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Bias Analysis and Reduction for Underwater Localization Problem

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Abstract: In the previous work, we proposed a generic method to analyze and reduce the bias in localization algorithms. The proposed method combines the Taylor series and Jacobian matrices to determine the bias and leads to an easily calculated analytical bias expression. However all the previous works focus on the terrestrial localization applications. In this paper we extend the method to underwater localization which is more complex than the terrestrial one. In order to reduce the underwater localization bias, an adjustment is required for the proposed bias-correction method. The Monte Carlo simulation results demonstrate the performance of the proposed bias-correction method in underwater localization situation.

Key Words: Bias; Taylor series; Underwater localization; Targeting; Tracking; TDOA

1 Introduction

Underwater Acoustic Sensor Networks (UWA-SNs) has attracted much attention, because it has many potential applications such as oceanographic data collection, early warning systems for natural disasters (like tsunamis), oil drilling, environment monitoring, structure monitoring, tactical surveillance and military surveillance, and so on[1,2]. For all these applications, localization is an indispensable issue. For example, energy-efficient geo-routing schemes require location information to make routing decisions. Further aquatic monitoring and underwater surveillance applications demand high-precision localization. [3]

It is well known that localization is widely studied for terrestrial sensor networks; however, most of the existing terrestrial techniques cannot be directly applied to UWA-SNs because of some unique characteristics.

First of all, it is difficult to deploy anchor nodes at precise locations in underwater environments. While it is reasonable to assume that nodes in terrestrial networks remain static, in contrast underwater nodes will inevitably drift due to some reason, such as underwater currents, winds, shipping activity, and so on. In fact, nodes may drift differently as oceanic current is spatially dependent.

Additionally, there are some unique characteristics in underwater acoustic communication channel, such as limited bandwidth capacity, limited battery power and half-duplex. The current available limit of bandwidth underwater is roughly 40km.kbps [4], which means that (as shown in table 1) a Long-range system, which operates over several tens of kilometers, may have a bandwidth of only a few kHz, while a short-range system operating over several tens of meters may have more than a hundred kHz of bandwidth.

Table 1: Available bandwidth for different ranges in UW-A channels [5]

	Range [km]	Bandwidth [kHz]
Very Long	1000	< 1
Long	10-100	2-5
Medium	1-10	10
Short	0.1-1	20-50
Very Short	< 0.1	> 100

Moreover, the underwater sensors are powered by battery and it is difficult or even impossible to recharge or replace the nodes' batteries because they are deployed underwater. Unlike RF modems, acoustic modems consume much more power (order of tens of watts) in transmit mode compared to receive mode (order of milliwatts). This asymmetry in power consumption makes it preferable for ordinary nodes to be localized through passive/ silent listening.[3]

Finally, distance measurements underwater suffer from large errors. It may cause most underwater localization schemes inaccurate. Underwater localization schemes often use Time of Arrival (TOA), Angle of Arrival (AOA), Time Difference of Arrival (TDOA), the latter range technique will be used later, to estimate the target position. Unlike the speed of light which is constant, the speed of sound underwater varies with water temperature, pressure and salinity, giving rise to refraction. Without measuring the sound speed, the accuracy of the target position estimated by time-of-arrival approaches may be degraded.[3]

From the above analysis, we know the underwater localization problem is more complex than terrestrial one. However, this makes the underwater localization problem more attractive to study. Despite there being significant differences between the UWA-SNs and terrestrial sensor networks, they face the same problem: a kind of systematic error which is called bias almost always exists, thereby decreasing the localization accuracy. Bias is a term in estimation theory and is defined as the difference between the expected value of a parameter estimate and its true value [6]. It can be removed under the condition that the bias of a particular estimation scheme can be estimated. It is expected to improve the quality of the estimate over a number of experiments, which is decreasing the mean square error.

