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The impact of energetic particles and rotation on tokamak plasmas

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Abstract.

We discuss two contributions that elucidate the impact of energetic particles and rotation on tokamak plasmas: FLOW-M (M. J. Hole and G. Dennis, Plasma Phys. Control. Fusion 51, 035014, 2009), a generalisation of the ideal MHD flow code FLOW to multiple quasi-neutral fluids, and recent work on steady poloidal and toroidal bulk flows in tokamak plasmas [K. G. McClements and M.J. Hole, Phys. Plasmas **17**, 082509 (2010)]. Hole and Dennis have generalized ideal MHD to consider multiple quasi-neutral fluids, each in thermal equilibrium and each thermally insulated from each other such that no population mixing occurs. Kinetically, such a model may be able to approximate the ion or electron distribution function in regions of velocity phase space with a large number of particles, at the expense of more weakly populated phase space, which may have uncharacteristically high temperature and hence pressure. As magnetic equilibrium effects increase with the increase in pressure, this work constitutes an upper limit to the effect of energetic particles. McClements and Hole have examined the effects of poloidal and toroidal flows on tokamak plasma equilibria in the MHD limit. Transonic poloidal flows, of the order of the sound speed multiplied by the ratio of poloidal magnetic field to total field B_θ/B , can cause the (normally elliptic) Grad-Shafranov (G-S) equation to become hyperbolic in part of the solution domain. The discontinuity in variables produced by this transition indicates a breakdown in the validity of the MHD model in tokamak plasmas. It is pointed out that the range of poloidal flows for which the G-S equation is hyperbolic increases with plasma beta and B_θ/B , thereby complicating the problem of determining spherical tokamak plasma equilibria with transonic poloidal flows. When the assumption of isentropic flux surfaces is replaced with the more tokamak-relevant one of isothermal flux surfaces, a simple expression can be obtained for the variation of density on a flux surface when poloidal and toroidal flows are simultaneously present. Combined with Thomson scattering measurements of density and temperature, this expression could be used to infer information on poloidal and toroidal flows on the high field side of a tokamak plasma, where direct measurements of flows are not generally possible.

1. Introduction

Neutral beam heating produces two lowest-order corrections to toroidal magnetically confined plasmas: toroidal and poloidal plasma flows, and anisotropy. Bulk plasma motion can be incorporated into ideal MHD by including flow terms in the equations of continuity, momentum and the ideal form of Ohm's law. [1, 2] When combined with a suitable closure condition, and the assumption of toroidal symmetry, the steady-state form of the equations yield a flow-modified

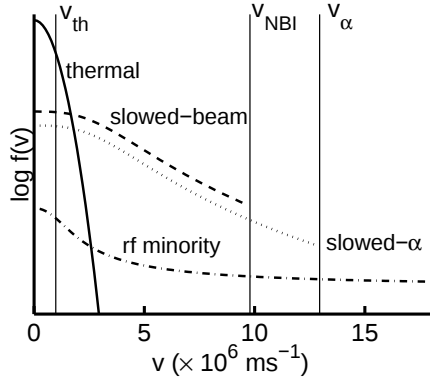


Figure 1. Illustrative 1D distribution function showing thermal, neutral beam slowing-down, α -particle slowing-down, and radio frequency minority heated populations. Also shown are the birth speeds of the beam ions and α -particles.

Grad-Shafranov equation. The effects of anisotropy are typically studied through the use of a single-fluid model with different parallel and perpendicular pressures. [3] A more recent approach that captures both bulk flows and anisotropy involves the use of energetically-resolved fluids that separate the thermal and beam populations. The effects of these energetic components increase with flow speed and anisotropy. If the poloidal flows become “transonic”, in the sense that they approach the sound speed multiplied by the ratio of poloidal magnetic field to total field B_θ/B , they cause the (normally elliptic) flow modified Grad-Shafranov equation to become hyperbolic in part of the solution domain. [1] Radial discontinuities in density and pressure can form at the interface between the elliptic and hyperbolic regions.

The aim of this paper is to illustrate some of the magnetic configuration changes that can be induced by the presence of energetic particles and rapid rotation. In Sec. II we introduce a multiple fluid model and describe a code that solves the model equations, presenting results for a spherical tokamak equilibrium with both toroidal and poloidal flows. In Sec. III we describe the effects of transonic poloidal flows on plasma equilibria, showing that the Grad-Shafranov equation is hyperbolic for a much greater range of flow Mach numbers in high performance spherical tokamaks than in conventional tokamaks. In Sec. IV we compute the anisotropy of a fast-ion distribution of a Mega Ampère Spherical Tokamak (MAST) discharge from a TRANSP [4] simulation. Our concluding remarks appear in Sec. V.

