

SOME CASES OF ISOSPIN MIXING AND
ISOSPIN DISTORTION IN LIGHT NUCLEI

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Thesis submitted for the degree of Doctor of
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To my family.

PREFACE

I collaborated with my supervisor, Dr. F.C. Barker, in developing the general formulae in chapter II, and in applying these to the cases discussed in chapters III and V. In each of these cases each of us carried out the calculations separately. I carried out the initial calculations described in chapter IV and these were checked by my supervisor.

The work reported in chapters III and V is published in the following papers:

" 3^+ states of ^8Be ", F.C. Barker and Nasima Ferdous, Aust. J. Phys., 1978, 31, 239-51.

"The low-lying states of ^{13}C and ^{13}N ", F.C. Barker and Nasima Ferdous, Aust. J. Phys., 1980, 33, 691-717.

Another project on which I have worked with Dr. F.C. Barker, not related to this thesis but related to the field of light nuclei, has been accepted for publication in the Journal of Physics G: Nuclear Physics, under the title "*A proposed test for time reversal invariance in nuclear elastic scattering*".

No part of this thesis has been presented for a degree at any other university.

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NASIMA FERDOUS.

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ABSTRACT

Some cases of isospin mixing and isospin distortion in light nuclei are considered in which the experimental information is compared with calculated values. In addition to contributions from the internal Coulomb matrix element the calculated values include external contributions obtained by using the Bloch operator technique, which enables the use of wavefunctions with correct asymptotic forms. In this approach the calculated values are functions of channel radius. The cases considered are isospin mixing and isospin distortion in the 3^+ states of ^8Be , isospin mixing in the 7.12 MeV state of ^{16}O and isospin distortion in the low-lying states of ^{13}C and ^{13}N .

The first case involves the pair of 3^+ states in ^8Be at about 19 MeV excitation which are interpreted as a mixture of states, one with isospin $T=0$ and the other with $T=1$. The experimental value of the isospin mixing matrix element is obtained by fitting primarily the $^7\text{Li}+p$ phase shifts and also various reaction cross-sections involving the 3^+ states of ^8Be . The calculated value of the isospin mixing matrix element is obtained by using shell model basis states of $J^\pi = 3^+$, $T=0$ and $J^\pi = 3^+$, $T=1$, and agreement is obtained with the experimental value for reasonable value of channel radius. Similar agreement is obtained for isospin distortion effects, evidence for which comes from the different level displacement energies of the corresponding 3^+ , $T=1$ states in ^8Be and ^8Li .

Isospin mixing in the 1^- , mainly $T=0$ state of ^{16}O at 7.12 MeV is attributed to several higher lying 1^- , $T=1$ states. The isospin mixing is calculated for various shell model interactions and compared with the experimental value. The results show appreciable dependence on the interaction.

Finally, the isospin distortion effects in the low-lying states of ^{13}C and ^{13}N have been considered. The asymmetries shown by the difference in the excitation energies of the corresponding levels and different strengths in some of the E1 transitions in the mirror nuclei ^{13}C and ^{13}N are effects of isospin distortion. A consistent account of the relevant properties of the $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states is given together with an account of two notable asymmetries, the 720 keV difference in the first excited states and the very different E1 transition strengths of the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transitions in ^{13}C and ^{13}N . The resultant parameters of the $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states obtained by fitting the experimental data and using standard R-matrix theory, agree with those obtained by using the shell model theory for reasonable values of channel radius.

CONTENTS

PREFACE	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iii
CHAPTER I. INTRODUCTION	1
CHAPTER II. ISOSPIN MIXING AND DISTORTION FORMULAE	10
CHAPTER III. ISOSPIN MIXING AND ISOSPIN DISTORTION IN 3^+ STATES OF ^8Be	17
A. 3^+ states of ^8Be	17
B. Two level fit to $^7\text{Li} + p$, $^5\text{P}_3$ phase shifts	21
1. Isospin mixing formulae for 3^+ states of ^8Be	21
2. R-matrix fit to $^7\text{Li} + p$, $^5\text{P}_3$ phase shifts	22
C. Predictions of cross-sections for other reactions	26
1. $^7\text{Li}(p,n)$	26
2. $^7\text{Li}(p,\gamma)$	27
3. $^{10}\text{B}(d,\alpha)^8\text{Be}$	29
4. $^9\text{Be}(d,t)^8\text{Be}$	31
5. $^9\text{Be}(p,d)^8\text{Be}$, $^9\text{Be}(^3\text{He},\alpha)^8\text{Be}$	31
6. $^7\text{Li}(d,n)^8\text{Be}$	32
D. Comparison with values from model calculations	33
1. Reduced width amplitudes	33
2. Experimental value of isospin mixing matrix element and energy of $T=1$ state	34
3. Shell model value of isospin mixing matrix element	34
4. Relative energies of $T=1$ states in ^8Be and ^8Li and isospin distortion	39
E. Discussion	44

CHAPTER IV.	ISOSPIN MIXING IN THE 7.12 MeV STATE OF ^{16}O	47
A.	The 7.12 MeV state of ^{16}O	47
B.	Experimental data	49
C.	Isospin mixing formulae	49
D.	Calculation of isospin mixing matrix elements	51
E.	Comparison with experimental values	55
F.	Discussion	58
CHAPTER V.	ISOSPIN DISTORTION IN LOW-LYING STATES OF ^{13}C AND ^{13}N	61
A.	Low-lying states of ^{13}C and ^{13}N	61
B.	Experimental data	64
C.	R-matrix analysis and fits to ^{13}C and ^{13}N data	70
	1. Formulae for level parameters	70
	2. Formulae for capture cross-sections and El radiative widths	74
	3. Fits to data	82
D.	Level parameters from shell model calculations and other experimental data	92
	1. Description of the states and spectroscopic amplitudes	92
	2. Values of the spectroscopic amplitudes	95
	3. El transition moments	97
E.	Level displacements of ^{13}C and ^{13}N	104
	1. Isospin distortion formulae for level displacement energies	106
	2. Calculation of point Coulomb and boundary condition contribution to level displacement energies	108
	3. Correction terms to the level displacement energies	111

4. Comparison with Thomas's calculations	117
5. Nolen-Schiffer anomaly	117
F. Discussion	119
REFERENCES	123

CHAPTER I

INTRODUCTION

Isospin of a nucleon was introduced into nuclear physics to write the nuclear Hamiltonian symmetrically with respect to the change of proton and neutron co-ordinates. In isospin formalism the generalized Pauli principle is used to describe the wavefunctions that are totally antisymmetric under the interchange of all co-ordinates (space, spin and charge) of any two nucleons. Strict charge independence of the forces between two nucleons would assume the three resulting physical systems neutron-neutron, neutron-proton and proton-proton, had identical energies if they have identical descriptions in space and spin.

Isospin formalism brings an extra quantum number to nucleons, the isospin t of magnitude $\frac{1}{2}$. This quantum number does not correspond to any physical angular momentum but it possesses all the algebraic properties (commutation relations, combination rules etc.) of the spin. Thus any state of a nucleus will be characterized by the total isospin T and M_T (value of the third component T_3) in isospin space along with total angular momentum J and M (value of the third component J_3) in ordinary space. The value of M_T is $\frac{1}{2}(N-Z)$, where N is the neutron number and Z is the proton number (where $N + Z = A$, is the mass number). M_T is always a good quantum number for our purpose. Corresponding states which differ only in their M_T

values therefore belong to different systems of the same mass and are said to form an isobaric multiplet. Since T behaves like an angular momentum $|M_T| \leq T$.

Isospin would be a good quantum number in nuclei if all forces were charge independent. Strict charge independence implies (Soper 1969)

- a. Exact energy degeneracy of isobaric multiplets.
- b. Pure isospin for all nuclear stationary states.
- c. Identical space-spin wavefunctions for all states of an isobaric multiplet. This is known as dynamic validity of isospin.

Charge dependent interactions can cause violation of one or more of these properties. The property (a) is not necessarily related to isospin mixing. Violations of (b) and (c) are caused by mixing between states of different isospin, and of (c) by charge dependent mixing between states of the same isospin. The latter is called the dynamic distortion (MacDonald 1960) or isospin distortion (Adelberger 1974). We call this isospin distortion.

One source of charge dependence is the Coulomb interaction between pairs of protons. Other charge dependent effects like electromagnetic spin-orbit interaction do not contribute greatly to isospin mixing. The Coulomb interaction between two point nucleons vanishes unless both the nucleons are protons. Such an asymmetrical potential violates the condition for the total isospin T to be a good quantum number (i.e. states having pure isospin).

The expression for the Coulomb interaction in the isospin formalism is

$$H^C = e^2 \sum_{i < j} (\frac{1}{2} - t_3(i)) (\frac{1}{2} - t_3(j)) r_{ij}^{-1}$$

where e is the charge of proton and r_{ij} is the relative distance between the nucleons i and j . This form of H^C can be used to verify that H^C does not commute with T^2 . The Coulomb interaction perturbs and mixes states of different isospin. In order to determine the extent to which the isospin is a good quantum number, if the nuclear force is charge independent, one needs to determine the nature and extent of this perturbation. This is done more easily by decomposing H^C into terms having well defined transformation properties for rotations of the isospin space (irreducible tensors in isospin space)

$$H^C = T^{(0)} + T^{(1)} + T^{(2)}$$

where

$$T^{(0)} = e^2 \sum_{i < j} \left[\frac{1}{4} + \frac{1}{3} \underline{t}(i) \cdot \underline{t}(j) \right] r_{ij}^{-1}$$

$$T^{(1)} = -\frac{e^2}{2} \sum_{i < j} [t_3(i) + t_3(j)] r_{ij}^{-1}$$

$$T^{(2)} = e^2 \sum_{i < j} \left[t_3(i) t_3(j) - \frac{1}{3} \underline{t}(i) \cdot \underline{t}(j) \right] r_{ij}^{-1}$$

The term $T^{(0)}$ is a scalar in isospin space and commutes with T^2 , and this is included in the charge independent part of the interaction. The term $T^{(1)}$ is a vector and produces isospin mixing and isospin distortion in nuclear states, and the term $T^{(2)}$ is a tensor whose effect is often neglected.

It may be mentioned that it is important to realize that the three consequences (a), (b) and (c) of charge independence discussed above can be independent of each other. The energy degeneracy can be upset by an effect which is charge dependent but not space or spin dependent. The neutron and proton mass difference is such an effect; it gives rise to no off-diagonal matrix elements in the potential and merely causes a displacement in the energy from state to state across the isobaric multiplet.

The second consequence of charge independence is isospin purity. The presence of charge dependent interactions causes isospin mixing. The nuclear states will be eigenfunctions of T^2 for a charge independent interaction. Isospin mixing will be appreciable if H^C has non-negligible matrix elements off-diagonal in isospin between states not too far apart in energy. Though magnetic interaction and some possible charge dependent nuclear forces can cause isospin mixing, we consider mixing by the Coulomb interaction only, because the effect of other charge dependent interactions is too small to be significant compared to the Coulomb interaction.

Radicati (1953) and MacDonald (1955, 1956) were the first to do systematic calculations of isospin mixing. Many numerical estimates of isospin mixing have used the statistical or Fermi gas model (see

Bertsch and Mekjian 1972). Although this model is only reliable for the order of magnitude of effects, its simplicity makes it invaluable for deriving the qualitative features of isospin mixing. However, this method does not take into account the shell effects or differences in the probability distributions for individual nucleons. In particular, no account is taken of the Coulomb fields acting on particles near the nuclear surface and deep inside the nucleus. Furthermore, realistic single-particle wavefunctions of the last bound particles are often peaked at the nuclear surface where the Coulomb field is stronger than average. To make a precise calculation of the isospin mixing strength of the Coulomb interaction we use the shell model descriptions of the states. Information on isospin mixing usually comes from comparison of cross-sections for reactions related through rotations in isospin space and from violation of one of the isospin selection rules on transition rates.

The third consequence of charge independence, the validity of isospin, refers to the equations describing the relations between the states of an isobaric multiplet. In charge independent situations the isospin lowering and raising operators $T_{\pm}$ behave in a similar fashion to those for the ordinary angular momentum lowering and raising operators. Thus the single nucleon operator simply changes proton into neutron or vice versa, the numerical factor being unity. These operators ($t_{\pm}$) give zero when applied to the "wrong particle"; this is a special case of an important property possessed by a state of pure T with $M_T = \pm T$ as $T_{\pm} \Psi(T, \pm T) = 0$. The isospin lowering or raising operator relates states of the charge independent Hamiltonian. The accuracy with which these relations are satisfied for real nuclear states is

a measure for the dynamic validity of the isospin. MacDonald (1960) has pointed out that it is possible to have systems where all the states have pure isospin but the simple isospin lowering and raising relations do not apply. This will occur if H^C has negligible matrix elements off-diagonal in isospin (between states reasonably close in energy) but possess appreciable matrix elements diagonal in isospin which depend on the third component of T_3 (i.e. varying from one member of isospin multiplet to the other) between fairly close-lying states. This is isospin distortion. Isospin distortion is thus associated with the properties of the corresponding energy levels of isobaric multiplets. Information on isospin distortion usually comes from the positions of the energy levels of the isobaric multiplets, their widths and various transition properties.

Theoretically, interpretations of isospin mixing and isospin distortion are hampered by the complexity of the nuclear states. Depending on the nucleus, the calculation of isospin mixing or isospin distortion may be straightforward or very complicated. For light nuclei simple interpretations are possible if the states are described by simple shell model states. In cases where the information on isospin mixing (and isospin distortion) from related experimental data differs considerably from the isospin mixing (and isospin distortion) calculated from the Coulomb interaction, it would be useful to have better nuclear wavefunctions to understand how serious the discrepancies are. Since the form of

charge dependent nuclear interaction if it exists is not known with certainty, we have not considered any effects of charge dependent nuclear interactions. However, for one case we have considered charge dependent interactions, like the electromagnetic spin-orbit interaction and other magnetic interactions, besides the Coulomb interaction.

We now mention the particular cases of isospin mixing and isospin distortion that are considered in this work.

1. ISOSPIN MIXING AND ISOSPIN DISTORTION IN ^8Be

Three pairs of levels in ^8Be around 16 to 19 MeV of excitation are known to show considerable isospin mixing (Marion 1965, Paul 1966, Barker 1966). The charge dependent matrix elements derived from fitting the experimental data on the 2^+ states at 16.63 and 16.93 MeV, 1^+ states at 17.64 and 18.15 MeV, and 3^+ states at 19.06 and 19.22 MeV, are -149, -120, and -63 keV respectively (Barker 1966). Barker (1966) attempted to calculate these matrix elements using Coulomb interaction and harmonic oscillator wavefunctions but obtained only about half of the experimental values. Several attempts had been made to overcome these discrepancies (see Barker 1978) but the problem for the 3^+ states still remained.

The level parameters for the 3^+ states of ^8Be are obtained from a two level R-matrix fit to the $^7\text{Li} + p$, $^5p_3(2^{S+1}L_J)$ phase shifts, using the restrictions of a two-state isospin mixing model. To check the consistency of these parameters, the predicted cross-sections for the reactions $^7\text{Li}(p,n)$, $^7\text{Li}(p,\gamma)$, $^{10}\text{B}(d,\alpha)$

and ${}^9\text{Be}(d,t)$ are calculated using the parameters obtained from the fit and compared with the experimental values. The isospin mixing matrix element for the 3^+ states is then calculated using the level parameters obtained from the two level fit to ${}^7\text{Li}+p$, ${}^5\text{P}_3$ phase shifts. The energy difference of the 3^+ states of ${}^8\text{Be}$ and ${}^8\text{Li}$ which is an effect of isospin distortion is calculated assuming that charge dependent interaction is purely Coulomb.

2. ISOSPIN MIXING IN 7.12 MeV STATE OF ${}^{16}\text{O}$.

In ${}^{16}\text{O}$ the 1^- state at 7.12 MeV is mainly a $T=0$ state in the lowest shell model configuration, but the observations show that this state has some amount of $T=1$ admixture. This admixture is assumed to arise from the five higher states of 1^- , $T=1$, having the same configuration as the $T=0$ state. The amount of admixture is calculated using various shell model interactions and compared with the experimental value.

3. ISOSPIN DISTORTION IN LOW-LYING STATES OF ${}^{13}\text{C}$ AND ${}^{13}\text{N}$.

Isospin distortion is found in the low-lying states of ${}^{13}\text{C}$ and ${}^{13}\text{N}$. If there is no charge dependent interaction then the energies of the corresponding states in the mirror nuclei ${}^{13}\text{C}$ and ${}^{13}\text{N}$ would be equal. The E1 transition strengths between the corresponding pairs of states would also be the same. But some observed properties of the states and their E1 transition strengths in these two nuclei show asymmetry. Among the asymmetries that have aroused interest in the past are the

marked difference in the excitation energies of the $\frac{1}{2}^+$ first excited states of ^{13}C and ^{13}N and the very different strengths of the El transitions from them to the $\frac{1}{2}^-$ ground states. These low-lying states have $T = \frac{1}{2}$ and the lowest $T = \frac{3}{2}$ state is at about 15 MeV excitation; and the effects of isospin mixing of the low-lying states is not believed to be important. Thus the asymmetries in ^{13}C and ^{13}N seem to be an effect of isospin distortion.

In the present work an attempt is made to give a consistent account of observed properties of the low-lying states of ^{13}C and ^{13}N . In the first stage of the analysis, least square fits to the available data are made using the R-matrix formulae in the one and two-level approximations; in the second stage the resultant parameter values are compared with shell model predictions. Fitted properties include level widths, neutron scattering data, El radiative widths and El capture cross-sections. The R-matrix formulae include external contributions to the El transition matrix elements calculated using the wavefunctions with correct asymptotic forms, as well as the internal contributions. A reasonable account is given of the two notable asymmetries in ^{13}C and ^{13}N .

CHAPTER II

ISOSPIN MIXING AND DISTORTION FORMULAE

We use the formulae and notation of Barker (1978) except when change is needed. Isospin mixing formulae derived by Barker (1978) have given satisfactory explanation of the isospin mixing between pairs of levels in ^8Be , ^{12}C and ^{16}O . Here we have obtained a more general formulae which could be used where the mixing involves more than two states as well as for two states. Let us first consider isospin mixing formulae.

Consider the eigenstates Ψ_λ of the Hamiltonian H defined in the whole configuration space

$$H\Psi_\lambda = E_\lambda \Psi_\lambda, \quad (1)$$

where Ψ_λ are written in terms of basis states Ψ_{Tk} of good isospin T

$$\Psi_\lambda = \sum_{Tk} A_{\lambda Tk} \Psi_{Tk}. \quad (2)$$

The Ψ_{Tk} are eigenstates of H^0 , the charge independent part of the total Hamiltonian and satisfy

$$H^0\Psi_{Tk} = E_{Tk}^0 \Psi_{Tk}, \quad (3)$$

then

$$H = H^0 + H^C, \quad (4)$$

and H^C is the charge dependent interaction. The wavefunctions Ψ_λ and Ψ_{Tk} are expressed in terms of single particle wavefunctions such as harmonic oscillator wavefunctions where there is no allowance for the presence of nearby nucleon channels and the consequent difference in the asymptotic forms of neutron and proton wavefunctions. Solution of equation (1) using equation (2) involves changing the order of the derivative in the kinetic energy term of the Hamiltonian and the summation of the infinite series (equation 2), which is permissible only if the series is absolutely convergent. Thus the Hamiltonian operator is not realizable in this space. These problems of wrong asymptotic behaviour and non-realizability can be overcome by the use of Bloch operators (Bloch 1957, Lane and Robson 1966) defined by

$$\mathcal{L}(S) = \sum_c |c\rangle \frac{\hbar^2}{2m_c} \delta(r_c - a_c) \left(\frac{\partial}{\partial r_c} - \frac{S(c)-1}{a_c} \right) \langle c|, \quad (5)$$

where $|c\rangle$ is the channel wavefunction which is a function of intrinsic and angular variables but not of radial distance r_c , $S(c)$ is a parameter the choice of which is discussed later and $r_c = a_c$ is the channel radius. The channel radius a_c separates the internal region $r_c < a_c$ from the external region $r_c \geq a_c$. As in R-matrix theory (Lane and Thomas 1958) the internal region is a region of the configuration space where all the nucleons are close together in a volume of nuclear dimension in physical space. The

external region, also known as the channel region, is a region of the configuration space where all the nucleons are separated into two groups (each group being bound and having certain quantum states). Then $H + \mathcal{L}(S)$ is realizable. S can be chosen to keep the continuity of the internal and external wavefunctions with correct asymptotic form in each channel. The states Ψ_λ are now defined in the internal region, and they are made to join smoothly onto the correct asymptotic wavefunctions at the surface of the internal region.

Since $H + \mathcal{L}(S_\lambda)$ is realizable it is convenient to introduce the boundary condition

$$\mathcal{L}(S_\lambda)\Psi_\lambda = 0, \quad (6)$$

with S_λ taken as the logarithmic derivative of the appropriate external wavefunctions in the channel c evaluated at the channel radius $r_c = a_c$ and the channel energy $E_{\lambda c} = E_\lambda - E_{th,c}$, where $E_{th,c}$ is the threshold energy of the channel c . Similarly the correct asymptotic forms of Ψ_{Tk} are obtained by imposing the boundary condition

$$\mathcal{L}(\bar{S}_{Tk})\Psi_{Tk} = 0, \quad (7)$$

where $\bar{S}_{Tk}(c)$ is similar to $S_\lambda(c)$ except that for the mirror neutron and proton channel it is evaluated at the average channel energy $\frac{1}{2}(E_{Tkn} + E_{Tkp})$ and for average charges of the neutron and proton channels. The essential point is that $\bar{S}_{Tk}(n) = \bar{S}_{Tk}(p)$. The states Ψ_{Tk} are

normalized and orthogonal in T , but not in k , because $\bar{S}_{Tk}$ depends on k .

Since $H + \mathcal{L}(S_\lambda)$ is realizable we can write

$$\begin{aligned} 0 &= \{H + \mathcal{L}(S_\lambda) - E_\lambda\} \Psi_\lambda \\ &= \sum_{T'k'} A_{\lambda T'k'} \{H + \mathcal{L}(S_\lambda) - E_\lambda\} \Psi_{T'k'} \\ &= \sum_{T'k'} A_{\lambda T'k'} \{E_{T'k'}^0 + H^c + \mathcal{L}(S_\lambda) - \mathcal{L}(\bar{S}_{T'k'}) - E_\lambda\} \Psi_{T'k'} \quad (8) \end{aligned}$$

which gives

$$\begin{aligned} \sum_{T'k'} A_{\lambda T'k'} \{E_{T'k'}^0 - E_\lambda\} \delta_{T'T} \langle \Psi_{Tk} | \Psi_{T'k'} \rangle + \\ \langle \Psi_{Tk} | H^c + \mathcal{L}(S_\lambda) - \mathcal{L}(\bar{S}_{T'k'}) | \Psi_{T'k'} \rangle = 0, \quad (9) \end{aligned}$$

for any λ and Tk . For the two state mixing model the states Ψ_{Tk} are orthogonal and equation (9) becomes equation (12) of Barker (1978).

The last term in equation (9) gives the isospin mixing matrix element between the basis states Ψ_{Tk} and $\Psi_{T'k'}$, and can be written as

$$V_{TkT'k'}^\lambda = H_{TkT'k'}^c + L_{TkT'k'}^\lambda, \quad (10)$$

where

$$H_{TkT'k'}^c = \langle \Psi_{Tk} | H^c | \Psi_{T'k'} \rangle \quad (11)$$

is the internal mixing matrix element and

$$L_{TkT'k'}^{\lambda} = \langle \Psi_{Tk} | \mathcal{L}(S_{\lambda}) - \mathcal{L}(\bar{S}_{T'k'}) | \Psi_{T'k'} \rangle \quad (12)$$

is the external mixing matrix element. The calculation of $L_{TkT'k'}^{\lambda}$ depends on the description of the channel region and we shall discuss them in detail for the problems considered. We calculate the internal contribution to the mixing matrix element by using standard shell model techniques (Elliott 1953) and assuming that H^C is the two-body (point) Coulomb interaction coupling pairs of protons, that Ψ_{Tk} is a shell model state with harmonic oscillator single particle wavefunctions and that the integration is extended over all space. These approximations should be accurate provided that the channel radii a_c can be chosen so that on the one hand they are sufficiently small that the harmonic oscillator wavefunctions adequately represent the true wavefunctions for $r_c < a_c$, and on the other hand they are sufficiently large that the oscillator wavefunctions are small in the additional region of integration. Agreement between calculated and experimental values of isospin mixing matrix elements can therefore be expected only for a small range of a_c value.

We shall apply the isospin mixing formulae discussed above in the problem of isospin mixing of the pair of 3^+ states in ${}^8\text{Be}$ and in the isospin mixing of the 7.12 MeV state of ${}^{16}\text{O}$.

Now we shall discuss the isospin distortion formulae. For this we consider the M_T dependence of the basis states Ψ_{Tk} and write

$$H\Psi_{M_T}(J^\pi) = E_{M_T}(J^\pi)\Psi_{M_T}(J^\pi) , \quad (13)$$

with

$$\mathcal{L}\{S_{M_T}(J^\pi)\}\Psi_{M_T}(J^\pi) = 0 . \quad (14)$$

The eigenfunctions $\Psi_{M_T}(J^\pi)$ are expanded in terms of states $\Psi_{T_k}(J^\pi, M_T)$ of good isospin T as

$$\Psi_{M_T}(J^\pi) = \sum_{T_k} A_{T_k}(J^\pi, M_T) \Psi_{T_k}(J^\pi, M_T) , \quad (15)$$

which satisfy

$$H^0\Psi_{T_k}(J^\pi, M_T) = E_{T_k}^0(J^\pi)\Psi_{T_k}(J^\pi, M_T) , \quad (16)$$

and

$$\mathcal{L}\{\bar{S}_{T_k}(J^\pi)\}\Psi_{T_k}(J^\pi, M_T) = 0 . \quad (17)$$

$E_{T_k}^0(J^\pi)$ and $\bar{S}_{T_k}(J^\pi)$ are independent of M_T , so that the dependence of $\Psi_{T_k}(J^\pi, M_T)$ on M_T is trivial, whereas the dependence of $\Psi_{M_T}(J^\pi)$ on M_T is all important. Proceeding in the same manner as in equation (9) we obtain

$$\begin{aligned} & \sum_{T', k'} A_{T', k'}(J^\pi, M_T) [\{E_{T', k'}^0(J^\pi) - E_{M_T}(J^\pi)\} \delta_{TT'} \langle \Psi_{T', k'}(J^\pi, M_T) | \Psi_{T_k}(J^\pi, M_T) \rangle \\ & + \langle \Psi_{T_k}(J^\pi, M_T) | H^C + \mathcal{L}\{S_{M_T}(J^\pi)\} - \mathcal{L}\{\bar{S}_{T', k'}(J^\pi)\} \Psi_{T', k'}(J^\pi, M_T) \rangle = 0 . \end{aligned} \quad (18)$$

In the absence of charge dependent effects $\Psi_{M_T}(J^\pi)$ would be one of the $\Psi_{T^p k^p}(J^\pi, M_T)$ which is a particular value of $\Psi_{T^p k^p}(J^\pi, M_T)$. Then in the first order we have

$$E_{M_T}(J^\pi) = E_{T^p k^p}^0(J^\pi) + V_{T^p k^p T^p k^p}^C(J^\pi, M_T), \quad (19)$$

where

$$V_{T^p k^p T^p k^p}^C = H_{T^p k^p T^p k^p}^C(J^\pi, M_T) + L_{T^p k^p T^p k^p}(J^\pi, M_T), \quad (20)$$

with

$$H_{T^p k^p T^p k^p}^C(J^\pi, M_T) = \langle \Psi_{T^p k^p}(J^\pi, M_T) | H^C | \Psi_{T^p k^p}(J^\pi, M_T) \rangle, \quad (21)$$

$$L_{T^p k^p T^p k^p}(J^\pi, M_T) =$$

$$\langle \Psi_{T^p k^p}(J^\pi, M_T) | \mathcal{L}\{S_{M_T}(J^\pi) - \mathcal{L}\{\bar{S}_{T^p k^p}(J^\pi)\} | \Psi_{T^p k^p}(J^\pi, M_T) \rangle, \quad (22)$$

Equation (21) is similar to equation (11) except that equation (21) gives the diagonal matrix elements of the Coulomb interaction whereas equation (11) gives the off-diagonal matrix elements. The assumptions for calculating (21) are similar to those for equation (11). Likewise the assumptions for calculating (22) are discussed separately for individual problems.

CHAPTER III

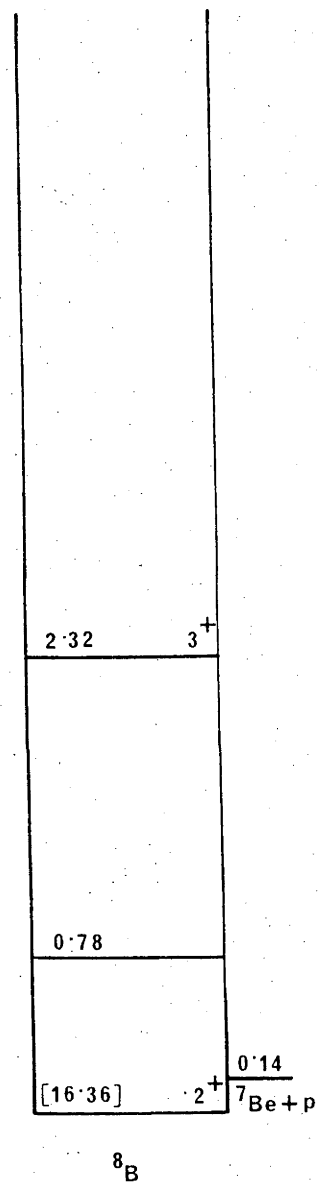
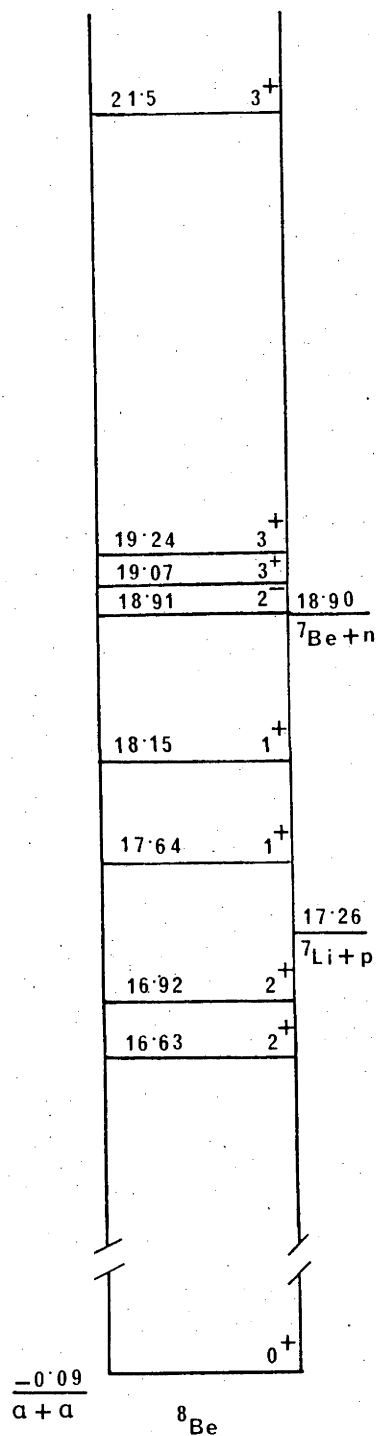
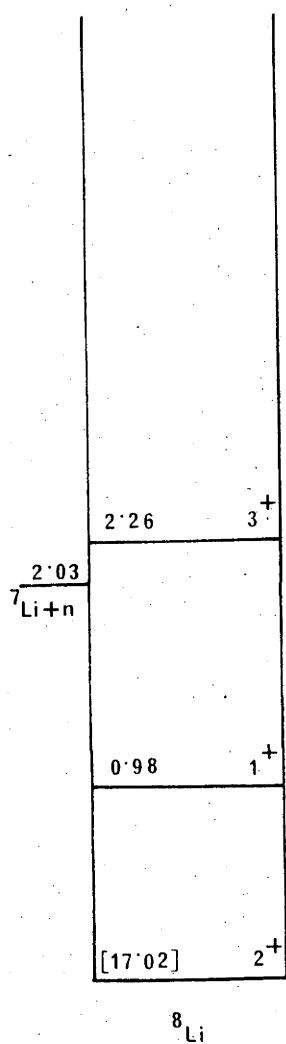
ISOSPIN MIXING AND ISOSPIN DISTORTION IN 3^+ STATES OF ^8Be

A. 3^+ STATES OF ^8Be

The most recent compilation of energy levels of ^8Be (Ajzenberg-Selove and Lauritsen 1979) gives a pair of 3^+ levels at 19.07 ± 0.02 MeV and 19.24 MeV, and a possible 3^+ level at 21.5 MeV. Fig. 1 shows these levels together with other nearby levels and the related levels of ^8Li and ^8B . The pair of levels has been interpreted as an isospin-mixed doublet (Barker 1966). The observed properties of the levels provide values for the matrix element of the charge-dependent interaction causing the isospin mixing, and the expectation energy of the pure $T=1$, 3^+ state of ^8Be . From the latter one may obtain the energy difference of the analogue 3^+ states of ^8Be , relative to the analogue 2^+ states. These values of the mixing matrix element and of the energy difference have been compared with calculated values (Barker 1978), and some disagreement was obtained with values calculated on the assumption that the charge-dependent interaction was purely Coulomb.

Here we investigate whether these discrepancies could be due to inaccurate values for some of the measured properties of the levels, in particular their energies and the ratios of neutron and

Fig. 1 Energy level diagram for $A=8$ isobars.



proton reduced widths. Population of the 19.07 and 19.24 MeV levels has been reported for many reactions (Ajzenberg-Selove and Lauritsen 1979), but most quantitative information has come from the three resonance reactions ${}^7\text{Li}(p,\gamma)$, ${}^7\text{Li}(p,n)$ and ${}^7\text{Li}(p,p)$.

The ${}^7\text{Li}(p,\gamma)$ reaction to the first excited state of ${}^8\text{Be}$ showed a resonance at $E_p = 2.06 \pm 0.02$ MeV (Riech 1963), corresponding to $E_x = 19.06 \pm 0.02$ MeV. Other groups have also observed a peak at about 2.1 MeV, either in the γ_1 transition (Fisher et al. 1976) or in the summed $\gamma_0 + \gamma_1$ transitions (Newson et al. 1957; Perry et al. 1963). It has been assumed that this peak is due to the lower 3^+ level.