In our previous work we proposed a method to determine and reduce the bias in localization algorithms [7, 8, 9]. These articles hypothesized the existence of a localization mapping g (which maps the measurements to a target position). However, in a localization problem it is normally very hard to express the derivatives of g analytically. In contrast, the inverse mapping of g (call it f), which maps the target position to the measurements, can often be obtained analytically, together with its derivatives. Therefore, we introduced the Jacobian matrix to identify the derivatives of the localization mapping g in terms of the derivatives of f, resulting in a simple calculation of bias.

In our previous work, the method proposed was only applied in the terrestrial localization. In this paper, we apply the bias correction method to reduce the bias in the underwater situation, which has not been analyzed before. In the process of applying the proposed method to underwater localization, the original bias correction method needs to be adjusted in a minor way to allow for certain correlations in the measurement errors.

The rest of this paper is organized as follows. In Section 2 the review of bias is presented. The generic bias correction method is briefly reviewed in Section 3. In Section 4, we apply the bias correction method to the underwater localization problem. Section 5 summarizes the main results of the paper.

2 Review of bias

A brief review of estimation bias will be presented in this section. Firstly, assuming $x = (x_1, x_2, . . . , x_n)T$ and $t = (t_1, t_2, . . . , t_N)T$ denote the coordinate vector of a target and the noiseless measurement vector respectively, n denotes the number of dimensions of the ambient space and N denotes the number of independent usable measurements which is often but not the same as the number of sensors; $f : x \rightarrow t$ denotes the associated mapping, which is almost always analytically computable; $g : t \rightarrow x$ denotes the inverse localization mapping; thus with $t = f(x)$, there holds $x = g(t)$. In order to obtain precise definition of this mapping, it is necessary that $N \geq n$. In the case where $N > n$, usually the set of equations $f(x) = t$ yielding x from t will be overdetermined, while in the case where $N = n$, there may be two or more solutions; in this case, the selection of the correct solution requires some additional information regarding the target position. In practice, noise in the measurements is inevitable. This noise is denoted as $\delta t = (\delta t_1, \delta t_2, ..., \delta t_N)T$. The δt_i is generally assumed to be a Gaussian random variable with zero mean and known variances σ_i^2. The error in the target position resulting from an estimation procedure using noisy data is denoted as $\delta x = (\delta x_1, \delta x_2, \ldots, \delta x_N)T$. If we define $\tilde{t} = t + \delta t$ and $\tilde{x} = x + \delta x$ the localization amounts to solving $f(\tilde{x}) = \tilde{t}$ for $\tilde{x}$. If the function g is known, then in effect we are implementing the following equation $\tilde{x} = x + \delta x = g(t + \delta t)) = g(\tilde{t})$ (1).

If g is a non-linear function then this process will lead to bias in the target position estimation[8]. Here we take x_i as an example: $E[\tilde{x_i}] = E[g_i(\tilde{t})]$ (2).

Now note that if g_i is nonlinear we would have:

$E[\tilde{x_i}] = E[g_i(\tilde{t})] \neq g_i(E[\tilde{t_i}]) = g_i(\text{t}) = x_i$

The bias in our estimation of x_i is defined as the difference between the expected value of x_i and the true value of x_i, i.e. $Bias(x_i) = E[\tilde{x_i}] - x_i = E[g_i(\tilde{t})] - x_i$

According to the above analysis, it can be seen that if the estimation mapping g is nonlinear and the sensor measurements are noisy, bias is present. We assume that the estimator mapping g is well defined; they expanded the function g by a Taylor series and truncated it at second order, here we take g_i for example:

$$x_i + \delta x_i = g_i(\tilde{t_1}, \tilde{t_2}, ..., \tilde{t_N}) = g_i(t_1 + \delta t_1, t_2 + \delta t_2, ..., t_N + \delta t_N) \approx g_i(t_1, t_2, ..., t_N) + \sum_{j=1}^{N} \frac{\partial g_i}{\partial t_j}\delta t_j + \frac{1}{2!}\sum_{j=1}^{N}\sum_{l=1}^{N} \delta t_j \delta t_l \frac{\partial g_i^2}{\partial t_j \partial t_l}$$

