



Australian
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Dynamic Metasurfaces: From Tunability to Parametric Modulation

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A thesis submitted for the degree of Doctor of Philosophy of
The Australian National University

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Declaration

This thesis presents research conducted between February 2022 and October 2025 at the Department of Fundamental and Theoretical Physics, Research School of Physics, The Australian National University and the Australian Research Council Centre of Excellence for Transformative Meta-Optical Systems (TMOS), during my candidature for the Doctor of Philosophy degree.

The research was carried out under the supervision of Prof. Ilya Shadrivov, Dr. Yana Izdebskaya, Prof. Mariusz Martyniuk and Prof. Andrey Miroshnichenko.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of the author's knowledge, it contains no material previously published or written by another person, except where due reference is made in the text.

Fedor Kovalev

October 2025

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In this work, ChatGPT was utilised for editing and writing code for calculating multipoles.

Explore the ways to
control and manipulate
propagating waves

— Fedor Kovalev

Best Haiku

ANU Research Fest 2024

Abstract

Metamaterials, artificially designed composite media made of subwavelength structural elements, enable new ways of controlling wave-matter interactions. Their elements, or meta-atoms, through tailored geometry, arrangement, and collective response, provide precise control over electromagnetic waves across different frequency ranges. Their planar analogues, metasurfaces, are particularly attractive because they combine subwavelength thickness with powerful wave-manipulation capabilities. Current developments in metamaterials and metasurfaces focus on tunability and approaches that enable dynamic changes of their parameters over time.

This thesis investigates methods for dynamically tuning metasurface properties and controlling their electromagnetic responses through spatial and temporal changes. While the advanced functionality of the dynamic metasurfaces studied here is realised through control of their resonant responses, the proposed parametric effects are based on temporal modulation of physical parameters induced by external energy sources.

Chapter 1 reviews the development of metamaterials and methods for their dynamic tuning, followed by an analysis of progress in tunable and time-varying metasurfaces. It concludes with the thesis overview.

Chapter 2 examines mechanical tuning of metasurface resonant responses, focusing on tunable filtering based on extraordinary optical transmission (EOT) and bound states in the continuum (BIC) enabled by microelectromechanical systems (MEMS). We first review mechanically tunable metasurfaces and then demonstrate spectrally tunable metasurface filters for hyperspectral infrared imaging, realised through adjustment of EOT resonances by varying the spacing between a dielectric membrane and a nanohole-patterned gold layer. We then discuss advances enabled by integrating metasurfaces into optical fibres and introduce EOT-based metasurface filters for fluorescence fibre-optic imaging. Finally, we introduce a new concept of dynamic control of quasi-BIC resonances through MEMS-enabled hybridisation of one-dimensional (1D) and two-

dimensional (2D) metasurfaces, achieving ultranarrow tunable filtering for spectroscopy, remote sensing, and imaging.

Chapter 3 explores control of resonances in toroidal metasurfaces using phase change materials for applications such as terahertz detectors and modulators. After reviewing recent progress in active toroidal metasurfaces, we demonstrate quasi-BIC resonances in gold split-ring resonator (SRR) arrays and analyse their dependence on geometrical parameters. We then examine the role of toroidal dipoles in scattering using multipole expansion and introduce tunability by integrating thin film patches of the phase-change material vanadium dioxide into meta-atoms.

Chapter 4 investigates parametric metasurfaces for amplification and frequency conversion of free-space electromagnetic waves in the microwave range. It begins with an introduction of time-varying metasurfaces based on SRRs with variable capacitances and an analytical model of wave amplification through electrical modulation of metasurface parameters. We then analyse the results of full-wave numerical simulations and compare them with analytical predictions. In the final section, we propose a dual-resonance parametric metasurface for amplified up-conversion, demonstrating significant efficiency beyond Manley-Rowe limits. The chapter concludes with experimental verification of amplified up-conversion efficiency and metasurface performance.

Chapter 5 summarises the research findings and concludes the thesis.

List of Abbreviations

BIC	bound states in the continuum
q-BIC	quasi-bound states in the continuum
EOT	extraordinary transmission
MEMS	microelectromechanical systems
NEMS	nanoelectromechanical systems
Q-factor	quality factor
PCB	printed circuit board
VO ₂	vanadium dioxide
GST	germanium-antimony-tellurium
PCM	phase change material
ENZ	epsilon near-zero
GaAs	gallium arsenide
FPGA	field-programmable gate array
PEC	perfect electric conductor
PMC	perfect magnetic conductor
ITO	indium tin oxide
PT	parity-time
VRM	virtually rotating metasurface
PTC	photonic time crystal
dB	decibel
dBm	decibel-milliwatts
OCT	optical coherence tomography
SPP	surface plasmon polariton
LIDAR	light detection and ranging
LWIR	long-wavelength infrared
SEM	scanning electron microscope
GMR	guided-mode resonance
TEM	transverse electromagnetic
SMT	semiconductor-to-metal transition
TCR	temperature coefficient of resistance
SRR	split-ring resonator
1D	one-dimensional
2D	two-dimensional

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Chapter 1: Introduction

1.1 Metamaterials and their evolution

Understanding and controlling material properties is fundamental to developing new electromagnetic technologies. Over time, researchers have learned not only to use materials available in nature but also to design artificial materials with tailored properties. Engineered electromagnetic materials enable regimes of light–matter interaction that are not readily accessible in naturally occurring media, allowing unprecedented control over electromagnetic wave propagation. This control arises from the ability to engineer the electromagnetic response of matter, which is determined by three fundamental parameters: permittivity, permeability, and conductivity. By manipulating these properties, it becomes possible to realise materials with desired electromagnetic behaviour.

Materials with simultaneously negative dielectric permittivity and magnetic permeability were first studied theoretically in 1968 [1]. This fundamental work predicted that such media would exhibit unusual phenomena, including negative refraction, the reversal of the Doppler effect, and the possibility of realising exceptional lenses. More than three decades later, in 1999, the concept of artificial structures capable of achieving such properties — now known as metamaterials — was proposed [2], and their first experimental realisation in the microwave regime was reported at the beginning of this century [3]. Since then, over the past twenty-five years, metamaterials have advanced significantly, enabling a wide range of unique electromagnetic functionalities, including cloaking [4] and perfect absorption [5].

Metamaterials are artificially engineered composite materials consisting of subwavelength structural elements, also called meta-atoms. Owing to the exceptional properties arising from their periodic arrangement, collective response, and geometry, metamaterials enable versatile control of electromagnetic waves [6]. Metasurfaces, the flat counterparts of metamaterials, are especially attractive because they combine extremely small thickness with powerful wave-manipulation capabilities [7–9]. A key

example of their potential is the commercial success of metalenses [10], which have been successfully integrated into new technologies such as machine vision systems and consumer electronics.

While static metamaterials and metasurfaces have already found practical applications [11], particularly in the miniaturisation of optical elements, the current focus has shifted towards reconfigurable metamaterials and approaches enabling the dynamic change of their properties [12,13]. This dynamic aspect represents an active area of research, holding the promise of being the next breakthrough in the evolution of metamaterials [14]. Figure 1-1 illustrates the schematic representation of static (a) and dynamic (b) metamaterials. In this thesis, the term “dynamic” is used to collectively refer to tunable and spatiotemporal metamaterials. Dynamic metamaterials rely on modulated or reconfigurable elements, in contrast to their static precursors. The most interesting effects are achieved in inhomogeneous metamaterials with the independent control of each meta-atom [15].

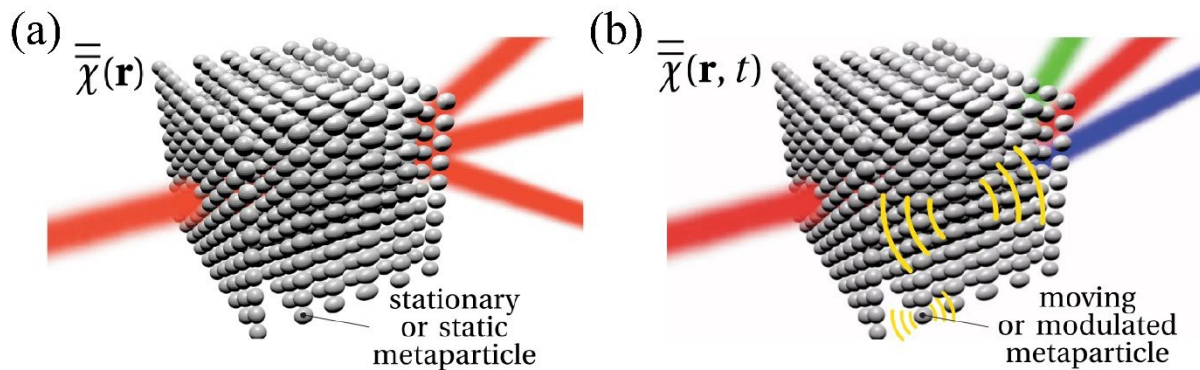


Figure 1-1 Schematic representation of static (a) and dynamic (b) metamaterials. Adapted from [16].

Tunability expands the range of applications and opportunities enabled by metamaterials and metasurfaces [17]. Tunable metamaterials provide flexible control over electromagnetic waves and their fundamental properties: phase, frequency, amplitude, polarisation and direction of propagation [13]. Reconfigurable metasurfaces let researchers to design metalenses with tunable focal lengths, dynamic beam steering elements, tunable filters and absorbers. Tunable metasurfaces have already found

important commercial applications, such as a metamaterial-surface antenna technology [18] for satellite communication. While implementing reconfigurability of metasurfaces is more challenging at higher frequencies due to small sizes of meta-atoms, researchers proposed different methods for pushing this field towards impactful solutions for further industrial realisation [19].

The current generation are dynamic metamaterials whose properties are partly due to the temporal variation of some of their physical parameters, induced by an external source of energy [15,16]. The time dimension takes metamaterials to a whole new level [20,21]. Time-varying metasurfaces with physical parameters modulated over time go beyond the limits of passive electromagnetic media [22] such as energy conservation and time-reversal symmetry. Modulation of metasurface parameters both in space and time provides complete control over electromagnetic waves [20]. Spatiotemporal metasurfaces enable a range of non-trivial physical effects, such as nonreciprocity [23–25], in which electromagnetic waves are transmitted differently when propagating in opposite directions.

Recently, time-varying metasurfaces have enabled the realisation of photonic time crystals (PTC) [26], a form of matter structured in time instead of regular crystals structured in space, providing a new platform for amplification of electromagnetic waves [27]. They have also facilitated the demonstration of a temporal analogue of the Young’s double-slit diffraction experiment [28]. Modulation of metasurface parameters over time gives rise to generation of new frequency harmonics when electromagnetic waves interact with such media, enabling frequency conversion [15,29]. Moreover, temporal interfaces recently demonstrated such exotic effects as time refraction and reflection [30–33], temporal counterparts of classical physical processes.

In the next two sections, we provide an overview of advances in dynamic metasurfaces, both tunable and time-varying, with a particular focus on the tuning mechanisms and functionalities explored in this thesis.

1.2 Tunable metasurfaces

There are multiple approaches to achieving dynamic control of metasurfaces. Tuning can be realised electrically, mechanically, optically, or thermally, while nonlinear, magnetic, and chemical mechanisms have also been employed to adjust their properties. The choice of tuning strategy depends on the design requirements, including the desired switching speed, material compatibility, and the ability to access individual meta-atoms [15].

Figure 1-2 shows an ideal active metasurface that allows efficient control of optical phase, amplitude, and polarisation by tuning its spatial, spectral, and temporal responses, thereby enabling remarkable light manipulation and diffraction control [13]. Such capabilities underpin a broad range of potential applications. Among these, precise control at the level of individual meta-atoms has attracted particular attention, as it allows tailoring of wavefronts on demand. However, implementing individual meta-atom control in the optical domain is especially challenging due to the nanometre-scale dimensions involved. For this reason, optical metasurfaces typically employ collective tuning of groups of resonators, whereas individual-element control is more feasible at lower frequencies such as microwaves or terahertz.

Control over fundamental properties of electromagnetic waves can be used for creating modulators [34], tunable filters and polarisers [35], beam shaping and steering devices [36,37], adaptive metalenses [38,39,10] and holographic displays [40,41] for communication, imaging, remote sensing, and other applications. Achieving full 2π continuous phase control is essential for enabling arbitrary wavefront manipulation required in beam steering, holography, and lensing applications. In the microwave regime, active metasurfaces incorporating varactor diodes based on semiconductor technology have enabled such control [42]. At optical frequencies, alternative approaches such as mechanically reconfigurable structures [43,44] have also demonstrated the feasibility of full continuous phase modulation.

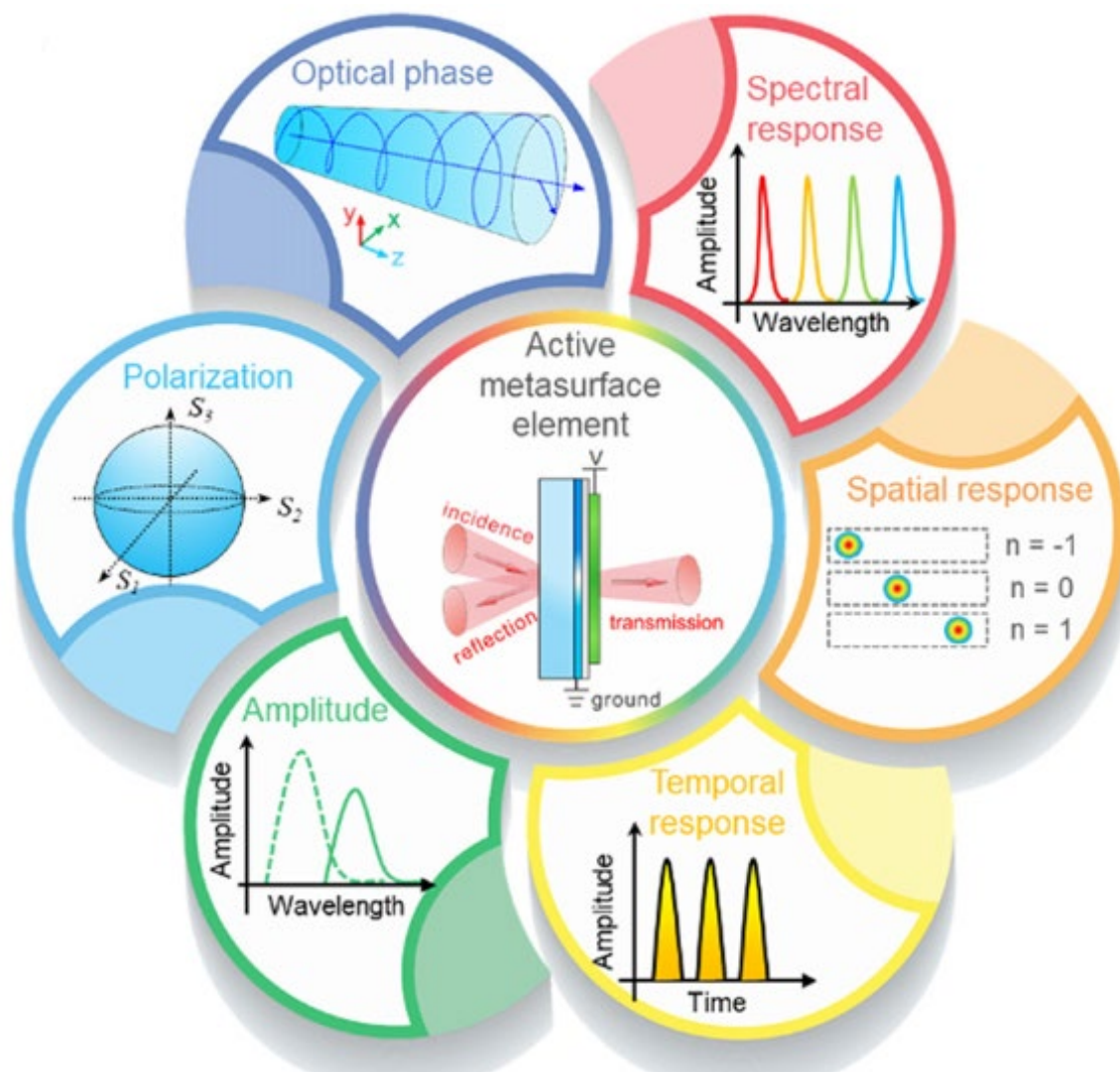


Figure 1-2 Ideal tunable metasurface able to perform control over fundamental characteristics of electromagnetic waves: amplitude, phase, polarisation, and direction of propagation through its spectral, spatial and temporal response. Adapted from [13].

Figure 1-3 illustrates different tuning methods and their speed. The fastest switching is typically achieved through optical tuning. Recent studies have demonstrated that optical pumping can induce refractive index changes on femtosecond time scales, corresponding to tuning speed comparable to the period of electromagnetic waves in the optical range. However, such approaches generally require high-power laser sources [32]. Electrical methods such as gating in two-dimensional materials [45–49] and conducting oxides [50–52] represent another highly promising route. They offer switching on picosecond time scales and can achieve parameter changes on the order

of unity, making them a highly effective tuning strategy. Other approaches, including mechanical actuation [53,54], phase-change materials [55–57], and liquid crystals [58–62], provide versatile knobs for reconfigurability but suffer from limited switching speeds, which restrict their use in applications requiring ultrafast tuning.

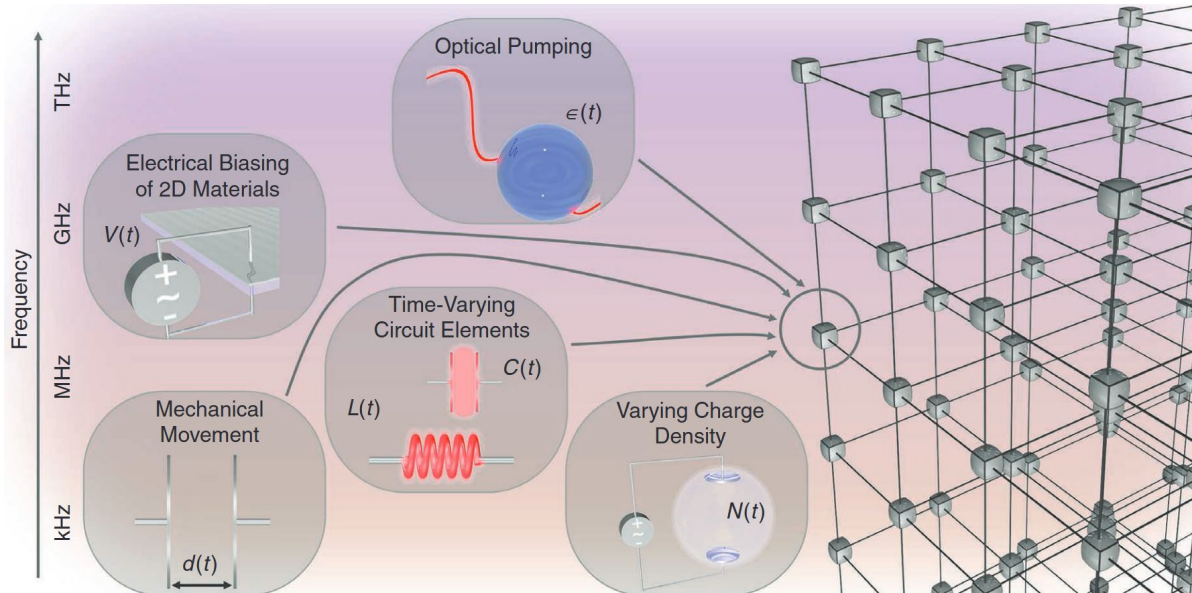


Figure 1-3 Tuning methods and their speed. Adapted from [63].

The suitability of a given tuning mechanism is closely related to the operating frequency range of the metasurface. Lumped electronic components, such as varactor and PIN diodes, are commonly employed at microwave and millimetre-wave frequencies, where circuit-based tuning remains effective, whereas optical and carrier-based tuning mechanisms are more suitable for infrared and optical metasurfaces due to fundamental scaling and integration constraints. Mechanical reconfiguration and phase-change materials can be applied across a wide frequency range, extending from the microwave to the infrared and optical regimes.

High-speed modulation is particularly critical for realising temporal control of metasurface parameters at frequencies comparable to those of optical electromagnetic waves. In this context, optical pumping of epsilon-near-zero (ENZ) materials is especially promising, as it can be used for realisation of time crystals at infrared wavelengths [32].

Nevertheless, even slower modulation rates, though well below the optical frequencies, can produce intriguing effects.

In this section, we focus on the manipulation of electromagnetic waves enabled by spatially tunable metasurfaces, without considering temporal modulation. Active spatial control of metasurface parameters provides powerful capabilities for tailoring the fundamental characteristics of electromagnetic waves. For many applications, achieving full control of phase, amplitude, or polarisation is more critical than ultrafast response. Accordingly, relatively slow tuning approaches remain highly attractive for practical implementations.

This thesis focuses on dynamic metasurfaces employing three principal tuning mechanisms: **mechanical reconfiguration**, **phase-change materials (PCMs)**, and **varactor diodes**. The remainder of this section reviews these approaches and highlights major advances achieved through their implementation in active metasurfaces.

Mechanical reconfiguration relies on the physical alteration of meta-atom geometry, relative spacing, or orientation. It is among the most direct and conceptually simple approaches to making metasurfaces tunable [64]. By changing structural degrees of freedom, mechanical actuation enables control over resonant frequencies, scattering characteristics, and near-field coupling, thereby allowing large spectral shifts, continuous phase tuning, and multifunctional switching.

A particularly powerful implementation of mechanical tunability is provided by microelectromechanical systems (MEMS), which integrate microactuators with metasurface elements to enable precise in-plane or out-of-plane displacements under electrical control [54]. MEMS-based metasurfaces combine relatively large tuning ranges with low optical loss and compatibility with planar fabrication. Both plasmonic and dielectric MEMS-tunable metasurfaces have demonstrated fast response, high tuning contrast, and strong potential for integration into miniaturised adaptive optical systems [53]. For example, MEMS-tunable plasmonic metasurfaces that modulate

nanoscale gap sizes enable efficient tuning of surface plasmon resonances and Fabry–Pérot modes within compact device footprints [65]. MEMS integration has also enabled ultrathin tunable absorbers with fast switching speeds and wide tuning ranges [66], as well as low-loss tunable dielectric metasurface lenses with reduced thickness and weight [67].

MEMS-based tuning typically operates on microsecond-to-millisecond time scales, depending on device geometry and actuation scheme. Faster operation can be achieved using small displacements and mechanically resonant structures. While microwave and millimetre-wave metasurfaces naturally scale to individually addressable elements, implementing per-pixel mechanical actuation at optical frequencies remains challenging due to wiring and routing constraints. Although repeated mechanical cycles can lead to deformations, MEMS devices can be designed to last for more than a million cycles through device encapsulation and material selection, which is critical to long-term performance.

PCMs form another major class of active media for reconfigurable metasurfaces [68]. Their key advantage lies in the large and reversible change in refractive index between distinct structural phases, which enables dynamic tuning of resonance frequencies, scattering amplitudes, and phase responses. PCMs support spatially localised switching, allowing metasurfaces to encode reconfigurable phase and amplitude distributions and to realise multiplexed functionalities within a single device platform.

Among PCMs, vanadium dioxide (VO_2) has emerged as the most extensively studied material for tunable metasurfaces. VO_2 undergoes a semiconductor–metal phase transition near 68 °C, accompanied by pronounced changes in permittivity and conductivity [55,57,69]. This transition can be triggered thermally, electrically via Joule heating, or optically through ultrafast photoexcitation. Optical excitation can induce switching on picosecond time scales, whereas thermal and electrical control typically operates on nanosecond-to-microsecond time scales, depending on heat dissipation and device design. Importantly, the phase transition occurs over a hysteretic

temperature window, enabling both binary and continuously tunable responses, and VO₂-based devices have demonstrated millions of reversible switching cycles without significant degradation.

In metasurface implementations, the VO₂ phase transition enables large modulation of conductivity and refractive index, supporting functionalities such as switchable broadband absorption, polarisation control, and beam steering[70,71]. Spatial patterning of VO₂ and localised temperature control across a metasurface allow the formation of reconfigurable phase gradients for dynamic beam steering[72]. VO₂ metasurfaces have also been proposed for adaptive radiative cooling, exploiting temperature-dependent emissivity modulation, as well as for dual-band thermal management using vertically aligned nanosheet architectures[73]. VO₂-based reconfigurable metasurfaces have been explored for analogue computing and image processing applications [74].

Varactor diodes, which act as voltage-dependent lumped capacitors, can be embedded in meta-atoms to enable continuous control of their local resonant response. By directly modifying the effective capacitance of each unit cell, varactor diodes enable smooth control of resonance frequency and scattering phase, making them particularly well suited to applications requiring continuous phase modulation in the microwave and millimetre-wave regimes [42,75,76]. Varactor integration has become a canonical approach for electronically reconfigurable metasurfaces [77].

Varactor diodes are based on reverse-biased pn-junctions, formed by two doped semiconductor regions, where voltage-controlled changes in the depletion region width modify the device capacitance. This mechanism enables fast electrical tuning, typically on sub-microsecond time scales. However, varactors and their bias networks introduce series resistance and parasitic inductance and capacitance, which can reduce efficiency and limit metasurface performance. Careful electromagnetic and circuit design is required to obtain large tuning while keeping insertion loss low and maintaining impedance matching. Furthermore, addressing large numbers of individually tunable

elements necessitates complex bias routing schemes that must be electromagnetically decoupled to avoid performance degradation. In addition, varactors have limited power handling due to their breakdown at high voltages.

Varactor-loaded metasurfaces have been central to the emergence of programmable and multifunctional metasurfaces in the microwave range. A foundational step was the introduction of the digital coding metamaterial concept, in which each unit cell can be switched between discrete reflection phase states under external bias control, effectively digitising wavefront manipulation [78]. Varactor-based metasurfaces have also enabled continuously tunable devices beyond simple phase control, including broadband polarisation rotators operating in transmission through anisotropic metasurface architectures [79].

Varactor-based metasurfaces remain the workhorse for electronically reconfigurable microwave and subterahertz devices due to their continuous tunability, fast response, and compatibility with printed circuit board technologies. However, their applicability diminishes at higher frequencies, where lumped elements become impractical. In many modern designs, varactors are combined with other tuning modalities to realise hybrid platforms in combination with PIN diodes [76,80] (based on the combination of one undoped intrinsic and two doped semiconducting materials) or optical control [81] that inherit the best properties of each approach. Varactor diodes have also emerged as a powerful platform for time-varying metasurfaces, which are discussed in the following section.

In summary, tunable metasurfaces have shown significant potential for transforming technologies from the microwave to the optical range. Among the various tuning strategies, MEMS, PCMs, and varactor diodes stand out as particularly promising approaches, enabling advanced control of electromagnetic waves and their fundamental properties. The advances achieved in enabling tunable metasurfaces provide a strong foundation for the development of time-varying designs.

1.3 Time-varying metasurfaces

1.3.1 Time-varying metasurfaces and their differences from nonlinear analogues

Time-varying media were first investigated more than fifty years ago [82–84] and have recently emerged as a highly promising area in metamaterials [20,85], driven by the development of materials capable of ultrafast parameter modulation. Figure 1-4 illustrates important functionalities enabled by time-varying metasurfaces [15]: frequency Doppler-like shifts, nonreciprocity and beam steering.

Control of the frequency of electromagnetic waves can only be achieved by changing the parameters of metasurfaces over time and remains unattainable for tunable metasurfaces. An equally significant feature of temporal modulation is its ability to inject energy into the system, enabling amplification. This process becomes possible when the modulation frequency exceeds that of the incident radiation [20].

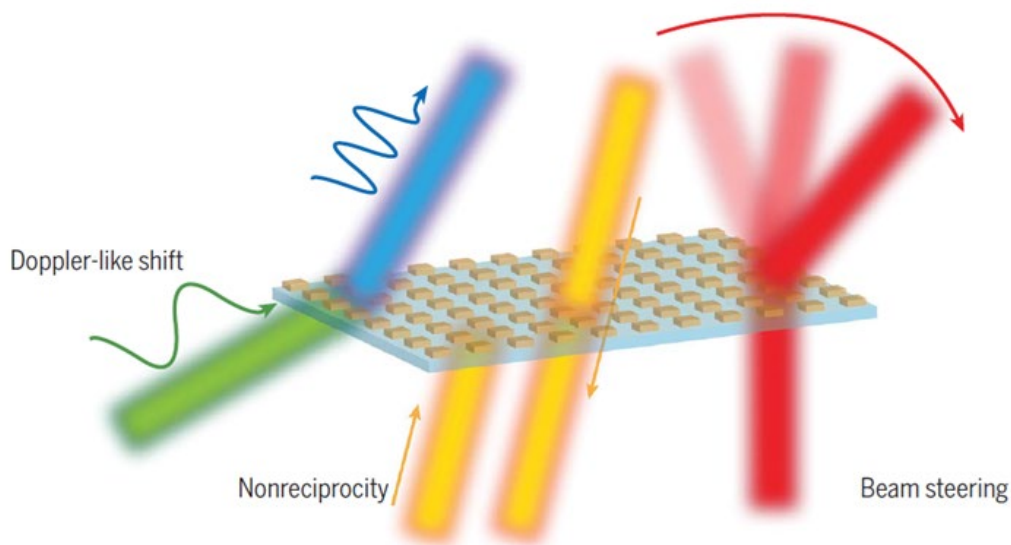


Figure 1-4 Key functionalities of time-varying metasurfaces. Adapted from [15].

Amplification and frequency conversion can be realised in combination with nonreciprocity [25], making the transmission of electromagnetic waves through time-varying media dependent on the direction of incidence. The breaking of time-reversal

symmetry in wave propagation using time-varying metasurfaces allow for realisation of nonreciprocity without requiring bulky magnets and magneto-optic materials.

In recent years, two complementary paradigms have been explored for enabling frequency conversion and amplification using metasurfaces: nonlinear, leveraging intrinsic material responses, and time-varying or parametric, utilising actively modulated system parameters. While both approaches rely on breaking the linear response of the medium, they are fundamentally distinct in their physical mechanisms and practical implications [86].

In nonlinear metasurfaces, frequency conversion arises from the intrinsic nonlinear polarisation of the constituent material [87]. When multiple waves interact with such a nonlinear medium, processes such as second-harmonic generation, sum- and difference-frequency generation, and four-wave mixing can occur [88]. In metasurfaces, strong local field confinement can significantly enhance these processes, enabling compact implementations of frequency mixers. However, the conversion efficiency remains inherently limited by the strength of material nonlinearities, and achieving efficient phase matching in ultrathin geometries poses an additional challenge. Moreover, relatively high input intensities are typically required to drive nonlinear responses, which restricts the use of such platforms in low-power and integrated applications.

In contrast, time-varying metasurfaces employ external modulation of their effective parameters, such as permittivity, permeability, capacitance or inductance, to achieve frequency conversion and amplification [20,85]. Here, the modulation injects energy into the system, enabling parametric amplification of a signal wave or the generation of sidebands. Time-varying metasurfaces can achieve strong amplification and efficient frequency translation without phase-matching constraints, and their tunability through the modulation waveform and frequency. Nevertheless, their implementation requires using active elements such as varactor diodes, semiconducting materials or conducting oxides, which significantly complicates the metasurface design.

While nonlinear metasurfaces offer compact and conceptually simple platforms for frequency mixing, their performance is fundamentally limited by material properties and power requirements. Time-varying metasurfaces, by contrast, provide a more flexible and powerful tool for amplification and frequency conversion, at the expense of greater fabrication complexity and dependence on external modulation of metasurface parameters.

Amplification and frequency conversion using time-varying metasurfaces have recently attracted significant attention owing to advances in ultrafast modulation techniques, which hold promise for their implementation at optical frequencies. Moreover, since temporal modulation at rates comparable to the frequency of electromagnetic waves in the microwave and terahertz domains is feasible with current technologies, this allows researchers to experimentally demonstrate functionalities that are only beginning to emerge at higher frequencies through the development of novel materials. Next, we review recent research on frequency conversion and amplification using time-varying metasurfaces, as the second half of this thesis focuses on these phenomena and their interplay.

1.3.2 Time-varying metasurfaces for frequency conversion

Frequency conversion represents one of the most fundamental functionalities enabled by time-varying metasurfaces, arising from temporal modulation of their electromagnetic parameters allowing to redistribute electromagnetic energy among different frequency components and enable nonreciprocal wave manipulation. In the following subsections, we review theoretical and experimental advances in frequency conversion using time-varying metasurfaces.

1.3.2.1 Theoretical predictions of frequency conversion using time-varying metasurfaces

Frequency conversion enabled by temporal modulation provided a route to nonreciprocity, generating significant interest in time-varying metasurfaces within the research community. A method for breaking Lorentz reciprocity using time-varying

metasurfaces was first introduced in 2015 [23]. Temporally modulated metasurfaces that enable frequency conversion for obliquely incident waves were shown to support the design of optical isolators without relying on magnetic materials. By introducing a time-gradient phase discontinuity, Snell's law was generalised to a nonreciprocal form, allowing asymmetric light propagation. This generalisation arises from the temporal modulation of the metasurface, which mediates energy exchange between the incident photons and the modulated structure, resulting in frequency shifts.

Plasma and metamaterials based on SRRs represent fundamental platforms for time-varying media, where modulation of the plasma frequency and SRR resonance frequency leads to time-dependent permittivity and permeability, respectively. Dynamically modulated homogeneous media with refractive index decreasing over time, such as ionised plasma with a decreasing dielectric permittivity or metamaterials composed of SRRs exhibiting a decreasing effective magnetic permeability, enable frequency up-conversion of electromagnetic waves [89]. However, it was analytically demonstrated that the fundamental limit of conversion efficiency in these systems cannot exceed 50% due to energy coupling to low-frequency modes.

Beyond these fundamental platforms, graphene and silicon were subsequently investigated as practical material systems for implementing temporally modulated media, leveraging their established roles in tunable photonic and metasurface-based devices.

Temporal modulation of the Fermi level in a graphene ribbon array enabled frequency comb generation in the terahertz range [90]. By adjusting the modulation frequency, the spacing and frequencies of the generated comb lines were precisely controlled. Notably, it was demonstrated that the required modulation amplitude is three orders of magnitude smaller than that needed for comparable frequency comb generation in modulated nonresonant screens, highlighting the efficiency gained from exploiting plasmonic resonances. Moreover, the proposed tunable graphene grating can operate effectively across the mid- to far-infrared spectrum due to its geometrical and material properties.

This approach can be extended to time-modulated cavities and other photonic structures, paving the way for advancements in dynamic photonic devices and time-varying metasurface design.

Electrically modulated all-dielectric metasurfaces based on one-dimensional arrays of silicon nanobars supporting guided-mode resonances can enable full phase control in the infrared range [91]. The proposed design relies on carrier injection and consequent refractive index modulation, achieved by applying voltage to silicon nanobars with PIN junctions formed through selective doping. Fast tuning enables temporal modulation of the metasurface parameters, allowing frequency conversion and related functionalities. Moreover, optimised temporal modulation waveforms enable generation of the desired harmonic spectrum, making this metasurface concept particularly suitable for multifunctional electromagnetic wave manipulation, including pure frequency mixing, dual-frequency generation in transmission, focusing and ultrafast beam scanning.

While early theoretical studies established the fundamental principles of temporal frequency conversion, later works began identifying material platforms suitable for experimental implementation. The following subsections review experimental realisations of time-varying metasurfaces for frequency conversion.

1.3.2.2 Varactor-based time-varying metasurfaces

Due to advantages such as PCB compatibility and straightforward electrical tuning, varactor-diode modulation has become one of the most widely adopted approaches for experimental implementation of time-varying metasurfaces in the microwave range. The following works demonstrate how such platforms enable controllable frequency conversion.

A mixer-duplexer-antenna system based on periodic spatiotemporal modulation in a leaky-wave structure similar to metasurfaces can function as a full transceiver, simultaneously enabling frequency conversion, duplexing via magnet-free nonreciprocity, and directional radiation of electromagnetic waves [92]. The system was

implemented in the microwave range using a microstrip line loaded with modulated varactor diodes, which dynamically alter the effective permittivity to achieve space-time modulation. Experimentally, the system demonstrated key functionalities of time-varying metasurfaces, including beam steering achieved by tuning the modulation frequency.

The scattering of electromagnetic waves can be precisely controlled by independently modulating the electric and magnetic polarisations of metasurfaces over time [93]. Time-varying metasurfaces enable nearly arbitrary control over the amplitudes and phases of generated parametric waves. Since this control can be applied locally to each meta-atom, it allows for the generation of waves possessing spatial phase gradients, leading to controlled steering, focusing, or defocusing of electromagnetic fields. Furthermore, complex phase gradients enable the metasurface to perform multiple functions simultaneously for different sidebands. The experimentally realised parametric metasurfaces achieved a high frequency conversion efficiency of approximately 75%, confirming the effectiveness of this approach.

Digital coding metasurfaces based on varactor-loaded patches provide another route to programmable frequency conversion. Time-domain digital coding metasurfaces consisting of metallic patches connected via varactor diodes enable independent phase control of each meta-atom via applied bias voltages [94]. Periodic temporal modulation of the reflection coefficient and precise time delays between modulation functions in different meta-atoms enable manipulation of both the intensity and phase of generated harmonics in the microwave range. Programmable time-domain digital-coding metasurfaces allow independent control of the amplitude and phase of generated harmonics by dynamically modulating the reflection coefficient of each meta-atom via applied voltages to varactors [95]. This functionality can be realised through a field-programmable gate array (FPGA), which governs spatiotemporal coding patterns over the metasurface to shape the harmonic spectrum. Precise manipulation of different harmonics was achieved through specially coded sequences of control signals applied to varactors, offering flexible spectrum engineering.

A closely related class of works exploits engineered temporal phase progression in varactor-based metasurfaces to achieve spurious-free frequency translation. Time-varying metasurfaces based on electrical tuning of varactor diodes and a sawtooth temporal modulation waveform provided a platform for the experimental demonstration of full phase control of transmitted waves and serrodyne frequency translation with conversion loss of 5 dB and sideband suppression near 12 dB in the microwave range [96]. Moreover, cascading two such metasurfaces with opposite modulation directions enables non-magnetic nonreciprocity. Time-varying metasurfaces can achieve controllable frequency shifts in reflection by modulating the effective surface admittance through electrically tunable phase delays [97]. A dynamic high-impedance surface composed of varactor-loaded mushroom-like metallic patches, providing appropriate electrical modulation of metasurface parameters, was proposed for enabling up- and down-conversion of frequencies. Transmissive time-varying metasurfaces consisting of metallic patch resonators, interconnected through varactor diodes whose capacitance is temporally modulated, achieve linear and spurious-free frequency conversion in the microwave regime [98]. Precise temporal phase progression, enabling efficient power transfer into a single target harmonic while fully suppressing spurious sidebands, was realised using the surface-interconnector-phaser-surface architecture, ensuring high conversion efficiency within a thin and compact platform.

Finally, spatiotemporally modulated reconfigurable metamaterials based on spiral resonators electrically controlled through varactor-loaded lumped circuits achieve nonreciprocal transmission and frequency conversion of magneto-inductive waves at radio frequencies [99]. Tunable cavity-mode waveguides, strong nonreciprocity, and frequency conversion with inherent suppression of unwanted harmonics enabled by the narrowband cavity resonances were experimentally demonstrated within this deep-subwavelength platform by combining spatial and temporal modulation.

These electrically modulated implementations established practical routes to frequency conversion using time-varying metasurfaces in the microwave regime, laying the

groundwork for subsequent investigations of amplification and higher-frequency implementations.

1.3.2.3 Time-varying metasurfaces based on PIN diodes and RF switches

Electrical modulation of metasurface parameters can be realised using PIN diodes and RF switches, which, like varactor diodes, are compatible with PCB-based implementations. While PIN diodes have been employed for camouflaging realised through frequency conversion, electrically controlled RF switches have enabled the realisation of photonic time interfaces at microwave frequencies.

Time-varying metasurfaces were also demonstrated for providing a π phase shift across a broad frequency range in the microwave domain, offering a new platform for spectral camouflage [100]. The proposed metasurface consists of metallic resonators with embedded PIN diodes that are modulated by applied voltages, enabling controlled switching between two resonance modes. Quasi-random time modulation can spread an incident narrowband signal into a broadband, noise-like spectrum, effectively masking its spectral signature and reducing reflected power by about 10 dB across a wide frequency band and range of incidence angles. Furthermore, spatiotemporal modulation of the metasurface parameters suppresses harmonic scattering in the direction of incidence.

Time-modulated metasurfaces can perform spectrum spreading of incident waves to achieve selective camouflaging [101]. Switching the metasurface between perfect electric conductor (PEC) and perfect magnetic conductor (PMC) reflection states through electrical modulation of PIN diodes embedded in an array of interconnected metallic patches and applying pseudorandom temporal modulation sequences enables conversion of narrowband incident signals into a broadband noise-like spectrum, redistributing power across numerous harmonics. Importantly, the camouflaging can be programmable and selective, masking only signals encoded with the chosen modulation sequence. The effectiveness of this approach, showcasing strong interference immunity

and reliable spectral camouflage, was demonstrated in simulations and microwave experiments.

Microwave transmission-line metamaterials have recently enabled the experimental demonstration of temporal reflection and broadband frequency translation [31]. Uniform and abrupt modulation of the effective capacitance through a synchronised array of voltage-controlled switches, in which the system parameters change faster than the period of the propagating wave, enabled the implementation of photonic time interfaces. At such interfaces, a portion of the input signal is time-reversed, while momentum is conserved and the frequency spectrum is shifted. Cascading two such temporal boundaries into a temporal slab creates interference between temporally refracted and reflected waves, producing a temporal analogue of the Fabry-Pérot effect.

These electrically modulated implementations demonstrated the versatility of time-varying metasurfaces in the microwave regime and highlighted the potential for leveraging ultrafast modulation techniques and novel materials to extend these concepts toward higher-frequency applications.

1.3.2.4 Time-varying metasurfaces based on ultrafast optical excitation

Optical pumping has been applied to realise temporal interfaces through ultrafast refractive index changes, enabling frequency conversion at terahertz and infrared frequencies.

A time-variant metasurface composed of SRRs was experimentally demonstrated to achieve frequency conversion of terahertz pulses [102]. The proposed metasurface exhibits two distinct resonances that, upon ultrafast optical excitation suddenly increasing the conductivity of the gallium arsenide (GaAs) region separating the SRRs, merge into a single resonance mode, thereby forming a temporal boundary. This sudden merging leads to the radiation of new frequency components, which was experimentally confirmed when the temporal boundary overlapped with the incident terahertz pulse. Moreover, full phase control over the converted waves was demonstrated, enabling

beam steering and focusing by introducing spatial variations in the time delay between the optical pump and terahertz probe pulses across the metasurface.

Frequency conversion in the near-infrared range was realised through all-optical temporal modulation of dielectric metasurfaces, where femtosecond pump pulses generate free carriers in high-Q GaAs nanoresonators, producing ultrafast refractive index changes [103]. A resonant wavelength shift of 10 nanometres over a 30-nanometre bandwidth under moderate optical pump intensities was demonstrated experimentally, confirming the developed time-dependent coupled-mode theory. As one of the first experimental demonstrations of frequency conversion in time-variant dielectric metasurfaces, this research established a foundation for extending time-modulation concepts to high-frequency photonics.

Time refraction was experimentally realised via single-cycle refractive index modulation in ENZ materials [32]. However, this demonstration required intense, ultrafast laser pulses for abrupt modulation of the refractive index in transparent conducting oxides such as indium tin oxide (ITO), producing time interfaces in the infrared range. Such rapid modulation produces an immediate red shift of the probe frequency spectrum during the index rise, followed by a blue shift as the index relaxes, achieved within a single optical cycle. Notably, refractive index changes of up to 0.5 were observed in ITO with rise times of only a few femtoseconds and relaxation times of about 10-20 femtoseconds, allowing reproducible modulation over multiple optical cycles.