Only the upper level appears to contribute to the ${}^7\text{Li}(p,n)$ reaction, with the peak energy well determined as $E_p = 2.25$ MeV, giving $E_x = 19.22$ MeV (probably within about 0.01 MeV). The data can be fitted with any value of the reduced width ratio γ_n^2/γ_p^2 between 3 and 10 (Macklin and Gibbons 1958).

Early measurements of the ${}^7\text{Li}(p,p)$ excitation function at $\theta_{\text{lab}} = 164^\circ$ showed a sharp peak at $E_p = 2.06$ MeV (Bashkin and Richards 1951). Later measurements at several angles suggested the presence of two interfering levels of ${}^8\text{Be}$ in this energy region, one corresponding to the resonance seen in ${}^7\text{Li}(p,\gamma)$, the other to the resonance in ${}^7\text{Li}(p,n)$, but no parametric fit to the data was given (Malmberg 1956). These measurements showed that the peak observed by Bashkin and Richards moved to lower energies at smaller angles. Recently Brown et al. (1973) have extracted phase shifts from ${}^7\text{Li}(p,p)$ measurements with both

polarized and unpolarized protons. The imaginary parts of the phase shifts were chosen to fit the ${}^7\text{Li}(p,n)$ and other data. They obtained trial values for their 5P_3 phase shift, which contains information about the 3^+ levels of ${}^8\text{Be}$, on the assumption that two such levels occur in the region, but did not obtain values of the level parameters by fitting the resultant phase shift.

Among the reactions that populate ${}^8\text{Be}$ as a product nucleus, rather than as a compound nucleus, only ${}^7\text{Li}(d,n){}^8\text{Be}$ has been reported to excite all three of the levels at 18.93, 19.05 and 19.24 MeV (Kerr 1967), but details of this work are unpublished. The 18.9 MeV level is 2^- (Ajzenberg-Selove and Lauritsen 1979). In ${}^{10}\text{B}(d,\alpha){}^8\text{Be}$, peaks observed at about 18.9 and 19.2 MeV were attributed to the 2^- level and the upper 3^+ level (Callender and Browne 1970). In all other such reactions, only a single peak appeared in the spectrum in this energy region; in the cases where the resolution was sufficient to separate the levels, the peak corresponded to a level at about 19.2 MeV in each of ${}^9\text{Be}(p,d){}^8\text{Be}$ (Kull 1967), ${}^9\text{Be}(d,t){}^8\text{Be}$ (Oothoudt and Garvey 1977), and ${}^9\text{Be}({}^3\text{He},\alpha){}^8\text{Be}$ (Ajzenberg-Selove et al. 1976).

Brown et al. (1973) have extracted 5P_3 ($2^{S+1}L_J$), ${}^7\text{Li}+p$ phase shifts which give accurate information about the pair of 3^+ levels, but they did not obtain the level parameters for the pair of levels. We have done a two-level fit to the phase shift data of Brown et al. using the R-matrix theory and the two state isospin mixing model to obtain the level parameters for the pair of levels. It may be mentioned here that the reason for choosing the R-matrix theory is that it enables the analysis to be made in terms of

constant parameters like eigenenergies and reduced widths. The values of these parameters can then be compared with the calculated values using nuclear shell model. For example in our case (for 3^+ states of ^8Be) we obtain values of certain parameters from the two level fit of the $^5\text{P}_3$ phase shifts obtained from $^7\text{Li}(p,p)$ measurements. Using these values of the parameters we then obtain experimental values of the isospin mixing matrix element for the pair of states and the expectation energy of the $T=1, 3^+$ state of ^8Be . These values are then compared with those obtained from the shell model calculations. Thus the R-matrix treatment separates the problem of relating the experimental data and models into two parts.

data $\leftrightarrow$ parameters, parameters $\leftrightarrow$ models.

In the following section we discuss the two level fit to the $^7\text{Li}+p$, $^5\text{P}_3$ phase shift and the extraction of relevant parameters. In section C we check if these parameters are consistent with the cross-sections observed in other reactions involving the 3^+ levels. Then in section D we use shell model techniques to calculate the values of some of the parameters and compare them with their experimental values. Finally in section E we discuss the results of the present work together with the results of previous analyses of isospin mixing of 3^+ states of ^8Be .

B. TWO LEVEL FIT TO ${}^7\text{Li} + p$, ${}^5\text{P}_3$ PHASE SHIFTS

1. Isospin Mixing Formulae for 3^+ States of ${}^8\text{Be}$

For the two levels a and b around 19 MeV in ${}^8\text{Be}$ equation (2) becomes

$$\left. \begin{aligned} \Psi_a &= \alpha \Psi_0 + \beta \Psi_1, & \Psi_b &= \alpha' \Psi_0 + \beta' \Psi_1 \\ \text{with } \alpha^2 + \beta^2 &= 1, & \alpha'^2 + \beta'^2 &= 1 \\ \text{and } \alpha' &\neq \alpha, & \beta' &\neq \beta \end{aligned} \right\}, \quad (23)$$

Since the levels are nearby we can assume Ψ_a and Ψ_b to be orthogonal to each other (Barker 1978):

$$\left. \begin{aligned} \Psi_a &= \alpha \Psi_0 + \beta \Psi_1, & \Psi_b &= \beta \Psi_0 - \alpha \Psi_1 \\ \text{with } \alpha^2 + \beta^2 &= 1 \end{aligned} \right\}. \quad (24)$$

Wavefunctions given in equation (24) are particular cases of equation (2) where $\lambda = a$ and b , $A_{aok} = \alpha$, $A_{alk} = \beta$ and $A_{bok} = \beta$ and $A_{blk} = -\alpha$, and α is known as the isospin mixing coefficient. Since the mixing is between two states the label k can be dropped. It may be mentioned here that due to the closeness of the levels $V_{01}^a = V_{01}^b = V_{01}$. Then solving equation (9), by using the two state mixing model (equation 24) we obtain an expression for the isospin mixing matrix element

$$V_{01} = -\alpha\beta(E_b - E_a). \quad (25)$$

The experimental value of the isospin mixing matrix element is obtained from equation (25) by using the observed level energies (E_a, E_b) , together with the isospin mixing coefficient α determined from the two level fit to the 5P_3 , ${}^7\text{Li}+p$ phase shift, and $\beta = \sqrt{1-\alpha^2}$. The energies of the two 3^+ states of pure isospin $T=0$ and $T=1$ are given by

$$\left. \begin{aligned} E_0^0 &= \alpha^2 E_a + \beta^2 E_b \\ E_1^0 &= \beta^2 E_a + \alpha^2 E_b \end{aligned} \right\}, \quad (26)$$

which are also obtained by solving equation (9).

2. R-Matrix Fit to ${}^7\text{Li}+p$, 5P_3 Phase Shifts

The energy dependence of the complex 5P_3 phase shift $\delta \equiv \delta^R + i\delta^I$ is given by the formulae of R-matrix theory (Lane and Thomas 1958) in terms of the eigenenergies E_λ and reduced width amplitudes $\gamma_{\lambda c}$ for the level λ and channel c . With the ${}^7\text{Li}+p$ channel denoted by p , we have

$$e^{2i\delta} \equiv \eta e^{2i\delta^R} = e^{-2i\theta_p} \left[1 + 2i \sum_{\lambda\mu} \gamma_{\lambda p} \gamma_{\mu p} A_{\lambda\mu} \right], \quad (27)$$

where $\eta = e^{-2\delta^I}$ is the absorption coefficient and the $A_{\lambda\mu}$ are elements of a matrix in level space, defined by its inverse

$$(\underline{A}^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - \sum_c L_c^0 \gamma_{\lambda c} \gamma_{\mu c} . \quad (28)$$

Here $L_c^0 = S_c^0 + i P_c$ and $S_c^0 = S_c - B_c$, where S_c , P_c and $- \phi_c$ are the shift factor, penetration factor and hard-sphere phase shift, evaluated at the channel radius a_c , and B_c is the boundary condition parameter.

In the two-level approximation, the level labels λ and μ have the values a for the lower level and b for the upper level (to avoid confusion with isospin labels). The sum over c in eq. (28) is in principle over all channels, open and closed. We approximate by neglecting all closed channels, and also the proton channel to the first excited state of ${}^7\text{Li}$, which would require f-wave protons. Then c takes on the values p and n only, corresponding respectively to the ${}^7\text{Li}$ and ${}^7\text{Be}$ ground state channels with p-wave nucleons. Following equation (24) the reduced width amplitudes are written as

$$\left. \begin{aligned} \gamma_{ap} &= 2^{-1/2}(\alpha\gamma_0 + \beta\gamma_1) , & \gamma_{bp} &= 2^{-1/2}(\beta\gamma_0 - \alpha\gamma_1) , \\ \gamma_{an} &= 2^{-1/2}(-\alpha\gamma_0 + \beta\gamma_1) , & \gamma_{bn} &= 2^{-1/2}(-\beta\gamma_0 - \alpha\gamma_1) , \end{aligned} \right\} \quad (29)$$

where $\gamma_T(T=0,1)$ is the reduced width amplitude for the pure T state (excluding the isospin Clebsch-Gordan coefficient). The relations (29) restrict the values of the four quantities $\gamma_{\lambda c}$ by expressing them in terms of three independent parameters, say γ_0 , γ_1 and α . These, with E_a and E_b , provide five

adjustable parameters, for given values of a_c and B_c .

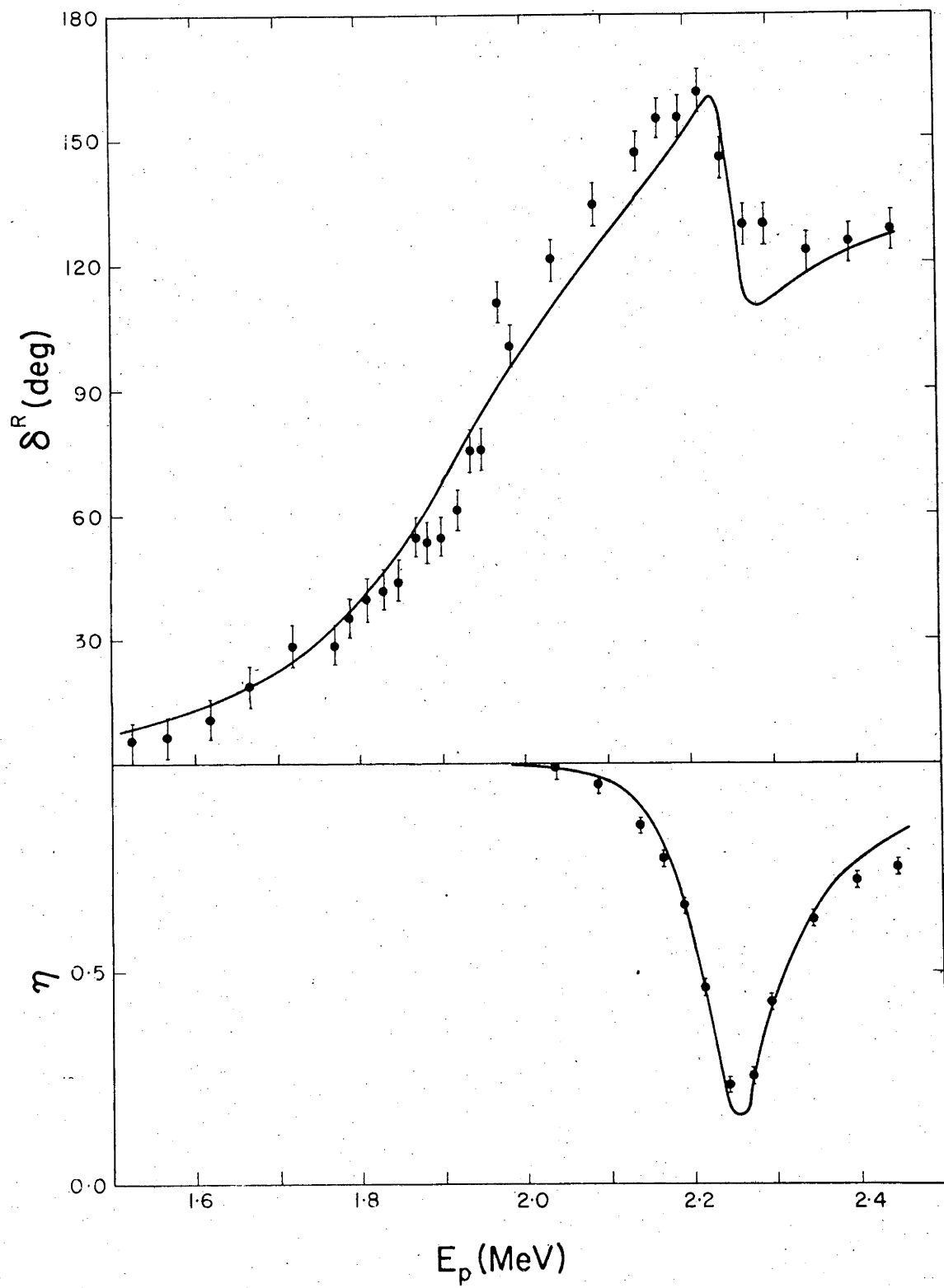
As reasonable values of the channel radii, we take $a_p = a_n = 5.0$ fm (Barker 1978). The effect of changing this value is discussed later. Equally good fits to the phase shift can be obtained for any choice of the B_c provided that the values of the E_λ and $\gamma_{\lambda c}$ are freely adjustable (Barker 1972); since $\sum_\lambda \gamma_{\lambda c}^2$ remains invariant under such changes of B_c , and since the restriction implied by equations(29) is that $\sum_\lambda \gamma_{\lambda c}^2$ should be independent of c , one can also get equally good fits for any B_c with only five adjustable parameters. For convenience, we choose each B_c equal to the average value of the shift factor S_c in the energy region fitted, giving $B_p = -0.53$ and $B_n = -0.81$.

Experimental values of the 5P_3 phase shift $\delta_{\text{exp}} \equiv \delta_{\text{exp}}^R + i\delta_{\text{exp}}^I$ have been given by Brown et al. (1973), and these values of δ_{exp}^R and values of $\eta_{\text{exp}} = \exp(-2\delta_{\text{exp}}^I)$ are shown in Fig. 2 for $E_p > 1.5$ MeV. A least squares fit to these is made by minimizing the quantity

$$X = \frac{1}{N^R + N^I} \left[\sum_{i=1}^{N^R} \left| \frac{\delta_{\text{exp}}^R(E_i) - \delta^R(E_i)}{\epsilon^R(E_i)} \right|^2 + \sum_{i=1}^{N^I} \left| \frac{\eta_{\text{exp}}(E_i) - \eta(E_i)}{\epsilon^I(E_i)} \right|^2 \right], \quad (30)$$

where the E_i are the energies at which measurements have been made, and ϵ^R and ϵ^I are the errors in the real part of the phase shift and in the absorption coefficient respectively. We fit the real phase shift for $E_p > 1.5$ MeV ($N^R = 30$) and the absorption coefficient for $E_p > 2.0$ MeV ($N^I = 12$), since η is necessarily

Fig. 2 Real part of 5P_3 phase shift for $^7\text{Li}+p$ elastic scattering and corresponding absorption coefficient as functions of proton energy. The experimental points are from Browne et al. (1973) and the error bars show the assumed errors. The curves are best fits from a two-level R-matrix approximation with parameters restricted by the two-state isospin mixing model.



unity below the neutron threshold at $E_p = 1.881$ MeV. We assign, rather arbitrarily, equal errors $\epsilon^R = 5^\circ$ and $\epsilon^I = 0.02$ at each of these energies.

The best fit is obtained with $\alpha = 0.53$ (for $\beta > 0$) and is illustrated in Fig. 2. If α is changed from this value and the other parameter values are optimised, the fit to δ^R remains good but the fit to η worsens, particularly in the region of the wings. Fits regarded as acceptable are obtained for α between about 0.4 and 0.7. The solid curves in Fig. 3 show these optimum values of X , E_λ and γ_T as functions of α . Corresponding values of various derived quantities are shown by the solid curves in Fig. 4. Of these, the values of $\gamma_{\lambda n}^2/\gamma_{\lambda p}^2$ are obtained from equations (29). Widths of the levels are calculated in an approximate way by using the parameter values E_λ and $\gamma_{\lambda c}$ in an independent one-level approximation for each level, so that the observed width in the Thomas approximation (Lane and Thomas 1958) is given by

$$\Gamma_\lambda^0 = \sum_c \Gamma_{\lambda c}^0, \quad (31)$$

$$\Gamma_{\lambda c}^0 = 2 \gamma_{\lambda c}^2 P_c / (1 + \sum_{c'} \gamma_{\lambda c'}^2 dS_{c'}/dE),$$

evaluated at the peak energy. The energies E_T^0 of the states of pure isospin T and the isospin mixing matrix element V_{01} are given by equations (26) and (25).

These parameter values all correspond to a particular choice of B_c values. The properties of the level λ ($\lambda = a$ or b) are

Fig. 3 Minimum values of X (eq. (30)) and corresponding values of the level parameters E_λ ($\lambda = a, b$) and γ_T ($T = 0, 1$) as functions of the isospin mixing parameter α . The solid curves are for $B_p = -0.53$ and $B_n = -0.81$. The dashed curves are values of $E_\lambda^{(\lambda)}$ ($\lambda = a, b$) corresponding to $B_c = B_c^{(\lambda)} = S_c(E_\lambda^{(\lambda)})$ ($c = p, n$).

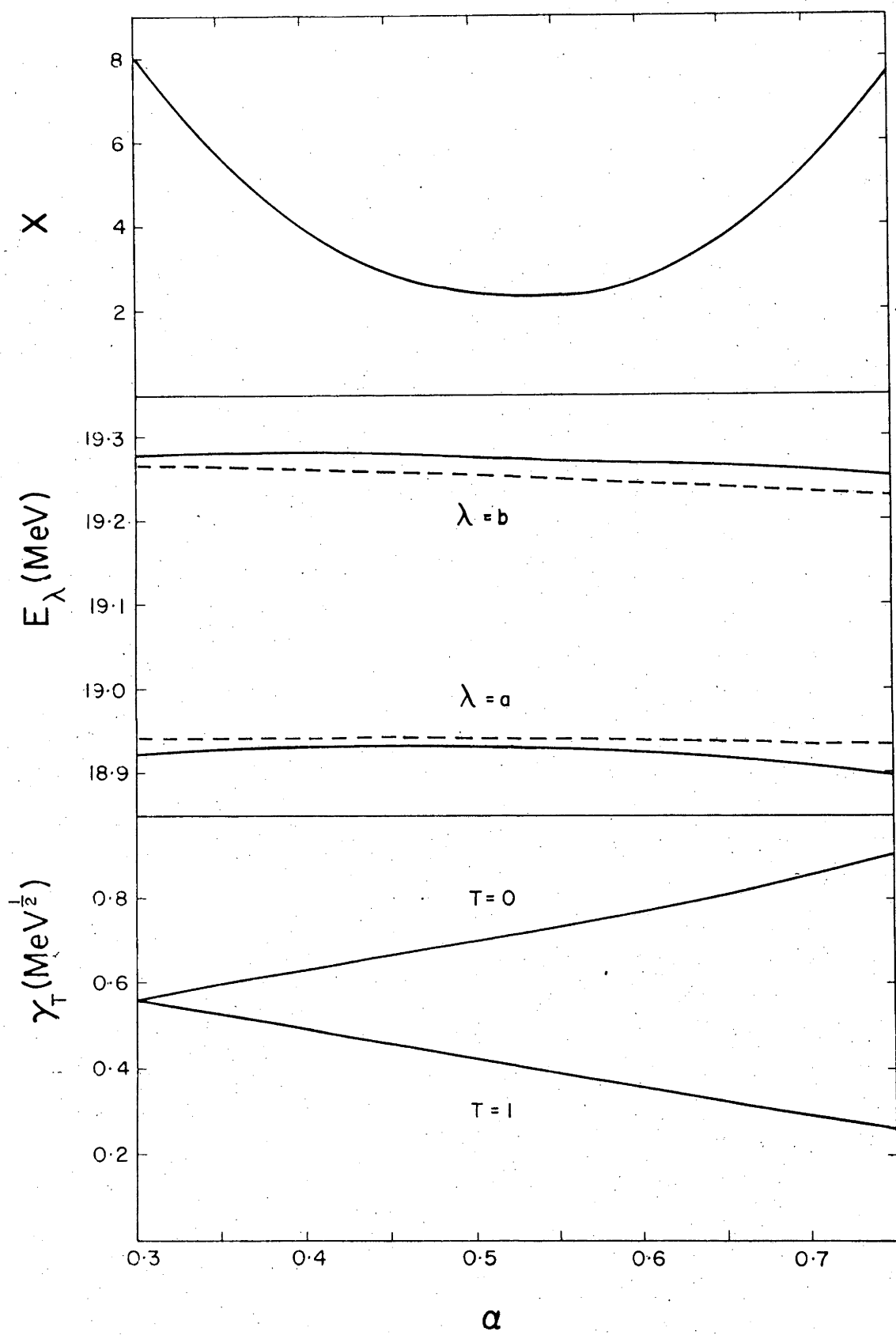
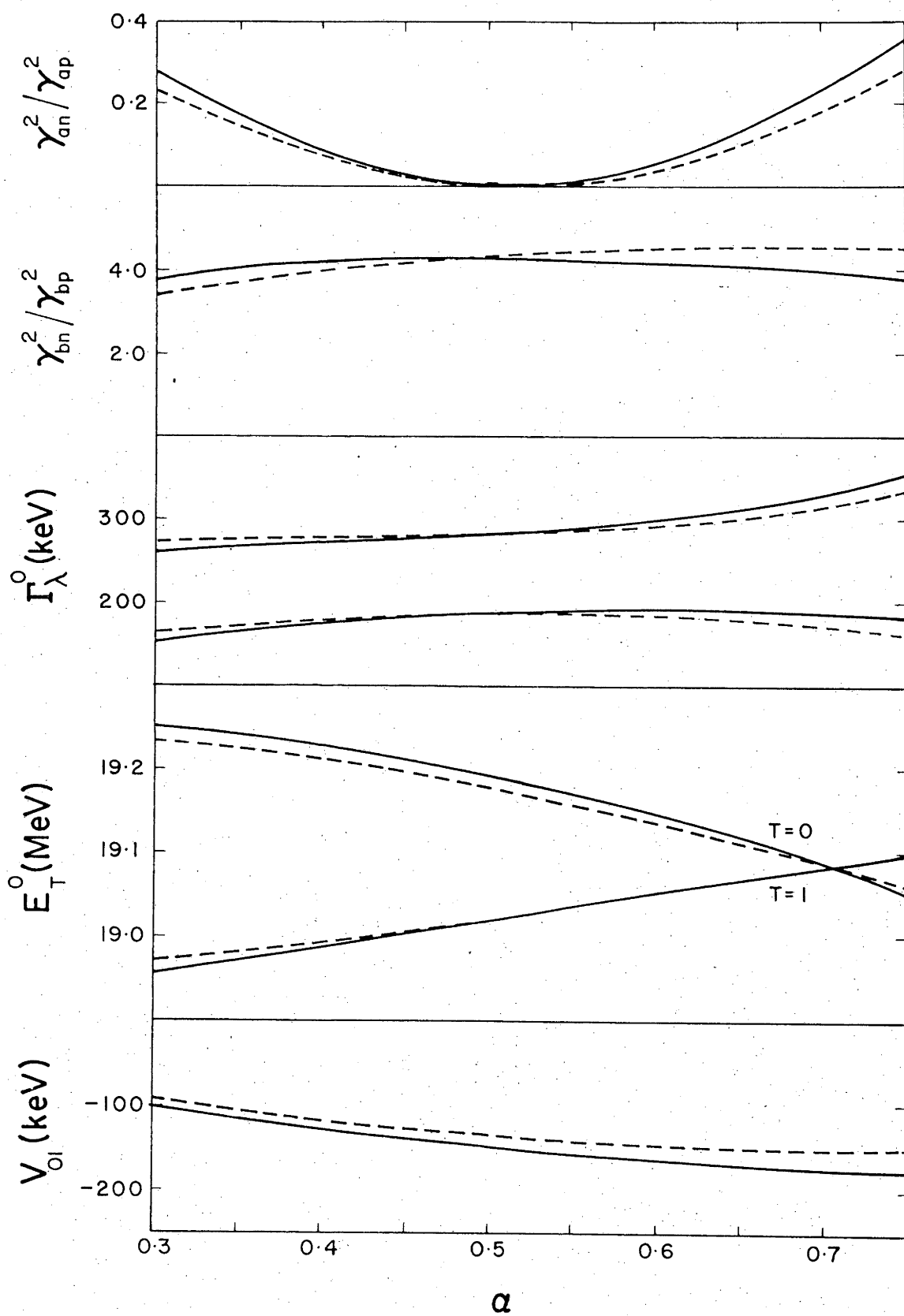


Fig. 4 Values of quantities derived from the parameter values
of Fig. 3 using eqs. (29), (31), (26) and (25).



probably best described by the parameter values for $B_c = S_c(E_\lambda)$; we write these values as $B_c^{(\lambda)}$ and the corresponding parameter values that give exactly the same fit to the data (Barker 1972) as $E_\mu^{(\lambda)}$ and $\gamma_{\mu c}^{(\lambda)}$, with $\mu = a, b$ and $c = p, n$. Values of $E_a^{(a)}$ and $E_b^{(b)}$ are shown by the dashed curves in Fig. 3. This procedure is not quite consistent, since the values of $\gamma_{\lambda c}^{(\lambda)}$ do not satisfy exactly equations like (29), but they satisfy them approximately with the original value of α . The dashed curves in Fig. 4 show the corresponding values of $(\gamma_{\lambda n}^{(\lambda)} / \gamma_{\lambda p}^{(\lambda)})^2$ and of Γ_λ^0 , E_T^0 and V_{01} obtained by using values of $E_\lambda^{(\lambda)}$ and $\gamma_{\lambda c}^{(\lambda)}$ in equations (31), (26) and (25).

C. PREDICTIONS OF CROSS SECTIONS FOR OTHER REACTIONS

Before comparing the derived values of V_{01} and E_1^0 with shell model values, we calculate the cross sections for various reactions involving the 3^+ levels, using the new values of the level parameters obtained from fitting the 5P_3 phase shift in $^7\text{Li} + p$ scattering, to check that consistent fits are possible and to see if the range of allowed values of α may be reduced. For some reactions, values of additional parameters are required, and these are obtained from shell model calculations or from isospin conservation.

1. $^7\text{Li}(p, n)$

The method used by Brown et al. (1973) to determine the imaginary part of the 5P_3 phase shift ensures that the predicted 3^+ contribution to the $^7\text{Li}(p, n)$ cross section will be consistent

with the measured values, that the values of E_b (or $E_b^{(b)}$) in Fig. 3 and of $\gamma_{bn}^2/\gamma_{bp}^2$ in Fig. 4 will agree with previous values, and that $\gamma_{an}^2/\gamma_{ap}^2$ will be small, supporting the view that only the upper 3^+ level contributes appreciably to the ${}^7\text{Li}(p,n)$ cross section.

2. ${}^7\text{Li}(p,\gamma)$

It is not immediately obvious why the ${}^7\text{Li}(p,\gamma_1)$ cross section should have only a single peak in the energy region of the 3^+ levels, as is observed, nor why this peak should be attributed to the lower level alone. One expects the 3^+ contribution to the cross section to be incoherent with the background, since the former requires channel spin 2 in the ${}^7\text{Li}+p$ channel, while the background should be mainly channel spin 1 (this assumes that the background is due to direct capture of s- and d-wave protons with E1 radiation, and that the ${}^7\text{Li}$ ground state and ${}^8\text{Be}$ first excited state are adequately described as the LS coupled states $[3]^{22}P_{3/2}$ and $[4]^{11}D_2$ respectively Barker 1966).

Since the angular distribution of the 3^+ contribution to the ${}^7\text{Li}(p,\gamma_1)$ cross section is unique for p-wave protons ($\propto 1 - \frac{9}{28} \cos^2 \theta_\gamma$), the shapes of the excitation functions measured at 0° and 90° may be compared directly with the calculated integrated cross section, which is given by

$$\sigma = \frac{7\pi}{2k_p^2} P_p E_\gamma^3 \left| \sum_{\lambda\mu} \gamma_{\lambda p} \gamma_{\mu\gamma} A_{\lambda\mu} \right|^2. \quad (32)$$

Here the M1 radiation width of the level μ is taken as

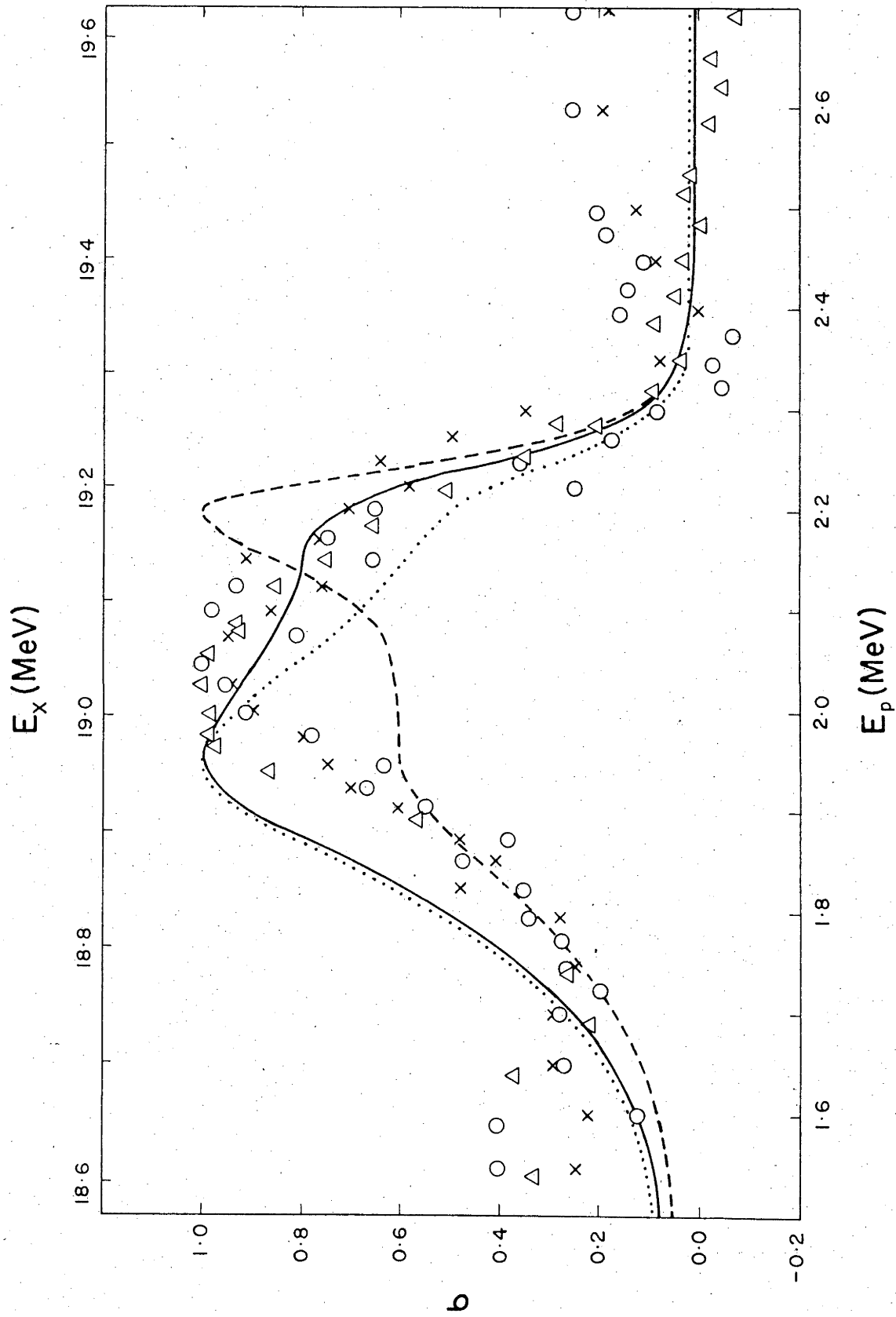
$\Gamma_{\mu\gamma} = E_\gamma^3 \gamma_{\mu\gamma}^2$, and $A_{\lambda\mu}$ is the same as in equation (27). From the two-state isospin mixing model, the reduced width amplitudes $\gamma_{\mu\gamma}$ are given by

$$\gamma_{a\gamma} = \alpha \gamma_{o\gamma} + \beta \gamma_{1\gamma} \quad , \quad \gamma_{b\gamma} = \beta \gamma_{o\gamma} - \alpha \gamma_{1\gamma} \quad , \quad (33)$$

where shell model values may be used for the $\gamma_{T\gamma}$. Since we do not consider absolute values of σ , we require only values of the ratio $\gamma_{o\gamma}/\gamma_{1\gamma}$, for which shell model calculations give 0.057 (Cohen and Kurath 1965), 0.059 (Barker 1966) and 0.058 (Kumar 1974).

Fig. 5 shows calculated values of σ , normalized to unity at the highest point, for $\gamma_{o\gamma}/\gamma_{1\gamma} = 0.058$ and for three values of α , the optimum value of 0.53 and values of 0.4 and 0.7. Even though both levels are contributing appreciably, the cross-section does not show two distinct peaks, owing to the constructive interference in the region between the two levels (since $\gamma_{ap}/\gamma_{bp} > 0$ and $\gamma_{a\gamma}/\gamma_{b\gamma} < 0$). The experimental points in Fig. 5 are the 0° and 90° excitation functions of Newson et al. (1957) and the 90° excitation function of Riech (1963), after subtraction of linear backgrounds chosen to make the resonant contribution resemble the calculated curves for E_p near 1.7 and 2.4 MeV and normalization to make the resonant contribution approximately unity at the peak. From the scatter of points in the region of the tails, it is clear that there are problems with this procedure; in fact Newson et al. comment on an interference dip at $E_p \approx 2.35$ MeV. However, the ratio of the normalization

Fig. 5 Normalized contribution to ${}^7\text{Li}(p,\gamma_1)$ cross-section due to 3^+ levels of ${}^8\text{Be}$ as a function of proton energy. The experimental points are the 0° and 90° excitation functions of Newson et al. (1957) and the 90° excitation function of Riech (1963), with backgrounds subtracted (denoted by $\circ$, $\times$ and Δ respectively). The curves are calculated using parameter values obtained from fits to the ${}^7\text{Li}+p$ ${}^5\text{P}_3$ phase shift and shell model values of the γ -ray reduced width amplitudes, for three different values of α (dotted curve, $\alpha = 0.4$; solid curve $\alpha = 0.53$; dashed curve, $\alpha = 0.7$).



factors required for the 0° and 90° excitation functions of Newson et al. is 1.39, which agrees with the ratio of $28/19 = 1.47$ expected for a pure 3^+ contribution. No allowance for experimental energy resolution is necessary since target thicknesses of 5 keV (Newson et al. 1957) and 20 keV (Riech 1963) were used.