Assuming, as is often the case, that the measurement errors are independent Gaussian random variables with zero mean and known variance (the variance of measurement errors would have to be obtained from the manufacturer or the experimental data), the approximate bias expression is:

$$E(\delta x_i) = \frac{1}{2!}\sum_{j=1}^{N} \sigma_j^2 \frac{\partial g_i^2}{\partial t_j^2} \quad (3)$$

For some estimators, the mapping g can be obtained analytically. However in some situations, including many localization problems, finding the analytic g analytically is very hard or even impossible. If g can not be obtained then equation (3) can not be evaluated, and we need a new method to analytically obtain the derivatives.

The key to doing this is to notice that g is the inverse of the mapping f for which an analytic form is often known. Below we show how to use the mapping f and its derivatives to calculate the derivatives of g using the Jacobian identity, ultimately resulting in an estimate of the bias.

3 Bias correction method

First of all, we assume $N = n$ (the number of measurements is equal to the dimension of space); and f is a known analytic function. Because f and g are inverse mappings, the Jacobian identity holds:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial t_1} & \frac{\partial g_1}{\partial t_2} & \cdots & \frac{\partial g_1}{\partial t_N} \\ \frac{\partial g_2}{\partial t_1} & \frac{\partial g_2}{\partial t_2} & \cdots & \frac{\partial g_2}{\partial t_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_n}{\partial t_1} & \frac{\partial g_n}{\partial t_2} & \cdots & \frac{\partial g_n}{\partial t_N} \end{bmatrix} = I_n \quad (4)$$

By solving the equation set (4), we can obtain expressions for the $\frac{\partial g_i}{\partial t_j}$ (i = 1, 2, ..., n; j = 1, 2, ...,N) in terms of $\frac{\partial f_i}{\partial x_j}$, which is analytic . For ease of exposition we use $g_j{}^i$ to denote the expressions of $\frac{\partial g_i}{\partial t_j}$ as functions of $x_1, x_2, ..., x_n$. To obtain the second derivatives of g, here we take $\frac{\partial g_1}{\partial t_1}$ as an example. Start with $\frac{\partial g_1}{\partial t_1} = g_1{}^1$ (5)

If we further differentiate the equation (5) with respect to $x_2, ..., x_n$ we can obtain an equation set as follows:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_2} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_1}{\partial x_n} & \frac{\partial f_2}{\partial x_n} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1^2}{\partial t_1 \partial t_1} \\ \frac{\partial g_1^2}{\partial t_1 \partial t_2} \\ \vdots \\ \frac{\partial g_1^2}{\partial t_1 \partial t_N} \end{bmatrix} = \begin{bmatrix} \frac{\partial g_1{}^1}{\partial x_1} \\ \frac{\partial g_1{}^1}{\partial x_2} \\ \vdots \\ \frac{\partial g_1{}^1}{\partial x_N} \end{bmatrix} \quad (6)$$

Note that the quantities on the right side of this equation are all expressible analytically of $x_1, x_2, ..., x_n$. And the entries $\frac{\partial f_i}{\partial x_j}$ in the matrix on the left are known functions of

$x_1, x_2, ..., x_n$. Hence by solving the equation set (6), we can obtain a formula for $\frac{\partial g_1^2}{\partial t_1 \partial t_1}$ as a function of $x_1, x_2, ..., x_n$. The formulas for $\frac{\partial g_1^2}{\partial t_j \partial t_j}$ for all *i, j* can be obtained in the same way. At first, the estimated position of the target will be obtained by using an existing localization algorithm. Then the estimated target location can be calculated with the obtained analytical expression for the bias.

Finally the accuracy of the localization can be improved by subtracting the obtained bias, viz. $\widetilde{x_i} - bias(x_i)$ (i = 1, 2, ..., *n*). (The process could in fact be iterated, but the effect of even one more iteration on the corrected position estimate will almost always be marginal).