These works demonstrate the potential of optical pumping for enabling ultrafast refractive index changes, paving the way for photonic time crystals, parametric optical phenomena, and amplification at terahertz and infrared frequencies, and indicating viable routes toward their implementation at visible wavelengths.

1.3.3 Time-varying metasurfaces for amplification

Beyond frequency conversion, time-varying metasurfaces can enable electromagnetic wave amplification by allowing energy transfer from the modulation process to the

propagating waves. Such active functionalities are of particular interest for overcoming the limitations of passive and nonlinear metasurfaces, and for enabling compact and reconfigurable sources and amplifiers. In this section, we review theoretical concepts and experimental demonstrations of amplification using time-varying metasurfaces.

1.3.3.1 Theoretical predictions of amplification using time-varying metasurfaces

Recent advances in ultrafast modulation techniques, particularly in materials such as ITO, have enabled refractive index changes on sub-optical-cycle timescales, motivating exploration of time-varying media at optical frequencies. Among the emerging functionalities, electromagnetic wave amplification has become a particularly compelling research direction.

Parity-time (PT) symmetric time-Floquet systems with periodic modulation of refractive index at twice the signal frequency were theoretically shown to enable parametric gain and exotic scattering responses [104]. Combining multiple parametrically pumped slabs with out-of-phase modulations yields PT-symmetric time-Floquet systems exhibiting coherent perfect absorption, anisotropic transmission resonances, and bidirectional invisibility, thereby enabling amplification without introducing unwanted reflections. Time-periodic modulation in these systems provides new avenues for wave manipulation beyond conventional static PT-symmetric structures.

Luminal metamaterials, where a travelling-wave modulation in permittivity moves at a velocity close to the wave speed in the medium, were proposed for broadband nonreciprocal amplification [105]. It was shown that modulation of acoustic plasmons in double-layer graphene enables strong wave amplification while breaking time-reversal symmetry, a key requirement for nonreciprocal devices. Unlike conventional parametric systems, which require high-frequency pumping and rapid modulation to achieve substantial amplification, luminal metamaterials operate efficiently at much lower modulation speeds. This key distinction makes them particularly attractive for practical implementations, where generating high-speed modulations can be technologically challenging. Additionally, such structures can enable harmonic generation. This novel

mechanism for nonreciprocal amplification with relaxed modulation speed requirements provides a foundation for the development of next-generation isolators, amplifiers, and frequency converters based on space-time modulated metamaterials. Periodic space-time modulation of permittivity and permeability at luminal velocities can lead to wave amplification without relying on energy transfer between distinct frequency modes [106,107]. This approach exploits compression of electromagnetic field lines in a luminally modulated medium, leading to the formation of amplified pulsed waves and offering an alternative route to achieving gain in photonic and electromagnetic systems.

Similar to parametric amplifiers, PTC enable wave amplification through temporal modulation [108]. They offer the distinct advantage of producing gain without relying on resonant conditions. Radiation within the broad momentum bandgap of a PTC, generated by embedded sources such as atoms or oscillators, can be exponentially amplified and acquire coherence due to the time-periodic modulation, which under certain conditions leads to the formation of pulsed emission. Non-resonant lasers based on PTCs can be realised by placing a temporally modulated dielectric medium between two mirrors, opening prospects for compact and tunable light sources. Introducing resonances into PTCs can expand their momentum bandgaps and enhance amplification [109]. High-quality resonant responses in time-varying metasurfaces reduce the required modulation amplitudes, which are difficult to achieve at high frequencies without powerful pump sources. Moreover, resonances enable broader bandgaps and more efficient amplification, providing a pathway toward the realisation of PTC and light amplification at optical frequencies.

Subwavelength spherical particles with dielectric permittivity modulated at twice the frequency of the incident light enable parametric amplification [110]. Such time-modulated scatterers can provide directional amplification via excitation of parametric Mie resonances, enhancing radiation in specific directions while suppressing it in others. Notably, a temporally modulated sphere exhibiting zero forward scattering allows for the detection of incident light through side-scattered signals. These findings pave the way for the development of parametric metasurfaces based on time-varying scatterers, offering

promising applications in the amplification and detection of extremely weak optical signals.

Time-dependent media was proposed for controlled compression, amplification, and frequency up-conversion of optical pulses [111]. Molecular modulators, such as impulsively aligned diatomic molecules whose rotational dynamics produce periodic revivals of molecular alignment, enable temporal modulation of refractive index over tens of picoseconds. This modulation allows for exponential transformation of a probe pulse's amplitude, frequency, and duration, while conserving photon number. Notably, this mechanism requiring only minimal changes in the refractive index enabled conversion of near-infrared input pulses centred at 1 micrometre with durations of about 30 femtoseconds into amplified ultrashort pulses at 550 nanometres with durations of approximately 20 femtoseconds. These findings not only support recent theoretical predictions on gain in time-dependent media but also extend them by accounting for medium dispersion and demonstrating concrete pulse transformations under standard ambient conditions, highlighting practical applications in ultrafast optics.

These theoretical works provided key insights into amplification mechanisms in time-varying media, directly motivating the experimental amplification platforms discussed in the following subsections.

1.3.3.2 Varactor-based time-varying metasurfaces

Electrically modulated metasurfaces demonstrate feasibility of electromagnetic wave amplification and experimental realisation of PTC in the microwave regime.

Virtually rotating metasurfaces (VRM) were proposed for frequency conversion and parametric amplification of electromagnetic waves [112]. These time-varying metasurfaces exhibit a temporally modulated surface susceptibility that emulates a rotating anisotropic response and can be realised using electrically tunable elements. When a circularly polarised wave is incident on a VRM, the scattered wave contains both the original and a frequency-converted component separated by twice the rotation

frequency. VRM-metal screen systems enable complete transfer of energy to either an up- or down-converted frequency. In the case of opposite rotation between the incident wave and the metasurface, the up-converted component experiences parametric amplification, unlike conventional parametric amplifiers that require pump frequencies exceeding the signal frequency. These findings highlighted the potential of VRM for high-efficiency frequency conversion, amplification, and nonreciprocal wave control in microwave and photonic applications.

Unlike conventional methods that rely on spatiotemporal modulations and cascaded phase modulators or frequency converters, bianisotropic systems can break reciprocity using solely temporal modulation [113]. The key mechanism behind this functionality is the temporal modulation of spatial dispersion effects, which had not been accounted for in prior studies. A single-layer metasurface with a temporally modulated capacitive layer on a dielectric substrate was proposed as a practical implementation of such a bianisotropic time-modulated medium. The resulting metasurface exhibits unidirectional amplification and phase-insensitive isolation behaviour while adhering to a generalised time-reversal symmetry and achieving strong nonreciprocal transmission, thereby providing a new platform for magnet-free nonreciprocal devices.

A metasurface-based PTC formed by a periodic array of metallic patches connected via varactor diodes and dynamically modulated surface impedance was experimentally demonstrated in the microwave range [114]. The time-varying metasurface exhibits a momentum bandgap shared by both surface and free-space electromagnetic waves. Exponential amplification of surface waves within this bandgap was observed experimentally, reaching gains of up to 25 dB at a modulation frequency twice the surface wave frequency. In addition, frequency up-conversion of incident free-space waves upon reflection was demonstrated within the momentum bandgap using a modulation frequency approximately two-thirds of the incident wave frequency.

These electrically modulated implementations establish time-varying metasurfaces as a promising platform for amplification and photonic time crystals in the microwave range,

motivating further research into ultrafast modulation mechanisms for implementing compact amplifiers at higher frequencies.

1.3.3.3 Time-varying metasurfaces based on integrated amplifiers

A more direct approach to electromagnetic wave amplification at microwave frequencies, compatible with PCB technology, is the integration of electrically controlled power amplifiers into metasurfaces.

Spatial-energy digital-coding metasurfaces integrated with active amplifiers were proposed to enable arbitrary control over the energy of spatially propagating electromagnetic waves [115]. The metasurface incorporating amplifier chips for energy modulation in the microwave frequency range demonstrated its effectiveness in both simulations and experiments. A fabricated coding metasurface equipped with nearly one hundred integrated amplifiers achieved tunable energy levels ranging from -10 to 20 dB, confirming the feasibility of programmable energy manipulation. These results highlight the potential for integrating metasurfaces with conventional radio-frequency components, paving the way for advanced applications in active and programmable wave control.

Amplifier-based transmissive space-time-coding metasurfaces achieved strong nonlinear control over electromagnetic waves in both space and frequency domains [116]. Unilateral power amplifiers between patches on both sides of the time-varying metasurface enabled forward transmission while effectively blocking backward propagation, thereby breaking Lorentz reciprocity. The metasurface's unit cells can be dynamically controlled using a FPGA, which switches the amplifiers between active and inactive states in both spatial and temporal dimensions. This configuration allows programmable manipulation of harmonic transmission coefficients, enabling precise control over the amplitude and phase of scattered harmonics.

These works demonstrate the potential of integrating power amplifiers into metasurfaces for active wave manipulation in the microwave regime. However, electrical tuning

approaches based on lumped circuit and active electronic components such as varactors and power amplifiers become impractical at higher frequencies, motivating the exploration of ultrafast optical excitation discussed in the next subsection.

1.3.3.4 Time-varying metasurfaces based on ultrafast optical excitation

Optical pumping offers a viable route toward amplification of electromagnetic waves at high frequencies, as it enables modulation on optical cycle timescales while avoiding embedded electrical circuitry in deeply subwavelength meta-atoms. Time boundaries implemented using Huygens' metasurfaces optically pumped by femtosecond near-infrared pulses enabled experimental observation of terahertz wave amplification [117]. The proposed time-varying metasurface consists of silicon resonators supporting degenerate electric and magnetic modes that yield broadband transmission close to unity in the static case. Ultrafast optical excitation via free-carrier injection breaks this degeneracy and creates a temporal boundary that induces rapid resonance switching. As a result, a transmission gain exceeding 20% was experimentally demonstrated at selected frequencies, with amplification controllable through adjustment of the pump–probe delay and persisting for up to 35 picoseconds due to the high quality factor (Q-factor) of the metasurface. These experimental results, supported by numerical simulations and time-dependent coupled-mode theory, confirm that the observed gain originates from the dual-mode resonant response of the time-varying Huygens' metasurface. This work provides a fundamental demonstration of metasurface-based temporal amplification in the terahertz regime and establishes a practical foundation for extending time-modulated metasurface concepts toward photonic time crystals and electromagnetic wave amplification at optical frequencies.

1.3.4 Summary on time-varying metasurfaces

Overall, recent advances in time-varying metasurfaces have established them as powerful platforms for dynamic wave manipulation, nonreciprocity, frequency conversion and amplification. While temporal modulation of metasurface parameters can generate harmonics and frequency shifts, precise modulation schemes enable full

control over harmonic amplitudes and phases. Recent research efforts have pushed these concepts toward amplification, where spatiotemporal modulation and resonant enhancement allow electromagnetic waves to efficiently extract energy from the modulation process. These results show that time-varying metasurfaces can not only convert and redistribute spectral content with high flexibility but also overcome the fundamental limitations of passive electromagnetic systems by providing controllable gain, paving the way for applications ranging from wireless communications to compact light sources.

1.4 Thesis overview

Over the past two decades, there has been rapid progress in dynamic metamaterials and metasurfaces, where spatial and temporal modulation enables complete control over electromagnetic waves and their fundamental properties. This thesis investigates multiple tuning mechanisms that enable dynamic metasurfaces to control the frequency and amplitude of their resonant responses across various spectral ranges, supporting functionalities such as tunable filtering, modulation, frequency conversion, and amplification. While the first half of this thesis focuses on tunable metasurfaces realised through the integration of MEMS and PCMs, the second half explores time-varying metasurfaces based on electrical modulation of embedded varactor diodes.

Chapter 2 is dedicated to mechanically reconfigurable metasurfaces for spectrally tunable filtering in the infrared range. The chapter opens with an introduction to mechanical tuning approaches and the EOT phenomenon. It then examines how the integration of MEMS into plasmonic metasurfaces supporting EOT resonances enables broadband tunability across the long-wavelength infrared (LWIR) range and discusses the potential of such devices for advancing hyperspectral infrared imaging. Furthermore, a concept for metasurface filters based on EOT integrated on the tip of double-clad optical fibres is proposed for fluorescence fibre-optic imaging. This approach enables selective collection of fluorescence signals from biological samples while suppressing unwanted laser and optical coherence tomography (OCT) beams. Finally, mechanical

tuning strategies are explored for controlling high-quality quasi-BIC (q-BIC) resonances. A MEMS-enabled hybridisation of 1D and 2D all-dielectric metasurfaces is proposed for spectral tuning of ultranarrow q-BIC resonances in transmission, an important functionality for infrared imaging, sensing, and spectroscopy applications.

Chapter 3 focuses on tunable toroidal metasurfaces that employ the phase-change material VO_2 for applications in terahertz bolometric detectors and modulators. The study investigates q-BIC resonances supported by such metasurfaces and their sensitivity to temperature-induced phase transitions in thin VO_2 film patches integrated with metallic resonators. Structural asymmetry is introduced to enhance tunability, enabling precise control and significant modulation of both the amplitude and frequency of q-BIC resonances with temperature variations of only 10 °C. The contribution of different multipoles to the metasurface response is analysed, revealing the dominant role of the toroidal dipole in resonance excitation. Multipole expansion methods are used to reconstruct the metasurface characteristics, confirming the accuracy of the analytical approach and providing deeper insight into the underlying physics of the observed resonances.

Chapter 4 presents research on amplification and frequency conversion of free-space electromagnetic waves using parametric metasurfaces. An analytical model is developed to describe electromagnetic wave amplification by a time-varying metasurface composed of SRRs with variable capacitances. The model is validated through numerical simulations, demonstrating excellent agreement between analytical and computational results. The study shows that the proposed parametric metasurfaces can achieve significant gain in various operating modes. Moreover, a dual-resonant parametric metasurface design consisting of connected SRRs with embedded varactors is introduced to enable amplified up-conversion of free-space electromagnetic waves. The analysis shows that such time-varying metasurfaces achieve high up-conversion efficiency, surpassing conventional Manley-Rowe limits through cascaded amplification. These theoretical predictions are experimentally verified using the proposed parametric

meta-atoms in a transverse electromagnetic (TEM) cell, confirming substantial gain across different operating regimes.

Finally, **Chapter 5** summarises the research results, outlines the key conclusions, and discusses promising future directions in the field of dynamic metasurfaces. The studies presented in this thesis establish a comprehensive framework for achieving electromagnetic wave manipulation, amplification, and frequency conversion using tunable and time-varying metasurfaces, highlighting pathways toward next-generation active photonic and electromagnetic platforms.

Chapter 2: Mechanical tuning of resonant responses in metasurfaces

In this chapter, we examine the mechanical tuning of metasurfaces, with a particular focus on MEMS-enabled control of their resonant responses. We begin with an overview of mechanically tunable metasurfaces based on MEMS, outlining their operating principles, applications, advantages, and limitations. Next, we discuss the theory and applications of EOT and present spectrally tunable metasurface filters for hyperspectral infrared imaging, which achieve control over EOT through the integration of patterned thin metallic films with MEMS membranes. We then extend this research to biomedical fibre-optic technologies by introducing a plasmonic metasurface filter designed for fluorescence imaging. Finally, we introduce the concept of mechanical tuning of BICs to achieve tunable ultranarrow spectral filtering, a highly promising approach for spectroscopic applications.

Results presented in this chapter are partially included in the published article listed below, where I am the first author and my contribution include formulating the initial idea, developing the theoretical framework, performing numerical modelling, designing the metasurfaces, drafting the manuscript, and preparing its final version:

F. Kovalev, M. Martyniuk, A. Miroshnichenko, and I. Shadrivov, Tunable bound states in the continuum through hybridization of 1D and 2D metasurfaces, *Nanophotonics* **14**, 4973–4981 (2025).

Additionally, findings presented in this chapter are partially included in the published article listed below, where I am the second author and my contribution is in theoretical aspects, numerical simulations, metasurface design and writing of the corresponding section of the paper:

O. Bannik, **F. Kovalev**, M. Liu, M. Zawierta, D. Silva, L. Faraone, M. Martyniuk, and I. Shadrivov, Micromachined Double-Membrane Mechanically Tunable Metamaterial for Thermal Infrared Filtering, *Advanced Photonics Research* **6**, 2400185 (2025).

2.1 Mechanically tunable metasurfaces

One of the most straightforward ways to design tunable metasurfaces is to mechanically alter their structure and geometry. A key advantage of mechanically tunable metasurfaces lies in their broad material applicability [64]. Unlike tunable metasurfaces that require specialised materials (e.g., semiconductors, liquid crystals, phase-change materials), mechanically reconfigurable metasurfaces utilise widely available materials. This flexibility allows for compatibility with various fabrication techniques and environmental conditions, from flexible and stretchable electronics to rigid and robust optical systems.

Structural changes in the meta-atoms of mechanically tunable metasurfaces occur through applying various external stimuli, including physical stretching, electrostatic forces, thermal effects, optical excitation, magnetic fields, pneumatic pressure, and microfluidic flows [53,64,118]. This wide range of external stimuli enhances the versatility of mechanically reconfigurable metasurfaces, enabling their use in diverse environmental conditions and garnering interest from researchers across a broad spectrum of disciplines.

Among the various approaches to mechanical tuning, electrostatic actuation emerges as a key method for dynamically reconfiguring structures by applying a voltage input. Electrostatic actuation relies on the attraction between oppositely charged electrodes to induce motion in the system. The motion enables altering the morphology or adjusting the distances between different parts of meta-atoms. The use of MEMS based on the electrostatic actuation in metasurfaces provides several advantages, including low power consumption, relatively fast operation speed, long-term repeatability, a wide range of operational frequencies, and compatibility, which facilitates integration with existing technologies. Additionally, electrostatic actuation operates without special requirements on the use environment, making it versatile for a variety of applications [64]. Nanoelectromechanical systems (NEMS) enable metasurfaces to achieve reconfigurable functions in the visible range, opening new opportunities for

active photonics and the development of compact, tunable optical components [64,118].

Despite its advantages, electrostatic actuation has inherent limits. Typically, the tuning range depends on both the applied voltage and the metasurface design. Moreover, electrostatic actuation offers a limited capacity to tune different meta-atoms separately. Additionally, achieving the desired functionality often necessitates complex structural systems, which makes design and fabrication more complex. Despite these limitations, electrostatic actuation remains a widely used and effective method for tuning the metasurface response.

Mechanically tunable metasurfaces offer diverse applications, leveraging their ability to dynamically adjust optical properties through physical reconfiguration. In beam focusing and steering, they enable precise control of light direction and intensity, which remains critical for light detection and ranging (LIDAR) systems, optical communication, and augmented reality technologies. For holographic imaging, mechanically tunable metasurfaces facilitate dynamic and reconfigurable holograms for advanced display and data storage applications. Their capability for polarisation conversion and optical switching forms a foundation for use in photonic circuits and dynamic optical networks. In tunable filtering, mechanically reconfigurable metasurfaces enable real-time wavelength selection, making them invaluable in spectrometry and remote sensing. Additionally, they enhance sensitivity and selectivity in sensing and detection.

The ability of mechanically tunable metasurfaces to control optical properties across diverse applications is fundamentally linked to their underlying resonant behaviour. By manipulating resonances, metasurfaces achieve precise control over light-matter interactions, enabling functionalities such as filtering and wavefront shaping. Understanding the physics behind different resonant phenomena is crucial for developing effective tuning strategies. In the following sections, we examine two types of resonant responses that arise in metasurfaces under incident radiation, EOT and q-BIC, and propose approaches for their mechanical control.

2.2 Tunable filtering based on extraordinary optical transmission

2.2.1 *Extraordinary optical transmission and its applications*

The phenomenon of EOT describes the enhanced transmission of light through subwavelength apertures in an opaque metallic film. This effect, which defies classical subwavelength aperture theory, was first systematically reported in 1998 [119]. Transmission of light through periodic arrays of subwavelength holes significantly exceeds predictions from the Bethe theory of diffraction, which dictates that it should rapidly diminish as the aperture size decreases below the wavelength of light [120]. This discovery sparked extensive research into the underlying physics and potential applications of EOT.

The key mechanism behind EOT arises from the interaction between incident light and surface plasmon polaritons (SPPs), coherent electron oscillations at the metal-dielectric interface, induced by the periodic arrangement of the apertures. SPPs mediate the coupling of incident light to subwavelength apertures, effectively enhancing transmission. In particular, the periodic arrangement of holes governs resonant conditions, where a collective effect related to an electromagnetic coupling between holes via SPPs leads to peaks in transmission spectra at specific wavelengths. These resonances depend on the array geometry, material properties, and the surrounding dielectric environment. Figure 2-1 illustrates the canonical system for EOT containing a metallic hole array deposited on a dielectric substrate which is illuminated with light and its important parameters: D is the array periodicity, d is the hole diameter, and h is the metal film thickness.

There are two primary mechanisms underlying EOT: surface plasmon resonances and localised waveguide modes. Surface plasmon resonances arise from the collective oscillations of conduction electrons at the metal-dielectric interface. They are highly sensitive to the periodicity of the aperture array, the dielectric environment, and the polarisation of incident light. At resonance, incident photons couple efficiently with SPPs, facilitating enhanced transmission. Subwavelength apertures function as

waveguides, supporting resonant modes when their dimensions align with specific wavelengths. These modes enhance the electromagnetic fields within the apertures, enabling efficient light coupling and transmission that exceeds classical expectations.

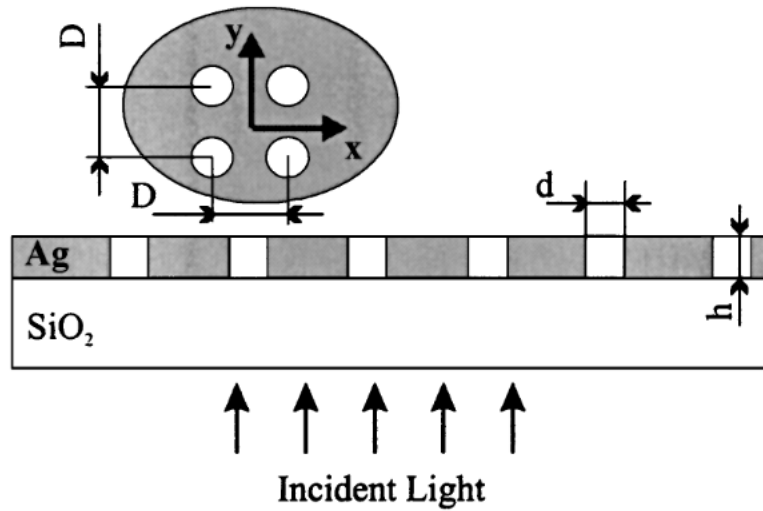


Figure 2-1 The geometry of a canonical system for EOT consisting of a metallic hole array deposited on a dielectric substrate which is illuminated with light.

Adapted from [121].

Several resonant conditions for SPP excitation in such systems for EOT must be satisfied such as a compensation of the difference of the wave vectors of SPPs and incident light by the diffraction on a periodic surface array, support of the corresponding Bloch-type waves at a two-dimensional periodic structure by a metal surface and a certain polarisation of the incident light. Additionally, coupling between the SPPs at interfaces and radiation damping due to SPP scattering on the holes must also be considered [121]. Moreover, the microscopic theoretical approach elucidated the critical role of localised electromagnetic fields within and around the apertures, showing that transmission peaks directly result from the excitation of resonant modes within the holes and their interaction with SPPs [122]; therefore, highlighting the complex nature of the EOT phenomenon [123–127].

This phenomenon finds applications in diverse fields, including sensing, energy conversion, sub-diffraction-limited optics and filtering [128,129]. EOT-based sensors exploit the high sensitivity of transmission spectra to changes in the refractive index of

the surrounding medium, enabling the detection of molecular binding events with high precision. Nanohole arrays serve as plasmonic lenses, concentrating optical energy and enabling sub-diffraction imaging and enhanced field localisation useful for focusing light [130].

Despite its promise, the practical implementation of EOT faces several challenges at optical frequencies. The most important among these is the inherent loss associated with plasmonic materials, which limits transmission efficiency and device performance. Advances in nanofabrication techniques (focused ion beam, electron beam lithography and photolithography), improvement of the quality of metal films and exploration of alternative materials such as superconductors [131] offer pathways to mitigate significant absorption losses. Further innovations involve all-dielectric nanostructures that replicate EOT phenomena without relying on lossy plasmonic materials [132].

EOT plays a critical role in the development of tunable optical devices, where transmission properties undergo dynamic modulation through alterations in the geometry or material composition of aperture arrays [133]. Recently, different tuning mechanisms based on semiconductors and phase change materials for electrical and thermal control of EOT have been studied [134–136]. However, the mechanical control of EOT remains unexplored.

2.2.2 Spectrally tunable metasurface filters for hyperspectral infrared imaging

The peak thermal radiation from objects at approximately room temperature lies in the LWIR range (8-12 μm), overlapping with an atmospheric transmission "window" particularly interesting for imaging and remote sensing applications. For a body at 300 K, the peak wavelength of emitted radiation falls within the LWIR range at approximately 10 μm . Detectors in spectral imaging and remote sensing applications require lower sensitivity when operating in the LWIR range. The higher photon flux from radiating bodies in this range enables the use of less sensitive equipment, which is particularly beneficial in environments with high noise levels. Beyond imaging and remote sensing applications, the LWIR range offers significant potential for optical satellite communication due to

reduced atmospheric losses [137]. However, current technologies in this range remain bulky, heavy and expensive, hindering the realisation of practical systems. This issue extends to infrared astronomy, where space missions like the James Webb Telescope rely on bulky filter wheels with multiple separate filter devices, increasing satellite weight and complicating their deployment in orbit [138].

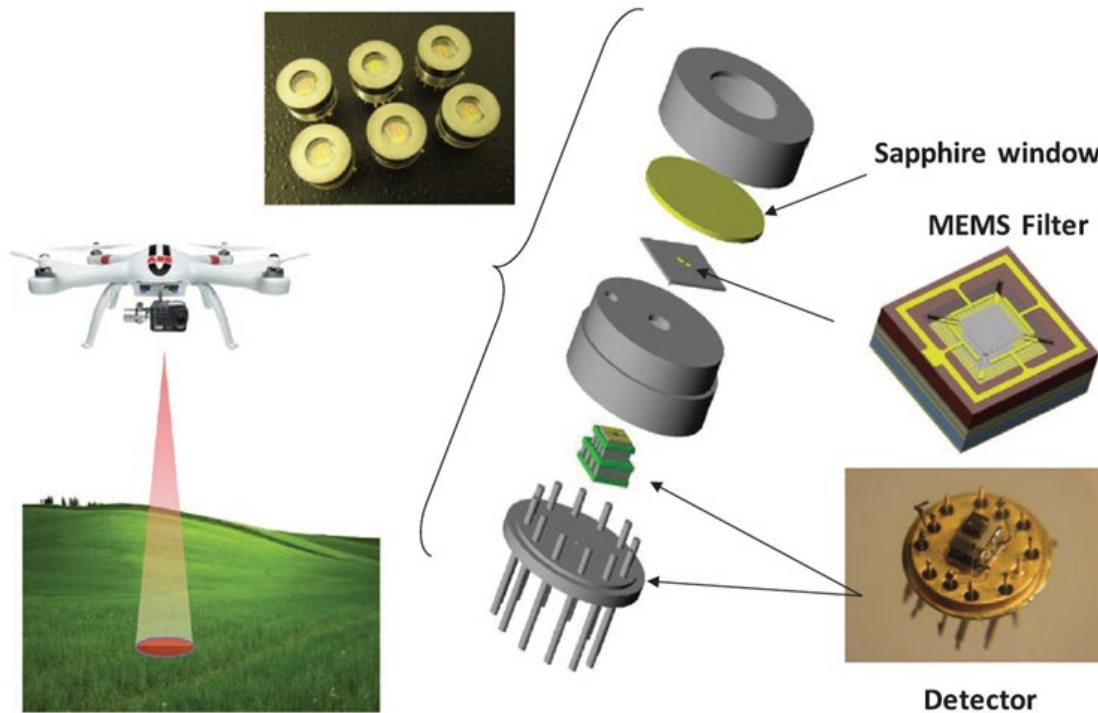


Figure 2-2 On-chip infrared spectrometer module for remote sensing application.
Adapted from [139].

The next generation of infrared imaging devices requires the ability to detect multiple wavelengths [139], similar to how modern cameras capture colour images through frequency-selective detection in the visible range. Hyperspectral imaging addresses this challenge by enabling the collection and processing of spectral information across a wide frequency range. A practical method for infrared hyperspectral imaging involves placing a tunable filter in front of a broadband detector, as illustrated in Figure 2-2. By tuning the filter's response, the system selectively detects signals from different parts of the LWIR range. Mechanically tunable filters play a crucial role in hyperspectral imaging systems, with MEMS-enabled designs offering promising control over resonant responses.

Current MEMS-based filter technology primarily relies on Fabry-Pérot resonances [139]. Figure 2-3 depicts the operating principle of these filters, which selectively reflect electromagnetic radiation outside a specific bandwidth. The central wavelength of this bandwidth can be shifted by adjusting the distance between two mirrors, equal to half a wavelength. This design, however, demands high-precision fabrication of perfectly flat and parallel mirrors, which must maintain alignment during operation. Moreover, the wavelength shift is directly proportional to the displacement between the mirrors. Achieving large spectral tuning ranges requires significant air gaps, necessitating high actuation voltages up to 160 V to tune the resonator cavity for Fabry-Pérot filtering in the LWIR range. Additionally, the broadest spectral tuning range reported for MEMS-based filters currently covers approximately 75% of the full LWIR band.

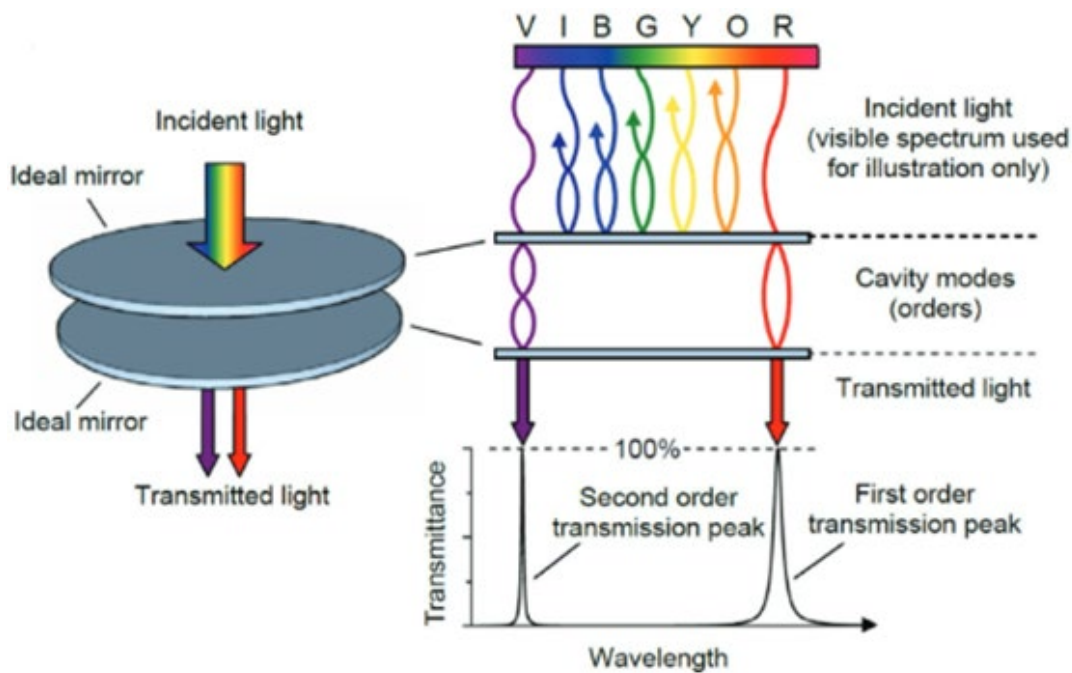


Figure 2-3 The operating principle of a Fabry-Pérot optical filter. Adapted from [139].

To address the limitations of current MEMS technology and reduce device thickness, we propose tunable infrared filters based on a thin metallic sheet perforated with an array of holes that exhibit EOT for one polarisation. The design incorporates a high-index dielectric MEMS membrane that moves in proximity to the metallic layer, enabling dynamic tuning of the transmission resonance by altering its position relative to the holes. Figure 2-4(a) shows a schematic of the tunable metasurface filter comprising two

parallel periodically perforated membranes made of gold and silicon separated by an adjustable air gap d . The lower gold membrane features a hole array, functioning as a bandpass filter. The upper silicon membrane introduces a controlled shift in the EOT resonance by modifying the effective dielectric environment of the gold membrane as the air gap d is adjusted. Silicon provides a balance between optical performance and CMOS-compatible MEMS fabrication, making it well suited for tunable metasurface devices. While higher-index materials such as germanium could potentially enhance tuning sensitivity in the LWIR, their integration typically requires non-standard fabrication processes, motivating the use of silicon for scalable and reliable device implementation.

The following meta-atom geometrical parameters were used for the electromagnetic modelling in CST Studio: the unit cell size (p_x and p_y) of $5.5\ \mu\text{m}$; the hole height (h_{Au}) and width (w_{Au}) in the gold layer of $3.3\ \mu\text{m}$ and $1.5\ \mu\text{m}$, respectively; the hole height (h_{Si}) and width (w_{Si}) in the silicon layer of $3.9\ \mu\text{m}$; the hole corners rounding radius in lower gold layer (r_{Au}) of $0.3\ \mu\text{m}$; the hole corners rounding radius in upper silicon layer (r_{Si}) of $0.9\ \mu\text{m}$; the gold membrane thickness (t_{Au}) of $0.4\ \mu\text{m}$; the silicon membrane thickness (t_{Si}) of $0.29\ \mu\text{m}$ and the air gap (d) of $0.2\ \mu\text{m}$. The rectangular holes in both layers feature rounded corners to address fabrication constraints. We used the refractive index of gold from [140] and CST Studio embedded data for lossy silicon in modelling.

We utilised parametric sweeps in CST Studio Suite to optimise the geometrical parameters of the meta-atom. The optimisation procedure was guided by physical considerations derived from EOT theory and by fabrication constraints. Specifically, the unit cell period and hole dimensions were chosen to position the fundamental EOT resonance at a wavelength of $7\ \mu\text{m}$. The thicknesses of the gold and silicon membranes were chosen based on fabrication constraints and mechanical stability considerations. The structural parameters were varied to maximise the achievable resonance tuning range while maintaining high transmission contrast and wide bandwidth. The final parameter set represents a compromise between spectral performance, mechanical stability, and fabrication constraints.

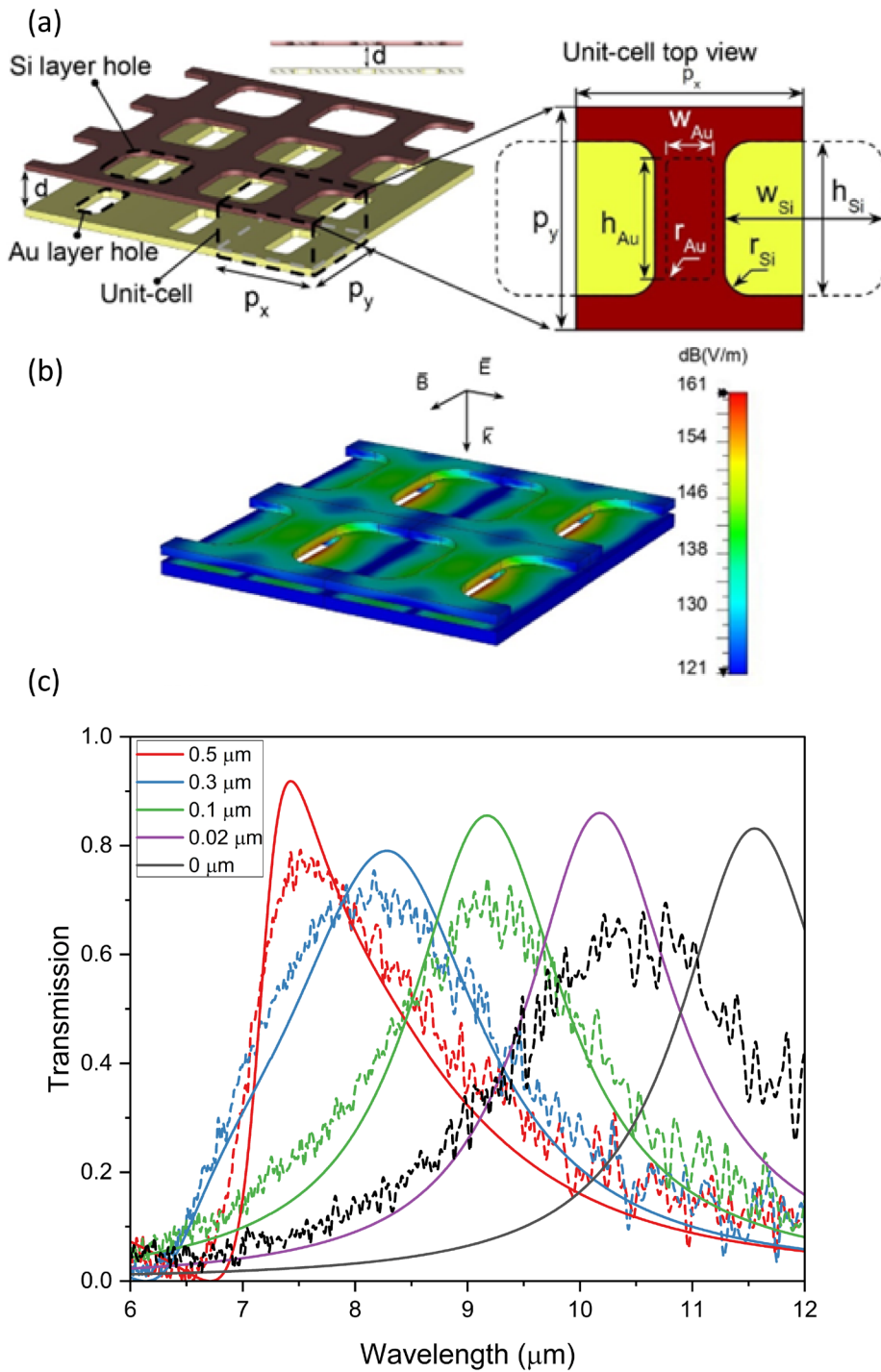


Figure 2-4 (a) Schematic of the spectrally tunable metasurface filter, comprising a silicon membrane suspended above a perforated thin gold film, with the air gap d controlling the transmission resonance wavelength. (b) Simulated electric field distribution for $d = 0.2 \mu\text{m}$, showing maximum field concentration near the hole edges. (c) Simulated (solid) and experimental (dashed, provided by Oleg Bannik from the University of Western Australia) transmission characteristics for varying air gap values d .

The perforations in the gold layer enable a polarisation-sensitive bandpass transmission response in the LWIR range. Light polarised along the short side of the rectangular holes is transmitted, while the orthogonal polarisation component is suppressed. The upper silicon membrane is also perforated to optimise signal transmission while tuning the filter's response. However, the MEMS membrane design is not restricted to this geometry and can take different shapes. The most critical factor is ensuring the presence of a dielectric near the metal perforations, as the EOT theory predicts that this significantly influences the resonance's central wavelength. Ideally, the dielectric should be positioned in the region of maximum electric field intensity to maximise the spectral tuning range of the resonance.

Figure 2-4(b) illustrates the electric field distribution at the EOT resonance, demonstrating strong field concentration at the edges of the holes in the gold membrane. An optimised MEMS membrane design could include protruding sections that, when the membrane is moved closer to the metal layer, extend into the perforations. However, implementing such complex volumetric MEMS structures experimentally is significantly more challenging compared to flat membrane designs.

Figure 2-4(c) presents the transmission characteristics for various air gap values d , ranging from 0.5 μm to 0 μm . The transmission peak wavelength shifts from 7.5 μm to 11.5 μm as the air gap value decreases. Notably, the EOT peak exhibits increased sensitivity to smaller changes in d . For instance, reducing d from 0.1 μm to 0.02 μm shifts the resonance by approximately 1 μm , while a further reduction from 0.02 μm to 0 μm results in a shift of approximately 1.5 μm . This heightened sensitivity arises from the evanescent nature of SPPs, which decay exponentially away from the surface of the gold layer. As a result, the tuning effect diminishes with larger membrane separations.

The nonlinear relationship between the air gap and the EOT peak position enables significant wavelength tuning with minimal adjustments to d . The proposed tunable metasurface filter spans the entire LWIR range while requiring only a 0.5 μm adjustment of the gap. This represents a substantial improvement over traditional LWIR micro-Fabry-

Pérot interferometers, which typically require gap tuning in the range of 6 μm to 4 μm . However, this approach demands precise membrane flatness and highly accurate control of the air gap, particularly as d approaches zero. To achieve this control, the metasurface design incorporates an electrostatic MEMS actuator for dynamic adjustment of the air gap. Unlike switch-based MEMS devices, the present actuator operates in a continuous displacement regime, enabling analogue tuning of the metasurface response.

As a proof of concept, our colleagues at the University of Western Australia realised microelectromechanical actuator designs featuring the developed tunable metasurface filter, as depicted in Figure 2-4(a). Several fabrication challenges were encountered, such as an inability to achieve a zero air gap due to surface roughness and the pull-in effect, which caused abrupt changes in the gap and uncontrolled shifts in the transmission peak. Despite these challenges, the fabricated devices with two suspended membranes showed satisfactory agreement between experimental and simulated transmission filtering performance. Figure 2-4(c) includes experimental data for one of the fabricated metasurface filters, confirming the feasibility of actively tuning the EOT peak wavelength by adjusting the air gap with actuation voltages below 10 V. Nevertheless, the inability to achieve a zero gap due to surface roughness remains a limiting factor for the tunable filter's performance.

To address the pull-in effect issue, we propose an alternative meta-atom design with a MEMS membrane capable of achieving the same distance between the dielectric structures and the holes in the gold film. Figure 2-5(a) illustrates the schematic of the meta-atom including a MEMS structure with a dimple over a subwavelength hole in a gold film. This shape of the MEMS membrane ensures operation in an elastic regime, preventing the membrane from sticking to the metal film.

Figure 2-5(b) shows the transmission characteristics of the alternative design for varying air gaps between the bottom of the dimple and the gold film. This design allows for tuning across the entire LWIR range by adjusting the separation distance by less than 1 μm ,

providing a robust solution for achieving precise spectral control while mitigating the limitations observed in flat MEMS membranes.

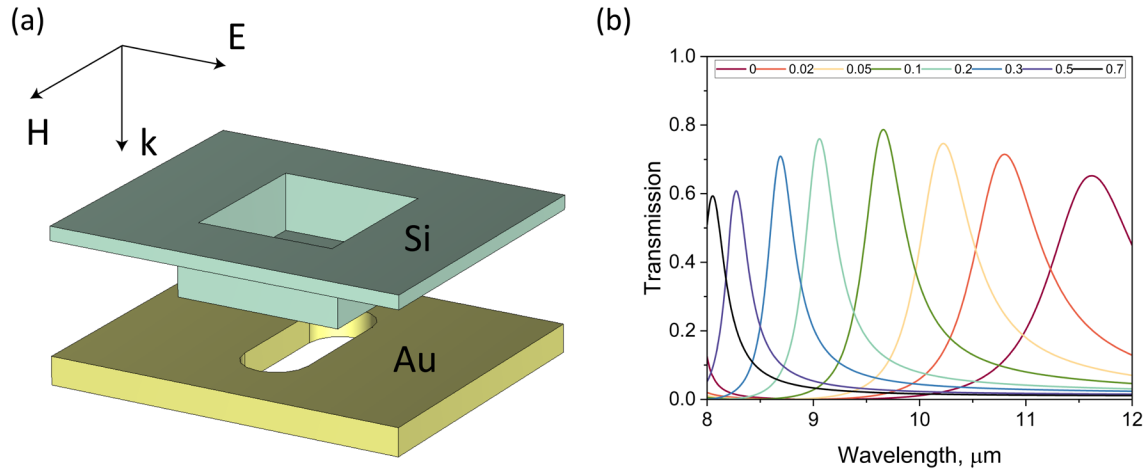


Figure 2-5 (a) The schematic of the meta-atom consisting of the volumetric MEMS structure placed over the perforated thin gold layer. (b) Transmission for various air gap values between the bottom of the dimple and the gold film (0-0.7 μm).