2. Multiple Fluid Equilibria

Rather than attempt to solve the equilibrium field given a distribution function for the electron and ions, we use the energetic fluid model of Hole and Dennis [5] to resolve different quasi-neutral fluids, and solve for the field configuration of a multiple fluid plasma in an axisymmetric toroidal configuration. Each fluid represents a different energetic component of the distribution function, with ions paired with electrons within each fluid to ensure quasi-neutrality. Figure 1 shows an illustrative one-dimensional distribution function in velocity space for a plasma comprising a thermal population, beam slowing-down & fusion α -particle slowing-down populations, and a minority radio-frequency heated population. Also shown are the birth speeds of the beam ions and α -particles.

The model used to describe the plasma is given by Eqs. (1)-(7) of Hole and Dennis [5]. That is, each quasi-fluid, labeled by subscript α , is described by ideal MHD, and governed in steady state by:

$$\nabla \cdot (\rho_\alpha \mathbf{v}_\alpha) = 0, \quad (1)$$

$$\rho_\alpha (\mathbf{v}_\alpha \cdot \nabla) \mathbf{v}_\alpha = -\nabla p_\alpha + \mathbf{J}_\alpha \times \mathbf{B}, \quad (2)$$

$$\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (4)$$

$$\nabla \times \mathbf{E} = 0, \quad (5)$$

$$\nabla \times \mathbf{B}_\alpha = \mu_0 \mathbf{J}_\alpha. \quad (6)$$

Here, γ_α is the ratio of specific heats and μ_0 the permeability of free-space, and the unknowns are: ρ_α , the mass-density; $\mathbf{v}_\alpha$, the fluid velocity; p_α , the isotropic partial pressure; $\mathbf{J}_\alpha$, the current density; $\mathbf{E}$, the electric field; $\mathbf{B}$, the total magnetic field; and $\mathbf{B}_\alpha$, the magnetic field produced by the current $\mathbf{J}_\alpha$. The set of equations is completed by a closure equation for the pressure. Highly collisional plasmas obey the standard steady state form of the ideal MHD adiabatic equation [6]

$$\mathbf{v}_\alpha \cdot \nabla \left(\frac{p_\alpha}{\rho_\alpha^{\gamma_\alpha}} \right) = 0, \quad (7)$$

Equations (1)-(6), together with Eq. (7) define the relationship between these unknowns for each fluid in the plasma.

To simplify this system it is convenient to introduce the poloidal flux functions $\Omega_\alpha(\Psi)$, a toroidal rotation frequency; $f_\alpha(\Psi)$, a modified toroidal flux function and $\sigma_\alpha(\Psi)$, an adiabatic flux function. These functions are defined by

$$\Omega_\alpha(\Psi) \equiv \frac{v_{\alpha,\phi}}{R} - \frac{B_\phi}{\rho_\alpha R} \zeta'_\alpha(\Psi), \quad (8)$$

$$f_\alpha(\Psi) \equiv B_{\alpha,\phi} R - \mu_0 v_{\alpha,\phi} R \zeta'_\alpha, \quad (9)$$

$$\sigma_\alpha(\Psi) \equiv \frac{p_\alpha}{\rho_\alpha^{\gamma_\alpha}}. \quad (10)$$

As $\rho_\alpha \mathbf{v}_\alpha$ is divergence free [see Eq. (1)] it can be defined by

$$\rho_\alpha \mathbf{v}_\alpha = \frac{1}{R} \nabla \zeta_\alpha(\Psi) \times \hat{\mathbf{e}}_\phi + \rho_\alpha v_{\alpha,\phi} \hat{\mathbf{e}}_\phi, \quad (11)$$

such that $\zeta(\Psi)$ is the poloidal mass flux of the fluid. The set of equations (1) - (7) can then be reduced to a Bernoulli relation for each fluid

$$H_\alpha(\Psi) = \frac{1}{2} v_\alpha^2 + \frac{\gamma_\alpha}{\gamma_\alpha - 1} \sigma_\alpha(\Psi) \rho_\alpha^{\gamma_\alpha - 1} - \Omega_\alpha(\Psi) v_{\alpha,\phi} R, \quad (12)$$

