Optimal Superannuation Guarantee for heterogeneous groups of Australians

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Abstract

The focus of this thesis is on retirement savings decisions and their impact on individual welfare in an Australian context. Chapter 2 examines the role of the mandatory contribution rate, known as the Superannuation Guarantee (SG). The research then turns to investigate the influence of housing arrangements. Chapter 3 builds a 'cascade model' that supports the simulation of economic outcomes including housing-related variables. This model is applied in Chapter 4 to analyse the implications of housing arrangements for individual consumption outcomes and savings decisions, including the optimal SG.

The key finding is that there is no single 'one-size-fits-all' optimal SG that suits all individuals; and that the optimal SG varies substantially with income levels and consumption objectives as well as other factors. This research develops a 'cascade model' that simulates integrated long-term projections for interest rates, incomes, house prices, rents, and (superannuation) fund returns intending to support life-cycle modelling. Allowing for house tenure and housing consumption choices, this research finds that the optimal SG increases with income levels for both renters and owners but is much lower for owners.

This analysis provides insights into the design of the superannuation system by highlighting how the optimal SG level varies considerably with various individual attributes and is impacted by the role that is intended for social security including the Age Pension, rental assistance, and related supplements and potentially individuals' behavioural decision-making effects. The analysis shows that purchasing a home can significantly reduce the savings required for retirement through the superannuation system. This research poses the question of whether purchasing a home can substitute mandatory saving schemes in the presence of the publicly funded Age Pension.

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1 Introduction

A shift from publicly managed, unfunded defined benefit (DB) schemes to privately managed, fully funded defined contribution (DC) schemes has been occurring in many countries (Holzmann 2013). This shift empowers individuals to choose their contribution rates and manage their asset allocations (Dahlquist et al. 2018), hence transferring the responsibility and the risks to the individuals. Many individuals have trouble starting and maintaining long-term savings programs (Thaler and Bernartzi 2004), which has led some policymakers to establish minimum compulsory retirement savings schemes (Bateman and Piggott 2001). It is thus vital to design private mandatory saving schemes to improve individuals' retirement outcomes to the greatest degree possible.

This thesis focuses on retirement savings decisions and their impact on individual welfare in an Australian context. Chapter 2 examines the role of the mandatory contribution rate, known as the Superannuation Guarantee (SG). The research then turns to investigate the influence of housing arrangements. Chapter 3 builds a 'cascade model' that supports the simulation of economic outcomes including housing-related variables. This model is used in Chapter 4 to analyse the implications of housing arrangements for individual consumption outcomes and savings decisions, including the optimal SG. The analysis is conducted in a life-cycle modelling context, also known as the life-cycle consumption-investment problem, which entails the simulation of economic circumstances throughout an individual's lifetime and the optimisation of their financial decisions. This approach stems from Merton (1969), who frames the problem around an intertemporal utility function defined over consumption with constant relative risk aversion and a personal discount rate. Researchers have enriched the model by adding more realistic factors and constraints (See Horneff et al. 2015; Blake et al. 2013; Blake et al. 2014; Yogo 2016).

Chapter 2 estimates the optimal SG for individuals operating in the Australian socio-economic and policy environment and examines how individual welfare changes with differing SG levels. The Superannuation Guarantee (SG) serves as the Australian mandatory saving scheme and was introduced in 1992 with the SG contribution rate originally set at 3%. It is now set as 10.5% in 2020 and is slated to increase to 12% from July 2025 (Parliament of Australia

2010 and 2014; ATO 2020). Whether the proposed SG increase should proceed has generated wide public debate[1].

This research investigates the influence of three important factors in determining the optimal SG level: the availability of the publicly funded Age Pension, investment strategy assumptions, and time preferences. The analysis shows that the combination of income level and consumption objective can both have an important influence on the optimal contribution (i.e., savings) rate.

The key finding is that there is no single 'one-size-fits-all' optimal SG that suits all individuals; and that the optimal SG varies substantially with income level and consumption objectives as well as other factors. This research finds that the optimal SG can range from 0.5% up to more than 20% under the baseline modelling.

Decisions on housing arrangements can potentially have a significant impact on the wellbeing of individuals over their lifetime. For many individuals, owner-occupied housing is their portfolio's largest and most important asset (see Cocco 2005; Yao and Zhang 2005). Homeownership effectively provides both imputed rental services, and a form of asset investment that is often leveraged through mortgage borrowings. The alternative arrangement is renting. Typically, more than 70% of households in OECD countries own a home before retirement (Chiuri and Jappelli 2010). Several authors have examined house tenure and housing consumption choices in a life-cycle framework (See Fernandez-Villaverde and Krueger 2011; Dotsey et al. 2014; Li et al. 2016; Shao et al. 2019). The importance of including a housing component when undertaking life-cycle modelling in an Australian context is heightened by the principal residence being excluded from the asset test for the Age Pension[2], which provides means-tested public retirement income support and thus enhances the attractiveness of purchasing a house.

To facilitate the analysis of housing arrangements, a 'cascade model' is developed in Chapter 3 that simulates integrated long-term projections for interest rates, incomes, house prices, rents and (superannuation) fund returns to support life-cycle modelling. The model is specifically designed to generate projections for the key variables consistently by maintaining economically plausible economic links between the variables within each simulated path while

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1 Further discussion of this debate is provided in Chapter 2 .

2 For more details see Services Australia 'Asset types', https://www.servicesaustralia.gov.au/asset-types?context=22526.

generating outcomes that broadly accord with those seen historically. This research ensures that this is the case by checking the distribution of projections.

This consistency occurs as the cascade model structure ensures that the impact of each variable acts as a driving force that transfers through to other variables when moving down the model tiers. This style of model is motivated by Wilkie (1986, 1995), who used long-term British data to develop a series of autoregressive distributed lag (ADL) models for long-term forecasts within actuarial applications[3]. This research is expanded to investigate house prices and rents in an Australian context. The model not only generates house prices and rents but also links them to other key variables that are central to life-cycle modelling, most notably wage income and interest rates (the latter are linked to asset returns).

The analysis in Chapter 2 treats the SG as excluding various features that may impact optimal SG levels. One of the major exclusions is the assumed absence of other saving mechanisms outside of the superannuation system, with the analysis effectively modelling the first pillar (i.e., mean-tested public-funded Age Pension) and the second pillar (i.e., privately managed defined contribution superannuation) of the Australian retirement system. It ignored the third ‘voluntary savings’ pillar, of which housing is the most important component.

Chapter 4 continues the research on how the SG contribution rate impacts individual welfare by allowing for house tenure and housing consumption choices. This research estimates the optimal SG for individuals who choose to purchase a home (‘owner’) and those who rent (‘renter’) over their lifetime and compare their financial welfare. This research investigates the influence of four factors on the optimal SG level and the benefit of home ownership versus renting. These include the utility function used to represent an individual’s preference; assumptions around the housing market and individual’s behaviours regarding the buy-versus-rent decision; an individual’s income; and the policy environment, specifically the rules around tax and social security.

The analysis uses stochastic projections from the model presented in Chapter 3, which allows us to evaluate the impact of fluctuations in personal incomes, inflation, interest rates, superannuation investment returns, rents, and house prices. The analysis considers the full range of tax, social security, and house purchase effects for Australia. The most noteworthy items

[3] Wilkie et al. (2010) update and refit his cascade model using the data up to 2009. They find that the residuals of many series are much fatter tailed than in a normal distribution.

include a tax on both personal and superannuation income, contribution tax, detailed mortgage loan terms, transaction costs for house purchase and eligibility for social security including the Age Pension, rental assistance, and related supplements.

This research finds that the optimal SG increases with income level for both renters and owners but is much lower for owners. The baseline results estimate an optimal SG for the renter that ranges from 5.5% at the 10th income percentile up to 14.0% for the 90th income percentile, while the optimal SG for the owner ranges from only 0.5% up to 2.0%. This research also notes that the gap between the optimal SG between renters and owners increases with income level, moving from 5.0% at the 10th income percentile to 12.0% at the 90th income percentile. These findings reinforce the result of Chapter 2 that there is no single 'one-size-fits-all' optimal SG that suits all individuals, emphasising that individual heterogeneity extends beyond characteristics like income levels to include decisions on house tenure. It indicates that purchasing a home can significantly reduce the savings required for retirement through the superannuation system. This research poses the question of whether purchasing a home can substitute for mandatory saving schemes in the presence of the publicly funded Age Pension, given the extremely low optimal SG found for the homeowner.

Chapter 5 concludes the findings from Chapter 2 to Chapter 4 and their potential policy implications. The scope for future research is also discussed.

2 The ‘Right’ Level for the Superannuation Guarantee: A Straightforward Issue by No Means[1]

Yifu Tang: 80%, Gaurav Khemka: 10%, Geoffrey J.Warren: 10%

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[1] This chapter is conducted in 2019, the information in this chapter is up-to-date at that date.

2.1 Introduction

Many countries have been reforming their multi-pillar pension systems, including shifting from publicly managed, unfunded defined benefit (DB) schemes to privately managed, fully funded defined contribution (DC) schemes (Holzmann 2013; Benartzi and Thaler 2007). This policy change gives pension plan members[2] greater responsibility to choose their contribution rates and manage their asset allocations (Dahlquist et al. 2018). However, many members display inertia (Madrian and Shea 2001), are susceptible to procrastination (Choi et al. 2003) and have difficulties in starting and maintaining long-term savings programs (Thaler and Bernartzi 2004). This has led some policymakers to put in place minimum compulsory retirement saving schemes (Bateman and Piggott 2001). Consequently, it is important to design private mandatory saving schemes so that they improve members' retirement outcomes to the greatest degree possible.

The Superannuation Guarantee (SG) is the key component of the second pillar of the Australian retirement income system and amounts to a mandatory contribution rate[3]. The SG requires employers to contribute a percentage of an employee's earnings to a superannuation fund on their behalf, and the employee generally cannot access these savings until they reach the superannuation preservation age[4]. SG policy was introduced in 1992 with the SG contribution rate originally set at 3%. The rate progressively increased from 3% to 9% between 1992 and 2002; then increased to 9.5% from 1 July 2014. It was initially slated to increase to 12% by July 2019, but it was later delayed by the government to July 2025 (Parliament of Australia 2010 and 2014; ATO 2020). Whether the proposed SG increase (from 9.5% currently) should proceed has generated wide public debate (see for example Daley and Coates 2018; Rice

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[2] Since the research focus in this chapter is on the individuals who only save via superannuation system the term 'member' is used. Where other chapters' research allows for the voluntary saving, the term 'individual' is used.

[3] Fund members can contribute more than the compulsory rate, although the availability of tax advantage is limited by concessional contribution caps, see https://www.ato.gov.au/Rates/Key-superannuation-rates-and-thresholds/?page=3.

[4] Fund members normally can't use their super until they reach their preservation age. To get their super released early fund members must meet one of the eligibility requirements, see https://www.servicesaustralia.gov.au/individuals/services/centrelink/early-release-superannuation/who-can-access-their-super-early. Alternatively, if fund members have been adversely financially affected by COVID-19, they may be able to access some of their superannuation early. See https://www.ato.gov.au/individuals/super/in-detail/withdrawing-and-using-your-super/covid-19-early-release-of-super/.

Warner 2019; Mercer 2019; Evans and Razeed 2019); and may be re-examined as a consequence of the Retirement Income Review currently being conducted by the Australian Treasury[5].

Against this background, this research examines the issue of how the contribution rate impacts plan member (‘member’) welfare within an Australian setting. This research estimates the optimal SG for the Australian socio-economic and policy environment and investigate how member welfare changes with differing levels of the SG. The analysis is also used to investigate the influence of three factors that are important in determining the optimal SG level: the availability of the publicly funded Age Pension, investment strategy assumptions and time preference assumption. In doing so, this research contributes to the academic literature on optimal savings via pension plans, while hopefully helping to improve member welfare through informing policy formulation.

This research uses a stochastic life-cycle model that trades off the utility benefit that savings generated through funding post-retirement consumption against the utility reduction from lower pre-retirement consumption. The model is used to identify the ‘optimal’ SG and the change in member welfare for a plausible range of SG levels, assuming that the member applies optimal investment and drawdown decisions. The model is solved numerically using stochastic dynamic programming techniques. The analysis captures different consumption objectives by using the power utility function to represent a consumption smoothing objective and reference dependent utility functions for members with explicit retirement consumption targets. The latter includes a replacement rate target and two fixed dollar amount targets expressed by ASFA standards. The model is run for members with nine representative pre-retirement income levels, imposing a hump-shaped deterministic age-earnings profile based on Freestone (2018). The analysis considers the full range of tax and social security rules that apply concerning the generation of retirement incomes. The most noteworthy items include a tax on both personal and superannuation income (allowing for Medicare levy and imputation credits), contribution tax, minimum drawdown rules for retirement saving accounts, and eligibility for the Age Pension and related supplements.

The first finding is that there is no single ‘one-size-fits-all’ optimal SG that suits all members. This research finds that the optimal SG varies substantially with income level and

[5] See https://treasury.gov.au/review/retirement-income-review.

consumption objectives, ranging from 0.5% up to more than 20% under the baseline modelling. The analysis shows that the combination of income level and consumption objective can both have an important influence on the optimal contribution (i.e., savings) rate.

An analysis of how member welfare changes with the SG levels further highlights how the implications of different SG levels can vary considerably across member types. This research finds that members with a consumption smoothing objective can experience large welfare losses when the SG is set below their optimal level, while they are not affected too much when it is too high. On the other hand, members with consumption target objectives can suffer a significant welfare loss when the SG is set either too low or too high relative to their optimal level. Under the baseline findings, increasing the SG from 9.5% to 12% would impose a large welfare loss for members with consumption target objectives while generating marginal welfare gain for members with a consumption smoothing objective. This indicates that increasing the SG may hurt some members to a greater extent than those it helps. This research also estimates the government net revenue per member for a range of SG levels and find that increasing the SG has only a modest impact on government revenue or outlays summed over a member’s lifetime.

The second contribution is to reinforce the importance of the availability of the Age Pension system for determining the welfare-maximizing contribution rate in an Australian context. There is considerable research into the role of mean-tested pensions within retirement savings systems, see Hulley et al. (2013) and Andreasson et al. (2017) for a detailed overview. For instance, Tran and Woodland (2014) and Kudrna (2016) highlights the importance of the taper rate in forming the direction of the welfare effects; Andreasson et al. (2017) investigate the optimal policies with a means-tested public pension only in retirement and find that the optimal allocation to risky assets increases when wealth decreases as the Age Pension works as a buffer against investment losses. This research is expanded by examining the impact of the availability of a mean-tested pension system on the optimal contribution rate in the context of a life-cycle model that considers welfare over a member’s entire life, rather than just welfare during retirement.

The results indicate that assuming no access to the Age Pension system leads to the most optimal SG estimates exceeding 10%. Low-income earners experience the largest increase in optimal SG, reflecting the fact that they receive the greatest relative benefit from the Age Pension as a substitute for private savings. The analysis confirms the important role of publicly funded pensions as a retirement income source and highlights a particular significance for Australian retirees with modest income and savings. It also suggests that the current and proposed SG of 9.5% and 12% respectively may be supported if the SG system aims to replace the Age Pension, where possible, by requiring members to save via their superannuation fund.

The third contribution is to highlight the relevance of assumptions about member behaviour for setting a compulsory contribution rate. There is growing evidence of behavioural decision-making effects on savings outcomes, including the research on framing effects (Card and Ransom 2011) and default effects (Choi et al. 2003; Madrian and Shea 2001). The baseline analysis makes two notable assumptions about member behaviour that is investigated further. First, the baseline model assumes that members invest optimally, in line with dynamic programming principles. In practice, many members use pre-determined investment strategies, either constant asset mixes or lifecycle (i.e., target date) strategies. Second, the baseline does not allow for any preference for earlier rather than later consumption, as reflected in a time preference of 1.00. See Frederick et al. (2002) for a detailed review of the effect of time preference.

To evaluate the impact of the assumptions about investment behaviour, this research estimates the impact on the optimal SG of two commonly adopted default investment strategies, namely a constant mix of 70% in the risky asset and 30% in the risk-free asset, and a lifecycle investment strategy that reduces the risky asset weight from 90% to 40% by retirement. Both strategies hold less risky asset exposure than optimal and confirm that default strategies tend to be more defensive than implied by members' risk preferences (Alserda et al. 2019). This research finds that 'optimal' SGs are higher when conditioned on sub-optimal investment strategies, increasing by between 0.5% and 4.5% relative to the baseline across most of the income percentiles and objectives.

To evaluate the relevance of the time preference assumption, this research produces estimates applying a time preference parameter of 0.98, equivalent to a discount rate of 2% per annum. This decreases the optimal SG by between 0.5% and 5% relative to the baseline across most of the income percentiles and objectives. This result is in line with the analysis of Angeletos et al. (2001), Diamond and Koszegi (2003) and Goda et al. (2015) that allowing for a discount rate can lead to lower savings for members by the time they retire.

These two sets of results highlight the need for policymakers to consider the appropriate behavioural assumptions when setting the compulsory contribution rate. The issue that arises is whether they should set the policy to cater for observed member behaviours or align the policy with the baseline assumption that members optimise their investment strategies and have no time preference. Admitting the pre-defined investment strategies assumes that the members do not have adequate financial literacy to optimise their investment and the policymakers need to set a relatively higher contribution rate to ensure attaining sufficient retirement funds. Whereas no time preference assumption indicates the paternalistic role of the policymakers that makes sure members do not suffer a form of regret when they have not saved enough at retirement.

Overall, the analysis finds that optimal contribution rates can vary considerably due to member heterogeneity based on characteristics like income levels and retirement consumption objectives, as well as potentially extending to behavioural effects that might influence members to varying degrees. This research adds to a growing body of literature that highlights the importance of allowing for heterogeneity in retirement planning. For example, Lusardi et al. (2013) show that theoretically there is substantial heterogeneity in member financial literacy, which leads to differing saving patterns and investment strategies among people. Dahlquist et al. (2018) find considerable heterogeneity among passive investors in stock market participation as well as variation in optimal equity holdings. Moreover, Alserda et al. (2019) emphasised that members’ heterogeneous risk preferences imply different optimal asset allocations. The research highlights the extent to which impacts from heterogeneity extend to the optimal contribution rate. This research finds that imposing a one-size-fits-all optimal contribution rate may lead to meaningful welfare losses for heterogeneous members.

The research also adds to the literature on the link between default contributions and better retirement savings plan design. Madrian and Shea (2001) and Choi et al. (2003) emphasised the importance of the default contribution rate to mitigating member inertia and procrastination. Chetty et al. (2014) highlight the effectiveness of automatic contributions in increasing saving rates. Blake et al. (2014) propose a compulsory minimum contribution rate at all ages with age-related additional voluntary contributions due to members’ reluctance to maintain long-term savings programs. This research extends their argument by investigating the optimal level of this minimum contribution rate. This research aligns with that of Munnell et al. (2014) and Poterba (2014). However, they focus on the savings rates required to obtain a target replacement rate, which fails to consider the impact of retirement saving on consumption throughout a member’s whole lifecycle. Evans and Razeed (2019) discuss the impact of the increase in SG on members’ whole-life consumption. However, their analysis is much more limited than ours, including using a utility function framework in a life cycle model, allowing for multiple objectives and a wide range of incomes.

This study sits in the large literature that considers optimal consumption-investment problems under a life-cycle model stemming from Merton (1969). Although there is considerable research in this area, only limited work in the Australian context can be found. Butt and Khemka (2015) examine the optimal consumption and portfolio choices under both shortfall and utility objectives. Iskhakov et al. (2015) investigate the optimal annuity purchases at retirement. The impact of the Australian means-tested pension system on consumption and investment strategy is studied by Hulley (2013), Ding (2014) and Andreason et al. (2017). Shao et al. (2019) incorporate reverse mortgages and long-term care insurance into a life cycle model and access their impact on a retiree's optimal consumption and investment. Butt et al. (2019) examine the dividend imputation system in Australia.

In the following parts of this chapter, the methodology and the assumptions are briefly discussed in Section 2.2; Section 2.3 presents the optimal SG, members' welfare change with SG and the impact of determinants on optimal SG; the sensitivity tests are listed in Section 2.4; and a conclusion is provided in Section 2.5.

2.2 Methodology and assumptions

This analysis is conducted in three steps. First, investment decisions throughout the members’ whole life and their drawdown strategies after their retirement under four consumption objectives and nine income levels are optimised. This step is repeated for a plausible range of SG levels, ranging from 0% up to 20% in 0.5% increments. The chosen maximum tested SG level of 20% is higher than observed global settings, and it is unlikely that the Australian Government will implement an SG above 20% considering the current debate around the 12% SG. Second, for each member with a specific objective and income, welfare across the range of SG levels examined by estimating expected utility and converting it into two types of certainty equivalents is compared, thus reporting utility in an economically meaningful way. This second step allows us to identify optimal SGs and establish the welfare loss from adopting a sub-optimal SG. Third, how the results change when the assumptions are varied is investigated. The prime focus is assumptions related to the availability of the Age Pension, the investment strategy and time preference; although this research also presents results for selected other assumptions including early retirement, self-insuring against longevity risk, lower asset returns, and so forth. The analysis is conducted in real terms and all real quantities are implicitly assumed to inflate at a common rate.

This research models an Australian male member who earns only wage income and saves only by contributing to his superannuation fund (i.e., retirement savings account) at the mandated SG level from age 25 to age 66. Contributions to the superannuation fund lower his take-home pay, which reflects the assumption that contributions come directly out of wages and hence lower net disposable income. There has been considerable debate over who bears the cost of the SG. Stanford (2019) argues that increases in the SG are largely borne by employers, and do not result in equivalent reductions in direct wages to covered employees. Coates et al. (2020) find that about 80 percent of the cost of increases in SG is passed to workers through lower wage rises.

The member retires at age 67 and transfers their balance to a retirement savings account (account-based pension) from which they withdraw until death, or the account is depleted. They are eligible for the Age Pension and supplements, which in Australia are means-tested under both an asset and income test (see Section 2.2.3). The member is assumed to obtain utility from consumption only and places no value on any bequest. The member optimises their financial decisions before and after retirement to maximise lifetime utility, making portfolio allocation decisions in both the pre-retirement and the post-retirement phases, and deciding the amount to withdraw from the retirement account in the post-retirement phase subject to minimum drawdown rules (see Section 2.2.3).

The research starts by introducing how members are characterised, with the utility functions that represent the consumption objectives outlined in Section 2.2.1 and the income model set out in Section 2.2.2. The Australian tax and social security rules incorporated in the model are explained in Section 2.2.3, the assumptions regarding investment return distributions in Section 2.2.4, and details of the mortality rates in Section 2.2.5. The dynamic programming technique is outlined in Section 2.2.6. The certainty equivalent metrics are presented in Section 2.2.7.

2.2.1 Utility functions

Two types of utility functions to represent member objectives and preferences regarding real consumption outcomes are used. See Warren (2019) for a discussion.

The first is power utility, which is broadly used within academic literature (for example, Cocco et al. 2005; Huang et al. 2008; Huang et al. 2012; Chiu and Wong 2013; Horneff et al. 2015). Its functional form is described by Equation (2.1):

$$U_{PU,t} = \frac{C_t^{1-\rho}}{1-\rho} \tag{2.1}$$

where $U_{PU,t}$ represents power utility function at time t; C_t is consumption at time t; ρ is the coefficient of constant relative risk aversion (CRRA). The baseline assumption is that ρ equals 4, which sits within the range used in the literature (for example, Ameriks et al. 2011; Yogo 2016; Butt et al. 2019). Higher and lower values of ρ are examined under sensitivity testing. The power utility is defined over consumption directly and represents members caring about the overall level of their lifetime consumption stream. Under risk aversion, it implies a preference for consumption smoothing.

The second function is reference dependent utility, which is based on the value function from prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992). Its use in this setting is similar to Blake et al. (2013). The utility is defined over the difference between consumption (C) and target consumption (C^*):

$$U_{RDU,t} = \mathbf{1}_{(C_t > C_t^*)}(C_t)(C_t - C_t^*)^{\gamma_1} - \mathbf{1}_{(C_t < C_t^*)}(C_t)(\lambda(C_t - C_t^*)^{\gamma_2}) + \mathbf{1}_{(C_t = C_t^*)}(C_t)0 \qquad (2.2)$$

where $U_{RDU,t}$ represents reference dependent utility at time t; C_t is consumption at time t; C_t^* is target consumption at time t; $\mathbf{1}$ is an indicator function that equals 1 when the condition is satisfied, and 0 otherwise; γ_1 is the curvature parameter on gains $(C_t > C_t^*)$; γ_2 is the curvature parameter on losses $(C_t < C_t^*)$; and λ is the loss aversion parameter $(C_t < C_t^*)$. In the baseline setting, this research tests the parameters from Blake et al. (2013) where $\gamma_1 = 0.44, \gamma_2 = 0.88, \lambda = 4.5$. Different parameter settings representing higher and lower loss aversion are tested in the sensitivity analysis. This utility function is used to represent members who prefer a specific level of consumption.

For target consumption, this research assumes the level differs in the pre-retirement (i.e., accumulation) and post-retirement (i.e., drawdown) phases given that the member's life situation changes (Skinner 2007). The target in the accumulation phase (C_{acc}^*) is set as the post-tax income when the SG is equal to zero, which represents the funds available to consume if there is no SG. For the drawdown phase, this research sets three consumption targets (C_{retire}^*) as follows:

- A replacement rate of 0.70 (OECD 2012), set concerning the average post-tax income with the SG at zero over the last five working years of age 62-66 (representing a higher target level relative to the situation with the SG set at 9.5%);

- A fixed consumption target equal to the modest retirement standard for single retirees published by ASFA in June 2019 of $27,814 (ASFA 2019);

- A fixed consumption target equal to the comfortable retirement standard for single retirees published by ASFA in June 2019 of $43,601 (ASFA 2019).

In the model, the target levels in the drawdown phase are lower than the target in the accumulation phase for most members, reflecting the observation that members consume less during retirement (see Skinner 2007; Olafsson and Pagel 2018).

The time preference β is assumed to be 1.00, which represents an assumption that the member values present and future equally (Butt et al. 2019). This does not allow for the possibility that the member may act myopically by placing more value on present consumption and thinking less about their future. However, the motivation behind imposing mandatory savings via the SG is to ensure that the members save enough for their retirement life. The baseline time preference of 1.00 aligns with this perspective, notwithstanding evidence that most people exhibit time preference (Frederick et al. 2002).

Power utility places equal weight on consumption during the pre-retirement and post-retirement phases, at least in absence of any time preference. It hence fails to capture a situation where many members would be accepting lower consumption post-retirement, as reflected in commonly observed consumption patterns. Thus, while this research presents the power utility results, it de-emphasises them as a basis for retirement planning.

2.2.2 Income model

This research applies a deterministic age-earnings profile using the model of Freestone (2018). This model generates a hump-shaped profile over a member’s working life with the peak at age 47[6]. The basic model of the real hourly pre-tax wage after SG at age t (h_t) is:

$$h_t = \exp\left(\alpha + \beta_1 t + \beta_2 t^2\right) \tag{2.3}$$

where α is the intercept of log hourly wage and β_1, β_2 are parameters for age and square of age respectively for a full-time male worker at age t. The parameters are calibrated using HILDA data from the Household, Income and Labour Dynamics in Australia (HILDA) survey, with $\alpha = 1.86769$, $\beta_1 = 0.06891$, and $\beta_2 = -0.000731$. Nine income levels are tested in this chapter, reflecting the 10th to 90th percentiles of wages for full-time employees in 2018 (ABS 2018). This research uses real annual pre-tax wages after the SG (equal to 9.5%) in 2018 as reference points, then apply the model of Freestone (2018) to identify the pre-tax incomes after SG at age 25 that are consistent with the real annual pre-tax wages after SG for each income percentile. The real annual pre-tax income after SG at age t for z^{th} income percentile ($e_{t,z}$) is obtained as follows:

$$e_{t,z} = i_{25,z}(h_t/h_{25}) \tag{2.4}$$

[6] The model implicitly assumes that different income percentiles have the same wage growth pattern. The relaxation of this assumption may lead to different wage growth rates with different slopes and peaks. It directly affects the pre-retirement consumptions and hence pre-retirement utility. In the post-retirement phase, the difference between superannuation balances due to the different contributions may also have impact on consumption patterns.

where $i_{25,z}$ represents the annual pre-tax income after SG the member of z^{th} income percentile earns at age 25; and z is 10 to 90. The equivalent gross annual pre-tax income before SG for zth income percentile is then estimated as $1.095e_{t,z}$.

Table 2.1 illustrates the main features of the income distribution. The average income for the 5-years before retirement is approximately 10% less than the average income over the pre-retirement phase from age 25 to age 66. This means that the 70% replacement rate objective of average income from age 62 to age 66 is equal to about a 63% replacement rate of average income over the full pre-retirement phase from age 25 to age 66.

Table 2.1: Wage Income Distribution

Income Percentiles	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Base Income, Age 25	$37,323	$42,649	$48,015	$53,745	$60,524	$68,432	$77,632	$89,454	$111,807
Maximum, Age 47	$53,395	$61,015	$68,692	$76,889	$86,587	$97,901	$111,062	$127,976	$159,955
Pre-Retirement Average, Age 25–66	$48,100	$54,964	$61,880	$69,264	$78,000	$88,192	$100,048	$115,284	$144,092
Average 5-years Prior Retirement, Age 62–66	$43,334	$49,518	$55,749	$62,401	$70,272	$79,454	$90,135	$103,862	$129,815

2.2.3 Australian tax and social security rules

In this section, this research introduces all the tax and social security rules incorporated in the model. Table 2.2 summarises these rules, with the rule names appearing in the left column and their main features in the right column.

Table 2.2: The Australian Tax and Social Security Rules

Rule name	Its main features
Australian income tax rates for 2019-20	The Australian personal income tax rates are zero for income below \$18,200, 19c for each \$1 between \$18,201 and \$37,000, then 32.5c for additional income up to \$90,000, 37c for additional income up to \$180,000, and 45c on any income above \$180,000.
Concessional contribution cap	The concessional contribution cap is \$25,000 in 2018-19 from the Australian Taxation Office.
Low income super tax offset (LISTO)	The LISTO is a government superannuation payment of up to \$500 to help low-income earners save for retirement. The LISTO is 15% of the concessional (before tax) superannuation contributions member or member's employer pays into his super fund if he earns \$37,000 or less a year. The formula used is: $LISTO_t = \min(0.15 e_t SG, 500)\, 1_{(e_t \leq 37000)}$.
Minimum drawdown rules in the retirement account	Members are required to drawdown a minimum portion of their retirement accounts at their withdrawals. The portions vary with ages: 4% when members are younger than 65, 5% at age of 65 to 74, 6% at age of 75 to 79, 7% at age of 80 to 84, 9% at age of 85 to 89, 11% at age of 90 to 94, and 14% after age of 95.
Age Pension	Asset test: The asset test is only applied where a person's assets do not exceed the upper limit for eligibility of an Age Pension. The reduction of pension under the asset test is calculated by the following method: divide the asset excess (a further asset in excess of \$258,500) by 250; round the result down to a whole number; multiply the whole number by 19.5, see https://guides.dss.gov.au/guide-social-security-law/4/2/3. Income test: The income test involves applying the deemed income rule. Deemed income is calculated by summing up 1.75% of the first \$51,200 of the account balance and 3.25% of any further balance of more than \$51,200. The income test will reduce the Age Pension received by 50 cents per dollar of additional deemed income earned.
Pension supplement	Pension supplements and energy supplements are an additional regular payment made to recipients of the Age Pension to help with the cost of prescriptions, rates, telephone, and energy.
Energy supplement	The energy supplement is a fixed amount of \$366.60 per annum. The maximum pension supplement is \$1,781.00, reducing to a minimum amount of \$954.20 at a rate that is proportionate to the Age Pension.

To estimate the post-tax disposable income that is available for consumption this research applies the Australian income tax rates for 2019-20 including the Medicare levy equal to 2% of taxable income.

The contribution tax of 15% (ATO 2019) is imposed on SG contributions and is made of pre-tax income subject to the concessional contribution cap. Contributions above this cap are added to the account balance as a non-concessional contribution, meaning that they are made of post-tax income. Members who earn a low income can benefit from the low-income super tax offset (LISTO) offered by the government[7].

Superannuation funds pay a tax rate on investment earnings of 15% on regular income and 10% on capital gains pre-retirement, and then zero tax in retirement. Superannuation funds also qualify for imputation tax credits (Butt et al. 2019). This research constructs and applies a blended tax rate on the risky asset (i.e., equities) of 6.74% pre-retirement and -6.05% post-retirement, with calculations being set out in Appendix 2A. The risk-free asset incurs an income tax rate of 15% during pre-retirement.

After retirement, withdrawals from a retirement account are subject to the minimum drawdown rules, which vary with age. The effect is that the member may be required to draw down more than what is considered optimal.

To determine Age Pension eligibility, this research applies both the asset test and income test concerning the retirement account balance. The calculations are based on the means testing arrangements in 2018-2019 for a single homeowner. This research also determines eligibility for the pension supplement and energy supplement. The sum of the basic Age Pension, pension supplement and energy supplement will be treated as the total Age Pension for which the member is eligible. Using the same simplification method as Hulley et al. (2013), the total pension received P_t as a function of the member's retirement account balance W_t at time t is calculated as follows:

$$P_t(W_t) = \begin{cases} 24{,}063, & \text{if } W_t < 161{,}231, \\ 24{,}063 - 0.016857(W_t - 161{,}231), & \text{if } 161{,}231 \le W_t < 284{,}097, \\ 24063 - 20.22813\left\lfloor \frac{W_t - 258{,}500}{250} \right\rfloor, & \text{if } 284{,}097 \le W_t < 539{,}700, \\ 0, & \text{if } W_t \ge 539{,}700 \end{cases} \tag{2.5}$$

Equation (2.5) is a piecewise linear function that results from the interaction between the asset test and income test. The first reduction in the Age Pension amount, occurring between account balances of $161,231 and $284,097, is due to the income test. The second reduction in

[7] This has no impact given income distribution.

the Age Pension amount, occurring between account balances of $284,097 and $539,700, is due to the asset test.

2.2.4 Asset returns

This research assumes that members can only invest in either a risky asset or a risk-free asset (Bikker et al. 2012). A risky asset generates real returns that follow a lognormal distribution with a geometric mean of 5.0% and a standard deviation of 17.4%, in line with historical returns on global equities as reported by Credit Suisse (2019). The real risk-free asset return is assumed to be 1.0%, which aligns with a neutral interest rate for Australia as estimated by McCririck and Rees (2017). This research uses a Gauss-Hermite quadrature method (see Hildebrand 1956; Khemka and Butt 2017) to discretise the continuous distribution of the risky return into 25 nodes.

This research applies both tax and management fees to generate net returns. Tax rates are described in Section 2.2.3. This research assumes the management fees comprise two parts: a fixed fee equal to $70 per annum and a variable fee equal to 0.80% of the account balance. The latter is deductible at the 15% rate during the pre-retirement phase. The assumed fees are comparable to those currently observed in the superannuation industry (see Australian Institute of Superannuation Trustees 2019). The total fee will typically sit somewhere between 0.80% and 0.90% per annum pre-tax for most balances. For simplicity, fees at the beginning of each year after accounting for any contribution or withdrawal are charged.

2.2.5 Mortality rates

Mortality from age 67 is based on male mortality rates with mortality improvement from the Australian Life Tables 2015-2017 in the publication of the Australian Government Actuary (2019). The mortality rate with improvement follows the formula below:

$$q_x(t) = q_x\left(1 + \frac{I_x}{100}\right)^{(t-2016)} \tag{2.6}$$

where $q_x(t)$ is the deterministic mortality rate age x in year t; q_x is the mortality rate reported for age x in the life tables; and I_x is the deterministic rate of improvement at age x with 125-year improvement factors being used to reflect the fact that the simulation spans across

86 years. In the model, the member is assumed to be at age 25 in 2020. The member dies with certainty at age 110. This research assumes that there is no mortality over the pre-retirement working period from age 25 to age 66. According to the Australian Life Tables, a male aged 25 will die before age 66 with a probability of only 8%. The assumption of zero mortality before retirement also reflects the likelihood that members will plan their retirement on the expectation that they will live through to their retirement age.

The treatment of mortality is an important factor in generating the results, as it results in a lower weight being attached to the expected utility generated post-retirement than on utility pre-retirement, as well as those weights declining with age. For example, the probability that a male member aged 25 survives to the age of 90 is 47.4%, which means the same amount of consumption at age 90 can only contribute 47.4% of utility compared to that pre-retirement. And the survival probability to age 100 is only 6.90%. Therefore, the model effectively trades off the certain reduction of consumption pre-retirement against the uncertain increase of consumption post-retirement caused by the consumption generated which might not be added to total utility in the case of a member's death. Consequently, taking into consideration of mortality rate post-retirement the model will locate an optimal SG that maximises the lifetime utility by attaching low weight on consumption post-retirement.

2.2.6 A stochastic dynamic programming model

At any time $t = 0, 1, \dots, T_R - 1, T_R, T_R + 1, \dots, T$[8], where T_R is retirement time and T is termination time, the member's value function V_t of the account balance W_t is given by maximizing member's lifetime expected utility from time t with respect to risky asset allocation rate π_t before retirement and both π_t and retirement drawdown amount D_t after retirement

The balance in the retirement account W_t is determined by Equation (2.7):

$$W_{t+1} = (W_t + S_t - D_t - F_t)\left[1 + \pi_t R_{E,t} + (1 - \pi_t) R_{C,t}\right] \tag{2.7}$$

where S_t is the total post-tax contribution to superannuation account[9]; F_t is the fee paid to a superannuation fund; $R_{E,t}$ is the post-tax real return on a risky asset (equities) and $R_{C,t}$

[8] In the model set-up, $t = 0$ corresponds to the age of 25.

[9] The calculations are detailed in Appendix 2C.

is the post-tax real return on a risk-free asset (cash)[10]. The consumption amount C_t is equal to a member's post-tax disposable income at age t before retirement ($t < T_R$), $C_t = D_t + P_t$ after retirement ($T_R \le t \le T-1$) and $C_T = W_T + P_T$ at termination time T.

The Bellman equation for any time $t = T-1, \dots, T_R+1, T_R, T_R-1, \dots, 0$ is expressed by:

$$V_t(W_t) = \sup_{\substack{\pi_t, D_t \\ D_t = 0 \text{ when } t<T_R; \\ m_t * W_t \le D_t \le W_t \text{ when } T_R \le t \le T-1; \\ \pi_t \in [0,1] \text{ when } 0 \le t \le T-1}} \left[U_{j,t}(C_t) + \beta \ {}_1p_{25+t} E_{t+1}[V_{t+1}(W_{t+1}) | W_t] \right],$$

$$\text{where } V_T(W_T) = U_{j,T}(C_T) \tag{2.8}$$

where $U_{j,t}$ is the utility function of type j in Section 2.2.1 (power utility or reference dependent utility); C_t is the consumption amount at time t, m_t denotes the minimum drawdown rates in Section 2.2.3; ${}_tp_x$ is the probability that a member aged x will be alive at age $x+t$; and β is the time preference. $E_{t+1}[V_{t+1}(W_{t+1})|W_t]$ calculates the conditional expectation of future value function with given the information at time t, noting that W_t is a function of $R_{E,t}$ by Equation (2.7) and $R_{E,t}$ follows lognormal distribution simulated by Gauss-Hermite quadrature method.

For a given SG level, this research recursively optimises each member's lifetime financial decisions at each age using the Bellman equation (Equation 2.8) under a dynamic programming framework (Butt and Khemka 2015). The state variable is the retirement account balance W_t, and is discretised in \$20,000 increments from \$0 to \$2,000,000. This research uses shape preserving Schumaker splines (Schumaker 1983; Judd 1998) for interpolation due to concavity and monotonicity of the recursive utility values. The optimisation process is performed using R (R Core Team 2019) with the DEoptim package (Ardia et al. 2016). Once optimisation is undertaken, this research projects 10,000 asset return paths for each age over member's entire lifetime (age 25-109) and simulate the member's outcomes using the optimal decisions. This research assumes that the returns between ages are independent and perform linear interpolation of the optimal decisions between wealth increments.

For a specific SG, the member's lifetime utility $\hat{V}_0^y(W_0, SG)$ in yth simulation out of K is calculated by adding utility through their lifetime as Equation (2.9):

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[10] The return calculations are detailed in Appendix 2B.

$$\hat{V}_0^y(W_0, SG) = \sum_{s=0}^{T} \beta^s \ {}_sp_{25} U_{j,s}(C_s^{y*}) \tag{2.9}$$

where C_t^{y*} represents the optimal consumption at time t in yth simulation.

Member's expected lifetime utility $\bar{V}(SG)$ is calculated by averaging member's lifetime utility across K simulations as Equation (2.10):

$$\bar{V}(SG) = \frac{1}{\mathrm{K}} \sum_{y=1}^{\mathrm{K}} V_y(SG) \tag{2.10}$$

where $K = 10{,}000$ is applied in this chapter.

For each member with a specific objective and income, this research defines that the 'optimal' SG (SG^*) is the one that can maximise their expected lifetime utility as Equation (2.11), recalling that the SGs tested are increased in increments of 0.5%:

$$\bar{V}(SG^*) = \max_{0\% \leq SG \leq 20\%} \bar{V}(SG) \tag{2.11}$$

2.2.7 Certainty equivalents

Certainty equivalents based on estimates of expected lifetime utility are calculated to evaluate member welfare. The measures include, 'certainty equivalent consumption' (CEC) for power utility, and a new measure called 'constant distance from reference level' (CDRL) for reference dependent utility.

CEC is the constant known consumption over a member's lifetime that generates the same total utility as the expected lifetime utility. It can be interpreted as the riskless consumption stream throughout the member's life that generates the same utility as the random consumption stream under the simulations. CEC is estimated by locating the CEC value that solves the following equations:

$$\sum_{s=0}^{T} \beta^s \ {}_sp_{25} U_{PU,s}(CEC) = \bar{V}(SG) \tag{2.12}$$

CEC and the concept of a constant consumption stream are not applicable under reference dependent utility where target consumption varies over time. This research reformulates the certainty equivalent calculations to generate CDRL, which is the constant deviation from the reference level across all ages that generates the same utility as the expected lifetime utility arising from the random consumption stream under the simulations. CDRL is estimated by locating the CDRL value that solves the following equation:

$$\sum_{s=0}^{T} \beta^{s} \ {}_{s}p_{25} U_{RDU,s}(C_t^* + CDRL) = \bar{V}(SG) \tag{2.13}$$

Figure 2.1 illustrates CDRL. The solid line represents the consumption reference level throughout a member’s whole life, which follows a hump-shaped curve pre-retirement in line with the income model and a horizontal line post-retirement represents a constant post-retirement consumption target. It is assumed there exists a lifetime consumption pattern represented by the dashed line that has the same shape as the reference level. This consumption pattern would produce a total utility equal to the expected lifetime utility in the simulation. This research finds this consumption pattern and reports the difference between the reference level and consumption level as CDRL. A positive CDRL means a consumption level above the reference level, while a negative CDRL represents lifetime consumption below the reference level. CDRL is typically negative in this study as any positive SG contributions will incur deterministic utility losses pre-retirement, which act to increase the probability of positive utility post-retirement subject to stochastic investment returns and mortality.

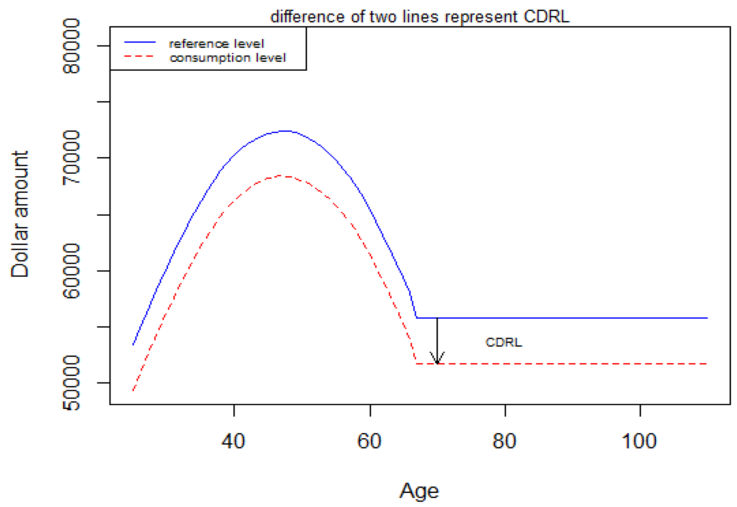


Figure 2.1: Constant Distance from Reference Level

2.3 Results

Here, the optimal SG estimates across nine income levels and four objectives, including consumption smoothing as represented by the power utility as well as three consumption target objectives (income replacement rate, ASFA Modest and ASFA comfortable) represented by the reference dependent utility functions are reported. First, the optimal SG estimates are presented in Section 2.3.1, emphasising heterogeneity in members' characteristics. How members' welfare changes with different SG levels is then gauged in Section 2.3.2, followed by the analysis of government budgetary impacts from the perspective of members in Section 2.3.3. Other determinants of the optimal SG are then investigated in Section 2.3.4 by reporting the revised optimal SG when there is no availability of the Age Pension system, when members take two commonly adopted investment strategies (fixed and life cycle), and when members have inter-temporal consumption preferences.

2.3.1 Optimal superannuation guarantee

Table 2.3 reports the matrix of the optimal SGs for the four consumption objectives and the nine income levels, as estimated using the method outlined in Section 2.2.6. Two stark findings emerge. First, there exist large variations in the optimal SG across income levels and objectives. Second, the optimal SG estimates for the power utility case are quite different from those for the three reference dependent utility cases. Specifically, the optimal SG for the power utility case varies from 11.5% to 20%, while the optimal SG for the three reference dependent utility cases is below 10%, varying from 0.5% to 8.5%. The reasons for these results are discussed in the following sub-sections, initially addressing the consumption smoothing results and then explaining the consumption target objectives.

Table 2.3: Optimal Superannuation Guarantee

	Income Percentiles								
Objective	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Consumption Smoothing	11.5%	13.0%	14.5%	15.5%	17.0%	18.0%	19.5%	20.0%*	20.0%*
70% Replacement Rate	1.0%	2.0%	3.5%	3.5%	4.0%	4.5%	5.0%	5.5%	6.5%
ASFA Comfortable	8.5%	7.5%	6.5%	6.0%	5.5%	4.5%	4.0%	3.5%	3.0%
ASFA Modest	2.0%	1.5%	1.5%	1.5%	1.0%	1.0%	1.0%	1.0%	0.5%

'*' indicates that an optimal SG of equal to, or larger than, the maximum tested of 20%.

2.3.1.1 Large variation in the optimal SG

Table 2.3 reveals that the optimal SG increases with income under the consumption smoothing and replacement rate objectives, while the optimal SG declines with income under the two ASFA objectives. These patterns can be explained by three factors:

- *Age Pension* - The Age Pension provides Australian members with a stable income stream at retirement. It is of greater value for members with lower incomes relative to those with higher incomes because it is a higher portion of their retirement income (Grattan 2018), and due to the influence of the asset and income means tests. The full Age Pension and other supplements in the model are equal to $24,063. Members who are eligible for the full pension are only $3,751 short if their objective is ASFA modest ($27,814), while low-income earners with a replacement rate objective have a post-retirement consumption target not far from the ASFA modest standard. The Age Pension is important in explaining the low SG under the ASFA modest and replacement rate objectives for low-income earners, as it lowers the savings required to make the target. For middle and high-income earners with the replacement rate and members with ASFA comfortable objectives, the Age Pension makes a lower contribution and hence a higher SG is required to attain the retirement consumption target.

- *Nature of the post-retirement consumption target* – Under fixed post-retirement consumption targets as represented by ASFA standards, the optimal SG declines as the income increases as a lower portion of income is needed to be saved to achieve the target when income is higher. On the other hand, for the consumption smoothing target and replacement rate where the post-retirement target scales up and down with income, the optimal SG increases with income primarily due to the reduction in relative benefit from Age Pension receipts.