A low-loss substrate can improve mechanical stability and simplifies fabrication. However, integrating a substrate requires optimising the metasurface design, as the EOT phenomenon is highly sensitive to the dielectric permittivity of adjacent materials. Additionally, the added substrate increases the overall thickness of tunable metasurface filters.

2.2.3 Metasurface filters for fluorescence fibre-optic imaging

This section addresses a dedicated sub-project on metasurface filters for fluorescence fibre-optic imaging, which represents an important first step towards implementing tunable metasurfaces in fibre-optic technologies. The operation of the proposed filter relies on EOT resonances supported by a periodic array of subwavelength apertures in a thin metallic film.

Optical fibre technologies play a fundamental role in modern optical communication, imaging, and sensing. Their compact form factor, high transmission efficiency, and compatibility with various optical systems make them essential for many applications, such as biomedical imaging and diagnostics. Despite these advantages, a fundamental

drawback of optical fibres is their fixed optical properties, which limit their adaptability [13], particularly in biomedical imaging where tunable responses are often needed. Moreover, achieving complex functionality in optical fibre-based devices remains challenging due to the difficulty of fabricating miniaturised optical elements within a fibre or on its tip. This limitation highlights the need for innovative approaches, such as metasurfaces, to enhance fibre-optic capabilities.

Metasurfaces offer a practical and flexible way to introduce advanced optical functions into fibre-based systems and are crucial for miniaturising fibre-integrated optical elements [141]. The need for miniaturised *in vivo* devices makes metasurfaces particularly valuable in biomedical fibre-optic technologies. By integrating metasurfaces, fibre-optic systems gain expanded functionality and, in the future, could enable active control over optical properties such as amplitude, phase, and polarisation through tunable solutions. Incorporating active metasurfaces with optical fibres could revolutionise optical technologies by enabling real-time tunability and adaptive optical responses. Dynamic fibre metasurfaces will rely on electrically tunable conducting oxides, phase-change materials, and nanoelectromechanical systems [13]. Active metasurfaces facilitate tunable focusing, beam steering, and filtering, all of which are crucial for next-generation fibre communication and biomedical imaging technologies.

To realise advanced optical functionalities in fibre-optic systems, extensive research has focused on developing reliable fabrication methods for metasurfaces on fibre facets. Various techniques have been proposed to achieve high-precision patterning and seamless integration, including decal transfer [142], nanoskiving [143], focused ion beam milling [144–146], electron-beam lithography [147], 3D laser nanoprinting [148], and transfer bonding [149]. These methods enable the fabrication of metasurfaces designed to realise specific optical functions on fibre faces. Recently, plasmonic metasurfaces based on EOT have been proposed for fibre-tip applications, enhancing optical transmission [144], filtering [150,151], beam steering [145], focusing [146], and refractive index sensing [152–154]. Beyond plasmonic structures, dielectric

metasurfaces offer additional advantages, particularly in endoscopic biomedical applications [147], focusing and imaging [148].

Despite these developments, most existing fibre-based metasurface designs focus on modifying light at the core of single-cladding fibres, primarily for laser beam manipulation. However, in fluorescence fibre-optic imaging, multimode or double-cladding fibres are often used, where fluorescence signals from biological specimens are collected through the multimode core or inner cladding [155–157]. A major challenge in such systems is that unwanted laser and OCT signals (when fluorescence imaging is combined with OCT) introduce noise, reducing fluorescence imaging accuracy. Addressing this issue requires an effective filtering mechanism that selectively transmits the fluorescence signal while blocking unwanted laser and OCT beams.

In this chapter section, we investigate a plasmonic metasurface integrated with the inner cladding at the fibre tip, functioning as a filter based on EOT, for enhancing the existing fluorescence imaging technique. Unlike previous approaches that focus on the fibre core, the proposed metasurface selectively suppresses unwanted signals in double-cladding fibres, improving signal clarity and performance of fluorescence imaging.

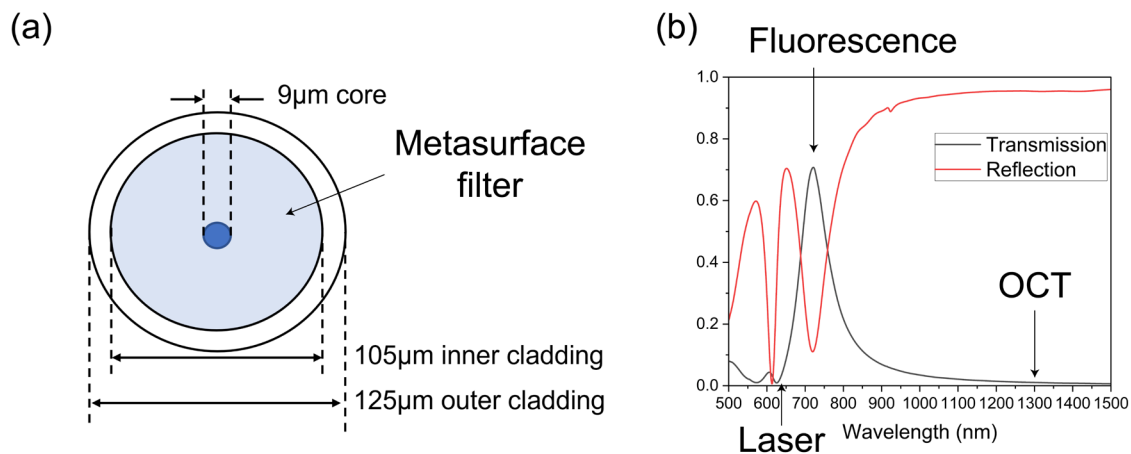


Figure 2-6 (a) The cross section of the double cladding fibre and its dimensions. The proposed metasurface filter is integrated with the inner cladding at the fibre tip. (b) Transmission and reflection characteristics of the metasurface filter that selectively transmits the fluorescence signal while blocking the unwanted laser and OCT beams.

Figure 2-6(a) shows the cross section of the double cladding fibre for biomedical applications along with the diameters of the core, inner and outer claddings. The metasurface filter has a hole in the area above the fibre core to avoid any unwanted effects on the laser and OCT signals. Figure 2-6(b) illustrates the transmission and reflection characteristics of a metasurface filter integrated with the inner cladding at the fibre tip. While allowing for collecting of the fluorescence signal from a biological environment in the range of 680-750 nm into the inner cladding, it achieves a significant level of suppression of the laser (615-650 nm) and OCT (1250-1360 nm) beams.

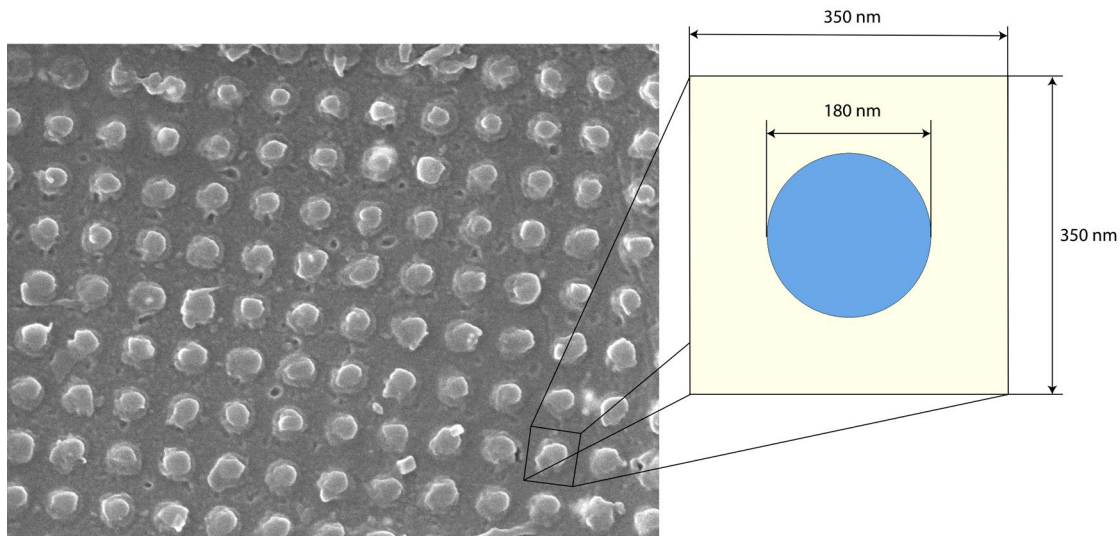


Figure 2-7 Microscope image of the fabricated metasurface filter, consisting of a perforated 120 nm-thick gold film supporting extraordinary optical transmission, along with a schematic of the meta-atom and its dimensions. The nanoholes are filled with OrmoClear, the encapsulation material used for transferring the metasurface filter onto the fiber tip. The microscope image was provided by Dr. Fei He from the University of Nottingham.

Figure 2-7 presents a scanning electron microscope (SEM) image of the fabricated fibre metasurface filter along with the dimensions of its meta-atom. The proposed filter consists of a periodic array of subwavelength holes in a thin gold film. The unit cell has a pitch of 350 nm in both directions, a hole diameter of 180 nm, and a gold layer thickness of approximately 100 nm. The holes are filled with an encapsulation material (Ormoclear) used to facilitate the transfer of the metasurface, which was fabricated using conventional electron beam lithography [158]. The following characteristics of the materials were used for electromagnetic modelling of the metasurface fibre filter: the

refractive index of gold from [159], the refractive index of the encapsulation material (Ormoclear) of 1.54, and we assume that the biological environment has the refractive index of water from [160].

2.3 Mechanical control of bound states in the continuum

2.3.1 *Bound states in the continuum in metasurfaces*

The concept of BICs represents a remarkable phenomenon in modern physics, where certain states remain localised within a system despite existing at energies or frequencies that typically allow for radiation [161]. Originating from quantum mechanics, BICs have found significant applications in photonics and metamaterials, enabling extreme confinement of electromagnetic waves and forming extremely high-Q resonances [162,163]. These unique properties of BICs make them invaluable for enhancing light-matter interactions, paving the way for developing advanced photonic devices [164].

Recent advancements have focused on the active control of BICs [165], where external stimuli, such as electric fields, temperature variations, or material phase transitions, are used to dynamically tune their properties [166]. This capability enables real-time reconfiguration [19], opening up opportunities for a wide range of applications, including tunable filters [167], sensors [168–172], and modulators [173–175], which are critical components in modern communication, imaging, and sensing technologies. The precise manipulation of q-BIC resonances in real-time could revolutionise photonic systems, enhancing device performance and functionality, and making this a rapidly growing field in applied physics and engineering [176].

Due to their compactness, precise control, and low power consumption, MEMS/NEMS have emerged as a leading technology in fields such as active nanophotonics, sensing, and telecommunications [139,177–179]. These systems offer dynamic, strong on-demand tuning of optical properties, which has revolutionized many applications by enabling integration into smaller, more efficient devices. One of the most significant

breakthroughs in this area is the control of high-Q resonances in metasurfaces through mechanical actuation [53,54,36]. This has led to significant advancements, particularly in imaging and active photonics, allowing for the real-time manipulation of optical properties with minimal energy input [64,118].

Recently, the control of guided-mode resonances (GMR) using MEMS/NEMS has opened new avenues for dynamic manipulation of light in nanophotonic devices [180–183]. By integrating MEMS/NEMS, the mechanical actuation of these structures allows for precise, real-time tuning of resonance characteristics, enabling modulation of amplitude, phase, and wavelength. Notably, NEMS technology has been employed to tune both GMR and q-BIC resonances in all-dielectric metasurfaces, enabling dynamic modulation of amplitude and phase [184]. The interference between these two resonant modes significantly enhanced the phase response of the metasurfaces, allowing for precise control with minimal input. In particular, a phase shift of 144° was achieved experimentally with just a 4 V bias change. However, this approach exhibited non-zero reflection at the q-BIC resonance, which is less than ideal for imaging applications that require minimal reflection for optimal performance. Exploring the q-BIC resonances in transmission is critical for applications such as spectroscopy, sensing and imaging, where high transmission and precise control of resonances are essential for accurate data acquisition [139].

More recently, static and thermally tunable BIC-based metasurfaces have been proposed for use in spectrometers, achieving sub-nm resolution [185,186]. While the static designs in the optical range rely on bandstop filtering and require multiple metasurfaces to cover the desired spectral range, leading to extra computations and increased spectrometer size, respectively, the designed ultracompact tunable solutions in the infrared range face limitations, including the inability to maintain a constant bandwidth and a relatively narrow spectral tuning range of only 6 nm.

2.3.2 Mechanical control of quasi-BIC resonances in all-silicon metasurfaces

In this chapter section, we introduce MEMS-enabled comprehensive control over q-BIC resonances in a transmission configuration within all-silicon metasurfaces. The q-BIC resonance is achieved via breaking the out-of-plane symmetry by combining a 2D MEMS structure that moves close to a 1D metasurface without direct contact. We demonstrate that the resonance Q-factor approaches infinity as the MEMS structure moves farther from a 1D metasurface, thereby reducing the asymmetry – a defining characteristic of q-BIC resonances. The vertical movement of a MEMS structure combined with horizontal displacement within a 1D metasurface enables the central wavelength to be shifted by more than 60 nm while maintaining a consistent resonance bandwidth and Q-factor. The proposed all-silicon metasurfaces achieve an ultranarrow resonance linewidth with dynamic tunability in the infrared range, making them promising candidates for advanced tunable filtering applications and highly relevant for spectroscopy, remote sensing, and imaging.

We begin by investigating high-Q q-BIC resonances excited in all-silicon metasurfaces by a plane wave under normal incidence. Figure 2-8(a) presents a conceptual design of a resonant structure composed of a 1D metasurface made of silicon bars and an overlying 2D array of cuboids. The structural parameters shown in Figure 2-8(a) are as follows: $w_c = 0.8 \mu\text{m}$, $h_c = 1 \mu\text{m}$, $w_r = 0.8 \mu\text{m}$, $h_r = 0.5 \mu\text{m}$, $d = 0.7 \mu\text{m}$. We model the transmission in CST Studio, where we employ Floquet ports and periodic boundary conditions. The edge length of the square unit cell (p) is $1.5 \mu\text{m}$. The polarisation of the incident plane wave is shown in Figure 2-8(a). The structural parameters were determined using parametric sweeps in CST Studio, guided by the goal of achieving a resonance linewidth close to 1 nm while enabling tunability and maintaining a low out-of-band transmission level over a wide spectral range.

Figure 2-8(b) illustrates the electric field distribution at the q-BIC resonance when the distance between the cuboids and bars (a) is $1 \mu\text{m}$. Transmission and reflection spectra for various distances between the bars and cuboids are depicted in Figures 2-8(c,d). This structure enables out-of-band transmission of near than or below 0.1 within the 3100–

3300 nm range when tuning a from 0.2 to 1 μm and less than 0.04 across 2700–3400 nm when a is set to 1 μm .

In the absence of cuboids, no q-BIC resonance appears, as the non-radiating BIC mode remains confined within the bars. Nevertheless, the BIC mode was identified using the eigenmode solver. Introducing asymmetry by positioning the cuboids above the bars enables radiation leakage, leading to the appearance of high-Q resonances in transmission.

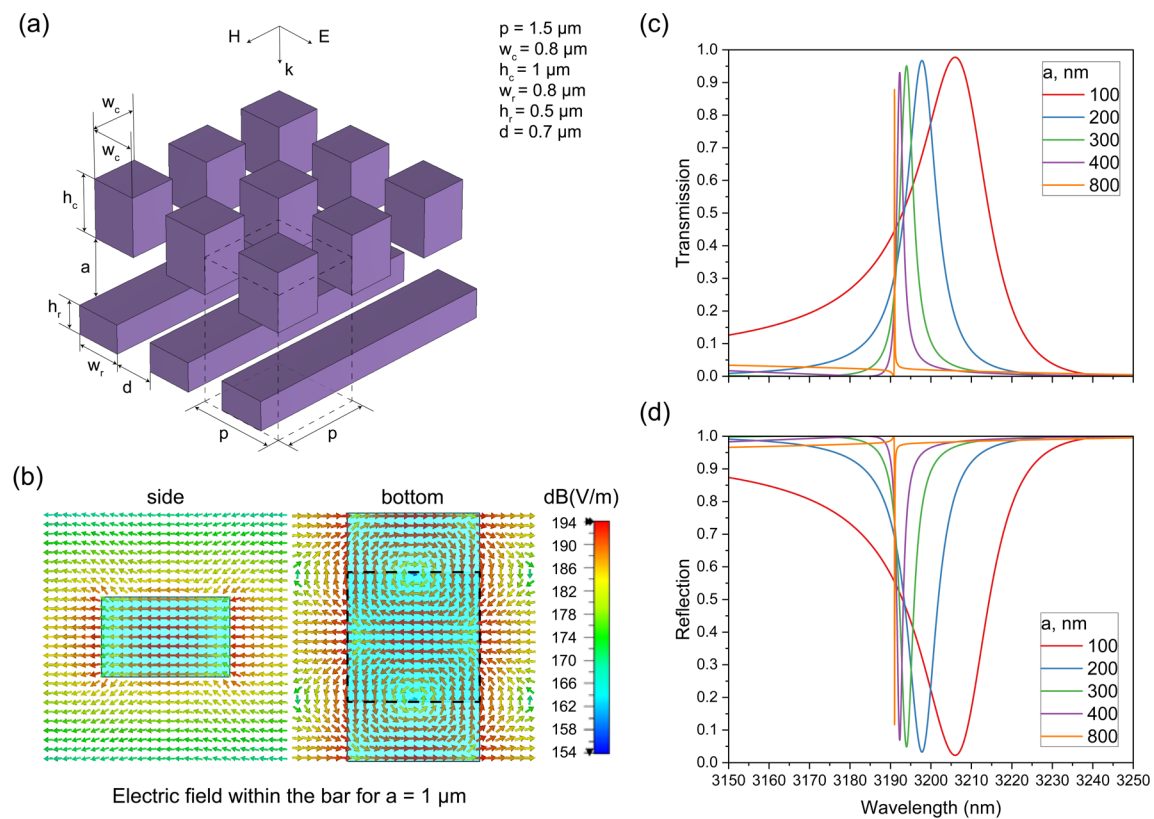


Figure 2-8 (a) Schematic of the tunable metasurface conceptual design: silicon bars and an array of cuboids, including the unit cell indicated by the dashed lines; (b) Maximum electric field amplitude of the BIC mode within the bar for $a = 1 \mu\text{m}$ (side and bottom views, the dashed lines show the position of the cube); (c,d) Transmission and reflection coefficients for various distances between silicon bars and cuboids (a).

We use the Fano formula [187,188] to fit the transmission curves shown in Figure 2-8(c), enabling accurate analysis of the characteristics of the excited q-BIC resonances while accounting for their asymmetry:

$$T_{Fano}(\omega) = t_{d_0} + t_d^2 \frac{\left[q + \frac{2(\omega - \omega_0)}{d\omega} \right]^2}{1 + \frac{4(\omega - \omega_0)^2}{d\omega^2}}, \quad (2.1)$$

where t_{d_0} is the minimum of transmission near resonance, t_d is the non-resonant transmission amplitude, q is the shape factor, ω_0 is the resonance frequency, $d\omega$ is the resonance width.

Figures 2-9(a,b) show the dependence of the q-BIC resonance wavelength and the Q-factor on the distance between the bars and the cuboids. The q-BIC resonance wavelength approaches the BIC value of approximately 3193.2 nm as the cuboids move away from the 1D metasurface. To accurately describe the relationship between the Q-factor of the excited q-BIC resonances and the distance between the cuboids and the bars depicted in Figure 2-9(b), we can use the exponential growth formula:

$$Q(a) = Q_0 + Q_1 e^{ka}, \quad (2.2)$$

where Q_0 is the offset parameter, Q_1 is the initial Q-factor value, k is the rate. The curve has the following parameters: $Q_0 = -10.36$, $Q_1 = 76.38$ and $k = 0.008$.

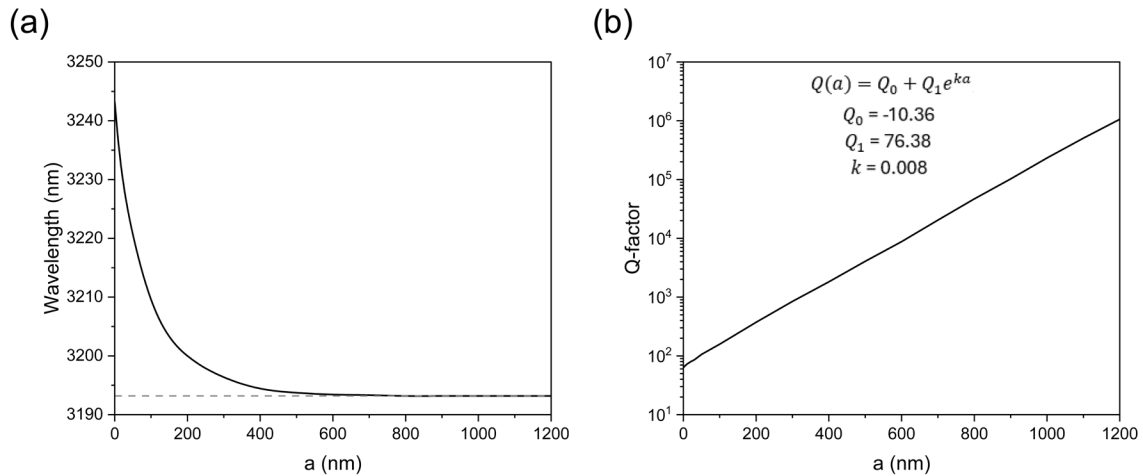


Figure 2-9 (a, b) Dependence of the q-BIC resonance wavelength and Q-factor on the distance between the bars and cuboids.

Figure 2-9(b) shows that the Q-factor exponentially increases as the gap widens, reaching a value of 10^6 when the gap is 1.2 μm . This behaviour confirms the q-BIC nature of the

resonance, as the Q-factor of the q-BIC mode theoretically approaches infinity with decreasing asymmetry. The proximity of the cuboids to the 1D metasurface introduces asymmetry into the field distribution excited by the incident wave. Conversely, increasing the distance between the cuboids and the 1D metasurface restores the system's symmetry, thereby reducing radiation leakage. Such control over the Q-factor of the q-BIC resonances, realised through out-of-plane symmetry breaking, offers an additional degree of freedom for tuning their characteristics. Figures 2-8(c,d) illustrate that, as the Q-factor increases, the transmission level decreases while the reflection level increases. This behaviour is also accompanied by higher losses at the resonance wavelength when approaching the BIC state. Additionally, the vertical movement of the cuboids does not allow significant tuning of the central wavelength unless the separation is reduced to less than 100 nm, as shown in Figures 2-8(c,d) and 2-9(a).

Next, we examine a more practical design featuring a 1D array of bars and a 2D MEMS membrane, as schematically shown in Figure 2-10(a), which enables control of the same q-BIC resonance. The MEMS membrane is flat and thin, offering a significant advantage over bulk cuboids that would require attachment to a supporting scaffolding structure for manufacturing, thereby complicating the process. Using flat membranes instead of volumetric structures greatly simplifies fabrication.

Figure 2-10(a) illustrates the schematic of the proposed tunable metasurface design, comprising silicon bars and a perforated MEMS membrane, along with its transmission characteristics for varying distances between the two layers (a). The structural parameters are as follows: $w_m = 0.8 \mu\text{m}$, $h_m = 0.3 \mu\text{m}$, $w_h = 0.7 \mu\text{m}$, $l_h = 1.1 \mu\text{m}$, $w_c = 0.4 \mu\text{m}$, $w_r = 0.8 \mu\text{m}$, $h_r = 0.25 \mu\text{m}$, $d = 0.7 \mu\text{m}$. The edge length of the square unit cell (p) is $1.5 \mu\text{m}$. The polarisation of the incident plane wave is shown using the triple of vectors in Figure 2-10(a). The membrane parameters were chosen to achieve control of the q-BIC mode comparable to that achieved with cuboids, while accounting for fabrication constraints associated with perforated MEMS membranes. Figure 2-10(b) demonstrates that this design enables the excitation of the q-BIC resonance, with the electric field distribution corresponding to the loops in the bars previously depicted in Figure 2-8(b).

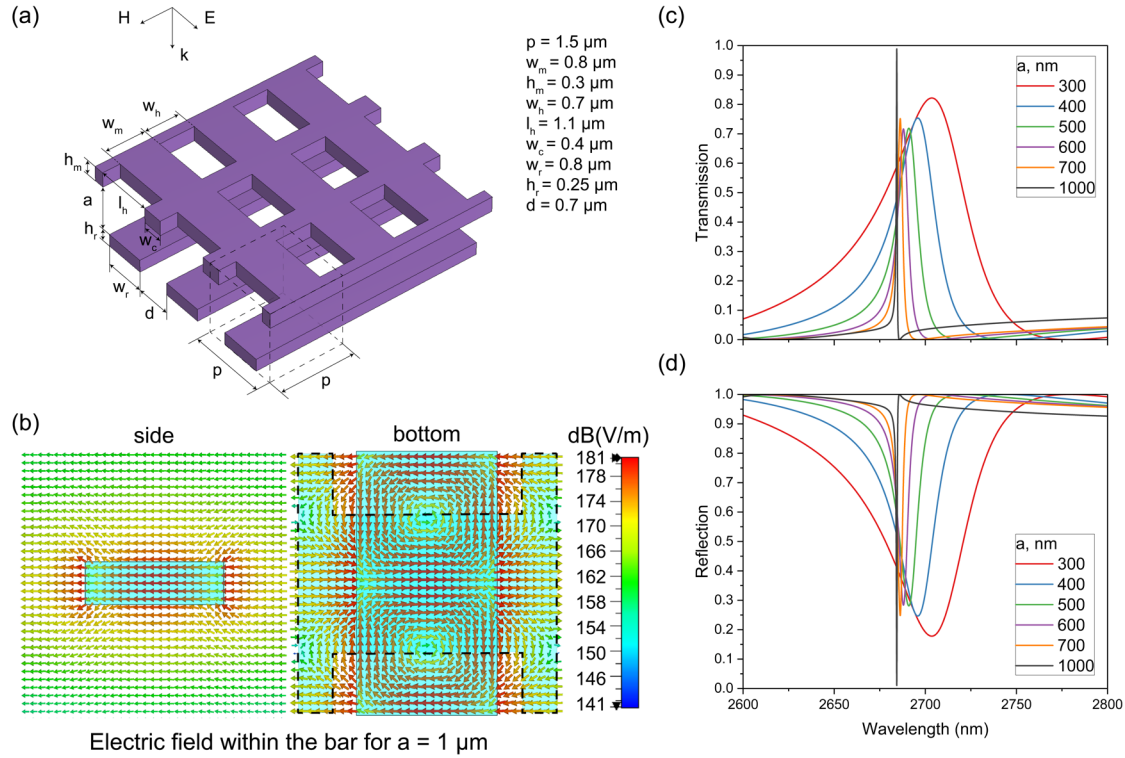


Figure 2-10 (a) Schematic of the tunable metasurface design consisting of silicon bars and a MEMS membrane, including the unit cell indicated by the dashed lines; (b) Maximum electric field amplitude of the q-BIC mode within the bar for $a = 1 \mu\text{m}$ (side and bottom views, the dashed lines show the position of the MEMS membrane); (c, d) Transmission and reflection coefficients for various distances between two layers (a).

Figures 2-10(c,d) show that the vertical movement of the MEMS membrane enables control over the Q-factor of the q-BIC resonances but does not provide substantial tuning of the resonance peak wavelength. The q-BIC resonance shifts by less than 20 nm when a is varied from 0.3 to $1 \mu\text{m}$, corresponding to less than 1% of the resonance wavelength. The Q-factor also varies with a , and the resonance exhibits an ultranarrow bandwidth of approximately 1 nm when the distance between the layers is $0.9 \mu\text{m}$. This indicates that the MEMS-based tuning method is effective for adjusting the bandwidth but is not suitable for applications requiring significant wavelength tunability.

The height of the bars was optimised to shift the q-BIC resonance into the range of maximum reflection. This adjustment accounts for the structural differences between the cuboids and the perforated MEMS membrane, achieving out-of-band transmission below 0.1 within the 2600–2900 nm range when a is tuned from $0.3 \mu\text{m}$ to $1 \mu\text{m}$. Such

performance makes the proposed system suitable for spectrometers employing broadband receivers in combination with tunable filters.

Interestingly, the peak transmission level does not decrease with increasing the Q-factor and a , suggesting a more complex behaviour of the q-BIC resonance compared to the previous case with cuboids and bars, where the peak transmission decreased as a increased. While losses remain proportional to the Q-factor, Figure 2-10(d) shows that the reflection at the resonance wavelength does not increase with a . This observation highlights the potential for optimising the 2D MEMS structure to maintain high transmission values at the resonance wavelength while achieving the desired ultranarrow bandwidth.

The bars and the overlying membrane can be attached to a supporting structure (not shown in Figure 2-10(a)). Alternatively, the bars may be placed on a quartz substrate, which can improve the mechanical stability of the proposed metasurfaces. However, this would also increase their thickness and limit the horizontal movement of the bars. As mentioned earlier, many tunable filter applications require maintaining a constant Q-factor while significantly shifting the resonance wavelength. To meet this requirement, we further investigate the horizontal movement of the bars to gain additional control over the resonance peak wavelength.

Figure 2-11(a) depicts the tunable metasurface design consisting of pairs of silicon bars that can move horizontally in the direction normal to their axis, while the membrane moves vertically above the bars. The structural parameters remain consistent with those in Figure 2-11(a), but we alter the distance between the silicon bar pairs (d), while keeping the period p constant. The polarisation of the incident plane wave is shown in Figure 2-11(a). Two independent orthogonal tuning mechanisms enable precise control of both the central wavelength and bandwidth (or Q-factor) of the q-BIC resonance.

Figure 2-11(b) shows the transmission coefficient for various distances between the bars (d) when $a = 900$ nm. The horizontal movement of the bars allows for a resonance

wavelength shift of approximately 65 nm. In this case, out-of-band transmission is less than 0.1 within the 2600–3000 nm range. The specular reflection remains almost constant as the distance between the bars increases. The reduction in transmission at the q-BIC resonance in Figure 2-11(b) arises from enhanced coupling of the incident wave into the first-order diffracted modes.

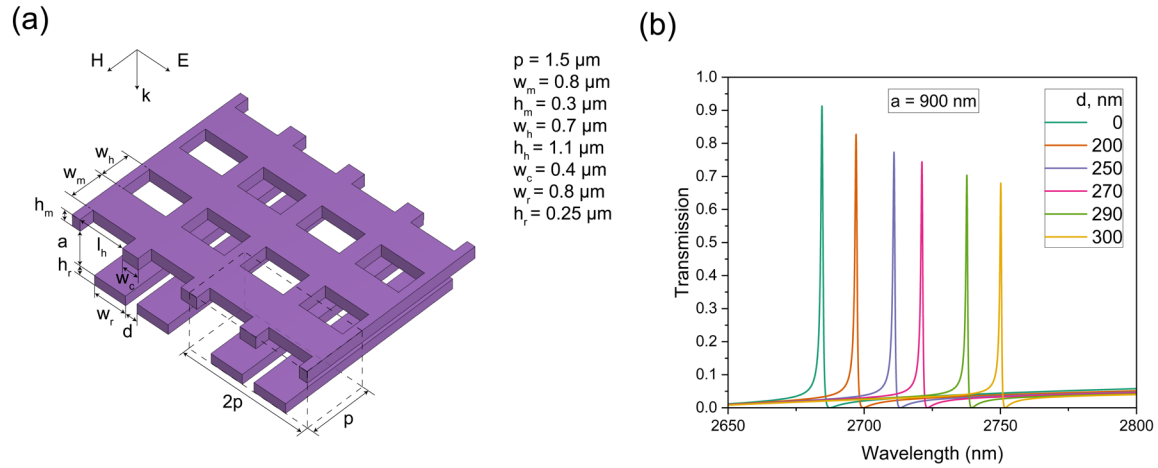


Figure 2-11 (a) Schematic of the tunable metasurface design consisting of pairs of silicon bars and a MEMS membrane, including the unit cell indicated by the dashed lines; (b) Transmission characteristics for varying distances between the bars (d), with the distance between the two layers (a) fixed at 900 nm.

This approach alone does not maintain a constant bandwidth or Q-factor. As previously demonstrated, precise bandwidth control is achieved through the vertical movement of the membrane. By combining two independent orthogonal tuning mechanisms, complete control over the q-BIC resonance characteristics can be realised.

Figures 2-12(a,b) illustrate the dependence of the resonance central wavelength and Q-factor on two parameters: the distance between two layers (a) and the distance between the bars (d). By precisely adjusting both parameters, as indicated by the black lines in Figure 2-12(b), it is possible to maintain a constant Q-factor while tuning the central wavelength by more than 60 nm.

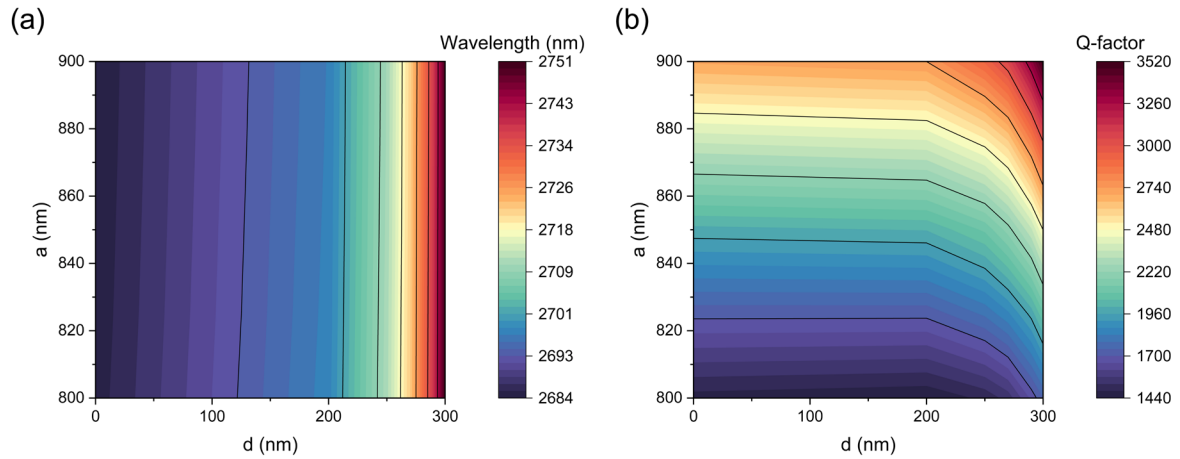


Figure 2-12 (a, b) Dependence of the resonance central wavelength and Q-factor on two parameters: the distance between the two layers (a) and the distance between the bars (d). The black lines indicate examples of curves with constant wavelength and Q-factor, respectively.

Table 2-1 summarises the characteristics of three investigated tunable metasurface designs:

1. Metasurface concept utilising vertically adjustable cuboids, as shown in Figure 2-8(a);
2. Metasurface employing a single tuning mechanism based on a vertically adjustable perforated membrane, as illustrated in Figure 2-10(a);
3. Metasurface incorporating two independent orthogonal tuning mechanisms, as depicted in Figure 2-11(a).

Table 2-1 highlights the benefits of utilising two independent orthogonal tuning mechanisms. These mechanisms enable the preservation of a consistent Q-factor of the q-BIC resonance while substantially extending the spectral tuning range. Additionally, the design ensures flat MEMS structures and achieves out-of-band transmission of less than 0.1 across a broad range near the q-BIC resonance.

Table 2-1 Comparison of characteristics of the proposed tunable metasurfaces.

Characteristic	Metasurfaces based on vertically adjustable cuboids	Metasurfaces based on the vertically adjustable membrane	Metasurfaces based on two independent orthogonal tuning mechanisms
Spectral tuning range for $\alpha > 100$ nm	15 nm	50 nm	65 nm
Constant Q-factor	-	-	+
Range in which transmission is less than 10% (for 1 nm resolution)	2700-3500 nm	2600-3000 nm	2600-3000 nm
Flat MEMS structure	-	+	+

In summary, this chapter section introduces a new way to control q-BIC resonances in all-silicon metasurfaces. This is achieved by breaking out-of-plane symmetry with a supplementary 2D structure on top of a 1D metasurface where the q-BIC mode is excited. By changing the distance between the 2D MEMS structure and the 1D metasurface, we gain precise control over the Q-factor of the q-BIC resonance. Moreover, by combining two independent orthogonal tuning mechanisms within the proposed tunable metasurface designs, we demonstrated a spectral tuning range of more than 60 nm for the ultranarrow q-BIC resonance in transmission while maintaining a constant Q-factor and out-of-band transmission below 0.1 across a broad range.

The proposed all-dielectric metasurfaces achieve an ultranarrow resonance linewidth with dynamic tunability in the infrared range, making them promising candidates for advanced tunable filtering applications highly relevant for spectroscopy, remote sensing, and imaging. These findings are particularly promising for mid-infrared molecular sensing, where many environmentally and industrially relevant gases exhibit strong and distinct absorption features. The demonstrated ability to mechanically tune ultranarrow resonances with high precision enables selective and sensitive detection of various molecules, offering strong potential for applications such as environmental monitoring, atmospheric science, industrial safety, astronomy, and defence.

This theoretical work outlines pathways for the experimental realisation of sub-nm resolution filters combined with a wide tuning range, enabling advancements in

spectroscopy and hyperspectral imaging. The proposed tunable metasurfaces can be fabricated using techniques similar to those employed for nano-electromechanical tunable suspended gratings [184]. In this approach, a buffered silicon oxide layer beneath a 1D metasurface is partially etched to achieve suspension, while the anchors remain supported by the oxide layer, ensuring structural stability.

However, we acknowledge that the experimental implementation of two independent orthogonal tuning mechanisms presents greater challenges. The bandwidth and Q-factor of the proposed metasurfaces are expected to be constrained by fabrication accuracy. Variations in the sizes of the bars, cuboids, or perforated holes in the membranes may lead to a broadening of the q-BIC resonances. The parallel motion of nanostructures at such small distances presents another practical challenge. Misalignments may cause additional broadening of resonances in experiments, which can be mitigated by positioning the MEMS structures farther from the 1D metasurface. Notably, the proposed tunable metasurface designs do not require MEMS membranes to approach the 1D metasurface closer than a few hundred nanometres, which greatly simplifies experimental realisation.

The horizontal movement of cuboids or membranes does not provide significant tunability of the q-BIC resonance wavelength while maintaining a 1 nm bandwidth. This suggests that achieving significant tunability of the q-BIC resonance wavelength within the studied metasurfaces requires modifications to the bars, where the maximum electric field is observed at resonance.

An alternative approach to realising q-BIC resonances could be based on introduction of asymmetry within the meta-atom by positioning structures near one section of the meta-atom while leaving the other section unaffected by any additional layers. This method resembles in-plane symmetry breaking [189,190], where geometric asymmetry enables resonance control. However, this approach will likely require MEMS structures to move close to 1D metasurfaces, which may pose challenges due to imperfections in the fabricated meta-structures.

This work extends the exploration of additional degrees of freedom for manipulating the characteristics of q-BIC resonances and can be integrated with previously proposed permittivity-asymmetric methods [191] that rely on the electric control of silicon properties, as well as other approaches utilising the thermal tuning of silicon [185]. MEMS structures that can be moved close to metasurfaces, particularly near high-symmetry points of undisturbed BIC modes, could act as perturbative elements and couplers to the radiation continuum [192]. The out-of-plane dimension can be leveraged to control optical chirality [193] and other key characteristics of metasurfaces, further enhancing the range of applications for mechanically tunable q-BIC resonances.

2.4 Conclusion

In this chapter, we examined various approaches for the mechanical tuning of metasurface characteristics, with a particular focus on MEMS-enabled control methods. We first introduced mechanically reconfigurable metasurfaces, highlighting their suitability for dynamic optical applications. We then discussed the EOT phenomenon and demonstrated how MEMS technology can be employed to actively modulate EOT resonances in plasmonic metasurfaces. We further showed that static EOT-based metasurfaces can enhance fluorescence imaging by serving as compact optical filters integrated onto fibre tips. Finally, we presented a new approach for dynamic control of q-BIC resonances in all-dielectric metasurfaces, utilising mechanical actuation to achieve precise modulation of their optical response.

Chapter 3: Control of resonances in toroidal metasurfaces using phase change materials

In this chapter, we investigate active toroidal metasurfaces that support high-Q resonances. We begin by introducing the concept of toroidal metasurfaces and discussing various approaches for achieving tunability. We then demonstrate how vanadium dioxide, a widely used phase-change material, enables dynamic control of q-BIC resonances in toroidal metasurfaces. This control is realised by heating vanadium dioxide thin-film patches integrated into a metasurface comprising gold SRRs on a sapphire substrate, allowing precise tuning of both the amplitude and frequency of the resonances by exploiting their sensitivity to material losses. Building on this approach, we further show that breaking the symmetry of the meta-atoms provides an additional mechanism for enhanced tunability. Using these methods, we achieve a change in the q-BIC resonance amplitude by more than an order of magnitude for a temperature variation of approximately 10 °C. In addition, we analyse the contribution of the toroidal dipole component to the excitation of q-BIC resonances through multipole expansion and reconstruct the reflection and transmission characteristics of the metasurfaces from the power scattered by the multipoles, thereby verifying the accuracy of our calculations. Finally, we discuss potential applications of the proposed active toroidal metasurfaces in room-temperature bolometric detectors, tunable filters and modulators.

The findings presented in this chapter are reported in the publication listed below, where I am the first author. My contributions include formulating the initial idea, developing the theoretical framework, performing numerical modelling, designing the metasurfaces, drafting the manuscript, and preparing its final version.

F. V. Kovalev, A. E. Miroshnichenko, A. A. Basharin, H. Toepfer, and I. V. Shadrivov, Active Control of Bound States in the Continuum in Toroidal Metasurfaces, *Advanced Photonics Research* **6**, 2400070 (2025).

3.1 Active toroidal metasurfaces based on bound states in the continuum

Over the past fifteen years, significant progress has been made in the study of toroidal metamaterials [194,195]. They were first introduced in 2007 [196], with experimental demonstrations of toroidal dipole response occurring in the microwave range in 2010 [197]. Subsequent investigations have extended their realisation to higher frequencies, including the optical range [198,199].

In metamaterials, the excitation of the toroidal dipole, a component of the multipole expansion [200], is intricately tied to the specific design and arrangement of subwavelength meta-atoms. Special design of the current distribution within meta-atoms is essential to induce the toroidal dipole moment. The required configuration of current loops can be achieved by carefully engineering the meta-atoms' geometry and material properties.

Toroidal metamaterials have found applications in many fields, including detectors, sensors, modulators, emitters, and waveguides [195]. One of their fascinating aspects lies in the ability to excite resonances with remarkably high Q-factors and narrow bandwidth [201–204]. By adjusting electric and toroidal dipole components, one can create an anapole state when their radiation is cancelled in the far field [198,201,205,206]. Metamaterials with anapole configurations can exhibit unique properties, such as near-zero scattering or enhanced field confinement, which is utilised in cloaking devices and advanced optics.

