EMPIRICAL INVESTIGATION INTO THE EFFECT OF NUPTIALITY AND FERTILITY VARIABLES ON THE CONVERGENCE AND DURATION OF STABILITY OF AGE-SEX DISTRIBUTION OF SINGAPORE'S POPULATION

by

U Yu Khin

A thesis submitted in partial fulfilment of the requirements for the degree of Master of Arts in Demography in the Australian National University

August, 1978

D E C L A R A T I O N

Except where otherwise indicated, this thesis is my own work.

August, 1978. U Yu Khin

CONTENTS

Contents (contd.)

ABSTRACT

The stable population is a tool that displays the implications for age composition, and the growth rates of specified schedules of mortality and fertility on the assumption that the schedules prevail long enough for other influences to be erased. In this study, the process of convergence from arbitrary circumstances to the stable form is analysed by using Singapore's populations - 1957 observed, 1957 stable and 1970 observed populations - with 1969-71 mortality incorporating nuptiality and fertility as separate inputs.

The first part of the study, drawing on experience of the Singapore 1970 situation, Sweden 1875 and Japan 1960 situations, derives the hypothetical data for nuptiality and fertility schedules. The second part of the study demonstrates the effect of nuptiality and fertility variables on the duration of convergence, which shows that nuptiality can have more effect on the duration of convergence than fertility although this is more pronounced in the shorter term, i.e. transition spread over 35 years, than in the longer term, i.e. transition spread over 65 years. Finally, the process of convergence is analysed in terms of the actual population change during the projection period.

LIST OF TABLES

List of Tables (contd.)

LIST OF FIGURES

List of Figures (contd.)

List of Figures (contd.)

CHAPTER 1

B A C K G R O U N D

1.1 INTRODUCTION

It is well accepted that the age distribution is a matter of major importance in demographic analysis, both because of its immediate economic and social consequences and because of its influence on trends of births, deaths and natural increase (Lorimer, 1951:31). The age distribution of a population is determined wholly by three major components - births, deaths and migration (Coale, 1956:79). But the age distribution of most national populations is not much affected by migration, especially today when almost everywhere international migration is restricted. In a closed population the age distribution is determined by births and deaths only.

If we have an age distribution and the schedule of fertility and mortality, the future age distribution can be computed by standard methods of population projection. If these schedules are kept constant and applied indefinitely to an initial population, the resulting population will have fixed proportions in different age groups. This is the concept of a stable population developed by Alfred J. Lotka (Lotka, 1939).

Such a population defined by constant fertility and mortality is hypothetical, because the vital rates of actual populations are never absolutely constant. How closely does an observed population match the stable population? The answer lies in how far the fixed schedules of fertility and mortality used in the construction of the stable population, remained constant. In most of the developing countries the fertility rates have fluctuated in a narrow range but the mortality rates have declined gradually in the light of improvements in the standards of living, health and hygienic conditions, and medical technology.

It has been shown that when fertility is constant, changes in mortality often have little effect on the age structure of the population (Coale, 1956:79-114). Consequently, when fertility is constant and mortality is declining, the relationship between the birth rate, the death rate, the natural increase and the age distribution is at all times the same as in a stable population (United Nations, 1956:31). The term quasi-stable population has been used for such a population in which fertility has been constant for a long period of time and mortality has been falling. The quasi-stable population is more common in actual populations of developing countries in the recent period. Also extremely young populations are found in the developing countries - those that have not incorporated modern industrial technology in their economies - the populations of Africa, Asia and Latin America. The per cent of children under 15 years of age of Togo in 1970 was 50, that of Pakistan in 1971 was 46 and of Brazil was 42 (Richardson, 1975).

A special case of the stable population, called a stationary population, has also been studied. In stationary population the rate of growth is zero and the age distribution of population is given by the L_x column of the life table. When mortality is very low, a stationary state can be attained only with low fertility. Ideally such a population will have a constant age distribution and constant age-specific mortality and fertility rates; its NRR (net reproduction rate) will be equal to 1.0. A few European countries are coming close to such a stationary population state (Frejka, 1973:33).

Comparing the age distribution of various national populations in the world today, the highly industrialized countries all have older populations than the developing countries, and also older populations than they did fifty to a hundred years ago. Since 1900 the median age rose in England from 24 to 36, in United States from 23 to 30, in Japan from 23 to 26, and in Russia from 21 to 27. In the developing countries, however, the age distributions changed only slightly and they have, if anything, become slightly younger.

In Taiwan, for example, the median age declined from 21 to 18 since 1915 (Peterson, 1972:116).

These differences and trends in the age distribution of population were accounted for by the different histories of fertility and mortality in the past. If the different age distributions are subject to the same histories of fertility and mortality for a sufficient time interval, their age distributions will eventually be the same. This is the property of human populations that A. Lopez (Lopez, 1961) called 'weak ergodicity'. However, Lotka's proof of convergence provides a framework for exploring the factors that control the duration and nature of the transition from arbitrary initial conditions to the stable state.

1.2 THE PRESENT STUDY: ITS OBJECTIVES AND SCOPE

A recent review of literature for this study can be found generally in 'Mathematical Demography" A Bibliographical Essay' (Keyfitz, 1976: 9-19), particularly in 'Convergence of Age-Sex Distributions and Population Change in the Presence of Migration' (Sivamurthy, 1970: 2-12). In view of the stable population and the related literature, little attention has been given to analysing the process of convergence from arbitrary initial circumstances to the stable form. Most of the studies have been concerned with proofs of existence - with the proof of ultimate convergence, and not with intermediates or the duration of convergence. The literature on the use of the stable population concepts for estimation and other pragmatic purposes has not included a systematic study of convergence (Coale, 1972:61).

So far, the effect of fertility and nuptiality on the duration of convergence has not been investigated. Hence, in this study, an attempt has been made to examine empirically the effects of fertility and nuptiality variables on the convergence and duration of stability of Singapore's population by using the nuptial-specific fertility projection method, presented in Appendix A.

The present study will examine the process of population change which is caused and determined by demographic forces only, especially by changes in fertility and nuptiality. This will be achieved by the use of the models, both theoretical and numerical. In the numerical models demographic data drawn from the present and past experience of Singapore's population development will be used. The availability of such information for Singapore's population will be discussed later in Chapter 2.

1.3 METHOD AND ORGANIZATION OF THE STUDY

The study employs the basic techniques for the computation of the population projections by the component method. The initial populations and their age structures are projected into the future on the basis of certain assumptions about the course of age-specific mortality rates, nuptial-specific fertility rates and the proportions of married women by age.

Since the limits of probable future mortality development are comparatively narrow, and the 1969-71 mortality in Singapore had been comparatively low, it will be assumed that the 1969-71 mortality level will remain constant into the future. Thus, the variation of fertility and nuptiality assumptions in the future development will constitute the basic framework of this study.

This study consists of five chapters. The first chapter is the general background to the present study. After presenting the fundamental data for the study in Chapter 2, the process of convergence of Singapore's population will be examined extensively both through theoretical and numerical investigations in Chapter 3. But the investigation in Chapter 3 will cover only the constant conditions of those hypothetical nuptiality and fertility schedules applied to Singapore's population. The experiments on the effect of changing nuptiality and fertility will be investigated in Chapter 4. Finally, the summary of findings and conclusion of the study will be presented in Chapter 5.

CHAPTER 2

THE BASIC DATA

2.1 INTRODUCTION

It is well known that the age-specific fertility rates are greatly affected by the marriage patterns and by the dissolution of marriages. But the conventional population projection which uses the age-specific fertility rates based on all women of childbearing ages, does not take into account these phenomena. Here, we wish to make projections on the basis of nuptial-specific fertility rates - the number of births per 1,000 married women. This is a valid procedure, and highly important where substantial changes are anticipated in the pattern of marriages, or in the dissolution of marriages (Bogue, 1974:42).

The basic data required for this study are : (a) some populations having significantly different initial age-sex distributions; (b) survival ratios; (c) marital status distributions by age; (d) nuptial-specific fertility rates; and (e) the sex ratio at birth. We shall present in this chapter a brief description of these data.

2.2 AVAILABILITY OF DATA

In this study, we have made use of the demographic data observed in Singapore during the period 1947-75 and some hypothetical data prepared on the basis of certain assumptions. The population of Singapore was used because Singapore is one of the few developing countries that has been able to achieve a transition from a very high fertility level to a remarkably low level within a short span of twelve years, 1957-70. Besides, demographic data have been known to be virtually complete in Singapore since the early post-war period.

The two most recent censuses of Singapore taken in 1957 and 1970 were conducted independently from the Malaysian censuses. The tabulation of age data in these two censuses has been in very fine groupings: single years up to 99, which permits an analysis of the digit preference of the population. This analysis was done in the 1970 census report by using Myer's index, which was 5.2 in 1957 and 4.2 in 1970 for total population (both sexes). It showed that there was relatively little digit preference in both censuses (Singapore, 1970:50-51).

Between the two censuses the first nation-wide sample survey of Singapore was conducted in 1966. It was comprised of a four per cent sample of the population for demographic, social and economic characteristics. The sample survey data were inflated to get the 1966 Singapore population, and it was published in 1967 (Seng, 1967).

Apart from the censuses and survey data, birth and death statistics have been available in Singapore from 1872 and do provide some indication of the order of magnitude of vital events from then on. Although the registration of births and deaths was introduced in 1872, it was fifteen years later that the statistics were published in the annual report on the registration of births and deaths for the year 1886; figures for the years prior to 1886 were not given (Saw, 1970:16). Over the course of time the name of the report changed many times, the current being the report on the registration of births and deaths, marriages and persons. From the beginning of 1959 an additional report known as 'Monthly Demographic Bulletin' was issued to make available the latest data on population and vital statistics (Singapore, 1959).

A system for compulsory registration for all marriages solemnized in Singapore was introduced on the 15th of September 1961. From then on marriage statistics were also made available. Prior to 1961, there was only partial registration of marriages and the published statistics were incomplete in coverage (Saw, 1970:18). The details of marriage data will be discussed later in this chapter.

Unfortunately, migration statistics in Singapore are relatively weak. But the 1970 census report evaluated the coverage of 1957 and 1970 censuses, and found the population increase during 1957 and 1970 to be 628,578 persons. Of this the natural increase was 595,614 persons accounting for 94.8 per cent of the total intercensal increase. The balance between population increase and natural increase was 32,964 persons amounting to 5.2 per cent of the total population increase, and was supposed to be an insignificant net migrational increase.

Indeed, today Singapore's population is closed and the natural increase is the sole determinant of the rate of growth (Singapore, 1970:46).

2.3 AGE-SEX DISTRIBUTIONS OF THE INITIAL POPULATIONS

In this section we shall discuss the differences in the initial populations, which will be used in the following chapter, to show the effect of initial age-sex distribution on the convergence and duration of stability of the population by using the actual population projection. Fortunately, we could have two census populations of Singapore as the 1957 observed and the 1970 observed populations. However, we would like to compare these two populations with a stable population so that we can understand more clearly about their differences. Here, we shall make use of the 1957 underlying stable population along with those two observed populations.

First of all we shall present the calculations of the intrinsic characteristics of Singapore population in 1957, given in tables 2.3.1 and 2.3.2. The age-specific fertility rates in respect of the female births were obtained by multiplying the age-specific fertility rates per woman by the proportion of female births to total births. Age-specific fertility rates were obtained by using the Singapore's vital statistics and the census population of 1957 (Singapore, 1970:36). The ratio of male to female births, i.e. the sex ratio at birth was 106 in 1957 (Saw, 1970:62). The abridged life table for Singapore for 1957, adopted from Saw Swee-Hock (Saw, 1970:178) and presented in Appendix B, was used.

Table 2.3.1

Calculation of Intrinsic Rate of Increase (5), Singapore 1957

Age of Mother	Annual Births of Pivotal (Central)		F	Products		
	Daughters per female $_5F_x$ (1)	Age $(x + 2.5)$ (2)	$\frac{_5L_x}{l_o}$ (3)	Calculation of Zero Moment (1) x (3) = (4)	Calculation of First Moment (2) x (4) = (5)	Calculation of Second Moment (2) x (5) = (6)
15 to 19 years	.03764	17.5	4.6700	.17576	3.0758	53.8265
20 to 24 years	.14385	22.5	4.6452	.66822	15.0349	338.2853
25 to 29 years	.16931	27.5	4.6125	.78096	21.4764	590.6010
30 to 34 years	.13837	32.5	4.5695	.63228	20.5492	667.8500
35 to 39 years	.09322	37.5	4.5090	.42031	15.7618	591.0680
40 to 44 years	.03909	42.5	4.4215	.17284	7.3457	312.1923
45 to 49 years	.00563	47.5	4.2998	.02419	1.1491	54.5802
Summation	.62711	(xx)	(xx)	2.87456	84.3929	2608.4033

GRR = 5 col. (1) = 5 x .62711 = 3.13555

NRR = $H_o = \Sigma$ col. (4) = (Zero Moment) = 2.87456

$H_1 = \Sigma$ col. (5) = (First Moment) = 84.3929

$H_2 = \Sigma$ col. (6) = (Second Moment) = 2608.4033

$$\chi = \frac{H_1}{H_o} = 29.3585, \quad \phi = \chi^2 - \frac{H_2}{H_o} = -45.4875, \quad \log_e H_o = 1.0559, \quad r = .3703$$

(xx) Not Applicable

Table 2.3.1 (contd.)

The solution of the integral equation:

$\int_0^w e^{-ra} P(a)\ m(a)\ da = 1$ is approximately close to the real root of the equation $\frac{1}{2} \phi\ r^2 + \chi r = l_n\ H_0 = 0$

$$\therefore\ r = \frac{-\chi \pm \sqrt{\ ^2 + 2\ \phi\ l_n\ H_o}}{}$$

where:

$$\chi = \frac{H_1}{H_0} = \text{mean age of net reproduction} = \bar{m}$$

$$\text{and } \phi = \chi^2 - \frac{H_2}{H_0}$$

$$\text{and } H_0 = \int_0^w m(a)\ p(a)\ da = \sum_a \text{ASFFR}_a \frac{L_a}{l_0} = \text{NRR}$$

$$H_1 = \int_0^w a.m(a)\ p(a)\ da = \sum_a a\ \text{ASFFR}_a \frac{L_a}{l_0}$$

$$H_2 = \int_0^w a^2.\ m(a)\ p(a)\ da = \sum_a a^2.\ \text{ASFFR} \frac{L_a}{l_0}$$

(Age 'a' is usually single year or five years age group and w is the highest age attained in a given mortality region. ASFFR is the age-specific fertility rate for female births only.)

Table 2.3.2

Computation of the Intrinsic Birth and Death Rates, and the Age Distribution of the Stable Population, for Singapore : 1957

Age interval (x to x+5)	Midpoint (x+2.5) (1)	r(x + 2.5) (2)	-r(x + 2.5) (3)	$\frac{_5L_x^F}{l_o^F}$ (4)	Female Stable Population derivative (3)x(4) = (5)	$\frac{_5L_x^M}{l_o^M}$ (6)	Male Stable Population derivative (3)x(6) = (7)	Stable Population Female $(5)x\frac{10,000}{(5)+(7)}$ = (8)	Stable Population Male $(7)x\frac{10,000}{-(5)+(7)}$ = (9)
0 to 4 years	2.5	0.09257	0.91158	4.7782	4.35572	5.01560	4.57213	926	973
5 to 9	7.5	0.27772	0.75750	4.7112	3.56876	4.94755	3.74779	759	797
10 to 14	12.5	0.46287	0.62947	4.6888	2.95147	4.91893	3.09633	628	659
15 to 19	17.5	0.64802	0.52308	4.6700	2.44277	4.89137	2.55857	520	544
20 to 24	22.5	0.83317	0.43467	4.6452	2.01912	4.85777	2.11151	429	449
25 to 29	27.5	1.01833	0.36120	4.6125	1.66603	4.81664	1.73977	354	370
30 to 34	32.5	1.20348	0.30015	4.5695	1.37153	4.76385	1.42987	292	304
35 to 39	37.5	1.38863	0.24942	4.5090	1.12463	4.73947	1.18211	239	251
40 to 44	42.5	1.57378	0.20726	4.4215	0.91641	4.56733	0.94663	195	201
45 to 49	47.5	1.75893	0.17223	4.2998	0.74055	4.37971	0.75432	157	160
50 to 54	52.5	1.94407	0.14312	4.1305	0.59116	4.09955	0.58673	126	125
55 to 59	57.5	2.12922	0.11893	3.8885	0.46246	3.67905	0.43755	98	93
60 to 64	62.5	2.31437	0.09883	3.5488	0.35072	3.09626	0.30600	75	65
65 to 69	67.5	2.49952	0.08212	3.0900	0.25376	2.36189	0.19397	54	41
70 to 74	72.5	2.68467	0.06824	2.4995	0.17057	1.57707	0.10762	36	23
75 to 79	77.5	2.86982	0.05671	1.8080	0.10253	0.89591	0.05081	22	11
80 to 84	82.5	3.05497	0.04712	1.1	0.05184	0.40227	0.01896	11	4
85+	87.5	3.24012	0.03916	0.6392	0.02503	0.14925	0.05844	5	1
Summation	(xx)	(xx)	(xx)	(xx)	23.16506	(xx)	23.89910	4926	5071

b_F = .04317, d_F = .00614, b_M = .04435, d_M = .00732, b_T = .04377, d_T = .00679, r = .03703

b_F and d_F are intrinsic birth and death rates of females, b_M and d_M that for males, b_T and d_T that for total, r is intrinsic growth rate.

Table 2.3.1 gives the intrinsic rate of natural increase, r, as 3.7 percent. This shows that the Singapore's population in 1957 had a comparatively high intrinsic rate of natural increase in comparison with other developing countries (Saw, 1970:25). Having calculated intrinsic rate of natural increase, we calculated the intrinsic birth rate, the intrinsic death rate, and the stable age distributions of male and female populations. For the total population, the intrinsic birth and death rates were 43.8 and 6.8 per thousand population respectively.

Having obtained the 1957 stable population, we shall study the differences in the age-sex distribution among the 1957 obs., the 1957 stb. and 1970 obs. populations in terms of five-year age groups. Sivamurthy (1970:48) says that the age-sex distributions of any two populations can be compared by two methods: '(1) by computing the percentages of the populations in the age-sex groups to the total population and then comparing these percentages for the two populations; (2) by computing the percentage of the age distributions for males and females separately and then comparing these age distributions in the two populations along with a comparison of the sex ratios in the same age groups in the two populations'. The latter is used in this study because it compares the age distributions as well as the sex composition in the populations.

The age distributions of males and females are presented in Fig. 2.3.1 and Fig. 2.3.2 separately, and the sex ratios in these age groups are also presented in Fig. 2.3.3. The curves of the age distributions of the 1957 observed and 1970 observed populations compared with the smooth curve of those of the 1957 stable population, reveal the impact of the demographic changes on the population of Singapore. The effect of rapid decline in fertility on the age distribution heavily reduces the proportion at young ages and therefore to make the proportion in adolescence (between 10-14 and 20-24 age groups) and the adulthood and old age groups (between 35-39 and 80+ age groups) of the 1970 observed population higher than the 1957 stable population. The influence of fertility change on the age structure can also be seen by comparing the 1970 age distribution with that of 1957 observed population. The 1957 observed age distribution

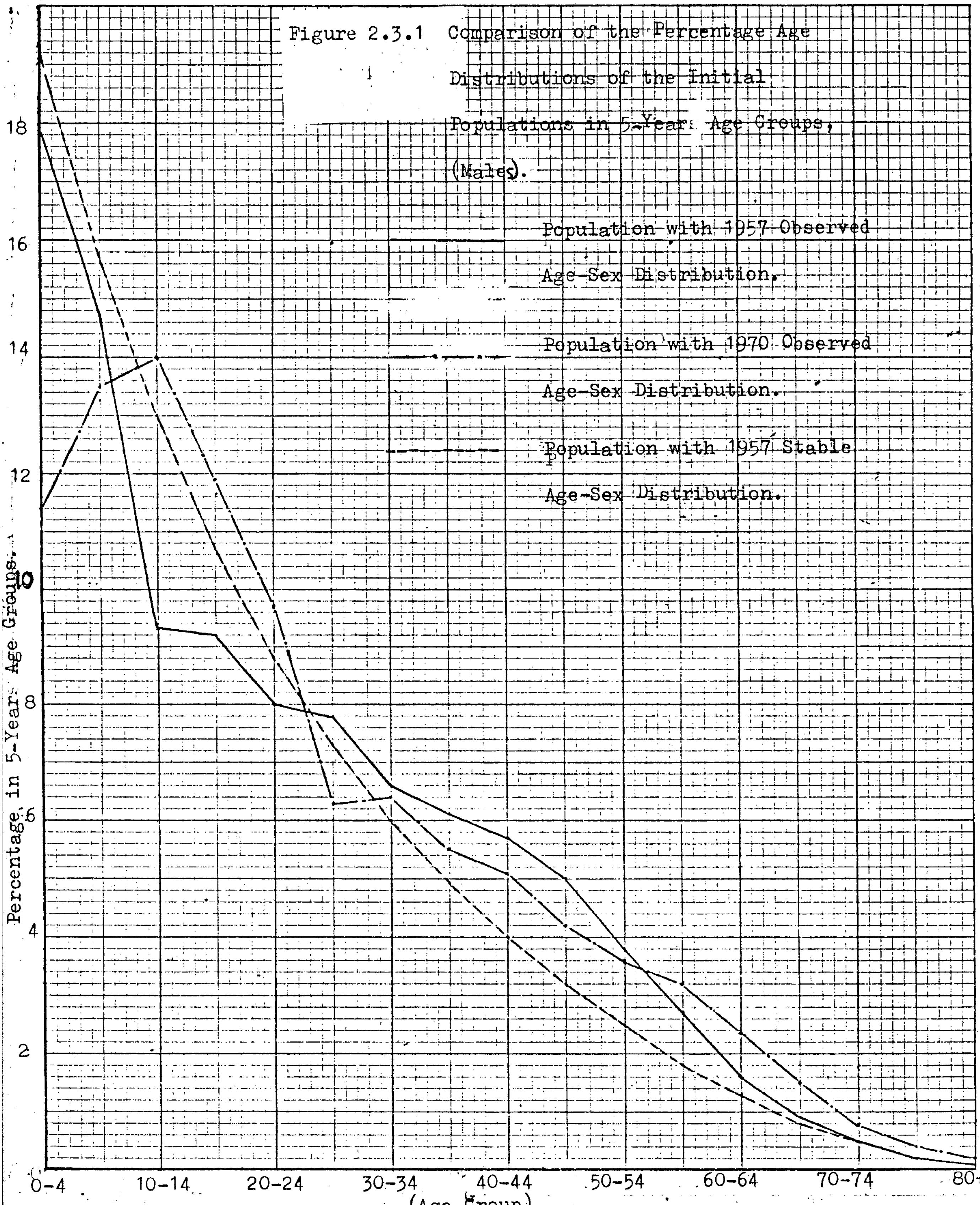


Figure 2.3.1 Comparison of the Percentage Age Distributions of the Initial Populations in 5-Years Age Groups, (Males).