- *Tax effects* – The difference between the income tax rate and the superannuation contribution tax rate in Australia generates differing benefits for members with low and high incomes, making saving via the SG less beneficial for low-income earners. Recall that the Australian personal income tax rates are zero for income below $18200, 19c for each $1 between $18,201 and $37,000, then 32.5c for additional income up to $90,000. Meanwhile, superannuation is taxed at 15% on contributions[11], 15% on income, and 10% on capital gains. Saving via SG would lead to lower effective tax rates for a high-income earner.

[11] The contribution tax rate is 30% for incomes above $250,000.

2.3.1.2 Difference between power utility and reference dependent utility

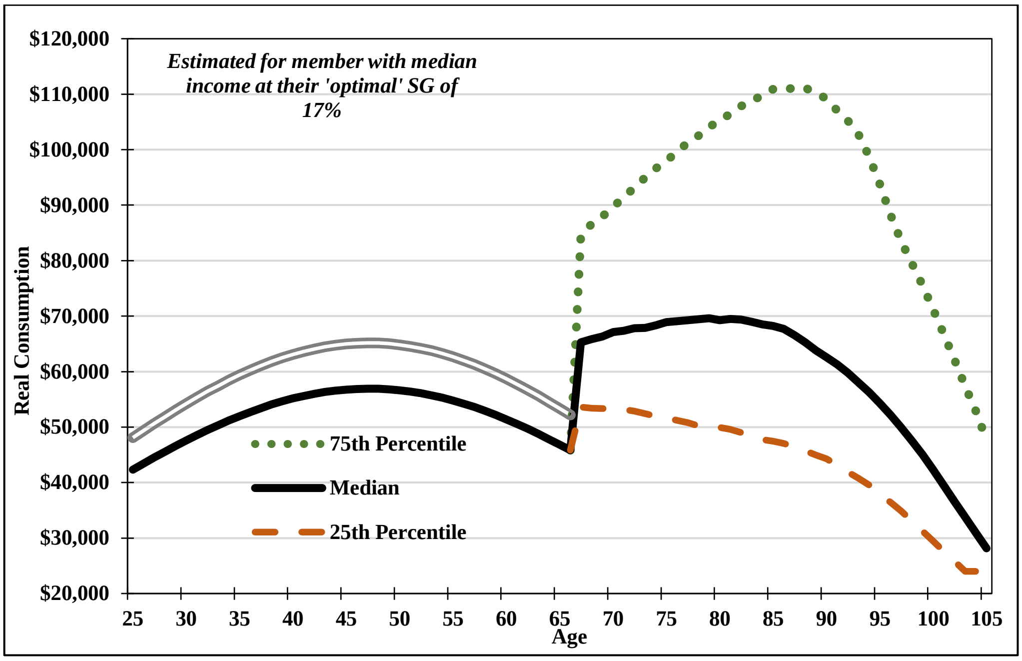
The large difference in optimal SG estimates between the consumption smoothing objective and the three retirement consumption target objectives stems from the relation between the model set-up and the member’s retirement objectives. For the consumption smoothing objective as expressed by the power utility, the members want to maximise their lifetime consumption and avoid poor outcomes in the lower tail of the distribution. As their pre-retirement consumption, which is equal to the post-tax income after contribution, is fixed and without any risk, any poor outcomes only appear in the post-retirement phase when investment returns are extremely low, or their balance is exhausted. The optimal SG for these members is the one that helps them build adequate retirement fund balances to mitigate the risk of potentially poor outcomes that can arise in the post-retirement phase. The preference to avoid poor outcomes and the inherent consumption smoothing pattern under power utility results in a relatively high SG. Moreover, the high investment returns also play a role, as they provide the opportunity to secure higher expected consumption post-retirement in the high SG situation, as well as avoid the risk of low outcomes.

Recall that any positive SG is treated as a loss of pre-retirement consumption relative to the target under the reference dependent model set-up. The member prefers an SG that effectively balances the utility loss incurred pre-retirement due to SG contribution against the utility loss post-retirement due to failure to achieve the consumption target. Moreover, the post-retirement consumption target is generally lower than the pre-retirement consumption target in the model set-up. A lower SG hence emerges as a balancing act between utility losses pre-retirement and post-retirement consumption target with the help of the Age Pension, without the more extreme aversion to lower consumption post-retirement that occurs under power utility with its high concavity.

2.3.1.3 Consumption patterns under different objectives

To illustrate the consumption patterns under the different objectives at the optimal SG level, the consumption profiles of the median income earner are plotted for the consumption smoothing objective expressed by power utility and replacement rate target expressed by reference dependent utility, as the consumption patterns under the two ASFA standard objectives are quite similar to that under the replacement rate objective.

Figure 2.2 illustrates the consumption profile of the median income earner with the consumption smoothing objective. The double grey line represents the post-tax income with 0% SG. Median and 75th percentile post-retirement consumptions follow a hump-shaped pattern and indicate an optimal SG rate higher than required to maintain the consumption level. This pattern is similar to Butt and Khemka's (2015) results. At the 25th percentile, relatively quick deterioration in consumption levels post-retirement are seen, providing an indication of the lower tail outcomes that the power utility setting is trying to avoid as discussed above. The main trade-off, in this case, is that members contribute 17% SG pre-retirement in exchange for consumption being higher post-retirement than pre-retirement with a more than 50% likelihood. The decline in consumption later in retirement reflects the influence of mortality weighting, which results in consuming earlier in retirement generating higher expected utility than consumption later in retirement.



The double grey line represents the post-tax income with 0% SG.

Figure 2.2: Consumption for Median Income Earner at the Optimal SG under Power Utility

Figure 2.3 depicts the consumption profile of the median income earner under the replacement rate objective. The double grey line represents the pre-retirement and the post-retirement targets: the post-tax income available with 0% SG before retirement, and a 70% income replacement rate after retirement. The consumption pattern in Figure 2.3 indicates that it is optimal to consume the post-retirement target in most cases until the retirement account balance runs out, which is in line with Butt et al. (2019). As the drawdown is subject to the minimum drawdown rule, the members may need to withdraw more than the target when the retirement account balance is boosted by investment returns. This helps explain the bumps in the 75th percentile line. The figure demonstrates the main trade-off in the reference dependent setting, namely the reduction of pre-retirement consumption against the increase of post-retirement consumption. The latter is expressed by the length of time that the member can achieve the target consumption. Specifically, the optimal SG of 4% could maintain the target consumption until a median age of 91, with a 25th percentile age of 87 and 75th percentile age of 102. It indicates that a member with median income and a replacement rate objective would be willing to sacrifice 4% of their pre-retirement consumption in exchange for consuming the target post-retirement until their 80’s with a 75% likelihood and until their age of 100 with more than 25% probability.

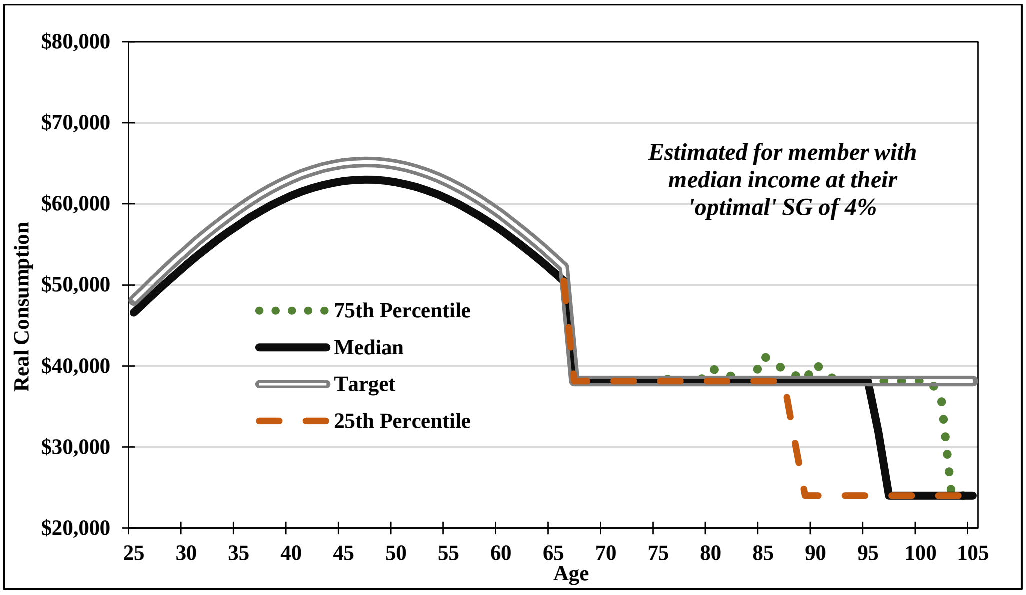


Figure 2.3: Consumption for Median Income Earner at the Optimal SG under Reference Dependent Utility with Replacement Rate Target

2.3.2 Welfare changes with differing Superannuation Guarantee levels

Using the expected lifetime utility estimated in the simulations for members with different objectives and income levels, how member welfare changes with the SG levels are now investigated. The outcomes also help us to test the impact of increasing the SG from 9.5% to 12% on members’ welfare. To make the analysis more tangible, the expected lifetime utility is converted into certain equivalents in terms of lifetime consumption as described in Section 2.2.7. This research calculates CEC in the power utility case and CDRL in the three reference dependent utility cases.

Figure 2.4 plots the respective certainty equivalents under the four consumption objectives for income percentiles of 30%, 50%, and 70%, except in the ASFA modest case where the research plots income percentiles of 10%, 20%, and 30% as this objective has limited relevance for members with higher incomes. The vertical double-line in each chart represents the current SG of 9.5%. The highest point in each curve is the optimal SG estimate presented in Table 2.3. The slope of the curves around the optimal SG indicates the degree of potential welfare loss incurred if a sub-optimal SG is imposed. In the power utility case, the three curves all exhibit a steep increase when the SG is small before flattening out at high SG levels. This indicates a larger welfare loss for imposing a very low SG if the member’s objective is to maximise lifetime consumption.

For all three reference dependent utility cases, the slope of the curve around the optimal SG is relatively steep, which means imposing a sub-optimal SG in either direction would lead to a meaningful welfare loss for the member. This feature seems reasonable. When there exists a penalty on any positive SG before retirement and on consumption below the post-retirement consumption target after retirement, member welfare would be harmed by saving too much or too little.

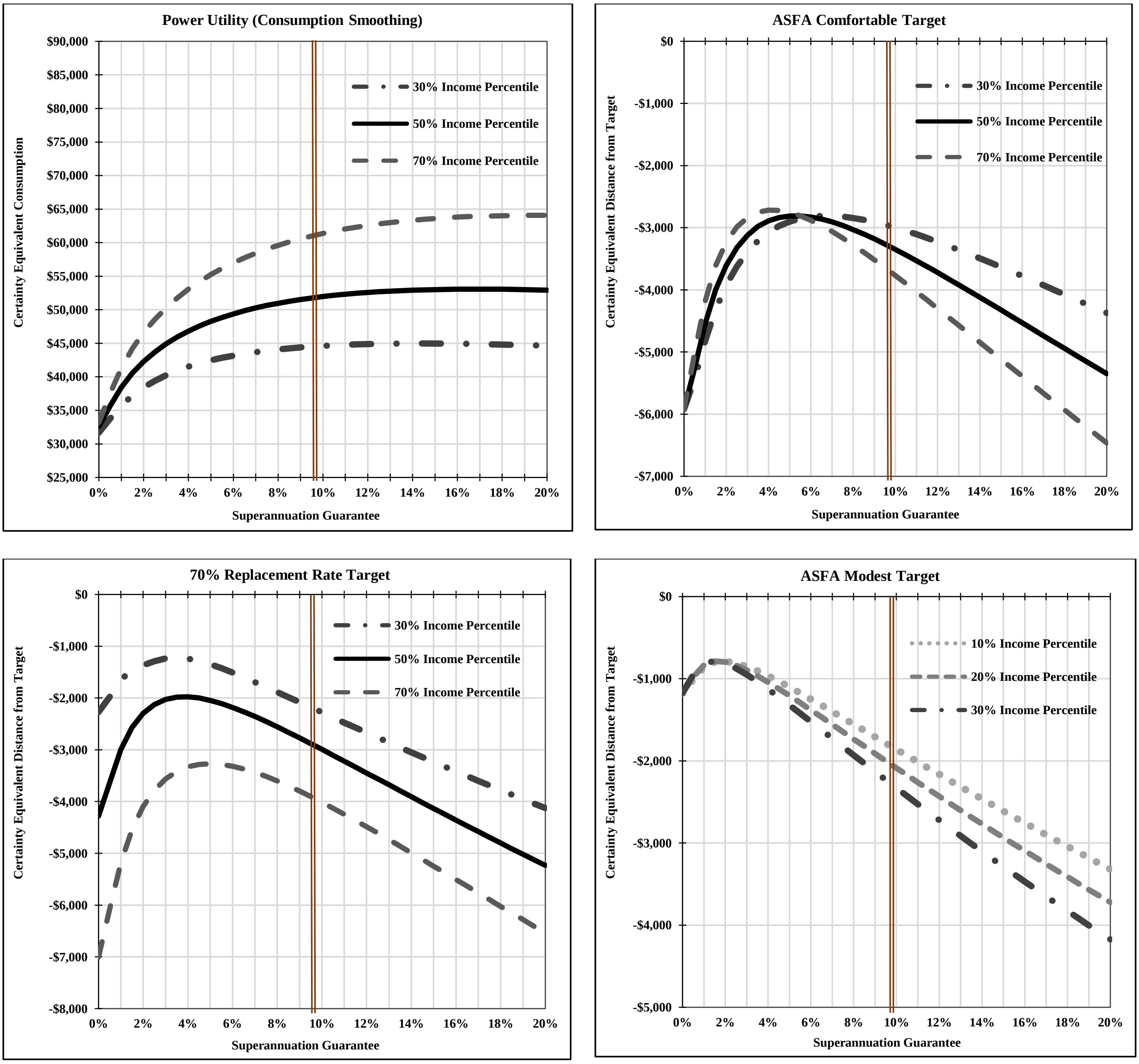


Figure 2.4: Certainty Equivalent Metrics at Three Income Levels

Table 2.4 provides the difference in certain equivalents for the current SG of 9.5% versus the optimal SG in Panel A, and the proposed SG of 12% versus the current SG of 9.5% in Panel B. The two panels report both real dollar differences in certainty equivalent, and then these differences are expressed as a percentage of income in the 5 years before retirement. Panel A reveals the welfare loss incurred by setting the current SG instead of the optimal SG. The results indicate that members with income percentiles of 10% to 40% suffer large losses under the replacement rate and ASFA modest objectives, where the losses vary from 1.6% to 2.9%, which reflects the relatively low optimal SG estimates in these cases. From the 50% to 90% income percentiles, welfare losses are relatively modest (0.3% to 1.6%) for the replacement rate and ASFA comfortable objectives, while they are large at around 2.9% for ASFA modest objective and up to 5.4% for the consumption smoothing objective.

Table 2.4: Certain Equivalent Gains (Losses) at 9.5% and 12% Superannuation Guarantees

Difference in Certainty Equivalent Consumption or Distance from Target	**Income Percentile**								
	10%	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
PANEL A: 9.5% vs Optimal SG									
Difference in Dollars (Real)									
Power Utility, Consumption Smoothing	-65	-226	-458	-797	-1,308	-2,031	-3,070	-4,473	-7,005
Reference Dependent, Replacement Rate	-1,275	-1,117	-962	-1,001	-903	-781	-634	-512	-354
Reference Dependent, ASFA Comfortable	-6	-54	-141	-271	-451	-677	-925	-1,302	-2,125
Reference Dependent, ASFA Modest	-997	-1,213	-1,446	-1,688	-1,987	-2,278	-2,583	-3,013	-3,923
% of Income 5-Year Prior Retirement									
Power Utility, Consumption Smoothing	-0.2%	-0.5%	-0.8%	-1.3%	-1.9%	-2.6%	-3.4%	-4.3%	-5.4%
Reference Dependent, Replacement Rate	-2.9%	-2.3%	-1.7%	-1.6%	-1.3%	-1.0%	-0.7%	-0.5%	-0.3%
Reference Dependent, ASFA Comfortable	0.0%	-0.1%	-0.3%	-0.4%	-0.6%	-0.9%	-1.0%	-1.3%	-1.6%
Reference Dependent, ASFA Modest	-2.3%	-2.5%	-2.6%	-2.7%	-2.8%	-2.9%	-2.9%	-2.9%	-3.0%
PANEL B: 12% SG vs. 9.5% SG									
Average Income, 5-Years Prior Retirement	43,334	49,518	55,749	62,401	70,272	79,454	90,135	103,862	129,815
Difference in Dollars (Real)									
Power Utility, Consumption Smoothing	60	209	368	562	816	1,145	1,573	2,117	3,144
Reference Dependent, Replacement Rate	-391	-429	-488	-532	-567	-595	-583	-587	-601
Reference Dependent, ASFA Comfortable	-107	-191	-276	-361	-460	-570	-658	-786	-1,032
Reference Dependent, ASFA Modest	-381	-433	-491	-552	-625	-709	-772	-874	-1,088
% of Income 5-Year Prior Retirement									
Power Utility, Consumption Smoothing	0.1%	0.4%	0.7%	0.9%	1.2%	1.4%	1.7%	2.0%	2.4%
Reference Dependent, Replacement Rate	-0.9%	-0.9%	-0.9%	-0.9%	-0.8%	-0.7%	-0.6%	-0.6%	-0.5%
Reference Dependent, ASFA Comfortable	-0.2%	-0.4%	-0.5%	-0.6%	-0.7%	-0.7%	-0.7%	-0.8%	-0.8%
Reference Dependent, ASFA Modest	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.8%	-0.8%

Panel B investigates the impact on member welfare if the SG increases as planned to 12% from the current level of 9.5%. For the power utility objective, members experience a welfare increase of 0.1% to 2.4% of income, with larger gains for higher-income earners. However, members with reference dependent objectives incur modest welfare losses of 0.2% to 0.9%. Overall, the results indicate that increasing the SG to 12% might harm the welfare of members with consumption target objectives, and only benefit members with a consumption smoothing objective. To the extent that the consumption target objectives are more relevant, it seems the planned increase in SG might harm more members than it helps, although the estimates of welfare losses are relatively modest compared to their incomes.

2.3.3 The net impact on the government budget

The total impact of differing SG levels on the government budget is calculated at the member level, i.e., this research estimates the aggregate flows between the government and a member over their lifetime. The main inflows to the government include income tax and the Medicare levy, the superannuation contributions tax, and net taxes on superannuation earnings after franking credits and fees; all of which occur in the pre-retirement phase. The outflows consist of Low-Income Superannuation Tax Offset (LISTO) in the pre-retirement phase, and the Age Pension and supplements and franking credits claimed during the post-retirement phase. All the elements are summed together to get the net revenue to the government derived from the member, with the post-retirement inputs weighted by survival probabilities to reflect uncertain life expectancy.

As the calculation focuses on the member level, it ignores the aggregate budget impacts. Therefore, it is more meaningful to examine how the net revenue to the government varies with the SG rather than the absolute numbers themselves.

Table 2.5 provides the estimates of net revenue to the Government when the SG is set at 0%, 9.5%, and 12% in Panel A; with the change in net revenue for an SG of 9.5% versus 0% and an SG of 12% versus 9.5% in Panel B. The estimates indicate that the government net revenues increase by $85,000 to $272,000 by imposing an SG of 9.5% relative to 0% SG across different income earners and different objectives, with high-income earners contributing greater increases in dollar terms. Further, government revenue benefits from changing the current SG of 9.5% to the proposed SG of 12%. There is an increase in net revenue across most objectives and income levels, ranging from $1,000 up to $42,000. The exceptions are at the income percentile of 90% under ASFA standards, where the government incurs a revenue loss of $13,000. Overall, the government benefits from the set-up of the SG system and the increase of the SG from 9.5% to 12%. However, the increase of government revenue from the increase of the SG is only modest considering the numbers in Table 2.5 is the total amount throughout a member's lifetime.

Table 2.5: Net Revenue to the Government – Tax Collected less Pension and Related Supplements

$'000 per Member over their Lifetime (Constant 2019 Dollars)	**Income Level**								
	L1	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**
PANEL A: Net Revenue Estimates									
Superannuation Guarantee of 0%	-193	-94	6	113	240	393	580	828	1,300
Superannuation Guarantee of 9.5%									
Power Utility, Consumption Smoothing	-84	35	153	278	424	591	791	1,053	1,547
Reference Dependent, Replacement Rate	-108	7	133	214	374	575	804	1,079	1,572
Reference Dependent, ASFA Comfortable	-92	27	146	272	418	585	783	1,042	1,521
Reference Dependent, ASFA Modest	-107	7	121	240	380	540	731	981	1,452
Superannuation Guarantee of 12%									
Power Utility, Consumption Smoothing	-52	68	187	312	457	622	818	1,075	1,561
Reference Dependent, Replacement Rate	-84	32	160	256	408	603	827	1,100	1,588
Reference Dependent, ASFA Comfortable	-59	61	181	306	450	613	804	1,051	1,508
Reference Dependent, ASFA Modest	-83	31	144	262	399	556	740	982	1,439
PANEL B: Changes in Net Revenue									
Superannuation Guarantee 9.5% vs. 0%									
Power Utility, Consumption Smoothing	110	128	147	165	184	198	210	225	247
Reference Dependent, Replacement Rate	85	101	127	101	134	183	224	251	272
Reference Dependent, ASFA Comfortable	101	121	140	158	178	192	203	214	221
Reference Dependent, ASFA Modest	87	101	114	127	140	147	150	153	153
Superannuation Guarantee 12% vs. 9.5%									
Power Utility, Consumption Smoothing	32	33	34	34	33	32	27	22	14
Reference Dependent, Replacement Rate	24	24	26	42	33	28	23	21	16
Reference Dependent, ASFA Comfortable	33	34	35	34	32	29	20	9	-13
Reference Dependent, ASFA Modest	24	24	23	22	20	16	9	1	-13

2.3.4 Some key considerations for setting an appropriate SG rate

In addition to member objectives and income levels, the research investigates the implications for the optimal SG of assumptions related to the Age Pension, investment strategies and inter-temporal consumption preference. This section reveals that changes to any of these assumptions can significantly change the estimates of the optimal SG. This research finds no access to the Age Pension has a large impact, with the investment strategy and time preference having more moderate but still meaningful effects.

2.3.4.1 Availability of the Age Pension

Section 2.3.1 explained the impact of the Australian Age Pension system on the optimal SG under heterogeneity. In this section, the optimal SG levels excluding the means-tested Age Pension and related supplements are estimated. This is equivalent to establishing the contribution rate that maximises expected utility without relying on access to the Age Pension. These estimates might be interpreted considering the primary objective for superannuation is ‘‘to provide income in retirement to substitute or supplement the Age Pension’’ as recommended by the Australian Government in Financial System Inquiry (The Treasury 2016). No access to the Age Pension indicates a situation where members become self-funded retirees, and hence an objective of replacing rather than supplementing the Age Pension. In this situation, members need to optimise savings via the superannuation mechanism to ensure they have enough funds to sustain consumption during retirement.

Table 2.6 presents the revised optimal SG estimates without the Age Pension along with the changes in optimal SG from the baseline estimates that are reported in Table 2.3. The impact is particularly large. The optimal SG estimates are all above 20% in the power utility case. Under the replacement rate objective, all the optimal SG estimates stand above 10%. Under the ASFA comfortable objective, members with 10% to 70% income percentiles need to save 9.5% or more of their incomes, while for the ASFA modest objectives, an optimal SG of more than 9.5% emerges for members in the 10% to 30% income percentile. (The other optimal SG estimates under the ASFA standards are below 9.5%). Moreover, the increases in the optimal SG from the baseline setting vary from 3.5% to 11.5% across the three reference dependent utility cases. It is the low-income earners that experience the largest increases in the optimal

SG, which also confirms the role of the Age Pension as a crucial component of members’ retirement income in the baseline setting for low-income earners. This finding is also in line with Poterba (2014) that social security is the primary source of retirement income for most members.

Table 2.6: Optimal SG without Age Pension System

	Income Percentile								
Objective	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
REVISED OPTIMAL SG									
Consumption Smoothing	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*
70% Replacement Rate	12.0%	11.5%	11.5%	11.0%	11.0%	10.5%	10.5%	10.5%	10.0%
ASFA Comfortable	20.0%*	18.5%	16.0%	14.5%	12.5%	11.0%	9.5%	8.5%	6.5%
ASFA Modest	13.0%	11.5%	10.0%	9.0%	7.5%	7.0%	6.0%	5.5%	4.0%
CHANGE vs. BASELINE SETTING									
Consumption Smoothing	8.5%	7.0%	5.5%	4.5%	3.0%	2.0%	0.5%	**	**
70% Replacement Rate	11.0%	9.5%	8.0%	7.5%	7.0%	6.0%	5.5%	5.0%	3.5%
ASFA Comfortable	11.5%	11.0%	9.5%	8.5%	7.0%	6.5%	5.5%	5.0%	3.5%
ASFA Modest	11.0%	10.0%	8.5%	7.5%	6.5%	6.0%	5.0%	4.5%	3.5%

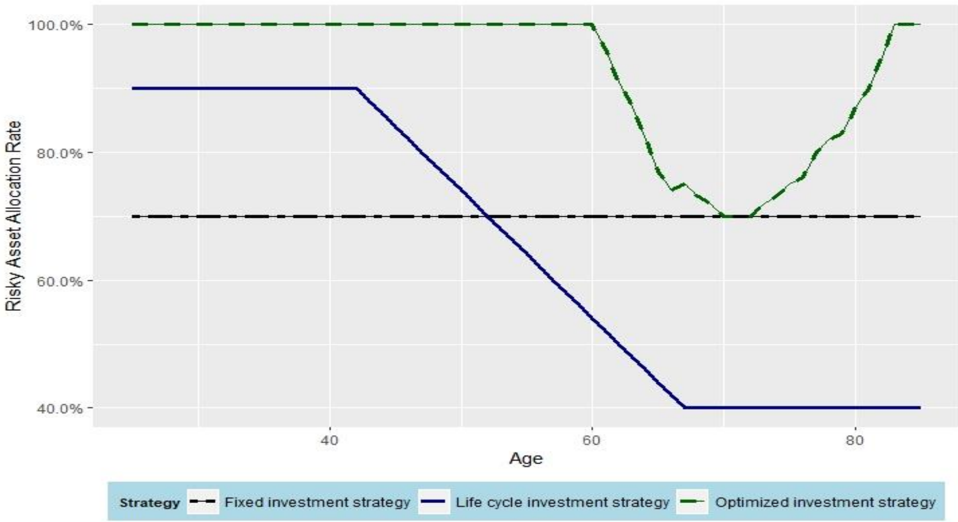
‘*’ indicates that an optimal SG of equal to, or larger than, the maximum tested of 20%
‘**’ indicates both baseline and revised optimal SG is equal to, or larger than, maximum tested of 20%, i.e., model captures no change

The revised findings indicate that the current SG of 9.5% and the proposed 12% are well-situated under the reference dependent objectives, which indicates that most members need a 9.5% or 12% SG level to attain their post-retirement consumption objectives if the Age Pension and supplements are excluded from consideration. It suggests that both the current and proposed SG are justified under an objective for the SG system of replacing the Age Pension system.

2.3.4.2 Investment strategy assumptions

The model effectively generates the optimal SG assuming the members rationally optimise their investment and drawdown decisions, in line with dynamic programming techniques. However, members may act differently in making financial decisions instead of performing rationally (Knoll 2010). In the following two sections, this research investigates the impact on the optimal SG of assumptions related to two commonly observed behavioural decision-making effects. The first is the adoption of pre-defined investment strategies. The second is to assume that the members have inter-temporal consumption preferences.

Alserda et al. (2019) find that asset allocations in pension plans impose welfare losses as these allocations are safer than implied by member risk preferences. Bhargava and Hyde (2022) document substantial risk aversion and choice diversity in the decisions of employees, while Guiso et al. (2018) find that the risk aversion increase substantially after the 2008 crisis. This research tests the impact of two common investment strategies that members often adopt, rather than optimizing their asset allocations. The first is a fixed investment strategy: 70% in the risky asset and 30% in the risk-free asset (Drew et al. 2014). This represents the ‘balanced’ investment pattern adopted by a wide range of members. The other is a life-cycle strategy starting with 90% weight in the risky asset until 25 years before retirement, then linearly reducing to 40% weight at retirement which is maintained thereafter. This represents the typical glide path observed for Australian life-cycle funds (Rice Warner 2016). Figure 2.5 compares risky asset weights under the fixed and life-cycle through member lifetime with the optimised strategy for the median



income earner with an ASFA comfortable objective at their optimal SG level of 5.5%.

Figure 2.5: Investment Strategy Comparison for Median Income Earner under Reference Dependent Utility with ASFA Comfortable Objective

Figure 2.5 illustrates that both the fixed and life-cycle strategies allocate much less to risky assets than the optimised strategy, and thus fail to take advantage of the higher compound returns of the risky asset. The more conservative asset mix occurs not only pre-retirement but continues into the post-retirement phase when the two pre-defined strategies still allocate less to the risky asset while the optimised strategy follows a u-shaped equity allocation. The increase in equity weight under the optimised strategy commences from an age of early 70s and stems from the combined effect of a decrease in members' private retirement saving and the effect of the Age Pension, where the latter provides a hedge against poor investment returns. The Age Pension allows members to take higher investment risks after retirement as it acts as a safety net in the case of poor investment returns, which is in line with the finding of Andreasson et al. (2017). The two commonly adopted investment strategies are consistent with members adopting investment strategies that are safer than implied by the member objectives and preferences underpinning the modelling, and hence may not be utility-maximizing.

Table 2.7 reports the revised optimal SG under fixed and life-cycle investment strategies, with the change of SG relative to the baseline set at the bottom. Adopting these two pre-defined investment strategies leads to increases of between 0.5% and 4.5% in the optimal SG across most of the income percentiles and objectives. The power utility case experiences the largest increase[12]. As in the baseline setting, the optimal SG in the power utility case is over 10% and implies that members are not taking full advantage of high compound returns available from risky assets, requiring members to save more to achieve their consumption smoothing targets and avoid the possibility of poor consumption outcomes post-retirement. In addition, the need for higher savings to attain the post-retirement target when returns are lower explains the relatively large increase in the optimal SG for high-income earners with a replacement rate objective and low-income earners with an ASFA comfortable objective. Meanwhile, members with an ASFA modest objective do not see any increase in their optimal SG as the Age Pension has already done most of the job of helping to attain the consumption targets.

[12] The increase of optimal SG for low-income earners with ASFA comfortable is also large. However, this objective has little relevance for low-income earners as it implies an increase in the consumption target after retirement.

Table 2.7: Optimal SG with Fixed and Life-Cycle Investment Strategies

	Income Percentile								
Objective	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
REVISED OPTIMAL SG									
Fixed 70/30 asset mix									
Consumption Smoothing	13.0%	15.0%	17.0%	19.0%	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*
70% Replacement Rate	0.5%	2.5%	3.5%	4.0%	5.0%	5.5%	6.5%	7.0%	8.5%
ASFA Comfortable	11.5%	10.0%	9.0%	8.0%	7.0%	6.0%	5.5%	4.5%	3.5%
ASFA Modest	2.0%	2.0%	1.5%	1.5%	1.5%	1.0%	1.0%	1.0%	1.0%
Life-cycle: 90/10 to 40/60									
Consumption Smoothing	14.0%	16.0%	18.5%	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*	20.0%*
70% Replacement Rate	0.5%	2.5%	4.0%	4.5%	5.5%	6.5%	7.0%	8.0%	9.5%
ASFA Comfortable	13.0%	11.0%	10.0%	8.5%	7.5%	7.0%	6.0%	5.0%	4.0%
ASFA Modest	2.0%	1.5%	1.5%	1.5%	1.0%	1.0%	1.0%	1.0%	0.5%
CHANGE vs. BASELINE SETTING									
Fixed 70/30 asset mix									
Consumption Smoothing	1.5%	2.0%	2.5%	3.5%	3.0%	2.0%	0.5%	**	**
70% Replacement Rate	-0.5%	0.5%	..	0.5%	1.0%	1.0%	1.5%	1.5%	2.0%
ASFA Comfortable	3.0%	2.5%	2.5%	2.0%	1.5%	1.5%	1.5%	1.0%	0.5%
ASFA Modest	..	0.5%	..	..	0.5%	..	..	..	0.5%
Life-cycle: 90/10 to 40/60									
Consumption Smoothing	2.5%	3.0%	4.0%	4.5%	3.0%	2.0%	0.5%	**	**
70% Replacement Rate	-0.5%	0.5%	0.5%	1.0%	1.5%	2.0%	2.0%	2.5%	3.0%
ASFA Comfortable	4.5%	3.5%	3.5%	2.5%	2.0%	2.5%	2.0%	1.5%	1.0%
ASFA Modest	..	..	..	..	..	..	..	..	..

'*' indicates that an optimal SG of equal to, or larger than, the maximum tested of 20%
'**' indicates both baseline and revised optimal SG is equal to, or larger than, maximum tested of 20%, i.e., model captures no change
'..' indicates there is no change.

2.3.4.3 *Time preference assumption*

Another commonly observed behavioural effect is that members have inter-temporal consumption preferences (myopia) valuing present consumption more than consumption in the future. This preference may lead to an issue whereby members consume too much today and do not save enough for retirement. In the baseline setting it is assumed the members have no discount rate (time preference parameter of 1) valuing the utility generated pre-retirement and post-retirement equally important. However, the time preference estimates within the literature show very wide variation, with the majority indicating a preference for current over future consumption (Frederick et al. 2002). This section estimates the optimal SG levels assuming members have a time preference of 0.98 (see Lockwood, 2018; Li et al., 2016).

Table 2.8: Optimal SG with Higher Discount Rate

	Income Percentile								
Objective	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
REVISED OPTIMAL SG									
Consumption Smoothing	7.5%	8.5%	9.5%	10.5%	12.0%	13.0%	14.5%	15.5%	17.5%
70% Replacement Rate	0.0%	1.0%	1.5%	2.0%	2.5%	2.5%	3.0%	3.0%	3.5%
ASFA Comfortable	5.0%	4.5%	4.0%	3.5%	3.0%	3.0%	2.5%	2.0%	1.5%
ASFA Modest	1.0%	1.0%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%
CHANGE vs. BASELINE	-	-	-	-	-	-	-	-	-
Consumption Smoothing	-4.0%	-4.5%	-5.0%	-5.0%	-5.0%	-5.0%	-5.0%	-4.5%	-2.5%
70% Replacement Rate	-1.0%	-1.0%	-2.0%	-1.5%	-1.5%	-2.0%	-2.0%	-2.5%	-3.0%
ASFA Comfortable	-3.5%	-3.0%	-2.5%	-2.5%	-2.5%	-1.5%	-1.5%	-1.5%	-1.5%
ASFA Modest	-1.0%	-0.5%	-1.0%	-1.0%	-0.5%	-0.5%	-0.5%	-0.5%	..

‘..’ indicates no change.

Table 2.8 depicts the optimal SG when the time preference is 0.98 and the comparison with the baseline setting where there is a time preference of 1. It illustrates that setting a time preference lower than 1 leads to a decrease in the optimal SG across all the income levels and objectives. To be more specific, the decrease of optimal SG is 2.5%-5% in the power utility case, 1%-3% in the replacement rate case, 1.5%-3.5% in ASFA comfortable case, and 0%-1% in ASFA modest case. Applying a 2% discount rate generates a preference for current utility over future utility. Members will make saving decisions valuing more current consumption, which leads to less saving for the future and hence the reduction of optimal SG.

From the analysis, acknowledging members being myopic generates a much smaller optimal SG. However, this result should not be treated as an indicator of the appropriate SG level. Instead, this preference should be considered as motivation in setting up an SG system, requiring members to save adequate funds to enjoy their retirement lives for good, which also relates to the paternalistic role of the SG system.

2.4 Sensitivity analysis

In this section, a wide range of assumptions has been changed to test their effects on the sensitivity of the optimal SG. It allows us to identify the important assumptions to determine the optimal SG in addition to the factors mentioned above. It turns out that changing the following assumptions would lead to a significant change in the estimates of optimal SG:

Higher SG indicated (in rough order of significance**)**

- Early retirement at age 62: members consume only their super balance until getting access to Age Pension at age 67.

- Members save knowing they would survive until age 102: self-insuring against longevity risk.

- Lower asset returns: the expected return of a risky asset reduces by 1.5%.

- Greater required drawdown: 80% replacement rate.

Lower SG indicated (in rough order of significance)

- Higher asset returns: the expected return of a risky asset increases by 1.5%.

- Imposing a minimum drawdown pattern: members only withdraw the minimum drawdown percentage after retirement (decreases the optimal SG in most cases, but not in all).

Table 2.9 reports an overview of the sensitivity. It presents the baseline assumptions and related sensitivity tests. In the final column, a summary of the impact of SG is provided for each test. The tests are separated into three categories by their characteristics[13].

[13] A summary of the sensitivity analyses is presented here. The complete tables of the analyses are available from the authors upon request.

Table 2.9: Overview of the Sensitivity Tests

Sensitivity Test	Input Assumptions		Impact on Optimal SG
	Baseline setting	Sensitivity Testing	
Utility Parameters			
Higher risk aversion (power utility)	CRRA = 4	CRRA = 6	Lower[14]
Higher loss aversion (reference dependent utility)	Slope 4.5, Curv(-) 0.88, Curv(+) 0.44	Slope 5.5, Curv(-) 0.88, Curv(+) 0.33	None
Lower risk aversion (power utility)	CRRA = 4	CRRA = 2	Higher
Lower loss aversion (reference dependent utility)	Slope 4.5, Curv(-) 0.88, Curv(+) 0.44	Slope 3.5, Curv(-) 0.88, Curv(+) 0.66	Very small, if any
Required Income Stream			
Higher replacement rate	70% of income age 61-66	80% of income age 61-66	Higher
Lower replacement rate	70% of income age 61-66	60% of income age 61-66	Lower
Minimum drawdown	Optimised, subject to minimum drawdown	Draw no more than the minimum drawdown	Higher for power utility, lower for ASFA comfortable, none for the rest
Draw the target	Optimised, subject to minimum drawdown	Draw the target until pension account exhausted	Higher
Longevity risk self-insured	Expected mortality	Assume death at age 102	Higher
Retire early	Retire at age 67	Retire age 62, draw income until access pension age 67	Higher
Investment Assumptions			
Higher risky asset return	5.0% compound	6.5% compound	Lower
Lower risky asset return	5.0% compound	3.5% compound	Higher, except for ASFA modest

[14] A higher risk aversion leads to lower optimal SG since the reduction of the super contribution let the individual be exposed to less investment risk.

2.5 Conclusion

This research uses a stochastic life-cycle model to investigate the optimal SG level and members' welfare changes with the SG for members across nine income levels and four consumption objectives. The analysis applies the power utility function to represent a consumption smoothing objective and the reference dependent utility function to capture either a replacement rate or fixed amount retirement consumption target objectives. This research also identifies the key considerations for setting an appropriate SG. They are the availability of the Age Pension system and two behavioural effects, where the latter includes pre-defined investment strategies that members commonly adopt and the time preference of consumption.

This research finds that there is no 'one-size-fits-all' optimal SG. Members' income levels and their consumption objectives can dramatically change the optimal SG. Specifically, the optimal SG for a consumption smoothing objective varies from 11.5% to more than 20%, while the optimal SG for the three consumption target objectives is all below the current SG level of 9.5% and the proposed one of 12% varying from 0.5% to 8.5%. These results emphasised the importance of specifying the objective of the SG system and of the cohorts of members being targeted. It also queries whether one single SG could meet the heterogeneous members' financial needs. The allowance for setting the contribution rate might be given to provide more flexibility.

The analysis also highlights how members' welfare changes with the SG levels. Members with consumption smoothing objectives experience large welfare losses when the SG is very low, while they are not affected too much when posing a very high SG. Members with consumption target objectives suffer a significant welfare loss when the SG is sub-optimal. Increasing the SG from 9.5% to 12% incurs a large welfare loss for members with consumption target objectives and generates modest welfare gains for members with a consumption smoothing objective, which indicates that increasing the SG may harm some member welfare to a greater extent than those that it helps. This research also estimates the government net revenue per member for a range of SG levels. The impact of variation of SG levels on government revenue is modest.

This research estimates the revised optimal SG assuming no Age Pension system, which can be treated as a test of the SG level needed if members prefer self-funding their retirement. The estimates indicate that assuming no access to the Age Pension system leads to most of the optimal SG levels being larger than 10%. It is the low-income earners who experience the largest increase in optimal SG. The analysis confirms the important role of the Age Pension as a retirement income source for most Australian retirees. It also suggests that the current SG of 9.5% and the proposed one of 12% might be rational if the objective of the SG system is to replace the Age Pension system.

This research also estimates the optimal SG levels when members adopt fixed or life-cycle investment strategies instead of optimizing them throughout their lifetimes and when members have inter-temporal consumption preferences. Taking these two pre-defined investment strategies leads to an increase of optimal SG between 0.5% and 4.5% across most of the income levels and objectives. Meanwhile, having the inter-temporal consumption preference leads to a decrease of the optimal SG by 0.5% to 5.0%. These optimal SG estimates should not be treated directly as the indicator of the SG level. Instead, acknowledging that members have certain behavioural decision-making effects should be considered in setting the SG level.

The analysis provides insights into the design of the superannuation system by highlighting member heterogeneity, analysing the welfare changes with different SG, and identifying the impact of Age Pension and members’ behavioural decision-making effects on optimal SG. The model is built to easily incorporate different countries’ taxes and social security rules to estimate the optimal contribution rate in their systems.

Also admitted in the analysis is that there exist limitations that may change the optimal SG levels. The major limitation is the absence of other saving mechanisms outside of the superannuation system. In the analysis, the research is effectively modelling the first and the second pillar of the Australian retirement system, not the third one. Housing is the most important saving form in the third pillar. It can provide retirees with an income stream in terms of rental services, and it is exempt from income and capital tax. For these reasons, housing is a genuine substitute for the superannuation system. Members with a house might need a lower SG to ensure their retirement consumptions than those without any house. In future research, an analysis of housing and its policy implication will be conducted. In this analysis, our focus on a member fails to capture the changes along his life path such as marriage, parenthood or the death of a spouse. These changes would significantly influence their expenses and their consumption patterns. Incorporating household behaviours will reflect members’ real financial situations throughout their lifetime, which might change the optimal contribution rates required.

3 Cascade Model for Australian House Prices

3.1 Introduction

Life-cycle modelling is also known as the life-cycle consumption-investment problem and has drawn much attention since the 1970s. It stems from Merton (1969), who considers constant risk aversion in the intertemporal utility functions for consumption with a personal discount rate. Since then, researchers have enriched the model by adding more realistic factors and constraints (See Horneff et al. 2015; Blake et al. 2013; Blake et al. 2014; Yogo 2016). Life-cycle modelling usually requires researchers to simulate the economic circumstances throughout the individual's lifetime and to optimise their financial decisions.

During the individuals' lifetime, decisions on their housing arrangements can potentially have a significant impact on their wellbeing. For many, owner-occupied housing is the largest and most important asset in their portfolio (see Cocco 2005; Yao and Zhang 2005). This effectively both provides imputed rental services and a form of asset investment, which is often leveraged through mortgage borrowings. The alternative is to pay rent. Several authors have examined house tenure and housing consumption choices in a life-cycle framework (See Fernandez-Villaverde and Krueger 2011; Dotsey et al. 2014; Li et al. 2016; Shao et al. 2019). This contribution is to develop a 'cascade model' that generates integrated long-term projections for interest rates, incomes, house prices, rents and (superannuation) fund returns in an Australian context with a view to support life-cycle modelling. The model is specifically designed to generate projections for the key variables in a consistent manner. The importance of including a housing component when undertaking life-cycle modelling in an Australian context is heightened by the principal residence being excluded from the asset test for Age Pension[1], which is Australian means-tested public retirement income support. It enhances the attractiveness of purchasing a house in constructing an individual's retirement saving portfolio.

Despite its importance, research that models Australian house prices over long horizons is sparse. Liu et al. (2008) and Ma and Liu (2013) use the vector auto-regression (VAR) model and error correction model (ECM) respectively to build a model for the house prices of

[1] For more details see Services Australia 'Asset types', https://www.servicesaustralia.gov.au/asset-types?context=22526.

Australian eight capital cities[2]. While their models operate over the long term, they do not necessarily have the interactions between the house price and other economic variables required to undertake life-cycle modelling. Saunders and Tulip (2019) build an empirical model of Australian house prices, quantifying the interrelationships between construction, vacancies, rents, and prices. Their model is also difficult to incorporate into the life-cycle framework as it consists of the real-estate market variables, which are not necessarily relevant to the life-cycle model. The new MARTIN model of the Reserve Bank of Australia (RBA) (Ballantyne et al. 2019) examines the housing market as part of its structural macro-econometric models. The model indicates that house prices are mainly driven by rents and real mortgage rates. However, the whole structure consists of more than 20 variables, with some of them being exogenous ones. And the exogenous variables require separate models, which cannot maintain the interrelationship with the key variables in the life-cycle model.

The cascade model provides a structure that generates not only house prices and rents, but also links them to other key variables that are central to life-cycle modelling, most notably wages (i.e., income) and interest rates (which are linked to asset returns) while doing so in a manner that ensures consistency within each simulated path (which is shown in model checks). This occurs as the impact of each variable acts as a driving force that transfers to other variables when moving down the model tiers. This style of model is motivated by Wilkie (1986, 1995), who used long-term British data, to develop a series of autoregressive distributed lag (ADL) models for long-term forecasts within actuarial applications[3]. Other cascade models include Huang et al. (2014) using Chinese data and Sneddon et al. (2016) for Australian data. The existing research on cascade models has been focused on investment options such as bonds and equities. This research is expanded to investigate house prices and rents. Chen et al. (2021) extends the model in Sneddon et al. (2016) by including the house price, where they focus only on using the autoregressive integrated moving average model (ARIMA) and autoregressive distributed lag model (ADL). This research incorporates error correction model (ECM) for rents and rental yields, which can maintain the long-term relationship between variables.

In this chapter, the concept of the 'fundamental' value of house prices is incorporated (See Fox and Tulip 2014; Ballantyne et al. 2019), defined as indifference between owning and

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[2] Liu et al. (2008) find that house prices in Canberra and Hobart are the driving forces in Australia, while Melbourne's house prices can be treated as driving force in Ma and Liu (2013).

[3] Wilkie et al. (2010) update and refit his cascade model using the data up to 2009. They find that the residuals of many series are much fatter tailed than in a normal distribution.

renting a house and housing affordability (See Gallin 2008; Gan and Hill 2009) where rent, house price and income maintain a long-run relationship. An autoregressive integrated moving average model (ARIMA), an autoregressive distributed lag model (ADL), and an error-correction model (ECM) to historical data to form candidate projection models for each variable are fitted. The final models are selected by considering a combination of statistical significance, consistency with economic theories and parsimony. The cascade model consists of eight variables which are shown in Figure 3.1: inflation (CPI), wages (AWE), official cash rate, mortgage rate, rent, rental yield, house price and fund returns.