together with the generalised Grad-Shafranov equation

$$\begin{aligned} \frac{1}{\mu_0} \nabla \cdot \left[\left(1 - \mu_0 \sum_\alpha \frac{1}{\rho_\alpha} \zeta_\alpha'^2 \right) \frac{\nabla \Psi}{R^2} \right] = & - \sum_\alpha \rho_\alpha H'_\alpha - R^2 \sum_\alpha \rho_\alpha \Omega_\alpha \Omega'_\alpha - \\ B_\phi R \left(\frac{1}{\mu_0} \frac{f'}{R^2} + \sum_\alpha (\Omega_\alpha \zeta'_\alpha)' \right) + & \sum_\alpha \frac{\rho_\alpha^{\gamma_\alpha}}{\gamma_\alpha - 1} \sigma'_\alpha - \sum_\alpha \frac{1}{R^2} \frac{\zeta'_\alpha \zeta''_\alpha}{\rho_\alpha} \left[|\nabla \Psi|^2 + (B_\phi R)^2 \right]. \end{aligned} \quad (13)$$

Hole and Dennis have modified the 2D flow equilibrium code **FLOW** [6] to solve the coupled set of N algebraic Bernoulli equations and the nonlinear PDE, Eq. (13). The new code, **FLOW-M**, takes as user input profiles the quasi-toroidal magnetic field $B_0(\Psi)$, together with the following variables for each fluid: the mass density $D_\alpha(\Psi)$; the pressure $P_\alpha(\Psi)$; the poloidal sonic Mach number $M_{\alpha,\theta}(\Psi)$ (defined as the poloidal flow normalised to the sound speed c_s and multiplied by the ratio of B to poloidal field B_θ); and the toroidal sonic Mach number $M_{\alpha,\phi}(\Psi)$. The

Function	Definition
$f(\Psi)$	$R_0 B_0(\Psi)$
$\zeta'_\alpha(\Psi)$	$\sqrt{\gamma_\alpha P_\alpha(\Psi) D_\alpha(\Psi)} \frac{M_{\alpha,\theta}(\Psi)}{B_0(\Psi)}$
$\Omega_\alpha(\Psi)$	$\sqrt{\gamma_\alpha \frac{P_\alpha(\Psi)}{D_\alpha(\Psi)}} \frac{M_{\alpha,\phi}(\Psi) - M_{\alpha,\theta}(\Psi)}{R_0}$
$H_\alpha(\Psi)$	$\gamma_\alpha \frac{P_\alpha(\Psi)}{D_\alpha(\Psi)} \left(\frac{1}{\gamma_\alpha - 1} + M_{\alpha,\theta}(\Psi) M_{\alpha,\phi}(\Psi) - \frac{1}{2} M_{\alpha,\phi}^2(\Psi) \right)$
$\sigma_\alpha(\Psi)$	$\frac{P_\alpha(\Psi)}{[D_\alpha(\Psi)]^{\gamma_\alpha}}$

Table 1. Relationship between FLOW-M constraint functions and quasi-variable user input profiles, with R_0 the radial position of the geometric axis.

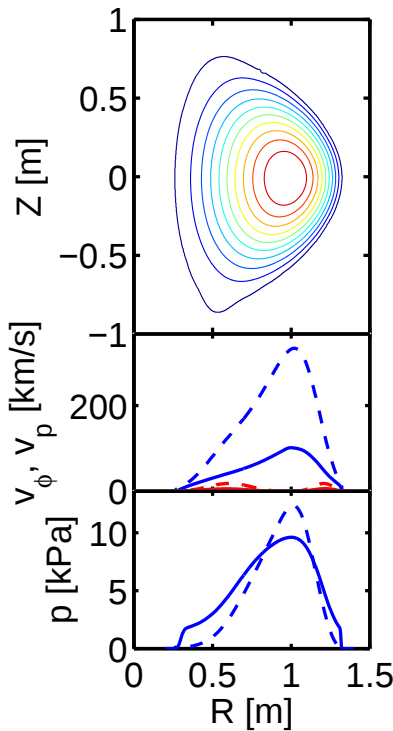


Figure 2. A MAST-like multi-fluid equilibrium with toroidal and poloidal rotation. The plasma boundary and thermal pressure profile is based on MAST discharge #7085 at 290ms, [7] and the flow profiles parametrized. The middle panel shows the toroidal (blue) and poloidal (red) rotation profiles for the thermal (solid) and energetic (dashed) fluids. The lower panel shows the pressure profile for the thermal (solid) and energetic (dashed) fluids.

relationship between FLOW-M constraint functions and quasi-variable user input profiles is shown in Table 1.