It is seen that there is qualitative agreement between the calculated and experimental results in Fig. 5. The main discrepancy, independent of the value of α , is that the calculated cross-section is too large on the low-energy side of the peak. Changing the value of $\gamma_{0\gamma}/\gamma_{1\gamma}$ does not significantly affect this; in fact changes of ± 0.03 in $\gamma_{0\gamma}/\gamma_{1\gamma}$ can be almost exactly compensated by simultaneous changes of ∓ 0.02 in α . The shape of the peak at higher energies probably limits α to between about 0.4 and 0.65.

3. $^{10}\text{B}(d,\alpha)^8\text{Be}$

For reactions of this type, in which ^8Be appears as a product nucleus, the dependence of the cross-section on ^8Be excitation energy should be given by a formula similar to (32), but with the reduced width amplitudes for the γ channel replaced by feeding amplitudes, which are dependent on the particular reaction (Barker 1967). More precisely, the contribution of the 3^+ levels to the cross-section is taken to be

$$\sigma_x \propto \sum_c P_c \left| \sum_{\lambda\mu} \gamma_{\lambda c} g_{\mu x} A_{\lambda\mu} \right|^2, \quad (34)$$

where x specifies the producing reaction and the sum is over both p and n channels, since the decay of the ^8Be is not

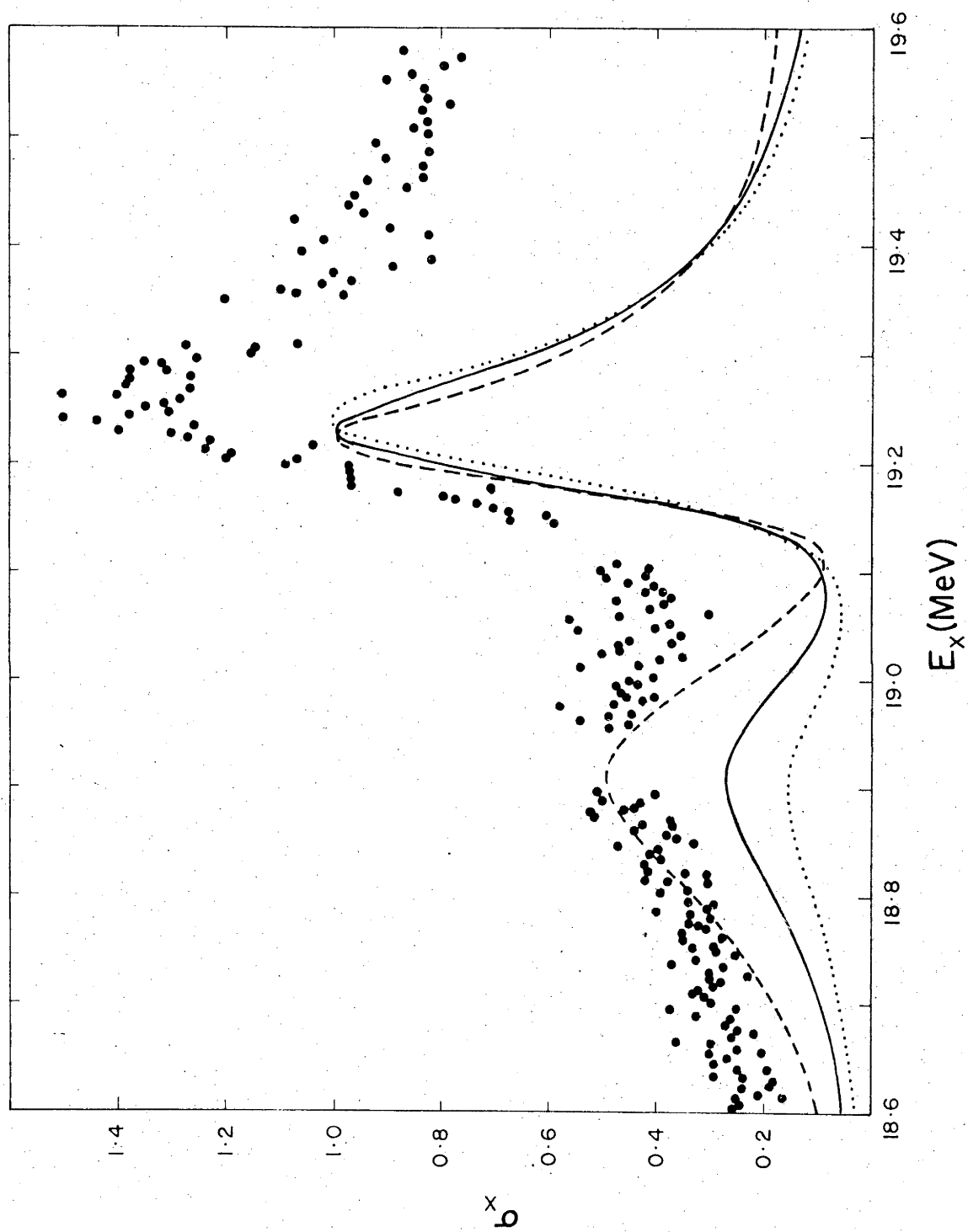
observed. A possible weak dependence on the energy of the emitted particle (the α -particle) has been omitted. The feeding amplitudes $g_{\mu x}$ are given by

$$g_{ax} = \alpha g_{ox} + \beta g_{1x} \quad , \quad g_{bx} = \beta g_{ox} - \alpha g_{1x} \quad (35)$$

in terms of the feeding amplitudes g_{Tx} for the pure T states.

For the $^{10}\text{B}(d,\alpha)^8\text{Be}$ reaction, isospin conservation requires that only the $T=0$ parts of the ^8Be states are fed, so that $g_{1x} = 0$. The calculated cross-section is shown by the curves in Fig. 6 for the same α values as in Fig. 5. In this case $g_{ax}/g_{bx} = \alpha/\beta > 0$, so that there is destructive interference in the region between the levels, which therefore appear as two distinct peaks. The experimental points are from Callender and Browne (1970). No background has been subtracted, because the 3^+ contribution could be coherent with background contributions coming from other levels of ^8Be . Comparable normalization has been used for the calculated and experimental values. The energy resolution was about 14 keV. It seems reasonable to interpret the peak observed at 18.9 MeV as being due to the lower 3^+ level instead of attributing it, as did Callender and Browne, to the 2^- level of ^8Be known to exist at this energy. The 2^- level should not be populated if the $^{10}\text{B}(d,\alpha)$ reaction proceeds as a direct transition and the ^{10}B ground state belongs to the lowest shell model configuration. The size of the 18.9 MeV peak is consistent with $\alpha \approx 0.5$.

Fig. 6 Normalized contribution to $^{10}\text{B}(d,\alpha)^8\text{Be}$ cross-section due to 3^+ levels of ^8Be as a function of ^8Be excitation energy. The experimental points are from Callender and Browne (1970) and include a background contribution (with a constant value of 0.3 subtracted from the ordinate). The curves are calculated using parameter values obtained from fits to the $^7\text{Li}+p$ $^5\text{P}_3$ phase shift and from isospin conservation, for α values as in Fig. 5.



4. ${}^9\text{Be}(d,t){}^8\text{Be}$

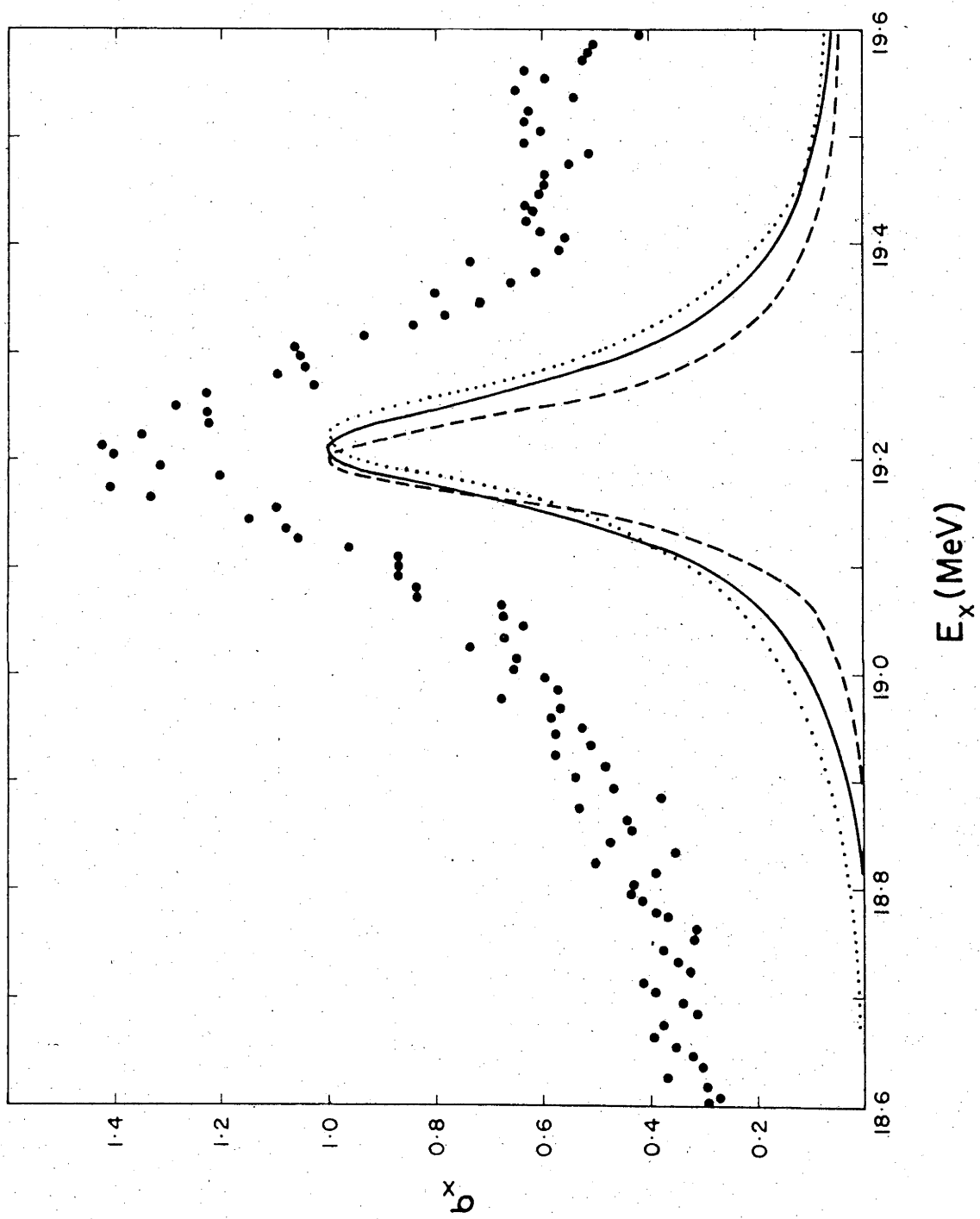
If the ${}^9\text{Be}(d,t)$ reaction proceeds by neutron pickup, then the feeding amplitude g_{Tx} is proportional to the spectroscopic amplitude (including the isospin Clebsch-Gordan coefficient) of ${}^9\text{Be}$ ground state for the p-wave neutron channel with ${}^8\text{Be}$ in its 3^+ state with isospin T . Shell model values of these spectroscopic amplitudes give $g_{ox}/g_{1x} = -0.88$ (Barker 1966), -0.78 (Cohen and Kurath 1967) and -0.95 (Kumar 1974).

Fig. 7 shows calculated values of σ_x for $g_{ox}/g_{1x} = -0.9$, for the same three values of α . For $\alpha \approx 0.53$ one has $g_{ax}/g_{bx} \approx -0.3$; this small negative value implies constructive interference between the levels and a single peak in the region of the upper level. The experimental points in Fig. 7 are from Oothoudt and Garvey (1977), without background subtraction because of possible coherence, and with suitable normalization. Again there is qualitative agreement, but here the width of the calculated peak, (≈ 150 keV) is less than the measured width (≈ 200 keV), the difference being too great to attribute to the experimental energy resolution ($\lesssim 40$ keV). Reasonable changes in the value of g_{ox}/g_{1x} have little effect on the shape of the cross section. The large width observed for the peak favours smaller magnitudes of g_{ox}/g_{1x} and also the smaller allowed values of α .

5. ${}^9\text{Be}(p,d){}^8\text{Be}$, ${}^9\text{Be}({}^3\text{He},\alpha){}^8\text{Be}$

If these reactions also proceed by neutron pickup, then the formulae and value of g_{ox}/g_{1x} are the same as for ${}^9\text{Be}(d,t)$.

Fig. 7 Normalized contribution to ${}^9\text{Be}(d,t){}^8\text{Be}$ cross-section due to 3^+ levels of ${}^8\text{Be}$ as a function of ${}^8\text{Be}$ excitation energy. The experimental points are from Oothoudt and Garvey (1977) and include a background contribution. The curves are calculated using parameter values obtained from fits to the ${}^7\text{Li}+p$ ${}^5\text{P}_3$ phase shift and shell model values of the feeding amplitudes, for α values as in Fig. 5.



Thus the calculated curves of Fig. 7 should also be valid for these reactions. In the ${}^9\text{Be}(p,d)$ reaction, with $E = 33.6$ MeV and an energy resolution in the deuteron spectrum of 100-130 keV, Kull (1967) observed a single peak corresponding to a level at 19.21 MeV with a width of 208 ± 30 keV. With $E_p = 185$ MeV, Sundberg and Källne (1969) observed a peak at 19.16 MeV with an intrinsic width of 500 keV but their energy resolution was about 350 keV. Ajzenberg-Selove et al. (1976) studied the ${}^9\text{Be}({}^3\text{He},\alpha){}^8\text{Be}$ reaction with a bombarding energy of 49.3 MeV and an energy resolution of about 50 keV, and observed a peak at 19.22 ± 0.03 MeV with a width of 265 ± 30 keV (Ajzenberg-Selove, private communication).

As for the ${}^9\text{Be}(d,t)$ reaction, these measured values of the width are much greater than the calculated value, although the peak positions agree. Fits to the phase shift, in which the level parameters were restricted so that they would give a larger width for the 19.2 MeV peak in these cross-sections, were acceptable only for widths less than about 160 keV.

6. ${}^7\text{Li}(d,n){}^8\text{Be}$

If the ${}^7\text{Li}(d,n){}^8\text{Be}$ reaction proceeds by stripping, then the feeding amplitudes g_{Tx} are proportional to the reduced width amplitudes γ_T used as parameters in the phase shift fit. Therefore additional parameters are not required in calculating the cross-section. Since the proton decay channel gives the main contribution there is destructive interference in the region between the levels, which produce a peak at about 18.93 MeV and a weaker peak at about 19.26 MeV. The former of these peaks would not be

resolvable from a peak due to the 2^- level of ^8Be at 18.9 MeV, so the origin of the peak reported by Kerr (1967) at 19.05 MeV is not apparent.

D. COMPARISON WITH VALUES FROM MODEL CALCULATIONS

In this section, we compare the parameter values obtained from fits to the $^5\text{P}_3$ phase shift, or quantities derived from them, with values obtained from model calculations.

1. Reduced Width Amplitudes

The reduced width amplitudes γ_T of the 3^+ states of ^8Be for the $A=7$ ground state channels may be written

$$\gamma_T = f_T^{1/2} [\theta_0^2(1p) \hbar^2/m_c a_c^2]^{1/2} . \quad (36)$$

We take values of the spectroscopic amplitudes $f_T^{1/2}$ from shell model calculations, and calculate the single-particle dimensionless reduced width

$$\theta_0^2(1p) = (a_c/2)u^2(a_c) / \int_0^{a_c} u^2(r)dr \quad (37)$$

using radial wavefunctions $u(r)$ in a Woods-Saxon potential (Barker 1978). With $a_c = 5.0$ fm, calculated values of γ_0, γ_1 (in $\text{MeV}^{1/2}$) are 0.46, 0.40 (Barker 1966), 0.42, 0.42 (Cohen and Kurath 1967) and 0.43, 0.40 (Kumar 1974). To avoid the uncertainty in the value of $\theta_0^2(1p)$, we may consider only values of γ_0/γ_1 ,

which are 1.16, 1.01 and 1.08 respectively. Comparison with the values of γ_0 and γ_1 in Fig. 3 favours the smaller allowed values of α .

2. Experimental Value of Isospin-Mixing Matrix Element and Energy of Pure $T=1$ State

The value of E_a (or $E_a^{(a)}$) from Fig. 3 is about 18.94 MeV, which is 130 keV below the accepted value for the lower 3^+ level as given by Ajzenberg-Selove (1979). This changed value of E_a leads to considerably different values of E_1^0 and V_{01} from those obtained or used previously, namely $E_1^0 = 19.07$ MeV and $V_{01} = -63$ keV (Barker 1966), and $E_1^0 = 19.09 \pm 0.03$ MeV and $V_{01} = -60 \pm 12$ keV (Barker 1978). Since our calculated values for the cross-sections and γ_T favoured values of α somewhat lower than the 0.53 obtained in the best fit to the phase shift, we take the acceptable range of α as 0.4 to 0.6; then from Fig. 4 the range of E_1^0 is 18.99 to 19.05 MeV and the range of V_{01} is -115 to -145 keV.

3. Shell Model Value of Isospin Mixing Matrix Element

For the two state mixing case the isospin mixing matrix element given in equation (10) can be written as

$$V_{01} = H_{01}^C + L_{01}^\lambda, \quad (38)$$

where H_{01}^C is the internal mixing matrix element which is calculated from equation (11) assuming that H^C is the two-body Coulomb interaction coupling the pairs of protons and Ψ_{Tk} is the shell

model state with harmonic oscillator single particle wavefunctions. The detailed assumptions of the calculation are described in chapter I. Taking the value of the harmonic oscillator length parameter as 1.65 fm (Barker 1978) the value of H_{01}^C obtained is -32 keV for the interaction of Barker (1966), -30 keV for the interaction of Cohen and Kurath (6-16 2BME, 1965) and -32 keV for the interaction of Kumar (1974).

L_{01}^λ in equation (38) gives the external contribution to the isospin mixing matrix element and is given by

$$\begin{aligned} L_{01}^\lambda &= \langle \Psi_0 | \mathcal{L}(S) - \mathcal{L}(\bar{S}_1) | \Psi_1 \rangle \\ &= \sum_c \langle \Psi_0 | c \rangle \frac{\hbar^2}{2m_c a_c} \delta(r_c - a_c) \{ \bar{S}_1(c) - \bar{S}_\lambda(c) \} \langle c | \Psi_1 \rangle . \end{aligned} \quad (39)$$

We consider the contributions from the nucleon channels only, with $c \equiv (\tilde{c}, m_t)$. Then

$$\langle c | \Psi_T \rangle = u_{T\tilde{c}}(r_c) f_{T\tilde{c}}^{\frac{1}{2}}(\tilde{T} \frac{1}{2} - m_t m_t | T_0) . \quad (40)$$

where $u_{T\tilde{c}}(r)/r$ is the normalized radial wavefunction

$$\int_0^{\tilde{a}_c} u_{T\tilde{c}}^2(r) dr = 1 , \quad (41)$$

and $f_{T\tilde{c}}^{\frac{1}{2}}$ is the spectroscopic amplitude, excluding the isospin Clebsch-Gordan coefficient which is written explicitly in equation (40). $\tilde{T}$ is the isospin of the residual nucleus in the channel $\tilde{c}$, which must be $\frac{1}{2}$ in the present case. Equation (39) then becomes

$$L_{01}^{\lambda} = \sum_{\tilde{c}} \frac{\hbar^2}{4m_{\tilde{c}}a_{\tilde{c}}} u_{0\tilde{c}}(a_{\tilde{c}})u_{1\tilde{c}}(a_{\tilde{c}}) \mathcal{J}_{0\tilde{c}}^{\frac{1}{2}} \mathcal{J}_{1\tilde{c}}^{\frac{1}{2}} \{S_{\lambda}(\tilde{c}, \frac{1}{2}) - S_{\lambda}(\tilde{c}, -\frac{1}{2})\} \quad (42)$$

In order to obtain numerical values of L_{01}^{λ} for given channel radii $a_{\tilde{c}}$, we take $u_{T\tilde{c}}(r_c)$ as wavefunctions at the channel energy $E_{T\tilde{c}} = \frac{1}{2}(E_{Tn} + E_{Tp})$ in a Woods-Saxon potential with the depth adjusted to make the logarithmic derivative equal to $\bar{S}_T(\tilde{c})$ at $r_c = a_{\tilde{c}}$. The channel radius $a_{\tilde{c}}$ must be chosen so that there is no polarizing interaction for $r_c > a_{\tilde{c}}$; then the external radial wavefunctions are Coulomb functions and $S_{\lambda}(\tilde{c}, m_t)$ are the shift factors of R-matrix theory (Lane and Thomas 1958). The $\mathcal{J}_{T\tilde{c}}^{\frac{1}{2}}$ are obtained from shell model calculations. For the calculation of L_{01}^{λ} we have included the contributions from the channels $\tilde{c}$ corresponding to the levels of ${}^7\text{Li}$ and ${}^7\text{Be}$ with $J^{\pi} = \frac{3}{2}^{-}, \frac{1}{2}^{-}, \frac{7}{2}^{-}, \frac{5}{2}^{-}, \frac{5}{2}^{-*}, \frac{7}{2}^{-*}, \frac{3}{2}^{-*}$ where the $*$ denotes higher levels with the same J^{π} . Thus we can write $\tilde{c} = \tilde{J}s$, where $\tilde{J}$ is the spin of the $\tilde{T} = \frac{1}{2}$, $A = 7$ level and s is the channel spin. We have assumed all nucleons are p wave. As the excitation energy increases, both $u_{T\tilde{c}}(a_{\tilde{c}})$ and $\{S_{\lambda}(\tilde{c}, \frac{1}{2}) - S_{\lambda}(\tilde{c}, -\frac{1}{2})\}$ decrease and $\mathcal{J}_{T\tilde{c}}^{\frac{1}{2}}$ also tends to decrease. This means that the corresponding L_{01}^{λ} tends to decrease so that contributions from higher unidentified levels become negligible. As mentioned earlier the levels (a,b) that we are considering are nearby and so we can assume them to be degenerate to the extent that $u_{0\tilde{c}} = u_{1\tilde{c}}$ and

$$S_a(\tilde{c}, \frac{1}{2}) - S_a(\tilde{c}, -\frac{1}{2}) = S_b(\tilde{c}, \frac{1}{2}) - S_b(\tilde{c}, -\frac{1}{2}) \quad (43)$$

Then we have $L_{01}^a = L_{01}^b = L_{01}$ with Ψ_a and Ψ_b orthogonal as described by equation (23). The external contribution is then

$$L_{01} = \sum_{\tilde{c}} L_{01}(\tilde{c}) \quad , \quad (44)$$

with contributions from different channel $\tilde{c}$, for different values of channel radii $a_{\tilde{c}}$ (assumed to be same for all channels). The wavefunctions $u_{T\tilde{c}}(r)$ are taken as lp wavefunctions in a Woods-Saxon potential with the standard parameter values $r_0 = 1.25$ fm, $a = 0.65$ fm and a uniform charge distribution with $r_{oc} = 1.25$ fm. Table 1 shows contributions to L_{01} , for 3^+ states of ${}^8\text{Be}$ for various values of the channel radii for the interaction of Barker (1966).

Table 1.

Contributions to L_{01} for 3^+ states of ${}^8\text{Be}$. The calculations use the shell model wavefunctions of Barker (1966), and are for various values of the channel radius $a_{\tilde{c}}$

$a_{\tilde{c}}$ (fm)	$\tilde{J}$	$\sum_s f_{0Js}^{1/2\sim} f_{1Js}^{1/2\sim}$	$\frac{\hbar^2}{4m_{\tilde{c}}a_{\tilde{c}}} u_{\tilde{c}}^2(a_{\tilde{c}})$ (keV)	$S(\tilde{c}, 1/2) - S(\tilde{c}, -1/2)$	$L_{01}(\tilde{c})$ (keV)
4	3/2	0.349	1104	-0.2082	-80.3
	7/2	0.741	541	-0.1253	-50.2
	5/2	0.0499	430	-0.1178	- 2.5
	5/2*	-0.724	406	-0.0678	19.9
	7/2*	-0.313	338	-0.0484	5.1
	3/2*	-0.0631	317	-0.0453	0.9
	Total				-107.1
5	3/2		513	-0.2716	-48.6
	7/2		153	-0.2006	-22.7
	5/2		107	-0.1840	- 1.0
	5/2*		97	-0.1178	8.3
	7/2*		71	-0.0895	2.0
	3/2*		65	-0.0848	0.3
	Total				-61.7
6	3/2		282	-0.3076	-30.3
	7/2		48	-0.2796	- 9.9
	5/2		29	-0.2527	- 0.4
	5/2*		26	-0.1702	3.2
	7/2*		16	-0.1324	0.7
	3/2*		14	-0.1259	0.1
	Total				-36.6

The calculated values of H_{01}^C and L_{01} and the resultant values of V_{01} are shown in Table 2 for various interactions (Barker (1966), Cohen and Kurath (1965), Kumar (1974)).

Table 2.

Values of isospin mixing matrix elements for 3^+ states

a_c (fm)	H_{01}^C (keV)	L_{01} (keV)	V_{01} (keV)
4	B -32.0	-107.1	-139.1
	K -31.7	- 97.2	-128.9
	C -30.0	-101.7	-131.7
5	B -32.0	- 61.7	- 93.7
	K -31.7	- 57.2	- 88.9
	C -30.0	- 59.3	- 89.3
6	B -32.0	- 36.6	- 68.6
	K -31.7	- 34.5	- 66.2
	C -30.0	- 35.5	- 65.5

B interaction of Barker (1966)

K interaction of Kumar (1974)

C interaction of Cohen and Kurath (1965)

It may be noted here that the previous value of -32 keV for V_{01} (Barker 1966) is only the internal contribution and the external contribution accounts for the larger part of V_{01} . Fig. 8a shows the comparison of the calculated and experimental values of V_{01} and their variation with channel radius. The experimental value obtained from fitted data discussed in the previous section is denoted by the hatching; the calculated values are based on three different shell model interactions that are

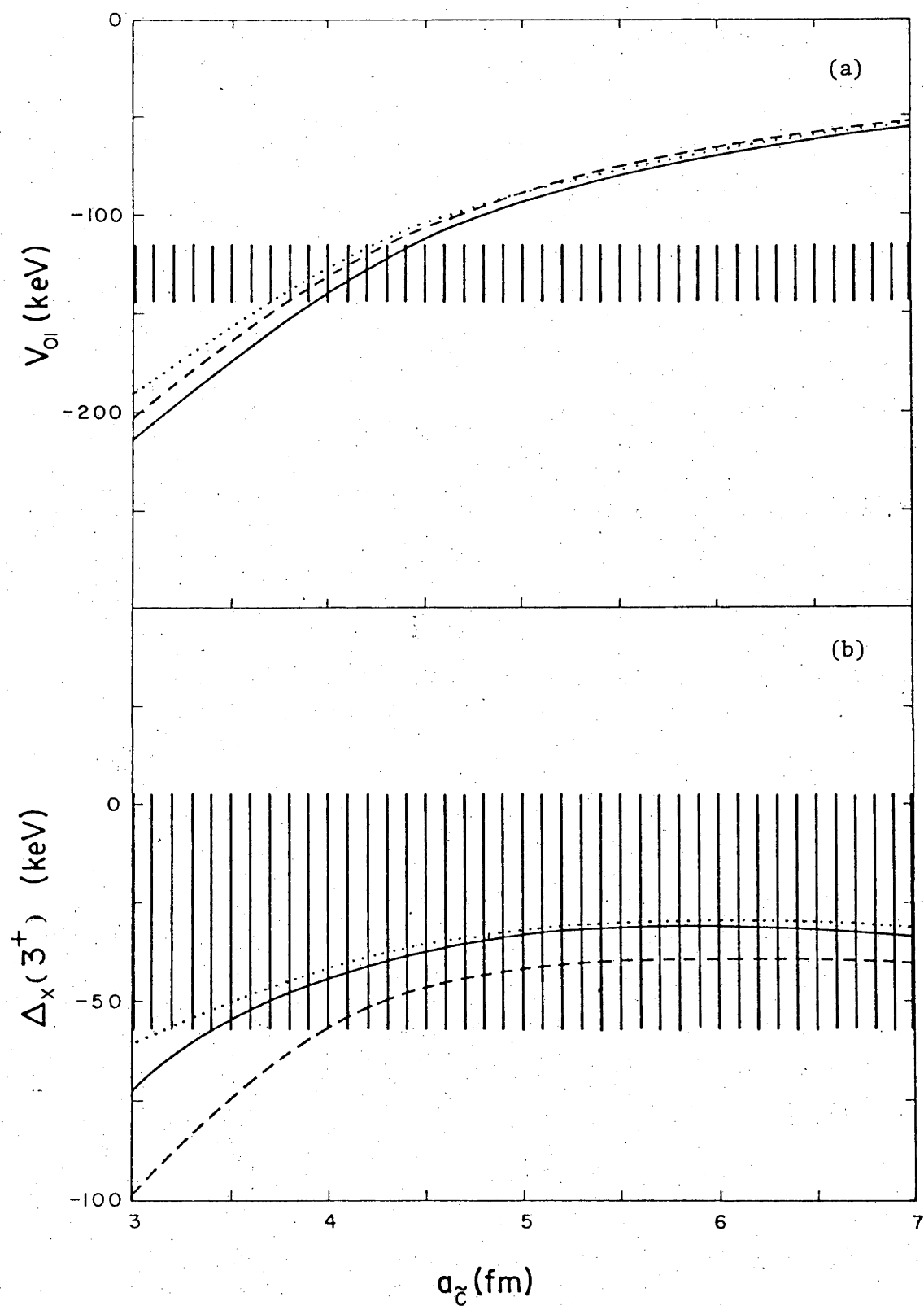
discussed in this section. The agreement between the experimental and calculated values of V_{01} is obtained for a channel radius of about 4 fm. This value of channel radius is a reasonable value when compared with the conventional value of the channel radius $1.45(7^{1/3} + 1^{1/3}) \approx 4.22$ fm and the channel radius required for other levels of ${}^8\text{Be}$. For example for the 2^+ states of ${}^8\text{Be}$ at 16.6 and 16.9 MeV, the experimental value of the isospin mixing matrix element coupling the $T=0$ and $T=1$ states is about -150 keV (Barker 1966, Oothoudt and Garvey 1977). The calculated values agree with this for a channel radius of about 5 fm (Barker 1978). Thus the present analysis of the 3^+ states of ${}^8\text{Be}$ shows an improvement over the earlier analysis (Barker 1978), which required a channel radius ≥ 6 fm.

4. Relative Energies of $T=1$ States in ${}^8\text{Be}$ and ${}^8\text{Li}$ and Isospin Distortion.

Since the energy of the 3^+ , $T=1$ state, in ${}^8\text{Li}$ is well known it is interesting to compare the energy difference of the 3^+ , $T=1$ states, in ${}^8\text{Li}$ and ${}^8\text{Be}$. For this purpose we consider the relative energies of 3^+ and 2^+ states in ${}^8\text{Be}$ and ${}^8\text{Li}$ and define a quantity $\Delta_x(3^+)$, which is the difference in the separation energies between 3^+ and 2^+ , $T=1$, states in ${}^8\text{Be}$ and ${}^8\text{Li}$. This quantity $\Delta_x(3^+)$ would be zero if there is no isospin distortion. From equation (19) the observed energy of a level J^π of ${}^8\text{Be}$ or ${}^8\text{Li}$ is given by

$$E_{M_T}(J^\pi) = E_{11}^0(J^\pi) + H_{1111}^C(J^\pi, M_T) + L_{1111}(J^\pi, M_T), \quad (45)$$

Fig. 8 Comparison of theoretical and experimental results for the variation with channel radius $a_{\tilde{c}}$ of the isospin mixing matrix element V_{01} for the 3^+ levels of ${}^8\text{Be}$, and of the excitation energy difference $\Delta_x(3^+)$ for the 3^+ levels of ${}^8\text{Li}$ and ${}^8\text{Be}$. The experimental results with estimated uncertainties as obtained in the present work are denoted by the hatching. The calculated values are for three different interactions (solid curves, Barker 1966; dashed curves, Cohen and Kurath 1965; dotted curves, Kumar 1974).



where we have used 11 for $T^P k^P$. $M_T = 0$ for ${}^8\text{Be}$ and 1 for ${}^8\text{Li}$. Since here we consider states with $T^P = 1$ only and also one such state $k^P = 1$ is considered in each nucleus we can drop the subscript 11, then

$$E_{M_T}(J^\pi) = E^O(J^\pi) + H^C(J^\pi, M_T) + L(J^\pi, M_T), \quad (46)$$

where

$$H^C(J^\pi, M_T) = \langle \Psi(J^\pi, M_T) | H^C | \Psi(J^\pi, M_T) \rangle, \quad (47)$$

and

$$L(J^\pi, M_T) = - \sum_{\tilde{c}} \frac{\hbar^2}{2m_{\tilde{c}} a_{\tilde{c}}} u_{1\tilde{c}}^2(J^\pi, a_{\tilde{c}}) \mathcal{J}_{1\tilde{c}}(J^\pi) \{S_{M_T}(J^\pi, \tilde{c}) - \bar{S}(J^\pi, \tilde{c})\}. \quad (48)$$

Equation (48) has been obtained in a similar manner to equation (42). The symbols appearing in equation (48) have the same significance as those appearing in equation (42) (except that here in (48) we have included J^π for the identification of the levels considered). Contributions to $L(J^\pi, M_T)$ come from the $\tilde{T} = \frac{1}{2}$ and $\tilde{T} = \frac{3}{2}$ channels. For ${}^8\text{Be}$, both ${}^7\text{Li} + p$ and ${}^7\text{Be} + n$ channels contribute and $S_0(J^\pi, \tilde{c})$ is the average shift factor for the proton and neutron channels. For ${}^8\text{Li}$ the $\tilde{T} = \frac{1}{2}$ channels are purely ${}^7\text{Li} + n$, while $\tilde{T} = \frac{3}{2}$ channels include both ${}^7\text{He} + p$ and ${}^7\text{Li} + n$, and $S_1(J^\pi, \tilde{c})$ is a weighted average of the shift

factors for the proton and neutron channels. In calculating $L(J^\pi, M_T)$ we have also included the contributions from unidentified channels of $A=7$. The unidentified channels are simulated by a fictitious level (labelled J in Table 3) with spectroscopic factors that exhaust the sum rule

$$\sum_{\tilde{c}} f_{T'c}^{\frac{1}{2}} f_{T\tilde{c}}^{\frac{1}{2}} = 4\delta_{T'T} \quad (49)$$

and situated 14 MeV above the lowest $\tilde{T}=\frac{1}{2}$ level (approximately the weighted mean energy of such levels from shell model calculations). The quantity $\Delta_x(3^+)$ is defined by

$$\begin{aligned} \Delta_x(3^+) &= [\{E_o(3^+) - E_o(2^+)\} - \{E_1(3^+) - E_1(2^+)\}] \\ &= [\{\Delta H^c(3^+) + \Delta L(3^+)\} - \{\Delta H^c(2^+) + \Delta L(2^+)\}] , \quad (50) \end{aligned}$$

where

$$\Delta H^c(J^\pi) = H^c(J^\pi, 0) - H^c(J^\pi, 1) , \quad (51)$$

$$\Delta L(J^\pi) = L(J^\pi, 0) - L(J^\pi, 1) \quad (52)$$

The numerical values of $H^c(J^\pi, M_T)$ and $L(J^\pi, M_T)$ needed to calculate $\Delta_x(3^+)$ are obtained assuming the same interactions and the same assumptions as those for H_{o1} and L_{o1} respectively (except that for $H^c(J, M_T)$ the tensor component of the Coulomb interaction contributes and is included). Table 3 shows contributions to $\Delta L(J^\pi)$ calculated using the interaction of Barker (1966) for a channel radius of 5 fm.