In the realm of metamaterials, BIC [161,162,164,207] refer to specific wave configurations that maintain localisation within the material, showcasing zero bandwidth and maintaining complete confinement without any radiation. This becomes possible due to several mechanisms [199,208], for example, due to the symmetry of the mode not allowing the coupling to propagating waves. Recent studies have shown that this phenomenon is one of the ways to achieve very high Q-factors [209,210]. There is a critical distinction between anapoles and non-radiating eigenmodes, such as bound

states in the continuum or embedded eigenstates. Unlike bound states in the continuum, anapole is not an eigenmode of the cavity, it cannot sustain oscillations on its own to meet boundary conditions, and can be observed only in scattering [208]. Instead, anapole represents a resonant distribution of fields that can be activated by a specific impinging excitation.

The concept of the toroidal dipole BIC was initially introduced in all-dielectric optical metasurfaces [211]. Symmetry breaking in silicon metasurfaces was demonstrated to excite ultrahigh-Q toroidal dipole leaky resonances. More recently, it was shown that high Q-factors of metal toroidal metamaterials can also be explained by using the concept of BIC [168,202]. Notably, the Q-factor exceeding 10^3 for a toroidal dipole q-BIC resonance in a metasurface composed of aluminium SRRs on a 50 μm -thick cyclic olefin copolymer substrate was recently reported in the terahertz range [202]. Toroidal dipole BIC metasurfaces stand out as promising candidates for ultrasensitive measurements due to their substantial changes in resonance frequency and amplitude with the slightest variation in the surrounding media. They provide a platform for a strong electric and magnetic field enhancement and can be used for creating compact lasers.

Active control of toroidal dipole q-BIC resonances represents a promising avenue for further exploration and research. Recently, dynamic tuning of toroidal metasurface characteristics has been achieved using thin films of silicon [212], graphene [213–216], phase-change materials such as GST (Germanium-Antimony-Tellurium) [217] and VO_2 (vanadium dioxide) [218–220], and other techniques, for example, integration of diodes [221] or utilisation of microelectromechanical systems [222,223]. Tunable toroidal metamaterials have unlocked new opportunities for resonant applications and multifunctional devices with remarkable potential [195].

Recently, a concept of metabolometers has been introduced, leveraging the unique properties of toroidal metamaterials [224]. The metabolometer combines a high Q-factor toroidal metamaterial with an embedded micro-pad superconductor, serving as an absorber for terahertz radiation. The practical implementation involves a terahertz beam

heating the absorber while a plane gigahertz wave wirelessly reads out the absorber's temperature, unveiling a novel approach with potential implications for advancing terahertz/millimetre-wave detectors in the future.

VO₂ stands out as a promising material in the realms of both bolometric detection [225–232] and active photonics [55,68,69] thanks to its distinctive semiconductor-to-metal transition (SMT) near room temperature. Bolometers are widely utilised devices for measuring incident electromagnetic radiation energy [226]. They rely on changes in the electrical resistance of a thermosensitive absorbing material when it is heated due to the absorption of an incident wave.

VO₂'s sharp transition and reversible phase change enable sensitive temperature-dependent measurements, facilitating rapid and accurate detection of incident radiation. The material's broad sensitivity across wavelengths makes it valuable for applications like infrared and terahertz imaging. In active photonics, VO₂'s tunable optical properties and switchable characteristics offer dynamic control over refractive indices and the modulation of light transmission or reflection.

VO₂ is a remarkable material known for its intriguing phase change transition. At temperatures below approximately 68 degrees Celsius, VO₂ is in its semiconducting phase, characterised by a distorted monoclinic crystal structure. As the temperature rises beyond this point, a rapid and reversible phase transition occurs, leading to the metallic phase due to a tetragonal crystal structure. This transition is accompanied by significant changes in electrical and optical properties, making VO₂ a valuable material for various applications. A crucial parameter in comprehending VO₂'s behaviour during the phase transition is the large temperature coefficient of resistance (TCR). The substantial TCR is pivotal for applications requiring sensitive temperature-dependent measurements, and underscores VO₂'s suitability for tunable devices.

In this chapter, we propose that thin film patches of VO₂ integrated into the toroidal metasurface can dramatically change the amplitude and frequency of q-BIC resonances

as the temperature changes. The proposed tunable metasurface comprises an array of meta-atoms formed by two gold SRRs on a sapphire substrate. The phase change transition of VO_2 occurs in a narrow temperature range, providing a large variation in its conductivity. This leads to a change in the transmission at the frequency of the q-BIC resonance and, accordingly, in the amplitudes of the multipole expansion components, demonstrating a high sensitivity to the losses present in the system. Additionally, we analyse the behaviour of q-BIC resonances when introducing asymmetry in the geometry of the meta-atom. The amplitude of resonances increases with the degree of asymmetry, corresponding to the excitation of symmetry-protected q-BIC resonances. The proposed tunable metasurface holds potential applications in various fields, including active photonic systems, room-temperature terahertz and infrared bolometers, as well as multifunctional metadevices.

3.2 Bound states in the continuum in toroidal metasurfaces

We study the metasurface in the form of a square array of meta-atoms illustrated in Figure 3-1(a). Meta-atoms are shown in Figure 3-1(b), and in this study we manipulate their central and outer gaps. The size of the unit cell p is $475 \mu\text{m} \times 475 \mu\text{m}$ with the central gap size $dc = 40 \mu\text{m}$. The outer radius of the resonator $R = 235 \mu\text{m}$, the track width $w = 30 \mu\text{m}$, the thickness of the resonator $t = 0.51 \mu\text{m}$. The outer gap size dl is used to tune the q-BIC resonance. The linearly polarised wave with the E-field parallel to the central microstrip is normally incident on the metasurface.

The proposed metasurface was modelled as an array of meta-atoms using the frequency domain solver and the Floquet boundary condition in CST Studio. We first study the symmetrical meta-atoms without a substrate. We use the Drude model for gold in the terahertz range [233]. Figure 3-1(c) shows the q-BIC resonance for different outer gaps of the resonators. The proposed metasurfaces can support q-BIC resonances by tuning structure parameters. When $dl = 50 \mu\text{m}$, the resonance disappears, corresponding to the accidental BIC, which has zero bandwidth and infinite Q-factor in the ideal case of perfect electric conductors. We analysed the vanishing resonance at 314.8 GHz for $dl = 50 \mu\text{m}$

using the Eigenmode solver in CST Studio. For this purpose, toroidal resonators were modelled as perfect electric conductors. Figure 3-2 shows surface currents of the BIC eigenmode forming two current loops with opposite flow directions in the meta-atom.

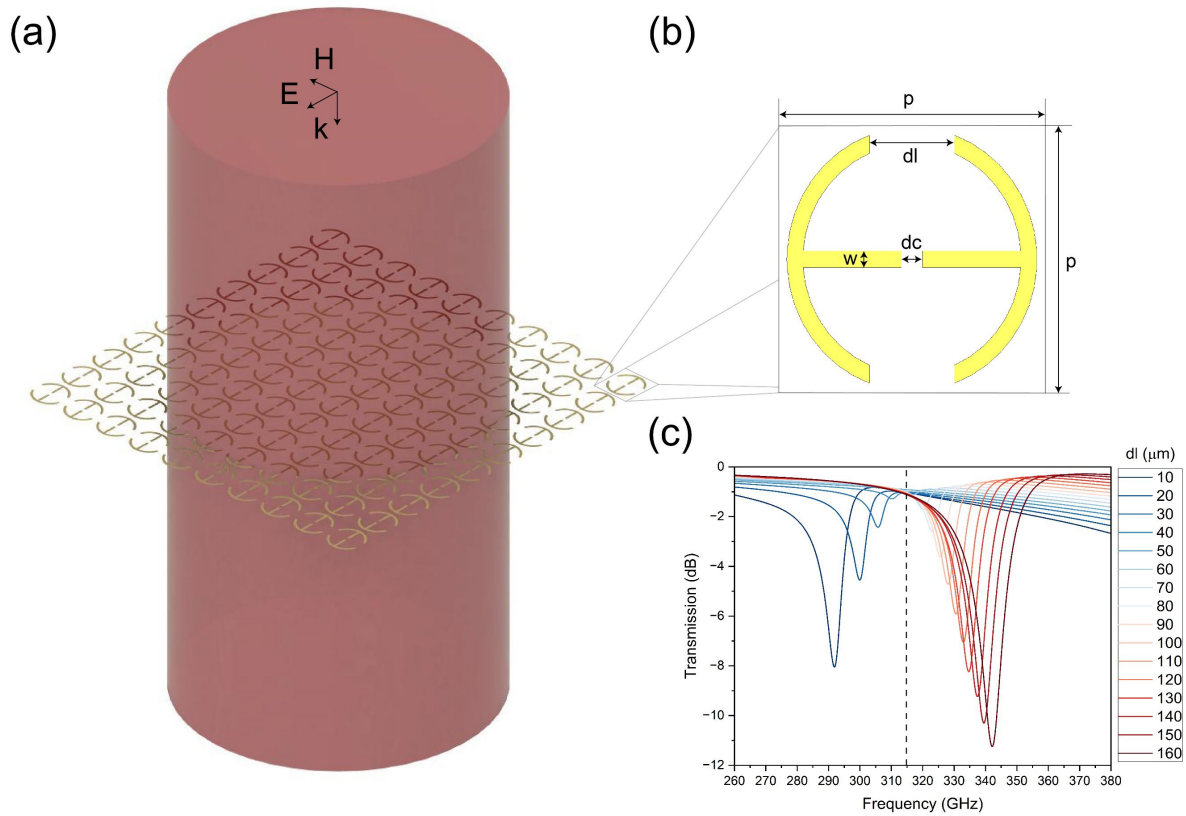


Figure 3-1 Schematic of the simulation setup showing a normally incident plane electromagnetic wave interacting with a metasurface (a); geometry of the meta-atom (b); transmission coefficient for different outer gap sizes (c). The dotted vertical line indicates the BIC at approximately 315 GHz.

Any losses dramatically reduce the amplitude of such q-BIC resonances that can have an extremely high Q-factor ($\geq 10^4$) in ideal metasurfaces composed of perfect electric conductors. We observe that losses in gold reduce the Q-factor, so the resonance band becomes much wider, and its amplitude decreases. For example, the transmission of the proposed metasurface at the q-BIC resonance frequency is around -6 dB in the case of gold and below -50 dB in the case of perfect electric conductors for the same size of the outer gap $dl = 100 \mu\text{m}$.

Figure 3-3(a) shows that the electric field at the resonant frequency (242 GHz) for the outer gap size of 160 μm is mostly localised in the central gap of the meta-atoms. Figure 3-3(b) shows the magnetic field at the resonant frequency. Figure 3-3(c) shows currents at the resonant frequency creating two loops with opposite flow directions.

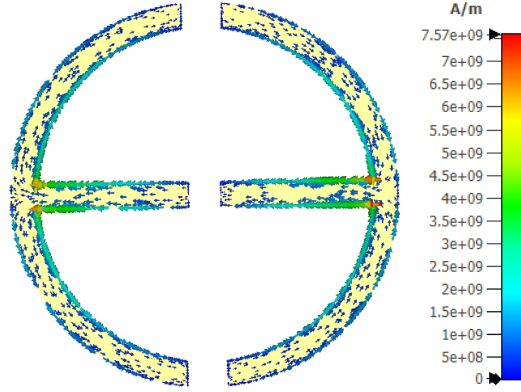


Figure 3-2 Surface currents of the BIC eigenmode in the meta-atom.

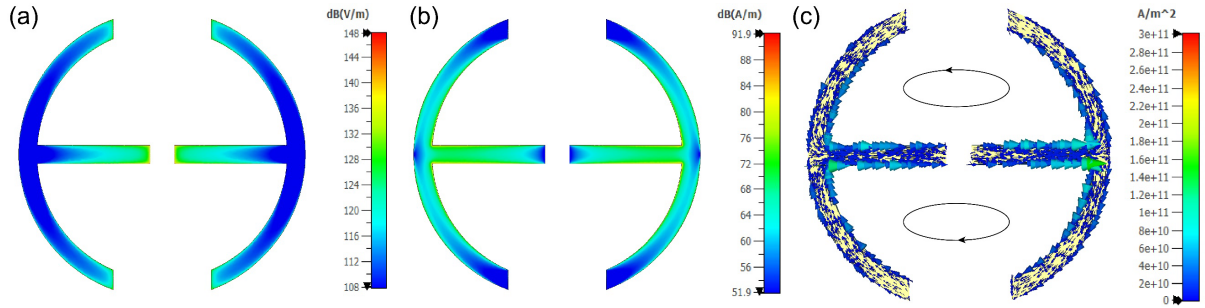


Figure 3-3 Electric (a) and magnetic (b) fields and current density (c) within the designed meta-atom at the q-BIC resonance frequency for the outer gap size (dl) equal to 160 μm .

Figures 3-4(a,b) show reflection and transmission of the metasurface for $dl = 50 \mu\text{m}$ and 160 μm . Power scattered by individual multipoles shown in Figures 3-4(c,d) was calculated by integrating the current density over the volume using the following formulas for the exact multipole moments [234–236]:

$$p_{\alpha} = -\frac{1}{i\omega} \int J_{\alpha} j_0(kr) d^3r, \quad (3.1)$$

$$m_{\alpha} = \frac{3}{2} \int (\mathbf{r} \times \mathbf{J})_{\alpha} \frac{j_1(kr)}{kr} d^3r, \quad (3.2)$$

$$T_{\alpha}^{(e)} = -\frac{k^2}{2i\omega} \int [3(\mathbf{r} \cdot \mathbf{J})r_{\alpha} - \mathbf{r}^2 J_{\alpha}] \frac{j_2(kr)}{(kr)^2} d^3r, \quad (3.3)$$

$$Q_{\alpha\beta}^{(e)} = -\frac{3}{i\omega} \int [3(r_{\alpha}J_{\beta} + r_{\beta}J_{\alpha}) - 2\delta_{\alpha\beta}(\mathbf{r} \cdot \mathbf{J})] \frac{j_1(kr)}{kr} d^3r, \quad (3.4)$$

$$T_{\alpha\beta}^{(Qe)} = -\frac{6k^2}{i\omega} \int [5r_{\alpha}r_{\beta}(\mathbf{r} \cdot \mathbf{J}) - (r_{\alpha}J_{\beta} + r_{\beta}J_{\alpha})\mathbf{r}^2 - \mathbf{r}^2(\mathbf{r} \cdot \mathbf{J})\delta_{\alpha\beta}] \frac{j_3(kr)}{(kr)^3} d^3r, \quad (3.5)$$

$$Q_{\alpha\beta}^{(m)} = 15 \int [(\mathbf{r} \times \mathbf{J})_{\alpha}r_{\beta} + (\mathbf{r} \times \mathbf{J})_{\beta}r_{\alpha}] \frac{j_2(kr)}{(kr)^2} d^3r, \quad (3.6)$$

$$O_{\alpha\beta\gamma}^{(e)} = \frac{15i}{\omega} \int (r_{\gamma}r_{\beta}J_{\alpha} + r_{\gamma}r_{\alpha}J_{\beta} + r_{\alpha}r_{\beta}J_{\gamma} - A_{\alpha\beta\gamma}) \frac{j_2(kr)}{(kr)^2} d^3r, \quad (3.7)$$

$$P_{scat} = \frac{k^4}{12\pi\epsilon_0^2 c \mu_0} |p + T^{(e)}|^2 + \frac{k^4}{12\pi\epsilon_0 c} |m|^2 + \frac{k^6}{1440\pi\epsilon_0^2 c \mu_0} |Q^{(e)} + T^{(Qe)}|^2 + \\ + \frac{k^6}{1440\pi\epsilon_0 c} |Q^{(m)}|^2 + \frac{k^8}{3780\pi\epsilon_0^2 c \mu_0} |O^{(e)}|^2, \quad (3.8)$$

where p_{α} is the electric dipole moment, m_{α} is the magnetic dipole moment, $T_{\alpha}^{(e)}$ is the toroidal electric dipole moment, $Q_{\alpha\beta}^{(e)}$ is the electric quadrupole moment, $T_{\alpha\beta}^{(Qe)}$ is the toroidal electric quadrupole moment, $Q_{\alpha\beta}^{(m)}$ is the magnetic quadrupole moment, $O_{\alpha\beta\gamma}^{(e)}$ is the electric octupole moment, $A_{\alpha\beta\gamma} = \delta_{\alpha\beta}V_{\gamma} + \delta_{\beta\gamma}V_{\alpha} + \delta_{\alpha\gamma}V_{\beta}$, $\mathbf{V} = \frac{1}{5}[2(\mathbf{r} \cdot \mathbf{J})\mathbf{r} - \mathbf{r}^2\mathbf{J}]$, $j_n(kr)$ are the spherical Bessel functions of the first kind of the order n , ω is the frequency, $\mathbf{J}$ is the current density, $\mathbf{r}$ is the radius vector, δ is the Kronecker delta symbol, k is the wave number, c is the speed of light, ϵ_0 and μ_0 are the electric and magnetic constants, indices α, β, γ correspond to x, y, z components. In this work, the center of coordinates for the purpose of the multipole expansion is chosen at the middle of the unit cell.

We observe four orders of magnitude increase in the scattered power by the toroidal dipole (T) near the q-BIC resonance in Figures 3-4(c,d) when changing the outer gap value dl from 50 to 160 μm , respectively. The power scattered by multipoles was calculated in Matlab using Eq. 3.1-3.8 and the current density exported from CST Studio. The power

scattered by the electric dipole (P) remains significantly higher than the power scattered by the toroidal dipole (T) and the magnetic quadrupole (Q_m). The magnetic quadrupole plays a parasitic role in such toroidal metamaterials [201]. We also found that the electric octupole (O_e), which has not been considered in previous studies, plays an important role in the excitation of the q-BIC resonances. The power scattered by other multipoles (the magnetic dipole M , the electric quadrupole Q_e and the toroidal electric quadrupole T_{qe}) is significantly lower. The obtained results agree with previous studies of toroidal metamaterials [201,202].

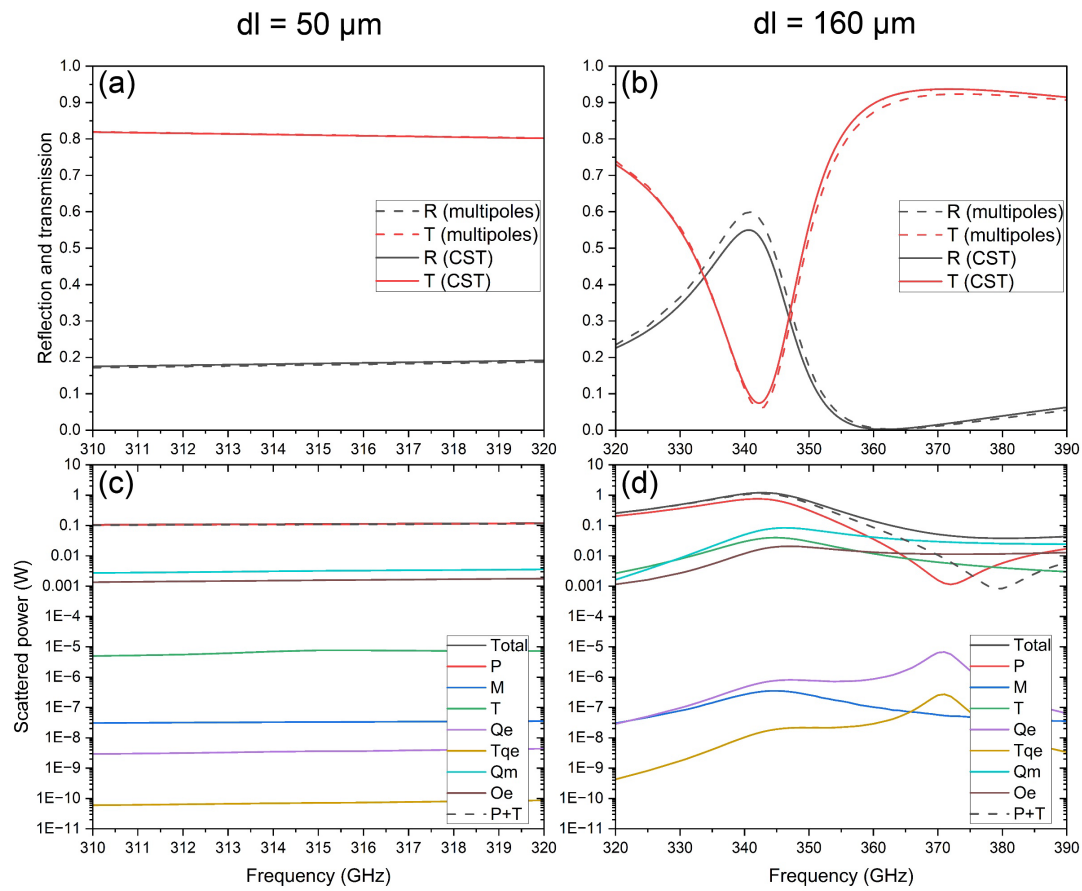


Figure 3-4 Reflection and transmission of the metasurface obtained in CST Studio (solid) and calculated from the multipole decomposition (dashed) and power scattered by multipoles for the outer gap size $dl = 50 \mu\text{m}$ (a, c) and $dl = 160 \mu\text{m}$ (b, d).

Comparing the reflection and transmission characteristics (directional) with the scattered power by multipoles (non-directional) does not allow for a clear conclusion about their contribution to the metasurface characteristics. However, the transmission

minimum frequency in Figure 3-4(b) corresponds to the maximum overall scattered power by multipoles in Figure 3-4(d). Additionally, we provide the reconstruction of the reflection and transmission characteristics from the multipole components [237] in Figures 3-4(a,b) and compare their contribution. Figure 3-5 shows the reflection and transmission characteristics of the metasurface with the outer gap size $dl = 160 \mu\text{m}$ obtained directly from numerical modelling in CST and calculated from the exact multipoles using the following expression for one of the incident plane wave polarisations (with the E-field parallel to the central microstrip):

$$r = \frac{ik}{2E_0 S_L \varepsilon_0} \left[p_x - \frac{m_y}{c} + t_x + \frac{ik(Q_{xz}^{(e)} + T_{xz}^{(Qe)})}{6} - \frac{ikQ_{yz}^{(m)}}{6c} - \frac{k^2 O_{xxx}^{(e)}}{6} \right], \quad (3.9)$$

$$t = 1 + \frac{ik}{2E_0 S_L \varepsilon_0} \left[p_x + \frac{m_y}{c} + t_x - \frac{ik(Q_{xz}^{(e)} + T_{xz}^{(Qe)})}{6} - \frac{ikQ_{yz}^{(m)}}{6c} - \frac{k^2 O_{xxx}^{(e)}}{6} \right], \quad (3.10)$$

where p – the electric dipole, m – the magnetic dipole, t – the toroidal electric dipole, $Q^{(e)}$ – the electric quadrupole, $T^{(Qe)}$ – the toroidal electric quadrupole moment, $Q^{(m)}$ – the magnetic quadrupole, $O^{(e)}$ – the electric octupole, k is the wave number, c is the speed of light, E_0 is the electric field of the normally incident plane wave at the geometric center of the meta-atom, S_L is the area of the lattice unit cell ($S_L = p^2$ for the square lattice, where p is the period), and ε_0 is the vacuum permittivity. The reflection and transmission coefficients are $R = |r|^2$ and $T = |t|^2$.

We compare the contribution of the dipoles, quadrupoles and electric octupole by plotting the reflection and transmission coefficients calculated from the corresponding multipole moments shown in Figure 3-5. By including only electric and magnetic dipoles in Eq. 3.9 and 3.10, we can qualitatively reconstruct the metasurface characteristics. To further analyse the toroidal dipole contribution, we added this component to the previously considered electric and magnetic dipoles. Figure 3-5(a) illustrates that the magnitude of the resonance in reflection increases by approximately 1.5 times in this case. The electric and magnetic quadrupoles reduce the amplitude of the resonance in reflection by nearly half. Finally, the electric octupole increases the reflection coefficient

to nearly the same value as when only considering the electric and magnetic dipoles. Figure 3-5(b) also shows the contributions of the above-mentioned multipoles to the transmission characteristics of the developed metasurface. These results offer important insights into the nature of the q-BIC resonance.

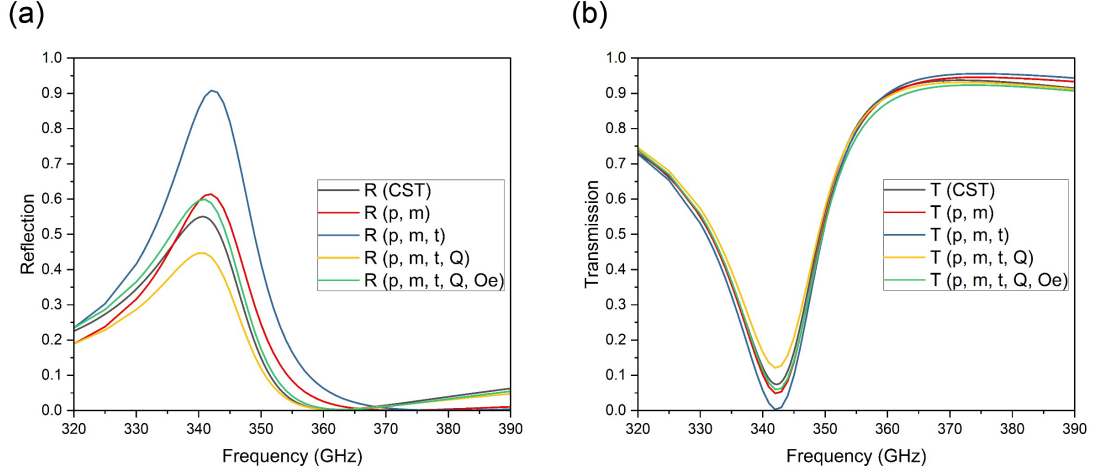


Figure 3-5 Reflection (a) and transmission (b) characteristics of the metasurface with the outer gap size (dl) of $160\ \mu\text{m}$ calculated from exact multipoles.

Additionally, Figures 3-4(a,b) demonstrate the reflection and transmission of the metasurface calculated from the exact multipole moments using Eq. 3.1-3.10. Good agreement with the results of full numerical simulations demonstrates the high accuracy of the exact expressions for multipole expansion.

The toroidal dipole component plays a substantial role in the excitation of the q-BIC resonance. It should be noted that the q-BIC resonances and the metasurface in this study can be referred to as toroidal due to the possibility of making the toroidal component dominant (the same order as the electric dipole) by carefully optimising the meta-atom structure and using metals with low losses [201,202]. This would also lead to a very high Q-factor and shift the frequency of the anapole state (the destructive interference of the electric and toroidal dipoles: P+T minimum in Figure 3-4(d)) close to the BIC shown in Figure 3-1.

We analysed the Q-factor of the excited q-BIC resonance by fitting the transmission spectrum with a Fano formula [188]:

$$T_{Fano}(\omega) = t_{d_0} + t_d^2 \frac{\left[q + \frac{2(\omega - \omega_0)}{d\omega} \right]^2}{1 + \frac{4(\omega - \omega_0)^2}{d\omega^2}}, \quad (3.11)$$

where t_{d_0} is the minimum of transmission near resonance, t_d is the non-resonant transmission amplitude, q is the shape factor, ω_0 is the resonance frequency, $d\omega$ is the resonance bandwidth. The Q-factor can be found as $Q = \frac{\omega_0}{d\omega}$ and is 27.5 when $dl = 160 \mu\text{m}$. The dependence of the Q-factor and bandwidth of the q-BIC resonance on the outer gap size dl and the collision frequency is detailed in Figure 3-6.

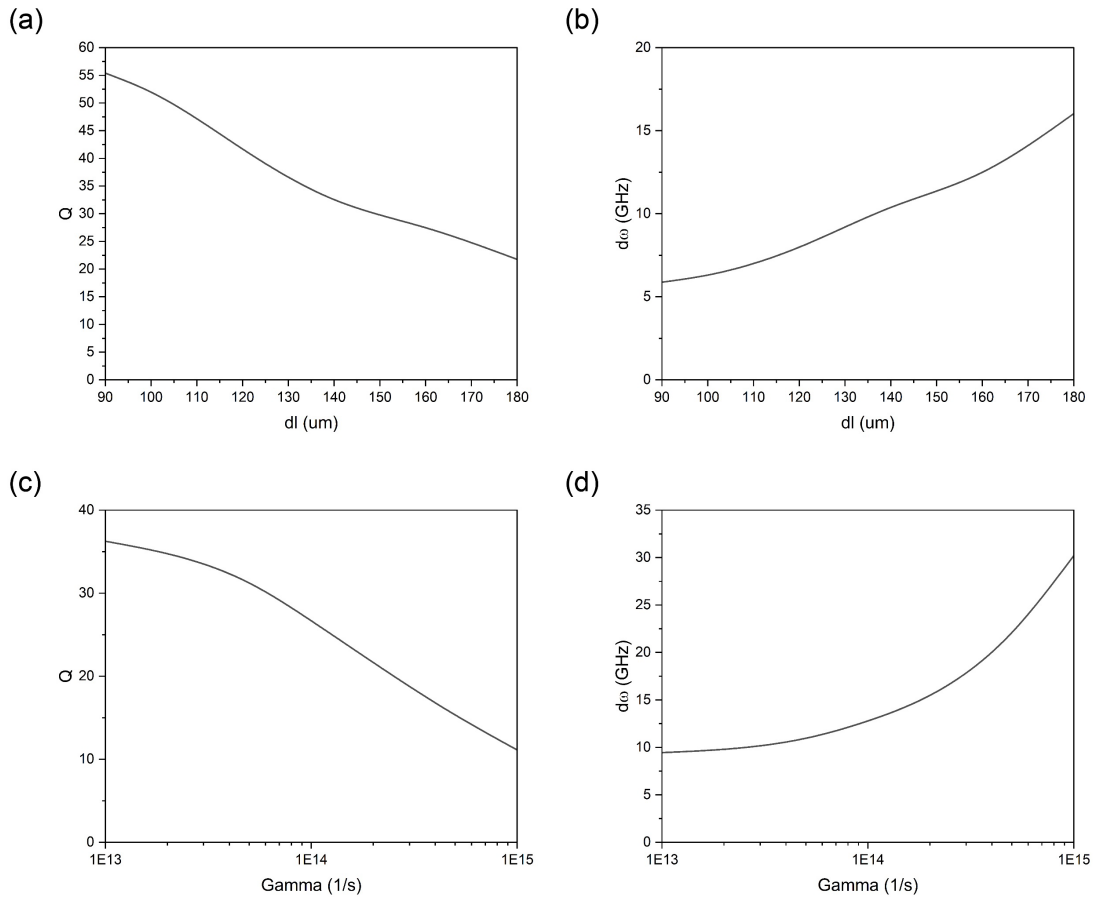


Figure 3-6 The q-BIC resonance Q-factor and bandwidth for different outer gap sizes (dl) when $dc = 40 \mu\text{m}$ (a, b) and collision frequencies (gamma) corresponding to different resonator losses when $dl = 160 \mu\text{m}$ (c, d).

Figures 3-6(a,b) illustrate the Q-factor and bandwidth of the q-BIC resonances for different outer gap sizes (dl) when $dc = 40 \mu\text{m}$ and collision frequencies (γ) corresponding to different resonator losses when $dl = 160 \mu\text{m}$. As the resonance disappears when approaching $dl = 50 \mu\text{m}$, the analysis relying on the fitting of the transmission spectrum with a Fano formula becomes inaccurate. Therefore, we considered the interval of dl from 90 to 180 μm . The Q-factor increases, while the bandwidth decreases, almost linearly with decreasing dl . Since we used the Drude model for gold, we also showed the Q-factor and resonance bandwidth for different collision frequencies to analyse their dependence on the resonator losses. Figures 3-6(c,d) show that the Q-factor increases, while the bandwidth decreases, with decreasing collision frequency, emphasising the need for optimisation of the meta-atom and using metals with lower losses to achieve ultrahigh Q-values of q-BIC resonances.

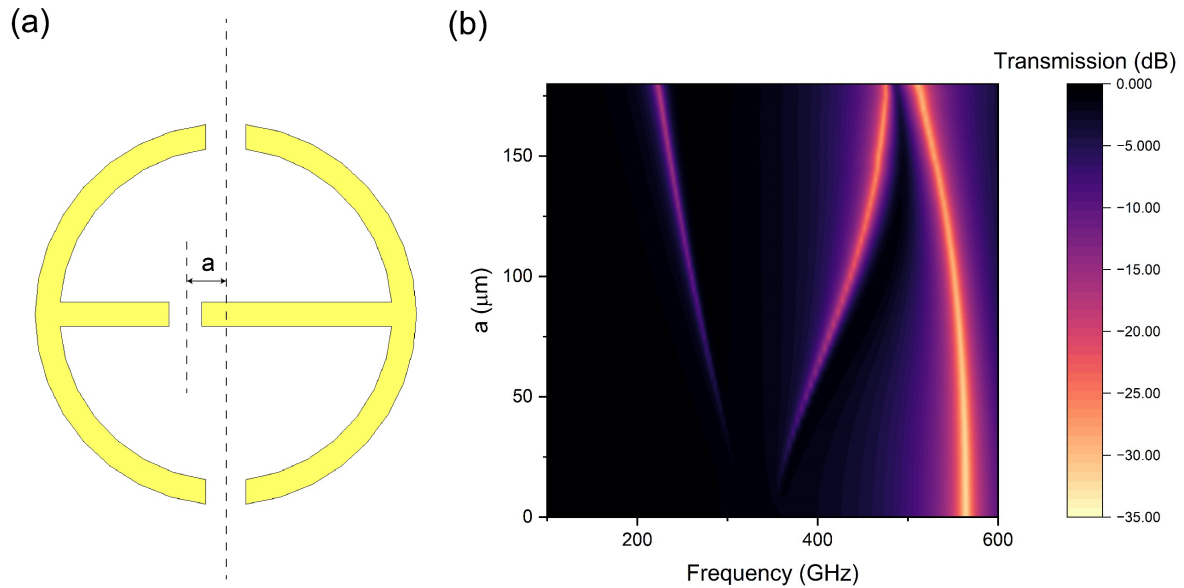


Figure 3-7 Asymmetric meta-atom with an offset parameter a (a) and the metasurface transmission spectrum for different a (b).

Next, we study the behavior of q-BIC resonances when the central gap is shifted from the center of the meta-atom, as shown in Figure 3-7(a). The transmission characteristics for the different offset a for the fixed outer gap size $dl = 50 \mu\text{m}$ are shown in Figure 3-7(b). When $a = 0 \mu\text{m}$, there is no q-BIC resonance. We found that two resonances appear in the asymmetric structure when $a > 0 \mu\text{m}$. Asymmetrical meta-atoms in gold metasurfaces

can induce q-BIC resonances through symmetry breaking, and their amplitude increases with the asymmetry parameter.

Figures 3-8(a,b) show the transmission and reflection characteristics of the metasurface with broken symmetry obtained in CST Studio and calculated from the exact multipole moments using Eq. 3.1-3.10 for two values of the asymmetry parameter: $50\ \mu\text{m}$ and $180\ \mu\text{m}$. Power scattered by multipoles is shown in Figures 3-8(c,d). Both resonances – the first at $289\ \text{GHz}$ ($a = 50\ \mu\text{m}$) and $223\ \text{GHz}$ ($a = 180\ \mu\text{m}$) and the second at $387\ \text{GHz}$ ($a = 50\ \mu\text{m}$) and $475\ \text{GHz}$ ($a = 180\ \mu\text{m}$) – notably feature a robust contribution from the electric quadrupole (Q_e).

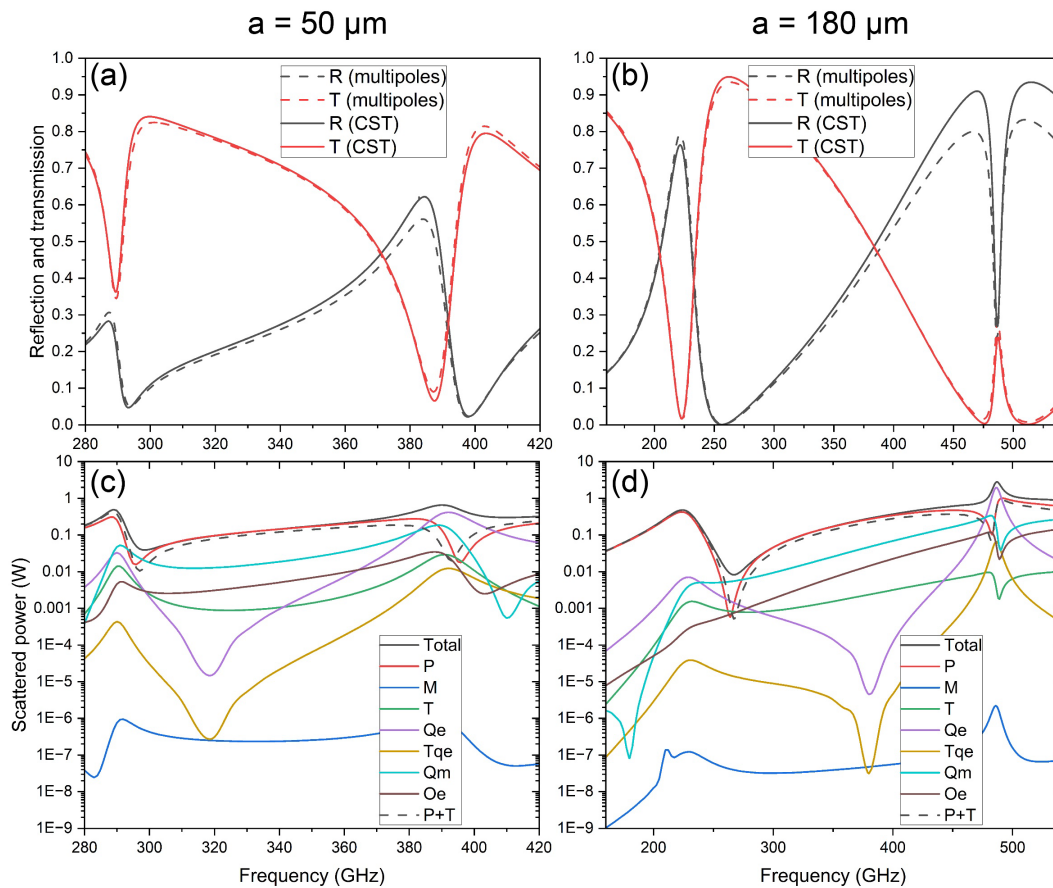


Figure 3-8 Transmission and reflection characteristics of the metasurface with broken symmetry obtained in CST Studio (solid) and calculated from the multipole decomposition (dashed) and power scattered by multipoles for two offset parameters a : $50\ \mu\text{m}$ (a,c) and $180\ \mu\text{m}$ (b,d).

Figure 3-9 illustrates the current density in the meta-atom with broken symmetry for two parameters of asymmetry a : $50\ \mu\text{m}$ at $289\ \text{GHz}$ (a) and $387\ \text{GHz}$ (b); $180\ \mu\text{m}$ at $223\ \text{GHz}$ (c) and $475\ \text{GHz}$ (d), respectively. The currents at the lower resonant frequency have a maximum in the right section, while those at the higher resonant frequency have a maximum in the left section. This behaviour can be explained by the difference in the length of the central microstrip when the symmetry is broken.

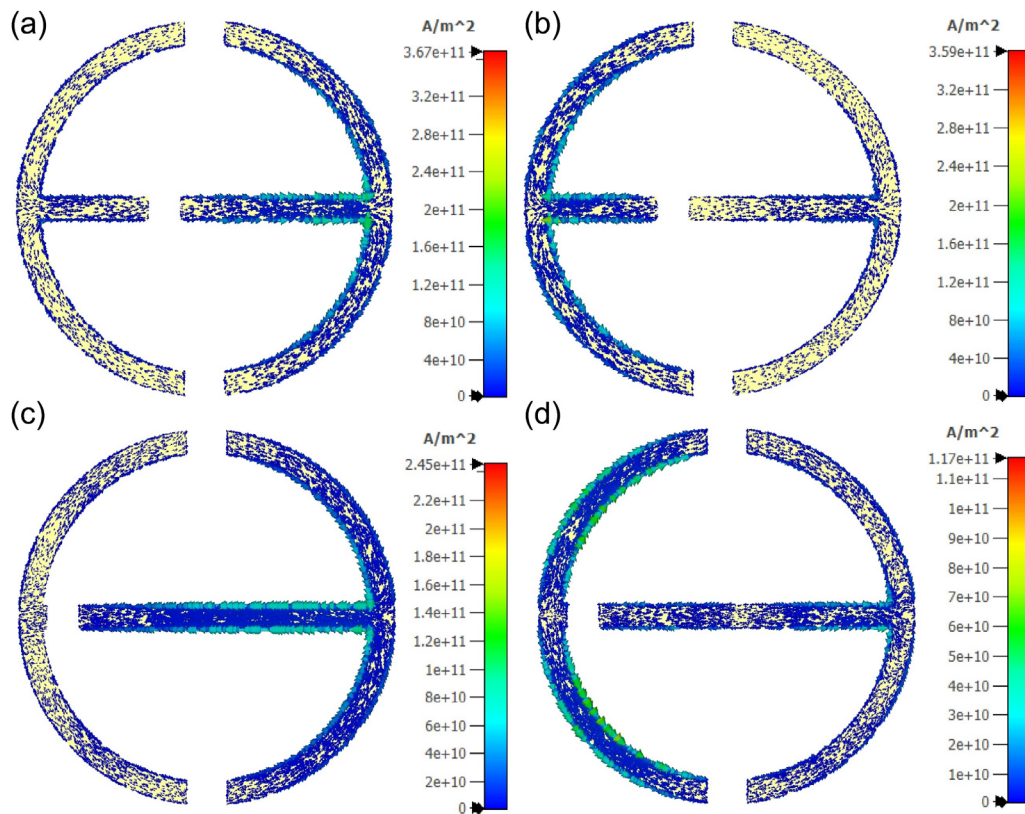


Figure 3-9 Current density in the meta-atom with broken symmetry for two offset parameters a : $50\ \mu\text{m}$ at $289\ \text{GHz}$ (a) and $387\ \text{GHz}$ (b), $180\ \mu\text{m}$ at $223\ \text{GHz}$ (c) and $475\ \text{GHz}$ (d), respectively.

3.3 Control of bound states in the continuum in toroidal metasurfaces using vanadium dioxide

Next, we study a metasurface consisting of gold SRRs with integrated thin film patches of VO_2 . The Drude model can be employed to characterise the permittivity of VO_2 within the terahertz range [218,238], providing an effective representation of charge carriers and aiding researchers in predicting dynamic changes in resistivity and conductivity associated with the phase transition:

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2(\sigma)}{\omega^2 + i\gamma\omega}, \quad (3.12)$$

where ε_{∞} is the permittivity at high frequencies, $\omega_p(\sigma)$ is the conductivity dependent plasma frequency and γ is the collision frequency. The plasma frequency can be approximately expressed as $\omega_p^2(\sigma) = \frac{\sigma}{\sigma_0} \omega_p^2(\sigma_0)$. The following constants were used in this modelling [238]: $\sigma_0 = 3 \times 10^5$ S/m, $\varepsilon_{\infty} = 12$, $\omega_p(\sigma_0) = 1.4 \times 10^{15}$ rad/s, $\gamma = 5.75 \times 10^{13}$ rad/s.