4 Underwater Localization Problem with Bias Correction Method

In this section, firstly we briefly review two well know underwater localization techniques. Next we apply the proposed bias-correction method to the underwater localization problem. Further, Monte Carlo simulation results are provided, which demonstrate the performance of the proposed method in underwater situation.

4.1 Review of Underwater Localization

A. R. Bucher Method

Consider an emitter at an unknown location vector E = (x, y, z), which we wish to locate. The source is within range of *N+1* receivers at known locations $P_0, P_1, ..., P_m, ..., P_N$. The subscript m refers to any one of the receivers: $P_m = (x_m, y_m, z_m)$ $0 \le m \le N$.

The TDOA equation for receivers m and 0 is

$v\tau_m = R_m - R_0$ (7)

Starting with equation (7), solve for R_m, square both sides, collect terms and divide all terms by $v\tau_m$: 0 = $(v\tau_m) + 2R_0 + \frac{(R_0^2 + R_m^2)}{v\tau_m}$ (8)

Removing the 2 R_0 term will eliminate all the square root terms. That is done by subtracting the TDOA equation of receiver *m = 1* from each of the others *(2 ≤ m ≤ N)*

$$0 = (v\tau_m) - (v\tau_1) + \frac{(R_0^2 + R_m^2)}{v\tau_m} - \frac{(R_0^2 + R_1^2)}{v\tau_1} \quad (9)$$

$$R_0^2 - R_m^2 = -x_m^2 - y_m^2 - z_m^2 + x2x_m + y2y_m + z2z_m \quad (10)$$

Combine equations (15) and (16), and write as a set of linear equations of the unknown emitter location *x, y, z*

$$0 = xA_m - yB_m - zC_m + D_m$$

$$A_m = \frac{2x_m}{v\tau_m} - \frac{2x_1}{v\tau_1} \quad B_m = \frac{2y_m}{v\tau_m} - \frac{2y_1}{v\tau_1} \quad C_m = \frac{2z_m}{v\tau_m} - \frac{2z_1}{v\tau_1} \quad D_m = (v\tau_m) - (v\tau_1) - \frac{(x_m)^2 + (y_m)^2 + (z_m)^2}{v\tau_m} + \frac{(x_1)^2 + (y_1)^2 + (z_1)^2}{v\tau_1}$$

(11)

Use equation (11) to generate the four constants A_m, B_m, C_m, D_m from measured distances and time for each receiver $1 \le m \le N$. This will be a set of *N* homogeneous linear equations.

B Least Squares Solution

One receiver has position (x, y, z), Suppose that there are N surface nodes, at positions (x_n, y_n, z_n), *n = 0, . . . , N*. We set the first surface node at the origin, i.e., $x_0 = y_0 = z_0 = 0$, $R_0^2 = (x)^2 + (y)^2 + (z)^2 = u^T u$(12)

such that the actual time of arrival is $t_n = R_n / v$, where v is the sound propagation speed.

The receiver needs to provide an estimate of the time of arrival. Let $\widehat{t_n}$ denote the estimate of t_n from the OFDM modem. It can be expressed as the sum of the real transmission propagation and the estimation noise w_n.

$\widehat{t_n} = t_n + w_n$ (13) $\Delta\widehat{t_{n0}} = \widehat{t_n} - \widehat{t_0}$, n = 2, . . . , N (14)

We use the least-squares solution.

$$x_n x_r + y_n y_r + z_n z_r = 1/2\left([x_n^2 + y_n^2 + z_n^2 - R_{n0}^2]\right) - R_{n0}R_0 \quad (15)$$