Figure 2 shows the equilibrium for a two quasi-neutral fluid plasma with toroidal and poloidal flow; with similar parameters to those of MAST plasmas. The boundary profile and thermal pressure profile $P_0(\Psi)$ were taken from discharge #7085 at 290ms [7], with the energetic pressure profile chosen to be of the form $P_1(\Psi) \propto (1 - \psi_n)^2$. The toroidal and poloidal flow profiles for both thermal and energetic species were parametrized as

$$M_\phi(\Psi) = M_{\phi,max}(1 - \psi_n)^{1/2} \quad (14)$$

$$M_\theta(\Psi) = 4M_{\theta,max}(1 - \psi_n)\psi_n \quad (15)$$

with $M_{0,\phi,max} = 0.30$, $M_{1,\phi,max} = 0.70$ and $M_{0,\theta,max} = 0.1$, $M_{0,\phi,max} = 0.25$. These rotation profiles are representative of MAST plasmas. [8]

The impact of these energetic quasi-neutral fluids on the Shafranov shift, the central safety factor, and the poloidal flux surfaces can be assessed by independently changing $M_{1,\phi,max}$,

$M_{1,\theta,max}$ and $p_1(0)$. As the toroidal rotation $M_{1,\phi,max}$ is increased from 0 to 1.3 the Shafranov shift increases by 60% due to the increased centrifugal force, and the central safety factor decreases from 1.7 to 0.8. The latter occurs because the toroidal current decreases with the toroidal Mach number and the magnetic axis moves outboard. As $M_{1,\theta,max}$ is increased from 0 to 0.4, the central safety factor increases from 1.3 to 1.6: this results from an increase of toroidal current with increasing $M_{1,\theta}$. Finally, as $p_1(0)$ is increased from 0 kPa to 8 kPa the Shafranov shift increases by $\approx 30\%$, and central safety factor drops from 2.5 to 1.0 as the magnetic axis moves outboard. In all cases however, the change in poloidal flux is less than 2%. We conclude that although changing the energetic profiles has little impact on the flux surface geometry and magnitude of the poloidal flux, it does have a significant effect on the Shafranov shift, and more importantly, the central safety factor.

3. Poloidal Rotation

It has long been recognised that “transonic” poloidal flows $v_\theta \approx c_s B_\theta/B$ can cause the normally elliptic Grad-Shafranov equation to become hyperbolic in part of the solution domain. The effect of this modification is to produce radial contact discontinuities in plasma properties such as density. The fact that internal transport barriers in tokamaks, which are characterised by sharp transitions in plasma profiles, are often associated with high poloidal flows suggests a possible link with elliptic/hyperbolic transitions in MHD equilibria, although it has been shown that no such transitions occur in the case of transonic flows when either two-fluid or kinetic effects are taken into account [3, 9]. McClements and Hole [10] have recently sought to clarify some of the outstanding issues raised by the presence of poloidal and toroidal flows in tokamaks, in particular with regard to the effect of poloidal flows on density profiles, the dependence of the elliptic/hyperbolic transition on the choice of closure condition, and whether there is evidence for such transitions occurring in existing tokamak experiments.

In a single-fluid model with isentropic flux surfaces the Bernoulli relation can be used to express total mass density ρ in terms of $\nabla\Psi$. The terms in the Grad-Shafranov equation involving second order derivatives of Ψ can then be written as

$$\left(1 - \mu_0 \frac{F'^2}{\rho}\right) \left(A_{RR} \frac{\partial^2 \Psi}{\partial R^2} + A_{RZ} \frac{\partial^2 \Psi}{\partial R \partial Z} + A_{ZZ} \frac{\partial^2 \Psi}{\partial Z^2} \right), \quad (16)$$

where $F = F(\Psi)$ is defined such that poloidal momentum density is $\nabla F \times \nabla \Psi$ and A_{RR}, A_{RZ}, A_{ZZ} depend on the flow components together with c_s , the local Alfvén speed and the local poloidal Alfvén speed. Equation (16) indicates that the Grad-Shafranov equation is elliptic (hyperbolic) if the quantity $D = A_{RZ}^2 - 4A_{RR}A_{ZZ}$ is negative (positive); it can be shown that

$$D = -4 \frac{1 - M_{s\theta}^2(1 + \beta)}{1 - M_{s\theta}^2(1 + \beta) + \beta M_{s\theta}^4 (B_\theta/B)^2}, \quad (17)$$

where $\beta = \gamma \mu_0 p / B^2$, γ being the single-fluid ratio of specific heats.

