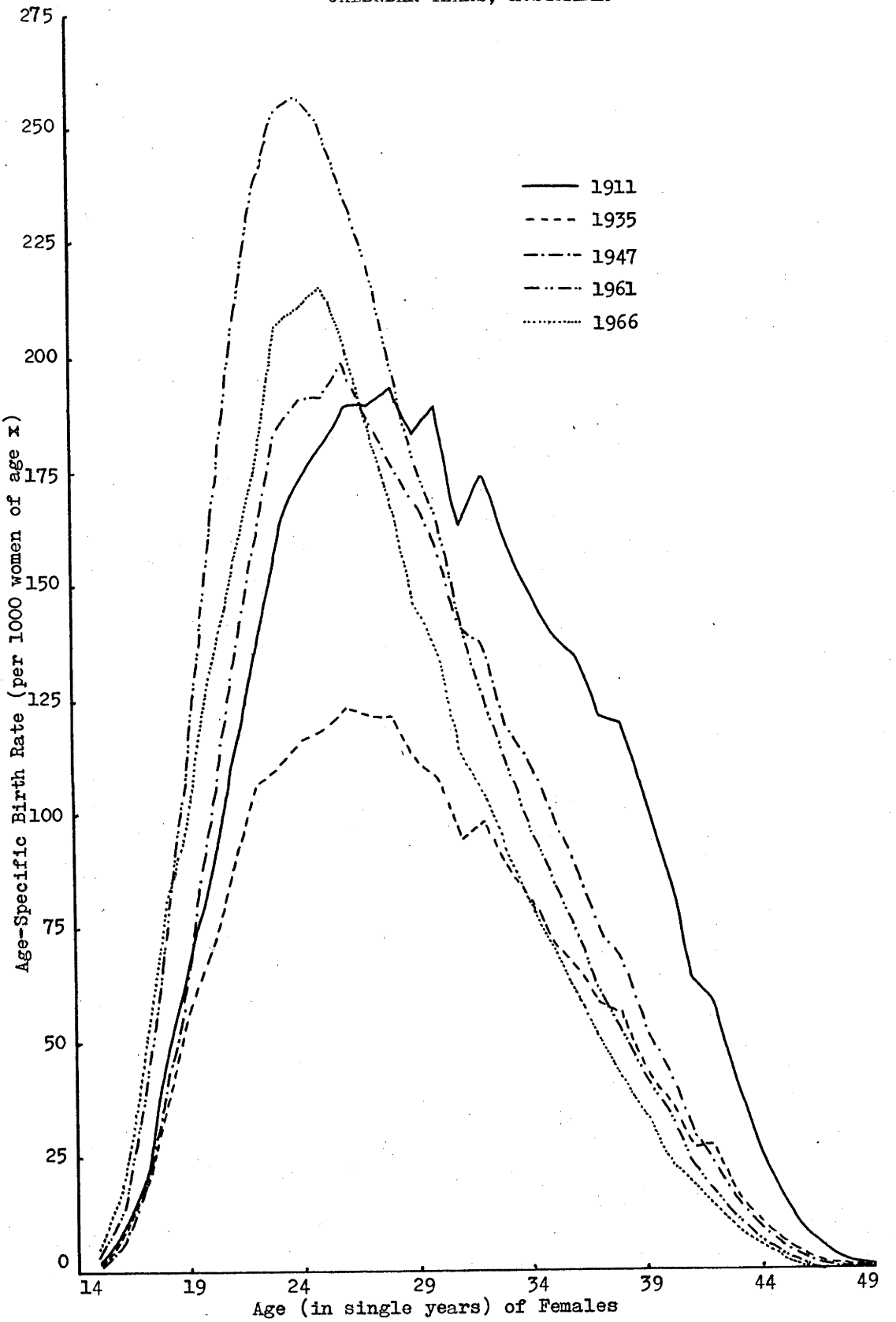


TABLE 2.2

PERCENTAGE CONTRIBUTION TO TFR BY FEMALES IN BROAD AGE GROUPS: SELECTED CALENDAR YEARS: AUSTRALIA, 1911-66

Calendar Year	Age Group 15-19	20-34	35-49	Total	TFR (per 1000 women)	Mean Age	Variance
1911	3.83	69.67	26.50	100.00	3512.47	30.49	42.71
1916	3.96	71.17	24.88	100.00	3203.95	30.13	41.84
1921	4.45	71.79	23.77	100.00	3105.11	29.90	42.14
1926	5.10	72.39	22.51	100.00	2843.22	29.62	42.27
1931	5.81	73.18	21.01	100.00	2576.09	29.33	41.70
1934	5.81	73.85	20.34	100.00	2161.34	29.28	41.18
1935	5.70	74.10	20.21	100.00	2101.44	29.25	40.92
1936	5.75	75.14	19.11	100.00	2107.66	29.09	39.83
1941	5.19	77.66	17.15	100.00	2244.92	28.80	36.79
1946	4.29	77.59	18.11	100.00	2733.12	29.07	36.15
1951	5.80	79.30	14.90	100.00	3048.90	28.22	35.45
1956	6.54	80.38	13.08	100.00	3275.95	27.70	34.39
1960	6.88	81.06	12.06	100.00	3466.30	27.50	33.34
1961	6.76	81.46	11.78	100.00	3473.74	27.48	32.86
1962	7.05	81.30	11.65	100.00	3456.71	27.48	32.85
1966	8.34	80.25	11.41	100.00	2887.82	27.34	32.13

[Source: Computed from the age specific fertility rates by single years of age of females.]

percentage contribution by women in these age groups to the total fertility in each calendar year by calculating the ratio of the sum of the fertility rates at these ages to the TFR. Such percentages for some selected years are presented in Table 2.2 along with TFR, mean and

variance of the respective fertility schedules. It is clear from the table that the contribution of women aged 20-34 years consistently increased while that of women aged 35-49 years consistently decreased over the years. The trend in the case of women aged 15-19 years also increased with minor fluctuations. These trends and the changes in the values of the mean age and the variance, depict clearly the increasing concentration of births in the early years of the reproductive life in recent years as compared to the situation in 1911. Borrie[7] and McArthur[8] came to the same conclusion through the analysis of the data on the completed family size and on the confinements by duration of the existing marriage.

These observed trends were caused by the decrease in the marital fertility rates, the decrease in the age at marriage, the increase in the proportion of women marrying and the increase in the births within the early years of marriage.[9]

From 1911 till about the late 1930s there appears to have been only small changes in the age pattern and in the concentration of births in the younger ages of reproduction (see Table 2.2). Therefore,

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7 W.D. Borrie, 'Recent Trends and Patterns in Fertility in Australia', Journal of the Biosocial Science, Vol. 1, No. 1, 1969, pp.57-70.

8 N. McArthur, 'Australia's Birth Rate in Perspective', Economic Record, Vol. 43, No. 101, March 1967, pp.57-64.

9 K.G. Basavarajappa, 'Trends in Fertility ...', Ph.D. Thesis, 1964. (Chapters 7 & 8.)

N. McArthur, 'Australia's Birth Rate ...', Economic Record, 1967.

the decline in the total fertility rate during this period was contributed by the declines in the rates at almost all ages. It was found that, during this period, there were only small changes in the age at marriage, in the proportion of the currently married among the females in the reproductive ages and hence the decline in TFR was largely the result of the declines in the marital fertility rates.[10]

During the Second World War period, 1939-45, the increasing trend in the TFR was the effect of the rise in the proportions currently married in the younger age group (15-24), and perhaps the effect of the 'making up' tendency of the females in the older age groups who had postponed their child bearing due to the economic depression of the early nineteen thirties.[11] A very similar trend was observed by Whelpton in respect of the fertility performance of the American women.[12]

At the close of the Second World War, there was the 'marriage boom'. The expected mean age at first marriage decreased sharply and there was a tremendous drop in the expected proportion of females remaining unmarried by the end of their reproductive age.[13] These, coupled with the increase in the fertility of married females, caused

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10 K.G. Basavarajappa, 'Trends in Fertility ...', Ph.D. Thesis, 1964, pp.138-144.

11 K.G. Basavarajappa, 'Trends in Fertility ...', Ph.D. Thesis, 1964, pp.205-206.

12 P.K. Whelpton, Cohort Fertility, 1954, pp.189-191.

13 M. Sivamurthy, 'Trends in First Marriages in Australia: 1933-66', Economic Record, (to appear).

the fertility rates at the younger and middle ages to rise. But the fertility rates for females in the older reproductive age range, 35 years and over, showed very little change or even declined further at some ages, indicating clearly the widespread practice of family limitation in Australia.

The reversal of the trend in fertility since 1961 was brought about by a general decline in the rates at almost all ages except at a few of the youngest ages. The increase at these youngest ages was governed by the increase in the ex-nuptial birth rate at these ages and in the proportion of pre-marital pregnancies.[14] The declines at other younger ages were probably caused by a caution in the early years of married life in response to the economic recession of 1961,[15] and at the older ages due to the effective use of contraceptives to plan the families and to limit the family size. The 'pill' is mentioned as responsible for this effective control of the family size. But it has been observed that the desire for small family size came much earlier than the 'pill'.[16] Perhaps the 'pill' has become a convenient instrument to fulfill that desire. With the increased knowledge of contraception it seems

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14 K.G. Basavarajappa, 'Pre-marital Pregnancies and Ex-nuptial Births in Australia, 1911-66', The Australian and New Zealand Journal of Sociology, Vol. 4, No. 2, 1968, pp.126-145.

15 W.D. Borrie, 'Recent Trends and Patterns ...', Journal of the Biosocial Science, 1969.

16 W.D. Borrie, 'Pills and Populations', Current Affairs Bulletin, Vol. 40, No. 1, June 1967, pp.3-16.

> ... for many, child-bearing within marriage has become an episode of life, not an end in itself, and exactly when children are born is increasingly a rational decision likely to be determined by immediate economic and social circumstances.[17]

Thus early marriage, early child bearing, restriction of effective reproductivity to a very short period and rearing of small families appear to have become ~~a~~ the widespread reproductive norms in Australia as in other economically developed countries.

(ii) Fertility Rates by Age of Males

We shall now briefly look at the fertility rates by age of males during the two years 1911 and 1966. These two years were selected to see the possible changes that have occurred in these rates over the period 1911-66. But only the values for 1911 were used in the illustrations. In calculating the fertility rates by age of males, a particular difficulty arose due to the lack of information regarding the age of fathers of ex-nuptial children. It was not possible to use any of the known relationships between the ages of fathers and those of mothers because the fathers of ex-nuptial children may have an entirely different age pattern. Hence, it was assumed arbitrarily that the ages of fathers of ex-nuptial children would be 2 years higher than those of mothers though, in some cases, it is possible that the fathers may be

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17 W.D. Borrie, 'Recent Trends and Patterns ...', Journal of the Biosocial Science, 1969, p.67.

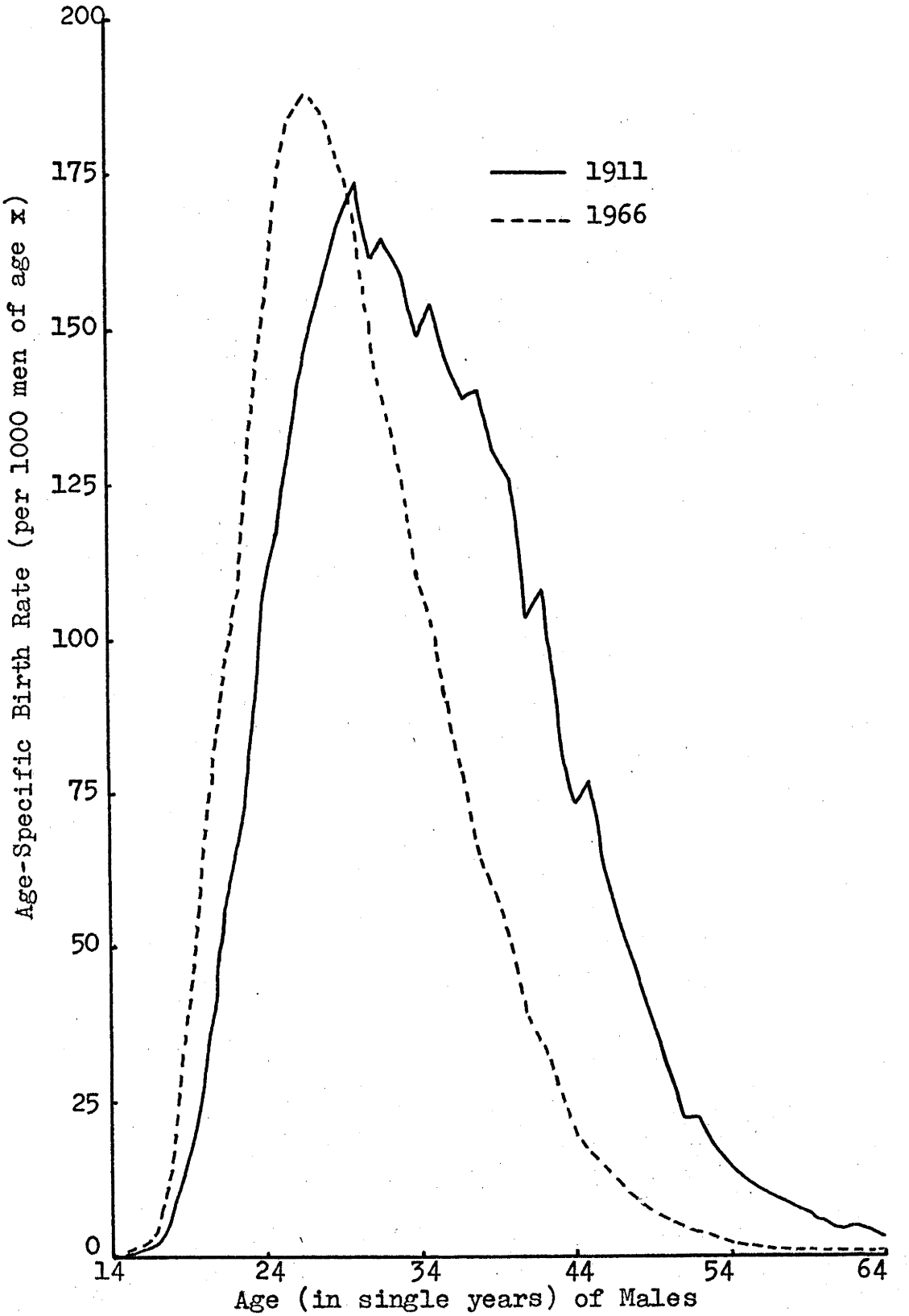
younger than mothers. The effect of this assumption would be to introduce only a small error in the average age of the fertility schedules since the ex-nuptial births constitute a small proportion (3 to 8 per cent) of all births, and occur at relatively young ages of females which reduces to some extent the possibility of fathers being younger than mothers. Nearly 58 to 66 per cent of mothers of ex-nuptial births were less than 25 years of age and hardly 3 per cent of them were above 40 years of age.

Figure 2.6 presents the fertility rates by age of males. It exhibits the same phenomenon of concentration of births into the younger age groups in recent years as was observed in the case of females. In fact, the variance of the fertility schedule for males was nearly 35 per cent less in 1966 than in 1911 (44.23 as against 68.68), while that of the fertility schedule for females was about 25 per cent less (32.13 as against 42.71). The mean age decreased from 35.15 years in 1911 to 30.44 years in 1966, and the TFR from 3.54 to 2.85.

2.3.3 MIGRATION DATA

The variations in the basic data on migration - the numbers of arrivals and departures - and in those derived from them, viz. the overall net migration rates, the age-sex compositions of net migrants and the age-sex-specific net migration rates, observed in Australia during 1911-66 are discussed in this section. In obtaining the derived data, the financial years were used because, in Australia, the census

FIGURE 2.6: VARIATIONS IN THE AGE-SPECIFIC FERTILITY RATES AT SINGLE YEARS OF AGE OF MALES: SELECTED CALENDAR YEARS, AUSTRALIA



was taken as at 30 June of the respective census years except in 1911 and 1921 when it was taken in April. Moreover, for this study the population as at 30 June 1911 was taken as the initial population and therefore, the projections always referred to the financial years June to June. The numbers of arrivals and departures for the financial years were obtained from those for the calendar years on the assumption that the movements were evenly distributed over the calendar years.

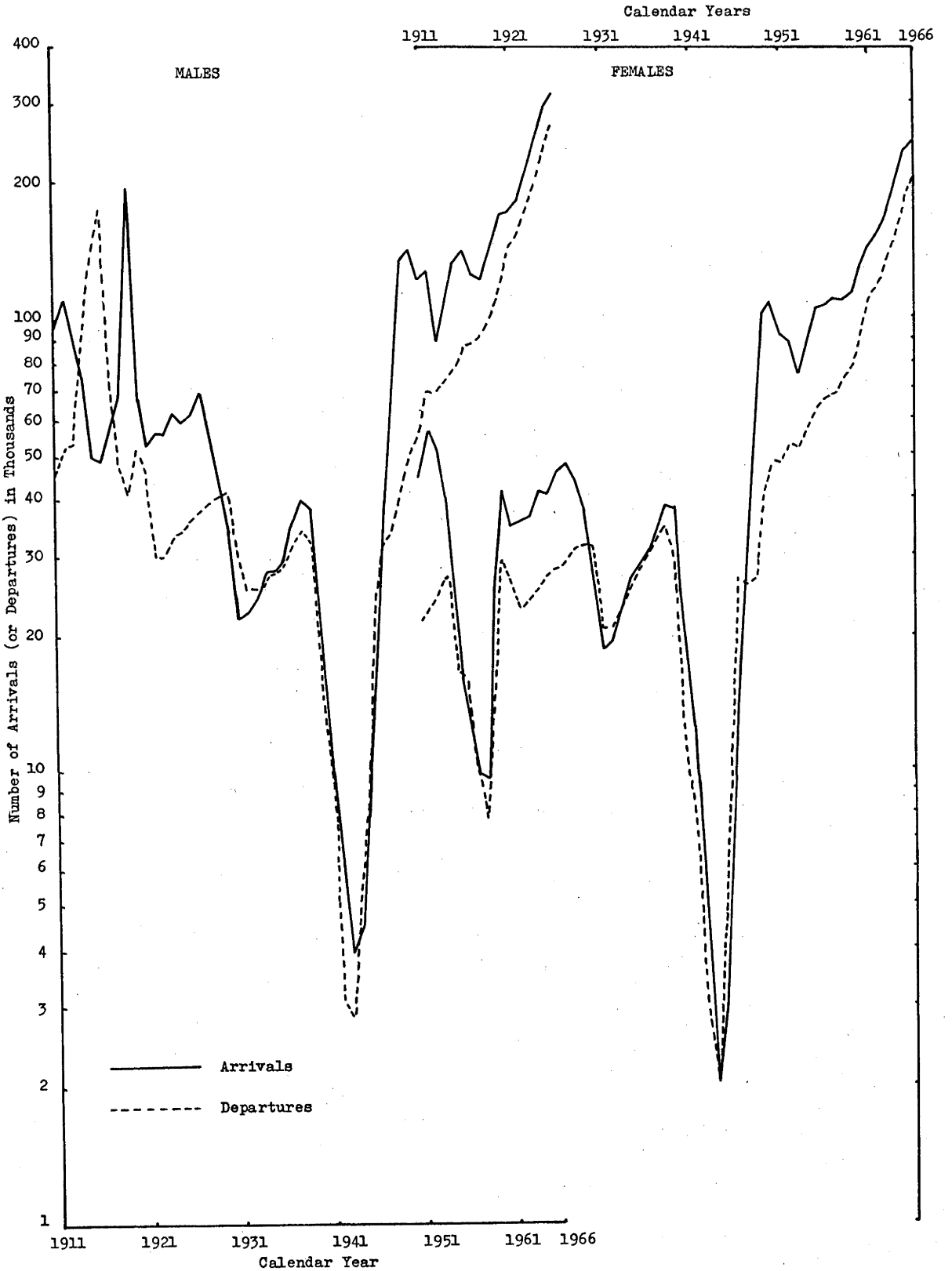
(i) Net Numbersof Migrants

The observed numbers of arrivals and departures during the calendar years 1911-66 are presented in Figure 2.7. It is clear from the graph that there were significant variations in the net numbers of migrants during this period. These variations were correlated with the economic conditions in the country and the policies pursued by the government.

Immigration into Australia which had virtually ceased as a result of the stringent economic conditions of the 1890s, began to recover by about 1909 and a big inflow occurred during 1910-12. This was mainly due to the efforts of the state governments to bring in migrants. But the outbreak of the First World War in 1914, cut the immigration down and even caused emigration during some of the war years. However, the large emigration and immigration observed respectively during 1914-17 and 1918-19, especially 1919, were the result of the inclusion of troop movements in the migration statistics.

At the close of the war, concerted efforts were made to

FIGURE 2.7: TOTAL NUMBER OF ARRIVALS AND DEPARTURES DURING EACH CALENDAR YEAR: AUSTRALIA 1911-66



Note: For 1914-19, the troop movements were included. For 1939-45, they were not included.

bring more people into the country. The idea that migration and colonization should be planned in a way which would mutually benefit both the mother country and the colonies, gained force after the war and was supported by the Empire Settlement (Tennyson) Committee of 1917 and by the Oversea Settlement Committee of 1918. In 1920 the Federal Government of Australia, which had only advisory status, gained full control over migration matters and it negotiated with the government of the United Kingdom 'an enduring policy of oversea settlement'. Accordingly, the Empire Settlement Act was passed in 1922 and the £34 million loan agreement was signed in 1925, both aiming to assist migrants to come and settle in Australia. The plans did not work satisfactorily, and in fact proved uneconomical.[18] However, due to such aids and the economic stability in the country net gain from migration improved. But soon came the economic depression of 1929-32 and set the migration back to an unfavourable position. No new state aid schemes were forthcoming during that period. By the time this situation changed and migration began to recover the Second World War broke out, and again migration showed a negative balance. Thus the net number of immigrants was the smallest during the intercensal period 1933-47 as can be seen from Table 2.3.

With the end of the Second World War, it was realised that a large population was not only of economic significance but also of

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18 K.M. Jupp, 'Factors Affecting the Structure ...', M.A. Thesis. She has analysed the causes for the failure. (pp.60-88)

TABLE 2.3

NET NUMBERS OF MIGRANTS IN BROAD AGE GROUPS: AUSTRALIA, 1911-66

Years	MALE						FEMALE					
	Age Groups					Average	Age Groups					Average
	0-14	15-44	45-64	65+	Total	per Year	0-14	15-44	45-64	65+	Total	per Year
						Absolute Numbers						
1911-21	23822	47047	-3215	-230	67424	6742	39279	69315	2931	62	111587	11159
1921-33	32929	131955	7766	-1517	171133	14261	28871	70044	9836	1195	109946	9162
1933-47	6403	17162	2478	-725	25318	1808	5167	7652	3626	1208	17653	1261
1947-54	86943	269676	24838	2341	383798	54828	79236	152372	25209	4777	261594	37371
1954-61	86661	199464	15680	2355	304160	43451	78278	158754	21092	4858	262982	37569
1961-66	68794	121999	13490	4106	208389	41678	63535	115737	18857	6072	204201	40840
						Percentage Distribution Sex Ratio						
1911-21	35.33	69.78	-4.77	-0.34	100.00	60.42	35.20	62.12	2.63	0.05	100.00	
1921-33	19.24	77.11	4.54	-0.89	100.00	155.65	26.26	63.71	8.95	1.09	100.00	
1933-47	25.29	67.78	9.79	-2.86	100.00	143.42	29.27	43.35	20.54	6.84	100.00	
1947-54	22.65	70.27	6.47	0.61	100.00	146.71	30.29	58.24	9.64	1.83	100.00	
1954-61	28.49	65.58	5.16	0.77	100.00	115.66	29.77	60.36	8.02	1.85	100.00	
1961-66	33.01	58.55	6.47	1.97	100.00	102.05	31.11	56.69	9.23	2.97	100.00	

Note:- The data refer to the period June to June of the respective years. The number for half a year is taken as equal to half of that for the whole year.

[Source: Computed from the single year age distributions of total arrivals and departures in Australia during the calendar years 1911-66.]

strategic importance. Even the Labour government which had looked upon immigration with grave suspicion, changed its policy towards organizing 'an immigration programme to fill the gaps in the younger ages of the work force without threatening employment opportunities of native Australians'.[19] Thus very optimistic targets were set and strenuous efforts were made to obtain migrants. As a result, net immigration was spectacular during the intercensal period 1947-54 and since then, has remained more or less steady and considerable except during the economic recession of 1953 when it was purposely cut down by the government. But during the economic recession of 1961, the government took care not to cut down the number of immigrants. Hence immigration continued to occur but, as a result of the recession, its growth decreased (see Fig. 2.7 and 2.8) to a small extent.

It is clear from this brief analysis that migration in Australia has been very sensitive to the economic situation in the country and the measures taken by the government to promote migration. In fact, 'large flows of immigration have always been associated with extensive passage and other state assistance to the immigrants'.[20]

A rough idea of the changes in the age and sex compositions of net migrants, may be obtained by examining the percentage distributions and the sex ratios given in Table 2.3. Until about 1954

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19 C.A. Price, 'Overseas Migration ...', Australian Outlook, 1962, p.162.

20 W.D. Borrie and G. Spencer, Australia's Population Structure and Growth, Committee for Economic Development of Australia, Melbourne, 1965, p.14.

there was a heavy concentration in the younger and middle age groups, especially in the age group (15-44); and there was heavy predominance of males among the net migrants. The position during 1911-21 was blurred by the inclusion of the troop movements in the migration statistics. When the numbers of troops were deducted from the numbers of arrivals and departures, the sex ratio during that period became 135.54 which was in line with the above contention though comparatively low. To an extent this lower value was caused by the coming in of war brides. The high proportion of males and the concentration in the adult and middle ages were mainly the result of the government policy to recruit workers rather than to bring in families.

After the recession of 1953, it was realised that it would be better to alter the composition of the intake at the time of difficult economic situations instead of cutting down the numbers - as was done during 1953 - and then trying to resume the immigration programme later. Accordingly, during the recession of 1961 it was decided to increase the proportion of females and to encourage the migration of families.[21] Thus during the period 1961-66, not only did the sex ratio among net migrants decrease but also the former concentration in the age distribution disappeared to a considerable extent.

(ii) The Overall Net Migration Rates and the Age-Sex Compositions of Net Migrants

A net migration rate may be defined in many different ways

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21 A.R. Downer, 'Our Story is One of Continuing Success', Digest of the Australian Citizenship Convention, Canberra, 1962, pp.12-14.

depending upon the practical and other considerations.[22] The definition given in Section 1.8, was adopted mainly because this definition makes it convenient to include the migration component into the process of population growth.

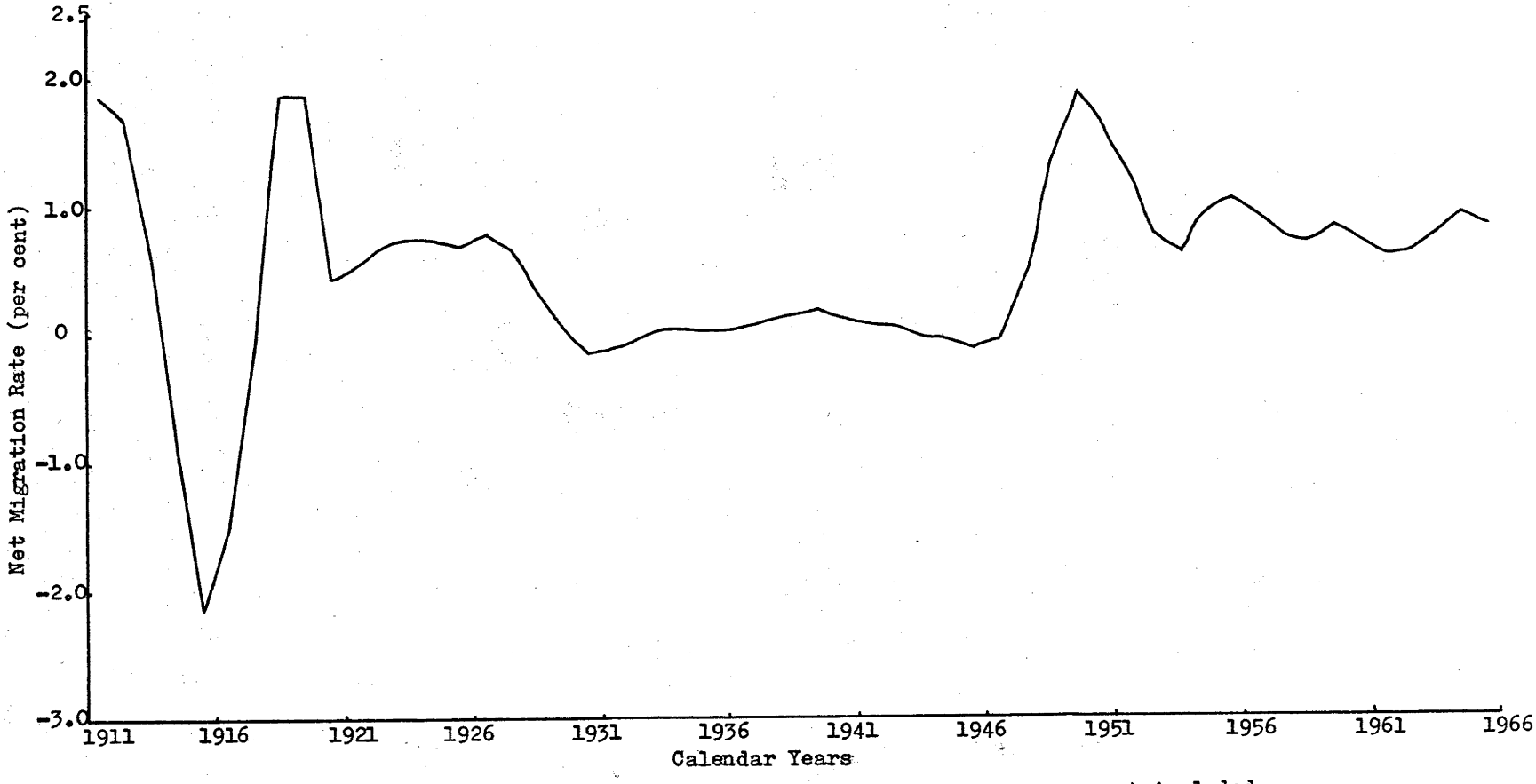
Figure 2.8 depicts the trend in the net migration rate in Australia during 1911-66. The graph describes the same features of fluctuations in migration in relation to the economic situation in the country and the policy adopted by the government, as revealed by the trend in the net numbers of migrants.

There were also large variations in the age-sex compositions of net migrants during this period. A rough idea of these changes has already been given. But Table 2.3 provides only an average picture for each of the intercensal periods and thus, the year to year variations are suppressed. Figure 2.9 presents the proportionate age distributions for four somewhat typical years and the corresponding sex distributions are given in Table 2.4. The years 1911-12 and 1965-66 were chosen to show the changes in these distributions over normal years. For both years, the age composition of net migrants contained positive values at all ages for males and females. The year 1915-16 was selected because during that year there was a large net emigration of males but a small net immigration of females which was mainly at ages below 28 years with net emigration at other ages; and 1945-46 was

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22 C.H. Hamilton, 'Practical and Mathematical Considerations in the Formulation and Selection of Migration Rates', Demography, Vol. 2, 1965, pp.429-443.

FIGURE 2.8: RATE OF NET MIGRATION DURING EACH OF THE FINANCIAL YEARS IN TERMS OF THE POPULATION AT THE BEGINNING OF THE YEAR: AUSTRALIA, 1911-12 TO 1965-66



Note: For 1914-19, the troop movements were included. For 1939-45, they were not included.

FIGURE 2.9: PERCENTAGE AGE DISTRIBUTIONS OF NET MIGRANTS BY SINGLE YEARS OF AGE: SELECTED YEARS, AUSTRALIA

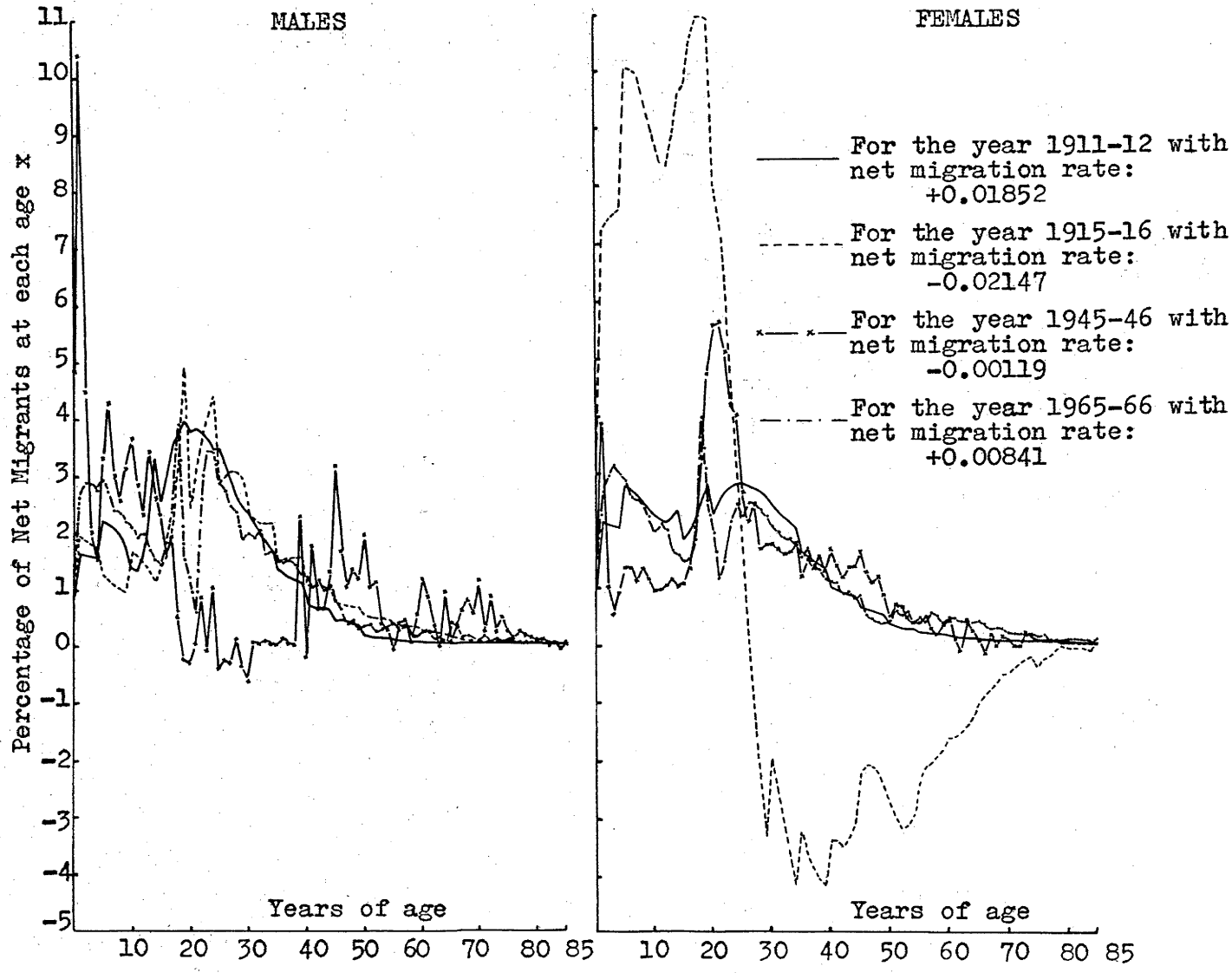


TABLE 2.4

NET MIGRATION RATE AND SEX PROPORTION AMONG NET MIGRANTS IN AUSTRALIA DURING SELECTED YEARS

Year	Net Rate	Male Proportion	Female Proportion
1911-12	+0.01852	0.65723	0.34277
1915-16	-0.02147	1.00921	-0.00921
1945-46	-0.00119	0.29150	0.70850
1965-66	+0.00841	0.51838	0.48162

[Source: Computed from the statistics of arrivals and departures for Australia.]

taken up because during that year there was net emigration at almost all ages for both males and females.

Table 2.4 brings out the practical difficulty inherent in using the net migration rate and the age-sex composition of net migrants for representing the migration situation such as the one during 1915-16. As far as the years 1911-12 and 1965-66 are concerned, there is little difficulty in interpreting the figures because both sexes showed net immigration during these years. Similarly, during 1945-46 both the sexes showed net emigration and the interpretation is quite clear. But the situation in 1915-16 needs a little care in interpreting the proportions.

(iii) Age-Sex-Specific Net Migration Rates

The age-specific net migration rates can also be defined in many alternative ways. The definition adopted in this study makes it

particularly easy to include the migration component into the process of population growth. The same definition has been used by other authors.[23] Figure 2.10 presents the age-specific net migration rates for males and females for the four selected years, viz. 1911-12, 1915-16, 1945-46 and 1965-66. It indicates that the effect of migration was more on the male population than on the female population. During 1915-16, due to the troop movements, the net migration rates for males were very high between the ages 15-40 years and were negative at all ages. The lessening of the concentration of migrants in the middle ages from 1911-12 to 1965-66 can be seen in this graph also.

2.3.4 THE SEX RATIO AT BIRTH

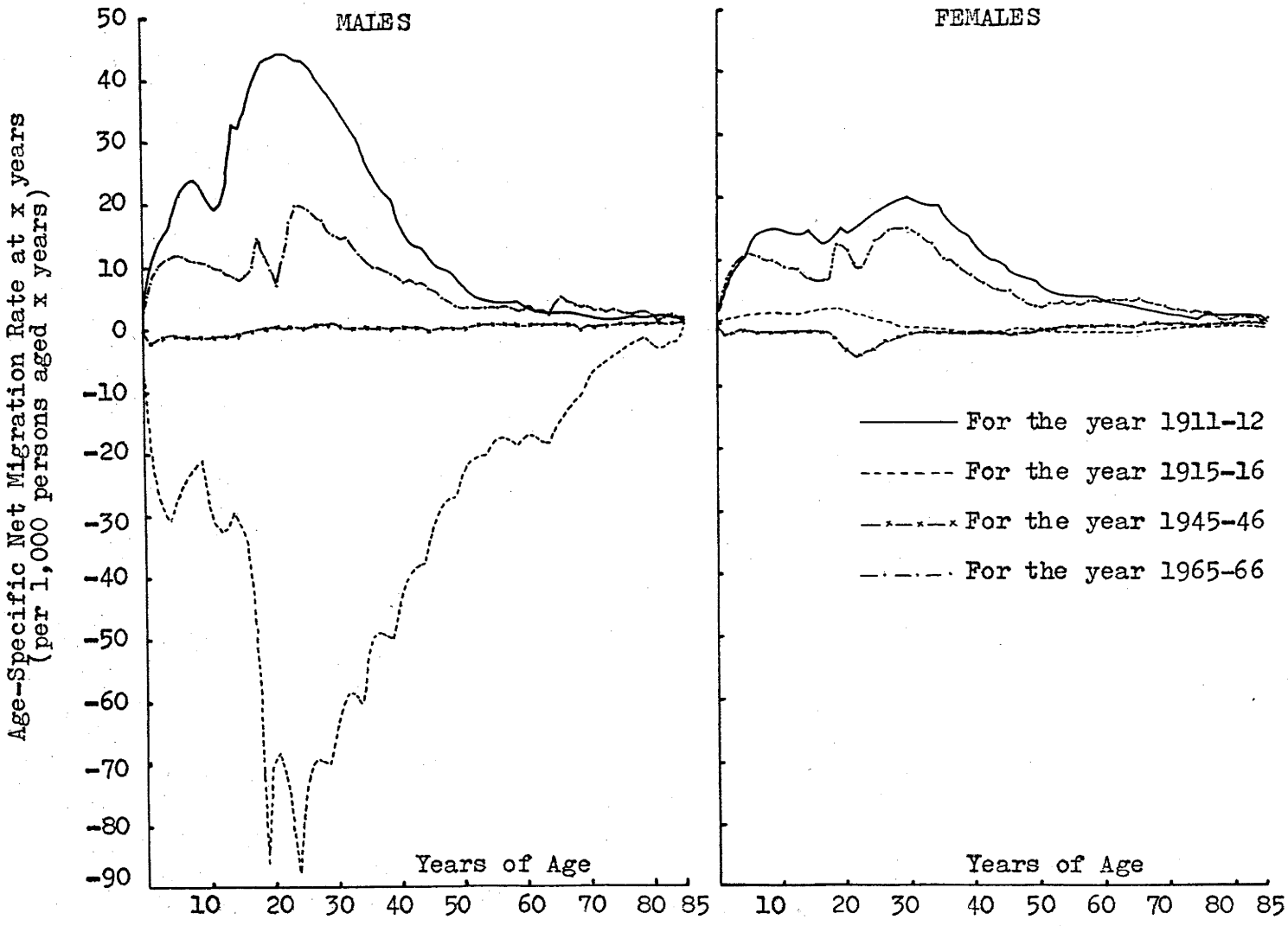
The other numerical data that was necessary for the construction of empirical models, was the sex ratio at birth. This would not be required if, instead of one set of age-specific fertility rates computed from all births, two sets of age-specific fertility rates - one using the male births and the other using the female births - are used. However, if the sex ratio at birth does not change with the age of the females or males, the two approaches would give the same results. In this study the sex ratio at birth was assumed to be constant over time and to be independent of the ages of females and

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23 A. Rogers, Matrix Analysis of ..., 1968.

L.O. Stone, 'Stable Migration Rates from the Multi-regional Growth Matrix Operator', Demography, Vol. 5, No. 1, 1968, pp.439-442.

FIGURE 2.10: AGE-SPECIFIC NET MIGRATION RATES AT SINGLE YEARS OF AGE: SELECTED YEARS, AUSTRALIA



males. Its value was taken as 105 males per 100 females.

A detailed study of the sex ratio at birth in Australia during 1902-65,[24] showed that it fluctuated around a mean value of 105.38 and did not show any apparent trend over the period. Though a decreasing linear relationship with increasing age of females was observed, the relationship was not strong, while no significant variation with the age of males was detected. In fact, the constancy of the sex ratio at birth has become a well recognized demographic phenomenon.

2.4 HYPOTHETICAL DATA USED IN THE STUDY

For examining the convergence of age-sex distributions under changing conditions of fertility, mortality and migration it was necessary to have a fairly long series of numerical data. The sex ratio at birth was assumed to be constant in these examples also and therefore, there was no need for any other assumptions. In the case of mortality, the survival rates observed in Australia during 1911-66, were taken for the first 56 years and those of 1966 were assumed to continue indefinitely with the presumption that significant changes in mortality are unlikely to occur in the absence of unforeseen natural or other calamities.

Hence, hypothetical data were derived only in the case of

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[24] G.N. Pollard, 'Factors Influencing the Sex Ratio at Birth in Australia', Journal of Biosocial Science, Vol. 1, No. 2, 1969, pp.125-144.

fertility and migration. The procedures adopted in obtaining the hypothetical data and a brief description of the characteristics of these data are given in this section.

(i) Fertility Rates

In the case of fertility also, the age-specific fertility rates by age of females observed in Australia during 1911-66 were used for the first 56 years. For the next two years, viz. 57th and 58th years, these were taken as observed in Australia during 1967-68, while for the later years these were derived by adopting the age pattern observed in 1968 to the values of TFR assumed as follows: for the 59th year, average of the TFR for the years 1918 and 1920; and for the 60th and the next 46 years, the values of TFR for the years 1920-66. The TFR for 1919 was not assumed for the 59th year because it was too low compared with the adjacent values. Finally, it was assumed that the sequence of fertility rates for the last 50 years, viz. from 57th to 106th years, would operate repeatedly so that the parabolic trend in TFR observed during 1911-61 would continue to occur over every fifty year period, but the age pattern would remain the same. This kind of estimation of fertility rates was used because it is very difficult to visualize the possible changes in fertility over such a long period of time. Also, the assumption of this kind of repetition enabled us to study the population change under a cyclical model. However, it must be mentioned that the sequence of fertility rates obtained is

completely hypothetical and should not be construed as a projection of fertility rates.

Now the question to be considered is whether the above assumptions regarding the annual fertility rates, lead to any impossible results in terms of the fertility of cohorts? In order to examine this question, cohort fertility schedules were prepared by using the annual age-specific fertility rates as obtained above. For the sake of comparison, an alternative sequence of annual fertility rates was also considered. In this sequence, it was assumed that the fertility rates for the first 56 years were as observed in Australia during 1911-66 and for all the later years as in 1966. Figure 2.11 depicts the trend in CTFR and the mean age of the cohort fertility schedules ($\overline{m}_c$) when the two assumptions were adopted. In analogy with TFR, the CTFR is defined as the total number of births a cohort would bear during its fertile lifetime if it is not affected by mortality during that interval, and is equal to the sum of the single year age-specific birth rates of the cohort fertility schedule.

It may be observed that both CTFR and $\overline{m}_c$ would have a parabolic trend when the annual TFR followed a parabolic trend and the age pattern of the annual fertility rates was assumed constant, as in the first assumption. On the other hand, when the annual fertility rates were assumed to become constant the CTFR and $\overline{m}_c$ also became constant after a comparatively short period of time. However, for the cohorts which commenced their fertility performance in 1961 or

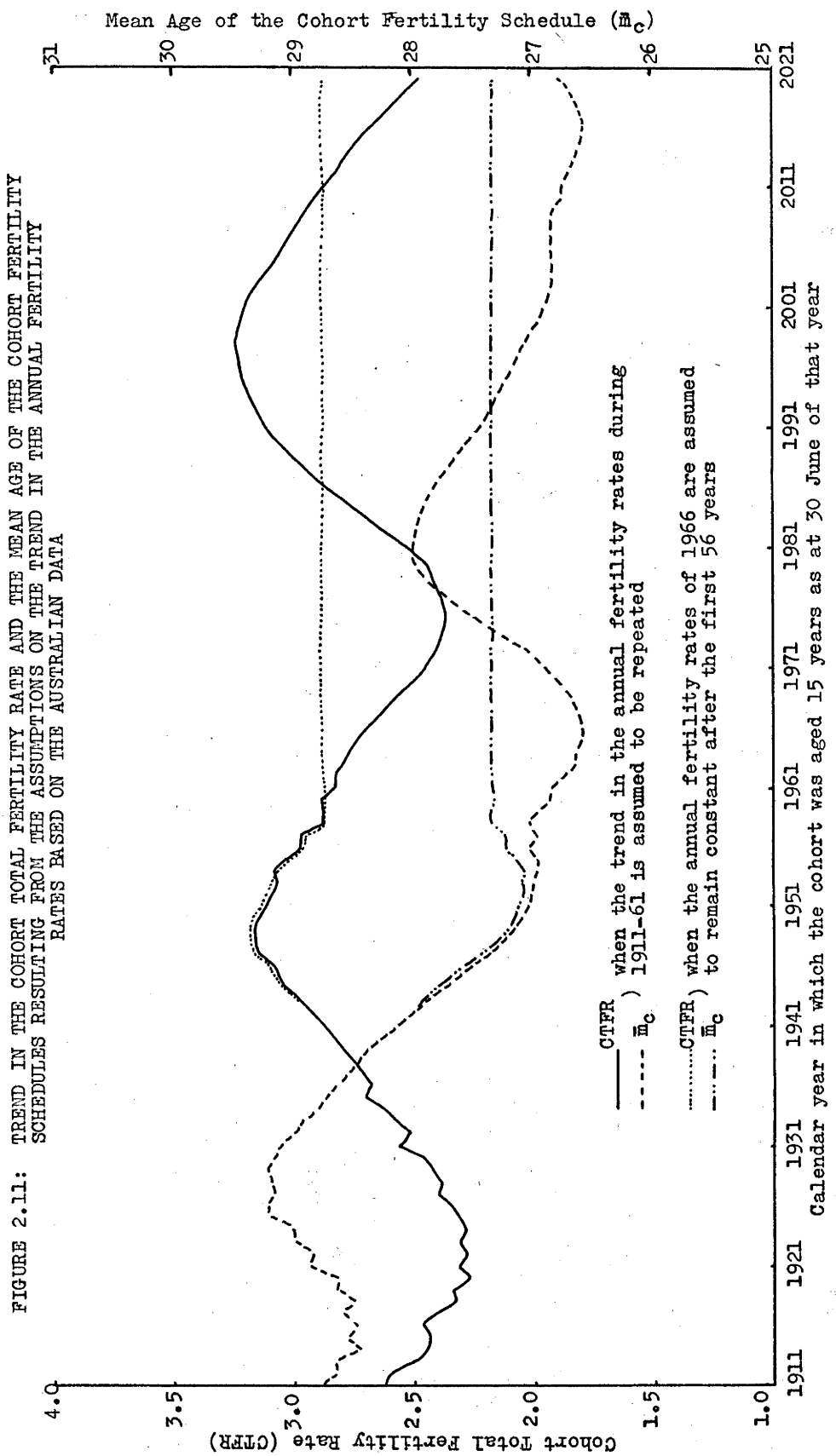


FIGURE 2.11: TREND IN THE COHORT TOTAL FERTILITY RATE AND THE MEAN AGE OF THE COHORT FERTILITY SCHEDULES RESULTING FROM THE ASSUMPTIONS ON THE TREND IN THE ANNUAL FERTILITY RATES BASED ON THE AUSTRALIAN DATA

earlier years both the assumptions indicated almost identical CTFR, though the first assumption suggested a lowering of the mean age of the cohort fertility schedule. Ryder[25] found a similar trend in CTFR from the data of the United States in respect of the cohorts born during 1891-95 to 1941-45. It may be noted that he used the year of birth to identify the cohorts whereas we have used the year in which the cohort was aged 15 years.