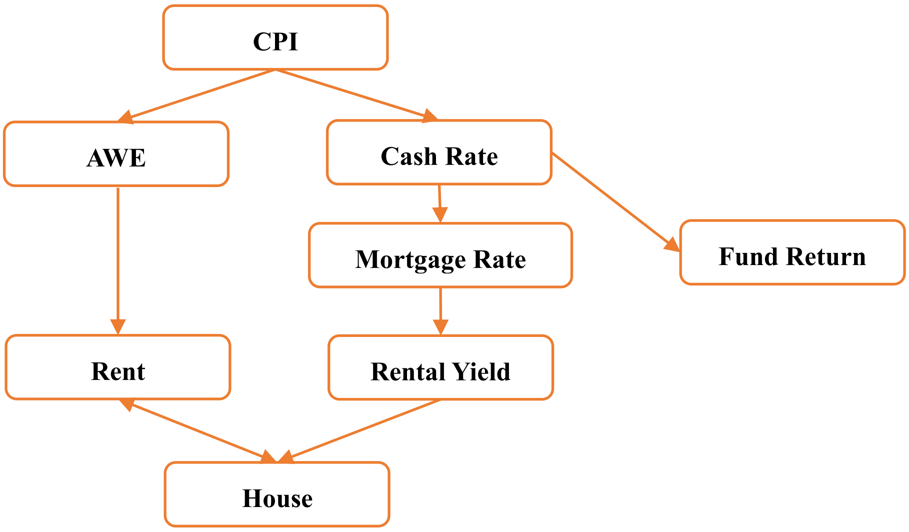

Figure 3.1: Cascade Structure

The model has two important features. The first one is how it captures the interrelationship between house prices and rents and other key economic variables. House prices and rents are driven by both shocks and changes in other economic variables. The second feature is that the model generates valid long-run projections (for instance, 85 years), which is enusred by examining the distribution of projections to make sure that they are economically plausible considering the historical range and interrelationship between the variables. Since the model includes all the influential economic variables in individual financial life, it is easy to incorporate into the life-cycle framework to perform the analysis of financial decisions including housing arrangements over an individual's life.

This chapter proceeds as follows. Data and the methodology are briefly discussed in Section 3.2 and Section 3.3; while Section 3.4 presents the model equations and projections for each variable. Model checks are conducted in Section 3.5. A conclusion is offered in Section 3.6.

3.2 Data

Table 3.1 summarises the important features of the data. All the series are transformed into quarterly data for modelling fittings. Quarterly data balances the need for adequate data points and reduces the unnecessary noises that may arise if more frequent data is used. For the monthly time series, data points for March, June, September, and December are extracted to form the quarterly series. Quarterly data is retained in its original form.

Table 3.1: Data information summary

Variable	Series	Frequency	Starting Date	Ending Date	Source	Details
Consumer Price Index (CPI)	All Groups CPI; Australia	Quarterly	Mar 1993	Dec 2020	ABS Cat No 6401.0 – Consumer Price Index, Australia	
Average Weekly Earnings (AWE)	Total Earnings; Persons; Full Time; Adult	Quarterly before 1995; Biannually from 1995	Mar 1982	Dec 2020	ABS Cat No 6302.0 – Average Weekly Earnings, Australia	Data before 1995 are extracted from historical publications.
Cash Rate	Interbank Overnight Cash Rate	Monthly	Mar 1993	Dec 2020	RBA Statistical Table F1.1 Interest Rates and Yields – Money Market	
Mortgage Rate	Lending rates; Housing loans; Banks; Variable; Standard; Owner-occupier	Monthly	Mar 1993	Dec 2020	RBA Statistical Table F5 Lending Rates	
Rent Index	Index Numbers; Rents; Australia	Quarterly	Mar 1982	Dec 2020	ABS Cat No 6401.0 – Consumer Price Index, Australia	
House Price Index	Hedonic Home Value Index	Quarterly	Mar 1982	Dec 2020	CoreLogic – Home value index	All dwellings, hedonic, weighted eight capital cities
Super Fund Return	70/30 Fund - Nominal Returns	Quarterly	Mar 1985	June 2019	Datastream	Monthly rebalancing assumed; quarterly returns estimated from monthly rebalanced series.

This research explains how to choose the data periods and the special treatments for all the variables in the following paragraphs. As discussed in the introduction, the research fits ARIMA, ADL and ECM models to form candidate projection models for each variable. All these models require the variables that appeared in the same model to have the same historical data lengths, which leads to the situation that the longest historical data period used is constrained by the variable with the shortest historical data period. In the Australian context, it is the AWE series that has the shortest data period starting from 1982.

Another consideration to determine the starting date is the changes in Australian economic circumstances, which leads to the structural changes for CPI, cash rate, and mortgage rate series. A detailed discussion is provided below. In the 1980s and 1990s, a series of financial reforms in Australia took place that had a large impact on the behaviours of these variables. In April 1986, the interest rate ceiling on new owner-occupied housing loans was removed (see De Brouwer 1999), so that the mortgage rate became determined in the open market. Australia then experienced financial instability in the early 1990s (Yanotti 2014), after which inflation shifted to lower levels in line with global trends. In 1993, RBA initiated Inflation Target Policy[4] with a target inflation rate of 2-3 percent. This research uses the data after 1993 to fit the models for these three variables on the assumption that the post-1993 environment and related policies will be sustained over the entire projection period.

The graphs of all the series and the continuously compound growth rates to illustrate the data characteristics are plotted. The research uses $\ln\left(\frac{I_t}{I_{t-1}}\right)$ to generate the continuously compound growth rates for any series I_t at time t, where ln is the natural logarithm.

CPI – Figure 3.2 shows the graph of the consumer price index (CPI)[5] from 1982 to 2020. Overall, it is an increasing curve. The 'V' shape bounce in 2020 is due to the fluctuating situation caused by Covid-19. Figure 3.3 illustrates the historical continuous growth rates for CPI. It is easy to identify the drop in the growth rate at the beginning of the 1990s, which is in line with the RBA's policy change discussed above. After 1993, the fluctuations are relatively stable with only limited outliers occurring in the early 2000s and 2020.

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[4] For more details see Reserve Bank of Australia 'Inflation Target', https://www.rba.gov.au/inflation/inflation-target.html.

[5] CPI and inflation are used interchangeably below.

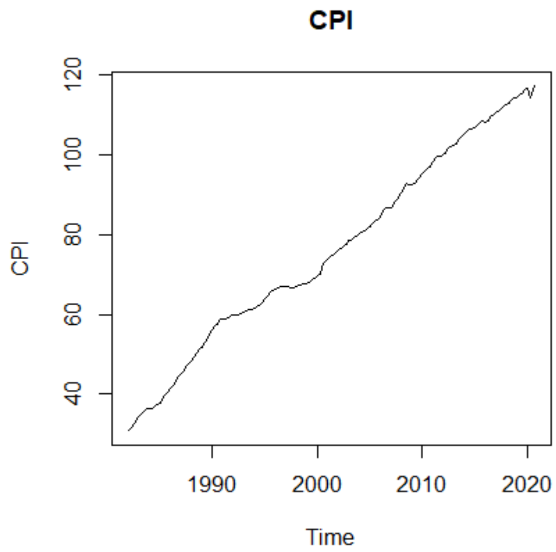

Figure 3.2: Consumer Price Index

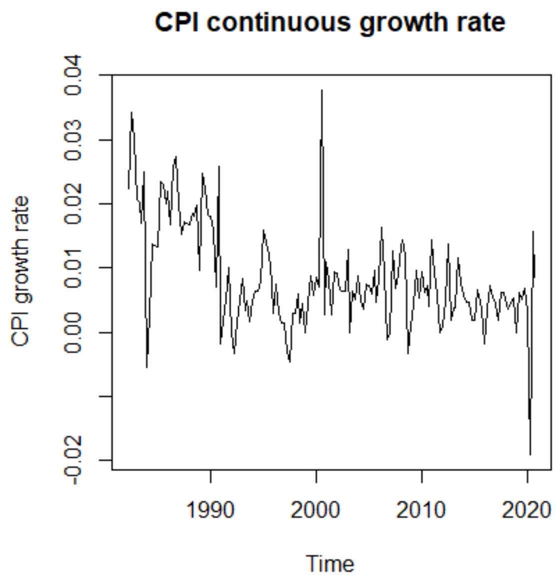

Figure 3.3: CPI Continuous Growth Rate

AWE – The average weekly earnings (AWE) was published by the Australian Bureau of Statistics (ABS) in May and November before 1995, then moved to quarter observations for February, May, August, and November after 1995. The AWE data points for March, June, September, and December are interpolated using shape-preserving Schumaker splines (Schumkaer 1983; Judd 1998). The earliest data point found for this series is one in 1982. It also leads to the starting date for the rent and house series to be 1982 for the benefit of investigating the interrelationship between variables.

Figure 3.4 illustrates the graph of AWE from 1982 to 2020. Visually it is an increasing curve without too many fluctuations. However, the growth rates in Figure 3.5 show the magnitudes of oscillations are relatively large, while the fluctuations seem stable over the whole historical period from 1982 to 2020. Since there are not any structural changes due to the economic reforms, this research uses the whole historical period for modelling fittings.

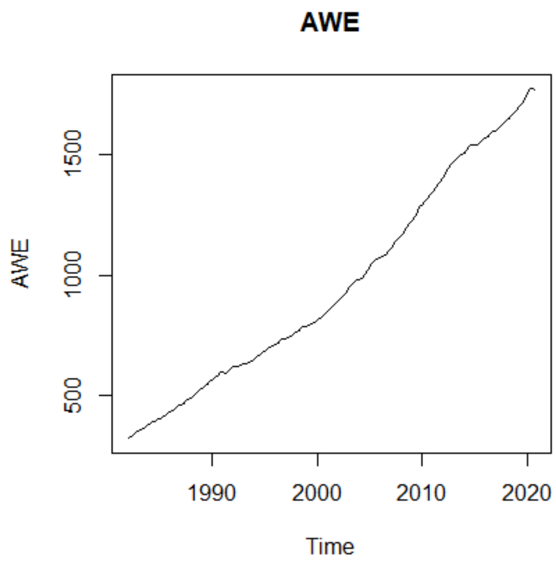

Figure 3.4: Average Weekly Earnings

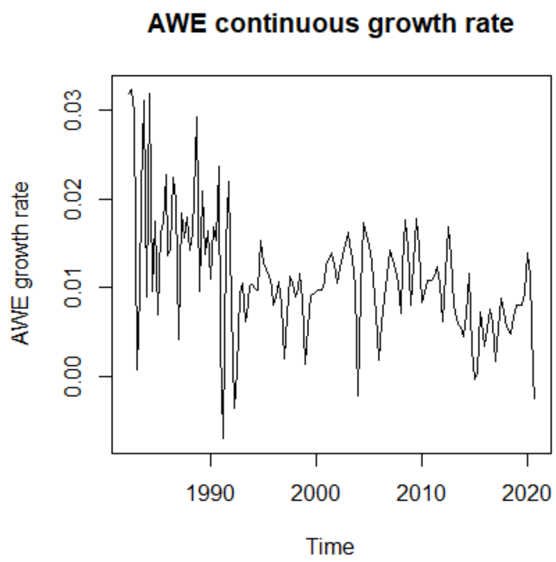

Figure 3.5: AWE Continuous Growth Rate

Cash rate and mortgage rate – these two series are discussed together due to their close interrelationship. The financial intermediates usually set up their mortgage rate considering the cash rate. Their relationship can be easily recognised by observing Figure 3.6 and Figure 3.7.

In these two figures, the cash rate and the mortgage rate follow almost the same pattern. Their values are different with the mortgage rates being higher than the cash rate. It is also noticeable that after the 1990s, these two series vary to a low level of under 10% per year, while they fluctuate over 10% per year before the 1990s. This is mainly due to the financial instability in the early 1990s discussed above. With reference to RBA's Inflation Target Policy in 1993, the data period from 1993 to 2020 is chosen for these two variables.

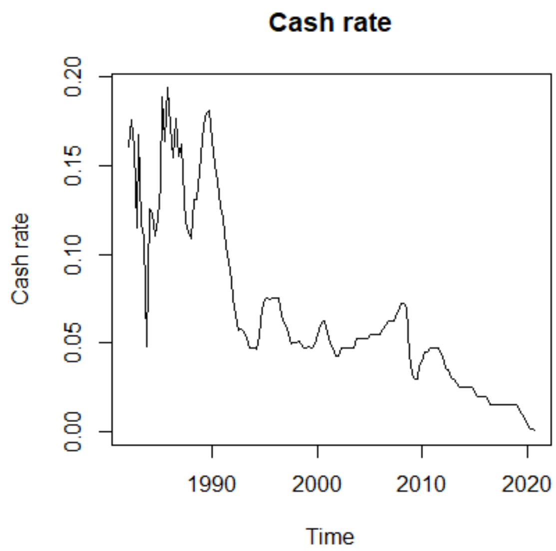

Figure 3.6: Cash Rate

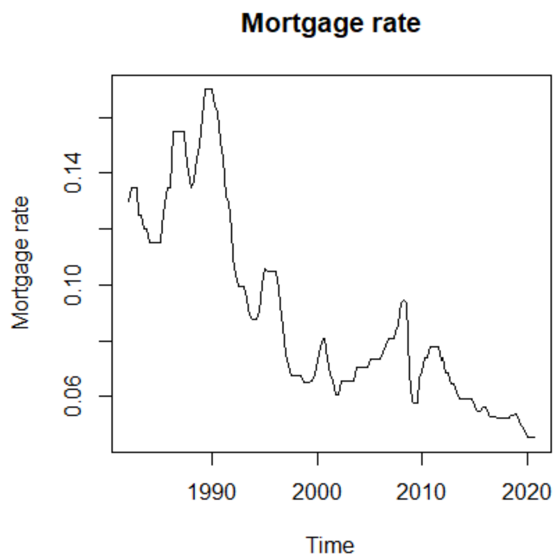

Figure 3.7: Mortgage Rate

Rent Index – Figure 3.8 depicts the historical values for the rent index from 1982 to 2020. It experiences two quick increases in the 1980s and in 2010. The slight decrease in 2020 is also due to the COVID-19 situation. Figure 3.9 shows the rent growth rate with small fluctuations most of the time. Although it does exist a large decrease at the beginning of the 1990s, considering the replications of the high growth rate in 2010, the data period from 1982 to 2020 for model fittings is kept.

Figure 3.8: Rent Index

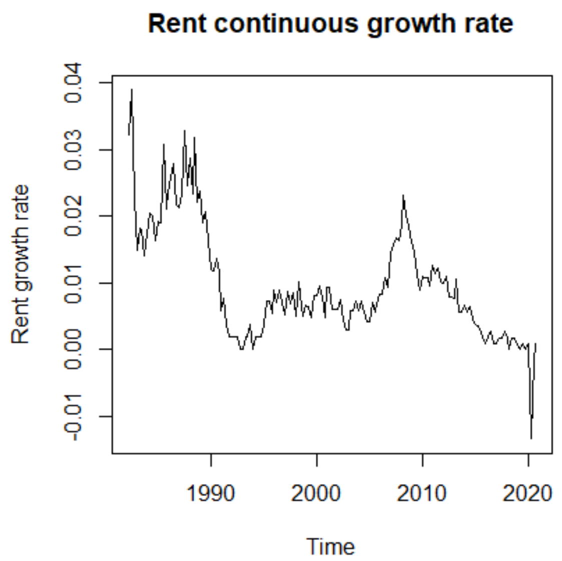

Figure 3.9: Rent Continuous Growth Rate

House prices index – Figure 3.10 illustrates the levels of the house prices index from 1982 to 2020. Although it is an increasing curve over the past 40 years, the large bumps from time to time can still be observed. In Figure 3.11 of house growth rates, these large oscillations are clearer, bumping between 4% per quarter to -3% per quarter in most of the historical time. Its fluctuations are stable from 1982 to 2020, which provides evidence for choosing the whole historical period for modelling fittings.

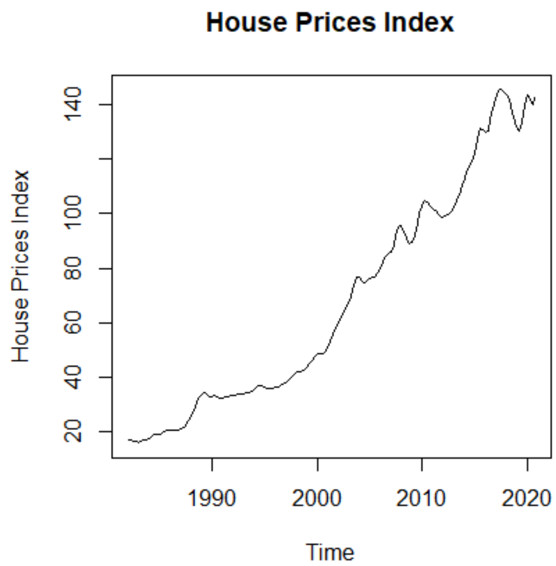

Figure 3.10: House Index

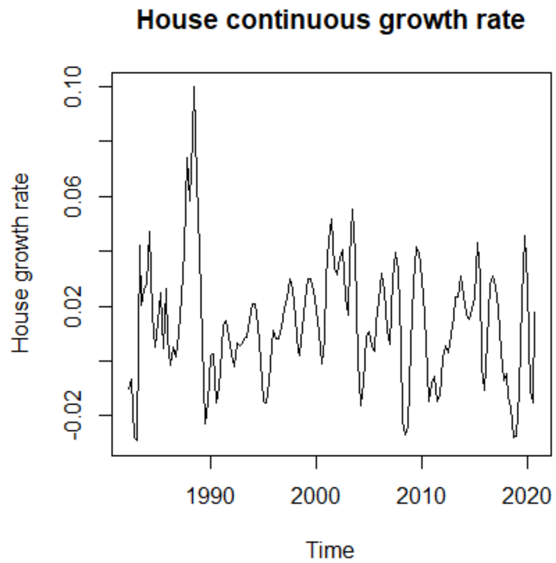

Figure 3.11: House Continuous Growth Rate

Superfund return – The chosen super fund return is a commonly used portfolio return in many super funds. The portfolio consists of 70% of growth assets and 30% of defensive assets. Concerning 'MySuper' product, the growth assets include Australian equities, world equities, and Australian property, where their weights are 44%, 44% and 12% respectively; the defensive assets consist of Australian fixed income, World fixed income, and cash, where their weights are 40%, 40% and 20% respectively. The series for all the assets are summarised in Table 3.2, where most of them are from Datastream and the same cash rate as discussed above is used in Table 3.1. It is assumed that the portfolio is rebalanced monthly, and its quarterly returns from this monthly rebalanced series is estimated.

Table 3.2: Assets Series Summary

	Growth assets			Defensive assets		
	Australian Equities	**World Equities**	**Australian Property**	**Australian Fixed Income**	**World Fixed Income**	**Cash**
Index Name	S&P/ASX300 Accumulation Index	MSCI World Ex Australia in $AU gross dividends	S&P/ASX 300 Property Trust Accumulation Index	Citi Australian Bond Accumulation Index	Salomon Smith Barney World Bond Accumulation (Hedged) Index	Refer to Table 3.1
Datastream Code	ASX300I(RI)	MSWXAU$(RI)*($AU/$US) /0.8929	ASG3PT1(RI)	CGBI WGBI AU ALL MATS. (L) – TOT RETURN IND	WDSBXADC	
Index ID	I-ASX300	I-MSCXAG	I-LPT300	SBADLII	I-SALWBH	

Figure 3.12 shows the quarterly super fund returns from 1985 to 2019, which is also the longest period that can be found in the data. The fluctuations of fund returns are relatively large varying between 15% per quarter to -10% per quarter for most of the historical period. However, the oscillations are stable over time, which allows us to use the whole period for modelling fittings.

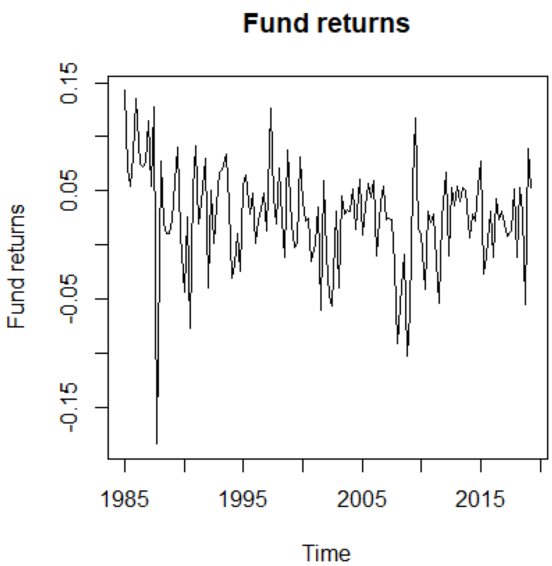

Figure 3.12: Fund Returns

3.3 Methodology

This section outlines the method for fitting and selecting the models. This research starts by fitting the autoregressive integrated moving average model (ARIMA), autoregressive distributed lag model (ADL), and error-correction model (ECM) to all variables. Certain models are then selected as potential options using the statistical criteria. The interrelationships between the variables from these models are then examined whether they are consistent with the economic literature. The consistency with the economic literature means that the relationship between the variables found from the data is in line with widely accepted economic literature. The final models are selected based on trading off statistical significance, consistency with economic literature and parsimony[6].

3.3.1 ARIMA and ADL

The ARIMA and ADL modelling process are explained together as they follow similar steps. Before fitting the models, a logarithmic transformation is applied to all series, and transformations are applied to ensure the series is stationary.

For index series W_t, such as CPI and AWE, this research calculates the continuously compounded (CC) rate of change at time t as $\ln\left(\frac{W_t}{W_{t-1}}\right)$, where $\ln$ is the natural logarithm. For the rate of return series V_t, such as the cash and mortgage rates, the CC rate at time t is expressed as $\ln\left(1+\frac{V_t}{100}\right)$. The benefit of logarithmic transformation is to dampen the oscillation and make the non-linear trend straight (See Huang et al. 2014; Enders 2015).

Augmented Dickey-Fuller (ADF) (Dickey and Fuller 1981)[7] and Phillips-Perron (PP) (Perron 1988) tests[8] are applied to test for unit roots. If a unit root exists, the series can be transformed into a stationary series by differencing. For the series l_t, the first difference of l_t is denoted as $(l_t - l_{t-1})$. If the differenced series is not stationary, the second difference of l_t as $(l_t - 2l_{t-1} + l_{t-2})$ may be taken. The process (Enders 2015) is continued until a stationary

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[6] Parsimony here means a short model with less terms in it is preferred. It usually functions better than longer models in the long-term projection. More detail is discussed in Section 3.3.1.

[7] For more details see Appendix 3A

[8] For more details see Appendix 3B.

series is produced. For the variables discussed in this chapter, most of them only need the first difference, the rent index is the only one that needs the second difference to become stationary series[9]. Canova-Hansen test (Canova and Hansen 1995)[10] is used to determine whether the series has seasonality and how many differences need to take to make the series stationary. If the series has seasonality, the difference taken is $(l_t - l_{t-4})$.

The main difference between the ARIMA model and the ADL model is whether exogenous variables in the model fitting are included. The ARIMA model only considers the relationship between the variable and its lags and innovations, while the ADL model allows for the variables of interest to be influenced by other variables. The models tested can be described as follows.

Suppose there is a group of n stationary series $(X_1, X_2, \dots, X_n)$, for any series at time t, $X_{j,t}, where\ j \in (1,2, \dots, n)$, the models fitted can be expressed as,

$$X_{j,t} = \alpha_0 + \sum_{i=0}^{N_j} a_i X_{j,t-i} + \mathbf{I}_1 \sum_{i=0}^{N_1} b_i X_{1,t-i} + \mathbf{I}_2 \sum_{i=0}^{N_2} c_i X_{2,t-i} + \cdots + \mathbf{I}_{j-1} \sum_{i=0}^{N_{j-1}} d_i X_{j-1,t-i} + \mathbf{I}_{j+1} \sum_{i=0}^{N_{j+1}} f_i X_{j+1,t-i} + \cdots + \mathbf{I}_n \sum_{i=0}^{N_n} g_i X_{n,t-i} + \sum_{i=1}^{N_\varepsilon} \varepsilon_{t-i} + \varepsilon_t \quad (3.1)$$

where $(N_1, N_2, \dots, N_n, N_\varepsilon)$ are the largest lags for each variable. The maximum number of lags considered is assumed to be 3 to cover all the impact of lags in that year, for the lags larger than one year, they can transfer their impact through the model one by one until reaching the present one; $(a_i, b_i, \dots, g_i)$ and α_0 are the parameters to be estimated; ε_t is the innovation; and $(\mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_{j-1}, \mathbf{I}_{j+1}, \dots, \mathbf{I}_n)$ are the indicator functions of one or zero for whether an exogenous variable is included or not in the model fitting. It is easy to observe that when all indicator functions $(\mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_{j-1}, \mathbf{I}_{j+1}, \dots, \mathbf{I}_n)$ are zero an ARIMA model is fit, while when any of the indicator functions is one, an ADL model is fit.

To estimate all the parameters, the quasi-maximum likelihood method (Hall and Yao 2003) is applied. This method does not require the innovations to be normally distributed and allows for other heavy-tailed distributions, for example, t-distribution.

To select the best potential models, different combinations for $(N_1, N_2, \dots, N_n, N_\varepsilon)$ are tried where each of them can take values from 0 to 3 and calculate the Bayesian information criterion (BIC) goodness-of-fit measures (Schwarz 1978). Since BIC imposes a penalty on the number of parameters, it favours the more parsimonious models, which are known to perform

[9] Please refer to Section 3.3.2 for detailed discussion.

[10] For more details see Appendix 3C.

better over long forecast periods such as more than 20 years (Enders 2015). For each variable, a model with minimum BIC as one of the potential models is selected.

The second potential model is selected by conducting out-of-sample validations (See Hyndman and Koehler 2006). The latest 10 data points are used as the forecast sample and all the rest data points as the estimation sample. The root mean square error (RMSE) is then used to evaluate the forecast performance of the estimated models[11]. This research follows the researchers' approach and select the model with the smallest RMSE. The parameters of the selected model are then re-estimated using the whole data points. One problem with out-of-sample validations in the current context is only evaluating forecast performance over the short term, i.e., two and half years. Considering the long-term forecast goal of more than 80 years, the out-of-sample validation method is less useful than BIC in selecting long-term forecast models.

Once the models with minimum BIC and minimum RMSE are obtained, the Ljung-Box test (Ljung and Box 1978)[12] is conducted to test the independence of residuals and the Jarque-Bera test is used (Jarque and Bera 1980)[13] for testing residuals' normality. For all the potential models, this research finds the residuals are independent of each other. If the residuals are normally distributed, a normal distribution is used to estimate the distribution of innovations. Otherwise, the statistical descriptive parameters are calculated for the residuals and generate a skewness-kurtosis plot following the method of Cullen and Frey (1999). The residuals' skewness and kurtosis are compared with those of all the theoretical distributions to determine the distribution of innovations. For the variables with non-normal residuals, t-distribution is the most suitable one.

3.3.1 Error Correction Model (ECM)

An Error Correction Model (ECM) is widely used to describe the long-run equilibrium relationship between two variables or a set of variables. This model stems from the presence of cointegrated variables. In Enders (2015), to say the variables are cointegrated requires all the

[11] RSME is the square root of the mean square prediction error (MSPE), noting many researchers select the model with the smallest MSPE (Enders, 2015).

[12] For more details see Appendix 3D.

[13] For more details see Appendix 3E.

variables are not stationary and integrated in the same order. By using the cointegrated relationship, ECM can model the long-run model and the short-run dynamics simultaneously.

This research applies a four-step procedure proposed by Engle and Granger (1987) to determine if the variables are cointegrated and to build the ECM.

Step 1: Test the variables and determine their order of integration. Since the cointegrated relationship requires the variables to be integrated in the same order, the first step of analysis is to test each variable and determine its order of integration. ADF test (Dickey and Fuller, 1981) is applied to infer the number of unit roots for each variable. Only the variables with the same order of integration to proceed to Step 2 are selected.

Step 2: Estimate the long-run equilibrium relationship. Supposing an attempt is made to fit an ECM for variable Y_t, from Step 1 it is found that Z_t has the same order of integration as Y_t, where Z_t can be either a single variable or a set of variables. For all the Z_t satisfy the condition, this research uses Ordinary Least Squares (OLS) regression (Wilkinson and Rogers 1973) to estimate the long-run equilibrium relationship as described in Equation (3.2),

$$Y_t = \beta_0 + \beta_1 Z_t + e_t \tag{3.2}$$

where β_0 and β_1 are the estimators of the cointegrating parameters and e_t is the error term.

To prove Y_t and Z_t are cointegrated, the research further tests whether e_t is stationary using the ADF test. If e_t is stationary, then Y_t and Z_t are cointegrated and can proceed to Step 3. If not, other potential series that have the same order of integration as Y_t are checked.

Step 3: Estimate the ECM. If Y_t and Z_t are cointegrated, then the ECM is estimated in the following form:

$$Y_t - Y_{t-1} = \alpha_1 + \alpha_Y (Y_{t-1} - \beta_1 Z_{t-1} - \beta_0) + \sum_{i=1}^{N_Y} \alpha_{Y,i} (Y_{t-i} - Y_{t-i-1}) + \sum_{i=1}^{N_Z} \alpha_{Z,i} (Z_{t-i} - Z_{t-i-1}) + \varepsilon_t \tag{3.3}$$

where α_1, α_Y, $\alpha_{Y,i}$, $\alpha_{Z,i}$ are the parameters; ε_t is the error term; N_Y and N_Z are the largest lag difference examined for Y_t and Z_t. In the model, the term $(Y_{t-1} - \beta_1 Z_{t-1} - \beta_0)$ represents the deviation from the long-run equilibrium relationship at the time $(t-1)$. α_Y is the speed of adjustment. This is always negative, with larger values indicating a greater response by Y_t to the previous period deviation. The term $\sum_{i=1}^{N_Y} \alpha_{Y,i} (Y_{t-i} - Y_{t-i-1}) +$

$\sum_{i=1}^{N_Z} \alpha_{Z,i}(Z_{t-i} - Z_{t-i-1})$ captures the short-run dynamics that cause short-term fluctuations in Y_t.

The maximum N_Y and N_Z tested is 3, which captures the impact of past values within a year. The model with the largest adjusted R^2 (Yin and Fan 2001) is chosen and with all the parameters being significant.

Step 4: Assess Model Adequacy. After the ECM model is built, the independence of ε_t is tested with the Durbin-Watson test (Durbin and Watson 1971)[14] and the normality of ε_t with Shapiro-Wilk Normality test (Royston 1982)[15]. Following the same method as ARIMA and ADL models, the distribution of the residuals is determined.

All the statistical analysis is performed using R studio (R Core Team 2019) with the 'tseries' package (Trapletti and Hornik 2021). The codes can be provided upon request.

3.3.2 Illustration of the process with rent index

This research uses the rent index to illustrate how the modelling fitting process works and how the economic literature help select the final models in this subsection.

Fitting ARIMA and ADL models finds the difference of the CC rate of rent index $\left(\ln\frac{Rent_t}{Rent_{t-1}} - \ln\frac{Rent_{t-1}}{Rent_{t-2}}\right)$ has seasonality. The stationary series is

$$\left[\left(\ln\frac{Rent_t}{Rent_{t-1}} - \ln\frac{Rent_{t-1}}{Rent_{t-2}}\right) - \left(\ln\frac{Rent_{t-4}}{Rent_{t-5}} - \ln\frac{Rent_{t-5}}{Rent_{t-6}}\right)\right] \tag{3.4}$$

The minimum BIC model for Term 4 is ARIMA (3,0,3) with the impact of AWE growth rate, while its minimum RMSE model is ARIMA (0,0,1) with the impact of CPI growth rate, AWE growth rate and the cash rate. It is easy to find that these two potential models are relatively long and difficult to interpret. Another concern is that since the goal is to find a model for long-term forecasts, the rent growth rate can be expected to be stationary. From the descriptions of these two models, none of them has a stationary rent growth rate.

This research also finds two potential ECM models. The first one is that the rent index has a long-run relationship with cash rate, mortgage rate, and AWE, while the second one indicates

[14] For more details see Appendix 3F.

[15] For more details see Appendix 3G.

that the rent index follows a long-run relationship with the house index and AWE. Both long-run relationships are statistically significant and can generate easy-to-explain models.

With all these four potential models, economic literature is refered to to finalise the rent index model. The research finds the concept of rental affordability is widely used to evaluate the rationality of rental cost. It indicates rent and income should hold some certain ratio (30 percent) (see Gabriel and Painter 2020 for details). This concept reinforces the relationship between rent and income. Gallin (2008) uses the long-run relationship between rent and house prices to model rent. In Australia, Saunders and Tulip (2019) specify a relationship between rent and income. This literature indicates that rent should hold a long-run relationship with income and house prices, which supporst the ECM model with the house index and AWE.

The rent model is then finalised as the ECM model with both statistical significances and being in line with the economic literature. For most of the variables, the same process as discussed above is followed to select the final models which are statistically significant and are also justified by the economic theories. There are admittedly some circumstances in both cannot be achieved. The judgement of this research is to choose the model backed up by the economic theory, like in the AWE case discussed in Section 3.4.

3.4 Model Descriptions

In this section, a cascade structure is firstly outlined before briefly discussing the interrelationship between the variables. The main equations for the models are then introduced, followed by the projections of each variable.

3.4.1 Cascade Structure

This research first goes through the cascade structure from the top tier down to the bottom tier. The relationships between the variables are then discussed briefly here, with a detailed explanation in Section 3.4.2.

Figure 3.13 describes the cascade structure. The left and middle branches of the structure depict the variables involved in the housing model. The right branch is the model of fund returns.

A CPI is chosen as the initial driving force of the system, which is in line with other cascade models in the literature such as Wilkie (1986, 1995), Huang et al. (2014), and Sneddon et al. (2016). The second tier includes models of the real AWE growth rate and the real cash rate, to which the inflation rate is added to formulate the gross rates. These two variables are critical to the ability to afford a house via determining income and the baseline interest rate for setting mortgage rates, where the income and mortgage rates together consider the ability to repay the mortgage. The long-run relationship between income and mortgage rates and house prices measures housing affordability (Gan and Hill 2009). This idea is followed and the relationship with rents is discussed in Section 3.3.3 to build the house prices model. To model the real AWE growth rate, the commonly accepted theory that the real income growth rate is equal to labour productivity (see Productivity Commission 2020) is adopted. The real cash rate is treated stationary and an ARIMA model is fitted. Moving to the next tier, rents are determined regarding the level of AWE as discussed in Section 3.3.3. The mortgage rate is modelled as a spread over the cash rate. The rental yield is subsequently modelled regarding its long-run relationship with the mortgage rate (See Fox and Tulip 2014). At the bottom tier, the house index is an outcome that emerges by dividing the rent index and rental yield. The fund return is modelled as a risk premium in excess of the cash rate with reference to the Capital Asset Pricing Model (See Sharpe 1964; Lintner 1965).

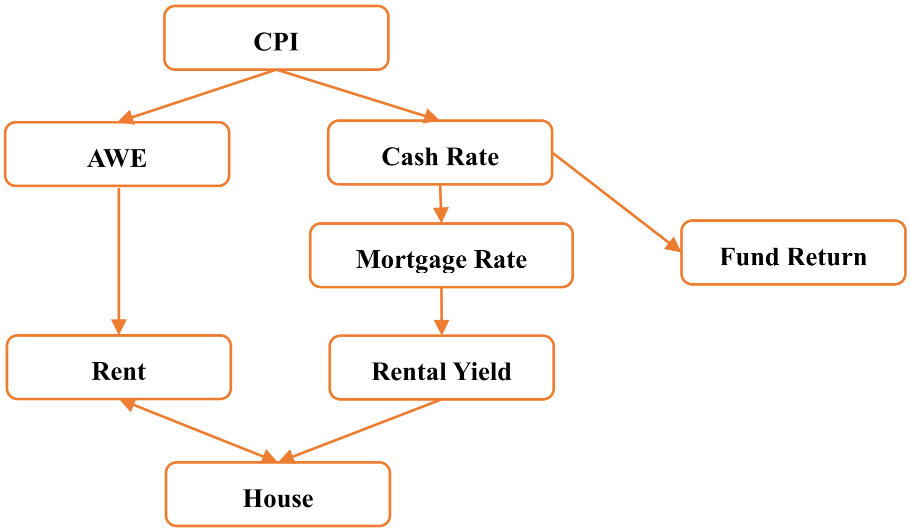

Figure 3.13: Cascade Structure

3.4.2 Models

This section introduces the model equations for each variable. It also presents projections over 85 years starting from 2021 to reveal how the models perform over the long period. To do so, for each variable, 10,000 paths are formed by simulating 340 quarters (i.e., 85 years × 4 quarters) in each path for the error terms, which are assumed to be independent of each other. The research then applies the model equations to generate a distribution of projections across 10,000 paths at each quarter for examination. The research then present time series plots of the 25th percentile, 50th percentile and 75th percentile across all paths for each variable, starting from March 1986 and ending in December 2105 thus showing the historical series for comparison.

3.4.2.1 CPI

As the driving force of the cascade system, the CPI is only driven by its own long-term mean and its error term. An ARIMA model is fitted to the data, following the widely used approach of, for example, Wilkie et al. (2010) and Huang et al. (2014). The CC rate of CPI $\left(\ln\frac{CPI_t}{CPI_{t-1}}\right)$ is stationary with no seasonality. The minimum BIC model is used, ARIMA (0,0,0), to model $\ln\frac{CPI_t}{CPI_{t-1}}$,

$$\ln\frac{CPI_t}{CPI_{t-1}} = 0.0059 + e_t, \text{ where } e_t \sim t(\nu = 2.761,\ \mu = 0, \sigma = 0.00318) \qquad (3.5)$$

The model generates the mean of 0.59% per quarter, which is 2.388% per year, in line with the RBA inflation target of 2% to 3% per year. For the error term e_t, since the normality test indicates the residuals are not normal and its skewness and kurtosis indicate t distribution, t distribution is used to fit them. In the parameters of t distribution, ν is the degree of freedom; μ is called the location parameter which is also the mean of the error and σ is the scale parameter which is not the standard deviation. The standard deviation is expressed as $\sqrt{\sigma^2 \frac{\nu}{\nu-2}}$.

Other variables show most of the error terms are not necessarily normal, which is in line with the findings of Wilkie et al. (2010). Over the past 20-30 years, Australia has experienced many significant financial events, like the financial reform and instability in the early 1990s and the Global Financial Crisis in 2008. These events generated some unusual and extreme values for the variables, which lead to heavy-tailed distributions.

Figure 3.14 illustrates the projection graph for the CPI quarterly growth rate, calculated as $\left(\frac{CPI_t}{CPI_{t-1}} - 1\right)$. The historical data from 1993 to 2020 reveals that most of time the series fluctuates around the long-term mean reflected in the model. There are some extreme values, for instance in the early 2000s caused by the introduction of the GST and in 2020, which contribute to the fat-tailed distributions of the error terms in the model. Over the long projection period, the distribution of the CPI growth rate is stable, which leads to straight lines for different percentiles.

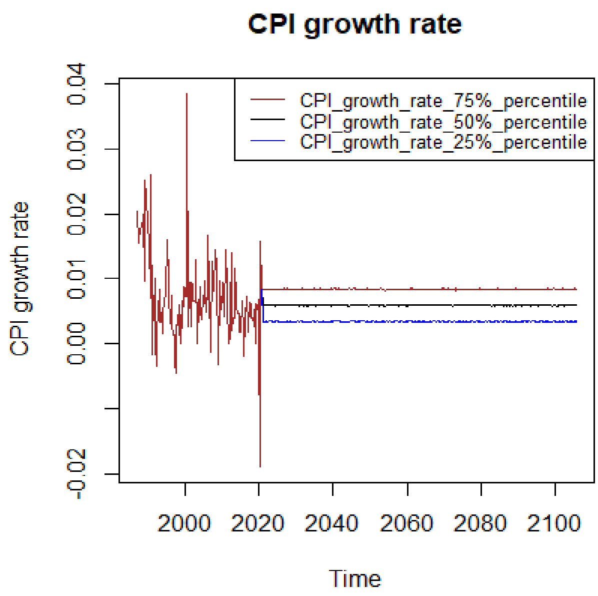

Figure 3.14: Projection graph of CPI growth rate

3.4.2.2 AWE

The selection of the AWE model involves a large difference between the model with the greatest statistical significance and the model that best accords with the economic theory. Using the data the calculated mean of the real AWE growth rate is at only 0.24% per quarter, which is mainly due to the stagnation of real wages in Australia since 2013. This issue and the potential policies to restore healthier wage growth are discussed by Stanford (2019) and the Productivity Commission (2020).

In the literature, it is widely accepted that labour productivity is the key source of long-term real income growth, and is estimated at 1.5% per year, or 0.373% per quarter according to the Productivity Commission (2020). While recognising that labour productivity growth has been lower over more recent times, this research nevertheless adopts the Productivity Commission's estimate as the mean of the AWE growth rate to form long-term projections. The model is written as:

$$CPI\ Growth_t = \frac{CPI_t}{CPI_{t-1}} - 1 \tag{3.6}$$

$$Labour\ Productivity = 0.00373 \tag{3.7}$$

$$Nominal\ AWE\ Growth_t = CPI\ Growth_t + Labour\ Productivity + e_t, \text{ where } e_t \sim t(\nu = 3.228,\ \mu = 0, \sigma = 0.00494) \tag{3.8}$$

The structure is relatively simple. The nominal AWE growth rate is the sum of the CPI growth rate and labour productivity[16]. Ballantyne et al. (2019) also use the same method to model real wage growth. The error term adds to the fluctuations around the mean.

Figure 3.15 depicts the projection of the nominal AWE growth rate. Notice that the growth rate after 2013 fluctuates at a lower level, reflecting the stagnation of real wages. Over the projection period, like the CPI growth rate, a stable distribution for AWE can be seen.

[16] It is admitted that forcing the labour productivity equal to the real AWE growth rate without data justifications has limitations, ideally, a model that allows for a gradual convergence to the labour productivity is a better option in the future research.

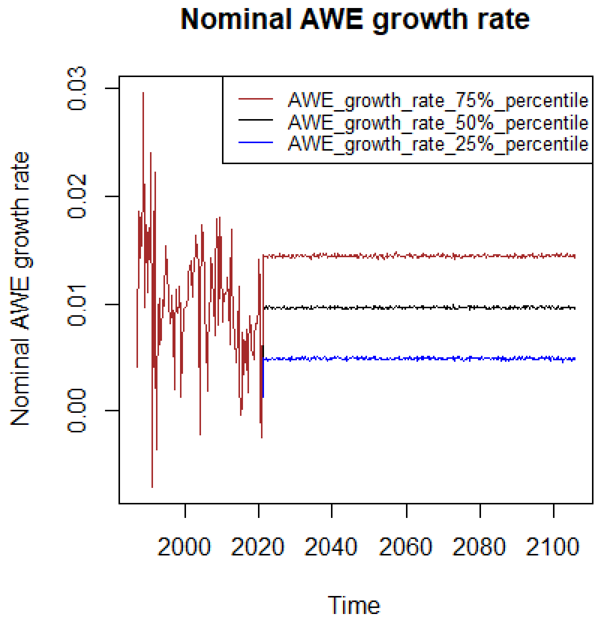

Figure 3.15: Projection of nominal AWE growth rate

3.4.2.3 Cash Rate

At the time of writing, Australia is experiencing a historically low official cash rate which the RBA has set at close to 0%, amounting to a negative real rate. Examining the history of real cash rates estimated as the difference between the nominal cash rate and the CPI growth rate reveals that the real cash rate is stationary over time and has averaged 1.5% per year. This research projects the nominal cash rate as the sum of inflation plus the real cash rate. The latter is estimated using an ARIMA(1,0,1) model as described by Equation (3.9)[17], which is selected on the basis of minimum BIC:

$$\ln(1 + Real\ Cash_t) = 0.015 + 0.968(\ln(1 + Real\ Cash_{t-1}) - 0.015) - 0.825e_{t-1} + e_t, \text{ where } e_t \sim t(\nu = 2.758,\ \mu = 0, \sigma = 0.0133) \quad (3.9)$$

$$\ln(1 + Nominal\ Cash_t) = \max\left(\ln\left(\frac{CPI_t}{CPI_{t-1}}\right) + \ln(1 + Real\ Cash_t), 0\right) \quad (3.10)$$

A ‘Zero Lower Bound’ for nominal cash rates is set to ensure they never go negative during the projection period. It is admitted that in response to the global financial crisis (GFC) of 2007-2009 and the deep recession, the central banks of several advanced economies have used negative nominal interest rates (See Campos 2019). Nevertheless, the RBA classifies the

[17] ADF and PP tests confirm the stationarity of the real cash. It is admitted that Figure 3.16 shows a visual decreasing trend which can lead to future research to study other models for the cash rate series.

negative interest rates as unconventional monetary policy and never used this before[18]. The RBA also discuss the adverse effects of negative interest rates, such as damaging bank profitability, allowing less productive firms to survive and fuelling excessive increases in asset prices. In addition, Campos (2019) uses American bank-level data and finds that negative cash rates are between 60% and 90% as effective as positive cash rates. All the literature back up the view that the non-negative nominal cash rates dominate the monetary policy in Australia during the projection period.

In the model, the cash rate is expressed as the annual effective rate to give us a direct impression of what the cash rate level is in each quarter. The model generates long-term means of the CC rate of 1.5% per year for the real cash rate and 3.86% per year for the nominal cash rate. Variation in the nominal cash rate around its mean reflects the sum of the error terms for real rates and inflation.

Figure 3.16 and Figure 3.17 illustrate the CC rate of real cash and nominal cash rate respectively. Figure 3.16 shows that even if starting with a negative real cash rate, the projections bounce back to the long-term mean at the beginning of the projections and are stable ever after. For the nominal cash rate, the same pattern can be seen. The model can correct the current low-interest rate and maintain the fluctuations around the long-term mean.

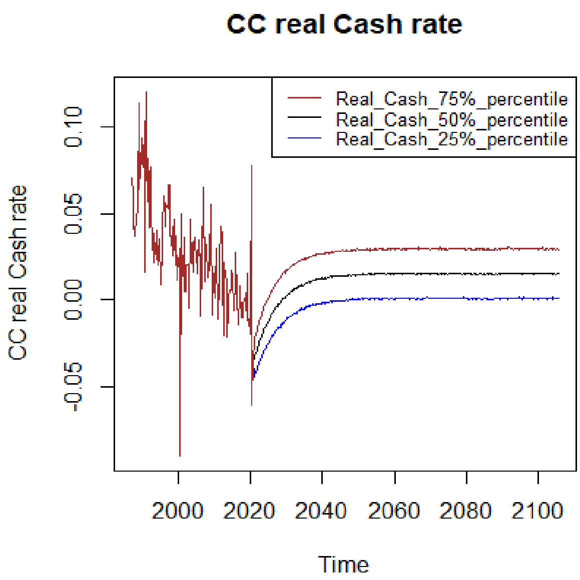

Figure 3.16: Projection of continuously compound real cash rate

[18] For details see Reserve Bank of Australia 'Unconventional Monetary Policy', https://www.rba.gov.au/education/resources/explainers/unconventional-monetary-policy.html.

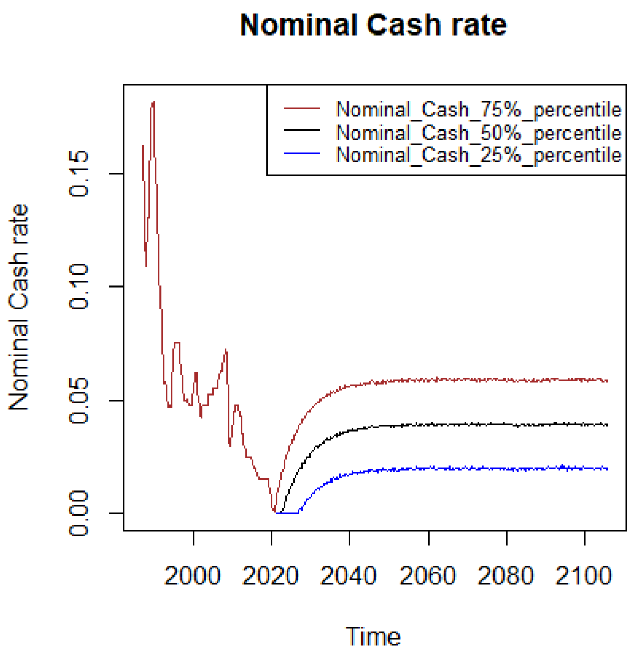

Figure 3.17: Projection of nominal cash rate

3.4.2.4 Mortgage rate

This research models mortgage rates as the sum of the cash rate plus a mortgage spread, with the historical series for the spread estimated as the difference between the nominal mortgage rate and the nominal cash rate as follows:

$$Mort\ Spread_t = Nominal\ Mortgage_t - Nominal\ Cash_t \tag{3.11}$$

where $Mort\ Spread_t$ represents the mortgage spread at time t; $Nominal\ Mortgage_t$ and $Nominal\ Cash_t$ are the nominal mortgage rate and the nominal cash rate at time t respectively.