Define the following matrix and vectors as

$$H = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_N & y_N & z_N \end{bmatrix} \quad v = \begin{pmatrix} -\hat{} R_{10} \\ -\hat{} R_{20} \\ \vdots \\ -\hat{} R_{N0} \end{pmatrix} \quad (16)$$

$$u = 1/2 \begin{bmatrix} x_1^2 + y_1^2 + z_1^2 - R_{10}^2 \\ x_2^2 + y_2^2 + z_2^2 - R_{20}^2 \\ \vdots \\ x_N^2 + y_N^2 + z_N^2 - R_{N0}^2 \end{bmatrix} \quad a = \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} \quad (17)$$

the equation (17) can be rewritten as H*a= u + v * R_0

The least-squares solution can be obtained as

$\hat{a} = R_0 H^\dagger v + H^\dagger u$ (18)

where † stands for pseudo-inverse. Substituting the entries of $\hat{a}$ into (12) yields a quadratic equation for R_0. Solving R_0 and substituting the positive root back into (18) provides the final solution for the receiver position a. [12]

4.2 Bias-Correction in Underwater Localization

In Section 3, we have already obtained the analytical expression for the bias. However, in the underwater localization problem, equation (3) requires some adjustments. Note that noise in the time-of-arrival (TOA) measurements can be modeled as follows:

$\widetilde{t_i} = t_i + \delta t_i$ i =0, 1, 2, 3 (19)

where the δt_i are assumed to be i.i.d Gaussian random variables with zero mean and known variance σ^2.

However in underwater localization, the physical measurements are replaced by quasi-measurements $\widetilde{t_{10}} = \widetilde{t_1} - \widetilde{t_0}$ $\widetilde{t_{20}} = \widetilde{t_2} - \widetilde{t_0}$ and $\widetilde{t_{30}} = \widetilde{t_3} - \widetilde{t_0}$ leading to

$\widetilde{t_{10}} = \widetilde{t_1} - \widetilde{t_0} = t_{10} + \delta t_{10}$ (20)

$\widetilde{t_{20}} = \widetilde{t_2} - \widetilde{t_0} = t_{20} + \delta t_{20}$ (21)

$\widetilde{t_{30}} = \widetilde{t_3} - \widetilde{t_0} = t_{30} + \delta t_{30}$ (22)

where δt_{10}, δt_{20} and δt_{30} are no longer independent and have covariance matrix Σ given by

$$\Sigma = 2\sigma^2 \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 1 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix} \quad (23)$$

where $2\sigma^2$ is the variance of an individual time-difference measurement. Note that the means of δt_{10}, δt_{20} and δt_{30} remain zero. Now the approximate bias expression for three receivers is as follows:

$$E(\delta x) = \frac{1}{2!}[2\sigma^2 \frac{\partial g_1^2}{\partial t_{10}^2} + 2\sigma^2 \frac{\partial g_1^2}{\partial t_{20}^2} + 2\sigma^2 \frac{\partial g_1^2}{\partial t_{30}^2} - 2\sigma^2 \frac{\partial g_1^2}{\partial t_{10}\partial t_{20}} - 2\sigma^2 \frac{\partial g_1^2}{\partial t_{10}\partial t_{30}} - 2\sigma^2 \frac{\partial g_1^2}{\partial t_{20}\partial t_{30}}] \quad (24)$$

$E(\delta y)$ and $E(\delta z)$ can be obtained in the same way. The remaining calculations are the same as those described in Section 3

4.3 Simulation Result

We firstly carried out simulation using a simple noise model to generate the TDOA measurements, which are mentioned in the 4.2. The variance of the measurement is varied between 0.01 and 0.2. In the R. Bucher method, coordinate z is assumed to be known, and we only need to estimate x and y coordinates. Four transmitters are placed at coordinates (0, 0), (750, 2250), (3000, 0), and (-1500, 0). One receiver is placed at the (750, 0) point. We can calculate x and y coordinates.

In the least square method, the four transmitters are placed at coordinates (0, 0, 0), (750, 3750，0), (3000, 0, 0), and (750, 0, 5250). One receiver is placed at the (750, 0, 0) point and we can calculate the x, y and z coordinates.