When the parabolic trend in the annual TFR was assumed, the CTFR fluctuated between 2.37 and 3.25 and the mean age between 26.59 to 28.01 years while, when the annual fertility rates became constant, these also became constant at 2.89 and 27.34, respectively. These fluctuations in CTFR and $\overline{m}_c$ do not appear to be abnormal as to invalidate the assumption.

(ii) Migration Data

In the case of migration, two alternative assumptions were considered. In the first one, the net migration rates and the age-sex compositions observed during 1911-12 to 1965-66 were assumed during the first 55 years and then the values for the year 1965-66 were taken to remain the same for all the later years. In the second one, it was assumed that the trend in the net migration rate would follow a Cosine curve with a maximum absolute value of 1 per cent. It was further

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25 N.B. Ryder, 'The Emergence of a Modern Fertility Pattern: United States, 1917-66', Fertility and Family Planning: A World View, University of Michigan, Michigan. (Memiographed) 1967.

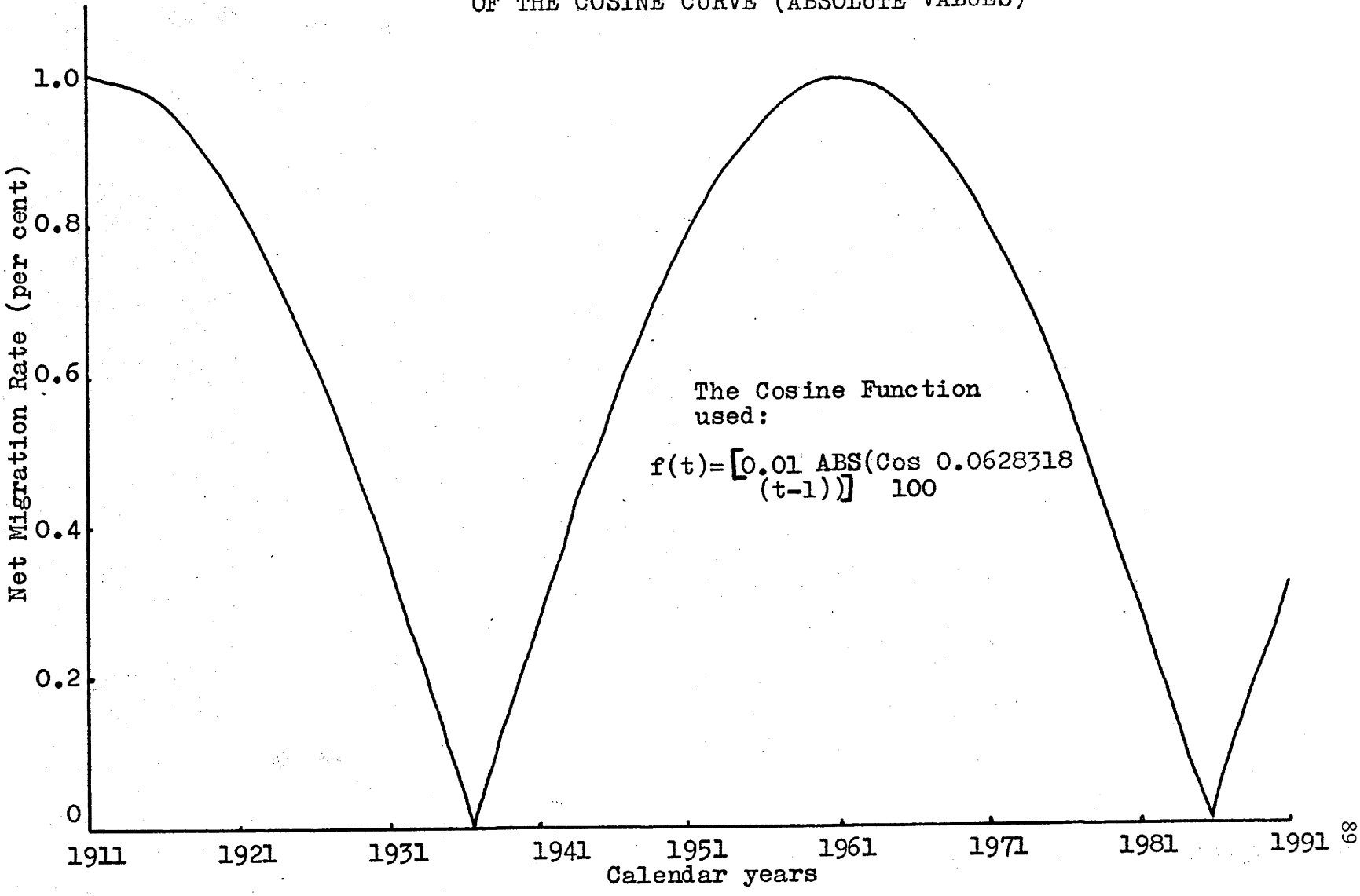
assumed that the period from peak to peak was 50 years so that the trend would resemble, very roughly, the trend observed in Australia during 1911-61, and would correspond to the trend in the fertility rates. Since migration is influenced by the economic conditions in the country and by the policies of the government, it is again very difficult to visualize the long term changes. Hence we have resorted to the use of mathematical curves. Further, the use of a mathematical curve such as the Cosine curve was advantageous because it became easy to manipulate the changes in the migration situation needed to test the analytical conclusions. For instance, we could easily increase the magnitude of migration by changing the absolute maximum value to 5 per cent. Similarly, we could change the trend in the net migration rate by assuming a Sine curve. The assumption of the Cosine curve also provided an example for the cyclical model of population change (see Section 5.3.3).

Figure 2.12 gives an idea of the trend in the net migration rate under the assumption of the Cosine curve. The trend when a Sine curve is assumed, would be exactly the opposite to that shown in the graph.

Along with these net migration rates, the age-sex composition of net migrants observed in Australia during 1962-66 was used. This was assumed to remain constant over all the years.

Several other age-sex distributions were also used as the age-sex compositions of net migrants in the different numerical

FIGURE 2.12: TREND IN THE NET MIGRATION RATE UNDER THE ASSUMPTION OF THE COSINE CURVE (ABSOLUTE VALUES)



experiments conducted under assumptions of unchanging conditions. Figure 2.13 and Table 2.5 compare these special age-sex compositions of net migrants.

TABLE 2.5

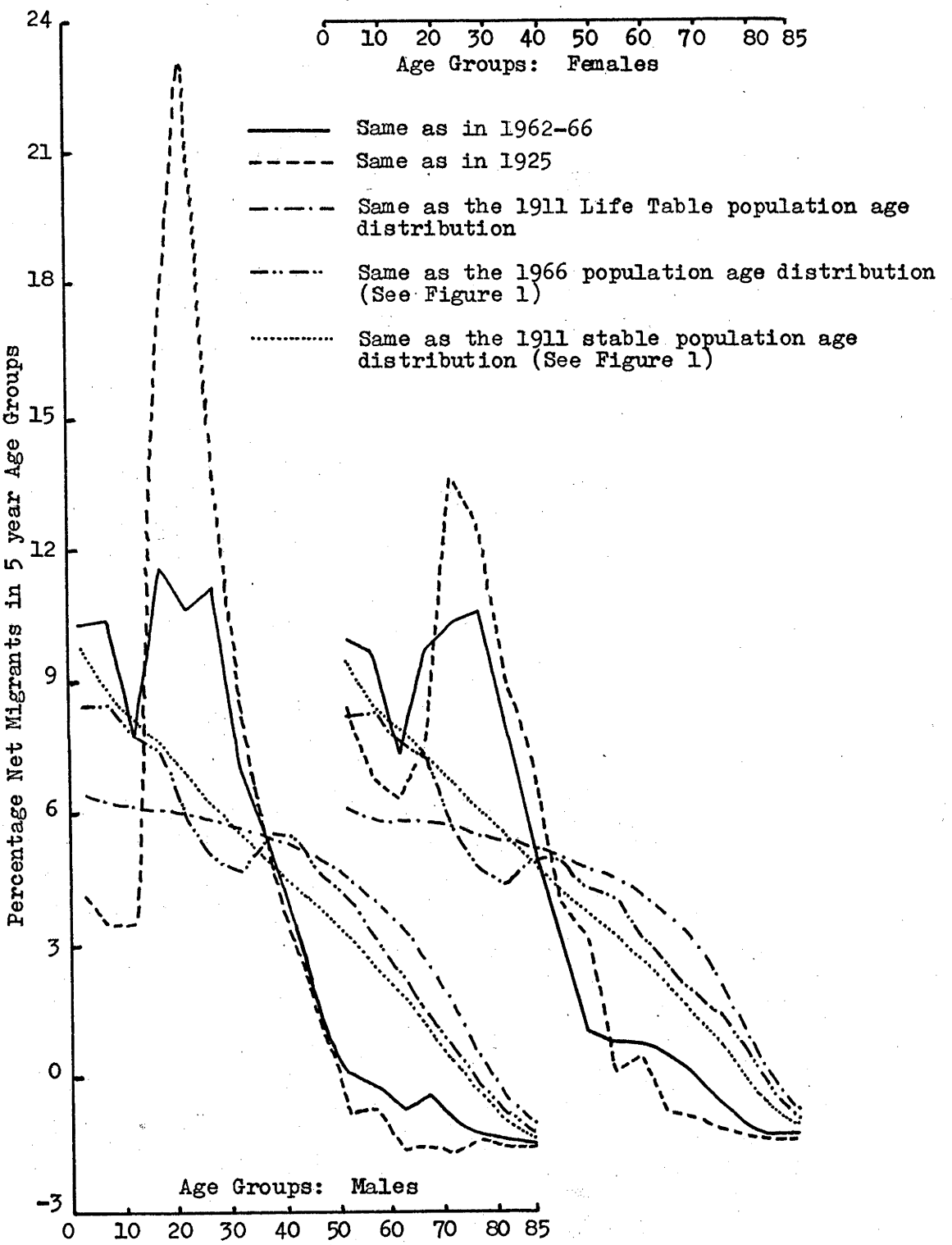
PROPORTION OF NET MIGRANTS BY SEX ASSOCIATED WITH THE SPECIAL AGE DISTRIBUTIONS OF NET MIGRANTS

Proportionate Age Distribution	Male Proportion	Female Proportion
Same as in 1962-66	0.50863	0.49137
Same as in 1925	0.68577	0.31423
Same as 1966 Obs. Population	0.50615	0.49385
Same as 1911 Stb. Population	0.50033	0.49967
Same as 1911 Life Table Population	0.49608	0.50392

[Source: Computed from the relevant data for Australia.]

It may be observed that the age-sex structures of net migrants during 1962-66 and in 1925 represented two distinct features of the age-sex compositions of net migrants. During 1962-66, there was only a small concentration in the age group (15-44) and a slight excess of males, whereas in 1925 there was a heavy concentration in that age group and a very high proportion of males. The other age-sex distributions were purely hypothetical, and were adopted for theoretical interest.

FIGURE 2.13: COMPARISON OF CERTAIN AGE COMPOSITIONS OF NET MIGRANTS, USED IN THE HYPOTHETICAL EXPERIMENTS



Since immigrants to one area are the emigrants from another, the age-sex composition of net migrants during 1962-66 was used both when net immigration and net emigration were assumed.

When the age-sex-specific net migration rates were used in the process of population growth, it was assumed that the age-sex-specific net migration rates observed in Australia during 1911-12 to 1965-66 would operate during the first 55 years and the values for 1965-66 would remain the same for all the later years. No hypothetical data were derived in this case.

2.5 SUMMARY

The results of the analysis of the basic numerical data given in this chapter, show that the age-sex distributions of the initial populations considered for the study were quite diverse, the changes in the survival rates observed in Australia during 1911-66 were not very significant except at the very young ages and at the old ages, and there were large variations in the fertility rates and the net migration. The sex ratio at birth did not show any trend over time or any significant relationship with the increase in the age of females or males.

The hypothetical fertility rates were derived such that the trend in TFR observed in Australia during 1911-61 repeated over every fifty year period, but the age pattern observed in 1968 remained unchanged after the first 56 years. On the other hand, the

hypothetical net migration rates were obtained by using Cosine and Sine curves and the age-sex composition of net migrants was kept the same as observed in Australia during 1962-66.

The different data on the components used in the numerical examples are displayed in Table 2.6 for the convenience of reference.

TABLE 2.6

NUMERICAL DATA USED IN THE STUDY

	Mortality	Fertility	Net Migration Rate	Migration: Age-Sex Composition of Net Migrants	Migration: Age-Sex-Specific Net Migration Rates
Constant Conditions	1911 1966	1911 1966	(i) 1911-12 1915-16 1945-46 1965-66 (ii) - 1.0% 0.5% 1.0% 5.0%	(i) 1911-12 1915-16 1945-46 1965-66 (ii) Same as in 1962-66 Same as in 1925 Same as 1911 Stb. Population Same as 1966 Obs. Population Same as 1911 Life Table Population	1911-12 1915-16 1945-46 1965-66
Changing Conditions	First 56 years: 1911-66 57th Year and Later: 1966	First 56 Years: 1911-66 57th and 58th Years: 1967-68 59th to 106th Years: Average TFR for 1918 and 1920, and TFRs for 1920-66 with Age Pattern as in 1968. 107th Year and Later: 57th to 106th Years Repeat.	(i) First 55 Years: 1911-12 to 1965-66 56th Year and Later: 1965-66 (ii) Cosine and Sine Curves	(i) First 55 Years: 1911-12 to 1965-66 56th Year and Later: 1965-66 (ii) Same as in 1962-66	First 55 Years: 1911-12 to 1965-66 56th Year and Later: 1965-66

Note: The calendar years in this table indicate that the data observed in Australia for those years were used in the study. The net numbers of migrants by age and sex for the period 1911-66 were also used.

PART II

CONVERGENCE OF AGE-SEX DISTRIBUTIONS

CHAPTER 3

CONVERGENCE OF AGE-SEX DISTRIBUTIONS TO AN EQUILIBRIUM STATE AGE-SEX DISTRIBUTION

3.1 INTRODUCTION

The classical theory of stable populations has shown that, starting from an arbitrary state, any human age-sex structure reaches, in the absence of migration, a stable form which is independent of the initial structure if the fertility and mortality rates operating on it remain constant for a sufficiently long period of time. Lopez[1] indicated that the same would hold good when migration is included into the process by incorporating a set of age-specific net migration rates into the survival rates. But it is not known whether such a convergence would occur when an overall net migration rate and an age-sex composition of net migrants are utilized in the process of population growth.

Among the recent studies connected with the stable population theory, Coale[2] has drawn attention to an aspect which was not properly treated so far, that is the study of the duration of the process of convergence of an age distribution to the stable form. McFarland[3]

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1 A. Lopez, Some Problems in Stable Population Theory, 1961, p.64.

2 A.J. Coale, 'Convergence of a Human Population to a Stable Form' Journal of the American Statistical Association, Vol. 63, No. 322, 1968, pp.395-435.

3 D.D. McFarland, 'On the Theory of Stable Populations: A New and Elementary Proof of the Theorems under Weaker Assumptions', Demography, Vol. 6, No. 3, 1969, pp.301-322.

examined this problem in the general case in which the fertility and mortality rates were assumed to be changing over time, and obtained a formula to compute the duration of convergence in terms of the youngest and the oldest ages at which child bearing takes place. But again, what effect the introduction of migration into the process of population change has on the duration of convergence, has not been investigated.

In this chapter, our main concern will be to discuss these two questions. First, we consider the one-sex case with no migration and derive, under certain approximation, a formula which would give the duration of convergence in terms of the characteristics of the net maternity function.

For the purpose of this chapter, we define the duration of convergence as the number of years needed for the difference between an arbitrary age-sex distribution and the equilibrium state age-sex distribution, to become less than a pre-assigned small quantity, from the time the specified schedules of fertility, mortality and migration start operating.

Then we show that the convergence to the equilibrium state age-sex distribution occurs when migration is included into the process of population growth using an overall net migration rate and an age-sex distribution of net migrants at the time of migration, and discuss the changes in the duration of convergence due to the inclusion of migration. Since the duration of convergence is very closely related

to the intrinsic growth rate, some comments on the changes in the latter are also included here, though, strictly, this topic belongs to Chapter 5. However, the relationship between the intrinsic growth rate and the equilibrium state age-sex distribution on the one hand, and the given schedules of fertility, mortality and migration on the other, will be examined in Chapter 5.

Finally, we extend the results to the two-sex model and present some numerical illustrations using the two-sex model.

The question whether the 'forgetting' of the initial shape of the age-sex distribution which happens under constant conditions, would occur when these conditions are changing over time, will be taken up in the next chapter.

3.2 ONE-SEX MODEL

3.2.1 ONE-SEX MODEL WITH NO MIGRATION

The process through time which leads to the stable state could be studied by successively projecting the population over time and comparing the percentage age distribution at time t with that at time (t-1) or with the one obtained by applying the stable population equation:

$$p(x) = b\, e^{-rx}\, s(x) \qquad (3.1)$$

where $p(x)$ is the proportion of the population at age x years, b and r

the constant birth and growth rates and s(x) the proportion surviving to age x years. The stable population is established when the difference between the two age distributions being compared, becomes smaller than a pre-assigned small quantity. The value of t, then, is the duration of the process of convergence.

Alternatively, we may decompose the sequence of births as the sum of a real exponential term and a series of relatively diminishing oscillatory terms and claim that

> ... the stable population is established no more than w years after the number of births last deviates by an arbitrarily small proportionate amount from (the) pure exponential sequence, where w is the highest age attained under the given mortality regime.[4]

We shall follow the procedure of population projections since it facilitates the inclusion of the migration component.

Let P(x,t) be the female (or male) population aged x years at time t. Then the population at time t can be obtained from that at time (t-1) by using the following equations:

$$P(0,t) = S(b) \sum_{\alpha}^{\beta} \frac{1}{2}[P(x,t-1) + P(x,t)]f(x) \qquad (3.2)$$

and $P(x,t) = S(x-1)\ P(x-1,t-1)$, for $x = 1, 2, \ldots, w$ (3.3)

where the absence of t in the symbols for fertility and survival rates indicates that these are constant over time.

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[4] A.J. Coale, 'Convergence of ...', Journal of the American Statistical Association, 1968, p.396.

Making use of the matrix notation the above equations can be written as:

$$(P_t) = L(P_{t-1}) \tag{3.4}$$

where (P_t) denotes a column vector giving the population by age which we shall call the population vector; and L is the matrix given below:

$$L = \begin{pmatrix} 0 & 0 & \cdots & 0 & m(\alpha) & m(\alpha+1) & \cdots & m(\beta) & 0 & \cdots & 0 & 0 \\ S(0) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & S(1) & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & S(w-1) & 0 \end{pmatrix}$$

in which $$m(x) = \frac{1}{2}[f(x) + S(x)\ f(x+1)]\ S(b)\ . \tag{3.5}$$

The matrix representation has an advantage in studying the process of population growth since some of the aspects of population growth could be studied conveniently by the properties of the matrix L.

The repeated application of the equation (3.4) gives:

$$(P_t) = L^t\ (P_0) \tag{3.6}$$

where (P_0) is the initial population vector.

We can partition the matrix L at the $(\beta+1)$th row and column, and write it as follows:

$$L = \begin{pmatrix} M & 0 \\ A & B \end{pmatrix}\ ,$$

then it can be shown that, in any power of L, the elements of the first row will not be affected by the elements in the second row and the resultant matrix will have the same form as L. Hence, the growth of the population at ages below β years (age β years is the oldest age at which reproduction occurs) at any time, is independent of the population at ages beyond β years.[5] But, in course of time, the populations at ages beyond β years become linear functions of the populations at ages below β years. The coefficients of the functions are determined entirely by the known fertility and mortality schedules. Therefore, the intrinsic growth rate obtained using the populations at ages below β years, must apply to the whole of the age range. In fact, it can be shown that the non-zero characteristic roots of the matrix L are the same as those of its sub-matrix M:

$$M = \begin{pmatrix} 0 & 0 & \cdots & 0 & m(\alpha) & m(\alpha+1) & \cdots & m(\beta-1) & m(\beta) \\ S(0) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & S(1) & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & S(\beta-1) & 0 \end{pmatrix} .$$

Hence it suffices, in this case, to study the sub-matrix M. This matrix is an irreducible non-negative square matrix which is primitive, i.e. when it is raised to some powers it becomes a matrix with all

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[5] N. Keyfitz, Introduction to the Mathematics ..., 1968, p.37.

elements positive. It has therefore a real positive characteristic root, λ_1, which is unique and whose value is greater than the absolute value of any other root, and corresponding to λ_1, there exists a characteristic vector whose elements are positive. The matrix M will be primitive whenever there are at least two consecutive ages at which the fertility rates are strictly positive.[6] As far as human populations are concerned, this requirement appears to be always satisfied.

Let λ_2 be another characteristic root of M, whose absolute value is greater than that of any other characteristic root except λ_1. We assume that λ_2 is also distinct. In fact, in demographic applications, it is observed that all the characteristic roots are distinct.[7]

Now, if we denote the population vector below the age β years as K, then we have:

$$(K_t) = M^t \ (K_0) \tag{3.7}$$

When t is sufficiently large, we assert that:

$$(K_t) \simeq \lambda_1^t \ C_1 \ (V_1) \tag{3.8}$$

which implies that the other terms become relatively negligible. Here

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6 J.H. Pollard, 'On the Use of Direct Matrix Product ...', Biometrika, 1966.

7 N. Keyfitz, Introduction to the Mathematics ..., 1968, p.51.

C_1 is called the stable equivalent and (V_1) the stable age distribution.[8] Since λ_2 is the root highest in absolute value next to λ_1 and is distinct, the ratio $[|\lambda_2|/|\lambda_1|]$ determines the value of t, the duration of convergence. If λ_2 is a complex root, then $|\lambda_2| = |\lambda_3|$ because λ_3 will be the complex conjugate of λ_2 and we can use any one of them to determine t. In fact, t can be calculated as follows:

$$[|\lambda_2|/|\lambda_1|]^t = \varepsilon \tag{3.9}$$

where ε is a small quantity chosen arbitrarily to ensure the required approximation. Hence

$$t = [(\log \varepsilon)/\{\log |\lambda_2| - \log |\lambda_1|\}] \tag{3.10}$$

and we may take the highest integer contained in the expression plus one as the value of t. It can easily be seen that the value of t is always positive as it should be. Thus the examination of the duration of convergence reduces to the study of λ_1 and λ_2 of the characteristic roots of the projection matrix. λ_1 also gives the intrinsic growth rate.

For the convenience of interpretation, it is better to transform the λs into r's by the transformation $\lambda = e^r$. Then r_1 corresponding to the real positive root λ_1 is the intrinsic rate of natural increase of the stable population and $r_2 = y + i\,z$ (or $r_2 = y - i\,z$), where y and z are real quantities, corresponding to the

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[8] N. Keyfitz, Introduction to the Mathematics ..., 1968, p.57.

complex root λ_2 (or λ_3) determines the rate of dampening of the waves of the oscillatory terms and their wavelength which is given by $(2\pi/z)$.[9] In terms of r_1 and r_2, equation (3.10) becomes:

$$t = [(\log_e \varepsilon)/(y - r_1)] \tag{3.11}$$

in which y will be negative in practice, or if positive it must be less than r_1, because the dampening will not occur otherwise. Again, it will have to be greater than r_1 whenever r_1 is negative because λ_1 will not be greater than the absolute value of λ_2 otherwise. Hence t is always positive as it should be. It may also be inferred that as $(y - r_1)$ increases in magnitude the value of t decreases and vice versa.

In demographic studies r_1 and r_2 can be approximated in terms of the moments or the cumulants of the net maternity function:

$$F(x) = [S(b)\ S(0)\ S(1)\ \ldots\ S(x-1)]\ f(x)\ . \tag{3.12}$$

To achieve this, different authors have used different curve fitting methods of which we shall choose here the Pearson Type III curve used by Wicksell[10] since it provides more flexible curves.

Let $R_i = \sum_{\alpha}^{\beta} x^i F(x)$. Then $\mu = [R_1/R_0]$ and $\sigma^2 = [R_2/R_0] - \mu^2$ are the mean and variance of the net maternity curve. Let $v = [\sigma/\mu]$ be the coefficient of variation. Adopting the Pearson Type III curve to

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9 N. Keyfitz, Introduction to the Mathematics ..., 1968, p.73.

10 N. Keyfitz, Introduction to the Mathematics ..., 1968, pp.147-157.

$F(x)$, we can write the values of r_1 and r_2 as follows:

$$r_1 = C[R_0^{1/k} - 1] \tag{3.13}$$

and

$$r_2 = C[R_0^{1/k}\, e^{2\pi i/k} - 1]$$

$$= C[R_0^{1/k}\, \mathrm{Cos}(2\pi/k) - 1] + i\, C\, R_0^{1/k}\, \mathrm{Sin}(2\pi/k) \tag{3.14}$$

where $C = [1/(\mu\, v^2)]$ and $k = [1/v^2]$.

Hence under this approximation, we have:

$$t = \frac{\log_e \varepsilon}{C\, R_0^{1/k}\, [\mathrm{Cos}(2\pi/k) - 1]} = \frac{(\log_e \varepsilon)\, \mu\, v^2}{R_0^{v^2}\, [\mathrm{Cos}(2\,\pi\, v^2) - 1]} \tag{3.15}$$

which is always positive for small ε.

When v is small (which is generally the case in human populations) we can write the equation (3.15) as:

$$t = -\frac{(\log_e \varepsilon)\, \mu}{R_0^{v^2}\, 2\pi^2\, v^2} \tag{3.15a}$$

If $R_0 = 1.0$, this equation becomes the same as that which would be obtained by fitting the Hadwiger curve to the net maternity function. From equation (3.15a) we can see that the duration of convergence is directly proportional to the mean and inversely proportional to the total density and the square of the coefficient of variation of the net maternity function. For a given R_0, if $v = 0$, t becomes infinity. In this case, the density of the net maternity function is concentrated at one age and the oscillations never die out. When v is large the above approximation may not be satisfactory.

On the other hand, for given μ and σ, which implies that the age pattern of the net maternity rates is unchanged, t decreases from ∞ to 0 as R_0 increases from 0 to ∞, for theoretical considerations.

The value of R_0, in practice, is rarely very near to 0 or greater than 4 for the human populations.[11]

In any actual situation, however, we can calculate the intrinsic growth rate (r_1), and the duration of convergence (t), from the given schedules of fertility and mortality by computing the total density R_0 (i.e. the net reproduction rate), the mean and the coefficient of variation of the net maternity function.

But it must be noted that the formula (3.15) does not take account of the nature of the initial age structure. Therefore, the number of years needed for the convergence of any particular age distribution, under the operation of the given fertility and mortality conditions, may be far less than the number obtained from this formula, depending on how near the initial age structure is to the stable one resulting from those fertility and mortality conditions. In fact, if the initial age distribution is the stable one itself, then the required t is zero. Further, the value of t, computed here, refers to the number of years required for the convergence of the age distributions below the age β years. Hence, unless this value of t is greater than (w+1) years which is the time required for the cohorts at all ages in the initial population to die out, it must be increased to make it greater than (w+1) years, when we consider the convergence of the age distributions over the whole age range (0-w) years.

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[11] United Nations, The Concept of a Stable Population ..., 1968, p.22.

In order to examine the nature of the process of convergence, we may construct the population series using the values of r_1 and r_2 calculated from the formulas (3.13) and (3.14); or in any particular case, we may draw the graph of the maximum (MaxGI) and minimum (MinGI) of the growth indexes at individual ages (see, for example, Fig. 3.1). A growth index at age x years is defined as the ratio of the population at that age at time t to the population at the same age at time (t-1). Lotka[12] used the stable populations corresponding to the MaxGI and MinGI at each point of time, to show that the resultant age distribution is stable. However, it is well known now that, for a stable population, the growth rates at all ages are constant and are equal to the growth rate of the total population. Therefore, when MaxGI and MinGI differ negligibly we may conclude that the age distribution under consideration has reached the stable state. In fact, this criterion was used to test the convergence of the age-sex distributions in the numerical examples.

3.2.2 ONE-SEX MODEL WITH MIGRATION

If N(x) denotes the net number of migrants of age x years arriving (or departing) into (or out of) the population, respectively, during a year, then the overall net migration rate n, is computed as:

$$n = \{\sum_{0}^{w} N(x)\}/P \qquad (3.16)$$

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[12] A.J. Lotka, 'The Stability of the Normal Age Distribution', Proceedings of the National Academy of Sciences, Vol. 8, 1922, pp.339-345.

where P is the size of the total population at the beginning of that year, and their age distribution at the time of migration as:

$$n(x) = N(x)/\{\sum_{0}^{w} N(x)\} \ . \qquad (3.17)$$

The absence of t in n and n(x) denotes that they are constant over time. Now the set of equations which enable us to calculate the population at time t in terms of the one at (t-1), are:

$$P(0,t) = S(b) \sum_{\alpha}^{\beta} \frac{1}{2} \{P(x,t-1) + S(x-1)\ P(x-1,t-1)\}\ f(x) + u \sum_{0}^{w} P(x,t-1) \qquad (3.18)$$

$$\text{and} \quad P(x,t) = S(x-1)\ P(x-1,t-1) + a(x-1) \sum_{0}^{w} P(x,t-1), \text{ for } x = 1, 2, \ldots, w, \qquad (3.19)$$

$$\text{where} \quad u = S(b) \sum_{\alpha}^{\beta} \frac{1}{2} [a(x-1)\ f(x)] + \frac{1}{4}[(1 + S(0))\ n(0)\ n]$$

$$\text{and } a(x-1) = \frac{1}{4} [(1 + S(x-1))\ n(x-1) + (1 + S(x))\ n(x)]n$$

in which the factor $\frac{1}{4}$ arises because of assumption (7) [see Section 1.8.].

In the case of the last age group, if it is an open end age group, a small adjustment may be made in the value of a(x) as for S(x). However, that will not affect the discussion here.

If we use separate schedules of immigrants and emigrants, we need to change only the values of [n(x)n] in the expressions for u and a(x), which will then be

$$[n(x)\ n] = [I(x)\ I] - [0(x)\ 0], \quad \text{for } x = 0, 1, 2, \ldots, w$$

where I and O denote the gross immigration and emigration rates in terms of the total population at the beginning of the year, and I(x) and O(x) the corresponding proportions at age x years.

We can write the growth equations (3.18) and (3.19) as the matrix equation:

$$(P_t) = L_M \, (P_{t-1}) \tag{3.20}$$

where L_M is the matrix:

$$\begin{pmatrix}
u & . & u & [m(\alpha)+u] & . & [m(\beta)+u] & u & . & u & u \\
[S(0)+a(0)] & . & a(0) & a(0) & . & a(0) & a(0) & . & a(0) & a(0) \\
a(1) & . & a(1) & a(1) & . & a(1) & a(1) & . & a(1) & a(1) \\
. & . & . & . & . & . & . & . & . & . \\
a(w-1) & . & a(w-1) & a(w-1) & . & a(w-1) & a(w-1) & . & [S(w-1)+a(w-1)] & a(w-1)
\end{pmatrix}$$

with m(x) same as in L.

The characteristic roots λ_1 and λ_2 of this matrix L_M, determine the ultimate growth rate and the time required to attain the equilibrium state. In this case also, we can show (see Section 4.2.2) that, in course of time, the populations at ages beyond β years, become linear functions of the populations at ages below β years. The coefficients of these functions are determined entirely by the known fertility, mortality and migration schedules. This means that the

elements in the first $(\beta+1)$ rows in the matrix L_M play an important role in the population growth. We shall make use of this fact at a later stage in our argument.

Now our problem is to investigate whether the matrix L_M has a positive dominant characteristic root and a corresponding characteristic vector of positive elements, and then to see how the value of t is affected. In this case, it is not possible to make an immediate general statement as in the case with no migration, because the effect of migration is not always in one direction. There may be immigration at all ages, emigration at all ages, or immigration at some ages and emigration at some others etc. Migration may also be zero at some of the ages. Hence we shall discuss certain important possibilities which would indicate the general effect of introducing migration.

First, let us assume that $n(x)$ are all positive. If n is also positive, then we have the case where there is net immigration at all ages. In this case, u and all $a(x)$ are positive so that the matrix L_M is itself a positive matrix. Hence, from the properties of positive square matrices, we can conclude that it has a unique real positive characteristic root λ_1 and a corresponding characteristic vector of positive elements exists.[13] The real dominant root gives the intrinsic growth rate and the characteristic vector yields the age distribution of the equilibrium state population.

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[13] F.R. Gantmacher, The Theory of Matrices, translated by K.A. Hirsch, Chelsea Publishing Co., New York, N.Y., 1960, Vol. II, p.53.

We assume again that λ_2, the characteristic root of L_M whose absolute value is greater than that of any other characteristic root except λ_1, is distinct. λ_2 may be complex. It was observed in our applications that all the characteristic roots of L_M were distinct and λ_2 was, in fact, complex. When λ_2 is a complex root, we may use either λ_2 or λ_3 for obtaining the duration of convergence. Since the characteristic equation of L_M is an equation with real coefficients, the complex roots occur in conjugate pairs.[14] Now, using the value of λ_1 and λ_2, we can compute the duration of convergence from formula (3.10).

Comparing the values of λ_1 and λ_2 in this case with those obtained in the case with no migration, we can infer the effect of immigration on the intrinsic growth rate and on the duration of convergence. We know from the properties of nonnegative square matrices that λ_1 increases when any element of a power positive matrix increases.[15] Since the value of λ_1 for the matrix L is the same as that for M, and since the elements of L_M are larger than those of L, we conclude that, in this case, the value of λ_1 for L_M is larger than for M. Thus it is easy to conclude that the intrinsic growth rate is greater when there is immigration than when there is no migration. But we cannot say much about the associated characteristic vectors except to say that they both contain positive elements. Accordingly,

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14 W.S. Burnside and A.W. Panton, The Theory of Equations, Dover Publications Inc., New York, 1960, Vol. I, p.26.

15 F.R. Gantmacher, The Theory of Matrices, 1960, p.63.

the comparative picture of the characteristics of the equilibrium state age distributions resulting in the two cases is not very clear. However, it is possible to find explicit relationships between these resultant age distributions and the operating schedules on which they depend. This question will be taken up in Chapter 5.

Also, we know very little about the second dominant root, λ_2. Therefore, it seems difficult to draw any conclusions regarding the effect of migration on the duration of convergence. But in demographic applications, it appears to be possible to infer from certain indirect evidences. The matrix L_M is a positive matrix whereas the corresponding matrix L is not and will not become a positive matrix even when raised to its powers. But we have seen that the convergence of the whole age distribution in the absence of migration, depends on the matrix M. The matrix M becomes positive only when it is raised to certain powers. Therefore, the averaging effect on the initial age distribution of the population, which, in the presence of immigration, starts happening at all ages from the beginning, can happen in the absence of migration only after M has become a matrix of positive elements. Further, the averaging effect at every age in the present case, involves the values at all the other ages and continues to occur throughout the whole age range. Whereas in the case with no migration, it includes only the values at ages below β years and continues only up to the age β years. Hence, we can conclude that the occurrence of immigration at all ages reduces the duration of convergence. We can

also infer that the effect of averaging would increase in this case, with the increase in the magnitude of the net migration rate because the dominance of one age group in determining the population in the next age group in the succeeding year, is further reduced. Therefore, the duration of convergence in this case, would decrease as the net immigration rate increases. These conclusions are supported by the empirical evidences given in Section 3.4.

When some of the a(x) are zero and all the rest are positive (i.e. n(x) are zero for some values of x and n is positive), we have the case where there is overall net immigration but it does not affect all ages. Then the matrix L_M ceases to be a positive matrix. However, if there is at least one age below β years at which a(x) is not zero, L_M will be an irreducible non-negative square matrix. Therefore, it has a unique real positive dominant root and associated with it there exists a characteristic vector of positive elements. This dominant root is, again, larger than the dominant root in the case with no migration. In order to comment on the changes in the duration of convergence, we may note that it is necessary in this case, to raise L_M to some power before it becomes a positive matrix. But the power to which the matrix L_M has to be raised to obtain positive elements in the first $(\beta+1)$ rows at least, is smaller than that needed to make the matrix M a positive matrix. This is because the row corresponding to the non-zero a(x) would contain all positive elements and every other row of the matrix contains at least one positive element at the same

place as in M. If the non-zero a(x) happens to be for some x which is between α and β years, then u is also positive and this increases the number of positive elements in the matrix. Further, in both cases, the populations at ages beyond β years, become linear functions of those at ages below β years. Hence in this case, we may infer that the duration of convergence will be greater than that in the case where immigration occurs at all ages; but is less than that in the case with no migration. If, however, the a(x) are zero at all ages below β years, then the matrix L_M is not irreducible and we shall consider this case later as a special case.

On the other hand, if n(x) are all positive but n is negative, then we have the case where there is net emigration at all ages. It may be mentioned here that the case in which n(x) are all negative and n is positive is not admissible. In this case, u and all a(x) are negative and therefore, the matrix L_M will not even be a non-negative matrix. But, it is not difficult to hypothesize that, when the matrix L_M is raised to its powers, at some stage it will become a matrix of positive elements because, by assumption (8) which is essential in demography, the values of u and a(x) whenever they are negative must be small compared to S(x) and some of the m(x) values. Hence, the matrix L_M can be assumed to be a power positive matrix. Then from the properties of power positive matrices[16] we can conclude

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16 A. Brauer, 'On the Characteristic Roots of Power Positive Matrices', Duke Mathematical Journal, Vol. 28, 1961, pp.439-445.

that, in this case also, the matrix L_M will have a unique real dominant characteristic root, and a corresponding characteristic vector with positive elements exists. The real dominant root will be positive if the row sums are all positive. This condition seems to be satisfied always in demographic applications.

Thus, we can conclude that the convergence to the equilibrium state age-sex distribution occurs in this case also. The duration of convergence will be more than in the absence of migration because the power to which the matrix L_M has to be raised to obtain all positive elements in the first $(\beta+1)$ rows at least, will be higher than that needed for the matrix M to become a positive matrix. But, the intrinsic growth rate will be less than that in the absence of migration because the row sums of the matrix L_M, in this case, are smaller than those of the matrix L.

If we now assume that some of the $n(x)$ are zero and the others are positive, then it implies that there is overall net emigration but it does not affect all ages. In this case, some $a(x)$ are zero and the corresponding rows in the matrix L_M will not contain negative values. Thus, only the number of negative elements in the matrix is reduced. Therefore, the convergence occurs. But, if there is at least one age below β years at which $a(x)$ is not zero, the matrix L_M cannot be partitioned as the matrix L. Hence, we may conclude that the duration is more than in the case with no migration but less than in the case with emigration at all ages, and the intrinsic growth rate

follows the opposite trend. If the a(x) are all zero below the age β years, then we have again the special case which will be considered later.

Sometimes, when the net emigration rate is large or when the age structure of the net emigrants is abnormally concentrated in some age groups or both, it may happen that the matrix L_M will not be a power positive matrix. In that case, either the dominant root would become negative or if it is positive, the characteristic vector associated with it contains negative elements. Such a result clearly indicates that the given net emigration conditions cannot prevail for a long time under the given set of fertility and mortality conditions. Therefore, we may call such a situation an incompatible situation. It may be noted that the same set of emigration conditions may be compatible with a certain set of fertility and mortality conditions but not with others.

Thus, by calculating the dominant characteristic root of the matrix L_M and the associated characteristic vector we can judge whether or not the given fertility and mortality conditions can sustain a given rate of net emigration with a certain age composition of net migrants. As a preliminary, we may check whether the sums of all the rows of the matrix L_M are positive or not. If any of the row sums are negative, the dominant root may be negative or if it is positive, the characteristic vector will have negative values.[17] Therefore, the

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[17] A. Brauer, 'On the Characteristic Roots ...', Duke Mathematical Journal, 1961.

situation will not be compatible. If the row sums are all positive, then we may proceed to calculate the characteristic roots and the characteristic vector of the matrix L_M. Alternatively, after computing the characteristic roots we may obtain the equilibrium state age distribution from the formulas derived in Section 5.2.1(b). When the equilibrium state age distribution contains all positive values, we conclude that the situation is compatible. The growth index and the age distribution of the resultant equilibrium state population may also be obtained using the iteration procedure suggested in Section 5.3.1(b). Then, the growth index itself is the dominant root and only the second dominant root need be obtained if the situation is found to be compatible.

So far, we have assumed that $n(x)$ are either positive or zero. But $n(x)$ may also take negative values. This happens if there is net immigration at some ages and net emigration at some others. In this case also, the matrix L_M will not be a non-negative matrix. Further, if there is at least one age below β years at which either net immigration or net emigration occurs, then the matrix L_M cannot be partitioned as the matrix L. But as long as the matrix L_M is a power positive matrix with positive row sums, the convergence to the equilibrium state age-sex distribution occurs, though the conclusion on the duration of convergence as well as the intrinsic growth rate depends on the age range affected by each of them and on the magnitude of the overall net migration rate. As in the other cases, the duration

of convergence and the intrinsic growth rate can be computed from the first two dominant roots - λ_1 and λ_2 - of the matrix L_M. We assume that λ_2 is also distinct. In all our applications, the characteristic roots of the matrix L_M were all found to be distinct.

(i) Special Case 1

We shall now examine some special cases. Suppose migration affects only the ages beyond β years, the oldest age at which reproduction occurs. Then the matrix L_M can be partitioned as in the absence of migration and will have the same matrix M in its top left hand corner and zero in the right. Hence, there will be no change in either the duration of convergence to the equilibrium state or in the intrinsic growth rate if we consider only the ages below β years. But, if we include all ages, the characteristic roots of the matrix L_M are not the same as those of the matrix M, as they are in the case of the matrix L. However, as the populations at ages beyond β years become, in course of time, linear functions of the populations below β years, the intrinsic growth rate must remain the same for all ages. Thus the first dominant root would remain the same. In Chapter 5, this will again be shown to be true through different considerations. But the associated characteristic vector and also the other roots will change. Hence, the duration of convergence changes. If there is immigration at all ages above β years, it introduces an averaging effect at these ages and the duration becomes less than that in the absence of migration, while if there is emigration at these ages, it acts as a decrement

factor and reduces to zero some of the contributions of the ages below β years so that the duration increases. On the other hand, if there is immigration at some ages and emigration at some others, again the duration may decrease, remain the same, or increase as compared to that in the absence of migration, depending on the number of ages affected by each and on the magnitude of the overall net migration rate.

(ii) Special Case 2

Another special case is of interest. Suppose the age composition of the net migrants is identical with that of the stable population resulting under the given set of fertility and mortality conditions. Then the duration of convergence and the growth rate would change compared to the case with no migration, because the elements of the matrix L would change as in any other case. But the age distribution of the equilibrium state population will remain the same as compared with the case with no migration, because the age distribution of the migrants that is added to or subtracted from the transient age distribution of the population remains the same when subjected to the given fertility and mortality conditions. If the initial population also has the same stable age structure, then the age distribution of the population as well as that of the migrants would not change by the operation of the vital rates and therefore, the duration of convergence is reduced to zero, and the age distribution of the equilibrium state population is not changed. But

the growth rate is changed to the extent of the net migration rate.

Before closing this discussion, we shall briefly refer to the effects of migration when the age-specific net migration rates are used. Though Lopez indicated that the convergence to an equilibrium state takes place in this case, no detailed analysis seems to have yet been presented regarding the effects of migration on the duration of convergence and on the intrinsic growth rate. Following the procedure adopted in the case with no migration we can show that, when the age-specific net migration rates are defined as in Section 1.8, the population growth depends on a matrix M' which is obtained by substituting (S(b) + u') and (S(x) + a'(x)) respectively for S(b) and S(x) in the matrix M. Hence, as long as u' and a'(x) are either positive or are less than S(b) and S(x) respectively, the whole theory of stable populations becomes applicable in this case also. Since the power to which M' is to be raised to obtain a positive matrix, is the same as that required for M, and the averaging effect does not also occur at the old ages, the duration of convergence would remain approximately the same as in the absence of migration whether there is immigration or emigration, or both. But the effects of migration on the intrinsic growth rate would be in the same direction as discussed earlier in this section. If we assume that the Pearson Type III curve could be used to approximate the modified net maternity function:

$$F'(x) = [(S(b) + u')(S(0) + a'(0)) \ldots (S(x-1) + a'(x-1))]\, f(x) \qquad (3.21)$$

the duration of convergence as well as the intrinsic growth rate can be calculated from the total density, R_0, the mean and the coefficient of variation of the modified net maternity function. However, if the age-specific net migration rates are the same at all ages, the duration of convergence and the age distribution of the equilibrium state population would be the same as in the absence of migration because the age-sex composition of net migrants would be changing in the same manner as the transient age distribution of the population, but the intrinsic growth rate changes to the extent of the common rate of net migration.[18] If the initial age distribution is identical with the stable one, then it is not affected by the occurrence of migration at an equal rate at all ages. Hence in this case, the duration of convergence is reduced to zero, but the intrinsic growth rate changes to the extent of the common net migration rate.

3.3 TWO-SEX MODEL

So far, we have assumed that the population consisted of only one sex - either males or females. We shall now extend the results to the case where the population has both males and females. First, we shall discuss the female dominance case and then examine briefly the case with equal dominance. If male dominance is assumed, the same conclusions as those in the case of the female dominance, would follow with the terms 'female' and 'male' suitably interchanged.

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[18] J.H. Pollard, 'On the Use of Direct Matrix Product ...', Biometrika, 1966.

3.3.1 TWO-SEX MODEL WITH NO MIGRATION

Using the subscripts m and f to distinguish the male and the female populations and assuming $f_f(x)$ to be the age-specific birth rate for females aged x years, taking births of both sexes and s to be the male proportion at birth, we can write the growth equations as follows:

$$P_f(0,t) = (1 - s)\, S_f(b) \sum_{\alpha}^{\beta} \frac{1}{2}[P_f(x,t-1) + S_f(x-1)\, P_f(x-1,t-1)]\, f_f(x)$$

$$P_f(x,t) = S_f(x-1)\, P_f(x-1,t-1), \quad \text{for } x = 1, 2, \ldots, w$$

$$P_m(0,t) = s\, S_m(b) \sum_{\alpha}^{\beta} \frac{1}{2}[P_f(x,t-1) + S_f(x-1)\, P_f(x-1,t-1)]\, f_f(x)$$

and $$P_m(x,t) = S_m(x-1)\, P_m(x-1,t-1), \text{ for } x = 1, 2, \ldots, w. \qquad (3.22)$$

If our population vector has, now, females by age as the first (w+1) elements and the males by age as the next (w+1) elements, the growth equations could be written as:

$$(P_t) = L' \, (P_{t-1}) \qquad (3.23)$$

where the matrix L' is

$$
\left[\begin{array}{cccccccccc|cccccc}
0 & 0 & \cdots & 0 & m_f(\alpha) & \cdots & m_f(\beta) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
S_f(0) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & S_f(1) & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & S_f(w-1) & 0 & 0 & 0 & \cdots & 0 & 0 \\
\hline
0 & 0 & \cdots & 0 & m'_f(\alpha) & \cdots & m'_f(\beta) & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & S_m(0) & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & S_m(1) & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & S_m(w-1) & 0
\end{array}\right]
$$

in which $m_f(x) = \frac{1-s}{2} [f_f(x) + S_f(x)\ f_f(x+1)]\ S_f(b)$

and $m'_f(x) = \frac{s}{2} [f_f(x) + S_f(x)\ f_f(x+1)]\ S_m(b)$.