The mortgage spread approach is widely used in the literature. Li et al. (2016) estimate the spread as a constant of 0.04 using American data. Fox and Tulip (2014) and Saunders and Tulip (2019) both use the Australian 10-year bond yield spread to model the Australian mortgage spread. Ballantyne et al. (2019) use an AR(1) model for mortgage spread.

The mean of mortgage spread from 1993 to 2020 is calculated and model the spread as a random walk.

$$Mort\ Spread_t = \min\left(\max\left(Mort\ Spread_{t-1} + e_t, MinMortSpread\right),\right.$$
$$\left. MaxMortSpread\right), \text{where } e_t \sim N(\mu = 0, \sigma = 0.0135) \tag{3.12}$$

$$Nominal\ Mortgage_t = \min\left(max(Nominal\ Cash_t + Mort\ Spread_t, 0), MaxMortgage\right) \tag{3.13}$$

where $MinMortSpread$ and $MaxMortSpread$ represent the minimum and the maximum of the historical mortgage spread; $MaxMortgage$ is the maximum value of the historical mortgage rates. The upper and lower bounds for both mortgage spread and mortgage rate are set up to ensure that it does not fall outside economically plausible levels, which is set in line with the minimum and maximum values observed for the historical data (See Wilkie and Sahin 2015).

The model produces a mean of mortgage spread 2.88% per year and a mean of mortgage rate 6.73% per year.

To project the mortgage spread, this research follows the approach in Wilkie (1986) and start from the mean of mortgage spread from 1993 to 2020. It is a common practice for the long-term projection (more than 20 years) as for the random walk model, the projected paths are sensitive to the starting point.

Figure 3.18 and Figure 3.19 show the projection of mortgage spread and nominal mortgage rate respectively. Figure 3.18 shows the distributions of spread are stable over the long projection period. In Figure 3.19, the nominal mortgage rate bounds back to the long-term mean at the beginning of the projection and maintain stability for the rest of the period. Both the mortgage spreads and mortgage rates fluctuate in economically plausible ranges.

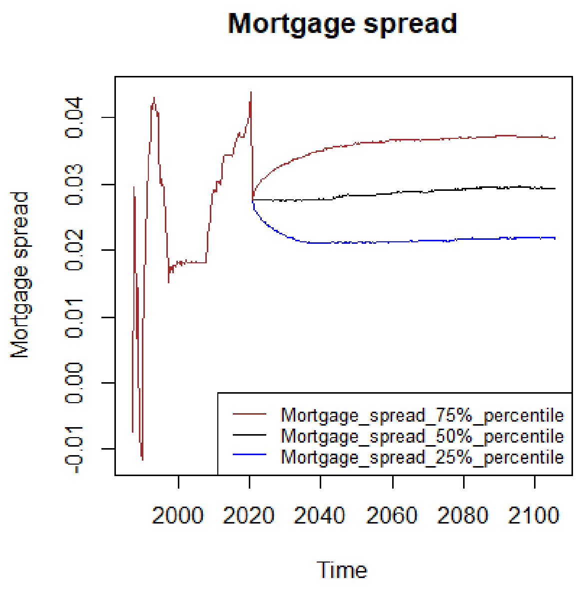

Figure 3.18: Projection of mortgage spread

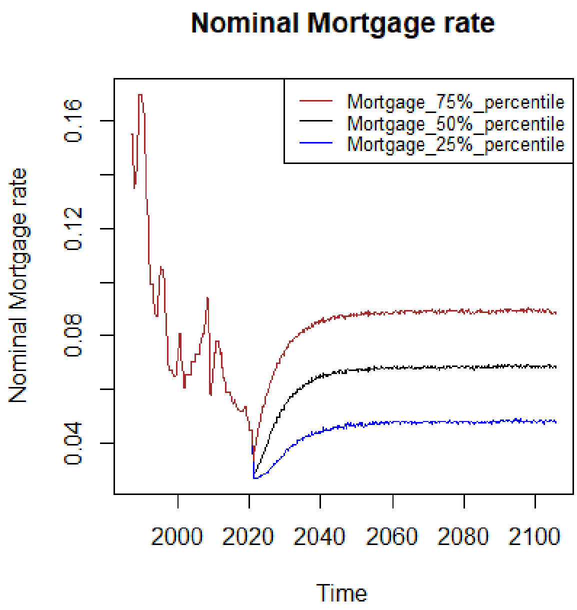

Figure 3.19: Projection of nominal mortgage rate

3.4.2.5 Rent

The research models the rents as cointegrated with AWE and the house prices. This relationship has been identified in the literature. Saunders and Tulip (2019) specify a relationship between rent and income, while Gallin (2008) finds that rent has a long-term relationship of rent with house prices. Using this relationship, ECM is used to model the rent index,

$$\begin{aligned} Rent_t - Rent_{t-1} &= 0.0567 - 0.00673\boldsymbol{error_{t-1}} + 0.586(Rent_{t-1} - Rent_{t-2}) \\ &+ 0.327(Rent_{t-2} - Rent_{t-3}) - 0.0165(House_{t-1} - House_{t-2}) + e_t, \end{aligned}$$

$$\text{where } e_t \sim t(\nu = 3.532,\ \mu = 0, \sigma = 0.129) \tag{3.14}$$

$$\boldsymbol{error_t} = Rent_t + 0.396House_t - \ 0.101AWE_t \tag{3.15}$$

where $Rent_t$, $House_t$, and AWE_t are the index values of rent, house, and AWE at time t; $\boldsymbol{error_t}$ represents the deviation from this relationship.

In Equation (3.14), $(\boldsymbol{error_{t-1}})$ is the deviation from the long run relationship at time $(t-1)$, the negative parameter before it ensures this deviation is corrected at time t. The rest of the terms add in the short-term fluctuations to the model, which are mainly driven by its own lags and house prices' lags.

The model generates a nominal growth rate for rent with a mean of 0.98% per quarter, or 3.979% per year. Figure 3.20 illustrates the projection of the nominal rent growth rate. The historical data indicates that the nominal rent growth rate has experienced large variations but has recently been in a low growth period. Growth rates increase sharply at the beginning of the projection period, then decrease to the long-term mean level and remain stable afterwards. The reason for this bump at the beginning of the projection is due to the small speed of adjustment -0.00673. It takes a relatively long time for the growth rate to correct to the long-term mean.

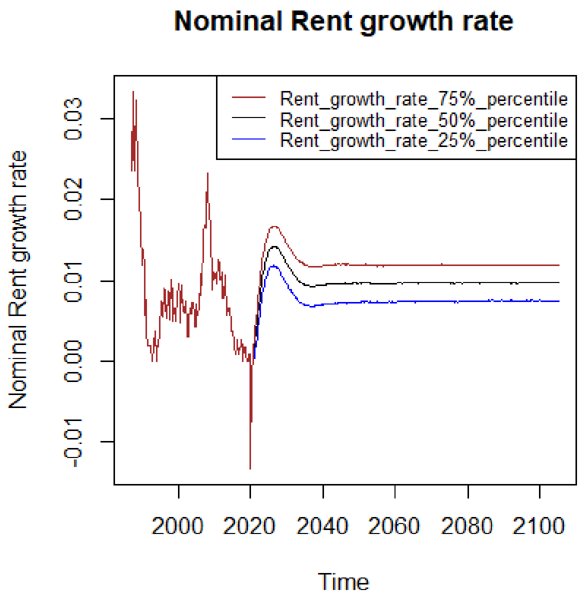

Figure 3.20: Projection of nominal rent growth rate

3.4.2.6 Rental Yield

This research initially investigated using an ECM to model the house index directly. The model shows statistical significance and generates outcomes that appear in line with economic theory. It has a similar model structure as the house price model in Ballantyne et al. (2019), where the house prices have a long-run relationship with rents, AWE, and mortgage rates. It nevertheless led to some implausible variations when used to perform projections. In the later projection period, the distributions of projected house prices are more centralised without any plausible economic explanations. The research hence focused on modelling the rental yield, to estimate house prices by capitalising rents at the rental yield.

The rental yield at time t is expressed as the ratio between the rent index and the house index, i.e., $\frac{Rent_t}{House_t}$. The literature has modelled the rental yield by examining its relationship with nominal mortgage rates. Fox and Tulip (2014) discuss the matching downtrends in the rental yield and the mortgage rate; Saunders and Tulip (2019) then use this relationship to model the rental yield. For the model, rental yield and the nominal mortgage rates demonstrate a cointegrated relationship in the data. The rental yield model using ECM is,

$$RY_t - RY_{t-1} = -0.000809 - 0.0154\boldsymbol{error_{t-1}} + 0.708(RY_{t-1} - RY_{t-2}) + 1.555(Nominal\ Mortgage_{t-1} - Nominal\ Mortgage_{t-2}) + e_t,$$

$$\text{where } e_t \sim t(\nu = 3.704,\ \mu = 0, \sigma = 0.0117) \tag{3.16}$$

$$\boldsymbol{error_t} = RY_t - 0.544 - 7.62 Nominal\ Mortgage_t \tag{3.17}$$

where RY_t is the rental yield at time t; $Nominal\ Mortgage_t$ is the nominal mortgage rate at time t; $\boldsymbol{error_t}$ is the deviation from the long run relationship at time t.

In Equation (3.16), $\boldsymbol{error_{t-1}}$ is the deviation at time $(t-1)$, the negative parameter -0.0154 is the speed of adjustment; the short-term fluctuations are caused by its own lags and mortgage rate lags.

As RY_t is the ratio of two index series, it does not have real meanings. To scale RY_t to real rental yield, the method in Saunders and Tulip (2019) is applied. They use the realistic rental yield in March 1983 as a reference point and scale all the historical raw ratios to the realistic rental yield. After scaling, the model produces a mean rental yield of 4.88%.

Figure 3.21 shows the scaled rental yield projections. Similar to the mortgage rate, the rental yield decreases considerably over the past 30 years in line with mortgage rates and Australia is experiencing a historically low rental yield at the time of writing. In the projections, the rental yield bounces back to the long-run mean and remains stable ever after.

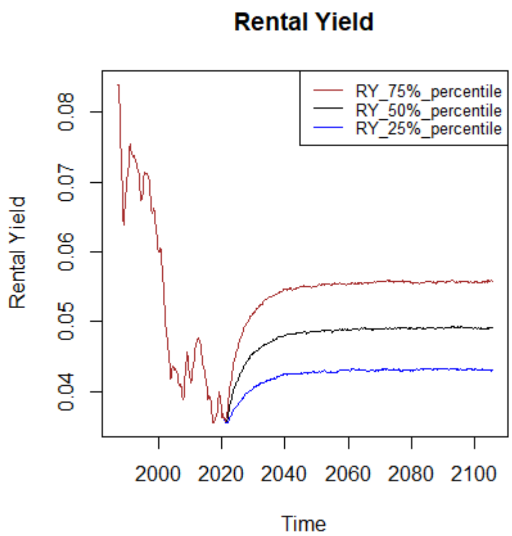

Figure 3.21: Projection of the scaled rental yield

3.4.2.7 House Prices

After obtaining the models for the rent index and rental yield, it is straightforward to calculate the house price index using the ratio of the rent index and rental yield. The house index at time t, $House_t$, is expressed as:

$$House_t = Rent_t/RY_t \tag{3.18}$$

The mean of nominal house growth rate calculated as $\left(\frac{House_t}{House_{t-1}} - 1\right)$, is equal to 1.01% per quarter, which is equivalent to 4.11% per year. This is close to the growth rate for AWE and reflects the modelling of house prices and is based on affordability concepts (See Gan and Hill 2009) coupled with a stationary process for the mortgage rate. In addition, it is also close to the growth rate for rents, which is in line with the 'fundamental' house price theory in Fox and Tulip (2014) where if the real mortgage rate is constant, nominal housing prices will grow in line with rents (See Ballantyne et al. 2019).

The projection of the nominal house growth rate is depicted in Figure 3.22. Historical data reveals that the fluctuations in the growth rate are relatively stable for the past 30 years, ranging from -0.02 to 0.06 per quarter. At the beginning of the projection period, the growth rate experiences a slight bump but remains stable from 2035. The 25% percentile and 75% percentile of the growth rates indicate that the magnitude of fluctuations broadly align with historical rates. This does not occur when house prices were modelled directly.

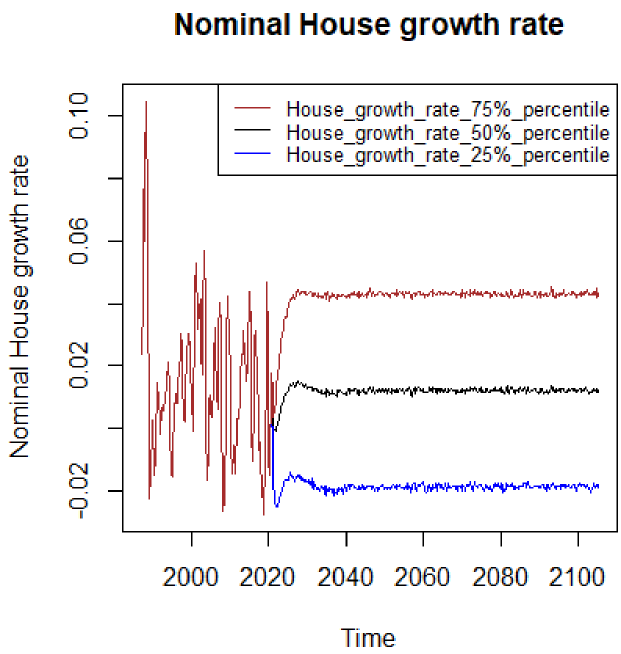

Figure 3.22: Projection of nominal house growth rate

3.4.2.8 Fund Return

This research models fund returns as the sum of the nominal cash rate plus a risk premium, which accords with the concept underpinning the Capital Asset Pricing Model (See Sharpe 1964; Lintner 1965). To facilitate model development, the historical risk premium was expressed as the difference between the fund return and cash rate,

$$Risk\ Premium_t = Fund_t - Cash_t \tag{3.19}$$

where $Risk\ Premium_t$, $Fund_t$, and $Cash_t$ represent the risk premium, fund return, and cash rate at time t respectively.

Statistical tests revealed that the risk premium series is stationary. ARIMA(0,0,0) is selected to model the risk premium, which has the minimum BIC. The model is written as:

$$Risk\ Premium_t = 0.0106 + e_t, \text{where } e_t \sim t(\nu = 4.26, \mu = 0, \sigma = 0.0349) \tag{3.20}$$

The model produces the mean of risk premium 1.06% per quarter (4.3% per year) and the mean of fund return of 1.9% per quarter (7.8% per year).

Figure 3.23 and Figure 3.24 show the projection of the risk premium and fund return respectively. The historical data for both figures are relatively volatile but fluctuate around the long-term mean. Most importantly, the distributions for these two variables are stable under the projections and provide a good representation of the distribution of accumulated compounded returns over long periods.

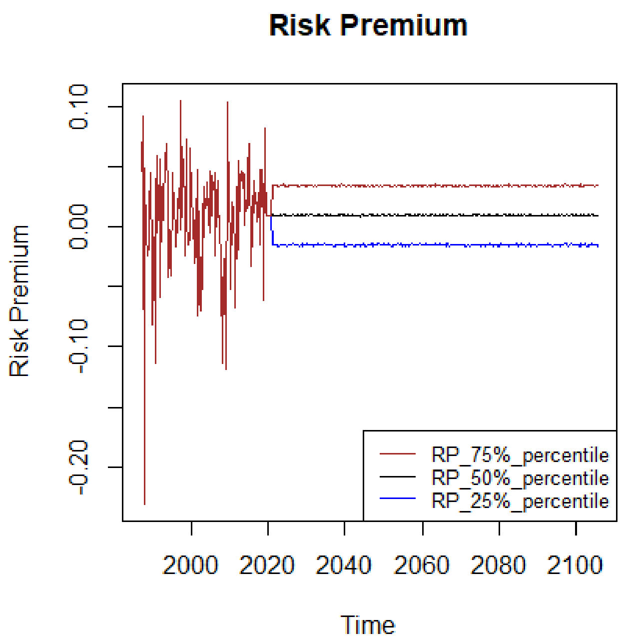

Figure 3.23: Projection of risk premium

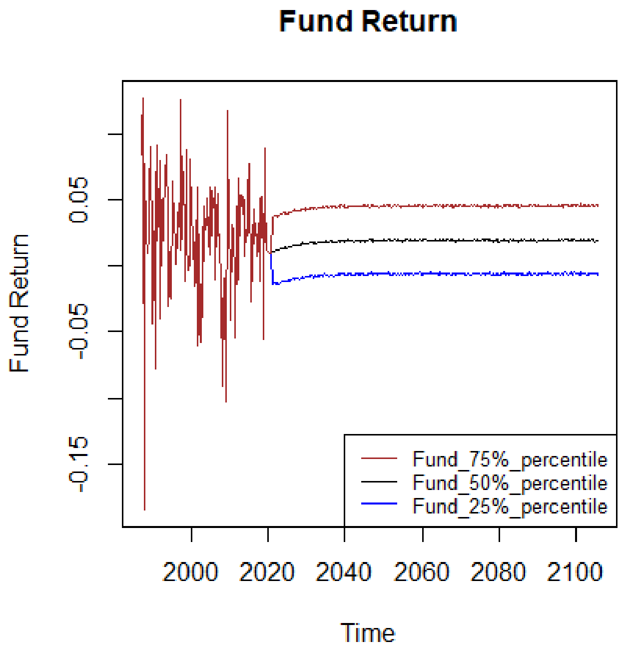

Figure 3.24: Projection of fund return

3.5 Model Checks

The model descriptions indicate the relationships between the variables. The models are further tested by investigating how it changes with respect to the increase and decrease of key input assumption, what are the distributions of key outputs over the long term, and the system's sensitivity to the initial starting point or how the system reacts to alternative initial economic circumstances.

3.5.1 Sensitivity to key input assumption

Scenarios are investigated where the CPI is 1% per year higher and lower (i.e., 0.25% per quarter), examining the changes in other variables. Table 3.3 summarises the changes in the other variables under these scenarios.

Table 3.3: Scenario analysis for lower and higher CPI growth rate

	+0.25% CPI growth rate	-0.25% CPI growth rate
Change of AWE growth rate	+0.251%	-0.250%
Change of Cash rate	+0.229%	-0.212%
Change of Mortgage rate	+0.227%	-0.211%
Change of Rent growth rate	+0.246%	-0.245%
Change of House growth rate	+0.230%	-0.235%
Change of Fund Return	+0.224%	-0.209%

For all the variables except for the cash rate, the mortgage rate and the fund return, the increase and decrease are almost of the same magnitude as the change in the CPI growth rate. It indicates that all the real variables are relatively stable in the models, implying that the relationship is retained under changes to CPI as the initial driving variables. For the AWE growth rate, this outcome can be easily explained by the modelling method where real values are simply added to the change in the CPI. The main exception is the cash rate. In this case, a 'zero lower bound' that the nominal cash rate never goes negative leads to a slightly lower sensitivity of the cash rate to CPI growth rate changes. Since, for mortgage rate and fund returns, this research models them as the summation of the cash rate and a loading over it, they are mortgage spread in the mortgage rate case and are risk premium in the fund return case. As the mortgage spread and the risk premium are stationary, mortgage rates and fund returns similarly adjust via a higher cash rate. For rent and house growth rates, the impact of CPI growth rates is not as direct. They inherit their connection to the CPI through their long-running relationship with other variables in the middle tier of the cascade model.

3.5.2 Path analysis

The research then investigates what are the distributions of key outputs over a long term by generating the outputs for 10-year rolling periods, 20-year rolling periods and 85-year rolling periods. The 20-year rolling period outputs are illustrated in Table 3.4, while the outputs for 10-year rolling periods and 85-year rolling periods are presented in Appendix 3H.

Table 3.4: Analysis of Distribution of Key Outputs Across Joint Paths for 20-year Rolling Periods

			Percentiles								
Simulated Outputs	**Mean**	**Standard Deviation**	**0%**	**5%**	**10%**	**25%**	**50%**	**75%**	**90%**	**95%**	**100%**
Compound growth rates											
Inflation	0.0239	0.0024	0.0115	0.0200	0.0209	0.0223	0.0239	0.0256	0.0270	0.0279	0.0358
Real wages	0.0150	0.0032	0.0001	0.0097	0.0109	0.0128	0.0150	0.0171	0.0190	0.0202	0.0299
Real rents	0.0152	0.0045	-0.0064	0.0077	0.0094	0.0121	0.0152	0.0182	0.0210	0.0227	0.0351
Real house prices	0.0137	0.0114	-0.0495	-0.0048	-0.0009	0.0058	0.0136	0.0214	0.0285	0.0326	0.0830
Real rent–- real wages	0.0002	0.0037	-0.0168	-0.0058	-0.0045	-0.0023	0.0002	0.0027	0.0050	0.0064	0.0189
Real house prices–- real wages	-0.0013	0.0111	-0.0658	-0.0192	-0.0155	-0.0090	-0.0014	0.0062	0.0131	0.0172	0.0681
Real house prices–- real rents	-0.0015	0.0125	-0.0689	-0.0219	-0.0177	-0.0102	-0.0016	0.0071	0.0148	0.0194	0.0691
Average yields over period											
Cash rate	0.0404	0.0108	0.0063	0.0237	0.0270	0.0330	0.0399	0.0472	0.0543	0.0591	0.0970
Real cash rate	0.0163	0.0106	-0.0186	-0.0003	0.0031	0.0090	0.0158	0.0229	0.0298	0.0345	0.0718
Rental yield	0.0494	0.0053	0.0374	0.0419	0.0431	0.0455	0.0488	0.0527	0.0566	0.0591	0.0903
Rental yield–- cash rate	0.0090	0.0090	-0.0390	-0.0064	-0.0026	0.0032	0.0093	0.0152	0.0202	0.0231	0.0527
Compound returns											
Real cash returns	0.0157	0.0103	-0.0180	-0.0004	0.0029	0.0087	0.0153	0.0222	0.0289	0.0334	0.0697
Real super fund return	0.0509	0.0233	-0.0521	0.0130	0.0212	0.0351	0.0507	0.0665	0.0809	0.0896	0.1620
Super fund premium over cash	0.0360	0.0211	-0.0557	0.0014	0.0090	0.0217	0.0360	0.0503	0.0631	0.0707	0.1324
Real housing return	0.0161	0.0111	-0.0441	-0.0018	0.0019	0.0084	0.0160	0.0237	0.0306	0.0347	0.0852
House return premium over cash	-0.0021	0.0155	-0.0801	-0.0283	-0.0222	-0.0123	-0.0017	0.0086	0.0174	0.0226	0.0851
Super fund–- house return	0.0381	0.0264	-0.0829	-0.0050	0.0044	0.0202	0.0379	0.0559	0.0722	0.0820	0.1761

For any r-year rolling periods, the compound growth rate is calculated for the index series I_t as $\left[\left(\frac{I_t}{I_{t-r}}\right)^{1/r} - 1\right]$, while the real compound growth rate is obtained by dividing the nominal growth rate with the inflation rate $\left[\left(\frac{\frac{I_t}{I_{t-r}}}{\frac{CPI_t}{CPI_{t-r}}}\right)^{1/r} - 1\right]$. The average yield over period for the rate series R_t is expressed as the average of $(R_t, R_{t-1}, \dots, R_{t-r})$. The compound return is calculated as $\left[\prod_{i=0}^{i=r-1}(1+R_{t-i})\right]^{1/r}$ with the real compound return deflated by the inflation rate. The calculations are performed for each path for all the plausible t values over the projection period to generate a pooled sample of r-year simulated outputs.

Table 3.4 reports the mean, the standard deviation and the percentiles of the outputs. Three important findings emerge. The first one is that the means in Table 3.4 are consistent with model descriptions in Section 3.4 and the output distributions are symmetric concerning the 50% percentile and the mean (they are almost the same). It indicates that the models do perform plausible outputs over a long-run projection period in line with the model equations. The second one is that the real wages, the real rent, and the real house grow together by observing the zero differences between their means. It reveals that the models incorporate the concept of housing affordability that the house prices and the rent grow in line with the income. The stable differences in outputs also indicates that the variables maintain long-term relationships. The third one is that the variations of inflation, real wages, and rent are small, while those of real house prices, cash rate, rental yield and real fund return are relatively large, which are consistent with the projection graphs in Section 3.4. By observing the percentiles of the differences, the models provide adequate fluctuations to simulate the risks and deviations between variables in addition to maintaining long-term relationships.

The research presents three joint paths of compound growth rates and compound return to illustrate how the model works over the long-run period. As introduced above, the compound growth rates for index series I_i at year i is calculated as $\left[\left(\frac{I_i}{I_{2020}}\right)^{1/i-2020} - 1\right]$, while the compound return for the rate series R_i is expressed as $\left[\prod_{j=0}^{j=i-2021}(1+R_{i-j})\right]^{1/(i-2020)}$. The three paths are selected as the 10th percentile, 50th percentile, and 90th percentile of CPI paths at the end of the projection period.

Figure 3.25 and Figure 3.26 depict the three sets of joint paths for house compound growth rates and rent compound growth rates[19]. Similar to the projection graphs in Section 3.4, rent and house inherit the growth rates from their historical trends, which lead to large variations at the beginning of the projection periods. Over the long run, their compound growth rates tend to their mean of around 4% per year. It can also be observed that the fluctuations for housing is

[19] Figures for other variables are in Appendix 3H.

larger than rent due to their standard deviation values. At the end of the projection period, their growth rates do not necessarily follow the same distribution as CPI with the 10th percentile path being at the bottom and the 90th percentile path being at the top. It is mainly due to the variations of their real growth rates, nevertheless, all three paths for these two variables end with plausible compound growth rates.

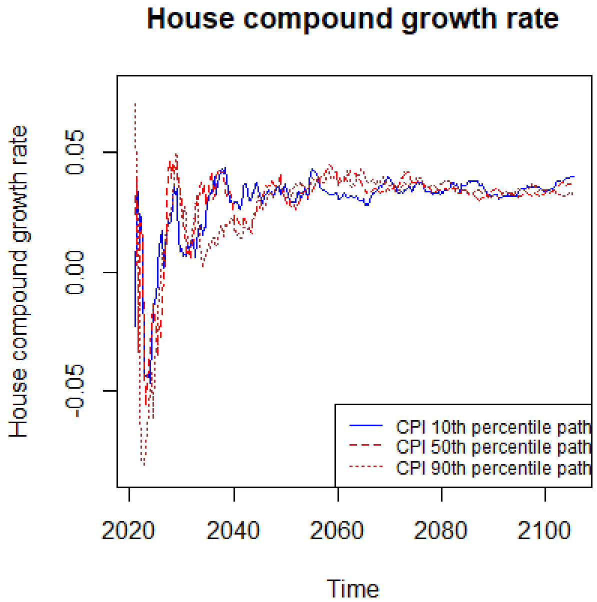

Figure 3.25: Path Analysis of House Compound Growth Rates

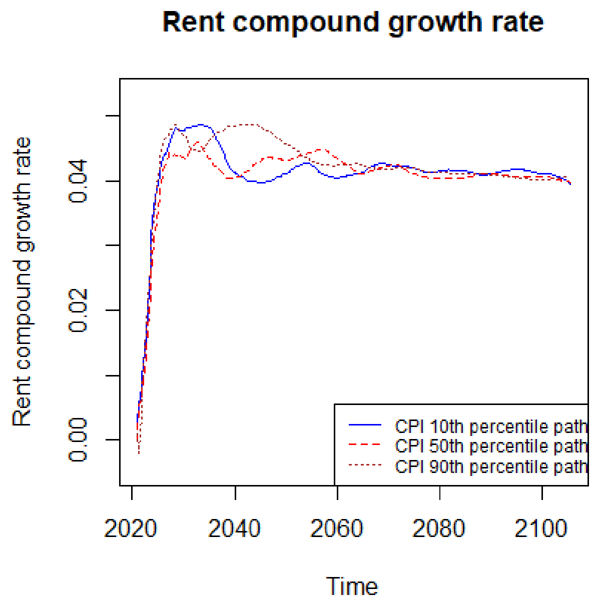

Figure 3.26: Path Analysis of Rent Compound Growth Rates

Figure 3.27 shows the same three paths of house index to AWE ratio. It illustrates how these two variables move together and hence the changes in housing affordability. For all three paths, it indicates that the projections replicate the ratio's historical variations and maintain stability over the projection period without upwards and downward trends. It confirms the long-run relationship between these two variables and generates adequate fluctuations to simulate the risks in housing affordability.

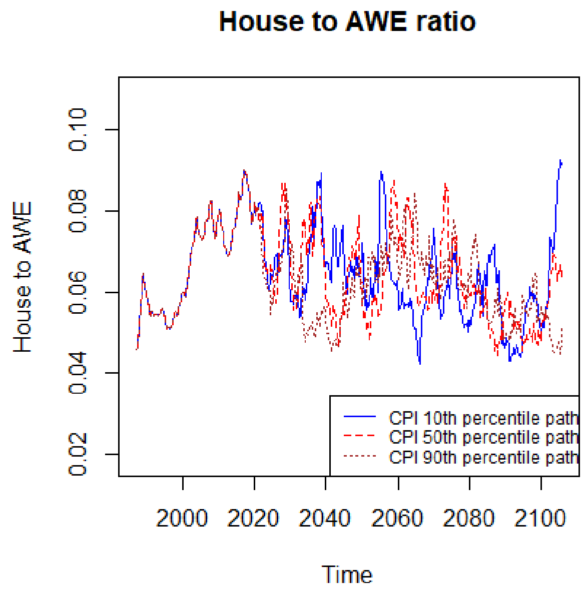

Figure 3.27: Path Analysis of House Index to AWE Ratio

Figure 3.28 compares CPI compound growth rates and cash compound returns in CPI 50th percentile path. Both start at a low level reflecting current economic circumstances. They then bounce to their long-run mean and maintain them afterwards with cash return being higher than the CPI growth rate. The difference between them represents the real cash return.

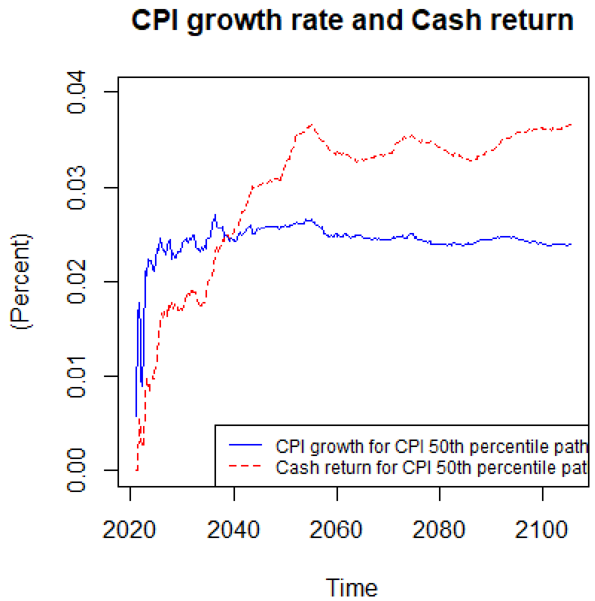

Figure 3.28: Path Analysis of CPI Growth Rates and Cash Returns

Figure 3.29 illustrate the comparison of real cash returns and rental yields in the CPI 50th percentile path. The rental yield keeps the fluctuations around its mean of 0.05 over the projection period, while the real cash return bounces to its mean of 1.5% per year from the negative initial values and makes slight fluctuations around its mean afterwards. The comparison indicates that both vary around their means in a long run and they keep relatively stable relationships over the projection period.

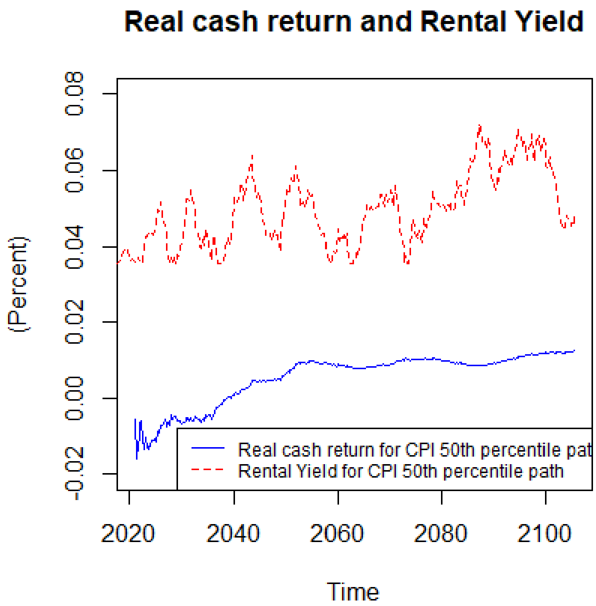

Figure 3.29: Path Analysis of Real Cash Returns and Rental Yields

3.5.3 Sensitivity to initial starting point

The models are tested by simulating the projections with a different initial starting point to illustrate how the models react to unexpected external shocks and how they impact the long-run relationships between the variables.

It can be affirmed that this test has no impact on the variables with ARIMA or ADL models since they are stationary. Altering the starting point hardly change their stationary property and their projection paths. It is the ECM models that require this test to evaluate whether their cointegrated relationships change.

The test is designed by starting with a 20% higher rental yield and evaluate its impact on rent and house compound growth rate over the projection period. This research sought to test whether this shock alters their long-run behaviours. Figure 3.30 and Figure 3.31 illustrate the comparison of the original and higher rental yield scenarios for rent and house compound growth rates in the CPI 50th percentile path respectively. At the beginning of the projection period, it is the house compound growth rate that drops significantly from around 0% to less than -40%, while it is hard to identify the change in rent growth rate with a slightly higher growth rate in higher rental yield cases. The shock from rental yield is quickly absorbed in the first five years of the projection period for both rent and house. The long-run growth rates keep almost the same for the rest of the projection period.

This result reveals that the models are insensitive to the initial starting point over the long run. The variables can quickly adjust to the unexpected shock and maintain long-term relationships afterwards.

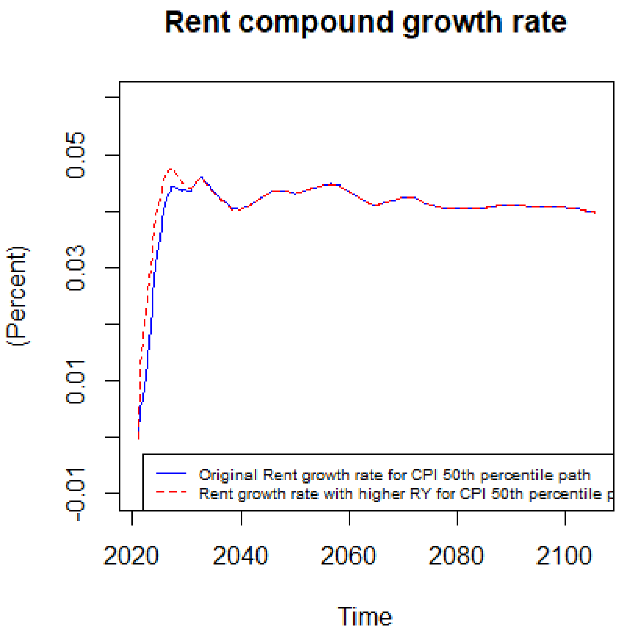

Figure 3.30: Rent Compound Growth rates for Different Rental Yield Starting Points

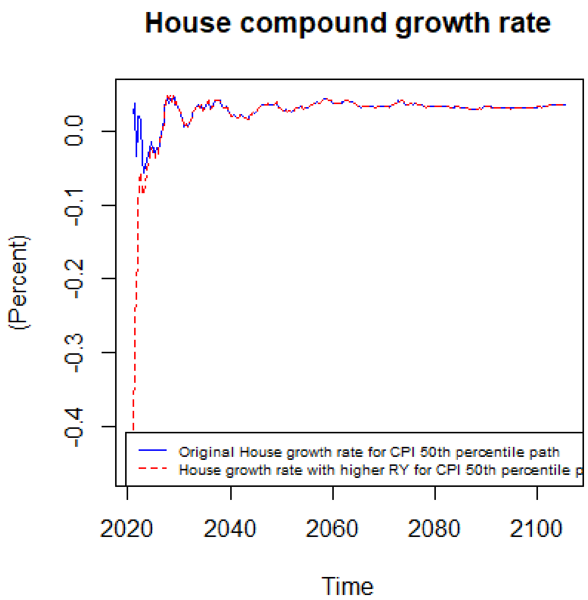

Figure 3.31: House Compound Growth rates for Different Rental Yield Starting Points

3.6 Conclusion

This chapter has built a cascade model for Australian house prices. For all the variables, ARIMA, ADL, and ECM models are fitted to the data. Statistical criteria were used to select some potential options, and economic theories are then considered to choose final models and construct the cascade structure. The model includes eight key variables, between which it maintains relationships in a way that supports forming coherent projections over long horizons that are suitable to incorporate into the life-cycle models that investigate individual financial decisions. The model should also be valid for long-term projections for actuarial applications.

We admit that there exist limitations that may change the model parameters and the relationships between variables. The most noticeable one is the inclusion of variables in the cascade structure. As the primary goal of our models is the application in the long-term projections and the life cycle models, we effectively exclude the variables that have the close relationship with the house prices but not useful in the individual's lifetime financial planning, for example, the vacancy rate and the population changes (See Ballantyne et al., 2019). In the future research, it would be useful to test different house price models for other purposes. Another thing we have not tested is the data period selection. We did not compare the performance of different models if we select different data period. We effectively choose the most sensible period for all the variables. It is worth of testing multiple data periods to fit the models and find the optimal ones.

In the next chapter, the cascade model will be applied to analyse the housing arrangements over an individual's lifetime.

4 Optimal Superannuation Guarantee and housing tenure arrangement over the life cycle[1]

4.1 Introduction

Many countries have been changing their publicly managed, unfunded defined benefit (DB) schemes to privately managed, fully funded defined contribution (DC) schemes (Holzmann 2013). This shift empowers individuals to choose their contribution rates and manage their asset allocations (Dahlquist et al. 2018), hence transferring the responsibility and the risks to the individuals. Many individuals have trouble starting and maintaining long-term savings programs (Thaler and Bernartzi 2004), which has led some policymakers to set up minimum compulsory retirement savings schemes (Bateman and Piggott 2001). It is vital to design private mandatory saving schemes to improve individuals' retirement outcomes to the greatest degree possible.

The Superannuation Guarantee (SG) serves as the Australian mandatory saving scheme and was introduced in 1992 with the SG contribution rate originally set at 3%. It is now set as 10.5% and is slated to increase to 12% from July 2025 (Parliament of Australia 2010 and 2014; ATO 2020). Whether the proposed SG increase should proceed has generated wide public debate[2].

Individuals' decisions on their housing arrangements can potentially have a significant impact on their lifetime wellbeing. Typically, more than 70% of households in OECD countries own a home before retirement (Chiuri and Jappelli 2010). For many, owner-occupied housing is the largest and most important asset in their portfolio (see Cocco 2005; Yao and Zhang 2005). This effectively both provides imputed rental services and a form of asset investment, which is often leveraged through mortgage borrowings. The alternative is to pay rent. Several authors have examined house tenure and housing consumption choices in a life-cycle framework (See Fernandez-Villaverde and Krueger 2011; Dotsey et al. 2014; Li et al. 2016; Shao et al. 2019).

1 This chapter was presented as a paper at the Australasian Actuarial Education and Research Symposium in Canberra on 24-25 November 2022.

2 Chapter 2 provides further details of this debate.

In Chapter 2, this research estimated the optimal SG for the Australian socio-economic and policy environment and investigate how individual welfare changes with differing levels of SG. However, that analysis excludes some features that may impact the optimal SG levels. One of the major exclusions is the assumed absence of other saving mechanisms outside of the superannuation system. The analysis in Chapter 2 effectively modelled the first pillar (i.e., means-tested public funded Age Pension) and the second pillar (i.e., privately managed defined contribution superannuation) of the Australian retirement system. It ignored the third 'voluntary savings' pillar, of which housing is the most important component. Homeownership not only provides retirees with an imputed income stream in terms of rental services. It is also made more favourable relative to other private savings as the principal home is not included in the asset test for determining Age Pension eligibility[3].

Against this background, this thesis continues the research on how the contribution rate impacts individual welfare by allowing for house tenure and housing consumption choices within the Australian setting. This research estimates the optimal SG for individuals who choose to purchase a home ('owner') and those who rent ('renter') over their lifetime and compare their financial welfare. This research investigates the influence of four factors on the optimal SG level and the benefit from home ownership versus renting: the utility function used to represent an individual's preference; assumptions around the housing market and individual's behaviours regarding buy-versus-rent decisions; individual income and the policy environment, specifically the rules around tax and social security.

This research uses a stochastic life-cycle model to simulate the financial outcomes for individuals who choose between renting a home throughout their lifetime and aiming to purchase a home once they are able to. The utility benefit is analysed from this choice, noting that home purchase leads to lower pre-retirement consumption but potentially higher post-retirement consumption and larger bequests relative to renting. The analysis captures the utility arising from both consumption and housing expenditure (either as rent or imputed rent), as well as bequests. The model is used to identify the 'optimal' SG for both renters and owners, and then compare the welfare arising from the two housing arrangements.

The stochastic projections have directly applied the model presented in Chapter 3. In doing so, exposure to fluctuations in personal incomes, inflation, interest rates, superannuation

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[3] For more details see Services Australia 'Asset types', https://www.servicesaustralia.gov.au/asset-types?context=22526.

investment returns, rent and house prices are modelled. The analysis considers the full range of tax, social security, and housing purchase effects for Australia. The most noteworthy items include the tax on both personal and superannuation income, contribution tax, detailed mortgage loan terms, transaction costs for house purchases and eligibility for the Age Pension, rental assistance, and related supplements.

It is found that the optimal SG increase with income level for both renters and owners but is much lower for owners. The renter's optimal SG ranges from 5.5% at the 10th income percentile up to 14.0% for the 90th income percentile, while the owner's optimal SG ranges from 0.5% up to 2.0%. This finding reinforces the result of Chapter 2 that there is no single 'one-size-fits-all' optimal SG that suits all individuals, emphasising that individual heterogeneity extends from characteristics like income levels to include decisions on house tenure. That the gap between the optimal SG between renters and owners increases with income level, moving from 5.0% at the 10th income percentile to 12.0% in the 90th income percentile is also noted.

This research adds to a growing body of literature that highlights the importance of allowing for heterogeneity in retirement planning. For instance, previous studies have highlighted the significance of heterogeneity in financial literacy (Lusardi et al. 2013), among passive investors in stock market participation (Dahlquist et al. 2018), and risk preferences and their impact on optimal asset allocations (Alserda et al. 2019; Butt et al. 2022). This research also adds to the literature on the link between default contributions and better retirement savings plan design by taking housing tenure into consideration (Madrian and Shea 2001; Choi et al. 2003; Chetty et al. 2014; Blake et al. 2014; Munnell et al. 2014; Evans and Razeed 2019). The findings indicate that purchasing a home can significantly reduce the need to save for retirement through the superannuation system under the current rules. The extremely low optimal SG for owners also poses the question that, if the SG moves to 12% as proposed and is combined with a house purchase, whether the SG of 12% might be intended to replace the need for generous public-funded Age Pension arrangement for those who own a home.

The analysis indicates that home ownership enhances individual welfare. SG levels are tested ranging from 0.5% to 20.0% and find that owners always enjoy higher utility than renters across all income and SG levels. The owner's higher utility mainly comes from overall higher consumption levels and bequests from his house relative to the renter. However, the research emphasises that these results arise under the assumption that the individuals choose and can access an affordable home in line with their income and borrowing capacity, and that they value their consumption, housing expenditure and bequest in line with the utility function settings. This study builds on a growing literature examining household tenure and housing consumption choices in a life-cycle framework (Li and Yao 2007; Fernandez-Villaverde and Krueger 2011; Dotsey et al. 2014; Li et al. 2016; Shao et al. 2019; Fehr and Hofmann 2020). This study investigates household tenure and housing consumption choices in a complete three-pillar retirement saving system and allows for a realistic policy environment with rules around tax and social security.

The study also undertakes its analysis assuming no access to the Age Pension. This leads to the most optimal SG estimates exceeding 10%. Specifically, the optimal SG for the renter is all above 20.0%, with a minimum increase of optimal SG from 14.5% for the 10th income level to 6.0%[4] for the 90th income level compared to the baseline case. The owner's optimal SGs exhibits a similar trend, with the optimal SG all above 10.0%, ranging from 13.0% at the lowest income to 10.0% for the highest income earner, versus the 0.5%-2.0% range under the baseline analysis. Low-income earners experience the largest increase in optimal SG, reflecting the fact that they receive the greatest relative benefit from the Age Pension as a substitute for private savings. This analysis reconfirms the importance of publicly funded Age Pension as a retirement income source, in particular for Australian retirees with modest income and savings, while highlighting the relative importance of the Age Pension for renters as a substitute for personal savings. These results contribute to the considerable research into the role of mean-tested pensions with retirement savings systems, see Hulley et al. (2013) and Andreasson et al. (2017) for a detailed overview.

This chapter proceeds as follows. Section 4.2 discusses the methodology and assumptions. Section 4.3 presents the results, including comparing the models in this chapter and Chapter 2, presenting the optimal SG for renters and owners, and comparing renter and owner lifetime total benefits. Section 4.4 conducts sensitivity tests, while Section 4.5 concludes the chapter.

4.2 Methodology and assumptions

This research simulates two types of living arrangements. The homeowner (owner) first rents the house they intend to buy, saves money to build a deposit and buys the house when they have sufficient savings and can service the home loan under standard rules. The renter

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[4] The maximum SG tested is 20.0%, so the increase is at least 6.0%.

decides to rent the house for their whole lifetime. Both owners and renters contribute to their superannuation fund in line with the SG. This research estimates consumption from regular spending and housing as well as bequests under these two arrangements over the entire life-cycle, which are evaluated using a utility function.

This section starts by introducing the general set-up for the model in Section 4.2.1, followed by the utility functions for consumption, housing, and bequest in Section 4.2.2 and the income model in Section 4.2.3. The initial house price and initial rental yield are calculated in Section 4.2.4 and Section 4.2.5. Section 4.2.6 to Section 4.2.8 explains the superannuation features, for instance, superannuation transfer balance cap, superannuation investment tax, and superannuation fund management fees. The mortality rates are described in Section 4.2.9. The Australian tax and social security rules incorporated in the model are explained in Section 4.2.10 and then detail the house purchase and mortgage loan in Section 4.2.11. The nominal terms are emphasised in Section 4.2.12. The simulation process and the certainty equivalents are presented in Section 4.2.13 and Section 4.2.14.

4.2.1 General set-up

At age 25, the individual's affordable house price is calculated according to their income level, as outlined in Section 4.2.4. The initial rent is calculated using the regression model of house price and rental yield appearing Section 4.2.5. The house price and rent growth are projected using the models from Chapter 3.

At the beginning of each pre-retirement year, the owner and renter receive the same income, put money into a superannuation account and pay the income tax and Medicare levy. Their post-tax income is allocated between savings, rent, and consumption, in line with the rules described below.

It is assumed that the owner and renter need to consume a minimum consumption equal to 30% of their post-tax income. This minimum consumption represents a basic daily living cost and plays the additional function of forming a consumption floor in the utility function as outlined in Section 4.2.2. They also need to pay the rent.