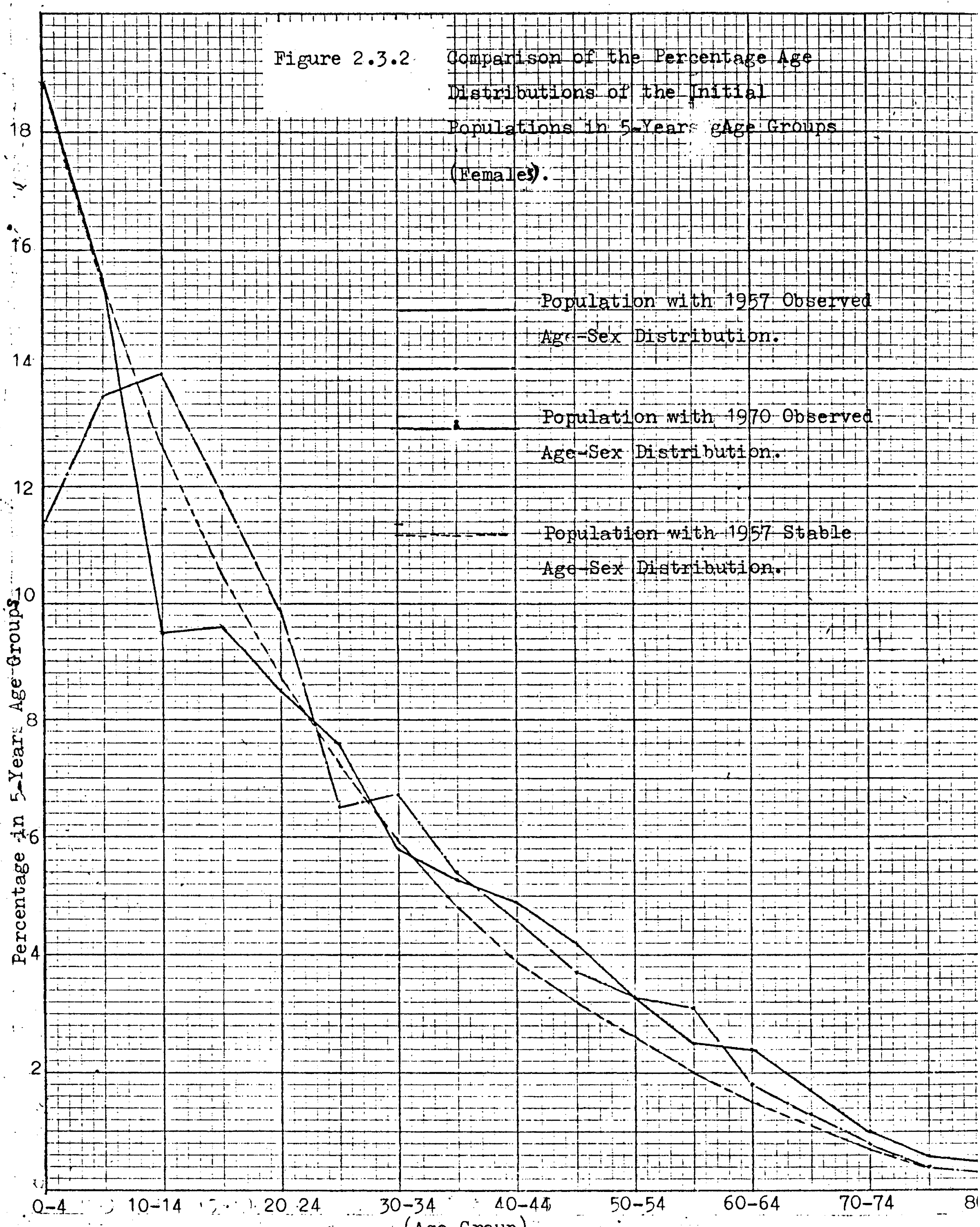
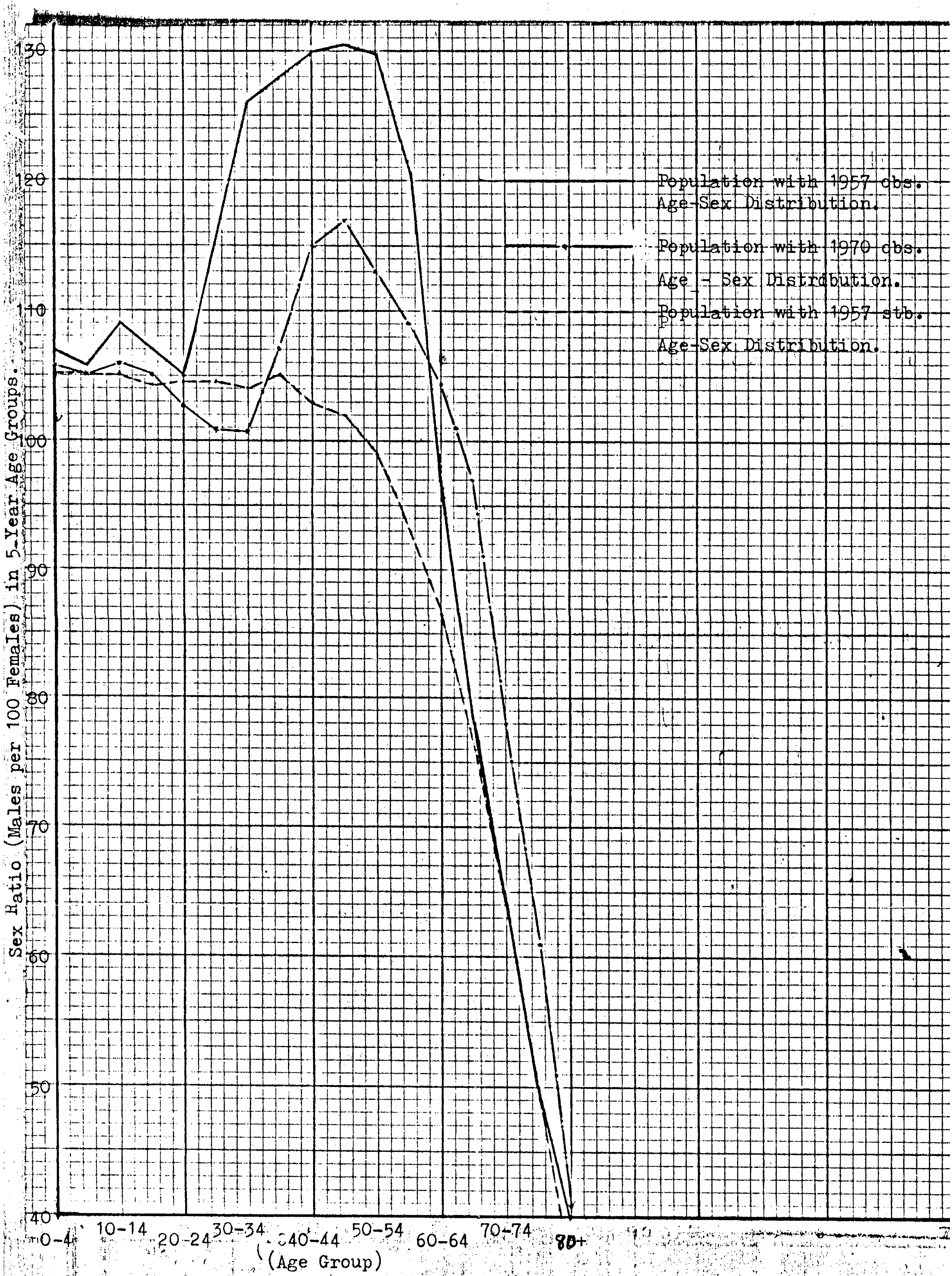


Figure 2.3.2
Comparison of the Percentage Age Distributions of the Initial Populations in 5-Years gAge Groups (Females).
Population with 1957 Observed Age-Sex Distribution.
Population with 1970 Observed Age-Sex Distribution.
Population with 1957 Stable Age-Sex Distribution.
Percentage in 5-Year Age Groups
18
16
14
12
10
8
6
4
2
0-4
10-14
20-24
30-34
40-44
50-54
60-64
70-74
(Age Group)

Figure 2.3.3 Sex Ratios in 5-Years Age Groups in the Initial Populations.



reflected an increasing population trend with the mode at the age group 0-4 years with 264,727 persons. With the fast decline in the volume of births occurring during the intercensal period the population aged 0-4 years in 1970 declined by 29,271 persons to 235,456 - a decrease of 11.1 per cent. The relative proportion of this age group had the largest decline from 18.3 per cent in 1957 to 11.4 per cent in 1970.

The very high sex ratios in the 1957 observed population in the age groups above 20 years, seen in Figure 2.3.3, show the effect of the large preponderance of males among the migrants prior to 1957. It can also be seen in the 1970 observed population in the age groups above 40 years. The new development of the Singapore's trading port by that time attracted immediately the large numbers of immigrants from the nearby territories. Due to the imbalance of sexes among the immigrants, i.e. more males than females, Singapore government had to adopt a new policy of encouraging female immigration. A trough in the middle age range, seen in the curve for the 1970 observed population is also a reflection of the strict migration policy in Singapore after 1957 (Singapore, 1970:33).

Furthermore, we shall investigate the differences of age-sex distribution in the 1957 observed, 1957 stable and 1970 observed populations by using dissimilarity indices. The dissimilarity in the age distributions can be measured by an age distribution dissimilarity index (ADI) which is defined as one-half of the sum of the absolute values of the percent differences in the two populations being compared (Keyfitz, 1964:8).

When we compared the age distributions of the 1970 observed population and 1957 observed population the value of ADI for males was 11.65 and that for females was 11.3; when we compared those of the 1957 observed population and the 1957 stable population the value of ADI for males was 8.5 and that for females was 7.2; and it was 11.2 for males and 10.25 for females between those of the 1970 observed population and 1957 stable population. One limitation of this index is that it does not show whether the differences are in the younger age groups or in the older age groups.

For this purpose we have to examine other indices such as the median age, proportion of aged persons, age dependency ratios, etc.. The median ages in the three populations 1970 observed, 1957 observed and 1957 stable populations - were respectively, 19.7, 19.4 and 16.0 for males and 19.7, 18.2 and 16.4 for females. The proportion of aged persons has also been regarded as an indicator of a young or an old population and of a population that is aging or becoming younger.

Table 2.3.3

Summary Measure of Age Composition for Various Populations: Singapore

Population	Median Age	Percent of Total Population: Under 15 Years	Percent of Total Population: 65 Years and over	Ratio of aged Person to Children (per 100) 100 x Col. (4)/ Col. (3)
(1)	(2)	(3)	(4)	(5)
1970 Obs.	19.7	38.8	3.3	8.5
1957 Obs.	18.8	42.8	2.1	4.9
1957 Stb.	16.2	47.4	2.1	4.4

Source: Basic data from Singapore: 1970 Census Report, 1957 Census Report and Table 2.3.2 of this Study.

It is clear from Table 2.3.3 that the 1970 observed population was the oldest compared with both the 1957 observed and 1957 stable populations, whereas the 1957 stable population was the youngest of all.

The sex composition in these three populations reveals that the overall sex ratios (number of males per 100 females) in 1970 observed, 1957 observed and 1957 stable populations were 104.9, 111.6 and 102.9 respectively. The dissimilarity in the sex ratio by age between any two populations can be measured by the sex ratio dissimilarity index (SRDI). It is defined as the sum of the absolute difference of the sex ratios in five-year age groups between the populations being compared (Sivamurthy, 1970:53). The value of SRDI was 143.6 between the sex ratios in the 1970 observed and 1957 stable populations; 157.9 between the sex ratios in the 1957 observed and 1957 stable populations; and 179.2 between those in the 1970 observed and 1957 observed populations. From these numerical comparisons and the graphs presented, it can be seen that the age-sex distributions of the three populations were considerably different from one another.

2.4 SURVIVAL RATIOS

The survival ratios used in the study were calculated from the life tables constructed on the basis of the 1957 census and the 1970 census populations (see Appendix B). It would be sufficient as well as convenient to examine the variations in the survival ratios observed at these points of time during the period 1957-70.

As the expectation of life at birth (e_o^o) is a composite index to represent the survival ratios we shall examine the expectation of life at birth, among these populations. The expectation of life at birth, which was 60.5 years for males and 66.6 for females in 1957, increased respectively to 65.9 and 72.3 in 1970. These figures show that the level of mortality had been lower in 1970 than in 1957, but the variations in e_o^o may not be a good guide to the variations in the survival ratios.

In fact, the measures of mortality are derived from the complex computations on mortality of the various age groups. There is a paradox that the different measures of mortality lead to different conclusions, about the relative levels of mortality in the compared populations (Frejka, 1973:49). For this study we are interested in the survival ratios by five-year age groups. We can examine the variations in the survival ratios by using the graphical method (Fig. 2.4.1 and Fig. 2.4.2).

To obtain a quantitative measure, we may assume that survival ratios at birth and the survival ratios for the last open end age group have the same width for their age intervals as the survival ratios at other ages (Sivamurthy, 1970:56). The age interval in the present study is five years. The sum of the product of the respective survival ratios and the width of the age interval gives the area under the survival ratios curve. This quantity is known as the total lifetime survived (TLS). The total lifetime survived which was 74.3 years for males and 77.8 years for females in 1957, increased respectively to 76.2 and 79.1 in 1970.

When we obtain a measure of location for this curve, say the median age ($\bar{S}_{md}$), it would indicate whether the weight of the improvements in the survival ratios was more on the older ages than on the younger ages, or vice versa. The median age for the survival ratios ($\bar{S}_{md}$) in 1957 was 32.7 for males and 34.4 for females. It increased to 33.3 for males and 34.8 for females in 1970 which means that the improvements in the survival ratios for both males and females were more on the older ages than on the younger ages.

The improvements in the survival ratios for both males and females were considerable only in the first few years of life - s(b) and s(0-4) - and in the old ages after 45 years. In fact, the teenage and the middle age groups had comparatively high survival ratios. The improvement in survival ratios at older ages for both sexes are probably due to greater reduction of the deaths from the causes of degenerative ailments (Saw, 1970:95). However, these figures show the slower improvement in the survival ratios for females than for males, and the improvement in the early childbearing period (between 15-19 and 25-29 for females) is not so much, as they have already reached the very high level of survivorship.

2.5 MARRIAGE DATA

On the assumption of no migration the proportion of married female population at various ages is determined by three aspects of nuptiality pattern prevailing in a society. These are the age

Figure 2.4.1 Survival Ratios at Five-Year Age Group for Males 1957 and 1970, Singapore.

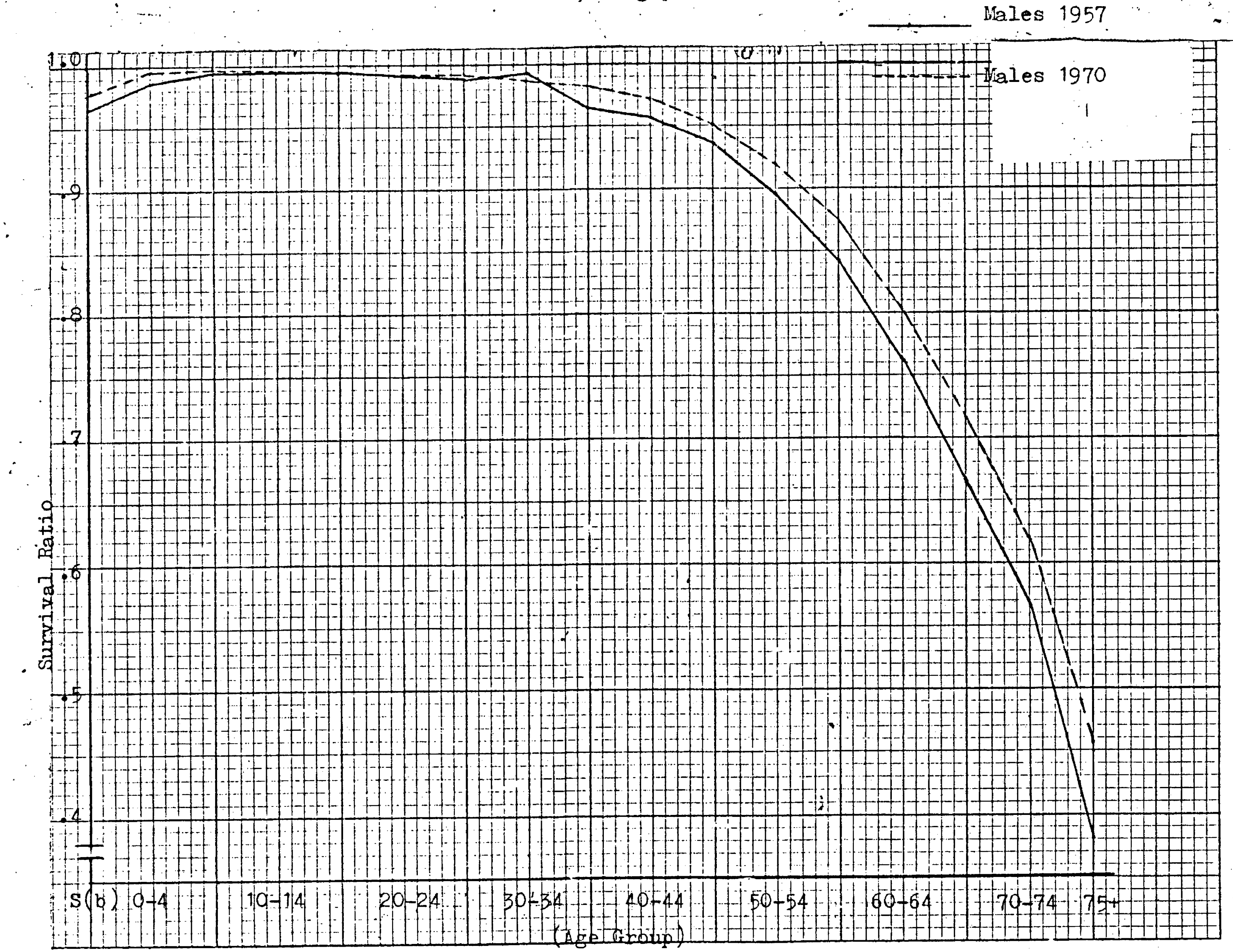
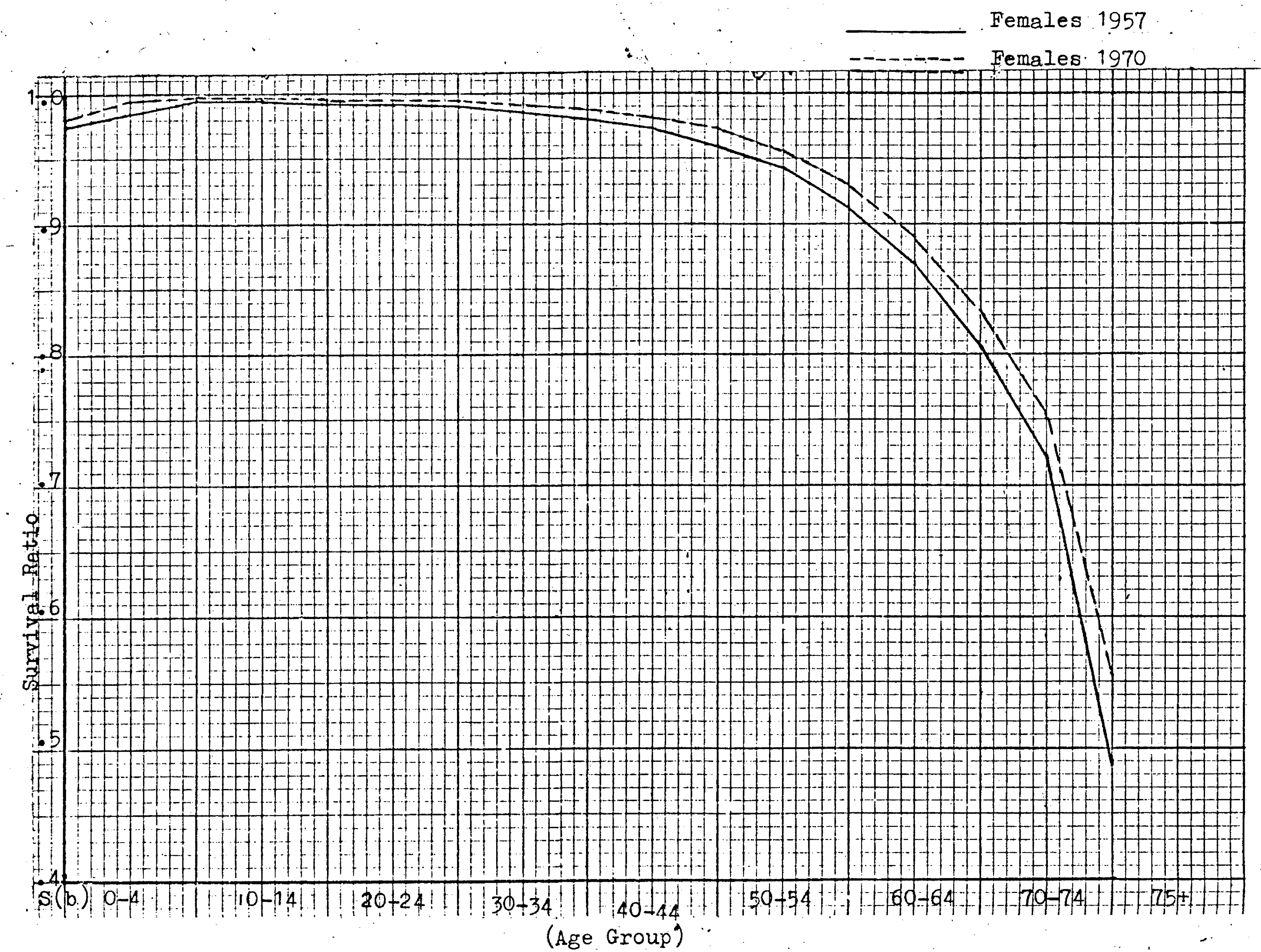


Figure 2.4.2 Survival Ratios at Five-Year Age Group for Females 1957 and 1970, Singapore.



pattern of marriage, which includes both the age at first marriage and the proportion ultimately marrying; the frequency of marital disruptions i.e. divorce and widowhood; and the frequency of remarriage among the divorced and widowed females.

In Singapore, the registered marriages are classified into three classes; Moslem marriages, Christian marriages and Civil marriages. Moslem marriages are solemnized and registered under the Moslem Ordinance, 1957, which came into force on the 25th of December 1958, and which replaced the old Ordinance of 1880. But up to the end of 1960 no comprehensive statistics had been published and only the yearly totals were available. Christian marriages are solemnized and registered under the Christian Marriage Ordinance, 1940, which came into force on January 1, 1941, and which replaced the previous ordinance of 1899. Similarly, civil marriages are solemnized and registered under the Civil Marriage Ordinance, 1940, which came into force on January 1, 1941, and which replaced the previous ordinance of 1899. Statistics on Christian and Civil marriages were first compiled and published in 1955 together with the Report on Births and Deaths (Singapore, 1958:11).

The enforcement of the Women's Charter, 1961 on September 15, 1961, made it compulsory to report all marriages to the registration offices other than Moslem marriages. Moslem marriages were first published for the year 1961. Therefore, these two sets of data give us the total marriages for Singapore's population.

Divorce among people married under Moslem marriages is registered under the Moslem Ordinance 1957. But no detailed data have been published. Divorce of other marriages have not been compiled in detail. Again, we have no data on the remarriage aspect of nuptiality. Thus our discussion will be centered on the age pattern of marriage and the dissolutions of marriage only.

In Table 2.5.1 we present the median age at first marriage for women who married in each year from 1961 to 1970. It will be observed that this median age increased continuously for several years from 22.5 years in 1961 to 23.2 years in 1966. This upward trend in median age became steady and later dropped to 23.0 in 1970.