Table 3.

Contributions to $\Delta L(J^\pi)$. Calculations use the shell model interaction of Barker (1966) and are for $a_c = 5.0$ fm

J^π	$\tilde{J}$	$\sum_s \mathcal{P}_{1Js}(\tilde{J}^\pi)$	$\Delta L(J^\pi, \tilde{c})$ (keV)
2^+	3/2	1.016	-73
	1/2	0.231	-14
	7/2	0.224	- 4
	5/2	0.032	0
	5/2*	0.703	- 8
	7/2*	0.256	- 2
	3/2*	0.060	0
	3/2 (T = 3/2)	1.059	- 6
	J	0.419	- 1
Total			-108
3^+	3/2	0.300	-35
	7/2	1.143	-37
	5/2	0.155	- 2
	5/2*	0.646	-12
	7/2*	0.241	- 3
	3/2*	0.087	- 1
	3/2 (T = 3/2)	1.007	- 9
	J	0.421	- 2
Total			-101

Values of $\Delta_x(3^+)$ (keV) for various channel radii are given in Table 4. Figure 8b shows the calculated and experimental values of $\Delta_x(3^+)$. The experimental value is obtained from the fitted data of section 2 and is shown with the estimated uncertainty denoted by hatching. The calculated values are for three different shell model interactions and are shown by three different curves as functions of channel radius. The agreement between the calculated and experimental values is obtained for a value of channel radius of $a_c \gtrsim 4$ fm ,

Table 4.

Contributions to $\Delta_x(3^+)$ (keV) for $a_c = 5$ fm

a_c (fm)	$\Delta H^C(3^+)$ (keV)	$\Delta L(3^+)$ (keV)	$\Delta H^C(2^+)$ (keV)	$\Delta L(2^+)$ (keV)	$\Delta_x(3^+)$ (keV)
4	B 1546	-225	1586	-221	-44
	K 1546	-227	1584	-223	-42
	C 1544	-230	1588	-218	-56
5	B 1546	-101	1586	-108	-33
	K 1546	-102	1584	-110	-30
	C 1544	-104	1588	-107	-41
6	B 1546	- 48	1586	- 56	-32
	K 1546	- 49	1584	- 56	-31
	C 1544	- 50	1588	- 55	-39

B interaction of Barker (1966).

K interaction of Kumar (1974).

C interaction of Cohen and Kurath (1965).

which is consistent with that obtained from the analysis of the isospin mixing matrix element V_{01} for the 3^+ states.

This also shows an improvement over the previous analysis where the agreement between the calculated and experimental values of $\Delta_x(3^+)$ was not obtained (Barker 1978). The previous discrepancy in $\Delta_x(3^+)$ has now been removed without the requirement of a charge dependent interaction other than the Coulomb interaction.

E. DISCUSSION

The method of obtaining the level parameters from the two level fit assuming a two-state isospin mixing seems to be satisfactory. The values of the level parameters obtained from the fits are in good agreement with those obtained from the shell model calculations. The value of the mixing coefficient α has been obtained by a different method than that previously employed (Oothoudt and Garvey 1977). The energy E_a of the lower level is appreciably less than the previously accepted value. These new values of α and E_a give experimental values of V_{01} and $\Delta_x(3^+)$ which agree with the calculated values for reasonable values of the channel radius.

The discrepancy between the calculated and experimental value of V_{01} has been of interest to many authors. The problem was first discussed by Barker (1966), where the calculated value of -32 keV, obtained by using shell model wavefunctions, was much smaller than the experimental value of -63 keV. Anderson and Goldhammer (1971) and Anderson et al. (1972) used correlated wavefunctions to calculate the isospin mixing matrix element for the 3^+ states in ^8Be and obtained good agreement with the experimental value. Later McCarthy and Walker (1974) repeated the calculation of Anderson et al. using Hamada Johnston potential and including the Pauli corrections which Anderson et al. omitted. However the result they obtained was about 10% smaller than that obtained by using the oscillator wavefunctions by Barker (1966). A further calculation was done by Goldhammer (1975) using the Sussex harmonic oscillator two-body matrix elements and agreement was obtained with the experimental value. Then Oothoudt and Garvey

(1977) suggested that the discrepancy between the calculated and experimental values of the isospin mixing matrix elements could be due to an uncertainty in the level parameters (γ_n^2, γ_p^2) which gives the value of the isospin mixing coefficient α . From the ratio of the cross-section for the reactions ${}^9\text{Be}(d,t){}^8\text{Be}$ and ${}^9\text{Be}(d,{}^3\text{He}){}^8\text{Be}$, which is connected to the isospin Clebsch-Gordon coefficients of the states, they obtained a range of values of α which is consistent with the value obtained by Barker (1966). A possible solution of the problem seemed to be the idea of Dalton and Robson (1966) who suggested that the mixing matrix element consists of an internal contribution and an external contribution. It was also suggested by Dalton and Robson that the large experimental value of the isospin mixing matrix element is mainly due to the external contribution; and then it became apparent that the mixing matrix element considered so far was only the internal mixing matrix element. Barker (1978) took up this idea and extended the method of Dalton and Robson (1966) to include the effects of many channel contributions to the external mixing matrix element. Applying this method Barker calculated the mixing matrix element which suggested a channel radius of more than 6 fm to account for the experimental value. It may be mentioned here that the conventional channel radius of nucleon channels in ${}^8\text{Be}$ is 4.22 fm and a similar analysis applied to the 1^+ and 2^+ states of ${}^8\text{Be}$ shows agreement between the calculated and the experimental values of the isospin mixing matrix elements for channel radii smaller than 6 fm.

Though some authors (Lind et al. 1977) suggested that such discrepancy between the calculated and experimental values of

isospin mixing matrix elements could be attributed to some nuclear charge dependent interaction, we have been able to give a reasonable account of the problem without assuming any charge dependent interaction other than the Coulomb interaction. Moreover, the parameters for the 3^+ doublet obtained from the two level fit to the ${}^7\text{Li} + p$, ${}^5\text{P}_3$ phase shift seems to be more accurate than previous values. This has the effect of allowing qualitative agreement to be obtained with the measured cross-sections for the ${}^7\text{Li}(p,\gamma_1)$, ${}^{10}\text{B}(d,\alpha){}^8\text{Be}$ and ${}^9\text{Be}(d,t){}^8\text{Be}$ reactions, involving re-interpretation of one of the peaks seen in the (d,α) reaction. It also improves the agreement with calculated values of the excitation energy difference for the 3^+ , $T=1$ levels of ${}^8\text{Li}$ and ${}^8\text{Be}$.

The fits and predictions are by no means perfect. The discrepancy in the ${}^7\text{Li}(p,\gamma)$ cross-section can be reduced by using a smaller channel radius, say $a_c = 4.0$ fm, which increases the energy of the lower level, but the effect is too small to remove the discrepancy altogether. This change in the channel radius does not produce significant changes in the fits to the phase shift, the cross-sections for the other reactions, and the values of E_1^0 and V_{01} . Part of the discrepancies may be due to the data, and part may be attributed to the use of the two-level approximation and of the two-state isospin mixing model for describing the 3^+ doublet, thus ignoring effects of other 3^+ levels. If the restrictions of this model as embodied in equation (29) were not imposed, then one could probably improve the agreement in most quantities by reducing the width of the lower level and increasing the width of the upper level. Without this model, however, we could not have calculated the various reaction cross-sections since the γ -ray reduced width amplitudes and the feeding amplitudes would not be obtainable in a simple way.

CHAPTER IV

ISOSPIN MIXING IN THE 7.12 MEV STATE OF ^{16}O

A. THE 7.12 MEV STATE OF ^{16}O

The 1^- state of ^{16}O at 7.12 MeV is mainly a $T=0$ state of the lowest shell model configuration, with one hole in the $1p$ shell and one particle in the $2s$ or $1d$ shell. But the measured lifetime (Ajzenberg-Selove 1977) and more recently the destructive interference in the experimental electro-excitation of the 7.12 Mev state (Miska et al. 1975) show that it has some $T=1$ admixture. This admixture is assumed to arise from the five higher 1^- , $T=1$ states of the lowest configuration. We do not consider the 1^- , $T=1$ states of higher configurations since these do not have large radiation widths.

Elliott and Flowers (1957) considered the isospin admixtures to the 7.12 MeV state due to the Coulomb interaction, calculating the Coulomb mixing matrix elements of the five 1^- , $T=1$ states with the lowest 1^- , $T=0$ state. In these mixing matrix elements the particle hole contribution was calculated using harmonic oscillator single particle wavefunctions and the rest was treated empirically by lumping together the Coulomb closed shell parts and the Thomas shift effects. They obtained the mixing coefficients using first order perturbation theory assuming that the mixing

coefficients are small. They also obtained the electric dipole transition width $\Gamma = 0.1 \text{ eV}$ for $1^-(7.12 \text{ MeV}) \rightarrow 0^+$ (ground state) which gives a mean lifetime τ_m of 7 fs, which is in good agreement with the then available experimental value of $3 \text{ fs} < \tau_m < 13 \text{ fs}$ and the present value of $12 \pm 1.2 \text{ fs}$ (Ajzenberg-Selove 1977). Glockner and Lawson (1975) treated the problem with a slightly different interaction than Elliott and Flowers (1957) and obtained $\tau_m = 15 \text{ fs}$. Arima et al. (1975) studied the influence of different radial dependence of the single particle wavefunctions on the isospin mixing of the 7.12 MeV state. They found that the use of harmonic oscillator wavefunctions gave the correct sign of the relative phase of the amplitudes of $T=0$ and $T=1$ parts of the state, whereas the use of Woods-Saxon wavefunctions gave an opposite sign to the sign determined by electron scattering experiment. It has been suggested by Barker (1976) that better agreement with Woods-Saxon wavefunctions could be obtained if a different criterion were adopted for choosing the potential depth. Heil and Stock (1976) investigated the problem using the continuum shell model and stated that any reliable explanation of the properties of the 7.12 MeV state in ^{16}O has to take the continuum into account properly.

One way to take proper account of the continuum is to use the Bloch operator technique (Lane and Robson 1966) which has been applied to isospin mixing of pairs of states (Barker 1978). This technique accounts for the isospin mixing, considering both internal and external mixing, where the internal mixing arises from the Coulomb interaction of all the pairs of protons and the external mixing

arises from contributions from nearby $^{15}\text{O} + n$ and $^{15}\text{N} + p$ channels. We have extended this method (chapter II) to account for the mixing which involves more than two states and applied it to the mixing of $T=1$ states in the 7.12 MeV state of ^{16}O .

B. EXPERIMENTAL DATA

The measured value of the lifetime of the 7.12 MeV state gives information about the intensity of the $T=1$ part of the state and the electron scattering about the values of both the $T=0$ and $T=1$ amplitudes. The experimental data were represented conveniently by Miska et al. (1975) in terms of parameters A_0 , A_1 and ϕ , which are the magnitudes of the coefficients of the amplitudes of the $T=0$ and $T=1$ parts of the form factor and their relative sign respectively. The values obtained by Miska et al. by fitting the electron scattering and lifetime data are $A_0 = 1.97 \pm 0.04$, $A_1 = (1.05 \pm 0.05) \times 10^{-2}$ and $\phi = -1$.

C. ISOSPIN MIXING FORMULAE

For the 7.12 MeV state, the five $T=1$ states are far away (the nearest being at 13.1 MeV) and their effects can be treated as perturbations, so we write

$$\psi_a = \psi_{01} + \sum_{k=1}^5 \alpha_k \psi_{1k}, \quad (53)$$

which is a particular case of equation (2) where $\lambda = a$,

$A_{aok} = \delta_{1k}$, $A_{alk} = \alpha_k$ and the Ψ_{Tk} are the 1^- states of good isospin of $T=0$ and 1 , belonging to the lowest shell model configuration. The α_k are isospin mixing coefficients (accounting for the amount of $T=1$ admixture to the 7.12 MeV state) which are small enough so that first order perturbation theory can be applied to calculate them. Using equation (53) and applying first order perturbation to equation (9), we obtain a set of equations for α_k .

$$V_{o11k'}^a = \sum_{k=1}^5 \alpha_k (E_{1k}^0 - E_a) \langle \Psi_{1k} | \Psi_{1k'} \rangle, \quad k' = 1, 5. \quad (54)$$

Here E_a is taken as 7.12 MeV, and E_{1k}^0 ($k = 1, 5$) are the energies of the five $T=1$ states that we consider. Following equation (10), the mixing matrix element is written as

$$V_{o11k'}^a = H_{o11k'}^c + L_{o11k'}^a, \quad (55)$$

where the internal and external mixing matrix elements $H_{o11k'}^c$ and $L_{o11k'}^a$ can be written from equations (11) and (12) as

$$H_{o11k'}^c = \langle \Psi_{o1} | H^c | \Psi_{1k'} \rangle, \quad (56)$$

$$L_{o11k'}^a = \langle \Psi_{o1} | \mathcal{L}(S_a) - \mathcal{L}(\bar{S}_{1k'}) | \Psi_{1k'} \rangle. \quad (57)$$

Using steps similar to those described in chapter III for obtaining L_{o1} , equation (57) can be written

$$L_{011k'}^a = \frac{\hbar^2}{2m_c a_c} \sum_{\tilde{c}} u_{01\tilde{c}}(a_c) u_{1k\tilde{c}}(a_c) \mathcal{J}_{01\tilde{c}}^{\frac{1}{2}} \mathcal{J}_{1k'\tilde{c}}^{\frac{1}{2}}$$

$$[\frac{1}{2} \{ S_a(c, \frac{1}{2}) - S_a(c, -\frac{1}{2}) \}] , \quad (58)$$

A similar procedure gives, for $k \neq k'$,

$$\langle \Psi_{1k} | \Psi_{1k'} \rangle = \frac{-\hbar^2}{2m_c a_c} \frac{1}{(E_{1k}^0 - E_{1k'}^0)} \sum_{\tilde{c}} u_{1k\tilde{c}}(a_c) u_{1k'\tilde{c}}(a_c) \mathcal{J}_{1k\tilde{c}}^{\frac{1}{2}} \mathcal{J}_{1k'\tilde{c}}^{\frac{1}{2}} [S_{1k}(\tilde{c}) - S_{1k'}(\tilde{c})] , \quad (59)$$

while

$$\langle \Psi_{1k} | \Psi_{1k} \rangle = 1 . \quad (60)$$

D. CALCULATION OF ISOSPIN MIXING MATRIX ELEMENTS

The internal mixing matrix elements $H_{011k'}^c$ given by equation (56) are calculated using harmonic oscillator radial wavefunctions and standard shell model techniques applied for the Coulomb interaction. Details of the calculation are described in chapter II. Table 5 shows the values of the internal mixing matrix elements between the 7.12 MeV $T=0$ and five higher $T=1$ states for different interactions. The harmonic oscillator length parameter is taken to be $b = 1.833$ fm which yields a ground state radius of ^{16}O of 2.718 fm (Miska et al. 1975).

Table 5.

Values of internal mixing matrix elements for different interactions

Interaction	$H_{011k'}^C$ (keV)				
	$k'=1$	2	3	4	5
Elliott and Flowers	-64.3	19.6	-60.0	4.7	-8.7
Gillett and Vinh Mau	-87.7	20.5	-19.7	14.4	-8.3
Harms (Green potential)	-85.5	-15.7	32.6	0.8	0.6

For calculation of $L_{011k'}^a$ and $\langle \psi_{1k} | \psi_{1k'} \rangle$ given by equations (58) and (59) respectively it is convenient to use the channel labels such as $\tilde{c} = \tilde{J} \ell j$, where ℓ and j are orbital and total angular momentum of the nucleon in the channel. The labels $\tilde{J}$ include positive and negative parity levels of mass 15 nuclei, i.e. 1p -2h and 1h states. The 1h states are the $\frac{1}{2}^-$ ground states and the $\frac{3}{2}^-$ states at ~ 6 MeV excitation. The spectroscopic amplitudes $f_{Tk\tilde{c}}^{\frac{1}{2}}$ for these states are obtained from the same shell model wavefunctions which are used in calculating the internal mixing matrix elements $H_{011k'}^C$. Among the positive parity levels we consider only the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states at about 5 MeV excitation, since the dominant contribution comes from the $\frac{1}{2}^-$ ground state. For the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states we approximate them by assuming that their only $A = 14$ parent state is the lowest $J = 0$, $T = 1$ state. Details of this approximation and the evaluation of spectroscopic amplitudes are discussed in Barker (1978).

In the calculation of $u_{Tk\tilde{c}}(a_{\tilde{c}})$, the channel energies depend on both k and $\tilde{c}$. We have used the wavefunctions obtained for a Woods-Saxon potential with standard parameters (see Chapter III) and with depths adjusted to make the logarithmic derivative equal to $\bar{S}_{Tk}(\tilde{c})$ at $r_c = a_{\tilde{c}}$. Table 6 shows the various contributions to L_{011k}^a , for a channel radius of 4 fm. The spectroscopic amplitudes are calculated using the wavefunctions of Elliott and Flowers (1957). Table 7 shows values of V_{011k}^a , for different channel radii for the same interactions that were used in Table 5. To obtain the values of $\langle \psi_{1k} | \psi_{1k'} \rangle$ given by equation (59), the quantities involved which are similar to those in equation (58) are calculated in the same manner as described above, while the values of $(E_{1k}^0 - E_{1k'}^0)$ are taken from the calculated values of the energies of the $T=1$ states.

In order to solve the equations (54) for α_k we still require the values of $E_{1k}^0 - E_a$. We have taken $E_{11}^0 = 13.1$ MeV from experiment and the energy differences $E_{1k}^0 - E_{11}^0$ ($k = 2, 4$) from the calculated values, since the experimental energies of the higher $T=1$ states are not known accurately. The values of the α_k depend on the channel radius due to the dependence of L_{011k}^a and $\langle \psi_{1k} | \psi_{1k'} \rangle$ on the channel radius.

Table 6.

Contributions to $L_{011k'}^a$ for channel radii 4 fm. Calculations use the shell model wavefunctions of Elliott and Flowers (1957)

k'	$\tilde{J}$	ℓ	$\sum_j f_{01\tilde{c}}^{1/2} f_{1k'\tilde{c}}^{1/2}$	$\frac{\hbar^2}{4m_c a_c^2}$	$u_{01\tilde{c}}(a_c) u_{1k'\tilde{c}}(a_c)$ (keV)	$S_a(\tilde{c}, \frac{1}{2}) - S_a(\tilde{c}, -\frac{1}{2})$	$L_{011k'}^a(\tilde{c})$ (keV)
1	$\frac{1}{2}^-$	0	0.7509		1331	-0.2142	-214.14
	$\frac{1}{2}^-$	2	-0.0041		744	-0.1592	0.48
	$\frac{1}{2}^+$	1	0.3100		455	-0.1377	- 19.41
	$\frac{5}{2}^+$	1	0.0010		"	"	0.06
	$\frac{3}{2}^-$	0	-0.0918		914	-0.1333	11.19
	$\frac{3}{2}^-$	2	0.0235		514	-0.1099	- 1.33
	Total -223.15						
2	$\frac{1}{2}^-$	0	-0.0410		1487		13.06
	$\frac{1}{2}^-$	2	-0.2174		929		32.15
	$\frac{1}{2}^+$	1	-0.0187		546	"	1.40
	$\frac{5}{2}^+$	1	0.0043		"		- 0.32
	$\frac{3}{2}^-$	0	-0.0380		1043		5.28
	$\frac{3}{2}^-$	2	0.1190		585		- 7.65
	Total 43.92						
3	$\frac{1}{2}^-$	0	0.1111		1424		- 33.89
	$\frac{1}{2}^-$	2	0.0173		1072		- 2.95
	$\frac{1}{2}^+$	1	0.0659		687	"	- 6.23
	$\frac{5}{2}^+$	1	0.0040		"		- 0.38
	$\frac{3}{2}^-$	0	0.4740		1209		- 76.39
	$\frac{3}{2}^-$	2	0.0103		661		- 7.48
	Total -127.32						
4	$\frac{1}{2}^-$	0	0.0976		1342		- 28.05
	$\frac{1}{2}^-$	2	-0.0696		1162		12.87
	$\frac{1}{2}^+$	1	0.0466		788	"	- 5.05
	$\frac{5}{2}^+$	1	-0.0105		"		1.14
	$\frac{3}{2}^-$	0	0.1415		1269		- 23.94
	$\frac{3}{2}^-$	2	-0.2386		747		19.58
	Total - 23.45						
5	$\frac{1}{2}^-$	0	-0.0103		1217		2.68
	$\frac{1}{2}^-$	2	0.0796		1261		- 15.97
	$\frac{1}{2}^+$	1	-0.0080		880	"	0.97
	$\frac{5}{2}^+$	1	0.0016		"		- 0.19
	$\frac{3}{2}^-$	0	-0.0884		1245		- 14.67
	$\frac{3}{2}^-$	2	0.0953		854		8.94
	Total - 6.81						

Table 7.

Values of V_{ollk}^a (keV) for the different shell model interactions

a_c (fm)	Interaction	V_{ollk}^a (keV)				
		$k'=1$	2	3	4	5
4	Elliott & Flowers	-287.5	63.5	-187.3	-18.7	-15.5
	Gillet & Vinh Mau	-366.4	57.9	- 53.4	13.4	-20.2
	Harms (Green pot.)	-356.5	-54.1	54.7	-27.3	-15.4
5	Elliott & Flowers	-174.6	43.3	-126.8	-14.3	-10.4
	Gillet & Vinh Mau	-223.5	39.8	- 33.6	8.3	-14.2
	Harms (Green pot.)	-217.3	-36.7	36.7	-16.3	-11.9
6	Elliott & Flowers	-113.7	31.0	- 88.8	- 3.8	- 9.4
	Gillet & Vinh Mau	-148.0	29.6	- 25.0	11.4	-10.8
	Harms (Green pot.)	-144.0	-25.0	33.1	- 6.8	- 4.7

E. COMPARISON WITH EXPERIMENTAL VALUES

In order to obtain results that can be compared with those of Miska et al. (1975) we need expressions for A_0 and A_1 in terms of the isospin mixing coefficients α_k . Miska et al. fitted their data with a form factor for the transition between the 7.12 MeV state and the ground state of ^{16}O .

$$|F(q)| = |[A_0 q^3 b^3 / 4\sqrt{10} + \phi A_1 \sqrt{2} q b (1 - q^2 b^2 / 8)] \exp(-q^2 b^2 / 4)]|, \quad (61)$$

obtained by assuming harmonic oscillator single particle wavefunctions. Here q is the momentum transfer and b is the harmonic oscillator length parameter. To obtain a comparable expression for the form factor in terms of our parameters α_k we write

$$F(q) = (\Psi_0 | 0 | \Psi_a), \quad (62)$$

where Ψ_0 , the ground state of ^{16}O belongs to the closed shell configuration, Ψ_a is given by equation (53) and O is the electromagnetic interaction operator causing the transition, which is defined as

$$O = \sum_{i=1}^A \epsilon_i e^{iq \cdot r_i}, \quad (63)$$

with ϵ_i as the charge of nucleon in units of proton charge. We assume harmonic oscillator single particle wavefunctions as did Miska et al. (1975) and expand the basis states in the LS coupled representation

$$\Psi_{Tk} = \sum_{\ell SL} b_T^k(\ell SL) \Psi(T \ell SL), \quad (64)$$

where ℓ is the orbital angular momentum of the last particle assumed to be either 0 or 2 and S and L are the total spin and orbital angular momenta of the 1^- states. With Ψ_a given by equation (53), equation (62) becomes

$$F(q) = M_{01} + \sum_{k=1}^5 \alpha_k M_{1k}, \quad (65)$$

where

$$M_{01} = \frac{iq^3 b^3}{4\sqrt{3}} [b_o^1(001) + \frac{2}{\sqrt{5}} b_o^1(201)] \exp(-q^2 b^2/4), \quad (66)$$

since the state Ψ_{01} is assumed to be non-spurious, and

$$M_{1k} = -\frac{iqb}{\sqrt{3}} [(1-q^2 b^2/4) b_1^k(001) + \sqrt{5}(1-q^2 b^2/10) b_1^k(201)] \exp(-q^2 b^2/4). \quad (67)$$

By comparing these equations with equation (61) and dropping terms of higher order in α_k we obtain

$$A_0 = \sqrt{\frac{10}{3}} [b_0^1(001) + \frac{2}{\sqrt{5}} b_0^1(201)] , \quad (68)$$

$$\phi A_1 = \frac{1}{\sqrt{6}} \sum_{k=1}^5 \alpha_k [b_1^k(001) + \sqrt{5} b_1^k(201)] . \quad (69)$$

Thus the coefficients A_0 and ϕA_1 are expressed in terms of the α_k and $b_T^k(\ell SL)$, both of which depend on the shell model interaction assumed.

The value of A_0 given by equation (68), calculated using the $1p-1h$ wavefunctions of Elliott and Flowers (1957) with Ψ_{01} as the lowest non-spurious state for a Rosenfeld interaction with $V_c = 44.5$ MeV and with $b = 1.833$ fm as used by Miska et al. (1975), is $A_0 = 0.942$, which is much smaller than the experimental value of 1.97 ± 0.04 . Other interactions of Gillet and Vinh Mau (1964) and Harms (1968) give still smaller values of A_0 as shown in Table 8. Although Elliott and Flowers (1957) wavefunctions were obtained by elimination of the 1^- , $T=0$ spurious state, other authors did not eliminate spurious state, so we have used their published eigenenergies and eigenfunctions to eliminate the spurious state, and take Ψ_{01} as the lowest non-spurious state.

Table 8.

Values of A_0 for various interactions

Interaction	Elliott & Flowers	Gillet & Vinh Mau	Harms (Green Pot.)	Experimental ^{a)}
A_0	0.942	0.826	0.846	1.97 ± 0.04

a) Miska et al. (1975).

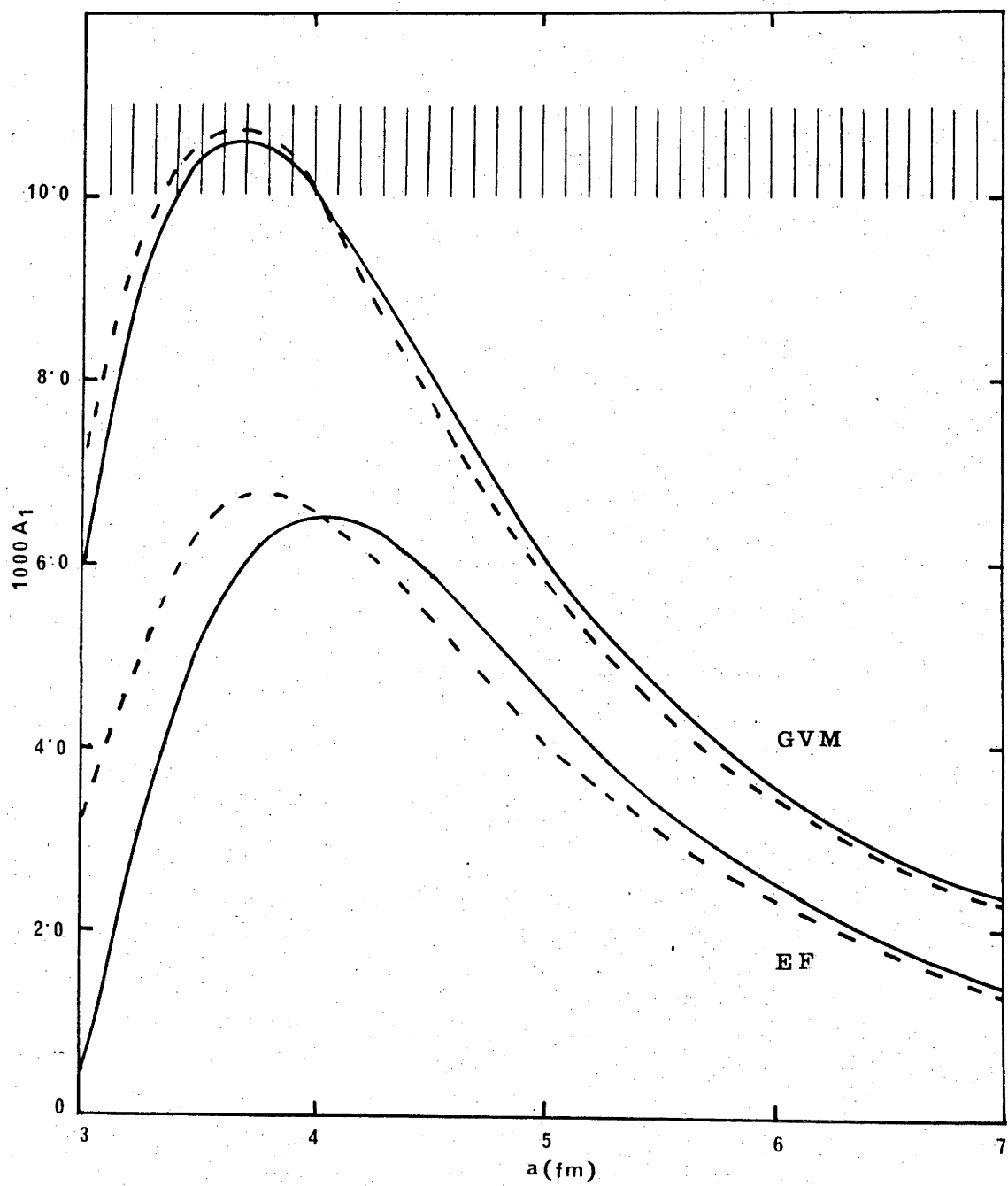
Values of ϕA_1 given by equation (69) are also calculated using the same interactions that were used to calculate A_0 . Since the α_k depend on the channel radius A_1 also depends on the channel radius. We find $\phi = -1$ which agrees with the experimental value. Values of A_1 for Elliott and Flowers (1957) (EF) interaction together with those for Gillet and Vinh Mau (1964) (GVM) interaction are shown by solid curves in Fig. 9, as functions of channel radius. Values for the interaction of Harms (1968) lie between these two curves. The dashed curves in Fig. 9 represent the values of A_1 calculated assuming that ψ_{1k} are orthogonal. It can be seen that this approximation becomes more accurate for the larger channel radii, which is to be expected. The hatching in Fig. 9 represents the experimental value of A_1 with estimated uncertainty. The best value of A_1 for Elliott and Flowers (1957) interaction is obtained for a channel radius of 4 fm as 0.65×10^{-2} which is about 50% of the experimental value. Values of A_1 obtained for the interaction of Gillet and Vinh Mau (1964) seem to agree better with the experimental value; for a channel radius of 4 fm $A_1 = 1.0 \times 10^{-2}$, which is in very good agreement with the experimental value. This shows the dependence of A_1 on the choice of interaction.

F. DISCUSSION

Values of A_0 obtained for various interactions are about half of the experimental value (Table 8). This problem has been discussed by Arima et al. (1975) who suggested that a possible explanation may be obtained by including $3\hbar\omega$ components in the

Fig. 9 Values of $1000 A_1$ as a function of channel radius a .

Solid curves give calculated values obtained by solving equation (54), and the dashed curves give values obtained by assuming the approximation that the Ψ_{Tk} are orthogonal. The experimental value of A_1 with estimated uncertainties is denoted by hatching.



7.12 MeV state. Heil and Stock (1976) have supported this suggestion.

To investigate the dependence of A_1 on the interaction we consider the contributions of various terms in the expression for A_1 (equation 69). Table 9 shows these for two different interactions, Elliott and Flowers (1957) (EF) and Gillet and Vinh Mau (1964) (GVM) and for a channel radius of 4 fm.

Table 9.

Contributions to A_1

k	EF			GVM		
	α_k	$[b_1^k(o01)+\sqrt{5}b_1^k(201)]$	$(1000A_1)$	α_k	$[(b_1^k(o01)+\sqrt{5}b_1^k(201)]$	$(1000A_1)$
1	0.0475	0.310	6.01	0.0614	0.431	10.80
2	-0.0083	0.313	-1.06	-0.0070	0.252	- 0.72
3	0.0140	0.121	0.69	0.0053	0.353	0.76
4	0.0019	2.00	1.55	-0.0003	2.02	- 0.25
5	0.0012	-1.33	-0.65	0.0011	-1.25	- 0.56
	Total		6.54	Total		10.03

It is clear from the above table that the main contribution to A_1 is from the lowest $T=1$ state at about 13.1 MeV. This is partly due to the dependence of α_1 on the interaction and partly due to the dependence of $[b_1^1(o01)+\sqrt{5}b_1^1(201)]$. α_1 is larger for the GVM interaction than for the EF interaction since both H_{o111}^c and L_{o111}^a and hence V_{o111}^a (equations 56, 57 and 54) are larger for GVM than for EF and this is due to the expansion coefficient $b_0^1(o11)$ which is larger for GVM (0.555) than for EF (0.330). Similarly

$[b_1^1(001) + \sqrt{5}b_1^1(201)]$ is larger for GVM than for EF and this is mainly due to the larger value of $b_1^1(001)$ for GVM (0.596) than for EF (0.419). Thus it seems that fitting of the experimental data available for the $T=1$ state at 13.1 MeV should give some information about the suitability of the interaction for the present problem. By fitting the observed radiative width of $\Gamma_{13.1}^0(E1) = 35$ eV (Ajzenberg-Selove 1976) for this state to the ground state of ^{16}O , and allowing for the appreciable isospin mixing in the pair of 1^- states at 12.4 and 13.1 MeV, we obtain a value of $[b_1^1(001) + \sqrt{5}b_1^1(201)] = \pm 0.28$. This is closer to the value of $[b_1^1(001) + \sqrt{5}b_1^1(201)]$ for EF interaction than for GVM interaction (Table 6). This suggests that the interaction of Elliott and Flowers (1957) is more appropriate than that of Gillet and Vinh Mau (1964), but it is not clear why the former interaction cannot account for the experimental value of A_1 whereas the latter interaction could.