For the saving, it is assumed that the renter and owner save 10% and 15% of their post-tax income respectively. The renter saves to build up a $10,000 precautionary saving, while the owner saves for the $10,000 precautionary saving and the deposit plus transaction costs to purchase the house. A 15% owner saving rate is chosen by referring to the observed Australian saving rate in 2022 OECD data[5]. The renter's saving rate of 10% is slightly lower to reflect the fact that he does not need to save the upfront payment and is in line with the Australian historical average saving rate (ABS 2022a). Individuals who are unable to save at the set level after paying rent and consuming the minimum consumption, save what they can. Both renter and owner stop saving until they achieve their goals and maintain their precautionary savings until their retirement.

On retirement at the age of 67, the renter's saving account and superannuation are combined, with the amount up to the transfer balance cap (Section 4.2.6) placed in a zero-tax pension account, with the amount over the cap placed in an accumulation account paying the same tax as pre-retirement. He receives Age Pension and rental assistance subject to asset and income tests. He also withdraws from his pension and accumulation account at the minimum drawdown rate. The total amount of income is the sum of withdrawals, Age Pension and rental assistance. This income is used to pay the rent and consumption. There is no minimum consumption after retirement, which aligns with the post-retirement consumption floor in the utility calculation as detailed in Section 4.2.2. No minimum consumption post-retirement along with 30% of their post-tax income pre-retirement reflects the observation that individuals consume less during retirement (see Skinner 2007; OECD 2012; Olafsson and Pagel 2018). An owner who has not been able to purchase the house is treated as a renter after retirement.

The renter needs to evaluate whether he can still afford the rent every year after retirement[6]. If he does not have sufficient income to pay the rent, it is assumed that he downsizes the house and moves to an alternative home and only pays 50% of the rent[7]. The 50% assumption is chosen to prevent a frequent change of homes, which is not a common practice for retirees.

Once the owner's savings exceed the sum of upfront payment, housing transaction cost and precautionary savings, he takes a mortgage loan and purchases the house. The rules of house purchase and mortgage loans are detailed in Section 4.2.11. After purchase, the owner

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[5] For more details see OECD 'Household Savings', https://data.oecd.org/hha/household-savings.htm.

[6] The thesis simulates an individual's lifetime financial decisions, which fails to capture the changes along his life path, like marriage, having children or death of his spouse. These changes would significantly influence his expenses and their decision of downsizing. In the future research, it is worth investigating the optimal SG in a household setting.

[7] Paying lower rent results in lower utility from housing expenditure, on the assumption that a cheaper home is less attractive and delivers less benefit.

ceases to pay rent but incurs a house maintenance cost equal to 0.5% of house price[8] and mortgage repayments. He consumes the residual of his income until the mortgage loan is paid back.

At retirement, the owner uses saving and super balances to pay back any remaining mortgage loan. If the balances are insufficient, he downsizes his house to pay back the remaining mortgage[9]. After settling his mortgage loan, he combines his savings and super accounts and allocates them between a pension account and an accumulation account (similar to the renter). The owner consumes the sum of the Age Pension, withdrawal from his accounts less the maintenance costs[10].

4.2.2 Utility functions[11]

Two types of utility functions are sued to represent individual objectives regarding consumption, housing and bequests.

The utility function for consumption and housing is a modified form of the power utility, with non-housing consumption evaluated relative to a consumption floor and housing expenditure treated as a form of consumption. It is motivated by utility functions broadly used within the academic literature that account for both non-housing consumption and housing expenditure (for example, Flavin and Nakagawa 2008; Yang 2009; Davidoff 2010; Li et al. 2016) and utility function with a consumption floor (for example Ding 2014; Iskhakov et al. 2015; Andréasson at el. 2017). Its functional form is described by Equation (4.1):

$$U_{CH}(C_t, H_t) = \frac{\left[\omega\left(C_t - C_{floor,t}\right) + (1-\omega)H_t\right]^{1-\rho}}{1-\rho} \tag{4.1}$$

8 For more details see Prosolution 'How much should you spend on property maintenance and improvements?', https://www.prosolution.com.au/property-maintenance/. This research chooses 0.5% as a middle number between 0.4% and 0.75% indicated in the article.

9 This results in living in a less attractive house that generates lower utility. If savings and the house value cannot cover the mortgage, the individual would be bankrupted. In this simulation, this situation never occurs. This is mainly due to the rational affordable house price calculation at age 25 in Section 4.2.4.

10 The research also checks if the owner can afford the maintenance cost. If not, he must downsize to a house with only half the price of original one. The 'half price' house follows the same logic as renter's downsizing to avoid the frequent house trade.

11 Since we did not find a reference-dependent utility function that has both non-housing consumption and housing consumption, in this chapter, only power utility are tested.

where U_{CH} represents consumption and housing utility function; C_t is the consumption at age t; H_t refers to housing service flow or housing expenditure at age t; $C_{floor,t}$ is the consumption floor at age t; ω is the preference parameter between consumption and housing expenditure. It is assumed that ω is 0.9, which is in line with the literature (e.g.: Yang 2009; Li et al. 2016). Sensitivity analysis is done for $\omega = 0.5$. ρ is the coefficient of constant relative risk aversion (CRRA). The baseline assumption is that ρ equals 4, which sits within the range used in the literature (for example, Ameriks et al. 2011; Yogo 2016; Butt et al. 2019). Higher and lower values of ρ are examined under sensitivity testing.

Housing expenditure H_t is typically measured by rent (See Davidoff, 010). For the renter, housing expenditure is simply the rent he pays. For the owner, housing expenditure is the imputed rent for his house less its maintenance costs.

The consumption floor $C_{floor,t}$ is 30% of post-tax income at age t before retirement hence altering every age, while it is set to be 0 after retirement. This formulation can be seen as mimicking a replacement rate of 70% (OECD 2012), noting that the same utility arises from consuming 100% of post-tax income prior to retirement and the equivalent of 70% of post-tax pre-retirement income after retirement.

The utility function for a bequest is consistent with the power utility, and follows Andréasson at el. (2017), Lockwood (2018), and Suari-Andreu et al. (2019):

$$U_B(B_t) = \frac{B_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho} \tag{4.2}$$

where U_B represents the bequest utility function; B_t is the bequest at age t; ρ is the same CRRA as the utility function for consumption and housing in Equation (4.1); ϕ is the strength of the bequest motive. For renters, B_t is equal to their outstanding balance in pension and accumulation account at age t. For owners, B_t is the sum of the value of their house and their outstanding balance in pension and accumulation account at age t; noting that higher ϕ means stronger bequest motive, it is assumed ϕ in the baseline case is 0.5, which is in line with Butt et al. (2022). A ϕ of 0.96 as per Lockwood (2018) is examined under sensitivity testing.

C_t, $C_{floor,t}$, H_t and B_t are all measured in real terms in the utility calculations. Real values are estimated by deflating nominal values at age t, back to the values at age 25 using the CPI.

4.2.3 Income model

This research follows the same method in Chapter 2 to generate the income distribution and the real annual pre-tax income: a detailed explanation can be found in Chapter 2. Nine income levels are tested in this chapter, reflecting the 10th to 90th percentiles of wages for full-time employees in May 2021 (ABS 2022b). This research uses real annual pre-tax wages after the SG (equal to 10%) in 2022 as reference points, then apply the model of Freestone (2018) to identify the pre-tax incomes after SG at age 25 that are consistent with the real annual pre-tax wages after SG for each income percentile. Table 4.1 summarises the income distribution at age 25, ranging from \$40,912 to \$120,456.

Table 4.1: Wage Income Distribution

Income Percentiles	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
Base Income, Age 25	\$40,912	\$46,977	\$52,677	\$58,702	\$66,069	\$74,333	\$84,266	\$97,130	\$120,456

The growth of nominal pre-tax income in this model consists of two parts: the determinist growth of real annual pre-tax income from the age-earnings profile of Freestone (2018) and the stochastic growth of nominal AWE from Chapter 3 subtracting the labour productivity (1.5% per year), where the labour productivity is the long-term real income growth. The reason to subtract labour productivity is that in Freestone (2018), the long-term real income growth has been reflected by the age-earnings profile. The nominal annual pre-tax income after SG at age t $(26 \leq t \leq 66)$ for zth income percentile $(e_{t,z})$ is obtained as follows:

$$e_{t,z} = i_{25,\mathrm{z}} \prod_{j=26}^{t} \left(\frac{h_j}{h_{j-1}} + \frac{AWE_{2022-25+j}}{AWE_{2022-25+j-1}} - LP \right) \tag{4.3}$$

where $i_{25,\mathrm{z}}$ represents the annual pre-tax income after SG the individual of zth income percentile earns at age 25; h_t is the real hourly wage from the deterministic age-earnings profile of Freestone (2018)[12]; $AWE_{2022-25+t}$ is the AWE value at age t since it is assumed that the individual is at age 25 in 2022; LP is the labour productivity equal to 1.5%; and z is 10 to

[12] The basic model of real hourly pre-tax wage after SG at age t (h_t) is:

$$h_t = \exp\left(\alpha + \beta_1 t + \beta_2 t^2\right) \tag{4.4}$$

where α is the intercept of log hourly wage and β_1, β_2 are parameters for age and square of age respectively for a full-time male worker at age t. The parameters are calibrated using HILDA data from Household, Income and Labour Dynamics in Australia (HILDA) survey, with $\alpha = 1.86769$, $\beta_1 = 0.06891$, and $\beta_2 = -0.000731$.

90. The nominal gross annual pre-tax income before SG for zth income percentile is then estimated as $1.1 * e_{t,z}$.

4.2.4 Initial house price

The approach to tying housing to income draws on the concept of housing affordability (Gan and Hill 2009), where rent and house price has a close relationship with income. It is assumed that the individuals initially occupy a home of the quality that aligns with their income and make the one-off decision to save to buy this home or to keep renting it throughout their lifetime.

This affordable house price is estimated by accounting for the individuals' borrowing capacity, income growth, and mortgage rates. This formula is similar to Gan and Hill (2009) except for adding the income growth rate. The following equation defines an individual's borrowing constraint,

$$\sum_{n=1}^{N}\left[\frac{\alpha X(1+r)^n}{(1+i)^n}\right] = H(1-u) \tag{4.5}$$

where N is the mortgage loan term; α is the percentage of pre-tax income that is used to pay the mortgage; X is the individual's income; r is the income growth rate; i is the mortgage rate; H is the house price that the individual can afford; u is the upfront payment ratio of the house price.

The affordable house price is calculated as:

$$HP = \frac{1}{(1-u)}\sum_{n=1}^{N}\left[\frac{\alpha X(1+r)^n}{(1+i)^n}\right] \tag{4.6}$$

It is assumed that the mortgage rate i[13] is 7.2% per year as the summation of the mean mortgage rate of 6.7% per year in Chapter 3 and 0.5% buffer, which is in line with the APRA interest rate buffers in RBA (2018)[14]. The income growth rate r is set as the mean AWE growth rate of 3.89% per year in Chapter 3. The upfront payment ratio u is 20% referring to APRA (2019), where the loan to house price ratio larger than 80% only account for less than 20% of

[13] The mortgage rate in Equation (4.6) is chosen as a long-term mortgage mean, while in the simulation process, this research varies the mortgage rates.

[14] For more details see Reserve Bank of Australia 'Box B: The Impact of Lending Standards on Loan Sizes', https://www.rba.gov.au/publications/fsr/2018/oct/box-b.html.

new lending. u is selected as 20% in line with RBA (2018) since lenders would generally set maximum loan sizes such that the required repayments did not exceed 30 per cent of pre-tax household income. N is 30, reflecting the common maximum loan term in Australia of 30 years.

The house price HP as calculated above is the maximum house price that the individual can afford considering his borrowing capacity. However, a majority borrow substantially less than their maximum borrowing capacity. Using the Household Income and Labour Dynamics Australia (HILDA) survey, RBA (2018) shows that the typical (median) owner-occupier borrower only borrowed about half of the maximum loan they could obtain, and more than two-thirds of households borrowed less than 70 percent of their maximum loan size. To reflect this fact, it is assumed the house that the individual occupies and aims to purchase is 70 percent of the house price HP.

4.2.5 Initial rental yield

The initial rental yield is assumed to be a function of the initial house price. Ordinary Least Squares (OLS) regression (Wilkinson and Rogers 1973) is used to estimate the relationship between rental yield and house price using the Australian real estate data (CoreLogic 2022). This research also sets up the upper and lower bounds using the observed maximum and minimum house prices. The following equations describe the rental yield function as,

$$RY = \begin{cases} -3.828 \times 10^{-8} \times 370335 + 0.06699, & H \leq 370335 \\ -3.828 \times 10^{-8} \times H + 0.06699, & 370335 < H < 1389948 \\ -3.828 \times 10^{-8} \times 1389948 + 0.06699, & H \geq 1389948 \end{cases} \tag{4.7}$$

where RY is the rental yield; H is the house price. Equation (4.7) indicates that the rental yield decreases with house prices. A minimum rental yield of 1.38% and a maximum of 5.28% is imposed to ensure the rental yield sits in a reasonable range. This setting results in that the rental yield of a house price less than \$370,335 is always equal to 5.28%, while the rental yield of a house price of more than \$1,389,948 is always equal to 1.38%. In this way, the rental yields for all house prices vary between 1.38% and 5.28%, representing a reasonable range.

The initial rent is calculated as the product of initial house price and initial rental yield.

4.2.6 Transfer balance cap

The Australian government set a transfer balance cap for retirees that limits how much of the superannuation balance from their accumulation account can be transferred to a tax-free pension account[15]. The cap is currently $1.7 million, and is subjected to proportional indexation in $100,000 increments, in line with the Consumer Price Index (CPI)[16].

4.2.7 Superannuation investment tax approximation

This research follows the superannuation investment tax calculations for growth and defensive assets in Chapter 2, where growth asset tax and defensive asset tax are 6.74% and 15% pre-retirement and -6.045% and 0% for pension accounts respectively. The negative tax rate on growth assets for pension accounts derives from imputation tax credits (see Butt et al. 2019). Since the research simulates the superannuation fund returns as a whole in Chapter 3 and the fund portfolio consists of approximately 70% growth assets and 30% defensive assets, the research approximates the superannuation investment tax as $(6.74\% \times 70\% + 15\% \times 30\%)$ pre-retirement and $(-6.045\% \times 70\% + 0\% \times 30\%)$ post-retirement.

4.2.8 Superannuation fund management fees

It is assumed that the superannuation fund management fees comprise two parts: a fixed fee equal to $78 per annum being inflated by CPI and a variable fee equal to 0.88% of the account balance. These two fees are applied to both the accumulation account and the pension account, with fees being deductible at the 15% rate for the accumulation account. The assumed fees are comparable to those currently observed in the superannuation industry[17]. When the account balance is close to zero (e.g., less than $100), it is assumed that the fees cease being deducted to ensure the balance keeps positive.

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[15] For retirees, their superannuation balance in their pension account benefit from 0% earnings tax, while their accumulation accounts are required to pay the same earnings tax as pre-retirement.

[16] For more details see Parliament of Australia 'Superannuation (Excess Transfer Balance Tax) Bill 2016 [Provisions] and Treasury Laws Amendment (Fair and Sustainable Superannuation) Bill 2016 [Provisions], Chapter 2', https://www.aph.gov.au/Parliamentary_Business/Committees/Senate/Economics/SuperReformbillsx2/Report/c02.

[17] The data is sourced from SuperRatings and can be provided upon request.

4.2.9 Mortality rate

This research follows the same mortality rate treatment as Chapter 2. Mortality from age 67 is based on the male rates with 125-year mortality improvement from the Australian Life Tables 2015-2017 in the publication of Australian Government Actuary (2019). This reflects the fact that the simulation spans 86 years. The modelling assumes the individual at age 25 in 2022. It is assumed that there is no mortality over the pre-retirement working period from age 25 to age 66, mortality risk between age 67 and age 109, and death with certainty at age 110. According to the Australian Life Tables, a male aged 25 will die before age 66 with a probability of only 8%. The assumption of zero mortality before retirement also reflects the likelihood that individuals will plan their retirement on the expectation that they will live through to their retirement age.

4.2.10 Australian tax and social security rules

This research incorporates the same tax and social security rules as Chapter 2 with the values updated to 2021-22. Table 4.2 summarises their main features.

Table 4.2: The Australian Tax and Social Security Rules

Rule name	Main features
Australian income tax rates for 2021-22	The Australian personal income tax rates are zero for income below $18,200, 19c for each $1 between $18,201 and $45,000, then 32.5c for additional income up to $120,000, 37c for additional income up to $180,000, and 45c on any income above $180,000.
Concessional contribution cap[18]	Concessional contribution cap is $27,500 in 2020-21 from Australian Taxation Office.
Low income super tax offset (LISTO)	The LISTO is a government superannuation payment of up to $500 to help low-income earners save for retirement. The LISTO is 15% of the concessional (before tax) superannuation contributions an individual or individual's employer pays into his super fund if he earns $37,000 or less a year. The formula used is: $LISTO_t = \min(0.15e_tSG, 500)\,1_{(e_t \leq 37000)}$.
Minimum drawdown rules, pension account	Individuals are required to drawdown a minimum portion of their retirement accounts at their withdrawals. The portions vary with ages: 4% when individuals are younger than 65, 5% at age of 65 to 74, 6% at age of 75 to 79, 7% at age of 80 to 84, 9% at age of 85 to 89, 11% at age of 90 to 94, and 14% after age of 95.
Age Pension	The maximum basic rate is $23,420.8 per year. Asset test: The asset test is only applied where a person's assets do not exceed the upper limit for eligibility of the Age Pension. The reduction of pension under the asset test is calculated by the following method: divide the asset excess (further assets in excess of $270,500 for homeowners and $487,000 for renters) by 250; round the result down to a whole number; multiply the whole number by 19.5, see the Australian Government 'Social Security Guide, Pensions & benefits assets tests' https://guides.dss.gov.au/guide-social-security-law/4/2/3. Income test: Income test involves applying the deemed income rule. Deemed income is calculated by summing up 0.25% of the first $53,600 of account balance and 2.25% of any further balance in excess of $53,600. The income test will reduce the Age Pension received by 50 cents per dollar of additional deemed income earned.
Pension supplement	A pension supplement and an energy supplement are additional regular payments made to recipients of the Age Pension, to assist with the cost of prescriptions, rates, telephone, and energy.
Energy supplement	The energy supplement is a fixed amount of $366.60 per annum. The maximum pension supplement is $1,890.2, reducing to a minimum amount of $1,016.6 at a rate that is proportionate to the Age Pension.
Rent Assistance	Rent assistance is provided to people who receive the Age Pension and pay rent. There is a minimum amount of rent people need to pay to get Rent Assistance. People receive 75c for every $1 of rent paid above this amount. For a single individual, without dependent children, the annual rent is at least $3,385.2. The maximum fortnightly payment is $3,790.8.

To estimate the post-tax disposable income available for consumption, the Australian income tax rates for 2021-22 including the Medicare levy equal to 2% of taxable income are applied where applicable. A partial Medicare levy applies to those whose taxable income is between $23,226 and $29,033[19]. The formula from Liu (2022) is used to calculate the partial

[18] This research assumes the concessional contribution cap is indexed annually with CPI and rounded down to the nearest $2,500. For more details see Carrickaland, Indexation of Super Contributions Cap', https://www.carrickaland.com.au/2021/04/08/indexation-of-super-contributions-caps.

[19] For more details see Australian Tax Office 'Medicare levy reduction for low-income earners', https://www.ato.gov.au/Individuals/Medicare-and-private-health-insurance/Medicare-levy/Medicare-levy-reduction-for-low-income-earners/.

Medicare levy,

$$Medi = \begin{cases} 0, & TI \leq 23226 \\ 0.1(TI - 23226), & 23226 < TI \leq 29033 \\ 0.02TI, & TI > 29033 \end{cases} \tag{4.8}$$

where TI is the taxable income and $Medi$ is the Medicare levy to be paid. It is assumed that individuals with more than $90,000 of income purchase private health insurance and therefore do not pay the Medicare levy surcharge[20], noting that failing to take private health insurance incurs a large Medicare levy surcharge penalty. It is assumed that the insurance premium comes from individuals' post-tax consumption rather than modelling it as a separate charge.

The contribution tax of 15% (ATO 2022) is imposed on SG contributions that are made of pre-tax income, subject to the concessional contribution cap. Contributions above this cap are added to the account balance as non-concessional contributions, meaning that they are made out of post-tax income.

After retirement, withdrawals from a retirement account are subject to the minimum drawdown rules, which vary with age (see Table 4.2).

To determine Age Pension eligibility, both the asset test and income test are applied with reference to the retirement account balance. The calculations are based on the means testing arrangements in 2021-2022 for a single homeowner and single renter. The research also determines eligibility for the pension supplement, energy supplement, and rent assistance. The sum of the basic Age Pension, pension supplement, energy supplement and rent assistance will be treated as the total Age Pension for which the individual is eligible.

4.2.11 House purchase and mortgage payment

The homeowners purchase the house they will live in when their savings balance is large enough to pay the upfront payment (20% of the house price) and the transaction cost[21], while keeping aside $10,000 of precautionary savings. The transaction cost is selected as 5% of the house price by reference to the stamp duty rates across states[22], adding some buffer for the legal

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[20] For more details see Australian Tax Office 'Medicare Levy Surcharge', https://www.ato.gov.au/Individuals/Medicare-and-private-health-insurance/Medicare-levy-surcharge/.

[21] The transaction cost mainly consists of legal and conveyancing fees and stamp duty.

[22] For more details see Conveyencing.com.au 'What is Stamp Duty?', https://conveyancing.com.au/articles/stamp-duty.

and conveyancing fees. A mortgage loan is taken to cover the rest of the house price.

The”mortgage payment m_t and the mortgage loan amount L_t at time t have the following relationship[23]:

$$m_t \sum_{k=1}^{n} \frac{1}{(1+i_t)^k} = L_t \tag{4.9}$$

where n is repayment periods; i_t is the mortgage rate.

Solving it gets,

$$m_t = \begin{cases} \frac{L_t(i_t(1+i_t)^n)}{((1+i_t)^n - 1)}, n > 1 \\ L(1+i_t), n = 1 \end{cases} \tag{4.10}$$

The outstanding mortgage loan at the start of time $(t+1)$ is calculated as follows:

$$L_{t+1} = L_t(1+i_t) - m_t \tag{4.11}$$

where L_t is the mortgage loan amount at time t; m_t is the mortgage payment amount calculated at time t but paid just prior the start of time $(t+1)$; i_t is the mortgage rate applied at time t.

It is assumed the mortgage loan term n is 30 years, in line with the mortgage term used in calculating the initial house price.

4.2.12 Nominal term

It is assumed all the tax, fees, and social security rules are in nominal terms and indexed annually by the CPI where required. This assumption unifies the indexation rules for different quantities by applying the CPI as a common inflator, e.g., CPI is also applied in estimating the concessional contribution and transfer caps[24]. This assumption is also in line with Chapter 2,

[23] For more details see United Nations Economic Commission for Europe (UNECE) ‘Formulae for calculation of interest, loan repayments and deposits’, https://unece.org/fileadmin/DAM/energy/se/pdfs/gee21/invest_ee_train_Almaty2013/Formulae_Calcul_e.pdf.

[24] For more details see Parliament of Australia ‘Superannuation (Excess Transfer Balance Tax) Bill 2016 [Provisions] and Treasury Laws Amendment (Fair and Sustainable Superannuation) Bill 2016 [Provisions], Chapter 2’, https://www.aph.gov.au/Parliamentary_Business/Committees/Senate/Economics/SuperReformbillsx2/Report/c02.

which inflated all quantities at a common rate.

The only exception is the Age Pension[25], which is inflated by AWE. One issue in allowing for the Age Pension to be inflated by AWE is that it leads to the Age Pension growing much faster than the other values, noting that under the economic model the expected growth rate of AWE consists of CPI plus labour productivity of 1.5%. Figure 4.1 below illustrates the mean growth of $1 when indexed to the CPI and AWE can result in very large value differences throughout an individual's life. This becomes problematic for the analysis as the Age Pension becomes implausible large relative to other quantities when indexed to AWE, and thus begins to dominate the results. To generate meaningful results that are not dictated by the Age Pension, this research follows the same assumption as the Retirement Standard of The Association of Superannuation Funds of Australia (ASFA)[26] and assume that the Age Pension is also indexed annually to the CPI.

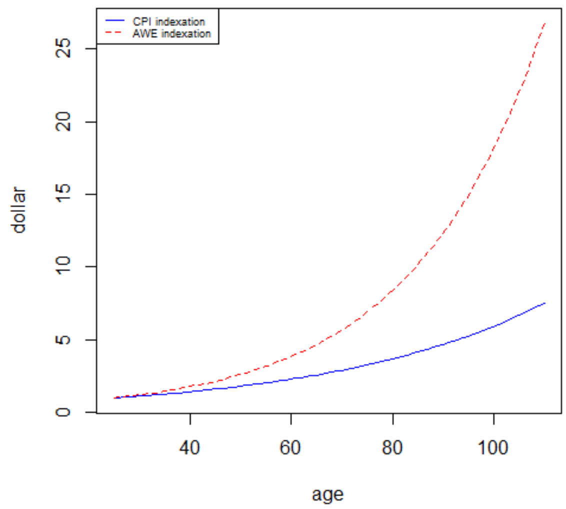

Figure 4.1: The comparison of CPI and AWE indexation

4.2.13 Simulation process

This research follows the same method used in Chapter 3 and simulate 10,000 paths over 85 years starting from 2022 for CPI, AWE, cash rates, mortgage rates, rent index, house price index and fund returns. This provides the distribution of economic circumstances that the owner

[25] For more details see Department of Veterans' Affairs 'Indexation of income support pensions and allowances', https://www.dva.gov.au/financial-support/payment-rates/indexation-income-support-pensions-and-allowances.

[26] For more details see ASFA 'ASFA Retirement Standard', https://www.superannuation.asn.au/resources/retirement-standard.

and renter could encounter over their lifetime.

Similar to Chapter 2, owner and renter outcomes are simulated for a range of SG levels extending from 0.5% up to 20% in 0.5% increments. For a specific SG, the individual's lifetime utility $\hat{V}_0^y(W_0, SG)$ in yth simulation out of K is calculated by adding utility through their lifetime using Equation (4.12)[27]:

$$\hat{V}_0^y(W_0, SG) = \sum_{s=0}^{T} \beta^s {}_{s-1}p_{25} \left({}_1p_s U_{CH}(C_s, H_s) + {}_1q_s U_B(B_s) \right) \tag{4.12}$$

where T is termination time; β is the time preference; ${}_tp_x$ is the probability that an individual aged x will be alive at age $x + t$, while ${}_tq_x$ is the probability that an individual aged x will die at age $x + t$; $U_{CH}(C_t, H_t)$ and $U_B(B_t)$ are the utilities generated from consumption and housing and bequest at time t, respectively. For the time preference parameter β, 0.98 is used as the baseline assumption (see Lockwood 2018; Li et al. 2016), while 0.96 (see Cocco 2005; Yao and Zhang 2005; Yogo 2016) and 1.00 (see Chapter 2; Andréasson at el. 2017) are tested in sensitivity analysis.

Individual's expected lifetime utility $\bar{V}(SG)$ is calculated by averaging individual's lifetime utility across K simulations using Equation (4.13):

$$\bar{V}(SG) = \frac{1}{\mathrm{K}} \sum_{y=1}^{\mathrm{K}} V_y(SG) \tag{4.13}$$

where $K = 10{,}000$ is applied in this chapter.

The 'optimal' SG (SG^*) is defined as the SG that maximises expected lifetime utility as per Equation (4.14), recalling that the SG is tested in increments of 0.5%:

$$\bar{V}(SG^*) = \max_{0.5\% \leq SG \leq 20\%} \bar{V}(SG) \tag{4.14}$$

4.2.14 Certainty equivalents

Certainty equivalents (CE) based on estimates of expected lifetime utility are calculated to evaluate renter and owner welfare and identify their differences in consumption, housing expenditure and bequests.

CE is defined as a constant amount at the beginning of the simulation that generates the same total utility as the expected lifetime utility. It can be interpreted as the riskless amount of

[27] The equations for evolution of state variables is in Appendix 4D.

money that achieves the same utility as the random consumption, housing expenditure or bequests stream under the simulations. CE is estimated by solving the CE value in the following equation:

$$\frac{CE^{1-\rho}}{1-\rho} = \bar{V}(SG) \tag{4.15}$$

In this chapter, six different CE is calculated to estimate renter and owner welfare from their consumption, housing expenditure and bequests and gauge the differences caused by their house tenure arrangements. They are CE for consumption only, CE for housing only, CE for consumption and housing, CE for a house bequest, CE for a superannuation balance bequest and CE for a total bequest. Table 4.3 summarises the lifetime utility formula for different CE.

Table 4.3 Lifetime utility formula for different CE

Certainty Equivalents	**Lifetime utility formula**
Consumption only	$V_1(SG) = \sum_{s=0}^{T} \beta^s \, {}_{s-1}p_{25} \left({}_1p_s \frac{\left[\omega\left(C_s - C_{floor,s}\right)\right]^{1-\rho}}{1-\rho} \right)$
Housing only	$V_2(SG) = \sum_{s=0}^{T} \beta^s \, {}_{s-1}p_{25} \left({}_1p_s \frac{[(1-\omega)H_s]^{1-\rho}}{1-\rho} \right)$
Consumption and housing	$V_3(SG) = \sum_{s=0}^{T} \beta^s \, {}_{s-1}p_{25} \left({}_1p_s \frac{\left[\omega\left(C_s - C_{floor,s}\right) + (1-\omega)H_s\right]^{1-\rho}}{1-\rho} \right)$
House bequest	$\mathrm{V}_4(SG) = \sum_{s=0}^{T} \beta^s \, {}_{s-1}p_{25} \left({}_1q_s \frac{(Bh_s)^{1-\rho}}{1-\rho} \left(\frac{\phi}{1-\phi}\right)^{\rho} \right)$ where Bh_s is the real house value at time s.
Superannuation balance bequest	$\mathrm{V}_5(SG) = \sum_{s=0}^{T} \beta^s \, {}_{s-1}p_{25} \left({}_1q_s \frac{(Bs_s)^{1-\rho}}{1-\rho} \left(\frac{\phi}{1-\phi}\right)^{\rho} \right)$ where Bs_s is the real superannuation balance value at time s.
Total bequest	$\mathrm{V}_6(SG) = \sum_{s=0}^{T} \beta^s \, {}_{s-1}p_{25} \left({}_1q_s \frac{(Bh_s + Bs_s)^{1-\rho}}{1-\rho} \left(\frac{\phi}{1-\phi}\right)^{\rho} \right)$

It may be noticed that the lifetime utility formula for each CE calculation is the decomposition of Equation (4.12) by calculating the utility generated by consumption, housing expenditure and bequests individually. Doing so allows for comparisons between the renter and owner welfare from consumption, housing expenditure and bequests from different assets. However, due to the concavity of utility function, the sum of CE for consumption only and from housing only is not necessarily equal to the CE from consumption and housing.

4.3 Results

Here, the optimal SG estimates across nine income levels and different housing tenures, i.e., renters and owners who save money to purchase a home are now reported. This research first compares the optimal SG estimates using the Chapter 2 model with similar settings in Section 4.3.1, emphasizing the changes of optimal SG under updated circumstances. The optimal SG for renters and owners is then presented under the revised model including housing in Section 4.3.2, followed by the investigation of whether renting or owning generates greater benefit for the individuals in Section 4.3.3. The welfare changes over the life cycle are then alalysed and their welfare quantified using the CE in Section 4.3.4.

4.3.1 Relationship between Chapter 2 and Chapter 4

Before presenting the baseline model results, it is worth first investigating how the optimal Superannuation Guarantee (SG) changes between the model in Chapter 2 and the model in this chapter. This investigation helps us shed light on the variations of the optimal SG with the changing economic circumstances and individual behaviours and helps us to understand the results emerging from the model used in this chapter.

As a quick reminder, this research highlights some assumptions in Chapter 2 that have been revised. The earlier model assumed that the individual owns a house at the age of 25 without a decision on housing tenure. The individual earns a deterministic income and only generates utility from his consumption while alive, i.e., no bequests. The utility functions tested are power utility and reference dependent utility, noting that the power utility is more relevant to this chapter. This research optimises post-retirement consumption and investment asset allocation over their lifetime. The Australian tax and social security rules reflect those in 2018. All the values are in real terms and inflated at the same rate.

This chapter updates the Australian tax and social security rules to 2022. Individual generates utility from consumption and housing expenditure while alive and bequest upon death. Only a variation on the power utility function is used in this Chapter. Unlike the deterministic income in Chapter 2, this research allows for stochastic income, inflation rates and superannuation fund returns.

To reconcile the models and understand their implications the research incorporates the new settings applied in this chapter step by step to the model from Chapter 2. In the first situation, it is assumed that the individual withdraws money from his pension account at minimum withdrawal rates instead of optimizing withdrawals as assumed in Chapter 2. The superannuation fund is assumed to be invested with approximately 70% in a risky asset and 30% in a risk-free asset in line with the previous chapter. instead of optimising. This first situation is called 'Chapter 2'.

In the second situation, the Australian tax and social security rules are updated from 2018 to 2022 for the model in Chapter 2. This second situation is called 'Chapter 2 with updated tax and social security'. In the third situation, in addition to the updated economic circumstance, the research allows for stochastic incomes and inflation rates. This third situation is called 'Chapter 2 with updated tax and social security and stochastic income and inflation rates'.

In the fourth situation, unlike the first three situations where the model used in Chapter 2 is reused, the model from this chapter is applied and the settings adjusted to make them comparable with the first three situations. The research assumes no housing tenure and related costs, e.g., rent expenditure and maintenance costs. The research lets the consumption floor $C_{floor,t}$ pre-retirement in Equation (4.1) equal to 0 and the preference parameter between consumption and housing expenditure ω equal to 1. These adjustments change the utility function for consumption and housing to the power utility function for consumption only in line with the model used in Chapter 2. The utility from bequests is removed by setting $\phi = 0$. The main differences between the fourth situation and the first three situations are the superannuation fund returns. This fourth situation is called 'Chapter 2 redo'.

Table 4.4: Optimal SG in model variants in Chapter 2 and Chapter 4

	Income Percentiles								
Model features	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
Chapter 2	7.0%	8.0%	8.5%	12.0%	18.5%	20.0%	20.0%	20.0%	20.0%
Chapter 2 with updated tax and social security	7.5%	8.0%	9.0%	13.5%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 with updated tax and social security and stochastic income and inflation rates	7.5%	8.0%	9.0%	11.0%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo	7.0%	7.5%	8.5%	19.5%	20.0%	20.0%	20.0%	20.0%	20.0%

An optimal SG of 20.0% means the optimal SG could be more than 20.0%.

Table 4.4 shows the optimal SG for the nine income percentiles in the four situations discussed above. In the first row, 'Chapter 2' situation, the optimal SG increase gradually from the low-income level to the high-income level, ranging from 7.0% for the 10th income percentile to more than 20.0% for the 60th – 90th income percentiles. If this result is compared with the sensitivity analysis with time preference 0.98 in Table 2.8 in Chapter 2, the optimal SG here is overall higher than those in Chapter 2. The reason is quite straightforward. In the ‘Chapter 2’ situation, it is assumed that the investment strategy is fixed and the withdrawal from a pension account follows the minimum withdrawal rates, while in Chapter 2 both are optimised. It again indicates that if the individual can plan optimal financial decisions over their lifetime, a relatively lower SG is optimal and consumption before retirement needs to be less constrained to generate sufficient savings. The application of the minimum drawdown rules, as they act to constrain income below what is affordable.

In the second and third rows, the optimal SG is almost the same as the ‘Chapter 2’ situation, with slight variations for the 40th – 50th income percentiles. These results show that the updated economic circumstance, e.g., the income tax, Age Pension, and the stochastic income and inflation rates have a trivial impact on the optimal SG. Revising these rules does not alter the trend for the SG to increase with income level.

Unlike the gradual increase of optimal SG for the first three situations, the ‘Chapter 2 redo’ situation generates relatively constant SGs at lower-income levels and a large jump at some median income level, usually around the 40th and 50th income percentiles. This new feature may be understood by focusing on the difference between ‘Chapter 2 redo’ and the third situation ‘Chapter 2 with updated economic circumstance and stochastic income and inflation rates’. The main difference between these two situations is superannuation fund returns. In the third situation, it is assumed that the real risky returns follow a normal distribution of 5% mean and 17.4% standard deviation, and the real risk-free rate is 1%. In the ‘Chapter 2 redo’ situation, the research adopts the superannuation fund returns model in the previous chapter, which is t-distribution. It is well-known that the t-distribution has more extreme values leading to more investment risks. This feature helps explain the SG’s jump behaviour.

The model trade-off the sacrifice of pre-retirement consumption in order to save for more post-retirement consumption. For low-income earners, they save less due to the tighter constraint on pre-retirement consumption combined with access to the Age Pension, which provides them with a significant level of post-retirement consumption. For median and high-income earners, compared to the third situation, they save more to hedge the greater investment risks imposed under the ‘Chapter 2 redo’, noting that power utility encourages consumption smoothing and a desire to avoid large reductions in consumption. High SG contributions along with the investment returns help them build sufficient account balances to enhance post-retirement consumption and hence compensate for the loss of utility from decreased pre-retirement consumption. Another reason contributing to this jump is the application of the minimum drawdown rates, which restrict the post-retirement consumption shown in Chapter 2. The individual with a higher income level usually needs a higher optimal SG to achieve consumption smoothing.

To better illustrate the trade-off between the sacrifice of pre-retirement consumption and savings for more post-retirement consumption and its impact on the utility calculations, plot the expected utility over the life cycle is plotted with an SG of 8.5% and 19.5% for both the 30th and 40th income percentiles, noting that 8.5% and 19.5% are the optimal SGs for the 30th and 40th income percentiles respectively in ‘Chapter 2 redo’ situation. Figure 4.2 and Figure 4.3 shows the comparison of mean utility at 8.5% and 19.5% SG for the 30th and 40th income percentiles respectively.

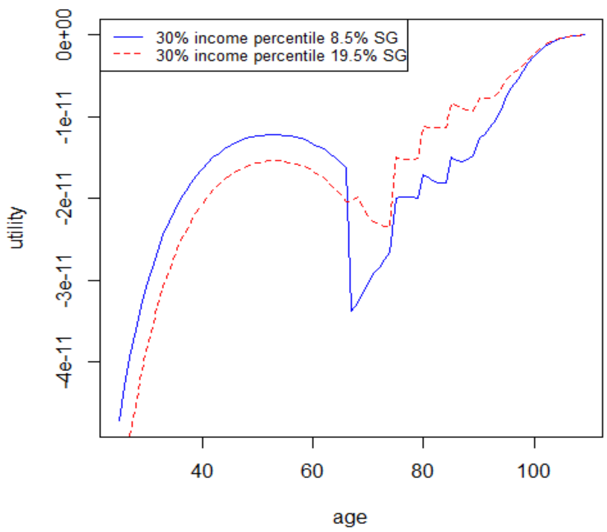

Figure 4.2: Comparison of utility for 30% income percentile with 8.5% and 19.5% SG

While the figure exhibits the same trend and shape for the mean utility curves of two different SG levels, the 8.5% SG generates higher utility than the 19.5% SG pre-retirement and lower utility post-retirement. This illustrates how a higher SG amounts to saving more for post-retirement consumption while sacrificing consumption and hence utility pre-retirement.

The hump-shape utility curves pre-retirement in both cases are due to the hump-shape nature of an individual’s age-earnings profile as discussed in Section 4.2.3. The large decrease in utility at retirement is due to the drop in post-retirement consumption relative to pre-retirement consumption. The sawtooth-shaped increase post-retirement is the combined effect of increasing minimum withdrawal rates and the compound effects of discounting utility at the 0.98 time preference.

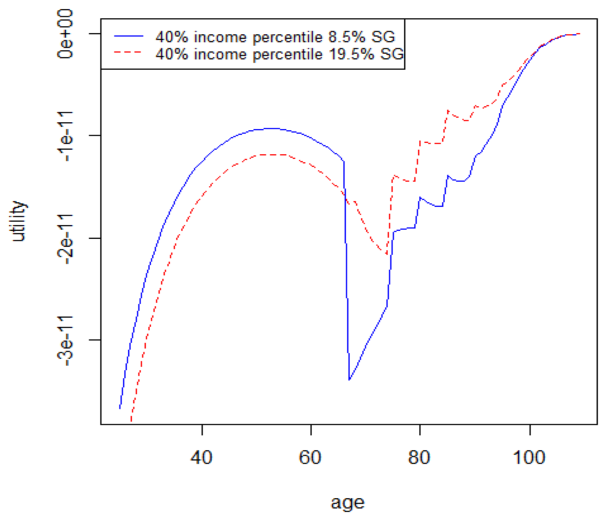

Figure 4.3: Comparison of utility for 40% income percentile with 8.5% and 19.5% SG

Visually, the utility difference between 8.5% and 19.5% SG for both income percentiles is almost the same before retirement, while the utility difference for the 40th income percentile in Figure 4.3 is significantly larger than that for the 30th income percentile post-retirement, especially in the years immediately post-retirement. This is mainly due to the much higher SG for the 40th income percentile, which when combined with investment returns accumulated over the work period, leaves them with a much larger balance at retirement. The evaluation of the utility loss from the pre-retirement consumption sacrifice against the resulting higher post-retirement consumption will determine the optimal SG across income levels. The combination of different income levels and access to the Age Pension combine with the high-risk nature of new superannuation fund returns stemming from t-distribution impact the trade-off and contributes to the individuals in the example having quite different retirement saving attitudes. For low-income earners, a relatively low optimal SG can secure higher pre-retirement consumption and avoid the investment risks, in part due to being able to rely on the Age Pension for consumption smoothing. For the median and high income earners, a high optimal SG helps them hedge against investment risks and take advantage of compound investment returns to boost post-retirement consumption. It is shown below that when the assumptions change, the jump in optimal SG occurs at differing income percentiles. These results can be explained using the trade-off discussed here.

Table 4.5: Optimal SG when adding new features in utility function in 'Chapter 2 redo'

	Income Percentiles								
Model features	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
Chapter 2 redo	7.0%	7.5%	8.5%	19.5%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo 0.9 omega	6.5%	7.0%	7.0%	16.0%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo maintenance	7.5%	8.0%	15.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo 0.5 phi	7.0%	7.5%	8.5%	19.5%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo with all 3	7.0%	7.5%	8.0%	18.5%	20.0%	20.0%	20.0%	20.0%	20.0%

An optimal SG of 20.0% means the optimal SG could be more than 20.0%.

Table 4.5 allows for the utility function form to gradually transition from those used in Chapter 2 to those applied in this chapter, with analysis based on the 'Chapter 2 redo' situation. The first row of Table 4.5 shows the 'Chapter 2 redo' optimal SG. From the second row to the fourth row, one feature is added at a time. A preference between consumption and housing expenditure with a parameter of $\omega = 0.9$ is added in the second row, house maintenance costs of 0.5% are added in the third row; bequest motive with $\phi = 0.5$ in the fourth row. In the last row, all three features are added together, and called 'Chapter 2 redo with all 3'.

The 'Chapter 2 redo 0.9 omega' situation shows a slight decrease in optimal SG compared to the 'Chapter 2 redo'. Consumption only counts for 90% rather than 100% in utility calculations, with the remaining 10% comprising housing expenditure that does not affect the cash flows in this setting. This dilutes the benefit of saving to cover the post-retirement consumption, so the individual saves a little less than in the 'Chapter 2 redo' situation.

In the 'Chapter 2 redo maintenance' situation, consumption is reduced by the house maintenance costs. To cover these extra costs post-retirement, individuals need to save much more than before. The optimal SG jump consequently occurs at a lower income percentile.

In the 'Chapter 2 redo 0.5 phi' situation, adding the bequest utility does not affect the cashflows, which are the same overall SG levels for any income percentile, as $\phi = 0.5$ leads to a minimal bequest utility value. The bequest utility is further downweighed by the mortality and time preference. Hence the same optimal SG as in the 'Chapter 2 redo' situation is seen.

In the bottom row where all three features are combined, the optimal SG drops a little at the 30th and 40th income percentiles but otherwise remains unchanged. These results can be seen as a combination of the large increase from the maintenance costs case and the small decrease from the 0.9 omega case, which keeps the jump that occurs at the 40th income percentile, the same as the 'Chapter 2 redo' case.

Table 4.6: Optimal SG with different consumption floors in 'Chapter 2 redo with all 3'

	Income Percentiles								
Model features	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
Chapter 2 redo with all 3	7.0%	7.5%	8.0%	18.5%	20.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo with all 3 cf 0.1	6.0%	6.5%	7.0%	7.5%	18.0%	20.0%	20.0%	20.0%	20.0%
Chapter 2 redo with all 3 cf 0.2	5.0%	5.5%	6.0%	6.0%	7.0%	17.0%	19.5%	20.0%	20.0%
Chapter 2 redo with all 3 cf 0.3	3.5%	4.5%	5.0%	5.0%	5.5%	6.0%	15.5%	18.0%	20.0%

An optimal SG of 20.0% means the optimal SG could be more than 20.0%.

Table 4.6 starts from 'Chapter 2 redo with all 3' and shows the optimal SG when allowing for a consumption floor $C_{floor,t}$, moving from 0% to 10%, 20% and then 30% of post-tax income at age t before retirement. The first result from this table is that the optimal SG decreases as the consumption floor increases. For instance, at the 30th income percentile, the optimal SG moved from 8.0% at a 0% consumption floor to 5.0% at a 30% consumption floor. Another result is that the large jump in the optimal SG occurs at higher income percentiles as the consumption floor increases. The jump moves from the 40th income percentile when the consumption floor is 0% to the 70th income percentile when the consumption floor is 30%. Both results can be explained by the argument of the trade-off between the sacrifice of pre-retirement consumption and saving for more post-retirement consumption. The consumption floor increases the curvature of the utility function pre-retirement for a given level of consumption relative to the no-consumption floor case. It means a heavier utility penalty for sacrificing pre-retirement consumption to save, resulting in lower optimal SGs. Individuals lower SG levels increase pre-retirement consumption such that the sacrifice of pre-retirement utility is minimised. On the other hand, increasing SG means a greater sacrifice of pre-retirement utility when a consumption floor exists. It requires higher post-retirement consumption from higher SG along with investment returns to compensate for this utility loss. Some median income percentiles that have high optimal SG in 0% consumption floor cases can no longer compensate for this pre-retirement utility loss in positive consumption floor situations with higher SG levels and turn to low optimal SG where they can secure higher pre-retirement consumptions and avoid the investment risks. For the high-income percentiles, a high optimal SG can still help them hedge the investment risks and take advantage of compound investment returns to achieve higher post-retirement consumption. This helps explain the jump of optimal SG moving to a higher income percentile when a consumption floor appears.

4.3.2 Optimal superannuation guarantee for owner and renter

The last section showed how the optimal Superannuation Guarantee (SG) changes from the model in Chapter 2 to the model in this Chapter. The decisions on housing tenure are now added, namely renting a house, or saving money to purchase a house and investigate the optimal superannuation guarantee for a renter and a house owner.

Table 4.7: Optimal SG for renter and owner

	Income Percentiles								
House choice	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
renter	5.5%	7.0%	8.0%	8.5%	8.5%	9.5%	9.5%	13.0%	14.0%
owner	0.5%	0.5%	0.5%	0.5%	0.5%	1.0%	1.0%	1.5%	2.0%

0.5% means the optimal SG is less than or equal to 0.5%, while 20.0% means the optimal SG is more than or equal to 20.0%.

Table 4.7 summarises the optimal SG for renters and owners across the 10th to 90th income percentiles. The first finding is that both renter and owner optimal SG increase with the income levels. The renter optimal SG moves from 5.5% for the 10th income percentile to 14.0% for the 90th income percentile. The owner optimal SG sits at the minimum of 0.5% until the 50th percentile, then moves up to 2.0% by the 90th percentile. The second finding is the large difference in optimal SG between renter and owner across income levels. This difference increases with income level, moving from 5.0% at the 10th income percentile to 12.0% at the 90th income percentile.