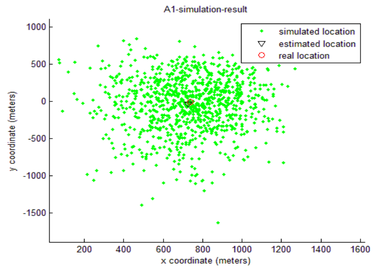

Figure 1 The simulation results of the R. Bucher localization algorithm with variance=0.2

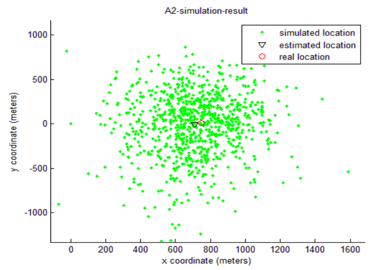

Figure 2 The simulation results of the least square localization algorithm with variance=0.2

The simulation results of R. Bucher localization algorithm are shown in Figure 1 as a map of estimated source locations. We can obtain that a huge bias exists in the solution of the R. Bucher localization algorithm, which is 31.7486. Further simulation results of least squares approach with fixed level of measurement noise (variance=0.2) are shown in Figure 2. As the measurement noise level remains the same, the bias is 37.7225m.

From the above simulation results we can conclude that bias exists in the both underwater localization algorithms.

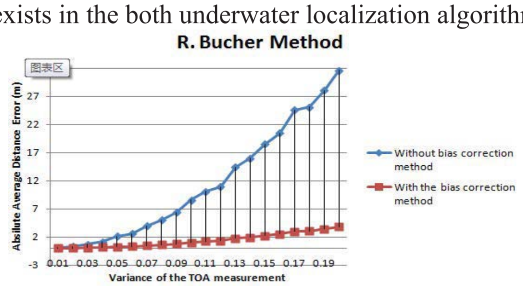

Figure 3 The original bias and the reduced bias of R. Bucher localization algorithm with variance from 0.01 to 0.2

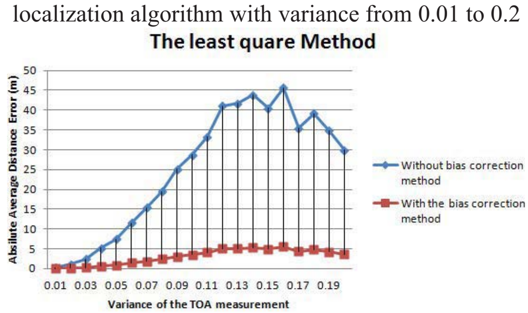

Figure 4 The original bias and the reduced bias of the least square localization algorithm with variance from 0.01 to 0.2

To exam the effect of various levels of measurement noise on the bias during localization process, however, the variance of the distance measurement error is varied from 0.01 to 0.2. From the Figure 3, we can obtain that the bias becomes larger when the variance of the measurement noise increases. And the bias correction method performs well on the R. Bucher localization algorithm as the variance of the measurement varied from 0.01 to 0.2. It is obvious that the method almost decreases 80% of the original bias. Figure 4 depicts the performance of the proposed bias correction in the least squares localization technique. Different from the R. Bucher method, the bias increases quickly when the variance is smaller than 0.09. When the variance increases from 0.12 to 0.19, the bias has a slowly fluctuate increase even a slight decrease. Again it is obvious that the bias correction method works effectively. As shown in Figure 4 the method almost reduces 80% of the original bias.