CHAPTER V

ISOSPIN DISTORTION IN LOW-LYING STATES OF

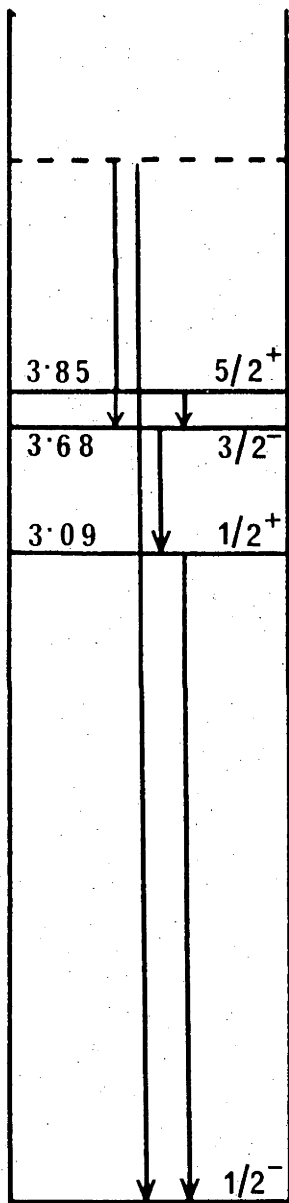
 ^{13}C AND ^{13}N A. LOW-LYING STATES OF ^{13}C AND ^{13}N

Fig. 10 shows the low-lying states of ^{13}C and ^{13}N that we have considered here. Nearly 30 years ago, Thomas (1952) summarized the then-known properties of the low-lying $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ levels of the mirror nuclei ^{13}C and ^{13}N , and used the nuclear reaction theories of Wigner and others to give a consistent account of them. Since that time, more experimental data concerning these levels and detailed shell model descriptions of them have become available. Some of this new information is not consistent with Thomas's assumptions or predictions. It thus seems appropriate to re-analyse the available data and compare the resultant parameter values with shell model calculations, to see if a consistent overall description is possible.

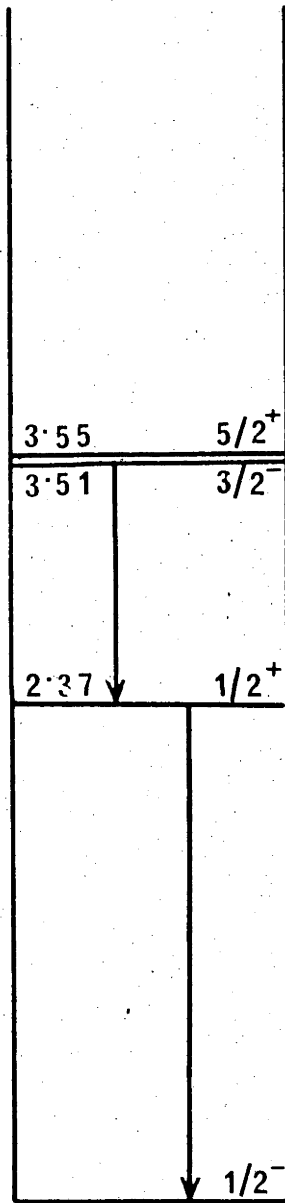
The data fitted by Thomas (1952) included level energy displacements and widths, and nucleon scattering and capture cross-sections. Thomas (mainly) used a one-channel approximation, describing the ^{13}C and ^{13}N levels as a single neutron or proton outside a ^{12}C ground-state core. Except for the $\frac{1}{2}^+$ levels, he also used a one-level approximation. The one-level approximation was not sufficient to describe the data on the $\frac{1}{2}^+$ levels, and Thomas

Fig. 10 Low-lying energy levels of ^{13}C and ^{13}N and E1 transitions between them.

$$\frac{4.95}{^{12}\text{C} + \text{n}}$$



^{13}C



^{13}N

$$\frac{1.94}{^{12}\text{C} + \text{p}}$$

included the effect of higher $\frac{1}{2}^+$ levels by using the representation of Feshbach et al. (1947) for the logarithmic derivative function.

We prefer to use the formalism of standard R-matrix theory (Lane and Thomas 1958) rather than that of Feshbach et al. This enables the description to be made in terms of constant parameters (eigenenergies E_λ and reduced width amplitudes $\gamma_{\lambda c}$) instead of an energy dependent quantity $Z(E)$, which is only restricted to be a monotonic increasing function of E although it is anticipated to have a fairly smooth energy dependence. The values of these and other constant parameters ($\mathcal{M}_{if}$, the internal transition moments between states i and f) required to fit the experimental data can then be compared with values calculated from the nuclear shell model. Thus as in chapter III the R-matrix treatment separates the problem of relating experimental data and models into two parts, data to parameters and parameters to models. In practice the separation may not be complete, since restrictions on the number of parameters used to fit the data may be based on model arguments, or the values of some parameters that are not well determined by the data may be taken from model calculations.

In Thomas's best fit to the available data, the reduced width for the channel ^{12}C ground state + p-wave nucleon was greater by a factor of about two in the excited $\frac{3}{2}^-$ states of the $A=13$ nuclei than in the $\frac{1}{2}^-$ ground states. Recent experimental values, however, as well as shell model calculations, suggest that the factor should be about 1/3. Also, the width of the $\frac{1}{2}^+$ first excited state of ^{13}C has recently been measured and is about 1/4 of the value predicted by Thomas's parameters. This small width makes the strength of the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ E1 transition in ^{13}C

less than one half of the corresponding strength in ^{13}N , although charge-symmetric forces would require such strengths in mirror nuclei to be equal. Shell model descriptions do not support the one-channel approximation for some of the low-lying levels of ^{13}C and ^{13}N , and it seems that a more general description is needed to explain the different strengths of the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transitions.

Another obvious evidence of departure from charge symmetry is the 720 keV difference in energies of the $\frac{1}{2}^+$ first excited states of ^{13}C and ^{13}N , the original and most pronounced example of the Thomas-Ehrman shift. Thomas calculated only two contributions to such level displacements, from the different external wavefunctions in the mirror nuclei and from the electromagnetic spin-orbit interaction; we include many other contributions and also do not make the one-channel approximation. Thomas used his fits to the level displacements in obtaining best values of the level parameters; we obtain the level parameters by fitting other data and use them to predict the level displacements. We do not attempt to fit the level displacements exactly since we do not include the effects of any charge-symmetry-breaking potential in the nuclear forces.

The experimental data that are fitted in order to determine values of the parameters E_λ , $\gamma_{\lambda c}$ and $\mathcal{M}_{if}$ are given in the next section. Not included are other data that do not yield direct information about these parameters, such as M1 matrix elements and log ft values, which are discussed by Cohen and Kurath (1965). The relevant R-matrix formulae and fits to the data are made in section C. In section D the resultant parameter values are compared with shell model values and with values deduced from other experimental

data. Coulomb displacement energies are calculated and compared with experimental values in section E. A discussion of these results and comments on earlier partial fits to the data are given in section F.

B. EXPERIMENTAL DATA

Experimental values of quantities relating to the lowest four levels in each of ^{13}C and ^{13}N are taken from Ajzenberg-Selove (1976), unless another reference is given. The relevant part of the energy-level diagrams of ^{13}C and ^{13}N is given in Fig. 10. This shows the energies of the levels and the E1 transitions that we consider.

Values of the fitted quantities are collected in Table 10. These are grouped according to the J^π values of the levels involved. Column A gives the values that Thomas (1952) used, column B the presently adopted values and column C the values obtained in our best fits. Of the quantities fitted, E_b and E_r are the energies of the ^{13}C and $^{13}\text{N } \frac{1}{2}^+$ levels, measured from the threshold of the ^{12}C ground-state channel. The scattering length a_s and effective range r_0 are obtained from the scattering of slow neutrons on ^{12}C . The Γ^o and Γ_γ^o are total and radiative widths in the c.m. system (the significance of the superscript, o, denoting observed width, is discussed in section C). The $\sigma_{n\gamma}$ and $\sigma_{p\gamma}$ are integrated cross-sections for the reactions $^{12}\text{C}(n,\gamma)^{13}\text{C}$ and $^{12}\text{C}(p,\gamma)^{13}\text{N}$. We comment here only on the adopted values that are not given in Ajzenberg-Selove (1976).

Table 10.

Values of quantities related to lowest levels of ^{13}C and ^{13}N

Relevant J^π value	Quantity	A	B	C
$\frac{1}{2}^+$	E_b (MeV)	-1.85	-1.858 ± 0.001	-1.858^*
	E_r (MeV)	0.42	0.421 ± 0.001	0.421^*
	a_s (fm)	6.11	6.142 ± 0.0012	6.142^*
	r_0 (fm)	2.9-3.6	3.42 ± 0.1	3.42^*
	$\Gamma^0(^{13}\text{N})$ (keV)	35	33 ± 2	33^*
$\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$	$\Gamma_Y^0(^{13}\text{C})$ (eV)		0.43 ± 0.04	0.472
	$\sigma_{n\gamma}$ (thermal) (mb)	1.8-3.5	2.31 ± 0.21	2.36
	$\sigma_{p\gamma}(E_r)$ (μb)	125 ± 15	102 ± 8	97.1
	$\sigma_{p\gamma}(E_p = 120 \text{ keV})$ (nb)	0.61 ± 0.09	0.61 ± 0.09	0.607
	$\sigma_{p\gamma}(E_p = 604 \text{ keV})$ (μb)		2.17 ± 0.19	2.03
$\frac{3}{2}^-$	$\Gamma^0(^{13}\text{N})$ (keV)	70 ± 10	60 ± 5	
$\frac{3}{2}^- \leftrightarrow \frac{1}{2}^+$	$\Gamma_Y^0(^{13}\text{C})$ (meV)		3.1 ± 0.3	3.11
	$\sigma_{n\gamma}$ (thermal) (mb)		1.09 ± 0.10	1.09
	$\Gamma_Y^0(^{13}\text{N})$ (eV)		0.054 ± 0.014	0.0512
$\frac{5}{2}^+$	$\Gamma^0(^{13}\text{N})$ (keV)	40	52 ± 6	52^*
$\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$	$\Gamma_Y^0(^{13}\text{C})$ (meV)		0.019 ± 0.001	0.019^*

A - Thomas (1952).

B Adopted value.

C Best fits for a channel radius $a = 5 \text{ fm}$.

* Exact fit.

The adopted value of a_s comes from the measurement of Koester and Nistler (1975), who gave $a_{\text{coh}} = 6.6572 \pm 0.0013$ fm. In a summary of previous measurements (but not including Koester and Nistler), Lachkar (1977) gave for the total elastic scattering cross-section for neutrons on ^{12}C the value

$$\sigma_T = (4.725 - 3.251 E + 1.316 E^2 - 0.227 E^3) \text{b} ,$$

where E is the neutron laboratory energy in MeV. From this one obtains $a_s = 6.132$ fm and the adopted value of r_0 . The uncertainty attributed to r_0 is based on the observation that Heaton et al. (1975) gave $r_0 = 3.33$ fm. The thermal-neutron cross-sections are obtained from the values of 3.4 ± 0.3 mb for the total capture cross-section and $68 \pm 1\%$ for the ground-state branching ratio. Absolute values of the $^{12}\text{C}(p,\gamma)$ peak cross-section have been given as $\sigma_{p\gamma}(E_r) = 120 \mu\text{b}$ (Fowler et al. 1948), $127 \mu\text{b}$ (Seagrave 1951, 1952) and $125 \pm 15 \mu\text{b}$ (Rolfs and Azuma 1974). Using the one-level approximation, Riess et al. (1968) gave $\Gamma_Y^0(^{13}\text{N}, \frac{1}{2}^+ \rightarrow \frac{1}{2}^-) = 0.45 \pm 0.05$ eV; taken with the total width value of $\Gamma^0(^{13}\text{N}, \frac{1}{2}^+) = 33$ keV, this gives $\sigma_{p\gamma}(E_r) = 92 \pm 10 \mu\text{b}$. Rolfs and Azuma quoted Vogl (1963) as giving a value of $130 \pm 4 \mu\text{b}$, but Vogl's error is a relative error only, and he normalized his cross-section to Seagrave's absolute value. By averaging the values of Rolfs and Azuma and of Riess et al., we get the adopted value given in Table 10. We also fit the $^{12}\text{C}(p,\gamma)$ cross-section at $E_p = 120$ keV, which Thomas took as being representative of early low-energy measurements. Cross-section measurements at higher energies by Vogl (1963) and by Rolfs and Azuma (1974) agree with each other (see Fig. 13). We fit Vogl's tabulated value at $E_p = 604$ keV, renormalized to a peak cross-section of $102 \mu\text{b}$.

Many measurements have been made of the total width of the $\frac{3}{2}^-$ level of ^{13}N . The adopted value is obtained from an average of these values, converted where necessary to the c.m. system:

68 ± 8 keV (Van Patter 1949), 65 ± 9 keV (Seagrave 1951), 53 keV[†] (Jackson and Galonsky 1953), 60 keV[†] (Armstrong et al. 1966), 60 ± 2.5 keV[‡] (Andreev et al. 1973), 54.8 ± 11.5 keV (Blatt et al. 1974) and 60 ± 3 keV (Rolfs and Azuma 1974). In order to obtain the radiative width for the $\frac{3}{2}^- \rightarrow \frac{1}{2}^+$ transition in ^{13}C we use the most recent branching ratio of $0.75 \pm 0.04\%$ (Warburton et al. 1980). The earlier branching ratios available are $0.65 \pm 0.1\%$ (Kane et al. 1960) and $1.6 \pm 0.3\%$ (Tryti et al. 1975). For the radiative width for the $\frac{3}{2}^- \rightarrow \frac{1}{2}^+$ transition in ^{13}N , we use measured values of the peak cross-section $\sigma_{\text{PY}_0\text{P}}(E_p = 1.7 \text{ MeV})$ of $35 \mu\text{b}$ (Seagrave 1952) and $37.5 \pm 7.5 \mu\text{b}$ (Young et al. 1963), and of the $\frac{3}{2}^- \rightarrow \frac{1}{2}^+$ branching ratio of $8 \pm 1\%$ (Rolfs and Azuma 1974). An earlier value of the branching ratio is $5 \pm 1\%$ (Woodbury et al. 1954). It may be noted

[†] These two values of the observed width Γ^0 are derived from the published values of the formal width Γ of 55 and 63 keV respectively, using the relation (81).

[‡] This value is based on the assumption that the published value of 65 ± 2.7 keV is a lab. value.

that the value of 0.04 eV for $\Gamma_Y^0(^{13}\text{N}, \frac{3}{2}^- \rightarrow \frac{1}{2}^+)$ given by Ajzenberg-Selove (1976) is based on the assumption by Young et al. (1963) of a total width of the $\frac{3}{2}^-$ level of 51.5 keV (lab), corresponding to a c.m. value of 47.5 keV.

Ajzenberg-Selove (1976) gives for the total width of the $\frac{5}{2}^+$ level of ^{13}N , $\Gamma^0(^{13}\text{N}, \frac{5}{2}^+) = 47 \pm 7$ keV, which comes entirely from the measurement of Blatt et al. (1974). Armstrong et al. (1966) gave $\Gamma = 74$ keV, but actually used R-matrix formulae and varied the reduced width $(\gamma^2 a)$ in fitting their phase shifts. From their values $\gamma^2 a = 3.55 \pm 0.18$ MeV fm and $a = 4.77$ fm, we find a formal width $\Gamma = 2P\gamma^2 = 68 \pm 3.4$ keV and an observed width $\Gamma^0 = 54 \pm 2.7$ keV. Thus, their value of 74 keV is presumably a value of the formal width. Similarly, the parameters of Jackson and Galonsky (1953) give $\Gamma^0 = 46$ keV. Our adopted value is an average of these.

It is of interest to compare the strengths of corresponding E1 transitions in ^{13}C and ^{13}N . The strengths from the adopted values in Table 10 are given as experimental values in Table 11. In this regard, the value of $\sigma_{p_Y}(E_r, \frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ corresponds to $\Gamma_Y^0(^{13}\text{N}, \frac{1}{2}^+ \rightarrow \frac{1}{2}^-) = 0.50 \pm 0.04$ eV. For charge-symmetric forces, one expects strengths of corresponding E1 transitions in mirror nuclei to be equal. It is seen from Table 11 that the experimental values of transition strengths in ^{13}C and ^{13}N are different. These asymmetries are due to the effect of isospin distortion of the corresponding states. Several attempts have been made in the past to explain the asymmetry in the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transition. Here we have attempted to explain the $\frac{3}{2}^- \rightarrow \frac{1}{2}^+$ asymmetry as well as the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ asymmetry. The calculated values in Table 11 correspond to our best fits (for 5 fm).

Table 11.

Comparison of strengths of E1 transitions in ^{13}C and ^{13}N

Values of Γ_γ^0 in Weisskopf units (1 W.u. = $0.376 E_\gamma^3$ eV, with E_γ in MeV).

Transition	Expt.			Calc. a)	
	^{13}C	^{13}N	$^{13}\text{N}/^{13}\text{C}$	^{13}C	$^{13}\text{N}/^{13}\text{C}$
$\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$	0.039 ± 0.004	0.101 ± 0.008	2.59 ± 0.34	0.043	0.096
$\frac{3}{2}^- \rightarrow \frac{1}{2}^+$	0.039 ± 0.004	0.095 ± 0.025	2.47 ± 0.68	0.039	0.091
$\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$	0.0103 ± 0.0005			0.0103^*	2.34

a) Best fits for a channel radius $a = 5$ fm.

* Exact fit.

C. R-MATRIX ANALYSIS AND FITS TO ^{13}C AND ^{13}N DATA

The general form of the results of R-matrix theory is based on physical principles but not on any special information or assumptions of quantum mechanical derivation. As mentioned earlier, this enables us to interpret the parameters such as $\gamma_{\lambda c}$ which are given a precise meaning as certain types of matrix elements. Thus one cannot only fit the necessary experimental data in terms of the parameters of R-matrix theory but also can calculate values and properties of these parameters on the basis of some theory of nuclear forces and nuclear structure. For this purpose in our work we use nuclear shell model theory.

1. Formulae for Level Parameters

Formulae and notation are taken from the paper on the R-matrix theory of nuclear reactions by Lane and Thomas (1958), unless otherwise noted. Only those formulae required for fitting the present data are given.

We are dealing with an energy region where there is for each J^π value at most one open channel, the ^{12}C ground-state channel. In extracting values of the parameters E_λ , $\gamma_{\lambda c}$ and $\mathcal{U}_{if}$ from experimental data we assume, as did Thomas (1952), a one-channel approximation, neglecting the contribution of all other (closed) channels (except in fitting the width of the $3/2^-$ level of ^{13}N). We do not make a similar approximation in relating these parameter

values to shell model values. The channel radius a separates the internal region ($r < a$) from the external or channel region ($r \geq a$). Properties of the internal region are described by an R-function which, for given J^π , is written $R_n(J^\pi, E)$ or $R_p(J^\pi, E)$, where the suffices n and p refer to the ^{13}C and ^{13}N system respectively, and E is the channel energy. For simplicity we drop the label J^π unless this could cause confusion, and we assume that $a_n = a_p = a$ for all J^π . Quantities measured experimentally are expressed in terms of the R-functions and certain Coulomb functions evaluated at the channel radius (the penetration factor P , the shift factor S and the hard-sphere phase shift $-\phi$), in addition to the boundary-condition parameter B . In principle equally good fits to the experimental data can be obtained for any choice of B and for any choice of a (provided it is greater than the range of the nuclear interaction). In practice, where a one- or two-level approximation is made, some choices of a may be better than others; the quality of fit is still independent of the choice of B , although the resultant parameter values will depend on B (Barker 1972).

For each of $J^\pi = \frac{1}{2}^-$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ we assume a one-level approximation

$$R_n(E) = \gamma_{1n}^2 / (E_{1n} - E), \quad R_p(E) = \gamma_{1p}^2 / (E_{1p} - E). \quad (70)$$

We use $B_n = S_n(E_{1n})$ and $B_p = S_p(E_{1p})$ so that E_{1n} and E_{1p} are just the observed energies of the levels as obtained from Fig. 10.

The γ_{1n} and γ_{1p} are treated as adjustable parameters in fitting the E1 transition probabilities.

For $J^\pi = \frac{1}{2}^+$, a one-level approximation is not sufficiently accurate, as Thomas (1952) pointed out, and we assume a two-level approximation

$$R_n(E) = \frac{\gamma_{1n}^2}{E_{1n} - E} + \frac{\gamma_{2n}^2}{E_{2n} - E}, \quad R_p(E) = \frac{\gamma_{1p}^2}{E_{1p} - E} + \frac{\gamma_{2p}^2}{E_{2p} - E}. \quad (71)$$

The level 1 is associated with the low-lying $\frac{1}{2}^+$ level (at 3.088 MeV in ^{13}C and 2.365 MeV in ^{13}N), while the level 2 represents a background due to all other $\frac{1}{2}^+$ levels.

The energy E_r of the $\frac{1}{2}^+$ level of ^{13}N is taken as the energy at which the resonant nuclear phase shift for scattering of s-wave protons on ^{12}C passes through $\frac{\pi}{2}$. From

$$\delta_p(E) = \arctan \frac{R_p}{1 - R_p(S_p - B_p)} - \phi_p, \quad (72)$$

we therefore obtain

$$R_p(E_r) = \frac{1}{S_p(E_r) - B_p}. \quad (73)$$

Similarly from the formula for the many-level density of states function (Barker 1967) applied to the $\frac{1}{2}^+$ states of ^{13}C , the energy E_b of the bound $\frac{1}{2}^+$ state of ^{13}C is given by

$$R_n(E_b) = \frac{1}{S_n(E_b) - B_n}. \quad (74)$$

The effective range expansion of the phase shift for scattering of s-wave neutrons on ^{12}C is

$$k \cot \delta_n(E) = -a_s^{-1} + \frac{1}{2} r_0 k^2 + \dots, \quad (75)$$

where k is the wave number ($k^2 = 2ME/\hbar^2$, where M is the reduced mass). Taken together with the R-function expression for the phase shift

$$\delta_n(E) = \arctan \frac{kaR_n}{1+R_n B_n} - ka, \quad (76)$$

this gives

$$R_n(0) = \left(\frac{a}{a-a_s} - B_n \right)^{-1}, \quad (77)$$

$$\left[\frac{d}{dE} R_n^{-1}(E) \right]_{E=0} = -\frac{2M}{\hbar^2} \frac{a^2 a_s^2}{(a-a_s)^2} \left[1 - \frac{a}{a_s} + \frac{1}{3} \frac{a^2}{a_s^2} - \frac{1}{2} \frac{r_0}{a} \right]. \quad (78)$$

The left hand side of equation (78) is just the zero-energy value of the quantity $[-\gamma_n^2(E)]^{-1}$, defined in equation (IV.2.8) of Lane and Thomas (1958)

$$\gamma_n^2(E) = R_n^2(E) \left[\frac{dR_n(E)}{dE} \right]^{-1}, \quad (79)$$

which is independent of the value of B .

The observed width of the $\frac{1}{2}^+$ level of ^{13}N can be simply expressed in terms of $\gamma_p^2(E_r)$, by assuming that P_p and ϕ_p are constant over the width of the level and S_p is a linear function of E ; then if the observed width Γ^0 is defined as the difference in the energies at which $\delta_p(E)$ is $\frac{\pi}{4}$ greater than or less than the resonance value, one has

$$\left[\frac{d}{dE} R_p^{-1}(E) \right]_{E=E_r} \equiv -\gamma_p^{-2}(E_r) = \frac{dS_p(E_r)}{dE} - \frac{2P_p(E_r)}{\Gamma^0} \quad (80)$$

We note that $\gamma_p^2(E_r)$ is the value of γ_{lp}^2 corresponding to $B_p = S_p(E_r)$. With this choice of B_p , equation (80) can be written

$$\Gamma^0 = \frac{\Gamma}{1 + \gamma_{lp}^2 \frac{dS_p(E_r)}{dE}}, \quad (81)$$

where $\Gamma = 2\gamma_{lp}^2 P_p(E_r)$ is the formal width. It is the observed width rather than the formal width that approximates the full width at half maximum of a resonance peak, which is usually quoted as the experimental width of a level.

Restrictions on the $\frac{1}{2}^+$ level parameters are then obtained by substituting from equation (71) into the equations (73), (74), (77), (78) and (80), and fitting the experimental data on the $\frac{1}{2}^+$ levels in Table 10.

2. Formulae for Capture Cross-Sections and El Radiative Widths

In order to discuss the transition probabilities, we now consider formulae for the capture cross-sections and El radiative widths. In their section XIII.3, Lane and Thomas (1958) dealt with the inclusion of photon channels in the R-matrix theory and showed that photons play a role in nuclear reactions similar to that of heavy particles. One difference is that the external region may contribute to the electromagnetic transition matrix elements.

Thomas (1952) showed the importance of such contributions in the case of El transitions in ^{13}C and ^{13}N . Thomas used a one-channel

approximation to described the internal as well as the external region. Lane and Thomas argued that even in many-channel cases, the external transitions are significant only in channels in which there are incident waves. We also assume this, limiting external contributions to the ^{12}C ground-state channel alone, with the extension that for transitions between bound states, we also include contributions from this channel only. In cases of radiative transitions in which a one-level approximation is assumed for the initial state, or the initial state is bound, we consider formulae for the radiative width, in other cases for the capture cross-section.

For simplicity we first assume the one-channel approximation for both the internal and external regions. The E1 capture cross-section in either ^{13}C or ^{13}N , from an initial continuum state i to a final bound state f , each with definite J^π values, can be written (Rolfs 1973)

$$\sigma_Y(i \rightarrow f) = \frac{2\pi M}{\hbar^2 k_i^3} (2J_i + 1) f_{if} \left| \int_0^\infty r u_i(r) u_f(r) dr \right|^2 / \int_0^\infty u_f^2(r) dr, \quad (82)$$

where

$$f_{if} = \frac{4}{3} \left(\frac{6}{13}\right)^2 e^2 \left(\frac{E_Y}{\hbar c}\right)^3 (\ell_i 100 | \ell_f 0)^2 U^2(1 \ell_f J_i \frac{1}{2}; \ell_i J_f) \quad (83)$$

Here J_q and ℓ_q ($q=i,f$) are the total and orbital angular momenta of the state q and $u_q(r)/r$ is its radial wavefunction. The long wavelength approximation for the E1 operator is assumed, and the spin part of the E1 operator is neglected (Blatt and Weisskopf 1952). The normalization of the continuum state is such that

$$u_i(r) = F_{\ell_i}(r)\cos\delta_i + G_{\ell_i}(r)\sin\delta_i, \quad (r \geq a) \quad (84)$$

where F_{ℓ_i} and G_{ℓ_i} are the regular and irregular Coulomb functions and δ_i is the nuclear phase shift for the state i , which is given by a formula such as (72) or (76), in terms of R-functions.

In order to obtain a formula for the radiative width of the transition $i \rightarrow f$, we then use the one-level approximation (70) for the R-function and equate the resulting expression for σ_Y with the one-level form of the cross-section

$$\sigma_Y(i \rightarrow f) = \frac{\pi}{k_i^2} \frac{2J_i + 1}{2} \frac{(2\gamma_i^2 P_i) \Gamma_Y(i \rightarrow f)}{[E_i - \gamma_i^2 (S_i - B_i) - E]^2 + (\gamma_i^2 P_i)^2}, \quad (85)$$

where γ_i^2 stands for $\gamma_{ln}^2(J_i^{\pi_i})$ or $\gamma_{lp}^2(J_i^{\pi_i})$ as appropriate, and P_i stands for $P_n(J_i^{\pi_i})$ or $P_p(J_i^{\pi_i})$, etc.

Holt (1978) et al. have shown that the elements of the collision matrix can be separated into internal and channel contributions while the channel contribution may again be separated into resonant and non-resonant parts; and only the internal contribution and the resonant part of the channel contribution should be considered when equating with the one level formula (equation 85) to obtain $\Gamma_Y(i \rightarrow f)$. For this one has to take u_i of the form of an outgoing wave such that

$$u_i(r) = \frac{F_{\ell_i}(a)\cos\delta_i + G_{\ell_i}(a)\sin\delta_i}{G_{\ell_i}(a) + i F_{\ell_i}(a)} [G_{\ell_i}(r) + i F_{\ell_i}(r)] \quad (r \geq a), \quad (86)$$

The form of equation (86) makes it obvious that $u_i(a)$ is the same as in equation (84). By using the relation

$$\gamma_i^2 = \frac{\hbar^2}{Ma^2} \frac{a}{2} \frac{u_i^2(a)}{\int_0^a u_i^2(r) dr}, \quad (87)$$

we obtain

$$\Gamma_Y(i \rightarrow f) = f_{if} \left| \int_0^\infty r u_i(r) u_f(r) dr \right|^2 / \left[\int_0^a u_i^2(r) dr \int_0^\infty u_f^2(r) dr \right]. \quad (88)$$

This is a formula for the formal radiative width, since the shift factor S_i is included explicitly in equation (85). The observed radiative width is given by (cf. equation (81))

$$\Gamma_Y^0(i \rightarrow f) = \frac{\Gamma_Y(i \rightarrow f)}{1 + \gamma_i^2 dS_i/dE} \quad (89)$$

For an E1 radiative transition between two bound states, one has (Blatt and Weisskopf 1952)

$$\Gamma_Y^0(i \rightarrow f) = f_{if} \left| \int_0^\infty r u_i(r) u_f(r) dr \right|^2 / \left[\int_0^\infty u_i^2(r) dr \int_0^\infty u_f^2(r) dr \right], \quad (90)$$

which can be written in the form of equations (88) and (89), since

$$\int_0^\infty u^2(r) dr = (1 + \gamma^2 dS/dE) \int_0^a u^2(r) dr \quad (91)$$

for a bound state (Lane and Thomas 1958).

A formula for the cross-section for E1 capture from one continuum state to another continuum state has been given by Rolfs and Azuma (1974). When one notes that the quantity Γ entering their expression (16) for the cross-section is the observed width of the final state, and if one makes a one-level approximation for the initial state, one finds that the formal radiative width is of the form (88) except that $\int_0^\infty u_f^2(r)dr$ is replaced by $(1+\gamma_f^2 dS_f/dE) \int_0^a u_f^2(r)dr$ (for a bound state f , these are identical because of equation (91)) and $\int_0^\infty r u_i(r)u_f(r)dr$ is replaced by $\int_0^\infty r u_i(r)u_f^*(r)dr$. Thus an expression for the observed radiative width, valid for the states i and f being either bound or continuum states, is

$$\Gamma_Y^O(i \rightarrow f) = f_{if} \frac{\left| \int_0^\infty r u_i(r) u_f^*(r) dr \right|^2}{(1+\gamma_i^2 dS_i/dE) (1+\gamma_f^2 dS_f/dE) \int_0^a u_i^2(r) dr \int_0^a u_f^2(r) dr} \quad (92)$$

Some authors (Christy and Duck 1961; Tombrello and Parker 1963) have replaced $\int_0^\infty u_f^2(r)dr$ in equation (82) by $\int_0^a u_f^2(r)dr$, which is equivalent to omitting the factor $1+\gamma_f^2 dS_f/dE$ in equation (92). The normalization given in equation (82) is also used by Thomas (1952) and Lane and Thomas (1958), although we note that in his actual fitting of the data, Thomas replaced his factor N by 1, thus effectively using the normalization of Christy and Duck.

As in Thomas (1952), we separate the radial integrals occurring in equation (82) into internal and external parts, by introducing the dimensionless internal transition moment $\mathcal{M}_{if}$ and other dimensionless quantities

$$\mu_{if} = \frac{1}{a} \frac{\int_0^a r u_i(r) u_f(r) dr}{\left[\int_0^a u_i^2(r) dr \int_0^a u_f^2(r) dr \right]^{1/2}}, \quad (93a)$$

$$\theta_q = u_q(a) \left[\frac{1}{2} a / \int_0^a u_q^2(r) dr \right]^{1/2}, \quad (93b)$$

$$J_{if} = \frac{1}{a^2} \int_a^\infty r w_i(r) w_f^*(r) dr, \quad w_q(r) = u_q(r)/u_q(a), \quad (93c)$$

and

$$N_q = 1 + \gamma_q^2 dS_q/dE \quad (93d)$$

Then, by making use of equation (91), equation (82) can be written

$$\sigma_Y(i \rightarrow f) = \frac{2\pi M}{\hbar^2 k_i^3} (2J_i + 1) f_{if} \frac{a^3 u_i^2(a)}{2N_f \theta_i^2} |\mu_{if} + 2\theta_i \theta_f J_{if}|^2. \quad (94)$$

The additional factor of 2 in the external contribution in (94) relative to Thomas's formula (40c) is due to the additional factor of $\frac{1}{2}$ in the definition of θ_q in equation (93b). This is introduced in order to retain the usual relationship between the θ_q and the reduced width amplitude γ_q (Lane and Thomas 1958)

$$\gamma_q = (\hbar^2/Ma^2)^{1/2} \theta_q. \quad (95)$$

In a similar way, equation (92) can be written

$$\Gamma_Y^o(i \rightarrow f) = f_{if} \frac{a^2}{N_i N_f} |\mu_{if} + 2\theta_i \theta_f J_{if}|^2. \quad (96)$$

Now we drop the assumption that the internal region can be described in the one-channel approximation. Since we still assume that the external contributions come from the ^{12}C ground-state channel alone, the form of equations (94) and (96) is unchanged, but the definitions (93a,b) of $\mathcal{M}_{if}$ and θ_q are changed. Shell model formulae for $\mathcal{M}_{if}$ and θ_q are given in section D.