Thus, in this case, the intrinsic growth rate and the duration of the convergence are determined by the characteristic roots of the matrix L'. We can partition this matrix into four parts by dividing at the (w+1)th row and at the (w+1)th column as shown above. Hence, at any time, the changes in the male population will not have any effect on the female population at any age. Further, it can easily be shown that in (w+1) years, the male populations at all ages and the female populations at ages beyond β years, become linear functions of the female populations at ages below β years. Thus the intrinsic growth rate and the duration of convergence, in this case, would be the same as for the female

population at ages below β years provided that the duration of convergence is greater than (w+1) years. Hence, they depend on the characteristic roots of the matrix M, and could be calculated from the matrix roots or from the approximate formulas (3.13) and (3.15). If, however, the initial age-sex distribution is the same as the stable age-sex distribution resulting from the given fertility and mortality conditions, then the duration of convergence is zero.

3.3.2 TWO-SEX MODEL WITH MIGRATION

In this case, the growth equations can easily be written by referring to the one-sex model and the case with no migration discussed in 3.3.1. The population projection matrix now becomes the following matrix L_M': (see page 124)

where $u_f = (1-s)\ S_f(b) \sum_{\alpha}^{\beta} \frac{1}{2}[a_f(x-1)\ f_f(x)] + n\ (1-s')\ [\frac{1+S_f(0)}{4}]\ n_f(0)$,

s' being the male proportion among the net migrants;

$$u_m = s\ S_m(b) \sum_{\alpha}^{\beta} \frac{1}{2}[a_f(x-1)\ f_f(x)] + n\ s'\ [\frac{1+S_m(0)}{4}]\ n_m(0)\ ;$$

$$a_f(x) = \frac{1}{4}\ [(1 + S_f(x))\ n_f(x) + (1 + S_f(x+1))\ n_f(x+1)]\ n(1-s')\ ;$$

and $$a_m(x) = \frac{1}{4}\ [(1 + S_m(x))\ n_m(x) + (1 + S_m(x+1))\ n_m(x+1)]\ n\ s'\ .$$

Since in this case also, the male populations at all ages become linear functions of the female populations at ages below β years in (w+1) years, migration, if it affects only the male population, will not have

$$
\begin{pmatrix}
u_f & \cdot & [m_f(\alpha) + u_f] & \cdot & [m_f(\beta) + u_f] & \cdot & u_f & u_f & u_f & \cdot & u_f & u_f \\
[S_f(0) + a_f(0)] & & a_f(0) & & a_f(0) & & a_f(0) & a_f(0) & a_f(0) & & a_f(0) & a_f(0) \\
\cdot & & \cdot & & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\
a_f(w-1) & & a_f(w-1) & & a_f(w-1) & & [S_f(w-1) + a_f(w-1)] & a_f(w-1) & a_f(w-1) & & a_f(w-1) & a_f(w-1) \\
u_m & \cdot & [m_f'(\alpha) + u_m] & \cdot & [m_f'(\beta) + u_m] & \cdot & u_m & u_m & u_m & \cdot & u_m & u_m \\
a_m(0) & & a_m(0) & & a_m(0) & & a_m(0) & a_m(0) & [S_m(0) + a_m(0)] & & a_m(0) & a_m(0) \\
\cdot & & \cdot & & \cdot & & \cdot & \cdot & \cdot & & \cdot & \cdot \\
a_m(w-1) & & a_m(w-1) & & a_m(w-1) & & a_m(w-1) & a_m(w-1) & a_m(w-1) & & [S_m(w-1) + a_m(w-1)] & a_m(w-1)
\end{pmatrix}
$$

any effect on the duration of convergence or on the intrinsic growth rate, provided the duration is greater than (w+1) years. But the age-sex distribution of the equilibrium state population changes due to the effect of migration on the male population as compared to the results in the absence of migration. However, if the initial age-sex distribution is the stable one, then the occurrence of migration into the male population, affects the duration of convergence to the extent of (w+1) years which is the time required for the male populations at all ages to become functions of the female populations at ages below β years. The age-sex distribution of the equilibrium state population is also changed, though the intrinsic growth rate itself is not changed. When migration affects the female population, the effects would be as discussed in the case of the one-sex model (see Section 3.2.2).

If the age-sex-specific net migration rates are used, the duration of convergence and the intrinsic growth rate depend on the characteristic roots of the matrix M' and could be calculated from the matrix roots or from the approximate formulas (3.13) and (3.15). The other results could easily be inferred from those discussed under the one-sex model.

3.3.3 TWO-SEX MODEL WITH EQUAL DOMINANCE

Before closing our analytical discussion for this chapter, we shall refer to the case where both the sexes are taken into consideration for calculating the number of births. Among the earlier

authors who attempted this, A.H. Pollard[19] related the female births to males and the male births to females, while Goodman[20] assumed female dominance in a portion δ of the year and male dominance in the remaining part $(1-\delta)$ of the year. Yntema[21] proposed the use of age-sex-specific birth rates instead of age-specific ones, by relating births to a combination of the male and the female populations at each age in the reproductive ages.

We argue in a manner similar to that of A.H. Pollard and state that each birth (male or female) will have a mother of certain age i, and a father of certain age j. Thus, the same event could be related once with the mother and again with the father. Hence, if we calculate the number of births during a year using the birth rates for males as well as for females we must get two times the number of births that would actually occur. Hence, half of the total number of births thus obtained, must be the actual number of births during the year. This procedure is called here, the equal dominance method because males and females are given equal weight in obtaining the births.

However, the procedure suffers from the defect that it gives some births even when persons of only one sex exist in the population.

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19 A.H. Pollard, 'The Measurement of Reproductivity', Journal of the Institute of Actuaries, Vol. 74, 1948, pp.288-305.

20 L.A. Goodman, 'On the Age-Sex Composition of the Population that would Result from given Fertility and Mortality Conditions', Demography, Vol. 4, No. 2, 1967, pp.423-441.

21 L. Yntema, Mathematical Models of Demographic Analysis, J.J. Groen and Zoon, Leiden, 1952. (Chapter 3)

But such a situation is an extreme one and rarely occurs in practice. Moreover, this artificial situation cannot remain long because the population of each sex at each age becomes in course of time, a function not only of the population of the same sex but also that of the other, unless the sex ratio at birth is abnormal so that births of the same sex occur always, to the existing population of one sex.

If we use the equal dominance method the growth equations change only in respect of the population at age 0 years for males and females. These equations can now be written as:

$$P_f(0,t) = (1-s)\, S_f(b)\, B(t)$$

and $$P_m(0,t) = s\, S_m(b)\, B(t) \qquad (3.24)$$

with $$B(t) = \sum_{\alpha'}^{\beta'} \frac{1}{2}\left[\frac{1}{2}\{P_f(x,t-1) + S_f(x-1)\, P_f(x-1,t-1)\}\, f_f(x) + \frac{1}{2}\{P_m(x,t-1) + S_m(x-1)\, P_m(x-1,t-1)\}\, f_m(x)\right]$$

where α' and β' are the youngest and the oldest ages at which births occur among males and/or females and $f_m(x)$ is the birth rate for males aged x years.

Accordingly, the elements of the first row and the (w+2)th row of the projection matrix L', change and they can now be written as follows:

First row:

$$0 \;.\; 0 \;\; m_f(\alpha') \;.\; m_f(\beta') \;\; 0 \;.\; 0 \;\; 0 \qquad 0 \;.\; 0 \;\; m_m(\alpha') \;.\; m_m(\beta') \;\; 0 \;.\; 0 \;\; 0$$

and the (w+2)th row:

$$0 \;.\; 0 \;\; m_f'(\alpha') \;.\; m_f'(\beta') \;\; 0 \;.\; 0 \;\; 0 \qquad 0 \;.\; 0 \;\; m_m'(\alpha') \;.\; m_m'(\beta') \;\; 0 \;.\; 0 \;\; 0$$

where $m_f(x)$ and $m_f'(x)$ are half of those in L', and

$$m_m(x) = \frac{1}{2}(1-s)\, S_f(b)\, [\tfrac{1}{2}\{f_m(x) + S_m(x)\, f_m(x+1)\}]$$

$$m_m'(x) = \frac{1}{2}\, s\, S_m(b)\, [\tfrac{1}{2}\{f_m(x) + S_m(x)\, f_m(x+1)\}] \quad .$$

The other elements of the matrix remain the same as in L'. Let us denote this matrix as L''.

We can partition the matrix L'' at $(\beta'+1)$th, $(w+1)$th and $(w+1+\beta'+1)$th rows and columns and write it as follows:

$$L'' = \begin{pmatrix} Q & 0 & R & 0 \\ A & B & 0 & 0 \\ Q_1 & 0 & R_1 & 0 \\ 0 & 0 & A_1 & B_1 \end{pmatrix} \quad .$$

It may be shown by matrix multiplication that the first row and the third row of any power of this matrix would be independent of the elements in the second and fourth rows. Thus, as in the case of the one-sex model, we can conclude that the changes in the population above β' years either among males or among females will not have any effect on the populations below that age. But the populations at ages above β' years become, in the course of time, the survivors of the populations below the age β' years. Hence the convergence in this case, depends on the matrix

$$\begin{pmatrix} Q & R \\ Q_1 & R_1 \end{pmatrix}$$

which is the projection matrix if we consider the female and the male populations up to the age β' years only. Since the fertility rate for males at age β' years can be assumed to be positive (because the oldest age at which reproduction occurs is generally higher in the case of males than in the case of females), the above matrix is a non-negative square matrix which will be power positive. It will have, therefore, a unique positive dominant characteristic root and an associated characteristic vector of positive elements. Thus, we conclude that the convergence to the equilibrium state age-sex distribution occurs. The duration of convergence and the intrinsic growth rate may be obtained from the dominant characteristic roots of the above matrix. Alternatively, these may be computed from the combined net maternity function using formulas (3.13) and (3.15), if we assume that the Pearson Type III curve will be a good fit in this case also.

If migration is introduced into the process, the values of u_f and u_m change as follows:

$$u_f = \frac{(1-s)}{2} S_f(b) \sum_{\alpha'}^{\beta'} \frac{1}{2}[a_f(x-1)\, f_f(x) + a_m(x-1)\, f_m(x)] + n(1-s')\left(\frac{1+S_f(0)}{4}\right) n_f(0)$$

and

$$u_m = \frac{s}{2} S_m(b) \sum_{\alpha'}^{\beta'} \frac{1}{2}[a_f(x-1)\, f_f(x) + a_m(x-1)\, f_m(x)] + n\, s' \left(\frac{1+S_m(0)}{4}\right) n_m(0) \quad .$$

The elements of the matrix L'_M can be written down easily by adding the migration coefficients - u_f, u_m, $a_f(x)$ and $a_m(x)$, to the respective elements of the matrix L'' as in the case of female dominance. It may be observed that the structure of the matrix L'_M remains the same as in the case of female dominance. Therefore, similar conclusions follow with the exception that, now, the duration as well as the intrinsic growth rate would change when migration affects the male or the female population at ages below β' years, where the corresponding fertility rates are not zero. They may be obtained from the dominant characteristic roots of the new matrix.

In this case also, the effects of migration when the age-sex-specific net migration rates are used, may easily be inferred.

3.4 NUMERICAL ILLUSTRATIONS

For purposes of illustration of the analytical conclusions, the three initial populations - 1911 obs., 1911 stb., and 1966 obs. (see Chapter 2) - were projected successively under the same constant conditions of fertility, mortality and migration. At each age x years, a growth index was computed as the ratio of the population at age x at time t to the population at the same age at time (t-1). These growth indexes were then compared among themselves and the maximum (MaxGI) and the minimum (MinGI) growth indexes were recorded for each of the three populations at every five year interval of time. When the difference between the MaxGI and the MinGI became less than 0.00009, the process

was terminated and each of the age-sex distributions was taken to have reached the equilibrium state. The limit of 0.00009 was arbitrarily chosen. It was observed that the other indicators such as the ADI and the SRDI between the percentages and the sex ratios in 5 year age groups at time t and those in the same age groups at time (t-1), reduced to zero long before the difference between MaxGI and MinGI became less than the specified limit. An advantage in computing the MaxGI and MinGI is that the graphs of MaxGI and MinGI over time would not only show the nature of the process of convergence but also the magnitude of the intrinsic growth rate resulting from the given fertility, mortality and migration conditions.

The value of t, thus obtained, was used to verify the conclusions arrived at by employing the matrix method or the approximate formulas resulting from it. It must be mentioned here, that the value of t obtained by the direct projection method is conditioned by the nature of the initial age-sex distribution while the value of t computed from the matrix roots or from formula (3.15) is not. This fact should be borne in mind in studying the illustrations presented here.

(i) Effect of Migration on the Duration of Convergence

Table 3.1 compares the results obtained under the different assumed conditions. The four situations of migration were purposely selected for reasons detailed in Section 2.3.3. The following points may be observed from the table. Except when the initial age-sex

TABLE 3.1

EFFECT OF MIGRATION ON THE DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS TO AN EQUILIBRIUM STATE AGE-SEX DISTRIBUTION

	Prevailing Conditions			From Actual Projections: Value of t (in No. of Years)			From Matrix Roots			From the Moments of the Net Maternity Function				
	Mortality (1)	Fertility (2)	Migration (3)	1911 Qbs. (4)	1911 Stb. (5)	1966 Obs. (6)	λ_1 (7)	λ_2 (8)	Value of t (in No. of Years) with ε=0.00009 (9)	R_0 (10)	μ (11)	σ^2 (12)	Value of t (in No. of Years) with ε=0.00009 (13)	Estimate of λ_1 (14)
							No Migration							
1	1911	1911	0	245	95	240	1.01219	0.95804 ±0.19791i	274	1.43783	30.28	42.21	308	1.01217
2	1966	1966	0	235	215	245	1.01159	0.94914 ±0.22559i	258	1.36608	27.31	33.13	289	1.01154
							Migration Specified by a Net Migration Rate and an Age-Sex Composition of Net Migrants							
3	1911	1911	1911-12	170	145	175	1.03044	0.95756 ±0.19801i	178					
4	1911	1911	1915-16	x	x	x	1.01292	0.95798 ±0.19797i	x					
5	1911	1911	1945-46	260	190	255	1.01002	0.95808 ±0.19789i	292					
6	1966	1966	1965-66	195	175	195	1.02180	0.94912 ±0.22561i	202					
							Migration Specified by Age-Sex-Specific Net Migration Rates							
7	1911	1911	1911-12	245	195	255	1.02723	0.97324 ±0.19995i	280	2.26943	31.04	42.84	320	1.02727
8	1911	1911	1915-16	245	160	240	1.01347	0.95907 ±0.19879i	273	1.49359	30.27	42.03	308	1.01346
9	1911	1911	1945-46	245	175	240	1.00997	0.95579 ±0.19741i	273	1.34640	30.18	42.22	306	1.00995
10	1966	1966	1965-66	235	215	245	1.02213	0.95940 ±0.22683i	258	1.82187	27.75	34.32	288	1.02214

Note:- (1) The figures in columns (1), (2), (3) are the calendar years to which the mortality and fertility rates, and the migration data relate. The respective data were those observed in Australia.
(2) x denotes that the situation was not compatible. Percentages in some age groups of the equilibrium state population became negative.

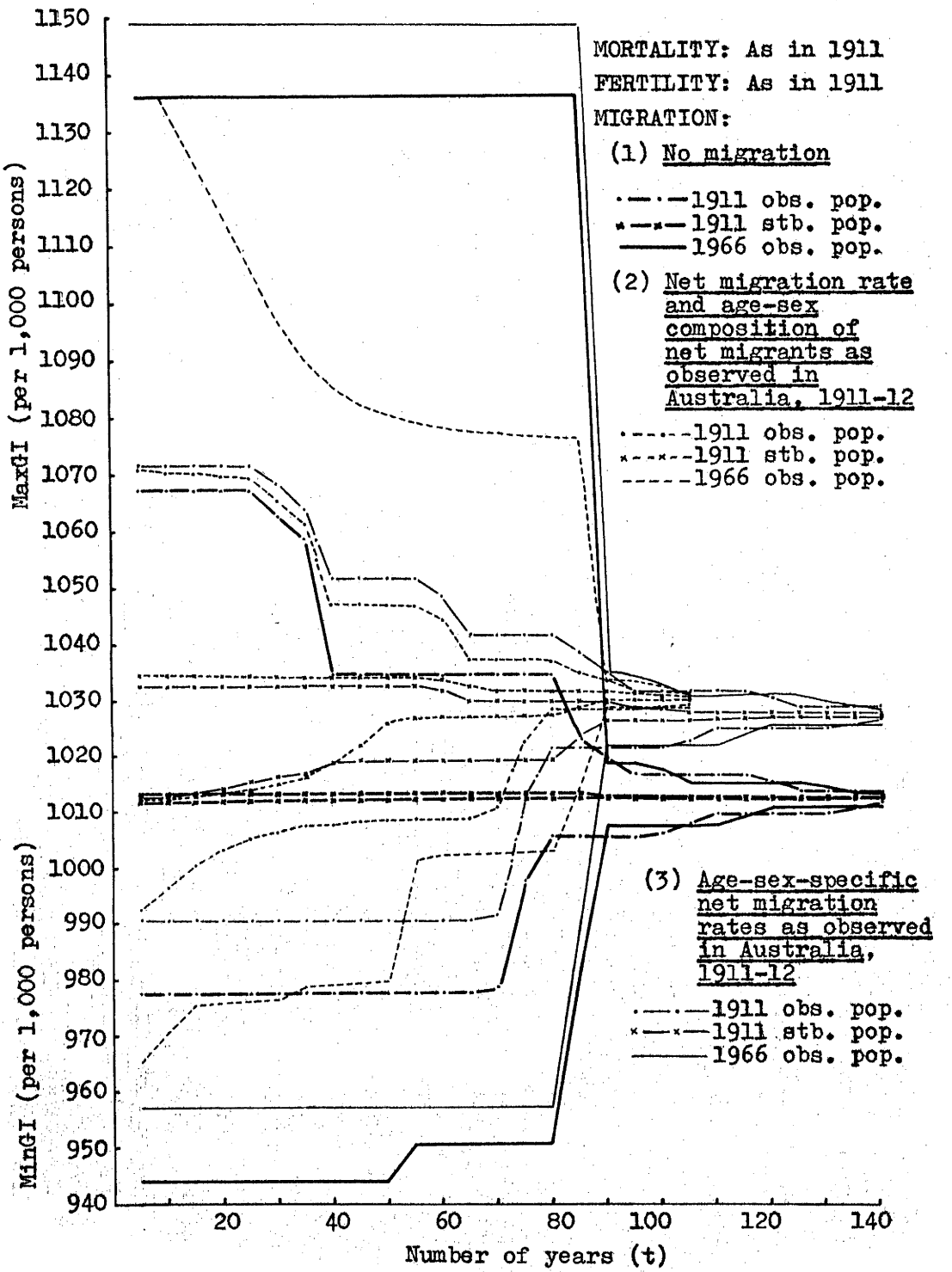
structure was the stable one, the values of t obtained from the matrix roots were fairly close to those observed in the actual projections with the same order of approximation. The approximate formula (3.15) gave quite satisfactory results as far as the estimation of the intrinsic growth rate was concerned. But it yielded somewhat higher values of t both when no migration was assumed and when the age-sex-specific net migration rates were used. That formula is not directly applicable in the other case. It appears that the time limit given by formula (3.15) may be taken as a safe value of t for any arbitrary age-sex distribution.

When the age-sex composition of net migrants contained positive values at all ages, there was a definite decrease in the value of t as could be seen in the cases when the 1911 vital rates and the 1911-12 migration condition or the 1966 vital rates and the 1965-66 migration condition were assumed. When the same migration situations were specified by age-sex-specific net migration rates, the duration increased in the first case and remained the same in the second case as compared to the case with no migration. The migration situation of the war years 1915-16 was shown to be incompatible with the vital rates situation in 1911, when migration was specified by the net migration rate and age-sex composition of net migrants. When the same situation was specified by the age-sex-specific net migration rates, the duration turned out to be the same as that in the absence of migration. But the resultant age-sex distribution became abnormal compared with that in

the absence of migration (see Chapter 5). In 1945-46, both the male and the female populations experienced emigration at almost all ages. In this case the duration increased, as expected, when migration was specified by the net migration rate and the age-sex composition of net migrants. On the other hand, the duration remained almost the same when the situation was specified by age-sex-specific net migration rates.

Figure 3.1 depicts the nature of the process of convergence as seen from the convergence of the MaxGI and MinGI when the fertility and mortality rates for 1911 and net migration conditions of 1911-12 were assumed. It shows a clear similarity in the processes when there was no migration and when migration was specified by age-sex-specific net migration rates. For the 1911 stb. population the difference between MaxGI and MinGI should have been negligible from the beginning when there was no migration. But since a slightly different sex ratio at birth (105.22 males per 100 females) was used in the computation of the 1911 stb. age-sex structure than the one (105 males per 100 females) used in the actual projections, a small difference is found to exist and this continues for one lifetime, i.e. 86 years. The figure also indicates that the large differences in the initial age-sex distributions reduced as soon as the cohorts alive at the initial point of time died out; but the smaller variations induced by the initial differences took a long time to disappear. When the migration was specified by the net migration rate and an age-sex composition of the

FIGURE 3.1: THE PROCESS OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS AS SEEN FROM THE CONVERGENCE OF MAXGI AND MINGI FOR FEMALES, UNDER CONSTANT SCHEDULES OF FERTILITY, MORTALITY AND MIGRATION, WHEN MIGRATION IS SPEFICIED IN TWO DIFFERENT WAYS



net migrants, the differences started getting reduced even in the lifetime of the cohorts alive at the initial point of time, and the convergence was accelerated. Thus the numerical examples support our analytical conclusions.

(ii) Changes in the Duration of Convergence Due to the Changes in the Net Migration Rate

A comparison of the values of t when migration observed during 1911-12 and 1965-66, were assumed to operate, suggested that the duration increased with the decrease in the value of the net immigration rate. But during these two years the age-sex composition of net migrants was also different. Hence, the increase could not be attributed specifically to the decrease in the net migration rate.

To examine this aspect, the age-sex composition of net migrants during 1962-66, which contained positive values at all ages for both sexes, was assumed to remain the same and different hypothetical net migration rates were used with it. It may, however, be noted that it is the values of u and a(x) derived from the net migration rate and the age-sex composition that are important in the study of the process of population growth. The results of these experiments are given in Table 3.2. It shows that the net immigration reduced the duration, while net emigration increased it. Also it can be seen that the duration decreased with the increase in the magnitude of the net migration rate. Figure 3.2 describes the processes of convergence when net immigration, net emigration and no migration were assumed. It supports the above conclusions.

TABLE 3.2

CHANGES IN THE DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS TO AN EQUILIBRIUM STATE AGE-SEX DISTRIBUTION DUE TO THE CHANGES IN THE NET MIGRATION RATE

Prevailing Conditions Mortality - 1911 Fertility - 1911 Net Migrants Age-Sex Composition - 1962-66 Net Migration Rate: (per cent)	Value of t (in Number of Years) from Actual Projections		
	1911 obs.	1911 stb.	1966 obs.
-1.0	365	280	360
0.0	245	95	240
0.5	210	145	210
1.0	185	145	195
5.0	110	100	115

(iii) Changes in the Duration of Convergence Due to the Changes in the Age-sex Composition of Net Migrants

Another question which arises from the above experimental results is whether the change in the age-sex composition of net migrants has any effect on the duration of the process of convergence. For studying this question, a net migration rate of 1 per cent was kept constant and several hypothetical age-sex compositions of net migrants were tried and the results are presented in Table 3.3. Again the conclusion that net immigration reduces the duration if it affected some ages below β years, is well supported. But the table indicates that the changes in the age-sex compositions had only a small effect on

FIGURE 3.2: COMPARISON OF THE EFFECT OF IMMIGRATION AND EMIGRATION ON THE PROCESS OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS AS SEEN FROM THE CONVERGENCE OF MAXGI AND MINGI FOR FEMALES, UNDER CONSTANT SCHEDULES OF FERTILITY, MORTALITY AND MIGRATION

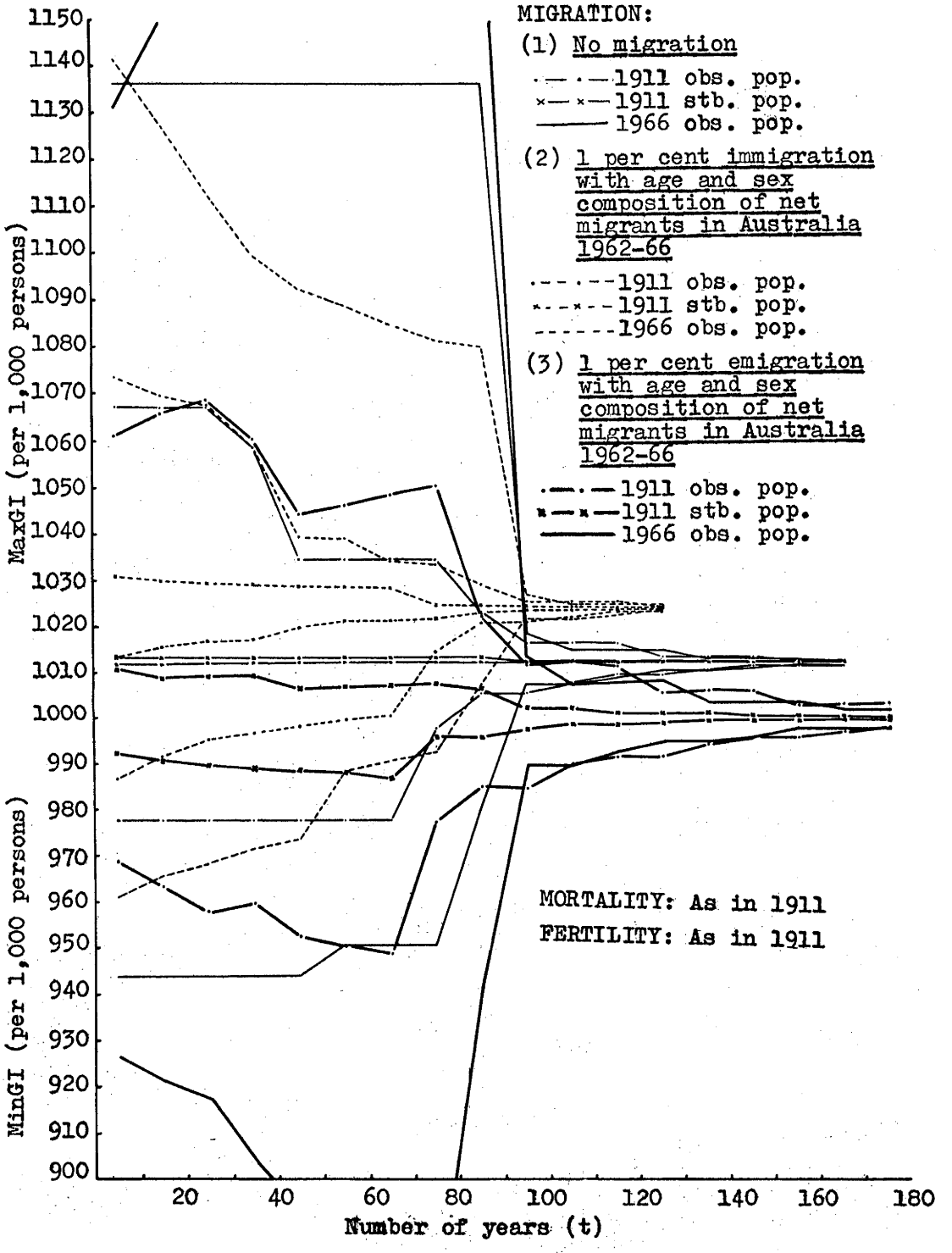


TABLE 3.3

CHANGES IN THE DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS TO AN EQUILIBRIUM STATE AGE-SEX DISTRIBUTION DUE TO THE CHANGES IN THE AGE-SEX COMPOSITION OF NET MIGRANTS

	Prevailing Conditions Mortality - 1911 Fertility - 1911 Net Migration Rate - 1 per cent Age-Sex Composition of Net Migrants:	Value of t (in Number of Years) from Actual Projections		
		1911 obs.	1911 stb.	1966 obs.
1	Same as in 1962-66	185	145	195
2	Same as in 1925	205	165	205
3	Same as 1911 Stb. Population	185	95	190
4	Same as 1966 Obs. Population	190	115	190
5	Same as 1911 Life Table Population	195	130	190
6	Same as 1911 Life Table Population with no Female Migrants	245	165 (95 females only)	240
7	Same as 1911 Life Table Population with no Male Migrants	200	160	195
8	1962-66, with no Female Migrants below the Age 49 Years	225	160	225
9	1962-66, with Female Migrants only at Age 0 Years below the Age 49 Years	225	185	210

Note:- A comparison of these age compositions of net migrants and the sex compositions associated with them is given in Chapter 2.

the duration unless they were such that the structure of the projection matrix changed, as it happened when the age-sex composition had special features such as no female migrants at all ages or at ages below β years, and so on.

The larger value of t observed when the 1925 age-sex distribution was assumed, needs some explanation. Though the age-sex composition in 1925 contained positive values at all ages for females, the proportions at about 15 ages in the old age range, i.e. above 60 years, were negative for males. Therefore the projection matrix was not really a positive matrix as it was when the other (for example, the 1962-66 or the 1911 stb. population, etc.) age-sex compositions were assumed. Hence, the duration increased in this case as compared with the other cases.

(iv) Duration of Convergence under Equal Dominance

The duration of convergence and the intrinsic growth rate change, in this case, to the extent that the fertility and mortality rates and the effect of migration are different for males from those for females. Table 3.4 shows that the duration decreased under equal dominance both when there was no migration and when there was migration. In the case of no migration, it was perhaps the slight increase in the value of the coefficient of variation of the combined net maternity function as compared to that of the net maternity function for females, that made the duration decrease. While in the case where migration was included, the decrease was caused by the

TABLE 3.4

DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS TO AN EQUILIBRIUM STATE AGE-SEX DISTRIBUTION WHEN EQUAL DOMINANCE IS ASSUMED

Prevailing Conditions			From Actual Projections Value of t			From Matrix Roots		
Mortality	Fertility	Migration	1911 obs.	1911 stb.	1966 obs.	λ_1	λ_2	Value of t (in Number of Years)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				No Migration				
1911	1911	0	235	180	225	1.01135	0.95742 ±0.18817i	260
				Migration Specified by a Net Rate and Age-Sex Composition of Net Migrants				
1911	1911	1911-12	155	155	165	1.03504	0.95661 ±0.18848i	156

Note:- See Note (1) to Table 3.1.

greater impact of immigration on the male population than on the female population. However, this observation need not be true in general. The duration, obviously, depends on the characteristics of the combined net maternity function and the effect of migration on both the male and the female populations.

3.5 SUMMARY

The process of convergence of arbitrary age-sex distributions to an equilibrium state age-sex distribution has been investigated in this chapter. In the case with no migration, a relatively simple formula is obtained to calculate the time required for the convergence (i.e., the duration of convergence) in terms of the characteristics of the net maternity function.

It is shown that the convergence occurs when migration is included into the process of population change using an overall net migration rate and an age-sex composition of net migrants. The duration of convergence is reduced when the migration schedule is such that the intrinsic growth rate is increased, as compared to the case with no migration. The opposite result holds good if the migration schedule is such that the intrinsic growth rate is decreased. But no definite conclusion follows if the migration schedule is such that the intrinsic growth rate remains the same as in the absence of migration. The duration of convergence and the intrinsic growth rate can be computed from the first two dominant roots of the projection matrix.

Further, it is observed that, for a given age-sex composition of net migrants containing positive values at all ages, the duration of convergence decreases as the net migration rate increases. On the other hand, for a given net migration rate, the changes in the age-sex composition of the net migrants produce only small changes in the duration of convergence unless the changes are such that the structure of the projection matrix is changed.

Alternatively, if age-sex-specific net migration rates are used in the process of population change, the duration of convergence remains approximately the same as in the absence of migration, though the intrinsic growth rate follows a similar trend as in the case where a net migration rate and an age-sex composition of net migrants are utilized.

CHAPTER 4

WEAK ERGODICITY OF THE AGE-SEX DISTRIBUTIONS

4.1 INTRODUCTION

Weak ergodicity of an age-sex distribution is its property to 'forget' and to become independent of its original shape in the remote past, when subjected to a set of age-specific fertility and mortality rates probably varying over time but satisfying certain conditions. It means that, after a sufficiently long period of time, the age-sex distribution of the population is determined entirely by the history of the age-specific vital rates. This theorem which was conjectured by Coale,[1] was proved by Lopez.[2] Recently, McFarland[3] has given a new heuristic proof of the theorem and has suggested a method of computing the time required to 'forget' the original shape of the age structure in terms of the oldest and the youngest ages at which child bearing takes place. These authors have concentrated mainly on closed populations. However, Lopez indicated that the weak ergodicity property holds good in the presence of migration, if migration is specified by a set of age-specific net migration rates, when it could be easily incorporated into the survival rates. Though McFarland made

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1 A.J. Coale, 'How the Age Distribution of a Human Population is Determined?', Cold Spring Harbor Symposia on Quantitative Biology, Vol. 22, 1957, pp.83-89.

2 A. Lopez, Problems in Stable Population Theory, 1961.

3 D.D. McFarland, 'On the Theory of Stable Populations ...', Demography, 1969.

a passing remark that, unless conscious efforts are made to maintain a particular initial age-sex distribution through selective migration, migration could only speed up the process by which the initial age distribution is 'forgotten', he did not attempt to demonstrate the same.

Our problem in this chapter is, therefore, to examine whether this property holds good when migration specified by a net migration rate and an age-sex composition of net migrants, is introduced into the process of population change and to investigate the possible effects of the presence of migration on the duration of the process of convergence, as compared to the case with no migration.

The weak ergodicity theorem is often stated in terms of two arbitrary populations as follows: Two populations with different age structures, when subjected to identical histories of age-specific vital rates, the rates possibly varying over time, will eventually, have age structures which remain arbitrarily close to one another even though both may be changing over time. They will have, therefore, the same current crude rates of birth, death and growth. The same formulation will be used in the present investigation as it is convenient for analytical exposition.

Hence, in this chapter, we define the duration of convergence as the number of years needed for the difference between any two arbitrary age distributions to become less than a pre-assigned small quantity, from the time both are subjected to identical histories of

fertility, mortality and migration conditions.

We start, as in the previous chapter, with the one-sex model with no migration, then introduce migration, and finally, extend the results to the two-sex model which is our ultimate concern. The process of population change is, again, represented by matrix multiplication, instead of by a birth series,[4] because it facilitates the inclusion of migration into the process. We shall also present a heuristic argument in Section 4.4.

4.2 ONE-SEX MODEL

4.2.1 ONE-SEX MODEL WITH NO MIGRATION

Let the two female (or male) populations be denoted by V and W so that V(x,t) and W(x,t) are the numbers at age x years and V(t) and W(t) the total numbers at all ages in the two populations. We assume that at the initial point of time, the age cohorts V(x,t) and W(x,t) are all positive. This is not a restrictive assumption because, if there is at least one non-zero cohort below the oldest age at which reproduction takes place, the whole age range gets filled up in course of time, and we can consider that point of time as the initial point. If no such cohort exists in any one of the populations, then that population becomes extinct and can not be considered further.

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[4] A. Lopez, 'Asymptotic Properties of a Human Age Distribution under a Continuous Net Maternity Function', Demography, Vol. 4, No. 2, 1967, pp.680-687.

The population growth of the two populations from t=0 to t=t years can be written down in matrix notation as:

$$\left.\begin{aligned}(V_t) &= L(t-1)\ L(t-2)\ \ldots\ L(1)\ L(0)(V_0)\\ \text{and}\qquad (W_t) &= L(t-1)\ L(t-2)\ \ldots\ L(1)\ L(0)(W_0)\end{aligned}\right\} \tag{4.1}$$

where (V_t) and (W_t) denote the population vectors at time t, and L(t) is the population projection matrix:

$$\begin{pmatrix} 0 & 0 & . & 0 & m(\alpha,t) & . & m(\beta,t) & 0 & . & 0 & 0 \\ S(0,t) & 0 & . & 0 & 0 & . & 0 & 0 & . & 0 & 0 \\ 0 & S(1,t) & . & 0 & 0 & . & 0 & 0 & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & 0 & 0 & . & 0 & 0 & . & S(w-1,t) & 0 \end{pmatrix}$$

where $m(x,t) = \frac{1}{2}[f(x,t) + S(x,t)\ f(x+1,t)]\ S(b,t)$.

Lopez considered first the ages up to β years and proved that:

$$\lim_{t \to \infty} \left\{\frac{V(x,t)}{W(x,t)} - \frac{V(y,t)}{W(y,t)}\right\} = 0 \tag{4.2}$$

for any two ages x and y years.

This means that the two populations tend to have identical age structures. The essential steps in his proof are presented below: (1) The following Lemma is proved: There is a sufficiently large positive integer k such that the product of k or more matrices M(t), is a strictly positive matrix where M(t) is the matrix:

$$\begin{pmatrix} 0 & 0 & . & 0 & m(\alpha,t) & . & m(\beta-1,t) & m(\beta,t) \\ S(0,t) & 0 & . & 0 & 0 & . & 0 & 0 \\ 0 & S(1,t) & . & 0 & 0 & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & 0 & 0 & . & S(\beta-1,t) & 0 \end{pmatrix} .$$

(2) The projection process is, then, carried in lumped steps of k years apart, k being the positive integer satisfying the Lemma. At each lumped step T, the ratios [V(x,T)/W(x,T)] are calculated for all x. Let r_T be the minimum and R_T the maximum of these ratios. Then it is proved that the sequence of minima in the successive lumped steps, i.e. r_0, r_1, ..., is bounded and monotonically increasing, whereas the sequence of maxima, i.e. R_0, R_1, ..., is bounded and monotonically decreasing. Both sequences are therefore convergent.

(3) Finally, it is shown that the limits to which these two sequences converge are the same, thereby proving the weak ergodicity theorem.

After establishing that the weak ergodicity theorem holds good for the truncated age distribution excluding ages above β years, he demonstrated that the same holds good for the entire age range, from the fact that the populations at ages above β years are the survivors of those below β years.

The details of the rigorous proofs of the three steps are given in Lopez[5] and to an extent in Keyfitz,[6] and will not be repeated

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5 A. Lopez, Problems in Stable Population Theory, 1961, pp.50-57.

6 N. Keyfitz, Introduction to the Mathematics ..., 1968, pp.89-94.

here. It can be inferred from the above that the duration of the process of convergence depends on the size of the lump (i.e. the value of k) needed to obtain a matrix, H(T,T+1), of positive elements, from the original matrix M(t). If k is small, the duration is short and if k is large the duration is long. From this, it follows that our problem when migration is introduced, is rather simple because it is enough if we examine whether a lumped matrix with positive elements could be obtained and see whether the size of the lump is less than, equal to, or greater than that required in the absence of migration. The other two steps would then follow from the results proved by Lopez.

4.2.2 ONE-SEX MODEL WITH MIGRATION

In this case the population growth can be represented by the matrix equations:

$$\left.\begin{aligned} (V_t) &= L_M(t-1)\ L_M(t-2)\ \ldots\ L_M(0)(V_0) \\ (W_t) &= L_M(t-1)\ L_M(t-2)\ \ldots\ L_M(0)(W_0) \end{aligned}\right\} \qquad (4.3)$$

where $L_M(t)$ is the matrix:

$$\begin{pmatrix} u(t) & u(t) & . & [m(\alpha,t) & . & [m(\beta,t) & . & u(t) & u(t) \\ & & & +u(t)] & & +u(t)] & & & \\ [S(0,t) & a(0,t) & . & a(0,t) & . & a(0,t) & . & a(0,t) & a(0,t) \\ +a(0,t)] & & & & & & & & \\ a(1,t) & [S(1,t) & . & a(1,t) & . & a(1,t) & . & a(1,t) & a(1,t) \\ & +a(1,t)] & & & & & & & \\ . & . & . & . & . & . & . & . & . \\ a(w-1,t) & a(w-1,t) & . & a(w-1,t) & . & a(w-1,t) & . & [S(w-1,t) & a(w-1,t) \\ & & & & & & & +a(w-1,t)] & \end{pmatrix}$$

with $u(t) = [S(b,t) \sum_{\alpha}^{\beta} \frac{1}{2}[a(x-1,t)\ f(x,t)] + (\frac{1+S(0,t)}{4})\ n(0,t)\ n(t)]$

and $a(x,t) = \frac{1}{4}[(1+S(x,t))\ n(x,t) + (1+S(x+1,t))\ n(x+1,t)]\ n(t)$.

Before proceeding further, we shall make it clear that, as in the case where there is no migration, the populations at ages beyond β years (the oldest age at which reproduction occurs) become in course of time linear functions of the populations at ages below β years. We made this claim in Section 3.2.2 also (see Chapter 3). We shall now give a justification for the claim. Let us assume that there is net migration at all ages above β years. Then the population at age $(\beta+1)$ years at $t = 1$ could be obtained as follows:

$$V(\beta+1,1) = S(\beta,0)\ V(\beta,0) + a(\beta,0) \sum_{0}^{w} V(x,0)$$

$$= [a(\beta,0) \sum_{0}^{\beta-1} V(x,0)] + [(S(\beta,0) + a(\beta,0))\ V(\beta,0)] + a(\beta,0) \sum_{\beta+1}^{w} V(x,0) \quad .$$

At t=2, we have:

$$V(\beta+1,2) = [a(\beta,1) \sum_{0}^{\beta-1} V(x,1)] + [(S(\beta,1) + a(\beta,1))\, V(\beta,1)]$$

$$+ a(\beta,1)\, V(\beta+1,1) + a(\beta,1) \sum_{\beta+2}^{w} V(x,1)$$

$$= [a(\beta,1) \sum_{0}^{\beta-1} V(x,1)] + [(S(\beta,1) + a(\beta,1))\, V(\beta,1)]$$

$$+ a(\beta,1)\, a(\beta,0) \sum_{0}^{\beta-1} V(x,0) + a(\beta,1)\, [(S(\beta,0) + a(\beta,0))\, V(\beta,0)]$$

$$+ a(\beta,1)\, a(\beta,0) \sum_{\beta+1}^{w} V(x,0) + a(\beta,1) \sum_{\beta+2}^{w} V(x,1) \quad .$$

Continuing further we can show that, as the original cohorts die out, the population at age (β+1) years becomes a linear function of the populations at ages below β years. Similarly, it follows that the populations at other ages above β years become functions of the populations at ages below β years. The coefficients of these functions are determined entirely by the mortality and migration schedules in the different periods of time.

Hence, if the populations below the age β years become eventually proportional in the two populations which are subjected to the same set of fertility, mortality and migration conditions, so do the populations at the ages above β years, in the two populations.

Now we shall examine the effects of introducing migration. Let us consider first the extreme cases. Suppose n(x,t) and n(t) are all positive for all x and t. This implies that there is net immigration at all ages in all successive years. Then u(t) and all

a(x,t) are positive for all x and t. Hence, the matrix $L_M(t)$ has all its elements positive and could be taken as the lumped matrix we are seeking. Thus the Lemma proved by Lopez as the first requirement for the convergence of the age distributions of the two populations V and W, is satisfied for k=1. Once the Lemma is satisfied, the other two steps needed to prove convergence of the age distributions follow from the proofs given by Lopez. Therefore we conclude that the theorem of weak ergodicity holds good in this case as well. Regarding the duration of convergence, we infer that it would be shorter in this case than when there is no migration because, in the absence of migration we would need a k definitely greater than 1 to obtain a matrix satisfying the fundamental Lemma.

On the other hand, if there is emigration at all ages during the successive years, i.e. all n(x,t) are positive but n(t) is negative, then u(t) and all a(x,t) become negative. It may be noted that the case where all n(x,t) are negative and n(t) is positive is not admissible. From assumption (8), it is necessary that n(t) be sufficiently small so that the population will be able to sustain this continuous emigration by which we mean that at no time, the population at any age becomes negative. This requires that there is at least one positive element in each of the rows of the matrix $L_M(t)$ for all t such that the vectors of the products $L_M(t)(V_t)$ and $L_M(t)(W_t)$ are strictly positive. By examining the matrix $L_M(t)$, it is not difficult to see that the positive elements are the sub-diagonal elements corresponding to S(0,t), S(1,t),

S(2,t), ..., S(w-1,t) as in the matrix L(t) and some elements, m(x,t), in the first row corresponding to the fertility rates. We assume that m(x,t) will remain strictly positive for at least two fixed consecutive values of x, for all t.

Thus the positive elements remain at fixed positions in all the matrices being multiplied, as in the case with no migration. Hence, from the results proved by Lopez, it is possible to infer, without actual multiplication, that we can find a number k for which the product of the elements passes through positive paths and thus a lumped matrix of positive elements could be obtained. But, in this case, we cannot partition the matrix $L_M(t)$ as we partitioned the matrix L(t) and consider only the ages below β years. Further, the matrix $L_M(t)$ may contain a smaller number of positive elements in the first row than the matrix M(t) because at some of the ages at the beginning and at the end of the reproductive life the fertility rates are very small and therefore, some of these elements may become negative. Hence, it would require a larger k to obtain a lumped matrix of positive elements than would be required in the case of the matrix M(t). The weak ergodicity theorem holds good in this case also, but the duration of convergence would be larger than it would be when there is no migration.

The question that remains to be settled in this case of continuous emigration, is whether the result would be applicable even when t is not a multiple of k as it does in the case when there is no migration or when there is continuous immigration. Following the

procedure adopted by Lopez,[7] let the product of the lumped matrices, H(T,T+1), tend to the matrix B which has the following form:

$$\begin{pmatrix} k_0\, b_t(0) & k_1\, b_t(0) & \cdot & \cdot & \cdot & k_w\, b_t(0) \\ k_0\, b_t(1) & k_1\, b_t(1) & \cdot & \cdot & \cdot & k_w\, b_t(1) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ k_0\, b_t(w) & k_1\, b_t(w) & \cdot & \cdot & \cdot & k_w\, b_t(w) \end{pmatrix}$$

when t is a multiple of k, the size of the lump. The numbers $b_t(x)$ keep changing in general as the number t of factor matrices varies. To see what happens when t is not a multiple of k, we shall pre-multiply the matrix B by a matrix of the type $L_M(t)$ with u(t) and a(x,t) being negative. Then we obtain the matrix B' given below:

$$\begin{pmatrix} k_0 b_{t+1}(0) & k_1 b_{t+1}(0) & \cdot & \cdot & \cdot & k_w b_{t+1}(0) \\ k_0 b_{t+1}(1) & k_1 b_{t+1}(1) & \cdot & \cdot & \cdot & k_w b_{t+1}(1) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ k_0 b_{t+1}(w) & k_1 b_{t+1}(w) & \cdot & \cdot & \cdot & k_w b_{t+1}(w) \end{pmatrix}$$

in which $b_{t+1}(0) = \sum_{\alpha}^{\beta} m(x,t)\, b_t(x) + u(t) \sum_{0}^{w} b_t(x)$, and

$b_{t+1}(x) = S(x-1,t)\, b_t(x-1) + a(x-1,t) \sum_{0}^{w} b_t(x)$, for x = 1, 2, ..., w.

The product matrix B is a matrix of positive elements and hence $b_t(x)$ are positive. Therefore, $b_{t+1}(x)$ must be positive from the requirements

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[7] A. Lopez, Problems in Stable Population Theory, 1961, pp.65-67.

of assumption (8). Hence, by comparing the matrices B and B' we conclude that when the emigration rates are such that condition (8) is always satisfied, then the matrix product $\prod_{i=0}^{t-1} L_M(i)$ will tend, as t becomes large, to a matrix of the form as B whether t is a multiple of k or not and therefore the weak ergodicity theorem holds good.

In the considerations so far, we have assumed that n(x,t) are positive for all x though varying over time while n(t) is either positive for all t or negative for all t. But if we allow n(x,t) to assume zeros for some x which means that we relax the condition that migration affects all ages, then there can arise several situations. For instance, if we consider an extreme case where migration occurs at only one age every year, this age may remain the same or change over time apart from the fact that the magnitude of the migration effect may change. In the case of the equilibrium state conditions considered in the previous chapter, the ages at which migration occurs are automatically fixed for all t, once the migration schedule is specified. But in the present case, this is not so, unless we introduce additional assumptions. This does not seem to be necessary because the sub-diagonal elements corresponding to the survival rates and at least two of the elements of the first row corresponding to the fertility rates which we have assumed to be at fixed ages will remain at fixed positions over time and are positive. Their magnitude may vary over time. Therefore, if only immigration occurs for all t at some ages including at least one age below β years, it would only increase the

number of positive elements in the matrices. Hence the convergence occurs and the duration is reduced as compared to that in the case with no migration. But the size of the lump, k, in this case, is obviously more than 1, so that the duration will be more than in the case with immigration at all ages.