For the final metasurface design we used the substrate with the following characteristics: $\varepsilon_r = 11.5$ and $\tan\delta = 0.0001$, which corresponds to sapphire. The thickness of the substrate was chosen to be 1 μm to avoid Fabry-Perot resonances, as well as to obtain a higher Q-factor. However, such an ultrathin substrate will require mechanical support by a frame. We chose the outer gap size of 160 μm for the integration of VO_2 thin film patches due to the significant q-BIC resonance amplitude (over 10 dB). The central gap can be loaded with a tuning element, a thin film patch (100 nm thick) of VO_2 sized at 40 μm x 5 μm . The tunable meta-atom is shown in Figure 3-10(a). Modulating the electrical conductivity of the film by varying the material's temperature induces changes in the transmission coefficient and phase of transmitted waves, as illustrated in Figures 3-10(c,e). This study was guided by the following experimental data obtained for 107-nm-thick VO_2 films on sapphire [230]: the conductivity values for temperatures of 70°C (close to the maximum TCR) and 80°C were measured as 1000 S/m and 100000 S/m, respectively.

Heating by the incident terahertz or infrared radiation can switch VO_2 films between two phases – semiconducting and metallic. This leads to a change in the electric and magnetic field at the probing frequency of the q-BIC resonance and, accordingly, in the amplitudes of the multipole expansion components when a subterahertz electromagnetic wave is incident on the developed toroidal metasurface. For this reason, the q-BIC resonance changes its amplitude, demonstrating a high sensitivity to the losses.

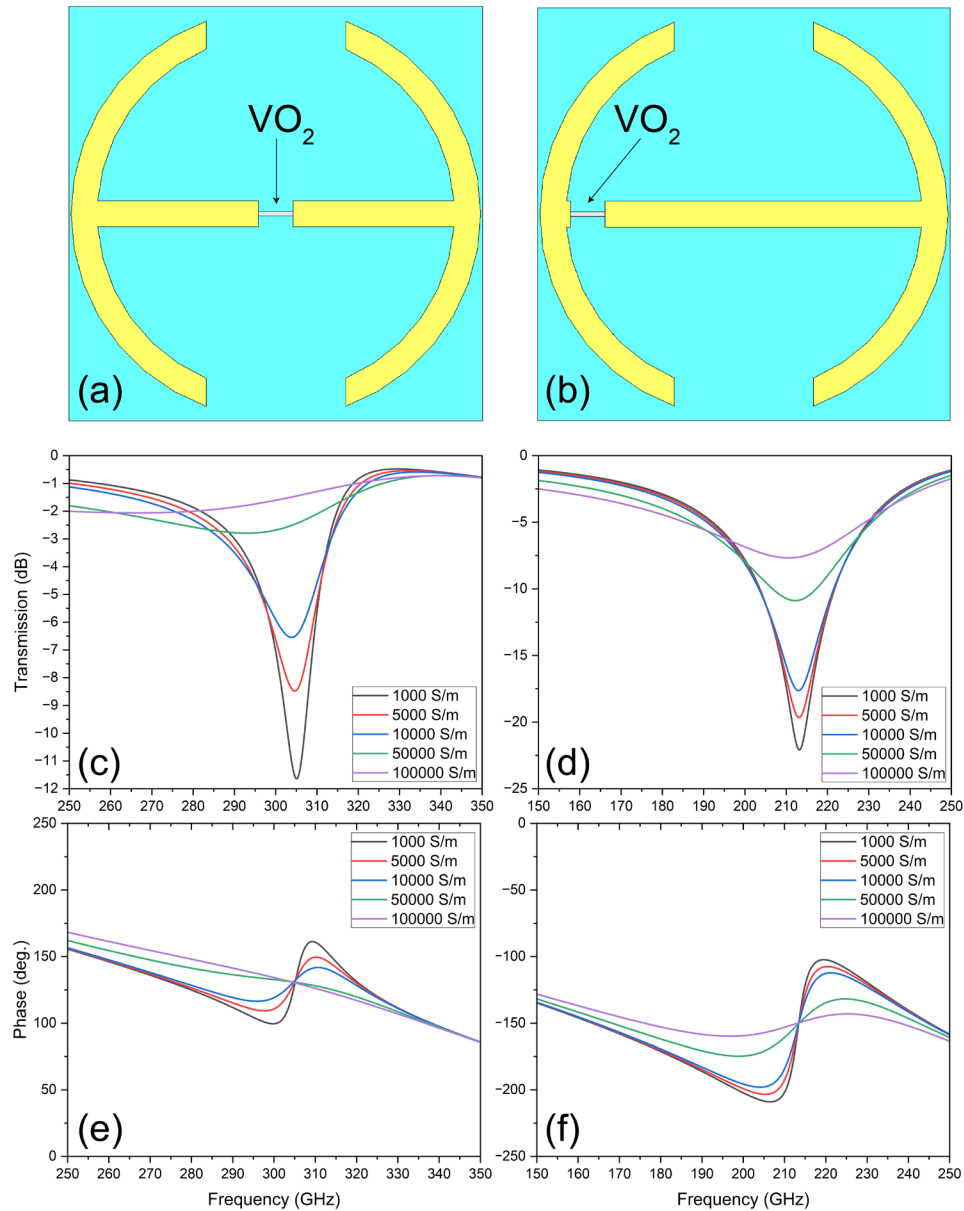


Figure 3-10 Tunable symmetric (a) and asymmetric (b) meta-atoms with integrated thin film patches of VO_2 in the central gap and simulated transmission characteristics (c,d) of the corresponding metasurfaces and phase of the transmitted waves (e,f) for different values of VO_2 conductivity.

Symmetry breaking can increase the q-BIC resonance amplitude. The asymmetric meta-atom of the tunable toroidal metasurface with the central gap shift a equal to $180 \mu\text{m}$ is shown in Figure 3-10(b). The asymmetry allows to increase the transmission change of the q-BIC resonance by more than 3 dB. The transmission coefficient and phase of transmitted waves for different values of electrical conductivity of the integrated VO_2 film is shown in Figures 3-10(d,f), respectively.

Figure 3-11 illustrates the current density within symmetric and asymmetric meta-atoms with integrated VO₂ patches, as shown in Figure 3-10, for different conductivity values of 1000 S/m (a, b) and 100000 S/m (c, d), corresponding to temperatures of 70°C and 80°C, respectively. We observe an increase in current penetration into the VO₂ patches with increasing conductivity, coinciding with the disappearance of the q-BIC resonance, which requires a central gap where the electric field can be concentrated in the absence of a conductor.

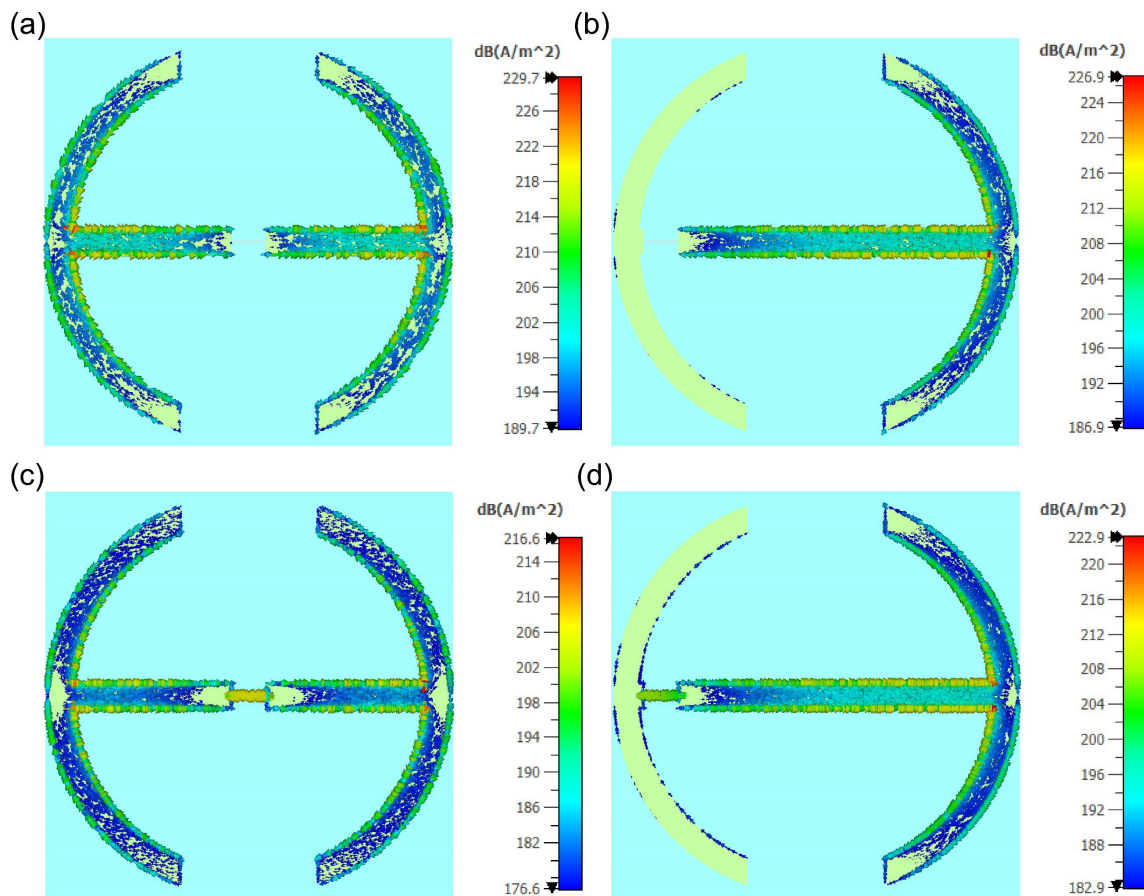


Figure 3-11 Current density within symmetric and asymmetric meta-atoms with integrated VO₂ patches for their different conductivity values – 1000 S/m (a, b) and 100000 S/m (c, d) corresponding to 70°C and 80°C, respectively.

We conclude that it is possible to use asymmetry to control the q-BIC resonance behaviour and improve the metasurface tunability. Change in the transmission spectrum for different asymmetry parameters is shown in Figure 3-12. It is calculated as the difference between the transmission coefficients when VO₂ switches from the

semiconducting to the metallic states. The difference reaches its maximum when the central gap is maximally shifted from the symmetrical position. The maximum change of the transmission coefficient is predicted to be 14 dB as the temperature changes by approximately 10 °C.

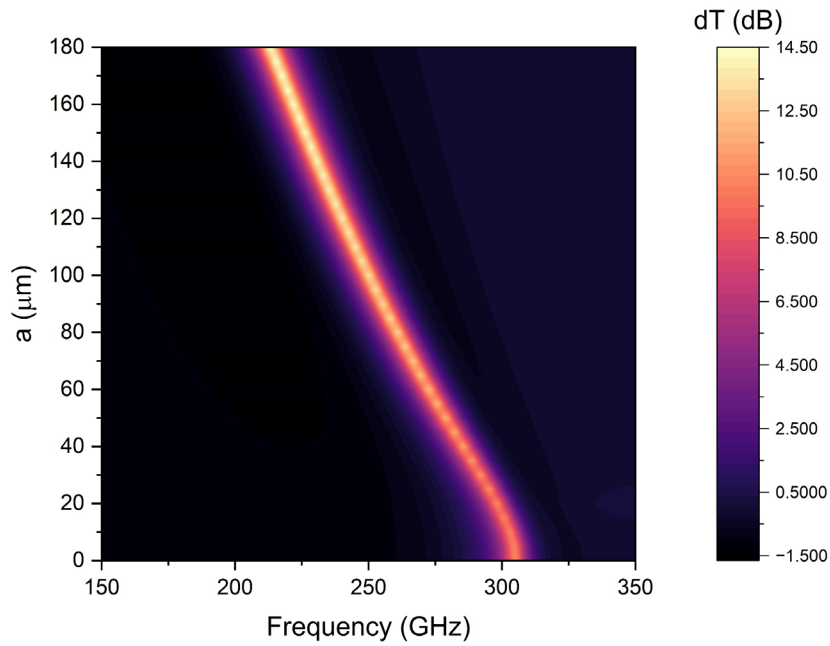


Figure 3-12 Transmission change for different asymmetry parameters.

Furthermore, we conduct an analysis of the active toroidal metasurface characteristics in scenarios where the integrated thin film patches of VO_2 lack a galvanic connection to the SRRs. In this configuration, the q-BIC resonance exhibits both frequency and amplitude shifts, shown in Figure 3-13, albeit with a smaller amplitude change at a given frequency compared to the previously discussed case. This design, despite the reduced amplitude change, holds promise for bolometric detection, especially given the imperative for thermal isolation of the absorber from other bolometer elements.

Figures 3-13(a,b) show tunable symmetric and asymmetric meta-atoms, incorporating thin film patches of vanadium dioxide in the central gap. The specified parameters include the outer gap size (dl) of 160 μm , the central gap size (dc) of 40 μm , and the absorber size of 30 μm x 30 μm . The remaining meta-atom parameters remain consistent

with those outlined in the chapter. Such design reduces the transmission change at a given frequency but allows us to read the state of the absorbers without their galvanic connection to the resonators.

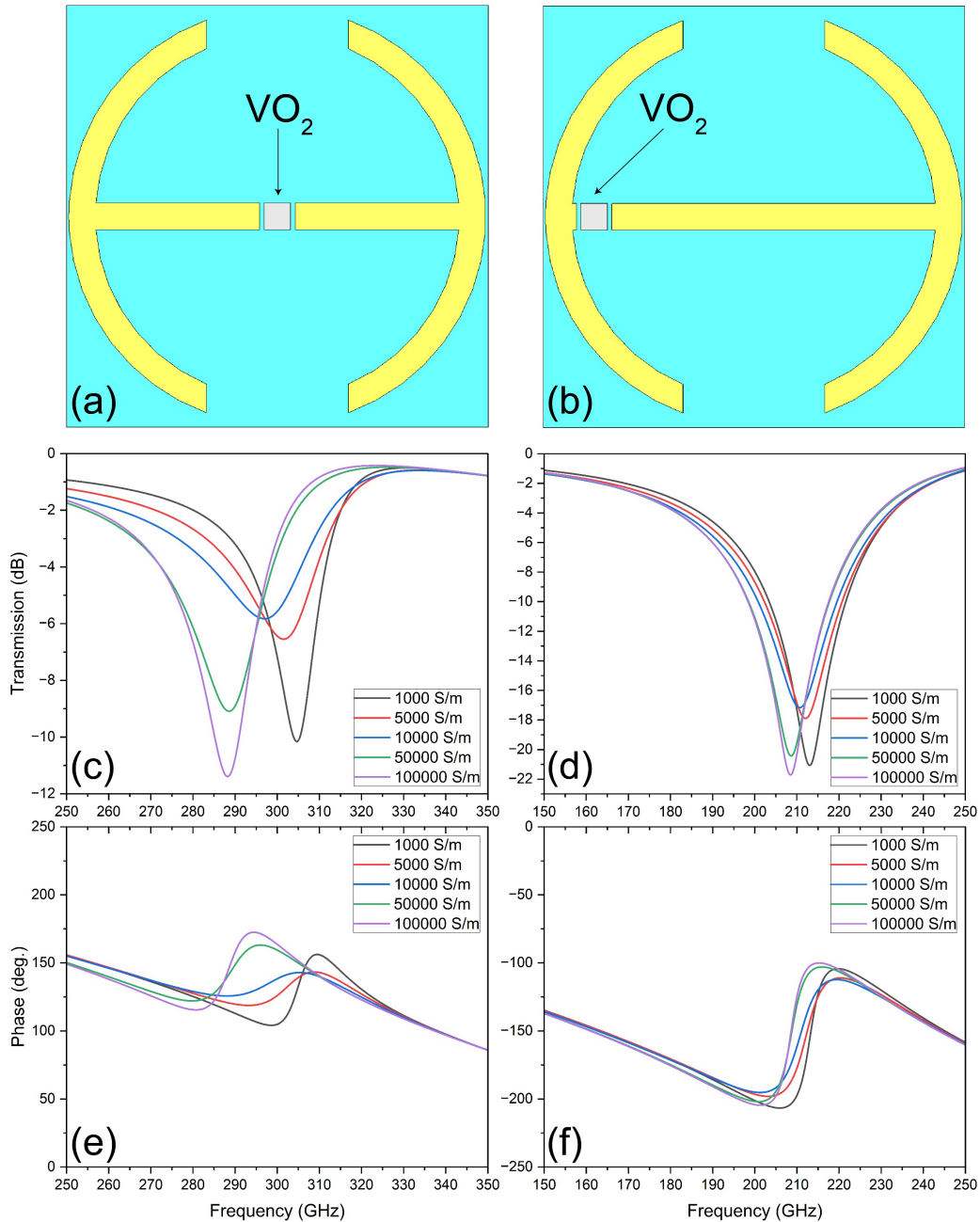


Figure 3-13 Tunable symmetric (a) and asymmetric (b) meta-atoms with the integrated thin film patches of vanadium dioxide in the central gap (without their galvanic connection to the resonators) and simulated transmission characteristics of the metasurface (c,d) and phase of transmitted waves (e,f) for different values of VO₂ conductivity.

Figures 3-13(c-f) display the transmission characteristics and phase of transmitted waves for both symmetric and asymmetric meta-atoms for different values of VO_2 conductivity. Notably, the q-BIC resonance shifts to lower frequencies with increasing the absorber conductivity, indicating material heating. This configuration not only enhances thermal isolation for the absorbers – a critical factor in bolometric detection – but also allows control over the q-BIC resonance frequency. This tunability also holds promising implications for other applications, such as reconfigurable filters.

3.4 Discussion of the proposed active toroidal metasurfaces

This chapter explores the intricate behaviour of toroidal metasurfaces, focusing on the effects of symmetry breaking and the integration of VO_2 thin film patches. The presented modelling results shed light on the principles governing q-BIC resonances, the impact of losses on their characteristics, and the tunability of toroidal metasurfaces.

The q-BIC resonances exhibited by the proposed metasurface are highly dependent on the geometry of the meta-atoms, specifically the outer and central gaps of the SRRs. Losses in gold are identified as a critical factor affecting the Q-factor of the q-BIC resonances, resulting in a wider bandwidth and reduced amplitude. Our observations confirm that losses play a significant role in the performance of toroidal metamaterials, emphasising the importance of careful meta-atom optimisation and the use of low-loss metals or superconductors for achieving ultrahigh Q-factors and the predominance of the toroidal dipole.

This work introduces the asymmetry in the toroidal metasurface design, revealing a fascinating connection to BIC. The amplitude of q-BIC resonances increases with the degree of asymmetry, presenting an avenue for manipulating resonance behaviour. This insight into symmetry breaking opens possibilities for tailored applications in active photonic systems and bolometric detection. This work aligns with recent toroidal dipole BIC metasurface studies, highlighting the potential for ultrasensitive measurements and applications in various fields.

The integration of VO₂ thin film patches introduces a dynamic tunability to the toroidal metasurface, driven by the semiconductor-to-metal transition of VO₂. This study emphasises the role of symmetry breaking in enhancing the tunability of q-BIC resonances. By introducing asymmetry in the central gap of the meta-atom, we show a substantial increase in transmission change, providing a means to control and improve metasurface tunability. The presented metasurface design, incorporating VO₂, demonstrates a significant change in q-BIC resonance amplitude with temperature variations, opening possibilities for applications in terahertz and infrared room-temperature bolometers and active photonics.

In contrast to earlier study [218], the q-BIC resonance is excited when VO₂ is in the semiconducting state. This allows the use of one absorber for the control of the resonance amplitude and frequency. This distinctive feature sets this metasurface design apart, making it particularly suitable for bolometric applications.

The integration of VO₂ thin film patches into the resonators allows the use of its dynamic and reversible SMT near room temperature. This transition, accompanied by significant changes in electrical and optical properties, enables the proposed metasurface to function effectively as a bolometric readout for the detection of terahertz or infrared radiation, as well as a tunable filter or modulator in future communication systems in the subterahertz range. By attaching antennas to the VO₂ film patches, it is possible to enhance the efficiency of terahertz or infrared radiation absorption. Alternatively, this can be achieved by using lenses placed on the absorbers for focusing the radiation onto them. This approach makes it possible to protect VO₂ film patches from oxidation.

The concept of metabolometers, as previously developed [224], faced limitations due to the requirement for cryogenic cooling when employing superconductors as absorbing materials. The proposed use of VO₂ instead of superconductors eliminates the need for cryogenic conditions and opens the possibility of designing room-temperature metabolometers. This advancement is crucial for practical applications in various scientific and technological domains where the demand for efficient room-temperature

detectors is high. Another significant improvement involves adjusting the size of meta-atoms. Specifically, we opted for the subterahertz range to excite the q-BIC resonance, departing from the previously suggested microwave readout. This choice aims to enlarge the effective absorption area for incident radiation, as smaller resonator sizes are achieved by elevating the frequency of the q-BIC resonance. This study also highlights the potential for using thin and low-loss substrates for the further experimental implementation of this device.

An experimental realisation could rely on standard planar fabrication, where a periodic array of meta-atoms with integrated VO₂ patches is patterned on a thin, thermally isolated substrate. To clearly observe the quasi-BIC response, a finite array containing tens of unit cells per side is typically sufficient, as edge effects then become negligible compared to the collective resonance.

Nonuniform temperature across the metasurface would mainly lead to inhomogeneous broadening and a reduction of the effective Q-factor, rather than suppressing the resonance entirely. Using a thermally insulating membrane and uniform incident radiation or electrical heating can therefore help maintain homogeneous switching and preserve the tunability predicted in the simulations.

In essence, this research not only contributes to the understanding of toroidal metamaterials but also offers a practical solution for advancing bolometric applications. The room-temperature capabilities of the designed metasurface pave the way for the development of metabolometers that are not only highly sensitive but also operationally feasible in a broader range of environments. This shift from cryogenic to room-temperature operation represents a significant step forward in the utilisation of the proposed toroidal metasurface for real-world applications, particularly in the field of terahertz detection and imaging.

3.5 Conclusion

This chapter demonstrates that control of q-BIC resonances in toroidal metasurfaces, specifically by incorporating thin-film patches made of vanadium dioxide, reveals a promising avenue for achieving tunable and highly sensitive responses. By tuning structure parameters, breaking symmetry in meta-atoms, and leveraging the temperature-dependent properties of vanadium dioxide, we demonstrate the ability to control the amplitude and frequency of high-Q q-BIC resonances dynamically. Breaking the symmetry of meta-atoms further expands the tunability range, offering a versatile platform for tailoring q-BIC resonances. The proposed tunable toroidal metasurface exhibits great promise for applications in active photonic systems, room-temperature bolometers, and multifunctional metadevices. The ability to manipulate high-Q q-BIC resonances opens doors to innovations in fields ranging from communication to sensing technologies.

Chapter 4: Parametric metasurfaces for amplification and frequency up-conversion of electromagnetic waves

In this chapter, we investigate parametric amplification and frequency up-conversion of electromagnetic waves using metasurfaces. We begin by proposing a variable capacitor-loaded metasurface capable of amplifying incident electromagnetic waves. We analyse different operational regimes and demonstrate that a single layer of the structure can achieve significant gain (over 10 dB), which can be tuned through parametric modulation. We study the instability threshold of this system and show that a simple theoretical model provides excellent agreement with full numerical simulations.

We then introduce parametric metasurfaces designed to perform both frequency up-conversion and amplification of free-space electromagnetic waves. This functionality is enabled by modulating variable capacitances embedded in specially designed dual SRRs. We explore various combinations of the pump and incident wave frequencies and investigate different operational regimes of the proposed parametric system. Theoretical analysis predicts high up-conversion efficiency, with amplification exceeding 20 dB through a cascaded process, thereby surpassing the limits set by the Manley-Rowe relations. Furthermore, we demonstrate precise control over the phase of the up-converted waves by tuning the pump phase.

Finally, we examine the optimised meta-atom design and the TEM cell employed in the experiments. The experimental setup is introduced, and the characteristics of the parametric meta-atoms obtained in the TEM cell are compared with numerical modelling results. Experimental findings confirm amplified up-conversion of electromagnetic waves using the fabricated meta-atoms, with gain exceeding 10 dB in the degenerate regime and 5 dB in the non-degenerate regime, despite the presence of additional losses. Importantly, the experimental results corroborate that the up-conversion efficiency is not constrained by the Manley-Rowe relations, owing to the multi-cascade amplification mechanism.

Results presented in this chapter are partially included in the published articles listed below, where I am the first author. My contributions include formulating the initial ideas, developing the theoretical framework, performing numerical modelling, designing the metasurfaces, drafting the manuscript, and preparing its final version.

F. V. Kovalev and I. V. Shadrivov, Parametric metasurfaces for electromagnetic wave amplification, *Opt. Mater. Express* **14**, 494 (2024).

F. Kovalev and I. Shadrivov, Parametric metasurfaces for amplified upconversion of electromagnetic waves, *Applied Physics Letters* **126**, 261702 (2025).

4.1 Control of electromagnetic waves using parametric metasurfaces

Metasurfaces, whose characteristics vary over time, are commonly referred to as time-varying [16,20] or parametric [93,110]. They offer opportunities to overcome the fundamental limitations of passive electromagnetism [22] and realise effects previously unattainable with conventional metamaterials, such as magnetic field-free nonreciprocity [23,239]. Parametric metasurfaces provided a platform for realising time crystals [26,114], a temporal analogue of the Young's double-slit diffraction experiment [28], beam steering [240,241], frequency conversion [102,100,96,98,242–244] and electromagnetic wave amplification [105,112,116,245,246]. In general, achieving a higher modulation rate corresponds to unlocking more intriguing effects [15], such as compression and spectral manipulation of pulses [247], as well as parametric amplification discussed in this chapter. With higher modulation speeds, it is possible to achieve complete control over light both in space and time [20].

Recently, several research groups used various methods to amplify incident radiation with the help of metasurfaces, such as rotation of the anisotropy of surface susceptibility [112], utilisation of active substrates [248], permittivity modulation [105,107], and integration of power amplifiers [115,116] or transistors [245,249]. Parametric amplification in left-handed transmission lines [250–252] and metamaterial waveguides [253,254] was studied at the beginning of this

century, but these methods have not yet been applied to the amplification of free-space electromagnetic waves.

Parametric amplification techniques were originally used in electronic circuits and became the leading technology for ultralow-noise microwave measurements in quantum computing [255]. Parametric processes were first studied in mechanics by Lord Rayleigh and later applied to radio-frequency conversion in systems with nonlinear capacitance by van der Ziel in the mid-20th century. Simple examples of parametric systems include a pendulum with a time-varying length or an oscillating circuit with variable capacitance or inductance. The mathematical description of such systems is considered in detail in the theory of oscillations [256]. Differential equations with time-dependent coefficients provide a simplified mathematical analysis of parametric metasurfaces. The solutions of these equations contain two dominant frequency components — up-converted and down-converted — demonstrating that parametric metasurfaces enable frequency conversion through temporal modulation of their parameters.

In parametric amplification, gain arises from frequency down-conversion of a high-frequency pump through temporal modulation of system parameters, rather than from direct power injection at the signal frequency. Conversely, conventional microwave amplifiers increase signal power by supplying additional energy at the same frequency via active electronic devices. In principle, frequency conversion can be realised using nonlinear elements, such as diode-based mixers, where static material nonlinearity leads to the generation of sum and difference frequencies. In contrast, time-varying metasurfaces rely on temporal modulation of system parameters, enabling controlled parametric energy transfer and allowing access to regimes of amplification and phase control that are not achievable with conventional nonlinear mixing.

There is a growing interest in leveraging time-varying media for combined amplification and up-conversion [257,111]. However, research on parametric amplification of free-space electromagnetic waves combined with frequency up-conversion using time-varying metasurfaces remains limited. Only a few theoretical works [112,246] have

predicted the feasibility of achieving parametric up-conversion with amplification, but the reported frequency conversion ratios and gains in these transmission line-based systems are relatively low. In these works, the conversion efficiency was enhanced by utilising reflective designs, making them bulky and less practical for integration into compact systems. For a metasurface without the reflector, the reported frequency conversion efficiency was only 23%.

4.2 Analytical model of electromagnetic wave amplification using parametric metasurfaces

In this chapter, we propose and investigate a new approach to electromagnetic wave amplification using metasurfaces. Specifically, we design a time-varying metasurface based on SRRs. By integrating a modulated variable-capacitance diode into the SRR, we transform it into a parametric amplifier. We utilise varactor diodes exploiting the voltage-dependent capacitance of a reverse-biased pn-junction in semiconductor materials. When the capacitance of a varactor diode is periodically modulated at approximately twice the frequency of the incident wave, the theory of parametric amplification predicts a unidirectional influx of energy into the SRR. To enhance the amplitude of oscillations excited in the SRR by incident electromagnetic waves, we use additional energy supplied to the circuit from an external source.

Figure 4-1 shows the proposed parametric metasurface (a) and a lumped element circuit diagram of its meta-atom (b). The meta-atom includes the copper SRR placed on the substrate (Rogers RO4003C) and the embedded varactor diode (SMV1405), which is modulated using the pump source via the filtering circuit ($C_F = 10$ pF, $L_F = 650$ uH, $R_F = 200$ k Ω). The SRR parameters are as follows: the SRR inner radius $r = 14$ mm, the SRR track width $w = 2$ mm, the SRR gap size $g = 0.5$ mm, the SRR track height $h = 35$ μ m. The gap size is selected to accommodate the varactor diode. The substrate thickness is 508 μ m and the unit cell size is 34 mm $\times$ 34 mm. The track height is fixed by the standard copper thickness available for Rogers RO4003C substrates. The geometrical and circuit parameters were selected to place the SRR resonance in the low-frequency microwave

range, ensure compatibility with standard fabrication and packaged varactor diodes, and enable efficient numerical modelling of the parametric metasurface.

The schematic in Figure 4-1(b) shows the effective circuit model of the SRR loaded by a varactor with a biasing network. The capacitor C_f together with the resistor R_f (a high-pass filter) protects the AC pump source from the DC voltage U_0 , which is used to set the operating bias voltage on the capacitance-voltage characteristic of the varactor diode $C(t)$. The inductor L_f along with the resistor R_f (a low-pass filter) is utilised to protect the DC voltage source from the AC currents at ω_s and $\omega_p = 2\omega_s$.

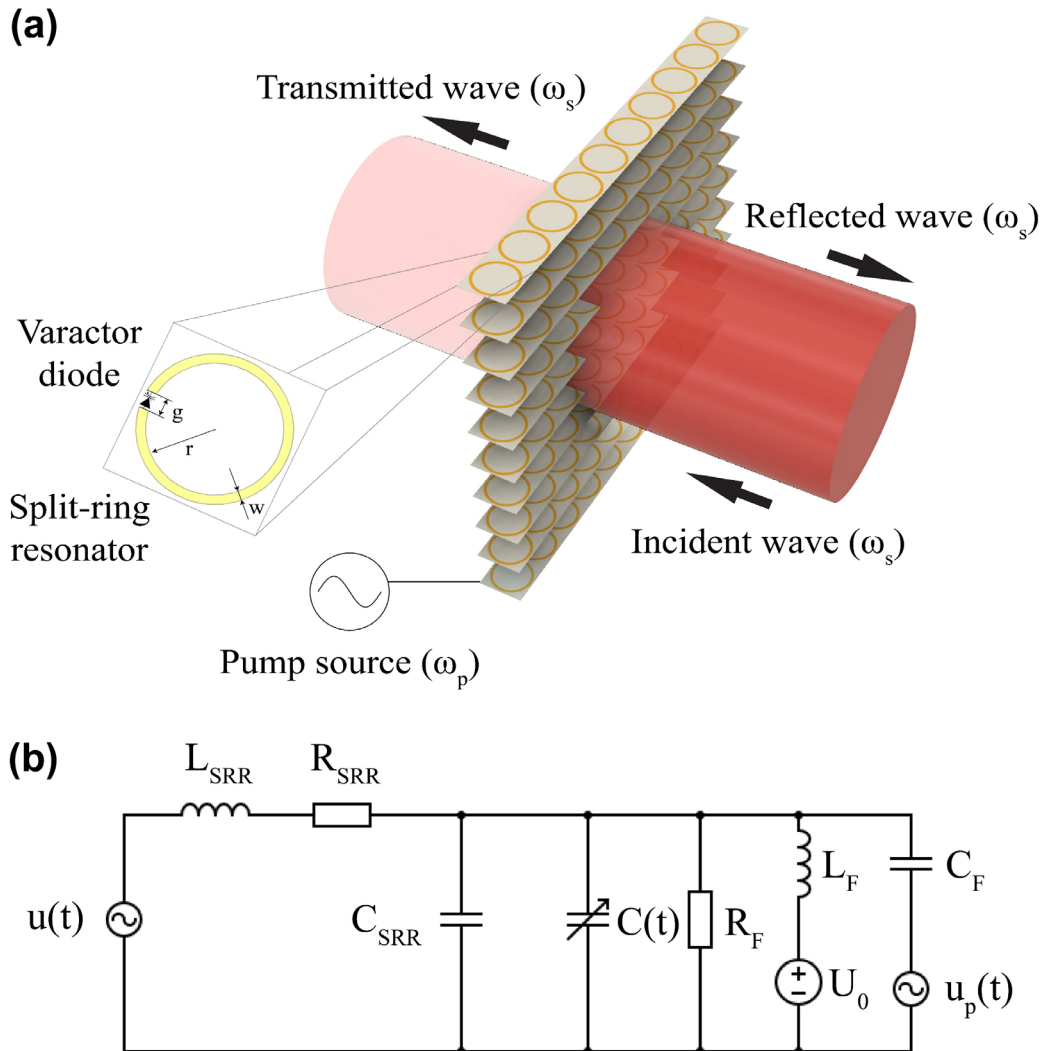


Figure 4-1 (a) Schematics of the parametric metasurface made of one layer of resonators loaded with variable capacitance diodes. (b) The lumped element diagram of the meta-atom.

In this work, we used the following formulas to calculate the capacitance C_{SRR} and inductance L_{SRR} of the SRR [258]:

$$C_{SRR} = C_{gap} + C_{surf}, \quad (4.1)$$

$$C_{gap} = \varepsilon_0 \left(\frac{hw}{g} + h + w + g \right), \quad (4.2)$$

$$C_{surf} = \frac{2\varepsilon_0(h+w)}{\pi} \log \left[\frac{4 \left(r_m - \frac{w}{2} \right)}{g} \right], \quad (4.3)$$

$$L_{SRR} = \mu_0 r_m \left[\log \left(\frac{8r_m}{h+w} \right) - \frac{1}{2} \right], \quad (4.4)$$

where $r_m = r + \frac{w}{2}$ is the SRR average radius, ε_0 and μ_0 is the free-space permittivity and permeability, respectively. The effective resistance R_{SRR} in Figure 4-1(b) includes both radiation and loss resistance of the SRR.

The magnetic field component of the incident wave couples efficiently to the SRR when it threads the loop area, inducing a circulating current. This current excites the fundamental resonance of the SRR, with the inductance provided by the loop and the capacitance by the gap. If instead the electric field is oriented across the gap, it can also excite the resonance via capacitive coupling. In this case, the electric field directly drives charge accumulation at the gap, leading to oscillating currents in the ring. Therefore, resonance excitation does not strictly require magnetic field threading. However, we excite the meta-atoms via the magnetic resonance of the split-ring resonators, as magnetic field coupling induces circulating currents that are largely insensitive to the local electric field distribution in the gap, ensuring robust excitation of the lowest-order mode even in the presence of a varactor diode, which can otherwise perturb electric field coupling.

Next, we consider in detail the SRR with an integrated varactor, which is excited via the electromotive force induced by the incident wave in the following way:

$$u(t) = U_s \cos \omega_s t, \quad (4.5)$$

where ω_s is the signal frequency equal to the SRR resonance frequency, $u(t) = -\frac{d\Phi}{dt}$ is the induced electromotive force in the SRR, and Φ is the magnetic flux through the SRR.

Modulation of the varactor response is performed by an independent source with a voltage given by

$$u_p(t) = U_0 + U_p \cos(\omega_p t + \varphi_p), \quad (4.6)$$

where U_0 is the constant bias voltage, U_p is the pump amplitude, ω_p is the pump frequency, φ_p is the pump phase.

By applying a modulated voltage to the embedded varactors in each SRR, we change their capacitance. The $C(U)$ characteristics of all real varactor diodes are nonlinear, and we consider the exact dependence in the numerical simulations in this chapter. For analytical considerations we assume that the capacitance also changes according to the simple cosine periodic law. If the pump amplitude is much greater than the signal amplitude ($U_p \gg U_s$), then it is possible to neglect the capacitance change under the action of the incident wave signal, and we end up with the expression:

$$C(t) = C_0 [1 - \beta \cos(\omega_p t + \varphi_p)], \quad (4.7)$$

where C_0 is the average value of the capacitance, $\beta = \frac{\Delta C}{C_0}$ is the parametric modulation coefficient, ΔC is the amplitude of the capacitance parametric modulation.

The charge on the varactor, as well as the current and voltage, are related by the following expressions:

$$q(t) = C(t)u_c(t), \quad (4.8)$$

$$i(t) = \frac{dq}{dt} = C(t) \frac{du_C}{dt} + u_C(t) \frac{dC}{dt}. \quad (4.9)$$

The linear parametric capacitance $C(t)$ includes the pump source action. Therefore, $u_C(t) = u(t)$ and the total current through capacitance $C(t)$ can be expressed as:

$$\begin{aligned} i(t) = & -\omega_s C_0 U_s \sin(\omega_s t) + \\ & + \frac{1}{2} (\omega_p + \omega_s) \Delta C U_s \sin[(\omega_p + \omega_s)t + \varphi_p] + \\ & + \frac{1}{2} (\omega_p - \omega_s) \Delta C U_s \sin[(\omega_p - \omega_s)t + \varphi_p]. \end{aligned} \quad (4.10)$$

If $\omega_p = 2\omega_s$, then only the first and third terms of Eq. 4.10 contribute to the total current oscillating with frequency ω_s , because the second component oscillates outside the band of interest (at $3\omega_s$). The first term is the response of the constant capacitor C_0 . As shown below, the third term can be represented by the effective conductance G_{eq} connected in parallel to C_0 . The third current component in Eq. 4.10 at ω_s is:

$$i_{\omega_s}(t) = \frac{1}{2} \omega_s \Delta C U_s \sin(\omega_s t + \varphi_p) = \frac{1}{2} \omega_s \Delta C U_s \cos\left(\omega_s t + \varphi_p - \frac{\pi}{2}\right). \quad (4.11)$$

For some phases of the pump source and incident wave, the parametric capacitance can amplify currents at ω_s , which can feed the currents excited by the incident wave and be re-radiated by the SRR. The power delivered to the incident wave can be calculated as follows:

$$P_{\omega_s} = \frac{U_s I_{\omega_s}}{2} \cos\left(\varphi_p - \frac{\pi}{2}\right) = \frac{1}{2} \omega_s \Delta C \frac{U_s^2}{2} \sin\varphi_p = \frac{G_{eq} U_s^2}{2}, \quad (4.12)$$

where $G_{eq} = \frac{\beta \omega_s C_0 \sin\varphi_p}{2}$ is the equivalent active conductance and $I_{\omega_s} = \frac{1}{2} \omega_s \Delta C U_s$ is the current amplitude. For $\varphi_p = -\frac{\pi}{2}$ the equivalent active conductance is negative: $G_{eq} = -\frac{\beta \omega_s C_0}{2}$. Thus, the total conductance of the circuit can be reduced or even become

negative. Therefore, the variable capacitance added to the SRR amplifies the currents induced by the incident wave.

To simplify the analysis, we replace the lumped element circuit with an equivalent dual circuit applying the principle of duality [259,260]. This circuit includes a current source loaded by the SRR and varactor impedances, as illustrated in Figure 4-2. Note that the dual equivalent inductance has the value C_0 and the dual equivalent capacitance $-L_{SRR}$, respectively. The overall capacitance $C_0 = C_{SRR} + C_{0\text{ var}} + C_f$ includes the capacitance of the SRR C_{SRR} , the filtering capacitance C_f and the average capacitance of the varactor diode $C_{0\text{ var}}$ corresponding to the value when a constant bias voltage U_0 is applied.

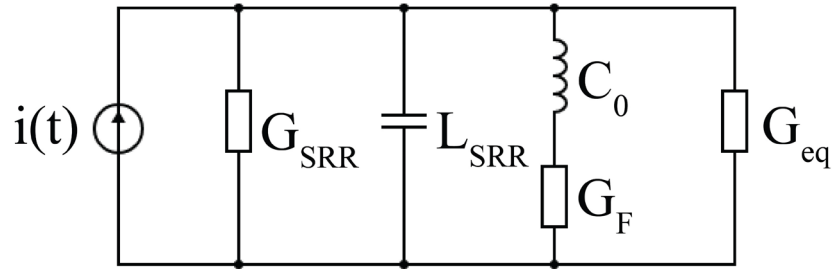


Figure 4-2 An equivalent dual circuit containing a current source loaded by SRR and varactor impedances. Note that the values of inductance and capacitance are defined by the values of capacitance and inductance, respectively, in the original circuit.

If there is no parametric modulation, the output voltage amplitude of the source is $U_s = \frac{I}{G_{SRR} + G_{add}}$, where $G_{add} = \frac{C_0^2 G_F \omega_s^2}{G_F^2 + C_0^2 \omega_s^2}$, $G_F = \frac{1}{R_F}$, $G_{SRR} = \frac{1}{R_{SRR}}$ and I is the current source amplitude ($G_{add} \approx G_F$ for our specific circuit parameters).

The active re-radiated power by the SRR can be determined as follows:

$$P_s = \frac{1}{2} \text{Re}\{U_s I^*\} = \frac{1}{2} \text{Re}\{U_s (U_s^* G_{SRR})\} = \frac{1}{2} U_s^2 G_{SRR} = \frac{1}{2} \frac{I^2 G_{SRR}}{(G_{SRR} + G_{add})^2}, \quad (4.13)$$

where U_s and I denote voltage and current in phasor form, $*$ denotes complex conjugation.

In the case of parametric modulation and $G_{eq} < 0$, the procedure is similar to the one described above, but the source output voltage becomes $U_s = \frac{I}{G_{SRR} + G_{add} + G_{eq}}$. Therefore, the re-radiated power in the parametrically modulated system can be found as:

$$P_s' = \frac{1}{2} \frac{I^2 G_{SRR}}{(G_{SRR} + G_{add} + G_{eq})^2}. \quad (4.14)$$

Then the power gain (compared to the SRR without any pump) is:

$$K_P = \frac{P_s'}{P_s} = \frac{1}{\left(1 + \frac{G_{eq}}{G_{SRR} + G_{add}}\right)^2}. \quad (4.15)$$

When the absolute value of the equivalent active conductance $|G_{eq}|$ in the system exceeds the total conductance of the circuit, the resonator becomes unstable, and it starts oscillating even when there is no incident wave at the signal frequency. The self-excitation threshold of the parametric amplifier or parametric generation condition can be determined from the following condition:

$$|G_{eq}| > G_{SRR} + G_{add}. \quad (4.16)$$

This implies that parametric generation appears in the SRR with the equivalent active conductance exceeding the total conductance of the circuit. The critical value of the parametric modulation coefficient is

$$\beta_{critical} = \frac{2 G_o}{\omega_s C_0} = \frac{2}{Q}, \quad (4.17)$$

where Q is the Q-factor of the SRR including all losses $G_o = G_{SRR} + G_{add}$.