In fitting the experimental data on capture cross-sections and radiative widths by means of equation (94) and (96), we treat the quantities $\mathcal{M}_{if}$ and θ_q as parameters, other quantities being calculable in terms of the known asymptotic forms of the radial wavefunctions. Among the latter quantities are the radial integrals J_{if} . The $u_q(r)$ for $r \geq a$, which occur in J_{if} , are either bound-state wavefunctions, determined by the binding energy of the state, or continuum wavefunctions given by equation (84) or equation (86) in terms of the nuclear phase shift δ_q . This phase shift is required only for $J^\pi = \frac{1}{2}^+$ or $\frac{3}{2}^-$ (see Fig. 10); for the $\frac{3}{2}^-$ case δ_q is needed only on resonance, where it has the value $\frac{\pi}{2} - \theta_q$, while for the $\frac{1}{2}^+$ case δ_q is given by equation (72) or (76) in terms of the level parameters of the $\frac{1}{2}^+$ levels. The integration in J_{if} can be performed analytically for the ^{13}C cases and numerically for the ^{13}N cases. For the $\frac{3}{2}^- \rightarrow \frac{1}{2}^+$ transition in ^{13}N , there is the problem that both the states i and f belong to the continuum, so that the integral cannot be evaluated in a straightforward manner. Faessler (1965) and Rolfs and Azuma (1974) have shown how the Ehrenfest theorem can be used to

evaluate the integral[†] in such a case when the limits of integration are 0 and ∞ . The method involves use of the Schrödinger equation for the initial and final radial wavefunctions, integration by parts, the assumption that the integrated parts vanish both at the upper and lower limits, and $\ell_f = \ell_i \pm 1$ which holds for E1 transitions. We have used the same method to evaluate the integral from a to ∞ that occurs in J_{if} ; the only difference is that the integrated parts do not now vanish at the lower limit. We find

$$\begin{aligned}
 \frac{1}{a^2} \int_a^\infty r w_i(r) w_f^*(r) dr = & \\
 \frac{1}{a^2} \frac{\hbar^2}{ME_Y^2} \left\{ \int_a^\infty H^{C'}(r) \frac{u_i(r) u_f^*(r)}{u_i(a) u_f^*(a)} dr - \frac{\hbar^2}{2M} \frac{u_i'(a) u_f'^*(a)}{u_i(a) u_f^*(a)} \right. & \\
 + \frac{a}{2} [E_Y + \{\ell_i(\ell_i+1) - \ell_f(\ell_f+1)\} \frac{\hbar^2}{2Ma^2}] \left[\frac{u_i'(a)}{u_i(a)} - \frac{u_f'^*(a)}{u_f^*(a)} \right] & \\
 - \frac{1}{2} [E(i) - E(f) - 2H^C(a) - \{\ell_i(\ell_i+1) + \ell_f(\ell_f+1)\} \frac{\hbar^2}{2Ma^2}] \left. \right\}, & \quad (97)
 \end{aligned}$$

where $H^C(r)$ is the Coulomb potential, $E(q)$ is the energy of the state q and $E_Y = E(i) - E(f)$. The right hand side of equation (97) depends only on $u_q(r)$ for $r \geq a$. Since $H^{C'}(r) \propto r^{-2}$, the integral on the right hand side of (97) can be evaluated numerically.

[†] This method works only for the integral $\int r u_i(r) u_f^*(r) dr$, which is obtained after the long wavelength approximation has been assumed, and justification of this approximation is difficult for an integral that is not convergent.

3. Fits to Data

The R-matrix formulae discussed so far are now used to fit the experimental data of Table 10 in order to derive values of the parameters E_λ , $\gamma_{\lambda c}$ (or γ_q) and $\mathcal{M}_{if}$. Written more fully, these are $E_{\lambda n}(J^\pi)$ or $E_{\lambda p}(J^\pi)$, $\gamma_{\lambda n}(J^\pi)$ or $\gamma_{\lambda p}(J^\pi)$ and $\mathcal{M}(J_i^{\pi_i} \rightarrow J_f^{\pi_f})$. We assume that the $\mathcal{M}_{if}$ depend only on $J_i^{\pi_i}$ and $J_f^{\pi_f}$, and do not depend on the particular energies of the states i and f or on whether the transition is in ^{13}C or ^{13}N . Then for ^{13}C , we note that $\mathcal{M}(\frac{3}{2}^- \rightarrow \frac{1}{2}^+) = \mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$. In some cases we consider θ_q rather than γ_q , since these are related by equation (95).

a. Determination of $\frac{1}{2}^+$ level parameters.

For the $\frac{1}{2}^+$ levels of ^{13}C and ^{13}N there are five pieces of data given in Table 10, but the formulae (71) for the R-functions contain eight unknowns. We therefore reduce the number of background parameters, which we cannot expect to be well determined by the data, by assuming that $E_{2p} - E_{1p} = E_{2n} - E_{1n}$ and that $\gamma_{2p}^2 = \gamma_{2n}^2$, provided that $B_p = B_n = B$, say (this makes the quality of fit dependent on the choice of B , since $\gamma_{1p}^2 \neq \gamma_{1n}^2$, but this dependence is not significant for reasonable values of B). We also take $E_{2n} = 10$ MeV; varying this value does not change significantly any of the results, except the value of γ_{2n}^2 . Thus for given values of a and B , the remaining five adjustable parameters E_{1n} , E_{1p} , γ_{1n}^2 , γ_{1p}^2 and γ_{2n}^2 can be determined. Table 12 shows the values of the five parameters obtained by fitting the best experimental values given in column B of Table 10 (called the standard set). These five parameters are obtained for a choice

of $B = S_n(E_b)$; this choice is not most appropriate for the values of E_{1p} and γ_{1p}^2 for which $B = S_p(E_r)$ is more appropriate. Changes in data allowed by the experimental errors do not change the parameter values appreciably; a change of E_{2n} to 20 MeV doubles the values of γ_{2n}^2 (see Table 12). Fig. 11 shows the values of γ_{1n}^2 for $B = S_n(E_b)$ and γ_{1p}^2 for $B = S_p(E_r)$ as functions of a . As a decreases, the values of γ_{2n}^2 decrease and become negative for $a < 4.2$ fm so that the solutions are inadmissible; for $a \approx 4.2$ fm, a one-level approximation for the $\frac{1}{2}^+$ states therefore gives an acceptable fit to the data.

Table 12.

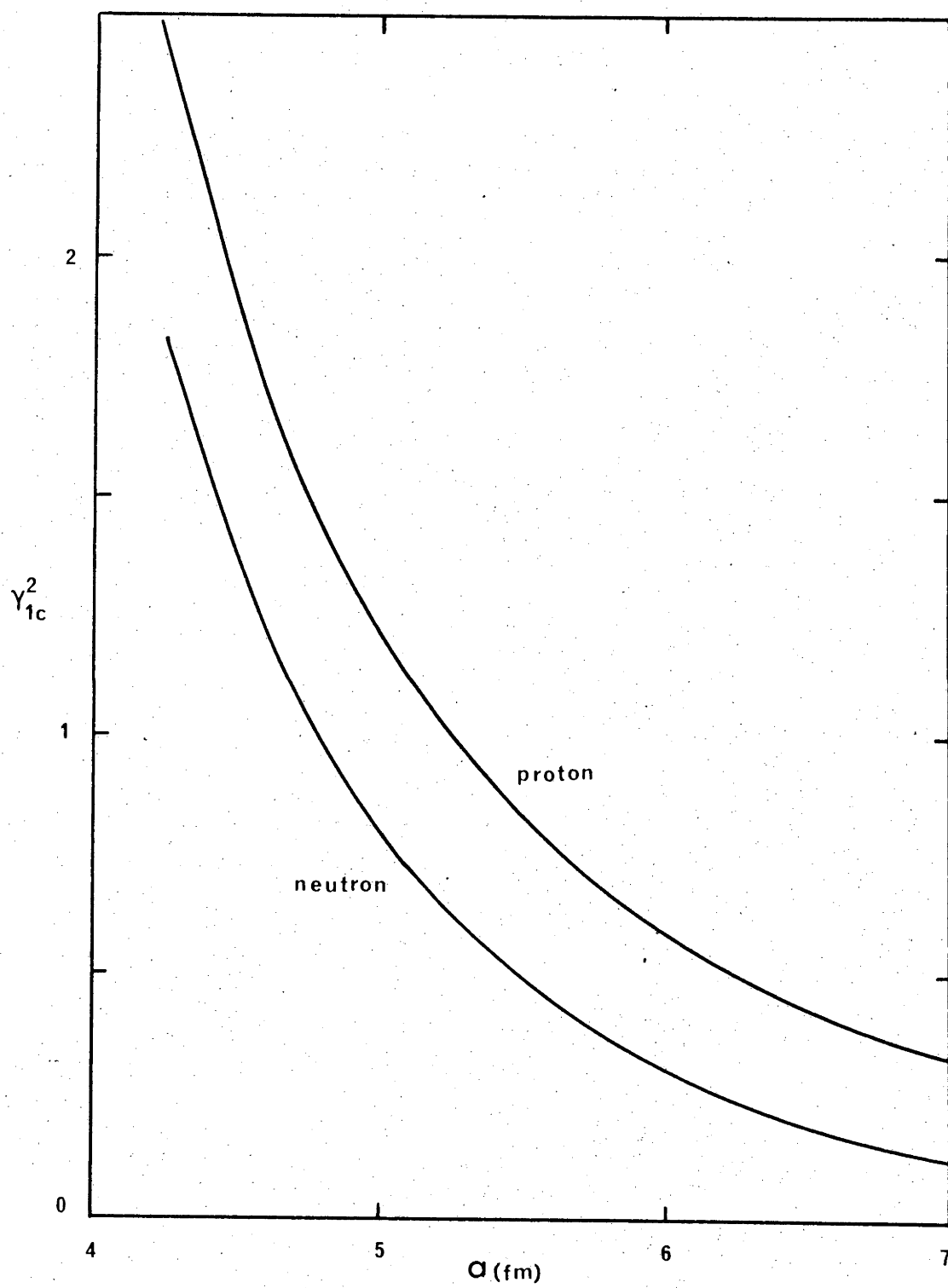
Values of $\frac{1}{2}^+$ level parameters (in MeV) for $a = 5$ fm and $B = S_n(E_b)$

Parameter Set	E_{1n}	γ_{1n}^2	E_{1p}	γ_{1p}^2	γ_{2n}^2
Standard	-1.858	0.802	1.053	1.115	0.915
Modified					
$r_0 = 3.33$ fm	-1.858	0.899	1.039	1.063	1.440
$\Gamma^0(^{13}\text{N}) = 31$ keV	-1.858	0.802	0.981	0.987	0.915
$E_{2n} = 20$ MeV	-1.858	0.816	1.048	1.095	1.986

b. Fits to El transitions

The formulae (94) and (96) for the El capture cross-sections and radiative widths involve parameters for the $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ levels and internal transition moments μ_{if} . Values of the $\frac{1}{2}^+$

Fig. 11 Values of the reduced widths γ_{1n}^2 and γ_{1p}^2 for the $\frac{1}{2}^+$ levels as functions of the channel radius a , obtained by fitting data in Table 10.



level parameters are known from the above fits. Thus in formula (94), which is required for $\frac{1}{2}^+$ to $\frac{1}{2}^-$ or to $\frac{3}{2}^-$ transitions, the value of θ_i is obtained from equations (95), (79) and (71), while $u_i(a)$ is obtained from equation (84) or (86) with δ_i given by (72) or (76) together with (71). For the levels with $J^\pi \neq \frac{1}{2}^+$ the level energies are known, so one is left with the adjustable parameters $\gamma_q \equiv \gamma_{\lambda c}$ (or $\theta_{\lambda c}$) for these levels and the $\mathcal{U}_{if}$. Because of the scarcity of data we take θ_{ln} and θ_{lp} as being related but not necessarily equal. We use the shell model expression for the $\theta_{\lambda c}$ (equation (104) below) and assume $f_{ln}^{\frac{1}{2}} = f_{lp}^{\frac{1}{2}}$, but allow $u_{ln}(r)$ and $u_{lp}(r)$ to be different by taking them as Woods-Saxon wavefunctions with appropriate boundary conditions. Then θ_{lp} may be expressed with sufficient accuracy as a multiple of θ_{ln} by

$$\frac{\theta_{lp}}{\theta_{ln}} = 1.0270 - 0.02817a + 0.01017a^2, \quad \text{for } \frac{1}{2}^- \text{ states,} \quad (98)$$

$$\frac{\theta_{lp}}{\theta_{ln}} = 1.1199 - 0.08950a + 0.02265a^2, \quad \text{for } \frac{3}{2}^- \text{ states,} \quad (99)$$

and only θ_{ln} and $\mathcal{U}_{if}$ are adjusted. They are chosen to minimize the quantity

$$X = \frac{1}{N-2} \sum_{i=1}^N \left| \frac{V_{\text{calc}}(i) - V_{\text{exp}}(i)}{\epsilon(i)} \right|^2, \quad (100)$$

where $V_{\text{calc}}(i)$, $V_{\text{exp}}(i)$ and $\epsilon(i)$ are the calculated and experimental values and the error of the quantity i , and N data

are fitted.

i. $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transition

Let us first consider the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transition. The experimental data available are the radiative width $\Gamma_Y^0(^{13}\text{C})$, thermal neutron cross-section $\sigma_{n\gamma}$, the resonance cross-section $\sigma_{p\gamma}(E_r)$ and the cross sections $\sigma_{p\gamma}$ at $E_p = 120$ and 604 keV. For these five pieces of data in Table 10 involving transitions between the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ levels, the best fit values of $\theta_{\text{ln}}(\frac{1}{2}^-)$, $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ and X_{min} are shown in Fig. 12 as functions of a . It is seen that the best fits occur for the smaller values of a , those for $a \lesssim 6$ fm being regarded as acceptable. Values for $a = 5$ fm are given in column C of Table 10 and in Table 11. Reasonable changes in the data in Table 10 involving the $\frac{1}{2}^+$ levels and the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transitions have little effect on either the values of the parameters or the quality of fit, except that smaller values of X_{min} are obtained if $\Gamma_Y^0(^{13}\text{C}, \frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ is increased or $\sigma_{p\gamma}(E_r)$ is decreased. Table 13 shows the values of $\mathcal{M}$, θ and X_{min} obtained by fitting the best experimental values given in column B of Table 10 (called the standard set) together with various changes in the $\frac{1}{2}^+$ and $(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ transition data for a channel radius of 5 fm.

The values $\theta_{\text{ln}}(\frac{1}{2}^-)$ and $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ in Fig. 12 do not agree with the values Thomas (1952) obtained, which correspond to region I of his Fig. 6, but do agree with his alternative solution in region II (see Barker 1961). Thomas's preference for region I was based mainly on his study of the displacements of corresponding

Fig. 12 Values of $\sqrt{2} \theta_{1n}(\frac{1}{2}^-)$, $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ and $\chi_{\min}$ as functions of the channel radius a , obtained by least squares fitting of data in Table 10.

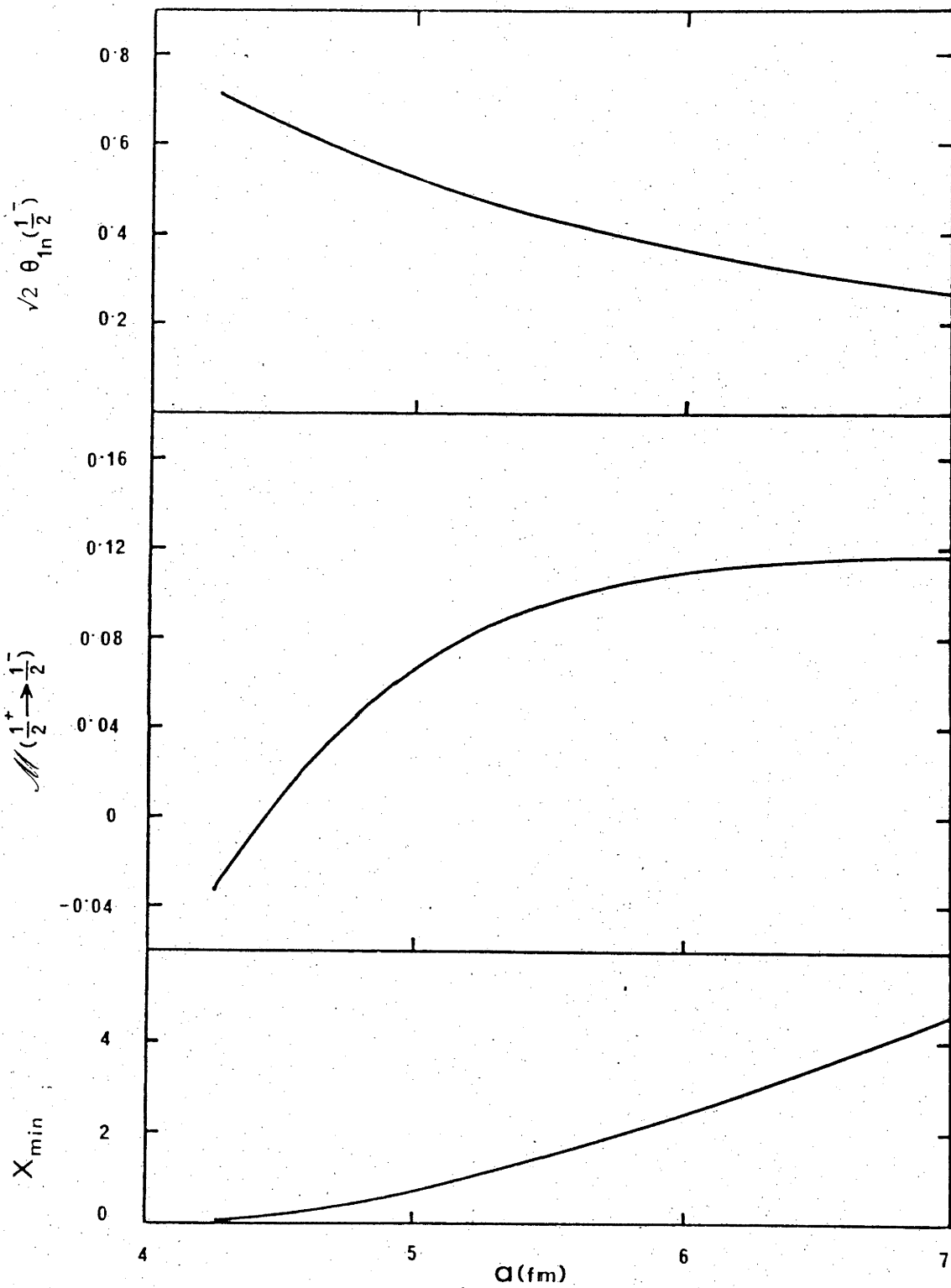


Table 13.

Values of $(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ transition parameters for $a = 5$ fm

Parameter Set	μ	θ	$X_{\min}$
Standard	0.0672	0.373	0.692
Modified			
$r_0 = 3.33$ fm	0.0634	0.374	1.13
$\Gamma^0(^{13}\text{N}) = 31$ keV	0.0693	0.376	1.79
$E_{2n} = 20.0$ MeV	0.0669	0.373	0.772
Modified			
$\Gamma_Y^0(^{13}\text{C}) = 0.47$ eV	0.0705	0.376	0.178
$\sigma_{n\gamma}(\text{thermal}) = 2.52$ mb	0.0633	0.380	0.545
$\sigma_{p\gamma}(E_r) = 94$ μb	0.0656	0.369	0.527
$\sigma_{p\gamma}(E_p = 120 \text{ keV}) = 0.52$ nb	0.0667	0.371	0.977
$\sigma_{p\gamma}(E_p = 604 \text{ keV}) = 1.98$ μb	0.0654	0.370	0.452

levels in ^{13}C and ^{13}N (see section E). It is our inclusion of the radiative width of the $\frac{1}{2}^+$ state of ^{13}C in the fit that excludes parameter values corresponding to region I.

Among the fitted data are the $^{13}\text{C}(p,\gamma_0)^{13}\text{N}$ cross-section at three energies ($E_p = 120, 456$ and 604 keV). From the parameter values we can calculate the cross-section at other energies. Fig. 13 shows the calculated cross-section for $a = 5$ fm, together with the renormalized experimental points of Vogl (1963) and of Rolfs and Azuma (1974). (The Rolfs and Azuma values were deduced from the S-factors given in Fig. 5 of Rolfs and Azuma and Fig. 4 of Fox et al. (1975), since accurate values could not be obtained from the 0° and 90° excitation functions plotted by Rolfs and Azuma). The agreement seems to be reasonable.

The $^{12}\text{C}(p,\gamma_0)^{13}\text{N}$ reaction is the first member in the CNO cycle and its cross-section is therefore of interest at stellar energies ($E_p \approx 20\text{--}50$ keV). The usual astrophysical S-factor is deduced from the experimental cross-section through the relation (Burbidge et al. 1957)

$$S = E_{\text{c.m.}} \sigma \exp(2\pi\eta) \quad (101)$$

where η is the Coulomb parameter and $E_{\text{c.m.}}$ is the projectile energy in the c.m. system. The value of S at $E_{\text{c.m.}} = 25$ keV has been given as $S = 1.33 \pm 0.15$ keV.b (Hebbard and Vogl 1960) and 1.45 ± 0.20 keV.b (Rolfs and Azuma 1974). Since their

Fig. 14 Values of $\theta_{1n}(\frac{3}{2}^-)$, $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$ and $X_{\min}$ as functions of the channel radius a , obtained by least squares fitting of data in Table 10.

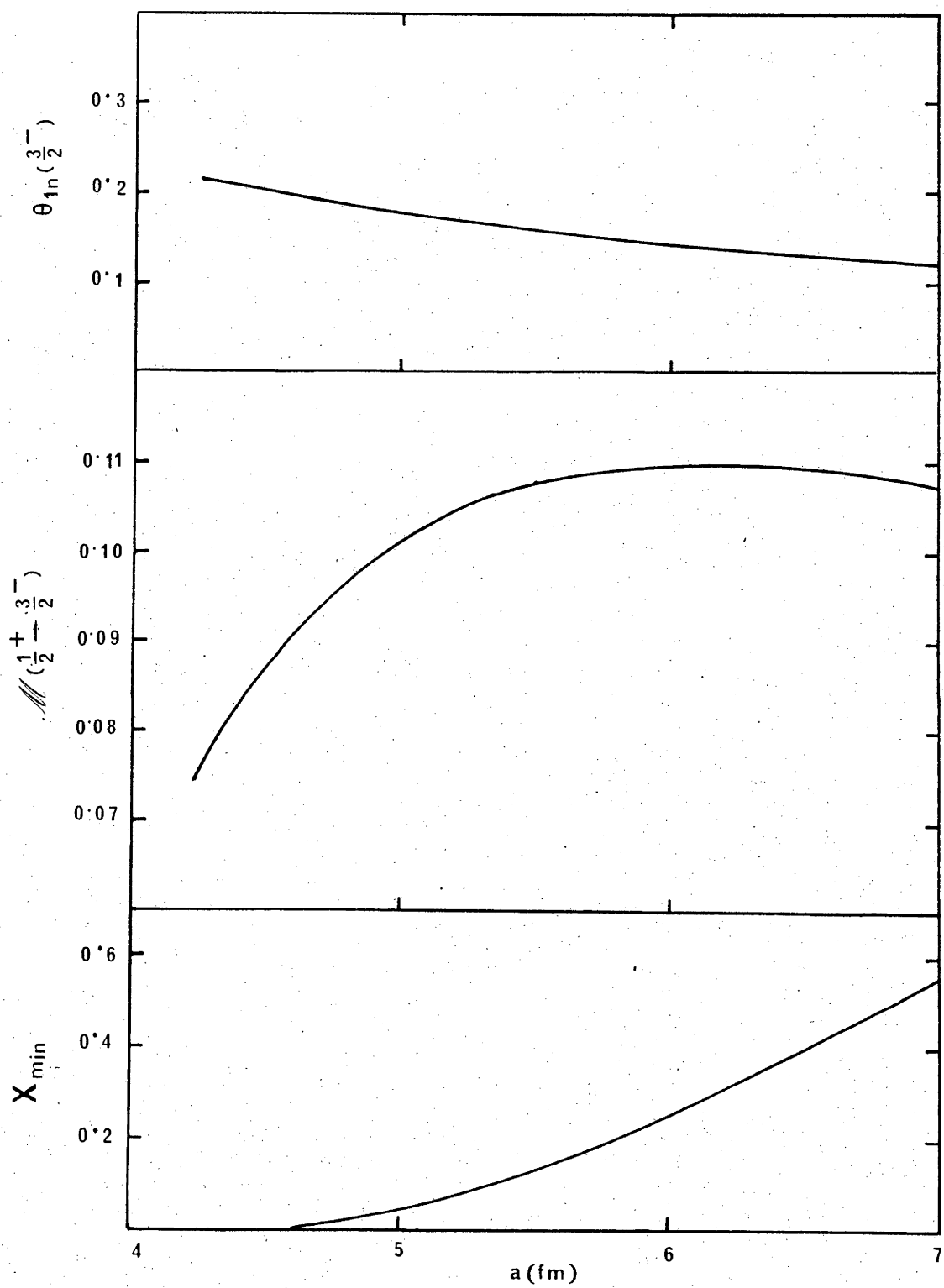


Table 14.

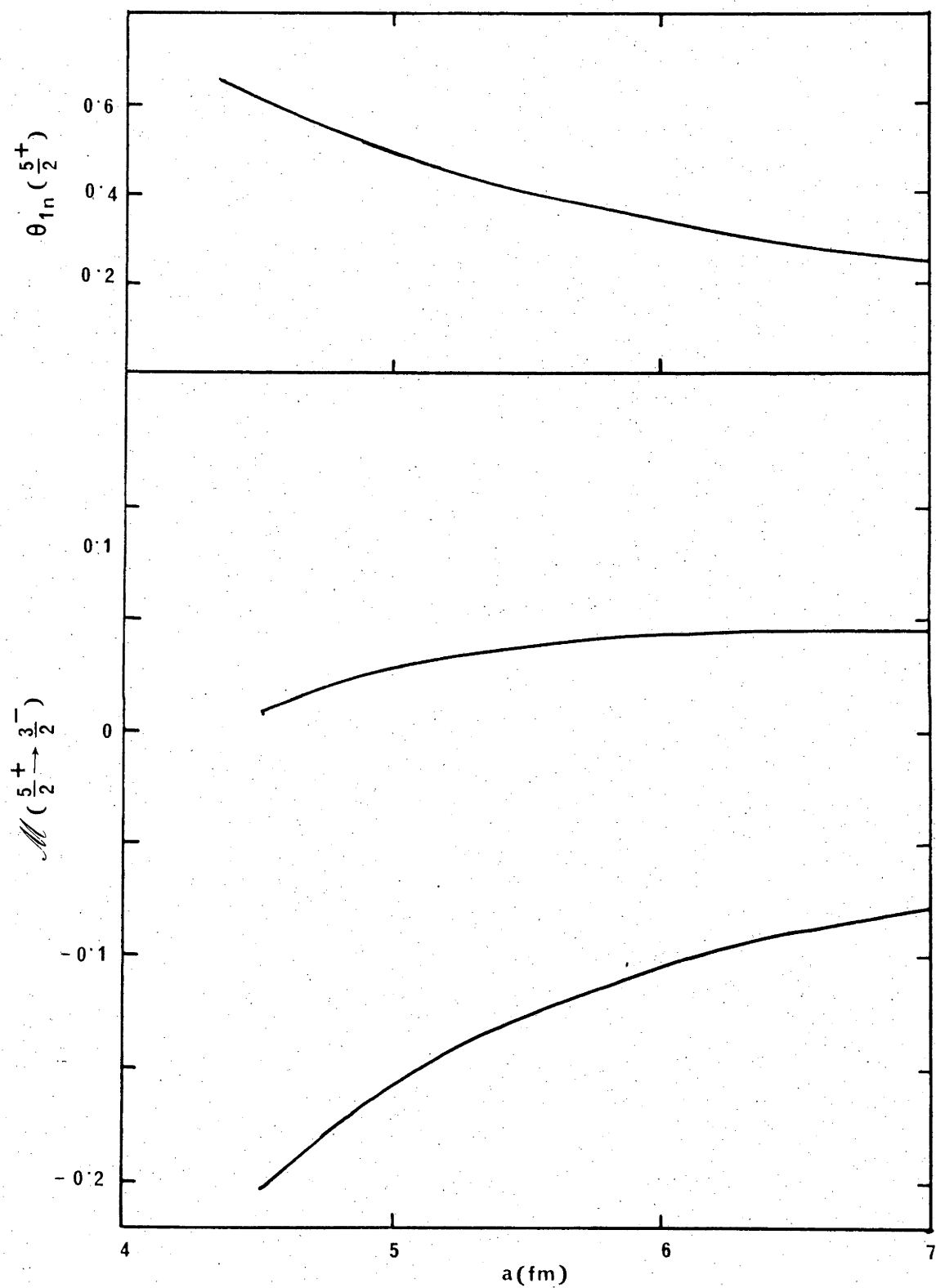
Values of $(\frac{1}{2}^+ \leftrightarrow \frac{3}{2}^-)$ transition parameters for $a = 5$ fm

Parameter Set		$\mathcal{U}$	θ	$\chi_{\min}$
Standard		0.101	0.178	0.042
Modified ($\frac{1}{2}^+$ data)	$r_0 = 3.33$ fm	0.098	0.178	0.054
	$\Gamma^O(^{13}\text{N}) = 31$ keV	0.101	0.178	0.120
	$E_{2n} = 20.0$ MeV	0.101	0.177	0.045
Modified ($\frac{3}{2}^- \leftrightarrow \frac{1}{2}^+$ data)	$\Gamma_Y^O(^{13}\text{C}) = 3.4$ meV	0.110	0.179	0.000
	$\sigma_{n\gamma}(\text{thermal}) = 1.19$ mb	0.098	0.184	0.003
	$\Gamma_Y^O(^{13}\text{N}) = 0.040$ eV	0.101	0.176	0.591

iii. $\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$ transition

Finally we discuss the fitting of the $\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$ transition data. The experimental data available are the width of the $\frac{5}{2}^+$ level in ^{13}N and the radiative width of the transition ($\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$) in ^{13}C . We have assumed one-level, one channel approximations to fit the observed width of the $\frac{5}{2}^+$ level in ^{13}N and this gives $\theta_{1p}(\frac{5}{2}^+)$ as a function of a . From this values of $\theta_{1n}(\frac{5}{2}^+)$ are obtained and using the values of $\theta_{1n}(\frac{3}{2}^-)$ from the previous fits the measured value of $\Gamma_Y^O(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-)$ in ^{13}C is fitted by adjusting the parameter $\mathcal{U}(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-)$. Fig. 15 shows the values of $\theta_{1n}(\frac{5}{2}^+)$ and

Fig. 15 Values of $\theta_{1n}({}^{5/2}^+)$ and $\mathcal{M}({}^{5/2}^+ \rightarrow {}^{3/2}^-)$ as functions of the channel radius a , obtained by fitting data in Table 10.



the two solutions for $\mathcal{M}(^{5/2^+} \rightarrow ^{3/2^-})$ as functions of a . The two solutions arise since we find the amplitudes $\mathcal{M}(^{5/2^+} \rightarrow ^{3/2^-})$ from the intensity $\Gamma_Y^0(^{5/2^+} \rightarrow ^{3/2^-})$.

c. Fits to $^{3/2^-}$ data

In the above fits to data, the observed width of the $^{3/2^-}$ second excited state of ^{13}N was not fitted, due to the expected importance of ^{12}C excited-state channels. In the one-level, many-channel approximation, the observed width is given by

$$\Gamma^0 = 2 \gamma_{1g}^2 P_g / [1 + \sum_c \gamma_{1c}^2 dS_c/dE] \quad (102)$$

where

$$\gamma_{1c}^2 = \frac{\hbar^2}{2ma} f_{1c} \frac{u_c^2(a)}{\int_0^a u_c^2(r) dr}, \quad (103)$$

and f_{1c} is the spectroscopic factor (see section D). The subscript $c = g$ indicates the ^{12}C ground-state channel, which is the only one open, and the sum over c includes all open and closed channels. For $c \neq g$, we use values of γ_{1c}^2 obtained from calculated values of f_{1c} (for the POT interaction of Cohen and Kurath 1965). Then the value of γ_{1g}^2 in equation (102) is adjusted for each value of a in order to fit the measured value $\Gamma^0(^{13}\text{N}, ^{3/2^-}) = 60 \pm 5 \text{ keV}$ (see Table 10). Table 15 gives the shell model values of f_{1c} for the more important excited-state channels, and the corresponding values of $\gamma_{1c}^2 dS_c/dE$ for a few values of a . The resultant values of $f_{1g}^{1/2} \equiv [f_{1g}(^{3/2^-})]^{1/2}$

Table 15.

Calculated quantities entering the expression (102) for the observed width of the $\frac{3}{2}^-$ state of ^{13}N

Channel c		f_{1c}	$\gamma_{1c}^2 \frac{dS_c}{dE}$		
E_x (MeV)	$J\bar{T}$		a=4 fm	a=5 fm	a=6 fm
4.44	20	1.143	0.250	0.093	0.036
12.71	10	0.206	0.015	0.003	0.001
15.11	11	0.803	0.046	0.009	0.001
16.11	21	0.120	0.006	0.001	0.000

are shown as a function of a by the dashed curve in Fig. 16, the uncertainty being of the order of 5% due to the experimental error in Γ^0 . There is good agreement between the value of $f_{1g}^{\frac{1}{2}}$ obtained in this way and that obtained from fitting the $\frac{3}{2}^- \leftrightarrow \frac{1}{2}^+$ transition data (see Fig. 16 below).

d. Isospin distortion in ^{13}C and ^{13}N transitions

From the least square fits discussed so far we can find various transition strengths for ^{13}C and ^{13}N . Table 11 shows the asymmetry between the E1 transitions in ^{13}C and ^{13}N obtained from the fits of the properties involving $\frac{1}{2}^+$, $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states. These asymmetries are due to the isospin distortion effects of these states. Values given in Table 11 are in Weisskopf units (1 W.u. = $0.376 E_\gamma^3$ eV, with E_γ in MeV). The agreement between the observed and calculated values is reasonable. We shall discuss these asymmetries in detail together with the explanations given by various authors in section F.

D. LEVEL PARAMETERS FROM SHELL MODEL CALCULATIONS AND FROM OTHER EXPERIMENTAL DATA

We now compare the parameter values obtained from fitting the experimental data with those obtained by using shell model calculations. Among the parameters for which we have obtained values are the reduced widths γ_{1n}^2 and γ_{1p}^2 for the $\frac{1}{2}^+$ states, dimensionless reduced width amplitudes θ_{1n} for the $\frac{1}{2}^-$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states and the internal transition moments $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$, $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$ and $\mathcal{M}(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-)$. Values of $\theta_{\lambda c}$ are also available from single particle transfer reactions, which are not considered in our earlier fits.

