



Mitigation of phase noise and Doppler-induced frequency offsets in coherent random amplitude modulated continuous-wave LiDAR

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Abstract: We present a detailed analysis of techniques to mitigate the effects of phase noise and Doppler-induced frequency offsets in coherent random amplitude modulated continuous-wave (RAMCW) LiDAR. The analysis focuses specifically on a technique which uses coherent dual-quadrature detection to enable a sum of squares calculation to remove the input signal's dependence on carrier phase and frequency. This increases the correlation bandwidth of the matched-template filter to the bandwidth of the acquisition system, whilst also supporting the simultaneous measurement of relative radial velocity with unambiguous direction-of-travel. A combination of simulations and experiments demonstrate the sum of squares technique's ability to measure distance with consistently high SNR, more than 15 dB better than alternative techniques whilst operating in the presence of otherwise catastrophic phase noise and large frequency offsets. In principle, the technique is able to mitigate any sources of phase noise and frequency offsets common to the two orthogonal outputs of a coherent dual-quadrature receiver including laser frequency noise, speckle-induced phase noise, and Doppler frequency shifts due to accelerations.

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1. Introduction

Light detection and ranging (LiDAR) is a remote sensing technology used to generate high-resolution three-dimensional maps of the environment. LiDAR is used in a variety of applications including aerial surveying [1], remote asset inspection [2], wind velocimetry [3–5], and autonomous vehicle navigation [6,7]. LiDAR sensors estimate distance by measuring the round-trip time-of-flight of a specific time-varying attribute of a laser.

LiDAR sensors which use incoherent (or *direct*) detection are sensitive only to the intensity of the received light, and thus the time-varying attribute for these types of systems is laser intensity. Examples of incoherent LiDAR include pulsed [8,9], amplitude-modulated continuous wave (AMCW) [10,11], and random modulation continuous wave (RMCW) [12,13].

Sensors that use coherent (or *interferometric*) detection are sensitive to not just intensity, but also the phase (and thus frequency) of the received light. Coherent LiDAR is therefore capable of measuring both distance *and* relative radial velocity caused by relative motion between the illuminated object and the sensor which introduces a measurable Doppler shift to the light. Examples of coherent LiDAR include amplitude modulated pulse train [14], frequency modulated continuous wave (FMCW) [15–19], and chirped pulse compression [12,20–22].

Random modulation continuous wave (RMCW) LiDAR using incoherent detection was first proposed by Takeachi et al. in 1983 [23], in which time-of-flight is measured by calculating the correlation between the received light and a pseudo-random pattern that is modulated onto the intensity of the outgoing light. Light returning to a receiver is captured via direct detection, for example using an avalanche photodetector or photomultiplier tube. One advantage of RMCW LiDAR over pulsed LiDAR is an improved immunity to interference, as the correlation measurement is relatively insensitive to spurious light (e.g., light from other LiDAR sensors, glint from the sun). A further advantage is RMCW LiDAR's ability to resolve multiple time-separated objects within a single measurement. Other examples of incoherent RMCW LiDAR can be found in [13,24,25].

RMCW LiDAR estimates distance by correlating the received signal with a local copy of the pseudo-random bit sequence used to encode the outgoing light [26]. This process is also referred to as matched-template filtering. Maximal length sequences (or *m-sequences* for short) are often used in RMCW LiDAR for their excellent auto-correlation properties including high peak-to-side lobe ratio (PSLR), and efficient generation in digital signal processing [27]. Other sequences including Legendre sequences [28,29] and A* sequences [25,30] have also been demonstrated.

The work presented in this article focuses on a technique called coherent Random Amplitude Modulated Continuous Wave (RAMCW) LiDAR in which a pseudo-random pattern is encoded onto the amplitude of the laser, and received light (referred to as the *probe*) is interfered with a local oscillator at a photodetector [31]. Interfering the probe with a bright local oscillator coherently amplifies the received probe signal, allowing measurement sensitivity to be limited by quantum effects rather than technical noise sources such as photodetector thermal noise. In this regime, the shot noise from the local-oscillator is the dominating noise source, and is referred to as shot-noise limited detection.

A key property that differentiates coherent RAMCW LiDAR from incoherent RMCW LiDAR is its sensitivity to the relative phase between the probe and local oscillator fields. This makes coherent LiDAR sensitive to relative motion between the sensor and the illuminated object, which introduces a Doppler frequency shift to the probe relative to the local oscillator. With knowledge of the laser wavelength, a measurement of the Doppler frequency shift can be used to estimate relative radial velocity. Sensitivity to phase also makes coherent LiDAR sensitive to laser frequency noise and random phase fluctuations in the optical system [32–34].

Whilst carrying potentially valuable information about relative velocity, frequency offsets are known to degrade the correlation properties of pseudo-random sequences and the reliability with which distance can be measured [28,35,36]. One approach proposed by Holmes et al. in [36] involves decoding the received signal at every delay in the pseudo-random bit sequence, and then calculating the fast Fourier transform (FFT) for each decoded signal to simultaneously identify the correct delay and frequency offset. In [28] Shemer et al. suggest calculating the root-mean-square sum of the four outputs of a dual-polarization 90 degree optical hybrid receiver, albeit without thorough analysis. In [37] Scotti et al. use a sum of squares approach to recover instantaneous power for a coherent RAMCW LiDAR sensor, but do not extend or analyse the technique's use for mitigating phase noise and frequency offsets. A similar solution is presented by Yang et al. in [21] to overcome random phase fluctuations in coherent amplitude pulse-compression LiDAR to improve SNR, reduce false alarm rate, and improve measurement precision.

In this article, we analyse four different techniques that can be used to overcome the effects of Doppler-induced frequency offsets and phase noise (e.g., due to laser frequency noise, and speckle-induced phase noise [34]) in coherent RAMCW LiDAR. The analysis converges on one technique in particular, in which the in-phase and quadrature signals produced by a dual-quadrature coherent optical receiver (e.g., a 90 degree optical hybrid) are squared and summed to remove sensitivity to carrier phase.

The paper is structured as follows: Section 2 provides an overview of different coherent RAMCW LiDAR configurations using both homodyne and heterodyne detection. Section 3 motivates the need for phase noise and frequency offset compensation in coherent RAMCW LiDAR. Section 4 introduces and analyses the performance of four different techniques that may be used to compensate for the effects of frequency offsets in correlation measurements. Section 5 introduces the sum of squares technique for mitigation of phase noise, accompanied by a detailed comparison with the techniques presented in Section 3. Section 6 presents results from a series of experimental demonstrations of the techniques introduced in Sections 4 and 5. The conclusion is presented in Section 7.

2. Coherent optical configurations

Coherent LiDAR is based on the principles of laser interferometry, which can be separated into two broad detection classifications: i) heterodyne detection; and ii) homodyne detection.

2.1. Heterodyne detection

Heterodyne detection works by introducing an *intentional* frequency shift ω_h to either the probe or local oscillator fields, forcing their interfered waveform to oscillate at the heterodyne frequency. The example optical configuration of a coherent random amplitude modulation LiDAR sensor with heterodyne detection is shown in Fig. 1, where an acousto-optic modulator (AOM) is used to shift the frequency of the local oscillator by $\omega_h = 2\pi f_h$.

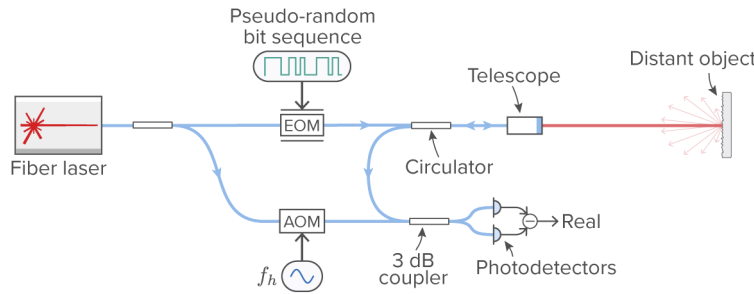


Fig. 1. Example optical configuration of a coherent RAMCW LiDAR sensor employing heterodyne detection. An electro-optic modulator is used to encode a pseudo-random bit sequence onto the amplitude of the probe signal (upper arm in the figure). An acousto-optic modulator (AOM) is used to shift the frequency of the local oscillator (lower arm) relative to the probe. The interference of the received probe electric field and the local oscillator at a balanced photodetector produces a heterodyne beatnote which oscillates at the heterodyne frequency $\omega_h = 2\pi f_h$.

The electric field equation describing the frequency shifted local oscillator as it arrives at the 3 dB coupler can be expressed as

$$E_{LO}(t) = E_{LO} e^{j(\omega_h t + \phi_{LO})},$$

where E_{LO} is the electric field magnitude, ω_h is the heterodyne frequency, and ϕ_{LO} represents its phase.

The amplitude of the probe electric field $E_P(t)$, which is encoded with a pseudo-random bit sequence denoted by $c(t) \in [0, 1]$ with modulation depth $\alpha \in [0, 1]$, travels to and from an object

before arriving at the 3 dB coupler after some time delay τ

$$E_P(t) = E_P(1 - \alpha c(t - \tau))e^{j(\Delta\omega t + \phi_P)},$$

where ϕ_P represents the phase of the received probe field and $\Delta\omega$ represents a constant Doppler frequency shift due by relative motion between the sensor and illuminated object.