The total conductance of the circuit is determined by the bandwidth of the resonance and the overall average capacitance:

$$G_o = \Delta\omega C_0. \quad (4.18)$$

The critical capacitance modulation value at which the instability occurs is then estimated as

$$\Delta C_{critical} = \frac{2G_o}{\omega_s} = \frac{2\Delta\omega C_0}{\omega_s}. \quad (4.19)$$

4.3 Amplification of electromagnetic waves using parametric metasurfaces

In this section, we discuss full-wave numerical modelling of the proposed metasurface using CST Studio. We perform numerical simulations of a periodic structure using finite-element method and output the results of 3D modelling into Schematic simulator to model the nonlinear lumped element circuit that includes the varactor diode and the biasing network. We employ the Spectral Lines task and the varactor SPICE model [261] within Schematic in CST Studio to conduct simulations of the nonlinear circuit in frequency domain. This task specifically facilitates Harmonic Balance analysis, enabling us to examine the pump and signal mixing processes within the designed circuit. This method encompasses mixing frequencies up to a specified limit of harmonics. To ensure accurate representation of the mixing effects, we utilise 20 harmonics for the signal source and 10 harmonics for the pump source, optimising the simulations for the most precise results. In this study, the highest-order pump and signal harmonics considered in the simulations occur at nearly the same frequency, ensuring that the harmonic balance analysis accurately captures their interaction.

Figure 4-3 shows the calculated transmission and reflection coefficients for the incident linearly polarised electromagnetic wave for different values of the DC voltage applied to the varactor diodes (U_0). We set the operating bias voltage $U_0 = 15$ V and modulate the varactor capacitance using the AC pump source as discussed earlier.

The critical capacitance modulation value, $\Delta C_{critical}$, can be determined as described in the previous section and is approximately 0.4 pF. For the considered varactor diode (SMV1405), this corresponds to a pump amplitude of $U_{p\ critical} \approx 11$ V at a DC bias voltage of 15 V. When the pump amplitude exceeds this value, the circuit becomes

unstable and enters a self-oscillating regime. The self-excitation threshold was verified through modelling of the proposed metasurface using the Spectral Lines task in CST Studio. In the context of parametric amplification, we interpret the onset of self-oscillations as a manifestation of circuit instability. As shown in Figure 4-4, the overall radiated power at ω_s grows exponentially with the pump amplitude once it exceeds the critical value, $U_{p\text{ critical}}$. This behaviour is observed in the presence of the pump signal and in the absence of propagating electromagnetic waves. The self-excitation threshold of the parametric amplifier, also referred to as the parametric generation condition, corresponds to a shift in the circuit's resonance frequency equal to its bandwidth [259,262].

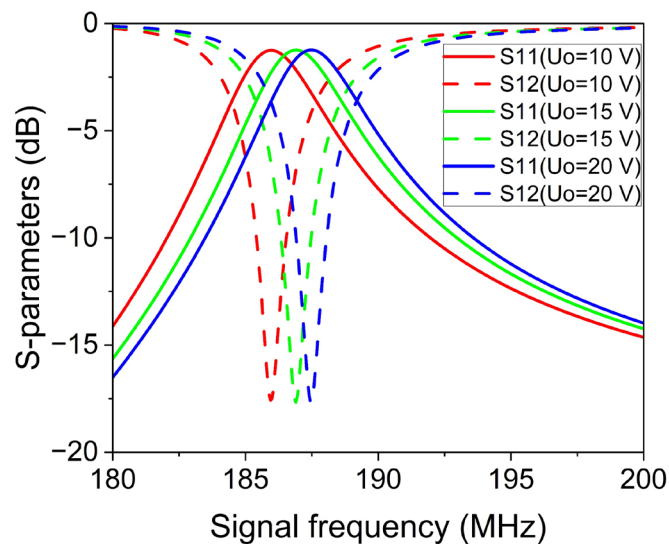


Figure 4-3 Transmission (S12) and reflection (S11) of the electromagnetic wave incident on the metasurface for one of the polarisations and different values of the applied DC voltage.

Parametric amplifiers operate substantially differently in the degenerate regime, when $\omega_p = 2\omega_s$, and in the non-degenerate regime, when $\omega_p \neq 2\omega_s$. In the degenerate regime, parametric amplifiers require strict phase relationships between the signal and pump. The maximum gain in the degenerate regime occurs at the pump phase for which the equivalent conductance (see Eq. 4.12) of the varactor reaches maximum negative value and the pump energy is transferred to the induced currents at ω_s . When the inserted resistance (equivalent conductance) of the varactor becomes positive, parametric

modulation introduces additional losses in the degenerate case. In contrast, the gain in the non-degenerate regime does not depend on the pump phase, but the phases of the reflected and transmitted waves depend on the phase of the pump. This is similar to the results reported in Ref. [93], where it was demonstrated that the phases of the pump waves control the phase of the transmitted and reflected signals.

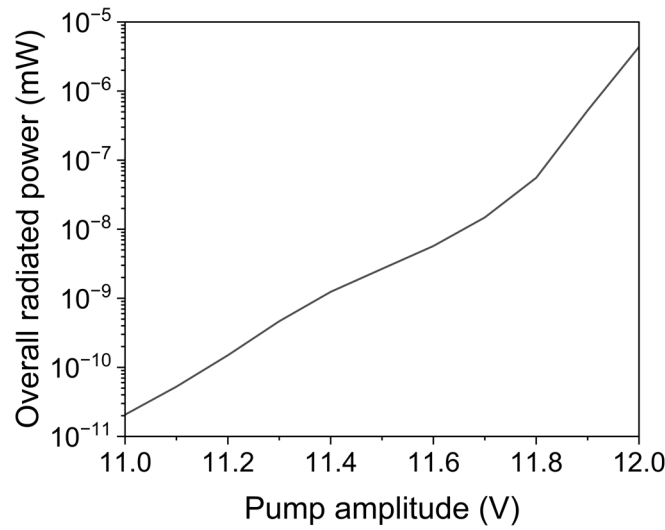


Figure 4-4 The dependence of the total power of the generated electromagnetic waves at ω_s on the pump amplitude in the absence of incident waves.

In addition to the phase dependence, the gain changes with the pump amplitude. When approaching the instability threshold, the gain increases with an increase in the pump amplitude, as shown in Figure 4-5. The maximum gain reaches approximately 16 dB in the degenerate regime (under conditions of phase locking) and 10 dB in the non-degenerate regime when the pump with the amplitude $U_p = 11$ V at the frequency of 372 MHz is applied to SRRs and the small-amplitude wave ($U_s = 0.01$ V) is incident on the metasurface at 186 and 186.1 MHz, respectively. The results are shown up to $U_p = 11$ V because this value corresponds to the instability threshold of the system described previously. Figure 4-5(a) also shows the theoretically predicted overall gain calculated using Eq. 4-15. As the exact ratio of the incident power and the induced electromotive force in the equivalent circuit cannot be found analytically with sufficient accuracy, we have normalised the obtained values, so that the gain without the pump ($U_p = 0$ V) matches the value obtained numerically.

In both the degenerate and non-degenerate cases, the reflected signal and the total scattered energy continuously increase with the pump amplitude. The signal transmitted through the metasurface first decreases as the pump amplitude increases to a certain threshold value, after which it also begins to increase. This occurs because the selected operating frequency is below the transmission resonance. Due to the nonlinearity of the capacitance-voltage characteristic, the average capacitance over a wave period increases with a rise in pump amplitude, similar to the results reported in Ref. [263]. The rise in capacitance results in a decrease in the resonant frequency of the circuit. Consequently, for frequencies below the resonance frequency when $U_0 = 15$ V, this initiates a reduction of the transmission coefficient. However, the parametric gain experiences exponential growth with increasing pump amplitude, eventually reaching a level sufficient to offset the decline in the transmission coefficient. This explains the initial drop in transmission, where the coefficient decreases while the parametric effect remains weak, unable to fully compensate for the decline. As the pump amplitude continues to rise, the parametric gain eventually surpasses the decrease in transmission, leading to an overall increase in gain.

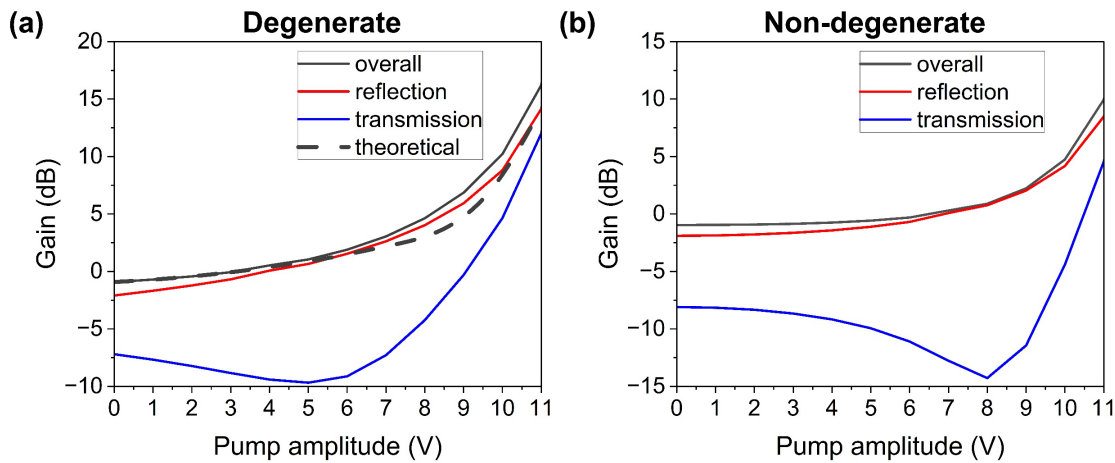


Figure 4-5 The dependence of the gain on the pump amplitude in the degenerate $\omega_s = 186$ MHz, $\omega_p = 372$ MHz (a) and non-degenerate $\omega_s = 186.1$ MHz, $\omega_p = 372$ MHz (b) cases. The dashed line shows the theoretically predicted overall gain from Eq. 4-15.

The dependence of the metasurface gain on the incident wave frequency in the degenerate case is shown in Figure 4-6(a). It is calculated in the regime of phase-locking

of the pump source with the small-amplitude incident wave ($U_s = 0.01$ V) and the pump amplitude $U_p = 10$ V, which is below the instability threshold. We maintain the degenerate condition $\omega_p = 2\omega_s$ by changing both the pump and signal frequencies in this regime. Figure 4-6 illustrates that maximum amplification occurs at frequencies slightly lower than the resonance frequency when the DC bias voltage is $U_0 = 15$ V (indicated by dotted vertical lines), approximately around 186 MHz.

Figure 4-6(b) shows the dependence of the gain on the signal frequency in the non-degenerate case for the pump amplitude $U_p = 10$ V. The gain does not depend on the phase; therefore, there is no need for phase-locking. The magnitude of the overall gain is slightly lower than the maximum possible gain in the degenerate case shown in the middle ($\omega_s = 186$ MHz, $\omega_p = 372$ MHz). The bandwidth of degenerate amplification is exceedingly narrow; hence, in practice, utilising this regime may pose challenges. With any frequency mismatch, some of the energy is converted to the idler frequency, thereby reducing the gain. We observe that the overall achievable gain is higher in the degenerate case. Most remarkably, the amplification mostly occurs for the reflected wave in the non-degenerate case, whereas the transmitted wave is further suppressed from its already low value near the resonance.

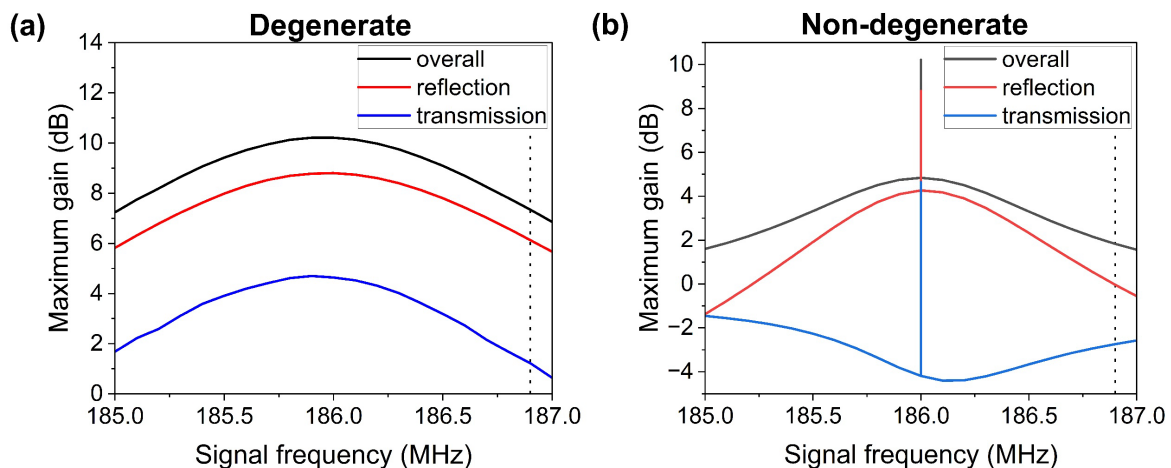


Figure 4-6 The dependence of the gain (overall, for reflection and transmission separately) on the frequency of the electromagnetic wave incident on the metasurface in the degenerate case (a), where we maintain $\omega_p = 2\omega_s$, and the non-degenerate case (b), where $\omega_p = 372$ MHz. The dotted vertical line shows the resonance position when $U_0 = 15$ V.

A comparison of the dependence of the gain on the signal amplitude in the degenerate and non-degenerate regimes for the pump amplitude $U_p = 8$ V is shown in Figure 4-7. Amplification shows no signs of saturation, and the gain is restricted by the breakdown voltage of the varactor diode in both cases. The signal amplitude becomes sufficiently high to significantly modulate the pump, so it reaches the breakdown value when $U_s \approx 1.5$ V.

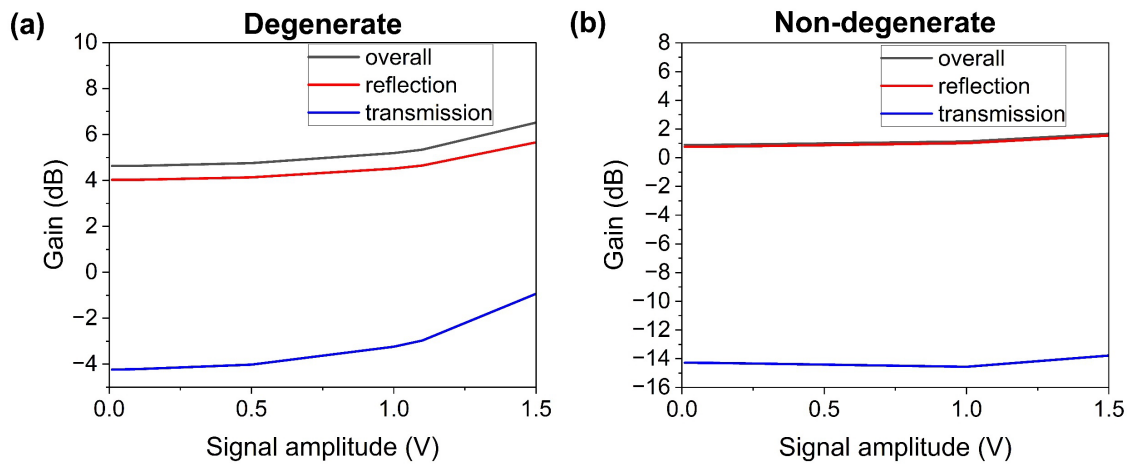


Figure 4-7 The dependence of the gain on the signal amplitude in the degenerate (a) and non-degenerate (b) cases.

4.4 Discussion of the proposed parametric metasurfaces for amplification

The obtained simulation results are consistent with the theory of parametric generation and amplification and the developed analytical model. We demonstrate a significant gain of more than 10 dB in both the degenerate and non-degenerate regimes for a small incident radiation power (tens of nanowatts). These results were confirmed for the incident radiation power up to approximately 0.75 mW. The theoretically calculated self-excitation threshold matches the results of numerical simulations of the parametric metasurface.

An important feature of degenerate parametric amplification is the need for phase locking of the pump source with the incident wave. In practice, this can only be realised if there is information about the phase of the incident radiation and if the frequency of the

pump source is fine-tuned to the condition $\omega_p = 2\omega_s$. Therefore, the proposed parametric metasurface is more practically useful in the non-degenerate regime.

The parametric amplification considered in this work is analysed for a continuous-wave excitation. In the case of a modulated incident wave, amplitude modulation is preserved during the amplification process in both degenerate and non-degenerate regimes, allowing amplification of amplitude-encoded signals. Phase modulation, however, is preserved only in the non-degenerate regime. In the degenerate regime, the gain depends explicitly on the phase difference between the pump and the incident wave. As a result, phase-modulated signals cannot be reliably processed in this regime unless the phase of the incident wave is known and synchronised with the pump prior to interaction with the metasurface.

Further improvements to the proposed metasurface include optimisation of filtering circuits or the use of non-radiating modes of resonators to avoid radiation at the pump frequency. It is also possible to use the pump without electrically applying parametric modulation but inducing currents in the resonators at $\omega_p \approx 2\omega_s$ by the incident electromagnetic wave. This requires the development of two-mode resonators to effectively couple the incident radiation.

We anticipate that the proposed structure can be used not only for amplifying electromagnetic waves but also for providing spatial control of the scattered waves by employing spatially inhomogeneous amplification. This integration of multiple functions can be useful in various applications, including multifunctional and programmable radomes or ground and orbit-based repeaters that can amplify incident signals. Other tuning methods that can modulate the parameters of metasurfaces at higher rates can extend these amplification techniques to the terahertz and infrared ranges.

Given that the scatterers in this study primarily reflect incident radiation at the resonance frequency, it is inherent that the amplified scenario is dominated by reflected waves. We

anticipate that employing a structure primarily designed for resonant forward radiation scattering would lead to a prevalence of transmitted waves.

These amplification techniques can be applied to quantum communications, such as the development of low-noise repeaters required for quantum channels over long distances. Given that parametric amplification finds extensive use in readouts for quantum computing, the proposed parametric metasurfaces hold the potential to enhance existing technologies in this area.

4.5 Amplified up-conversion of electromagnetic waves using parametric metasurfaces

In this section, we introduce a dual-resonance metasurface, which up-converts and amplifies incident electromagnetic waves. Each meta-atom of the proposed parametric metasurface comprises two connected SRRs with an embedded varactor diode that is electrically modulated at the pump frequency. One resonator is dedicated to resonantly exciting currents by the incident wave at the signal frequency, while the second SRR resonates at the sum-frequency. The current at the signal frequency flowing through the capacitor, whose capacitance changes over time at the pump frequency, generates currents at combination frequencies. Resonant excitation of currents at interacting frequencies significantly enhances the effect compared to non-resonant structures [264]. We optimise this structure and predict that our parametric metasurface achieves significant up-conversion efficiency with amplification exceeding 20 dB by utilising a cascaded process, surpassing the limitations imposed by the Manley-Rowe relations. Moreover, by tuning the phase of the modulation, we gain precise control over the phase of the up-converted waves, offering additional possibilities for advanced wave manipulation. Our findings are particularly relevant for applications in the microwave and terahertz frequency ranges, with the potential for extending to higher frequencies through ultrafast modulation techniques [265,266].

Dual-resonance parametric metasurfaces enable the amplification and conversion of incident radiation by employing auxiliary oscillations excited at the combination

frequencies $f_{comb} = mf_p \pm nf_s$, where f_s is the incident wave (signal) frequency, f_p is the pump frequency, m and n are integers. This study introduces a parametric metasurface with variable capacitance designed to amplify incident waves and up-convert them to the combination frequencies $f_{comb} = mf_p \pm nf_s$.

Figure 4-8 shows the meta-atom concept of the proposed dual-resonance parametric metasurface comprising two connected SRRs with an embedded varactor diode that is electrically modulated at the pump frequency f_p . The developed parametric system utilises the nonlinear capacitance of varactor diodes to mix currents excited by the incident wave and the pump source. One resonator serves to effectively excite currents by incident electromagnetic waves, while another resonator is designed for resonant radiation at one of the combination frequencies $f_{comb} = mf_p \pm nf_s$.

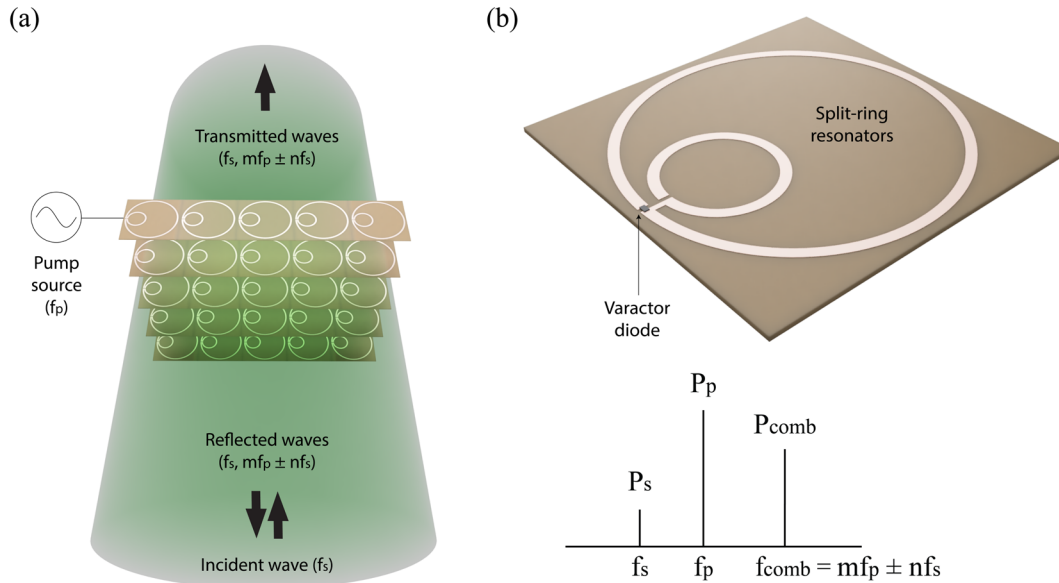


Figure 4-8 (a) Schematic representation of parametric amplification and frequency conversion by a metasurface. An electromagnetic wave at frequency f_s is incident on a parametric metasurface modulated by a pump source at frequency f_p . (b) The meta-atom structure consisting of two connected SRRs with an embedded varactor diode. The meta-atom utilises parametric interaction of the incident wave at f_s and the pump source at f_p , achieving up-conversion to the combination frequencies f_{comb} .

Amplified up-conversion is achieved by using varactor diodes with parametrically modulated capacitance, which connect two resonators in each meta-atom. We assume

that the capacitance oscillates periodically with the pump frequency, as specified in Eq. 4.7. Modulation of the varactor is achieved by the pump source with a voltage given in Eq. 4.6. In this section, we explore several regimes of amplification and identify the required amplitude, frequency and phase of the pump source.

The maximum possible conversion efficiency for the combination frequency f_{comb} is given by the Manley-Rowe relations [267,268]:

$$\begin{cases} \frac{P_s}{f_s} + \frac{mP_{comb}}{f_{comb}} = 0 \\ \frac{P_p}{f_p} + \frac{nP_{comb}}{f_{comb}} = 0 \end{cases} \quad (4.20)$$

where P_{comb} is the generated power at f_{comb} , P_s is the power of the incident wave and P_p is the power of the pump source. Here, we assume that the power of the incident wave and the pump source is fully utilised in the mixing process.

For $m = n = 1$ the Manley-Rowe relations can be simplified as follows:

$$\begin{cases} \frac{P_s}{f_s} + \frac{P_{sum}}{f_{sum}} = 0 \\ \frac{P_p}{f_p} + \frac{P_{sum}}{f_{sum}} = 0 \end{cases} \quad (4.21)$$

In this case, the maximum gain is determined by the frequency ratio:

$$K_P = \frac{-P_{sum}}{P_s} = \frac{f_{sum}}{f_s}, \quad (4.22)$$

where $-P_{sum} = P_s + P_p$ is the generated power at $f_{sum} = f_s + f_p$. We adopt the following notation [267]: the output power at the sum-frequency is considered negative, while the input powers of the incident wave and the pump source are taken as positive, as both contribute to the generation of power at the combination frequencies.

Figure 4-9(a) depicts the meta-atom comprising two connected copper SRRs positioned on the Rogers RO4003C substrate, with the embedded varactor diode MGV100-08 shown

as a black square. The selection of this varactor diode is due to its wide capacitance adjustment range, with a very low minimum capacitance of less than 0.1 pF, enabling the necessary tuning of the system's resonant frequencies. The varactor diode is modulated by the pump source through the filtering circuit shown in Figure 4-10: the passband LC filter at f_p serially connected to the AC source and the lowpass RL filter with a cutoff frequency of 16 MHz connected to the DC source ($R_{dc} = 1 \text{ M}\Omega$ and $L_{dc} = 10 \text{ mH}$), which is used to set the operating bias voltage (U_0) on the capacitance-voltage characteristic of the varactor diode. The connection of the circuit to the resonators is implemented using a discrete port in the gap. To avoid additional losses in the pump circuit, we keep the following relation when changing the pump frequency:

$$L_{AC} = \frac{1}{(2\pi f_p)^2 C_{AC}}, \quad (4.23)$$

where L_{AC} is the inductance and $C_{AC} = 0.1 \text{ pF}$ is the capacitance used in the bandpass LC-filter. We also take into account parasitic capacitance and inductance of the package 0805-02 and the varactor diode's resistance [269]: $R_s = 0.3 \text{ }\Omega$, $C_p = 0.06 \text{ pF}$, $L_p = 0.4 \text{ nH}$.

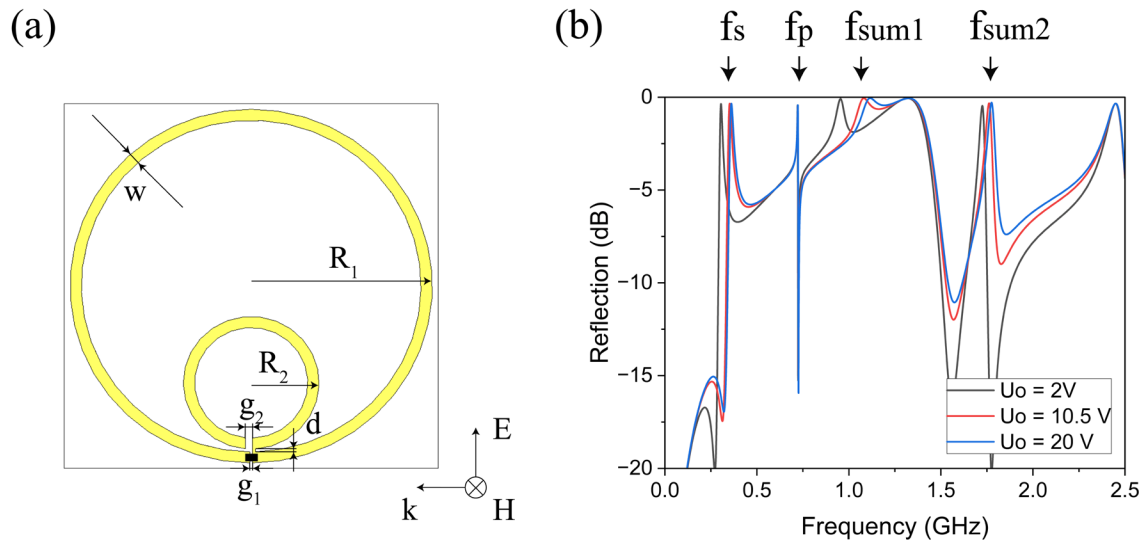


Figure 4-9 (a) The meta-atom design consisting of two copper SRRs placed on the substrate and the embedded varactor diode shown as a black square. (b) The reflection characteristics of the proposed metasurface for three DC voltage values (U_0) applied to the varactor diodes.

The SRRs have the following parameters shown in Figure 4-9(a): $R_1 = 34$ mm, $R_2 = 14$ mm, $w = 2$ mm, $g_1 = 0.5$ mm, $g_2 = 1.25$ mm, $d = 0.5$ mm. The thickness of the SRRs is 35 μm , the substrate thickness is 508 μm , and the unit cell size is 70 mm x 70 mm.

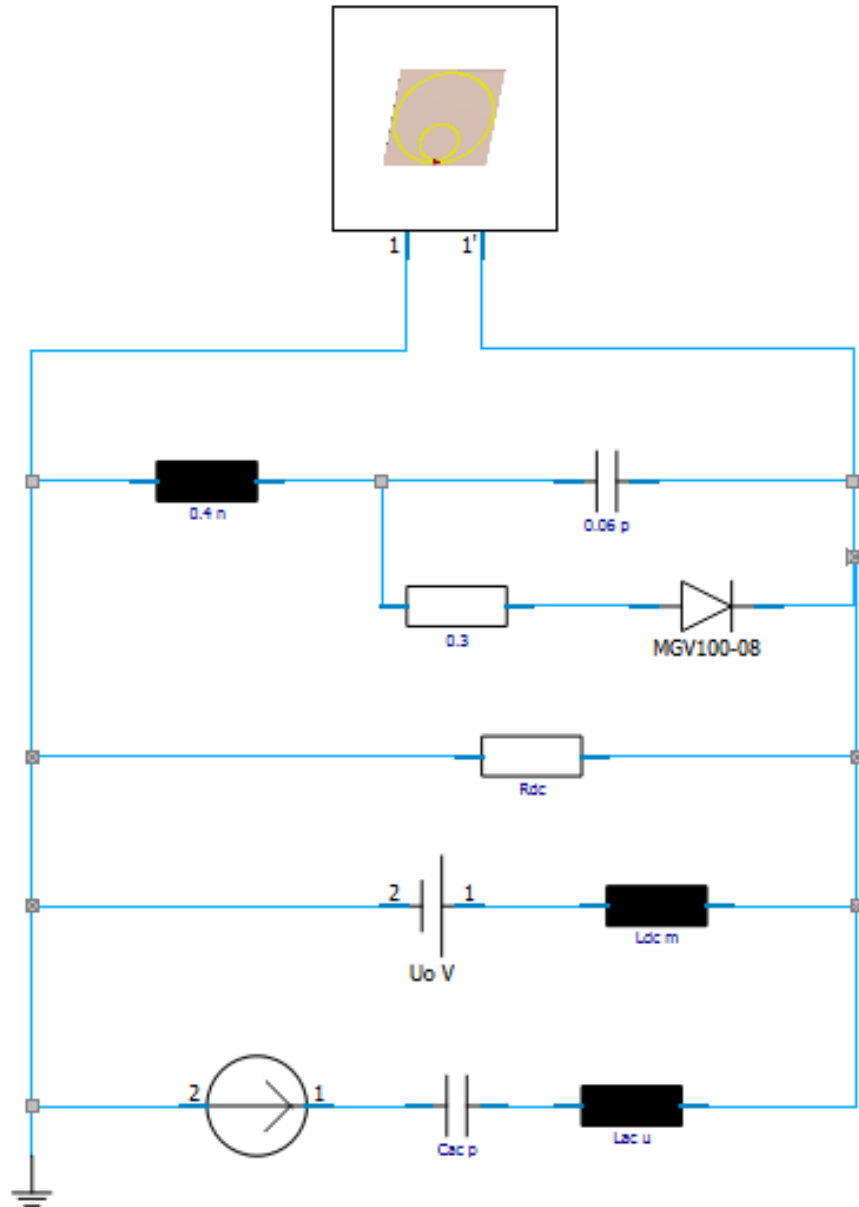


Figure 4-10 The pump circuit attached to the meta-atom.

The metasurface is an array of meta-atoms shown in Figure 4-8(a), supporting multiple resonances. We conduct full-wave numerical simulations of the periodic structure using the finite-element method and transfer the 3D modelling results to the Schematic

simulator in CST Studio to model the nonlinear lumped element circuit, which includes the varactor diode and the biasing network. Figure 4-9(b) illustrates the reflection characteristics of the proposed metasurface for three different DC voltage values (U_0) applied to the varactor diodes when the incident plane wave with one of the polarisations propagates along one of the substrate axes. In this work we utilise the resonances at f_s , f_{sum1} and f_{sum2} shown in Figure 4-9(b). The resonance at f_p is not related to the SRRs and their structure, it appears due to a bandpass LC-filter connecting the pump source and varactors in meta-atoms. The geometrical and circuit parameters of the SRRs were selected to realise the desired metasurface resonant responses in the low-frequency microwave range (below 2 GHz), while ensuring sufficient tunability, compatibility with standard fabrication processes and packaged varactor diodes, and feasibility of numerical modelling and experimental implementation.

The proposed parametric metasurface operates in the degenerate regime when

$$f_p = 2f_s, \quad (4.24)$$

$$f_{sum1} = f_p + f_s = 3f_s, \quad (4.25)$$

$$f_{sum2} = 2f_p + f_s = f_p + 3f_s = 5f_s. \quad (4.26)$$

We found that using a pump at twice the signal frequency and tuning the SRRs to resonate near integer multiples of the signal frequency enables the generation of higher-power up-converted electromagnetic waves compared to using frequencies that deviate significantly from these multiples. This occurs because of a cascaded (multi-stage) amplification and up-conversion process, in which the pump source amplifies the signal and up-converts it to the sum-frequencies. As a result, it is possible to exceed the maximum conversion efficiency determined by the frequency ratio in Eq. 4.22.

The cascaded process begins with parametric amplification of currents at the signal frequency, induced by the incident electromagnetic waves in the SRRs. These currents

are further mixed with the currents excited at the pump frequency, producing currents at the sum-frequencies, which then radiate up-converted electromagnetic waves.

We utilise the Spectral Lines task and the varactor SPICE model [269] within the Schematic module of CST Studio to simulate the nonlinear circuit in the frequency domain. This approach leverages Harmonic Balance analysis, allowing us to investigate the interaction between the pump and signal within the designed circuit. The method accounts for frequency mixing up to a defined harmonic limit. To accurately capture the mixing effects, we consider 20 harmonics for the signal source and 10 harmonics for the pump source.

First, we determine the frequency at which the maximum power of the radiated waves occurs in the degenerate mode. Figures 4-11(a-c) depict the dependence of the power of the amplified electromagnetic waves at f_s , f_{sum1} and f_{sum2} on the frequency for $U_0 = 10.5$ V and $U_p = 8$ V, when both f_s and f_p change under the condition of phase-locking in the degenerate regime (the pump phase is fixed relative to the phase of the incident wave to achieve optimal gain). The blue and red lines represent the power radiated in different directions relative to the propagation of the incident wave at f_s , denoted as P_t (transmission) and P_r (reflection), respectively.

Figures 4-11(d-f) show the dependence of the power of the amplified electromagnetic waves at $f_s = 0.336$ GHz, $f_{sum1} = 1.008$ GHz and $f_{sum2} = 1.68$ GHz on the pump amplitude at $f_p = 0.672$ GHz for $U_0 = 10.5$ V under the condition of phase-locking in the degenerate regime. The horizontal dotted line shows the incident electromagnetic wave power level (0.25 μ W). The area when the varactor model becomes inaccurate according to the varactor manufacturer ($U_p > 8.5$ V) is shaded. We predict amplification of more than 300 times (25 dB) at f_s , with the power of the radiated electromagnetic waves at f_{sum1} being more than 500 times (27 dB) and at f_{sum2} being more than 50 times (17 dB) higher than the power of the incident wave in the degenerate regime.

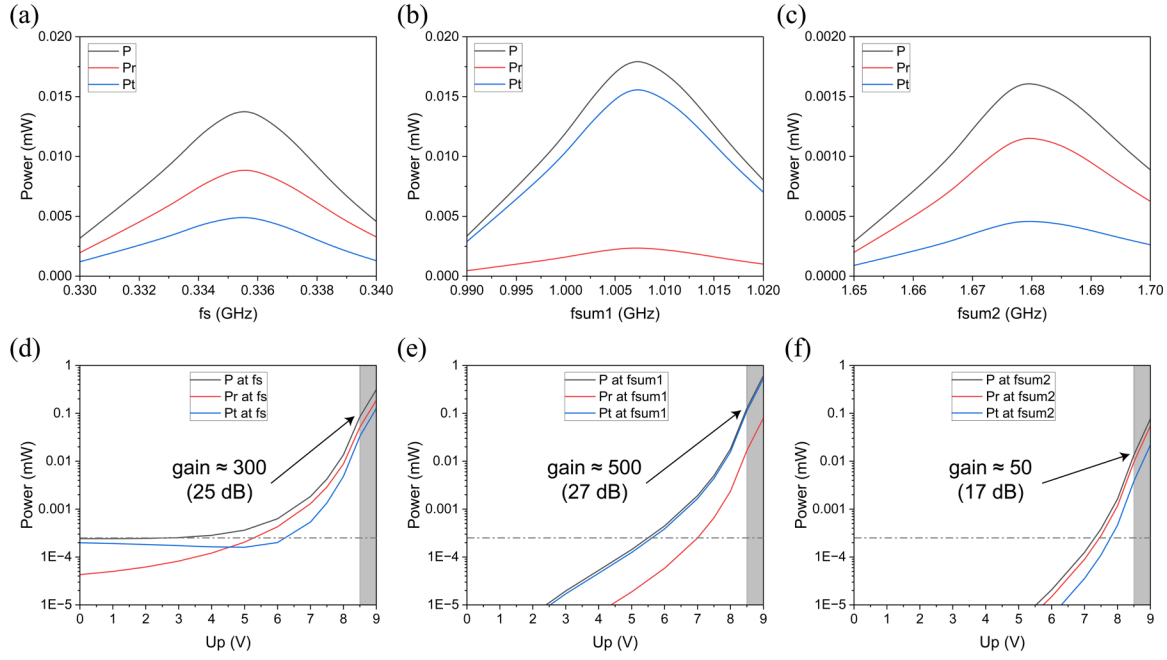


Figure 4-11 (a-c) The dependence of the power of the amplified electromagnetic waves at f_s , f_{sum1} and f_{sum2} on the frequency for $U_0 = 10.5$ V and $U_p = 8$ V under the condition of phase-locking in the degenerate regime. **(d-f)** The dependence of the power of the radiated electromagnetic waves at $f_s = 0.336$ GHz, $f_{sum1} = 1.008$ GHz and $f_{sum2} = 1.68$ GHz on the pump amplitude (U_p) at $f_p = 0.672$ GHz for $U_0 = 10.5$ V under the condition of phase-locking in the degenerate regime (logarithmic scale).

The blue and red lines indicate the power radiated to different directions. The horizontal dotted line shows the incident electromagnetic wave power level ($0.25 \mu W$). The area when the varactor model becomes inaccurate (for $U_p > 8.5$ V) is shaded.

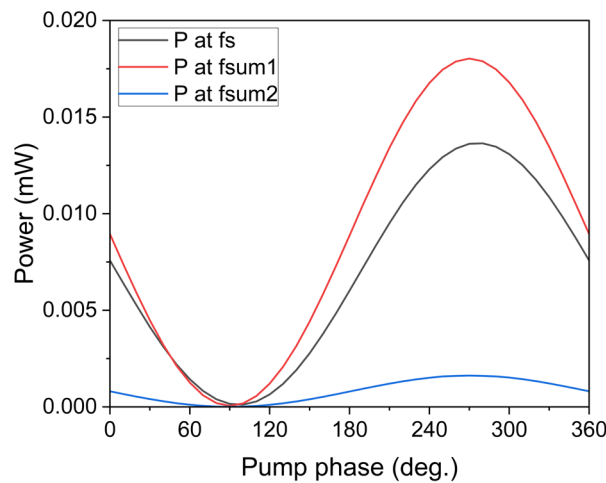


Figure 4-12 The dependence of the radiated power at $f_s = 0.336$ GHz, $f_{sum1} = 1.008$ GHz and $f_{sum2} = 1.68$ GHz on the pump phase at $f_p = 0.672$ GHz for $U_0 = 10.5$ V and $U_p = 8$ V in the degenerate regime.

The radiated power at f_s , f_{sum1} and f_{sum2} depends on the pump phase in the degenerate regime as it is shown in Figure 4-12 for $U_0 = 10.5$ V and $U_p = 8$ V. In our work, we use a pump phase of 270 degrees to achieve maximum radiated power in the degenerate regime, referring to this condition as phase-locking — where the pump phase is fixed relative to the incident wave phase to ensure optimal gain.

Next, we analyse the operation of the proposed parametric metasurface in the non-degenerate regime when the frequency ratio given in Eq. 4-24 – 4-26 is not satisfied. This process is less efficient due to the additional generation of electromagnetic waves at idler frequencies f_i , f_{sum1i} and f_{sum2i} . The non-degenerate regime is characterised by the following frequency ratio: $f_i = f_p - f_s$, $f_{sum1} = f_p + f_s$, $f_{sum1i} = f_p + f_i$, $f_{sum2} = 2f_p + f_s$, $f_{sum2i} = 2f_p + f_i$.

In this section, we analyse two non-degenerate regimes: a) operating near the degenerate regime in Eq. 4-24 – 4-26; when $f_{p1} = 0.671$ GHz; b) operating at frequencies far from those of the degenerate regime when $f_{p2} = 0.74$ GHz. These scenarios are labelled as 1 and 2 in Figures 4-13(a-c) for clarity. We demonstrate that higher conversion efficiency can be achieved at frequencies close to the degenerate case, while also exceeding the converted power limits imposed by the Manley-Rowe relations in Eq. 4-22.

Figures 4-13(a-c) show the dependence of the power of the radiated electromagnetic waves at f_s , f_{sum1} , f_{sum2} (solid lines), f_i , f_{sum1i} and f_{sum2i} (dotted lines) on the frequency in the non-degenerate regime for $U_0 = 10.5$ V, $U_p = 8.5$ V and $f_s = 0.336$ GHz. The horizontal dotted line shows the incident electromagnetic wave power level (0.25 μ W). Although the degenerate regime (corresponds to the narrow peaks when $f_p = 0.672$ GHz) allows for achieving significantly higher level of output power at both sum-frequencies and the signal frequency, its practical realisation is challenging due to its extremely narrow band. In addition to phase locking, fine tuning of the pump and signal sources is necessary. While the power of the radiated electromagnetic waves in the non-degenerate regime is lower, it is still possible to achieve a level at both sum-frequencies that exceeds the incident wave power.

Figures 4-13(a-c) show that the non-degenerate regime operating near the degenerate resonance frequency ratio allows for achieving significant level of the radiated power at f_s , f_{sum1} , f_{sum2} (solid lines), f_i , f_{sum1i} and f_{sum2i} (dotted lines). The 3-dB bandwidth of the up-conversion at f_{sum1} (close to 1.008 GHz) and at f_{sum2} (close to 1.68 GHz) is around 3 and 6 MHz respectively.