1. Description of the States and Spectroscopic Amplitudes

General formulae for calculating the reduced width amplitudes for a level λ and channel c are given by Lane (1960):

$$\theta_{\lambda c} = f_{\lambda c}^{\frac{1}{2}} u_c(a) [1/2a / \int_0^a u_c^2(r) dr]^{\frac{1}{2}}, \quad (104)$$

where $\theta_{\lambda c}$ is the dimensionless reduced width amplitude (defined in equation 93b for one channel), and $f_{\lambda c}^{\frac{1}{2}}$ is the spectroscopic amplitude. The negative and positive parity states are treated separately.

a. Negative parity states

For the negative (normal) parity $A=13$ states, we use the L-S coupling representation in the lowest $(1s^4 1p^9)$ configuration

$$\Psi_{13}(TM_T J^-) = \sum_{[\lambda]SL} a([\lambda]TSLJ) \Psi(1s^4 1p^9 [\lambda]TSL M_T J), \quad (105)$$

where $T = \frac{1}{2}$, $M_T = +\frac{1}{2}$ or $-\frac{1}{2}$ for ^{13}C and ^{13}N respectively, $J = \frac{1}{2}$ or $\frac{3}{2}$, and $[\lambda]$ is the orbital symmetry. For describing the channels consisting of an $A=12$ nucleus plus a nucleon, we need the $A=12$ wavefunctions, which we take to be normal-parity states of the lowest configuration

$$\Psi_{12}(\bar{T}\bar{M}_T\bar{J}) = \sum_{[\bar{\lambda}]\bar{S}\bar{L}} \bar{a}([\bar{\lambda}]\bar{T}\bar{S}\bar{L}\bar{J}) \Psi(1s^4 1p^8 [\bar{\lambda}]\bar{T}\bar{S}\bar{L} \bar{M}_T\bar{J}) \quad (106)$$

For the present purpose, this description is required only for the ^{12}C ground and first excited states ($\bar{T}=\bar{M}_T=0$, $\bar{J}=0$ or 2), but a more general formalism is needed for use in section E. The spectroscopic amplitudes are given by

$$\begin{aligned} \mathcal{J}_{\lambda c}^{\frac{1}{2}} &\equiv \mathcal{J}_{\lambda c}^{\frac{1}{2}(J^-M_T, \bar{J}\bar{T}\bar{M}_Tj)} \\ &= \sum_{[\lambda]SL[\bar{\lambda}]\bar{S}\bar{L}} a([\lambda]\frac{1}{2}SLJ) \bar{a}([\bar{\lambda}]\bar{T}\bar{S}\bar{L}\bar{J}) \mathcal{J}^{\frac{1}{2}}([\lambda]\frac{1}{2}SLM_TJ, [\bar{\lambda}]\bar{T}\bar{S}\bar{L}\bar{M}_T\bar{J}j), \end{aligned} \quad (107)$$

where

$$\begin{aligned} &\mathcal{J}^{\frac{1}{2}}([\lambda]TSLM_TJ, [\bar{\lambda}]\bar{T}\bar{S}\bar{L}\bar{M}_T\bar{J}j) \\ &= 3(\bar{T}\frac{1}{2}\bar{M}_T\bar{M}_T-\bar{M}_T|TM_T) < 1p^9 [\lambda]TSL\{ | 1p^8 [\bar{\lambda}]\bar{T}\bar{S}\bar{L}, 1p > \\ &[(2\bar{J}+1)(2j+1)(2S+1)(2L+1)]^{\frac{1}{2}} \left\{ \begin{array}{ccc} \bar{S} & \bar{L} & \bar{J} \\ \frac{1}{2} & 1 & j \\ S & L & J \end{array} \right\}, \end{aligned} \quad (108)$$

involving a Clebsch-Gordan coefficient, a fractional parentage coefficient and a 9-j symbol. The label j is here the angular momentum of the odd lp nucleon. At present $f_{\lambda c}^{1/2}$ is required only for the ^{12}C ground-state channel, for which $j=J$.

b. Positive parity states

For the positive (non-normal) parity $A=13$ states we use the jj coupling representation, where the states are obtained by coupling a $2s$ or $1d$ nucleon to a core which is either the 0^+ ground or 2^+ first excited state of ^{12}C and then antisymmetrizing (Barker 1961). The states are written as

$$\Psi_{13}(T M_T J^+) = \sum_{\bar{J} j} b(T \bar{J} j J) \Psi((\bar{J} j) T M_T J) \quad (109)$$

where T and M_T are as before and $J=1/2$ or $5/2$. The $\bar{J}$ in Ψ represents the core states $\Psi_{12}(00\bar{J})$ of equation (106) with $\bar{J}=0$ or 2 only, and j is an abbreviation for $n\ell j$, with $n\ell = 2s$ or $1d$. We omit other basis states that are included in some of the more elaborate shell model calculations for the $A=13$ positive-parity states (e.g. Jäger et al. 1971). For simplicity we take $\Psi_{12}(00\bar{J})$ to be the LS coupled state with $S=0$ and $L=\bar{J}$. The spectroscopic amplitudes are given by

$$f^{1/2}(J^+ M_T, \bar{J} \bar{M}_T j) = b(1/2 \bar{J} j J) \delta(\bar{T}, 0) \delta(\bar{M}_T, 0) \quad (110)$$

2. Values of the Spectroscopic Amplitudes

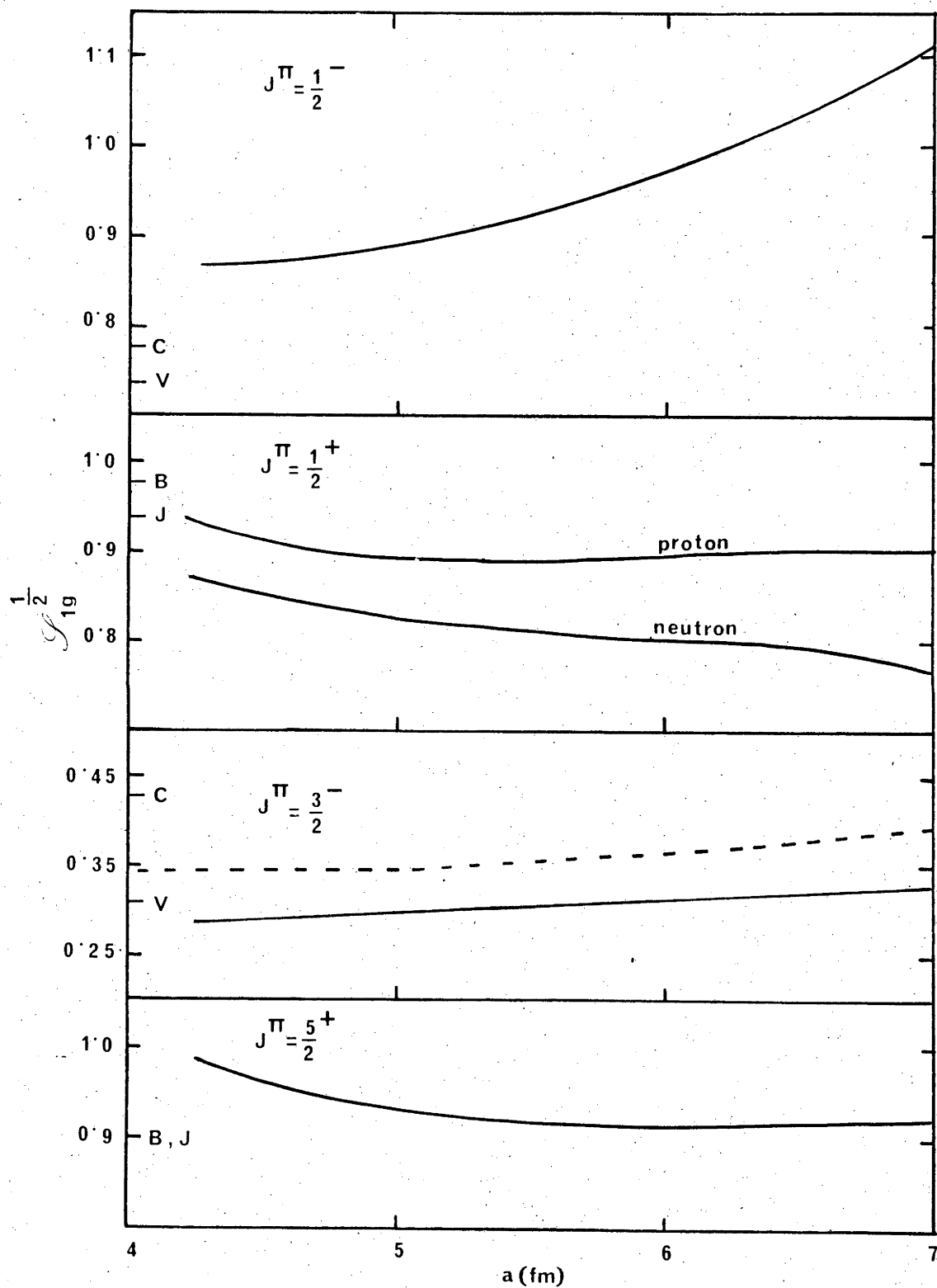
a. Experimental values

We consider the spectroscopic amplitudes $f_{1g}^{1/2}$ for the level labelled $\lambda = 1$ and for the ^{12}C ground-state channel labelled $c = g$. The experimental values of $f_{1g}^{1/2}$ for the various values of J^π are obtained by making use of equations (95) and (104). The wavefunction $u_g(r)$ ($u_c(r)$ of equation (104)) is taken as a wavefunction in a Woods-Saxon potential with conventional parameter values $r_0 = 1.25$ fm and $a = 0.65$ fm, and a uniform charge distribution with radius 1.25 fm, the depth in each case being chosen to fit the observed separation energy for a wavefunction with the correct ℓ value and the appropriate number of nodes. We also assume that $f_{1g}^{1/2} > 0$ and $u_g(a) > 0$ which is consistent with the positive values of $\theta_{\lambda c}$ assumed in section C.3.

b. Calculated values

The calculated values of $f_{1g}^{1/2}$ are obtained from equations (107), (108) and (110). The assumptions $f_{1g}^{1/2} > 0$ and $u_g(a) > 0$ imply a certain sign convention for the shell model states which must be retained in calculating the values of $\mathcal{M}_{if}$ (sub-section 3). For the negative parity states shell model values of f_{1g} are given by Cohen and Kurath (1967) and by Varma and Goldhammer (1969). For the positive parity levels we give only two sets of values; those of Barker (1961) represent weak-coupling calculations, while those of Jäger et al. (1971) are from complete calculations within the space of all $1\hbar\omega$ excitations. These values are given in Fig. 16 and in Table 16. Other calculated values are not essentially different from these.

Fig. 16 Values of the spectroscopic amplitudes $f_{1g}^{1/2}$ as functions of the channel radius a , for levels with the J^π values indicated and for the ^{12}C ground-state channel. The solid curves are experimental values derived from the fits shown in Figs. 11, 12, 14, and 15. The dashed curve is obtained by fitting $\Gamma^0(^{13}\text{N}, 3/2^-)$ using equation (102). Calculated shell model values are indicated along the ordinate axis, with labels C (Cohen and Kurath 1967), V (Varma and Goldhammer 1969), B (Barker 1961) and J (Jäger et al. 1971).



c. Comparison between experimental and calculated values

of $f_{lg}^{1/2}$

The curves in Fig. 16 show the experimental values of $f_{lg}^{1/2}$ plotted as functions of a , while the calculated values which are independent of a , are indicated on the left. There is reasonable agreement between the calculated and experimental values.

For the $1/2^+$ case, the experimental value of $f_{lg}^{1/2}$ obtained from the best $^{13}\text{N}(\text{proton})$ data is somewhat different from that obtained from $^{13}\text{C}(\text{neutron})$ data, but equal values can be achieved for $a \lesssim 6$ fm when allowance is made for the experimental uncertainties in r_0 and $\Gamma^0(^{13}\text{N}, 1/2^+)$.

The experimental value for the $3/2^-$ case agrees better with the shell model value of Varma and Goldhammer (1969) than with that of Cohen and Kurath (1967), suggesting that the former gives a better description of the $3/2^-$ states. It may be mentioned here that assuming the interaction of Goldhammer et al. (1968) which includes two body and three body matrix elements, we were unable to reproduce some of their values for the eigenenergies. We have also calculated the spectroscopic amplitude $f_{lg}^{1/2}(3/2^-)$ using Goldhammer's interaction and it does not agree with the values given by Varma and Goldhammer (1969), and it is even higher than the values given by Cohen and Kurath (1967). However, the values of $f_{lg}^{1/2}(3/2^-)$ obtained by fitting the observed width (section C) agrees reasonably with the experimental values.

d. Values from DWBA calculations

Analysis of single-nucleon transfer reactions by DWBA or an equivalent formalism provides values of the spectroscopic factor f_{1g} . Some of these values are collected in Table 16. It is seen that they vary widely, particularly for the excited ^{13}C and ^{13}N states, and in general their agreement with the theoretical values is much poorer than that indicated in Fig. 16. It is interesting to note that the recent work of Franey et al. (1979) on the $^{12}\text{C}(^{17}\text{O}, ^{16}\text{O})^{13}\text{C}$ reaction, although it does not quote a value of $f_{1g}^{(\frac{1}{2}^-)}$ as such, obtains results in excellent agreement with the $^{12}\text{C}(^{13}\text{C}, ^{13}\text{C})^{12}\text{C}$ study by Gubler et al. (1977) whose value of $f_{1g}^{(\frac{1}{2}^-)}$ in Table 16 which corresponds to $f_{1g}^{(\frac{1}{2}^-)} = 0.90$ is in good agreement with the experimental values in Fig. 16.

3. E1 Transition Moments

Now we require a shell model formula for $\mathcal{M}_{if}$, which appears in equations (94) and (96). The radiative width for an E1 transition from a positive-parity bound state i given by equation (109) to a negative-parity bound state f given by equation (105) is (Blatt and Weisskopf 1952)

$$\Gamma_Y^0(i \rightarrow f) = \frac{16\pi}{9} \left(\frac{E_Y}{\hbar c} \right)^3 | \langle J_i || Q_1 || J_f \rangle |^2, \quad (111)$$

where

$$Q_{1\mu, \text{op}} = \frac{e}{2} \sum_{i=1}^A \left[\frac{N-Z}{A} - \tau_3(i) \right] r(i) Y_{1\mu}(\Omega_i). \quad (112)$$

Table 16.

Spectroscopic factors of ^{13}C and ^{13}N low-lying levels
for ^{12}C ground-state channels

f_{1g}				Reaction	Reference ^A
$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$		
Calculated					
0.61		0.19			a
0.56		0.10			b
	0.97		0.81		c
	0.89		0.81		d
Experimental					
0.8	0.9	0.26	0.8	$^{12}\text{C}(\text{d}, \text{p}) ^{13}\text{C}$	e
1.16		0.22		"	f
0.58±0.15	0.36±0.02			"	g
1.1, 1.4	1.1, 1.2	0.10, 0.20	1.1, 1.4	$^{12}\text{C}(\vec{\text{d}}, \text{p}) ^{13}\text{C}$	h
0.53±0.12				$^{12}\text{C}(\text{d}, \text{n}) ^{13}\text{N}$	g
0.82				$^{13}\text{C}(\text{p}, \text{d}) ^{12}\text{C}$	i
0.7-1.48	0.25	~0.02	~0.14	$^{12}\text{C}(^3\text{He}, \text{d}) ^{13}\text{N}$	j
0.68				"	k
0.52				$^{13}\text{C}(\text{d}, \text{t}) ^{12}\text{C}$	l
0.80	0.44	0.17	0.74	$^{12}\text{C}(^7\text{Li}, ^6\text{Li}) ^{13}\text{C}$	m
0.72				$^{12}\text{C}(^7\text{Li}, ^6\text{He}) ^{13}\text{N}$	m
0.25, 0.40	0.38, 0.61		0.23, 0.37	$^{12}\text{C}(^{10}\text{B}, ^9\text{Be}) ^{13}\text{N}$	n
0.66				$^{12}\text{C}(^{13}\text{C}, ^{12}\text{C}) ^{13}\text{C}$	o
0.81				"	p
0.59±0.12				"	q
0.81±0.04				$^{12}\text{C}(^{13}\text{C}, ^{13}\text{C}) ^{12}\text{C}$	r
0.72			0.57	$^{12}\text{C}(^{14}\text{N}, ^{13}\text{N}) ^{13}\text{C}$	s
0.62	0.09		0.49	$^{12}\text{C}(^{14}\text{N}, ^{13}\text{C}) ^{13}\text{N}$	s

^AReference:

- | | |
|--------------------------------|----------------------------------|
| a. Cohen and Kurath (1965) | k. Karban et al. (1976) |
| b. Varma and Goldhammer (1969) | l. Ludwig et al. (1974) |
| c. Barker (1961) | m. Zeller et al. (1979) |
| d. Jäger et al. (1971) | n. Nair et al. (1974) |
| e. Glover and Jones (1966) | o. DeVries (1973) |
| f. Schiffer et al. (1967) | p. Von Oertzen and Bohlen (1975) |
| g. Pearson et al. (1972) | q. Bennett (1976) |
| h. Darden et al. (1973) | r. Gubler et al. (1977) |
| i. Taketani et al. (1968) | s. Nair et al. (1975) |
| j. Fortune et al. (1969) | |

Substituting from equations (105) and (109), and making use of the formulae (107), (108), (110), we obtain

$$\langle J_i \| Q_1 \| J_f \rangle = e^{(\frac{3}{4\pi})^{\frac{1}{2}} (\frac{12}{13} M_T)} \sum_{J_j i j_f} (\ell_i 100 | 10) U(11 j_i \frac{1}{2}; \ell_i j_f) U(1 j_f J_i \bar{J}; j_i J_f)$$

$$\mathcal{J}^{\frac{1}{2}}(J_i^+, \bar{J} j_i) \mathcal{J}^{\frac{1}{2}}(J_f^-, \bar{J} j_f) \frac{\int_0^\infty r u_{\bar{J} j_i}(r) u_{\bar{J} j_f}(r) dr}{[\int_0^\infty u_{\bar{J} j_i}^2(r) dr \int_0^\infty u_{\bar{J} j_f}^2(r) dr]^{\frac{1}{2}}}, \quad (113)$$

where $\mathcal{J}^{\frac{1}{2}}(J^\pi M_T, \bar{J} 00 j)$ has been written $\mathcal{J}^{\frac{1}{2}}(J^\pi, \bar{J} j)$. With θ_q given by equation (104) we find that equation (111) is of the form (96) provided that

$$\mathcal{U}_{if} = \frac{1}{(\ell_g 100 | 10) U(11 J_i \frac{1}{2}; \ell_g J_f)} \left| \frac{\int_0^\infty u_{0 J_i}^2(r) dr \int_0^\infty u_{0 J_f}^2(r) dr}{\int_0^a u_{0 J_i}^2(r) dr \int_0^a u_{0 J_f}^2(r) dr} \right|^{\frac{1}{2}}$$

$$\sum_{\bar{J} j_i j_f} (\ell_i 100 | 10) U(11 j_i \frac{1}{2}; \ell_i j_f) U(1 j_f J_i \bar{J}; j_i J_f)$$

$$\mathcal{J}^{\frac{1}{2}}(J_i^+, \bar{J} j_i) \mathcal{J}^{\frac{1}{2}}(J_f^-, \bar{J} j_f) \frac{1}{a} \frac{\int_0^a r u_{\bar{J} j_i}(r) u_{\bar{J} j_f}(r) dr}{[\int_0^\infty u_{\bar{J} j_i}^2(r) dr \int_0^\infty u_{\bar{J} j_f}^2(r) dr]^{\frac{1}{2}}}. \quad (114)$$

Then (114) is the shell model generalization of the one-channel formula (93a).

In evaluating $\mathcal{M}_{if}$ from equation (114), we make the approximations that the $u_{\bar{j}}(r)$ are harmonic oscillator single-particle wavefunctions $u_{n\ell}(r)$ and that the upper limits of the integrals in (114) can be extended from a to ∞ (cf. similar approximations in Chapter II) and assume that the same formula can be applied to transitions involving continuum states. Then

$$\mathcal{M}_{if} = \frac{1}{(\ell_g 100 | 10) U(11J_i \frac{1}{2}; \ell_g J_f)} \sum_{\bar{j}_i \bar{j}_f} (\ell_i 100 | 10) U(11j_i \frac{1}{2}; \ell_i j_f) U(1j_f J_i \bar{j}; j_i J_f) \mathcal{J}^{\frac{1}{2}}(J_i^+, \bar{j} j_i) \mathcal{J}^{\frac{1}{2}}(J_f^-, \bar{j} j_f) \frac{1}{a} I_{n_i \ell_i}, \quad (115)$$

where

$$I_{2s} = b, \quad I_{1d} = \left(\frac{5}{2}\right)^{\frac{1}{2}} b, \quad (116)$$

and b is the harmonic oscillator length parameter[†].

[†] For consistency with our convention that $u_c(a) > 0$ for the ^{12}C ground-state channel, we have to choose the sign of $u_{2s}(r)$ opposite to that in Barker (1961); otherwise our sign conventions are the same regarding the order of coupling of angular momenta.

a. $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transition

For this transition equation (115) becomes

$$\begin{aligned} \mu(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-) = & \frac{b}{a} [f^{\frac{1}{2}}(\frac{1}{2}^+, 0\frac{1}{2}) f^{\frac{1}{2}}(\frac{1}{2}^-, 0\frac{1}{2}) \\ & + \{ -\frac{1}{\sqrt{10}} f^{\frac{1}{2}}(\frac{1}{2}^+, 2\frac{3}{2}) + \frac{6}{\sqrt{15}} f^{\frac{1}{2}}(\frac{1}{2}^+, 2\frac{5}{2}) \} f^{\frac{1}{2}}(\frac{1}{2}^-, 2\frac{3}{2})] \quad (117) \end{aligned}$$

Values of $f^{\frac{1}{2}}(J^-, \bar{J}j)$ for the POT interaction of Cohen and Kurath (1965) and $f^{\frac{1}{2}}(J^+, \bar{J}j)$ from Jäger et al. (1971) are given in Table 17.

Table 17.

Shell model values of spectroscopic amplitudes $f^{\frac{1}{2}}(J^\pi, \bar{J}j)$

	$\bar{J} = 0$				$\bar{J} = 2$	
	$j = \frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
J^π						
$\frac{1}{2}^-$	0.783				-1.059	
$\frac{3}{2}^-$		0.433		0.910	0.561	
$\frac{1}{2}^+$	0.943				0.120	0.270
$\frac{5}{2}^+$			0.899	0.149	-0.095	-0.373

Using these values of spectroscopic amplitudes and $b = 1.666$ fm, from electron scattering on ^{13}C (Ajzenberg-Selove 1979), we obtain

$$\mu(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-) = 0.560/a \quad (118)$$

This value of $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ is plotted as a dashed curve in Fig. 17a as a function of a , while the solid curve gives the best fit value from section C.

It is seen that there is agreement between calculated and experimental values of $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{1}{2}^-)$ for $a \approx 5.5$ fm, but this particular value is probably not very significant. The calculated values are very sensitive to the values of $f^{\frac{1}{2}}(\frac{1}{2}^+, \bar{J}j)$; for example they are increased by a factor of 1.6 if these f -values are taken from Barker (1961), and by a factor of 2.3 if the one-channel approximation is used for the $\frac{1}{2}^+$ level ($f^{\frac{1}{2}}(\frac{1}{2}^+, 0\frac{1}{2}) = 1$). Thus, agreement could be obtained for smaller values of a , which are favoured from the fits in section C.3, if $f^{\frac{1}{2}}(\frac{1}{2}^+, 0\frac{1}{2})$ were somewhat smaller than the value of Jäger et al. Additional support for this comes from Fig. 16.

b. $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ transition

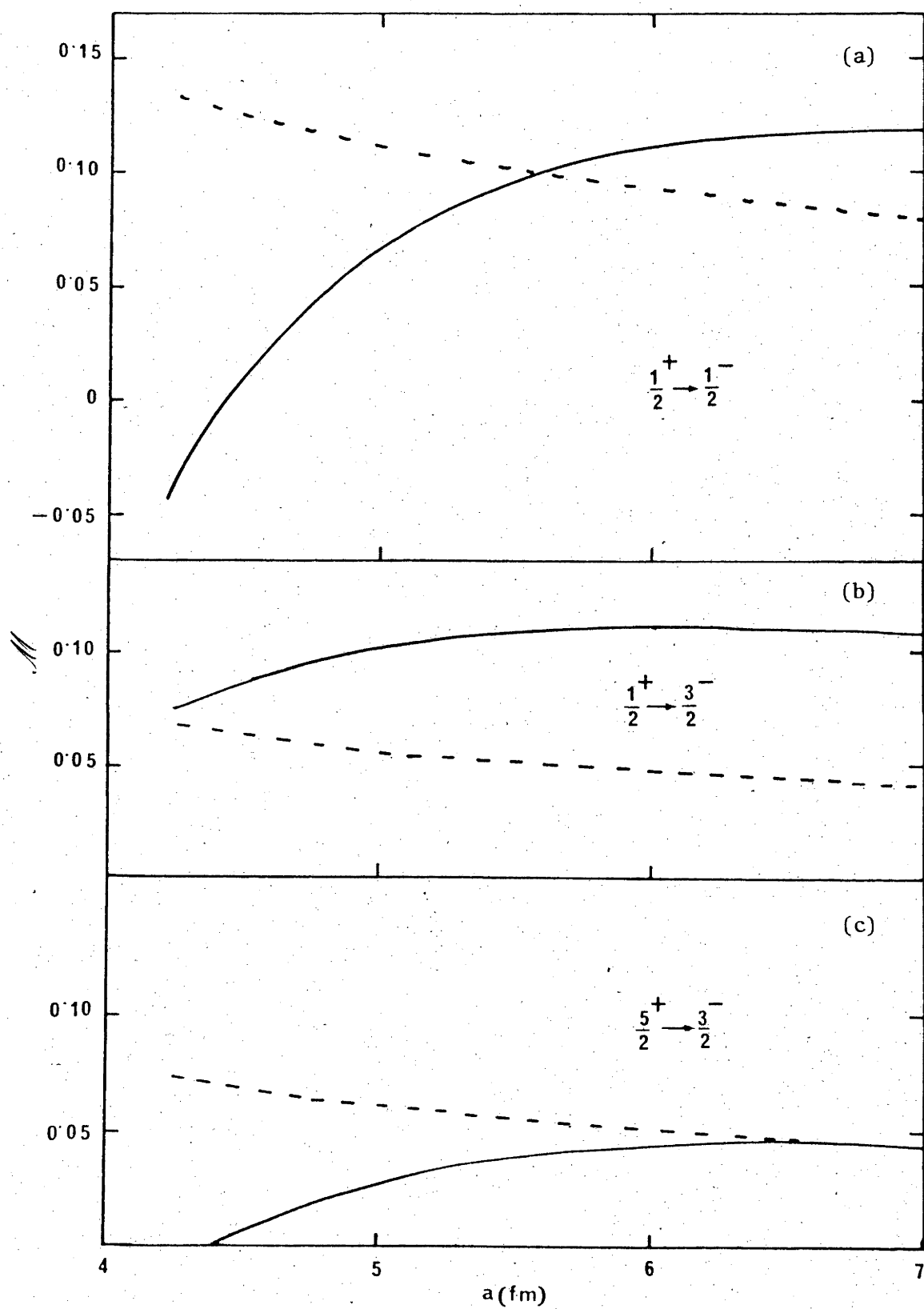
From equation (115) we obtain the formula for the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ transition as

$$\begin{aligned} \mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-) = & \frac{b}{a} [f^{\frac{1}{2}}(\frac{1}{2}^+, 0\frac{1}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 0\frac{3}{2}) - \frac{\sqrt{5}}{2} f^{\frac{1}{2}}(\frac{1}{2}^+, 2\frac{3}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2\frac{1}{2}) \\ & - \frac{1}{\sqrt{5}} f^{\frac{1}{2}}(\frac{1}{2}^+, 2\frac{3}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2\frac{3}{2}) - \frac{3}{\sqrt{30}} f^{\frac{1}{2}}(\frac{1}{2}^+, 2\frac{5}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2\frac{3}{2})] . \end{aligned} \quad (119)$$

Using spectroscopic amplitudes from Table 17 together with

$b = 1.666$ fm equation (119) becomes

Fig. 17 Values of the E1 internal transition moments $\mathcal{M}$ as functions of the channel radius a , for the transitions indicated. The solid curves are experimental values derived from the fits shown in Figs. 12, 14 and 15, using the upper solution in the $5/2^+ \rightarrow 3/2^-$ case since this gives better agreement. The dashed curves give the calculated shell model values.



$$\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-) = 0.288/a \quad (120)$$

Fig. 17b shows the calculated values of $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$ (dashed curve) together with the experimental values (solid curve). The small calculated value of $\mathcal{M}(\frac{1}{2}^+ \rightarrow \frac{3}{2}^-)$ is due to cancellation between the terms having ^{12}C ground-state as parent and those having ^{12}C excited states as parent, being only about 40% of the value in the one-channel approximation.

c. $\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$ transition

The formula for the $\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$ transition is obtained in a similar manner to that discussed earlier. Thus

$$\begin{aligned} \mathcal{M}(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-) = \frac{b}{a} [& \frac{\sqrt{10}}{2} f^{\frac{1}{2}}(\frac{5}{2}^+, 0 \frac{5}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 0 \frac{3}{2}) - \frac{2}{3} f^{\frac{1}{2}}(\frac{5}{2}^+, 2 \frac{1}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2 \frac{1}{2}) \\ & - \frac{1}{3} f^{\frac{1}{2}}(\frac{5}{2}^+, 2 \frac{3}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2 \frac{3}{2}) + \frac{\sqrt{35}}{6} f^{\frac{1}{2}}(\frac{5}{2}^+, 2 \frac{3}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2 \frac{1}{2}) \\ & - \frac{\sqrt{35}}{15} f^{\frac{1}{2}}(\frac{5}{2}^+, 2 \frac{3}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2 \frac{3}{2}) + \frac{\sqrt{35}}{5} f^{\frac{1}{2}}(\frac{5}{2}^+, 2 \frac{5}{2}) f^{\frac{1}{2}}(\frac{3}{2}^-, 2 \frac{3}{2})] \end{aligned} \quad (121)$$

giving

$$\mathcal{M}(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-) = 0.310/a, \quad (122)$$

The small calculated value of $\mathcal{M}(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-)$ is again due to the cancellation between the ground-state and excited state channel

contributions. Fig. 17c shows the calculated and experimental values of $\mathcal{M}(\frac{5}{2}^+ \rightarrow \frac{3}{2}^-)$.

E. LEVEL DISPLACEMENTS OF ^{13}C AND ^{13}N

In Fig. 10 the level energies of ^{13}C and ^{13}N are adjusted to make the ground-state energies the same. The actual binding-energy difference between analogue states of the same J^π , known as the Coulomb displacement energy, is defined by

$$\Delta E_c(J^\pi) = M(^{13}\text{N}, J^\pi) - M(^{13}\text{C}, J^\pi) + \delta_{np}, \quad (123)$$

where δ_{np} is the neutron-proton mass difference and all masses are atomic masses. δ_{np} has a value of 782 keV. The experimental values of ΔE_c for the $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states shown in Fig. 10 are 3003, 2280, 2830 and 2696 keV respectively (Ajzenberg-Selove 1976). The Coulomb displacement energies can be attributed to the electro-magnetic interaction if we assume charge symmetry of the specifically nuclear forces i.e. neutron-neutron interaction equals proton-proton interaction for the same space spin state. $\Delta E_c(J^\pi)$ would be zero if there were no isospin mixing or distortion of the states involved.

Thomas (1952) attempted to fit the net displacements of these levels, i.e. the differences of the Coulomb displacement energy for each pair of excited states from that for the ground-states. These are the energy differences apparent in Fig. 10. Thomas considered three contributions to the net displacements, due to the internal Coulomb interaction, the electromagnetic spin-orbit interaction and the different external wavefunctions in ^{13}C and ^{13}N . The last contribution he also called the boundary-condition level displacement. The internal Coulomb energies were not calculated but were estimated to vary by not more than ± 200 keV in the different states. The other two contributions were calculated assuming that each state could be represented as a single nucleon outside an inert ^{12}C ground-state core. The depression of the $\frac{1}{2}^+$ state in ^{13}N relative to that in ^{13}C was attributed mainly to the boundary-condition contribution, the Thomas-Ehrman effect.

Coulomb displacement energies have been studied in other mirror systems, and many different contributions to them have been considered. In a recent review article, Shlomo (1978) lists and discusses these contributions, in particular for systems with one particle or one hole outside closed shells. He finds that, in first-order perturbation theory, the inclusion of various correction terms (see Table 6 of Shlomo 1978) does not remove the discrepancy between calculated and experimental Coulomb displacement energies that exists for the point Coulomb interaction alone - this is the Okamoto - Nolen - Schiffer anomaly (Okamoto 1964, Nolen and Schiffer 1969). Shlomo considers also higher-order perturbation effects, such as isospin mixing in the core, and suggests that the discrepancy of about 7% still remaining may, for the lighter mirror nuclei, be attributed largely to charge-symmetry-breaking nuclear potentials.

We calculate the Coulomb displacement energies for the $^{13}\text{N} - ^{13}\text{C}$ pairs of states including the point Coulomb contribution, boundary condition contribution and contributions from various correction terms in the first order perturbation theory. The Coulomb contribution to the Coulomb displacement energy is calculated by assuming the point Coulomb interaction between protons, while the boundary condition contribution is assumed to arise from the nearby nucleon channels. We have neglected higher-order perturbation effects and the effects of charge-symmetry-breaking nuclear potentials. Thus we do not expect to obtain quantitative agreement with the experimental values of the level displacements as Shlomo did. We have used the shell model wavefunctions of Cohen and Kurath (1965) for the POT interaction for the negative parity states and of Jäger et al. (1971) for the positive parity states, including only the ^{12}C , 0^+ and 2^+ $T=0$ states as core states. The harmonic oscillator length parameter is taken to be $b = 1.666$ fm as before. Because of the complexity of the wavefunctions the calculation of some of the contributions is more complicated than in the cases considered by Shlomo. In particular, the boundary condition contribution, which Shlomo (1978) obtained by taking the difference of matrix elements of the Coulomb interaction calculated with harmonic oscillator and Woods-Saxon potentials, is here calculated using the Bloch operator (Bloch 1957, Lane and Robson 1966).