Interfering the probe and local oscillator fields at a balanced photodetector produces a voltage waveform

$$V_{PD}(t) \propto P_{LO}P_P(1 - \alpha c(t - \tau)) \cos((\omega_h - \Delta\omega)t + \phi_{LO} - \phi_P),$$

where P_{LO} and P_P represent the power of the local oscillator and probe at the detector. Distance information is contained within the time delay τ , whereas velocity information is contained within the Doppler frequency component $\Delta\omega$.

The velocity and direction-of-travel of an illuminated object can be unambiguously resolved by observing the frequency of the beatnote produced by the photodetector *relative* to the heterodyne frequency.

2.2. Homodyne detection

With homodyne detection, no intentional frequency difference is introduced between the probe and local oscillator. The optical configuration for a coherent RMCW LiDAR sensor with homodyne detection is shown in Fig. 2. A dual-quadrature receiver (e.g., a 90 degree optical hybrid) is used to produce waveforms that are out of phase by 90 degrees, allowing velocity and direction-of-travel to be unambiguously resolved as demonstrated by Abari et al. in [5]. Dual-quadrature detection is also referred to as complex detection (e.g., [5,18,20]). Other techniques for dual-quadrature detection such as the time-separated dual-quadrature detection scheme presented by Sutton et al. in [38] may also be used.

The electric field equations describing the local oscillator and probe signals arriving at the dual-quadrature receiver are

$$E_{LO}(t) = E_{LO}e^{j\phi_{LO}}$$

and

$$E_P(t) = E_P(1 - \alpha c(t - \tau))e^{j(\Delta\omega t + \phi_P)}$$

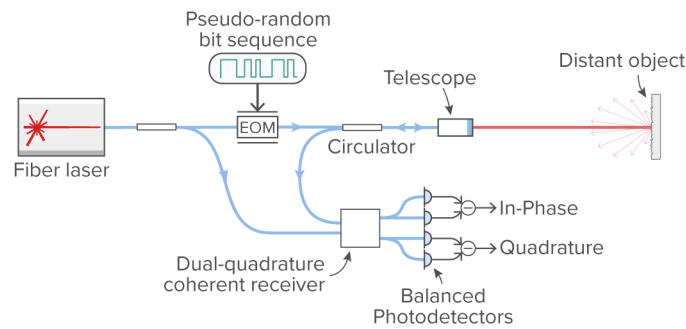


Fig. 2. Example optical configuration of a coherent RMCW LiDAR sensor with dual-quadrature detection. The amplitude of the probe signal (upper arm in the figure) is encoded with a pseudo-random bit sequence. The interference of the received probe light and local oscillator by a pair of balanced photodetectors located at the output of a dual-quadrature coherent receiver (e.g., a 90 degree optical hybrid) produces in-phase and quadrature projections of the probe field relative to the local oscillator. A dual-quadrature coherent receiver is required with homodyne detection to resolve both the velocity and direction of travel caused by relative motion between the sensor and a distant object.

respectively. Interfering the two outputs of the dual-quadrature receiver at a pair of balanced photodiodes produces voltage waveforms proportional to the in-phase and quadrature projections of the probe field relative to the local oscillator:

$$\begin{aligned} V_I(t) &\propto P_{LO}P_P(1 - \alpha c(t - \tau)) \cos(\Delta\omega t + \phi_{LO} - \phi_P), \\ V_Q(t) &\propto P_{LO}P_P(1 - \alpha c(t - \tau)) \sin(\Delta\omega t + \phi_{LO} - \phi_P). \end{aligned}$$

As with heterodyne detection, distance can be estimated by measuring the delay τ , and velocity can be estimated by measuring the Doppler frequency component $\Delta\omega$.

3. How frequency offsets affect measurement performance

Random amplitude modulated continuous wave LiDAR estimates distance using matched-template filtering, in which the received signal is effectively correlated with a locally stored *template* waveform. The output of the matched-template filter is an estimate of signal correlation. Distance is extracted from the correlation by identifying the delay that corresponds to a specific correlation peak.

Matched-template filtering requires a minimum degree of similarity between the received signal and template waveform. Distortion of the received waveform due to amplitude and phase noise degrade the performance of matched-template filters. In particular, large frequency offsets due to relative motion between the LiDAR sensor and the illuminated object can have a significant impact on correlation, since the frequency modulated received signal no longer *matches* the template. In addition to Doppler, disturbances in optical phase due to laser frequency noise, fiber phase noise and random phase fluctuations imposed by diffuse scattering all contribute to a degradation in matched-template filtering performance.

The degradation in correlation due to frequency offsets is well described by the ambiguity function $\chi(k, \theta)$ of a pseudo-random bit sequence, which represents correlation as a function of both delay and offset frequency. The normalised ambiguity function for a particular 10-bit m-sequence is shown in Fig. 3 (top), along with constrained illustrations of correlation as a function of delay with zero frequency offset (bottom left) and correlation as a function of normalised offset frequency at zero delay (bottom right).

For zero offset frequency, the normalised auto-correlation function resembles that of an N-bit m-sequence:

$$A(\tau) = \begin{cases} 1 & \text{for } \tau = 0 \\ -1/(2^N - 1) & \text{elsewhere} \end{cases}$$

At zero relative delay the normalised correlation function as a function of relative offset frequency follows a sinc profile which is expected because correlation acts as a filter and thus has a predictable frequency response. The correlation $R(\tau)$ of a sequence with zero frequency offset $x[nT_s]$ and the same sequence with a relative frequency offset $y[nT_s]$ is

$$R(kT_s) = \sum_{k=0}^{RT_s} x[nT_s]y[(n - k)T_s], \quad (1)$$

where the index $k \in \mathbb{Z}$ represents relative delay, T_s represents the discrete-time sampling period, and R represents the length of the sequence. Equation 6 shows that the correlation calculation includes a summation over the period RT_s , which is mathematically equivalent to a constant coefficient averaging filter. Normalized correlation as a function of offset frequency, assuming a

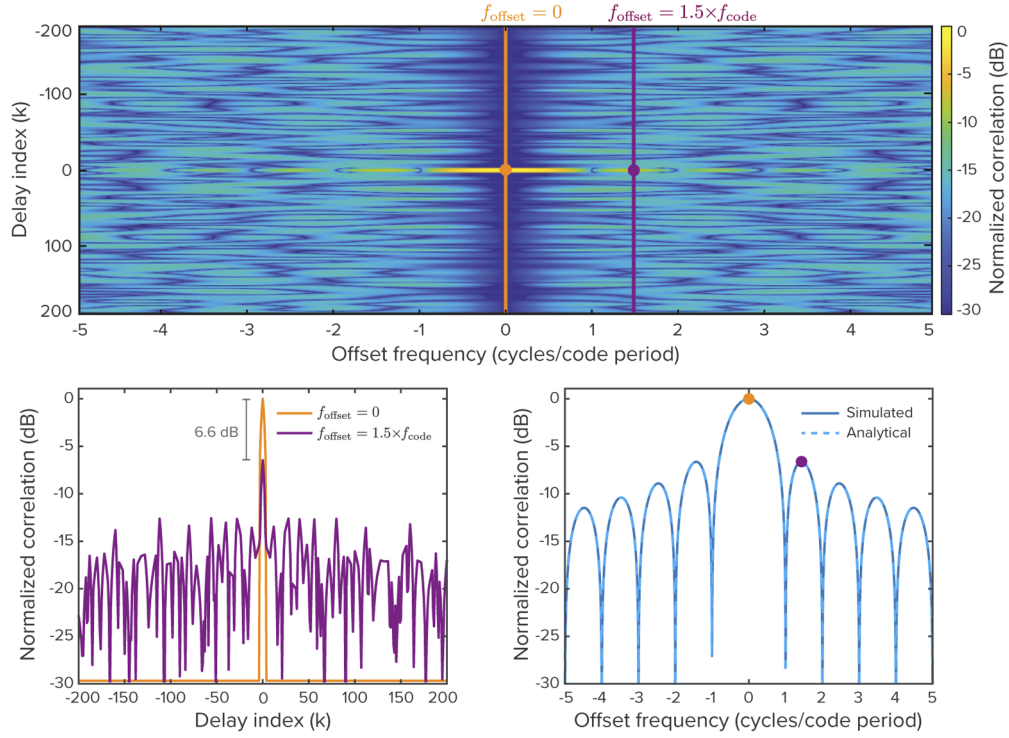


Fig. 3. (Top) Correlation ambiguity function of a 10-bit m-sequence as a function of delay index k and relative offset frequency θ . The frequency axis is normalised to the period of the m-sequence (f_{code}) in units of cycles per code period. (Bottom left) Normalized correlation profiles extracted from the ambiguity function for frequency offsets of zero and $1.5f_{\text{code}}$ as indicated by the solid vertical lines in the top figure. (Bottom right) Numerically simulated and analytically predicted normalized correlation as a function of offset frequency at a relative delay of $k = 0$. The green and red markers at offset frequencies 0 and 1.5 correspond to the equivalent markers in the top plot.

relative delay $\tau = 0$, can thus be described by the equation

$$A(f)_{\tau=0} = \left| \frac{\sin(\pi R f / f_s)}{\sin(\pi f / f_s)} \cdot e^{-j\pi(R-1)\frac{f}{f_s}} \right|, \quad (2)$$

where $f_s = 1/T_s$ is the sampling frequency and f represents offset frequency. According to Eq. (2), the magnitude of the correlation equals zero whenever $\sin(\pi R f / f_s) = 0$. This occurs whenever the offset frequency is an integer multiple of the code repetition frequency:

$$f_{\text{code}} = \frac{f_s}{R}.$$

As an example, consider a pseudo-random bit sequence with duration $T_{\text{code}} = 1/f_{\text{code}} = 10 \mu\text{s}$. According to Eq. (2), the correlation function will experience nulls at relative offset frequencies equal to integer multiples of $f_{\text{code}} = 100 \text{ kHz}$. At a laser wavelength of $\lambda = 1550 \text{ nm}$, a Doppler frequency shift of $\Delta f = 100 \text{ kHz}$ corresponds to a relative radial velocity of

$$\begin{aligned} \Delta v &= \frac{\lambda f_{\text{code}}}{2} = \frac{1550 \text{ nm} \times 100 \text{ kHz}}{2} \\ &= 0.0775 \text{ m/s} \end{aligned}$$

highlighting just how sensitive coherent RAMCW LiDAR is to relative motion. The Doppler response of the sensor is dependent on f_{code} , which is proportional to the length of the code and inversely proportional to code chip rate. Increasing f_{code} will provide more inherent immunity against Doppler since the null in the Doppler response will be increased. However, this requires either a shorter code (which would decrease maximum unambiguous sensing range) or faster code rate (which will also require faster front-end electronics and sampling rates). Equation 2 reveals that even small frequency offsets can produce large side-lobes in the correlation measurement. It is therefore crucial to compensate for frequency offsets due to laser frequency noise and Doppler to maintain an acceptable measurement quality.