On the other hand, if only emigration occurs for all t at some ages including at least one age below β years, we can infer from a similar argument that the convergence takes place and the duration is more than that in the case with no migration, but less than that in the case with emigration at all ages for all t.

Even if $n(x,t)$ and $n(t)$ assume positive, negative and/or zero values for any x and for any t, which implies that immigration, emigration and no migration can occur at any age and at any time, the fact that in each of the projection matrices we have at least one positive element in a fixed position in each row except in the first one where, actually, two are always positive, suggests that the convergence should take place sooner or later, though it is difficult to conclude whether the duration of convergence will be less than, equal to, or greater than that in the case with no migration. The duration would depend on the number of years each of the situations prevails and the age range affected by immigration or emigration or no migration in each case.

It may also be noted that the actual number of years needed for the convergence of any two age distributions is dependent as much

on the initial differences in the two age distributions as on the nature of the variations in the components.

(i) Special Case 1

We shall now consider the special case in which the net migration affects only the ages beyond the oldest age of reproduction. Then we cannot find a lumped matrix with positive elements for any value of k, as can be seen from the following. Let the matrix $L_M(t)$ be written as follows:

$$\begin{pmatrix} M(t) & 0 \\ A(M,t) & B(M,t) \end{pmatrix} .$$

Multiplying two matrices of this form we have:

$$\begin{pmatrix} M(2) & 0 \\ A(M,2) & B(M,2) \end{pmatrix} \begin{pmatrix} M(1) & 0 \\ A(M,1) & B(M,1) \end{pmatrix} = \begin{pmatrix} M(2)M(1) & 0 \\ [A(M,2)M(1) & B(M,2)B(M,1) \\ +B(M,2)A(M,1)] & \end{pmatrix} . \tag{4.4}$$

We can see that the right hand side of (4.4) is of the same form as $L_M(t)$ and continuing in this manner it can be shown that we cannot get a lumped matrix containing all positive elements. But, if we multiply the matrix $L_M(t)$ in the form (4.4) by the initial population vector, we can see that, as in the case with no migration, the population change above the age β years has no effect on the population growth below the age β years. We can, therefore, consider first the convergence of the truncated age distribution below β years as we did in the case with no

migration and then extend to the other ages. As far as the truncated age distribution is concerned, the situation is the same as in the case with no migration and the convergence occurs.

The question is therefore, whether the weak ergodicity property holds good in this case if we include all the ages as it does when there is no migration. To examine this aspect we may recall that the populations at ages beyond β years become linear functions of the populations at ages below β years, and the coefficients of these functions are determined entirely by the mortality and migration conditions. Since these schedules are common to the populations V and W and since the cohorts at ages below β years become proportional with the increase in t, the populations at ages beyond β years also become proportional. Hence, the theorem holds good for the whole age range.

Regarding the duration of convergence, we can infer that, if there is immigration at ages beyond β years for all t, the time needed for the convergence would be less than that in the case with no migration because the averaging effect occurs at these ages every year. But the duration would obviously be greater than in the case where there is immigration at all ages. On the other hand, when there is emigration at these ages we can infer that the convergence occurs, but it is not clear how the duration would change. Since the effect of emigration is similar to the effect of death, we may safely conclude that the duration will not be less than that in the case with no migration. But a logical argument following the proof of the weak

ergodicity theorem given by McFarland (see Section 4.4), suggests that the duration would actually be greater. The one empirical example we have tried (see Section 4.6), supports this conclusion.

(ii) Special Case 2

Let us consider another special case in which migration affects only the ages below β years. In this situation we may note that, in course of time, the populations at ages above β years, become the survivors of the populations at ages below β years. The situation remains the same as in the general case except that, now, the averaging effect of immigration or the deductions due to emigration occur only up to age β years. Hence, we can infer, from the earlier discussion, that convergence occurs. If there is immigration, the duration of convergence will be smaller than when there is no migration but larger than when immigration occurs at all ages. The opposite results follow if there is emigration.

If, alternatively, we use the age-specific net migration rates in the process of population change, the convergence of the age distributions depends on the matrices $M'(t)$ which have the same structures as the matrices $M(t)$. Hence, the weak ergodicity theorem holds good and the duration of convergence remains the same as in the case with no migration.

4.3 TWO-SEX MODEL

From the results observed in the case of the one-sex model it becomes very easy to infer the convergence in the case of the two-sex

model. We shall consider first the female dominance and then refer to the case with equal dominance. It may be noted that for the convergence of the age-sex distributions, it is enough if the ratios for the male populations and for the female populations converge separately to a limit. The two limits need not be equal as is the case under constant conditions.

4.3.1 TWO-SEX MODEL WITH NO MIGRATION

Using the subscripts m and f to distinguish the male and the female populations respectively, and assuming $f_f(x,t)$ to be the age-specific birth rate (taking births of both sexes) for females and s_t to be the male proportion at birth for the year t, we have, in this case, the projection matrix L'(t) as follows:

$$\begin{pmatrix}
0 & \cdots & 0 & m_f(\alpha,t) & \cdots & m_f(\beta,t) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
S_f(0,t) & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & S_f(w-1,t) & 0 & 0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & m'_f(\alpha,t) & \cdots & m'_f(\beta,t) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & S_m(0,t) & \cdots & 0 & 0 \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & S_m(w-1,t) & 0
\end{pmatrix}$$

where $m_f(x,t)$ and $m'_f(x,t)$ are similar to m(x,t) in L(t), but multiplied by $(1-s_t)\ S_f(b,t)$ and by $s_t\ S_m(b,t)$ respectively, instead of by S(b,t).

If the population vectors (V_t) and (W_t) are formed in such a way that females by age are respectively in the first (w+1) places followed by the males by age in the other (w+1) places, we have a similar situation as in the one-sex model. In fact, it is not difficult to see that L'(t) can be partitioned as follows:

$$\begin{pmatrix} L(t) & 0 \\ A'(t) & B'(t) \end{pmatrix}$$

where L(t) is the same matrix as in the one-sex model. By matrix multiplication we can see that, in any product, the elements of the first row are not affected by the elements of the second row. Thus the growth of the female population is not affected by the changes in the male population. Further, in course of time, the male population at each age becomes a linear function of the female populations at ages below the highest age at which reproduction occurs and the coefficients of that function are determined entirely by the fertility and survival rates which are common to both the populations. Hence, the convergence of the age-sex distributions takes place as the convergence of the female age distributions below the age β years happens. The duration of convergence is also the same as required for the convergence of the female age distribution, provided that duration is greater than (w+1) years which is the time needed for the male populations at all ages to become functions of the female populations at ages below β years. In fact, the sex ratios in the two populations become arbitrarily close

when the male populations become functions of the female populations and as the convergence of the female age distributions occurs, automatically the male age distributions also converge.

4.3.2 TWO-SEX MODEL WITH MIGRATION

When migration is included into the process of population change, the projection matrix can be obtained, as in the one-sex model, by adding the migration coefficients to each of the elements in the respective rows of the matrix $L'(t)$.

The new matrix $L'_M(t)$, thus obtained, is presented on page 163,

where $u_f(t) = [(1-s_t)\ S_f(b,t) \sum_{\alpha}^{\beta} \frac{1}{2}\ a_f(x-1,t)\ f_f(x,t)]$

$$+ \frac{1}{4}(1+S_f(0,t))(1-s'_t)\ n_f(0,t)\ n(t)$$

s'_t being the male proportion among the net migrants;

$$u_m(t) = s_t\ S_m(b,t) \sum_{\alpha}^{\beta} \frac{1}{2}\ a_f(x-1,t)\ f_f(x,t) + \frac{1}{4}(1+S_m(0,t))\ s'_t\ n_m(0,t)\ n(t)$$

$$a_f(x,t) = \frac{1}{4}[(1+S_f(x,t))\ n_f(x,t) + (1+S_f(x+1,t))\ n_f(x+1,t)](1-s'_t)\ n(t)$$

and $$a_m(x,t) = \frac{1}{4}[(1+S_m(x,t))\ n_m(x,t) + (1+S_m(x+1,t))\ n_m(x+1,t)]\ s'_t\ n(t)\ .$$

Examining the matrix $L'_M(t)$ we see that, when net immigration occurs at all ages for both males and females, the age-sex distributions converge and the duration will be less than that in the absence of migration. On the other hand, if there is emigration at all ages for both sexes, we have only one element per row, which is positive except

$$
\begin{pmatrix}
u_f(t) & \cdots & u_f(t) & [m_f(\alpha,t)+u_f(t)] & \cdots & [m_f(\beta,t)+u_f(t)] & u_f(t) & \cdots & u_f(t) & u_f(t) & u_f(t) & u_f(t) & u_f(t) \\
[S_f(0,t)+a_f(0,t)] & \cdots & a_f(0,t) & a_f(0,t) & \cdots & a_f(0,t) & a_f(0,t) & \cdots & a_f(0,t) & a_f(0,t) & a_f(0,t) & a_f(0,t) & a_f(0,t) \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_f(w-1,t) & \cdots & a_f(w-1,t) & a_f(w-1,t) & \cdots & a_f(w-1,t) & a_f(w-1,t) & \cdots & [S_f(w-1,t)+a_f(w-1,t)] & a_f(w-1,t) & a_f(w-1,t) & a_f(w-1,t) & a_f(w-1,t) \\
u_m(t) & \cdots & u_m(t) & [m'_f(\alpha,t)+u_m(t)] & \cdots & [m'_f(\beta,t)+u_m(t)] & u_m(t) & \cdots & u_m(t) & u_m(t) & u_m(t) & u_m(t) & u_m(t) \\
a_m(0,t) & \cdots & a_m(0,t) & a_m(0,t) & \cdots & a_m(0,t) & a_m(0,t) & \cdots & a_m(0,t) & a_m(0,t) & [S_m(0,t)+a_m(0,t)] & a_m(0,t) & a_m(0,t) \\
\cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_m(w-1,t) & \cdots & a_m(w-1,t) & a_m(w-1,t) & \cdots & a_m(w-1,t) & a_m(w-1,t) & \cdots & a_m(w-1,t) & a_m(w-1,t) & a_m(w-1,t) & [S_m(w-1,t)+a_m(w-1,t)] & a_m(w-1,t)
\end{pmatrix}
$$

in the first row and the (w+2)th row in each of which there are at least two positive elements at fixed positions. As seen in the case of the one-sex model, convergence takes place but the duration needed will be greater than that in the case where there is no migration.

If migration affects only the males, then migration will have no effect on the female population because the matrix $L'_M(t)$ can be partitioned as the matrix $L'(t)$, and the changes in the male population will have no effect on the changes in the female population. But, as in the case with no migration, the male population at each age becomes, in course of time, a function of the female populations at ages below β years because we have assumed female dominance. The coefficients of these functions are determined entirely by the fertility, mortality and migration schedules which are common to the two populations. Thus as the female age distributions in the two populations come closer, the male age distributions also converge and therefore the age-sex distributions converge. The duration needed will depend on the time required for the male populations at all ages to become entirely functions of the female populations at ages below β years, and on the time needed for the age distributions for females to converge. It may be noted that (w+1) years will be necessary for the male populations at all ages to become functions of the female populations at ages below β years. Hence, by (w+1) years the differences in the sex ratios in the two populations again vanish. The differences in the age distributions vanish as the female age distributions converge.

Once migration affects the female population, the results would be as in the case of the one-sex model. Similarly, if age-sex-specific net migration rates are used, the results would depend entirely on the convergence of the female age distributions as in the absence of migration.

4.3.3 TWO-SEX MODEL WITH EQUAL DOMINANCE

In this case, only the growth equations for the male and the female populations at age 0 years change, so that the projection matrix L'(t) will remain the same except for the first row and the (w+2)th row which become now, as follows:

First row:

$$0 \;.\; 0 \; m_f(\alpha',t) \;.\; m_f(\beta',t) \; 0 \;.\; 0 \;\; 0 \;\; 0 \;.\; 0 \; m_m(\alpha',t) \;.\; m_m(\beta',t) \; 0 \;.\; 0 \; 0$$

and the (w+2)th row:

$$0 \;.\; 0 \; m'_f(\alpha',t) \;.\; m'_f(\beta',t) \; 0 \;.\; 0 \;\; 0 \;\; 0 \;.\; 0 \; m'_m(\alpha',t) \;.\; m'_m(\beta',t) \; 0 \;.\; 0 \; 0$$

where $m_f(x,t)$ and $m'_f(x,t)$ are half of the corresponding values in L'(t),

$$m_m(x,t) = \tfrac{1}{2}(1-s_t)\; S_f(b,t)\; [\tfrac{1}{2}\{f_m(x,t) + S_m(x,t)\; f_m(x+1,t)\}] \;,$$

and $$m'_m(x,t) = \tfrac{1}{2}\; s_t\; S_m(b,t)\; [\tfrac{1}{2}\{f_m(x,t) + S_m(x,t)\; f_m(x+1,t)\}] \;.$$

Let this new matrix be denoted by L''(t). Partitioning the matrix at the (β'+1)th, where β' is the oldest age at which reproduction occurs among females or among males, (w+1)th and (w+1+β'+1)th rows and

columns, we can write the matrix L''(t) as follows:

$$\begin{pmatrix} Q(t) & 0 & R(t) & 0 \\ A(t) & B(t) & 0 & 0 \\ Q_1(t) & 0 & R_1(t) & 0 \\ 0 & 0 & A_1(t) & B_1(t) \end{pmatrix} .$$

By multiplying L''(t) in this form for various values of t, we find that the population change below age β' years is independent of the population change above that age at any time, for males and females. Hence, we first consider the female and the male populations at ages up to β' years only. We can then write down the projection matrix as follows:

$$\begin{pmatrix} Q(t) & R(t) \\ Q_1(t) & R_1(t) \end{pmatrix} .$$

Since the fertility rate for males at age β' years can be assumed to be positive (because the oldest age of reproduction is generally higher in the case of males than in the case of females) for all t, we can now find a lumped matrix of positive elements, for some k, by multiplying the matrices of the form given above. Thus, we conclude that the convergence of the truncated age-sex distributions occurs. Consequently, the convergence of the entire age-sex distributions also takes place because the populations at ages above β' years become, in course of time, the survivors of those below that age.

When migration is introduced into this process, the results obtained earlier hold good with the change that the duration will be

affected by the migration of males or females, or both. For instance, the duration is shortened if there is immigration and is lengthened if there is emigration as compared to the case with no migration, whether migration affects only males or only females, or both.

4.4 CONVERGENCE SEEN THROUGH A HEURISTIC APPROACH

Our main procedure so far, has been to deal with matrices and their products. To understand what demographic changes lead to the faster convergence in the presence of immigration and to the slower convergence in the presence of emigration as compared to the case with no migration, it would be worthwhile to look at the problem in a descriptive manner. In general, 'forgetting' of the initial shape of the age or age-sex distribution happens gradually as the initial cohorts die out and are replaced by cohorts which are not replica of the old ones. In fact, each cohort replacing an older cohort, in course of time, arises from a combination of the earlier cohorts.

When there is no migration the cohorts replacing the older ones start from the age 0 years and proceed successively age by age, year after year. But when immigration is assumed to occur, we are artificially making every cohort to contribute to all age cohorts including itself and the new-born cohort. Thus the averaging effect which should have come through the new-born cohorts in the absence of migration, takes place at all ages for which the age-sex composition of the net migrants contains non-zero elements and during every year when

immigration occurs. Therefore, the 'forgetting' becomes faster. On the other hand, when emigration occurs, it reduces the contribution of some cohorts to the other cohorts and delays the process. When migration is specified by age-sex-specific net migration rates, the situation will be exactly as in the case with no migration and the averaging effect must come through the new-born cohorts only.

We shall consider these statements in a more rigorous manner. For simplicity of exposition, let us consider the one-sex case. Our interest throughout is in the changes that take place in the population ratios $[V(x,t)/W(x,t)]$, for $x = 0, 1, \ldots, w$. If we consider a cohort alive at time t, it can easily be seen that the ratio for that cohort will remain the same in the absence of migration, till the cohort dies out. Therefore, the change must come through the new-born cohorts in the successive years after the two populations start experiencing the same vital rates. Let us consider the cohort born in the year t. If R(t) and r(t) are the maximum and the minimum of the population ratios at time t, we have:

$$V(0,t+1) = S(b,t) \sum_{\alpha}^{\beta} \frac{1}{2}[V(x,t) + S(x-1,t)\ V(x-1,t)]\ f_f(x,t)$$

$$\leq S(b,t) \sum_{\alpha}^{\beta} \frac{1}{2} R(t)[W(x,t) + S(x-1,t)\ W(x-1,t)]\ f_f(x,t)$$

$$\leq R(t)\ W(0,t+1) \quad .$$

Similarly $\quad V(0,t+1) \geq r(t)\ W(0,t+1) \quad . \qquad (4.5)$

Thus, $R(t+1) \leq R(t)$ and $r(t+1) \geq r(t)$. Hence, we have a sequence of maxima which is decreasing and a sequence of minima which is increasing. Therefore, the sequence $d(t) = R(t) - r(t)$ decreases. When $d(t)$ tends to zero the population ratios become equal at all ages showing thereby that the two populations have the same age distributions.

Now we introduce migration and examine how the ratios change. If there is immigration at all ages we have:

$$V(x+1,t+1) = S(x,t)\, V(x,t) + a(x,t) \sum_{0}^{w} V(x,t)$$

$$\leq S(x,t)\, R(t)\, W(x,t) + a(x,t) \sum_{0}^{w} R(t)\, W(x,t)$$

$$\leq R(t)\, W(x+1,t+1)$$

Similarly $\quad V(x+1,t+1) \geq r(t)\, W(x+1,t+1)$. $\qquad (4.6)$

Thus, the ratio at each age changes every year but the maximum of any year will not be greater than that of its previous year and the mimimum will not be less than the minimum of the previous year. The inequalities apply more strictly in the case of the new-born cohorts. This is the reason why in the presence of immigration the duration of convergence becomes smaller than in the absence of migration.

On the other hand, if there is emigration the above considerations do not apply. The cohorts continue as in the absence of migration but with greater decrements at every age affected by migration.

Since we have assumed that the size of the cohorts cannot become negative at any time, the situation becomes similar to the one with no migration. Hence, we conclude that the convergence occurs. The question that remains to be answered is: How does the duration of convergence change? McFarland[10] has shown that the convergence takes place w years after each cohort at time t has some descendents in each of the cohorts alive at time t' where t' > t. Therefore, when immigration occurs at all ages, each cohort contributes to all the other age cohorts in the (t+1)th year itself. If there is no immigration at some of the ages it takes some time before all the age cohorts have contributions to all the other age cohorts and the duration increases from that in the case with immigration at all ages. Similarly, when emigration occurs at all ages or some ages, the contribution of some of the age cohorts is offset by the deductions caused by emigration so that the duration becomes longer than that in the case with no migration.

When the age-specific net migration rates are used in the process of population change, the situation becomes exactly similar to the one with no migration because in that case, we have

$$\frac{V(x+1,t+1)}{W(x+1,t+1)} = \frac{(S(x,t) + a(x,t))V(x,t)}{(S(x,t) + a(x,t))W(x,t)} = \frac{V(x,t)}{W(x,t)} \qquad (4.7)$$

and the ratio for a cohort will not change till that cohort dies. Hence, as in the case with no migration, the changes in the ratios must

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10 D.D. McFarland, 'On the Theory of Stable Populations ...', Demography, 1969.

come through the new-born cohorts only. Thus, the duration of convergence will be approximately the same as in the case with no migration.

4.5 COROLLARIES

As a corollary to the weak ergodicity theorem, McFarland showed that the two populations subjected to identical fertility and mortality conditions will tend to have the same birth, death and growth rates. It is not difficult to see that the same holds good when migration is introduced into the process.

Further, we shall show that the age-sex cohorts in the two populations will have the same growth rates. We know from the above theorem that, when t is sufficiently large,

$$\frac{V(x,t)}{W(x,t)} = R', \quad \text{for all } x.$$

Then,

$$\frac{V(x+1,t+1)}{V(x+1,t)} = \frac{S(x,t)\ V(x,t) + a(x,t) \sum_{0}^{w} V(x,t)}{V(x+1,t)}$$

$$= \frac{S(x,t)\ R'\ W(x,t) + a(x,t) \sum_{0}^{w} R'\ W(x,t)}{R'\ W(x+1,t)}$$

$$= \frac{W(x+1,t+1)}{W(x+1,t)} \quad . \tag{4.8}$$

The proofs in the case with no migration and when age-sex-specific net migration rates are used, follow easily.

Thus, we can use the equality of the growth rates for each age as the criterion for recognising the weak ergodicity in any practical situation. We can also make use of the graphs of the MaxGI and MinGI to depict the process of convergence as in the case of constant conditions (see, for instance, Fig. 4.1). It may be noted, however, that, in the present case, MaxGI and MinGI which coincide and become constant over time when the operating conditions are constant (see Chapter 3), will not only be different from each other but also will be changing over time. Therefore, if we are looking at only MaxGI and MinGI but not at the growth indexes at all ages, it may happen that the age distributions might not have converged even though MaxGI and MinGI for the two populations differ by an arbitrarily small quantity. This happens because the growth indexes at different ages may actually be different within the same maximum and minimum limits. Hence, in practice, we must either test for the equality of growth indexes at all ages or use other indicators such as the ADI and SRDI in conjunction with the MaxGI and MinGI to test the occurrence of convergence.

4.6 NUMERICAL ILLUSTRATIONS

For numerical illustrations, the same three populations, 1911 obs., 1911 stb. and 1966 obs., which were considered in the case of constant conditions, were projected under identical assumptions of varying schedules of fertility, mortality and migration. The survival

rates of the life tables interpolated for each year from the life tables for Australia at the census dates between 1911-66, were assumed to operate during the first 56 years and the values for 1966 were assumed to remain constant thereafter (see Section 2.4). As regards fertility, the sequence of hypothetical fertility rates derived for the study (see section 2.4), was assumed. On the other hand, different assumptions were used for net migration.

(i) Effects of Migration on the Duration of Convergence of Age-sex Distributions

Figure 4.1 compares the process of convergence in three situations - when there was no migration, when migration was specified by overall net migration rates and age-sex compositions of net migrants, and when the same migration situation was specified by age-sex-specific net migration rates. The legend accompanying the graph reveals the actual assumptions. It is clear from the graph that in all the three cases the initial differences between the given age-sex distributions vanished in course of time, indicating that the weak ergodicity theorem holds good.

Regarding the duration of convergence, the graph indicates that the duration was smaller when migration was specified by the overall net rates and age-sex compositions of net migrants, than when there was no migration or when migration was specified by the age-sex-specific net migration rates. But this is not very clear from the graph. Hence, Table 4.1 and Figure 4.2 were prepared. The values in

FIGURE 4.1: THE PROCESS OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS AS SEEN FROM THE CONVERGENCE OF MAXGI AND MINGI FOR FEMALES, UNDER CHANGING SCHEDULES OF FERTILITY, MORTALITY AND MIGRATION, WHEN MIGRATION IS SPECIFIED IN TWO DIFFERENT WAYS

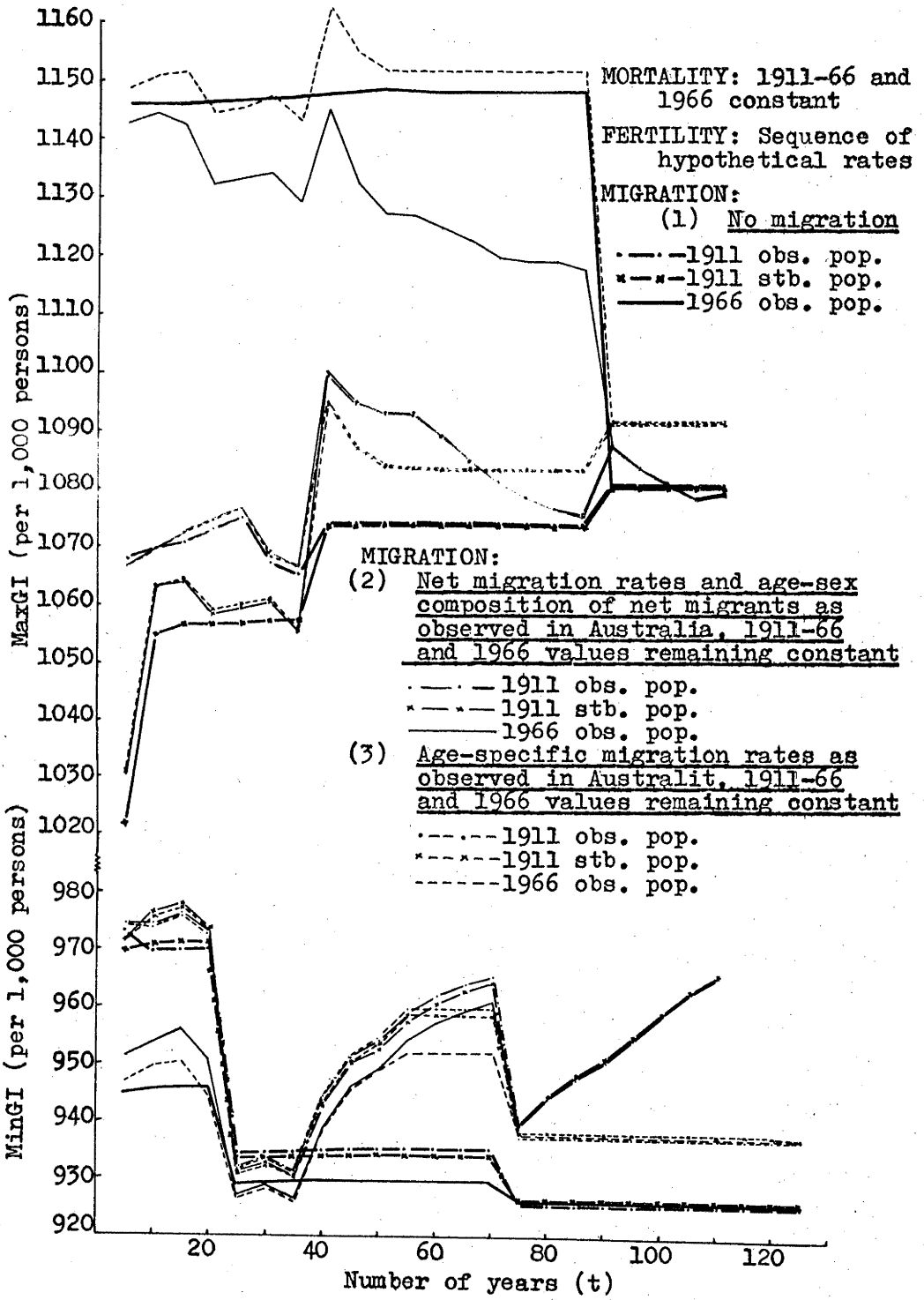
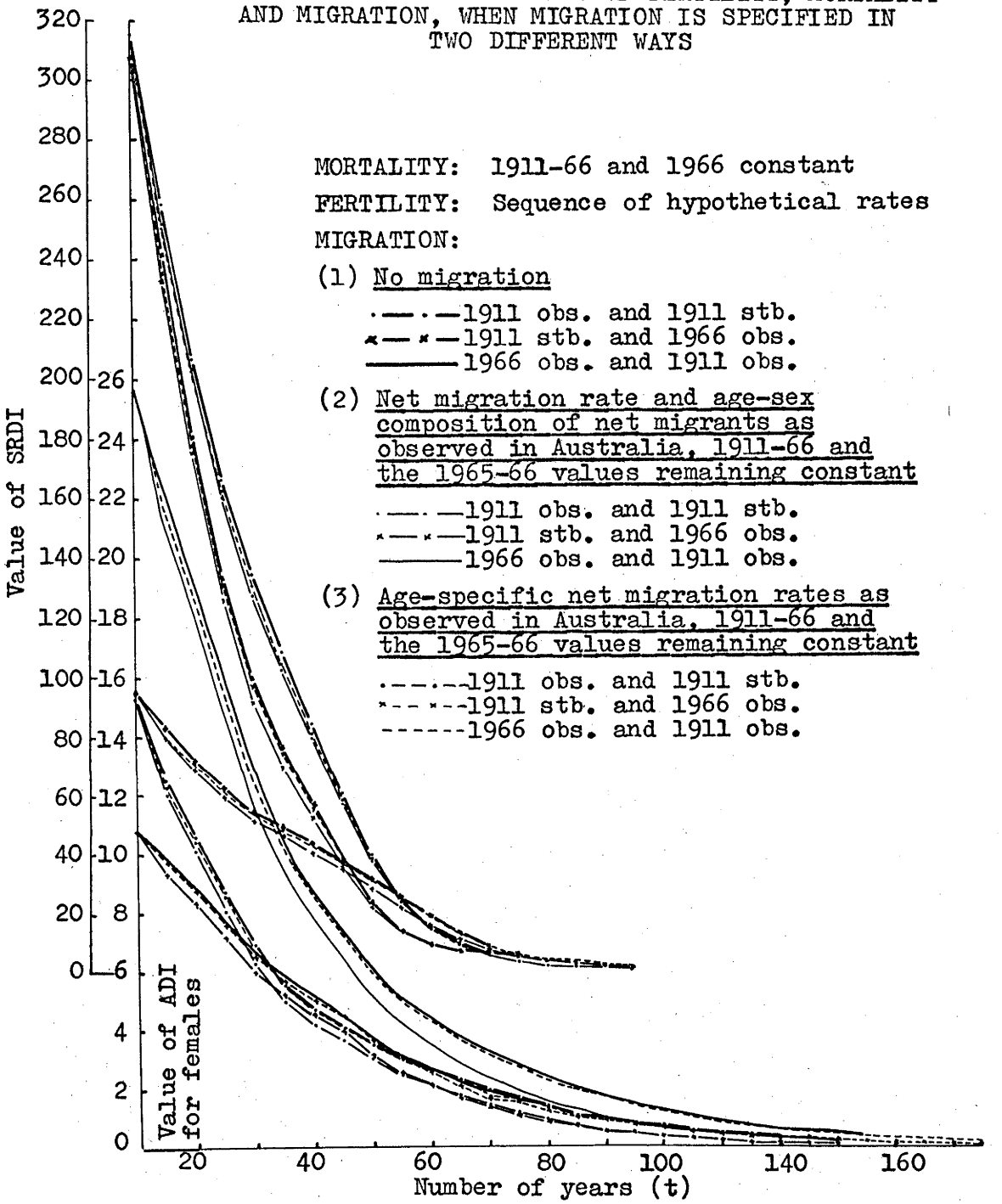


TABLE 4.1

EFFECT OF MIGRATION ON THE DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS UNDER VARYING CONDITIONS OF FERTILITY, MORTALITY AND MIGRATION

Prevailing Conditions	From Actual Projections: Value of t in Number of Years		
	1911 obs. and 1911 stb.	1911 stb. and 1966 obs.	1966 obs. and 1911 obs.
1 Mortality: As observed in Australia during 1911-66 and the 1966 values remaining constant Fertility: Hypothetical fertility rates having the continuing trend similar to the one observed during 1911-61 Migration: No migration	225	210	225
2 Mortality: Same as in 1 Fertility: Same as in 1 Migration: Net migration rates and the age-sex compositions of net migrants as observed in Australia during 1911-66 and the 1965-66 values remaining constant	150	150	165
3 Mortality: Same as in 1 Fertility: Same as in 1 Migration: Age-sex-specific net migration rates as observed in Australia during 1911-66 and the 1965-66 values remaining constant	225	220	225

FIGURE 4.2: CONVERGENCE OF AGE-SEX DISTRIBUTIONS AS SEEN FROM THE DECREASE IN THE VALUES OF ADI AND SRDI UNDER CHANGING SCHEDULES OF FERTILITY, MORTALITY AND MIGRATION, WHEN MIGRATION IS SPECIFIED IN TWO DIFFERENT WAYS



Note: At larger values of t, the values which could not be distinguished from those already plotted, have been omitted. But the merging of the respective curves can easily be seen.

the table and the decrease in the values of ADI and SRDI seen in the graph, depict very clearly the expected changes in the duration of convergence. Figure 4.2 also supports the conclusion that the differences in the sex ratios vanish as soon as the cohorts which were alive at the initial point of time, die out. This happens in all the three cases, as can be observed from the graph.

(ii) Effects of Immigration and Emigration on the Duration of Convergence

As we have seen in Chapter 2, net migration in Australia during 1911-66 which was assumed in the previous examples, was not always of one type - net immigration or net emigration - though, generally, there was net immigration into the country. Hence, to examine the effects of immigration and emigration operating continuously, the age-sex composition of net migrants was kept constant and was taken to be as in Australia during 1962-66, and the net migration rate was obtained using a Cosine curve. The age compositions of net migrants in 1962-66, had positive values at all ages for both sexes. These were purposely reduced to zero at all ages up to β years in the case of two of the experiments. The actual situations of fertility, mortality and migration conditions assumed, are indicated in Figures 4.3 and 4.4 as well as in Table 4.2 which summarizes the results of these experiments.

The results support the conclusion that in the presence of immigration, the duration becomes less than and in the presence of

FIGURE 4.3: COMPARISON OF THE EFFECT OF IMMIGRATION AND EMIGRATION ON THE PROCESS OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS AS SEEN FROM THE CONVERGENCE OF MAXGI AND MINGI FOR FEMALES, UNDER CHANGING SCHEDULES OF FERTILITY, MORTALITY AND MIGRATION

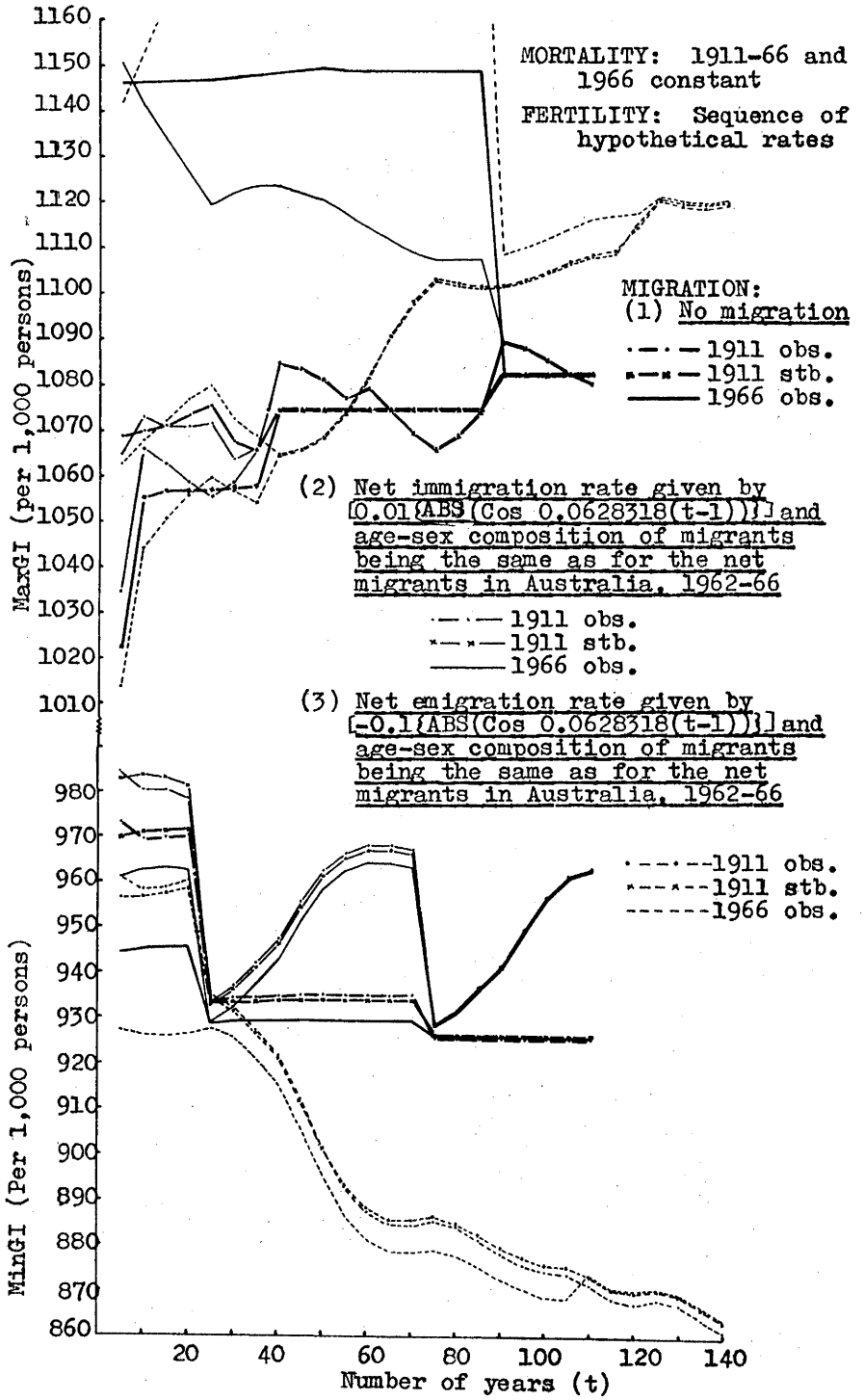
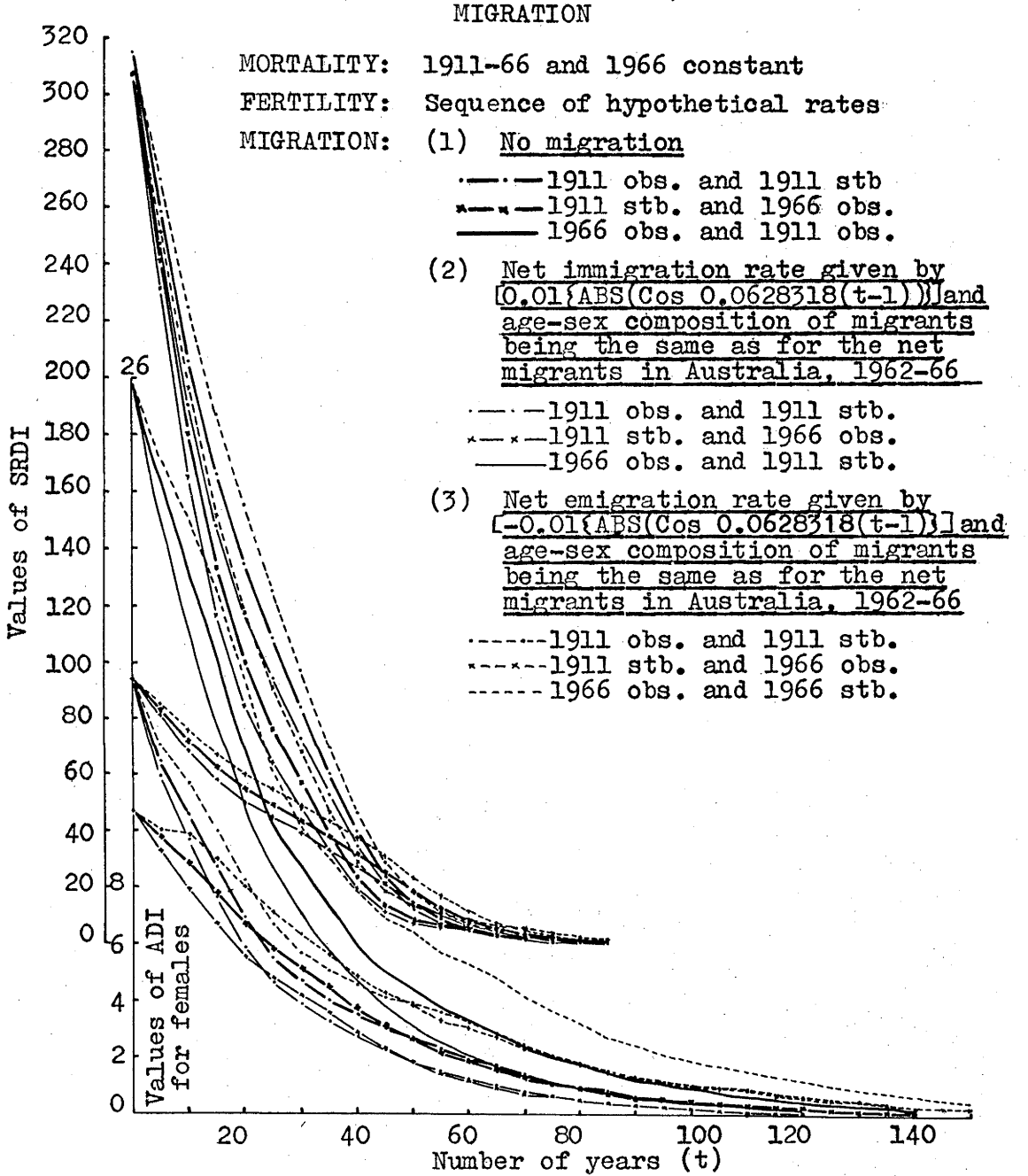


FIGURE 4.4: COMPARISON OF THE EFFECT OF IMMIGRATION AND EMIGRATION ON THE CONVERGENCE OF AGE-SEX DISTRIBUTIONS AS SEEN FROM THE DECREASE IN THE VALUES OF ADI AND SRDI UNDER CHANGING SCHEDULES OF FERTILITY, MORTALITY AND MIGRATION



Note: At larger values of t, the values which could not be clearly distinguished from those already plotted, have been omitted. But the merging of the respective curves can easily be seen.

TABLE 4.2

COMPARISON OF THE EFFECT OF IMMIGRATION AND EMIGRATION ON THE DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS UNDER VARYING CONDITIONS OF FERTILITY, MORTALITY AND MIGRATION

Prevailing Conditions	From Actual Projections		
Mortality: As observed in Australia during 1911-66 and the 1966 values remaining constant Fertility: Hypothetical fertility rates having the continuing trend similar to the one observed during 1911-61	Value of t in Number of Years		
	1911 obs. and 1911 stb.	1911 stb. and 1966 obs.	1966 obs. and 1911 obs.
1 Age-sex composition of net migrants as observed in Australia during 1962-66			
(a) Net immigration for all years: Net rate given by 0.01 ABS(Cos 0.0628318(t-1))	155	155	165
(b) Net emigration for all years: Net rate given by -0.01 ABS(Cos 0.0628318(t-1))	275	275	275
(c) Net immigration and emigration alternating over time: Net rate given by 0.01 Cos 0.0628318(t-1)	225	210	245
2 Age-sex composition of net migrants as observed in Australia during 1962-66 but with no (female or male) migrants below the age 50 years			
(i) Net immigration for all years: Net rate given by 0.01 ABS(Cos 0.0628318(t-1))	195	185	200
(ii) Net emigration for all years: Net rate given by -0.01 ABS (Cos 0.0628318(t-1))	250	250	250

emigration more than that in the absence of migration. It must be mentioned here that the use of the Cosine curve resulted in the occurrence of no migration for one year in each of the cycles and this perhaps, increased the observed duration to some extent in the case of immigration and decreased it in the case of emigration. Further, Table 4.2 shows that when net immigration and emigration alternated over time, the duration remained the same as in the absence of migration in two of the cases considered while in the third, it increased. Figure 4.4, again, makes it clear that the differences in the sex ratios vanished in (w+1) years, i.e. in 86 years in our examples.

(iii) Duration of Convergence in a Few Special Cases

The main purpose in continuing with these exercises, was to see whether a change in the magnitude of the net migration rate or a change in the time path of net migration rate would have an effect on the duration of convergence. The age-sex composition of net migrants was kept unchanged in these experiments also. The details of the assumptions in these exercises are given in Table 4.3. The results indicate that the difference in the time path had very little effect on the duration provided the magnitude of the net migration rate reached similar levels during a period of the same length. For example, the use of the Sine curve which gave the opposite trend in the net migration rate to that given by the Cosine curve, showed no change in the duration of convergence. This seems to happen because the

TABLE 4.3

DURATION OF CONVERGENCE OF AGE-SEX DISTRIBUTIONS UNDER CERTAIN SPECIAL CONDITIONS OF MIGRATION

Prevailing Conditions Mortality: As observed in Australia during 1911-66 and the 1966 values remaining constant Fertility: Hypothetical fertility rates having the continuing trend similar to the one observed during 1911-61	From Actual Projections Value of t in Number of Years 1911 obs. and 1911 stb.	1911 stb. and 1966 obs.	1966 obs. and 1911 obs.
1 Age-sex composition of net migrants as observed in Australia during 1962-66			
(a) Net immigration for all years: Net rate given by 0.05 ABS(Cos 0.0628318(t-1))	95	100	110
(b) Net rate given by 0.05 ABS(Cos 0.3141590(t-1))	105	105	105
(c) Net rate given by 0.05 ABS(Sin 0.0628318(t-1))	100	105	110
(d) Net rate given by 0.05 ABS(Sin 0.3141590(t-1))	105	105	105
(e) Net rate constant at 0.01 per year	140	140	155
2 Age-sex composition of net migrants as observed in Australia during 1962-66 with no migrants (females or males) at ages 50 years and above			
Net rate given by 0.01 ABS(Cos 0.0628318(t-1))	160	160	165
3 Age-sex composition of net migrants as observed in Australia during 1962-66 with no female migrants at any age			
Net rate given by 0.01 ABS(Cos 0.0628318(t-1))	225	210	225

structures of the matrices in the product, remain unchanged. But the change in the net rate of migration to a higher level indicated a reduction in the duration of convergence.

When the net immigration rate was kept constant at 1 per cent, the duration was found to be less than that under the assumption of the Cosine curve. At least a part of this decrease has come due to the fact that for none of the years under this assumption the net migration rate became zero.

The last two examples presented in Table 4.3 show that, when the female population was not affected by migration, the duration remained the same as in the absence of migration as we expected it to be, and when there were no migrants above the oldest age at which reproduction occurred, the duration decreased as compared to the case with no migration, but increased as compared with that in the case with immigration at all ages in at least two of the three cases and remained the same in the other.

4.7 SUMMARY

Thus, the investigations in this chapter reveal that, when migration is included into the process of population growth using an overall net migration rate and an age-sex distribution of net migrants, the weak ergodicity theorem holds good. The time required for the convergence of any two arbitrary age distributions becomes less if there is continuous net immigration and becomes more if there is

continuous net emigration, than if there is no migration. The actual number of years decreased or increased depends on the age distributions of the net migrants and the magnitude of the net migration rates. If net immigration, net emigration and zero migration affect the population for different years, the property still holds good, but the duration of convergence may decrease, remain the same, or increase as compared to the case with no migration, depending on the number of years each of the situations prevails and the age range affected by immigration, emigration or no migration in each case. On the other hand, if age-sex-specific net migration rates are used in the process of population change, the duration of convergence remains the same as in the absence of migration.

PART III

POPULATION CHANGE

CHAPTER 5

POPULATION CHANGE UNDER CONSTANT CONDITIONS

5.1 INTRODUCTION

In Part II, the main concern has been to see whether the convergence of age-sex distributions occurs or not, and how the duration of convergence is affected, when migration is introduced into the process of population change. But, the relationship between population change - the changes in the growth and age-sex composition - and the components of change - fertility, mortality and migration - has not been investigated. In this part, Part III, we propose to explore the above relationship. While the present chapter will consider the population change under constant conditions, the next will examine the population change under changing conditions.

When a set of single schedules of fertility, mortality and migration operates constantly over time, we know from the investigations in Chapter 3 that a constant growth rate and an unchanging age-sex distribution are evolved. Though these can be obtained from the dominant characteristic root and the associated characteristic vector of the population projection matrix, the relationship between these and the given schedules is not explicitly shown in that case. Hence, by using an elementary approach which avoids the use of matrix algebra, we shall derive formulas (in Section 5.2) that would reveal clearly the nature of this relationship. Numerical examples are presented to illustrate the

the formulas and to study the changes in these characteristics of the equilibrium state population in relation to the changes in the operating schedules of fertility, mortality and migration.

Namboodiri[1] has shown that, when a sequence of k schedules of fertility and mortality operates repeatedly over time on a closed population, a stable set of k growth rates and age-sex distributions is evolved, and has examined the relationship of this set with the operating conditions. He has named this set as the 'cyclical' model of population change. In this chapter (in Section 5.3), the procedure adopted by Namboodiri is slightly modified to facilitate the inclusion of migration and the population change resulting from the repeated operation of a sequence of k schedules of fertility, mortality and migration, is studied. The set of growth rates and age-sex distributions which results when migration is introduced as one of the components in the process of population change, is called here the equilibrium state cycle to distinguish it from the 'cyclical' model obtained in the absence of migration. The model seems to be useful in investigating the implications of the operation of certain observed or assumed conditions and also in obtaining the growth rate and the age-sex distribution of the equilibrium state population by iteration.

