

Direct Measurement of the Spectral Distribution of Thermal Noise

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Declaration

This thesis is an account of research undertaken between February 1999 and March 2003 at The Department of Physics, Faculty of Science, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Important contributions made by others are appropriately referenced in the text.

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Abstract

This thesis investigates the direct measurement of the thermal noise spectral distribution. Long base line gravitational wave detectors, being commissioned around the world, are limited in sensitivity in the intermediate frequencies by the thermal noise. These detectors are utilising suspended test mirrors for the detection of gravitational waves by measuring their relative displacement. One of the fundamental noise sources in these detectors is the thermally induced displacement of the suspension onto and within the mirrors. This thermally induced motion of the test mirrors limits the displacement sensitivity of the gravitational wave detectors. Knowledge of the spectral behavior of thermal noise over a wide frequency range will improve predictions and understanding of the behavior of the suspension and test mirrors.

In this thesis the direct measurement of the thermal noise spectral distribution of a mechanical flexure resonator is described. The mechanical flexure resonator is an unidirectional 'wobbly table' made from copper-beryllium, which hinges around four thin flexures 15 mm wide, 1 mm high and $\sim 116 \mu\text{m}$ thick. The mechanical flexure resonator has a resonant frequency of 192 Hz, with a quality factor of ~ 3000 .

The thermal noise induced displacement of the mechanical flexure resonator was measured using an optical cavity. The end mirror of a two mirror optical cavity was mounted on the mechanical flexure resonator. A laser was made resonant with the test cavity by use of a locking control system. Thermal noise induced displacement moved the test cavity away from resonance. By measuring the error-signal in the control system, the equivalent thermal noise displacement was obtained.

The thermal noise induced displacement of the mechanical flexure resonator was predicted to be in the order of 10^{-12} to $10^{-17}\text{m}/\sqrt{\text{Hz}}$ over a frequency range of 10 Hz to 10 kHz. All other external noise sources needed to be suppressed to below this level. A major noise source was the laser frequency fluctuations. When the test cavity was locked to the laser, the laser frequency fluctuations dominated the read out signal. To suppress the frequency fluctuations, the laser was locked to a rigid long optical reference cavity. This allowed the frequency fluctuations to be suppressed to below the equivalent thermal noise displacement of the test cavity over the frequency range of interest.

Acoustic noise was suppressed by placing the whole experiment inside a vacuum chamber, and evacuating the air inside the chamber down to a pressure level of 10^{-4} mbar. A seismic vibration isolation system was used to suppress the seismic noise in the laboratory to below $10^{-14}\text{m}/\sqrt{\text{Hz}}$ at frequencies above 4 Hz.

With the experimental set up, the thermal noise displacement of the mechanical flexure resonator has been measured. Due to the degradation of the isolator performance, measurement of the thermal noise behavior over a wide frequency range of the mechanical flexure resonator was unsuccessful. By using an analytical curve fitting routine around the fundamental and first order resonant modes of the resonator,

a loss factor of $(3.5 \pm 1.5 - 3.7 \pm 1.5) \times 10^{-4}$ for the copper-beryllium mechanical flexure resonator was obtained and structural damping was inferred.

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