4. Compensating for frequency offsets

In this section, we present an analysis of three different methods that can be used to compensate for frequency offsets, presented in comparison to the case where offsets are not compensated. These techniques include: 1) dual-quadrature demodulation of the input signal; 2) modulation of the template; and 3) complex conjugate demodulation of the input signal. For the purposes of this analysis, we assume total and perfect *a priori* knowledge of the received signal's phase and frequency.

Figure 4 illustrates the signal processing diagram for the reference case when frequency offsets are not compensated. In this case, the input signal is correlated against the pseudo-random bit sequence that is encoded onto the amplitude of the received signal.

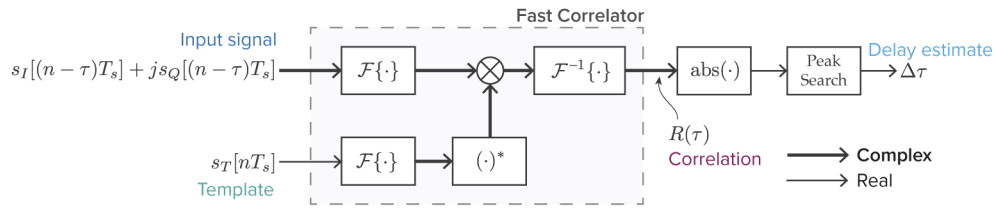


Fig. 4. Signal processing diagram of a matched-template filter implemented using a fast correlator [39] where the in-phase and quadrature input signals are correlated against a template pseudo-random bit sequence.

4.1. Input signal demodulation (1)

The first method that can be used to compensate for frequency offsets is to demodulate the input signal at the carrier's offset frequency as shown in Fig. 5. This method relies on precise knowledge of the carrier frequency to down-shift the input signal to baseband (which in this case is DC).

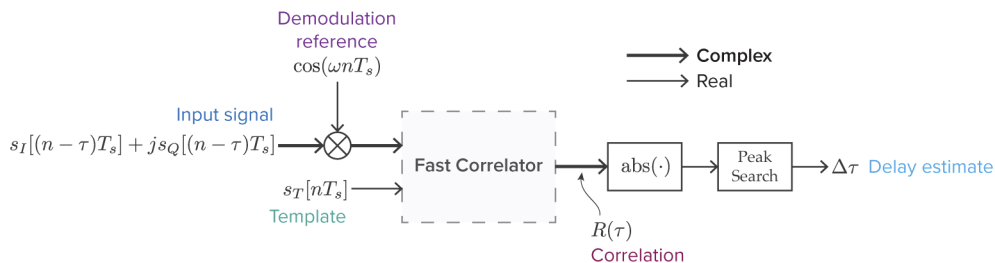


Fig. 5. Signal processing diagram illustrating input signal demodulation (technique 1) of the input signal at the offset carrier frequency ω .

Consider two discretely sampled orthogonal in-phase and quadrature signals with pseudo-random bit sequence $c[nT_s] \in [0, 1]$ encoded into the amplitude with modulation depth α , angular frequency ω and phase ϕ , and where n and T_s represent sample index and period respectively:

$$\begin{aligned} s_I[nT_s] &= (1 - \alpha c[(n - \tau)T_s]) \cos(\omega nT_s + \phi), \\ s_Q[nT_s] &= (1 - \alpha c[(n - \tau)T_s]) \sin(\omega nT_s + \phi). \end{aligned}$$

Demodulating these signals with a reference signal $s_R[nT_s] = \cos(\omega nT_s)$ yields:

$$s_I[nT_s]s_R[nT_s] = \frac{1}{2}(1 - \alpha c[(n - \tau)T_s])(\cos(\phi) + \cos(2\omega nT_s + \phi)), \quad (3)$$

$$s_Q[nT_s]s_R[nT_s] = \frac{1}{2}(1 - \alpha c[(n - \tau)T_s])(\sin(\phi) + \sin(2\omega nT_s + \phi)). \quad (4)$$

A consequence of demodulation is the generation of an up-shifted component containing half the total amplitude of the demodulated signal, and which carries the same pseudo-random bit sequence encoded into its amplitude but at a higher frequency. We refer to the decrease in amplitude of the down-shifted component by a factor of two as heterodyne loss. When correlated with a baseband template of the pseudo-random bit sequence, the up-shifted component will introduce side-lobes to the correlation measurement (as described in Section 3.) and thus contribute to a reduction in signal-to-noise ratio.

4.2. Template modulation (2)

The second method to compensate for frequency offsets is to modulate the template pseudo-random bit sequence at the carrier's offset frequency. The modulated template is then correlated with the modulated input signal as shown in Fig. 6. The phase of the modulated template signal is unimportant as the in-phase and quadrature components of the input signal are available. The modulated template signal thus has the form

$$s_T[nT_s] = p[nT_s] \cos(\omega nT_s),$$

where ω represents the offset frequency and $p[nT_s] \in [-1, 1]$ is the pseudo-random bit sequence. The template signal $s_T[nT_s]$ uses an antipodal sequence with a range from -1 to $+1$ to maximise energy in code harmonics by minimising power at the carrier frequency ω to maximise correlation.

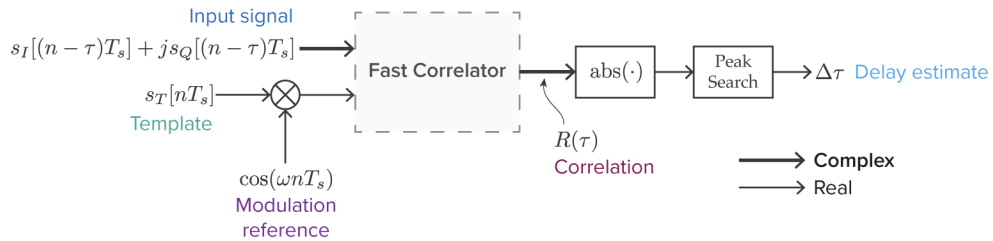


Fig. 6. Signal processing diagram illustrating template modulation (technique 2) at the offset carrier frequency ω .

4.3. Complex demodulation (3)

The final way to compensate for frequency offsets (and any form of phase noise) is to demodulate the complex representation of the input signal with a complex conjugate of the unencoded carrier

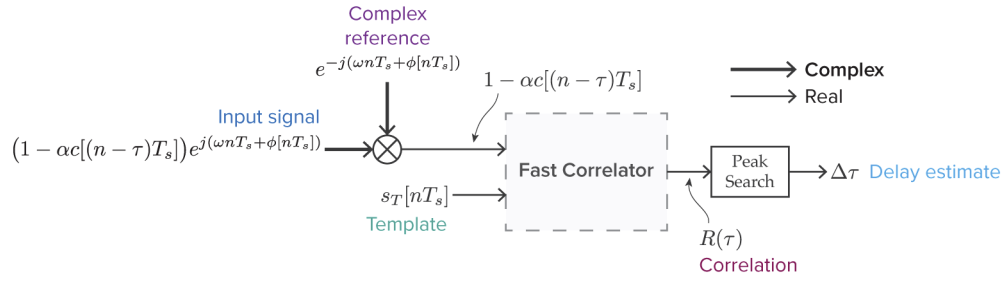


Fig. 7. Signal processing diagram showing the demodulation of the input signal with a complex carrier (technique 3) assuming total knowledge of both offset frequency ω and signal phase ϕ .

[Fig. 7]

$$(s_I[nT_s] + js_Q[nT_s]) e^{-j(\omega n T_s + \phi)} = 1 - \alpha c[(n - \tau)T_s],$$

where

$$s_I[nT_s] + js_Q[nT_s] = (1 - \alpha c[(n - \tau)T_s]) e^{j(\omega n T_s + \phi)}.$$

This technique has the advantage of not producing an up-shifted frequency component and the side lobes that come with it. Despite this advantage, this technique requires total knowledge of the input signal's frequency and phase which may be difficult to measure depending on, for example, input carrier-to-noise ratio.

4.4. Performance comparison

Numerical simulations of the techniques presented above were performed to investigate and compare their normalised correlation as a function of frequency offset with assumed perfect knowledge of the carrier's offset frequency and phase.