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[1] N.K. Namboodiri, 'On the Dependence of Age Structures ...', Demography, 1969.

5.2 EQUILIBRIUM STATE POPULATIONS

An equilibrium state population as defined in this study, possesses a constant growth rate and an unchanging age-sex distribution. We shall use this property as the basis for deriving the required relationships. Following the practice in the previous chapters, we shall deal with the one-sex model first and then, extend the results to the two-sex case.

5.2.1 ONE-SEX MODEL

5.2.1(a) One-Sex Model with No Migration

When there is no migration, the equilibrium state population is the same as the stable population which is already well known in the literature. Its age composition could be computed from the following formulas derived by Goodman:[2]

$$[P(x,t)/\sum_{0}^{w} P(x,t)] = [s(x)/\lambda^{x}]/[\sum_{0}^{w} (s(x)/\lambda^{x})] \qquad (5.1)$$

where $s(x) = 1$, if $x = 0$, and

$$= S(0)\ S(1)\ \ldots\ S(x-1), \text{ if } x > 0 \qquad (5.2)$$

and λ is obtained from the equation:

$$\lambda^{\beta+1} - \sum_{\alpha}^{\beta} m(x)\ s(x)\ \lambda^{\beta-x} = 0 \quad . \qquad (5.3)$$

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[2] L.A. Goodman, 'An Elementary Approach to the Population Projection Matrix ...', Demography, 1968, pp.394-396.

It may be noticed that the equation (5.3) is the same as the characteristic equation of the matrix M (see Section 3.2.1). This equation has only one real positive root since there is only one change of sign in the coefficients.[3] Hence, either by using the matrix algebra or by solving the equation (5.3) by any other iterative methods, we can obtain the unique positive root of this equation and thus obtain the age composition. Then r, the intrinsic growth rate of the stable population may be calculated from the relation $\lambda = e^{rd}$ where d is the interval of the age groups. Usually, d is taken as equal to 1 or 5 years of age.

5.2.1(b) One-sex Model with Migration

In this case, the population growth is given by the set of equations:

$$P(0,t) = \sum_{\alpha}^{\beta} m(x)\, P(x,t-1) + u \sum_{0}^{w} P(x,t-1) \qquad (5.4)$$

$$P(x,t) = S(x-1)\, P(x-1,t-1) + a(x-1) \sum_{0}^{w} P(x,t-1),$$

$$\text{for } x = 1, 2, \ldots w \qquad (5.5)$$

where m(x), u and a(x) are as defined in Section 3.2.2.

Since we are interested in the equilibrium state population, we have:

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[3] W.S. Burnside and A.W. Panton, The Theory of Equations, Dover Publications Inc., New York-14, N.Y., 1960, Vol. I, pp.28-31.

$$p(x) = [P(x,t)/\sum_{0}^{w} P(x,t)] = [P(x,t-1)/\sum_{0}^{w} P(x,t-1)]$$

$$\text{for } x = 0, 1, 2, \ldots, w; \tag{5.6}$$

$$\text{and } [\sum_{0}^{w} P(x,t)]/[\sum_{0}^{w} P(x,t-1)] = \lambda, \text{ a constant independent of } t. \tag{5.7}$$

Now our problem is to find the values of $p(x)$ and λ in terms of the $S(x)$, $m(x)$, u, and $a(x)$ which are known and are assumed to remain constant over time. From equations (5.4), (5.6) and (5.7), we can show that:

$$[P(0,t)/P(0,t-1)] = [\sum_{0}^{w} P(x,t-1)]/[\sum_{0}^{w} P(x,t-2)] = \lambda \quad . \tag{5.8}$$

By successive application of the equation (5.8), we get:

$$[P(0,t)]/[P(0,t-x)] = \lambda^{x} \quad . \tag{5.9}$$

Now substituting for $P(x-1,t-1)$ in equation (5.5), we have:

$$P(x,t) = S(x-1)\ S(x-2)\ P(x-2,t-2) + S(x-1)\ a(x-2) \sum_{0}^{w} P(x,t-2)$$

$$+ a(x-1) \sum_{0}^{w} P(x,t-1) \tag{5.10}$$

and continuing in a similar manner, we obtain:

$$P(x,t) = s(x)\ P(0,t-x) + \sum_{\xi=0}^{x-1} a(\xi)\ S(\xi+1)\ \ldots\ S(x-1)\ [\sum_{0}^{w} P(x,t-x+\xi)]$$

$$\text{for } x = 1, 2, \ldots, w. \tag{5.11}$$

Dividing both sides of equation (5.11) by P(0,t) and using the fact that p(0) is constant over time, we have:

$$[P(x,t)/P(0,t)] = [s(x)/\lambda^x] + \frac{1}{p(0)} \left[\sum_{\xi=0}^{x-1} A(\xi,x)/\lambda^{x-\xi}\right] \qquad (5.12)$$

where $A(\xi,x) = a(\xi)\ S(\xi+1)\ S(\xi+2) \ldots S(x-1)$, for $x = 1, 2, \ldots, w$. But p(0) is unknown. To obtain p(0), we use the definition of p(0) and write:

$$[1/p(0)] = 1 + \sum_{1}^{w} \left[s(x)/\lambda^x + \frac{1}{p(0)} \sum_{\xi=0}^{x-1} A(\xi,x)/\lambda^{x-\xi}\right] \quad .$$

This yields the value of p(0) as:

$$p(0) = \frac{1 - \sum_{1}^{w} \sum_{\xi=0}^{x-1} [A(\xi,x)/\lambda^{x-\xi}]}{1 + \sum_{1}^{w} [s(x)/\lambda^x]} \quad . \qquad (5.13)$$

The proportions of populations at other ages can be calculated from the following relationship which is easily established:

$$[P(x,t)/\sum_{0}^{w} P(x,t)] = [P(x,t)/P(0,t)]\ p(0) \quad . \qquad (5.14)$$

Now, if we know the value of λ, our problem is solved. The value of λ can be computed by matrix methods, since it is the characteristic root of the population matrix L_M (see Section 3.2.2). However, we can obtain λ from equation (5.4). On dividing both sides of that equation by P(0,t), we get:

$$1 = \sum_{\alpha}^{\beta} m(x) \left[\frac{P(x,t-1)}{P(0,t-1)} \frac{P(0,t-1)}{P(0,t)}\right] + u \sum_{0}^{w} \left[\frac{P(x,t-1)}{P(0,t-1)} \frac{P(0,t-1)}{P(0,t)}\right]$$

which on substitution for the terms on the right hand side, and simplification, gives the polynomial equation in λ:

$$\lambda^{\beta+1} - [u/p(0)]\ \lambda^{\beta} - \sum_{\alpha}^{\beta} [m(x)\ s(x)\ \lambda^{\beta-x} + \{1/p(0)\} \sum_{\xi=0}^{x-1} m(x)\ A(\xi,x)\ \lambda^{\beta-x+\xi}] = 0 \quad . \qquad (5.15)$$

The last term in equation (5.15) is:

$$-[s(\beta) + \{A(0,\beta)/p(0)\}]\ m(\beta)$$

which can be written as:

$$-[\{S(1)\ S(2)\ \ldots\ S(\beta-1)\}/p(0)]\ [S(0)\ p(0) + a(0)]\ m(\beta)$$

and will be negative under assumption (8) whether a(0) is positive or negative. Thus the equation (5.15) has at least one positive real root.[4] In particular cases, however, there may be more than one positive root. Then the positive root which has the largest value is taken as the value of λ.

Thus by solving the equation (5.15) we can obtain the value of λ. But in this equation p(0) is also not known and it is dependent on λ. Hence, we start with a trial value of λ, e.g. the value obtained without including migration plus the net migration rate, calculate p(0) and estimate λ; and continue the process of iteration till we obtain

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[4] W.S. Burnside and A.W. Panton, The Theory of Equations, 1960, Vol. I, pp.20-21.

constant values of λ and $p(0)$ to the desired approximation. Alternatively, we may obtain the value of λ from the matrix methods and use it in computing the other values. In practice, it may be preferable to obtain λ from the matrix methods since the coefficients of the above polynomial change in every iteration and the calculation of λ from the polynomial itself involves an interation procedure.

It may be noticed from equation (5.15) that, if migration affects only ages beyond β years, the values of u, $A(\xi,x)$ in the equation become zero and hence, the equation becomes the same as equation (5.3) and the value of λ would remain the same as in the case with no migration (see Section 3.2.2).

5.2.2 TWO-SEX MODEL

The formulas for the two-sex case can be derived by a simple extension of the argument given for the one-sex case. In the two-sex model we note that the age-sex distribution is unchanging and the growth rates for the male and the female populations are the same. Hence, we have to obtain in this case not only the age distributions of each sex but also the sex composition in the age groups as well as in the total population. We shall first consider the two-sex model with female dominance and then discuss the case with equal dominance. We use the suffixes m and f to distinguish males and females.

5.2.2(a) Two-sex Model with No Migration

Goodman[5] has discussed in detail the case when there is no

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[5] L.A. Goodman, 'On the Age-sex Composition of the Population ...', Demography, 1967.

migration and has obtained the following formulas:

$$[P_f(x,t)/P_f(0,t)] = [s_f(x)/\lambda^x], \text{ for } x = 1, 2, \ldots, w \tag{5.16}$$

$$[P_m(0,t)/P_f(0,t)] = SR(0) = \sum_{\alpha}^{\beta} m'_f(x)\ s_f(x)/\lambda^{x+1} \tag{5.17}$$

and $$[P_m(x,t)/P_f(0,t)] = [s_m(x)/\lambda^x]\ SR(0), \text{ for } x = 1, 2, \ldots, w, \tag{5.18}$$

where $m'_f(x) = (s/2)\ [f_f(x) + S_f(x)\ f_f(x+1)]\ S_m(b)$ with s as the male proportion at birth and SR(0) the sex ratio in the age group 0 years. The sex ratios at other ages are obtained as follows:

$$SR(x) = SR(0)\ [s_m(x)/s_f(x)] \tag{5.19}$$

and the sex ratio in the total population is shown to be:

$$SR = SR(0)\ [\sum_{0}^{w} s_m(x)/\lambda^x]/[\sum_{0}^{w} s_f(x)/\lambda^x] \quad . \tag{5.20}$$

From equations (5.16) - (5.18), the age structures for males and females can easily be computed provided the value of λ is known. The value of λ can either be obtained as the dominant characteristic root of the projection matrix L', or by solving the following equation:

$$\lambda^{\beta+1} - \sum_{\alpha}^{\beta} m_f(x)\ s_f(x)\ \lambda^{\beta-x} = 0 \tag{5.21}$$

where $$m_f(x) = \frac{1}{2}\ (1-s)\ [f_f(x) + S_f(x)\ f_f(x+1)]\ S_f(b) \quad .$$

5.2.2(b) Two-sex Model with Migration

For convenience, let us write down the growth equations for this model:

$$P_f(0,t) = \sum_{\alpha}^{\beta} m_f(x)\, P_f(x,t-1) + u_f \sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)] \qquad (5.22)$$

$$P_m(0,t) = \sum_{\alpha}^{\beta} m'_f(x)\, P_f(x,t-1) + u_m \sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)] \qquad (5.23)$$

$$P_f(x,t) = S_f(x-1)\, P_f(x-1,t-1) + a_f(x-1) \sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)] \qquad (5.24)$$

and

$$P_m(x,t) = S_m(x-1)\, P_m(x-1,t-1) + a_m(x-1) \sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)] \qquad (5.25)$$

for $x = 1, 2, \ldots, w$.

Assuming that the population is an equilibrium state population, we can show that:

$$\frac{P_f(0,t)}{P_f(0,t-1)} = \frac{P_m(0,t)}{P_m(0,t-1)} = \frac{\sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)]}{\sum_{0}^{w} [P_f(x,t-2) + P_m(x,t-2)]} = \lambda \quad . \qquad (5.26)$$

By the repeated application of (5.26), we obtain:

$$[P_f(0,t)/P_f(0,t-x)] = [P_m(0,t)/P_m(0,t-x)] = \lambda^x \quad . \qquad (5.27)$$

Proceeding as in the one-sex model, we get from equation (5.24):

$$[P_f(x,t)/P_f(0,t)] = [s_f(x)/\lambda^x] + (1/f_p(0)) \sum_{\xi=0}^{x-1} [A_f(\xi,x)/\lambda^{x-\xi}]$$

$$\text{for } x = 1, 2, \ldots, w, \qquad (5.28)$$

where $f_p(0)$ is the proportion of the female population at age 0 years to the total population of both sexes, and is constant over time for the equilibrium state population.

From equation (5.25) we can similarly obtain:

$$[P_m(x,t)/P_f(0,t)] = SR(0)\ [s_m(x)/\lambda^x] + \frac{1}{f_p(0)} \sum_{\xi=0}^{x-1} [A_m(\xi,x)/\lambda^{x-\xi}]$$

$$\text{for } x = 1, 2, \ldots, w. \qquad (5.29)$$

On dividing both sides of the equation (5.23) by $P_f(0,t)$ we get:

$$[P_m(0,t)/P_f(0,t)] = SR(0) = \sum_{\alpha}^{\beta} [m'_f(x)\ s_f(x)/\lambda^{x+1}$$

$$+ \frac{1}{f_p(0)} \sum_{\xi=0}^{x-1} m'_f(x)\ A_f(\xi,x)/\lambda^{x-\xi+1}] + \frac{1}{\lambda} \left(\frac{u_m}{f_p(0)}\right) \quad . \qquad (5.30)$$

Equations (5.28), (5.29) and (5.30) give the population in all the age-sex groups in terms of the population in the first age group for females. From these we can easily compute the age distributions for males and females, the sex ratios in the age groups and the sex ratio in the total population, as we did in the case with

no migration. It may be noted that the formulas given by Goodman are particular cases of the above ones and could be derived by putting the migration coefficients equal to zero.

But, all these formulas are dependent on λ and $f_p(0)$ which are still unknown. We shall derive, now, expressions to compute these. To obtain $f_p(0)$ we use the relationship:

$$\sum_{0}^{w} [P_f(x,t) + P_m(x,t)] = P_f(0,t)\ [1 + \frac{P_m(0,t)}{P_f(0,t)} + \sum_{1}^{w} \left(\frac{P_f(x,t)}{P_f(0,t)} + \frac{P_m(x,t)}{P_f(0,t)} \right)].$$

Substituting the respective expressions for the terms in parenthesis on the right hand side and simplifying we get:

$$f_p(0) =$$

$$\frac{1-[\sum_{\alpha}^{\beta} \sum_{\xi=0}^{x-1} \frac{m'_f(x)\ A_f(\xi,x)}{\lambda^{x-\xi+1}} + \frac{u_m}{\lambda}](1+ \sum_{1}^{w} \frac{s_m(x)}{\lambda^x}) - \sum_{1}^{w} \sum_{\xi=0}^{x-1} [\frac{A_f(\xi,x)+A_m(\xi,x)}{\lambda^{x-\xi}}]}{[1+ \sum_{1}^{w} \frac{s_f(x)}{\lambda^x}]+[1+ \sum_{1}^{w} \frac{s_m(x)}{\lambda^x}](\sum_{\alpha}^{\beta} \frac{m'_f(x)\ s_f(x)}{\lambda^{x+1}})} \qquad (5.31)$$

For calculating the value of λ, we obtain using (5.22) the polynomial equation:

$$\lambda^{\beta+1} - (\frac{u_f}{f_p(0)})\ \lambda^{\beta} - \sum_{\alpha}^{\beta} [m_f(x)\ s_f(x)\ \lambda^{\beta-x} + \frac{1}{f_p(0)} \sum_{\xi=0}^{x-1} m_f(x)\ A_f(\xi,x)\ \lambda^{\beta-x+\xi}] = 0 \quad . \qquad (5.32)$$

As in the one-sex model, we can show that this equation has at least one positive root provided that $S_f(0)\ f_p(0) > a_f(0)$ whenever $a_f(0)$ is negative. Hence, the values of λ and $f_p(0)$ can be iterated from (5.31) and (5.32). Again, λ may preferably be obtained from the matrix roots. It can also be seen from equation (5.32) that, if migration does not affect the female population at any of the ages below β years, it will not have any effect on the intrinsic growth rate.

5.2.2(c) Two-sex Model with Equal Dominance

When we assume equal dominance the number of births changes and accordingly the equations for SR(0), $f_p(0)$ and λ change. The new expressions for these can easily be derived by proceeding as in the case with female dominance.

When the migration is specified by a net migration rate and an age-sex composition of net migrants, the growth equations for the female and the male populations at age 0 years are as given below:

$$P_f(0,t) = \sum_{\alpha'}^{\beta'} \frac{1}{2}[m_f(x)\ P_f(x,t-1) + m_m(x)\ P_m(x,t-1)] + u_f \sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)]$$

and

$$P_m(0,t) = \sum_{\alpha'}^{\beta'} \frac{1}{2}[m'_f(x)\ P_f(x,t-1) + m'_m(x)\ P_m(x,t-1)] + u_m \sum_{0}^{w} [P_f(x,t-1) + P_m(x,t-1)] \quad .$$

Consequently, the respective formulas for SR(0), $f_p(0)$ and λ are as follows:

Let $$D = 1 - \frac{1}{2}\sum_{\alpha'}^{\beta'}[m'_m(x)\ s_m(x)/\lambda^{x+1}]$$

Then, $$SR(0) = \frac{1}{D}\left[\frac{1}{2}\sum_{\alpha'}^{\beta'}\frac{m'_f(x)\ s_f(x)}{\lambda^{x+1}} + \frac{1}{f_p(0)}\left\{\frac{1}{2}\sum_{\alpha'}^{\beta'}\sum_{\xi=0}^{x-1}\left(\frac{m'_f(x)\ A_f(\xi,x)+m'_m(x)\ A_m(\xi,x)}{\lambda^{x-\xi+1}}\right) + (u_m/\lambda)\right\}\right] \quad (5.33)$$

$f_p(0) =$

$$\frac{\left[1-\frac{1}{D}\left\{1+\sum_{1}^{w}\frac{s_m(x)}{\lambda^x}\right\}\left\{\frac{1}{2}\sum_{\alpha'}^{\beta'}\sum_{\xi=0}^{x-1}\left(\frac{m'_f(x)\ A_f(\xi,x)+m'_m(x)\ A_m(\xi,x)}{\lambda^{x-\xi+1}}\right)+\left(\frac{u_m}{\lambda}\right)\right\} - \sum_{1}^{w}\sum_{\xi=0}^{x-1}\frac{A_f(\xi,x)+A_m(\xi,x)}{\lambda^{x-\xi}}\right]}{\left\{1+\sum_{1}^{w}\frac{s_f(x)}{\lambda^x}\right\} + \frac{1}{D}\left\{1+\sum_{1}^{w}\frac{s_m(x)}{\lambda^x}\right\}\left[\frac{1}{2}\sum_{\alpha'}^{\beta'}\frac{m'_f(x)\ s_f(x)}{\lambda^{x+1}}\right]} \quad (5.34)$$

and the polynomial equation in λ:

$$\lambda^{\beta'+1} - \left(\frac{u_f}{f_p(0)}\right)\lambda^{\beta'} - \frac{1}{2}\sum_{\alpha'}^{\beta'}\left[\{m_f(x)\ s_f(x) + SR(0)\ m_m(x)\ s_m(x)\}\ \lambda^{\beta'-x} + \frac{1}{f_p(0)}\sum_{\xi=0}^{x-1}\{m_f(x)\ A_f(\xi,x) + m_m(x)\ A_m(\xi,x)\}\ \lambda^{\beta'-x+\xi}\right] = 0 \quad (5.35)$$

where α' and β' are the youngest and the oldest ages at which reproduction occurs among males and/or among females.

From the considerations given in the case of the one-sex model, the equation (5.35) also has at least one positive root provided $S_f(0)\ f_p(0) > a_f(0)$, and $S_m(0)\ f_p(0) > a_m(0)$ whenever $a_f(0)$ and/or $a_m(0)$ are negative. Therefore, the values of $SR(0)$, $f_p(0)$ and λ may be obtained by iteration. Alternatively, perhaps preferably, we may obtain λ from the matrix roots and compute $SR(0)$ and $f_p(0)$ from the above expressions. Once these three quantities are known, we proceed as in the case with the female dominance to calculate the age distributions for males and females, the sex ratios in the age groups and the sex ratio in the total population.

The corresponding formulas when no migration is assumed, are obtained by substituting zero for the migration coefficients in the above expressions. They may be seen to be the same as those given by Goodman with $\delta = \frac{1}{2}$.

5.2.3 NUMERICAL ILLUSTRATIONS

We shall use the two-sex models to test the formulas derived here. Since the formulas for the case with no migration are well known and in fact, could be taken as particular cases of those derived here, we do not attempt to illustrate them. Though the case in which the age-sex-specific net migration rates are used in the process of population growth, is not considered in the theoretical investigations because the formulas applicable in the case with no migration are directly applicable in that case, we do compare the equilibrium state populations resulting from the use of the two procedures for specifying the migration situation.

Further, the effect of changing the set of fertility, mortality and migration schedules on the characteristics of the equilibrium state population, an aspect which we have not considered in the analytical work, is examined using the results of empirical experiments. Though expressions could be obtained to take account of these effects of the changes of schedules, we have not attempted to do so because the resulting formulas will be too cumbersome and in any case, the changes in the equilibrium state populations depend on the observed changes in the components and will have to be studied separately in each case.

Finally, the changes in the characteristics of the equilibrium state populations are separated into the effects of the changes in the schedules of fertility, mortality and migration, adopting a method similar to the one used by Keyfitz to decompose the changes in the characteristics of the stable populations.[6] Such a decomposition indicates in a given situation, the relative weights of the effects of the changes in the schedules of the components, on the characteristics of the equilibrium state populations.

(i) Age-sex Distributions and the Growth Rates Resulting from Given Setsof Fertility, Mortality and Migration Conditions

From the actual projections which were carried out to illustrate the convergence of the age-sex distributions of the three

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[6] N. Keyfitz, 'Changing Vital Rates and Age Distributions', Population Studies, Vol. 22, No. 2, 1968, pp.235-251.

initial populations (see Section 3.4), the age-sex distributions and the growth rates of the equilibrium state populations resulting from the given schedules, when female dominance was assumed and when equal dominance was assumed, were obtained. Then the formulas derived in Section 5.2.2(b) and 5.2.2(c) were used to compute the corresponding age-sex structures of the equilibrium state populations utilizing the values of λ from the characteristic roots of the matrix L_M of Sections 3.3.2 and 3.3.3.

Table 5.1 compares the equilibrium state age-sex distributions obtained by the two procedures. It can be seen that the values are quite close, both when the female dominance is assumed and when the equal dominance is assumed. The significant difference found in the sex ratio in the age group (85+), may be due to the different procedures used in obtaining the proportion in that age group in the actual projections and in applying the formulas. For applying the formulas, it was necessary to keep the width of the age interval the same throughout the age range. Hence, single year survival rates up to 99 years were used in the formulas to get the population proportion and the sex ratio in the age group (85+), whereas in the actual projections the group (84+) was projected as a group to (85+). In those old age groups the survival rates for females were higher than for males.

TABLE 5.1

THE AGE-SEX DISTRIBUTION OF THE EQUILIBRIUM STATE POPULATION THAT WOULD RESULT FROM THE FERTILITY AND MORTALITY RATES OF 1911, AND THE NET MIGRATION RATE AND THE AGE-SEX COMPOSITION OF NET MIGRANTS DURING 1911-12 IN AUSTRALIA

Age Groups	When Female Dominance is Assumed						When Equal Dominance is Assumed					
	From Actual Projections			From the Formulas Derived in Section 5.2.2(b)			From Actual Projections			From the Formulas Derived in Section 5.2.2(c)		
	Male	Female	Sex Ratio (per 100)	Male	Female	Sex Ratio (per 100)	Male	Female	Sex Ratio (per 100)	Male	Female	Sex Ratio (per 100)
0- 4	9.22	11.52	104.34	9.20	11.50	104.66	10.66	13.11	104.18	10.64	13.08	104.50
5- 9	8.64	10.47	107.65	8.63	10.45	107.96	9.65	11.55	107.10	9.64	11.52	107.40
10-14	8.24	9.72	110.64	8.24	9.71	110.94	8.91	10.40	109.86	8.91	10.39	110.15
15-19	8.36	9.00	121.18	8.35	8.99	121.48	8.76	9.37	119.82	8.75	9.36	120.11
20-24	8.88	8.45	137.05	8.88	8.45	137.34	9.04	8.57	135.23	9.04	8.57	135.52
25-29	9.13	8.04	148.15	9.13	8.04	148.42	9.09	7.96	146.38	9.09	7.96	146.65
30-34	8.84	7.57	152.29	8.84	7.58	152.53	8.63	7.34	150.74	8.63	7.34	150.98
35-39	8.14	6.94	152.97	8.14	6.94	153.19	7.80	6.59	151.57	7.80	6.60	151.79
40-44	7.18	6.16	152.00	7.18	6.17	152.21	6.75	5.74	150.69	6.76	5.75	150.90
45-49	6.14	5.35	149.67	6.14	5.36	149.86	5.66	4.89	148.40	5.67	4.90	148.60
50-54	5.08	4.54	146.12	5.09	4.55	146.31	4.60	4.07	144.87	4.61	4.08	145.05
55-59	4.07	3.75	141.53	4.07	3.76	141.71	3.61	3.30	140.24	3.61	3.30	140.41
60-64	3.12	3.00	135.67	3.13	3.01	135.84	2.71	2.58	134.31	2.71	2.59	134.48
65-69	2.24	2.28	128.52	2.25	2.28	128.68	1.90	1.92	127.13	1.91	1.93	127.29
70-74	1.45	1.58	119.89	1.46	1.59	120.04	1.21	1.31	118.53	1.21	1.31	118.68
75-79	0.80	0.95	109.31	0.80	0.95	109.45	0.65	0.77	107.98	0.65	0.77	108.12
80-84	0.35	0.47	96.37	0.35	0.47	96.50	0.28	0.37	95.14	0.28	0.37	95.26
85+	0.13	0.21	83.52	0.13	0.21	79.03	0.10	0.16	82.48	0.10	0.17	78.06
Total	100.00	100.00	130.46	100.00	100.00	130.73	100.00	100.00	128.18	100.00	100.00	128.45
Growth Index	1.03050	1.03050	-	1.03044	1.03044	-	1.03512	1.03512	-	1.03504	1.03504	-

Note:- The growth index used in the formulas of Section 5.2.2(b) was the matrix root given in Table 3.1; and the one used in the formulas of Section 5.2.2(c) was the matrix root given in Table 3.4.

(ii) Effect of Migration on the Characteristics of the Equilibrium State Population

The comparison of the age distributions presented in Table 5.2 and that of the sex distributions and the growth rates presented in Table 5.3, reveal that the use of the net migration rate and the age-sex composition of net migrants, made a substantial difference in the characteristics of the equilibrium state populations as against the use of the age-sex-specific net migration rates, when the net migrants age-sex composition was very much different from that of the stable population resulting from the given vital rates, as it was during 1911-12. Otherwise, the differences were small, as revealed by the situation in 1965-66. It must, however, be mentioned that the magnitude of the difference is also dependent on the magnitude of the net migration. For instance, during 1945-46, the net migration was small and thus the differences were quite small, though the age-sex composition of the net migrants was not close to that of the stable population. For the case in which the 1915-16 net migration situation was assumed, the use of the net migration rate and the age-sex composition of net migrants showed that the situation was incompatible with the vital rates of 1911. But when the same situation was specified by the age-sex-specific net migration rates, the age-sex distribution of the equilibrium state population became very unusual showing exceptionally high proportion of males under 15 years of age and abnormally low sex-ratios in all age groups. In fact, negative values would not, generally, result in this case.

TABLE 5.2

COMPARISON OF THE AGE DISTRIBUTIONS OF THE EQUILIBRIUM STATE POPULATIONS RESULTING FROM THE SPECIFIED FERTILITY, MORTALITY AND MIGRATION CONDITIONS

Prevailing Conditions			Male Age Distribution (per cent)				Mean Age	Female Age Distribution (per cent)				Mean Age
Mortality	Fertility	Migration	0-14	15-44	45-64	65+		0-14	15-44	45-64	65+	
					No Migration							
1911	1911	0	31.54	44.73	17.38	6.35	29.16	30.61	43.68	17.84	7.86	30.23
1966	1966	0	29.17	44.02	19.17	7.64	30.88	27.55	42.13	19.45	10.87	32.92
			Migration Specified by a Net Migration Rate and an Age-Sex Composition of Net Migrants									
1911	1911	1911-12	26.10	50.52	18.41	4.97	30.31	31.71	46.16	16.64	5.48	28.60
1911	1911	1915-16	x	x	x	x	x	x	x	x	x	x
1911	1911	1945-46	30.48	44.67	18.06	6.79	29.83	30.59	43.49	17.81	8.11	30.34
1966	1966	1965-66	29.66	45.43	18.51	6.41	30.05	28.59	43.63	18.66	9.12	31.65
			Migration Specified by a Set of Age-Sex-Specific Net Migration Rates									
1911	1911	1911-12	23.84	49.16	20.94	6.07	32.17	31.19	45.40	17.29	6.12	29.19
1911	1911	1915-16	62.81	32.94	3.37	0.88	14.70	31.17	44.40	17.20	7.24	29.61
1911	1911	1945-46	30.48	44.61	18.10	6.82	29.86	30.51	43.43	17.90	8.17	30.41
1966	1966	1965-66	29.45	45.51	18.65	6.39	30.16	28.34	43.67	18.85	9.14	31.79

Note:- See Note (1) and (2) to Table 3.1

TABLE 5.3

COMPARISON OF THE SEX COMPOSITIONS AND THE GROWTH RATES OF THE EQUILIBRIUM STATE POPULATIONS RESULTING FROM THE SPECIFIED FERTILITY, MORTALITY AND MIGRATION CONDITIONS

Prevailing Conditions			Sex Ratios in Age Groups (Males per 100 Females)				Sex Ratio in Total Population	Growth Rate* (per 1000)
Mortality	Fertility	Migration	0-14	15-44	45-64	65+		
			No Migration					
1911	1911	0	102.97	102.32	97.35	80.70	99.93	12.10
1966	1966	0	104.43	103.06	97.20	69.32	98.63	11.50
Migration Specified by a Net Migration Rate and an Age-Sex Composition of Net Migrants								
1911	1911	1911-12	107.37	142.78	144.34	118.25	130.46	30.05
1911	1911	1915-16	x	x	x	x	x	x
1911	1911	1945-46	102.84	105.98	104.61	86.37	103.18	10.02
1966	1966	1965-66	104.54	104.93	100.00	70.76	100.78	21.65
Migration Specified by a Set of Age-Sex-Specific Net Migration Rates								
1911	1911	1911-12	106.99	151.60	169.47	138.74	139.99	26.91
1911	1911	1915-16	85.23	31.38	8.30	5.15	42.29	13.38
1911	1911	1945-46	102.82	105.73	104.11	85.93	102.94	9.94
1966	1966	1965-66	104.52	104.81	99.49	70.35	100.58	21.92

Note:- See Note (1) and (2) to Table 3.1
* This was computed from actual projections, using the mid-year population as the base.

(iii) Changes in the Characteristics of the Equilibrium State Populations Due to the Changes in the Net Migration Rate and in the Age-sex Composition of Net Migrants

In attempting an empirical investigation of this kind, it is necessary to mention at the outset, that the effects observed will be true to the particular situations under consideration.

The results presented in Table 5.4, indicate a slowly increasing trend in the proportion in the age group (0-14) and in the sex ratio in the total population, and a decreasing trend in the old age proportion and in the mean age, associated with an increasing trend in the magnitude of the net migration rate from minus one per cent to five per cent, when the age-sex composition of net migrants was as in Australia during 1962-66. It may be recalled that this age-sex composition of net migrants was comparatively nearer to the smooth age-sex distribution of the stable population resulting from the vital rates of 1911 (see Section 2.4). But,even in this case, very heavy net migration such as the uncommon rate of immigration of five per cent, changed the age-sex distribution of the equilibrium state population to a marked extent as compared to that in the case with no migration.

On the other hand, when the age-sex composition of the net migrants was significantly different from the stable age-sex distribution resulting from the given vital rates, as in the experiments 2, 6, 7, 8 and 9 under (ii) in Table 5.4, the age-sex distribution of the resulting equilibrium state population was also significantly different. When the age-sex distribution of the net migrants was

TABLE 5.4

CHANGES IN SOME CHARACTERISTICS OF THE EQUILIBRIUM STATE POPULATIONS DUE TO THE CHANGES IN THE NET MIGRATION RATE AND IN THE AGE-SEX COMPOSITION OF NET MIGRANTS

Prevailing Conditions		Proportion in Young Age Group (0-14 Years)		Proportion in Old Age Group (65 years and over)		Mean Age		Sex Ratio (per 100) in Total Population	Growth Rate* (per 1000)
Mortality - 1911 Fertility - 1911 Migration -		Male	Female	Male	Female	Male	Female		
(i) Age-Sex Composition of 1962-66 and the Net Rate (per cent) as:	-1.0	31.28	29.75	7.74	9.49	29.95	31.34	97.59	00.01
	0.0	31.54	30.61	6.35	7.86	29.16	30.23	99.93	12.10
	0.5	31.73	30.97	5.76	7.23	28.77	29.77	100.68	17.95
	1.0	31.93	31.30	5.25	6.68	28.40	29.36	101.24	23.71
	5.0	33.31	32.89	2.96	4.41	26.30	27.34	103.01	66.43
(ii) Net Rate of 1per cent and Age-Sex Composition									
1. Same as in 1962-66		31.93	31.30	5.25	6.68	28.40	29.36	101.24	23.71
2. Same as in 1925		26.14	30.78	6.31	6.99	31.32	29.83	122.27	20.26
3. Same as 1911 Stb. Population		31.53	30.61	6.36	7.87	29.17	30.24	99.98	21.98
4. Same as 1966 Obs. Population		31.04	30.29	6.76	8.30	29.60	30.58	100.58	21.24
5. Same as 1911 Life Table Population		30.68	29.68	7.27	9.01	30.03	31.22	99.59	20.47
6. Same as 1911 Life Table Population with No Female Migrants		21.01	30.61	11.65	7.87	36.26	30.23	171.08	12.10
7. Same as 1911 Life Table Population with No Male Migrants		41.34	29.17	3.32	9.59	23.72	31.75	66.63	26.18
8. 1962-66, with No Female Migrants below Age 49 Years		24.39	26.00	8.19	16.73	33.03	36.13	120.84	12.10
9. 1962-66, with Female Migrants only at Age 0 Years below Age 49 Years		25.65	27.90	7.61	14.74	32.20	34.43	116.02	14.10

* This was computed from actual projections using mid-year population as base.

identical with the stable one, the age-sex distribution of the resulting equilibrium state population was also identical with the stable one. But the growth rate changed to the extent of the net migration rate.

These experiments indicate that a migration stream in which the balance of the sexes is not too uneven and in which there is not very heavy concentration of net migrants in certain age groups, is more beneficial from the point of view of the growth of the population and in obtaining a normal age-sex distribution in the population. Perhaps, this is desirable from the social point of view as well.

(iv) Decomposition of the Changes in the Characteristics of the Equilibrium State Populations into the Effects of the Changes in the Components

The fertility, mortality and migration schedules in Australia changed substantially from 1911 to 1966. If we compute the equilibrium state populations corresponding to these two situations observed at two points of time, their characteristics would naturally be different. For instance, the mean age for males which was 30.31 under the 1911 conditions decreased to 30.05 years under the conditions of 1966, while the mean age for females increased from 28.60 years to 31.65 years. Just by comparing these results, we are tempted to conclude that, probably, the changes in none of the components had any significant effect on the mean age for males. But, this observed change is the sum of all the changes produced by the differences in the values of the components in 1911 and in 1966. Hence, it would be interesting to decompose this observed change in the characteristics of the

equilibrium state populations into the effect of the change in each component separately and the effects of the interaction of the changes in the different components. This is attempted here.

The details of the procedure adopted for decomposing the observed change are given in Chapter 6 and will not be presented here.

Table 5.5 gives the decomposition of the change in the mean age and the proportion in the old age group (65+), for males and females. The proportion in the old age group was chosen for no particular reason except that the relative variation was large in that group. It is clear from the table that the large increase in the mean age for males, produced by the decrease in fertility was offset by a large decrease due to the change in the migration conditions and thus, the net result was a very small change in the mean age. Change in the mortality conditions decreased the mean age for males to some extent, but did not have any effect on the mean age for the females. It may be observed that, in the case of the females, the changes in all the components and hence their interactions, had an effect of increasing the mean age and the proportion in the old age group with fertility playing the most important part. However, the analysis of the proportion in the old age group for females and that of the mean age for males, showed that migration depending upon its characteristics, could be as important a factor in changing the age distributions as fertility is. Interaction effects, in general, were found to be comparatively small.

TABLE 5.5

DECOMPOSITION OF THE CHANGES IN THE MEAN AGE AND IN THE PROPORTION IN THE OLD AGE GROUP (65 YEARS AND OVER) OF THE EQUILIBRIUM STATE AGE-SEX DISTRIBUTION INTO THOSE DUE TO CHANGES IN FERTILITY, MORTALITY AND MIGRATION

Source of Change	Mean Age				Proportion in the Old Age Group (65+) Years			
	Male		Female		Male		Female	
	Absolute value (Number of Years)	Percentage	Absolute value (Number of Years)	Percentage	Absolute value (per 100)	Percentage	Absolute value (per 100)	Percentage
Value under Initial Conditions	30.31	-	28.60	-	4.97	-	5.48	-
Change in Mortality	-0.45	-173.08	0.01	0.33	-0.16	-11.11	0.58	15.93
Change in Fertility	1.74	669.23	1.87	61.31	0.94	65.28	1.10	30.22
Change in Migration	-1.68	-646.15	0.82	26.88	0.50	34.72	1.34	36.81
Interaction of the Changes in:								
Fertility and Mortality	0.00		0.06	1.97	-0.01	-0.69	0.13	3.57
Mortality and Migration	-0.07	-26.92	0.06	1.97	-0.04	-2.78	0.14	3.85
Fertility and Migration	0.22	84.61	0.22	7.21	0.23	15.97	0.32	8.79
Fertility, Mortality and Migration	-0.02	-7.69	0.01	0.33	-0.02	-1.39	0.03	0.83
Total Change	-0.26	-100.00	3.05	100.00	1.44	100.00	3.64	100.00
Value under Final Conditions	30.05	-	31.65	-	6.41	-	9.12	-

TABLE 5.6

DECOMPOSITION OF THE CHANGES IN THE PROPORTION OF MALES AND IN THE INTRINSIC GROWTH RATE IN THE EQUILIBRIUM STATE AGE-SEX DISTRIBUTION INTO THOSE DUE TO CHANGES IN FERTILITY, MORTALITY AND MIGRATION

Source of Change	Proportion of Males		Growth Rate	
	Absolute Value (per 100)	Percentage	Absolute Value (per 1000)	Percentage
Value under Initial Conditions	56.61	-	30.06	-
Change in Mortality	-0.21	-3.27	4.55	54.04
Change in Fertility	0.48	7.48	-4.31	-51.19
Change in Migration	-6.17	-96.11	-8.38	-99.53
Interaction of the Changes in:				
Fertility and Mortality	-0.08	-1.25	0.00	0.00
Mortality and Migration	0.12	1.87	0.09	1.07
Fertility and Migration	-0.56	-8.72	-0.39	-4.63
Fertility, Mortality and Migration	0.00	0.00	0.02	0.24
Total Change	-6.42	-100.00	-8.42	-100.00
Value under Final Conditions	50.19		21.64	

From the analysis presented in Table 5.6, we observe that the sole reason for the decrease in the proportion of males, was the change in the migration conditions, i.e. the decrease in the sex ratio among net migrants, though fertility and mortality contributed to a small

extent to increase it. The decline in mortality increased the growth rate, but the decline in fertility decreased it almost to the same extent so that the growth rate became smaller in 1966 than in 1911 to the extent of the decrease in the effect of migration. The interaction effects were, again, comparatively small.

Thus, the decompsotion of the changes in the characteristics of the equilibrium state populations gives a better understanding of the mechanisms effecting the change.

5.3 CYCLICAL MODEL OF POPULATION CHANGE

In this section we shall examine how the growth rate and the age-sex distributions change when a sequence of k schedules of fertility, mortality and migration conditions operates repeatedly over a long period of time. This model is called 'cyclical' because the growth rates and the age-sex distributions show, after a sufficiently long period of time, a fixed pattern of change, i.e. a pattern which will repeat itself cyclically.

Let $f(x,i)$ be the age-specific fertility rate at age x in the i-th fertility schedule; $S(b,i)$ and $[S(x,i),\ x = 0, 1, \ldots, (w-1)]$ the survival rates in the i-th mortality schedule; and $u(i)$ and $a(x,i)$ respectively, the values of u and $a(x)$ as defined in Section 3.2.2, in the i-th migration schedule.

From a practical point of view, it would be unrealistic to assume that the survival rates change in a cyclical manner. While

fertility and migration have shown trends which can roughly be considered as cyclical, mortality has rarely followed such a pattern. However, if migration is included into the survival rates using a set of age-sex-specific net migration rates, then the survival rates may show this kind of cyclical change. Hence, the formulas obtained in the case with no migration using these survival rates, could be employed without change in the case where age-sex-specific net migration rates are used, so that there would be no need to consider that case separately.

5.3.1 ONE-SEX MODEL

5.3.1(a) One-sex Model with No Migration

Namboodiri[7] proposed this model and demonstrated

> ... that for any arbitrary female population, closed to migration, the age distribution below the end of the childbearing period and the growth rates of the different age groups will eventually show a fixed pattern of change, i.e. one which will repeat itself cyclically.

We shall follow his argument briefly and slightly modify the procedures to make them suitable for the present purpose.

The age structure at the end of the k-th period in the first cycle is given by:

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[7] N.K. Namboodiri, 'On the Dependence of Age Structures ...', Demography, 1969, p.288.

$$P^*(1,k) = M(k)\ M(k-1)\ \ldots\ M(1)\ P^*(0,0) \tag{5.36}$$

where M(i), i = 1, 2, ..., k, represent the projection matrices corresponding to the k schedules of fertility and mortality; P*(T,i) a column vector representing the distribution by age of the population at the end of the period in which the i-th schedules are operating in the T-th cycle; and P*(0,0) is the population vector giving the distribution by age of the initial population.

If we continue beyond the k-th schedule, then by our assumption, the first schedule follows and the others in the cycle are repeated. Hence after m repetitions of the cycle, we have:

$$P^*(m,k) = (M^*)^m\ P^*(0,0) \tag{5.37}$$

where M* = M(k) M(k-1) ... M(2) M(1).

Since M(i) are the population projection matrices, it follows from the considerations given in Chapter 4 that M* is a power positive matrix. Hence, it has a unique real positive characteristic root whose value is greater than the absolute value of any other characteristic root.[8]

As m becomes large the vector P*(m,k) becomes proportional to a constant column vector C. In fact, we can write:

$$P^*(T,k) = [a^T\ r\ P^*(0,0)]\ C \tag{5.38}$$

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[8] A. Brauer, 'On the Characteristic Roots of ...', Duke Mathematical Journal, 1961.

where T is one large value of m and a^T is a scalar, and r is a row vector and thus, $[a^T\ r\ P^*(0,0)]$ is a scalar multiplier.[9] From this it is easy to prove that:

$$P^*(T+t,i) = a^t\ P^*(T,i) \tag{5.39}$$

which means that the proportionate age distributions at the end of the different periods within the (T+1)-th, (T+2)-th, etc., cycles, are the same as those within the cycle T. Thus the age structures repeat themselves in all the subsequent cycles.

If P(T+t,i,x) is the x-th element of the vector P*(T+t,i), then this denotes the population at age x years at the end of the i-th period in the (T+t)-th cycle and from (5.39), it can be written as:

$$P(T+t,i,x) = a^t\ P(T,i,x), \quad \text{for } x = 0, 1, 2, \ldots, \beta$$
$$i = 1, 2, 3, \ldots, k$$
$$t = 0, 1, 2, \ldots. \tag{5.40}$$

To extend this result to the ages beyond β years, we have to simply make use of the fact that the populations at ages above β years become linear functions of those at ages below β years. The coefficients of these functions being determined entirely by the given fertility and mortality conditions, would repeat in the respective periods within each cycle. Hence, the age distributions including the whole age range repeat within each cycle.

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[9] N.K. Namboodiri, 'On the Dependence of Age Structure ...', Demography, 1969, p.289.

But equation (5.40) does not suggest anything about the relationship between the populations at the end of the different periods within the cycle (T+t). To obtain this relationship which would enable the computation of the age structures at the end of the different periods if one is known, Namboodiri[10] defined k growth multipliers and using those multipliers, he obtained the relative numbers of females in the different age groups at the end of the i-th period in the (T+t)-th cycle. He has also suggested an iteration procedure to compute the growth multipliers.[11]

At this stage we depart slightly from Namboodiri and define R(i+1) as the growth index at age 0 years, i.e. the ratio of the population at age 0 years at the end of the (i+1)-th period to the population at the same age at the end of the i-th period in the T-th cycle, viz:

$$R(i+1) = [P(T,i+1),0)/P(T,i,0)],\ i = 1, 2, \ldots, (k-1)$$

$$\text{and} \quad R(1) = R(k+1) = [P(T+1,1,0)/P(T,k,0)] \quad . \tag{5.41}$$

It may be noted that R(i) as defined here would be the same as the growth multipliers used by Namboodiri if the survival rates are assumed to be the same for all the periods within the cycle. In fact,

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10 N.K. Namboodiri, 'On the Dependence of Age Structures ...', Demography, 1969.

11 N.K. Namboodiri, 'On the Cyclical Model of Population Change and its Applications', a paper submitted to the I.U.S.S.P. Conference in London, 1969.

Namboodiri made this assumption. But, for our purpose here, we cannot assume this.

Now, we shall obtain the age structures and the growth multipliers at the end of each period within a cycle in terms of the given sequence of fertility and mortality schedules. For this purpose we may proceed in two alternate ways. We may make use of the fact that the equilibrium state age structure which results from a set of fertility and mortality conditions is independent of the initial age structure with which we begin, and employ an iteration procedure to obtain the age structures and the growth indexes at the end of the different periods within any equilibrium state cycle. Or else, we may first obtain the matrix multiplication M* and calculate using this matrix, the equilibrium state age structure at the end of the k-th period and use this in computing the age structures and growth indexes at the end of the other periods. In either case, therefore, we can assume that one age distribution is known.

Hence, let $\phi(T,i,x) = [P(T,i,x)/P(T,i,0)]$ be the ratio of the population at age x years to the population at age 0 years at the end of the i-th period in the T-th cycle. Obviously, $\phi(T,i,0) = 1.0$.

The equations of population growth from period i to (i+1) are:

$$P(T,i+1,0) = \sum_{\alpha}^{\beta} [m(x,i+1)\ P(T,i,x)]$$

and $P(T,i+1,x) = S(x-1,i+1)\ P(T,i,x-1)$, for $x = 1, 2, \ldots, w$, (5.42)

where m(x,i+1) are computed from the fertility rates f(x,i+1) as in Section 3.2.1.

On dividing both sides of the first equation in (5.42) by P(T,i+1,0), we have:

$$\phi(T,i+1,0) = 1.0$$

and
$$R(i+1) = \sum_{\alpha}^{\beta} m(x,i+1)\ \phi(T,i,x) \qquad (5.43)$$

and on dividing the second equation in (5.42) by P(T,i+1,0), we get:

$$\phi(T,i+1,x) = [S(x-1,i+1)\ \phi(T,i,x-1)]/R(i+1) \quad . \qquad (5.44)$$

From the $\phi(T,i+1,x)$ values we can easily obtain the age distribution of the population at the end of the (i+1)-th period in the T-th cycle. It may be noted that equation (5.44) is true for any age x years and not merely for ages below β years.