The renter benefits from not saving for the upfront payment to purchase the house pre-retirement. After paying his rent, the rest of his post-tax income can be used to consume or save via the SG. In addition, due to the assumption of no mortality until retirement at the age of 67, there is also no bequest utility generated pre-retirement. Hence, all utility pre-retirement is from consumption and housing expenditure (rents), which makes the renter's life before the age of 67 comfortable and simple. After he retires, he relies on savings in the superannuation account along with any Age Pension to cover consumption and rent. Moreover, he may want to enhance his bequest utility, which comes from his remaining balance in the pension account. Substantial savings via the SG is required to achieve this under the set-up, resulting in higher optimal SG for the renter that in turn increase with income as the benefit from the Age Pension becomes less relevant. This higher optimal SG is further encouraged for the high-income percentiles since they can benefit from lower tax rates and compound superannuation investment returns.

The owner, at an early age, needs to pay rent and sacrifice consumption to save for the upfront payment for the house. After purchasing the house, rent payments cease but mortgage repayments and house maintenance costs begin. Once the mortgage is repaid, he benefits from being able to consume all income less the maintenance cost, plus the house supports a bequest once he passes away post-retirement. His superannuation contributions along with his Age Pension are used for post-retirement consumption and maintenance costs. Under the utility assumptions (see Section 4.2.2), the preference parameter between consumption and housing expenditure of $\omega = 0.9$ means that around 90% of utility is generated from pre-retirement consumption and housing expenditure only accounts for 10%. In addition, the compound effects of the time preference of 0.98 and mortality rates post-retirement depreciate the utility from consumption, housing, and bequests post-retirement. A low optimal SG maximises utility due to the combined effect of reducing the consumption sacrifice pre-retirement, the house acting to provide bequest utility post-retirement, and reliance on the Age Pension to help cover his consumption.

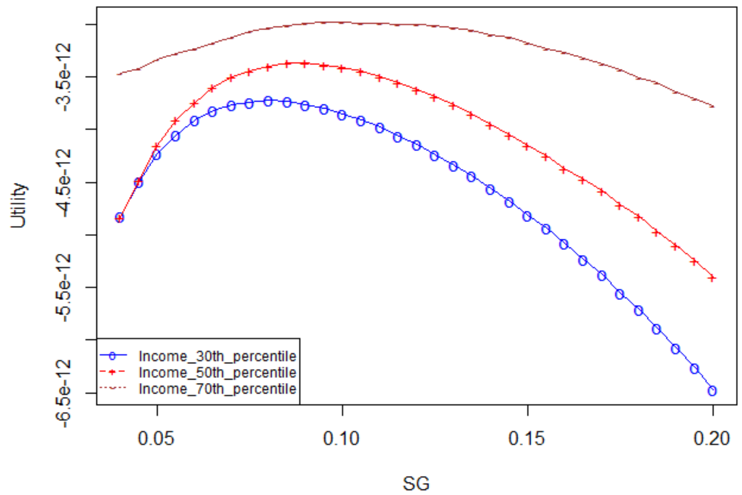

Figure 4.4: Renter expected lifetime utility across SG at the 30th, 50th, 70th income percentiles

Figure 4.4 uses renters at the 30th, 50th and 70th income percentiles as examples to illustrate how the renter expected lifetime utility changes with SG. The relative flat curve at the top is for the 70th income percentile, followed by the curve for the 50th income percentile in the middle and the curve for the 30th income percentile at the bottom. It indicates that higher income generally brings renters a higher benefitmeasured by the expected lifetime utility regardless of SG level. The exception appears at the very low SG level for the 30th and 50th income percentiles where their total expected utility is almost the same, indicating that with the Age Pension, renters with a low income percentile can enjoy a comparable life to those with median income percentile if they do not save too much via SG. The curves for the 30th and 50th income percentiles start to increase sharply with SG from almost the same level until they reach their peaks at the SG of 8.0% and 8.5% respectively. The SG levels at their peaks represent the optimal SG levels in Table 4.7. The curves then drop with SG at a slower speed until the SG reaches its maximum at 20.0%. Their change pattern indicates that renters with low- and medium-income percentiles benefit from the large utility increase when the SG increases from a low level to their optimal SG, while they also suffer a large utility decrease when the SG increases from their optimal level to an even higher level. Since their optimal SG is both less than 9.0%, the current SG of 10.5% and the proposed SG of 12.0% lead to a large loss of benefits for the low- and median-income renters. For the renter with a 70th income percentile, even though his curve follows a similar trend to that of the 30th and 50th income percentiles, his curve remains relatively flat. This indicates that the variations of SG do not affect the high-income renter too much, noting that his total expected utility keeps almost the same when the SG move from his optimal level of 9.5% to the current SG of 10.5% and the proposed SG of 12.0%.

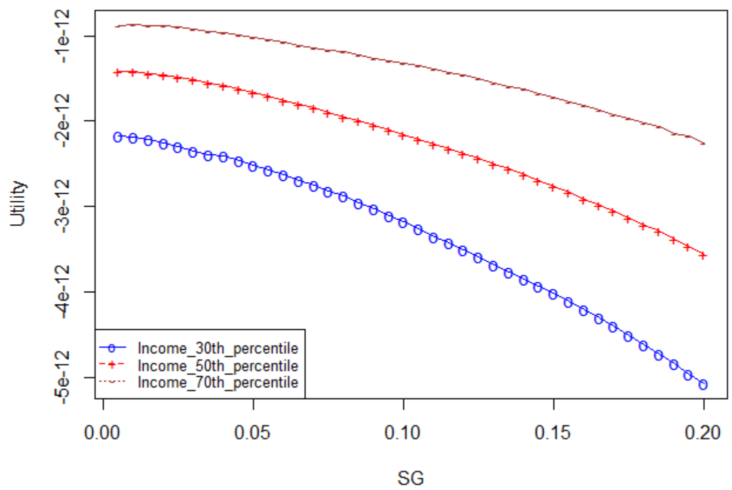

Figure 4.5: Owner expected lifetime utility across SG at the 30th, 50th, 70th income percentiles

Figure 4.5 illustrates the owner expected lifetime utility variations with SG for the 30th, 50th and 70th income percentiles. The three curves exhibit a similar decreasing trend with the increase of SG, with the curve of the 70th income percentile at the top, the curve of the 50th income percentile at the middle and the curve of the 30th income percentile at the bottom, indicating that owner with a higher income also enjoys a higher benefitrelative to the owner with a lower income. Since the optimal SG for owners are very low, approaching 0.5% for most of the income percentiles, any increase in SG leads to the loss of utility for all the income percentiles. It is the low-income percentile that experiences a larger utility loss, while the high-income percentile's utility loss is relatively small. When the SG increases from the current 10.5% to the proposed 12.0%, the low-income owners suffer a large loss of benefits, while the increase of SG has a limited impact on the high-income owners.

The results above show that the optimal SGs for most renters and owners across all income percentiles are lower than the current SG of 10.5% and the proposed SG of 12.0%. An SG such as 12.0% is required only for high-income individuals who rent. Low-income renters can rely on the Age Pension to an extent, while owners can rely on their home plus the Age Pension under the assumption that the policy of the Age Pension stays the same for a relatively long time. It indicates the SG of 12.0% only suits a portion of people; a high SG plus tax benefits for homeowners is a dubious combination.

4.3.3 Who achieves a higher lifetime utility, the renter or the owner?

After finding the optimal SG for renter and owner across the income levels, the research also investigates for a certain level of income, whether renting throughout an individual's lifetime or purchasing a house is better off. By comparing the lifetime utility between the renter and owner, the research summarises who achieves a higher lifetime utility in Table 4A.1 shown in Appendix 4A.

Table 4A.1 notes the housing tenure, the renter or the owner, that generates a higher utility across nine income percentiles and the forty SG levels. Surprisingly, the owner is always the winner no matter what the income percentile and the SG level are. It indicates that an individual who purchases a house always enjoys a higher satisfaction than those who rent throughout their lifetime if they evaluate their consumptions, housing expenditure, and bequest in the same way as the baseline assumptions. The owner may need to sacrifice his consumption at the early stage to save for the upfront payment for purchasing the house. His sacrifice brings more pleasures in his later life in terms of more funds to consume, less housing expenditure, and more assets for the bequest.

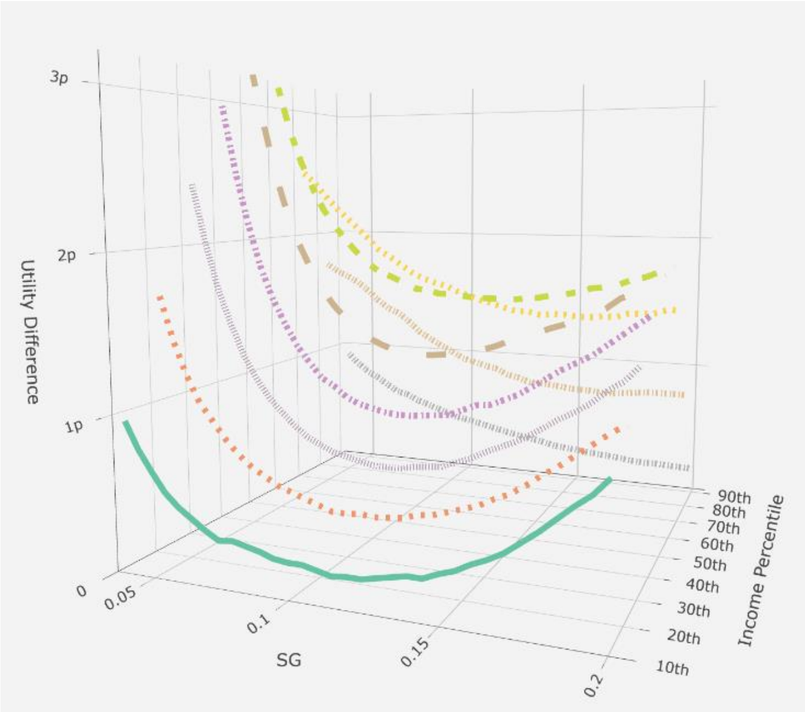

Figure 4.6: Owner expected lifetime utility difference over the renter across income percentiles and SG levels[28]

To further investigate the lifetime utility difference between the renter and owner, Figure 4.6 calculates the owner lifetime utility difference over the renter across the nine income percentiles and the forty SG levels. There are in total nine curves representing the lifetime utility difference for the nine income percentiles. Each curve calculates the owner utility difference over the renter for the forty SG levels from 0.5% to 20.0% for a certain income percentile. Since the owner always achieves a higher lifetime utility, the utility differences are all positive. For the 10th-60th income percentile, the curves exhibit a similar shape. The utility difference is at its highest point at SG of 0.5%, then starts to drop sharply to its lowest level at the SG of 11.0%-13.0% and increases until the SG reaches 20.0%. It indicates that for the low- and median-income percentiles, the lifetime benefit difference between owner and renter is minimal when the SG is between the current 10.5% and the proposed 12.0% since these SG levels can help the renter build sufficient funds for post-retirement and delay an owner saving for a house purchase. This lifetime benefit difference becomes larger for a low and high SG level, noting that a low SG prevents renter to save for retirement while a high SG level is a constraint of their pre-retirement consumption. For the 70th-90th income percentiles, the curves keep decreasing and reach their lowest point at the SG of 20.0%, indicating that for a high-income percentile,

[28] In ‘Utility Difference’ axis, ‘p’ represents 10^{-12}.

the lifetime benefit difference between renter and owner is minimal when they save at the highest SG level tested of 20.0%. Similar to the low- and median-income percentiles, an owner who has a home enjoys a far more favourable life than a renter when they do not save for retirement.

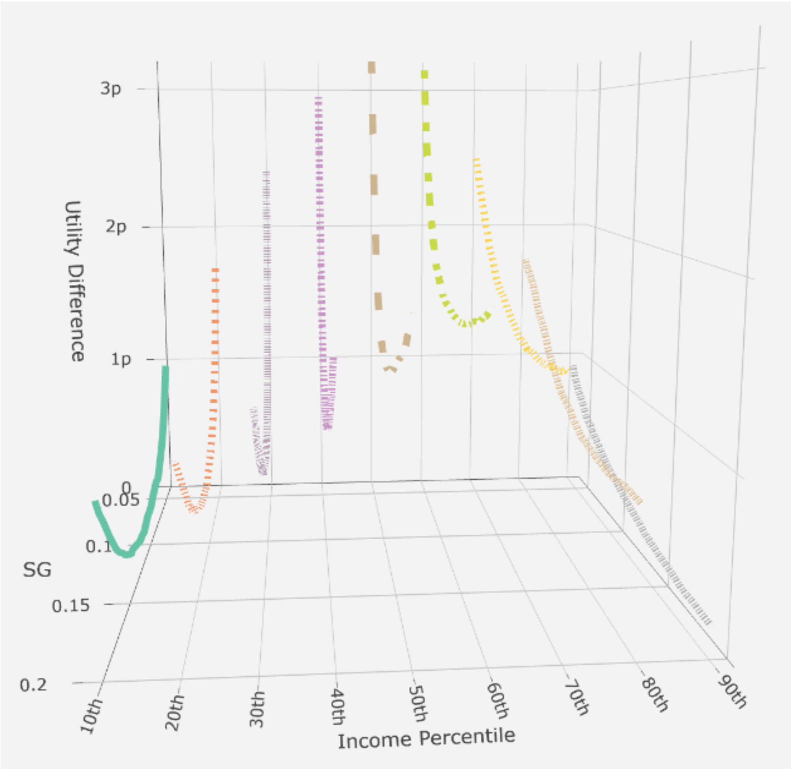

Figure 4.7: Another angle of Figure 4.6

Figure 4.7 shows another angle of Figure 4.6, comparing the utility differences across all income percentiles. The curves increase from the 10th-60th income percentiles and then decrease to lower levels for the 70th-90th income percentile. It is the 60th income percentile that has the largest utility difference. This pattern indicates that individuals with the median income percentile should seriously consider purchasing a home since home ownership brings them a much higher benefitwhen measured by utility. While for the low- and high-income percentiles, since the gain from home ownership is relatively small, the individuals can add in their preference when they consider house tenure choices.

4.3.4 Welfare changes for renter and owner over the life cycle

In this section, the welfare changes over the life cycle at the 50th income percentile for the renter and owner are examined. This study helps us better understand how the SG and housing tenure interact to affect individual welfare.

4.3.4.1 Utility difference: the optimal SG vs the current SG, renter vs owner

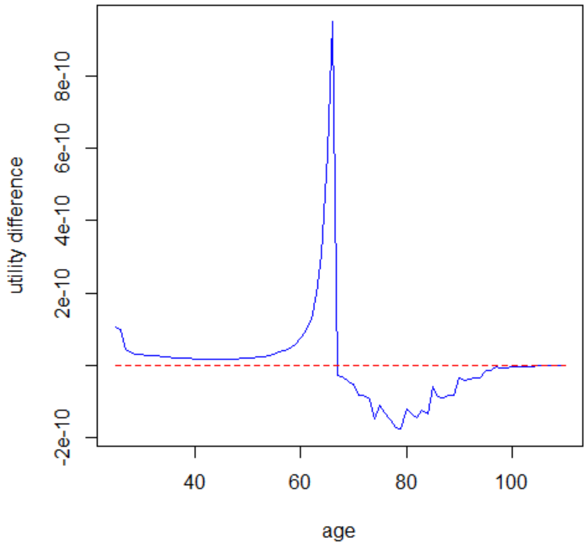

Figure 4.8: Utility difference for median income renter: 8.5% SG over 10.5% SG

Figure 4.8 plots the difference in utility for the median income renter at the optimal SG of 8.5% relative to the current SG of 10.5%. A positive difference means a utility gain, while a negative difference means a utility loss.

The optimal SG yields utility gains throughout the pre-retirement phase compared to the current SG, and a utility loss after retirement. It is easy to understand this pattern. The optimal SG is lower than the current SG, which results in less saving and greater pre-retirement consumption. It then changes to utility loss since the higher current SG combined with investment returns generates a higher pension balance and hence post-retirement consumption. Taking a closer look at Figure 4.8, the pre-retirement utility gain shows a slight drop at first as precautionary savings are built, and then maintains a relatively low level until surging after age 60. The utility loss after retirement grows to its largest value at age 80 and then decreases to close to zero after age 100 with irregular bumps. The jump at age 60 reflects the hump-shaped characteristics of real income combined with consumption being powered by $(1-\rho)$ under the utility form, which magnifies the utility difference. After retirement, the positive pre-retirement utility difference becomes negative. The bumps are caused by the mixed effect of the minimum drawdown rules and the different downsizing moments due to financial constraints.

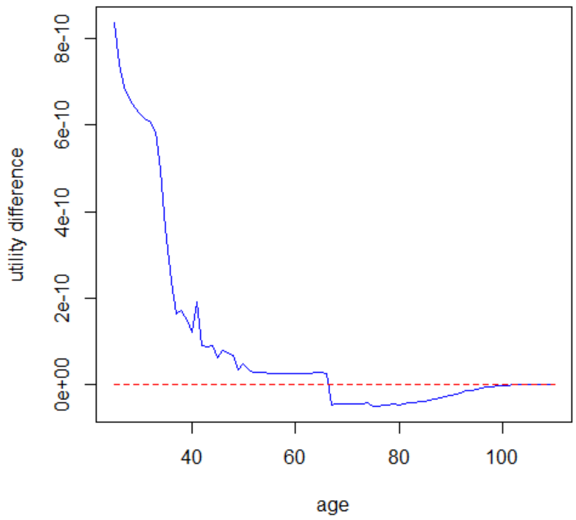

Figure 4.9: Utility difference for median income owner: 0.5% SG over 10.5% SG

Figure 4.9 shows the utility difference for the median income owner at the optimal SG of 0.5% over the current SG of 10.5%.

As for the renter, the lower optimal SG brings utility gains throughout the pre-retirement phase compared to the current SG, which turns to utility loss after retirement. The pre-retirement utility gain decreases sharply from age 25 to age 37, tapering off at age 34. From age 25-34, the owner rents the house, saves for an upfront payment, and consumes. In this phase, the difference in utility associated with the 0.5% SG versus 10.5% SG reduces as income increases, while a house purchase changes in the reduction speed from age of 34-37. After age 37, the utility gain keeps reducing until age 50 (with some bumps due to the different mortgage repayment amounts[29]), then maintains a low level from the age 50-66. Post retirement, utility loss relates to the lower superannuation balance under optimal SG. This utility difference becomes smaller over time, gradually trending to zero at age 100.

Similar to Figure 4.8 and Figure 4.9, the median income percentile is used as an example and show the owner's utility gain/loss over the renter to investigate how the housing tenure

[29] The mortgage repayment amount decreases with the reduction of outstanding mortgage loan. The different SG levels results in the different time to purchase the house and the different mortgage loans, which leads to a difference between the mortgage repayment amounts.

impacts the individual's utility over his life. It also illustrates the trade-off between the sacrifice of pre-retirement consumption and savings for more post-retirement consumption and bequest under the baseline assumptions.

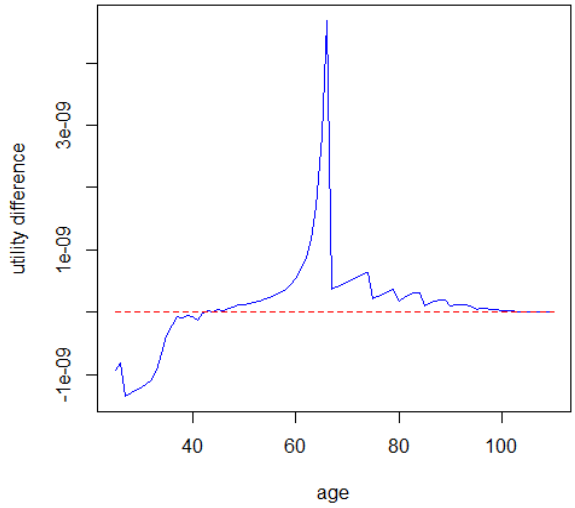

Figure 4.10: Utility difference for median income percentile at 10.5% SG: owner over renter

Figure 4.10 shows the owner's utility gain at the current SG of 10.5% over the renter for the 50th income percentile. It is calculated as the owner's expected utility minus the renter's expected utility at the current SG of 10.5%. When this difference is positive, it means the utility gain for the owner, while it is the owner's utility loss at the negative difference.

The curve in Figure 4.10 first experiences negative values from the age of 25 to 35 increasing to zero, indicating the renter achieves a higher utility than the owner. The large bump at the age of 25-27 is due to the different saving behaviours between the renter and owner. The renter saves 10% of his post-tax income to build the precautionary saving, while the owner saves 15% to build both the precautionary savings and upfront payment to purchase the house. Once the renter achieves $10,000 precautionary saving, he does not save anymore, this helps explain the large bump around the age of 27. The negative difference keeps becoming smaller from age of 27-35 to reflect the increase in income. After the owner purchases the house at age of 35, he does not pay rent any longer. Instead, he only needs to repay the mortgage. From age of 35-48, the utility difference can be seen as close to zero, which indicates that the mortgage repayment and the rent are almost the same. It becomes a utility gain at the age of 48 and surges to its highest levels right before retirement. During this life stage, the owner's mortgage loan is reduced, which leads to higher consumption for the owner, while the renter still needs to pay rent, restricting his consumption. After retirement, with the joining of a bequest utility, the owner's utility is always larger than that of the renter since they have the same superannuation balance, while the owner only pays a lower amount for the maintenance cost than the rent paid by the renter. The utility gain decreases to zero with bumps after retirement, where the bumps are mainly due to the minimum withdrawal rules and the compound discount effect of the time preference helping to explain the convergence to zero.

4.3.4.2 Real consumption, housing expenditure and bequest: renter vs owner

Figure 4.11-4.14 compare the real consumption, housing expenditure and bequest between renter and owner for the 50th income percentile at the current SG of 10.5%, illustrating their different consumption and bequest patterns due to the housing tenure choices[30]. All the values are in real terms by deflating nominal values at each age back to the values at age 25 using the CPI.

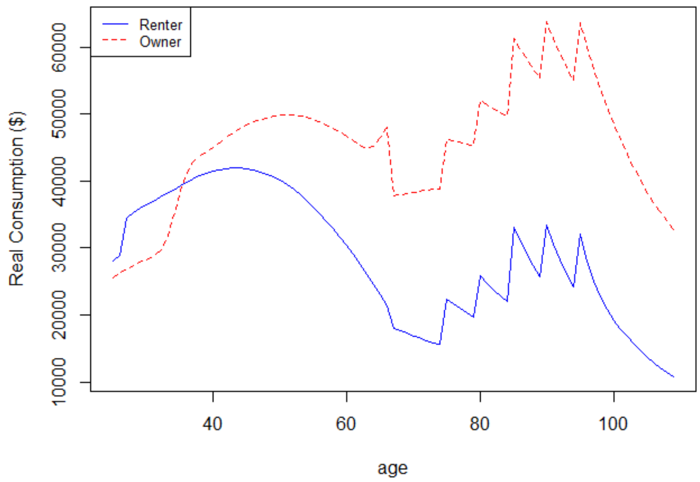

Figure 4.11: Real consumption for median income percentile at 10.5% SG: renter vs owner

Figure 4.11 shows the real consumption of the renter and owner throughout their lifetime for the median income percentile at the current SG of 10.5%. Both the renter and owner pay rent from the age of 25 to 35 before the owner purchases his home. The renter enjoys higher consumption than the owner due to the different saving behaviours, where the renter saves 10% of his post-tax income for precautionary savings while the owner saves 15% for both precautionary savings and an upfront payment to purchase a house. The large increase of renter consumption at the age of 25-27 reflects the fact that the renter finishes building their

[30] Please see Appendix 4C for comparisons of renter and owner both being at optimal SG.

precautionary savings and consume all post-tax income after paying rent. This situation changes after the age of 35 when the owner purchases a house and no longer pays rent. The owner's consumption increases faster than the renter's and overtakes the renter at around the age of 37. The owner then enjoys higher consumption than the renter until retirement as the owner only pays the mortgage and the maintenance cost while the renter still pays rent. The hump-shaped consumption for both renter and owner comes from the hump-shaped income profile (Freestone 2018). The increase in owner consumption right before retirement indicates that the mortgage loan is fully paid back. After retirement, the consumption patterns for the renter and owner are the same with the owner consumption level being higher than the renter, noting that the renter and the owner have the same superannuation withdrawal and therefore a similar Age Pension. The higher owner post-retirement consumption is mainly due to lower maintenance costs, while the renter must pay rent. The sawtooth-shaped consumption curve comes from the increasing withdrawal rates in the minimum withdrawal rules that alter renter and owner consumption patterns. It is also unusual that owner post-retirement consumption is higher than pre-retirement consumption (See Olafsson and Pagel 2018), indicating that the current SG is more a constraint on owner pre-retirement consumption than a benefit.

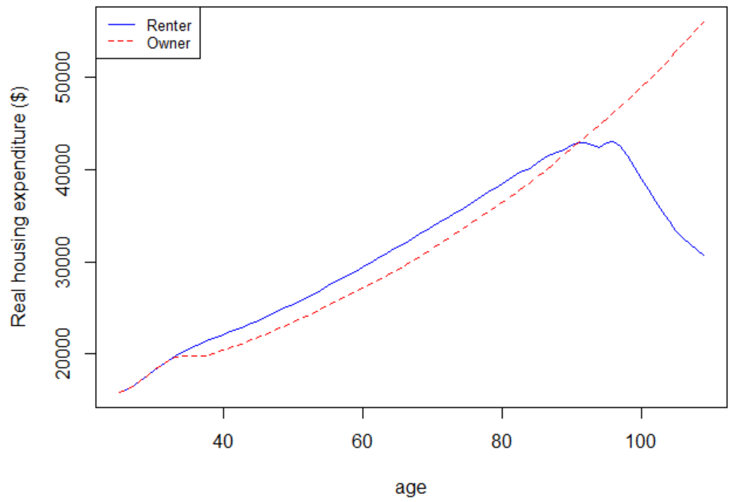

Figure 4.12: Real housing expenditure for median income percentile at 10.5% SG: renter vs owner

Figure 4.12 illustrates the real housing expenditure of the renter and owner throughout their lifetime for the median income percentile at the current SG of 10.5%. Their trends are relatively simple. Both curves are the same from the age of 25-35 since the renter and the owner pay the same rent. After the owner purchases the house, although both curves increase at a similar rate until the age of 90, the renter's housing expenditure is slightly larger than the owner's since the imputed rent less the maintenance cost is used for the owner housing expenditure. After the age of 90, due post-retirement financial constraints, the renter must downsize his home, leading to a decrease in housing expenditure, while the owner's housing expenditure keeps increasing until he passes away.

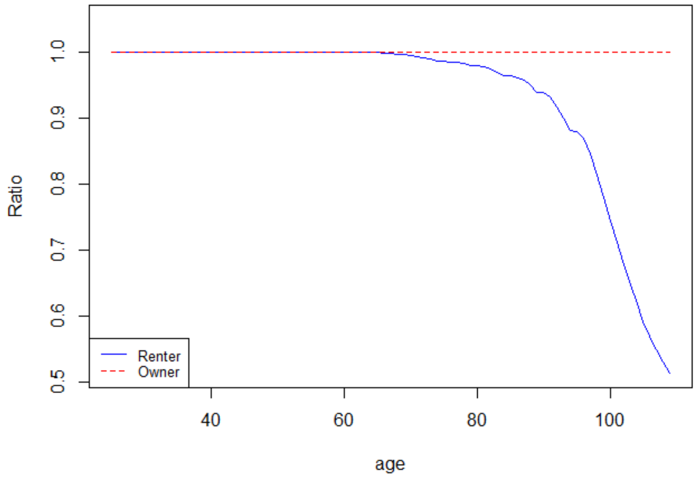

Figure 4.13: Ratio of the average house value and the initial house value: renter vs owner

Figure 4.13 shows the ratio of the average house value at any age and the initial house value for the renter and the owner over their lifetime. It facilitates an understanding of the renter and owner housing expenditure patterns in Figure 4.12. Before retirement, both renter and owner keep living in the initial house as their income can help them enjoy the benefits of the affordable house for as long as possible. The owner keeps living in the same house post-retirement, while the renter starts to downsize his house for some simulation paths and this process expands to almost all the simulation paths when he passes away.

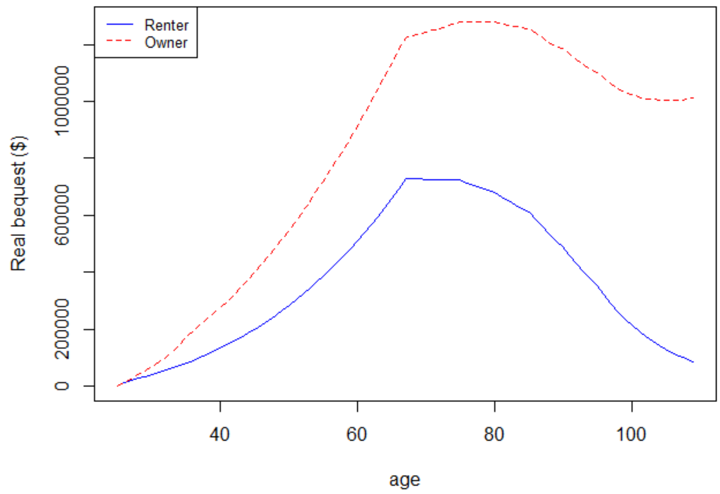

Figure 4.14: Real bequest for median income percentile at 10.5% SG: renter vs owner

Figure 4.14 illustrates the real bequest of the renter and the owner throughout their lifetime for the median income percentile at the current SG of 10.5%. Both bequest curves continue increasing pre-retirement, with the owner's bequest being always higher than the renter's, noting that the owner saves with higher rates before the house purchase and the house value less the mortgage loan is added into the bequest after the house purchase. The bequest difference becomes larger with the increase of the house value and the paying back of the mortgage loan. At retirement, the owner's real bequest is around $1,200,000, while the renter's bequest is only $700,000. The renter's bequest gradually decreases post-retirement due to withdrawal from the superannuation balance, while the owner's bequest keeps increasing until the age of 80 and then decreases post retirement. The owner bequest changes post-retirement are mainly due to the trade-off between the increase in house value and the withdrawals from the superannuation balance.

The results above show that housing, being the most important component of voluntary savings, does significantly increase the owner's lifetime benefits in terms of higher consumption and bequests compared to the renter, especially in the post-retirement phase. Homeownership not only allows the owner to pay lower maintenance costs instead of paying higher rent, hence providing for higher consumption, it also delivers the house value on top of the remaining superannuation balance as the bequests.

4.3.4.3 Certainty Equivalents: renter vs owner

Table 4.8 Certainty Equivalents for median income percentile at 10.5% SG

	Certainty Equivalent					
House choice	**Consumption only**	**Housing only**	**Consumption and housing**	**House bequest**	**Superannuation balance bequest**	**Total bequest**
renter	$277	$675	$4,590	$0	$154,759	$154,759
owner	$738	$650	$5,322	$931,370	$152,891	$1,558,634

Table 4.8 calculates the six CE for the renter and owner with a median income percentile at 10.5% SG using the method in Section 4.2.14. It shows that there exists a large difference between renter and owner CE for consumption only, where owner CE is $738 being more than

2 times of renter CE of $277[31]. It indicates that although the owner may need to sacrifice his consumption at the early stage, he enjoys an overall higher level of consumption than the renter by saving the rent after his house purchase. Owner CE for housing of $650 is slightly less than renter CE of $675 since the imputed rent is applied to the owner after the house purchase by deducting the maintenance cost. The difference in the first two CE helps explain the CE for consumption and housing in the third column. The owner enjoys higher consumption and housing expenditure than the renter, where the owner's CE for consumption and housing is $5,322 and the renter's is $4,590. Moving to the bequest, since the renter never purchases a house, his CE for the house bequest is defined as $0, while the owner's CE for the house bequest reaches $931,370. It indicates that even though the research chooses a relatively small strength of bequest motive $\phi = 0.5$ and account for the discount effect of the time preference, purchasing a house contributes an extremely large proportion of lifetime utility. CE for a superannuation balance bequest does not see any difference between renter and owner, being around $154,000. The reason is that both follow the same SG of 10.5% contribution rate and the minimum withdrawal rules. The slight difference comes from the owner behaviour of using the superannuation balance to pay back the remaining mortgage at retirement. The difference in CE for the total bequest between renter and owner is also significant due to the house bequest; the renter CE for the total bequest is $154,759, while the owner's reaches $1.5 million.

The CE analysis in Table 4.8 identifies the welfare difference in consumption, housing expenditure and bequest between the renter and The owner. The owner enjoys a much higher consumption coming from the rent savings. Purchasing a house also brings a significant increase in the bequest, leading to the owner enjoying higher welfare than the renter.

[31] The values are relatively small as a positive pre-retirement consumption floor in the utility calculation is set.

4.4 Sensitivity analysis

The model and its parameters are varied to evaluate the significance and direction of the impacts on the optimal SG and the variations of individual welfare. The utility parameters, housing assumptions and economic and social environment are examined[32].

4.4.1 Utility function parameters

Table 4.9 reports the change in the optimal SG case under alternative utility parameters. A positive number means that the revised optimal SG is larger than the baseline optimal SG, and vice versa.

[32] The individual welfare variations only apply to housing assumptions and social environment sensitivity analysis since the utility parameters do not change individual consumption, housing expenditure and bequests.

Table 4.9: Sensitivity analysis of optimal SG for utility function parameters

	Income Percentile								
Objective	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
BASELINE OPTIMAL SG									
Renter	5.5%	7.0%	8.0%	8.5%	8.5%	9.5%	9.5%	13.0%	14.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.5%	2.0%
REVISED OPTIMAL SG									
Lower CRRA $\rho = 3$									
Renter	5.5%	7.0%	8.0%	8.5%	8.5%	8.5%	9.5%	10.5%	12.5%
Owner	0.5%	0.5%	0.5%	1.0%	1.0%	1.5%	1.5%	2.0%	2.0%
Higher CRRA $\rho = 5$									
Renter	5.5%	7.0%	8.0%	8.0%	8.5%	8.5%	9.5%	13.0%	14.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.5%
$\omega = 0.5$									
Renter	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	8.0%	14.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.0%	1.5%
$\phi = 0.96$									
Renter	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
Owner	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	1.5%	1.5%	2.0%
$\beta = 0.96$									
Renter	4.0%	5.5%	6.5%	7.5%	7.5%	8.0%	8.5%	10.5%	12.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.0%
$\beta = 1$									
Renter	7.0%	8.5%	9.5%	9.5%	10.0%	11.0%	12.0%	13.5%	15.5%
Owner	0.5%	0.5%	0.5%	0.5%	1.5%	1.5%	2.0%	2.5%	3.0%
Lower consumption Floor									
Renter	8.0%	10.0%	11.5%	13.0%	14.5%	18.0%	19.0%	19.5%	20.0%
Owner	0.5%	0.5%	1.0%	2.5%	2.0%	2.0%	2.5%	2.5%	2.5%
CHANGE vs. BASELINE SETTING									
Lower CRRA $\rho = 3$									
Renter	..	..	..	..	..	-1.0%	..	-2.5%	-1.5%
Owner	**	**	**	0.5%	0.5%	0.5%	0.5%	0.5%	..
Higher CRRA $\rho = 5$									
Renter	..	..	..	-0.5%	..	-1.0%	..	..	..
Owner	**	**	**	**	**	-0.5%	-0.5%	-1.0%	-0.5%
$\omega = 0.5$									
Renter	-2.0%	-3.0%	-3.5%	-3.5%	-3.0%	-3.5%	-3.0%	-5.0%	..
Owner	**	**	**	**	**	-0.5%	-0.5%	-0.5%	-0.5%
$\phi = 0.96$									
Renter	14.5%	13.0%	12.0%	11.5%	11.5%	10.5%	10.5%	7.0%	6.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	..	0.5%	..	..
$\beta = 0.96$									
Renter	-1.5%	-1.5%	-1.5%	-1.0%	-1.0%	-1.5%	-1.0%	-2.5%	-2.0%
Owner	**	**	**	**	**	-0.5%	-0.5%	-1.0%	-1.0%
$\beta = 1$									
Renter	1.5%	1.5%	1.5%	1.0%	1.5%	1.5%	2.5%	0.5%	1.5%
Owner	**	**	0.5%	2.0%	1.5%	1.0%	1.0%	0.5%	0.5%
Lower consumption Floor									
Renter	2.5%	3.0%	3.5%	4.5%	6.0%	8.5%	9.5%	6.5%	6.0%
Owner	**	**	0.5%	2.0%	1.5%	1.0%	1.5%	1.0%	0.5%

0.5% means the optimal SG is less than or equal to 0.5%, while 20.0% means the optimal SG is more than or equal to 20.0%.

'**' indicates both baseline and revised optimal SG are at the boundaries of SG range, 0.5% or 20.0%, i.e., model captures no change

'..' indicates there is no change.

A higher CRRA of $\rho = 5$ and a lower CRRA $\rho = 3$ are compared to the baseline case of $\rho = 4$. The changes are modest at best, with no change often observed. A lower CRRA reduces the optimal SG for renters with high-income levels by around 1.5%, as they save less money to hedge against the uncertainties from post-retirement consumption, housing expenditure, and bequests. The optimal SG for the owner with high-income increases by around 0.5%. A higher CRRA leads to a 0.5% and 1.0% optimal SG decrease for renters at the 40th and 60th income percentiles, and an 0.5% optimal SG decline for owners from the 60th to 90th income percentiles. Otherwise, no other changes are observed.

For the preference parameter between consumption and housing expenditure, the research tests $\omega = 0.5$ instead of 0.9, consistent with consumption and housing expenditure being equally important. Renters experience a significant decrease of 2.0%-5.0% in the optimal SG across all the income levels except for the 90th income percentile. Increasing the weight of house expenditure relative to consumption in the utility calculation leads the renter to consume more pre-retirement and save less for the post-retirement phase, noting that the pre-retirement housing expenditure is the same as before but increases the utility. It allows the renters to sacrifice the uncertain post-retirement utility increase due to the mortality hazard and discount effect of time preference and reallocate their income to pre-retirement consumption to compensate for its reduced weight. For the owner, the optimal SG changes for the 60th to 90th income percentiles, where a decrease of 0.5% is seen. This can be explained by the fact that the owner wants to purchase the house earlier and save the rent to consume or pay back the mortgage.

Increasing the strength of the bequest motive from $\phi = 0.5$ to $\phi = 0.96$ convert the bequest motive from incidental (i.e., valued the same as consumption) to strong. This boosts the optimal SG for the renter to 20.0% across all income levels, as they desire to save as much as possible to build up a bequest as well as fund post-retirement consumption, noting that saving through the superannuation system is the only way for the renter to leave a bequest under this set-up. For the owner, the change in the strength of the bequest motive does not greatly affect the optimal SG. The 10th to 70th income level sees a 0.5% increase in optimal SG.

The research also reduces the consumption floor $C_{floor,t}$ from 30% to 20% of post-tax income pre-retirement. This is similar to Table 4.6, which suggests that a lower consumption floor induces a higher SG as the reduction of utility function curvature leads to more pre-retirement consumption being added to utility calculations. Consistent with these results, a consumption floor of 20%, leads to a significant increase in optimal SG for the renter, especially at the median and high-income levels, where the optimal SG increase by 6.0%-9.5%. The owner optimal SG increase more moderately by 1.5% on average.

The time preference parameter is vairied $\beta = 0.98$ to $\beta = 0.96$ and $\beta = 1$. A parameter of $\beta = 0.96$ has the effect of reducing the utility weight on outcomes occurring later in the lifecycle, while $\beta = 1$ means the individual cherishes current and future outcomes equally. For $\beta = 0.96$, the optimal SG for the renter decreases by 1.0%-2.5% to reflect the fact that the consumption in the future is further discounted, so that the renter prefers to consume more pre-retirement and save less for the post-retirement phase. The owner sees a decrease in optimal SG of 0.5% in the 60th to 70th income levels and 1.0% in the 80th to 90th income levels, noting that their optimal SG was already at the minimum of 0.5% at lower income percentiles. For $\beta = 1$, the opposite trend is seen, with both the renter and owner increasing their optimal SG by 0.5%-2.5% as the utility discount on future outcomes is decreased.

4.4.2 Housing assumptions

The housing assumption changes made vary the house in which the renter and owner live and their financial decisions related to a house purchase, which in turn affect owner saving and mortgage repayment patterns and hence cash flows. Table 4.10 summarises the revised optimal SG under alternative housing assumptions and how they change compared to the baseline case, while Table 4.11 investigate median income earner welfare variations at SG of 10.5% by showing the revised CE and the changes compared to the baseline case.

Table 4.10: Sensitivity analysis of optimal SG for housing assumptions

	Income Percentile								
Objective	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
BASELINE OPTIMAL SG									
Renter	5.5%	7.0%	8.0%	8.5%	8.5%	9.5%	9.5%	13.0%	14.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.5%	2.0%
REVISED OPTIMAL SG									
50% house price									
Renter	4.5%	6.5%	7.5%	9.0%	10.5%	12.5%	16.0%	20.0%	20.0%
Owner	0.5%	1.0%	1.0%	1.5%	2.0%	2.0%	2.5%	2.5%	2.5%
100% house price									
Renter	3.0%	3.0%	3.0%	4.0%	4.5%	5.5%	6.5%	7.0%	7.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%
First Home Loan Deposit Scheme									
Renter	5.0%	7.0%	8.5%	9.5%					
Owner	0.5%	0.5%	0.5%	1.0%					
10% saving rate for owner									
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	1.5%	1.5%	2.0%	3.0%
20% saving rate for owner									
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.0%	2.0%
20 years mortgage loan term									
Renter	5.0%	6.5%	8.0%	9.5%	11.0%	12.5%	17.5%	20.0%	20.0%
Owner	0.5%	0.5%	0.5%	0.5%	2.0%	2.0%	2.5%	2.5%	3.0%
CHANGE vs. BASELINE SETTING									
50% house price									
Renter	-1.0%	-0.5%	-0.5%	0.5%	2.0%	3.0%	6.5%	7.0%	6.0%
Owner	**	0.5%	0.5%	1.0%	1.5%	1.0%	1.5%	1.0%	0.5%
100% house price									
Renter	-2.5%	-4.0%	-5.0%	-4.5%	-4.0%	-4.0%	-3.0%	-6.0%	-7.0%
Owner	**	**	**	**	**	-0.5%	-0.5%	-1.0%	-1.5%
First Home Loan Deposit Scheme									
Renter	-0.5%	..	0.5%	1.0%					
Owner	**	**	**	0.5%					
10% saving rate for owner									
Owner	**	**	**	**	**	0.5%	0.5%	0.5%	1.0%
20% saving rate for owner									
Owner	**	**	**	**	**	-0.5%	-0.5%	-0.5%	..
20 years mortgage loan term									
Renter	-0.5%	-0.5%	..	1.0%	2.5%	3.0%	8.0%	7.0%	6.0%
Owner	**	**	0.5%	1.0%	1.5%	1.0%	1.0%	1.0%	0.5%

0.5% means the optimal SG is less than or equal to 0.5%, while 20.0% means the optimal SG is more than or equal to 20.0%.

'**' indicates both baseline and revised optimal SG are at the boundaries of SG range, 0.5% or 20.0%, i.e., model captures no change

'..' indicates there is no change.

The baseline case assumes that the house price that they aim to purchase is equal to 70% of their maximum borrowing capacity, which is in line with RBA (2018) that more than two-thirds of households borrowed less than 70 per cent of their maximum loan size. The research tests how the optimal SG and welfare change when the owner purchases a house using 50% and 100% of their maximum borrowing capacity. These alternative settings lead to changes in rent, mortgage loan size, the age of house purchases and bequest values.

Decreasing the house price to 50% of the maximum borrowing capacity generates mixed effects on renter optimal SG. At the low-income levels of the 10th-30th percentile, the optimal SG decrease 0.5%-1.0% since the reduction of price relieves their rent pressure during the post-retirement phase. They can hence consume more pre-retirement, with the Age Pension sufficient to support most of their post-retirement consumption and housing expenditure. At the 40th-90th income percentiles, the renter optimal SG increases by 0.5%-7.0%. This can be explained by the reduction in their rent pressure. However, they also save more to generate income from their superannuation account on top of the Age pension to maintain their median to high post-retirement consumption levels. For owners, a reduction in house prices facilitates the house purchase, allowing their optimal SG for most of the income levels to increase by around 1% on average.

In terms of renter and owner welfare, a lower house price leads to a boost of consumption by an increase of 300% for the renter and 806% for the owner, while these large increases come along with a relatively modest decrease of 29% for housing. Combined with their impacts, the CE for consumption and housing increases by 49% for the renter and 36% for the owner. It indicates that a lower house price relieves their rent and mortgage payment pressure, resulting in an increase in their utility from consumption and housing expenditure. The lower house price also reduces the owner's house bequest by 29%, leading to a decrease of 20% for the owner's total bequest. There are nearly no changes in CE from the superannuation balance bequest for both renter and owner, noting that the owner's 1% increase is mainly due to the reduction in mortgage repayments by using the superannuation balance at retirement.

Table 4.11: Sensitivity analysis of CE for housing assumptions

Objective	Certainty Equivalent					
	Consumption only	Housing only	Consumption and housing	House bequest	Superannuation balance bequest	Total bequest
BASELINE OPTIMAL SG						
Renter	$277	$675	$4,590	$0	$154,759	$154,759
Owner	$738	$650	$5,322	$931,370	$152,891	$1,558,634
REVISED OPTIMAL SG						
50% house price						
Renter	$1,105	$484	$6,817	$0	$154,653	$154,653
Owner	$6,683	$461	$7,222	$665,265	$154,646	$1,248,266
100% house price						
Renter	$134	$807	$3,391	$0	$154,653	$154,653
Owner	$179	$896	$2,750	$1,330,529	$109,718	$1,970,523
10% saving rate for owner						
Owner	$642	$657	$5,723	$931,370	$130,219	$1,525,753
20% saving rate for owner						
Owner	$741	$645	$4,717	$931,370	$154,636	$1,562,328
20 years mortgage loan term						
Renter	$958	$517	$6,458	$0	$154,653	$154,653
Owner	$2,159	$494	$6,854	$711,805	$154,653	$1,304,410
CHANGE vs. BASELINE SETTING						
50% house price						
Renter	299%	-28%	49%	0%	0%	0%
Owner	806%	-29%	36%	-29%	1%	-20%
100% house price						
Renter	-52%	19%	-26%	0%	0%	0%
Owner	-76%	38%	-48%	43%	-28%	26%
10% saving rate for owner						
Owner	-13%	1%	8%	0%	-15%	-2%
20% saving rate for owner						
Owner	0%	-1%	-11%	0%	1%	0%
20 years mortgage loan term						
Renter	246%	-23%	41%	0%	0%	0%
Owner	193%	-24%	29%	-24%	1%	-16%

When the house price increases to 100% of the individual's maximum borrowing capacity, the optimal SG for both the renter and owner decreases. The renter optimal SG is lower by a significant 2.5%-7.0%. This reflects the renter suffering from higher rent pressure pre-retirement, leading them to prefer a lower SG level at the cost of a decrease in their post-tax income for consumption and rent. The fact that post-retirement utility is discounted by the time preference encourages them to boost their pre-retirement utility and give up some of their post-retirement utility. For the owner, the optimal SG drops to the minimum 0.5% at the 60th to 90th income percentiles, recalling that it was already at 0.5% in the baseline case for the other income percentiles. The renter and owner CE show the opposite trends compared to the house price of 50% of maximum borrowing capacity. The CE from consumption only reduces by 52% for the renter and 76% for the owner, while a higher house price leads to an increase in CE for housing only for the renter and owner by 19% and 38% respectively. However, if the consumption and housing expenditure is evaluated, a higher house price decreases them by 26% for the renter and 48% for the owner. It indicates that using 100% of the individual's maximum borrowing capacity harms the renter and owner's overall consumption and housing, leading to a decrease in utility from consumption and housing. In return, the owner CE from house bequest and from total bequest increase by 43% and 26% respectively.