Table 2.5.1

Percentage Distribution of Women Marrying by Age, 1961-70

Years	Age Group:							Median Age at First Marriage
	<15	15-19	20-24	25-29	30-34	35+	Total	
1961	3.6	21.1	50.8	20.6	3.3	0.7	100.1* (4668)	22.5
1962	0.2	21.9	50.3	22.4	4.1	1.2	100.1* (5566)	22.8
1963	0.1	21.1	49.7	23.8	4.1	1.2	100.0 (6515)	22.9
1964	0.1	20.3	49.5	24.2	4.6	1.3	100.0 (7348)	23.0
1965	0.1	21.3	46.4	25.7	5.1	1.4	99.9* (8540)	23.2
1966	0.0	21.7	44.1	27.1	5.6	1.4	99.9* (9809)	23.2
1967	-	19.3	47.7	26.3	5.1	1.5	99.9* (10590)	23.2
1968	-	18.5	51.0	24.1	4.6	1.7	99.9* (11182)	23.1
1969	-	16.4	53.5	22.7	4.6	1.9	100.0 (12284)	23.1
1970	-	15.3	56.5	20.8	5.3	2.1	100.0 (14197)	23.0

* Due to rounding.

Source: Based on Singapore: Report on the Registration of Births and Deaths, Marriages and Persons for 1961-70.

As an indication of the trend in age at marriage of the population, the usefulness of such annual marriage statistics is rather limited. This is so because the age composition of women marrying in each year, from which we derive the median age at first marriage, is determined not only by the inclination of marrying early or late on the part of single women but also by the age composition of female population. The levelling off of the upward trend of the annual median age at first marriage is affected by the changing percentage distribution by age of women marrying during the past few years.

It will also be observed from Table 2.5.1 that since 1966 although the median age at first marriage stopped climbing up and actually declined slightly, the proportion of women marrying below age 20 to all women marrying each year has nevertheless continued to decline.

This is indeed very significant in view of the fact that the proportion of women of ages 15 to 19 to the total population increased substantially during the past several years as a result of the post-war baby boom. In 1957 this proportion was only 4.5 per cent; in 1970 it increased to as high as 5.8 per cent (Singapore, 1970:38). We may further note that during the same period the proportion of women aged 20 to 24 also showed an increase from 4.0 per cent to 4.8 per cent. Women 25 to 29 years of age, on the other hand, decreased from 3.6 per cent in 1957 to 3.2 per cent in 1970. Thus, if the proportion of women marrying below 20 had not continued to decline, the median age at first marriage registered in recent years could have dropped at least moderately as a consequence of this pattern of changes in the age composition of the female population.

The rising proportion of women remaining single in the younger age groups reflected an increasing postponement of marriage among these women (see Table 2.5.2). In 1957 the proportion single was 80 per cent for women aged 15 to 19 and 33 per cent for women aged 20 to 24. In 1970 these two proportions increased to 95.2

Table 2.5.2

Percentage of Women Remaining Single by Age, 1957-1970

Year	Age Group:						
	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1957	80.0	33.0	9.2	4.5	4.0	4.9	5.7
1966	94.3	55.8	19.9	7.2	3.8	2.9	3.0
1970	95.2	64.6	22.6	9.6	5.1	3.3	3.1

Source: Based on Singapore; 1957 Census, 1970 Census and 1966 Sample Survey

per cent and 64.6 per cent respectively. At the same time the women in their forties who remained single were decreasing. Since very few women have their first marriage as late as beyond age 50, the proportion remaining single of women in their forties may conveniently be taken as a negative index of the proportion of women ultimately marrying.

While the proportion of currently married women in a population is determined on the one hand by the age pattern of first marriages it is affected, on the other hand, by the frequency of marital disruption as a result of widowhood and divorce and their remarriages. The greater the element of widowhood and divorce the more likelihood there is that a population will maintain a high proportion of remarriages at any point in time. However, the overall effect on proportion of currently married, especially in the childbearing ages may not be too much.

Having noted that the proportion of currently married declined during 1957 and 1970 in Singapore, we shall now present the trends in widowhood and divorce in Singapore. As mortality has declined substantially since the end of the Second World War, the chance of widowhood for women has decreased remarkably from then on. The proportion of widowed to married females, although affected by the remarriage patterns, can give an idea of the decline in widowhood over time.

The downward trend, in terms of the absolute change in the proportion of widowed to married women (Table 2.5.3) was pronounced in the case of women over 25 years of age. In 1957 the ratio was as high as 36.5 per cent for women 45 to 49 years old and it had dropped to 17.7 by 1970. For the first two age groups within the childbearing period the proportion of widows to married women showed a significant decline from 1957 to 1966, and then increased again during 1966 and 1970. It should be noted here, that the 1966 data were collected by the sample survey and the two others were collected by the censuses, and so they may not be strictly comparable.

Table 2.5.3

Proportion of Widowed to Married Women by Age, 1957-1970

Year	Age Group							
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	15-49
1957	0.5	0.8	1.8	3.6	8.3	18.4	36.5	8.2
1966	0.0	0.4	1.4	1.8	4.5	11.5	21.1	5.5
1970	0.4	0.5	0.7	1.6	4.1	9.3	17.7	4.7

Source: Same as in Table 2.5.2.

The proportion of divorced to currently married women rose between 1957 and 1966 for practically all age groups except 15-19 and 20-24 within the childbearing period and then decreased during 1966 and 1970 except in age group 15-19 (see Table 2.5.4). However, these trends may have been due to the fact that the three sets of data may not be comparable.

Table 2.5.4

Proportion of Divorced to Married Women by Age, 1957-1970

Year	Age Group							
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	15-49
1957	0.8	0.8	0.6	0.7	0.6	0.6	0.7	0.7
1966	0.0	0.7	1.1	1.8	2.4	2.8	1.7	1.7
1970	0.5	0.7	0.8	1.2	1.5	1.7	1.5	1.2

Source: Same as in Table 2.5.2.

2.6 FERTILITY RATES

The changes in fertility rates can be examined by their level and the age pattern. The level can be measured by crude birth rate (CBR), general fertility rate (GFR), total fertility rate (TFR) and gross reproduction rate (GRR), but the last two are the only refined measures of fertility. The total fertility rate is the sum of the age-specific fertility rates in the reproductive age groups. This is a very useful and refined measure of fertility and provides an indication of the average number of children a woman, alive during entire reproductive age span, would bear during the whole of her reproductive period on the basis of the existing fertility pattern. The gross reproduction rate (GRR) is a special case of the total fertility rate in that it only measures the number of daughters the same woman would bear in her life.

The total fertility rate in Singapore declined from 6.5 in 1957 to 2.1 in 1975, and the gross reproduction rate declined from 3.2 in 1957 to 1.0 in 1975. The total fertility rate depicts in a more dramatic way the nature of the decline and shows that the average size of family for a woman during her reproductive years had been reduced from about seven to two during this period. The rate of decline accelerated from an annual average of 3.3 per cent for the first three years 1957-60, to about 9.5 per cent for the years 1966-69, and then slowed again to a figure of 2 per cent for the years 1969-72. The last three years 1972-75 showed a decline of 10 per cent.

For the age pattern of fertility we can examine the age-specific fertility rates which measure specifically the fertility of each age group of females. As Table 2.6.1 shows the age-specific fertility rates declined in all the age groups with major reductions in the 15-19 and old age groups (35 years and over). One feature of the change in the pattern of age-specific fertility rates has been the concentration of fertility in the narrow range of ages. This can be seen in Figure 2.6.1.

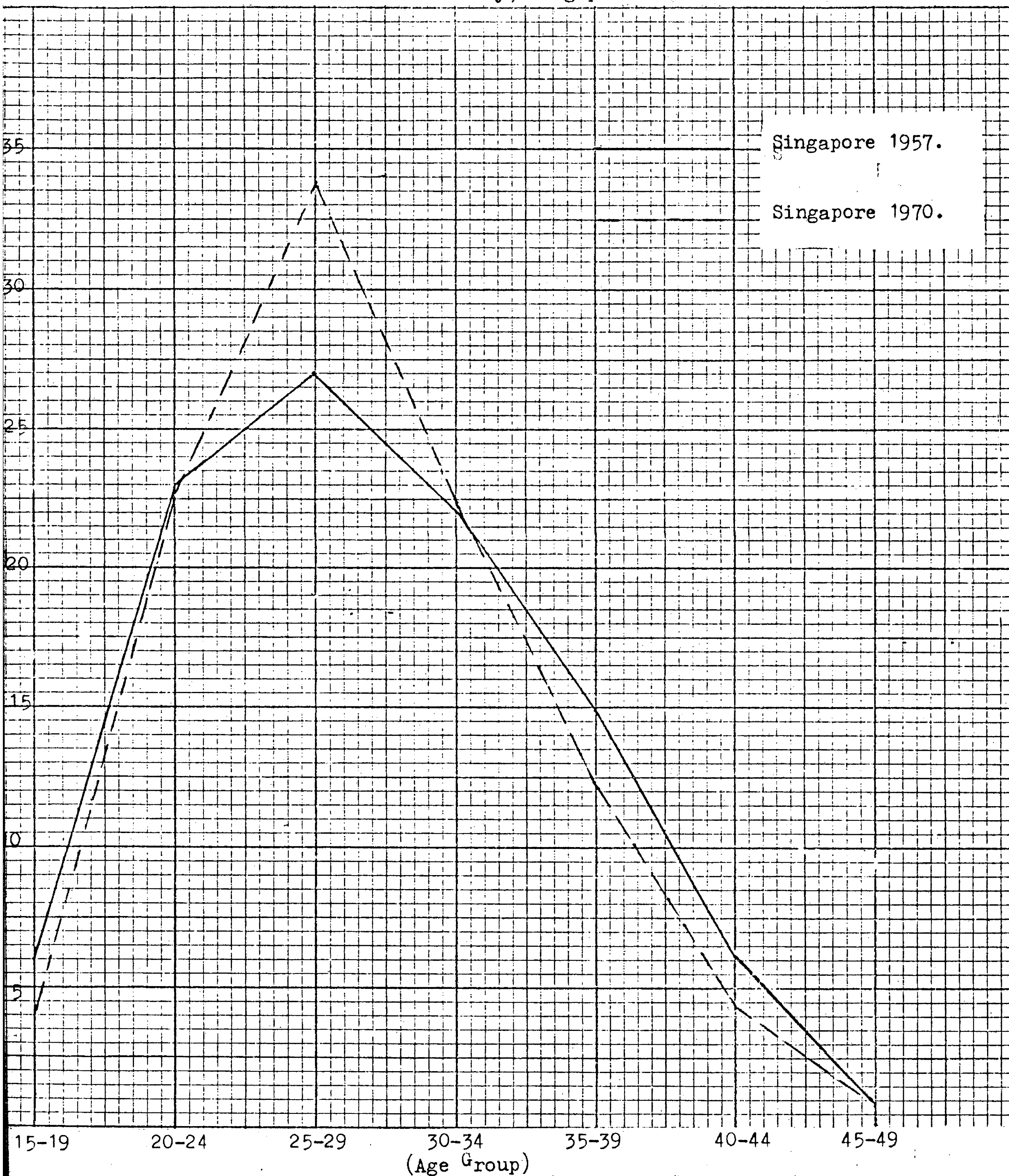
Table 2.6.1

Age-Specific Fertility Rates, 1947 & 1956-75 with Index (1957 = 100)

Year	Age Specific Fertility Rates (Index 1957 = 100)							Total Fertility Rates
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	
1947	131	106	96	95	102	103	91	101
1956	99	106	100	98	97	101	91	100
1957	100	100	100	100	100	100	100	100
Actual level	(77.6)	(296.6)	(347.1)	(285.3)	(192.2)	(80.6)	(11.6)	(6.5)
1958	103	96	97	97	96	97	107	97
1959	104	89	94	93	95	91	96	93
1960	90	84	93	91	92	88	99	90
1961	82	81	87	84	88	80	79	84
1962	68	83	84	81	81	81	78	81
1963	59	84	82	80	81	81	72	80
1964	49	81	80	79	77	78	73	77
1965	47	77	74	76	72	68	71	73
1966	43	74	75	71	65	64	76	70
1967	46	66	70	58	50	53	66	61
1968	40	56	68	53	44	44	60	55
1969	35	51	65	47	39	36	48	50
1970	34	47	60	48	39	33	41	48
1971	33	47	62	48	35	26	27	47
1972	33	47	63	49	34	26	26	47
1973	31	45	58	45	30	22	20	43
1974	27	41	50	36	22	16	15	37
1975	22	35	45	34	19	12	10	32

Source: Figures for pre-1970 periods are taken from Singapore, 1970 census report, Table 5.6, p.36. Figures for 1971-75 are computed from Singapore, Year Book of Statistics, 1975/76, Table 2.5, p.18.

Figure 2.6.1 Relative Contribution of Women in Each Age Group to ~~Gross~~ Total Fertility, Singapore 1957–1970.



The variation in the age pattern can also be measured by the variation in the mean age of the fertility schedule ($\bar{m}$) which is a measure of location for the fertility rates curve. Apart from this measure ($\bar{m}$), the variance of the curve can be found to show the concentration of fertility. The mean of the age-specific fertility rates declined from 29.5 in 1957 to 28.3 in 1975. The variance in the fertility rates declined from 45.9 in 1957 to 30.2 in 1975.

2.7 HYPOTHETICAL DATA

Before we describe the population projections we shall discuss our hypothetical data used in this study. As we discussed in Chapter 1, the survival ratios observed in Singapore for the period 1969-71 will be used throughout our study, with the presumption that significant changes in mortality are unlikely to occur in the absence of unforseseen natural or other calamaties. The sex ratio at birth was assumed to be constant with the value of 106 males per 100 females in this study.

Hence, hypothetical data were derived only in the case of proportions married and nuptial-specific fertility rates. The procedures adopted in obtaining the hypothetical data and a brief description of the characteristics of these data are given in this section. In fact, Coale has recently treated this subject for finding the patterns of nuptiality and fertility with sufficient sophistication to yield very satisfactory results, and his methods of characterizing these schedules are used here (Coale, 1971:193-214, and Coale, 1974:185-258).

2.7.1 HYPOTHETICAL NUPTIALITY

Coale has found that the first marriage frequencies observed in a number of diverse societies can be scaled very closely by a double exponential curve, i.e.,

$$r_s(x) = 0.174 \ e^{-4.411} \ e^{-0.309 \, x} \quad \ldots \quad \ldots \quad 2.7.1$$

where $r_s(x)$ is the standard risk of first marriage and x a standard time schedule. This standard nuptiality schedule can be fitted to the observed populations by using three parameters. They are a_o, k and C.

a_o is the lowest age at which a noticeable proportion in the population first enter marriage;

k is a measure of the time scale at which first marriages occur, i.e. the pace of first marriages;

C is a factor determined by the ultimate proportion ever married. The value of a_o, k and C can be calculated by using the standard schedules derived from the standard distribution (Coale, 1971:200-202, 210-213). In actual populations, a_o varies from 12.5 to 18.0 years, and k from 0.2 to 1.7 (Coale, 1974:191). In some populations the final proportion ever married, C, comes to unity, whereas in other (Ireland), it is nearly as low as 0.75.

This technique can be used in the development of our nuptiality schedules. On the basis of the first set of data (shown in Table 2.7.1 and Figure 2.7.1), a model nuptiality schedule characterized by a_o = 17.6 years, k=0.58 and C=.968 was assigned to represent the pre-transition behaviour (Schedule I). This schedule was obtained by using the 1970 census population of Singapore (see Appendix C). It represents a slightly older age pattern of marriage than is reported for 1957 census population.

After having this set of schedules a transition was computed by allowing Schedule I to shift to a second schedule; Schedule II with a_o=18 years, k=0.78 and C remains 0.968. The transition takes place through a linear change over an intermediate period.

From Table 2.7.1 we can see that the selected schedules represent three different levels of proportion ever-married. The singulate mean age at marriage from Schedule I is 24.0, that from Schedule M 25.4 and for Schedule II it is 26.8.

However, the fit of the standard schedules to the nuptiality of an actual population is, of course, only approximate (Coale, 1971:214). Again, the proportion ever married for women aged 15-19

Table 2.7.1

Percent Ever-Married at Various Age Groups

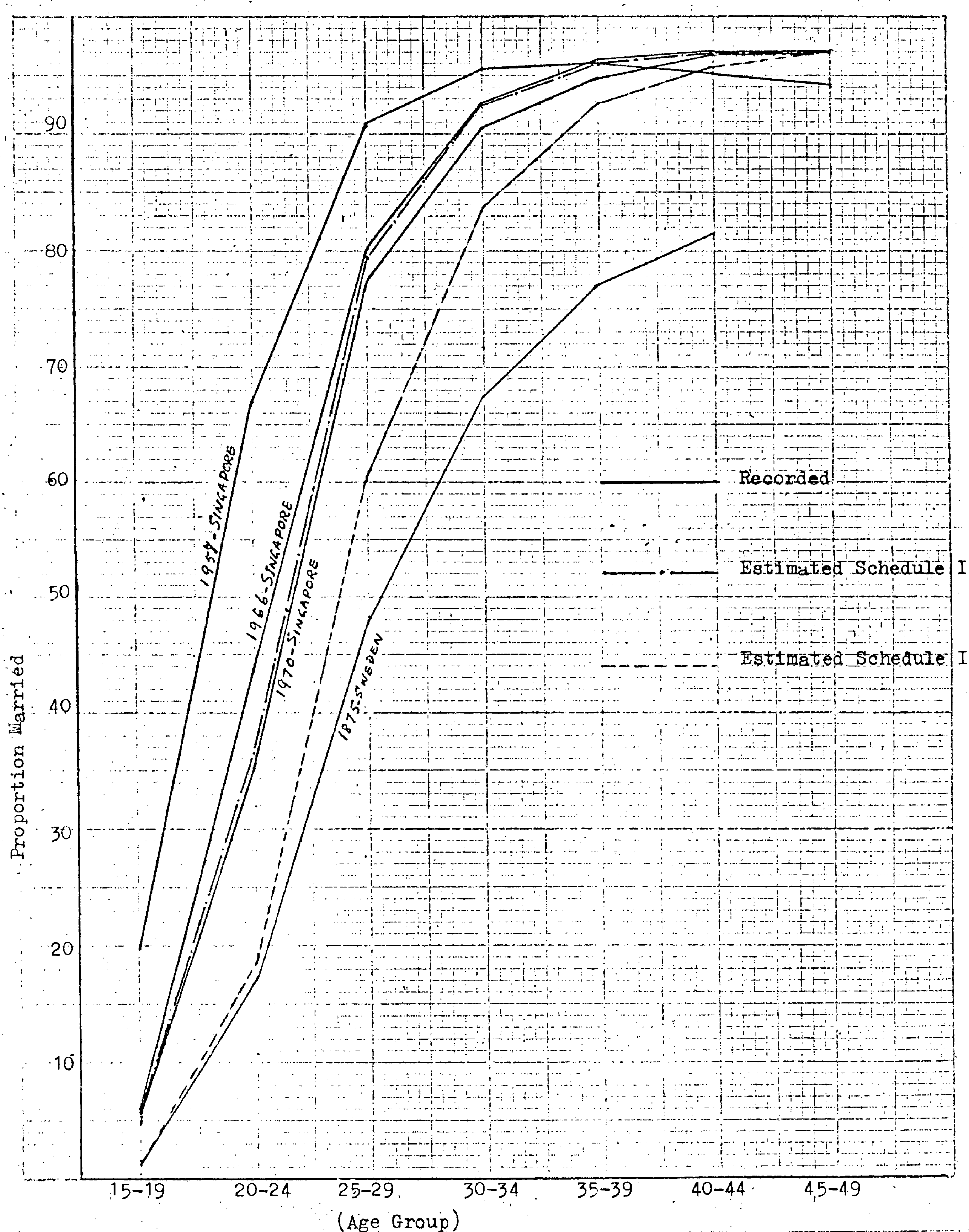
Census Years 1957, 1966, and 1970, and Selected Schedules

Census Years	Age Group						
	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1957	20.0	67.0	90.8	95.5	96.0	95.1	94.3
1966	5.7	44.1	80.0	92.8	96.2	97.0	97.0
1970	4.8	35.4	77.4	90.3	94.9	96.7	96.9
Selected Schedules:							
Pre-transitional (Schedule I with a_o=17.6, k=.58, C=.968).	4.8	36.3	79.5	92.7	96.1	96.8	96.8
Intermediate (Schedule M).	2.9	27.4	69.9	88.3	94.3	96.1	96.8
Post-transitional (Schedule II with a_o=.18, k=.78, C=.968).	1.0	18.5	60.3	83.8	92.5	95.4	96.8

Note: Values are rounded up to one decimal place.

has been substituted in Schedule I and Schedule II, by those of Singapore observed value in 1970 and Sweden observed value in 1875 respectively, because the standard proportion ever married for women aged 15-19 given from the estimation are far below those of the actual populations, and they cannot be representative of the actual schedules. Swedish 1875 data are chosen because the proportion ever married in Sweden at that time was one of the lowest ever recorded in actual population.

Figure 2.7.1 Proportions Ever-Married in Selected Years, Singapore Female Populations Compared to the Schedules used in the Simulation.



In the absence of data for marriages and divorces, it is assumed that the schedules of proportion ever married by age can also be converted into proportions currently married on the basis of the observed ratios of currently married to ever married proportions obtained for the 1970 Singapore population. These values are shown in Table 2.7.2 and Table 2.7.3 below.

Table 2.7.2

Ratios of Proportion Currently Married to Proportions Ever-Married in Singapore, 1970

Age Group						
15-19	20-24	25-29	30-34	35-39	40-44	45-49
1.000	0.989	0.985	0.972	0.946	0.900	0.839

Source: Calculated from the data given in 1970 Census Report, Singapore: Table 7, p.10.

Table 2.7.3

Proportion of Currently Married by Age Group for Selected Schedules

	Age Group						
Selected Schedules	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Pre-transitional (Schedule I)	4.8	35.9	78.3	90.1	90.9	87.1	81.3
Intermediate (Schedule M)	2.9	27.1	68.9	85.8	89.2	86.5	81.3
Post-transitional (Schedule II)	1.0	18.3	59.4	81.4	87.5	85.9	81.3

Note: Values are rounded up to one decimal place.

From the Table 2.7.2, the ratio of the first age group 15-19, for Singapore 1970 is one, where no divorce or widowhood occurred, and then other ratios decline gradually to the value of 0.839 at the last age group 45-49. Because of this effect the reduction of divorce and widowhood from the proportion of ever married is none at the age group 15-19, but significant in the later childbearing age groups. This can be seen by comparing Table 2.7.1 and Table 2.7.3.

2.7.2 HYPOTHETICAL MARITAL FERTILITY

This part of our hypothetical data derivation deals with the estimation of the age-specific marital fertility rates during the pre and post transitional stages. For this purpose technique developed by Coale (Coale, 1971:193-214), briefly discussed below, has been used. According to the technique, a schedule of high marital fertility (Hutterites) is chosen, and the desired schedules are obtained with the application of the following formula and suitably selecting values of M, m and V(a) which are also defined below.

$$\frac{m(a)}{n(a)} = M \exp (m.\ v(a)) \quad .. \quad .. \quad 2.7.2$$

where: m(a) = the observed schedule of marital fertility rates;

n(a) = the schedule of natural marital fertility (Hutterites, 0.550, 0.502, 0.447, 0.406, 0.222 and 0.061, for age groups 20-24 to 45-49);

M = the scale factor expressing the ratio m(a)/n(a) at some arbitrarily chosen age 'a';

v(a) = the characteristic age-pattern of deviation from the natural fertility schedule;

m = the degree to which the standard deviation pattern applies (Coale, 1974:187).