Equation (17) indicates that in the usual conventional tokamak ordering ($\beta \ll 1$, $B_\theta \ll B$) the flow-modified Grad-Shafranov equation is hyperbolic only for a very narrow range of values of $M_{s\theta}$ [6]. However, on the low field side of high performance spherical tokamak plasmas B_θ/B and β can both be of order unity; in this case the flow-modified Grad-Shafranov equation is hyperbolic at lower $M_{s\theta}$ and for a greater range of values of this parameter. Figure 3, which illustrates this point, shows D versus $M_{s\theta}$ for two pairs of values of β and B_θ/B corresponding to typical conditions in (a) a conventional tokamak, and (b) a high performance spherical tokamak.

When toroidal and poloidal flows v_ϕ, v_θ are both present there exists a flux function Ω such that [2]

$$\Omega = \frac{v_\phi}{R} - \frac{v_\theta}{R} \frac{B_\phi}{B_\theta}, \quad (18)$$

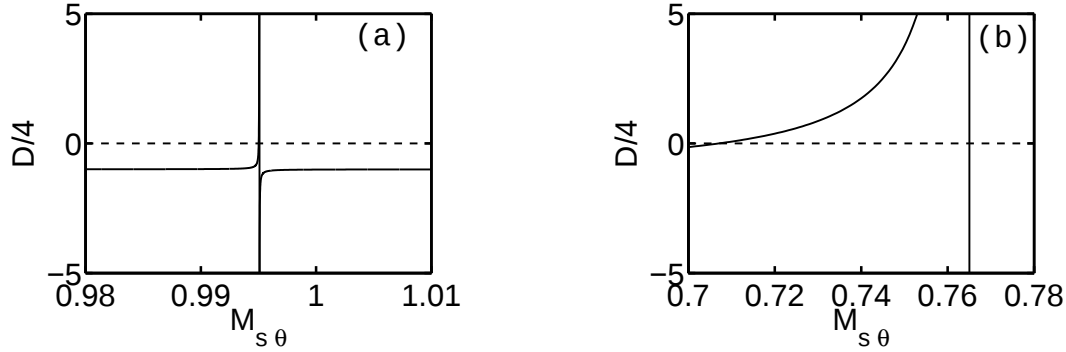


Figure 3. Discriminant D of the second order derivatives of the Grad-Shafranov equation plotted versus poloidal sonic Mach number $M_{s\theta}$ for (a) $\beta = 0.01, B_\theta/B_\phi = 0.1$, and (b) $\beta = 1, B_\theta/B_\phi = 1$. Negative (positive) values of D indicate that the Grad-Shafranov equation is elliptic (hyperbolic).

and the Bernoulli relation for isothermal flux surfaces can be written as

$$H(\Psi) = \frac{2T(\Psi)}{m_i} \ln \left(\frac{\rho}{\rho_0} \right) + \frac{v_\phi^2 + v_\theta^2}{2} - \Omega R v_\phi, \quad (19)$$

where H is a flux function, T is the single-fluid temperature, m_i is ion mass, and ρ_0 is an arbitrary constant. Eliminating v_ϕ from Eqs. (18) and (19) we obtain the following expression for particle density $n \approx \rho/m_i$:

$$n = n_1(\Psi) \exp \left[\frac{\Omega^2(\Psi)}{2v_i^2(\Psi)} (R^2 - R_0^2) - \frac{1}{2} M_{s\theta}^2 \right], \quad (20)$$

where n_1 is a flux function and $v_i = (2T/m_i)^{1/2}$. The quantities Ω , v_i and $M_{s\theta}$ can be measured on the low field side of the plasma using neutral beam diagnostics (charge exchange and motional Stark effect). Because of beam attenuation and injection geometry, such measurements are generally not possible on the high field side. Thomson scattering data can be used to infer n_e and electron temperature T_e across the midplane; measurements of inboard/outboard density asymmetry, combined with Eq. (20), can then yield $M_{s\theta}$ on the high field side. Moreover Eq. (18) can be used to determine v_ϕ on the high field side, provided that $B_\theta^2 \ll B^2$. The above expressions could thus provide a more complete picture of global tokamak dynamics.