The First Home Loan Deposit Scheme[33] is an Australian Government initiative to support eligible first-home buyers to purchase their first home sooner. Usually, first-home buyers with less than a 20% deposit need to pay lenders' mortgage insurance. Under the scheme, eligible first home buyers can purchase a modest home with a deposit of as little as 5%, with the Government guaranteeing to participating lenders up to 15% of the value of the property. Since the Government imposes a house price cap under which it will assist in the purchase, the table only presents the revised optimal SG for 10th-40th income percentiles. There are no changes to the optimal SG from the minimum 0.5% level except for the 40th income percentile which jump to 1.0%, noting it remains optimal for the owner to limit saving and try to purchase the house sooner. However, the average age of house purchase under the current SG level of 10.5% decreased significantly from the age of 34 in the baseline case to the age of 29 (see Table 4B.2 in Appendix 4B). The reduction of 6 years is in line with the Scheme's goal.

The owner's saving rate is also varied to 10% and 20% versus the baseline of 15% to test how the optimal SG changes when less or more savings are directed at building an upfront payment. A post-tax income saving rate of 10% is in line with the Australian historical average saving rate (ABS 2022a), while 20% is the Australian saving rate observed in 2020. The revised saving rates only lead to a slight change in optimal SG. A lower saving rate increases the optimal

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[33] For more details see NHFIC 'First Home Loan Deposit Scheme Fact Sheet', https://www.nhfic.gov.au/media/1684/first-home-loan-deposit-scheme-fact-sheet-19-june-2021.pdf.

SG for the 60th-90th income percentiles by 0.5%, while a higher saving rate reduces the optimal SG for the 60th-80th income percentiles by 0.5%. The changes in saving rates do not necessarily alter the desire in the owner to save as little as possible and purchase the house as soon as they can. Table 4B.3 and Table 4B.4 in Appendix 4B reveal that a lower saving rate delays the purchase to age 40.2, while a higher saving rate pushes the average purchase age to 32.8. The logic is simple: the more you save every year, the earlier you can purchase a house.

In terms of welfare, the changes in saving rates lead to modest variations in owner CE. The changes that are worth mentioning in the 10% saving rate are the decrease of CE for consumption only by 13% and CE for superannuation balance bequest by 15%. The reduction of consumption is mainly due to a longer rent-paying period before house purchase that restricts consumption, while the decrease in superannuation balance is from the mortgage repayment behaviour using superannuation balance at retirement.

The mortgage loan term is assumed to be 30 years in the baseline case, which is generally the longest available term in Australia. It is assumed a shorter loan term of 20 years, to test the impact of the reduction of loan term on the optimal SG. This reduction leads to a decrease in the house price the owner can purchase under the initial house price calculation outlined in Section 4.2.4. It also changes the mortgage repayment amounts and hence alters the utility patterns pre-retirement. Similar to 50% of the house price test, the impact on the renter's optimal SG is mixed. For the low-income levels of the 10th-20th percentiles, the optimal SG decreases by 0.5% due to lower rent, which the Age Pension can cover more readily in the post-retirement period. For the renter with 40th-90th income percentiles, their optimal SG increase by 1.0%-8.0%. After benefiting from the reduction in rent, they have more room to save to maintain median-to-high consumption post-retirement on top of Age Pension. The owner has more room to save due to the reduction in house prices, which leads to their optimal SG for most of the income levels increasing by around 1.0% on average. In addition, Table 4B.5 in Appendix 4B shows that the average age of purchase is reduced to 32 due to lower house prices. For the changes of welfare, CE at a shorter mortgage loan term mimics the situation of lower house prices, with a large increase in CE for consumption only and a decrease in CE for housing only for renter and owner along with a decrease in the owner's house bequest.

4.4.3 Changes of economic and social environment

This research investigates the impact of the removal of the means-tested Age Pension and related supplements, lower and higher investment returns, as well as AIA 'rule of thumb' withdrawal rates[34] (see De Ravin et al. 2019). Table 4.12 presents the revised optimal SG estimates along with the changes from the baseline estimates, while Table 4.13 presents the revised CE and its changes compared to the baseline case.

Table 4.12: Sensitivity analysis of optimal SG for changes of economic and social environment

	Income Percentile								
Objective	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
BASELINE OPTIMAL SG									
Renter	5.5%	7.0%	8.0%	8.5%	8.5%	9.5%	9.5%	13.0%	14.0%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	1.5%	2.0%
REVISED OPTIMAL SG									
No Age Pension									
Renter	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
Owner	13.0%	12.5%	13.0%	12.0%	11.5%	11.0%	11.5%	10.5%	10.0%
Lower investment returns									
Renter	8.0%	9.5%	10.5%	10.5%	11.0%	10.5%	11.5%	13.0%	14.0%
Owner	0.5%	0.5%	0.5%	0.5%	2.0%	2.0%	2.5%	3.0%	3.0%
Higher investment returns									
Renter	4.0%	5.0%	6.0%	6.5%	7.0%	7.5%	9.0%	10.5%	12.0%
Owner	0.5%	1.0%	1.0%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
AIA drawdown rules									
Renter	6.0%	7.0%	7.0%	8.5%	9.0%	9.0%	9.0%	8.5%	15.5%
Owner	0.5%	0.5%	0.5%	0.5%	0.5%	1.0%	1.0%	1.5%	2.0%
CHANGE vs. BASELINE SETTING									
No Age Pension									
Renter	14.5%	13.0%	12.0%	11.5%	11.5%	10.5%	10.5%	7.0%	6.0%
Owner	12.5%	12.0%	12.5%	11.5%	11.0%	10.0%	10.5%	9.0%	8.0%
Lower investment returns									
Renter	2.5%	2.5%	2.5%	2.0%	2.5%	1.0%	2.0%	**	**
Owner	**	**	**	**	1.5%	1.0%	1.5%	1.5%	1.0%
Higher investment returns									
Renter	-1.5%	-2.0%	-2.0%	-2.0%	-1.5%	-2.0%	-0.5%	-2.5%	-2.0%
Owner	**	0.5%	0.5%	1.0%	1.0%	0.5%	0.5%	0.0%	-0.5%
AIA drawdown rules									
Renter	0.5%	0.0%	-1.0%	0.0%	0.5%	-0.5%	-0.5%	-4.5%	1.5%
Owner	**	**	**	**	**	..	..	..	..

0.5% means the optimal SG is less than or equal to 0.5%, while 20.0% means the optimal SG is more than or equal to 20.0%.

'**' indicates both baseline and revised optimal SG are at the boundaries of SG range, 0.5% or 20.0%, i.e., model captures no change

'..' indicates there is no change.

[34] For more details see Actuaries Digital 'Spend your decimal age: A rule of thumb for retirement', https://www.actuaries.digital/2019/07/17/spend-your-decennial-age-a-rule-of-thumb-for-retirement/.

Excluding the Age Pension and supplements amounts to a situation where individuals become self-funded retirees, requiring them to optimise their savings via a superannuation mechanism to ensure they have enough funds to sustain consumption during retirement. This leads to large increases in optimal SG. The optimal SG for the renters are all above 20.0%, amounting to minimum increases ranging from 14.5% for the 10th income percentile to 6.0% for the 90th income percentile. The owner-optimal SGs exhibit the same trend. Instead of close to 0.5% in the baseline estimates, the optimal SG for the owner are all above 10.0%, ranging from 13.0% for the lowest income earner to 10.0% for the highest income earner. Low-income earners see the largest optimal SG increase, which confirms the role of the Age Pension as a crucial component of retirement income in the baseline setting for low-income earners in particular. This finding is also in line with Poterba (2014) that social security is the primary source of retirement income for most individuals. For the CE, the renter suffers a decrease in consumption and housing, with decreases of 15% in consumption, 20% in housing and 44% for consumption and housing. Due to the exclusion of the Age Pension, the renter can only use his superannuation balance to cover his post-retirement consumption and rent, leading him to consume less and move to a smaller house post-retirement. While for the owner, since he does not need to pay the rent post retirement, he only suffers a slight decrease of 3% in consumption. In addition, he can downsize his home and generate additional funds in his pension account if he cannot afford maintenance costs. It also helps explain the increase in his superannuation balance bequest.

Table 4.13: Sensitivity analysis of CE for changes of economic and social environment

Objective	Certainty Equivalents Consumption only	Housing only	Consumption and housing	House bequest	Superannuation balance bequest	Total bequest
BASELINE OPTIMAL SG						
Renter	$277	$675	$4,590	$0	$154,759	$154,759
Owner	$738	$650	$5,322	$931,370	$152,891	$1,558,634
REVISED OPTIMAL SG						
No Age Pension						
Renter	$234	$539	$2,549	$0	$154,759	$154,759
Owner	$716	$650	$5,175	$928,713	$214,725	$1,553,179
Lower investment returns						
Renter	$260	$672	$4,251	$0	$60,072	$60,072
Owner	$738	$650	$5,312	$931,370	$59,063	$1,321,334
Higher investment returns						
Renter	$286	$676	$4,807	$0	$374,767	$374,767
Owner	$738	$650	$5,337	$931,370	$371,545	$1,966,666
AIA drawdown rules						
Renter	$269	$674	$4,626	$0	$87,599	$87,599
Owner	$738	$650	$5,337	$931,370	$86,534	$1,368,035
CHANGE vs. BASELINE SETTING						
No Age Pension						
Renter	-15%	-20%	-44%	0%	0%	0%
Owner	-3%	0%	-3%	0%	40%	0%
Lower investment returns						
Renter	-6%	0%	-7%	0%	-61%	-61%
Owner	0%	0%	0%	0%	-61%	-15%
Higher investment returns						
Renter	3%	0%	5%	0%	142%	142%
Owner	0%	0%	0%	0%	143%	26%
AIA drawdown rules						
Renter	-3%	0%	1%	0%	-43%	-43%
Owner	0%	0%	0%	0%	-43%	-12%

The research tests 1.5% per annum lower and higher superannuation fund returns. The impact on the renter optimal SG is moderate, with lower fund returns leading to an increase of 2.0% across the income percentiles and higher returns resulting in a decrease by approximately the same 2.0%. The result indicates that the renter is sensitive to return variations as they take advantage of this compounding to accumulate retirement savings on top of the Age Pension. The impact of superannuation fund return changes on owner optimal SG is relatively small, where only some income levels respond to these changes by around 1.0%. The owner still tries to use a very low SG level to purchase the house earlier to decrease the sacrifice of pre-retirement consumption, as discussed in Section 4.3.2. In addition, noting that their optimal SG is low, the changes in fund returns do not significantly influence their superannuation fund growth.

For renter and owner welfare, the lower and higher superannuation fund returns have a large impact on their superannuation balance bequest, leading to a 61% decrease in the lower fund returns and a 142% increase in the higher fund returns in their superannuation balance bequest. The impact on their consumption is relatively small, only resulting in a 6% of decrease and a 3% increase in renter consumption and no changes for the owner. The result reinforces the compounding effect of the superannuation fund returns on renter and owner superannuation fund growth. The variations in the fund returns can significantly affect the accumulation of their superannuation balances, while the discounted effect of the time preference on the post-retirement consumption mitigates the impact of the fund returns on the CE from consumption.

The research tests an alternative drawdown rule (AIA drawdown rule) from the Rule of Thumb working group at the Actuaries Institute (see De Ravin et al. 2019), where the group developed a simple rule of thumb that is easy to apply for the Australian retirees. The drawdown rate consists of two parts: a baseline drawdown rate that is equal to the first digit of the individual's age (e.g., age 67 then 6%; age 70s then 7% etc) and an additional 2% to the baseline drawdown rate if the individual's real account balance is between $250,000 and $500,000[35]. This rule is also subject to the minimum drawdown rules and is triggered after age 85 at 9%.

The impact on the renter optimal SG is moderate, with a decrease of around 1.0% for the 20th to 80th income percentiles and an increase of 0.5% and 1.5% for the 10th and 90th income percentiles. The result indicates that the simple rule of thumb drawdown rates reduces the individual's saving rates by optimising their retirement consumption pattern, increasing their drawdown rates at their early stage of post-retirement when the utility is less discounted by the time preference and depending more on the Age Pension at the late stage of post-retirement. The increase of optimal SG for the 10th and 90th income percentiles can be treated as a re-allocation of their consumption in this higher drawdown rate for the extremely low- and high-income percentiles. The owner's optimal SG keeps the same, due to their low initial optimal SG and their intention to purchase their home sooner pre-retirement.

[35] The two account balance values are derived based on the Age Pension asset test thresholds. This research updates the thresholds to $270,500 and $599,750 by referring to the Age Pension asset test in 2022.

The AIA drawdown rules do not cause large variations of consumption and housing in terms of CE, with a 3% decrease in renter consumption and a 1% increase in renter consumption and housing. Its impact on owner consumption and housing is trivial with changes of less than 0.5% that cannot be reflected in the result. On the other hand, the larger withdrawal rates in AIA rules decrease the superannuation balance bequest by around 43% for both renter and owner, with only a 12% decrease for the owner total bequest.

Table 4.14 shows the comparison of mean utility at the current SG of 10.5% for applying the minimum drawdown rules and the AIA drawdown rules. For the renters, the minimum drawdown rules bring a higher lifetime utility for the 10th-40th and 90th income percentile, while the AIA rules are more favourable for the 50th-80th income percentile. The main trade-off in this situation is whether the renter enjoys higher post-retirement consumption under the AIA rates or leaves larger bequests under the minimum drawdown rates. For the low-income renters, their post-retirement consumption relies more on the Age Pension since their superannuation balance is relatively low, also noting that their superannuation balance is the only source for their bequests. These features lead them to prefer the minimum drawdown rates to keep enough assets for bequests. For the high-income renters, since their superannuation balance is high, the minimum drawdown rules with lower withdrawal rates enable them to balance their post-retirement consumption and bequests. On the other hand, the AIA rules dominate the owners across the income percentiles. Since their house constitutes a large proportion of their bequests, the owners apply the larger drawdown rates from the AIA rules to boost their consumption during the early post-retirement period and depend on the Age Pension at the late stage of post-retirement, noting that the consumption at the late stage of post-retirement would be discounted by the low survival probability and the time preference.

Table 4.14: Comparison of utility for drawdown rules at current SG of 10.5%

	Income Percentiles								
House choice	**10th**	**20th**	**30th**	**40th**	**50th**	**60th**	**70th**	**80th**	**90th**
renter	Min	Min	Min	Min	AIA	AIA	AIA	AIA	Min
owner	AIA	AIA	AIA	AIA	AIA	AIA	AIA	AIA	AIA

‘Min’ means the minimum drawdown rules achieve higher utility, while ‘AIA means AIA drawdown rules achieve higher utility.

4.5 Conclusion

A stochastic life-cycle model is used to simulate the individuals' financial outcomes with two types of house tenures, renting and owning. The model trades off the utility benefit from lower pre-retirement consumption due to saving for either purchasing a house or building up a retirement account, against the utility gains from higher post-retirement consumption. The 'optimal' SG is identified for both renters and owners and the welfare of the different house tenure arrangements is compared. The analysis considers the benefits from consumption, housing expenditure, and bequests by calculating the utility that they generate. The individuals are exposed to risks related to inflation, income, interest rates, investment return, rents, and house prices. The full range of tax, mortgage loans, and social security rules in Australia are considered.

The research finds that the optimal SG for both renter and owner increases with income level, with the renter optimal SG moving from 5.5% for the 10th income percentile to 14.0% for the 90th income percentile, while the owner optimal SG moves from 0.5% up to 2.0%. The research finds that there exists a large difference in the optimal SG between the renter and owner across all income levels, with the renter optimal SG being significantly larger than owner optimal SG due to a greater need to save to cover consumption and rent post-retirement. The findings reinforce the result of Chapter 2 that there is no single 'one-size-fits-all' optimal SG that suits all individuals. They also indicate that purchasing a home can significantly reduce the savings required for retirement through the superannuation system. The research poses the question of whether purchasing a home can perfectly replace the mandatory saving schemes in the presence of the publicly funded Age Pension, given the extremely low optimal SG found for the homeowner.

This research compares renter and owner lifetime utilities arising from consumption, housing expenditure (i.e., services) and bequests. The research finds that the owner enjoys higher benefits across all income and all SG levels tested, which range from 0.5% to 20.0%. The owner's higher utility mainly comes from an overall higher consumption level and a bequest from his house than the renter. This result indicates that purchasing a home is always beneficial for the individual. Given the extremely low optimal SG found for the homeowner, the policyholders should further think about whether the planned increase of SG to 12.0% truly helps the individuals in the retirement planning phase. However, the research emphasises that this conclusion is drawn under the assumptions that the individuals choose an affordable home that is in line with their income and borrowing capacity and that they value their consumption, housing expenditure, and bequest in the same way as the utility function settings.

The results also indicate that assuming no access to the Age Pension leads to most optimal SG estimates exceeding 10%. The optimal SG for the renter is all above 20.0%. The owner optimal SG also increase markedly, moving from close to 0.5% in the baseline estimates to above 10.0% and ranging from 13.0% for the lowest income earner to 10.0% for the highest income earner. Low-income earners experience the largest increase in optimal SG, reflecting the fact that they receive the greatest relative benefit from the Age Pension as a substitute for private savings. The analysis confirms the important role of a publicly funded pension as a retirement income source and highlights a particular significance for Australian retirees with modest income and savings.

The research indicates that the renter with low and medium income are sensitive to the SG changes. The current and the proposed SG of 10.5% and 12% lead to a large decrease of their lifetime consumptions. They are also the most vulnerable people during their retirement phase due to their double burdens from rent and consumption. The government should increase the rent assistance or design other social welfare to assist these people. In the comparison between renter's and owner's utilities, it is the median income earners that have the largest utility difference, while the low- and high-income earners' difference is not significant. We hence propose the median income earners should consider purchasing a home for the sake of their lifetime consumption and bequest. The current SG definitely does not help the owner at all due to the constraint of their pre-retirement consumptions and deferring their house purchase age. The government should evaluate the interactions of the Superannuation Guarantee and the house purchase behaviours by introducing more related policies, like First home super saver scheme[36], to help the individuals purchase their house more easily and with the help of SG.

The research indicates that in the interactions of the Age Pension, the Superannuation Guarantee and the house tenure arrangements, the Age Pension is a key retirement income source for Australian retirees with modest income and savings. The current and the proposed SG of 10.5% and 12% respectively can be justified as optimal for the owner and is far less than optimal for the renter in the absence of the Age Pension. This raises the question of whether the

[36] Please see https://www.ato.gov.au/Individuals/Super/Withdrawing-and-using-your-super/First-Home-Super-Saver-Scheme/ for more details.

aim of the SG system is to replace the Age Pension, requiring individuals to become self-funded retirees. The current SG of 10.5% also restricts the owner's savings for home purchase and his mortgage payments pre-retirement and limitedly contributes to the owner's post-retirement consumption, noting that the Age Pension constitutes a large proportion of his post-retirement consumption, furthermore he can also downsize his house to solve the liquidity issue. In addition, these favourable features of home ownership always allow the owner to enjoy higher benefits than the renter.

In terms of limitations, the most noticeable one is due to the application of the cascade model in Chapter 3. Since the simulations of future values depend on the current and historical values, the cascade model can not be applied to the dynamic programming process to generate the optimal consumption and asset allocation decisions similar to Chapter 2. We failed to investigate the optimal consumption and the optimal decisions on house purchases like the best purchase age, the optimal mortgage loan amount and repayments, etc. In our set-up, the different levels of SG lead to differing post-tax income and hence to the different house affordability. Our sensitivity analysis of different house prices touch this in some extent. It is beneficial to extend this research in the future. In addition, our model does not allow for the change of house needs over life cycle, for example, having children and downsizing post retirement, and the age care at the very old age. These features will reflect more realistic life path which may alter our research results.

5 Conclusion

The thesis uses its three chapters to investigate retirement savings decisions and their impact on individual welfare in an Australian context. Chapter 2 examines the role of the mandatory contribution rate, known as the SG in isolation, and Chapter 3 and Chapter 4 introduce the implications of housing arrangements in the modelling.

Chapter 2 uses a stochastic life-cycle model to investigate the optimal SG level and how an individual's welfare changes with the SG across nine income levels and four consumption objectives. The analysis applies the power utility function to represent the consumption smoothing objective and the reference dependent utility function to capture either replacement rate or fixed consumption target objectives.

This confirms that there is no 'one-size-fits-all' optimal SG. Individuals' income levels and their consumption objectives can dramatically change the optimal SG. Specifically, the optimal SG for the consumption smoothing objective varies from 11.5% to more than 20%, while the optimal SG for the three consumption target objectives is all below the current SG level of 9.5% and the proposed one of 12% varying from 0.5% to 8.5%. This identifies the key considerations for setting an appropriate SG as the availability of the Age Pension system, whether individuals follow optimal or commonly adopted strategies such as a fixed investment strategy (70% in risky assets and 30% in risk-free assets), and the time preference of consumption. These results emphasise the importance of specifying the objective of the SG system and clearly defining individual cohorts. It questions whether one single SG could meet the heterogeneous individuals' financial needs. There is a case for allowing for more flexibility in setting the contribution rate.

The analysis also highlights how individuals' welfare changes with the SG levels. Individuals with a consumption smoothing objective experience large welfare losses when the SG is very low, while they are not greatly affected when facing a very high SG. Individuals with consumption target objectives suffer a significant welfare loss when the SG is sub-optimal. Increasing the SG from 9.5% to 12% incurs a large welfare loss for individuals with a consumption target objective and generates a minor welfare gain for individuals with a consumption smoothing objective. This suggests that increasing the SG may reduce the welfare of some individuals to a greater extent than it helps others. The research also estimates the government net revenue per individual for a range of SG levels. The impact of varying SG levels seems to have modest impacts on the government net revenue extracted from individuals.

The research estimates a revised optimal SG assuming no Age Pension system, which can be treated as a test of the SG level needed if individuals prefer self-funding their retirement. The estimates indicate that assuming no access to the Age Pension system mostly leads to an optimal SG greater than 10%, with low-income earners experiencing the largest increase in optimal SG. The analysis confirms the important role of the Age Pension as a retirement income source for most Australian retirees. It also suggests that the proposed SG of 12% might be rational if the objective of the SG system is to replace the Age Pension system.

The research also estimates the optimal SG levels when individuals adopt pre-defined investment strategies that follow fixed or life-cycle growth/defensive allocations instead of optimising them throughout their lifetimes. Taking these two pre-defined investment strategies leads to an increase of optimal SG between 0.5% and 4.5% across most of the income levels and objectives. Further, if individuals' have inter-temporal consumption preferences, applying a discount rate of 2% leads to a decrease of the optimal SG by 0.5% to 5.0%. These alternative 'optimal' SG estimates indicate how certain behavioural decision-making effects may impact the appropriate SG, and thus might be taken into account when setting the SG level.

In summary, Chapter 2 provides insights into the design of the superannuation system by highlighting how the optimal SG level varies considerably with various individual attributes and is impacted by the role that is intended for the Age Pension and potentially individuals' behavioural decision-making effects.

Chapter 3 and Chapter 4 incorporate housing decisions into the analysis of the SG. Chapter 3 builds a cascade model for Australian house prices. Autoregressive integrated moving average (ARIMA) models, autoregressive distributed lag (ADL) models and error-correction model (ECM) models are fitted to eight variables of interest, including inflation, wages, official cash rate, mortgage rate, rent, rental yield, house price and fund returns. Model selection is based on a combination of statistical criteria and economic theory. The models maintain relationships between the variables in a way that supports forming coherent projections over long horizons that are suitable for incorporation into the life-cycle models that investigate individual financial decisions. The model should also be valid for long-term projections for actuarial applications.

Chapter 4 incorporates the cascade model developed in Chapter 3 into a stochastic life-cycle model to simulate individuals' financial outcomes with two types of house tenures; renting and owning. Individuals are exposed to risks related to inflation, income, interest rates, investment return, rents, and house prices. The research calculates the utility generated from consumption, housing expenditure, and bequests, considering Australia's full range of tax, mortgage loans, and social security rules. The model is used to trade-off the utility benefit from lower pre-retirement consumption due to saving by either purchasing a house or building up a retirement account, against the utility gains from higher post-retirement consumption. This allows us to identify the 'optimal' SG for both renters and owners and compare the welfare of the different house tenure arrangements.

It is found that the optimal SG for both renters and owners increases with income level, with renter optimal SG moving from 5.5% for the 10th income percentile to 14.0% for the 90th income percentile, while the owner optimal SG moves from 0.5% up to 2.0%. The substantially larger optimal SG for renters versus owners across all income levels indicates that housing tenure is an important determinant of the appropriate SG level. It indicates that purchasing a home can significantly reduce the savings required for retirement through the superannuation system. The question of whether purchasing a home can substitute for mandatory saving schemes in the presence of a publicly funded Age Pension is posed, given the extremely low optimal SG found for the homeowner. The findings further reinforce the result of Chapter 2 that there is no single 'one-size-fits-all' optimal SG that suits all individuals.

This research compares renter and owner lifetime utilities arising from consumption, housing expenditure (i.e., services) and bequests. The research finds that the owner enjoys higher utility across all income levels and all SG levels tested, which range from 0.5% to 20.0%. The owner's higher utility mainly comes from an overall higher consumption level and the bequest from his house relative to the renter. This result indicates that purchasing a home is always beneficial for the individual under the model set-up. Given the extremely low optimal SG found for the homeowner, policymakers might give further consideration as to whether a planned increase in the SG to 12.0% truly helps all individuals. However, it is emphasised that this conclusion emerges under the assumptions that the individuals choose an affordable home that is in line with their income and borrowing capacity, and that they value their consumption, housing expenditure and any bequest in line with the utility function settings.

Chapter 4 confirms the finding of Chapter 2, that assuming no access to the Age Pension leads to the majority of optimal SG estimates exceeding 10%. This finding also holds when housing tenure is incorporated into the analysis. The optimal SG for the renter all sit above 20.0%. The owner optimal SG also increases markedly, moving from close to 0.5% in the baseline estimates to above 10.0%, ranging from 13.0% for the lowest income earner to 10.0% for the highest income earner. Low-income earners experience the largest increase in optimal SG, reflecting the fact that they receive the greatest relative benefit from the Age Pension as a substitute for private savings. The analysis confirms the important role of a publicly funded pension as a retirement income source, and further highlights the particular significance for Australian retirees with modest income and savings.

The research indicates that interactions between the SG, the Age Pension and the house tenure arrangements are important and that both the Age Pension and homeownership are quite influential in determining what might be an appropriate SG level. The proposed SG of 12% (or possibly more) might be justified for renters with higher income, or if the objective of superannuation is viewed as substituting for the Age Pension requiring individuals to become self-funded retirees. Otherwise, an SG of 12% may reduce welfare by restricting the ability to purchase a home, which is always shown as welfare-enhancing under the modelling, especially in conjunction with access to the Age Pension.

Also admitted in the analysis is that there exist limitations that may change the optimal SG levels and housing arrangements. One major limitation is utility function forms. In Chapter 2, two types of utility functions are used to represent individual objectives and preferences regarding real consumption outcomes: power utility representing a preference for consumption smoothing and reference dependent utility representing a preference for a specific level of consumption. In Chapter 4, the utility function for consumption and housing is a modified form of the power utility, with non-housing consumption evaluated relative to a consumption floor and housing expenditure treated as a form of consumption. However, these utility functions cannot capture all the individuals' preferences for consumption and housing. In future research, it would be beneficial to test other utility functions and their impact on optimal saving rates and housing arrangements.

In Chapter 3 and Chapter 4, this research is effectively building a cascade model for Australian house prices and modelling the impact of housing arrangements on the optimal SG. The cascade model provides a structure that links house prices and rents to other key variables that are central to life-cycle modelling, most notably wages (i.e., income) and interest rates (which are linked to asset returns) while doing so in a manner that ensures consistency within each simulated path. Since the simulations of future values depend on current and historical values, the cascade model cannot be applied to the dynamic programming process to generate the optimal consumption and asset allocation decisions as demonstrated in Chapter 2. It is beyond the scope of the current research to investigate the optimal decisions on house purchases like the best purchase age, the optimal mortgage loan amount, optimal repayments, and so forth. Continuing the analysis initiated in Chapter 4 may prove to be a useful direction for future research.

In the post-retirement phase, it is assumed that individuals do not purchase any financial products to cover risks. The most noticeable ones are the annuity that covers the longevity risk and reverse mortgages to generate cash flows for consumption by using the house they purchase. With the models built, it would be interesting to investigate further the interactions between these financial products and housing arrangements.

In addition, the scope of the current research focuses only on the individual, and thus cannot account for the changes along his life path, such as like marriage, parenthood, or the death of a spouse. These changes would significantly influence their expenses and their consumption patterns. Incorporating household behaviours will reflect individuals' real financial situations throughout their lifetime, which might change the optimal contribution rates required, and their housing decisions.

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Appendix

2A. Investment tax

Appendix 2A. explains how to estimate tax rates on investment earnings in a retirement savings account, especially the tax on equities. Generally, tax rates on investment earnings in a retirement savings account are 15% on regular income and 10% on (long-term) capital gains in the accumulation phase, and 0% in the drawdown phase.

The following equation is used to calculate the tax on equities (tax_{ra})[1]:

$$tax_{ra} = p \times CGT + q \times DT \tag{2A.1}$$

where CGT is defined as the capital gains tax and DT is the dividends tax; p and q are the proportion of equities returns and the proportion of dividend returns respectively.

The estimation of p and q requires multiple assumptions on equity returns. The equity investments of Australian superannuation funds include not only Australian Equities (AE) but also World Equities (WE). It is assumed that the weight of Australian equities and World equities are $50:50$, which is close to that observed in practice (see Australian Institute of Superannuation Trustees 2019).

According to Credit Suisse (2019), the real pre-tax return of WE is 6.5% per annum. The real pre-tax return of AE is 8.3% per annum (Credit Suisse 2019), which is the second-highest return globally. However, it is unlikely that this high equity returns in Australia will continue. Therefore, it is assumed the real pre-tax returns of WE and AE are all 6.5%. The inflation rate ($Infla$) is assumed to be the widely accepted value of 2.5%. Then the total nominal return (real pre-tax return plus inflation rate) of WE and AE are all 9.0%. It is assumed that the net dividend yield rate of WE is 2.5% (MSCI website) and that of AE is 4.25%[2]. It indicates that the capital gains of WE and AE are 6.5% and 4.75% respectively (total nominal return minus net dividend yield rate). Thus, the weighted average of capital gains is 5.625% (i.e.,

[1] The capital gains less than 12 months attract the full 15% tax rate.

[2] For details see Australian Taxation Office 'Average Dividend and Franking Credit Yields', https://www.ato.gov.au/Rates/Company-tax---imputation--average-franking-credit---rebate-yields/.

$6.5\% \times 50\% + 4.75\% \times 50\%$) and the weighted average of dividend yield is 3.375%, which leads to p equal to 62.5% ($\frac{5.625\%}{5.625\%+3.375\%}$) and q equal to 37.5%.

In terms of CGT, according to the capital gains tax concession guide[3], there is a $33\frac{1}{3}\%$ CGT discount for complying superannuation funds. Therefore, the CGT applied to total equity investment is 10% in the accumulation phase and 0% in the drawdown phase.

To calculate DT of AE, the gross yield from dividends must first be calculated ($GrossDiv$). It follows the equation:

$$GrossDiv = NetDiv + FrankingCredits \tag{2A.2}$$

where $NetDiv$ represents the net dividend return; $FrankingCredits$ is the Franking Credits rate from Butt et al. (2018), which is 1.37% in this chapter. This number is also consistent with a 4.25% dividend yield and a 75% franked dividend component. For the WE, which do not have Franking Credits, the formula (14) can be used with zero Franking Credits.

Then the effective dividend tax rate is then calculated in the following equations:

$$NetTax = GrossDiv \times IncomeTax - FrankingCredits \tag{2A.3}$$

$$DT = NetTax/NetDiv \tag{2A.4}$$

where $IncomeTax$ is the tax rate applied on gross dividend: 15% in the accumulation phase and 0% in the pension phase. The dividend tax rates of WE and AE are equal to 15% and -12.4% before retirement and 0% and -32.24% after retirement. Then the DT rates applied to total equity investment is 1.3% $(15\% \times 50\% - 12.4\% \times 50\%)$ before retirement and -16.12% after retirement.

Substituting all the values into Equation (2.15) gives the tax of equities in the accumulation phase to 6.74% and -6.045% in the pension phase.

[3] For details see Australian Taxation Office ‘Advanced guide to Capital Gains Tax Concessions for Small Buisness 2012-13’, https://www.ato.gov.au/individuals/tax-return/2013/in-detail/publications/advanced-guide-to-capital-gains-tax-concessions-for-small-business-2012-13/.

2B. Post-tax real risky and risk-free returns

As mentioned in Section 2.2.6, $R_{E,t}$ and $R_{C,t}$ are used in calculating the relationship of superannuation balance between two successive years. The following formulas explain how to get $R_{E,t}$ and $R_{C,t}$:

$$R_{E,t} = (RR_{ra} + Infla)(1 - MTR_{ra}) - Infla \tag{2B.1}$$

$$R_{C,t} = (RR_{rf} + Infla)\left(1 - MTR_{rf}\right) - Infla \tag{2B.2}$$

where MTR_{ra} and MTR_{rf} are the marginal tax rate for a risky asset and for a risk-free asset[4] in Appendix 2A.

[4] Risk free earnings is taxed as regular income: 15% in the accumulation phase and 0% in the drawdown phase.

2C. Calculation of total post-tax superannuation contribution

Appendix 2C. explains how to calculate the pre-retirement consumption C_t and the Total Superannuation Contribution S_t.

The following relations apply between model variables:

$$taxable_t = e_t(1 - \mathrm{SG}) \tag{2C.1}$$

$$C_t = taxable_t - income.tax(taxable_t) - medicare_t(taxable_t) \tag{2C.2}$$

$$CC_t = \min\left(e_t SG, CC_{cap}\right) \tag{2C.3}$$

$$nonCC_t = \left(e_t - CC_{cap}\right) - income.tax\left(e_t - CC_{cap}\right) - medicare_t\left(e_t - CC_{cap}\right) - C_t \tag{2C.4}$$

$$S_t = CC_t(1 - super_{tax}) + nonCC_t 1_{(e_t * SG \geq CC_{cap})} + \mathrm{LISTO_t} \tag{2C.5}$$

where CC_t is concessional contribution amount and $nonCC_t$ is non-concessional contribution amount; $LISTO_t$ is Low Income Super Tax Offset; F_t is the fee paid to member's superannuation fund; $R_{E,t}$ is the post-tax real return on equity and $R_{C,t}$ is the post-tax real return on cash (see Appendix 2B.); $medicare_t$ is the Medicare levy, which is 2% of your taxable income; and $super\_tax$ is the concessional contribution tax rate (15%). Most of the formulas are quite straightforward, in terms of $nonCC_t$, it only appears when SG of earning is higher than CC cap.

3A. Augmented Dickey-Fuller test

An augmented Dickey-Fuller test (ADF) tests the null hypothesis that a unit root is present in a time series sample. The testing procedure for the ADF test is applied to the model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \tag{3A.1}$$

where α is a constant, β the coefficient on a time trend and p is the lag order of the autoregressive process.

The unit root test is then carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$. The test statistic is $\frac{\hat{\gamma}}{SE(\hat{\gamma})}$.

It is then compared with the Dickey-Fuller t-distribution.

3B. Phillips-Perron test

The Phillips-Perron test is a unit root test. It is used in a time series analysis to test the null hypothesis that a time series is integrated of order 1. It builds on the Dickey-Fuller test of the null hypothesis $\delta = 0$ in

$$\Delta y_t = \delta y_{t-1} + \varepsilon_t \tag{3B.1}$$

where y_t is the variable of interest, t is the time index, δ is a coefficient, and ε_t is the error term.

The test statistic is $\frac{\hat{\delta}}{SE(\hat{\delta})}$.

It has a specific distribution known as the Dickey-Fuller t-distribution.

3C. Canova-Hansen test

The test developed by Canova-Hansen (1995) takes as the null hypothesis that the seasonal pattern is deterministic. The model is

$$y_t = \alpha y_{t-1} + \sum_{i=1}^{S-1} D_{it}\beta_{it} + \varepsilon_t \tag{3C.1}$$

where y_t is the variable of interest, t is the time index, S is the number of seasons, for quarterly data, $S = 4$, D_{it} is the seasonal dummy variable in Season i, it takes on the value 1 in the ith season, ε_t is the error term, β_{it} is the coefficient on the seasonal dummy variables.

The idea is to test for instability of the β_{it} parameters.

$$\beta_{it} = \beta_{i(t-1)} + \varepsilon_t \tag{3C.2}$$

And test that $Var(\varepsilon_t) = 0$

The null hypothesis in the Conava-Hansen test is rejected in case the seasonality of a series is not constant. After seasonal adjustment, the Canova-Hansen test should not reject.

3D. Ljung-Box test

The Ljung-Box test is a statistical test of whether any of a group of autocorrelations of a time series are different from zero. Its null hypothesis is the data are independently distributed. The test statistic is

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k} \tag{3D.1}$$

where n is the sample size; $\hat{\rho}_k^2$ is the sample autocorrelation at lag k; and h is the number of lags being tested. Under null hypothesis, the statistic Q asymptotically follows Chi-square $\chi^2_{(h)}$.

3E. Jarque-Bera test

The Jarque-Bera test is a goodness-of-fit test of whether sample data have the skewness and kurtosis matching a normal distribution.

The test statistic JB is defined as

$$JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2 \tag{3E.1}$$

where n is the number of observations; S is the sample skewness, K is the sample kurtosis. The test statistic JB asymptotically follows Chi-squared distribution with two degrees of freedom.

3F. Durbin-Watson test

The Durbin-Watson statistic is a test statistic used to detect the presence of autocorrelation at lag 1 in the residuals from a regression analysis. If e_t is the residual given by $e_t = \rho e_{t-1} + \nu_t$, the Durbin-Watson statistic states that null hypothesis: $\rho = 0$, alternative hypothesis $\rho \neq 0$, then the test statistic is

$$d = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} {e_t}^2} \qquad (3F.1)$$

where t is the number of observations. $d = 2$ indicates no autocorrelation. The value of d always lies between 0 and 4.

3G. Shapiro-Wilk Normality test

The Shapiro-Wilk test is a test of normality. It tests the null hypothesis that a sample $X_1, \dots, X_n$ came from a normally distributed population. The test statistic is

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{3G.1}$$

where $x_{(i)}$ is the ith order statistic; $\bar{x}$ is the sample mean. The coefficients a_i are given by: $(a_1, \dots, a_n) = \frac{m^T V^{-1}}{C}$ where C is a vector norm: $C = \|V^{-1}m\| = (m^T V^{-1} V^{-1} m)^{1/2}$ and the vector m, is made of the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution; V is the covariance matrix of those normal order statistics. There is no name for the distribution of W. The cutoff values for the statistics are calculated through Monte Carlo simulations.

3H. Path Analysis

Table 3H.1: Analysis of Distribution of Key Outputs Across Joint Paths for 10-year Rolling Periods

			Percentiles								
Simulated Outputs	**Mean**	**Standard Deviation**	**0%**	**5%**	**10%**	**25%**	**50%**	**75%**	**90%**	**95%**	**100%**
Compound growth rates											
Inflation	0.0239	0.0034	0.0056	0.0184	0.0196	0.0217	0.0239	0.0262	0.0283	0.0296	0.0414
Real wages	0.0150	0.0045	-0.0069	0.0076	0.0092	0.0120	0.0150	0.0180	0.0207	0.0223	0.0381
Real rents	0.0156	0.0080	-0.0187	0.0026	0.0055	0.0102	0.0156	0.0209	0.0259	0.0288	0.0587
Real house prices	0.0136	0.0218	-0.1188	-0.0216	-0.0141	-0.0014	0.0133	0.0283	0.0419	0.0501	0.1281
Real rent - real wages	0.0006	0.0076	-0.0317	-0.0117	-0.0090	-0.0045	0.0005	0.0057	0.0105	0.0133	0.0434
Real house prices - real wages	-0.0013	0.0217	-0.1360	-0.0363	-0.0289	-0.0163	-0.0017	0.0133	0.0269	0.0349	0.1195
Real house prices - real rents	-0.0020	0.0239	-0.1352	-0.0407	-0.0326	-0.0186	-0.0023	0.0142	0.0292	0.0380	0.1168
Average yields over period											
Cash rate	0.0399	0.0133	0.0016	0.0191	0.0233	0.0308	0.0393	0.0480	0.0566	0.0623	0.1149
Real cash rate	0.0157	0.0130	-0.0265	-0.0046	-0.0004	0.0069	0.0152	0.0237	0.0319	0.0375	0.0893
Rental yield	0.0492	0.0066	0.0362	0.0403	0.0416	0.0443	0.0483	0.0533	0.0583	0.0614	0.1139
Rental yield - cash rate	0.0094	0.0111	-0.0603	-0.0094	-0.0048	0.0023	0.0098	0.0170	0.0232	0.0267	0.0762
Compound returns											
Real cash returns	0.0152	0.0126	-0.0254	-0.0045	-0.0005	0.0066	0.0147	0.0230	0.0309	0.0364	0.0868
Real super fund return	0.0506	0.0319	-0.1068	-0.0014	0.0099	0.0289	0.0503	0.0719	0.0916	0.1036	0.2019
Super fund premium over cash	0.0362	0.0298	-0.1084	-0.0126	-0.0019	0.0162	0.0362	0.0563	0.0744	0.0853	0.1801
Real housing return	0.0185	0.0214	-0.1064	-0.0159	-0.0087	0.0037	0.0181	0.0330	0.0463	0.0544	0.1349
House return premium over cash	-0.0016	0.0257	-0.1305	-0.0444	-0.0345	-0.0185	-0.0012	0.0160	0.0309	0.0396	0.1262
Super fund - house return	0.0379	0.0395	-0.1415	-0.0265	-0.0125	0.0111	0.0375	0.0642	0.0887	0.1035	0.2244

Table 3H.2: Analysis of Distribution of Key Outputs Across Joint Paths for 85-year Rolling Periods

Simulated Outputs	Mean	Standard Deviation	Percentiles 0%	5%	10%	25%	50%	75%	90%	95%	100%
Compound growth rates											
Inflation	0.0239	0.0012	0.0201	0.0220	0.0224	0.0231	0.0239	0.0247	0.0254	0.0258	0.0295
Real wages	0.0150	0.0015	0.0091	0.0125	0.0130	0.0139	0.0149	0.0160	0.0169	0.0175	0.0212
Real rents	0.0155	0.0016	0.0093	0.0128	0.0134	0.0144	0.0155	0.0165	0.0175	0.0181	0.0212
Real house prices	0.0118	0.0024	0.0028	0.0079	0.0087	0.0102	0.0118	0.0134	0.0148	0.0156	0.0198
Real rent - real wages	0.0005	0.0006	-0.0016	-0.0004	-0.0003	0.0001	0.0005	0.0009	0.0013	0.0015	0.0028
Real house prices - real wages	-0.0032	0.0019	-0.0104	-0.0063	-0.0056	-0.0045	-0.0032	-0.0018	-0.0007	-0.0001	0.0017
Real house prices - real rents	-0.0037	0.0021	-0.0130	-0.0072	-0.0064	-0.0052	-0.0037	-0.0021	-0.0008	-0.0002	0.0002
Average yields over period											
Cash rate	0.0387	0.0057	0.0216	0.0298	0.0316	0.0348	0.0385	0.0424	0.0462	0.0486	0.0635
Real cash rate	0.0146	0.0056	-0.0023	0.0058	0.0076	0.0106	0.0143	0.0182	0.0220	0.0243	0.0382
Rental yield	0.0488	0.0029	0.0408	0.0445	0.0453	0.0468	0.0486	0.0507	0.0526	0.0539	0.0626
Rental yield - cash rate	0.0101	0.0048	-0.0108	0.0019	0.0038	0.0070	0.0102	0.0134	0.0162	0.0178	0.0253
Compound returns											
Real cash returns	0.0140	0.0054	-0.0023	0.0055	0.0073	0.0102	0.0138	0.0176	0.0212	0.0235	0.0370
Real super fund return	0.0491	0.0115	0.0001	0.0304	0.0344	0.0413	0.0490	0.0566	0.0638	0.0684	0.0940
Super fund premium over cash	0.0359	0.0102	-0.0052	0.0188	0.0228	0.0290	0.0359	0.0427	0.0489	0.0524	0.0738
Real housing return	0.0124	0.0023	0.0040	0.0086	0.0093	0.0108	0.0124	0.0139	0.0153	0.0161	0.0202
House return premium over cash	-0.0023	0.0062	-0.0280	-0.0126	-0.0104	-0.0064	-0.0021	0.0020	0.0054	0.0074	0.0184
Super fund - house return	0.0382	0.0121	-0.0120	0.0186	0.0228	0.0301	0.0382	0.0463	0.0537	0.0584	0.0868

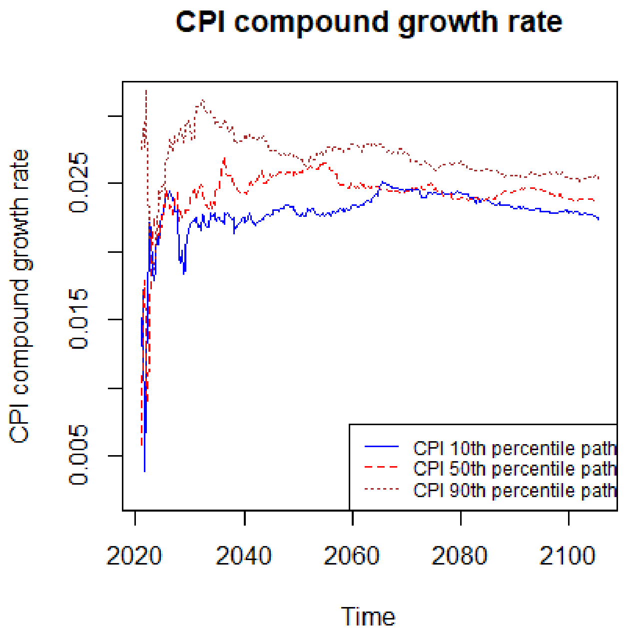

Figure 3H.1: Path Analysis of CPI Compound Growth Rates

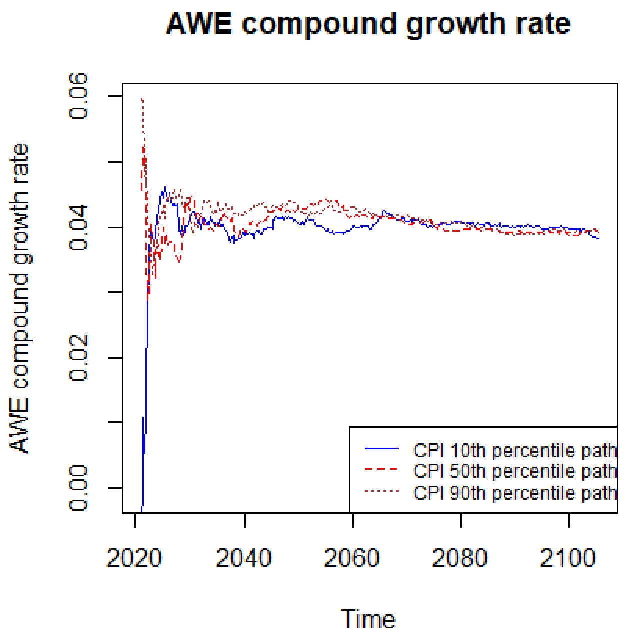

Figure 3H.2: Path Analysis of AWE Compound Growth Rates

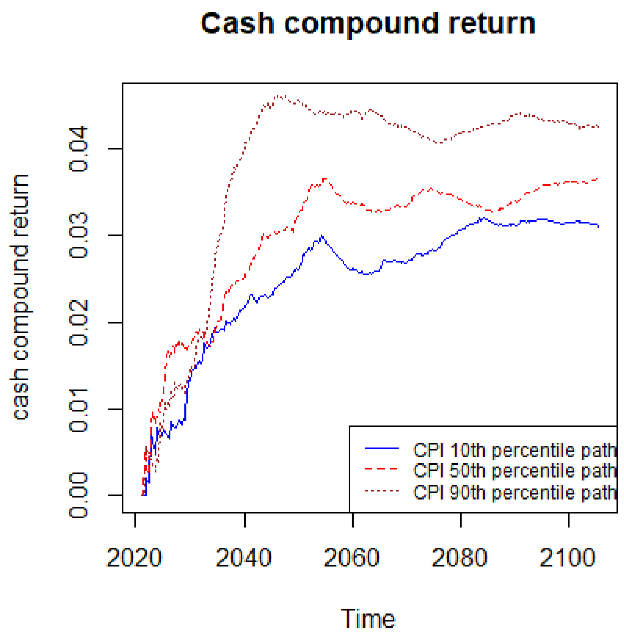

Figure 3H.3: Path Analysis of Cash Compound Returns

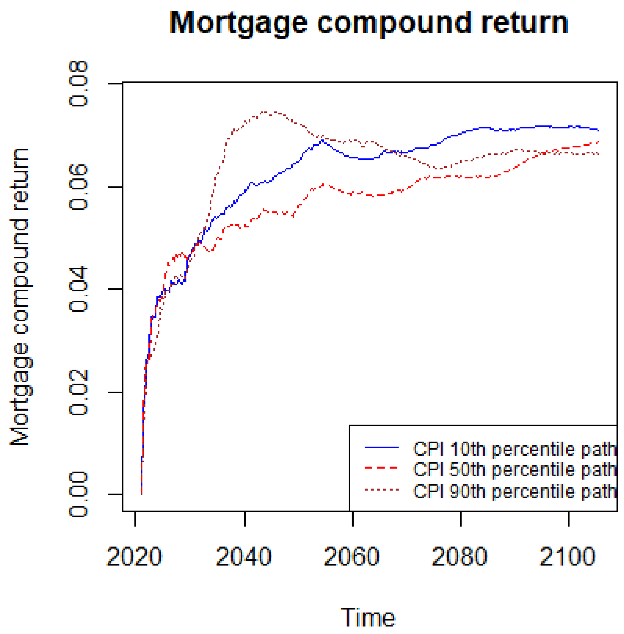

Figure 3H.4: Path Analysis of Mortgage Compound Returns

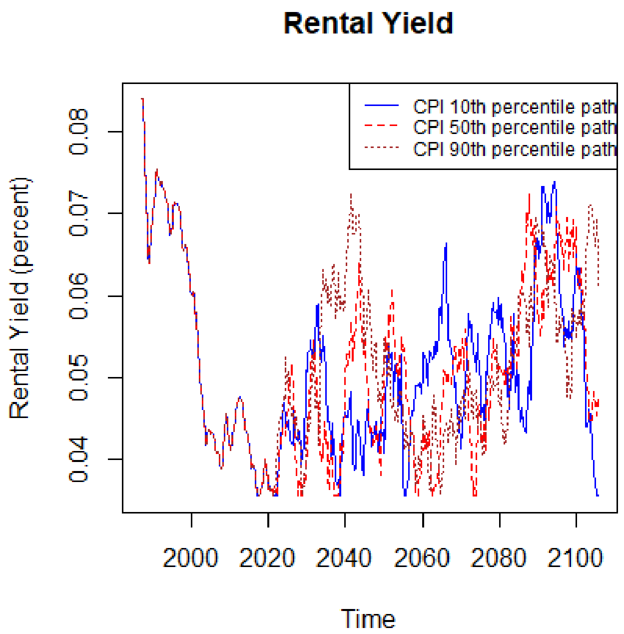

Figure 3H.5: Path Analysis of Rental Yields

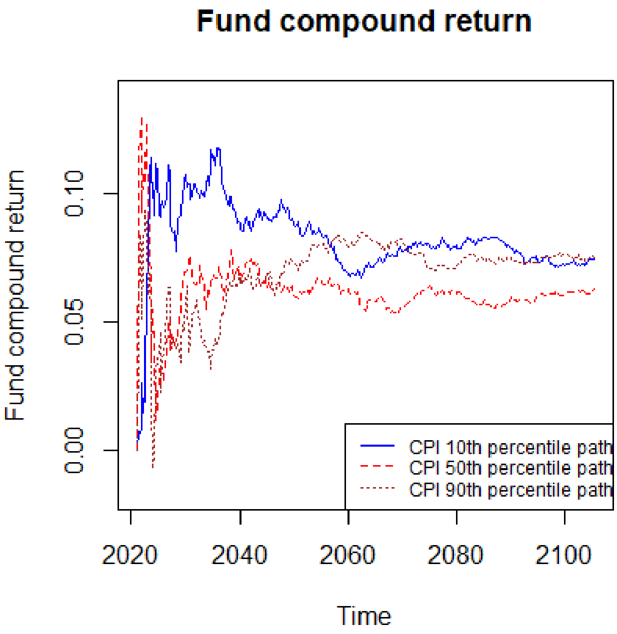

Figure 3H.6: Path Analysis of Fund Compound Returns

4A. Who achieves a higher lifetime utility, the renter or the owner?