When M=1 and m=0, equation 2.7.2 describes the observed marital fertility schedule as the natural fertility schedule (Hutterites). The higher the value of m, the greater the departure from the natural fertility schedule. The highest value of m observed in actual population is about 2.0 (Hungary 1965, Japan 1970); practically all the populations of industrialized nations possess values of m above unity (United Nations, 1965:568-573, United Nations, 1975:776-779). These measures can be used in the development of nuptial-specific fertility schedules (here age-specific marital fertility rates). The initial pre-transitional schedule was chosen by M=0.722 and m=1.15, which reflected the 1970 Singapore situation and the final post-transitional schedule was assumed to be characterized by M=0.62 and m=1.91 which represented the situation of Japan 1960 (see Appendix D). The transition in marital fertility is developed through linear changes in the parameters from those of the pre-transitional schedule to post-transitional schedule. Again, the transition takes over an intermediate period (see Table 2.7.4 and Figure 2.7.2).

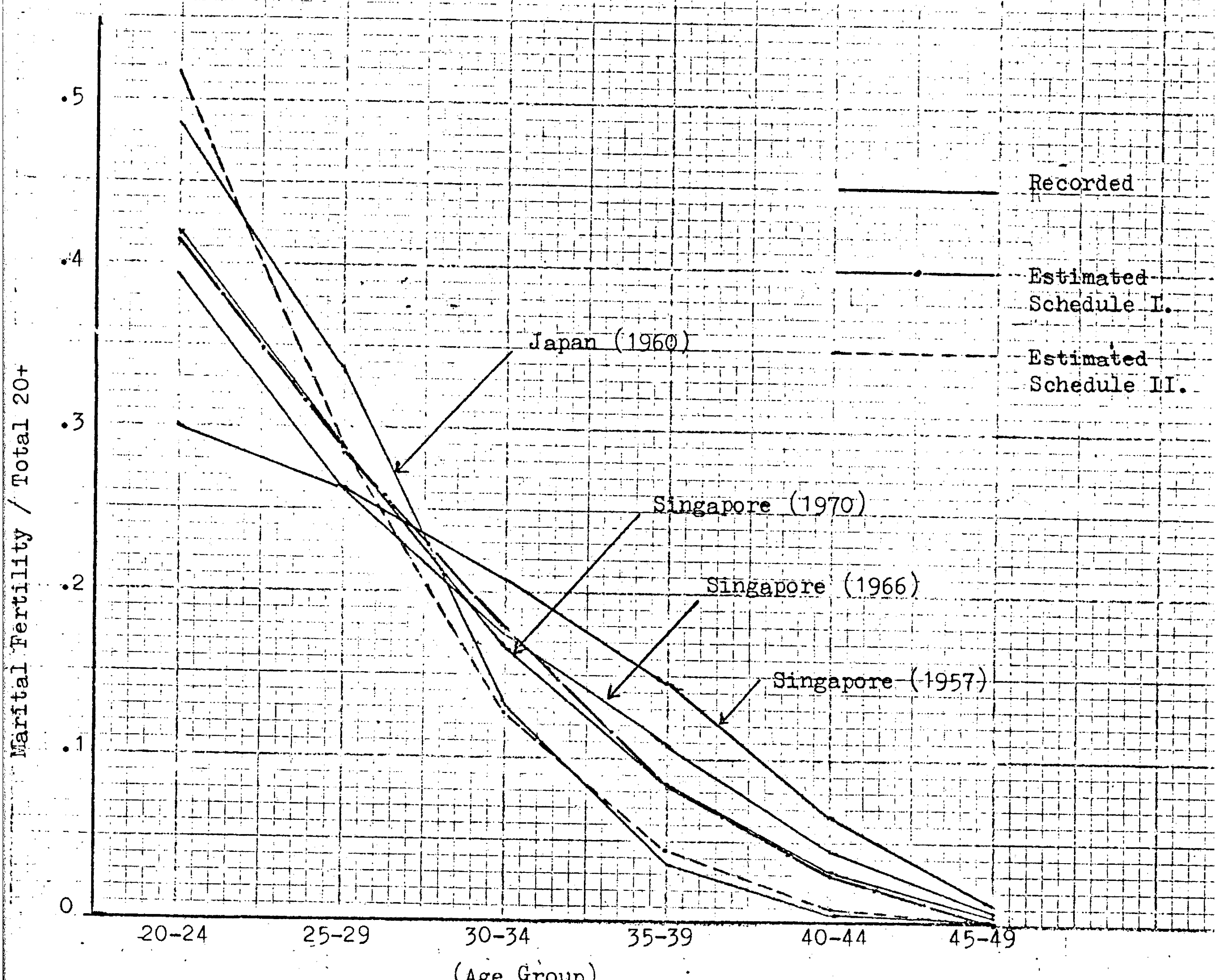
Table 2.7.4

Age-Specific Marital Fertility
Rates for 1957, 1966, 1970 and Selected Schedules

Census Years	Age Group						
	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1957	394.6	452.0	393.3	310.6	218.3	101.5	17.1
1966	582.3	499.9	334.5	225.4	138.7	60.9	11.2
1970	544.7	397.1	274.1	157.2	83.0	30.7	5.7
Selected Schedules:							
Pre-transitional (Schedule I with M=0.722, m=1.15)	544.7	397.1	271.3	177.1	83.3	27.9	2.7
Intermediate (Schedule M with M=0.671, m=1.54)	431.9	369.1	228.5	114.8	50.5	14.3	0.9
Post-transitional (Schedule II with M=0.620, m=1.91)	319.1	341.0	192.6	84.3	31.1	7.6	0.4

Note: Values are rounded up to one decimal place.

Figure 2.7.2 Marital Fertility in Each Age Interval Relative to Total Marital Fertility Over 20: Selected Years for Female Populations Compared to the Schedules Used in the Simulation.



From the Table 2.7.4 we can see that the selected schedules have three different levels of total marital fertility: 7.52 for Schedule I, 6.05 for Schedule M and 4.88 for Schedule II.

However, it is impossible to infer a marital fertility rate for the age group 15-19, since this value is often severely inflated by pre-marital conceptions precipitating in marriage before the expected births actually occur (Coale, 1974:188). But in cultural settings which place a high premium on virginity before marriage this marital fertility rate is lower than that observed for the age group 20-24. During a process of nuptiality restriction like that postulated in this approach, pre-marital conceptions might increase in such populations, leading to a rise in marital fertility rate in the youngest age group. Any estimate of the magnitude of this effect is bound to be somewhat speculative; the level of marital fertility assigned for the age group 15-19 in the post-transitional schedule is therefore, rather arbitrarily chosen. We chose here as observed marital fertility rate at age group 15-19, based on Japan's 1960 experience. Then the marital fertility at age group 15-19 for the transitional schedules are interpolated between those values of Singapore 1970 and Japan 1960 observed populations.

2.7.3 HYPOTHETICAL AGE-SPECIFIC-FERTILITY RATES

Following adaptation of the nuptiality and marital fertility behaviour for transversal schedules, these two sets were multiplied to yield age-specific fertility rates which were then used in our population projections. These schedules are presented in Table 2.7.5. It should be noted that fertility outside matrimony is rejected. In this table the three sets of fertility and three sets of nuptiality schedules were combined in nine different ways. Here, we call the marital fertility of pre-transitional schedule (Schedule I) as S(I), that of post-transitional schedule (Schedule II) as S(II), and that of middle transitional schedule (Schedule M) as S(M), and the proportion married of these as S'(I), S'(II) and S'(M) respectively.

Table 2.7.5

Age Specific Fertility Rates for Selected Schedules

Prevailing Conditions		Age Group									
Marital Fertility	Proportion Married	15-19	20-24	25-29	30-34	35-39	40-44	45-49	Total Fertility Rate	Mean ($\bar{m}$)	Variance (σ^2)
S(I)	S'(I)	26.1	142.6	212.4	159.6	75.7	24.3	2.2	3.2	29.0	35.1
S(I)	S'(M)	15.3	107.7	186.8	151.9	74.3	24.1	2.2	2.8	29.7	34.0
S(I)	S'(II)	5.6	72.7	161.2	144.2	72.9	24.0	2.2	2.4	30.5	31.7
S(M)	S'(I)	20.7	132.6	178.9	103.5	45.9	12.5	0.7	2.5	28.1	31.3
S(M)	S'(M)	12.6	100.1	157.3	98.5	45.1	12.4	0.7	2.1	28.7	30.9
S(M)	S'(II)	4.5	67.6	135.8	93.5	44.2	12.3	0.7	1.8	29.5	29.2
S(II)	S'(I)	15.3	122.5	150.8	76.0	28.3	6.6	0.3	2.0	27.6	27.3
S(II)	S'(M)	9.3	92.5	132.6	72.3	27.7	6.6	0.3	1.7	28.1	27.3
S(II)	S'(II)	3.3	62.4	114.4	68.7	27.2	6.5	0.3	1.4	28.8	26.2

Note: Values are rounded to one decimal place.

It can be clearly seen in Table 2.7.5, that the combination of the schedules of marital fertility and the schedule of proportion married affected the total fertility rate. When we compare the effect of changes in proportion married and marital fertility separately on the total fertility rate, we can also notice that the reduction of marital fertility is more serious than the reduction in proportion married. From Table 2.7.5 the reduction in proportion married without accompanying change of marital fertility reduced the total fertility from 3.2 to 2.8 and then to 2.4, while the reverse situation reduced the total fertility rates from 3.2 to 2.5 and then to 2.0. In the former case the mean age of fertility schedule was increased from 29.0 to 29.7 and then to 30.5, while the variance reduced from 35.09 to 34.01 and then to 31.67. In the latter case the mean age of fertility schedule decreased from 29.0 to 28.1 and then to 27.6, while the variance reduced very fast from 35.09 to 31.25 and then to 27.34. The effect of these changes in transitional form will be discussed later in Chapter 4.

CHAPTER 3

CONVERGENCE OF AGE-SEX DISTRIBUTION TO AN EQUILIBRIUM STATE

3.1 INTRODUCTION

Having obtained the basic data for studying the effects of nuptiality and fertility variables on the duration of convergence of population, in this chapter, we shall first explain the theoretical considerations of the convergence of the age-sex distribution to an equilibrium state. Secondly, the duration of convergence will be examined based on Singapore's population and the hypothetical data obtained in Chapter 2. We shall also make a comparison between the results of the duration of convergence based on theoretical consideration and the results obtained from the actual projections.

An equilibrium state population is defined here as a population which has an unchanging age-sex distribution and grows at a constant rate under the operation of a specified set of mortality, fertility and nuptiality schedules that remain constant over time. The classical theory of stable population which is well known in the literature, is an equilibrium state population under the operation of a given set of constant age-specific fertility and mortality schedules, but no recognition is given to marriage.

The stable population is established from the moment the proportion of population at age x falls within some arbitrary small margin of the population stipulated in the well known equation $p(x) = be^{-rx} S(x)$, where $p(x)$ is the proportion at ages x to x+1, b the constant birth rate in the stable population, r the constant rate of increase, and $S(x)$ the proportion surviving from birth to age x.

For the purpose of this chapter, we define the duration of convergence as the number of years needed for the difference between an arbitrary age-sex distribution and the equilibrium state age-sex distribution, to become less than a pre-assigned small quantity, from the time the specified schedules of fertility, mortality start operating. This can be investigated by two procedures corresponding to two methods:- a method in which population dynamics is defined in terms of the product of a sequence of matrices, and a method in which the sequence of births is defined by means of an integral equation (Lopez, 1961:8-41). The latter makes the exposition of the classical stable model rather complicated from the mathematical point of view, because of the continuity of the birth series as a function of time.

Being an empirical investigation we shall use the first approach in this study. We shall present a discussion of the analytical framework of this approach in Appendix E, which is in fact the work of Sivamurthy (1970: 97-103, 121-123). It should be noted that the unit of time interval in this theoretical consideration was single year. Unfortunately we did not have the complete data for the analysis of this study by single years of age. Since we have to deal with data in five-year age groups, the following empirical investigation will be in terms of five years age intervals.

3.2 EMPIRICAL INVESTIGATION

The three initial populations - 1957 obs., 1957 stb., and 1970 obs. populations were used in our empirical investigation. Projections were made successively under the same constant conditions of 1969-71 mortality, with selected fertility and nuptiality, presented in Table 3.2.1. In all projections, at each age group a growth index was computed as the ratio of the population at ages x to x+4 at time t to the population at the same age group at time t+5. The maximum growth index (MAGI) and the minimum growth index (MIGI) were then chosen and recorded for each of the three populations at every five-year interval of time.

Table 3.2.1

Projection Matrices $\underline{M}$ for Singapore 1970, with Constant Mortality, Fertility and Nuptiality Assumption

Age	Subdiagonal *	First Rows **								
		S(I)xS'(I)	S(I)xS'(M)	S(I)xS'(II)	S(M)xS'(I)	S(M)xS'(M)	S(M)xS'(II)	S(II)xS'(I)	S(II)xS'(M)	S(II)xS'(II)
0 - 4	.99551	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
5 - 9	.99795	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
10 -14	.99791	.03087	.01899	.00713	.02494	.01544	.00475	.01781	.01069	.00356
15 -19	.99706	.20012	.14717	.09375	.18278	.13411	.08544	.16259	.11986	.07713
20 -24	.99604	.42095	.35015	.27768	.37041	.30507	.24210	.32414	.26711	.20889
25 -29	.99472	.44165	.40243	.36202	.33491	.30282	.27191	.26964	.24348	.21732
30 -34	.99281	.28017	.26829	.25759	.17690	.16978	.16264	.12351	.11875	.11400
35 -39	.98878	.11867	.11629	.11510	.06885	.06767	.06647	.04155	.04155	.04036
40 -44	.98264	.03089	.03089	.03089	.01545	.01545	.01545	.00949	.00949	.00949
45 -49	-	.00238	.00238	.00238	.00119	.00119	.00119	.00119	.00119	.00119

* ${}_5L_{x+5} / {}_5L_x$ ** $(L_o/2l_o) \left\{ F_x + F_{x+5} \quad ({}_5L_{x+5}/{}_5L_x) \right\}$

Note: F_x is the age-specific female fertility rates; L_x and, l_0 are the same as life table notations.

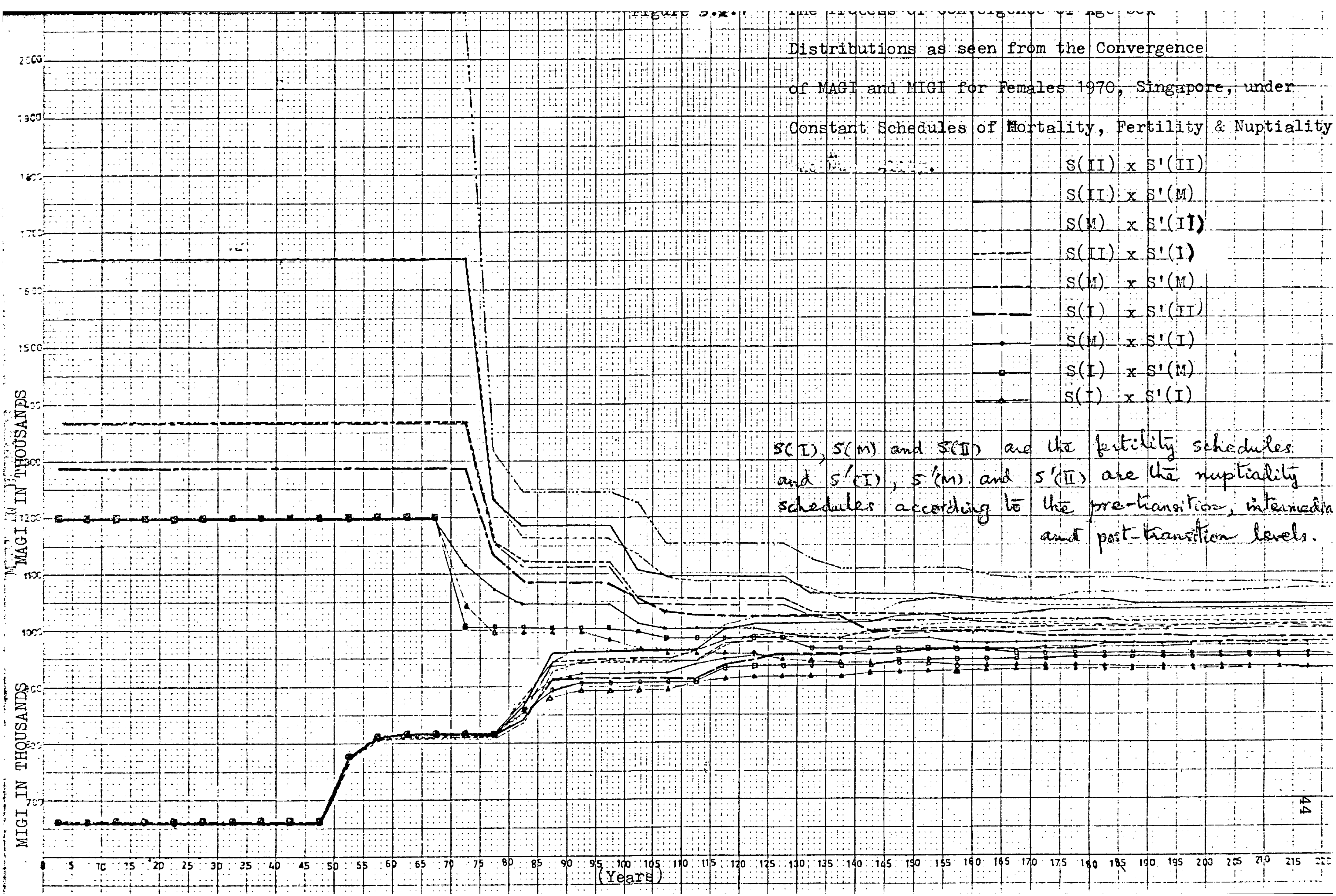
When the difference between MAGI and MIGI became less than 0.0009, each of the age-sex distributions was accepted to have reached the equilibrium state. The limit of 0.0009 was arbitrarily chosen. The computation of the MAGI and MIGI over time would not only show the nature of the process of convergence but also the magnitude of the intrinsic growth rate resulting from the given nuptiality and fertility conditions (Sivamurthy, 1970: 131).

The nature of the process of convergence can be seen in Fig. 3.2.1. In this figure we can see clearly the convergence of MAGI and MIGI when the fertility and nuptiality rates for 9 different combinations of the schedules, presented in Section 2.7.3 of Chapter 2, were assumed. The process of convergence of MAGI and MIGI seems to have a clear similarity, i.e. the way they converge is quite similar. In all the cases MAGI and MIGI remain at the same level for a number of years before they start to converge, and the large differences in the combination of fertility and nuptiality schedules reduce as soon as the cohorts alive at the initial point of time die out; but smaller variations induced by the initial differences take a long time to disappear.

The duration of convergence, t, thus obtained was compared with the one obtained from the matrix method. It is also mentioned in Sivamurthy's study (Sivamurthy, 1970:131) that the value of t obtained by the direct projection method is conditioned by the nature of the initial age-sex distribution while the value of t computed from the matrix roots is not conditioned by the initial age-sex structure. The projection matrices $\underline{M}$ presented in Table 3.2.1 are used for the calculation of the first stable matrices, often called the spectral components (Z_1), which are constant matrices, and not a function of time.

The matrix M is discussed in Appendix E and will not be repeated here. This matrix is of such a character that when taken to powers it stabilized. Our computer program presented in Appendix F, calculated the matrix M^{64} by successive squarings of M and also calculated the M^{65}. The matrix M was raised to these

Figure 3.[illegible] The Process of Convergence of Age-Sex Distributions as seen from the Convergence of MAGI and MIGI for Females 1970, Singapore, under Constant Schedules of Mortality, Fertility & Nuptiality



powers following the example of United States males, 1964, given in 'Introduction to the Mathematics of Population' (Keyfitz, 1968:64). $_1$ was computed by:

$$\lambda_1 = m_{21}^{65} / m_{21}^{64} \quad .. \quad .. \quad .. \quad 3.2.1$$

where $m_{21}^{t'}$ is the first element of the second row of matrix $M^{t'}$. Later, Z_1 was obtained by dividing all terms of M^{64} by λ_1^{64}.

Using the analog of $Z = M^{t'}/\lambda_1^{t'}$ for finding the next pair of components Z_2, Z_3 we first remove $\lambda_1 Z_1$, that is, calculate the deflated M such as:

$M - \lambda_1 Z_1 = N$, and take N to a high power. Although we need not go as high as for the first latent root, λ_1, here we use the same power of 64 again.

As the λ_2 and λ_3 are of the same absolute value they cannot be separated by taking a single high power of N. We need all together three powers, say $N^{t'}$, $N^{(t'+1)}$ and $N^{(t'+2)}$. Then λ_2 and λ_3 can be calculated from a quadratic equation:

$$x^2 \left[(n_{11}^{(t'+1)} . n_{21}^{(t')}) - (n_{21}^{(t'+1)} . n_{11}^{(t')})\right] - x \left[(n_{11}^{(t'+2)} . n_{21}^{(t')}) - (n_{21}^{(t'+2)} . n_{11}^{(t')})\right] + \left[(n_{11}^{(t'+2)} . n_{21}^{(t'+1)}) - (n_{21}^{(t'+2)} . n_{..}^{(t'+1)})\right] .. \quad .. \quad (3.2.2)$$

where n_{ij} are the elements of the matrix N, whose roots in x are λ_2 and λ_3 (Keyfitz, 1968:66). After we get these roots we could have the value of t, the duration of convergence of age-sex distribution to an equilibrium state, by using the formula E.10, presented in Appendix E.

Table 3.2.2 compares the results obtained under the different assumed conditions. The sets of fertility and nuptiality schedules were selected by the procedure presented in the previous chapter. The value of t obtained from the matrix roots were compared with those observed in the actual projections in Table 3.2.2. From the results obtained by the three different populations - 1957 obs., 1957 stb., and 1970 obs. populations, it shows that 1970 obs. population has the shortest duration of convergence compared with both the 1957 obs. and 1957 stb. populations, whereas the 1957 obs. population has the longest duration of convergence of all. These results are within the limits of the values of t obtained by the two values of ϵ, given in columns 10 and 11 of Table 3.2.2. However, the results do not show the clear relationship between initial age-sex distribution and the duration of convergence, because the age structure of the 1957 obs. population, which has the longest duration of convergence, is older than the 1957 stb. population, but younger than the 1970 obs. population (see Section 2.3 of Chapter 2).

When the proportion married or marital fertility declined, i.e. a decline in age-specific fertility rates, there was a definite increase in the value of t as could be seen in the cases when three schedules of proportion married combined with three schedules of marital fertility. From Table 3.2.2, in all the three different populations, the duration of convergence affected due to change in proportions married alone is higher than the effect due to the change in marital fertility alone. For example, taking the 1970 observed population the duration was increased from 235 to 255 and then on to 295 years by changing the proportion married alone and it was increased from 235 to 245 and then on to 260 by changing only marital fertility. Therefore, the change in proportion married alone had a more pronounced effect on the duration of convergence than the change in marital fertility alone.

In Table 3.2.2, the λ s are less readily interpreted than their logarithms. So we have to calculate the principal characteristic roots of the Singapore female population in 1970 with our hypothetical nuptiality and fertility schedules. Again, the duration of convergence can also be computed from these principal characteristic roots by using the formula E.11 of Appendix E.

