Correspondence

Comments on "Distributed event-triggered control of multi-agent systems with combinational measurements"☆

1. Comments on Fan, Feng, Wang, and Song (2013) and discussions on Zeno triggering

1.1. The existence of Zeno triggering

This note points out some incorrect statements in Fan et al. (2013) concerning the triggering behavior of the event-triggered consensus algorithm for multi-agent systems. We show that, in contrast to the claims in Fan et al. (2013), the two event-based consensus algorithms proposed in Fan et al. (2013) cannot guarantee a Zeno-free triggering for the multi-agent system. A remedy is also suggested to achieve a truly Zeno-free event-triggered consensus for multi-agent systems. In this note we follow the same notations as in Fan et al. (2013).

Lemma 4 in Fan et al. (2013) claims that if $q_i(0) \neq 0$ and t_k^i exists with $q_i(t_k^i) \neq 0$, then agent i will not exhibit Zeno triggering for all $t > t_k^i$. In the following we show, however, that this is not true. In the proof of Lemma 4 in Fan et al. (2013), a lower bound on the inter-event interval $t_{k+1}^i - t_k^i$ is derived, which is proportional to $\|q_i(t_k^i)\|$ (see Eq. (16) of Fan et al. (2013)). Note that if $q_i(t^*) \to 0$ with $t \to t^*$ from below, then such a lower bound also converges to zero, which thus cannot guarantee the exclusion of Zeno behavior. Since Lemma 4 is the basis of the two main results (Theorems 5 and 6) in Fan et al. (2013), the statements in the main results of Fan et al. (2013) should be properly adjusted. That is, even if one assumes that no agent is initially located at the center of its neighbors, Zeno triggering may still occur. In this note we show that if $q_i(t^*) = 0$ for some finite t^*, then *there exists Zeno triggering*.

Lemma 1. *With the same consensus controller, event condition and event functions as in* Fan et al. (2013), *if at a finite time t^*, there exists $q_i(t^*) = 0$ for agent i, then agent i exhibits Zeno triggering for $t \to t^*$.*

Proof. Define t^* to be the first time on $[0, \infty)$ at which $q_i(t^*) = 0$. (The case that there are multiple isolated time instants $t_1^*, t_2^*, \ldots, t_\ell^*, \ldots$ at which $q_i(t_\ell^*) = 0$ occurs can be treated in a similar manner by redefining the time interval as $[\tilde{t}, \infty)$ in which t_ℓ^* with $t_\ell^* > \tilde{t}$ is the first time with $q_i(t_\ell^*) = 0$ in the interval $[\tilde{t}, \infty)$. Since these
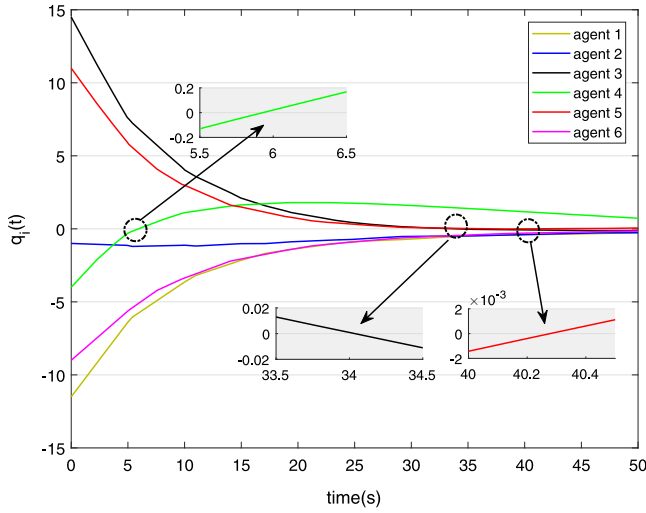
finite time instants are isolated, such an interval is well defined.) The statement that $q_i(t^*) = 0$ (or equivalently $\|q_i(t^*)\| = 0$) for agent i at $t = t^*$ implies that as $t \to t^*$ from below there holds $q_i(t^*) \to 0$ by the *continuity* of $q_i(t^*)$. Note that between two successive triggering time instants, the measurement error magnitude $\|e_i(t)\|$ increases from zero and evolves *continuously* until the next triggering condition (i.e. the equality (4) in Fan et al. (2013)) is met. We now prove there is an *infinite sequence* of triggering times (i.e. $k \to \infty$) when time approaches t^* from the left.