The results for each technique are presented in terms of correlation signal-to-noise ratio, which is defined as the ratio of the power in the signal S^2 (in this case the peak correlation) to the total power in the noise (which is defined as the mean-square value of the noise floor in the delay space) $E[\eta^2(\tau)]$, as is standard for matched-template filtering [40]:

$$\text{SNR} = \frac{S^2}{E[\eta^2(\tau)]}.$$

The simulation assumes a 10-bit maximal-length sequence of length $2^{10} - 1 = 1023$ and a chip frequency of $f_{\text{chip}} = 62.5$ MHz. The code repetition frequency is $f_{\text{code}} = 61.095$ kHz.

Monte-Carlo simulations were performed with a carrier-to-noise ratio of 0 dB, with additive white Gaussian noise used to simulate shot-noise limited detection. The results are presented with no averaging, however we acknowledge that averaging the measurement N times is expected to improve SNR by a factor of $\sqrt{N}$.

The results of these simulations is presented in Fig. 8 where SNR is normalised to the length of the pseudo-random bit sequence.

At a frequency offset of 100 Hz (which is close to DC), the four techniques are mathematically equivalent and thus result in the same SNR. The SNR of each technique begins to diverge as the frequency offset approaches the repetition frequency of the pseudo-random bit sequence, f_{code} .

In the reference case (i.e., no compensation), nulls appear at integer multiples of $f_{\text{code}} = 61.095$ kHz as predicted by Eq. (2). Noise power increases as the frequency offset increases beyond f_{code} due to the presence of sidelobes introduced by the correlation of a frequency offset carrier with a DC template as presented in Fig. 3.

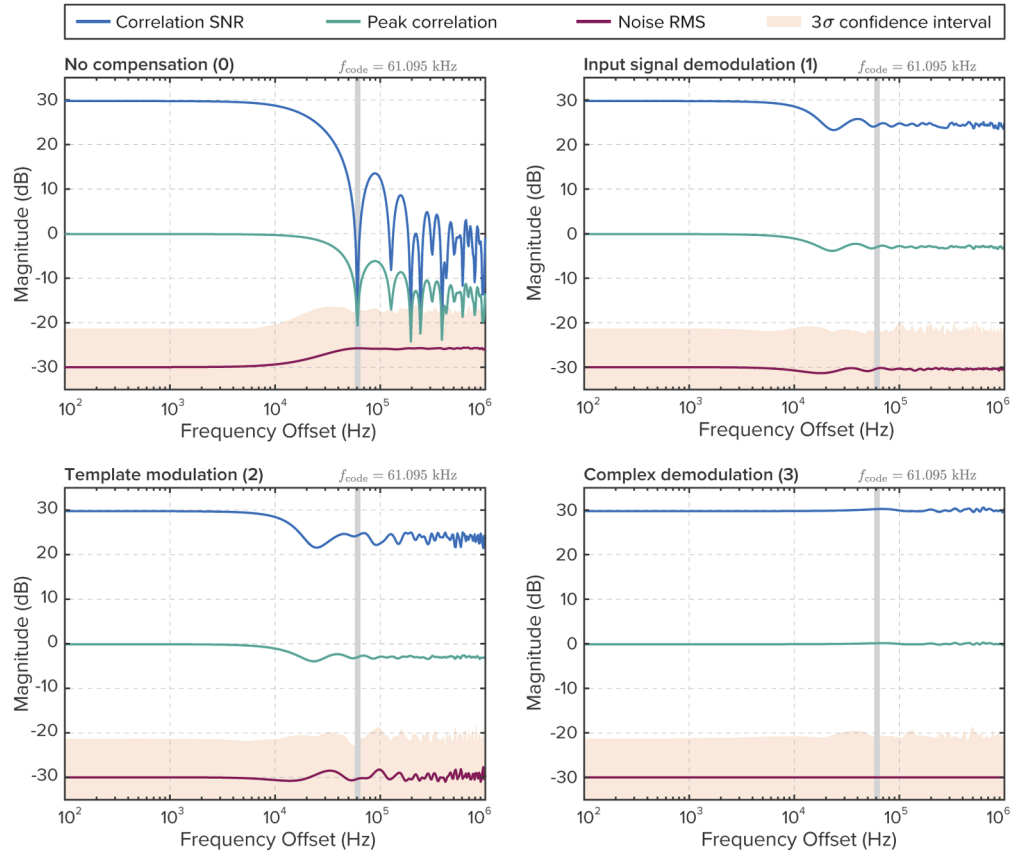


Fig. 8. Simulation results for the first 3 potential frequency correction techniques in comparison to the reference case with no compensation were undertaken with complete *a priori* knowledge of the signal phase, frequency and amplitude to allow for a perfect demodulation template to be constructed for each technique to identify the optimum signal-to-noise ratio. All simulation were undertaken at a carrier-to-noise ratio of 0 dB, 10-bit M-sequence modulated at a chip rate of 62.5 MHz, shot-noise limited detection and a sampling rate of 250 MS/s. The frequency axis is presented in Hz, but can also be considered in terms of integer multiple of the FFT period (PRBS code period) which is $f_{code} = 61.094$ kHz or 0.047 m/s of relative radial velocity.

When the input signal is demodulated (technique 1) using a frequency offset carrier (Section 4.1), the peak correlation power decreases by 6 dB (a factor of two in amplitude) as the frequency offset increases beyond f_{code} due to heterodyne loss. Oscillations in the observed SNR around -6 dB are caused by the correlation of the up-shifted product of the demodulation with a DC template as expected from Fig. 3.

In the case where the template is modulated (technique 2) with a frequency shifted carrier (Section 4.2), we observe the same reduction in peak power due to heterodyne loss as for input signal demodulation. However, because this approach effectively correlates two carrier-modulated pseudo-random bit sequences, it does not fully exploit the sequence's auto-correlation properties. As a consequence, the peak correlation power fluctuates unpredictably due to sidelobes introduced by the correlation of two up-shifted sequences.

In the optimal case when the input signal is demodulated using a complex carrier (technique 3) with full knowledge of the input signal's frequency and phase (Section 4.3), there is no generation of an up-shifted component and thus no heterodyne loss. The peak correlation power therefore remains uniform across all simulated offset frequencies.

5. Mitigation of phase noise and frequency offsets via sum of squares (4)

An alternative approach to overcoming the effects of frequency offsets is to remove any dependence the dual-quadrature input signal has on phase and frequency. With a complex representation of the input signal, this can be done by summing the squares of the in-phase and quadrature components as shown in Figs. 9 and 10:

$$\begin{aligned} s_I^2[nT_s] + s_Q^2[nT_s] &= (1 - \alpha c[(n - \tau)T_s])^2 (\cos^2(\omega nT_s + \phi) + \sin^2(\omega nT_s + \phi)), \\ &= (1 - \alpha c[(n - \tau)T_s])^2. \end{aligned} \quad (5)$$

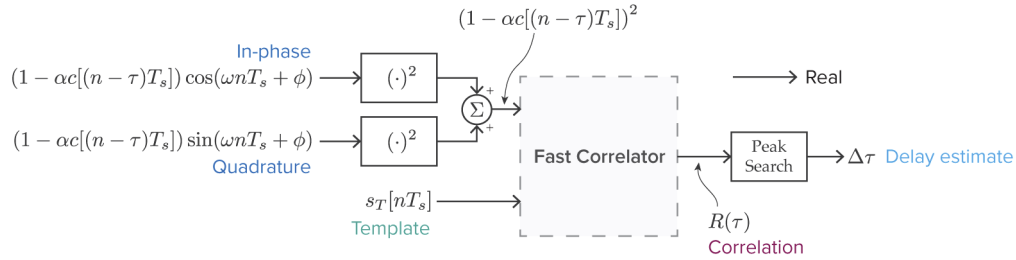


Fig. 9. Signal processing diagram for the sum of squares frequency mitigation technique which is used to eliminate common phase terms shared by the in-phase and quadrature input signals.

The sum of squares technique was originally proposed in 1988 by Kazovsky et al. in [41] to enable wide-bandwidth phase-diverse coherent homodyne reception for coherent laser communication systems. The same approach has since been used to overcome sensitivity to phase in coherent LiDAR sensors—for example by Scotti et al. in [37]—however they did not present an analysis of the technique's performance under different phase noise conditions and frequency offsets. A similar concept was demonstrated in 2019 by Yang et al. in [20] to overcome sensitivity to phase fluctuations in coherent amplitude pulse-compression LiDAR which was shown to increase SNR, reduce the probability of false alarm, and improve measurement precision.

Summing the squares of the orthogonal input signals is similar in principle to complex demodulation presented in Section 4.3, however it has the considerable advantage of not requiring prior knowledge of the phase and frequency of the input carrier.

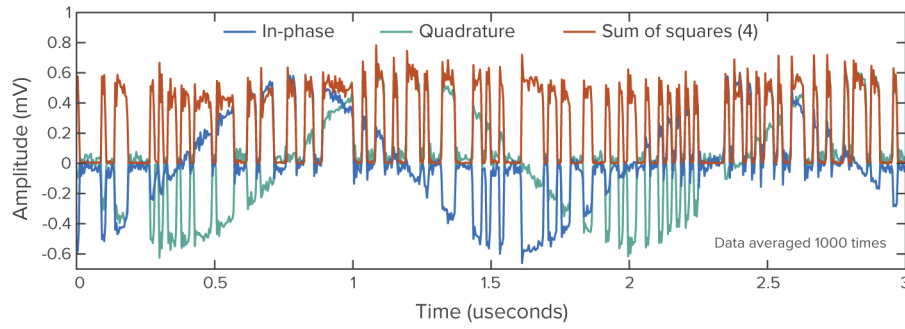


Fig. 10. Time-domain measurement of in-phase (light blue) and quadrature (dark blue) waveforms modulated at approximately 10 times the code repetition frequency, $f_{\text{offset}} \approx 10f_{\text{code}}$. The data was averaged on device over 1000 simultaneous correlation measurement frames to clearly resolve the pseudo-random bit sequence. Taking the sum of squares of the in-phase and quadrature signals produces the unmodulated pseudo-random bit sequence (orange).