4. Pressure anisotropy of fast ions

The inclusion of pressure anisotropy is also known to have an effect on flow equilibrium. [3] For instance, the characteristic poloidal flow speeds at which the MHD picture is hyperbolic is changed for large anisotropy. The centrifugal shift can also be increased or diminished/reversed with larger perpendicular or parallel temperature, respectively.

In preliminary working we have also computed pressure anisotropy using the TRANSP [4] NBI fast ion distribution output for MAST # 18696. Figure 4 shows fast ion density and parallel mean flow speed as well as the computed parallel and perpendicular pressures. For this discharge $p_\perp/p_\parallel \approx 1.7$. Work is underway to establish how this level of anisotropy changes the equilibrium.

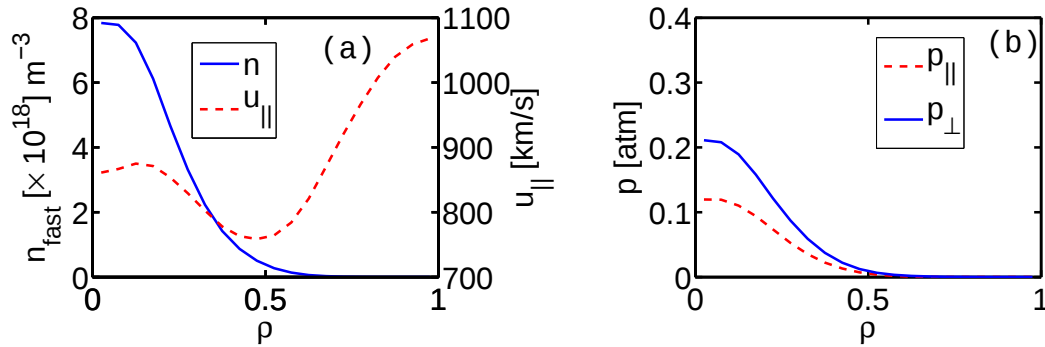


Figure 4. Profiles of (a) fast ion density and parallel mean flow speed and (b) parallel and perpendicular pressures, computed from a TRANSP distribution function for MAST discharge #18696. The horizontal axis is normalised radius.

5. Conclusions

In this paper we have explored the effect of energetic particles and rotation on tokamak plasmas. A multiple energetically resolved fluid model was described, and related to an illustrative distribution function for a plasma with neutral beam, α , and radio frequency heating. Using the new model, and its accompanying numeric solver FLOW-M, we have computed a MAST-like equilibrium comprising a slowly rotating thermal population and a rapidly rotating neutral beam population, broadly consistent with MAST data. We have also summarised earlier parameter scans that explore how the plasma configuration changes with varying fast ion flow speed and pressure. These show that although the poloidal flux has only a weak dependence on the energetic fluid, the dependency with the safety factor is strong. Resolving the different energetic components of the distribution function may thus be important for assessing MHD stability, even of the thermal fluid.

As the poloidal flow approaches transonic speeds, we have also re-examined the change in behaviour of the normally elliptic flow-modified Grad-Shafranov equation, which becomes hyperbolic in part of the solution domain. Associated with the spatial transitions from elliptic to hyperbolic behaviour are radial discontinuities in the plasma parameters, such as density. By computing the discriminant of the second order derivatives in the flow-modified Grad-Shafranov equation for both large aspect ratio tokamaks and the spherical tokamak, we have shown that the transition from elliptic to hyperbolic regimes occurs at lower values of the poloidal Mach number, and for a wider range. In the framework of the MHD model such transitions imply the presence of discontinuities in variables such as density, and so the profiles have gradients that are not consistent with the MHD ordering of length scales. One would thus expect discontinuities in MHD equilibria to be replaced with smooth gradients (for example, on length scales of the order of the ion skin depth or Larmor radius) in two-fluid or kinetic models [3, 9]. Indeed it has been shown explicitly that no transition to hyperbolic behaviour occurs at transonic values of the poloidal flow when either two-fluid [9] or kinetic [3] effects are taken into account. Moreover there appears to be no clear experimental evidence of density discontinuities of this type associated with elliptic/hyperbolic transitions occurring in present-day tokamak plasmas [10]. Finally, we report on the calculation of the pressure anisotropy of a TRANSP simulation of a MAST discharge, showing that the effects of anisotropy in MAST plasmas may be quite significant. In ongoing work we plan to compute the impact of this level of anisotropy on the

plasma.

Acknowledgments

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