Table 3.2.2

Effect of Fertility and Nuptiality on the Duration of Convergence of Age-Sex Distribution to an Equilibrium State Age-Sex Distribution

Prevailing Conditions			From Actual Projections			From Matrix Roots			Value of t in no. of years	
Mortality	Marital Fertility	Proportion Married	Value of t in no. of years 1957 obs.	1957 stb.	1970 obs.	λ_1	λ_2	λ_2/λ_1	$\epsilon = .0009$	$\epsilon = .0001$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)*	(11)*
1970	S(I)	S'(I)	250	240	235	1.07340	$-.43040 \pm .79203i$	.83978	201	264
1970	S(I)	S'(M)	275	265	255	1.04772	$-.44855 \pm .77437i$	.85414	223	293
1970	S(I)	S'(II)	310	300	295	1.01960	$-.46954 \pm .75669i$	.87341	259	340
1970	S(M)	S'(I)	260	250	245	1.02654	$-.38701 \pm .77143i$	.84075	203	266
1970	S(M)	S'(M)	285	270	265	0.99934	$-.40560 \pm .74905i$	.85238	220	288
1970	S(M)	S'(II)	315	320	295	0.97041	$-.42681 \pm .72888i$	.87040	253	332
1970	S(II)	S'(I)	275	275	260	0.98813	$-.36064 \pm .75731i$	.84887	215	282
1970	S(II)	S'(M)	295	285	280	0.96120	$-.37714 \pm .73237i$	.85702	228	299
1970	S(II)	S'(II)	330	320	315	0.93184	$-.39717 \pm .70833i$	.87148	255	335

*$t = \log \epsilon / (\log |\lambda_2| - \log |\lambda_1|)$ is the formula for single year unit of time interval.

However, the results are the same as those in Table 3.2.2 when we use the same value of ϵ as before.

From Table 3.2.3 the real root $r_1 = 0.2\, l_n \lambda_1$, is the rate of natural increase, compounded continuously, at which the female population of Singapore 1970 would increase if the mortality, fertility and nuptiality of the year in question were maintained long enough for the initial age distribution to wear off. When we use the marital fertility rates of Schedules S(M) and S(II), apart from the S(M) and S'(I) combination, all other assumptions yield negative growth rates.

Table 3.2.3

Principal Characteristic Roots $r = 0.2\, l_n \lambda$, Singapore Female Population 1970, with Different Schedules of Fertility and Nuptiality

Prevailing Conditions			$r_1 = 0.2\, l_n \lambda_1$	$r_2, r_3 = 0.2\, l_n (\lambda_2, \lambda_3)$
Mortality	Marital Fertility	Proportion Married		$(x \pm iy)^*$
1970	S(I)	S'(I)	0.014166	-.020757 + .21460i
1970	S(I)	S'(M)	0.009323	-.022209 + .20916i
1970	S(I)	S'(II)	0.003882	-.023187 + .20309i
1970	S(M)	S'(I)	0.005239	-.029453 + .22116i
1970	S(M)	S'(M)	-0.000132	-.032077 + .21490i
1970	S(M)	S'(II)	-0.006007	-.033767 + .20821i
1970	S(II)	S'(I)	-0.002388	-.035157 + .22527i
1970	S(II)	S'(M)	-0.007915	-.038772 + .21905i
1970	S(II)	S'(II)	-0.0014119	-.041631 + .21195i

* For a complex root $\lambda_2 = u + iv$ of Table 3.2.2, col. 8,
$x = 0.2\, l_n(\sqrt{u^2+v^2})$ and $y = 0.2 \arctan(v/u)$ were computed.

For the real part x of the complex root $r_2 = x + iy$ moves from -0.020757 to -0.023187 when we change the proportion married from S'(I) to S'(II), with the same marital fertility rates of S(I), from -0.020757 to -0.035157 when we change the marital fertility from S(I) to S(II), with the same proportion married of S'(I), and from -0.020757 to -0.041631 when we change the marital fertility as well as proportion married from S(I) and S'(I) to S(II) and S'(II); this corresponds to the widening of the range of ages within which reproduction takes place: the wider this range of ages, the more rapid the dampening of waves arising from disturbances in the age distribution (Keyfitz, 1968:73).

The imaginary part y, on the other hand, has decreased from 0.21460 in S(I) and S'(I) to 0.20309 in S(I) and S'(II), but increased from 0.21460 in S(I) and S'(I) to 0.22527 in S(II) and S'(I), and not significantly changed from 0.21460 in S(I) and S'(I) to 0.21195 in S(II) and S'(II): thus the period of the waves caused in later generations by a disturbance in the age distribution, which is equal to $2\pi/y$ years, increased from 29.3 years to 30.9 years in the case of the declining proportion married with the same marital fertility rates S(I), reduced from 29.3 years to 27.9 years in the case of changing marital fertility rates from S(I) to S(II) with the same proportion married S'(I), and also changed from 29.3 years to 29.6 years in the case of declining marital fertility and proportion married at the same time from S(I) and S'(I) to S(II) and S'(II).

From the above observation we can conclude that although the change in proportion married has a rapid dampening of waves arising from disturbances in the age distribution, it increases the wave length; this creates a significantly longer duration of convergence. Again, the change in marital fertility has more rapid dampening of waves and also the wave length tends to diminish; thus creating a very much shorter duration of convergence. Nevertheless, the changes in marital fertility and proportion married at the same time produce a very long duration though it has a rapid dampening of waves and very little change in wave length, because the drastic decline in overall fertility creates a very high initial magnitude of waves arising from disturbances in the age distribution.

CHAPTER 4

EXPERIMENTS ON THE EFFECT OF CHANGES IN NUPTIALITY AND FERTILITY

4.1 INTRODUCTION

The analysis up to this point has consisted only in the use of fixed schedules of nuptiality and fertility variables. For this chapter, we shall continue our experiments with varying fertility and varying nuptiality during the transition periods. Comparing the results from these experiments we can understand the effects of changes in nuptiality and fertility on the duration of convergence. Further, we shall investigate the changes in the projected populations, in terms of age composition, birth rates, death rates and growth rates, from time to time during the process of convergence.

Since the analysis incorporating changing fertility and nuptiality will be made through the population projection technique as before, we can make use of matrix formulation of those projections. But the projected population after two cycles will not be the product of the square of the projection matrix and the initial population vectors. It will be the product of the first and second projection matrices and the initial population vectors. However, the analysis of the matrix roots, as in the previous chapter, reveals little about the factors that make convergence slow or fast, or about the factors causing the transient deviation from the stable situation to be large or small. Therefore, we shall investigate the results of the actual projection of Singapore's 1970 population in a less technical form so that we can understand the process of convergence clearly.

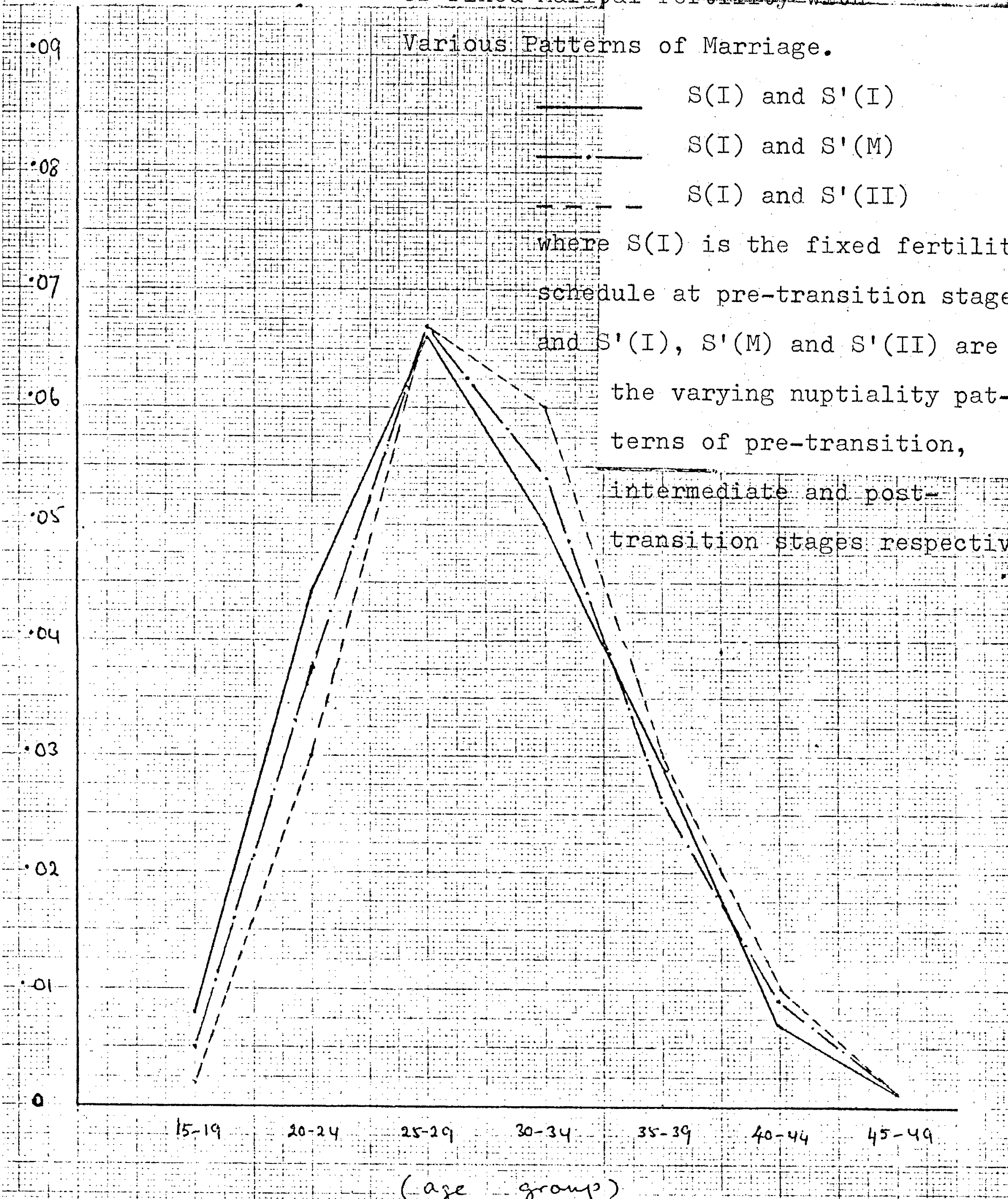
In all the projections the survival ratios of the 1969-71 Singapore life table are used as we mentioned in Chapter 1. The sequence of hypothetical fertility and nuptiality rates derived in Chapter 2 were set up according to the prevailing conditions; such as nuptiality varying with three sets of fertility, fertility varying with three sets of nuptiality, and both varying at the same time. Again, we have done our experiments in two transition periods, which take place quickly and slowly, over 35 years and over 65 years respectively.

4.2 THE EFFECT OF VARYING NUPTIALITY ON CONVERGENCE OF THE AGE-SEX DISTRIBUTION

The experiments were started with the process of convergence in three situations - when marital fertility was assumed to be constant at the level of Schedule S(I), and nuptiality was varying from Schedule S'(I) to Schedule S'(II), when marital fertility was constant with Schedule S(M) and nuptiality was varying from Schedule S'(I) to Schedule S'(II) and when marital fertility was constant with S(II) and nuptiality was varying from Schedule S'(I) to S'(II). The transitional age-specific fertility rates are the combinations of the said schedules, and their interpolation for the intermediate periods. Convenience is served here, by supposing .001 (1 per thousand) is the minimum age specific fertility rate for the age group 45-49, because our computer program (Shorter, 1968:18) allows us to use the age specific fertility rates to three decimal places only, and some schedules obtained by the outlined assumptions yield very low age-specific fertility rate at age group 45-49 to four decimal places.

The fertility schedules used in experiment I for the three transition periods such as pre-transition, intermediate and post-transition are presented in Figure 4.2.1, which reveals the nature of experiments II and III as well. The transitional patterns of experiments I, II, and III which are affected by the reduction of proportion married only, have a serious reduction on age-specific fertility rates in the early period of childbearing (age groups

Figure 4.2.1 Patterns of Fertility Schedules, Total Fertility = 1.0. Combinations of Fixed Marital Fertility with Various Patterns of Marriage.



———— S(I) and S'(I)

——·—— S(I) and S'(M)

— — — S(I) and S'(II)

where S(I) is the fixed fertility schedule at pre-transition stage, and S'(I), S'(M) and S'(II) are the varying nuptiality patterns of pre-transition, intermediate and post-transition stages respectively.

Source: Table 2.7.5 of Chapter 2 of this study.

A4 1 mm

between 15-19 and 25-29). Although the later age groups (i.e. from 30-34 to 45-49) have a reduction on the age-specific fertility rates to some extent, the figure shows the reverse feature since we have fixed the total fertility of 1.0 in every schedule. However, it shows the movement of mean age of fertility schedule from early period to later period. The figure illustrates the distribution of fertility by age, but it does not show the differences in levels of fertility associated with age patterns. The figure shows the structure of age-specific fertility that results when entry into cohabitation is early of late and gradual.

The nature of changes in fertility schedules caused by the transition of nuptiality patterns can also be pictured when we study the means and variances, presented in Table 4.2.1. This table compares the process of convergence in three situations with Singapore's 1970 observed population. From the Table, the decline in the level of fertility in experiments I, II and III can be seen according to the total fertility rates and gross reproduction rates. The changes in mean and variance of fertility schedules show the movement of age-specific fertility rates to the later peak and the concentration of those rates around the mean age. The means of fertility schedules under these experiments increased while the variances decreased very slowly. Again, the increase in mean age of fertility schedule makes the mean length of generation increase and consequently the intrinsic growth rate decreases further (Lesthaeghe, 1971:426), and also causes the duration of convergence to be longer probably due to the slower dampening of waves arising from disturbances in the age distribution during the process of convergence, and the period of waves caused in later generations by a disturbance in the age distribution, which is increasing. We can visualize the nature of oscillation during the process of convergence when we study the population change in the later part of this chapter.

Table 4.2.1

Effect of the Decline in Nuptiality on the

Duration of Convergence of the Age-Sex Distribution

Mortality Assumption	Fertility Assumption: Hypothetical Condition	Fertility Assumption: Total Marital Fertility	Nuptiality Assumption: Hypothetical Condition From	Nuptiality Assumption: Hypothetical Condition To	Singulate Mean Age From	Singulate Mean Age To	Total Fertility Rate (TFR) From	TFR To	Gross Reproduction Rate (GRR) From	GRR To	Mean Age of Fertility Schedule ($\bar{m}$) From	$\bar{m}$ To	Variance of Fertility Schedule (σ^2) From	σ^2 To	Duration of Convergence from Actual Projection of 1970 Obs. Population with nuptiality transition: no. transition*	35 years transition	65 years transition
Experiment (I)																	
1969-71	S(I)	7.52	S'(I)	S'(II)	24.0	26.8	3.2	2.4	1.55	1.17	29.0	30.5	35.1	31.7	235	285	270
Experiment (II)																	
1969-71	S(M)	6.05	S'(I)	S'(II)	24.0	26.8	2.5	1.8	1.21	0.87	28.1	29.5	31.3	29.2	245	285	280
Experiment (III)																	
1969-71	S(II)	4.88	S'(II)	S'(II)	24.0	26.8	2.0	1.4	0.97	0.68	27.6	28.8	27.3	26.2	260	295	295

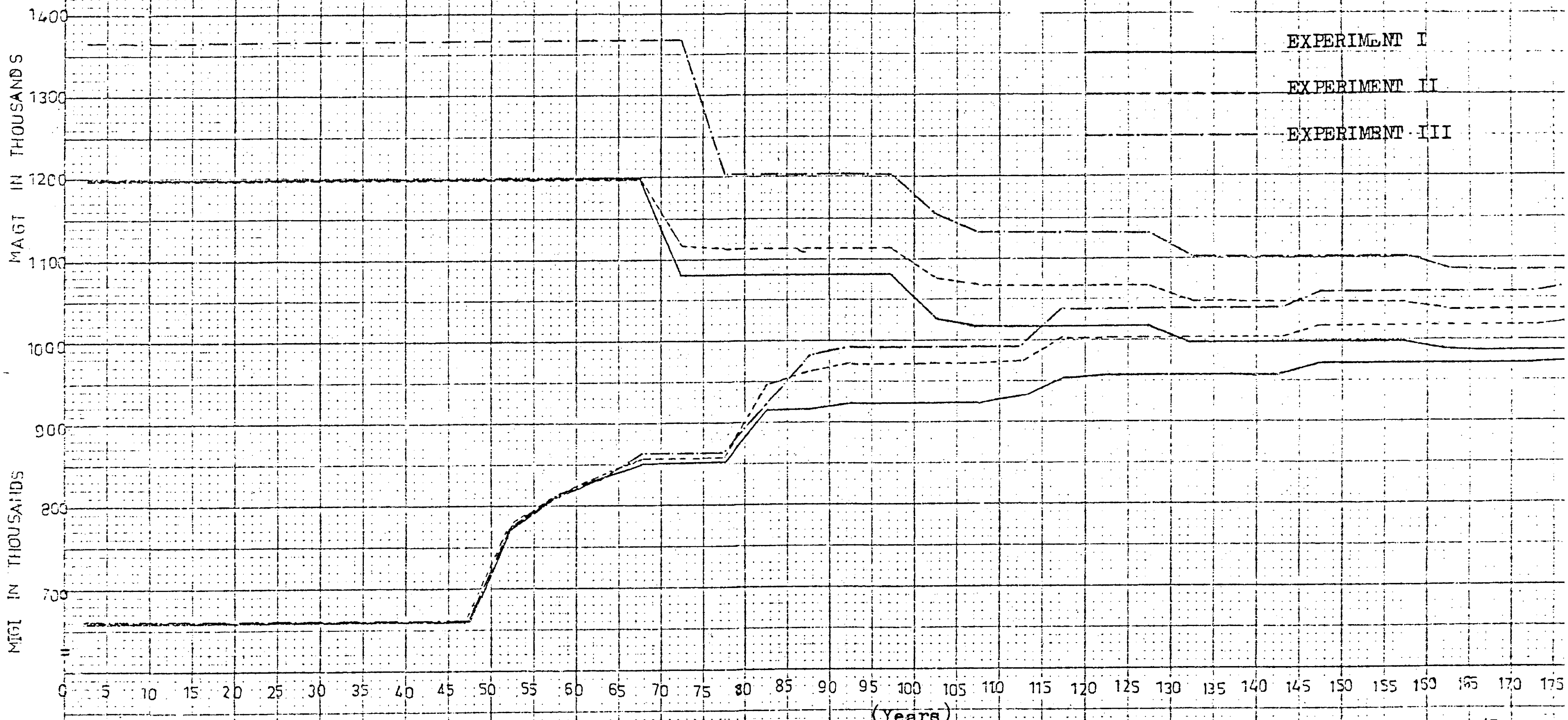
* Since no transition is allowed the schedules remain constant at pre-transition (from) states throughout the projection (see Table 3.2.2).

From Table 4.2.1 we can see that when the nuptiality alone was varying over the period of 35 years the duration of convergence increased by 21 per cent, i.e. from 235 years to 285 years. When the transition spread over 65 years, the duration of convergence increased by 15 per cent only. When we use Schedule S(M) instead of S(I) for the fixed hypothetical marital fertility rates with all other assumptions the same, the effect of nuptiality variable over a transition period of 35 years on the duration of convergence produces an increase of 16 per cent. When the transition is spread over 65 years the duration of convergence is increased by 14 per cent, and again it is less than that of the transition spread over 35 years. When we use the schedule S(II) instead of S(I) for the fixed marital fertility rates with all other assumptions the same, the effect of the nuptiality variable on the duration of convergence over the transition period of 35 years produces an increase of 13 per cent and it is the same with the transition spread over 65 years. The increase in duration of convergence for Experiment II is less than that for Experiment I because the former has the higher duration of convergence in the projection with no transition. Similarly, that for Experiment III is less than Experiment II.

In Experiments I and II, the 65 year transition has a shorter duration of convergence than the 35 year transition, because the reduction of the fertility level in the 65 year transition is slower than that in the 35 year transition. Moreover, the increase in mean age of the fertility schedule for 65 year transition is also slower than that for the 35 year transition. Nevertheless, in Experiment III, the 65 year transition has the same duration of convergence as the 35 year transition because of its very low level of fertility before the transition.

Further, the nature of the process of convergence for the above experiment as seen from the MAGI and MIGI are presented in Figure 4.2.2. It can be seen in this Figure that the differences between the age-sex distributions in the initial stages vanished in

Figure 4.2.2. The Process of Convergence of Age-Sex Distribution as seen from the Convergence of MAGI and MIGI for 1970 obs. Female Population, under Changing Schedules of ~~Fertility &~~ Nuptiality. (Experiments I, II and III with 35 Year Transition).



the course of time, and they converged to the different levels. The Figure indicates that the large differences of age-sex distributions reduced as soon as the cohorts alive at the initial point of time died out; but the smaller variations induced by the initial differences took a long time to disappear. Maximum growth indexes (MAGI) in all the experiments started to decline after 65 years, while their minimum growth indexes (MIGI) started to incline before 50 years. This is significant because MAGI started from very early childhood, i.e. 0-4 or 5-9 age group, and MIGI started from the middle of the childbearing period, i.e. 25-29 age group, at the beginning of the process of convergence. Again, we may say here, that the level of fertility reflected upon the level of convergence, which we measured by the growth indexes, i.e. the point where MAGI and MIGI converged, because Experiment I converged to the lowest level among these experiments, and Experiment III was the highest of all.

From the above observation we can conclude that a decline in nuptiality, without any accompanying change in marital fertility, increases the duration of convergence considerably. Apart from the extremely low level of fertility before transition, such as in Experiment III, the duration of convergence becomes shorter when the transition period is longer. Although a low marital fertility assumption reduces total fertility rates further, the effect on the duration of convergence is very little. There is no change in duration of convergence when we compare the results of Experiments I and II with the 35 year transition period. So, we may conclude here again that the duration of convergence does not depend upon the level of fertility. It would seem to me that the duration of convergence depends upon the frequency of oscillatory terms and their wave length, which will be visualized later in this Chapter.