The growth indexes of the population at each of the other ages and of the total population could be computed from the following equations:

$$GI(i+1,x) = P(T,i+1,x)/P(T,i,x) = [\phi(T,i+1,x)/\phi(T,i,x)]\ \{R(i+1)\},$$

$$\text{for } x = 1, 2, \ldots w,$$

and
$$GI(i+1) = \frac{\sum_{0}^{w} P(T,i+1,x)}{\sum_{0}^{w} P(T,i,x)} = \frac{\sum_{1}^{w} \phi(T,i+1,x)}{\sum_{1}^{w} \phi(T,i,x)}\ R(i+1) \quad . \qquad (5.45)$$

It is not difficult to see that, as Namboodiri has shown, the growth indexes for age 0 years are R(1), R(2), ..., R(k); for age 1 year R(k), R(1), R(2), ..., R(k-1); for age 2 years R(k-1), R(k), R(1), ..., R(k-2); and so on, provided the survival rates are the same for all the periods. Otherwise these would be affected to the extent of the product of the ratios of the survival rates at the different ages from one period to the other. For example, the growth index at age 1 year would be

$$GI(i+1,1) = \left[\frac{\phi(T,i+1,1)}{\phi(T,i,1)}\right] R(i+1) = \frac{S(0,i+1)}{S(0,i)} R(i) \qquad (5.46)$$

at age 2 years would be

$$GI(i+1,2) = \left[\frac{\phi(T,i+1,2)}{\phi(T,i,2)}\right] R(i+1) = \frac{S(1,i+1)}{S(1,i)} \frac{S(0,i)}{S(0,i-1)} R(i-1) \qquad (5.47)$$

and so on.

Thus, starting with any arbitrary age structure we can obtain the set of growth indexes R(2), R(3), ..., R(k), R(k+1) and the corresponding age structures. Then R(1) is put equal to R(k+1) and the initial age distribution is replaced by the age distribution corresponding to R(k+1), and the process is repeated. If R(k+1) now obtained, differs from R(1) by less than an assumed small quantity, then we say that the equilibrium state cycle has evolved and the set of growth indexes and the age distributions already obtained, is the set we are looking for. If R(k+1) is not equal to R(1), then R(1) is again replaced by the new value of R(k+1) and the initial age distribution by

the age distribution corresponding to the new value of R(k+1), and the whole process is repeated. When the effect of the initial age structure is thus eliminated, the equilibrium state cycle evolves. However, in testing for the attainment of the equilibrium state, it would be necessary to compare the age distributions of the first period with that of the (k+1)-th period or to compare the growth indexes at all ages in the two periods, because R(i) are dependent only on the age distributions up to the age β years.

We can conveniently use this iteration procedure to compute the intrinsic growth rate and the associated age distribution of the stable population without using the matrix methods. The model could also be useful in studying the implications of a sequence of k fertility and mortality schedules.

5.3.1(b) One-sex Model with Migration

In the proof of the cyclical model given by Namboodiri, the matrix M* could be any general power positive matrix, as he has indicated at the end of his paper. Hence, if we replace M(i) by the projection matrix $L_M(i)$, which is as in Section 3.2.2, we can write:

$$M^* = L_M(k)\ L_M(k-1)\ \ldots\ L_M(2)\ L_M(1) \qquad (5.48)$$

and from the results discussed in Chapters 3 and 4, and those given by Namboodiri we conclude that, in this case also, a cyclical model evolves. The only problem we have to resolve, therefore, is to obtain expressions which would enable us to compute the age distributions at

the end of the different periods within an equilibrium state cycle T. For this purpose, we assume, as in Section 5.3.1(a), that one age structure is known.

$$\text{Let} \qquad f_p(T,i,0) = [P(T,i,0)/\{\sum_{0}^{w} P(T,i,x)\}] \quad ,$$

be the proportion of the population at age 0 years and $\phi(T,i,x) = [P(T,i,x)/P(T,i,0)]$, the ratio of the population at age x years to the population at age 0 years at the end of the i-th period in the T-th cycle. Obviously $\phi(T,i,0) = 1.0$. Also let $R(i+1) = [P(T,i+1,0)/P(T,i,0)]$, be the growth index for the age 0 years. It may be noted that, when the ratio of the number of migrant children aged 0 years to the number of births during the period, is constant from period to period and the survival rates are also the same, this growth index would again, be equal to the growth multiplier defined by Namboodiri.

If we write down the equations of population growth for convenience, we have:

$$P(T,i+1,0) = \sum_{\alpha}^{\beta} m(x,i+1)\, P(T,i,x) + u(i+1) \sum_{0}^{w} P(T,i,x)$$

$$\text{and} \quad P(T,i+1,x) = S(x-1,i+1)\, P(T,i,x-1) + a(x-1,i+1) \sum_{0}^{w} P(T,i,x) \quad . \quad (5.49)$$

We have on dividing both sides of the first equation by P(T,i+1,0):

$$\phi(T,i+1,0) = 1.0$$

$$\text{and} \quad R(i+1) = \sum_{\alpha}^{\beta} m(x,i+1)\ \phi(T,i,x) + u(i+1)/f_p(T,i,0) \tag{5.50}$$

and on dividing both sides of the second equation by P(T,i+1,0), we get:

$$\phi(T,i+1,x) = [S(x-1,i+1)\ \phi(T,i,x-1) + a(x-1,i+1)/f_p(T,i,0)]/R(i+1)\ . \tag{5.51}$$

The growth index of the population at age x years and that of the total population are given by:

$$GI(i+1,x) = \frac{P(T,i+1,x)}{P(T,i,x)} = \left[\frac{\phi(T,i+1,x)}{\phi(T,i,x)}\right] R(i+1) \tag{5.52}$$

$$\text{and} \quad GI(i+1) = \frac{\sum_{0}^{w} P(T,i+1,x)}{\sum_{0}^{w} P(T,i,x)} = \left[\frac{f_p(T,i,0)}{f_p(T,i+1,0)}\right] R(i+1)\ . \tag{5.53}$$

Thus, all values required to continue the procedure to the (i+2)-th period are available if we know $f_p(T,i+1,0)$. This can be obtained using the equation which follows from the defintiion of $f_p(T,i+1,0)$, i.e.

$$f_p(T,i+1,0) = \left[P(T,i+1,0)/\sum_{0}^{w} P(T,i+1,x)\right] = \left[1/\sum_{0}^{w} \phi(T,i+1,x)\right]\ . \tag{5.54}$$

From these equations we can compute the complete set of age structures and the growth indexes for all the periods within any cycle T. It is not difficult to see that the expressions given by Namboodiri can be obtained from those given here, by putting the migration coefficients

equal to zero and assuming constant survival rates for the different periods. Since it has been shown that the age distributions repeat in all equilibrium state cycles we can see from equations (5.50), (5.52) and (5.53) that the growth indexes also repeat in these cycles. We make use of this fact to obtain, simultaneously the parameters R(1), R(2), ..., R(k) of the equilibrium state cycle and the age structures within that cycle using the method of iteration already described.

5.3.2 TWO-SEX MODEL

In order to extend our results to the two-sex model we merely note that the matrix M(i) can be replaced by a projection matrix appropriate to the two-sex case and still the results will hold good. Hence, as in 5.3.1(b), we shall deal here only with the problem of obtaining the growth indexes and the age-sex structures at the end of the different periods within any cycle T. We assume that one age-sex distribution is known and derive formulas which can be used to obtain the required age-sex structures and the growth indexes by the iteration procedure. We shall use the suffixes m and f to distinguish the male and the female populations.

5.3.2(a) Two-sex Model with No Migration

Let $R(i+1) = [P_f(T,i+1,0)/P_f(T,i,0)]$, be the growth index for the female population at age 0 years, and let $\phi_f(T,i,x)$ and $\phi_m(T,i,x)$ be the ratios of the female and the male populations at age x years to

the female population at age 0 years at the end of the i-th period. Let SR(x,i) be the sex ratio at age x years at the end of the i-th period.

The equations of population growth in this case, are:

$$P_f(T,i+1,0) = \sum_{\alpha}^{\beta} m_f(x,i+1)\, P_f(T,i,x)$$

$$P_m(T,i+1,0) = \sum_{\alpha}^{\beta} m'_f(x,i+1)\, P_f(T,i,x)$$

$$P_f(T,i+1,x) = S_f(x-1,i+1)\, P_f(T,i,x-1)$$

and $P_m(T,i+1,x) = S_m(x-1,i+1)\, P_m(T,i,x-1)$, for $x = 1, 2, \ldots, w$, (5.55)

where $m_f(x,i+1)$ and $m'_f(x,i+1)$ are obtained from the corresponding fertility rates, $f_f(x,i+1)$, as in Section 3.3.1.

Proceeding as in the case of the one-sex model, we get from the first equation:

$$\phi(T,i+1,0) = 1.0$$

and $$R(i+1) = \sum_{\alpha}^{\beta} m_f(x,i+1)\, \phi_f(T,i,x) \qquad (5.56)$$

From the second equation:

$$\phi_m(T,i+1,0) = SR(0,i+1) = [\sum_{\alpha}^{\beta} m'_f(x,i+1)\, \phi_f(T,i,x)]/R(i+1) \qquad (5.57)$$

and from the last two equations:

$$\phi_f(T,i+1,x) = [S_f(x-1,i+1)\ \phi_f(T,i,x-1)]/R(i+1) \tag{5.58}$$

and $$\phi_m(T,i+1,x) = [S_m(x-1,i+1)\ \phi_m(T,i,x-1)]/R(i+1), \tag{5.59}$$

for x = 1, 2, ..., w.

From the values of $\phi_f(T,i+1,x)$ and $\phi_m(T,i+1,x)$ we can compute the age distributions for females and males.

Now the sex ratios by age can be calculated from the equation:

$$SR(x,i+1) = [\phi_m(T,i+1,x)/\phi_f(T,i+1,x)], \text{ for } x = 1, 2, \ldots, w \tag{5.60}$$

and the sex ratio in the total population by:

$$SR(i+1) = [\sum_0^w \phi_m(T,i+1,x)]/[\sum_0^w \phi_m(T,i+1,x)]\ . \tag{5.61}$$

The growth indexes for the populations at age x years are obtained as:

for females: $$GI_f(i+1,x) = \frac{\phi_f(T,i+1,x)}{\phi_f(T,i,x)} R(i+1) \tag{5.62}$$

for males: $$GI_m(i+1,x) = \frac{\phi_m(T,i+1,x)}{\phi_m(T,i,x)} R(i+1) \tag{5.63}$$

and for the female and the male populations at all ages, respectively:

$$GI_f(i+1) = \frac{\sum_0^w \phi_f(T,i+1,x)}{\sum_0^w \phi_f(t,i,x)} R(i+1) \tag{5.64}$$

and
$$GI_m(i+1) = \frac{\sum_0^w \phi_m(T,i+1,x)}{\sum_0^w \phi_m(T,i,x)} R(i+1) \quad . \tag{5.65}$$

Finally, the growth index for the total population is given by:

$$GI(i+1) = \frac{\sum_0^w [\phi_m(T,i+1,x) + \phi_f(T,i+1,x)]}{\sum_0^w [\phi_m(T,i,x) + \phi_f(T,i,x)]} R(i+1) \quad . \tag{5.66}$$

As in the case of the one-sex model we can use these equations to continue the process and thus, obtain the growth indexes and the corresponding age-sex distributions at the end of the different periods within an equilibrium state cycle. From the equations derived here, it is easy to conclude that, since the same set of one schedule of each of the components operates repeatedly in the case of the equilibrium state population, the age-sex distributions remain constant and hence all the growth indexes become equal. Further, if the survival rates and the sex ratio at birth, are assumed to be constant for all the periods within a cycle, we can infer from the discussion given in the case of the one-sex model, that the growth index at each age is one of the growth indexes R(1), R(2), ..., R(k) and is the same for males and females. Hence, the sex ratios at all ages would remain identical in all the periods.

5.3.2(b) Two-sex Model with Migration

The expressions for this case, can be obtained without

difficulty from those derived in the one-sex case and those given in 5.3.2(a) above. Let

$$f_p(T,i,0) = P_f(T,i,0) / \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)]$$

be the proportion of the female population at age 0 years to the total population, and $\phi_f(T,i,x)$ and $\phi_m(T,i,x)$ the ratios of the female and the male populations at age x years to the female population at age 0 years at the end of the i-th period.

The population growth equations, in this case, are as follows:

$$P_f(T,i+1,0) = \sum_{\alpha}^{\beta} m_f(x,i+1)\, P_f(T,i,x) + u_f(i+1) \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)]$$

$$P_m(T,i+1,0) = \sum_{\alpha}^{\beta} m'_f(x,i+1)\, P_f(T,i,x) + u_m(i+1) \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)]$$

$$P_f(T,i+1,x) = S_f(x-1,i+1)\, P_f(T,i,x-1) + a_f(x-1,i+1) \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)]$$

and

$$P_m(T,i+1,x) = S_m(x-1,i+1)\, P_m(T,i,x-1) + a_m(x-1,i+1) \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)],$$

$$\text{for } x = 1, 2, \ldots, w. \qquad (5.67)$$

As in the case of the one-sex model, on dividing both sides of each of the above equations by $P_f(T,i+1,0)$, we get:

from the first equation: $\phi_f(T,i+1,0) = 1.0$

and $$R(i+1) = \sum_{\alpha}^{\beta} m_f(x,i+1)\ \phi_f(T,i,x) + [u_f(i+1)/f_p(T,i,0)] \quad (5.68)$$

from the second equation:

$$\phi_m(T,i+1,0) = SR(0,i+1) = [\sum_{\alpha}^{\beta} m'_f(x,i+1)\ \phi_f(T,i,x) + u_m(i+1)/f_p(T,i,0)]/R(i+1) \quad (5.69)$$

from the third equation:

$$\phi_f(T,i+1,x) = [S_f(x-1,i+1)\ \phi_f(T,i,x-1) + a_f(x-1,i+1)/f_p(T,i,0)]/R(i+1) \quad (5.70)$$

and from the fourth equation:

$$\phi_m(T,i+1,x) = [S_m(x-1,i+1)\ \phi_m(T,i,x-1) + a_m(x-1,i+1)/f_p(T,i,0)]/R(i+1). \quad (5.71)$$

Finally, using the values of $\phi_f(T,i+1,x)$ and $\phi_m(T,i+1,x)$ calculated above, we obtain the proportion of the female population at age 0 years as:

$$f_p(T,i+1,0) = [1/\sum_{0}^{w} \{\phi_f(T,i+1,x) + \phi_m(T,i+1,x)\}] \quad . \quad (5.72)$$

Thus, we have the set of quantities necessary to continue the process to the (i+2)-th period and so on.

From the values of $\phi_f(T,i+1,x)$ and $\phi_m(T,i+1,x)$ we can get the age distributions for females and males, the sex composition and the growth indexes using the respective formulas in the case with no migration.

The formulas derived here can be used to obtain the intrinsic growth rate and the age-sex distribution of the equilibrium state population by the method of iteration described in 5.3.1(a). From equation (5.68), it is clear that the growth rate of the equilibrium state population will not be changed by migration if migration does not affect the female ages below β years.

5.3.2(c) Two-sex Model with Equal Dominance

When equal dominance is assumed the only growth equations that change are those which depend on the fertility rates. Hence, considering the case in which migration is included into the process of population growth, we have:

$$P_f(T,i+1,0) = \frac{1}{2} \sum_{\alpha'}^{\beta'} [m_f(x,i+1)\, P_f(T,i,x) + m_m(x,i+1)\, P_m(T,i,x)] + u_f(i+1) \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)]$$

and

$$P_m(T,i+1,0) = \frac{1}{2} \sum_{\alpha'}^{\beta'} [m'_f(x,i+1)\ P_f(T,i,x) + m'_m(x,i+1)\ P_m(T,i,x)]$$

$$+ u_m(i+1) \sum_{0}^{w} [P_f(T,i,x) + P_m(T,i,x)] \qquad (5.73)$$

where $m_m(x,i+1)$ and $m'_m(x,i+1)$ are obtained from the fertility rates by age of males; $u_f(i+1)$ and $u_m(i+1)$ are suitably modified as explained in Section 3.3.3. From the first equation of (5.73), we have:

$$\phi_f(T,i+1,0) = 1.0$$

$$\text{and} \quad R(i+1) = \frac{1}{2} \sum_{\alpha'}^{\beta'} [m_f(x,i+1)\ \phi_f(T,i,x) + m_m(x,i+1)\ \phi_m(T,i,x)]$$

$$+ [u_f(i+1)/f_p(T,i,0)] \quad . \qquad (5.74)$$

From the second we get:

$$\phi_m(T,i+1,0) = SR(0,i+1) = [\frac{1}{2} \sum_{\alpha'}^{\beta'} \{m'_f(x,i+1)\ \phi_f(T,i,x)$$

$$+ m'_m(x,i+1)\ \phi_m(T,i,x)\} + \{u_m(i+1)/f_p(T,i,0)\}]/R(i+1) \qquad (5.75)$$

The other formulas needed for continuing the process are the same as in the case where female dominance is assumed. The set of formulas applicable in the case with no migration can be obtained simply by putting the migration coefficients equal to zero in the above equations.

5.3.3 NUMERICAL ILLUSTRATIONS

We use the numerical examples to examine whether the formulas derived lead in particular cases, to the conclusions we have drawn from

them, and utilize the iteration procedure to compute the age-sex distribution and the growth rate of an equilibrium state population. In these experiments, the 1911 obs. age-sex distribution was taken as the arbitrary initial age-sex distribution. The fertility, mortality and migration conditions were assumed as follows: (1) the fertility rates as in the sequence of hypothetical fertility rates (see Section 2.4); (2) the survival rates as observed in Australia in 1966 remaining constant; and (3) the net migration rate as given by the Cosine function [0.01 ABS(Cos 0.0628318 (i-1))] with the age-sex composition of net migrants as observed in Australia during 1962-66. These assumptions meant that the mortality schedule remained constant, the fertility and the migration schedules repeated every 50 years so that the rates for the 1st year and the 51st year were the same, and so on. Thus we expect that a cycle of 50 years would evolve from these conditions. Though the formulas are strictly applicable only when all the age groups have the same interval, in the numerical calculations we may make a small adjustment as in the usual population projections, to take account of the last open end age interval. Such an adjustment was made in the examples presented here. The process was discontinued when the difference between R(1) and R(51) became less than $\varepsilon = 0.00009$.

Table 5.7 gives the 50 values of the growth indexes for the female population at age 0 years, which we may call the parameters of the cyclical model, because the other quantities are functions of these indexes and the known values of fertility, mortality and migration

TABLE 5.7

GROWTH INDEXES FOR THE FEMALE POPULATION AT AGE 0 YEARS AND FOR THE TOTAL POPULATION IN THE DIFFERENT PERIODS WITHIN AN EQUILIBRIUM STATE CYCLE

	No Migration		With Migration			No Migration		With Migration	
Period i	Female Age 0	Total Population	Female Age 0	Total Population	Period i	Female Age 0	Total Population	Female Age 0	Total Population
1	1.00577	1.01312	1.02252	1.02508	26	0.97268	1.00688	0.97540	1.00904
2	1.01057	1.01310	1.02737	1.02520	27	0.98912	1.00655	0.99121	1.00806
3	0.99843	1.01281	1.01475	1.02499	28	1.01132	1.00660	1.01405	1.00871
4	0.99080	1.01236	1.00599	1.02454	29	1.01462	1.00670	1.01822	1.00943
5	0.97563	1.01161	0.98945	1.02370	30	1.00962	1.00673	1.01390	1.01006
6	0.96825	1.01074	0.98069	1.02267	31	1.01180	1.00679	1.01663	1.01073
7	0.97763	1.01009	0.98880	1.02182	32	1.01796	1.00697	1.02321	1.01149
8	0.99644	1.00983	1.00631	1.02131	33	1.04048	1.00751	1.04612	1.01262
9	1.01773	1.00996	1.02665	1.02118	34	1.02928	1.00788	1.03511	1.01355
10	1.05505	1.01080	1.06295	1.02174	35	1.06383	1.00887	1.07003	1.01508
11	1.05479	1.01164	1.06149	1.02224	36	1.05009	1.00964	1.05645	1.01638
12	1.03469	1.01209	1.04038	1.02229	37	1.05249	1.01049	1.05915	1.01773
13	1.03124	1.01246	1.03635	1.02221	38	1.08046	1.01192	1.08766	1.01965
14	1.01524	1.01249	1.01990	1.02175	39	1.05634	1.01292	1.06390	1.02110
15	1.01251	1.01246	1.01700	1.02120	40	1.01593	1.01304	1.02384	1.02163
16	1.01357	1.01244	1.01797	1.02065	41	1.01091	1.01304	1.01953	1.02203
17	1.00030	1.01214	1.00458	1.01980	42	1.01996	1.01325	1.02949	1.02262
18	1.00269	1.01190	1.00698	1.01899	43	1.00972	1.01322	1.02007	1.02295
19	1.01074	1.01184	1.01504	1.01835	44	1.02351	1.01350	1.03495	1.02358
20	1.00560	1.01166	1.00982	1.01759	45	1.01183	1.01350	1.02417	1.02392
21	0.98601	1.01109	0.99010	1.01641	46	1.00405	1.01332	1.01734	1.02406
22	0.98902	1.01060	0.99303	1.01531	47	1.01082	1.01330	1.02519	1.02433
23	0.95629	1.00948	0.96006	1.01357	48	1.01018	1.01327	1.02545	1.02458
24	0.94532	1.00823	0.94883	1.01168	49	1.01675	1.01338	1.03288	1.02495
25	0.96928	1.00750	0.97249	1.01032	50	1.00577	1.01325	1.02228	1.02503

Note:- The mortality, fertility and migration schedules assumed were: survival rates of 1966 remaining constant, hypothetical fertility rates having the continuing trend similar to the one observed during 1911-61 and net migration rate given by 0.01 ABS(cos 0.0628318(i-1)) with the age-sex composition of net migrants in Australia during 1962-66.

schedules. The growth indexes for the total population (males and females together) are also presented for a comparison of the trend in the two sets of growth indexes. It may be observed that there was a small lag in the trend in the parameters and that in the growth indexes for the total population both when there was no migration and when there was migration.

Comparing the corresponding values of the parameters in Table 5.7, with the growth indexes of the populations at selected ages given in Table 5.8, we observe that, when migration was absent, the growth indexes at the respective ages were equal to the parameters in the years in which these cohorts were born. For instance, if we take the 1st period, the growth indexes for ages 0, 10, 20, 30 and 40 years were the parameters respectively, for the 1st (i.e. 51st), 41st, 31st, 21st, and 11th periods; and the index for age 50 years was the same as for age 0 years, for 60 years the same as for 10 years and so on. These results are true to the approximation we have used. Thus taking a still smaller value of ε, we could obtain the values as close to each other as we desire. It may also be seen that the growth indexes, as we expected, were the same for males and females when there was no migration. The effect of this was that, in this case, the sex ratios at all ages remained unchanged from period to period within the cycle (see Table 5.10).

When migration was introduced, none of these results followed. This consequence was also expected from the formulas derived here. At

TABLE 5.8

GROWTH INDEXES FOR THE POPULATIONS AT SINGLE YEARS OF AGE FOR SELECTED AGES DURING CERTAIN PERIODS WITHIN AN EQUILIBRIUM STATE CYCLE

Age	Period i						Period i					
	1	11	21	31	41	51	1	11	21	31	41	51
	MALE						FEMALE					
	With No Migration											
0	1.00577	1.05479	0.98600	1.01180	1.01090	1.00577	1.00578	1.05479	0.98600	1.01180	1.01091	1.00578
10	1.01090	1.00574	1.05479	0.98600	1.01180	1.01090	1.01090	1.00574	1.05479	0.98600	1.01180	1.01090
20	1.01180	1.01093	1.00574	1.05478	0.98600	1.01180	1.01180	1.01093	1.00574	1.05478	0.98600	1.01180
30	0.98600	1.01180	1.01093	1.00574	1.05478	0.98600	0.98600	1.01180	1.01093	1.00574	1.05478	0.98600
40	1.05478	0.98594	1.01180	1.01093	1.00574	1.05478	1.05478	0.98595	1.01180	1.01093	1.00574	1.05478
50	1.00574	1.05492	0.98594	1.01180	1.01093	1.00574	1.00574	1.05493	0.98595	1.01180	1.01093	1.00574
60	1.01093	1.00559	1.05492	0.98594	1.01180	1.01093	1.01093	1.00559	1.05493	0.98595	1.01180	1.01093
70	1.01180	1.01097	1.00559	1.05493	0.98594	1.01180	1.01180	1.01096	1.00559	1.05493	0.98595	1.01180
80	0.98594	1.01206	1.01097	1.00559	1.05492	0.98594	0.98595	1.01206	1.01096	1.00559	1.05492	0.98595
85+	1.01862	0.97779	1.03210	1.01777	0.99181	1.01862	1.01835	0.97934	1.03006	1.01848	0.99318	1.01835
Total	1.01325	1.01150	1.01101	1.00675	1.01320	1.01325	1.01299	1.01178	1.01117	1.00683	1.01289	1.01299
	With Migration											
0	1.02251	1.06152	0.99012	1.01659	1.01951	1.02251	1.02252	1.06149	0.99010	1.01663	1.01953	1.02252
10	1.02241	1.02088	1.05249	0.98883	1.02427	1.02241	1.02233	1.02095	1.05282	0.98884	1.02402	1.02233
20	1.02722	1.02029	1.01495	1.05222	0.99688	1.02722	1.02675	1.02037	1.01549	1.05251	0.99620	1.02675
30	1.00344	1.02381	1.01338	1.01282	1.05740	1.00344	1.00265	1.02346	1.01360	1.01374	1.05750	1.00265
40	1.05678	1.00385	1.01793	1.01167	1.01811	1.05678	1.05690	1.00321	1.01737	1.01147	1.01923	1.05690
50	1.01978	1.05418	1.00187	1.01586	1.01472	1.01978	1.02077	1.05443	1.00136	1.01546	1.01439	1.02077
60	1.01564	1.01938	1.05191	1.00142	1.01766	1.01564	1.01574	1.02009	1.05103	1.00099	1.01814	1.01574
70	1.01858	1.01537	1.01789	1.05145	1.00311	1.01858	1.01958	1.01542	1.01790	1.04994	1.00358	1.01958
80	1.00406	1.01835	1.01431	1.01747	1.05231	1.00406	1.00454	1.01941	1.01444	1.01711	1.05084	1.00454
85+	1.02651	0.99998	1.02874	1.01979	1.00788	1.02651	1.02544	1.00141	1.02841	1.02036	1.00804	1.02544
Total	1.02524	1.02224	1.01633	1.01057	1.02213	1.02524	1.02492	1.02224	1.01649	1.01088	1.02193	1.02492

Note:- See note to Table 5.7.

age 0 years the sex ratio remained the same even in the presence of migration because the effect of migration was comparatively small on this age and because a constant sex ratio among the net migrants was used in the model. A similar comment holds good for some of the old age groups. If we had taken different age-sex compositions of net migrants for the different periods the results would have been different.

The age distributions for males and females resulting from the cyclical model in the absence and in the presence of migration, are presented in Table 5.9 at intervals of 10 years of age for selected periods, for convenience of presentation. Comparing the two results, we can see that the effect of introducing migration, in this case, was to increase the proportions in the younger age groups and to decrease those in the old age groups. However, it must be noted that this effect of migration is not true in general, and in fact, the effect depends as in the case of the equilibrium state populations, on the magnitudes of the net migration rates and the natures of the age-sex compositions of the net migrants within the cycle.

Finally, the procedure of iteration using the cyclical model, was used to compute the equilibrium state age-sex distribution resulting from the operation of the fertility and mortality rates observed in Australia in 1911 and the migration condition as in Australia during 1911-12. It was observed that the percentage age distributions and the sex ratios in 5 year age groups and the intrinsic growth rate (per 1000

TABLE 5.9

AGE STRUCTURES AT THE END OF CERTAIN PERIODS WITHIN AN EQUILIBRIUM STATE CYCLE

Age Groups	MALE Period i						FEMALE Period i					
	1	11	21	31	41	51	1	11	21	31	41	51
	With No Migration											
0- 4	11.13	9.73	10.49	8.29	10.90	11.13	10.52	9.19	9.89	7.80	10.27	10.52
5- 9	10.51	9.96	10.06	8.77	8.46	10.51	9.94	9.41	9.48	8.26	7.97	9.94
10-14	9.48	9.89	8.59	9.68	7.44	9.48	8.97	9.35	8.11	9.12	7.02	8.97
15-19	7.35	9.34	8.79	9.28	7.87	7.35	6.97	8.85	8.31	8.76	7.44	6.97
20-24	6.43	8.37	8.68	7.87	8.63	6.43	6.13	7.97	8.25	7.47	8.20	6.13
25-29	6.78	6.47	8.16	8.02	8.24	6.78	6.49	6.19	7.79	7.65	7.87	6.49
30-34	7.43	5.66	7.32	7.93	7.00	7.43	7.14	5.43	7.01	7.58	6.70	7.14
35-39	7.08	5.95	5.65	7.44	7.12	7.08	6.83	5.74	5.43	7.14	6.84	6.83
40-44	5.97	6.48	4.90	6.63	6.98	5.97	5.79	6.29	4.74	6.40	6.75	5.79
45-49	5.98	6.09	5.08	5.04	6.46	5.98	5.86	5.96	4.96	4.91	6.30	5.86
50-54	5.73	5.01	5.40	4.26	5.61	5.73	5.70	4.98	5.36	4.22	5.57	5.70
55-59	5.08	4.80	4.86	4.23	4.09	5.08	5.21	4.93	4.98	4.33	4.19	5.21
60-64	4.12	4.30	3.73	4.20	3.23	4.12	4.47	4.66	4.04	4.54	3.49	4.47
65-69	2.70	3.42	3.20	3.40	2.87	2.70	3.19	4.05	3.79	4.01	3.40	3.19
70-74	1.82	2.38	2.45	2.23	2.44	1.82	2.43	3.18	3.28	2.97	3.26	2.43
75-79	1.27	1.23	1.55	1.51	1.56	1.27	2.01	1.94	2.43	2.38	2.46	2.01
80-84	0.78	0.59	0.77	0.83	0.74	0.78	1.48	1.12	1.46	1.57	1.39	1.48
85+	0.38	0.33	0.31	0.40	0.39	0.38	0.87	0.77	0.71	0.90	0.89	0.87
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	With Migration											
0- 4	11.38	10.15	10.54	8.34	10.97	11.38	10.86	9.70	10.07	7.94	10.44	10.86
5- 9	10.60	10.36	10.24	8.75	8.64	10.60	10.10	9.89	9.77	8.33	8.21	10.10
10-14	9.54	10.00	8.94	9.61	7.58	9.54	9.08	9.54	8.52	9.16	7.20	9.08
15-19	7.61	9.27	9.02	9.33	7.87	7.61	7.20	8.80	8.59	8.89	7.46	7.20
20-24	6.87	8.49	8.74	8.12	8.63	6.87	6.53	8.08	8.33	7.76	8.24	6.53
25-29	7.20	7.09	8.21	8.20	8.43	7.20	6.85	6.73	7.83	7.84	8.07	6.85
30-34	7.65	6.39	7.53	7.92	7.33	7.65	7.37	6.11	7.20	7.59	7.06	7.37
35-39	7.24	6.38	6.22	7.41	7.23	7.24	7.03	6.17	5.97	7.10	6.99	7.03
40-44	6.16	6.50	5.49	6.72	6.85	6.16	6.02	6.35	5.31	6.47	6.62	6.02
45-49	5.85	5.94	5.31	5.46	6.24	5.85	5.74	5.85	5.20	5.30	6.05	5.74
50-54	5.27	4.84	5.17	4.68	5.45	5.27	5.25	4.85	5.17	4.63	5.38	5.25
55-59	4.54	4.33	4.48	4.32	4.23	4.54	4.68	4.49	4.64	4.43	4.33	4.68
60-64	3.69	3.63	3.40	3.93	3.37	3.69	4.06	4.00	3.75	4.27	3.68	4.06
65-69	2.58	2.80	2.72	3.05	2.78	2.58	3.11	3.39	3.30	3.65	3.33	3.11
70-74	1.76	1.95	1.95	1.98	2.17	1.76	2.41	2.68	2.68	2.70	2.93	2.41
75-79	1.14	1.08	1.19	1.25	1.33	1.14	1.83	1.75	1.93	2.02	2.12	1.83
80-84	0.63	0.53	0.60	0.64	0.62	0.63	1.21	1.02	1.15	1.25	1.19	1.21
85+	0.29	0.26	0.25	0.30	0.30	0.29	0.67	0.61	0.59	0.70	0.70	0.67
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Note:- See Note to Table 5.7.

TABLE 5.10

SEX STRUCTURES AT THE END OF CERTAIN PERIODS WITHIN AN EQUILIBRIUM STATE CYCLE

Age Groups	Males per 100 Females											
	No Migration Period i						With Migration Period i					
	1	11	21	31	41	51	1	11	21	31	41	51
0- 4	104.50	104.50	104.49	104.50	104.50	104.50	104.50	104.50	104.49	104.50	104.50	104.50
5- 9	104.42	104.42	104.42	104.41	104.42	104.42	104.68	104.66	104.55	104.46	104.65	104.68
10-14	104.34	104.34	104.34	104.34	104.34	104.34	104.81	104.80	104.68	104.42	104.70	104.81
15-19	104.14	104.13	104.13	104.13	104.12	104.14	105.47	105.25	104.88	104.43	104.90	105.47
20-24	103.59	103.59	103.59	103.59	103.58	103.59	104.97	104.95	104.66	104.04	104.13	104.97
25-29	103.13	103.15	103.14	103.14	103.14	103.13	104.77	105.25	104.68	104.03	103.95	104.77
30-34	102.78	102.78	102.79	102.78	102.79	102.78	103.59	104.47	104.41	103.90	103.32	103.59
35-39	102.37	102.36	102.38	102.37	102.37	102.37	102.63	103.25	103.94	103.82	102.91	102.63
40-44	101.81	101.80	101.80	101.81	101.80	101.81	102.18	102.38	103.18	103.35	102.84	102.18
45-49	100.85	100.87	100.84	100.88	100.86	100.85	101.74	101.47	101.93	102.45	102.48	101.74
50-54	99.21	99.23	99.20	99.21	99.24	99.21	100.21	99.68	99.88	100.56	100.66	100.21
55-59	96.17	96.12	96.17	96.08	96.22	96.17	96.88	96.19	96.32	97.04	97.11	96.88
60-64	91.11	91.03	91.09	91.01	91.04	91.11	90.89	90.45	90.53	91.41	91.22	90.89
65-69	83.43	83.32	83.21	83.33	83.13	83.43	82.61	82.44	82.29	83.25	83.15	82.61
70-74	73.72	73.83	73.71	73.80	73.68	73.72	73.01	72.81	72.69	73.19	73.62	73.01
75-79	62.42	62.81	62.67	62.53	62.68	62.42	62.23	61.87	61.81	61.83	62.52	62.23
80-84	52.00	52.04	52.14	52.03	52.13	52.00	52.02	51.60	51.43	51.32	51.75	52.02
85+	43.03	42.64	42.96	43.17	42.87	43.03	43.03	42.82	42.61	42.59	42.51	43.03
Total	98.76	98.68	98.45	98.31	98.45	98.76	99.77	99.88	99.81	99.49	99.48	99.77

Note:- See Note to Table 5.7.

persons) were almost identical with those obtained from the actual projections up to the second decimal point. The main advantage in using this iteration procedure is that we may start with any arbitrary age-sex distribution and obtain the equilibrium state age-sex distribution and the growth rate, simultaneously, to any desired degree of approximation without using the matrix methods. If the initial age-sex distribution chosen is close to the equilibrium state one, the iteration will be faster.

5.4 SUMMARY

The problem of population change under the operation of a constant set of fertility, mortality and migration conditions has been studied in this chapter in two situations: (1) when the set contains a single schedule of each of the components of change - the equilibrium state model; and (2) when the set contains k schedules of each of the components of change - the cyclical model. Since the formulas under the assumption of no migration in the equilibrium state model have been derived by Goodman, more emphasis is given to the case where migration is included into the process of population growth using a net migration rate and an age-sex composition of net migrants.

In the cyclical model, the formulas derived by Namboodiri are slightly modified to facilitate the inclusion of migration and then extended to the two-sex case.

In each case, numerical illustrations are presented to check the formulas and to compare the structures of the resultant populations. From the numerical illustrations, it is observed that the age-sex composition of migrants plays an important role in determining the shape of the age-sex distributions of the equilibrium state population. For instance, the closer the age-sex composition of net migrants is to the age-sex distribution of the stable population resulting from the given schedules of fertility and mortality rates, the closer will be the resultant age-sex distribution of the equilibrium state population to the stable one.

The case in which age-sex-specific net migration rates are used in the process of population growth, is not presented separately, because the formulas applicable in the case with no migration can be used without change in that case.

CHAPTER 6*

POPULATION CHANGE UNDER VARYING CONDITIONS: THE CASE OF POPULATION GROWTH IN AUSTRALIA DURING 1911-66

6.1 INTRODUCTION

In any actual population the fertility, mortality and migration conditions rarely remain constant. Hence the analysis of the characteristics of the equilibrium state populations or of those resulting under a cyclical model is of theoretical interest and is useful mainly in examining the implications of certain observed or assumed conditions. When the fertility, mortality and migration schedules are changing over time, no definite age-sex distribution or a fixed growth rate is evolved. In fact, both are modified at each period of time due to the changes in the fertility, mortality and migration schedules during the period. It is of interest therefore to examine the contributions of the observed changes in the components in changing the characteristics of a population during a given period. This question is taken up in this chapter.

In a recent paper, Keyfitz[1] suggested that the effects of the changes in fertility and mortality, and those of the interactions of these changes, on the characteristics of a population, could be

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* The material presented in this chapter, except the analyses for the periods 1933-66 and 1947-66, has been accepted for publication in the Australian Journal of Statistics.

1 N. Keyfitz, 'Changing Vital Rates ...', Population Studies, 1968.

separated by constructing stable populations under four different assumptions which could be considered to conform to a 2 x 2 factorial design of experiments. We have already used a similar procedure to decompose the changes in the characteristics of the equilibrium state populations into the effects of the changes in the components (see Section 5.2.3(iv)). Such an analysis will not, however, give the decomposition of the observed change in an actual population during a given period.

Some authors have used other numerical population models to study the effects of the changes in the components on the population characteristics. Schwartz[2] examined the influence of natality and mortality on the age composition and on the population growth by constructing numerical population models under assumptions comparable to those of Keyfitz. Notestein[3] tried to assess the contribution of the actual changes in fertility, mortality and migration to the population growth in the United States during 1930-55 through the construction of the population projection models.

Our attempt in this chapter is to present a method, similar to the one adopted by Notestein, to decompose the changes in the characteristics of a population during a certain period into the effects of changes in the components and those of the interactions of

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2 K. Schwartz, 'Influence de la Natalite ...', Population, 1968.

3 F.W. Notestein, 'Mortality, Fertility, the Size-Age Distribution ...', 1960.

these changes. The method is applied to analyse the changes in the characteristics of the population of Australia which had experienced considerable changes not only due to changes in the vital rates, but also due to those in migration. The analysis covers the period 1911-66.

The basic step in the method proposed here is to project the population as at the beginning of the period under study to the end of the period - from 30 June 1911 to 30 June 1966 in the case of the Australian population - under different assumptions which conform to a 2 x 2 x 2 factorial design. The method is therefore called the factorial projections method. It must be mentioned, however, that the design is used here not for the purpose for which the designs of experiments are commonly used in the Statistical Designs of Experiments and to that extent the analysis adopted is also different from the usual one.

The decomposition of the changes in the population characteristics during a certain period, as suggested here, would help to give a better understanding of the effects of the changes in the components during that period.

6.2 THE FACTORIAL PROJECTIONS METHOD

In demographic analysis, our interest is to assess separately the contribution of the main effects and the effects of the interactions of the changes in the components from a certain initial

situation to the observed changes in the population characteristics. Hence, the term interaction has the following meaning. Suppose we are considering a period during which both fertility and mortality declined and there was no migration. Due to the decline in mortality, more women would be in the reproductive period than would have been if mortality had not declined. These extra women saved, would have given birth to more children if fertility had not declined than if it did. The resultant addition or subtraction due to the simultaneous decline of mortality and fertility is termed as the effect of the interaction of the decline in mortality and fertility. Similarly, we can define the effects of the interactions between the changes in the other components.

Let P(t) be the total population of a country at time t. If we start with a population P(0) at time t = 0, it will change through the operation of fertility, mortality and migration. These components, in turn, may remain the same as at t = 0 or change during the period 0 to t. To analyse the effects of the changes in these three components on the population characteristics, we shall use a 2 x 2 x 2 factorial design. Let the factors in the design be represented as follows:

A : 1 - Mortality specified by age-specific survival rates, remains as at t = 0.

a - Mortality changes as observed in a country or as assumed.

B : 1 - Fertility specified by age-specific fertility rates,

remains as at t = 0.

b - Fertility changes as observed in the country or as assumed.

C : 1 - No migration or as assumed.

c - Migration given by the net numbers of migrants by age and sex or as assumed.

Let $P_t(A,B,C)$ denote the total population as obtained by projecting the initial population P(0) to time t with the respective assumptions regarding A, B, C and a constant sex ratio at birth. We may also assume this sex ratio to change. But as we have seen in Chapter 2, the change in this sex ratio is not generally significant. Thus $P_t(a,b,c)$ is the projection of P(0) to t with the observed changes in fertility, mortality and migration. Hence, it must be equal to P(t), the observed population at time t, except for some observational errors in the components and in P(t) itself, and a small variation in the sex ratio at birth.

The assumptions needed for constructing the population projection models from the initial population P(0) according to the 2 x 2 x 2 factorial design, could be set out in the form of an effect matrix given below:

Factor Combinations	Representation by Signs			Resultant Projections
	A	B	C	
A = 1, B = 1, C = 1	-	-	-	$P_t(1,1,1) = 1$
A = a, B = 1, C = 1	+	-	-	$P_t(a,1,1) = a$
A = 1, B = b, C = 1	-	+	-	$P_t(1,b,1) = b$
A = a, B = b, C = 1	+	+	-	$P_t(a,b,1) = ab$
A = 1, B = 1, C = c	-	-	+	$P_t(1,1,c) = c$
A = a, B = 1, C = c	+	-	+	$P_t(a,1,c) = ac$
A = 1, B = b, C = c	-	+	+	$P_t(1,b,c) = bc$
A = a, B = b, C = c	+	+	+	$P_t(a,b,c) = abc$

Note:- The signs - and + denote respectively the absence and occurrence of change in the component concerned.

The population projections are carried out by the usual component method of projections with female dominance. Of course, the method of equal dominance could also be used. The study would be more interesting if the fertility and mortality rates are available separately for migrants and the native population. But this raises an important question: when should the migrants be considered as one with the native population? This question by itself needs a detailed investigation. However, in the absence of such detailed information and in view of the analysis of the fertility and mortality experience

of the native born and the outside born populations given in Chapter 1, we shall continue with the assumption that the migrants experience the same fertility and mortality rates as the general population.

Now to decompose the observed change in the total population during the period 0 to t into the effects of the changes in the components during that period, we set up the following model:

$$TC = [E(1) + E(a) + E(b) + E(c) + E(ab) + E(ac) + E(bc) + E(abc)]$$

where TC denotes the total effect on the population of the changes in all the components during the period. The terms on the right hand side have the meaning given below and their values could be calculated using the equations given along with the explanations. The term E(abc) is the residual after all the effects and the interaction effects are taken out from the total change.

(i) E(1) is the effect on the total population of the vital rates remaining constant at the initial value with no migration or with migration at the level described as C = 1. This could be estimated by the equation: $E(1) = 1 - P_0$.

(ii) E(a) denotes the effect of the change in mortality only with no change in fertility and migration, and is given by: $E(a) = a - 1$.

(iii) E(b) stands for the effect of the change in fertility only with no change in mortality and migration and is

estimated by: $E(b) = b - 1$.

(iv) $E(c)$ represents the effect of migration (or change in it) with no changes in vital rates and is given by: $E(c) = c - 1$.

(v) $E(ab)$ is the effect of the interaction between the changes in mortality and in fertility with no change in migration. This is estimated by: $E(ab) = (ab - b) - (a - 1)$.

(vi) $E(ac)$ is the effect of the interaction between the changes in mortality and migration with no change in fertility and is computed from the equation: $E(ac) = (ac - c) - (a - 1)$.

(vii) $E(bc)$ is the effect of the interaction between the changes in fertility and migration with no change in mortality and this could be calculated by using the equation: $E(bc) = (bc - c) - (b - 1)$.

(viii) $E(abc)$ denotes, similarly, the effect of the interaction between the changes in all the components and is estimated by the equation: $E(abc) = [(abc - c) - (ac - c) - (bc - c)] - [E(ab)]$.

The sum of these effects is the total effect, TC, and hence it is equal to the change $(P_t(a,b,c) - P_0)$ which is approximately the actual observed change, $P_t - P_0$. The proportionate contribution of the changes in the components and their interactions are obtained by

expressing the individual effects calculated above as the percentage of the total effect. However, the interpretation of the effects of the changes in the components will be clearer if the effects of the interactions are small.

It may be noted that the analysis detailed here has assumed P_0, the initial population, to be given and therefore the effects observed will be relevant only in connection with that initial size and age distribution of the population. If we wish to examine the effects of the changes in the initial population simultaneously with the changes in the components, then we will have to obtain population projections under different assumptions according to a 2 x 2 x 2 x 2 factorial design or any other more elaborate one. However, for the analysis of the population dynamics in any country, it can be assumed that the initial population is given.

Though the total population has been used in the above analysis, the same procedure could be applied for analysing the change in any other population characteristics such as the mean age, proportion in the old age, the labour force, or the school age population, etc.

6.3 LIMITATIONS OF THE METHOD

An advantage of this method over the one suggested by Keyfitz, is that it could be used to study the changes during short periods as well as long periods. But the main drawback of the method

is that it is based on the assumption that the variations in one component are independent of those in the others. It may be recalled that this assumption is basic for the whole of the present study, and does not appear to be a very serious assumption. Also, the method cannot reveal the many indirect effects of the changes in the components which are mainly socio-economic in nature, except those that are reflected in the variations of the measures used for representing the components. Further, the estimates of the effects of the changes and those of the interactions of the changes depend upon the magnitudes of the variations in the components themselves and also on the length of the period of analysis.

6.4 ANALYSIS OF THE CHANGES IN THE SIZE AND THE AGE-SEX DISTRIBUTION OF THE POPULATION OF AUSTRALIA DURING 1911-66

During the period 1911-66, there were significant changes in fertility and migration conditions in Australia. Mortality declined almost continuously during the period, but it was already at a comparatively low level by 1911. From the analysis of the changes observed in these components given in Chapter 2, we see that it would be interesting to study the contributions of the variations in the components to the changes in the characteristics of the population during the three periods: 1911-66, 1933-66 and 1947-66. The period 1911-66 experienced compensating variations in the components such as decline and increase in both fertility and migration, whereas the

period 1933-66 had very little effect of migration over the first 14 years, and had a continuous effect of immigration thereafter, with fertility increasing from a very low level throughout the period, except during the last few years since 1961. On the other hand, there was a significant immigration throughout the period 1947-66, with fertility also increasing from a considerably higher level than that in 1933, except during the last few years since 1961.

The consideration of these three periods would also show how the contributions of the variations in the components estimated by this method, are dependent on the magnitudes of the variations in the components during the period and on the length of the period under study.

It must be mentioned here that the populations projected from 1911 to 1933 and to 1947 under the assumption of the observed changes in fertility, mortality and migration, were used as the initial populations respectively in the analyses for the periods 1933-66 and 1947-66. Hence, the population in 1966 obtained by projecting the three initial populations at 1911, 1933 and 1947, assuming the observed conditions of fertility, mortality and migration during the respective periods, was identical. However, the populations as at 30 June 1933 and 1947 obtained from the projections did not differ very much from the corresponding populations enumerated in the censuses (see Appendix C).

The method suggested in Section 6.2 is used to analyse the

changes in the following population characteristics:

(a) Population growth: (i) the total sizes of the male and female populations, and (ii) the total number of births, deaths and natural increase;

(b) Changes in the age-sex distribution: (i) the mean ages of the male and female populations, (ii) the proportion of the population in the old age group (65+) for males and females, and (iii) the proportion of males in the total population; and

(c) Changes in the sizes of certain segments of the population: (i) population in the school going age group (6-14), (ii) population in the working age group (15-64), and (iii) population in the old age group (65+).

For all the analyses presented here, the actual change in any of the characteristics refers to the total change between the value at the initial time and the value obtained from the projected population under the assumption of the observed changes in mortality, fertility and migration.

6.4.1 POPULATION GROWTH

In analysing the growth of the population, it is necessary to consider the change in the size of the total population as well as the natural increase because, while the changes in fertility and their interactions with the changes in mortality and with migration, affect the population growth through the natural increase, the changes in

mortality or migration and their interactions affect the populations at all ages and thus cause the change in the total population. Hence we have chosen to analyse these two separately.