Table 4A.1: Who achieves a higher lifetime utility, the renter or the owner?

SG	10th income percentile	20th income percentile	30th income percentile	40th income percentile	50th income percentile	60th income percentile	70th income percentile	80th income percentile	90th income percentile
0.005	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.01	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.015	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.02	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.025	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.03	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.035	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.04	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.045	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.05	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.055	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.06	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.065	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.07	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.075	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.08	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.085	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.09	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.095	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.1	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.105	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.11	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.115	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.12	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.125	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.13	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.135	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.14	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.145	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.15	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.155	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.16	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.165	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.17	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.175	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.18	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.185	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.19	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.195	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner
0.2	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner	Owner

4B. Expected age when owner purchases the house

The expected age when the owner purchases the house is reported in Table 4B.1 for the nine income percentiles across all SG levels. Taking the 50th income percentile as an example, the expected age starts at an age of 34.69 for a 0.5% SG level and ends at age of 36.14 for a 20.0% SG level, reflecting a lower SG level allows higher disposable income to help save the upfront payment. At the current SG of 10.5%, the expected age of purchase varies from 35.21 to 35.79. This result is in line with the observed situation in Australia, where the average age of the first home buyers was 34 in 2015-16 from ABS[5] and the average age of the Australian first-time buyer is 36[6] in 2020.

[5] For details see Australian Bureau of Statistics ‘Across the Generations: Twenty Years of Housing’, https://www.abs.gov.au/ausstats/abs@.nsf/Lookup/by%20Subject/4130.0~2015-16~Main%20Features~Across%20the%20Generations:%20Twenty%20years%20of%20housing~10003#.

[6] For details see Money.co, ‘Age of the First Time Buyer’, https://www.money.co.uk/guides/first-time-buyers-around-the-world.

Table 4B.1: Expected ages when the owner purchases the house

SG	10th income percentile	20th income percentile	30th income percentile	40th income percentile	50th income percentile	60th income percentile	70th income percentile	80th income percentile	90th income percentile
0.005	34.45	34.51	34.57	34.63	34.69	34.74	34.78	34.84	34.98
0.01	34.49	34.54	34.61	34.67	34.72	34.77	34.82	34.88	35.03
0.015	34.53	34.58	34.65	34.70	34.76	34.81	34.87	34.93	35.07
0.02	34.57	34.62	34.68	34.74	34.80	34.85	34.91	34.97	35.11
0.025	34.61	34.65	34.72	34.77	34.83	34.89	34.95	35.01	35.15
0.03	34.64	34.68	34.75	34.81	34.88	34.93	34.99	35.05	35.19
0.035	34.68	34.72	34.79	34.85	34.91	34.97	35.03	35.09	35.23
0.04	34.71	34.75	34.82	34.89	34.95	35.01	35.07	35.13	35.26
0.045	34.75	34.78	34.86	34.92	34.99	35.05	35.12	35.17	35.31
0.05	34.78	34.82	34.90	34.97	35.03	35.09	35.15	35.21	35.35
0.055	34.82	34.86	34.93	35.00	35.07	35.13	35.19	35.25	35.39
0.06	34.86	34.90	34.97	35.04	35.11	35.17	35.23	35.30	35.43
0.065	34.90	34.93	35.01	35.08	35.15	35.21	35.27	35.34	35.47
0.07	34.94	34.97	35.04	35.12	35.19	35.25	35.31	35.38	35.51
0.075	34.98	35.01	35.08	35.16	35.22	35.28	35.35	35.41	35.55
0.08	35.02	35.04	35.12	35.19	35.26	35.33	35.39	35.46	35.59
0.085	35.06	35.08	35.16	35.23	35.30	35.37	35.43	35.50	35.63
0.09	35.10	35.12	35.19	35.26	35.34	35.41	35.47	35.54	35.67
0.095	35.14	35.15	35.22	35.30	35.37	35.44	35.51	35.58	35.71
0.1	35.18	35.19	35.26	35.34	35.41	35.48	35.55	35.62	35.75
0.105	35.21	35.22	35.30	35.37	35.45	35.52	35.59	35.66	35.79
0.11	35.25	35.26	35.33	35.41	35.49	35.56	35.62	35.70	35.83
0.115	35.29	35.30	35.37	35.44	35.52	35.59	35.67	35.74	35.87
0.12	35.33	35.33	35.40	35.48	35.56	35.63	35.71	35.78	35.91
0.125	35.37	35.37	35.44	35.52	35.60	35.67	35.74	35.82	35.95
0.13	35.41	35.40	35.47	35.55	35.63	35.71	35.78	35.86	35.99
0.135	35.44	35.44	35.51	35.59	35.67	35.75	35.82	35.90	36.03
0.14	35.48	35.47	35.54	35.62	35.71	35.79	35.86	35.94	36.07
0.145	35.52	35.51	35.57	35.66	35.74	35.82	35.90	35.98	36.11
0.15	35.56	35.55	35.61	35.69	35.78	35.86	35.94	36.02	36.15
0.155	35.60	35.58	35.65	35.73	35.82	35.90	35.98	36.06	36.19
0.16	35.64	35.62	35.68	35.77	35.86	35.94	36.02	36.09	36.22
0.165	35.67	35.66	35.71	35.80	35.89	35.98	36.06	36.13	36.26
0.17	35.71	35.69	35.75	35.84	35.93	36.02	36.09	36.17	36.30
0.175	35.75	35.72	35.78	35.88	35.97	36.05	36.13	36.21	36.34
0.18	35.79	35.76	35.82	35.91	36.00	36.08	36.16	36.25	36.37
0.185	35.83	35.80	35.86	35.95	36.04	36.12	36.20	36.29	36.41
0.19	35.86	35.83	35.89	35.98	36.08	36.15	36.24	36.32	36.45
0.195	35.90	35.87	35.93	36.02	36.11	36.19	36.28	36.36	36.49
0.2	35.94	35.91	35.96	36.05	36.14	36.23	36.31	36.40	36.53

Table 4B.2: Expected ages when the owner purchases the house for 'First Home Loan Deposit Scheme' test

SG	10th income percentile	20th income percentile	30th income percentile	40th income percentile	50th income percentile	60th income percentile	70th income percentile	80th income percentile	90th income percentile
0.005	29.11	29.00	28.93	28.87	28.82	28.77	28.71	28.66	28.60
0.01	29.13	29.01	28.95	28.89	28.84	28.78	28.73	28.68	28.62
0.015	29.15	29.03	28.97	28.91	28.85	28.80	28.75	28.70	28.63
0.02	29.17	29.05	28.98	28.92	28.87	28.82	28.77	28.71	28.65
0.025	29.19	29.07	29.00	28.94	28.88	28.83	28.78	28.73	28.67
0.03	29.21	29.08	29.02	28.96	28.90	28.85	28.80	28.75	28.69
0.035	29.23	29.10	29.04	28.98	28.92	28.87	28.82	28.77	28.71
0.04	29.25	29.11	29.05	29.00	28.94	28.89	28.83	28.78	28.72
0.045	29.27	29.13	29.07	29.01	28.95	28.90	28.85	28.80	28.74
0.05	29.29	29.15	29.09	29.03	28.97	28.92	28.87	28.82	28.76
0.055	29.31	29.16	29.10	29.05	28.99	28.94	28.88	28.83	28.77
0.06	29.33	29.18	29.12	29.07	29.01	28.95	28.90	28.85	28.79
0.065	29.35	29.20	29.14	29.08	29.03	28.97	28.92	28.87	28.80
0.07	29.37	29.22	29.15	29.10	29.04	28.99	28.94	28.88	28.82
0.075	29.39	29.24	29.17	29.12	29.06	29.01	28.95	28.90	28.84
0.08	29.41	29.26	29.19	29.13	29.08	29.03	28.97	28.92	28.85
0.085	29.43	29.27	29.20	29.15	29.09	29.04	28.99	28.94	28.87
0.09	29.45	29.29	29.22	29.17	29.11	29.06	29.01	28.96	28.89
0.095	29.47	29.31	29.24	29.18	29.13	29.08	29.03	28.97	28.91
0.1	29.49	29.33	29.26	29.20	29.15	29.09	29.05	28.99	28.92
0.105	29.51	29.35	29.27	29.22	29.16	29.11	29.06	29.01	28.94
0.11	29.53	29.36	29.29	29.24	29.18	29.13	29.08	29.03	28.96
0.115	29.54	29.38	29.31	29.25	29.20	29.15	29.10	29.05	28.98
0.12	29.56	29.40	29.33	29.27	29.22	29.17	29.11	29.06	28.99
0.125	29.58	29.42	29.34	29.29	29.23	29.18	29.13	29.08	29.01
0.13	29.60	29.43	29.36	29.31	29.25	29.20	29.15	29.10	29.03
0.135	29.62	29.45	29.37	29.33	29.27	29.22	29.17	29.12	29.05
0.14	29.64	29.47	29.39	29.34	29.29	29.24	29.19	29.14	29.07
0.145	29.66	29.49	29.41	29.36	29.31	29.25	29.21	29.15	29.08
0.15	29.68	29.50	29.42	29.37	29.32	29.27	29.22	29.17	29.10
0.155	29.70	29.52	29.43	29.39	29.34	29.29	29.24	29.19	29.12
0.16	29.72	29.54	29.45	29.40	29.36	29.31	29.26	29.21	29.14
0.165	29.74	29.56	29.46	29.42	29.37	29.33	29.28	29.23	29.16
0.17	29.76	29.58	29.48	29.43	29.39	29.34	29.30	29.25	29.18
0.175	29.78	29.59	29.50	29.45	29.40	29.36	29.31	29.26	29.19
0.18	29.80	29.61	29.51	29.47	29.42	29.37	29.33	29.28	29.21
0.185	29.82	29.63	29.53	29.48	29.43	29.39	29.34	29.30	29.23
0.19	29.84	29.65	29.54	29.50	29.45	29.41	29.36	29.32	29.25
0.195	29.86	29.67	29.56	29.51	29.46	29.42	29.38	29.33	29.27
0.2	29.88	29.68	29.58	29.53	29.48	29.44	29.40	29.35	29.29

Table 4B.3: Expected ages when the owner purchases the house for '10% saving rate for owner' test

SG	10th income percentile	20th income percentile	30th income percentile	40th income percentile	50th income percentile	60th income percentile	70th income percentile	80th income percentile	90th income percentile
0.005	38.84	38.91	38.99	39.07	39.14	39.21	39.27	39.36	39.56
0.01	38.88	38.96	39.05	39.12	39.20	39.26	39.32	39.41	39.62
0.015	38.93	39.01	39.09	39.17	39.25	39.31	39.38	39.47	39.68
0.02	38.99	39.06	39.15	39.22	39.30	39.37	39.44	39.53	39.73
0.025	39.04	39.11	39.20	39.28	39.35	39.43	39.49	39.59	39.79
0.03	39.09	39.16	39.25	39.33	39.41	39.48	39.55	39.65	39.85
0.035	39.14	39.21	39.30	39.38	39.46	39.53	39.61	39.70	39.90
0.04	39.19	39.26	39.35	39.44	39.51	39.60	39.67	39.76	39.96
0.045	39.24	39.31	39.40	39.48	39.57	39.65	39.72	39.81	40.01
0.05	39.29	39.36	39.45	39.53	39.63	39.70	39.78	39.87	40.07
0.055	39.34	39.41	39.50	39.59	39.68	39.76	39.84	39.93	40.12
0.06	39.40	39.46	39.55	39.65	39.73	39.82	39.90	39.98	40.18
0.065	39.45	39.51	39.61	39.70	39.79	39.87	39.95	40.04	40.23
0.07	39.50	39.56	39.66	39.75	39.84	39.93	40.01	40.09	40.28
0.075	39.55	39.61	39.71	39.80	39.90	39.98	40.06	40.15	40.34
0.08	39.60	39.66	39.76	39.86	39.95	40.03	40.11	40.20	40.40
0.085	39.66	39.71	39.81	39.91	40.00	40.08	40.16	40.25	40.45
0.09	39.71	39.76	39.87	39.96	40.05	40.13	40.21	40.30	40.50
0.095	39.76	39.82	39.91	40.01	40.10	40.18	40.27	40.36	40.56
0.1	39.81	39.86	39.96	40.06	40.15	40.23	40.32	40.41	40.62
0.105	39.86	39.91	40.01	40.10	40.20	40.29	40.38	40.47	40.67
0.11	39.91	39.96	40.06	40.15	40.25	40.34	40.43	40.53	40.73
0.115	39.96	40.00	40.10	40.20	40.30	40.40	40.48	40.58	40.78
0.12	40.01	40.06	40.15	40.25	40.35	40.44	40.54	40.64	40.84
0.125	40.06	40.10	40.20	40.30	40.40	40.50	40.59	40.69	40.89
0.13	40.12	40.15	40.25	40.35	40.45	40.55	40.65	40.75	40.95
0.135	40.17	40.20	40.29	40.40	40.51	40.60	40.70	40.81	41.00
0.14	40.22	40.25	40.34	40.45	40.56	40.66	40.76	40.86	41.06
0.145	40.26	40.29	40.39	40.50	40.61	40.71	40.81	40.91	41.12
0.15	40.32	40.34	40.44	40.55	40.66	40.77	40.86	40.97	41.18
0.155	40.37	40.39	40.49	40.60	40.71	40.82	40.92	41.02	41.23
0.16	40.42	40.44	40.53	40.65	40.77	40.87	40.97	41.08	41.28
0.165	40.47	40.49	40.59	40.70	40.82	40.92	41.02	41.14	41.34
0.17	40.53	40.54	40.63	40.75	40.87	40.98	41.08	41.20	41.39
0.175	40.58	40.59	40.68	40.80	40.92	41.03	41.13	41.25	41.45
0.18	40.63	40.64	40.74	40.85	40.97	41.08	41.19	41.30	41.50
0.185	40.68	40.69	40.78	40.90	41.02	41.13	41.25	41.36	41.55
0.19	40.74	40.75	40.83	40.95	41.07	41.18	41.30	41.41	41.61
0.195	40.79	40.79	40.88	41.00	41.12	41.24	41.35	41.46	41.67
0.2	40.84	40.84	40.93	41.05	41.17	41.29	41.40	41.51	41.73

Table 4B.4: Expected ages when the owner purchases the house for '20% saving rate for owner' test

SG	10th income percentile	20th income percentile	30th income percentile	40th income percentile	50th income percentile	60th income percentile	70th income percentile	80th income percentile	90th income percentile
0.005	32.07	32.10	32.16	32.21	32.26	32.31	32.35	32.40	32.49
0.01	32.10	32.13	32.19	32.24	32.29	32.34	32.38	32.43	32.53
0.015	32.13	32.16	32.22	32.27	32.33	32.37	32.42	32.46	32.56
0.02	32.16	32.19	32.25	32.31	32.36	32.40	32.45	32.49	32.59
0.025	32.19	32.22	32.28	32.34	32.39	32.43	32.48	32.52	32.63
0.03	32.22	32.25	32.31	32.37	32.42	32.46	32.51	32.55	32.66
0.035	32.26	32.28	32.34	32.40	32.45	32.49	32.54	32.59	32.69
0.04	32.29	32.31	32.37	32.43	32.48	32.52	32.57	32.63	32.73
0.045	32.32	32.34	32.40	32.45	32.51	32.56	32.61	32.66	32.76
0.05	32.35	32.37	32.43	32.48	32.54	32.59	32.64	32.69	32.79
0.055	32.39	32.40	32.46	32.51	32.56	32.62	32.67	32.72	32.82
0.06	32.42	32.43	32.48	32.54	32.60	32.65	32.71	32.75	32.85
0.065	32.45	32.45	32.52	32.57	32.63	32.68	32.74	32.79	32.89
0.07	32.48	32.48	32.54	32.60	32.66	32.72	32.76	32.82	32.92
0.075	32.51	32.51	32.57	32.63	32.69	32.74	32.80	32.85	32.96
0.08	32.54	32.54	32.60	32.66	32.72	32.77	32.83	32.89	32.99
0.085	32.57	32.56	32.63	32.69	32.75	32.81	32.86	32.92	33.02
0.09	32.60	32.59	32.66	32.72	32.78	32.84	32.89	32.96	33.05
0.095	32.63	32.62	32.69	32.75	32.81	32.87	32.93	32.99	33.08
0.1	32.67	32.65	32.72	32.77	32.84	32.90	32.96	33.02	33.11
0.105	32.70	32.68	32.74	32.81	32.87	32.93	32.99	33.05	33.15
0.11	32.73	32.71	32.77	32.84	32.90	32.96	33.02	33.08	33.18
0.115	32.76	32.74	32.80	32.86	32.93	32.99	33.05	33.11	33.22
0.12	32.79	32.77	32.83	32.89	32.96	33.03	33.08	33.15	33.25
0.125	32.82	32.80	32.85	32.92	32.99	33.05	33.11	33.18	33.28
0.13	32.85	32.83	32.88	32.95	33.03	33.08	33.15	33.21	33.31
0.135	32.88	32.86	32.91	32.98	33.05	33.11	33.18	33.25	33.35
0.14	32.91	32.88	32.94	33.01	33.08	33.15	33.21	33.28	33.38
0.145	32.95	32.91	32.97	33.04	33.11	33.18	33.24	33.31	33.41
0.15	32.98	32.94	33.00	33.07	33.14	33.21	33.27	33.34	33.45
0.155	33.01	32.97	33.02	33.10	33.17	33.24	33.30	33.37	33.47
0.16	33.04	33.01	33.05	33.12	33.20	33.27	33.34	33.41	33.51
0.165	33.07	33.04	33.08	33.15	33.23	33.30	33.37	33.44	33.54
0.17	33.10	33.06	33.10	33.18	33.26	33.33	33.40	33.47	33.57
0.175	33.13	33.09	33.13	33.21	33.29	33.36	33.44	33.50	33.60
0.18	33.17	33.12	33.16	33.24	33.32	33.39	33.46	33.54	33.63
0.185	33.20	33.15	33.19	33.27	33.35	33.42	33.49	33.57	33.67
0.19	33.23	33.18	33.22	33.30	33.38	33.45	33.53	33.60	33.70
0.195	33.26	33.21	33.25	33.33	33.41	33.48	33.56	33.63	33.73
0.2	33.30	33.25	33.28	33.35	33.44	33.51	33.58	33.66	33.76

Table 4B.5: Expected ages when the owner purchases the house for '20 years mortgage loan term' test

SG	10th income percentile	20th income percentile	30th income percentile	40th income percentile	50th income percentile	60th income percentile	70th income percentile	80th income percentile	90th income percentile
0.005	32.61	32.60	32.63	32.65	32.67	32.70	32.71	32.73	32.80
0.01	32.64	32.63	32.66	32.68	32.71	32.73	32.74	32.76	32.84
0.015	32.67	32.66	32.69	32.71	32.74	32.76	32.77	32.80	32.87
0.02	32.70	32.69	32.72	32.74	32.77	32.79	32.81	32.83	32.90
0.025	32.74	32.73	32.75	32.77	32.80	32.82	32.85	32.87	32.94
0.03	32.77	32.75	32.78	32.81	32.83	32.86	32.88	32.90	32.97
0.035	32.80	32.78	32.82	32.84	32.87	32.89	32.91	32.93	33.01
0.04	32.84	32.81	32.84	32.87	32.89	32.92	32.94	32.97	33.04
0.045	32.87	32.85	32.87	32.90	32.93	32.95	32.98	33.01	33.07
0.05	32.90	32.87	32.90	32.93	32.96	32.99	33.02	33.04	33.11
0.055	32.93	32.90	32.93	32.96	33.00	33.02	33.05	33.07	33.14
0.06	32.96	32.94	32.97	33.00	33.03	33.05	33.08	33.11	33.18
0.065	33.00	32.97	33.00	33.03	33.06	33.09	33.11	33.14	33.21
0.07	33.03	33.00	33.03	33.06	33.09	33.12	33.15	33.18	33.25
0.075	33.07	33.03	33.06	33.09	33.12	33.15	33.18	33.21	33.28
0.08	33.10	33.06	33.09	33.12	33.15	33.18	33.22	33.25	33.31
0.085	33.13	33.09	33.11	33.15	33.18	33.22	33.25	33.28	33.35
0.09	33.16	33.12	33.15	33.18	33.22	33.25	33.28	33.31	33.38
0.095	33.20	33.15	33.18	33.22	33.25	33.29	33.31	33.35	33.42
0.1	33.23	33.18	33.21	33.25	33.29	33.32	33.35	33.39	33.45
0.105	33.27	33.22	33.24	33.28	33.31	33.35	33.39	33.42	33.48
0.11	33.30	33.24	33.27	33.31	33.35	33.38	33.42	33.45	33.52
0.115	33.33	33.28	33.31	33.34	33.38	33.42	33.45	33.49	33.55
0.12	33.37	33.31	33.33	33.37	33.41	33.45	33.49	33.52	33.58
0.125	33.40	33.34	33.36	33.40	33.45	33.48	33.52	33.55	33.61
0.13	33.43	33.37	33.39	33.44	33.48	33.52	33.55	33.58	33.65
0.135	33.47	33.40	33.42	33.47	33.51	33.54	33.58	33.62	33.68
0.14	33.50	33.43	33.46	33.49	33.54	33.57	33.61	33.65	33.71
0.145	33.54	33.46	33.48	33.53	33.57	33.60	33.64	33.68	33.74
0.15	33.57	33.49	33.51	33.55	33.59	33.63	33.67	33.71	33.77
0.155	33.60	33.52	33.54	33.58	33.62	33.67	33.70	33.74	33.81
0.16	33.63	33.56	33.57	33.61	33.66	33.70	33.74	33.78	33.84
0.165	33.66	33.58	33.60	33.64	33.69	33.73	33.77	33.81	33.87
0.17	33.69	33.61	33.62	33.67	33.72	33.76	33.80	33.84	33.90
0.175	33.73	33.64	33.65	33.70	33.75	33.79	33.82	33.87	33.93
0.18	33.76	33.67	33.68	33.73	33.78	33.82	33.86	33.90	33.96
0.185	33.79	33.70	33.71	33.76	33.80	33.84	33.89	33.93	33.99
0.19	33.82	33.74	33.74	33.79	33.83	33.87	33.92	33.96	34.03
0.195	33.86	33.77	33.77	33.81	33.86	33.90	33.95	33.99	34.06
0.2	33.89	33.80	33.80	33.84	33.89	33.93	33.98	34.02	34.09

4C. Real consumption, housing expenditure and bequest: renter vs owner when both are at optimal SG

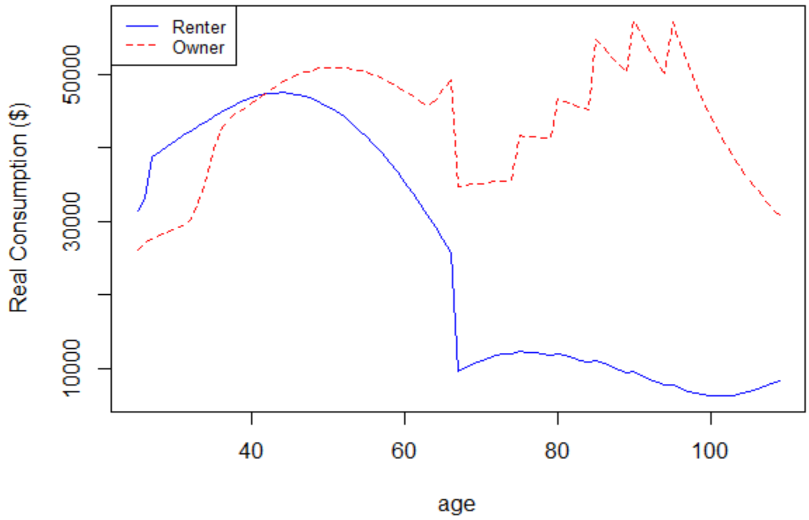

Figure 4C.1:Real consumption for median income percentile both at optimal SG: renter vs owner

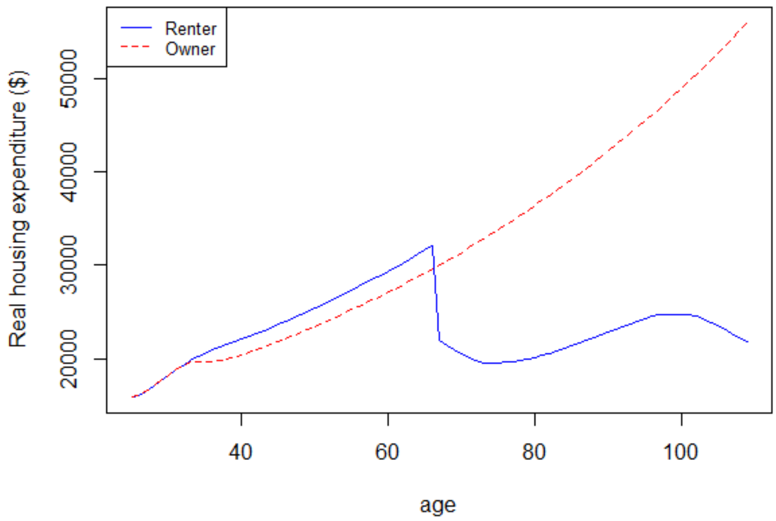

Figure 4C.2: Real housing expenditure for median income percentile both at optimal SG: renter vs owner

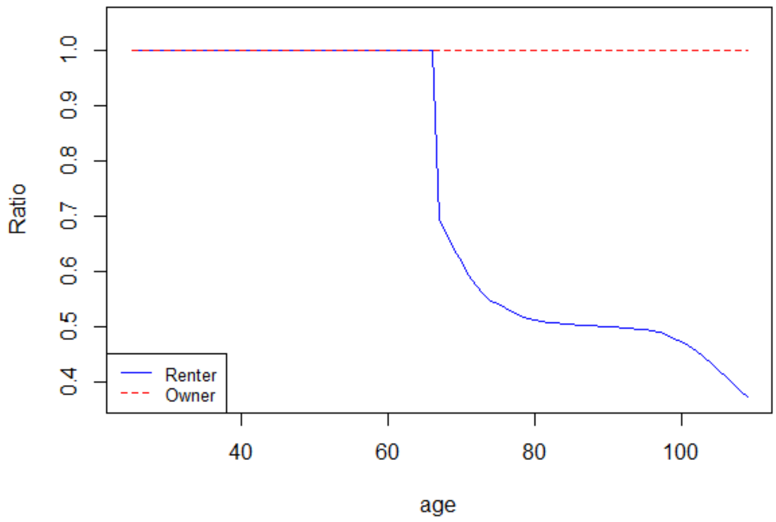

Figure 4C.3: Ratio of the average house value and the initial house value both at optimal SG: renter vs owner

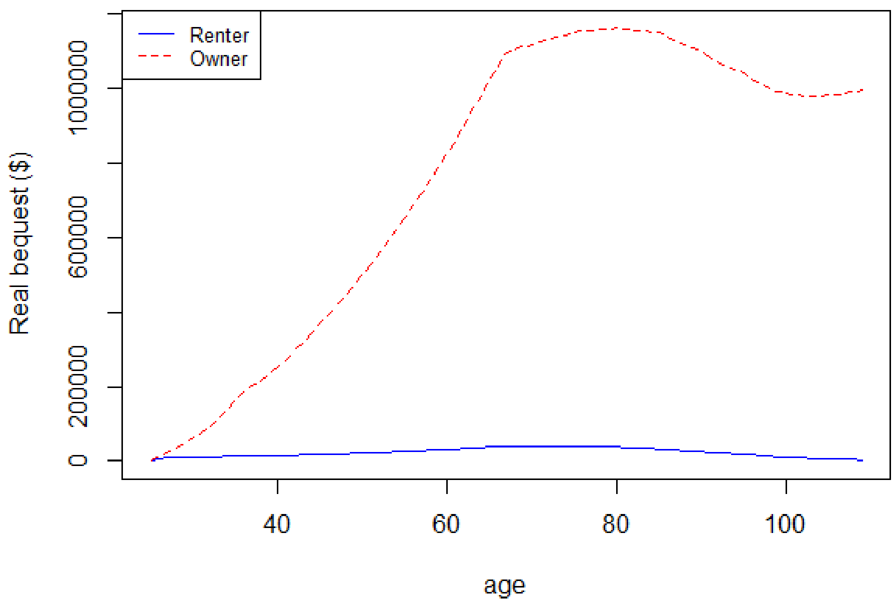

Figure 4C.4: Real bequest for median income percentile both at optimal SG: renter vs owner

4D. Evolution of state variables in Chapter 4

Appendix 4D explains the evolution of state variables in Chapter 4.

The following equations calculate the post-tax income $post.tax_t$ and the total Superannuation Contribution S_t:

$$taxable_t = e_t(1 - \mathrm{SG}) \quad (4D.1)$$

$$post.tax_t = taxable_t - \mathrm{income.tax}(taxable_t) - medicare_t(taxable_t) \quad (4D.2)$$

$$CC_t = \min\left(e_t SG, CC_{cap}\right) \quad (4D.3)$$

$$nonCC_t = \left(e_t - CC_{cap}\right) - \mathrm{income.tax}\left(e_t - CC_{cap}\right)$$

$$-medicare_t\left(e_t - CC_{cap}\right) - post.tax_t \quad (4D.4)$$

$$S_t = CC_t(1 - super_{tax}) + nonCC_t 1_{(e_t * SG \geq CC_{cap})} + \mathrm{LISTO_t} \quad (4D.5)$$

where CC_t is concessional contribution amount and $nonCC_t$ is non-concessional contribution amount; $LISTO_t$ is Low Income Super Tax Offset; $medicare_t$ is the Medicare levy in Equation (4.8). Most of the formulas are quite straightforward, in terms of $nonCC_t$, it only appears when SG of earning is higher than CC cap, S_t is the total superannuation contribution at time t.

At any time $t = 0, 1, \ldots, T_R - 1, T_R, T_R + 1, \ldots, T$[7], where T_R is retirement time and T is termination time, the evolution of state variables for renter and owner change before retirement, at the retirement and after retirement.

Before retirement, renter's consumption, house expenditure and bequest, along with the evolutions of his superannuation and saving balances are shown in the following equations:

$$H.renter_t = rent_t \quad (4D.6)$$

$$sav.renter_t = \mathbf{1}_{(sav.bal.renter_t \leq precaution.sav)}(sav.bal.renter_t) sav.rate.renter * post.tax_t$$

[7] In the model set-up, $t = 0$ corresponds to the age of 25.

$$+\mathbf{1}_{(sav.bal.renter_t > precaution.sav)}(sav.bal.renter_t) * 0 \quad (4D.7)$$

$$C.renter_t = post.tax_t - sav.renter_t - rent_t \quad (4D.8)$$

$$B.renter_t = super.bal.renter_t + sav.bal.renter_t \quad (4D.9)$$

$$super.bal.renter_{t+1} = (super.bal.renter_t + S_t - fee_t)$$

$$[1 + fund.return_t(1 - inv.tax.super_t)] \quad (4D.10)$$

$$sav.bal.renter_{t+1} = (sav.bal.renter_t + sav.renter_t)(1 + cash.rate_t) \quad (4D.11)$$

$$pension.bal.renter_{t+1} = pension.bal.renter_t = 0 \quad (4D.12)$$

where $C.renter_t$, $H.renter_t$ and $B.renter_t$ are renter's pre-retirement consumption, house expenditure and bequest taken into utility calculations at time t; $rent_t$ is the rent cost at time t calculated by Section 4.2.4, Section 4.2.5 and Chapter 3; $sav.renter_t$ is the renter's saving amount at time t; $\mathbf{1}$ is the indicator function that equals 1 when the condition is satisfied, and 0 otherwise; $precaution.sav$ is the precautionary saving in Section 4.2.1; $inv.tax.super_t$ is the superannuation investment tax at time t in Section 4.2.7; $cash.rate_t$ is the cash rate at time t in Chapter 3; $sav.rate.renter$ is the renter's saving rate equal to 10% at time t; $sav.bal.renter_t$ is the renter's saving account balance at time t; $super.bal.renter_t$ is the renter's superannuation account balance at time t; $pension.bal.renter_t$ is the renter's zero-tax pension account balance.

At retirement T_R, the renter's saving and superannuation accounts are combined, and then separated into pension and accumulation accounts following the transfer balance cap rule in Section 2.4.6:

$$total.bal.renter = super.bal.renter_{T_R} + sav.bal.renter_{T_R} \quad (4D.13)$$

$$super.bal.renter.adj = 1_{(total.bal.renter \leq trans.bal.cap)}(total.bal.renter) * 0$$

$$+1_{(total.bal.renter > trans.bal.cap)}(total.bal.renter)$$

$$(total.bal.renter - trans.bal.cap) \quad (4D.14)$$

$$pension.bal.renter.adj = 1_{(total.bal.renter \leq trans.bal.cap)}(total.bal.renter) *$$

$$+1_{(total.bal.renter > trans.bal.cap)}(total.bal.renter)trans.bal.cap \quad (4D.15)$$

$$H.renter_{T_R} = rent_{T_R} \tag{4D.16}$$

$$C.renter_{T_R} = min.withdral.rate_{T_R}(super.bal.renter.adj + pension.bal.renter.adj)$$
$$+age.pension_{T_R} + rent.assist_{T_R} - rent_{T_R} \tag{4D.17}$$

$$B.renter_{T_R} = total.bal.renter \tag{4D.18}$$

$$super.bal.renter_{T_R+1} = \left[super.bal.renter.adj\left(1 - min.withdral.rate_{T_R}\right) - fee_{T_R}\right]$$
$$\left[1 + fund.return_{T_R}\left(1 - inv.tax.super_{T_R}\right)\right] \tag{4D.19}$$

$$pension.bal.renter_{T_R+1} = [pension.bal.renter.adj(1 - min.withdral.rate_{T_R}) - fee_{T_R}]$$
$$\left[1 + fund.return_{T_R}\left(1 - inv.tax.pension_{T_R}\right)\right] \tag{4D.20}$$

where $total.bal.renter$ is the summation of renter's saving and superannuation account balances; $super.bal.renter.adj$ and $pension.bal.renter.adj$ are the accumulation and pension account balances after split; $trans.bal.cap$ is the transfer balance cap in Section 2.4.6; $min.withdral.rate_t$ is the minimum drawdown rules at time t in Section 4.2.10; $age.pension_t$ and $rent.assist_t$ are the Age Pension and the Rent Assistance that the renter is eligible to at time t; fee_t is the fund management fees at time t in Section 4.2.8; $fund.return_t$ is the fund return simulated in Chapter 3; $inv.tax.pension_t$ is the pension investment tax at time t in Section 4.2.7;

The evolution of renter's state variables after retirement are similar to the ones at retirement:

$$H.renter_t = rent_t \tag{4D.21}$$

$$C.renter_t = min.withdral.rate_t(super.bal.renter_t + pension.bal.renter_t)$$
$$+age.pension_t + rent.assist_t - rent_t \tag{4D.22}$$

$$B.renter_t = super.bal.renter_t + pension.bal.renter_t \tag{4D.23}$$

$$super.bal.renter_{t+1} = [super.bal.renter_t(1 - min.withdral.rate_t) - fee_t]$$
$$[1 + fund.return_t(1 - inv.tax.super_t)] \tag{4D.24}$$

$$pension.bal.renter_{t+1} = [pension.bal.renter_t(1 - min.withdral.rate_t) - fee_t]$$

$$[1 + fund.return_t(1 - inv.tax.pension_t)] \quad (4D.25)$$

Before retirement, owner's consumption, house expenditure and bequest, along with the evolution of his superannuation and saving balances are shown in the following equations:

$$H.owner_t = rent_t 1_{(renting\ home\ at\ t)}$$
$$+(rent_t - maintenance.rate * house_t)1_{(owning\ home\ at\ t)} \quad (4D.26)$$

$$sav.owner_t$$
$$= 1_{(renting\ home\ at\ t)} 1_{(sav.bal.owner_t \leq precaution.sav + (upfront.rate + transaction.rate)*house_t)}$$
$$(sav.bal.owner_t) sav.rate.owner * post.tax_t$$
$$+1_{(renting\ home\ at\ t)} 1_{(sav.bal.owner_t > precaution.sav + (upfront.rate + transaction.rate)*house_t)}$$
$$(sav.bal.owner_t) * 0$$
$$+1_{(owning\ home\ at\ t)} 1_{(sav.bal.owner_t \leq precaution.sav)}$$
$$(sav.bal.owner_t) sav.rate.owner * post.tax_t$$
$$+1_{(owning\ home\ at\ t)} 1_{(sav.bal.owner_t > precaution.sav)} (sav.bal.owner_t) * 0 \quad (4D.27)$$

$$C.owner_t = 1_{(renting\ home\ at\ t)} (post.tax_t - sav.owner_t - rent_t)$$
$$+1_{(renting\ home\ at\ t-1)} 1_{(owning\ home\ at\ t)}$$
$$* (post.tax_t - sav.owner_t - maintenance.rate * house_t)$$
$$+1_{(owning\ home\ at\ t-1)} *$$
$$(post.tax_t - sav.owner_t - m_t - maintenance.rate * house_t) \quad (4D.28)$$

$$B.owner_t = super.bal.owner_t + sav.bal.owner_t$$
$$+1_{(owning\ home\ at\ t)} (house_t - L_t) \quad (4D.29)$$

$$super.bal.owner_{t+1} = (super.bal.owner_t + S_t - fee_t)$$
$$[1 + fund.return_t(1 - inv.tax.super_t)] \quad (4D.30)$$

$$sav.bal.owner_{t+1} = 1_{(renting\ home\ at\ t)}(sav.bal.owner_t + sav.owner_t)(1 + cash.rate_t) + 1_{(renting\ home\ at\ t-1)}1_{(owning\ home\ at\ t)}precaution.sav(1 + cash.rate_t) + 1_{(owning\ home\ at\ t-1)}(sav.bal.owner_t + sav.owner_t)$$

$$(1 + cash.rate_t) \tag{4D.31}$$

$$pension.bal.owner_{t+1} = pension.bal.owner_t = 0 \tag{4D.32}$$

where $C.owner_t$, $H.owner_t$ and $B.owner_t$ are owner's pre-retirement consumption, house expenditure and bequest taken into utility calculations at time t; $house_t$ is the house price at time t calculated by Section 4.2.4 and Chapter 3; $maintenance.rate$ is the house maintenance cost rate in Section 4.2.1; $sav.owner_t$ is the owner's saving amount at time t; $upfront.rate$ and $transaction.rate$ are the house purchase upfront payment rate and transaction cost rate in Section 4.2.11; $sav.rate.owner$ is the owner's saving rate equal to 15% at time t; m_t and L_t are the mortgage payment and mortgage loan amount at time t in Section 4.2.11; $sav.bal.owner_t$ is the owner's saving account balance at time t; $super.bal.owner_t$ is the owner's superannuation account balance at time t; $pension.bal.owner_t$ is the owner's zero-tax pension account balance.

At retirement T_R, the owner's saving and superannuation accounts are combined and minus the outstanding mortgage loan if there is any, and then separated into pension and accumulation accounts following the transfer balance cap rule in Section 2.4.6:

$$total.bal.owner = 1_{(renting\ home\ at\ T_R)}(super.bal.owner_{T_R} + sav.bal.owner_{T_R})$$

$$+1_{(owning\ home\ at\ T_R)}(super.bal.owner_{T_R} + sav.bal.owner_{T_R} - L_{T_R}) \tag{4D.33}$$

$$super.bal.owner.adj = 1_{(total.bal.owner \le trans.bal.cap)}(total.bal.owner) * 0$$

$$+1_{(total.bal.owner > trans.bal.cap)}(total.bal.owner)$$

$$(total.bal.owner - trans.bal.cap) \tag{4D.34}$$

$$pension.bal.renter.adj = 1_{(total.bal.owner \le trans.bal.cap)}(total.bal.owner) *$$

$$+1_{(total.bal.owner > trans.bal.cap)}(total.bal.owner)trans.bal.cap \tag{4D.35}$$

$$H.owner_{T_R} = rent_{T_R}1_{(renting\ home\ at\ T_R)}$$

$$+\left(rent_{T_R} - maintenance.rate * house_{T_R}\right)1_{(owning\ home\ at\ T_R)} \quad (4D.36)$$

$$C.owner_{T_R} = 1_{(renting\ home\ at\ T_R)}\Big(min.withdral.rate_{T_R}(super.bal.owner.adj$$

$$+ pension.bal.owner.adj) + age.pension_{T_R} + rent.assist_{T_R} - rent_{T_R}\Big)$$

$$+1_{(owning\ home\ at\ T_R)}(min.withdral.rate_{T_R}$$

$$(super.bal.owner.adj + pension.bal.owner.adj)$$

$$+age.pension_{T_R} - maintenance.rate * house_{T_R}) \quad (4D.37)$$

$$B.owner_{T_R} = 1_{(renting\ home\ at\ T_R)}(super.bal.owner_{T_R} + sav.bal.owner_{T_R})$$

$$+1_{(owning\ home\ at\ T_R)}(super.bal.owner_{T_R} + sav.bal.owner_{T_R} + house_{T_R} - L_{T_R}) \quad (4D.38)$$

$$super.bal.owner_{T_R+1} = \left[super.bal.renter.adj\left(1 - min.withdral.rate_{T_R}\right) - fee_{T_R}\right]$$

$$\left[1 + fund.return_{T_R}(1 - inv.tax.super_{T_R})\right] \quad (4D.39)$$

$$pension.bal.owner_{T_R+1}$$

$$= \left[pension.bal.owner.adj\left(1 - min.withdral.rate_{T_R}\right) - fee_{T_R}\right]$$

$$\left[1 + fund.return_{T_R}(1 - inv.tax.pension_{T_R})\right] \quad (4D.40)$$

where $total.bal.owner$ is the summation of owner's saving and superannuation account balances and minus the mortgage loan is there is any; $super.bal.owner.adj$ and $pension.bal.owner.adj$ are the owner's accumulation and pension account balances after split.

The evolution of owner's state variables after retirement are similar to the ones at retirement:

$$H.owner_t = rent_t 1_{(renting\ home\ at\ t)}$$

$$+(rent_t - maintenance.rate * house_t)1_{(owning\ home\ at\ t)} \quad (4D.41)$$

$$C.owner_t = 1_{(renting\ home\ at\ t)}(min.withdral.rate_t(super.bal.owner_t + pension.bal.owner_t) + age.pension_t + rent.assist_t - rent_t) + 1_{(owning\ home\ at\ t)}(min.withdral.rate_t(super.bal.owner_t + pension.bal.owner_t) + age.pension_t - maintenance.rate * house_t) \quad (4D.42)$$

$$B.owner_t = 1_{(renting\ home\ at\ t)}(super.bal.owner_t + sav.bal.owner_t) + 1_{(owning\ home\ at\ t)}(super.bal.owner_t + sav.bal.owner_t + house_t) \quad (4D.43)$$

$$super.bal.owner_{t+1} = [super.bal.owner_t(1 - min.withdral.rate_t) - fee_t][1 + fund.return_t(1 - inv.tax.super_t)] \quad (4D.44)$$

$$pension.bal.owner_{t+1} = [pension.bal.owner_t(1 - min.withdral.rate_t) - fee_t][1 + fund.return_t(1 - inv.tax.pension_t)] \quad (4D.45)$$