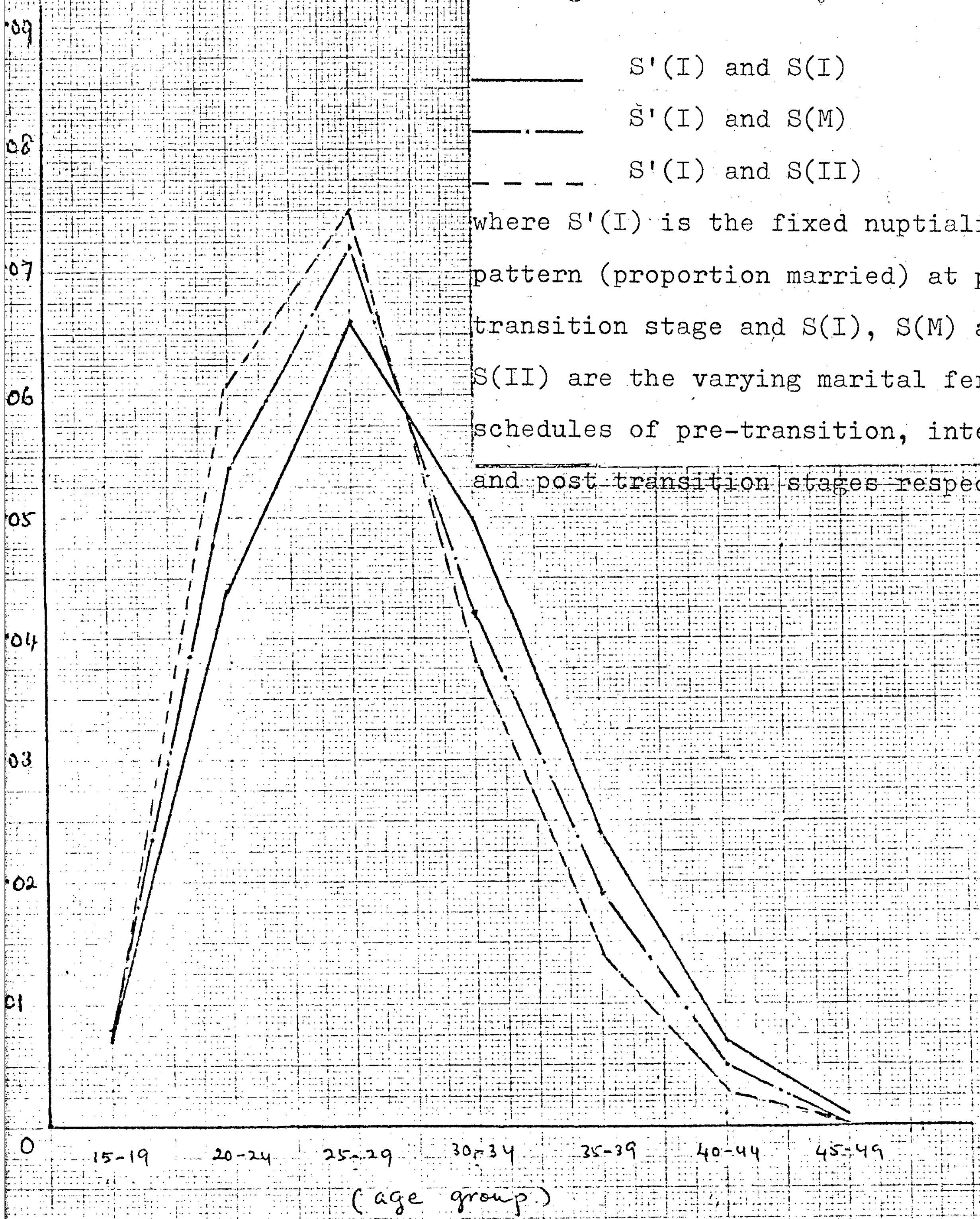
4.3 THE EFFECT OF VARYING FERTILITY ON CONVERGENCE OF THE AGE-SEX DISTRIBUTION

The experiments in this section were done to study the process of convergence in three different situations - when nuptiality was assumed to be constant at level of Schedule S'(I) and fertility was varying from Schedule S(I) to S(II), when nuptiality was constant with S'(M) and fertility was varying from Schedule S(I) to S(II), and when nuptiality was constant with Schedule S'(II) and fertility was varying from Schedule S(I) to S(II). The transitional age-specific fertility rates, were the combination of these schedules and their interpolation for the intermediate periods. As before, we supposed the minimum age-specific fertility rate for the age group 45-49 as 0.001 (1 per thousand).

The fertility schedules used in Experiment IV for the three situations - pre-transition, intermediate and post-transition - are presented in Figure 4.3.1, which also reveals the nature of Experiments V and VI. It shows the structure of age-specific fertility that results when fertility control is practised with varying degrees of control, i.e. according to M and m (see Appendix D). In this case the age-specific fertility rates are seriously reduced at the later period of childbearing (i.e. between 30-34 and 45-49), and so also raised at the early part of the schedules (i.e. between 15-19 and 25-29) since we fixed the total fertility of 1.0 to every schedule.

The nature of changes in fertility schedules caused by the transition in marital fertility can also be seen when we study the means and variances, presented in Table 4.3.1, which compare the process of convergence in three situations with Singapore's 1970 observed population. From this Table, the means of these experiments decreased slowly while the variances decreased very fast. The variances in these experiments started with very high

Figure 4.3.1 Patterns of Fertility Schedules, Total Fertility = 1.0. Combinations of Fixed Proportion Married with Various Degree of Fertility Control.



S'(I) and S(I)

S'(I) and S(M)

S'(I) and S(II)

where S'(I) is the fixed nuptiality pattern (proportion married) at pre-transition stage and S(I), S(M) and S(II) are the varying marital fertility schedules of pre-transition, intermediate and post-transition stages respectively.

Source: Table 2.7.5 of Chapter 2 of this study.

Table 4.3.1

Effect of the Decline in Fertility on the Duration of Convergence of the Age-Sex Distribution

Mortality Assumption	Nuptiality Assumption		Fertility Assumption				Total Fertility Rate (TFR)		Gross Reproduction Rate (GRR)		Mean Age of Fertility Schedule ($\bar{m}$)		Variance of Fertility Schedule (σ^2)		Duration of Convergence from Actual Projection of 1970 Obs. Population with fertility transition		
	Hypothetical Condition	Singulate Mean Age	Hypothetical Condition		Total Marital Fertility												
			From	To	From	To	From	To	From	To	From	To	From	To	no.* transition	35 years transition	65 years transition
Experiment (IV)																	
1969-71	S'(I)	24.0	S(I)	S(II)	7.52	4.88	3.2	2.0	1.55	0.97	29.0	27.6	35.1	27.3	235	245	255
Experiment (V)																	
1969-71	S'(M)	25.4	S(I)	S(II)	7.52	4.88	2.8	1.7	1.36	0.83	29.7	28.1	34.0	27.3	255	260	285
Experiment (VI)																	
1969-71	S'(II)	26.8	S(I)	S(II)	7.52	4.88	2.4	1.4	1.17	0.68	30.5	28.8	31.7	26.2	295	295	295

* Since no transition is allowed the schedules remain constant at pre-transition (from) states throughout the projection (see Table 3.2.2).

values and lowered to values less than their means after the transition. Again, the decrease in mean age of the fertility schedule makes the mean length of generation decrease and consequently a substantial part of the growth rate is increased. It also makes the duration of convergence shorter, probably due to the faster dampening of waves that occur during the process of convergence, and the diminishing of waves.

Table 4.3.1 also explained that when the fertility alone is varying over the 35 year transition, the duration of convergence increases by 4.0 per cent, i.e. from 235 years to 245 years. This shows that the reduction of marital fertility alone does not affect the duration of convergence. The duration of convergence is more when the transition is spread over 65 years. The figures show that during the transition period of 65 years the duration of convergence increased by 8.5 per cent, i.e. from 235 years to 255 years. It may be affected by the slower movement of the mean age of fertility schedule.

When we use the Schedule S'(M) instead of S'(I) for the fixed hypothetical proportion married with all other assumptions the same, the effect of the fertility variable over a transition period of 35 years, on the duration of convergence produced an increase of 2.0 per cent, i.e. from 255 years to 260 years, and 12 per cent, i.e. from 255 years to 285 years, for the transition spread over 65 years. When we changed the Schedule S'(II) in place of S'(I), the effect of varying marital fertility on the duration of convergence produced none at all, because the two transition periods - 35 year transition and 65 year transition - produced the same duration of convergence with no transition, that is, 295 years.

The increase in duration of convergence for Experiment V is less than that for Experiment IV, because the former has the higher duration of convergence in the projection with no transition. Similarly, Experiment VI has no change in duration of convergence because of the marital fertility transition. In Experiment IV and V, the 65 year transition has longer duration of convergence than the 35 year transition, because the reduction of mean age in the 65 year transition is slower than that in the 35 year transition.

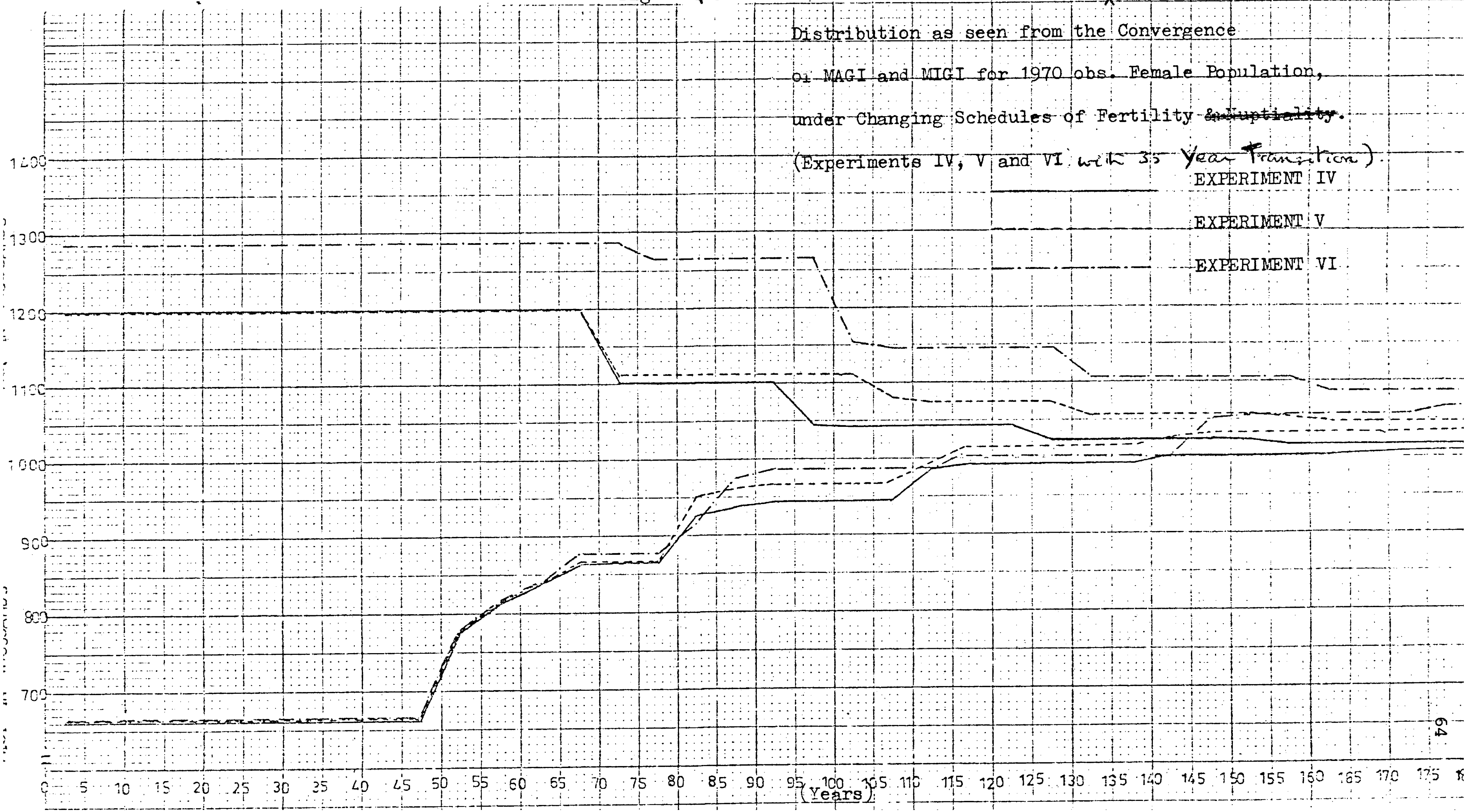
Nevertheless, it is hard to conclude that the duration of convergence increases when the marital fertility decreases. This is because the marital fertility transition from a very low level to an extremely low level does not change the duration of convergence in Experiment VI. However, the low nuptiality assumption reduces the total fertility even further and also increases the duration of convergence. It also increases the mean age of the fertility schedule for the initial stage, and this high mean age is reduced rapidly during the transition period. This reduction shortens the mean length of generation and consequently reduces the growth rate. Moreover, it cancels out a substantial part of gain in increasing the duration of convergence from the reduction of the fertility level.

Further, the nature of the process of convergence for the above experiments as seen from the MAGI and MIGI are presented in Figure 4.3.2. This Figure can also be interpreted as Figure 4.2.2. In fact, the nature of the process of convergence is the same for all projections experimented in this study. However, the points where MAGI and MIGI approached, are different from one experiment to another according to their level of fertility. Since Experiments III and IV have the same total fertility rate at the end of the transition they approach the same limit of convergence. Again, Experiments III and VI possess the lowest total fertility rate for the post-transition period and approach the highest level of convergence. Total fertility rates of 1.7, 1.8, 2.0 and 2.4 belong to the Experiments V, II, IV and I respectively, and they follow in this order for measuring the level of convergence.

4.4 THE EFFECTS OF VARYING FERTILITY AND NUPTIALITY ON CONVERGENCE OF AGE-SEX DISTRIBUTION

In this experiment we investigate the combined effect of fertility and nuptiality on the duration of the convergence of the

Figure 4.3.2 The Process of Convergence of the Age-Sex Distribution as seen from the Convergence of MAGI and MIGI for 1970 obs. Female Population, under Changing Schedules of Fertility ~~& Nuptiality~~. (Experiments IV, V and VI with 35 Year Transition).

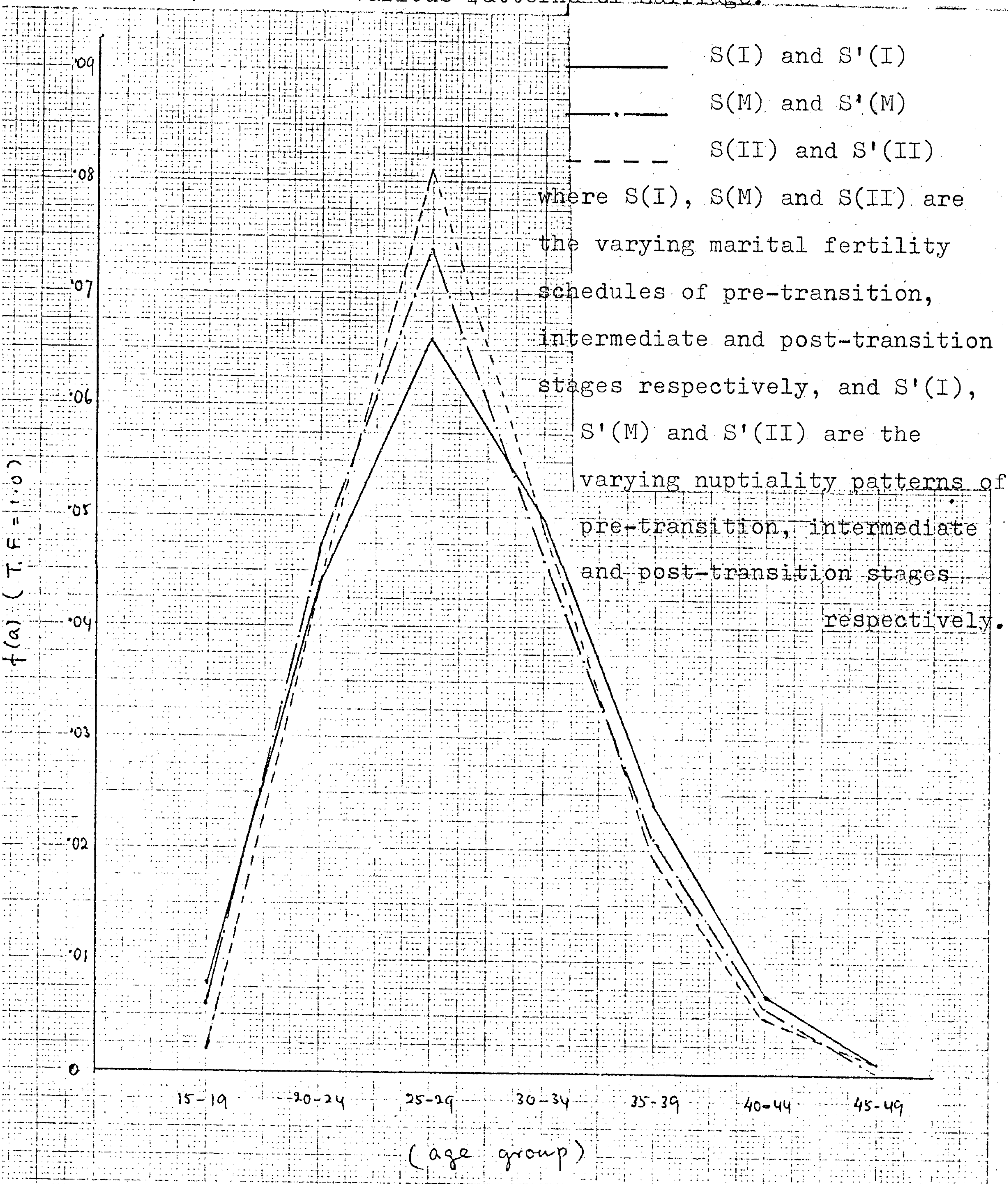


age-sex distribution. We have already set up the combinations of the Fertility Schedule S(I) and Nuptiality Schedule S'(I), Fertility Schedule S(M) and Nuptiality Schedule S'(M), and Fertility Schedule S(II) and Nuptiality Schedule S'(II), in the previous experiments. Now, we shall investigate the transition of these combinations from the first condition to the last situation over the transition periods of 35 years and 65 years as before.

The transitional age-specific fertility rates used for the experiment over the period of 35 years are the combination of these schedules and their interpolation for the intermediate periods. We again suppose the minimum age-specific fertility rate for the age group 45-49 as 0.001 (1 per thousand). The nature of both the proportion married and the marital fertility simultaneously affects the pattern of age-specific fertility rates and can be seen clearly in Figure 4.4.1. The Figure shows the structure of fertility that results when entry into cohabitation is early and rapid or late and gradual, combined with low fertility control or with high fertility control. The degree of fertility control is measured by M and m (see Appendix D), which have been explained earlier in Chapter 2.

The changes in fertility schedules caused by the transition of marital fertility and nuptiality at the same time, can be pictured when we study the changes in mean and variance of fertility schedule presented in Table 4.4.1. In this Experiment the fertility level reduced by a large amount from 3.2 to 1.4 though the mean age of fertility schedule ($\bar{m}$) changed only a small amount from 29.0 to 28.8. On the other hand, variance declined very fast from a very high level of 35.1 to the very low level of 26.2. Besides, the increase in duration of convergence has been very high in this experiment. It shows that when both fertility and nuptiality declined over the transition period of 35 years or 65 years, the duration of convergence increased by 23 per cent, i.e. from 235 years to 290 years. In this case, the transition

Figure 4.4.1 Patterns of Fertility Schedules, Total Fertility = 1.0. Combinations of Various Marital Fertility with Various Patterns of Marriage.



Source: Table 2.7.5 of Chapter 2 of this study.

F A4 1mm

Table 4.4.1

Effect of the Decline in Fertility and Nuptiality on the Duration of Convergence of the Age-Sex Distribution

Mortality Assumption	Fertility Assumption: Hypothetical Condition		Fertility Assumption: Total Marital Fertility		Nuptiality Assumption: Hypothetical Condition		Nuptiality Assumption: Singulate Mean Age		Total Fertility Rate (TFR)		Gross Reproduction Rate (GRR)		Mean Age of Fertility Schedule ($\bar{m}$)		Variance of Fertility Schedule (σ^2)		Duration of Convergence from Actual Projection of 1970 Obs. Population with fertility and nuptiality transition		
	From	To	From	To	From	To	From	To	From	To	From	To	From	To	From	To	no. transition*	35 years transition	65 years transition
Experiment (VII)																			
1969-71	S(I)	S(II)	7.52	4.88	S'(I)	S'(II)	24.0	26.8	3.2	1.4	1.55	0.68	29.0	28.8	35.1	26.2	235	290	290

* Since no transition is allowed the schedules remain constant at pre-transition (from) states throughout the projection (see Table 3.2.2).

period does not affect the duration of convergence, since the mean age of fertility schedule has not changed so much.

Further, the nature of the process of convergence for the Experiment VII as seen from MAGI and MIGI are presented in Figure 4.4.2. This Figure can also be interpreted as Figure 4.2.2, for the nature of convergence is similar in all the experiments. However, the decline of MAGI started only after 95 years in this case, because the great reduction in fertility caused a new series of MAGI after 25 years of processing. Since Experiment VII has the same level of fertility with Experiments III and VI for the post-transition period, it approached the same limit of convergence at the highest level among the experiments presented in this Chapter. Compared to the individual effect of decline in nuptiality and fertility, the combined effect has the longest duration of convergence in all transitions.

4.5 POPULATION CHANGE

The aim of the use of a stable population model is to trace the dynamic characteristics of a population which starts off with an arbitrary age structure and is submitted from that moment on to the specified demographic regime. In particular, what is wanted is to find the long run effects on the age distribution, the birth and death rates, and the pattern of growth of the population. Based on the experiments we have done so far, in this section we are going to study the changes in the characteristics of the population under these experiments over time. The experiments extracted for this section are labelled by Types A to G in consecutive (see Table 4.5.1).

The first of these experiments, called here Type 'A', is based on constant Schedules S(I) and S'(I) for fertility and nuptiality respectively. The second experiment, called Type 'B', embodies a transition of nuptiality without any accompanying change in marital fertility patterns, whereas the third, called here Type

Figure 4.4.2 The Process of Convergence of the Age-Sex Distribution as seen from the Convergence of MAGI and MIGI for 1970 obs. Female Population, under Changing Schedules of Fertility and Nuptiality (Experiment VII with 35 Year Transition).