We prove this by a contradiction argument. Suppose to the contrary there is not an infinite sequence of triggering times when time approaches t^* from the left, which implies that either there is a last trigger time before t^* or there are no trigger times at all before t^*. The latter case is impossible since singular triggering is excluded as is established in Lemma 2 of Fan et al. (2013). We now focus on the former case and denote the last trigger time before t^* as $\bar{t}$ with $\bar{t} < t^*$ to deduce a contradiction. Furthermore, in order to deal with the discontinuity issue of functions $e_i(t)$ and $g_i(t)$ at a triggering time instant, we also use notations $\bar{t}^+$ and t^{*-} to denote the time arguments defining limiting values at time $\bar{t}$ and t^* of functions which may be discontinuous at these points, the limits being computed by letting t approach $\bar{t}$ from above and t^* from below, respectively. On the interval $(\bar{t}, t^*)$, we have that $\|e_i(\bar{t}^+)\|$ assumes the value 0 at the left hand end, while $\|e_i(t^{*-})\|$ assumes the value $\|q_i(\bar{t})\|$ as t approaches t^{*-} (by noting the definition $e_i(t) = q_i(\bar{t}) - q_i(t^*)$ in the interval). On the other hand, $\|q_i(t)\|$ (which is actually continuous) assumes the value $\|q_i(\bar{t}^+)\|$ at the left hand end and approaches 0 as $t \to t^*$. Because t^* is the first time on $[0, \infty)$ at which $q_i(t) = 0$, within the interval $(\bar{t}, t^*)$, $\|q_i(t)\|$ remains nonzero. It follows that $g_i(t) = \|e_i(t)\| - \beta_i \|q_i(t)\|$ is nonzero at $\bar{t}^+$ and t^{*-} and takes values $-\beta_i \|q_i(\bar{t}^+)\|$ and $\|q_i(\bar{t}^+)\|$, which are of *opposite sign* at these points. Also, it is continuous in the interval $(\bar{t}, t^*)$, since both $e_i(t)$ and $q_i(t)$ are continuous in the same interval. Hence $g_i(t)$ equals zero at some intermediate point in the interval $(\bar{t}, t^*)$. But such a point would define a trigger time, and this contradicts the definition of $\bar{t}$ as the last trigger time before t^*. Hence this leads to a contradiction, which implies as $t \to t^*$, the inter-event time interval would be arbitrarily small and converges to zero at $t = t^*$. Therefore, as $t \to t^*$ an accumulation of the number of trigger times occurs, which results in an infinite number of trigger times during a finite time interval. In conclusion, for agent i there exists Zeno behavior with $\lim_{k \to \infty} t_k^i = \sum_{k=0}^{\infty} (t_{k+1}^i - t_k^i) = t_\infty^i < \infty$ for $q_i(t^*) \to 0$ as $t \to t^*$. □

As a consequence of Lemma 1, several further comments on the Zeno behavior of the event-triggered consensus dynamics in Fan et al. (2013) can be made:

- Any single agent has the possibility of exhibiting Zeno triggering, and the occurrence of Zeno behavior may also depend on the choice of initial positions.

Fig. 1. Trajectories of $q_i(t)$ under the event-triggered function of Fan et al. (2013). The embedded sub-figures show the zero-crossing points for $q_3(t)$, $q_4(t)$ and $q_5(t)$, respectively.