The correlation SNR of the sum of squares technique was simulated using the same parameters as in Section 4.4 for offset frequencies ranging from 100 Hz to 1 MHz at an input carrier-to-noise ratio of 0 dB. As can be seen in the results presented in Fig. 11, summing the squares of the in-phase and quadrature input signals effectively mitigates the effects of frequency offsets. The results show consistently high SNR across all simulated offset frequencies, with fluctuations caused by noise being more pronounced at higher frequencies due to the logarithmic scale on which the data is plotted. However, whilst summing the squares is demonstrably a promising technique, a comparison with the results shown in Fig. 8 reveals an SNR penalty of approximately 4 dB compared to the techniques presented in Section 4 due to this technique's increased susceptibility to noise at low input carrier-to-noise ratio.

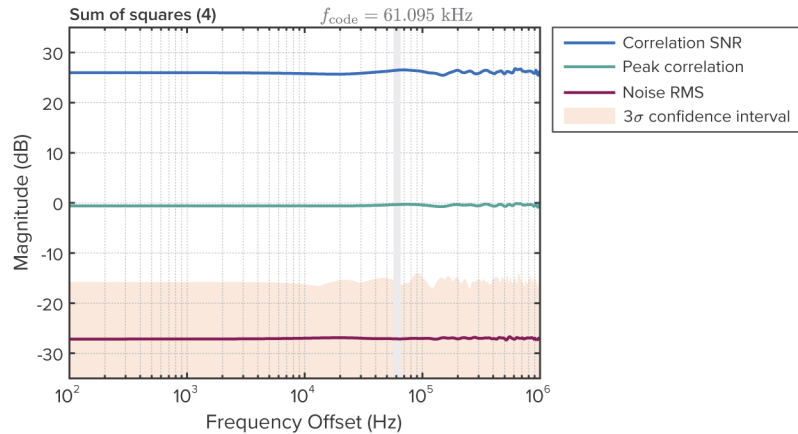


Fig. 11. Simulation results with novel frequency correction using sum of squares (4) showing uniform pass-band response without prior knowledge of the signal phase, frequency and amplitude. Simulation was undertaken at a carrier-to-noise ratio of 0 dB, 10-bit M-sequence modulated at a chip rate of 62.5 MHz, shot-noise limited detection and a sampling rate of 250 MS/s with a resultant $f_{\text{code}} = 61.095$ kHz.

5.1. Sensitivity to input carrier-to-noise ratio

The sum of squares method presented was compared to the techniques presented in Section 4 using Monte-Carlo simulations. The correlation SNR for each technique (including the reference case without compensation) was measured for a range of carrier-to-noise ratios spanning -10 dB to 30 dB at two specific offset frequencies: DC and 100 times f_{code} . The DC frequency offset was selected for analysis as it provides valuable insight into comparative performance at low carrier-to-noise ratios. A frequency offset of 100 times the code repetition frequency was selected because it is arbitrarily far away from DC. The results of the Monte-Carlo simulations is presented in Fig. 12.

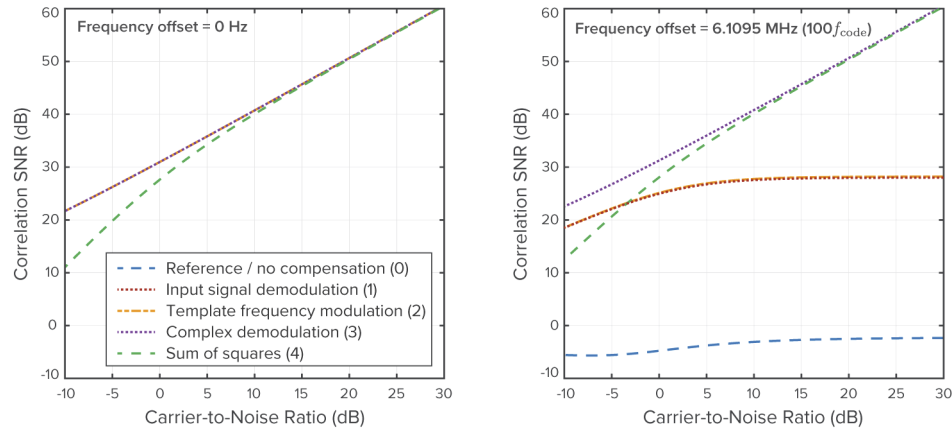


Fig. 12. Simulation results for the comparison of the correlation signal-to-noise ratio performance of the novel sum of squares frequency correction technique (4) and the aforementioned techniques as a function of input carrier-to-noise ratio. Frequency analysis at 0 Hz offset is shown to demonstrate the negative SNR effects of the squaring operation, with a frequency offset of 100 times the code repetition frequency selected because it is arbitrarily far away from DC. For the 0 Hz offset analysis, techniques (0-3) appear overlaid since they perform the same. The complex demodulation (3) technique is assumed to have priori knowledge of the signal frequency and phase for optimal correction.

At zero frequency offset, $f_{\text{offset}} = 0$ Hz, signal demodulation, template modulation and complex demodulation all yield the same correlation SNR across all input carrier-to-noise ratios, whereas the SNR for the sum of squares frequency mitigation technique begins to decrease more rapidly for CNRs below ~ 10 dB. Assuming shot-noise limited detection, a CNR of 0 dB corresponds to approximately to ~ 3 pW of received input signal power. This power level was inferred based on the correlation noise profile that had been calibrated against prior higher received optical powers.

At $f_{\text{offset}} = 100f_{\text{code}}$, the signal demodulation and template modulation compensation techniques plateau due to the presence of correlation side-lobes introduced by modulated code terms (e.g., the up-shifted higher frequency components in Eqs. (3) and 4). The reference technique has low correlation SNR that is due to additive white Gaussian noise for CNR below ~ 5 dB, and code side-lobes for CNR above 5 dB. In contrast, both the complex demodulation and sum of squares techniques result in ideal correlation SNR at high input CNRs. However, as was observed for zero frequency offset, the correlation SNR of the sum of squares technique degrades more rapidly compared to the other techniques as input CNR falls below 10 dB.

The divergence is caused by an effect referred to as squaring loss [42] which results from the summing and squaring of the in-phase and quadrature input signals. Whilst the sum of squares technique reduces sensitivity to frequency offsets and phase noise, the correlation SNR decreases

at lower carrier-to-noise ratio relative to the other techniques. This implies that the sum of squares technique will be less sensitive at lower received optical powers.

6. Experimental demonstration

The frequency correction and mitigation techniques presented in Sections 4 and 5 were demonstrated experimentally using the optical configuration for a coherent RAMCW LiDAR sensor shown in Fig. 13.

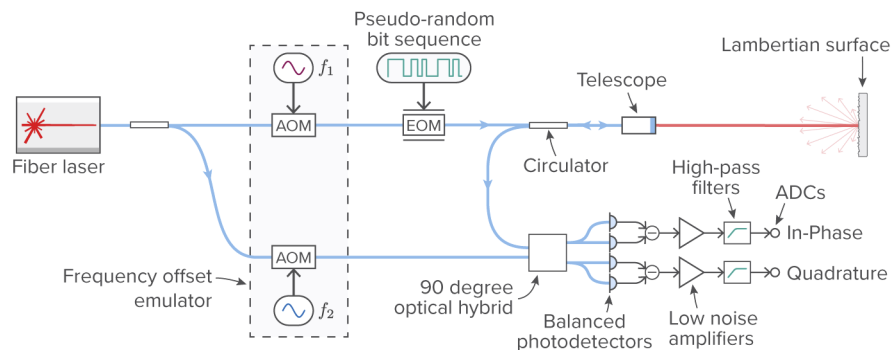


Fig. 13. Experimental configuration used to investigate the sum of squares technique's ability to mitigate large frequency offsets. A fiber laser with a <1 kHz spectral linewidth is separated into two arms: probe (top) and local oscillator (bottom). Acousto-optic modulators (AOMs) are placed in each arm to simulate positive and negative frequency offsets, and are not required for normal operation of the sensor. An electro-optic modulator (EOM) is used to encode the amplitude of the probe with a pseudo-random bit sequence. An optical circulator is used to separate transmitted and received light. A 40% Lambertian surface was interrogated at a distance of ~ 8.5 meters. The in-phase and quadrature voltage signals produced by the balanced photodetectors are amplified, high-pass filtered and then digitized at 250 MSa/s for processing on a field-programmable gate array.

Light from a narrow line-width 1550 nm laser (Koheras Basik Mikro E15) is separated into two paths, one serving as a probe and the other as a local oscillator. The amplitude of the light propagating in the probe arm (upper arm in Fig. 13) is encoded with a 10-bit m-sequence using an fiber-coupled electro-optic modulator (Thorlabs LN82S-FC). The amplitude modulated light passes through an optical circulator to a 1" telescope where it propagates into free-space. Light scattering back from a distant object is collected by the same telescope and passes back through the circulator to a 90-degree complex coupler (Kylia COH28). The received probe signal and local oscillator are then interfered at a pair of balanced photodetectors (Insight BPD-1 with bandwidth from DC-400 MHz and a noise-equivalent power of $< 5\text{pW}/\sqrt{\text{Hz}}$) to produce voltage waveforms proportional to the real and imaginary projections of the probe signal relative to the local oscillator. Balanced detection removes any common-mode noise present in both fields such as relative intensity noise. The photodetector outputs are amplified using low-noise amplifiers (Stanford Research Systems model SIM914) with an input voltage noise of $6.4\text{ nV}/\sqrt{\text{Hz}}$.