6.4.1(i) Changes in the Size of the Male and Female Populations

The population of Australia consisted of 2334 thousand males and 2156 thousand females as at 30 June 1911. It increased to 5881 thousand males and 5738 thousand females by 30 June 1966. At 30 June 1933, the population had 3394 thousand males and 3280 thousand females while at 30 June 1947 its size was made up of 3848 thousand males and 3780 thousand females.

The analysis of the changes in the size from these initial values to that in 1966, is presented in Table 6.1. The table shows clearly that the changes during 1911-66 had compensating effects of increasing and decreasing the size while those during 1933-66 and during 1947-66 were all towards increasing the size. Consequently, about 66 per cent out of the total change of 3547 thousands in the case of males, and about 71 per cent of the total change of 3582 thousands in the case of females, would have occurred even if migration had not contributed to the population growth, but if fertility and mortality conditions as in 1911 had remained unchanged. On the other hand, the continuation of the low fertility level of 1933 and the corresponding mortality with no migration during that period, would have caused only one-fourth of the actual change of 2487 thousands during 1933-66 in the case of males and about 28 per cent of

TABLE 6.1

ANALYSIS OF THE CHANGES IN THE TOTAL SIZES OF MALE AND FEMALE POPULATIONS IN AUSTRALIA

Source of Change	During 1911-66 Absolute Value ('000)	During 1911-66 Per cent to TC	During 1933-66 Absolute Value ('000)	During 1933-66 Per cent to TC	During 1947-66 Absolute Value ('000)	During 1947-66 Per cent to TC
			M A L E			
Value in the initial year	2333.8		3393.9		3847.7	
E(1)	2359.2 (4693.0)*	66.51 (79.80)(a)	599.0 (3993.0)*	24.09 (67.90)(a)	780.5 (4628.2)*	38.39 (78.70)(a)
E(a)	785.1	22.13	132.8	5.34	41.8	2.06
E(b)	-1049.0	-29.57	601.1	24.17	119.1	5.86
E(c)	1646.2	46.41	1029.5	41.40	1068.8	52.57
E(ab)	-180.4	-5.09	31.5	1.27	1.5	0.07
E(ac)	144.4	4.07	16.7	0.67	5.9	0.29
E(bc)	-136.0	-3.83	73.2	2.94	15.3	0.75
E(abc)	-22.4	-0.63	3.1	0.12	0.2	0.01
TC	3547.1	100.00	2486.9	100.00	2033.1	100.00
Value in 1966	5880.9	-	5880.9	-	5880.9	-
			F E M A L E			
Value in the initial year	2155.8		3279.8		3780.0	
E(1)	2539.0 (4694.8)*	70.88 (81.82)(a)	692.0 (3971.8)*	28.16 (69.22)(a)	829.1 (4609.1)*	42.35 (80.33)(a)
E(a)	804.7	22.46	210.5	8.56	87.3	4.46
E(b)	-1022.8	-28.55	580.1	23.60	114.5	5.85
E(c)	1447.1	40.40	856.8	34.86	903.2	46.13
E(ab)	-168.2	-4.70	27.1	1.10	1.0	0.05
E(ac)	135.6	3.79	18.5	0.75	8.0	0.41
E(bc)	-132.5	-3.70	70.6	2.87	14.7	0.75
E(abc)	-20.7	-0.58	2.5	0.10	0.1	0.01
TC	3582.2	100.00	2458.1	100.00	1957.9	100.01
Value in 1966	5738.0	-	5738.0	-	5738.0	-

Note:- The symbols in the column 'Source of Change' have the following explanations:

- E(1) - No change in mortality and fertility conditions and no migration;
- E(a) - Observed change in mortality conditions, no change in fertility conditions and no migration;
- E(b) - No change in mortality conditions, observed change in fertility conditions and no migration;
- E(c) - No change in mortality and fertility conditions and observed migration;
- E(ab) - Observed change in mortality and fertility conditions and no migration;
- E(ac) - Observed change in mortality conditions, no change in fertility conditions and observed migration;
- E(bc) - No change in mortality conditions, observed change in fertility conditions and observed migration;
- E(abc) - Observed change in mortality and fertility conditions and observed migration;
- * - This is the value of the characteristic when there is no change in mortality and fertility conditions and no migration;
- (a) - Ratio (per cent) of the value * to the value in 1966.

the actual change of 2458 thousands in the case of females. During 1947-66, the population would have increased by about 780 thousands only as against the actual increase of 2033 thousands (i.e. about 38 per cent) in the case of males and by about 829 thousands instead of 1958 thousands (about 42 per cent) in the case of females, if the fertility and mortality as in 1947 had not changed and no migration had occurred.

After almost a continuous decline till about 1935, fertility recovered and its level remained less than that in 1911 for the whole of the period 1911-66. The effect of this was that the whole effect of mortality decline which caused an increase in the sizes of the male and female populations was completely offset and even a part of the effect of migration was reduced. Thus most of the extra increase in the population size during 1911-66, which could be attributed to the changes in the components, came from immigration.

If we study the changes during 1933-66 and 1947-66, we observe that the increase due to the improvement in mortality became comparatively smaller, especially for males, and for the period 1947-66. This is in accord with the observation that the improvement in mortality slowed down during this period and even a slight retardation had occurred at some ages after 1961. Fertility improvements from the 1933 level contributed about one-fourth of the total change during 1933-66, whereas their contribution to the population growth during 1947-66 reduced to about one-twelfth. In

fact, immigration played a major role in the growth of the male and female populations during 1947-66. It may be recalled that during this period there was a consistent inflow of migrants. The comparison of the magnitudes of the interaction effects indicates that the length of the period as well as the magnitudes of the changes in the components during the period have an effect on their value.

6.4.1(ii) Analysis of the Total Number of Births, Deaths and Natural Increase

Variations in the mortality, fertility and migration schedules affect the total number of births and deaths, and thus the natural increase of the population during a period. Therefore, it is of interest to examine the part played by the changes in each of these components in changing the number of births and deaths, and natural increase. Such an analysis in respect of births and deaths is presented in Table 6.2 and that for the natural increase in Table 6.3.

It could be seen that the births contributed by immigrants and due to the improvement in mortality could not compensate the reduction due to the decline in fertility from the level in 1911 so that during 1911-66 there would have been nearly 10 per cent more births than the actual number occurring even in the absence of migration, if fertility and mortality had remained at the same level as in 1911. But in that case, the number of deaths would have gone up by about 26 per cent as compared with the actual number observed so that the natural increase would have been less by about 2 per cent of that actually observed (see Table 6.3).

TABLE 6.2

CONTRIBUTION OF THE CHANGES IN THE COMPONENTS TO THE NUMBER OF BIRTHS AND DEATHS IN AUSTRALIA

Source of Change	During 1911-66 Absolute Value ('000)	During 1911-66 Per cent to Tc	During 1933-66 Absolute Value ('000)	During 1933-66 Per cent to TC	During 1947-66 Absolute Value ('000)	During 1947-66 Per cent to TC
			Births			
E(1)	9583.6	109.79	4026.8	69.04	3296.2	82.28
E(a)	487.0	5.58	48.21	0.83	9.8	0.24
E(b)	-2417.9	-27.70	1258.0	21.57	242.5	6.05
E(c)	1485.6	17.02	322.5	5.53	425.2	10.61
E(ab)	-142.4	-1.63	20.6	0.35	-0.1	-0.00
E(ac)	61.1	0.70	3.0	0.05	1.3	0.03
E(bc)	-310.5	-3.56	152.2	2.61	31.1	0.78
E(abc)	-17.2	-0.20	1.1	0.02	0.0	0.00
TC	8729.3	100.00	5832.4	100.00	4006.0	100.00
			Deaths			
E(1)	4685.3	125.67	2735.7	107.06	1686.5	102.84
E(a)	-1102.8	-29.58	-295.2	-11.55	-119.2	-7.27
E(b)	-346.0	-9.28	76.8	3.01	9.0	0.55
E(c)	520.5	13.96	104.4	4.09	78.3	4.77
E(ab)	206.2	5.53	-38.0	-1.49	-2.7	-0.16
E(ac)	-218.9	-5.87	-32.0	-1.26	-12.7	-0.77
E(bc)	-42.0	-1.13	8.4	0.33	1.1	0.07
E(abc)	25.9	0.70	-4.6	-0.18	-0.3	-0.02
TC	3728.2	100.00	2555.3	100.00	1640.0	100.00

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

TABLE 6.3

CONTRIBUTION OF THE CHANGES IN THE COMPONENTS TO THE NATURAL INCREASE OF THE POPULATION IN AUSTRALIA

Source of Change	Natural Increase = Births - Deaths					
	During 1911-66		During 1933-66		During 1947-66	
	Absolute Value ('000)	Per cent to TC	Absolute Value ('000)	Per cent to TC	Absolute Value ('000)	Per cent to TC
E(1)	4898.3	97.94	1291.1	39.40	1609.6	68.03
E(a)	1589.8	31.79	343.4	10.48	129.1	5.46
E(b)	-2071.9	-41.43	1181.2	36.05	233.6	9.87
E(c)	965.1	19.30	218.2	6.66	347.0	14.67
E(ab)	-348.6	-6.97	58.6	1.79	2.6	0.11
E(ac)	280.0	5.60	35.1	1.07	13.9	0.59
E(bc)	-268.5	-5.37	143.8	4.39	29.9	1.26
E(abc)	-43.1	-0.86	5.6	0.17	0.3	0.01
TC	5001.1	100.00	3277.0	100.00	2366.0	100.00

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

During 1933-66, the increase in the level of fertility from the low level in 1933 contributed more than one-fifth of the actual number of births, while during 1947-66 its contribution was around only 6-7 per cent. This was expected because by 1947 the level of fertility had become considerably higher than that in 1933 and the increase in the level of fertility slowed down in later years after 1951. The increase in the number of deaths due to the increase in the

number of births as a result of the changes in fertility was not very much - about 3 per cent during 1933-66 and about half a per cent during 1947-66. It may also be observed that the effect of migration was more on the number of births than on the number of deaths during all the periods, especially during 1947-66. This was because the migrants were concentrated in the age group (15-44), except during the later years after 1961; and in this age range the change in mortality was very little. Again the interaction effects were significant only during the long period 1911-66.

From Table 6.3 we can see that the effect of the improvement in mortality and of net migration during all the three periods was to increase the natural growth of the population while the change in fertility reduced it during 1911-66 but increased it during 1933-66 and 1947-66. These consequences could easily be expected from the study of the changes in these components during the respective periods. But a quantitative estimation of the actual effects of their operation could only be made by an analysis similar to the one presented here. It may be noted that the effect of the improvement in mortality which contributed a larger portion of the total natural increase than that by migration during 1911-66 and 1933-66, became considerably smaller during the period 1947-66, and its place was taken by the effect of migration. The total effect of the increase in fertility contributed over 40 per cent of the actual natural increase during 1933-66 while its contribution during 1947-66 was only about 11 per cent. As could

be seen, the effects of the interactions of the changes in the components were prominent mainly in the period 1911-66.

6.4.2 CHANGES IN THE AGE-SEX DISTRIBUTION OF THE POPULATION

In order to examine the part played by the changes in each of the components in changing the age-sex distribution of the population of Australia, we shall examine the changes in the mean age and the proportion in the old age group (65+) separately for males and females; and the changes in the proportion of males in the total population. The proportion of males is preferred to the analysis of the ratio of males per females, as this proportion would have a finite bound in any population and the analysis would be comparable to that of the analysis of the proportion of population in a particular age group such as the old age group (65+).

Though it is difficult to answer the question: which of the components is more powerful in changing the age distribution (or even any other characteristic of the population)?, since the nature of their operation is different,[4] the importance of the effect of the observed variations in any of the components during a certain period on the characteristics of the population, could be assessed with the analysis suggested here. We may also note that mortality and migration are similar in nature in their effects on the age structure

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[4] A.J. Coale, 'How the Age Distribution of a Human Population is Determined?', Cold Spring Harbor Symposia on Quantitative Biology, Cold Spring Harbor, N.Y., 1957, pp.83-89. (Comment by Muhsam)

because they can affect all ages in each year, though with different intensity, whereas the effect of a change in fertility must pass age by age starting from the age zero. Thus it may be expected that the effect of the changes in fertility would be more prominent than that of mortality or migration. However, the relative effects depend on the magnitudes of the actual variations in the components during the period under consideration. With these preliminary remarks in mind we shall examine what has happened in the population of Australia.

6.4.2(i) Mean Ages of the Male and Female Populations

Table 6.4 gives the analysis of the observed change in the mean ages for males and females during the three periods. An examination of the results reveals that the mean ages of both males and females increased during 1911-66 and 1933-66, but decreased during 1947-66. The increase was more for females whereas the reduction was more for males. The effect of the variation in mortality in all the three periods was to decrease the mean age for males and to increase it for females. This was due to the fact that the improvements in mortality were more for females and occurred in the old age groups.

During 1911-66, the reduction in fertility from the level in 1911 was the main cause for the increase in the mean ages, while during 1933-66 the increase in fertility from the low level in the beginning years of the period effected a big reduction in the increase of the mean age which would have occurred otherwise.

The total effect of migration was comparatively small during

TABLE 6.4

ANALYSIS OF THE CHANGES IN THE VALUES OF THE MEAN AGES OF MALE AND FEMALE POPULATIONS IN AUSTRALIA

Source of Change	During 1911-66 Absolute Change	During 1911-66 Per cent to TC	During 1933-66 Absolute Change	During 1933-66 Per cent to TC	During 1947-66 Absolute Change	During 1947-66 Per cent to TC
			M A L E			
Value in the initial year	27.67		30.10		32.30	
E(1)	1.49 (29.16)*	43.70 (93.82)(a)	5.30 (35.40)*	540.82 (113.90)(a)	0.20 (32.50)*	16.39 (104.57)(a)
E(a)	-0.37	-10.85	-0.29	-29.59	-0.02	-1.64
E(b)	2.76	80.94	-3.29	-335.71	-0.64	-52.46
E(c)	-0.06	-1.76	-1.17	-119.39	-0.83	-68.03
E(ab)	-0.00	0.00	-0.02	-2.04	0.00	0.00
E(ac)	0.04	1.17	0.06	6.12	0.00	0.00
E(bc)	-0.52	-15.25	0.40	40.82	0.07	5.74
E(abc)	0.07	2.05	-0.01	-1.02	0.00	0.00
TC	3.41	100.00	0.98	100.01	-1.22	-100.00
Value in 1966	31.08	-	31.08	-	31.08	-
			F E M A L E			
Value in the initial year	26.64		30.43		33.22	
E(1)	3.58 (30.22)*	61.83 (93.19)(a)	6.01 (36.44)*	300.50 (112.37)(a)	0.54 (33.76)*	68.35 (104.10)(a)
E(a)	0.14	2.42	0.36	18.00	0.35	44.30
E(b)	2.94	50.78	-3.34	-167.00	-0.64	-81.01
E(c)	-0.48	-8.29	-1.31	-65.50	-1.04	-131.65
E(ab)	0.10	1.73	-0.03	-1.50	-0.01	-1.27
E(ac)	0.03	0.52	-0.01	-0.50	-0.04	-5.06
E(bc)	-0.54	-9.33	0.33	16.50	0.05	6.33
E(abc)	0.02	0.34	-0.01	-0.50	0.00	0.00
TC	5.79	100.00	2.00	100.00	-0.79	-100.01
Value in 1966	32.43	-	32.43	-	32.43	-

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

1911-66, while during the other two periods it contributed considerably to decrease the mean ages. In fact, during 1947-66, the part played by migration in reducing the mean ages was more prominent than that of the variations in fertility.

6.4.2(ii) Proportion of the Population in the Age Group (65+) Years

We have observed in the analysis of mortality that, except at the two youngest ages, the variation in mortality was more at the old ages. The male-female differences were also very significant at these ages. Hence, it was considered worthwhile to analyse the proportion in this old age group. In all the three periods considered here, this proportion increased both for males and females (see Table 6.5). The increase was less for males than for females. Mortality decline was more prominent in increasing the proportion for females than for males. Migration played a very important role in reducing the proportion, and its effect was more on the proportion for males than on that for females. While the declines in fertility from that in 1911 caused an increase in the proportion during 1911-66, the increases in fertility from that in 1933 and in 1947 effected a decrease in the proportion during the respective periods both for males and females. The effect of the interaction of the variation in fertility and net migration was the only interaction effect which was prominent.

6.4.2(iii) Proportion of Males in the Total Population

Analysis of this proportion was taken up to examine the effects of the variations in the components on the sex distribution in

TABLE 6.5

ANALYSIS OF THE CHANGE IN THE PROPORTION OF THE POPULATION IN THE OLD AGE GROUP (65 YEARS AND OVER) IN AUSTRALIA

Source of Change	During 1911-66 Absolute Value (per 100)	During 1911-66 Per cent to TC	During 1933-66 Absolute Value (per 100)	During 1933-66 Per cent to TC	During 1947-66 Absolute Value (per 100)	During 1947-66 Per cent to TC
			M A L E			
Value in the initial year	4.34		5.95		7.09	
E(1)	2.13 (6.47)*	71.00 (88.15)(a)	4.42 (10.37)*	317.99 (141.28)(a)	1.78 (8.87)*	712.00 (120.84)(a)
E(a)	0.07	2.33	-0.35	-25.18	0.02	8.00
E(b)	1.87	62.33	-1.36	-97.84	-0.22	-88.00
E(c)	-0.65	-21.67	-1.76	-126.62	-1.37	-548.00
E(ab)	0.02	0.67	0.03	2.16	0.00	0.00
E(ac)	0.05	1.67	0.09	6.47	0.00	0.00
E(bc)	-0.53	-17.67	0.34	24.46	0.05	20.00
E(abc)	0.04	1.33	-0.02	-1.44	-0.01	-4.00
TC	3.00	100.00	1.39	100.00	0.25	100.00
Value in 1966	7.34	-	7.34	-	7.34	-
			F E M A L E			
Value in the initial year	4.20		6.29		8.70	
E(1)	3.77 (7.97)*	63.68 (78.75)(a)	6.05 (12.34)*	157.96 (121.94)(a)	2.44 (11.14)*	171.83 (110.08)(a)
E(a)	0.88	14.86	0.76	19.84	0.60	42.25
E(b)	2.22	37.50	-1.58	-41.25	-0.27	-19.01
E(c)	-0.62	-10.47	-1.55	-40.47	-1.31	-92.25
E(ab)	0.22	3.72	-0.08	-2.09	-0.01	-0.70
E(ac)	-0.03	-0.51	-0.06	-1.57	-0.07	-4.93
E(bc)	-0.52	-8.78	0.29	7.57	0.04	2.82
E(abc)	0.00	0.00	0.00	0.00	0.00	0.00
TC	5.92	100.00	3.83	99.99	1.42	100.01
Value in 1966	10.12	-	10.12	-	10.12	-

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

the total population. Since we have assumed a constant sex ratio at birth, the variations in this proportion will be accounted for wholly by the variations in the components. It is to be expected from the nature of the changes that the effects of migration and the improvements in mortality would be in opposite directions - migration tending to increase the proportion and the variations in mortality tending to decrease it. This is supported by the results presented in Table 6.6. In fact, without the counteracting effect of migration, the proportion of males in 1966 would have been much less than it was. The effect of the variations in fertility was to decrease the proportion during 1911-66 and to increase it during the other two periods.

6.4.3 CHANGES IN THE SIZES OF CERTAIN SEGMENTS OF THE POPULATION

From the point of view of the study of population change in any actual situation, it is necessary to analyse the population change in certain important segments of the population. The choice of the segments depends on the purpose of the analysis. Since we are concerned here with the general analysis of the population change in Australia, we have selected the three segments which are of general interest.

6.4.3(i) The Population in the School Going Age Group (6-14 Years)

The age range 6-14 years is selected as the school age on the basis of the legislation for compulsory school attendance in the

TABLE 6.6

ANALYSIS OF THE CHANGE IN THE PROPORTION OF MALES IN THE POPULATION OF AUSTRALIA

Source of Change	During 1911-66		During 1933-66		During 1947-66	
	Absolute Value	Per cent to TC	Absolute Value	Per cent to TC	Absolute Value	Per cent to TC
Value in the initial year	51.98		50.86		50.44	
E(1)	-1.99 (49.99)*	-146.32 (98.77)(a)	-0.73 (50.13)*	-304.17 (99.05)(a)	-0.34 (50.10)*	-212.50 (98.99)(a)
E(a)	-0.09	-6.62	-0.47	-195.83	-0.25	-156.25
E(b)	-0.18	-13.24	0.10	+41.67	0.02	+12.50
E(c)	0.80	+58.82	0.85	+354.17	0.72	+450.00
E(ab)	-0.07	-5.15	0.08	+33.33	0.01	+6.25
E(ac)	-0.05	-3.68	0.05	+20.83	0.02	+12.50
E(bc)	0.22	+16.18	-0.11	-45.83	-0.02	-12.50
E(abc)	0.00	0.00	-0.01	-4.17	0.00	0.00
TC	-1.36	-100.01	-0.24	-100.00	0.16	100.00
Value in 1966	50.61	-	50.61	-	50.61	-

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

different states of Australia.[5] Though this has not always been uniform in all the states, the age range considered here more or less covers most of the ages specified in all the cases. Since the age group contains ages up to 15 years of age, the variations in the components during the fifteen years previous to the date at which the analysis is made, are of importance.

From Table 6.7, it may be observed that, during 1911-66, the total change in the school age population would have been about 28 per cent less in the case of males and about 26 per cent less in the case of females if fertility and mortality conditions had remained as in 1911 and no migration had occurred. Improvements in mortality compensated for this deficit, but the fertility rates during the 6-15 years previous to 1966 as compared to those in 1911 being lower, caused a larger deficit. Hence, if net migration had not contributed nearly four-ninths of the actual change, the size of the school going age population would have been smaller than what it actually was. The interaction effects were not insignificant but they operated in opposite directions so that the net effect was very little.

Because the fertility rates 6-15 years prior to 1933 were higher than in 1933, because there was a decline in mortality, and because considerable immigration occurred during that period, the size of the population in this segment would have been lower in 1966 than

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5 Commonwealth Bureau of Census and Statistics, Year Book of the Commonwealth of Australia, No. 51, 1965, p.687. [Recently the upper age limit has been changed to 15 years.]

TABLE 6.7

ANALYSIS OF THE CHANGE IN THE SIZE OF THE SCHOOL AGE POPULATION (6-14 YEARS) IN AUSTRALIA

Source of Change	During 1911-66 Absolute Change ('000)	During 1911-66 Per cent to TC	During 1933-66 Absolute Change ('000)	During 1933-66 Per cent to TC	During 1947-66 Absolute Change ('000)	During 1947-66 Per cent to TC
			M A L E			
Value in the initial year	400.9		572.9		509.4	
E(1)	446.6 (847.5)*	72.18 (83.11)(a)	-38.2 (534.7)*	-8.55 (52.44)(a)	255.1 (764.6)*	49.99 (74.98)(a)
E(a)	174.1	28.14	30.2	6.76	10.6	2.08
E(b)	-224.4	-36.27	260.1	58.20	64.3	12.60
E(c)	273.2	44.15	144.9	32.42	170.9	33.49
E(ab)	-45.2	-7.31	13.5	3.02	1.2	0.24
E(ac)	32.4	5.23	3.0	0.67	1.8	0.35
E(bc)	-31.5	-5.09	31.6	7.07	6.8	1.33
E(abc)	-6.4	-1.04	1.8	0.40	-0.4	-0.08
TC	618.8	100.00	446.9	99.99	510.3	100.00
Value in 1966	1019.7	-	1019.7	-	1019.7	-
			F E M A L E			
Value in the initial year	392.6		554.3		491.0	
E(1)	431.8 (824.4)*	74.16 (84.56)(a)	-38.0 (516.3)*	-9.03 (52.96)(a)	244.1 (735.2)*	50.43 (75.41)(a)
E(a)	155.6	26.72	25.3	6.02	8.8	1.82
E(b)	-218.5	-37.53	250.7	59.61	61.7	12.75
E(c)	261.5	44.90	136.0	32.33	161.2	33.31
E(ab)	-40.2	-6.91	11.9	2.83	0.4	0.08
E(ac)	28.3	4.87	3.1	0.74	0.6	0.12
E(bc)	-30.6	-5.26	30.7	7.30	6.1	1.26
E(abc)	-5.6	-0.96	0.9	0.21	1.1	0.23
TC	582.3	100.00	420.6	100.01	484.0	100.00
Value in 1966	974.9	-	974.9	-	974.9	-

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

in 1933 both for males and females if the low rates of fertility in 1933 had continued, with mortality also remaining as in 1933 and with no migration (see Table 6.7). It is mainly because of the recovery of fertility rates since 1935 and because of immigration that this situation was averted and an increase of about 447 thousands in the case of males and about 421 thousands in the case of females, was obtained. Mortality improvements contributed to only a small extent to increase the size of this group.

At 1947, the size of this group was small since it consisted of the cohorts born during the years when fertility was at its lowest levels during the period 1911-66 and when migration was considerably small. But, during 1947-66, fertility increased and immigration became a continuous force in the population growth. Hence, the increase in this segment during this period was more than that in the period 1933-66 both for males and females. The improvements in mortality contributed very little. The proportion of the total change due to the variations in fertility also became considerably less during this period than during the other periods, so that immigration contributed as much as one-third of the total change for males and females. Thus, in the absence of migration since 1947, the sizes of this segment for males and females in 1966 would have been much smaller than they actually were.

6.4.3(ii) Population in the Working Age Group (15-64 Years)

Analysis of this segment is of particular importance because

it gives an indication of the contribution of the variations in the components to the labour force. Table 6.8 presents the relevant analysis for the three periods. A point which may be noticed from the table is that the effect of the changes in fertility on this group had become negligible during 1947-66. This is simply because the changes in fertility had just reached this age group.

During 1911-66, about 37 per cent of the total change of about 2210 thousands in the case of males and about 29 per cent of the total change of about 2141 thousands in the case of females, were contributed by the changes in fertility and mortality, and due to migration. In fact, migration was a major factor in increasing the size of this group. It accounted for as much as about 50 per cent of the total increase for males and about 42 per cent for females. The declines in mortality contributed a little less than one-fifth of the total change in the case of both males and females. But, the declines in fertility brought down the increase by about one-third of the total change for both males and females.

Out of the total change of about 1475 thousands for males and of about 1345 thousands for females during 1933-66, nearly 70 per cent in the case of males and 65 per cent in the case of females were accounted for by the changes in fertility and mortality, and due to migration. The effect of the interactions of the changes became smaller and the contributions of the variations in fertility and mortality also reduced to a considerable extent, as compared to their

TABLE 6.8

ANALYSIS OF THE CHANGE IN THE SIZE OF THE POPULATION IN THE WORKING AGE GROUP (15-64 YEARS) IN AUSTRALIA

Source of Change	During 1911-66 Absolute Change ('000)	During 1911-66 Per cent to TC	During 1933-66 Absolute Change ('000)	During 1933-66 Per cent to TC	During 1947-66 Absolute Change ('000)	During 1947-66 Per cent to TC
			M A L E			
Value in the initial year	1512.7		2247.6		2628.5	
E(1)	1400.1 (2912.8)*	63.36 (78.25)(a)	448.7 (2696.3)*	30.42 (72.43)(a)	318.4 (2946.9)*	29.11 (79.17)(a)
E(a)	416.6	18.85	82.1	5.57	17.1	1.56
E(b)	-611.7	-27.68	156.2	10.59	2.4	0.22
E(c)	1096.1	49.60	768.6	52.12	753.5	68.88
E(ab)	-88.3	-4.00	6.4	0.43	0.0	0.00
E(ac)	74.8	3.39	9.9	0.67	2.6	0.24
E(bc)	-68.4	-3.09	2.8	0.19	-0.1	-0.01
E(abc)	-9.5	-0.43	0.1	0.01	0.0	0.00
TC	2209.7	100.00	1474.8	100.00	1093.9	100.00
Value in 1966	3722.4	-	3722.4	-	3722.4	
			F E M A L E			
Value in the initial year	1364.2		2160.9		2540.3	
E(1)	1522.4 (2886.6)*	71.09 (82.34)(a)	470.0 (2630.9)*	34.95 (75.05)(a)	334.3 (2874.6)*	34.63 (82.00)(a)
E(a)	410.6	19.17	108.6	8.08	33.2	3.44
E(b)	-598.3	-27.94	151.1	11.24	2.3	0.24
E(c)	903.5	42.19	595.9	44.31	591.4	61.27
E(ab)	-85.5	-3.99	5.7	0.42	0.0	0.00
E(ac)	64.8	3.03	10.5	0.78	4.1	0.42
E(bc)	-66.7	-3.11	2.8	0.21	-0.1	-0.01
E(abc)	-9.4	-0.44	0.1	0.01	0.1	0.01
TC	2141.4	100.00	1344.7	100.00	965.3	100.00
Value in 1966	3505.6	-	3505.6	-	3505.6	-

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

values during 1911-66. Hence, migration was again the major contributor to the increase in the size of the population in this segment.

During 1947-66, almost the whole of the extra growth in the working age population, which was the result of the changing conditions during the period, came from migration.

The policy pursued by the Australian Governments throughout the period 1911-66, except during the last few years, was responsible for the observed contribution of migration to the growth of the population in this segment. Comparing the results of Table 6.7 and 6.8, we can observe that the proportionate contribution due to mortality declines was less on the population in the working age group than on the school age population in all the periods in the case of males, while in the case of females this was higher during the two periods 1933-66 and 1947-66. This was due to the smaller changes in the survival rates in the age group (15-64) than in the younger age group (6-14) in the case of males; actually there were small decreases in the survival rates at some of the ages 45 years and above in 1947 as well as in 1966 compared respectively with the values observed in 1933 and 1961. As far as females were concerned, there were comparatively larger improvements, especially in the age groups 35 years and above, and this explains the higher proportionate contribution of mortality change in the case of females, during the two periods.

6.4.3(iii) Population in the Old Age Group (65 Years and Above)

Table 6.9 shows the analysis of the change in the old age populations during the three periods. Since the effect of the changes in fertility had not reached this age group during any of these periods, all the terms involving fertility change have become zero. Hence, the whole of the change in this population due to the changes in the conditions, during the respective periods, came from the changes in mortality and migration.

During 1911-66, about 39 per cent of the total change of 330 thousands in the case of males was due to the change in mortality and due to migration. Out of this, migration contributed about 20 per cent, the improvements in mortality about 17 per cent, and the remaining 2 per cent was accounted for by their interaction. In the case of females, out of nearly 42 per cent of the total change attributable to the changes occurring in the components, about 23 per cent came from the decline in mortality, about 16 per cent from migration, and the rest from their interaction.

The changes in the size of this population during the other two periods due to the changes in the mortality conditions and migration, were comparatively small, and were contributed almost wholly by migration in the case of males. On the other hand, in the case of females, these changes were quite considerable, though smaller than during 1911-66, and variations in mortality contributed a larger share than migration. Again, the consequences observed here are in

TABLE 6.9

ANALYSIS OF THE CHANGE IN THE SIZE OF THE POPULATION IN THE OLD AGE GROUP (65 YEARS AND OVER) IN AUSTRALIA

Source of Change	During 1911-66 Absolute Change ('000)	During 1911-66 Per cent to TC	During 1933-66 Absolute Change ('000)	During 1933-66 Per cent to TC	During 1947-66 Absolute Change ('000)	During 1947-66 Per cent to TC
			MALE			
Value in the initial year	101.3		201.9		272.9	
E(1)	202.4 (303.7)*	61.26 (70.35)(a)	212.2 (414.1)*	92.34 (95.92)(a)	137.8 (410.7)*	86.78 (95.14)(a)
E(a)	54.6	16.53	-0.6	-0.26	4.5	2.83
E(b)	0.0	0.00	0.0	0.00	0.0	0.00
E(c)	65.4	19.79	18.3	7.96	16.5	10.39
E(ab)	0.0	0.00	0.0	0.00	0.0	0.00
E(ac)	8.0	2.42	-0.1	-0.04	0.0	0.00
E(bc)	0.0	0.00	0.0	0.00	0.0	0.00
E(abc)	0.0	0.00	0.0	0.00	0.0	0.00
TC	330.4	100.00	229.8	100.00	158.8	100.00
Value in 1966	431.7	-	431.7	-	431.7	
			FEMALE			
Value in the initial year	90.6		206.3		328.9	
E(1)	283.6 (374.2)*	57.84 (64.42)(a)	283.7 (490.0)*	75.73 (84.35)(a)	184.6 (513.5)*	73.25 (88.40)(a)
E(a)	112.4	22.92	57.9	15.46	38.0	15.08
E(b)	0.0	0.00	0.0	0.00	0.0	0.00
E(c)	77.3	15.77	30.8	8.22	28.1	11.15
E(ab)	0.0	0.00	0.0	0.00	0.0	0.00
E(ac)	17.0	3.47	2.2	0.59	1.3	0.52
E(bc)	0.0	0.00	0.0	0.00	0.0	0.00
E(abc)	0.0	0.00	0.0	0.00	0.0	0.00
TC	490.3	100.00	374.6	100.00	252.0	100.00
Value in 1966	580.9	-	580.9	-	580.9	-

Note:- See the Note to Table 6.1 for explanations of the symbols in the column 'Source of Change'.

accordance with the observed variations in the components.

6.5 SUMMARY

Analysis of the population change in Australia during three selected periods using the method of factorial projections suggested here, provides a complete picture of the dynamics of population change during the three periods taken for study. It is observed that the effects and their interactions depend on the magnitudes of the changes in the components and the length of the period of analysis.

During the period 1911-66, the effects of fertility decline and those of mortality decline acted in opposite directions and almost cancelled their mutual effects so that migration formed the major determinant in the growth of the total population for males and females and in changing the sex distribution. The changes in fertility and their interactions with the changes in other components maintained their usual upper hand in changing the number of births and the age distributions. During 1911-66, migration contributed nearly 44 per cent to the school age population, 50 per cent to the population in the working age group, and 22 per cent to the old age group in the case of males, while in the case of females, the corresponding percentages were about 44, 42 and 19.

During the other two periods, 1933-66 and 1947-66, improvements in mortality had a larger share in the changes in the female population than in those in the male population. The effect of

fertility improvements was more prominent in the period 1933-66 and that of migration during 1947-66.

Due to the decrease in the length of the period of analysis, the effect of the change in fertility on the population in the working ages decreased from one period to the other. Its effect on the old age population was zero in all the periods due to the same reason.

It appears from this analysis of the changes in the population of Australia that the proposed decomposition of the observed changes in the characteristics of a population during a given period is useful in studying the relative impact on them of the variations in the components during that period.

PART IV

SUMMARY OF FINDINGS

SUMMARY OF FINDINGS

1. A review of literature on the population change revealed that analytical research including streams of migration seems not to have developed to the same extent as that on other components of population change, viz. fertility and mortality. Though the results applicable in the absence of migration, become applicable without change when migration is specified by age-sex-specific net migration rates, this procedure is not suited to investigate questions such as: What would be the effect of a given rate of net migration on the growth and age-sex distribution of the population? or, How would a given age-sex composition of net migrants affect the growth and the age-sex distribution of the population?, etc. These can be studied if migration is specified by an overall net migration rate and an age-sex composition of net migrants.

Hence, in this study, an attempt has been made to examine analytically the effects of introducing migration using an overall net migration rate and an age-sex composition of net migrants. The analysis is carried through the use of one-sex and two-sex deterministic models of population change, considering time and age as discrete variables. It is assumed that the fertility and mortality rates for the migrant and the non-migrant populations are the same. This assumption is supported by the analysis of the survival rates and the numbers of confinements for the Australian born and overseas born populations enumerated in the

censuses of Australia during 1911-66. It is also assumed that the variations in the components occur independently.

2. The fertility, mortality and migration data observed in Australia during 1911-66, and some hypothetical rates derived from them were used in the numerical illustrations. An analysis of the variations in the data observed in Australia showed that the survival rates by single years of age did not change very much during the period, except at the very young ages and at the old ages. Improvements in the survival rates for females were more than those for males and occurred at old ages. On the other hand, fertility which was at a relatively high level (with the total fertility rate (TFR) of 3.51) in 1911, decreased to a low level (with TFR of 2.10) in 1935, but increased thereafter till 1961. From 1961, fertility has experienced a reversal in the trend. The mean age of the fertility schedule declined almost continuously during this period, but showed very little change in the recent years. Net migration varied widely during this period due to the World Wars, the economic depression of the early nineteen thirties and the policies of the Australian government. There were also large changes in the age-sex composition of net migrants.

The hypothetical data on fertility were derived such that the approximate parabolic trend in TFR observed during 1911-61 repeats itself every 50 years with the age pattern becoming constant after the 56th year. The hypothetical data on migration were derived by assuming certain specific net migration rates - as, for example, 1.0 per cent or

0.5 per cent, etc. - or by using a Cosine (or Sine) curve to obtain the net migration rates and adopting certain observed (e.g. as in Australia during 1962-66 or 1925) or hypothetical (e.g. 1911 stb. or 1911 life table populations) age-sex compositions of net migrants.

Three populations with different age-sex distributions - the graduated age-sex distribution of the 1911 census (1911 obs.), the 1911 stable population (1911 stb.) and the age-sex composition obtained by projecting the 1911 population to 1966 (1966 obs.) - but having the same size at the initial point of time, were used to examine the convergence of age-sex distributions. The common size of the populations was taken as that of Australia as at 30 June 1911. Of course, the 1911 obs. population was taken as the initial population for the analysis of the population change in Australia during 1911-66. The sex ratio at birth was assumed to be constant at 105 males per 100 females in all the examples.

3. In Part II, the convergence of age-sex distributions is examined. First, the question of the convergence of arbitrary age-sex distributions to an equilibrium state age-sex distribution, when subjected to constant schedules of fertility, mortality and migration, is taken up in Chapter 3.

An equilibrium state population is defined as a population which has an unchanging age-sex distribution and grows at a constant rate under the operation of a specified set of fertility, mortality and migration schedules which remain constant over time. The constant age-

sex distribution is called the equilibrium state age-sex distribution and the constant growth rate the intrinsic growth rate.

When migration is not included into the process of population change, a relatively simple formula is derived by representing the net maternity function by a Pearson Type III curve, to obtain the duration of convergence (i.e. the number of years required for the difference between an arbitrary age-sex distribution and the equilibrium state age-sex distribution to become less than a specified small quantity) in terms of the characteristics of the net maternity function.

Then, migration is introduced into the process using an overall net migration rate and an age-sex composition of net migrants. It is seen that the convergence occurs and the change in the duration of convergence as compared to that in the absence of migration depends on the nature of migration. If the age-sex composition of net migrants contains positive values at all ages, then the duration is reduced if there is net immigration (i.e. the net migration rate is positive) and increased if there is net emigration (i.e. the net migration rate is negative). On the other hand, if the age-sex composition of net migrants contains positive, negative and/or zero values which would be the case when immigration, emigration and/or no migration occur at different ages, the duration may be less than, equal to or greater than that in the absence of migration depending on the age range being affected by each type and the magnitude of the net migration rate. The

intrinsic growth rate generally follows the opposite trend. The duration of convergence and the intrinsic growth rate can be calculated from the first two dominant roots of the projection matrix. Also, for a given age-sex composition of net migrants containing positive values at all ages, the duration of convergence decreases as the net migration rate increases. But, for a given net migration rate, the changes in the age-sex composition of net migrants produce only small changes in the duration of convergence unless the changes are such that the structure of the projection matrix is changed.

Alternatively, if migration is included into the process of population change using age-sex-specific net migration rates, the duration is seen to remain approximately the same as in the absence of migration. But the intrinsic growth rate follows a similar trend as in the case where a net migration rate and an age-sex composition of net migrants are used. In this case too, if the net maternity function is represented by a Pearson Type III curve, the duration and the intrinsic growth rate can be obtained from the formulas derived in the case with no migration.

When the schedules of fertility and mortality are changing over time, it is known, in the case of an arbitrary closed population, that the initial shape of the age-sex distribution is 'forgotten' in course of time, and the resulting age-sex distribution depends entirely on the history of the fertility and mortality rates. This is called the weak ergodicity theorem. In other words, the theorem states that

the difference between any two age-sex distributions subjected to identical schedules of fertility and mortality rates, the rates varying over time, would become in course of time, less than a pre-assigned small quantity. This theorem continues to hold good when migration is introduced into the process of population change using net migration rates and age-sex compositions of net migrants which are changing over time. The duration of convergence of any two age-sex distributions is lessened if there is continuous net immigration (i.e. the net migration rate is positive for all t), and is lengthened if there is continuous net emigration (i.e. the net migration rate is negative for all t) than if there is no migration, provided the age-sex compositions of the net migrants contain positive values for all ages and for all t. But, if there is net immigration for some years and net emigration for other years (or no migration occurs in some of the years), the duration of convergence may decrease, remain the same or increase as compared to the case with no migration, depending on the number of years each of the situations prevails.

On the other hand, if the age-sex compositions of net migrants for various values of t contain positive, negative and/or zero values, the duration of convergence may, again, be less than, equal to or greater than that in the absence of migration, depending on the nature of the changes in the age-sex compositions of net migrants and in the magnitudes of the net migration rates.

However, if the age-sex-specific net migration rates are used in the process of population change, the theorem holds good, but the duration of convergence would remain approximately the same as in the absence of migration.

4. Part III deals with the relationship between population change - the growth and the changes in the age-sex distribution - and the changes in the schedules of fertility, mortality and migration. When a set of one schedule of each of the components operates constantly over time, an equilibrium state population is evolved. In Chapter 5, formulas connecting the growth rate and the age-sex distribution of the equilibrium state population with the specified schedules of fertility, mortality and migration are derived. These bring out clearly the relationship between the two sets.

Through numerical analysis, it is observed that the closer the age-sex composition of the net migrants is to the age-sex distribution of the stable population resulting from the given schedules of fertility and mortality, the closer will be the age-sex distribution of the resulting equilibrium state population to that of the stable population. But the difference in the intrinsic growth rate between the equilibrium state and the stable populations is determined by the magnitude of the net migration rate unless the age-sex composition of the net migrants is such that migration affects only the ages beyond β years, the oldest age of reproduction. Some of the numerical experiments assuming hypothetical age-sex compositions of net migrants, and the decomposition of the changes in

the characteristics of the equilibrium state populations resulting under different sets of fertility, mortality and migration schedules into the effects of the differences in the operating schedules of each of the components, indicate that migration, depending upon its characteristics, could be as important a factor in changing the age-sex distribution of the equilibrium state population as fertility is.

If a set of k schedules of each of the components, instead of a set of one as in the above case, operates repeatedly over a sufficiently long period of time, a constant set of k growth rates and k age-sex distributions, is evolved. This set is called the cyclical model of population change (or the equilibrium state cycle) because the resulting set of k growth rates and k age-sex distributions repeats cyclically over every k years. In Section 5.3, formulas are derived which show explicitly the relationship between these growth rates and age-sex distributions on the one hand, and the specified schedules of fertility, mortality and migration on the other. An iteration procedure is suggested to obtain the resulting set of growth indexes (which give the growth rates) and the age-sex distributions, and is illustrated by a numerical example. This iteration procedure may conveniently be used to compute the intrinsic growth rate and the equilibrium state age-sex distribution. It is shown that, if the survival rates in the k mortality schedules are the same, the sex ratio at birth constant, and if there is no migration, the sex ratios in the resulting cyclical model will be the same in all the k populations

though the age distributions of males and females would be different due to the differences in the k fertility schedules. In the particular situation studied, the effect of migration was to increase the proportions in the younger age groups and consequently, to decrease those in the older age groups as compared with those resulting in the absence of migration.

Finally, in Chapter 6, the relationship between population change and the variations in the schedules of fertility, mortality and migration (when they are varying over time), is studied. In this case no fixed set of growth rates or age-sex distributions is evolved. Hence, the relationship has to be examined in particular populations. This is done by decomposing observed changes in the characteristics of the population - the growth and the changes in the age-sex distributions - during a certain period, into the effects of changes in each of the components during that period and the effects of the interactions of these changes. For this purpose, a method called the factorial projections method, is suggested and is applied to analyse the changes in the characteristics of the population of Australia during three periods, viz. 1911-66, 1933-66 and 1947-66.

From these analyses, it is observed that, during 1911-66, the effects of the declines in fertility and mortality acted in opposite directions and almost cancelled their mutual effects so that migration was the major determinant in the growth of the total population of males and females. In fact, during this period,

migration contributed nearly 44 per cent to the change in the size of the school age population, about 50 per cent to that of the working age group population and about 22 per cent to that in the old age group population in the case of males, while in the case of females the corresponding contributions were 44, 42 and 19 per cent. The changes in fertility and their interactions with the changes in mortality and with migration played a major role in changing the number of births and the age distributions of males and females.

During the other two periods - 1933-66 and 1947-66 - improvements in mortality had a larger share in the changes in the female population than in those in the male population. The effects of improvements in fertility were prominent in the period 1933-66 and those of migration during 1947-66. For instance, during 1947-66, migration accounted for about 69 per cent of the total change of 3722 thousands in the case of males and about 61 per cent of the total change of 3506 thousands in the case of females.

The effects of the interactions between the changes in the components were, generally, small except during the long period 1911-66. The analyses also reveal that the observed effects depend both on the magnitude of the changes in the components during the period under study and on the length of the period of study.

5. Thus, the investigations undertaken in this study show that the dynamics of population change traverses through parallel paths if migration is included into the process of population change. The

decomposition of the changes in the characteristics of the equilibrium state populations or those observed in an actual population during a certain period, as suggested here, seem to give a better understanding of the mechanisms effecting the changes.

APPENDIX A

ESTIMATION OF THE DISTRIBUTION OF THE MIGRANTS BY SINGLE YEARS OF AGE AND THE FERTILITY RATES BY SINGLE YEARS OF AGE OF FEMALES FOR THE PERIOD 1911-20

1. Though most of the data needed for the study were available for Australia in a published or unpublished form, the distributions by age of the number of arrivals and departures as well as the age-specific birth rates by single years of age of females were not available for the decade 1911-20. The survival rates were also not available for all the intercensal periods, but were interpolated from the official life tables and the life table presented in Appendix B. We shall discuss in this appendix the procedures adopted to obtain the migration data and the fertility rates for the period 1911-20.

2. Estimation of the age distributions of arrivals and departures

The total number of arrivals and departures for males and females were available for every year during this period. Hence it was necessary only to obtain the proportionate distribution by single years of age for arrivals and departures in the case of males and females.

To achieve this, two procedures were possible. One was to utilize the age distribution of the arrivals and departures during a period similar to the one under consideration; and the other was to make use of the information available in the census, i.e. the

statistics on the duration of stay in Australia. Each procedure has some advantages and disadvantages, and involve certain assumptions which are sufficiently well known and need no further elucidation.

Whichever procedure we adopt, it is necessary that the net effect on the population must be such that the components put together should yield approximately the population enumerated in a subsequent census. In Australia, the estimates of the total population of males and of females were published for each year during 1911-20;[1] and the population by age as at 30 June 1921 was estimated by adopting the age distributions observed in the census of 1921 which was taken on 3 and 4 April 1921, to the published total population of males and females. Hence it was prescribed that the age distributions of arrivals and departures should be such that, when these data on migration were used along with the data on births and the interpolated survival rates, (1) the resultant total population must be tolerably close to the published value for each of the years 1911-20; and (2) the resultant populations in 5 year age groups as at 30 June 1921 must not differ very much from the corresponding populations estimated on the basis of the census of 1921.

Using these two criteria for testing the estimates, some trials were made adopting the age distributions observed during other periods for which the age data were available. The results obtained

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[1] Commonwealth Bureau of Census and Statistics, Australian Demography, Bulletin No. 41, 1923, pp.12-13

from these trials did not satisfy the above criteria. Hence it was decided to try and estimate the age distributions of arrivals from the statistics on the duration of stay in Australia, as reported at the census of 1921; and using them, to obtain the age distributions of the departures.

In the census of 1921, males and females born outside Australia were classified in 5 year age groups and in single years of duration of stay in Australia for the first five years and then in groups of five years of duration of stay.[2] These data were used, as described below, to obtain the age distributions of the male and female arrivals. The procedure adopted was the same in the case of males and females.

First, the populations with the duration of stay of (0-4) years, in 5 year age groups, were subtracted from the total populations in those age groups. Thus two populations - one including the immigrants and another excluding them - were obtained as at the census date. These were then reverse survived using the five year survival rates obtained by interpolation from the official life tables. In computing the survival rates for this purpose the fact that the census was taken on 4 April 1921 was taken into account. By computing the simple averages of the respective populations at 1921 and the reverse survived ones, two populations - one including immigrants and another excluding them - were obtained at the mid-point of the period 1916-21.