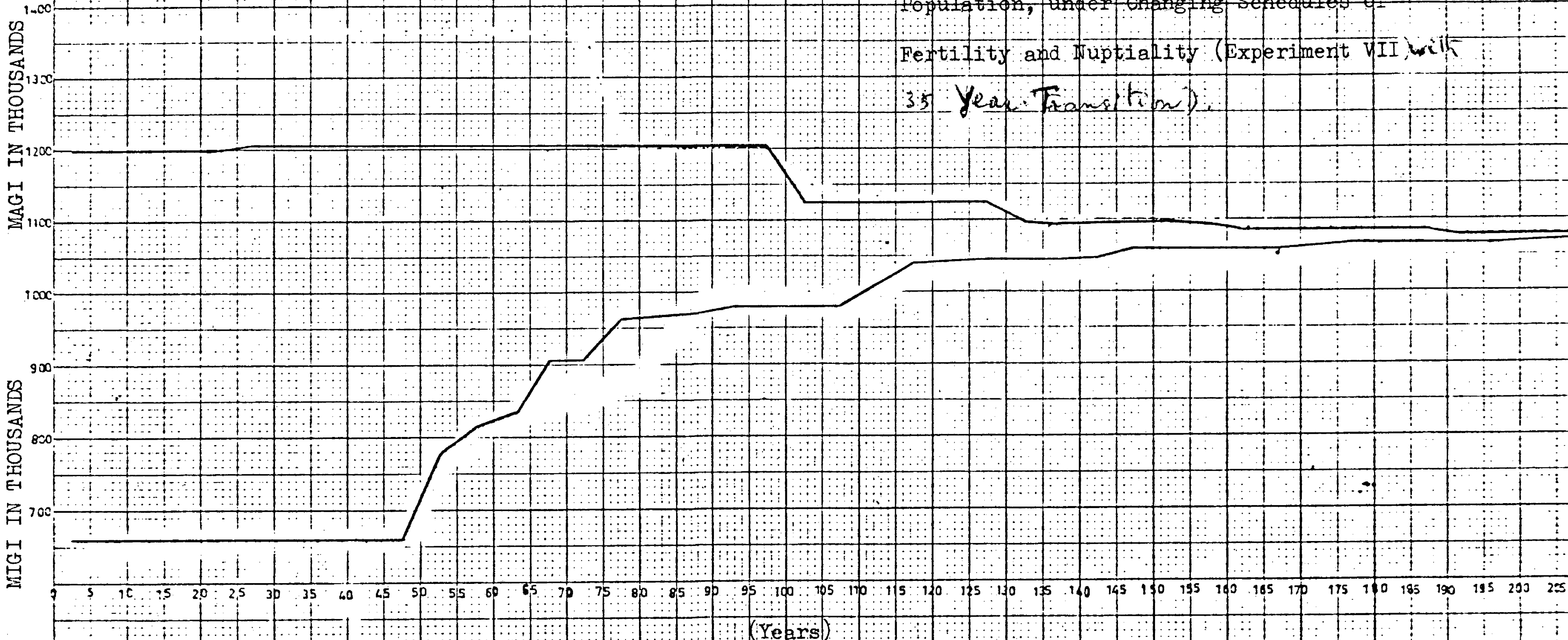


Table 4.5.1

Percentage Age Distribution at the End of 35 Year Transition, at the End of 65 Year Transition and at the Stable State (Population size in thousands)

Type	Prevailing Conditions	Age Group	Populations in Percentage 1970	2005	2035	Stable State
A	Constant Condition	< 15	38.80	30.49	30.24	30.53
	of Marital Fertility	15-64	57.86	64.15	62.22	61.95
	and Nuptiality with	65 +	3.35	5.36	7.54	7.52
	S(I) and S'(I) (TFR	Total	100.0*	100.0	100.0	100.0
	3.2) Duration					
	235 years	P.Size	2074.2	4022.3	6294.3	68389.2
B	Nuptiality Decline	< 15		25.95	23.59	23.53
	from S'(I) to	15-64		68.14	66.04	64.93
	S'(II) in 35 Year	65+		5.92	10.37	11.55
	Transition (TFR	Total		100.0*	100.0	100.0*
	3.2 to 2.4)					
	Duration 285 years	P.Size		3643.3	4682.1	10562.9
C	Marital Fertility	< 15		23.57	19.96	19.66
	Decline from S(I)	15-64		70.20	67.81	65.76
	to S(II) in 35	65+		6.23	12.23	14.58
	Year Transition	Total		100.0	100.0	100.0
	(TFR 3.2 to 2.0)					
	Duration 245 years	P.Size		3461.8	3883.4	2604.0
D	Nuptiality and	< 15		19.07	13.67	13.00
	Marital Fertil-	15-64		74.03	69.72	64.97
	ity Declines	65+		6.89	16.60	22.03
	from S(I) and	Total		100.0*	100.0*	100.0
	S'(I) to S(II) and					
	S'(II) in 35 Year	P.Size		3126.8	2860.0	55.3
	Transition (TFR					
	3.2 to 1.4)					
	Duration 290 years					

Table 4.5.1 (continued)

Type	Prevailing Conditions	Age Group	Populations in Percentage			
			1970	2005	2035	Stable State
E	Nuptiality Decline	< 15	38.80	28.30	24.70	23.53
	from S'(I) to	15-64	57.86	66.05	66.02	64.92
	S'(II) in 65 Year	65+	3.35	5.65	9.28	11.55
	Transition (TFR	Total	100.0*	100.0	100.0	100.0
	3.2 to 2.4)					
	Duration 270	P.Size	2074.2	3816.9	5117.0	11722.1
F	Marital Fertility	< 15		26.62	21.72	19.66
	Decline from S(I)	15-64		67.59	67.95	65.76
	to S(II) in 65 Year	65+		5.79	10.32	14.58
	Transition (TFR	Total		100.0	100.0*	100.0
	3.2 to 2.0)					
	Duration 255 years	P.Size		3723.9	4599.1	3283.8
G	Nuptiality and	< 15		24.19	16.10	13.00
	Marital Fertility	15-64		69.67	71.18	64.97
	Declines from	65+		6.14	12.72	22.03
	S(I) and S'(I)	Total		100.0	100.0	100.0
	to S(II) and S'(II)					
	in 65 Year	P.Size		3512.4	3731.8	181.2
	Transition					
	(TFR 3.2 to 1.4)					
	Duration 290 years					

N.B.* rounded up to one decimal place.

Duration means Duration of Convergence

P. Size means Population Size in Thousands

'C', incorporates the reverse situation. The fourth, called Type 'D', in which both transitions occur together gives a pattern of change affected by declining nuptiality and marital fertility simultaneously. The transition period used in Experiments 'B, 'C' and 'D' is 35 years and the same conditions of these Experiments with the 65 year transition period are presented here as Types 'E', 'F' and 'G' respectively.

According to the Table 4.5.1 age distribution of the stable population depends upon the post-transition level of fertility, i.e. the highest total fertility rate at the post-transition achieved the youngest age structure, while the lowest fertility level at the post-transition achieved the oldest age structure, and the same total fertility rate at that time achieved the same age structure, at the stable state. But the durations of convergence are not consistent with the age structures according to the Experiments presented here. The detailed comparison of the age distributions are also presented in Figures 4.5.1, 4.5.2, and 4.5.3. These figures show that in all the experiments the populations are ageing during the first century of the process. It is levelling off after that with a low frequency oscillation. Since the experiment Type D has the fastest decline in the level of fertility among the experiments, it possesses the highest magnitude of oscillation.

From the above observations it may be concluded that the population is ageing because of the decline in total fertility rate, which is affected by the nuptiality and marital fertility changes. Post-transitional level of fertility decides the age structure of the stable population. However the duration of convergence depends neither on these age distributions nor on the level of fertility.

Again, the changes in the birth rates, presented in Figure 4.5.4, especially at the beginning of the processing, about a century in all experiments, are those due to the effects of the ageing of the population. From Figure 4.5.4, it is clear that

Figure. 4-5-1. Changes in Projected Populations of Persons of Age Group 0-14, During the Process of Convergence with Singapore 1970 Observed Population.

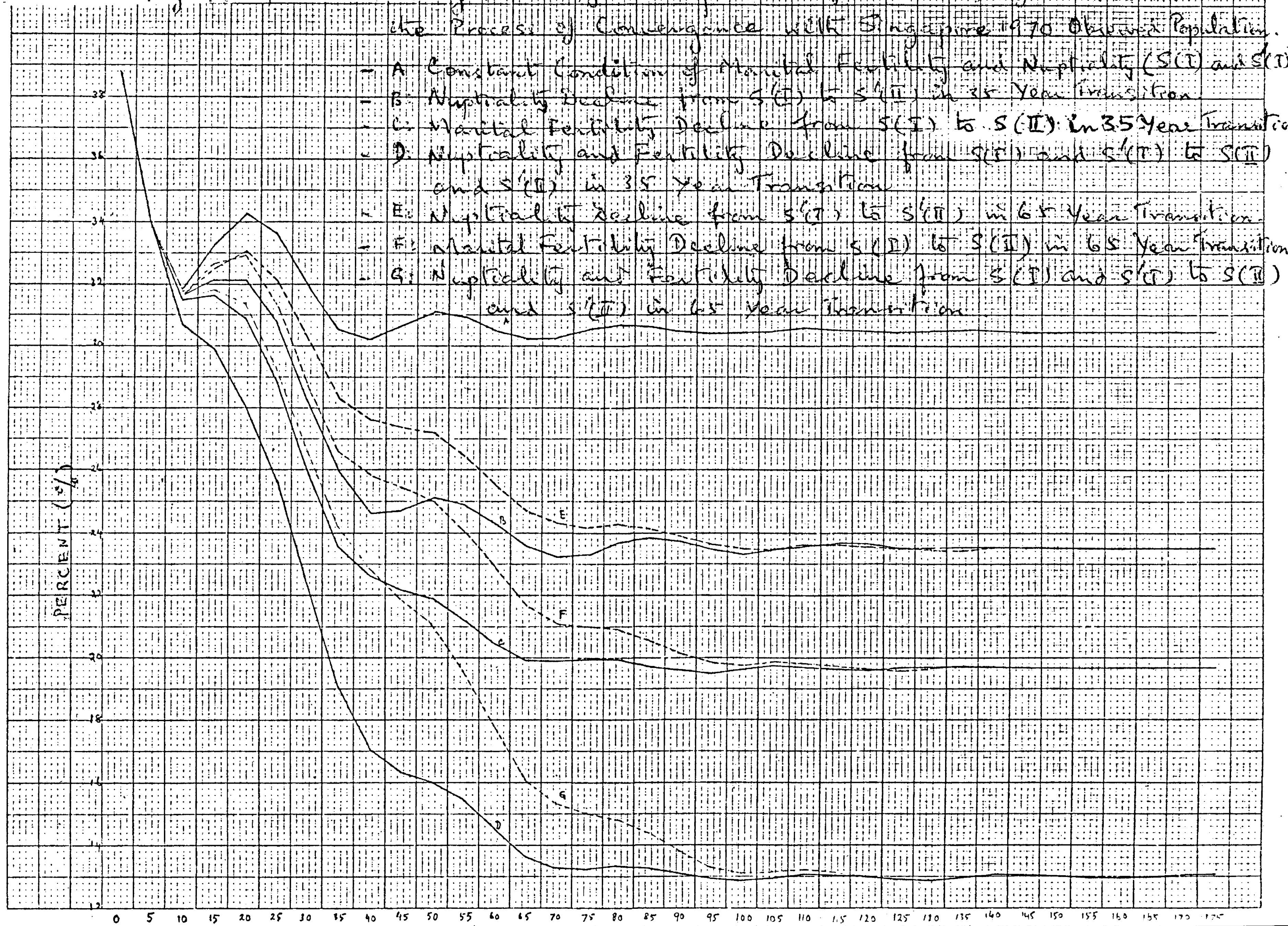
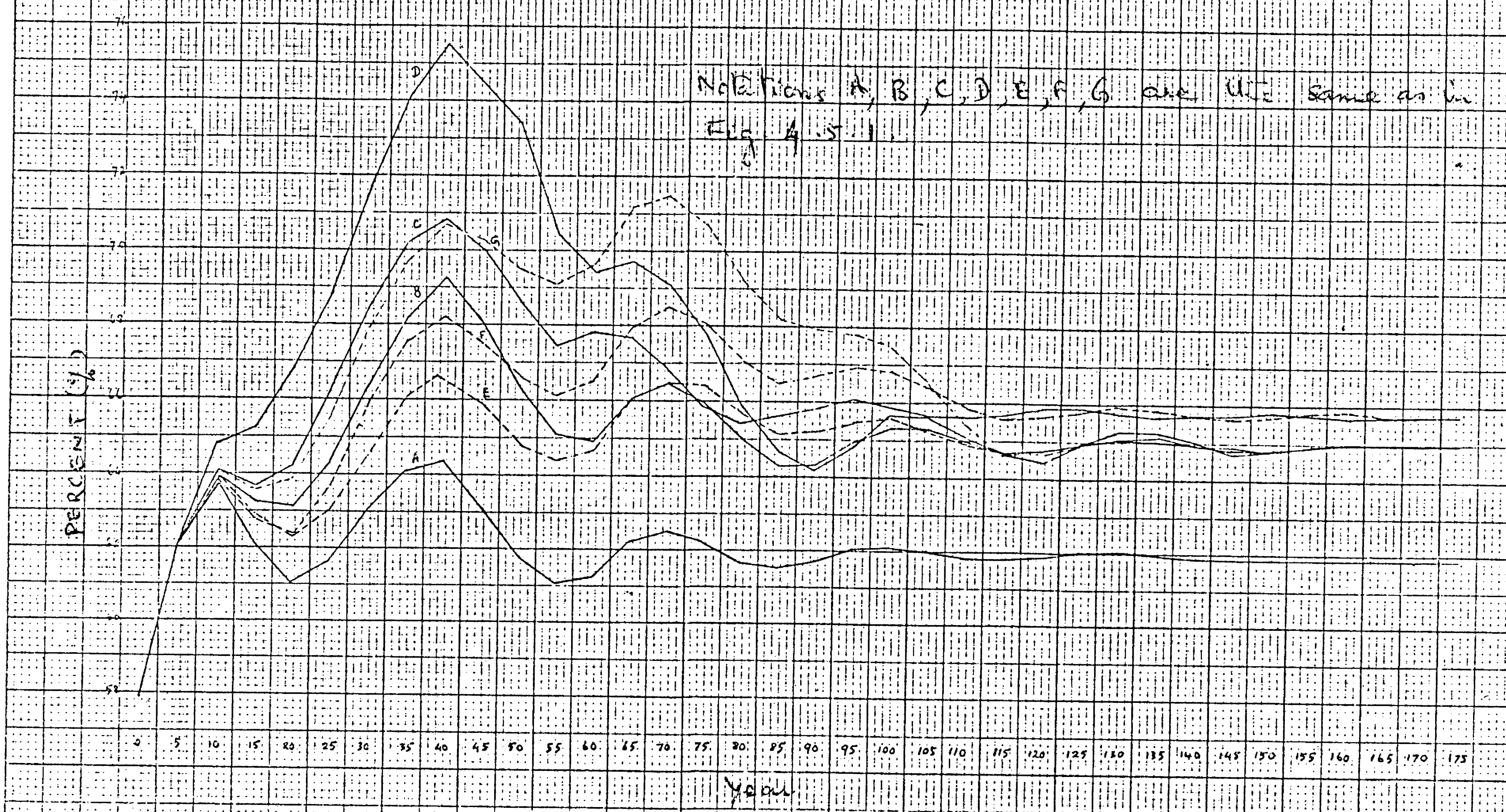


Fig. 4.5.2 Changes in Projected Populations of Persons of Age Group 15-64 During the Process of Convergence with Singapore 1970 Observed Population



Notations A, B, C, D, E, F, G are the same as in Fig. 4.5.1

Fig 4.5.3 Changes in Projected Population of Persons of Age Group 65+ During the Process of Convergence with Singapore 1970 Observed Population.

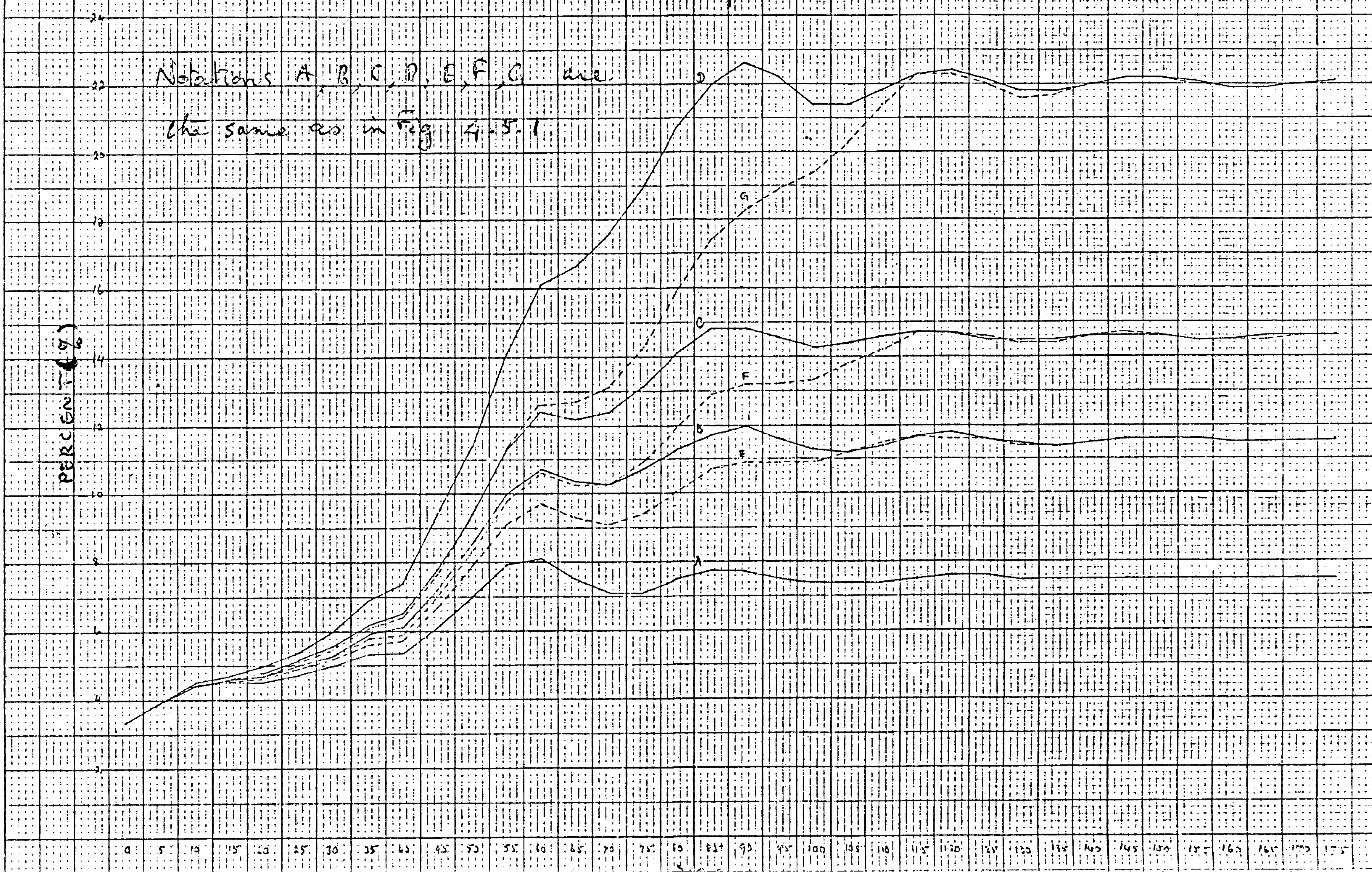
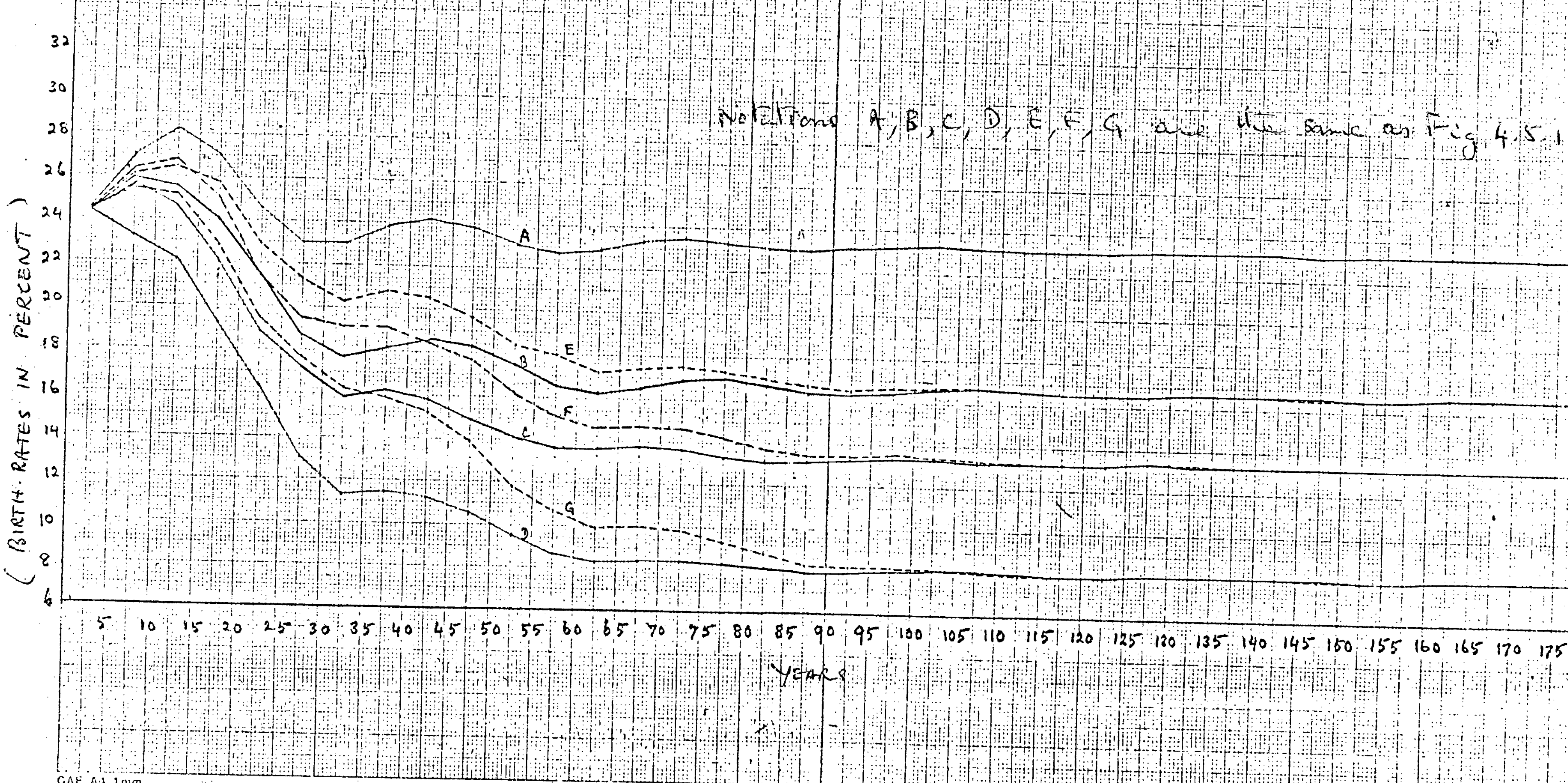


Fig 4.5.4 Changes in the Birth Rates During the Process of Convergence with Singapore 1970 Observed Population.



the change in nuptiality decline alone over the 35 year transition period (B) can produce a faster decline in the short-term than the marital fertility decline over the 65 year transition period (F). Moreover, nuptiality decline alone increased the mean age of the fertility schedule, which decreased in the case of marital fertility decline alone. These values have been presented earlier in this Chapter. Because of the reduction of the mean age of the fertility schedule in the case of marital fertility decline alone, a substantial part of the gain in reducing the birth rate is lost by the shortening of the mean length of generation.

It can also be seen that the death rates, which would increase only slightly in the first 35 years of fertility decline, would thereafter rise more rapidly and steadily to approach the long-term level after a century. Fluctuation in death rates would be significantly lower than those in birth rates, as can be seen from Figure 4.5.5.

The differences between the birth rates and death rates are also presented in Figure 4.5.6. In this Figure the growth rates for the two projections representing nuptiality decline alone over 35 years and marital fertility alone over 65 years, reach the same levels after 20-25 years and the latter is higher than the former again during 20-25 years and 40-45 years. At the end of the projections the reduction in growth rate of the declining marital fertility alone is lower than in the case of declining nuptiality alone. In the case in which both transitions occur together, growth rates decline much more rapidly.