The voltage waveforms produced by the two balanced photodetectors were digitized using two 14-bit, 250 MSa/s analog-to-digital converters (NI-5782) connected to a National Instruments FlexRIO FPGA (PXIe-7966R). The ADCs were DC coupled with a -3 dB analog input bandwidth of approximately 330 MHz. One of two 500 MSa/s digital-to-analog converters (DAC) was used to generate the 10-bit m-sequence at a chip frequency of 62.5 MHz. The DACs were DC coupled with a -3 dB bandwidth of 180 MHz.

Fiber-coupled acousto-optic modulators (AOMs) (Gooch & Housego Fiber-Q T-M080-0.4C2J-3-F2P and T-M080-0.5C8J-3-F2P) were installed in both the probe and local oscillator arms of

the interferometer to emulate arbitrary frequency offsets. The AOMs are not required for normal operation of the sensor. Both AOMs had a centre frequency of 80 MHz and a bandwidth of approximately 8 MHz. The AOM in the local oscillator was driven at fixed frequency of 80 MHz by a waveform generator (Agilent A33250). The AOM in the probe arm was driven by the second 500 MSa/s DAC using a numerically controlled oscillator implemented on the FPGA. The 180 MHz bandwidth of the DAC allowed the relative frequency offset between the probe and local oscillator fields to be tuned over the full ± 8 MHz bandwidth of the AOMs. A consequence of using dual AOMs as frequency offset emulators is that the reflected light due to prompt leakage through the optical circulator and Fresnel reflections from the telescope are also frequency shifted. This would not occur if it were not for the frequency offset emulator architecture.

Light exiting the telescope was directed onto a 40% reflectivity Lambertian reference target (Lake Photonics) located approximately 8.5 meters from the telescope.

6.1. Simultaneous measurement of distance and frequency

An example of a single measurement of correlation and frequency offset is presented in Fig. 14. The measurement shows an estimated range of 8.4 meters in the presence of a carrier frequency offset of approximately 180 kHz. A smaller peak is visible at 0 meters which is caused by prompt leakage through the optical circulator and Fresnel reflections from the end of the optical fiber (FC/APC) and telescope. The distance axis has been shifted such that the output of the telescope appears at 0 meters.

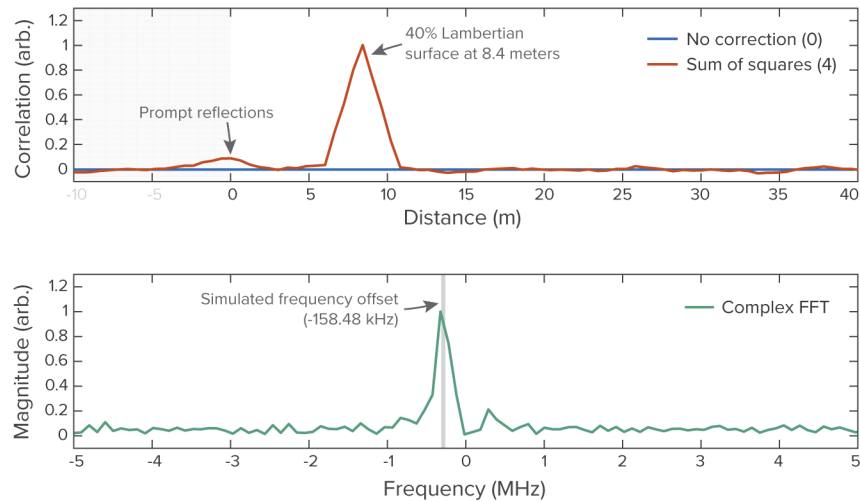


Fig. 14. Single measurement of correlation (top) and frequency (bottom) using a Lambertian surface with 40% reflectivity located ~ 8.5 m from the sensor and with a simulated Doppler shift of -158.48 kHz. The sum of squares (4) frequency mitigation produces a clear peak at a range of 8.4 meters (orange), with a smaller peak visible at 0 m range caused by leakage through the optical circulator and Fresnel reflection from the telescope optics. Without correction there is no discernible peak (blue). The distance axis has been shifted such that the telescope reflection appears at 0 meters. A frequency offset of -183.284 kHz was measured by calculating the two-sided FFT of the in-phase and quadrature input signals.

The resolution of the distance measurement in air is to a first approximation related to the sampling frequency of the ADC f_{ADC} by

$$\delta R = \frac{c}{2f_{\text{ADC}}}$$

where c represents the speed of light. The coarse measurement resolution for $f_{\text{ADC}} = 250$ MSa/s is $\delta R \approx 0.6$ meters. There exists a number of ways to improve the range resolution of the sensor. The first is to increase the sampling rate of the ADC and associated bandwidth of the sampling electronics. Increasing the ADC sampling frequency to 5 GSa/s will lead to 3 cm distance resolution. Alternatively, signal processing techniques that manipulate the shape of the correlation profile coupled with interpolation functions have been shown to further increase ranging resolution [43]. A delay-locked loop could also be used as demonstrated by Sutton et al. in [44].

A frequency offset of -183.284 kHz was measured by calculating the fast Fourier transform of the complex signal formed by the in-phase and quadrature input signals, demonstrating dual-quadrature homodyne detection's ability to resolve absolute relative frequency as originally demonstrated by Abari et al. in [5]. In a practical setting, this allows the sensor to resolve both velocity and direction of travel due to relative motion between the LiDAR sensor and the illuminated object.

The resolution of the frequency measurement is, to a first approximation, equal to the code repetition frequency, f_{code} . In this measurement $f_{\text{code}} = 61.095$ kHz. Frequency resolution can be improved via interpolation or by increasing the time period over which the FFT is calculated.

6.2. Correlation SNR vs. frequency offset

The relative performance of the various frequency correction techniques presented in Sections 4 and 5 was investigated by performing a series of correlation measurements over a wide range of frequency offsets. This was done by stepping the frequency of the numerically controlled oscillator driving the probe AOM from 100 Hz to 1 MHz in 600 logarithmically spaced increments. At each frequency, 100 correlation measurements with zero dead-time between them was recorded to obtain useful measurement statistics. The data was not averaged.

The results of this experiment are presented in Fig. 15, showing the mean correlation SNR for no correction (blue), signal demodulation (red), template modulation (orange), and sum of squares (green). Results for complex demodulation (Section 4.3) are not presented as we did not have accurate knowledge of the carrier's phase.

The shaded regions indicate $\pm\sigma$ (one standard deviation) error bounds. The results for signal demodulation and template modulation represent the best case scenario as they assume perfect knowledge of the exact frequency offset for each measurement, which is held constant for the duration of each measurement. Whilst it would be difficult to obtain perfect knowledge of the carrier frequency in a realistic setting, it is possible (at least in principle) to compensate for errors in offset frequency estimation by using a bank of matched-template filters to interrogate multiple frequencies simultaneously.

As predicted by the simulated results in Section 4.4, the correlation SNR of the reference technique decreases with increasing frequency offset, with the first null at the code period f_{code} of 61.094 kHz with a shape described by Eq. (2).

Signal demodulation (1) and template modulation (2) converge toward a 6 dB reduction in correlation SNR due to heterodyne loss. The SNR of the template modulation technique fluctuates over a wider range than signal demodulation, primarily because the correlation is taking place at the modulated carrier frequency and not at DC, thus resulting in lower correlation efficiency and greater sidelobe activity.

The sum of squares (4) frequency mitigation technique yields relatively uniform correlation SNR above 20 kHz, rolling off below this frequency due to AC-coupling caused by DC blocks placed at the input to the ADCs. According to the results in Fig. 12, a correlation SNR of 32 dB corresponds to a carrier-to-noise ratio of approximately 4 dB.

We speculate that the increased variance in the sum of squares correlation SNR (depicted by the shaded green region in Fig. 15) is caused by the inter-modulation between the received light

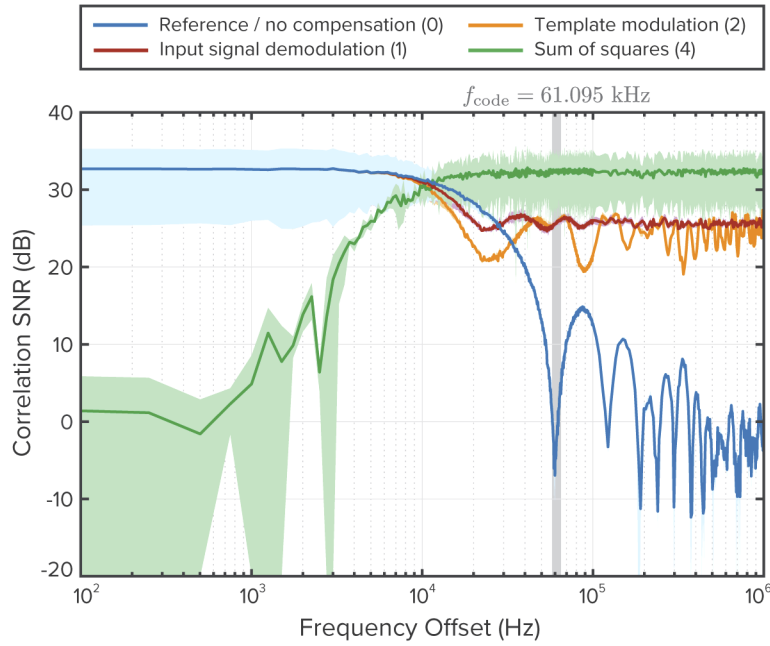


Fig. 15. Experimental correlation SNR results for ranging to a Lambertian surface with 40% reflectivity at a range of 8.5 m using a narrow line-width 1550 nm laser. Correlation SNR using sum of squares (4) is uniform across all tested Doppler frequencies. Signal demodulation (1) and template modulation (2) converge toward a 6 dB reduction in correlation SNR due to heterodyne loss. The mean correlation SNR is calculated over 100 consecutive measurement frames, with the standard deviation over the sequences represented by the shaded regions. Results for complex demodulation (Section 4.3) are not presented as we did not have accurate knowledge of the carrier's phase.

from the distant Lambertian surface and spurious returns caused by leakage through the optical circulator and Fresnel reflection from the telescope optics. We refer to other sources of light that originate within the optical system and arrive at the detector as *prompt* reflections.