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[2] Commonwealth Bureau of Census and Statistics, Census of the Commonwealth of Australia, 3 and 4 April 1921, Vol. 1, Part 4, pp.270

The population without the immigration component was then subtracted from the one with it and the percentage age distribution of immigrants at the mid-point of the period 1916-21 was obtained. The persons born outside Australia and enumerated in the census are referred here as immigrants to distinguish them from arrivals during the different years. Similarly emigrants and departures are distinguished.

A similar procedure was used to obtain the age distribution of immigrants at the mid-point of the period 1911-16, taking the total population exluding those with the duration of stay (0-4) years and the population excluding those with duration of stay (0-9) years at the census of 1921.

But it was observed that the percentages in the older age groups 75 years and over for males and females, obtained from the above procedure, were small compared to the age distributions of arrivals during the years for which data were available. To examine whether this had arisen due to the use of the duration of stay data which show only the survivors of the arrivals and also exclude the arrivals who returned before the census date, the populations with duration of stay of 0 years in the censuses of 1911 and 1921, were used. The populations in 5 year age groups, excluding the population with 0 yearsduration of stay, were distributed into single years of age by using the Karup-King interpolation formula for the age groups (5-9) to (70-74) years and by adopting the single years age distribution of the total population for the age groups (0-4) years, (75-79) and (80-84)

years. The population with 0 years of duration of stay was negligible in the age group (85+) years.

The populations by single years of age enumerated in the censuses of 1911 and 1921 and the ones excluding the populations of 0 years duration of stay, obtained as above, were then reverse survived and the age distributions of the immigrants at the mid-point of the periods 1910-11 and 1920-21, were obtained. From these estimates also, it was observed that the percentages in the older age groups 75 years and above, were small in the earlier estimates. Since the percentages in the age groups (75-79) and (80-84) years obtained in this case appeared more reasonable when compared with those of age distributions of arrivals in other years for which data were available and since the data for 0 years of duration represent a situation nearer to the dates of arrival of immigrants, it was decided to utilize these proportions for the age groups (75-79) and (80-84) years. But it was found that these proportions were sufficiently different for 1910-11 and 1920-21 in the case of females, while they were practically the same for males in the two years. Hence, for the females, the proportions in these age groups in the age distributions at the mid-points of the periods 1911-16 and 1916-21, were obtained by a linear interpolation of the respective proportions for 1910-11 and 1920-21. The percentages in the other age groups were proportionately adjusted to make the sum equal to 100. While this adjustment altered the percentages in the other age groups to a very

small extent, it made the percentages in the age groups (75-79) and (80-84) years look reasonable as compared with the known age distributions of arrivals. Thus the age distributions in 5 year age groups of the immigrant males and females at the mid-point of the periods 1911-16 and 1916-21, were calculated.

Then the age distributions in 5 year age groups of the emigrants were computed separately for males and females by using the ratios of the percentages in the age distributions of the departures to those in the age distributions of the arrivals during the period 1925-29, the earliest period for which data on the age distribution of arrivals and departures were available and during which the effect of the economic depression was still not very great. The values of the percentages in 5 year age groups, thus obtained, were adjusted to make the sum equal to 100.

Finally the male and female arrivals and departures in 5 year age groups were obtained by adopting the estimated age distributions of immigrants and emigrants, at the mid-point of the period 1911-16 to the published number of arrivals and departures of males and females during each of the five calendar years 1911 to 1915. Similarly the age distributions at the mid-point of the period 1916-21 were used to distribute by 5 year age groups the total number of arrivals and departures of males and females during the five calendar years 1916 to 1920. These were then distributed into single years of age with the help of the estimated single years of age distributions of arrivals and departures during 1925-29.

These estimated arrivals and departures by single years of age were used along with the published data on births and the interpolated survival rates to project the population from 30 June 1911 to 30 June 1921. The resultant total populations for each of the calendar years were compared with the totals published by the Commonwealth Bureau of Census and Statistics, and were found to be quite satisfactory in the case of females. The comparison of the projected population as at 30 June 1921 with the one obtained by adopting the census age distribution also showed that the female population was obtained within reasonable limits (see Tables A.3 and A.4, Females).

But in the case of males, the results were not satisfactory. This was expected because the effect of troop movements was greater on the male population than on the female population during the years 1914-19. In fact, as could be seen from Table A.1, the proportions of the troop movements among the total arrivals and departures were comparatively insignificant for females. Hence, the arrivals and departures by single years of age as estimated above, were kept for the females.

In the case of males some adjustment was needed to take account of the troop movements. If we had excluded the troop movements, it would have become necessary to adjust the total populations and the population at 30 June 1921 so as to make them comparable. Hence, it was decided to try and make some adjustments in

TABLE A.1

NUMBER OF TROOP MOVEMENTS INCLUDED IN THE MIGRATION STATISTICS: AUSTRALIA, 1914-19

Year	Arrivals		Departures	
	Males	Females	Males	Females
1914	-	-	34355	186
			(37.17)	(0.69)
1915	8423	102	95097	615
	(16.63)	(0.50)	(68.65)	(3.64)
1916	15739	273	143566	542
	(32.17)	(1.88)	(81.51)	(3.38)
1917	28244	205	45553	935
	(51.10)	(2.08)	(62.44)	(9.38)
1918	43714*	437	23124	381
	(62.87)	(4.65)	(50.51)	(4.91)
1919	162498	1258	1336	44
	(82.18)	(4.98)	(3.24)	(0.28)

Note:- The figures in parentheses are the ratios (per cent) of the troop movements to the total arrivals and departures.
* Corrected for misprint on the basis of the total figure published in Quarterly Summary of Australian Statistics, 1920, p.8; and in the official Year Book, 1901-1919, p.1101.
[Source: Commonwealth Bureau of Census and Statistics, Australian Demography, Bulletin No. 38, 1920, pp.20-21.]

the age distributions for males. After some trial and error, the 5 year age distributions were adopted as shown below.

Male Arrivals

(i) For each of the calendar years 1911 to 1915 the estimated age distribution of immigrants at the mid-point of the period 1911-16;

(ii) For each of the calendar years 1916 to 1918 the average of the estimated age distribution of immigrants at the mid-point of the period 1916-21 and that of the arrivals during 1925-29;

(iii) For the calendar year 1919 the estimated age distribution of immigrants at the mid-point of the period 1916-21;

(iv) For the calendar year 1920 the age distributions of arrivals during 1925-29.

Male Departures

(i) For each of the calendar years 1911 to 1913 the estimated age distribution of emigrants at the mid-point of the period 1911-16;

(ii) For the calendar year 1914 the average of the estimated age distribution of emigrants at the mid-point of the period 1911-16 and the estimated age distribution of immigrants at the mid-point of the period 1916-21;

(iii) For each of the calendar years 1915 to 1918 the estimated age distribution of immigrants at the mid-point of the period 1916-21;

(iv) For the calendar year 1919 the estimated age distribution of emigrants at the mid-point of the period 1916-21;

(v) For the calendar year 1920 the age distribution of departures during 1925-29.

The actual percentage age distributions in 5 year age groups adopted for the arrivals and departures during 1911-20 are shown in Table A.2. The distributions of the arrivals and departures of males by single years of age, were obtained by utilizing the estimated single years age distributions of arrivals and departures during 1925-29. Thus the complete set of arrivals and departures by single years of age of males and females, was estimated for the period 1911-20.

The projected populations at 30 June of the calendar years 1911-20, obtained from these estimated data on migration, the

TABLE A.2

PERCENTAGE AGE DISTRIBUTIONS OF ARRIVALS AND DEPARTURES: AUSTRALIA, 1911-20

Age Groups	Arrivals						Departures						
	Males				Females		Males					Females	
	1911-15	1916-18	1919	1920	1911-15	1916-20	1911-13	1914	1915-18	1919	1920	1911-15	1916-20
0- 4	6.51	6.02	7.65	4.39	8.13	7.15	5.63	6.64	7.65	6.56	3.31	6.59	5.77
5- 9	8.66	5.06	5.92	4.21	10.87	5.50	6.89	6.41	5.92	4.68	2.92	8.22	4.14
10-14	7.80	5.31	6.96	3.66	9.19	4.91	5.40	6.18	6.96	4.78	2.21	6.78	3.60
15-19	10.95	11.12	12.62	9.62	9.39	12.24	4.40	8.51	12.62	5.03	3.37	7.04	9.13
20-24	15.84	15.81	16.98	14.64	12.16	21.12	12.96	14.97	16.98	13.79	10.45	11.14	19.28
25-29	16.05	14.25	15.00	13.50	14.20	18.28	17.57	16.29	15.00	16.30	12.90	15.10	19.38
30-34	12.36	10.50	11.10	9.90	12.52	11.23	15.12	13.11	11.10	13.48	10.58	14.40	12.87
35-39	8.71	8.38	7.89	8.87	8.87	6.64	11.82	9.86	7.89	10.64	10.51	10.92	8.15
40-44	5.79	6.58	5.54	7.62	5.79	4.03	8.60	7.07	5.54	8.18	9.89	7.47	5.17
45-49	3.42	5.38	3.83	6.94	3.66	2.77	4.95	4.39	3.83	5.50	8.76	4.71	3.55
50-54	1.88	3.96	2.60	5.31	2.20	2.24	3.13	2.86	2.60	4.29	7.70	3.23	3.27
55-59	1.01	3.02	1.77	4.26	1.35	1.73	1.68	1.72	1.77	2.94	6.20	1.94	2.49
60-64	0.51	2.29	1.16	3.42	0.82	1.10	0.92	1.04	1.16	2.06	5.36	1.23	1.65
65-69	0.26	1.39	0.60	2.18	0.42	0.61	0.46	0.53	0.60	1.05	3.34	0.62	0.89
70-74	0.11	0.64	0.24	1.04	0.19	0.26	0.21	0.22	0.24	0.45	1.74	0.29	0.39
75-79	0.09	0.22	0.09	0.35	0.17	0.13	0.17	0.13	0.09	0.17	0.59	0.23	0.19
80-84	0.05	0.07	0.05	0.09	0.07	0.06	0.09	0.07	0.05	0.10	0.17	0.09	0.08
85+	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

published data on births and the interpolated survival rates, are presented in Table A.3. It may be observed that the total populations

TABLE A.3

COMPARISON OF THE TOTAL POPULATION OF AUSTRALIA AT 30 JUNE OF EACH CALENDAR YEAR: PUBLISHED AND PROJECTED, 1911-21

Year	MALE			FEMALE		
	Published (E) ('000)	Projected (P) ('000)	$(\frac{E-P}{E})100$	Published (E) ('000)	Projected (P) ('000)	$(\frac{E-P}{E})100$
1911	2333.8	-	-	2155.8	-	-
1912	2428.9	2425.2	+0.15	2224.8	2225.3	-0.02
1913	2517.3	2512.2	+0.20	2302.8	2298.7	+0.18
1914	2584.0	2562.5	+0.83	2365.0	2361.7	+0.14
1915	2565.0	2550.1	+0.58	2420.6	2412.8	+0.32
1916	2482.0	2481.7	+0.01	2461.2	2456.5	+0.19
1917	2438.2	2447.5	-0.38	2502.6	2497.0	+0.22
1918	2485.0	2486.5	-0.06	2544.4	2537.6	+0.27
1919	2610.9	2610.2	+0.02	2582.2	2581.3	+0.03
1920	2727.9	2732.8	-0.18	2632.6	2632.9	-0.01
1921	2771.9	2784.2	-0.44	2683.2	2687.2	-0.15

[Source: The published figures were taken from Commonwealth Bureau of Census and Statistics, Australian Demography, Bulletin No. 41, 1923.]

for males, as well as for females, were well within 1 per cent of the published values. The comparison of the populations in 5 year age groups from the projections and those obtained by adopting the census age distributions, presented in Table A.4, shows that the percentage difference was far below 5 per cent in each age group for each sex, except for the age groups (15-19), (20-24), (25-29), (50-54) and (85+) years in the case of males and for the age groups (60-64) and (85+)

TABLE A.4

COMPARISON OF THE POPULATION OF AUSTRALIA IN 5 YEAR AGE GROUPS AS AT 30 JUNE 1921: ENUMERATED AND PROJECTED

Age Groups	MALE			FEMALE		
	Enumerated (E) ('000)	Projected (P) ('000)	$(\frac{E-P}{E})100$	Enumerated (E) ('000)	Projected (P) ('000)	$(\frac{E-P}{E})100$
0- 4	306.5	310.3	-1.22	295.8	295.3	+0.18
5- 9	302.6	299.9	+0.88	295.3	299.7	-1.48
10-14	268.9	267.5	+0.52	262.0	265.5	-1.32
15-19	235.7	248.6	-5.49	230.2	239.4	-4.03
20-24	221.5	236.1	-6.62	234.2	225.1	+3.88
25-29	226.2	236.6	-4.61	237.9	234.1	+1.62
30-34	228.6	227.8	+0.33	222.4	225.5	-1.42
35-39	197.8	196.3	+0.77	190.7	193.2	-1.32
40-44	170.9	163.8	+4.14	161.8	160.5	+0.82
45-49	145.3	142.5	+1.91	136.2	137.6	-1.05
50-54	136.6	129.8	+4.95	120.4	119.4	+0.84
55-59	116.7	116.3	+0.35	99.9	101.1	-1.20
60-64	91.0	87.8	+3.52	79.0	74.5	+5.68
65-69	56.3	54.3	+3.56	49.1	48.5	+1.22
70-74	33.5	32.6	+2.69	31.9	31.7	+0.63
75-79	19.7	19.8	-0.51	20.5	20.8	-1.46
80-84	9.6	9.7	-1.04	10.4	10.6	-1.92
85+	4.7	4.4	+6.38	5.5	4.8	+12.73
Total	2771.9	2784.2	-0.44	2683.2	2687.2	-0.15

Note:- The enumerated values were obtained by adopting the age distributions observed in the census of 1921, to the published total populations of males and females as at 30 June 1921.

[Source: Commonwealth Bureau of Census and Statistics, Census of the Commonwealth of Australia, 3rd and 4th April 1921, Vol. I; Australian Demography, Bulletin No. 41, 1923.]

years in the case of females. The discrepancy could have arisen due to: (1) the arbitrary adjustments in the age distributions for arrivals and departures of males; (2) the influence of the epidemic of 1919 not being accounted for in obtaining the survival rates; (3) the errors of enumeration in the census data; and (4) the variations in the sex ratio at birth, however small. But the results appear to be satisfactory for practical purposes. Hence these data on migration were used in the study.

3. Estimation of the age-specific fertility rates for females 1911-20

The data on births by single years of age of females were published for all the calendar years 1911-20.[3] The mid-year population, i.e. the population as at 30 June of each year, was already obtained by projecting the population at 30 June 1911 for testing the consistency of the estimated age distributions of arrivals and departures. Hence the age-specific fertility rate at age x years was obtained by dividing the number of births of both sexes during each year to females aged x years by the total number of females at that age. The number of births to females whose ages were not known was allocated proportionately. The age-specific fertility rates thus obtained were negligible at ages below 15 years and above 49 years and were, therefore, assumed to be zero. For the calendar year 1912 the number of births registered was suspected to have been boosted up due

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[3] Commonwealth Bureau of Census and Statistics, Commonwealth Demography, Bulletin Nos. 29-38, 1911-20.

to the maternity allowances granted during that year.[4] Therefore, the average number of births for the three years 1910-12 was used for the year 1911 and the average of those for 1911-13 was used for the year 1912. For the other years the number of births as published for each year was utilized. Thus the gap in the age-specific fertility rates by single years of age of females was filled in.

From the procedure of estimation of the fertility rates, it is clear that these fertility rates used in conjunction with the estimates of migration and the interpolated survival rates would approximately give the same results as presented in Tables A.3 and A.4.

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[4] Commonwealth Bureau of Census and Statistics, Official Year Book of the Commonwealth of Australia, 1901-13, No. 7, 1914, p.146.

APPENDIX B

A COMPLETE LIFE TABLE FOR AUSTRALIA: 1965-67

1. The official life table for Australia based on the 1966 census was not available. Hence to obtain the survival rates required for the study, a life table with single years of age, was constructed using the data on the deaths and the populations for the three years 1965-67. It is intended here to give an account of the procedure used in the construction of the life table.

2. The data used

The basic data used in the construction of the life table were the following:

The populations of males and females by single years of age were taken for 1966 from the census publication: Census of the Commonwealth of Australia, 30 June 1966, Census Bulletin No. 9.1, Summary of Population: Australia (p.6), and for 1965 and 1967 from the estimates prepared by the Commonwealth Bureau of Census and Statistics. The data on deaths by single years of age, for the years 1965 and 1966 and on infant deaths, were published in Demography Bulletins 1965 and 1966; and the same for the year 1967, were supplied by the Commonwealth Bureau of Census and Statistics. They also made available the data on births registered in each calendar month and those on arrivals and departures at infant ages 0 and 1 years, in six monthly periods.

3. Method of calculation

In constructing a life table, as is well known, the primary step is the calculation of the probability of survival p_x or the probability of dying $q_x = 1 - p_x$. Then the other columns of the life table follow.

3(a). Probability of survival at infant ages

The method adopted in calculating these was essentially similar to the one used in the construction of the official life tables for Australia, 1960-62.[1] Since the deaths of infants under 1 year were available by months of age and the births were available by calendar months, a slight modification was made in the formulas given in the Australian life tables, 1960-62. These were as given below. The same procedures were applied separately for males and females.

For age 0 years

q_0 = infant mortality rate or the probability of dying before attaining the age of 1 year

$$= q_0^0 + q_0^1 + q_0^2 + \ldots\ldots\ldots + q_0^{11}$$

where the indexes 0, 1, 2, ..., 11 refer to the months of age at which death occurred.

q_0^0 was calculated as:

$$q_0^0 = \frac{\text{total deaths under 1 month of age during 1965, 1966 and 1967}}{P^0_{(0)}} \qquad \text{(B.1)}$$

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[1] Commonwealth Bureau of Census and Statistics, Australian Life Tables, 1960-62, Commonwealth Government Printer, Canberra (Appendix G).

where $P^0_{(0)} = [\frac{1}{4} B^{12}_{64} + B^{1-12}_{65} + B^{1-12}_{66} + B^{1-12}_{67} - \frac{1}{4} B^{12}_{67}]$

$$+ \frac{1}{24}[\frac{1}{6} M^{(2)}_{64}(0) + (M^{(1)}_{65}(0) + M^{(2)}_{65}(0)) + (M^{(1)}_{66}(0) + M^{(2)}_{66}(0))$$

$$+ (M^{(1)}_{67}(0) + M^{(2)}_{67}(0)) - \frac{1}{6} M^{(1)}_{67}(0)]$$

in which the indexes for B_t - the births, refer to the calendar months and for $M_t(0)$ - the net migrants at age 0, refer to the half years.

q^1_0 was calculated as:

$$q^1_0 = \frac{\text{total deaths aged 1 month during 1965, 1966 and 1967}}{P^1_{(0)}} \qquad (B.2)$$

where $P^1_{(0)} = [\frac{1}{2} B^{11}_{64} + B^{12}_{64} + B^{1-12}_{65} + B^{1-12}_{66} + B^{1-12}_{67} - B^{12}_{67} - \frac{1}{2} B^{11}_{67}]$

$$+ \frac{1}{12}[\frac{1}{6} M^{(2)}_{64}(0) + (M^{(1)}_{65}(0) + M^{(2)}_{65}(0)) + (M^{(1)}_{66}(0) + M^{(2)}_{66}(0))$$

$$+ (M^{(1)}_{67}(0) + M^{(2)}_{67}(0)) - \frac{1}{6} M^{(1)}_{67}(0)] + \frac{1}{24}[\frac{1}{6} M^{(2)}_{64}(0) - \frac{1}{6} M^{(1)}_{67}(0)]$$

These formulas were derived on the assumption that the births were uniformly distributed over the month in which they were registered and the net number of migrants were evenly distributed over the year of age, as well as over the half year to which they refer. The formulas for q^2_0, etc., were similar to that for q^1_0 with suitable changes in the births and the net migration values. The formula for q^0_0 was made slightly different in order to take account of the higher risk of dying in the early weeks of life. The correction for the effect of migration did not make a significant difference to the q^i_0 (i = 0, 1,

...11) values. After computing these q_0^i values, q_0 was obtained by adding them and the probability of survival p_0 was then computed by using the relation $p_0 = 1 - q_0$.

For age 1 year

To obtain the probability of dying between the age of 1-2 years the formula adopted in the construction of the Australian Life Table 1960-62 was used. Since the formula as given there did not present the adjustment for migration the one used in the present study is given below:

$$q_1 = \frac{\text{total deaths aged 1 year during 1965, 1966 and 1967}}{P(1)} \qquad (B.3)$$

where $P(1) = [(\frac{1}{8} B_{63}^{1} + \frac{3}{8} B_{63}^{2} + \frac{5}{8} B_{63}^{3} + \frac{7}{8} B_{63}^{4}) + (B_{64}^{1-4} + B_{65}^{1-4})$

$+ (\frac{7}{8} B_{66}^{1} + \frac{5}{8} B_{66}^{2} + \frac{3}{8} B_{66}^{3} + \frac{1}{8} B_{66}^{4})$ - (total deaths aged 0 years during 1964, 1965 and 1966)] + $[\frac{1}{4} M_{64}^{(1)}(0)$

$+ \frac{3}{4} M_{64}^{(2)}(0) + \frac{1}{4} M_{64}^{(2)}(1) + \frac{1}{2} (M_{65}^{(1)-(2)}(0) + M_{66}^{(1)-(2)}(0)$

$+ M_{65}^{(1)-(2)}(1) + M_{66}^{(1)-(2)}(1)) + \frac{3}{4} M_{67}^{(1)}(0) + \frac{1}{4} M_{67}^{(2)}(0)$

$+ \frac{1}{2} M_{67}^{(1)}(1) + \frac{1}{4} M_{67}^{(2)}(1)]$

in which the indexes for B - the births, indicate the quarters and those for M - net migration, refer to the half year.

The probability of survival was then obtained as $p_1 = 1 - q_1$.

3(b). Age 2 years and above

In the construction of the official life tables the ages (0-4) years were considered as infant ages and separate formulas were adopted. But the calculations in the present case showed that even at age 1 year the difference in the value of p_1 obtained by the elaborate method given above and the simple procedure given here, was small. Hence it was decided to adopt the same procedure from age 2 years onwards.

In the life tables of 1960-62, the populations enumerated in the census at the different ages were corrected for the effect of net migration by taking the figures of net movement in six monthly periods. As an alternative to this method, the averages of the populations in the three years 1965, 1966 and 1967 were used. Thus for all ages 2 years and above the procedure followed was as given below:

The age specific death rate at age x years was computed as:

$$m_x = \frac{\text{total deaths of persons aged x years during 1965, 1966 and 1967}}{\text{total population aged x years as at 30 June 1965, 1966 and 1967}} \quad \text{(B.4)}$$

From the m_x values the probability of survival from age x to (x+1) years was then obtained from the following simple formula:

$$p_x = \frac{2.0 - m_x}{2.0 + m_x}, \quad \text{for } x = 2, 3, \ldots, 84 \quad \text{(B.5)}$$

An examination of the mortality rates, m_x values, showed that a pronounced peak occurred at ages 19-20 years in the case of males and a similar peak of lesser magnitude occurred around the same ages in the

case of females. A similar feature was detected by the actuary in the 1960-62 mortality rates. To preserve this distinct feature of the mortality rates no graduation was applied for the mortality rates from ages (5-30) years in the official life tables of 1960-62. However, the mortality rates for ages 31 years and above were graduated. Even then considerable doubts were expressed regarding the values of m_x only at ages 89 or 90 years and above. Since the progression of the mortality rates from age to age in the present case was not too irregular even at ages 30 years and above except at very few ages, and since the effect of graduation would have caused only small changes in the probabilities of survival, it was decided to use the unsmoothed mortality rates in computing the probability of survival, p_x. Hence the p_x values as obtained from the above formula were used without any further graduation.

3(c). Computation of the other life table values

The other life table values were computed by the usual techniques employed in constructing life tables. The survivors at exact age x years were calculated by taking a radix $\ell_0 = 100{,}000$ and using the relation: $\ell_{x+1} = \ell_x \, p_x$. From the ℓ_x values the life table population or the number of years survived, L_x, was obtained from the following formulas:

$$L_0 = (1 - a)\, \ell_0 + a\, \ell_1 \qquad \text{(B.6)}$$

where a = 0.88186 for males and = 0.86692 for females, is the

proportion of deaths to persons born in the same year. The numerical values for males and females given here were the average values for the three years 1965-67.

$$L_x = \frac{1}{2}(\ell_x + \ell_{x+1}) \ , \quad \text{for } x = 1, 2, \ldots, 84 \tag{B.7}$$

and

$$L_{85+} = \ell_{85} \log \ell_{85} \tag{B.8}$$

Then the survival rates were calculated from the relations:

$$\left.\begin{aligned} S(b) &= (L_0/\ell_0) \\ S(x) &= (L_{x+1}/L_x) \ , \quad \text{for } x = 0, 1, 2, \ldots, 83 \\ \text{and} \quad S(84+) &= [L_{85+}/(L_{84} + L_{85+})] \ . \end{aligned}\right\} \tag{B.9}$$

Finally the expectation of life at age x years was computed as under:

$$^{o}e_x = \frac{1}{\ell_x} \left[\sum_{t=x}^{w} L_t \right] \tag{B.10}$$

4. The life tables for males and females

The complete life tables for males and females thus obtained are presented in Tables B.1 and B.2. A comparison of the mortality situation in 1965-67 as revealed by these life tables with those of the previous ones, is given in Chapter 2.

TABLE B.1

AUSTRALIAN LIFE TABLES, 1965-67: MALES

Age x	Survivors ℓ_x	Probability of Survival p_x	Probability of Dying $q_x=1-p_x$	Number of Years Lived L_x	Survival Rates $S(x)=\frac{L_{x+1}}{L_x}$	Expectation of Life at Age x $^{o}e_x$
					(0.98195)*	
0	100000	0.97953	0.02047	98195	0.99667	67.71
1	97953	0.99825	0.00175	97868	0.99863	68.12
2	97782	0.99900	0.00100	97733	0.99907	67.24
3	97685	0.99915	0.00085	97643	0.99928	66.30
4	97602	0.99942	0.00058	97573	0.99947	65.36
5	97545	0.99953	0.00047	97522	0.99952	64.40
6	97499	0.99952	0.00048	97476	0.99954	63.43
7	97452	0.99955	0.00045	97430	0.99957	62.46
8	97408	0.99959	0.00041	97388	0.99959	61.49
9	97368	0.99960	0.00040	97349	0.99960	60.51
10	97329	0.99960	0.00040	97310	0.99962	59.54
11	97290	0.99965	0.00035	97273	0.99958	58.56
12	97256	0.99952	0.00048	97233	0.99952	57.58
13	97210	0.99952	0.00048	97186	0.99946	56.61
14	97163	0.99940	0.00060	97134	0.99935	55.63
15	97105	0.99930	0.00070	97071	0.99919	54.67
16	97037	0.99909	0.00091	96992	0.99885	53.70
17	96948	0.99862	0.00138	96881	0.99845	52.75
18	96814	0.99828	0.00172	96731	0.99817	51.83
19	96648	0.99806	0.00194	96554	0.99808	50.91
20	96460	0.99810	0.00190	96369	0.99817	50.01
21	96277	0.99824	0.00176	96192	0.99827	49.11
22	96108	0.99831	0.00169	96026	0.99846	48.19
23	95945	0.99861	0.00139	95878	0.99854	47.27
24	95812	0.99848	0.00152	95739	0.99850	46.34
25	95666	0.99852	0.00148	95595	0.99853	45.41
26	95524	0.99854	0.00146	95455	0.99859	44.47
27	95385	0.99865	0.00135	95321	0.99857	43.54
28	95256	0.99850	0.00150	95185	0.99846	42.60
29	95113	0.99843	0.00157	95039	0.99849	41.66
30	94964	0.99855	0.00145	94895	0.99854	40.72
31	94826	0.99853	0.00147	94756	0.99849	39.78
32	94687	0.99845	0.00155	94613	0.99834	38.84
33	94540	0.99824	0.00176	94457	0.99816	37.90
34	94373	0.99808	0.00192	94283	0.99797	36.97
35	94192	0.99787	0.00213	94092	0.99777	36.04
36	93991	0.99767	0.00233	93882	0.99770	35.11
37	93772	0.99774	0.00226	93666	0.99753	34.19
38	93560	0.99731	0.00269	93435	0.99729	33.27
39	93309	0.99727	0.00273	93181	0.99711	32.36
40	93054	0.99696	0.00304	92913	0.99681	31.44
41	92771	0.99667	0.00333	92617	0.99631	30.54
42	92462	0.99595	0.00405	92275	0.99587	29.64
43	92088	0.99580	0.00420	91894	0.99571	28.76
44	91701	0.99562	0.00438	91500	0.99533	27.88

* This is the value of $S(b) = (L_0/\ell_0)$.

TABLE B.1

AUSTRALIAN LIFE TABLES, 1965-67: MALES (cont'd)

Age	Survivors	Probability of Survival	Probability of Dying	Number of Years Lived	Survival Rates	Expectation of Life at Age x
x	ℓ_x	P_x	$q_x=1-p_x$	L_x	$S(x)=\frac{L_{x+1}}{L_x}$	$^{o}e_x$
45	91299	0.99504	0.00496	91073	0.99484	27.00
46	90846	0.99465	0.00535	90603	0.99438	26.13
47	90360	0.99410	0.00590	90094	0.99363	25.27
48	89827	0.99316	0.00684	89520	0.99275	24.41
49	89213	0.99234	0.00766	88871	0.99207	23.58
50	88529	0.99180	0.00820	88166	0.99148	22.76
51	87803	0.99116	0.00884	87415	0.99032	21.94
52	87027	0.98947	0.01053	86569	0.98885	21.13
53	86111	0.98823	0.01177	85604	0.98778	20.35
54	85097	0.98732	0.01268	84558	0.98703	19.59
55	84018	0.98674	0.01326	83461	0.98584	18.83
56	82904	0.98493	0.01507	82279	0.98402	18.08
57	81654	0.98309	0.01691	80964	0.98219	17.35
58	80274	0.98127	0.01873	79522	0.98032	16.64
59	78770	0.97935	0.02065	77957	0.97878	15.95
60	77143	0.97820	0.02180	76303	0.97699	15.27
61	75462	0.97576	0.02424	74547	0.97409	14.60
62	73632	0.97237	0.02763	72615	0.97090	13.95
63	71598	0.96939	0.03061	70502	0.96819	13.34
64	69406	0.96695	0.03305	68259	0.96455	12.74
65	67112	0.96207	0.03793	65840	0.96210	12.16
66	64567	0.96214	0.03786	63345	0.95915	11.62
67	62122	0.95604	0.04396	60757	0.95514	11.06
68	59391	0.95419	0.04581	58031	0.95234	10.54
69	56671	0.95040	0.04960	55265	0.94890	10.02
70	53860	0.94733	0.05267	52441	0.94499	9.52
71	51023	0.94252	0.05748	49557	0.93832	9.02
72	48090	0.93386	0.06614	46500	0.93098	8.54
73	44910	0.92790	0.07210	43291	0.92548	8.11
74	41672	0.92288	0.07712	40065	0.91974	7.70
75	38458	0.91633	0.08367	36849	0.91359	7.30
76	35240	0.91059	0.08941	33665	0.90911	6.93
77	32089	0.90748	0.09252	30605	0.90267	6.56
78	29120	0.89737	0.10263	27626	0.89261	6.17
79	26132	0.88731	0.11269	24659	0.88381	5.82
80	23187	0.87986	0.12014	21794	0.87888	5.50
81	20401	0.87777	0.12223	19154	0.86895	5.18
82	17908	0.85891	0.14109	16644	0.85415	4.83
83	15381	0.84861	0.15139	14217	0.84036	4.55
84	13052	0.83064	0.16936	11947	0.78549*	4.27
85+	10842	-	-	43748	-	4.04

* This is the value of $S(84+) = (L_{85+}/L_{84+})$.

TABLE B.2

AUSTRALIAN LIFE TABLES, 1965-67: FEMALES

Age x	Survivors ℓ_x	Probability of Survival p_x	Probability of Dying $q_x=1-p_x$	Number of Years Lived L_x	Survival Rates $S(x)=\frac{L_{x+1}}{L_x}$	Expectation of Life at Age x $^{o}e_x$
					(0.98603)*	
0	100000	0.98389	0.01611	98603	0.99711	74.11
1	98389	0.99856	0.00144	98319	0.99890	74.32
2	98248	0.99924	0.00076	98211	0.99932	73.43
3	98173	0.99941	0.00059	98144	0.99945	72.48
4	98116	0.99950	0.00050	98091	0.99953	71.52
5	98066	0.99957	0.00043	98045	0.99960	70.56
6	98024	0.99964	0.00036	98007	0.99967	69.59
7	97989	0.99970	0.00030	97974	0.99970	68.62
8	97960	0199970	0.00030	97945	0199969	67.64
9	97930	0.99968	0.00032	97914	0.99971	66.66
10	97899	0.99975	0.00025	97886	0.99976	65.68
11	97874	0.99977	0.00023	97863	0.99976	64.69
12	97852	0.99975	0.00025	97839	0.99974	63.71
13	97827	0.99973	0.00027	97814	0.99970	62.72
14	97801	0.99968	0.00032	97785	0.99965	61.74
15	97769	0.99963	0.00037	97751	0.99957	60.76
16	97733	0.99950	0.00050	97709	0.99947	59.78
17	97684	0.99944	0.00056	97657	0.99941	58.81
18	97630	0.99938	0.00062	97599	0.99937	57.85
19	97569	0.99936	0.00064	97538	0.99937	56.88
20	97507	0.99938	0.00062	97476	0.99936	55.92
21	97446	0.99935	0.00065	97414	0.99941	54.95
22	97383	0.99947	0.00053	97357	0.99938	53.99
23	97331	0.99930	0.00070	97297	0.99934	53.02
24	97263	0.99939	0.00061	97233	0.99937	52.05
25	97204	0.99936	0.00064	97172	0.99932	51.08
26	97141	0.99928	0.00072	97106	0.99929	50.12
27	97071	0.99930	0.00070	97037	0.99932	49.15
28	97003	0.99934	0.00066	96971	0.99923	48.19
29	96939	0.99913	0.00087	96897	0.99915	47.22
30	96855	0.99917	0.00083	96815	0.99916	46.26
31	96774	0.99915	0.00085	96733	0.99903	45.30
32	96692	0.99891	0.00109	96639	0.99890	44.33
33	96587	0.99889	0.00111	96533	0.99895	43.38
34	96479	0.99902	0.00098	96432	0.99888	42.43
35	96385	0.99874	0.00126	96324	0.99868	41.47
36	96263	0.99863	0.00137	96197	0.99860	40.52
37	96131	0.99857	0.00143	96063	0.99853	39.58
38	95994	0.99849	0.00151	95921	0.99823	38.63
39	95849	0.99798	0.00202	95752	0.99802	37.69
40	95655	0.99806	0.00194	95562	0.99802	36.77
41	95470	0.99799	0.00201	95374	0.99776	35.84
42	95278	0.99753	0.00247	95160	0.99754	34.91
43	95042	0.99756	0.00244	94926	0.99748	33.99
44	94810	0.99741	0.00259	94688	0.99716	33.08

* This is the value of $S(b) = (L_0/\ell_0)$.

TABLE B.2

AUSTRALIAN LIFE TABLES, 1965-67: FEMALES (cont'd)

Age	Survivors	Probability of Survival	Probability of Dying	Number of Years Lived	Survival Rates	Expectation of Life at Age x
x	ℓ_x	p_x	$q_x=1-p_x$	L_x	$S(x)=\frac{L_{x+1}}{L_x}$	^oe_x
45	94565	0.99690	0.00310	94418	0.99675	32.16
46	94272	0.99660	0.00340	94111	0.99638	31.26
47	93951	0.99616	0.00384	93771	0.99594	30.36
48	93590	0.99572	0.00428	93390	0.99552	29.48
49	93190	0.99533	0.00467	92972	0.99523	28.60
50	92754	0.99513	0.00487	92528	0.99503	27.74
51	92303	0.99493	0.00507	92069	0.99457	26.87
52	91835	0.99421	0.00579	91569	0.99399	26.00
53	91303	0.99376	0.00624	91018	0.99325	25.15
54	90733	0.99274	0.00726	90404	0.99293	24.31
55	90074	0.99312	0.00688	89764	0.99246	23.48
56	89455	0.99179	0.00821	89087	0.99159	22.64
57	88720	0.99139	0.00861	88338	0.99103	21.82
58	87956	0.99067	0.00933	87546	0.99019	21.01
59	87136	0.98971	0.01029	86687	0.98926	20.20
60	86239	0.98881	0.01119	85756	0.98861	19.41
61	85274	0.98840	0.01160	84779	0.98756	18.62
62	84285	0.98672	0.01328	83725	0.98596	17.83
63	83165	0.98519	0.01481	82549	0.98434	17.07
64	81934	0.98347	0.01653	81256	0.98234	16.31
65	80579	0.98119	0.01881	79821	0.98099	15.58
66	79064	0.98079	0.01921	78304	0.97993	14.87
67	77545	0.97906	0.02094	76733	0.97792	14.15
68	75921	0.97675	0.02325	75038	0.97524	13.44
69	74156	0.97370	0.02630	73180	0.97177	12.75
70	72205	0.96978	0.03022	71114	0.96944	12.08
71	70023	0.96908	0.03092	68941	0.96628	11.44
72	67858	0.96339	0.03661	66616	0.96080	10.79
73	65374	0.95812	0.04188	64005	0.95620	10.18
74	62636	0.95420	0.04580	61202	0.95189	9.61
75	59767	0.94947	0.05053	58257	0.94730	9.04
76	56747	0.94502	0.05498	55187	0.94383	8.50
77	53627	0.94258	0.05742	52088	0.93755	7.96
78	50548	0.93222	0.06778	48835	0.92777	7.42
79	47122	0.92299	0.07701	45307	0.91966	6.92
80	43493	0.91606	0.08394	41668	0.91377	6.46
81	39842	0.91127	0.08873	38075	0.90409	6.00
82	36307	0.89622	0.10378	34423	0.88912	5.54
83	32539	0.88119	0.11881	30606	0.87463	5.12
84	28673	0.86719	0.13281	26769	0.80326*	4.75
85+	24865	-	-	109296	-	4.40

* This is the value of $S(84+) = (L_{85+}/L_{84+})$.

APPENDIX C

CONSISTENCY OF THE DATA ON THE COMPONENTS OF POPULATION CHANGE: AUSTRALIA, 1911-66

1. The data on the components - fertility, mortality and migration, viz. the fertility rates by single years of age of females, the survival rates by single years of age for males and females, and the numbers of arrivals and departures of males and females by single years of age - were taken for this study from different sources, published and unpublished, and were also estimated for some period. Hence, it was felt necessary to check whether they would yield results consistent with those observed in the different censuses. It must, however, be noted that the consistency tests which are applied, show only whether the sets of data - one on the components and the other on the population - are in agreement within a tolerable limit or not; but it would not mean that the two sets are accurate. In fact, there may be compensating errors. However, for practical purposes, such consistency checks do give some confidence in the numerical data used to represent the situation in the country. The other data used in the study were derived from these basic data. Hence, there was no need to examine them separately.

With this purpose in mind, two tests were applied to check the consistency. One was to compare the total population obtained by projecting the population as at 30 June 1911 successively for each

calendar year with the values published by the Commonwealth Bureau of Census and Statistics.[1] Another was to examine the differences between the projected populations in 5 year age groups and the corresponding populations enumerated in the censuses.

2. Comparison of the total population for males and females

In Appendix A, the data on the components for the period 1911-20 were obtained to make the projected populations consistent with the published values. Hence, the projected total populations from 1921 onwards were to be compared with the published values. However, for convenience of comparison, the values from 1911 to 1966 are presented in Table C.1 at five year intervals. The table shows that the populations were generally overestimated in the projections as compared with the published values. The differences for females were very small for all the years, almost negligible for many of the years. But in the case of males, the differences were considerable, especially after 1936. The deficiencies in the migration statistics during the war years 1939-45 and the exclusion of the troop movements in those statistics seem to have increased the differences as could be observed in 1946. However, in none of the years the overestimation in the projections was very high. In fact, the differences were only around 1 per cent of the published values. Hence it was concluded that

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[1] In the Demography Bulletin published by the Bureau, the estimated total populations of males and females at the end of each quarter in each year are given. These estimates are revised after every census to make them consistent with the census enumerations.

TABLE C.1

RATIO (PER CENT) OF THE DIFFERENCE BETWEEN THE PUBLISHED (E) AND THE PROJECTED (P) TOTAL POPULATIONS TO THE PUBLISHED POPULATION: AUSTRALIA, 1911-66

Year	MALES			FEMALES		
	Published (E) ('000)	Projected (P) ('000)	$(\frac{E-P}{E})100$	Published (E) ('000)	Projected (P) ('000)	$(\frac{E-P}{E})100$
1911	2333.8	-	-	2155.8	-	-
1916	2482.0	2481.7	+0.01	2461.2	2456.5	+0.19
1921	2771.9	2784.2	-0.44	2683.2	2687.2	-0.15
1926	3091.5	3104.9	-0.43	2964.8	2969.4	-0.16
1931	3321.2	3342.3	-0.64	3205.3	3215.6	-0.32
1936	3433.8	3467.4	-0.98	3344.6	3366.5	-0.65
1941	3584.5	3621.5	-1.03	3525.4	3542.9	-0.50
1946	3739.5	3800.1	-1.62	3725.6	3734.6	-0.24
1951	4253.7	4307.4	-1.26	4168.0	4167.4	+0.01
1956	4776.0	4831.7	-1.17	4649.5	4651.2	-0.04
1961	5312.3	5359.8	-0.89	5195.9	5190.4	+0.11
1966	5816.4	5880.8	-1.11	5734.1	5737.9	-0.07

Note:- For convenience of presentation, only values at 5 year intervals are shown here. The published values are from the Demography Bulletins.

[Source: Commonwealth Bureau of Census and Statistics, Demography, 1938, 1950, 1966.]

the data on the components which were used in the study were consistent with the published values of the total population.

3. Populations in 5 year age groups at the census date

In the comparison of the total population, the age distributions were not taken into account. The only way of seeing how best the projected age distributions were, was to compare them with

those observed in the respective censuses. This could be done by comparing the percentage age distributions or by comparing the populations in each of the age-sex groups. Since the second procedure would show how much difference there was in the populations in each age-sex group, irrespective of their relative magnitude in the total population, it was considered that this procedure would be better than the first one and was therefore utilized.

The differences between the enumerated and the projected populations in the respective age-sex groups as percentages of the enumerated populations in those age-sex groups are presented in Table C.2. A quick glance at the table shows that the values for females were considerably smaller than for males, suggesting that the projected values for females were more consistent with the census values than those for males. The total populations were obtained within reasonable limits for both sexes, though the differences for females were again smaller. The larger differences in the case of males may have arisen due to the following factors: the effects of migration and troop movements were significantly greater on the male population; arbitrary adjustments were made in obtaining the age distributions for the arrivals and departures of males during the period 1911-20; and a constant sex ratio at birth was used in the projections.

A striking feature in the table is the large differences in the age group (85+) years, both in the case of males and females.

TABLE C.2

COMPARISON OF THE ENUMERATED (E) AND THE PROJECTED (P) POPULATIONS IN 5 YEAR AGE GROUPS, AUSTRALIA 1921-66: VALUES OF [(E-P)/E]100 AT CENSUS DATES

Age Groups	Male						Female					
	1921	1933	1947	1954	1961	1966	1921	1933	1947	1954	1961	1966
0- 4	-1.22	-5.88	+4.03	+0.46	+0.21	0.00	+0.18	-6.46	+3.83	+0.37	-0.11	-0.67
5- 9	+0.88	-0.39	+1.95	+2.69	+1.11	+0.08	-1.48	-0.55	+2.00	+2.29	+0.81	+0.49
10-14	+0.52	+0.30	-0.10	+2.57	+3.00	+1.11	-1.32	+0.56	+0.15	+2.88	+2.66	+0.79
15-19	-5.49	-0.57	-4.41	+0.69	+2.50	+1.99	-4.03	+0.06	-3.93	+1.35	+2.59	+1.95
20-24	-6.62	+1.66	-3.36	-4.54	+1.65	+1.39	+3.88	+0.44	+0.73	-4.25	+1.75	+2.39
25-29	-4.61	-1.31	-4.56	-3.27	-2.18	+0.88	+1.62	+0.10	+1.53	-0.14	-1.44	+1.47
30-34	+0.33	-5.65	-2.93	-3.19	-4.02	-3.37	-1.42	+0.90	+0.26	+1.06	-1.97	-2.09
35-39	+0.77	-7.60	+0.65	-3.69	-2.56	-3.83	-1.32	+1.79	+1.36	+1.13	+1.57	-1.55
40-44	+4.14	-1.80	-3.23	+0.09	-4.54	-3.59	+0.82	-2.47	-1.71	-0.17	+1.23	+0.62
45-49	+1.91	+2.27	-4.34	-1.09	-0.88	-4.50	-1.05	-1.62	+2.53	-1.15	-0.45	+0.64
50-54	+4.95	+3.03	-7.76	-3.44	-0.32	-1.79	+0.84	-1.84	-1.25	+0.71	-0.58	-0.92
55-59	+0.35	-0.58	-5.28	-11.72	-5.25	-0.78	-1.20	-4.80	-4.52	-4.96	-5.25	-1.79
60-64	+3.52	+4.15	+0.77	-4.12	-6.25	-5.37	+5.68	+1.95	-3.12	-2.45	+1.18	-4.33
65-69	+3.56	+3.58	+2.56	-0.48	-8.43	-6.87	+1.22	+1.88	-1.65	-4.26	-2.97	+0.64
70-74	+2.69	+4.70	+1.03	+0.96	-0.63	-9.08	+0.63	+3.27	-2.25	-3.22	-4.19	-3.74
75-79	-0.51	+10.31	+5.87	+2.24	+1.39	+0.11	-1.46	+7.44	+0.37	-3.58	-5.21	-4.42
80-84	-1.04	+12.23	+4.35	+4.73	+0.34	+1.52	-1.92	+5.83	+0.31	+0.40	-1.12	-4.10
85+	+6.38	+16.67	+12.66	+9.71	+8.84	+9.28	+12.73	+15.97	+15.99	+12.07	+12.92	+14.92
Total	-0.44	-0.80	-1.32	-1.42	-0.89	-1.11	-0.15	-0.52	+0.05	-0.16	+0.11	-0.07

Note:- A -ve sign shows that the enumerated population was less than that obtained in the projections and a +ve value shows that it was greater. The values for 1921 were adjusted to refer to 30 June 1921.

[Source: Commonwealth Bureau of Census and Statistics, Census of the Commonwealth of Australia 30 June, 1947 Vol. I, Part IX; 1961 Vol. VIII, Part I; and 1966 Bulletin No. 9.1.]

These were not given serious consideration because the numbers involved were comparatively small. Apart from this group, the values for the age groups (75-79) and (80-84) years were significantly high in 1933, especially in the case of males. This discrepancy is hard to explain except through the deficiencies in the data on the departures. It may be mentioned that this source of error was considered a major source and often corrections were effected accordingly.[2] Since only the projected populations were used in the present study, no adjustment was made in the data on departures.

Another point which can easily be noticed in the table is the large overestimation in 1933 and the considerable underestimation in 1947 of the population in the age group (0-4) years. When we recall that the female dominant method was used to obtain the projected populations and note that the differences for females were not large, it appears that these discrepancies could have arisen mainly due to the enumeration errors and the errors in the migration data.

It may also be observed that the overestimation in the age groups (15-29) years for males, which was the result of the arbitrary adjustments made in the age distributions of the arrivals and departures of males, was carried through the age range at the successive censuses. But the large value in the age group (55-59) years was probably the combined result of this error and the errors of

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2 Commonwealth Bureau of Census and Statistics, Census of the Commonwealth of Australia, 30 June 1947, Vol. III, Statistician's Report, p.29.

age reporting in the census. This idea is somewhat confirmed when we look at the values for females in the same age group at the successive censuses.

The deficiencies in the migration data during the Second World War seem to have caused the observed overestimation in the age group (15-29) years for males at the census of 1947 and this again has passed into the older ages in the subsequent censuses.

4. In view of the long period of time under consideration, and considering the disturbances that occurred during this period, the differences on the whole were not too large to invalidate the assumption that the data on the components which were used in the study were consistent with the published estimates of the total populations for the calendar years 1911-66 and with the results of the census enumerations.

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