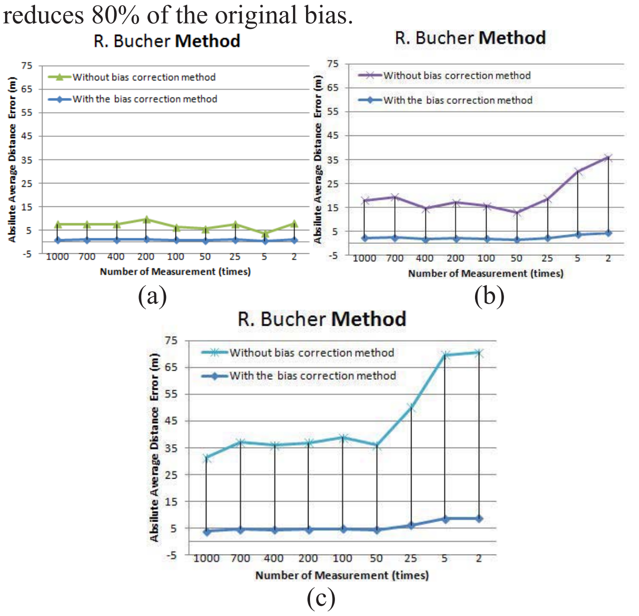

Figure 5 The bias of R. Bucher localization algorithm with different number of measurement with different noise level (a) variance=0.1 (b) variance=0.15 (c) variance= 0.2

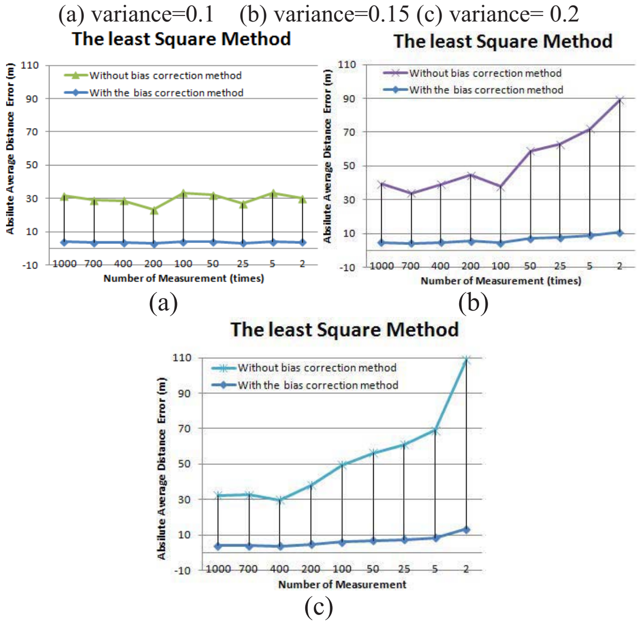

Figure 6 The bias of least square localization algorithm with different number of measurement with different noise level (a) variance=0.1 (b) variance=0.01 (c) variance= 0.2

The next simulation presents the bias in R. Bucher method with various levels of measurement noise when the number of measurement varies from 2 to 1000. It is obvious that the bias starts to fluctuate when the variance increases to 0.1 in Figure 5(a). Moreover, when the variance is larger than 0.15(shown in Figure (b) (c)), the bias has a significant increase when the number of measurement decreases. Additionally, it becomes stable when the number of measurement is bigger than 100 times. No matter how large the bias is, the bias correction method can reduce the bias to about 20% of the original.

The last simulations demonstrate the bias in the least square method with various levels of measurement noise when the number of measurement varies from 2 to 1000. From the Figure 6(a), the bias starts to fluctuate when the variance increases to 0.1. When the variance is larger than 0.15(shown in the Figure 6(b) (c)), similarly the bias has a significant increase when the number of measurement decreases. Additionally, the bias becomes stable when the number of measurement is bigger than 400 times. The bias in different situations all can be significantly reduced to about 20% of the original bias by using the proposed bias correction method.

5 Conclusion and future work

In the previous work, a bias-correction method is presented in [7, 8, 9]. This method mixes the Taylor series and Jacobian matrices, to express and reduce the bias in the terrestrial localization problem. In this paper, from the analysis we obtained that the bias also exists in the underwater localization situation which is more complex than the terrestrial one. Moreover, accurate localization is also an important issue in underwater situation. Therefore in this paper, we extend the bias-correction method to the underwater localization problem. First a review of two well-known underwater localization algorithms is provided. In order to apply the bias-correction method to these two underwater localization techniques, an adjustment is needed. To demonstrate the performance of the proposed method in underwater situation, Monte Carlo simulation results are provided in the paper. From the simulation results we can conclude the proposed method can reduce the bias, thereby improving the localization accuracy in underwater situation. Our future work is to analyze the bias in the mobile source localization problem.

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