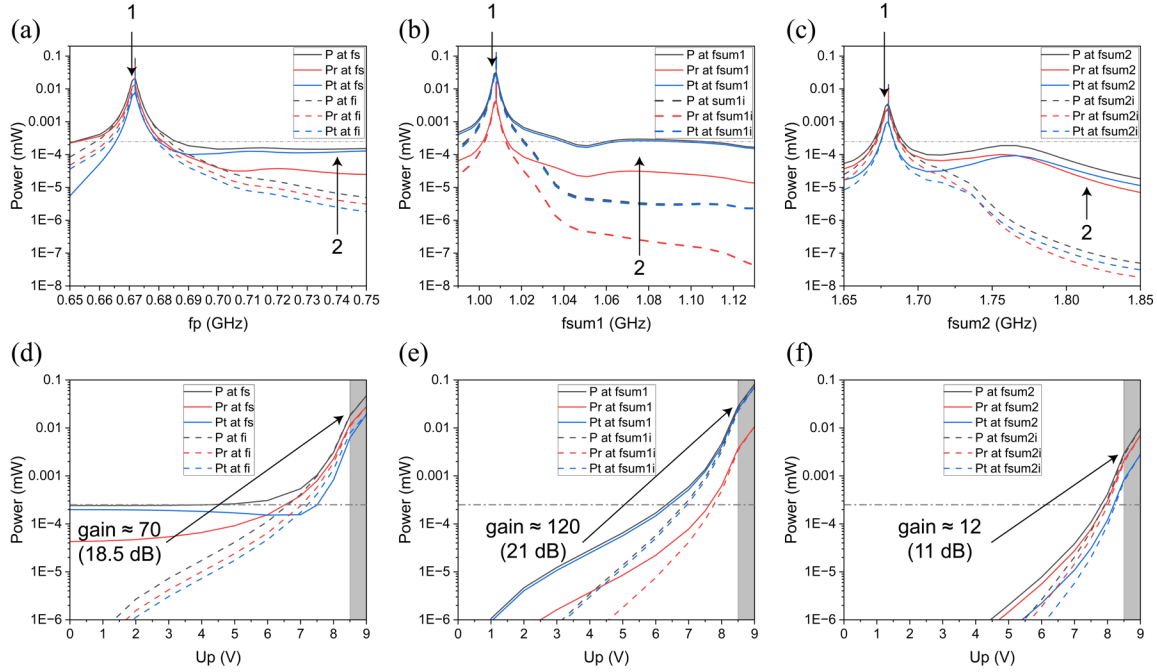


Figure 4-13 (a-c) The dependence of the power of the radiated electromagnetic waves at f_s , f_{sum1} , f_{sum2} (solid lines), f_i , f_{sum1i} and f_{sum2i} (dotted lines) on the frequency in the non-degenerate regime for $U_0 = 10.5$ V and $U_p = 8.5$ V when $f_s = 0.336$ GHz and f_p changes (logarithmic scale). The blue and red lines indicate the transmitted and reflected power (radiated by the metasurface in different directions), while the horizontal dotted line shows the incident electromagnetic wave power level ($0.25 \mu W$). (d-f) The dependence of the power of the radiated electromagnetic waves at $f_s = 0.336$ GHz, $f_i = 0.335$ GHz, $f_{sum1} = 1.007$ GHz, $f_{sum1i} = 1.006$ GHz, $f_{sum2} = 1.678$ GHz and $f_{sum2i} = 1.677$ GHz on the pump amplitude (U_p) at $f_{p1} = 0.671$ GHz for $U_0 = 10.5$ V in the non-degenerate regime (logarithmic scale). The area when the varactor model becomes inaccurate (for $U_p > 8.5$ V) is shaded.

Figures 4.13(d-f) show the dependence of the power of the radiated electromagnetic waves at $f_s = 0.336$ GHz, $f_i = 0.335$ GHz, $f_{sum1} = 1.007$ GHz, $f_{sum1i} = 1.006$ GHz, $f_{sum2} = 1.678$ GHz and $f_{sum2i} = 1.677$ GHz on the pump amplitude at $f_{p1} = 0.671$ GHz for $U_0 = 10.5$ V in the non-degenerate regime. The area when the varactor model becomes

inaccurate (for $U_p = 8.5$ V) is shaded. We predict amplification of more than 70 times (18.5 dB) at f_s , with the power of the radiated electromagnetic waves at f_{sum1} being more than 120 times (21 dB) and at f_{sum2} being more than 10 times (10 dB) higher than the power of the incident wave in the non-degenerate regime within the limits of the varactor model.

Figure 4-13(b) also shows that the power of the radiated electromagnetic waves around $f_{sum1} = 1.076$ GHz is higher than the power of the incident wave. This case corresponds to the sum-frequency value close to the reflection maximum when $U_0 = 10.5$ V shown in Figure 4-9(b) (red curve). We expect that the precise tuning of the resonant frequencies of the SRRs will result in the absence of this additional peak in the up-conversion spectra and allow us to achieve higher power levels of radiated electromagnetic waves at frequencies close to the degenerate resonance frequency ratio in Eq. 4-24 – 4-26. This slight detuning allows us to use the same system for comparing two cases of the non-degenerate regimes as discussed above.

Next, we analyse the non-degenerate regime when $f_{p2} = 0.74$ GHz and $f_s = 0.336$ GHz. In this case, there is no amplification of the electromagnetic waves at f_s , and the power of the incident waves is utilised for up-conversion. Figure 4-14 illustrates a decrease in the radiated power at f_s (a) and an increase in the radiated power at both sum-frequencies (b) as the pump amplitude increases, with the power at f_{sum1} exceeding the incident power when $U_p > 7.5$ V. This contrasts with the previously discussed non-degenerate regime operating near the degenerate resonance frequency regime, when we observe the exponential growth of the radiated power at f_s , f_{sum1} and f_{sum2} with increasing the modulation amplitude due to the cascaded amplification process. The dependence of the radiated power at f_{sum1} on the pump amplitude shows a decrease in its growth rate when U_p approaches 8.5 V.

The maximum radiated power at f_{sum1} in the non-degenerate regime operating at frequencies far from the degenerate regime is approximately 0.3 μ W for $U_0 = 10.5$ V, $U_p = 8.5$ V, $f_{p2} = 0.74$ GHz and $f_s = 0.336$ GHz. This results in a conversion efficiency of

1.2 (not exceeding 1 dB), which is significantly lower compared to the non-degenerate regime operating near the degenerate resonance frequency ratio with the same pump amplitude, where the conversion efficiency is approximately 120 (exceeding 20 dB) for $U_0 = 10.5$ V, $U_p = 8.5$ V, $f_{p1} = 0.671$ GHz and $f_s = 0.336$ GHz due to the cascaded amplification and up-conversion process.

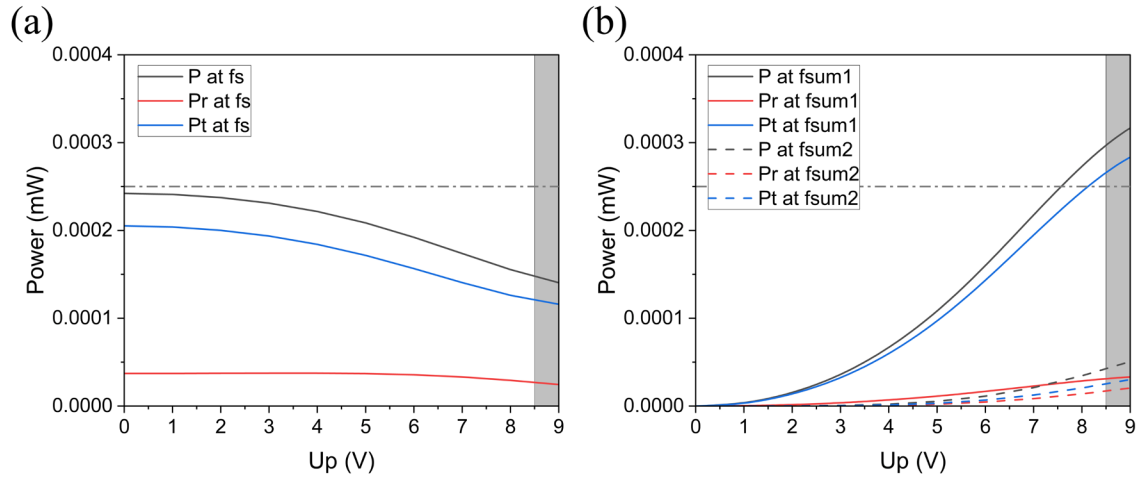


Figure 4-14 The dependence of the power of the radiated electromagnetic waves at $f_s = 0.336$ GHz (a), $f_{sum1} = 1.076$ GHz and $f_{sum2} = 1.816$ GHz (b) on the pump amplitude (U_p) at $f_{p2} = 0.74$ GHz for $U_0 = 10.5$ V in the non-degenerate regime. The blue and red lines indicate the power radiated to different directions. The area when the varactor model becomes inaccurate (for $U_p > 8.5$ V) is shaded.

Additionally, we study the dependence of the phase of the radiated electromagnetic waves at both sum-frequencies on the pump phase. We observe the linear relationship between the phases in the non-degenerate regime when $f_{p1} = 0.671$ GHz and $f_{p2} = 0.74$ GHz, as depicted in Figures 4-15(a,b), respectively. This suggests that by choosing an appropriate pump we can control the phase of the up-converted waves. Since the phase modulation can be made non-uniform across the metasurface, the proposed approach can be utilised not only for amplifying and up-converting electromagnetic waves but also for spatial control of the converted waves. The phase gradient across the array of the meta-atoms opens possibilities for beam steering and other forms of control useful in communication applications.

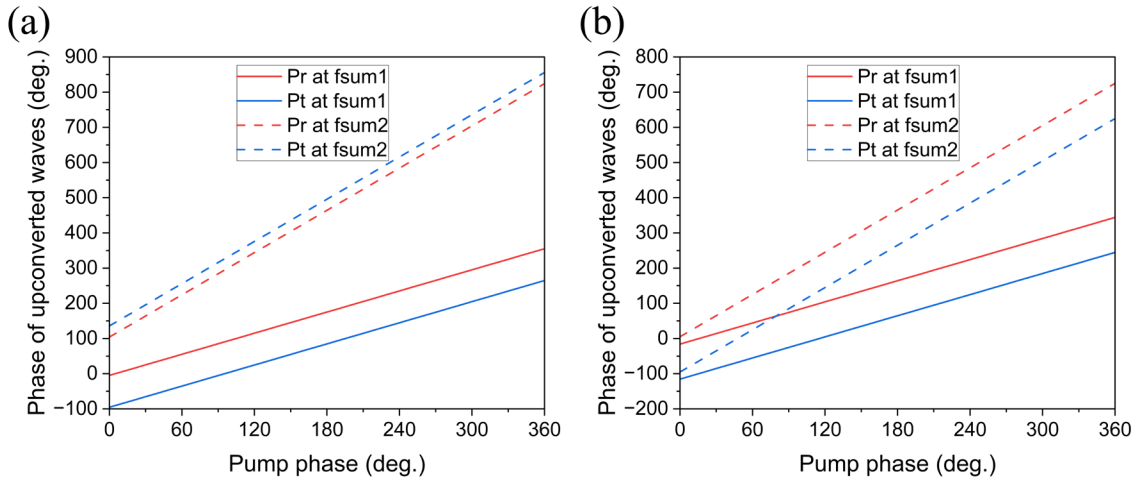


Figure 4-15 The dependence of the radiated electromagnetic wave phase at both sum-frequencies f_{sum1} and f_{sum2} on the pump phase in two non-degenerate regimes: when $f_{p1} = 0.671$ (a) and $f_{p2} = 0.74$ GHz (b).

4.6 Experimental confirmation of amplified up-conversion of electromagnetic waves using parametric metasurfaces

This section describes the optimised meta-atom design and the TEM cell used in the experiments. We first present additional modelling results based on the optimised design and demonstrate the feasibility of amplified up-conversion in both degenerate and non-degenerate regimes using a realistic meta-atom model. The diagrams of the experimental setup are then introduced, followed by a comparison between the characteristics of the parametric meta-atoms in the TEM cell obtained from modelling and experiments. Finally, we analyse the experimentally measured amplified up-conversion of electromagnetic waves using the fabricated meta-atoms in the TEM cell.

The results provide experimental confirmation of the modelling predictions discussed earlier in the chapter. Specifically, the parametric meta-atoms achieve an up-conversion efficiency (gain) exceeding 10 dB in the degenerate regime and 5 dB in the non-degenerate regime, despite the additional losses revealed in experiments. These findings highlight the accuracy of the parametric metasurface modelling. Furthermore, the experimental results confirm that the up-conversion efficiency is not constrained by the Manley-Rowe relations, owing to the multi-cascade amplification mechanism. Notably, the observed power of the radiated waves at the sum frequency is more than 5 dB (over

three times) higher than the amplified and radiated power at the signal frequency when the pump power approaches the value corresponding to the self-excitation threshold in both degenerate and non-degenerate regimes.

First, we present the optimised geometry of the meta-atoms together with the pump and biasing circuit used in additional modelling. Figure 4-16 illustrates the schematic of the varactor diode MGV100-20 SPICE model [269], along with the pump and biasing circuit attached to the meta-atom. The SPICE model incorporates the parasitic capacitance and inductance of the 0805-02 package and the resistance of the varactor diode: $R_s = 0.3 \text{ Ohm}$, $C_p = 0.06 \text{ pF}$, $L_p = 0.4 \text{ nH}$. The varactor diode is modulated through a filtering circuit consisting of a passband LC filter at f_p ($C_{ac} = 0.1 \text{ pF}$ and $L_{ac} = 370 \text{ nH}$, with the parasitic capacitance $C_{pac} = 0.0685 \text{ pF}$) connected in series with an AC source of 50Ω impedance, and a choke connected to the DC source ($L_{dc} = 500 \text{ nH}$ with the parasitic capacitance $C_{pdc} = 0.063 \text{ pF}$).

The passband filter at f_p is designed for protecting the microwave generator from the DC and AC currents at f_s , f_{sum1} and f_{sum2} . The optimised design incorporates the parasitic capacitances of the inductors, which significantly affect the characteristics of the parametric meta-atoms due to the small capacitance of the varactor diodes. The varactor diodes employed in this schematic differ slightly from those used in the previous modelling. The MGV100-08 diode exhibits lower capacitance values compared to the MGV100-20, while all other parameters remain identical [269]. Furthermore, no resistor is connected in parallel with the varactor diode. Instead, a separate choke was implemented for filtering, replacing the RL low-pass filter. This adjustment was necessary because embedding a high-impedance resistor together with an inductor of large inductance is challenging for the experimental realisation of the parametric meta-atoms.

Figure 4-17 presents the meta-atom design used for additional modelling. Figure 4-17(a) illustrates the configuration consisting of two perpendicular circuit boards with discrete ports. The additional horizontal circuit board serves two purposes: it delivers the pump

power to the varactor diode via an AC microstrip line and connects the SRRs to the ground through a via. This arrangement provides the required isolation between the propagating electromagnetic wave and the pump circuit, ensuring that their influence on wave propagation is negligible, since the electric field is normal to the horizontal circuit board.

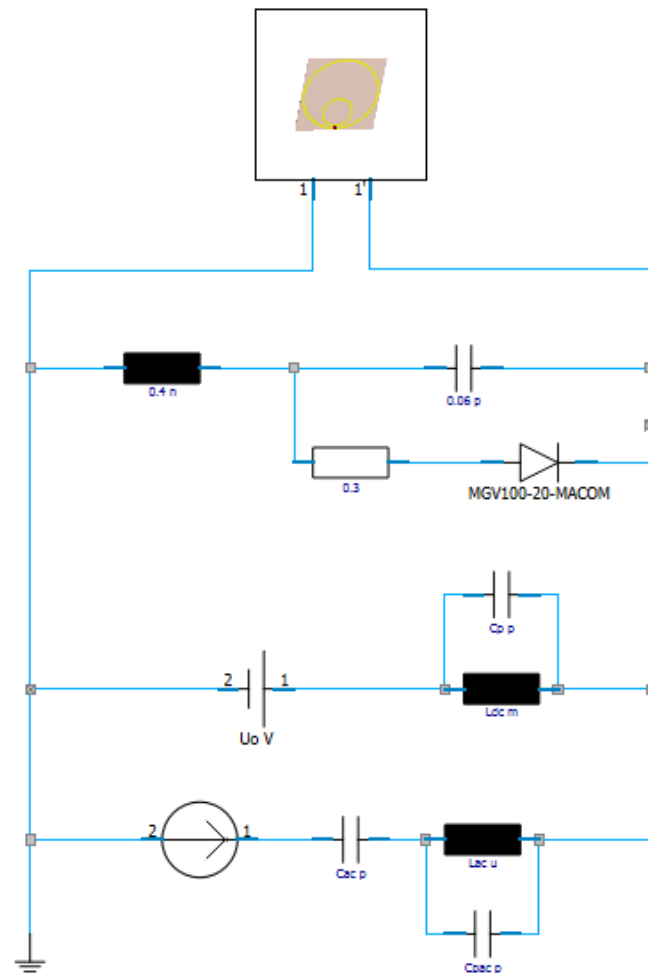


Figure 4-16 Schematic of the varactor diode MGV100-20 SPICE model, along with the pump and biasing circuit attached to the meta-atom.

Three discrete ports, instead of the single port placed in the SRR gap in the earlier modelling, were introduced for more realistic representation. On the vertical circuit board, a discrete port was used to model the varactor diode together with its lumped elements and the DC source with the choke and its parasitic capacitance. On the horizontal board, a microstrip line with two discrete ports was included: one for modelling the LC passband filter and the other for the AC pump source with a 50-Ω

impedance. The optimised design accounts for parasitic capacitances and the influence of the AC microstrip connected to the meta-atom, thereby providing more accurate results for subsequent experimental implementation. However, at this stage of the modelling we have not considered the DC power line.

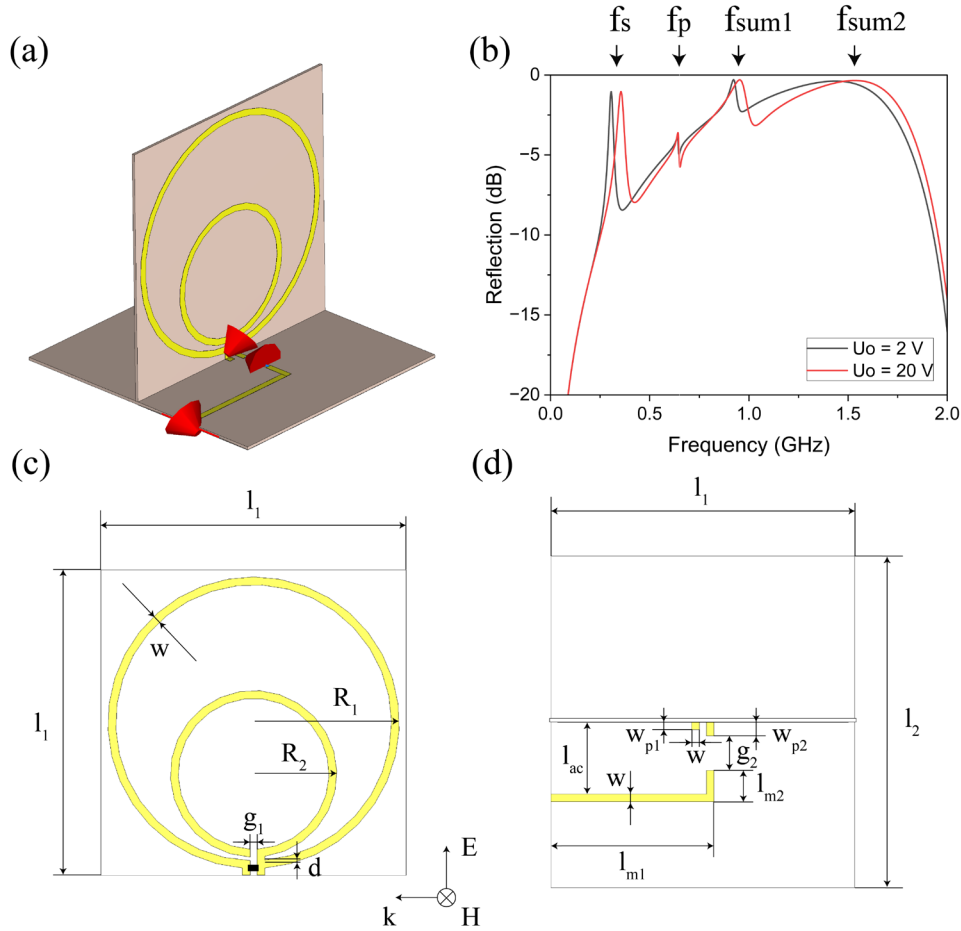


Figure 4-17 Meta-atom design used for additional modelling and corresponding metasurface reflection. (a) Meta-atom comprising two circuit boards with three discrete ports (red cones). (b) Reflection characteristics of the metasurface for two applied DC voltage values (U_0). (c) Geometry of the vertical circuit board with two copper SRRs on the substrate and an embedded varactor diode (black square). (d) Geometry of the horizontal circuit board with a microstrip line for pumping the varactor diode and a pad connecting to ground through a via. The two central lines indicate the position of the vertical circuit board for clarity.

Figure 4-17(b) shows the reflection characteristics of the metasurface for two applied DC voltages (U_0). Two narrow resonances at f_s and f_{sum1} shift with variations in the applied voltage. The meta-atom geometry was optimised so that these resonances satisfy the approximate relation $f_{sum1} = 3f_s$. Although no narrow resonance is observed at $f_{sum2} =$

$5f_s$, a broad resonance close to $5f_s$ can be seen whose central frequency shifts with the change of the applied DC bias. The peak at f_p does not originate from the meta-atom geometry itself; rather, it appears in reflection due to the LC passband filter connected to the SRRs.

Figures 4-17(c,d) provide the geometrical details of the meta-atom. Figure 4-17(c) depicts the vertical circuit board and the incident plane wave vectors, while Figure 4-17(d) shows the horizontal circuit board together with the position of the vertical board for clarity. The circuit board dimensions are: $l_1 = 44$ mm and $l_2 = 48$ mm. Rogers RO4350B substrates of thickness 0.51 mm were used in the modelling, with dielectric permittivity $\epsilon_r = 3.66$ and loss tangent $\tan\delta = 0.0037$. The thickness of all copper components is 35 μm . The SRRs have the following parameters: $R_1 = 21$ mm, $R_2 = 12$ mm, $w = 1.116$ mm, $g_1 = 1$ mm, $d = 0.543$ mm. The geometrical parameters were optimised by accounting for the influence of parasitic capacitances and microstrip interconnections on the metasurface resonant response, in order to ensure the required tunability and the target frequency ratios discussed in the preceding sections. The microstrip line dimensions are: $w = 1.116$ mm, $g_2 = 5$ mm, $l_{ac} = 10.456$ mm, $l_{m1} = 23.616$ mm, $l_{m2} = 4.558$ mm. The AC microstrip width was selected to realise a 50- Ω transmission line at f_p . The pad connecting the SRRs to ground has dimensions $w = 1.116$ mm and $w_{p1} = 1.081$ mm, while the pad connecting the SRRs to the microstrip has dimensions $w = 1.116$ mm and $w_{p2} = 1.965$ mm. The via diameter is 0.6 mm. The substrate of the horizontal circuit board is copper-coated and functions as a ground plane.

Figures 4-18(a–c) show the dependence of the radiated power at $f_s = 320$ MHz, $f_{sum1} = 960$ MHz and $f_{sum2} = 1600$ MHz on the pump amplitude U_p at $f_p = 640$ MHz, for $U_0 = 10$ V under the condition of phase locking in the degenerate regime. The dashed horizontal line indicates the incident wave power at f_s , equal to 0.25 μW . The shaded region marks the range where the varactor model becomes inaccurate according to the manufacturer ($U_p > 7.5$ V). However, it should be noted that the selected varactor diodes operate in reverse bias from 0 to 22 V, with the latter value corresponding to breakdown. The results shown in Figure 4-18 lie within this range.

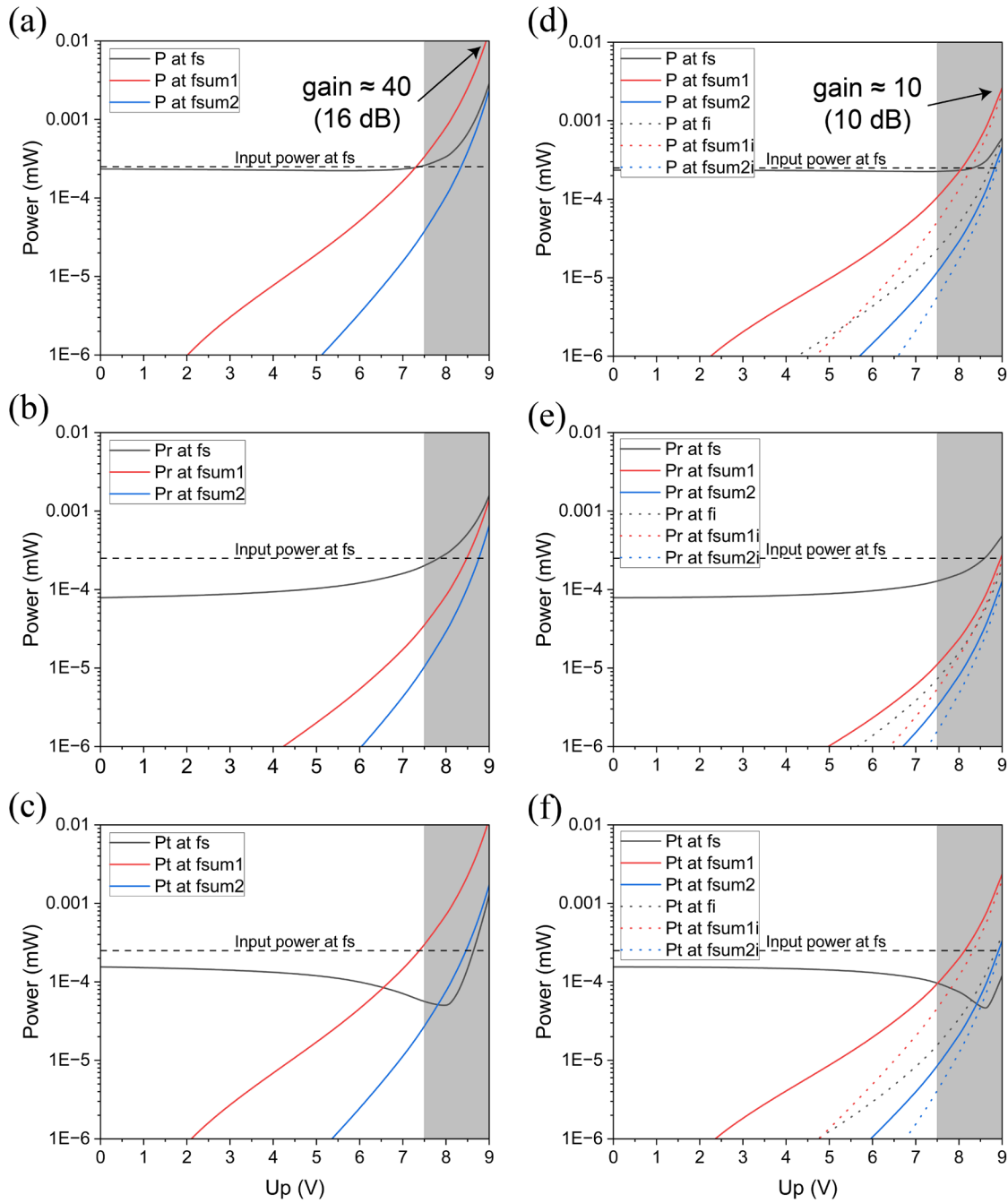


Figure 4-18 Radiated power versus pump amplitude for $U_0 = 10$ V (logarithmic scale). (a–c) Degenerate regime under phase-locking conditions with $f_s = 320$ MHz, $f_p = 640$ MHz, $f_{sum1} = 960$ MHz and $f_{sum2} = 1600$ MHz: (a) total radiation, (b) reflection, and (c) transmission relative to the incident wave at f_s . (d–f) Non-degenerate regime with $f_s = 320$ MHz, $f_i = 319.9$ MHz, $f_p = 639.9$ MHz, $f_{sum1} = 959.9$ MHz, $f_{sum1i} = 959.8$ MHz, $f_{sum2} = 1599.9$ MHz and $f_{sum2i} = 1599.8$ MHz: (d) total radiation, (e) reflection, and (f) transmission. The input power level at f_s is indicated by the horizontal dashed line. The shaded region corresponds to $U_p > 7.5$ V, where the varactor model becomes inaccurate.

The obtained characteristics are consistent with those discussed earlier, though the radiated power levels are lower. This reduction arises from the additional parasitic elements, which decrease the tunability of the system and thereby reduce the efficiency of modulation for amplification and up-conversion. Nevertheless, Figure 4-18(a) demonstrates that the overall radiated power at f_{sum1} exceeds the incident power level within the range of validity of the SPICE model and significant amplification and up-conversion efficiency can be achieved at a pump amplitude of 9 V. At this point ($U_p = 9$ V, $U_0 = 10$ V), the overall radiated power at f_{sum1} is more than 40 times higher than the incident power at f_s , corresponding to a gain of 16 dB.

Figure 4-18(a) also shows that the overall power radiated at f_s grows exponentially once the pump amplitude exceeds 7 V, whereas it remains nearly constant until this value is reached. The radiated power at both sum-frequency components increases rapidly with pump amplitude. Notably, the power at f_{sum1} at a pump amplitude of 9 V is more than four times greater than the radiated power at f_s , indicating that the Manley-Rowe limit can be surpassed not only with respect to the input power at f_s but also relative to the amplified and radiated power at f_s in the degenerate regime.

Figures 4-18(b,c) further demonstrate that the metasurface radiates more power at the sum frequencies in transmission than in reflection. While the reflected power at f_s increases monotonically with pump amplitude, the transmitted power at f_s initially decreases until $U_p \approx 8$ V, after which it rises sharply with further increases. This behaviour is consistent with previously discussed parametric amplification phenomena.

Figure 4-18(d) demonstrates that, in the non-degenerate regime, the overall radiated power at f_{sum1} exceeds the input power at f_s by more than an order of magnitude when the pump amplitude reaches 9 V and the pump frequency is set near the degenerate case ($f_p = 639.9$ MHz). However, when the pump amplitude remains within the range of validity of the varactor model (below 7.5 V), the radiated power at both sum frequencies is lower than the incident power at f_s . Figures 4-18(d–f) illustrate the amplification and up-conversion characteristics, which closely resemble those obtained in the degenerate

case. In the non-degenerate regime, however, the efficiency is reduced due to the simultaneous generation of power at the idler frequencies, as shown in Figures 4-18(d–f).

Figure 4-19 shows the dependence of the radiated power at $f_s = 320$ MHz, $f_{sum1} = 960$ MHz and $f_{sum2} = 1600$ MHz on the pump phase at $f_p = 640$ MHz for $U_0 = 10$ V and $U_p = 9$ V in the degenerate regime, in both reflection and transmission directions relative to the incident wave at f_s . The results in Figures 4-18(a–c) correspond to a pump phase of 190° , which yields the maximum radiated power at f_{sum1} . Figure 4-19 demonstrates that the radiated power at f_s , f_{sum1} and f_{sum2} follows a nearly sinusoidal dependence on the pump phase in the degenerate regime.

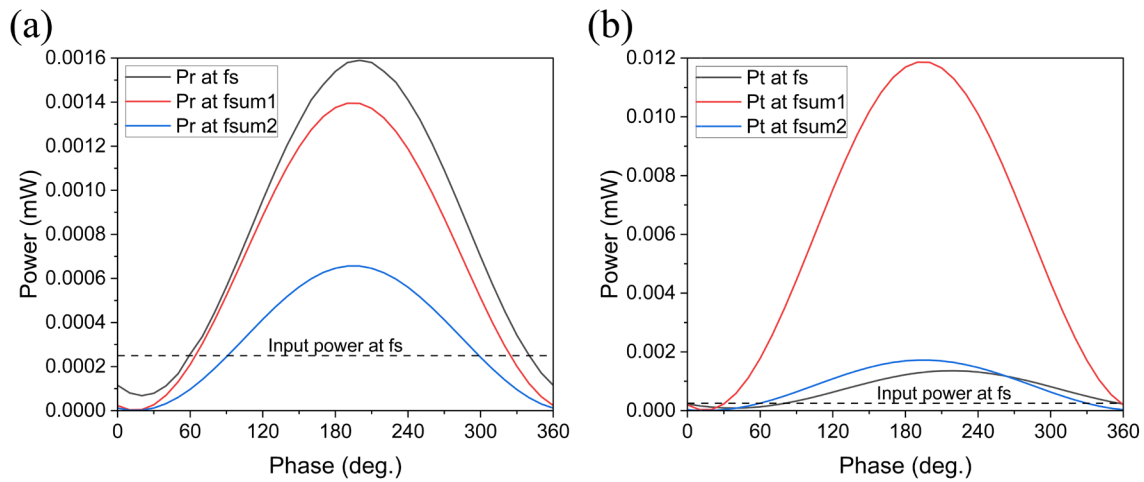


Figure 4-19 Radiated power at $f_s = 320$ MHz, $f_{sum1} = 960$ MHz and $f_{sum2} = 1600$ MHz versus pump phase at $f_p = 640$ MHz for $U_0 = 10$ V and $U_p = 9$ V in the degenerate regime: (a) reflection and (b) transmission relative to the incident wave at f_s .

Next, we describe the meta-atom design used in the experiments. Figure 4-20 presents the structure, which consists of two microwave circuits fabricated on Rogers RO4350B substrates. The vertical board incorporates two connected copper SRRs with an embedded MGV100-20 varactor diode and an additional blocking capacitor C0603C102J5GACTU to protect the DC source connection to ground via the outer ring. The resonator parameters, shown in Figure 4-20, are $R_1 = 21$ mm, $R_2 = 12$ mm, $w = 1.116$ mm, $g_1 = 1$ mm, $g_2 = 0.6$ mm, $d = 0.543$ mm. The AC microstrip line provides

pumping of the varactor diode through a passband filter consisting of an inductor LQW18ANR13G00D and a capacitor 06035J0R1PBSTR, driven by a 50 Ω microwave generator. A smaller inductor than in modelling was selected to improve pumping efficiency at f_p after experimental verification of the passband filter, since the resonance frequency shifted between the modelled and fabricated designs. The DC microstrip line includes an inductor LQW15CAR50J00D to block AC currents from reaching the DC source.

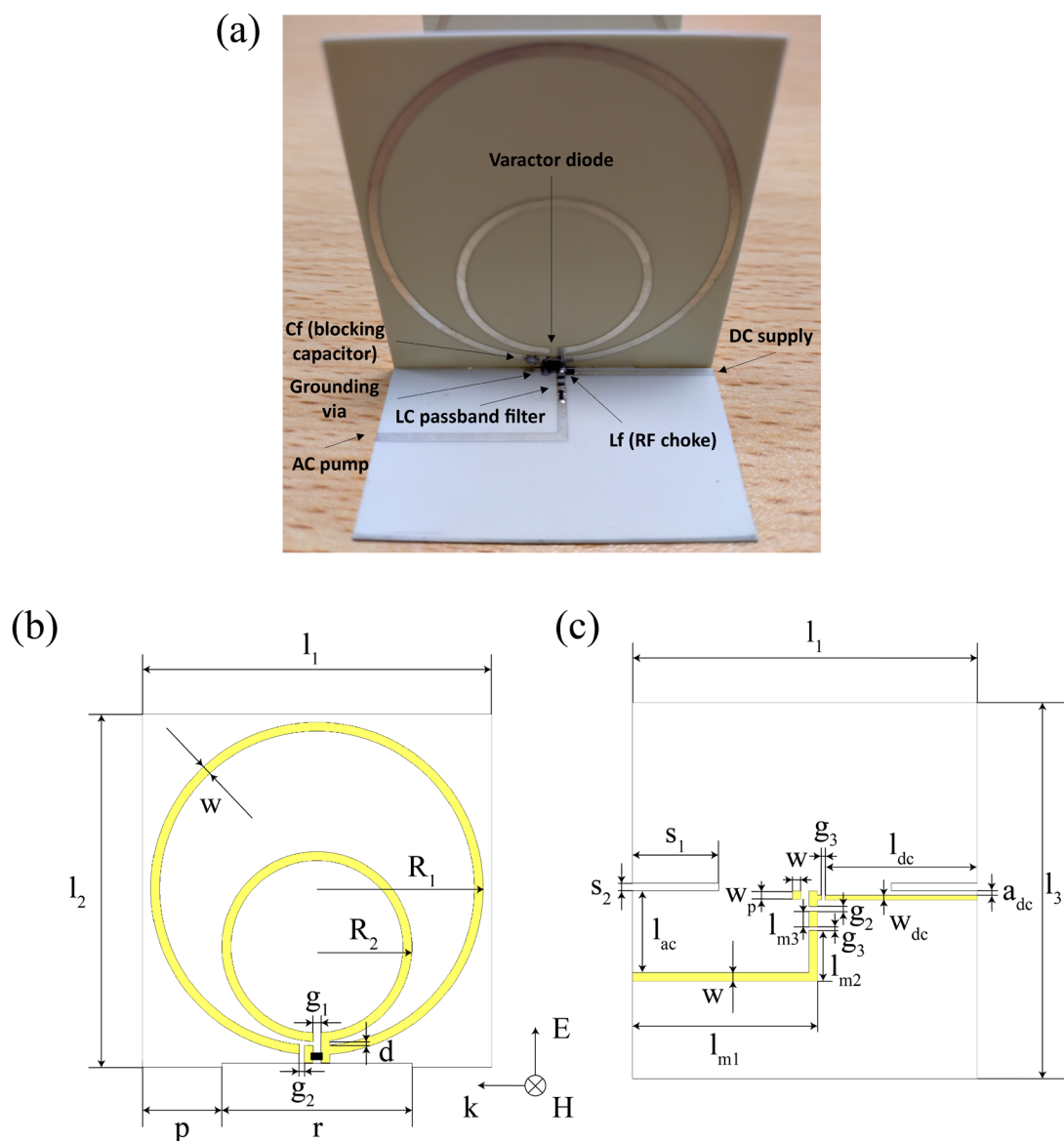


Figure 4-20 Meta-atom design used in experiments: (a) photograph of the fabricated meta-atom with soldered lumped elements; (b) vertical circuit board with two connected SRRs and an embedded varactor diode (black square); (c) horizontal circuit board with the microstrip and DC line.

The microstrip dimensions are $w = 1.116$ mm, $g_2 = 0.6$ mm, $g_3 = 0.45$ mm, $w_{dc} = 0.64$ mm, $l_{dc} = 19.434$ mm, $a_{dc} = 0.78$ mm, $l_{ac} = 10.456$ mm, $l_{m1} = 23.616$ mm, $l_{m2} = 6.508$ mm, $l_{m3} = 2$ mm. The pad connecting the resonators to ground has the following size: $w = 1.116$ mm and $w_p = 1.081$ mm, with via diameter of 0.6 mm. Copper thickness is 35 μ m throughout.

The circuit boards have dimensions $l_1 = 44$ mm, $l_2 = 44.51$ mm, $l_3 = 48$ mm, and a thickness of 0.51 mm. The horizontal board includes two slots ($s_1 = 11$ mm, $s_2 = 1.123$ mm) for inserting two protrusions of the vertical board ($p = 10$ mm) spaced by $r = 24$ mm. After soldering the lumped elements, the two circuit boards were glued using hot-melt adhesive, and finally the pads of the horizontal board were soldered to the SRRs.

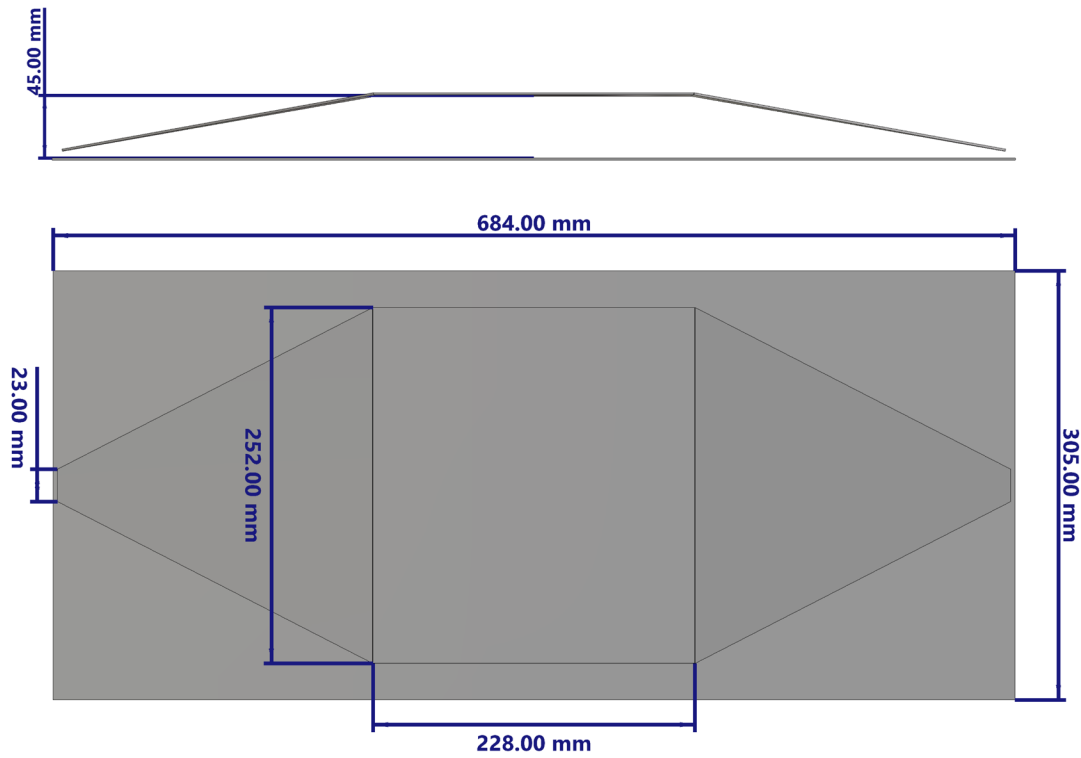


Figure 4-21 TEM cell and its dimensions.

Next, we provide details of the TEM cell used to emulate the response of the meta-atoms under plane-wave incidence. TEM cells offer a controlled environment for generating plane waves in a compact setting and for detecting radiation from meta-atoms. Functioning as waveguides, they convert electrical signals into nearly homogeneous

electromagnetic fields with a transverse mode distribution, closely resembling free-space conditions in modelling.

Figure 4-21 presents a schematic of the TEM cell designed in collaboration with Prof. Stanislav Maslovski at the University of Aveiro. It consists of two parts: the ground and the stripline. The ground plane has dimensions of 684 mm × 305 mm. The stripline comprises three segments: a central part (228 mm × 252 mm) and two side parts (252 mm × 255.14 mm × 23 mm). The distance between the central stripline and the ground is 45 mm, while the shortest facets of the side parts are separated from the ground by 5 mm. The dimensions of the ground and the stripline were selected in a way to maintain approximately uniform characteristic impedance along the whole TEM cell.

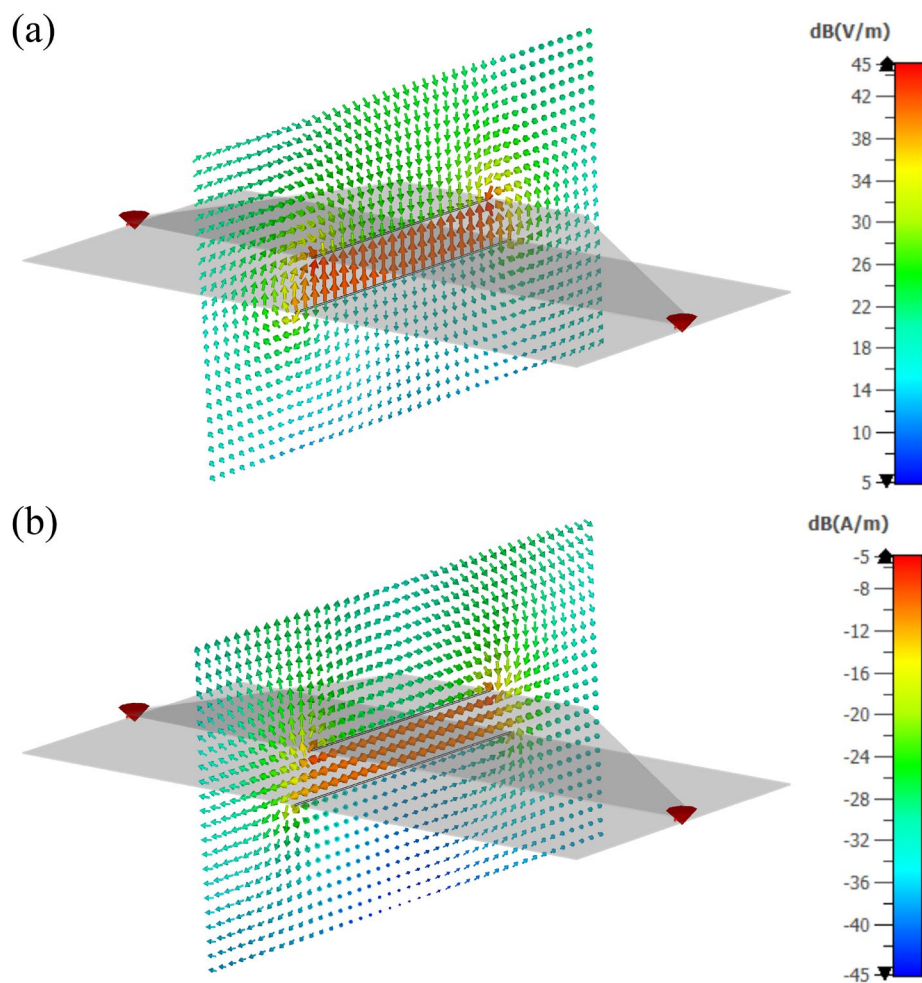


Figure 4-22 (a, b) Electric and magnetic field in the plane perpendicular to the TEM cell stripline and ground when excited at 320 MHz from one of its ports, shown as red cones.