In the optical system used in this experiment, the two prompt reflections were separated by ~30 cm of optical fiber, and as such were located within one chip length of the pseudo-random bit sequence which is related to the chip frequency f_{chip} by

$$L_{\text{chip}} = \frac{c}{nf_{\text{chip}}},$$

where c is the speed of light in vacuum and n represents the effective refractive index of the glass fiber. Because these signals overlap within what is referred to as the *range gate* of the PRBS [45], their electric fields coherently interfere at the photodetector producing fluctuations in detected optical power. These prompt reflections appear as a secondary peak in the correlation measurement, the amplitude of which changes over time. It was observed that environmental noise (e.g., temperature fluctuations, vibrations) caused the interference power of the prompt reflections fluctuate, most likely due to fiber phase noise.

One way to alleviate issues caused by the interference between optical circulator leakage and telescope Fresnel reflections is to increase the length of optical fiber between the circulator and telescope beyond L_{chip} . This would prevent the two prompt reflections from coherently interfering, thus producing two distinct peaks with relatively consistent magnitude in the

correlation measurement. Another solution is to use polarization sensitive optics (e.g., a polarizing beamsplitter in place of the optical circulator).

6.2.1. Sensitivity to DC offsets

Sum of squares (4) frequency mitigation was observed to be sensitive to DC offsets on the input signals. This sensitivity is due to the mixing between the frequency shifted encoded signals ($s_I(t)$ and $s_Q(t)$) and their corresponding DC offsets during the sum of squares calculation:

$$\begin{aligned}\hat{S}(t) &= \sqrt{(s_I(t) + I_{DC})^2 + (s_Q(t) + Q_{DC})^2}, \\ &= \sqrt{s_I^2(t) + s_Q^2(t) + 2s_I(t)I_{DC} + 2s_Q(t)Q_{DC} + I_{DC}^2 + Q_{DC}^2}.\end{aligned}\quad (6)$$

As can be seen in Eq. (6), the presence of a DC offset produces additional signal terms that maintain their original frequency offsets. When these terms are correlated with a DC template they produce side lobe noise (as described in Section 3.) which degrade SNR.

To illustrate the effect that DC offsets has on the sum of squares measurement, the correlation SNR was simulated over a range of frequency offsets (100 Hz to 1 MHz) for six DC offset ratios (i.e., DC offset divided by the amplitude of the PRBS encoded signal). The simulations assumed an input carrier-to-noise ratio of 0 dB and a carrier phase of 0 radians (i.e., all of the signal power is in the in-phase component). The results from this simulation are presented in Fig. 16.

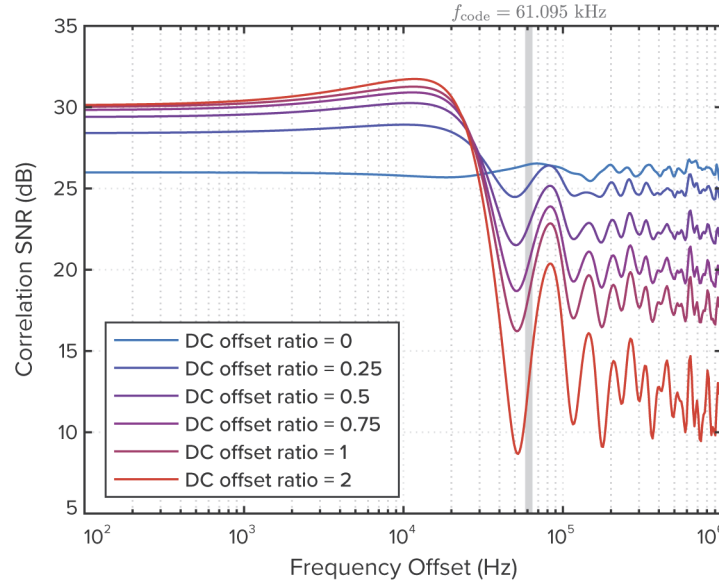


Fig. 16. Simulation results showing the effects of different relative DC offsets on the correlation SNR for the sum of squares (technique 4) with a carrier-to-noise ratio of 0 dB. The term offset ratio is defined as the ratio of the DC offset to the zero-to-peak amplitude of the signal. The presence of a DC offset produces additional signal terms that maintain their original frequency offsets, which when correlated with a DC template contribute additional noise. These terms coherently combine for frequencies below f_{code} which results in improved correlation SNR in that frequency region.

At frequency offsets much lower than f_{code} , the modulated terms introduced due to DC offsets combine with the sum of squares term $s_I^2(t) + s_Q^2(t)$, leading to an SNR gain that depends on the magnitude of the DC offset, as well as the amplitude and phase of $s_I(t)$ and $s_Q(t)$. Despite the SNR gain at low frequencies, the SNR penalty at higher frequencies is unacceptable.

A DC block with a specified high-pass cutoff frequency of approximately 10 kHz was placed at the inputs to the ADCs to remove DC offsets introduced by the balanced photodetectors and amplifiers. The DC blocks mitigated the SNR penalty caused by DC offsets at higher frequencies, but decreased SNR at frequencies below the DC block cutoff frequency. This is due to the loss of information carried at low frequencies which is necessary for the sum of squares technique to function. This effect can be explained mathematically by recognising that when high-pass filtered, the amplitude modulated pseudo-random bit sequence $c[nT_s] \in [0, 1]$ becomes an antipodal sequence $p[nT_s] \in [-0.5, 0.5]$. This means that Eq. (5) becomes

$$s_I^2[nT_s] + s_Q^2[nT_s] \rightarrow (\alpha p[nT_s])^2 = 0.25\alpha$$

where $p[nT_s]^2 \in [-0.5, 0.5]^2 = 0.25$. At carrier offset frequencies greater than the high-pass filter cutoff there is no conversion from $c[nT_s]$ to $p[nT_s]$ which is why the correlation SNR plateaus at the expected value above this frequency. One way to compensate for the loss in correlation SNR at low frequencies when using the sum of squares (4) frequency mitigation technique is to calculate a second correlation without frequency compensation as described in Fig. 4. The selection between the two correlation results can be made using a combination of estimated SNR and a measurement of detected frequency offset as shown in Fig. 14.

6.3. Suppression of laser frequency noise

According to Eq. (5), the sum of squares technique should mitigate any sources of phase noise that are common to both the in-phase and quadrature outputs of the dual-quadrature receiver. Sources of common-mode phase noise include laser frequency noise coupling in due to relative optical path length differences between the local oscillator and received probe; fiber phase noise caused by environmental disturbances; and atmospheric phase noise introduced by scintillation and speckle [34]. Sources of uncommon phase noise include shot noise and electronic noise, which are not mitigated by the sum of squares technique and thus limit SNR.

To demonstrate the sum of squares technique's ability to mitigate common-mode phase noise, the narrow line-width ($\Delta\nu = 1$ kHz) laser used in the previous experiments was replaced by a polarization maintaining distributed feedback (DFB) laser (Gooch & Housego AA1401) with a line-width of $\Delta\nu = 1.2$ MHz as measured using a delay-line interferometer. The optical power of the DFB laser was configured such that ~ 10 mW was transmitted from the telescope.

The experimental configuration shown in Fig. 17 was used to measure the distance to a 10% reflective Lambertian surface located ~ 8.5 meters from the telescope over a range of frequency offsets. To simulate extreme laser frequency noise coupling, a 500 m spool of optical fiber (SMF-28e) was added to the local oscillator path to produce a large relative optical path length difference far exceeding the coherence length of the laser ($L_{\text{coh}} = c/(\pi\delta\nu) = 80$ meters). A polarization controller was included in the local oscillator path to optimise interference power at the detectors.

To overcome the issues of prompt reflections, the optical circulator was replaced with a high return loss polarizing beamsplitter and quarter-wave plate located at the output of the telescope. The combination of polarizing beamsplitter and quarter-wave plate reduces the received power from the Lambertian surface by on average 3 dB by rejecting one polarization, but was observed to reduce the prompt reflected power to below the measurement noise-floor.

Figure 18 shows the correlation profile for no correction and sum of squares (middle); and two-sided FFT of the carrier frequency (bottom) at a carrier offset frequency of approximately 600 kHz. These results demonstrate the sum of squares technique's ability to mitigate both severe phase noise and large frequency offsets. The resolution to which velocimetry measurements may be resolved is still limited by the laser line-width, as can be seen by the frequency broadening of the peak shown in Fig. 18 (bottom).