1. Isospin Distortion Formulae for Level Displacement Energies

The energy of level J^π to the first order in the charge-dependent effects (H^C and $S_{M_T} - \bar{S}_{T P_K P}$), is given by equation (19)

$$E_{M_T}(J^\pi) = E_{\frac{1}{2}}^O(J^\pi) + V_{\frac{1}{2}\frac{1}{2}}^C(J^\pi, M_T) \quad (124)$$

Since all the levels we consider have $T^P = \frac{1}{2}$ and only one such level is considered for each J^π , i.e. $k^P = 1$, we can drop the subscript $\frac{1}{2}^1$, as in chapter III. Then

$$E_{M_T}(J^\pi) = E^O(J^\pi) + V^C(J^\pi, M_T) \quad (125)$$

The Coulomb displacement energy is given by the energy difference between the corresponding levels of ^{13}N and ^{13}C with $M_T = -\frac{1}{2}$ and $M_T = \frac{1}{2}$ respectively

$$\begin{aligned} \Delta E_C(J^\pi) &\equiv E_{-\frac{1}{2}}(J^\pi) - E_{\frac{1}{2}}(J^\pi) \\ &= \Delta H^C(J^\pi) + \Delta L(J^\pi) \end{aligned} \quad (126)$$

where

$$H^C(J^\pi) = \langle \Psi(J^\pi, -\frac{1}{2}) | H^C | \Psi(J^\pi, -\frac{1}{2}) \rangle - \langle \Psi(J^\pi, \frac{1}{2}) | H^C | \Psi(J^\pi, \frac{1}{2}) \rangle \quad (127)$$

is the internal contribution to ΔE_C and

$$\begin{aligned} \Delta L(J^\pi) &= \langle \Psi(J^\pi, -\frac{1}{2}) | \mathcal{L}\{S_{-\frac{1}{2}}(J^\pi)\} - \mathcal{L}\{\bar{S}(J^\pi)\} | \Psi(J^\pi, -\frac{1}{2}) \rangle - \\ &\quad \langle \Psi(J^\pi, \frac{1}{2}) | \mathcal{L}\{S_{\frac{1}{2}}(J^\pi)\} - \mathcal{L}\{\bar{S}(J^\pi)\} | \Psi(J^\pi, \frac{1}{2}) \rangle \end{aligned} \quad (128)$$

is the boundary condition contribution to ΔE_C .

2. Calculation of Point Coulomb and Boundary Condition

Contribution to Level Displacement Energies

The main contribution to ΔE_c (equation 126) is the contribution from the point Coulomb interaction. Details of the calculation of the matrix elements between two shell model basis states for the point Coulomb interaction is given in chapter II. Using harmonic oscillator length parameters $b = 1.666$ fm we calculate the values of the contribution due to the point Coulomb interaction for the J^π states we have considered. These values are shown in Table 18 together with other contributions.

Table 18.

Contributions (in keV) to Coulomb displacement energies for ^{13}C and ^{13}N levels

Contribution	$J^\pi = \frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$
Point Coulomb	2923	2835	2936	2713
Boundary condition ($a_c = 5$ fm)	-130	-626	-215	-215
Centre-of-mass-motion	-84	-67	-82	-75
Finite size of nucleon	59	39	54	51
Magnetic interactions	48	16	61	-54
Vacuum polarization	17	13	16	15
p-n mass difference	26	36	26	36
Short-range correlation	59	24	52	30
Total (calculated)	2918	2270	2848	2501
Experimental	3003	2280	2830	2696
Discrepancy	85	10	-18	195

For $\Delta L(J^\pi)$ (equation 128) we include the contributions from nucleon channels $c \equiv \tilde{c}m_t$, where $m_t = -\frac{1}{2}$ for a proton and $\frac{1}{2}$ for neutron. Then

$$(c|\psi(J^\pi, M_T)\rangle = u_{\tilde{c}}(J^\pi, r_c) \mathcal{J}_{\tilde{c}}^{\frac{1}{2}}(J^\pi) (\tilde{T}^{\frac{1}{2}} M_T - m_t m_t | \frac{1}{2} M_T) . \quad (129)$$

The symbols appearing in equation (129) have the same significance as those appearing in equation (39) except that the labels J^π and M_T are given explicitly. Proceeding in the same manner as in equation (42) we obtain

$$\Delta L(J^\pi) = - \sum_{\tilde{c}} \frac{\hbar^2}{2m_{\tilde{c}} a_{\tilde{c}}} u_{\tilde{c}}^2(J^\pi, a_{\tilde{c}}) \mathcal{J}_{\tilde{c}}(J^\pi) \sum_{m_t} [(\tilde{T}^{\frac{1}{2}-\frac{1}{2}-m_t m_t} | \frac{1}{2} - \frac{1}{2})^2 S_{-\frac{1}{2}}(J^\pi, \tilde{c}m_t) - (\tilde{T}^{\frac{1}{2}-\frac{1}{2}-m_t m_t} | \frac{1}{2} \frac{1}{2})^2 S_{\frac{1}{2}}(J^\pi, \tilde{c}m_t)] . \quad (130)$$

Terms in $\Delta L(J^\pi)$ containing $\bar{S}(J^\pi)$ cancel. We include contributions to ΔL from the channels involving $A=12$ levels identified as belonging to the lowest shell model configuration (Cohen and Kurath 1965), i.e. the lowest 0^+ , 2^+ and 1^+ , $T=0$ states of ^{12}C at 0.0, 4.44 and 12.71 MeV and the lowest two $T=1$ states (1^+ and 2^+) of the ^{12}B , ^{12}C and ^{12}N triad (at 15.11 and 16.11 MeV in ^{12}C).

The calculation of $\Delta L(J^\pi)$ is similar to the calculation of L_{01} for ^8Be and L_{011k}^k for ^{16}O , as described in chapters III and IV. Table (18a) shows the contributions to $\Delta L(J^\pi)$ for the various channels considered and for channel radii of 4, 5 and 6 fm.

Table 18a.

Boundary condition contributions to ΔE_c (in keV) for various values of the channel radius

a_c (fm)	Channel	$J^\pi = \frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$
4	$0^+, T=0$	-171	-909	-169	-323
	$2^+, T=0$	-148	- 6	-230	- 31
	$1^+, T=0$	- 35	0	- 13	0
	$1^+, T=1$	19	0	19	0
	$2^+, T=1$	30	0	2	0
	Total	-305	-915	-391	-354
5	$0^+, T=0$	- 80	-623	-111	-201
	$2^+, T=0$	- 52	- 3	-105	- 14
	$1^+, T=0$	- 8	0	- 4	0
	$1^+, T=1$	4	0	4	0
	$2^+, T=1$	6	0	1	0
	Total	-130	-626	-215	-215
6	$0^+, T=0$	- 35	-398	- 73	-120
	$2^+, T=0$	- 17	- 1	- 44	- 5
	$1^+, T=0$	- 2	0	- 1	0
	$1^+, T=1$	1	0	1	0
	$2^+, T=1$	1	0	0	0
	Total	- 52	-399	-117	-125

The total boundary condition contributions for a channel radius of 5 fm are also shown in Table 18. The sum of the point Coulomb and boundary condition contributions is called ΔE_c^0 , which gives the main contribution to ΔE_c . Contributions to ΔE_c from the correction terms are discussed in the next section.

3. Correction Terms to the Level Displacement Energies

Contributions of various correction terms to ΔE_c for different mirror nuclei have been discussed by many authors. Among them, recently Shlomo (1978) has discussed the Coulomb energy differences of the $T = \frac{1}{2}$ mirror states of $A = 3, 15, 17, 39$ and 41 i.e. nuclei with $1p$ or $1h$ outside a closed shell, and the $T = 1$ ground-states of $A = 18$ and 42 nuclei which are nuclei with $2p$ outside a closed shell. In this section we have estimated the contributions of the correction terms to ΔE_c of the low-lying $T = \frac{1}{2}$ mirror states in $A = 13$, which are not described as single particle or hole states relative to closed shells.

As in Shlomo (1978) the corrections due to centre-of-mass motion and the finite size of the proton and neutron charge distributions are made by changing b , the harmonic oscillator length parameter. The finite nucleon size also gives an additional contribution due to the changed interaction between nucleons. We have therefore corrected for the approximation that the electromagnetic interaction is given by the point Coulomb interaction H^c by taking into account the effect of the finite size of the nucleon and adding the magnetic and vacuum polarization terms. We have also taken into account the dynamical effect of the neutron-proton mass difference, and the two-body short range correlations. Some of the matrix elements for various corrections have been calculated with harmonic oscillator radial wavefunctions and some of them have been estimated as percentages of the point Coulomb contributions or ΔE_c^0 .

a. Centre-of-mass motion

In evaluating the harmonic oscillator length parameter b , for the shell model charge distribution $\rho_{SM}(r)$ we assumed that the centre of mass of the nucleus (in particular the centre of mass of the protons) coincides with the centre of mass of the single particle potential well. We used a value of $b = 1.666$ fm obtained from (Elton 1961)

$$\langle r^2 \rangle_{SM} = \left(\frac{Z-2}{Z} + \frac{3}{2} \right) b^2 = \frac{13}{6} b^2, \quad (131)$$

where $\langle r^2 \rangle_{SM}^{1/2}$ is the root mean square (r.m.s) radius for the harmonic oscillator potential well which we equate to the experimental r.m.s charge radius $\langle r^2 \rangle_I^{1/2} = 2.45$ fm obtained from $^{13}\text{C}(e,e)^{13}\text{C}$ (Ajzenberg-Selove 1976). To take into account the effect of the centre of mass motion of the protons one should instead use

$$\langle r^2 \rangle_I = \langle r^2 \rangle_{SM} - \frac{1}{A} \frac{3}{2} b^2. \quad (132)$$

With $A=13$ this gives $b = 1.711$ fm. The effect of the centre of mass motion is therefore taken to be a decrease of ΔE_C^0 by 3.0% and this is shown in Table 18.

b. Finite size-effects of the proton and neutron charge

In calculating the contributions due to the point Coulomb interaction we have assumed an interaction between point protons. But experiments of electron scattering on protons show that the proton charge distribution is given by (Elton 1961)

$$\rho_{pc} = \frac{1}{8\pi a^3} e^{-r/a}, \quad \text{with } a = a_p/\sqrt{12}, \quad (133)$$

where $a_p = 0.80$ fm is the r.m.s radius, i.e. $a_p^2 = \langle r^2 \rangle_{pc}$.

This charge distribution of protons affects the calculation of ΔE_c in two ways.

- (i) in the size of the potential well
- (ii) in the modification of the interaction between two protons at short distances ($r \leq 2$ fm).

Shlomo (1978) has shown that to take account of the charge distribution of protons the formula for the r.m.s radius should be replaced by

$$\langle r^2 \rangle_c = \langle r^2 \rangle_{SM} + \langle r^2 \rangle_{pc}. \quad (134)$$

Using $\langle r^2 \rangle_c^{1/2} = 2.45$ fm as before, equation (134) gives a value of $b = 1.573$ fm. As a result of this, the effect of the finite size of the proton is to increase the point Coulomb contribution by 5.7%. Similarly for the finite size of the neutron the R.H.S of equation (134) is replaced by

$$\langle r^2 \rangle_c = \langle r^2 \rangle_{SM} + \langle r^2 \rangle_{pc} + \frac{N}{Z} \langle r^2 \rangle_{nc}, \quad (135)$$

With $\langle r^2 \rangle_{nc} = -0.12$ fm² we obtain $b = 1.598$ fm. Thus the effect of the finite size of neutrons is to decrease the point Coulomb interaction by 1.2%. Column A of Table (18b) shows the values of the contributions to ΔE_c due to the effect of the finite nucleon size on the size of the potential well.

In addition to its effects on the size of the potential well, the nucleon charge distribution also modifies the interaction between two nucleons. Using equations (4.12) and (4.15) of Shlomo we obtain contributions to ΔE_c due to proton-proton (besides the point Coulomb interaction) and neutron-neutron interactions. It may be mentioned here that since in ^{13}C and ^{13}N there are equal numbers of neutron-proton pairs there is no contribution to ΔE_c due to the neutron-proton interaction. Column B of Table 18b shows contributions to ΔE_c due to the modified proton-proton and neutron-neutron interactions. In Table 18 the combined effects due to the

Table 18b.

Contributions to ΔE_c (in keV) of finite size effects of the proton and neutron charge

Level	A		B		Combined A and B
	Size of the potential well Proton	Neutron	Interaction Proton	Neutron	
$\frac{1}{2}^-$	166	-34	-58	-15	59
$\frac{1}{2}^+$	161	-33	-72	-17	39
$\frac{3}{2}^-$	166	-34	-62	-16	54
$\frac{5}{2}^+$	154	-31	-58	-14	51

modification in the size of the potential well and the modification due to the interaction are shown.

(c) Magnetic interaction

Now we shall discuss the contribution of the magnetic interactions (also known as the relativistic electromagnetic terms) to ΔE_c . The contributions from various terms are calculated using equations (4.19, a-d) of Shlomo (1978) and applying standard shell model techniques (Elliott 1953). Calculation of these terms is similar to the calculation of Coulomb matrix elements described in chapter II, except that the terms containing spin operators are more complicated. Table 18c shows the contributions of various terms to ΔE_c due to the magnetic interactions. The tensor term in Table 18c for positive parity states vanishes due to the assumption that the core has $S = 0$.

Table 18c.

Contributions to ΔE_c (in keV) due to magnetic interactions

Interaction	$J^\pi = \frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$
Orbit-orbit	4.5	18.9	3.9	4.6
Spin-orbit	42	-6	56	-63
Tensor	1.5	0	0.5	0
$\rho_p(r)$ term	0.3	3.2	0.8	4.3
Total	48	16	61	-54

In Table 18 the total contributions due to the magnetic interaction are given.

d. Vacuum Polarization

Vacuum polarization is another correction to the electromagnetic interaction between two protons. The virtual emission and absorption of electron-positron pairs calculated to lowest order in the fine structure constant gives an additional repulsive potential. The effect of this is to increase ΔE_C^0 by 0.6% (Shlomo 1978). The contributions to ΔE_C due to vacuum polarization for different states are shown in Table 18.

e. Dynamical effect of neutron-proton mass difference

In evaluating ΔE_C^0 we have assumed the proton mass is equal to the neutron mass. We now correct this assumption since the neutron mass is 0.14% greater than the proton mass. According to Shlomo (1978) this changes the point Coulomb contribution by 0.07% and the contributions for various states are shown in Table 18.

f. Short-range two-body correlations

Shlomo (1978) has shown that the main effect of the short-range correlations is to modify the $\ell=0$ matrix elements. In particular the $1s$ correlated matrix elements are about 12% larger than the corresponding harmonic oscillator values. With this we obtain the contributions to ΔE_C as shown in Table 18.

4. Comparison with Thomas's Calculations

Some comments may be made on the values in Table 18 in relation to the calculations of Thomas (1952). Although Thomas did not consider many of the terms that give large contributions to the Coulomb displacement energies, he did include those that contribute most to the net displacements. He included the boundary condition contribution only for the ^{12}C ground-state channels, obtaining values close to those in Table 18 except for the $\frac{1}{2}^-$ states. For these, in order to obtain a better fit to the net displacement for the $\frac{3}{2}^-$ levels, he preferred a solution with a small reduced width of the $\frac{1}{2}^-$ states (corresponding to his region I) rather than the solution with larger reduced width (region II), and so obtained a contribution of only -27 keV. Such a small reduced width is, however, in conflict with more recent experimental and calculated values (see section C.3 and Fig. 16). The spin-orbit contributions in Table 18c agree reasonably with the values obtained by Thomas, except for the $\frac{3}{2}^-$ states for which his value was -20 keV. This disagreement indicates the deficiency of Thomas's model for the $\frac{3}{2}^-$ states of a $p_{3/2}$ nucleon outside a spin-zero core.

5. Nolen-Schiffer Anomaly

The total contributions to ΔE_c are given at the bottom of Table 18 for each J^π value. The discrepancies between the calculated and experimental values of Coulomb displacement energies shown at the bottom of Table 18 are examples of the Nolen-Schiffer anomaly. However it seems that these discrepancies which range from

-0.6% to 7% of the total values are somewhat less systematic than the 3% to 9% deviation obtained by Shlomo (1978) for other mirror systems (see Shlomo's Table 6). A change of $\pm 2\%$ in the value of the harmonic oscillator length parameter b , which is allowed by the experimental error in the ^{13}C r.m.s radius, would change the discrepancies by about $\pm 2\%$. There is also a sensitive dependence of our calculated values on the choice of channel radius, since one of the main contributions to the calculated value is the boundary condition contribution and this depends sensitively on the channel radius (Table 18a). If the channel radius is changed from 5 to 4.5 fm, the discrepancies in Table 18 are changed to 167, 166, 66 and 267 keV for $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^-$ and $\frac{5}{2}^+$ states respectively, which are 2% to 11% of the total values. In this regard it may be mentioned here that though the dependence of ΔL (finally ΔE_c) on channel radius arises due to the use of Bloch operators, one of the main advantages of this method is that it allows the use of correct asymptotic forms of the neutron and proton wavefunctions. This is important because of the long-range character of the Coulomb forces, which give the main contribution to ΔE_c . Moreover, it has been observed in the R-matrix analysis (section C) that a value of channel radius of about 4 to 5 fm is reasonable. The sensitive dependence of ΔE_c on channel radius in our calculation is not apparent in Shlomo's calculation where the boundary condition contribution is calculated in a different way. If we use our approach for the $A=17$ cases considered by Shlomo with his simple wavefunctions, then we get agreement with his boundary-condition contributions of about -100 and -570 keV for $\frac{5}{2}^+$ and $\frac{1}{2}^+$ states,

for channel radii of 5.2 fm and 4.6 fm respectively. It may be mentioned that these are not values of Shlomo's Coulomb perturbation effect, which he refers to as the Thomas-Ehrman effect.

Thus it seems that for reasonable values of the channel radius, the discrepancies between the calculated and experimental values of the Coulomb displacement energies for these pairs of states in ^{13}C and ^{13}N are not inconsistent with those found by Shlomo in other light nuclei, and presumably they can likewise be attributed to a charge-symmetry-breaking part of the nuclear potential.

F. DISCUSSION

We have attempted to fit the properties of the low-lying states of ^{13}C and ^{13}N using the R-matrix theory of nuclear reactions. Among the properties of these states which show an effect of isospin distortion are the marked difference in excitation energies of the $\frac{1}{2}^+$ first excited states (Fig. 10) and the very different strengths of the E1 transition between these $\frac{1}{2}^+$ states and the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states (Table 11). These differences would not exist if there were exact charge symmetry of all nuclear forces.

We have attempted to fit these and other properties of the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states in a consistent R-matrix description with a two-level approximation for the $\frac{1}{2}^+$ levels and a one-level approximation for the $\frac{1}{2}^-$ levels, and to relate the resultant level parameters with those obtained from shell model calculations. Departures from charge symmetry were obtained by including effects of the Coulomb and other

electromagnetic interactions and of binding energy differences. An exact fit to the $\frac{1}{2}^+$ excitation energy difference was not sought, since we did not include the effects of a charge-symmetry-breaking nuclear potential, which is believed to contribute to Coulomb displacement energies in other cases of light mirror nuclei (Shlomo 1978); nevertheless, the main part of the energy difference appears to be attributable to the boundary condition contribution (Thomas-Ehrman effect), as found previously (Thomas 1952).

A least-squares fit of all the properties involving the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states gives a good account of the different E1 strengths of the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transitions (Table 11), with level parameters agreeing reasonably with shell model values (Figs. 16, 17). In previous discussions of the different E1 strengths, it was initially suggested (Robinson et al. 1968; Warburton and Weneser 1969) that the asymmetry was due to differences in the external radial wavefunctions for the $\frac{1}{2}^+$ states, which are indicated by the large Thomas-Ehrman shift. However Marrs et al. (1975), using a simple one-body model for the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states, with Woods-Saxon wells generating the radial wavefunctions, found no difference in the strengths of the two transitions. They concluded that simple binding energy effects of this kind could not explain the large asymmetry, and that charge-dependent configuration mixing was required. Kurath (1975) took up this suggestion and assumed that the expansion coefficients in the description (109) of the $\frac{1}{2}^+$ states were different for ^{13}C and ^{13}N , the difference being due to their different binding energies. Kurath, however, used inconsistent definitions of $B(\text{E1})$ in deriving his calculated and experimental values. The experimental values he quotes are in units of fm^2 . For consistency the factor e^2 in his equation (2)

should then be replaced by $(6/13)^2$. Fitting the experimental values then requires $\alpha_s = 0.99$, $\alpha_d = 0.15$ for ^{13}C and $\alpha_s = 0.99$, $\alpha_d = -0.11$ for ^{13}N , and Kurath's argument does not explain the difference in sign of the α_d values. According to Kurath (private communication) this difficulty can be overcome by extending the weak coupling model for the $\frac{1}{2}^+$ states by a third component, a $d_{5/2}$ nucleon coupled to the lowest 2^+ $T=1$ state. It is not clear why the expansion coefficients are assumed to be sensitive to binding energy effects whereas the radial wavefunctions are not. Fox et al. (1975) used a coupled-channel approach, but this work is still unpublished. In the present treatment, the source of the different strengths is best discussed in terms of equation (96), applied to the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transitions in ^{13}C and ^{13}N . The factor f_{if} gives no difference in the strengths, which are obtained by dividing Γ_Y^0 by E_Y^3 . The internal transition moment μ_{if} is assumed to be the same for each transition, also the factors N_f and θ_f for the $\frac{1}{2}^-$ state and θ_i for the $\frac{1}{2}^+$ state are approximately the same. The difference comes from the factors N_i (for the $\frac{1}{2}^+$ state) and J_{if} . For example, for $a = 5$ fm, one has $N_i(^{13}\text{N})/(N_i(^{13}\text{C})) = 0.66$ and $J_{if}(^{13}\text{N})/J_{if}(^{13}\text{C}) = 2.25$. In order to fit the observed asymmetry, one then needs the internal contributions to be small compared with the channel contributions. This is as expected from the shell model calculations of Cohen and Kurath (1965) and Jäger et al. (1971), which predict a small value of μ_{if} due to the terms involving ^{12}C excited state as parent largely cancelling the term involving ^{12}C ground-state (see equation (117)).

Similarly for the $\frac{3}{2}^-$ to $\frac{1}{2}^+$ transition the asymmetry between the transitions in ^{13}C and ^{13}N could be explained in terms of equation (96) applied to the $\frac{3}{2}^-$ to $\frac{1}{2}^+$ transition. For this transition N_i and θ_i for the $\frac{3}{2}^-$ state and θ_f for the $\frac{1}{2}^+$ state are approximately the same and the difference comes from N_f and J_{if} . For example, for a channel radius of 5 fm $N_f(^{13}\text{N})/N_f(^{13}\text{C}) = 0.66$ and $J_{if}(^{13}\text{N})/J_{if}(^{13}\text{C}) = 1.60 - 2.61i$. The observed asymmetry requires small internal contributions which is what the shell model calculations predict.

In summary, acceptable fits have been obtained to observed properties of low-lying states of ^{13}C and ^{13}N , including the asymmetries in the excitation energies of the $\frac{1}{2}^+$ states and in the E1 strengths of the $\frac{1}{2}^+$ to $\frac{1}{2}^-$ and $\frac{3}{2}^-$ transitions, which are indications of isospin distortion in the states. The resultant parameter values agree reasonably with shell model predictions.

REFERENCES

- Adelberger, E.G., (1974). Proc. Internat. Conf. on Nuclear Structure and Spectroscopy, Amsterdam, ed. H.P. Blok and A.E.L. Diepernik (Scholar's Press, Amsterdam) p. 641.
- Ajzenberg-Selove, F. (1976). Nucl. Phys. A268, 1.
- Ajzenberg-Selove, F., Maguire, C.F., Hendrie, D.L., Scott, D.K., and Mahoney, J. (1976). Phys. Rev. C13, 46.
- Ajzenberg-Selove, F. (1977). Nucl. Phys. A281, 1.
- Ajzenberg-Selove, F. (1979). Nucl. Phys. A320, 1.
- Anderson, R.K., and Goldhammer, P. (1971). Phys. Rev. Lett. 26, 978.
- Anderson, R.K., Wilson, M.R., and Goldhammer, P. (1972). Phys. Rev. C6, 136.
- Andreev, G.B., Klyucharev, A.P., and Slabospitskii, R.P. (1973). Izv. Akad. Nauk SSSR (ser. fiz.) 37/1, 183 (English translation: Bull. Acad. Sci. USSR Phys. Ser. 37/1, 160).
- Armstrong, J.C., Baggett, M.J., Harris, W.R., and Latorre, V.A. (1966). Phys. Rev. 144, 823.
- Arima, A., Manakos, P., and Strottman, D. (1975). Phys. Lett. 60B, 1.
- Barker, F.C. (1961). Nucl. Phys. 28, 96.
- Barker, F.C. (1966). Nucl. Phys. 83, 418.
- Barker, F.C. (1967). Aust. J. Phys. 20, 341.
- Barker, F.C. (1972). Aust. J. Phys. 25, 341.
- Barker, F.C. (1976). Phys. Lett. 64B, 251.
- Barker, F.C. (1978). Aust. J. Phys. 31, 27.
- Bashkin, S., and Richards, H.T. (1951). Phys. Rev. 84, 1124.

- Bennett, L.A. (1976). Ph.D. Thesis, Australian National University.
- Bertsch, G.F., and Mekjian, A.Z. (1972). *Ann. Rev. Nucl. Sci.* 22, 25.
- Blatt, J.M., and Weisskopf, V.F. (1952). *Theoretical Nuclear Physics* (J. Wiley, New York).
- Blatt, S.L., Marolt, G.L., and Goss, J.D. (1974). *Phys. Rev.* C10, 1319.
- Bloch, C. (1957). *Nucl. Phys.* 4, 503.
- Brown, L., Steiner, E., Arnold, L.G., and Seyler, R.G. (1973). *Nucl. Phys.* A206, 353.
- Burbidge, E.M., Burbidge, G.R., Fowler, W.A., and Hoyle, F. (1957). *Rev. Mod. Phys.* 29, 547.
- Callender, W.D., and Browne, C.P. (1970). *Phys. Rev.* C2, 1.
- Christy, R.F., and Duck, I. (1961). *Nucl. Phys.* 24, 89.
- Cohen, S., and Kurath, D. (1965). *Nucl. Phys.* 73, 1.
- Cohen, S., and Kurath, D. (1967). *Nucl. Phys.* A101, 1.
- Dalton, B.J., and Robson, D. (1966). *Phys. Lett.* 20, 405.
- Darden, S.E., Sen, S., Hiddleston, H.R., Aymar, J.A. and Yoh, W.A. (1973). *Nucl. Phys.* A208, 77.
- DeVries, R.M. (1973). *Phys. Rev.* C8, 951.
- Elliott, J.P. (1953). *Proc. Roy. Soc.* 218, 345.
- Elliott, J.P. and Flowers, B.H. (1957). *Proc. Roy. Soc.* A242, 57.
- Elton, L.R.B. (1961). *Nuclear Sizes* (Oxford, Clarendon Press).
- Faessler, A. (1965). *Nucl. Phys.* 65, 329.
- Feshbach, H., Peaslee, D.C., and Weisskopf, V.F. (1947). *Phys. Rev.* 71, 145.
- Fisher, G.A., Paul, P., Riess, F., and Hanna, S.S. (1976). *Phys. Rev.* C14, 28.

- Fortune, H.T., Gray, T.J., Trost, W., and Fletcher, N.R. (1969).
 Phys. Rev. 179, 1033.
- Fowler, W.A., Lauritsen, C.C., and Lauritsen, T. (1948). Rev.
 Mod. Phys. 20, 236.
- Fox, G., Polchinski, J.G., Rolfs, C., and Tombrello, T.A. (1975).
 Caltech Lemon Aid Preprint LAP-144 "Charge Symmetry for
 Mirror γ -ray Transitions in ^{13}N and ^{13}C ".
- Franey, M.A., Lilley, J.S., and Phillips, W.R. (1979). Nucl. Phys.
A324, 193.
- Gillet, V. and Vinh Mau, N. (1964). Nucl. Phys. 54, 321.
- Glockner, D.H., and Lawson, R.D. (1975). Phys. Lett. 56B, 301.
- Glover, R.N., and Jones, A.D.W. (1966). Nucl. Phys. 84, 673.
- Goldhammer, P. (1975). Phys. Rev. C11, 1422.
- Goldhammer, P., Hill, J.R., and Nachamkin, J. (1968). Nucl. Phys.
A106, 62.
- Gubler, H.P., Plattner, G.R., Sick, I., Traber, A., and Weiss, W.
 (1977). Nucl. Phys. A284, 114.
- Harms, U. (1968). Z. Naturf. 23a, 453.
- Heaton, H.T., Menke, J.L., Schrack, R.A., and Schwartz, R.B. (1975).
 Nucl. Sci. and Engin. 56, 27.
- Hebbard, D.F., and Vogl, J.L. (1960). Nucl. Phys. 21, 652.
- Heil, V., and Stock, W. (1976). Phys. Lett. 65B, 412.
- Holt, R.J., Jackson, H.E., Laszewski, R.M., Monahan, J.E., and
 Specht, J.R. (1978). Phys. Rev. C18, 1962.
- Jackson, H.L., and Galonsky, A.I. (1953). Phys. Rev. 89, 370.
- Jäger, H.U., Kissener, H.R., and Eramzhian, R.A. (1971). Nucl.
 Phys. A171, 16.

Kane, J.V., Pixley, R.E., and Wilkinson, D.H. (1960). Phil. Mag.

5, 365.

Karban, O., Basak, A.K., England, J.B.A., Morrison, G.C., Nelson, J.M.,

Roman, S., and Shute, G.G. (1976). Nucl. Phys. A269, 312.

Kerr, R.G. (1967). Thesis (University of Wisconsin).

Koester, L., and Nistler, W. (1975). Zeit. f. Phys. A272, 189.

Kull, L.A. (1967). Phys. Rev. 163, 1066.

Kumar, N. (1974). Nucl. Phys. A225, 221.

Kurath, D. (1975). Phys. Rev. Lett. 35, 1546.

Lachkar, J.C. (1977). Proc. Int. Specialists Symp. on Neutron

Standards and Applications, Gaithersburg, Maryland, March 28-31,

1977. NBS Special Publication 493, p. 93.

Lane, A.M. (1960). Rev. Mod. Phys. 32, 519.

Lane, A.M., and Thomas, R.G. (1958). Rev. Mod. Phys. 30, 257.

Lind, J.M., Garvey, G.T., and Tribble, R.E. (1977). Nucl. Phys.

A276, 25.

Ludwig, E.J., Busch, C.F., Clegg, T.B., Datta, S.K., and Watkins, A.C.

(1974). Nucl. Phys. A230, 271.

McCarthy, R.J., and Walker, G.E. (1974). Phys. Rev. C9, 809.

MacDonald, W.M. (1955). Phys. Rev. 100, 51.

MacDonald, W.M. (1956). Phys. Rev. 101, 271.

Macklin, R.L., and Gibbons, J.H. (1958). Phys. Rev. 109, 105.

Malmberg, P.R. (1956). Phys. Rev. 101, 114.

Marion, J.B. (1965). Phys. Lett. 14, 315.

Marrs, R.E., Adelberger, E.G., Snover, K.A., and Cooper, M.D.

(1975). Phys. Rev. Lett. 35, 202.

Miska, H., Gräf, H.D., Richter, A., Schüll, D., Spamer, E., and

Titze, O. (1975). Phys. Lett. 59B, 441.

- Nair, K.G., Voit, H., Hamm, M., Towsley, C., and Nagatani, K.
(1974). Phys. Rev. Lett. 33, 1588.
- Nair, K.G., Voit, H., Towsley, C.W., Hamm, M., Bronson, J.D.,
and Nagatani, K. (1975). Phys. Rev. C12, 1575.
- Newson, H.W., Williamson, R.M., Jones, K.W., Gibbons, J.H.,
and Marshak, H. (1957). Phys. Rev. 108, 1294.
- Nolen, J.A. Jr., and Schiffer, J.P. (1969). Ann. Rev. Nucl. Sc.
19, 471.
- Okamoto, K. (1964). Phys. Lett. 11, 150.
- Oothoudt, M.A., and Garvey, G.T. (1977). Nucl. Phys. A 284,
41.
- Paul, P. (1966). Z. Naturf. 21a, 914.
- Pearson, C.A., Covan, J.M., Zissermann, D., Miller, T.G.,
Gibson, F.P., Haglund, R., Morrison, W., and Westley, G.
(1972). Nucl. Phys. A191, 1.
- Perry, R.R., Mainsbridge, B., and Rickards, J. (1963). Nucl.
Phys. 45, 586.
- Radicati, L.A. (1953). Proc. Phys. Soc. A66, 139; A67, 39.
- Riech, V. (1963). Phys. Lett. 6, 267.
- Riess, F., Paul, P., Thomas, J.B., and Hanna, S.S. (1968).
Phys. Rev. 176, 1140.
- Robinson, S.W., Swann, C.P., and Rasmussen, V.K. (1968).
Phys. Lett. 26B, 298.
- Rolfs, C. (1973). Nucl. Phys. A217, 29.
- Rolfs, C., and Azuma, R.E. (1974). Nucl. Phys. A227, 291.
- Schiffer, J.P., Morrison, G.C., Siemssen, R.H., and Zeidman, B.
(1967). Phys. Rev. 164, 1274.

- Seagrave, J.D. (1951). Phys. Rev. 84, 1219.
- Seagrave, J.D. (1952). Phys. Rev. 85, 197.
- Shlomo, S. (1978). Rep. Prog. Phys. 41, 957.
- Soper, J.M. (1969). Isospin in Nuclear Physics, ed.
D.H. Wilkinson (North-Holland Publishing Company,
Amsterdam), p. 229.
- Sundberg, O., and Källne, J. (1969). Ark. Fys. 39, 323.
- Taketani, H., Muto, J., Yamaguchi, H., and Kokame, J. (1968).
Phys. Lett. 27B, 625.
- Thomas, R.G. (1952). Phys. Rev. 88, 1109.
- Tombrello, T.A., and Parker, P.D. (1963). Phys. Rev. 131, 2582.
- Tryti, S., Holtebekk, T., and Ugletveit, F. (1975). Nucl. Phys.
A251, 206.
- Van Patter, D.M. (1949). Phys. Rev. 76, 1264.
- Varma, S., and Goldhammer, P., (1969). Nucl. Phys. A125, 193.
- Vogl, J.L. (1963). Ph.D. Thesis, California Institute of Technology.
- Von Oertzen, W., and Bohlen, H.G. (1975). Physics Reports 19, 1.
- Warburton, E.K., Alburger, D.E., and Millener, D.J. (1980). Phys.
Rev. C22, 2330.
- Warburton, E.K., and Weneser, J. (1969). Isospin in Nuclear Physics,
ed. D.H. Wilkinson (North-Holland Publishing Company, Amsterdam),
p. 173.
- Woodbury, H.H., Tollestrup, A.V., and Day, R.B. (1954). Phys. Rev.
93, 1311.
- Young, F.C., Armstrong, J.C., and Marion, J.B. (1963). Nucl. Phys.
44, 486.
- Zeller, A.F., Kemper, K.W., Weisser, D.C., Ophel, T.R., Hebbard, D.F.,
and Johnston, A. (1979). Nucl. Phys. A323, 477.