- A single agent (say agent i) may exhibit multiple Zeno points if there are multiple crossing-zero points t_ℓ^* that lead to $q_i(t_\ell^*) = 0$.
- However, one can prove that at any time instant, at least one agent in the group will not exhibit Zeno triggering by following the proof in Dimarogonas, Frazzoli, and Johansson (2012, Theorem 4).

1.2. Simulations

In this subsection we show a simulation example to further illustrate the Zeno triggering issue at the zero-crossing time instants with $q_i(t) = 0$. For the purpose of comparison we use the same simulation example as that in Fan et al. (2013), but here we show that Zeno triggering indeed exists if a different set of initial states is chosen. For illustration purposes we assume that each agent lives in $\mathbb{R}^1$, but the results also extend to the higher-dimensional case. The parameters are set as $\epsilon_i = 0.1$ and $\beta_i = 0.9$ for all agents (the same to that in Fan et al. (2013)). The graph topology and system parameters are identical to the simulation settings in Fan et al. (2013), while we choose the initial states for all agents as $x_1(0) = 3$, $x_2(0) = -0.5$, $x_3(0) = -5$, $x_4(0) = -3$, $x_5(0) = -5$, $x_6(0) = 4$.[1]
Simulation results are shown in Figs. 1 and 2. It can be seen from Fig. 2 that agents 3, 4 and 5 exhibit Zeno triggering at some finite time instants. The time instants at which the Zeno triggering occurs are exactly the instants when the term $q_i(t)$ crosses zero as shown in Fig. 1. These time instants are the Zeno-triggering time instants.

2. Ensuring Zeno-free triggering

The main reason for the existence of the Zeno point when using the event-triggering algorithm of Fan et al. (2013) is that there may exist a finite time $t^* < \infty$ such that $q_i(t^*) = 0$ for some i. An event-based triggering scheme is proposed in Sun, Huang, Anderson, and Duan (2016) to circumvent this issue which achieves a Zeno-free multi-agent consensus. The main idea in Sun et al. (2016) is to include an additional positive and convergent signal in the event comparison function in addition to $\beta_i \|q_i(t)\|$. We refer the readers to Sun et al. (2016) for more details on the design of consensus controllers and event functions. As a final remark, we also mention
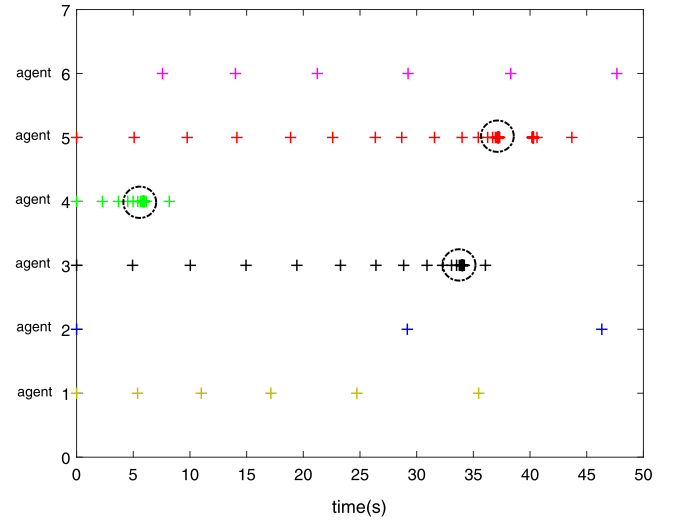
Fig. 2. Event time instants under the event-triggered function of Fan et al. (2013). The circled parts indicate accumulated triggering points where a Zeno triggering occurs, which correspond to the time instants that the corresponding $q_i(t)$'s ($i = 3, 4, 5$) cross zero, as shown in Fig. 1.

that other event-triggering consensus algorithms with truly Zeno-free behavior are also available; see e.g. Fan, Liu, Feng, and Wang (2015), Meng and Chen (2013) and Nowzari and Cortés (2016).

References

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[1] In Fan et al. (2013) the initial states in the simulation were not given.

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