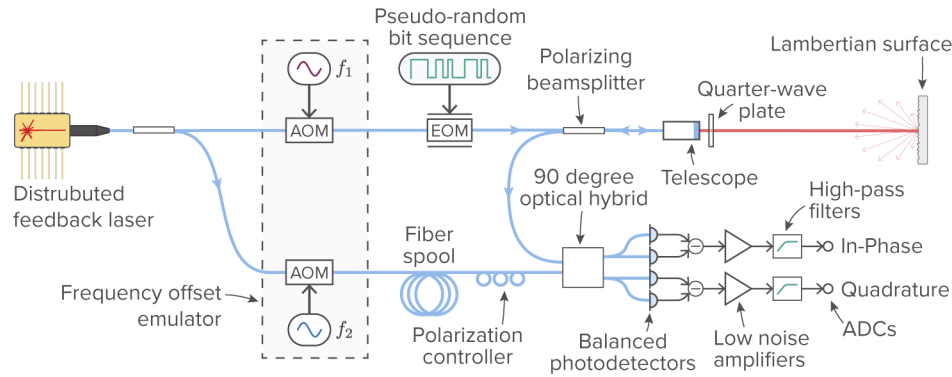


Fig. 17. Experimental configuration used to demonstrate the sum of squares technique's ability to mitigate phase noise. A distributed feedback (DFB) laser with a 1.2 MHz spectral linewidth is separated into two arms: probe (top) and local oscillator (bottom). Acousto-optic modulators (AOMs) are placed in each arm to simulate positive and negative frequency offsets, and are not required for normal operation of the sensor. An electro-optic modulator (EOM) is used to encode the amplitude of the probe with a pseudo-random bit sequence. A polarizing beamsplitter is used in combination with a quarter-wave plate to serve as an optical circulator to reduce the magnitude of prompt reflections at the detectors. A 10% Lambertian surface was interrogated at a distance of ~ 8.5 meters. The in-phase and quadrature voltage signals produced by the balanced photodetectors are amplified, high-pass filtered and then digitized at 250 MSa/s for processing on a field-programmable gate array.

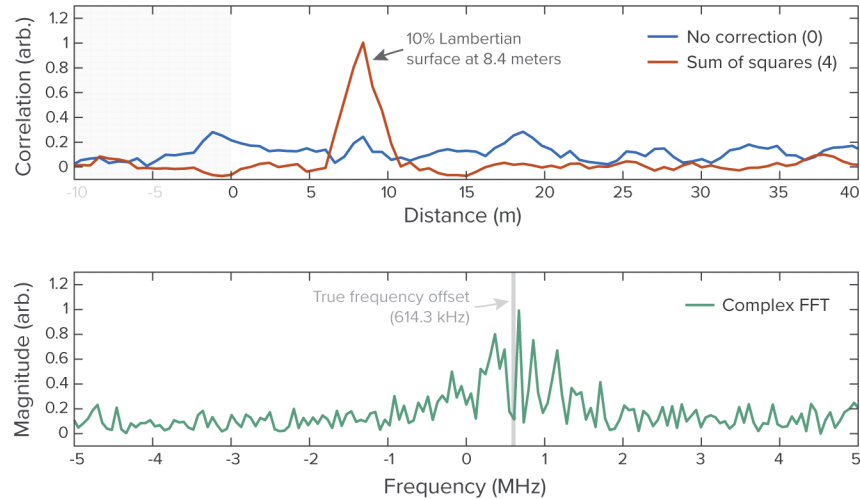


Fig. 18. Single measurement of correlation (top) and frequency (bottom) using a DFB laser with a spectral linewidth of 1.2 MHz to range to a 10% Lambertian surface. The frequency offset for this measurement was 614.3 kHz. The correlation measurement (top) reveals a clear peak at the expected distance of 8.4 meters. The two-sided FFT (bottom) reveals the 1.2 MHz linewidth of the laser.

The frequency offset was stepped from 100 Hz to 5 MHz in 600 logarithmically spaced increments across three separate measurements. The results of these measurements have been stitched together and are presented in Fig. 19, showing correlation SNR vs. frequency offset for no compensation (0), signal demodulation (1), template modulation (2), and sum of squares (4)

techniques. Complex demodulation is not presented as we did not have accurate knowledge of the carrier's phase for the duration of the each measurement. Both signal demodulation and template modulation techniques were calculated assuming perfect knowledge of the true frequency offset at each measurement (i.e., the best case scenario).

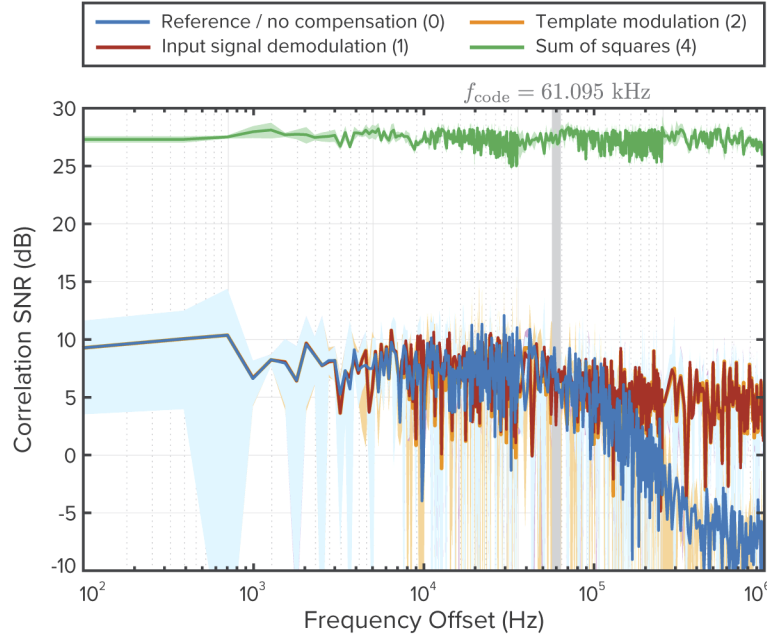


Fig. 19. Experimental results of the broad line-width DFB laser with injected Doppler frequency to a free-space 10% Lambertian target at 8.4 m. The shading represents $\pm\sigma$ of the correlation SNR as observed over 100 repetitions. A 500 m relative path length difference between the LO and probe paths which exceeds the laser's coherence length was used which resulted in a measured line-width of 1.2 MHz. A 10-bit M-sequence modulated at a chip rate of 62.5 MHz, shot-noise limited detection and a sampling rate of 250 MS/s is used. The frequency axis is presented in Hz, but can also be considered in terms of integer multiple of the FFT period (PRBS code period) which is 61.094 KHz or 0.047 m/s of relative radial velocity.

Figure 19 shows that even in the presence of significant phase noise and large frequency offsets, the sum of squares technique produces a consistently high correlation SNR, ~ 15 dB higher than signal demodulation and template modulation. The shaded regions represent the $\pm\sigma$ (one standard deviation) error bounds estimated over 100 consecutive measurements captured with zero dead time.

Both input signal demodulation (1) and template modulation (2) techniques had near uniform correlation SNR below 10 dB. This is due to the large line-width of the laser spreading the PRBS over a much larger frequency range than the single injected Doppler frequency. The large variance in correlation SNR for these techniques may be due to demodulation at the injected frequency offset, which may not be the true instantaneous frequency due to laser frequency noise. This would result in a reduction in correlation SNR due to correlation occurring at the incorrect frequency. The reference technique (0) has reduced correlation SNR than was observed with the narrow line-width laser in Fig. 15 due to power being spread out. Conversely, the signature nulls at multiples of f_{code} are not visible since some of the power at these frequencies is pulled down near DC where some correlation against the reference can occur. To reduce the correlation SNR degradation for techniques 0-2 in broad line-width environments the correlation bandwidth could

be increased by increasing the PRBS code repetition frequency. A trade-off of this is a reduction in the ambiguity range of the sensor.

The decrease in correlation SNR present in Fig. 15 due to the DC block was not observed in these measurements since the severe coupling of laser frequency noise into the measurement effectively spread the signal's energy outside the DC block's high-pass cutoff frequency. As such, only a small fraction of the signal's energy is suppressed by the high-pass filter.

7. Conclusion

We have presented a signal processing technique to mitigate the effects of frequency offsets and phase noise in coherent random amplitude modulated continuous wave (RAMCW) LiDAR. The technique uses a sum of squares calculation to remove the input signal's sensitivity to phase fluctuations and frequency offsets, requiring the use of a coherent dual-quadrature receiver to produce in-phase and quadrature waveforms which share common phase and frequency information. By eliminating correlated phase noise from the dual-quadrature input signal, it is possible to measure distance using a single matched-template filter without needing to actively compensate for frequency offsets (e.g., due to Doppler) and phase noise (e.g., due to laser frequency noise, speckle, and environmental noise).

A combination of simulations and experiments demonstrated the sum of squares technique's ability to overcome frequency offsets and phase noise under a variety of conditions. Impressively, the technique was used to mitigate severe laser frequency noise while the sensor was operating far beyond the coherence length of a distributed feedback laser, revealing a measurement SNR that was consistently better than alternative techniques by more than 15 dB across a wide range of frequency offsets. One implication of this result is that the sum of squares technique should also be capable of achieving high SNR measurements of absolute distance in the presence of otherwise catastrophic phase noise caused by speckle, accelerations, and environmental noise. It also demonstrates that the technique can support high precision ranging beyond the coherence length of the laser, assuming the bandwidth of the detection system is greater than the laser's spectral linewidth and maximum expected frequency shift.

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