Figures 4-22(a,b) illustrate the electric and magnetic fields in a plane perpendicular to the stripline and the ground plane when the TEM cell is excited at one port (shown as red cones) at 320 MHz. The fields are predominantly confined between the stripline and the ground plane, with the electric field oriented normal to these conductors and the magnetic field parallel to them. The stripline and the ground plane impose electric-wall boundary conditions, while the open lateral boundaries provide an approximate magnetic-wall symmetry. These symmetries are equivalent to the use of electric and magnetic mirror planes commonly employed to simulate infinite periodic structures. Consequently, when the meta-atom array is appropriately positioned with respect to these symmetry boundaries, the excitation conditions inside the TEM cell emulate those of an infinite periodic metasurface under plane-wave illumination.

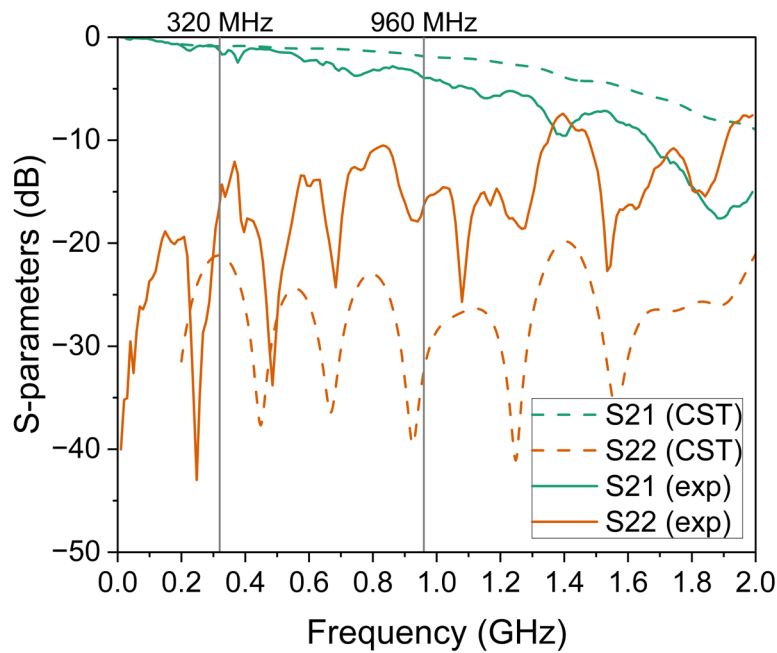


Figure 4-23 The simulated (dashed) and measured (solid) S-parameters of the TEM cell. The vertical lines at 320 and 960 MHz highlight the TEM cell performance around f_s and f_{sum1} .

Figure 4-23 shows the modelled and measured S-parameters of the TEM cell. The measurements were performed using a Keysight PNA-X N5242A vector network analyser. The results confirm the expected performance: although the measured reflection (S22) is slightly higher at high frequencies and the transmission (S21) at 960 MHz is -4 dB

compared to -2 dB in simulations, the reflection remains below -10 dB for frequencies up to 1350 MHz, which is sufficient to accurately measure the up-conversion characteristics at f_{sum1} .

The developed TEM cell model was further used to obtain the scattering characteristics of seven parametric meta-atoms excited at f_s from one of its ports, as illustrated in Figure 4-24(a). For the numerical implementation, lumped ports with equivalent inductance, capacitance, and resistance were employed to represent the optimised meta-atom's lumped elements. Figure 4-24(b) shows the simplified two-port meta-atom used for lumped-element modelling within the TEM cell. The lumped port in the SRR gap included $L_{ac} = 500$ nH and C_0 (the sum of the varactor diode capacitance and parasitic capacitances) in parallel. The lumped port connected to the horizontal microstrip line incorporated $C_{ac} = 0.1$ pF, $L_{ac} = 370$ nH, and $R_g = 50 \Omega$ in series. The number of meta-atoms and their spacing were selected such that, in combination with the electric- and magnetic-wall symmetries of the TEM cell, the array emulates the response of an infinite periodic metasurface, ensuring in-phase excitation of all SRRs at f_s and preserving the required tunability of the resonant responses.

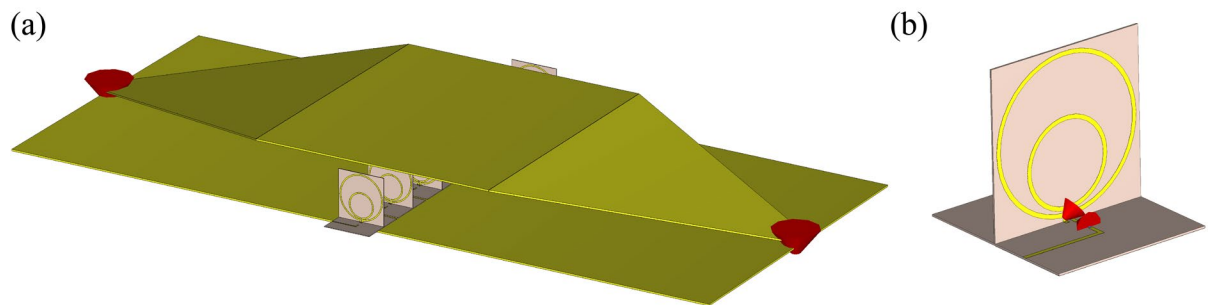


Figure 4-24 (a) Model of the TEM cell with 7 meta-atoms placed on the ground plane. (b) The meta-atom design with two corresponding ports containing equivalent lumped elements.

Figure 4-25(a) shows a photograph of the fabricated meta-atoms placed inside the TEM cell, along with biasing cables connected to a DC source (MX100QP) and coaxial connectors for applying the pump signal at f_p . The S-parameters of the parametric meta-atoms were measured using a Keysight PNA-X N5242A, as shown in Figure 4-25(b). TEM cell connections were made via vertical coaxial connectors mounted on the ground

plane, with their central conductors soldered to the stripline. For these measurements, the coaxial pump connectors were terminated with 50 Ω loads. Figure 4-25(b) also indicates the electric field direction and the wavevector of the excited TEM mode.

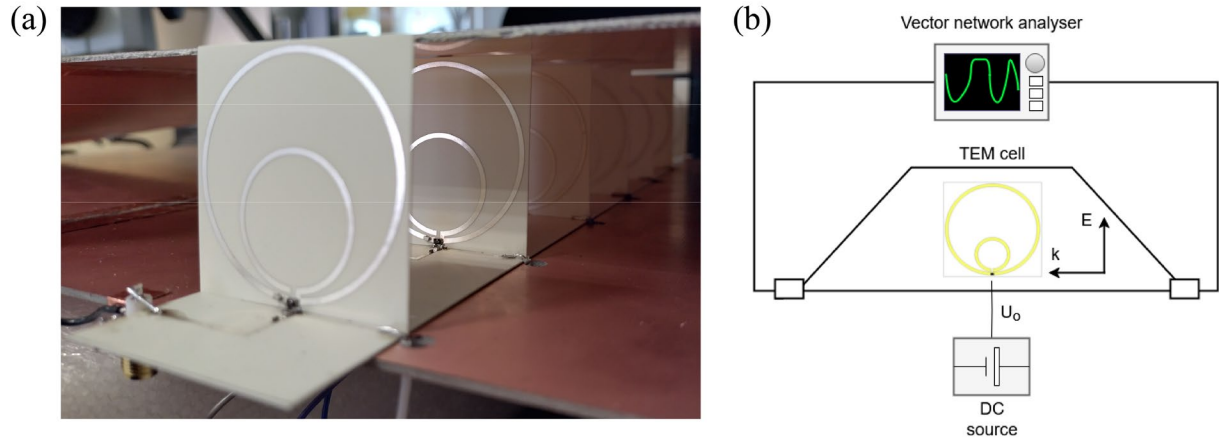


Figure 4-25 (a) Photograph of the fabricated meta-atoms inside the designed TEM cell. b) Diagram showing the measurement scheme.

Figure 4-26 presents the reflection characteristics of the parametric meta-atoms obtained from both simulations and experiments. Two resonances are observed at f_s and f_{sum1} (around 320 and 960 MHz, respectively, indicated by vertical lines for clarity). These resonances shift with changes in the equivalent capacitance C_0 , which takes values of $C_{min} = 0.4$ pF and $C_{max} = 1.2$ pF, and with the applied DC voltage (2 V and 16 V) in the experiments. Both simulation and experimental results show that the reflection level at f_{sum1} is relatively low, primarily due to additional losses in the TEM cell at this frequency. The reflection spectra also reveal parasitic resonances, present in both modelling and experiments. The resonance at f_s exhibits a reflection level more than 3 dB lower in experiments compared to simulations. This discrepancy is most likely caused by imperfect positioning of the meta-atoms within the TEM cell, inaccuracies in grounding and other connections.

Figures 4-27(a,b) present the experimental setup used to measure up-conversion and amplification of electromagnetic waves with the parametric meta-atoms in the TEM cell, in transmission and reflection configurations, respectively. The system comprised a Rohde & Schwarz FSQ8 signal analyser, a Rohde & Schwarz SMW200A two-port signal

generator, a ZHL-30W-252-S+ power amplifier (providing 50 dB gain), a ZBSC-8-82+ power splitter, and an MX100QP DC source. A directional coupler FDC-20-5-S+ was employed to measure the power of the radiated waves at the same port used to excite the incident wave, as it is shown in Figure 4-27(b). To enable this configuration, the opposite port was terminated with a 50 Ω load to absorb propagating waves. Since only one port could be monitored at a time, the measurements in reflection and transmission had to be performed separately.

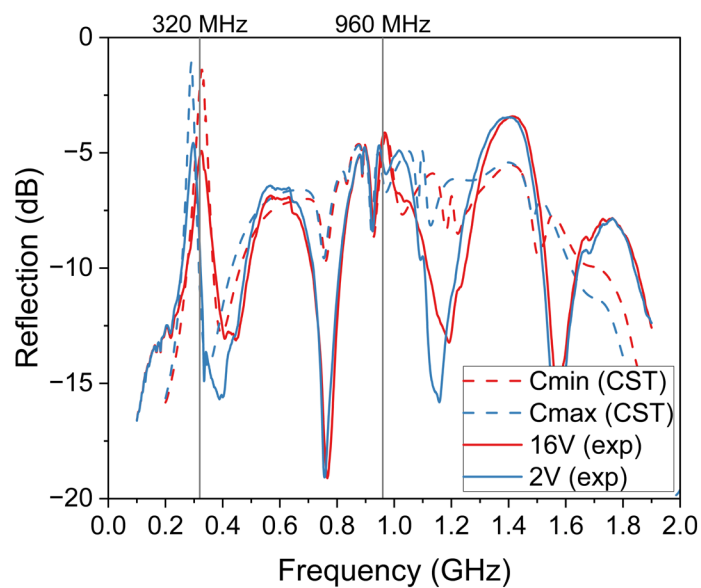


Figure 4-26 Reflection characteristics of the parametric meta-atoms in the TEM cell: simulations vs. experiments.

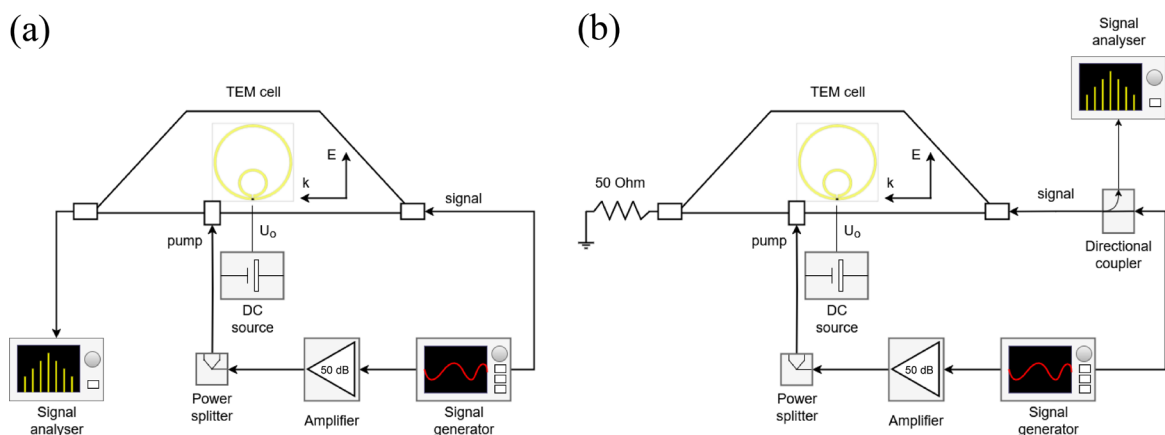


Figure 4-27 Microwave experimental setup for measurements of up-conversion and amplification characteristics of the parametric meta-atoms in transmission (a) and reflection (b).

We used lower values of the DC bias voltage in the experiments than in the modelling, owing to pump inefficiency observed during measurements. The achieved modulation levels were insufficient for operating at a DC bias of $U_0 = 10$ V, as the available pump power was not enough to reach the modulation amplitude corresponding to $U_p = 8-9$ V used in the modelling of the optimised meta-atom. Nevertheless, significant up-conversion efficiency was obtained experimentally with $U_0 = 3-4$ V and pump power levels up to 37 dBm at the output of the power amplifier. This choice of the DC voltage also enabled investigation of the self-excitation threshold and parametric generation, which were examined in the first half of this chapter but not considered in our modelling of dual-resonance metasurfaces.

Figure 4-28 shows the dependence of the overall radiated power at $f_s = 316$ MHz, $f_{sum1} = 948$ MHz and $f_{sum2} = 1580$ MHz on the pump power at $f_p = 632$ MHz (at the output of the power amplifier) for $U_0 = 4$ V under phase-locking in the degenerate regime (logarithmic scale, dBm): (a) illustrates the overall response; (b) highlights the distinction between parametric amplification and generation; (c) and (d) present results in the reflection and transmission directions, respectively, relative to the propagation of the incident wave at f_s .

In Figure 4-28, the solid lines correspond to measurements where the pump power was increased from 30 to 37 dBm, while the dotted lines show the reverse sweep from 37 to 30 dBm. The self-excitation threshold, at 35.7 dBm, is marked by a vertical line. The input power at f_s (−20 dBm at the generator output) is indicated by a horizontal dashed line. A dashed rectangle in Figure 4-28(a) highlights the section shown in Figure 4-28(b) for comparison with the generated power when no incident wave at f_s was applied and the pump power exceeded the self-excitation threshold.

The overall radiated power at f_s is approximately −26 dBm at a pump level of 30 dBm. It increases slightly with pump power, reaching −25 dBm at a pump level of 35 dBm, before rising sharply up to −17 dBm. This corresponds to a gain of ~3 dB relative to the input at f_s . The growth saturates around 35.5 dBm, close to the self-excitation threshold, and

reaches -16 dBm at 37 dBm. Figure 4-28(b) demonstrates the hysteresis loop: when decreasing the pump power from 37 dBm, the dotted curve follows the solid curve until the self-excitation threshold, after which the loop becomes evident.

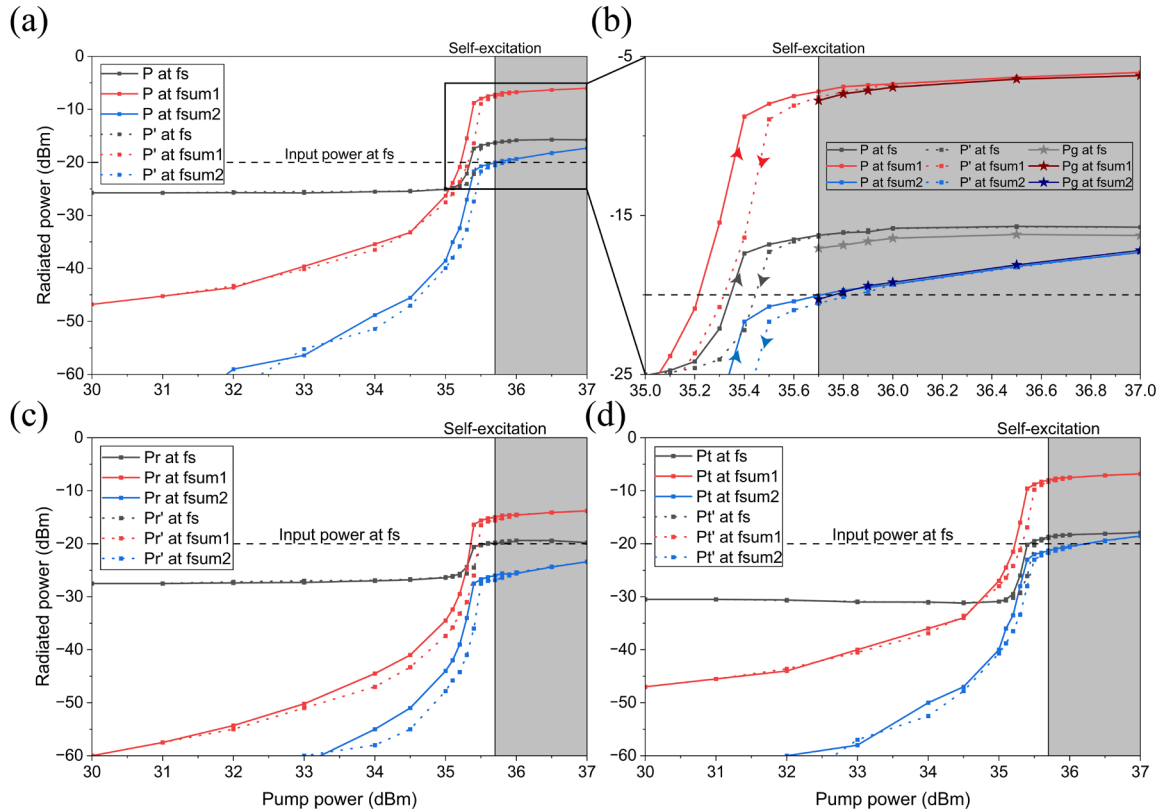


Figure 4-28 Dependence of the radiated power at $f_s = 316$ MHz, $f_{sum1} = 948$ MHz and $f_{sum2} = 1580$ MHz on the pump power at $f_p = 632$ MHz for $U_0 = 4$ V under phase-locking in the degenerate regime (logarithmic scale, dBm). (a) Overall radiated power; (b) overall radiated power distinguishing parametric amplification and generation; (c, d) power radiated in the directions of reflection and transmission, respectively, relative to the incident wave at f_s . Solid lines correspond to increasing pump power, while dotted lines correspond to decreasing pump power. The self-excitation threshold is indicated by a vertical line, and the input power at f_s is shown as a horizontal dashed line.

The radiated powers at f_{sum1} and f_{sum2} exhibit exponential growth with pump power. At f_{sum1} , the power surpasses the input level at ~ 35.25 dBm and approaches -8 dBm near the self-excitation threshold, corresponding to 12 dB amplification relative to the input at f_s . Notably, the radiated power at f_{sum1} is about 9 dB higher than that at f_s . At the self-excitation threshold, the powers at f_{sum1} and f_{sum2} reach -7 dBm and -20 dBm, respectively. Beyond threshold, the amplified up-conversion at f_{sum1} saturates, while the

radiated power at f_{sum2} continues to increase slightly. The dotted lines reveal hysteresis in both sum-frequency powers, consistent with the behaviour at f_s .

Figure 4-28(b) highlights the hard self-excitation process, characterised by an abrupt onset of radiation (P_g) at f_s , f_{sum1} and f_{sum2} when the pump exceeds 35.7 dBm. Interestingly, Figure 4-28(b) shows that in the absence of an incident signal at frequency f_s , the generated power P_g above the self-excitation threshold follows a similar dependence on the pump power, with the power radiated at f_{sum1} being an order of magnitude higher than the power radiated at f_s .

Figures 4-28(c,d) present the directional dependence. In reflection, the power at f_s grows with pump power, mirroring the overall trend. In transmission, however, the power at f_s initially decreases by ~ 1 dB before increasing rapidly above 35 dBm. The sum-frequency behaviour is similar to that discussed above. Most of the power at f_{sum1} and f_{sum2} is radiated in transmission: at 35.5 dBm, the power levels in reflection are -16 dBm and -27 dBm, compared with -9 dBm and -22 dBm in transmission.

Figures 4-29(a–c) show the dependence of the radiated power at $f_s = 316$ MHz, $f_{sum1} = 947.9$ MHz and $f_{sum2} = 579.9$ MHz on the pump power at $f_p = 631.9$ MHz for $U_0 = 4$ V in the non-degenerate regime (logarithmic scale, dBm): the overall power, and the powers measured in reflection and transmission relative to the incident wave at f_s , respectively.

Figure 4-29(a) shows that the radiated power at f_s remains nearly constant (-26 dBm) until the pump power reaches 35 dBm. Beyond this point, the power grows rapidly, reaching a maximum of about -20 dBm just below the self-excitation threshold (35.7 dBm), where it drops abruptly by 3 dB. This sharp decrease indicates that part of the pump power is diverted to generate an additional component at $f_p/2 = 315.95$ MHz due to the self-excitation process. Figures 4-29(b,c) further demonstrate that amplification at f_s occurs primarily in transmission, while the reflected power slightly fluctuates as the instability threshold is approached.

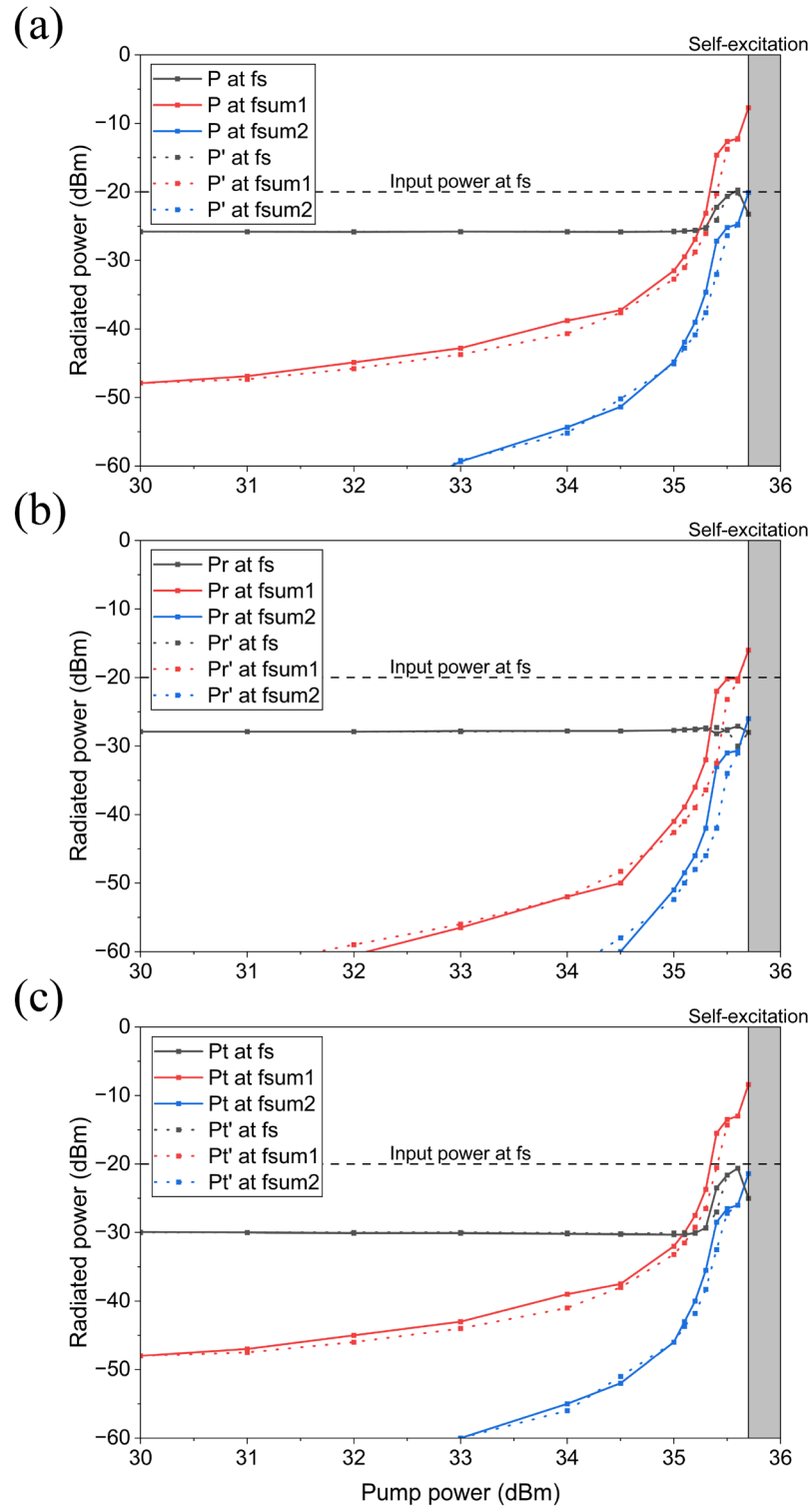


Figure 4-29 Radiated power at $f_s = 316$ MHz, $f_{sum1} = 947.9$ MHz and $f_{sum2} = 1579.9$ MHz as a function of the pump power at $f_p = 631.9$ MHz for $U_0 = 4$ V in the non-degenerate regime (logarithmic scale, dBm). (a) Overall power; (b, c) power measured in reflection and transmission, respectively, relative to the propagation of the incident wave at f_s . Solid lines indicate increasing pump power, while dotted lines indicate decreasing pump power. The vertical line marks the self-excitation threshold, and the horizontal dashed line denotes the input power level at f_s .

Meanwhile, the powers radiated at f_{sum1} and f_{sum2} increase exponentially with pump power up to about 35.5 dBm, after which they begin to saturate. At the instability threshold, both show an abrupt increase of approximately 4 dB. Remarkably, the overall radiated powers at f_{sum1} and f_{sum2} reach -8 dBm and -20 dBm, respectively, at the self-excitation threshold — values comparable to those observed in the degenerate regime. Although contributions are present in both reflection and transmission, the transmitted powers at both sum-frequencies dominate. At a pump power of 35.5 dBm, the overall radiated powers are about -21 dBm at f_s , -13 dBm at f_{sum1} , and -25 dBm at f_{sum2} . This indicates that the radiated power at f_{sum1} exceeds that at f_s by more than 7 dB in the non-degenerate regime. At this pump level, the radiated powers at f_{sum1} and f_{sum2} are -20.2 dBm and -31 dBm in reflection, and -13.5 dBm and -26.5 dBm in transmission, respectively.

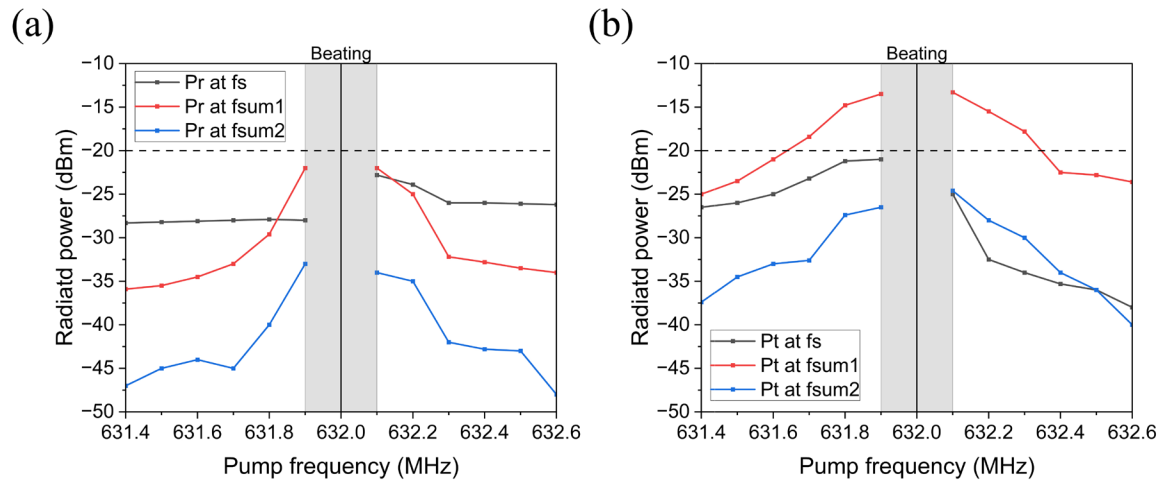


Figure 4-30 (a, b) Radiated power at $f_s = 316$ MHz, f_{sum1} and f_{sum2} as a function of pump frequency for $U_0 = 4$ V and a pump power of 35.5 dBm in the non-degenerate regime (logarithmic scale, dBm). The degenerate condition ($f_p = 316$ MHz) is indicated by a vertical line, while the input power at f_s (-20 dBm) is shown as a horizontal dashed line. The shaded region near the degenerate regime marks the frequency range where beating occurs in the non-degenerate regime.

Figure 4-30 presents the dependence of the radiated power at $f_s = 316$ MHz, f_{sum1} and f_{sum2} on the pump frequency for $U_0 = 4$ V and a pump power of 35.5 dBm in the non-degenerate regime. When the pump frequency is close to twice the signal frequency ($f_p \approx 2f_s$), the radiated power at f_s , f_{sum1} and f_{sum2} exhibits periodic temporal variations. These

arise from beating between the signal and idler waves, with the beat frequency equal to their frequency difference. This beating modulates the amplitude of the radiated waves, producing fluctuations in their measured power at the beat frequency. The beating range is shown as the shaded areas in Figure 4-30. Away from this condition, the up-conversion efficiency decreases as the pump frequency deviates from the degenerate value. As shown in Figure 4-30(b), the transmitted power at f_{sum1} exceeds the input power level within the frequency range 631.65–632.35 MHz, corresponding to an amplified up-conversion bandwidth of less than 1 MHz. Furthermore, the experiments verified a linear dependence of the phase of the radiated waves at f_s , f_{sum1} , and f_{sum2} on the pump phase.

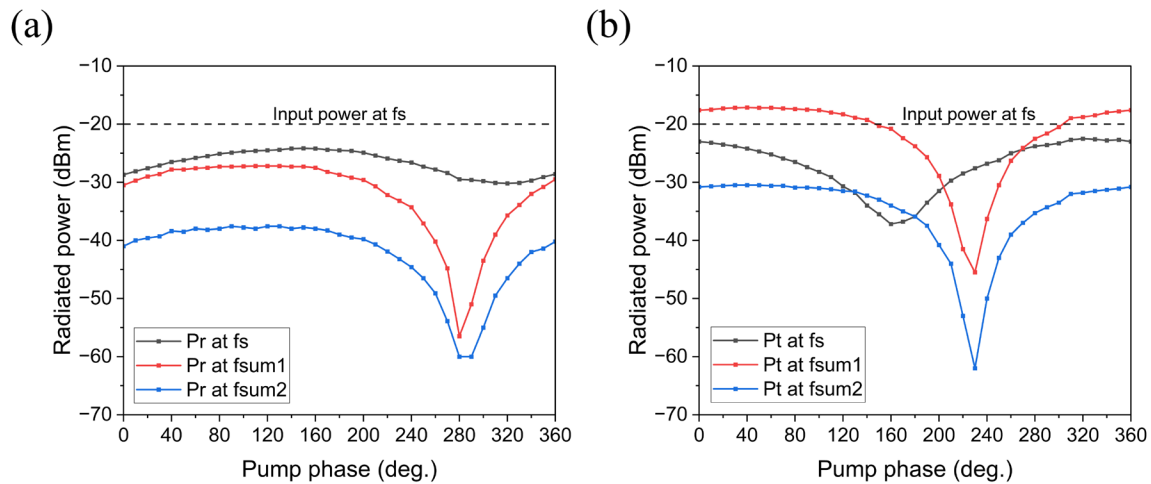


Figure 4-31 a, b) Dependence of the radiated power at $f_s = 313$ MHz, $f_{sum1} = 939$ MHz, and $f_{sum2} = 1565$ MHz on the pump phase for $U_0 = 3$ V and a pump power of 32.7 dBm at $f_p = 626$ MHz in the degenerate regime. The horizontal dashed line indicates the input power level at f_s .

Finally, we analyse the phase dependence of the radiated power in the degenerate regime. Figure 4-31 shows the radiated power at $f_s = 313$ MHz, $f_{sum1} = 939$ MHz, and $f_{sum2} = 1565$ MHz in the directions of reflection and transmission as a function of the pump phase for $U_0 = 3$ V and a pump power of 32.7 dBm at $f_p = 626$ MHz. The observed phase dependence is consistent with modelling predictions. While the radiated power in reflection remains below the incident wave power (shown as a horizontal dashed line), the radiated power at f_{sum1} in transmission can exceed this level when the pump phase is properly aligned with the signal phase. In addition, the minima and maxima of the radiated waves at f_s are shifted differently relative to those at the sum-frequencies in both

reflection and transmission. Although the experimental setup does not allow a direct comparison of reflection and transmission phase dependence (the directional coupler introduced an additional phase shift in reflection measurements), the results nevertheless confirm the dependence of the radiated waves on the pump phase in the degenerate regime and show good agreement with numerical modelling.

4.7 Discussion of the proposed parametric metasurfaces for amplified up-conversion

According to the theory of parametric amplification [267], the maximum gain is proportional to the ratio of the combination frequency to the incident wave frequency. However, we found that in the case of the pump and sum-frequencies near integer multiples of the signal frequency, the power of the radiated waves at the sum-frequencies can be increased compared to the case of the arbitrary choice of the pump frequency due to the cascaded amplification and up-conversion process. This allows us to overcome the converted power limits imposed by the Manley-Rowe relations. In this case, the pump frequency should be close to twice the signal frequency, which necessitates up-conversion to a sum-frequency approximately three times higher than the signal frequency. Additionally, the proposed parametric system has several features, including beatings in the non-degenerate regime when operating at frequencies near the degenerate frequency ratio, as well as the requirement to strictly maintain the frequency ratio and a fixed phase difference between the incident wave and the pump in the degenerate regime.

The frequency conversion ratio can be enhanced by employing modulation at several pump frequencies, optimising resonator design, and stacking multiple metasurface layers. One of the possible ways for increasing the power and bandwidth of the up-converted electromagnetic waves in the non-degenerate regime is using dual-tone modulation techniques with two pump sources at frequencies close to the degenerate case $f_p = 2f_s$. This will also allow us to utilise the power radiated at idler frequencies for further improvement of the up-conversion efficiency. Another approach to enhancing conversion efficiency is to design meta-atoms that suppress radiation at the pump

frequency while simultaneously implementing more efficient pump circuits. By minimising leakage of pump power into radiation at f_p , a larger fraction of the pump energy can be utilised for the conversion process. This would reduce the overall pump power required, which was a limiting factor in the experiments. Possible solutions include integrating a band-stop filter directly into the SRR structure or exploiting non-radiating modes at the pump frequency, both of which would suppress unwanted pump radiation and improve the effective energy transfer to the desired frequencies.

The integration of up-conversion and amplification within a single metasurface is highly promising for communication systems. Furthermore, as demonstrated, amplified conversion can be achieved at multiple sum-frequencies using a single signal frequency of the incident electromagnetic wave. This capability is particularly useful for designing frequency synthesizer systems in communication networks, enabling the utilisation of multiple radio channels. Sum-frequency generation using parametric metasurfaces is particularly attractive for the implementation of compact terahertz radiation sources. Due to the widespread application of parametric amplification in quantum computing readouts and receivers in radio astronomy and deep space communication, the proposed metasurfaces could significantly advance current technologies in this domain [255,270,271]. The obtained results can also be applied in the design of time crystals [265,266].

4.8 Conclusion

This chapter reveals the potential of amplifying free-space electromagnetic waves through the application of parametric metasurfaces. This study successfully demonstrates the capability to achieve significant gain exceeding 10 dB. The robust validation of the developed theoretical model, conducted through comprehensive full-wave numerical simulations, underscores the reliability and applicability of the proposed parametric amplification method.

Furthermore, we propose and investigate parametric metasurfaces enabling amplified up-conversion of free-space electromagnetic waves. The modelling and experimental

results presented in this chapter highlight the potential of using a pump at twice the signal frequency and resonances near integer multiples of the signal frequency for utilising a cascaded process, which allows to overcome the up-conversion efficiency limitations imposed by the Manley-Rowe relations.

This research provides valuable insights into the dynamic field of time-varying metasurfaces, paving the way for advancements in the manipulation of electromagnetic waves. The proposed parametric methods can be applied across the radio, microwave, and subterahertz ranges, currently constrained by the properties of modern varactor diodes, with the potential to extend to higher frequencies using ultrafast modulation techniques.

Chapter 5: Conclusion

In this thesis, we investigated dynamic metasurfaces that enable advanced control of electromagnetic waves for tunable filtering, bolometric detection, modulation, amplification, and frequency conversion. Several tuning mechanisms suitable for dynamic reconfiguration and modulation of metasurface parameters were explored, including mechanical actuation, phase-change materials, and varactor diodes. We examined different metasurface designs supporting resonant responses that can be tuned or modulated to achieve desired functionalities. The proposed dynamic metasurfaces offer promising applications across a wide spectral range, from microwave to optical frequencies, with relevance to communication, detection, imaging and sensing technologies.

Chapter 1 introduced the concept of dynamic metasurfaces and discussed their functionalities along with the mechanisms enabling tunability, focusing particularly on MEMS, VO₂, and varactor-based designs. Recent progress in tunable and time-varying metasurfaces was also reviewed.

Chapters 2 and 3 presented tunable metasurface designs based on MEMS and VO₂. In addition, we introduced a metasurface filter for fluorescence fibre-optic imaging, which can potentially be reconfigured using the tuning methods discussed in these chapters.

We proposed MEMS-based tunable metasurface filters exploiting EOT in the LWIR range, achieving continuous tuning of the central wavelength from 8 to 12 μm by varying the separation between a dielectric MEMS membrane and a nanohole array in a gold film. The total device thickness is below a quarter wavelength, and the required displacement of less than one micrometre can be achieved with actuation voltages below 10 V. These filters are particularly suitable for hyperspectral imaging and remote sensing applications, offering low power consumption and compactness for integration into portable systems.

Building on this foundation, we extended our research to fluorescence fibre-optic imaging by designing metasurface filters on the tips of double-clad fibres. These filters isolate the inner cladding from laser and OCT beams while allowing fluorescence signals from biological tissues to pass through for further detection and analysis. Such integration can enhance current bioimaging systems used for diagnosing conditions such as plaque accumulation within arteries.

Furthermore, we proposed a MEMS-enabled hybridisation of 1D and 2D metasurfaces for controlling ultranarrow q-BIC resonances excited via out-of-plane symmetry breaking. We demonstrated tuning of the q-BIC resonance bandwidth by varying the distance between the perforated MEMS membrane and the 1D metasurface. Moreover, by employing two independent orthogonal tuning mechanisms, we maintained a constant Q-factor of ultranarrow resonances in transmission while shifting them by more than 60 nm in the infrared range. These results are highly relevant for applications in tunable filtering, spectroscopy, remote sensing and imaging.

We also studied q-BIC resonances in toroidal metasurfaces exhibiting extreme sensitivity to introduced losses, enabling strong tunability through the integration of thin VO_2 film patches within meta-atoms. Introducing symmetry breaking further enhanced the tunability, yielding a 10 dB change in resonance amplitude for a temperature variation of only 10 °C. By analysing contributions of multipoles, we demonstrated that the toroidal dipole plays a dominant role in the metasurface response. Reconstruction of transmission and reflection spectra from multipole moments confirmed the validity of our analysis. These findings are applicable to the development of sensitive readouts for room-temperature bolometric detectors operating in the infrared and terahertz ranges, as well as terahertz filters and modulators.

Chapter 4 investigated free-space electromagnetic wave amplification and frequency conversion using parametric metasurfaces composed of varactor diodes integrated into SRRs. We developed an analytical model describing the parametric amplification of plane waves incident on the time-varying metasurface and validated it through numerical

simulations, demonstrating gain exceeding 10 dB across different regimes. Additionally, we showed that the electromagnetic wave phase can be controlled via the pump phase, paving the way for beam steering through spatiotemporal modulation.

A dual-resonance parametric metasurface was also proposed for amplified up-conversion of free-space electromagnetic waves. We demonstrated that the metasurface achieves high up-conversion efficiency and surpasses the Manley-Rowe limits through a cascaded amplification process. By tuning the resonances for maintaining a specific frequency ratio, we achieved up-conversion gains exceeding 20 dB.

Experimental studies of parametric meta-atoms in a TEM cell confirmed the accuracy of the modelling results. Despite additional losses revealed in experiments, the measured up-conversion efficiency remained high, and the radiated power at the sum frequency exceeded three times that of the incident wave at the signal frequency. These results experimentally verify that the up-conversion efficiency of the proposed time-varying metasurfaces is not constrained by the Manley-Rowe relations due to the multi-cascaded amplification mechanism. The developed parametric metasurfaces have potential applications in low-noise measurement and communication systems, quantum information processing, as well as in the design of ultracompact electromagnetic sources and generators, active radomes, ultrathin repeaters and transponders.

Tunable metasurfaces demonstrate remarkable potential for transforming a wide range of technologies and are now approaching the stage of industrial implementation. Future developments will focus not only on the ability to control electromagnetic waves but also on improving commercial viability, efficiency, durability, and integrability of reconfigurable metasurfaces. Research will continue toward achieving individual meta-atom control and enhancing the tuning range and switching speed.

The metasurface concepts proposed in this thesis can be further advanced through hybrid reconfiguration schemes combining multiple tuning mechanisms. The orthogonal mechanical actuation scheme demonstrated here can also be applied to other types of resonances and should be explored in the context of practical MEMS fabrication technologies. Fibre-integrated metasurfaces will continue to be an active area of research, and dynamic tuning will be critical to their further development. Methods such as MEMS actuation and phase-change materials can be utilised to create multifunctional reconfigurable metasurfaces integrated into fibre-optic systems. High-quality q-BIC resonances offer exciting opportunities for fibre-optic sensors, including biomedical applications, where their extreme sensitivity to environmental changes can enable precise quantitative analysis. The proposed VO_2 -based readout systems can be implemented in optical or infrared ranges, allowing metasurfaces to function as individual pixel detectors with potential for miniaturisation and integration on portable platforms or optical fibres.

Time-varying and parametric metasurfaces hold a highly promising future driven by the development of ultrafast modulation techniques using novel materials. The approaches presented in this work can be extended to higher frequencies by exploiting these emerging materials. Furthermore, combining parametric amplification and frequency conversion with nonreciprocal wave propagation and beam steering through spatiotemporal modulation presents a compelling direction for future research. The parametric phenomena discussed in this thesis are relevant to fundamental quantum processes such as the generation of entangled photons and the dynamic Casimir effect. Exploring these effects using time-varying metasurfaces represents an exciting frontier. Another promising avenue involves temporally modulated metamaterials with negative dielectric permittivity and magnetic permeability to study exotic effects such as the reversed Doppler effect and superlensing, potentially opening new horizons in electromagnetic wave manipulation.

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