There is another consideration, and this is the nature of the changes in birth rate which has reflected upon the nature of changes in the projected population of persons age group 0-14, and the nature of changes in the death rate which reflected upon the changes in projected population of persons of ages 65 years and over. This can be seen by comparing Figures 4.5.1 and 4.5.4, and Figures 4.5.3 and 4.5.5. When we compare the two Figures 4.5.5

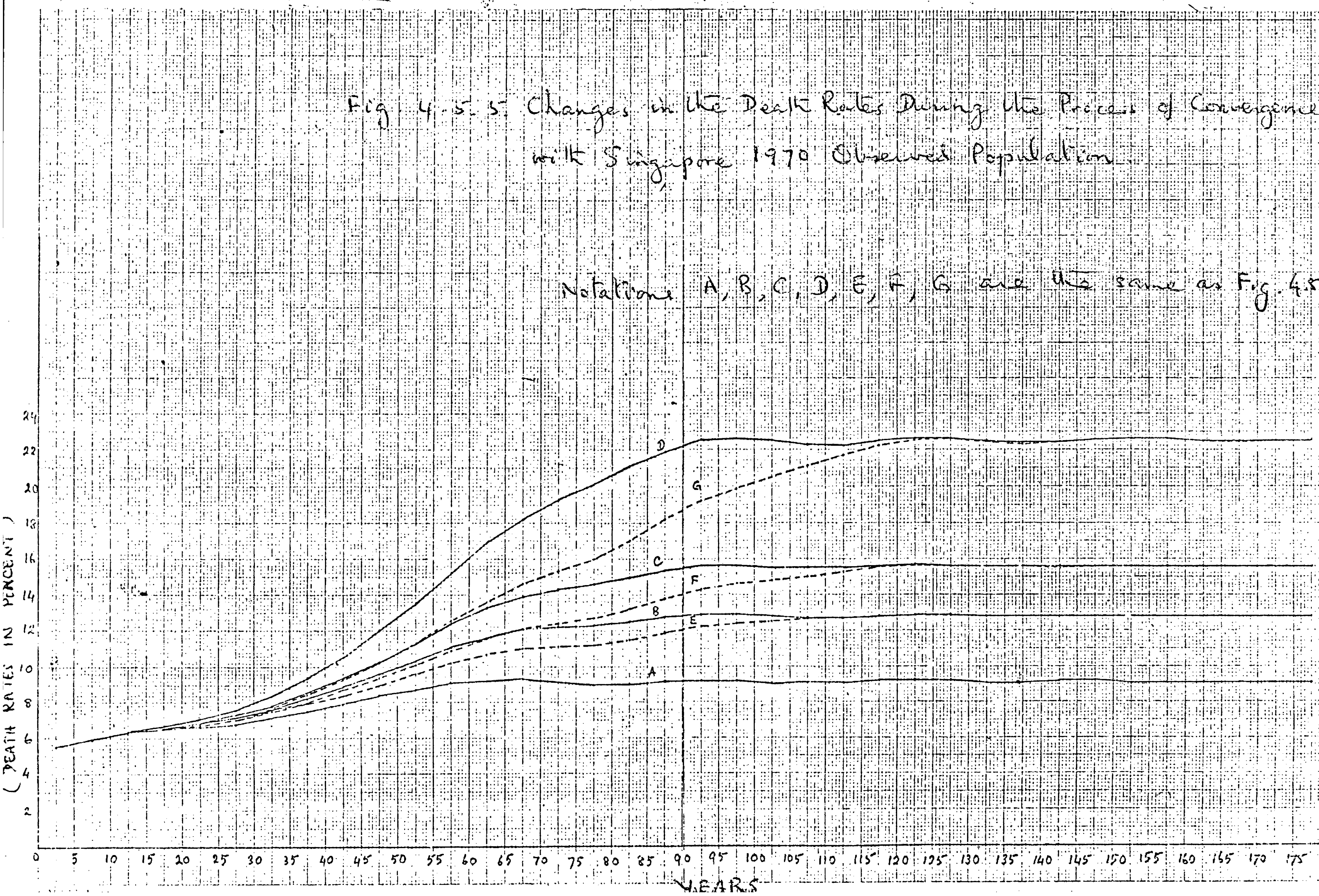


Fig. 4.5.5 Changes in the Death Rates During the Process of Convergence with Singapore 1970 Observed Population

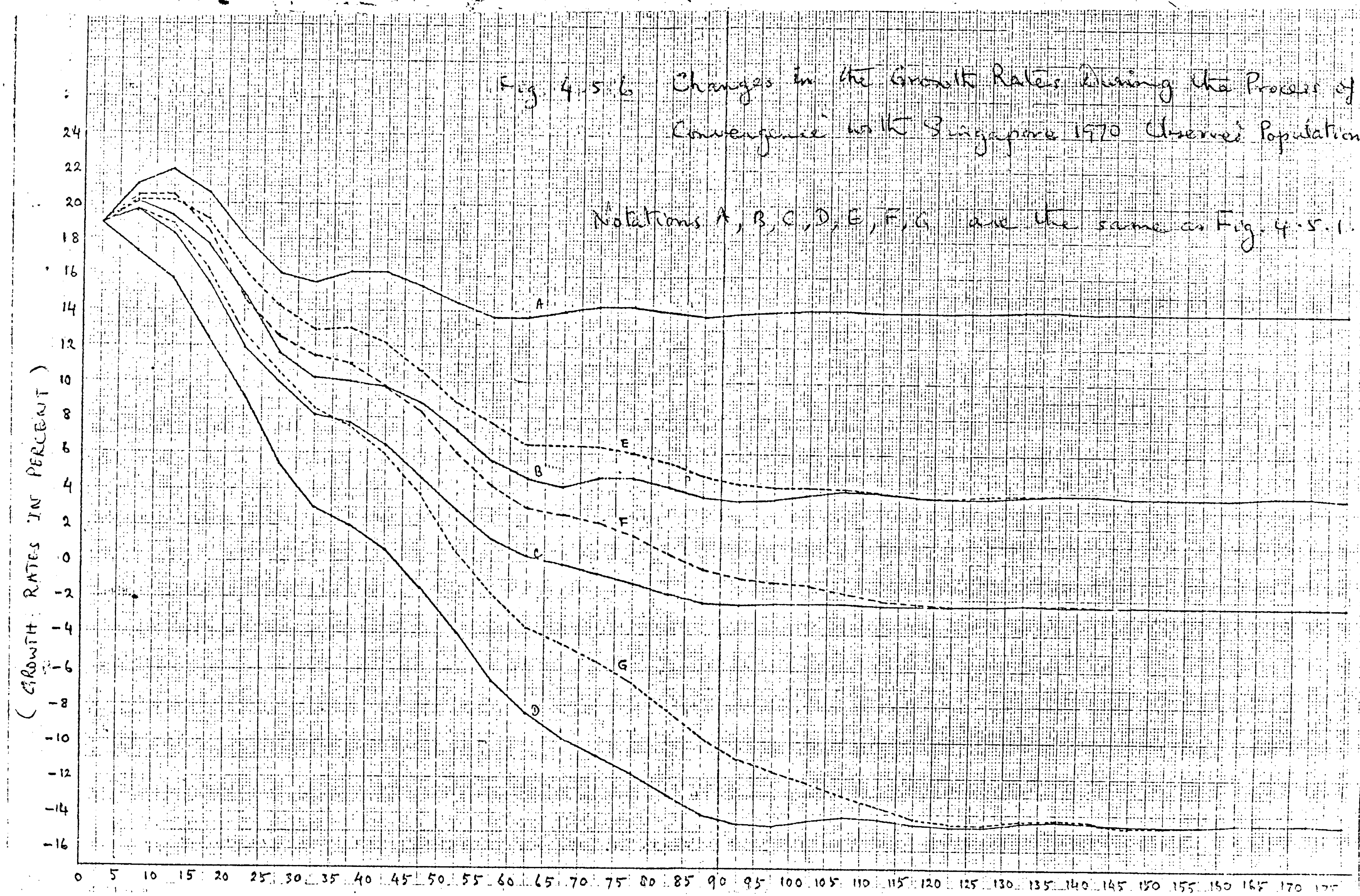


Fig. 4.5.6 Changes in the Growth Rates During the Process of Convergence to the Singapore 1970 Observed Population.

Notations A, B, C, D, E, F, G are the same as Fig. 4.5.1

and 4.5.6 we can say that the changes in the growth rates during the process of convergence followed the pattern of changes in the birth rates.

From the Figures presented above, we can notice the nature of the curves which showed the slow and fast dampening of the low frequency oscillations. Again, in these Figures projections B and D have the nature of slow convergence, while projections A and C have the fast convergence nature. At the end of the projections, Experiments E, F and G which have the 65 year transition, reached the same levels of birth rates, death rates and growth rates achieved by Experiments B, C and D respectively. Although these compared Experiments have the same level of fertility in the post-transition period they have different durations of convergence.

From the above observations we can say here that changes in nuptiality are able to produce the same effects as those brought about by serious changes in marital fertility and can, therefore, add as much to the reduction of population growth, especially as their effects become apparent sooner. Further, we can conclude that when there is a transition of age-specific fertility rates, the duration of convergence depends not only upon the level of fertility, but also upon the mean age and variance of the fertility schedule, which are also the determinants of the mean length of generation.

CHAPTER 5

SUMMARY OF FINDINGS

1. The major premise of this study is the theory of stable population, which has been discussed in Chapter 1. From our discussions the process of convergence from arbitrary initial circumstances to the stable form is obviously an important aspect of the theory of stable population. We have also discussed the distinctive roles of empirical investigation, in which the most useful tools are those numerical indicators of the main phenomena which are related to the state and change of a human population.

In this study, we are interested in two indicators, known as the nuptial-specific fertility and the proportion married, which are the representatives of fertility and nuptiality. In fact, the latter are too complex to give a complete understanding of their effects on the real demographic processes involved. However, we have investigated empirically the effects of these two variables on the duration of convergence of the age-sex distribution of Singapore's population by using the nuptial-specific fertility projection method. The use of the nuptial-specific fertility projection allowed us to understand more about the relationship between fertility and nuptiality.

The analysis is carried through the use of one-sex and two-sex deterministic models of population change, considering time and age as discrete variables. It is assumed that the mortality rates for all projections will be the same as for the 1969-71 Singapore situation. It is also assumed that the variations in the components occur independently.

2. The mortality, fertility and nuptiality data observed in Singapore during 1947-75, and some hypothetical data derived from them were used in the study. An analysis of the variations

in the data observed in Singapore showed that the survival ratios by five year age groups did not change very much during the period 1957-70, except at the very young ages and at the old ages. Improvements in the survival ratios were very narrow.

Singapore's overall fertility, which was very high (with a total fertility rate of 6.5) in 1957, decreased to a moderate level (with TFR of 3.1) in 1970, and continued to a low level (with TFR of 2.1) in 1975. The mean age of the fertility schedule also declined from 29.5 in 1957 to 29.1 in 1970 and reached at 28.3 in 1975. On the other hand, nuptiality followed the same pattern as fertility during the period 1957-70 (with the proportion married varying from 67.6 in the 15-49 age group in 1957 to 58.7 in that age group in 1966, continuing up to 54.9 in that age group in 1970).

Hypothetical data on fertility and nuptiality were derived such that the approximate standard schedules for the 1970 Singapore situation as a pre-transitional state, move from a comparatively low fertility and nuptiality to very low levels. We also use three different populations for Singapore namely 1957 obs., 1957 stb. and 1970 obs., to examine the duration of the convergence of the age-sex distributions. The sex ratio at birth was assumed to be constant at 106 males per 100 females in all projections.

3. From our empirical investigations it is found that the duration of convergence, which is obtained by using the direct projection method, is conditioned by the nature of the initial age-sex distribution while the value of t computed from the matrix roots is not conditioned by the initial age-sex distribution. It is hard to conclude what is the relationship between the duration of convergence and the initial age-sex distribution. However, Keyfitz (Keyfitz, 1968:90) has pointed out that the ultimate age distribution is fixed and depends neither on the original age distribution nor even on the age-specific rates of fertility, except as these latter are reflected in the intrinsic rate r_1 or the real root λ_1 (Chapter 3).

We shall conclude here, that the age distribution of the stable population is achieved according to the level of fertility which will continue throughout the process of convergence. The population with the highest total fertility rate at post-transition will achieve the youngest age structure and that with the lowest total fertility rate by that time will achieve the oldest age structure. Again, we can say that the two sequences of MAGI and MIGI converge to the same limit, determined by the level of fertility.

When the proportion married declined there was a definite increase in the duration of convergence as in the case of the marital fertility and nuptiality declining at the same time. However, it is hard to say that the duration of convergence increases when marital fertility alone decreases. The duration of convergence is affected not only by the initial magnitude of waves arising from disturbances in the age distribution, but also the range of ages within which reproduction takes place, and the wave lengths caused in later generations by a disturbance in the age distribution.

Furthermore, the changes in the proportion married can produce the same effect on birth and growth rates as changes in marital fertility; even their effects during transition become apparent sooner since they increase the mean length of generation. In the case of marital fertility alone, the mean age of the fertility schedule is reduced and a substantial part of the gain in reducing the birth rate is lost by this shortening of the mean length of generation. The conclusion to be made here is clear: changes in nuptiality are able to produce the same effects as those brought about by serious changes in marital fertility and can, therefore, add as much or more to the reduction of population growth since their effects become apparent sooner.

REFERENCES

Bogue, D.J.

1974 Techniques for Making Population Projections: How to Make Age-Sex Projections by Electronic Computer. Family Planning Research and Evaluation Manual, No. 12, Community and Family Study Center, University of Chicago.

Coale, Ansley J.

1956 'The Effects of Changes in Mortality and Fertility on Age Composition', Milbank Memorial Fund Quarterly, Vol. 34, 79-114.

-----------,

1971 'Age Patterns of Marriage', Population Studies, Vol. 25, No. 2, 193-214.

------------,

1972 The Growth and Structure of Human Populations: A Mathematical Investigation. New Jersey, Princeton: Princeton University Press.

--------------,

1974 'Model Fertility Schedules: Variations in the Age Structure of Childbearing in Human Populations', Population Index, Vol. 40, No. 2, 185-258.

Frejka, T.

1973 The Future of Population Growth: Alternative Paths to Equilibrium. A Population Council Book, New York.

Henry, L.

'Some Data on Natural Fertility', Eugenics Quarterly, Vol. 8, June 1961, 81-91.

Keyfitz, N.

1968 Introduction to the Mathematics of Population. Addison - Wesley Publishing Company, Reading, Massachusetts.

-----------,

1976 'Mathematical Demography: A Bibliographical Essay, Population Index, Vol. 42, No. 1, 9-19.

-----------, and E.M. Murphy

1964 Comparative Demographic Computations. Population Research and Training Centre, Chicago: University of Chicago.

Lesthaeghe, R.

1971 'Nuptiality and Population Growth', Population Studies, Vol. 25, No. 3, 415-432.

Lopez, Alvaro

1961 Problems in Stable Population Theory. New Jersey, Princeton: Office of Population Research.

Lorimer, F.

1951 'Dynamics of Age Structure in a Population with Initially High Fertility and Mortality', United Nations Population Bulletin, No. 1, 31-41.

Lotka, A.J.

1939 Théorie Analytique des Associations Biologiques, Part II. Analyse Démographique Avec Application Particuliere a L'espèce Humaine. (Actualité's Scientifiques et Industrielles, No. 780). Paris: Hermann et Cie.

Peterson, W.

1972 Readings in Population. New York: Macmillan Company.

Richardson, E.L.

1975 World Population 1975: Recent Demographic Estimates for the Countries and Regions of the World. U.S. Department of Commerce, Bureau of the Census.

Saw, Swee-Hock

1970 Singapore Population in Transition. Philadelphia: University of Pennsylvania Press.

Seng, You Poh, Stephen H.K. Yeh, et al.

1967 Singapore Sample Household Survey, 1966. Report No. 1, Singapore.

Shorter, F.C. and P. David

1968 Computational Methods for Population Projections: with Particular Reference to Development Planning. The Population Council, New York.

Shryock, H.S. and J.S. Siegel,

1975 The Methods and Materials of Demography. U.S. Department of Commerce, Bureau of the Census, Vol. 1.

Singapore,

1957 Report on the Census of Population, 1957. Singapore: Government Printing Office. Published in 1964.

----------,

1958 Report on the Registration of Births and Deaths, Marriages and Persons. Singapore: Government Printing Office.

------------,

1959 Monthly Demographic Bulletin, Department of Statistics, No. 1, Singapore.

------------,

1970 Report on the Census of Population, 1970. Singapore: Government Printing Office, Published in 1971.

Sivamurthy, M.

1970 'Convergence of Age-Sex Distributions and Population Change in the Presence of Migration', unpublished Ph. D. thesis, Australia: Department of Demography, Australian National University.

United Nations,

1956 The Ageing of Populations and its Economic and Social Implications, ST/SOA/Series A, Population Studies No. 26, United Nations, New York.

-----------,

1965 Demographic Yearbook, with Special Reference to Natality. United Nations, New York.

------------,

1975 Demographic Yearbook, with Special Reference to Natality. United Nations, New York.

Appendix A

NUPTIAL-SPECIFIC FERTILITY PROJECTIONS*

When nuptial-specific fertility projections are made, it becomes necessary to specify two parameters instead of one:

(a) The nuptial-specific fertility rate (for currently married or ever-married females).

(b) The proportion of women of each childbearing age who are currently married or ever married.

The age-specific nuptial fertility rate for a future projection cycle is defined as :

$$ {}^{n}f_x^{t+i} = \frac{B_x^{t+i}}{N_x^{t+i}} \cdot 1000 \qquad \ldots \quad \ldots \quad \ldots \qquad (1)$$

where ${}^{n}f_x^{t+i}$ is the nuptial age-specific fertility rate for some future cycle.

B_x^{t+i} are all births that are anticipated to occur in cycle t+i

N_x^{t+i} is the number of women who are anticipated to be currently married or ever-married at the midpoint of cycle t+i.

The usual procedure for computing the proportions currently married or ever married for each age group is as follows:

$$n_x = \frac{N_x}{F_x} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

where n_x is the proportion of persons aged x who are currently married or ever married.

N_x is the total number of women aged x who are currently married or ever married.

F_x is the total number of women aged x.

The number of married women anticipated in future years is calculated by multiplying the projected number of women of each age by the proportion anticipated to be married or ever married at that age. $N_x^{t+i} = F_x^{t+i} \cdot n_x^{t+i}$ .. .. (3)

Where F_x^{t+i} is the total number of women aged x anticipated at the midpoint of some future cycle t+i.

N_x^{t+i} is the total number of currently married or ever married women anticipated at the same midpoint of the cycle.

n_x^{t+i} is the proportion of women aged x who are currently married or ever married at time t+i.

Finally, the anticipated number of births at some future date is estimated by combining the result of equation (1) and (3) - multiplying the anticipated number of married women (currently married or ever married) by the projected nuptial-specific birth rate:

$$B_x^{t+i} = N_x^{t+i} \cdot {}_nf_x^{t+i} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4)$$

N.B.* This method was adopted from 'Techniques for Making Population Projections' by Donald J. Bogue (1974: 43-44), but in this study we used the currently married women only.

Appendix B

ABRIDGED LIFE TABLES FOR SINGAPORE

1956-58 and 1969-71

The life table for Singapore, based on the 1970 census was constructed here, whereas the abridged life table of 1956-58 was adopted from Saw Swee-Hock (Saw, 1970:179). The abridged life table for the year 1969-71 was constructed using the data on deaths for the years, 1969, 1970 and 1971, and also the population enumerated in the 1970 census on 19/22th June.

ADJUSTMENT OF DATA

The method adopted here was essentially similar to the one used in the construction of the abridged life table for Singapore 1956-58, by Saw Swee-Hock. The population of males and females by single year of age was not adjusted to bring the data forward to June thirtieth because the population increase in the fortnight or so in Singapore should not result in any significant error in the life table. The data for infant deaths are available in single days from 0 to 7 days, single weeks from 2 to 4 weeks, and single months from 2 to 11 months, while those for the post-infant deaths are broken down into quinary ages. It is this classification that is used in obtaining the abridged life table.

The first adjustment concerns the pro-rating of the 1969-71 deaths with unknown ages to the various age groups from 0 to 85 and over, then the adjusted deaths and the 1970 census population data are used to compute the age specific death rates, except that it was decided to adopt the customary practice of deriving the mortality rate for age 0 from infant deaths and births directly.

METHOD OF CALCULATION

The corrected ${}_nq_x$ values given in the abridged life table form the basis of calculating the various functions of the abridged life table. Since the deaths of infants under 1 year were available by month of age and the births were available by calendar months, q_o was directly calculated from infant death and birth records in four of the separate computations according to the following formula, where for instance, B^{68}, B_4^{68}, B_D^{68} denote births in the year 1968, births in the quarter of 1968, and births in December of 1968 respectively. The same procedures were applied separately for males and females.

q_o = infant mortality rate (or) the probability of dying before attaining the age of 1 year.

$$q_o = q_o^{(0\text{-}1\ mth)} + q_o^{(1\text{-}3\ mth)} + q_o^{3\text{-}6\ mth)} + q_o^{(6\text{-}12\ mth)} \quad \text{where;}$$

$$q_o^{(0\text{-}1\ mth)} = \frac{\text{Deaths in 1969-71 (age 0-1 mth)}}{1/2\ B_D^{68} + B^{69} + B^{70} - 1/2\ B_D^{71}}$$

= the probability of dying during the first three months after birth.

$$q_o^{(1\text{-}3\ mth)} = \frac{\text{Deaths in 1969-71 (age 1-3 mth)}}{1/2(B_O^{68}+B_N^{68})+B_D^{68}+B^{69}+B^{70}+B^{71} - 1/2(B_O^{71}+B_N^{71})-B_D^{71}}$$

= the probability of dying during the next three months after birth.

$$q_o^{(3\text{-}6\ mth)} = \frac{\text{Deaths in 1969-71 (age 3-6 mth)}}{1/2B_3^{68}+B_4^{68}+B^{69}+B^{70}+B^{71} - 1/2\ B_3^{71} - B_4^{71}}$$

= the probability of dying during the third three months after birth.

$$q_o^{(6\text{-}12\ mth)} = \frac{\text{Deaths in 1969-71 (age 6-12 mth)}}{1/2(B_1^{68}+B_2^{68})+B_3^{68}+B_4^{68}+B^{69}+B^{70}+B^{71}-1/2(B_1^{71}+B_2^{71}-B_3^{71})-B_4}$$

The value of ${}_nq_x$ from age one upwards is computed by the formula, ${}_nq_x = \dfrac{2n\ .\ {}_nm_x}{2+n.\ {}_nm_x}$

The values of l_x refer to the number of survivors at the beginning of the age interval, and are obtained by a direct mathematical procedure on the basis of the life table death rates according to the formula, $l_x = l_{x-n} - (l_{x-n} \times q_{x-n})$

The ${}_nL_x$ column represents the number of years that will be lived collectively within any one age interval by a cohort numbering 100,000 at birth and subject to the given mortality conditions. Owing to the very uneven distribution of deaths in the first year of life, it is necessary to drive L_o in separate stages by summing the results of the following calculations:

$$L_{(0\text{-}1\ mth)} = 1/2(l_{0\ mth} + l_{1\ mth}) \times 1/12$$

$$L_{(1\text{-}3\ mth)} = 1/2(l_{1\ mth} + l_{3\ mth}) \times 2/12$$

$$L_{(3\text{-}6\ mth)} = 1/2(l_{3\ mth} + l_{6\ mth}) \times 3/12$$

$$L_{(6\text{-}12\ mth)} = 1/2(l_{6\ mth} + l_{12\ mth}) \times 6/12$$

where the values of $l_{0\ mth}$, $l_{1\ mth}$, $l_{3\ mth}$, and $l_{6\ mth}$ may be obtained in the usual manner from the form corresponding values of q_o.

It is also known that the distribution of deaths in the second year of life is still uneven, and it has been ascertained that by equating L_1 to $0.45l_1 + 0.55\ l_2$, fairly satisfactory results can be attained. For ages two to four, L_x is taken as the average of l_x and l_{x+1}, and for the other quinary age groups

$_5L_x$ is equated to $\frac{5\ (l_x + l_{x+1})}{2}$

The T_x column indicates the number of years that will be lived collectively from the given age upwards, by the survivors to that age from the cohort of 100,000 births. This function is obtained by cumulative addition of the l_x function from the bottom up, and $T_{80} = \frac{l_{80}}{m_{80}}$ in the last age group. In UN Manual III: Methods for Population Projections by Sex and Age, ST/SOA/Series A, Population Studies No. 25, it is stated that L_{80} is easily obtained by multiplying l_{80} by its own logarithm.

The last function is e^o_x, which represents the individual expectation of life. If, as in this life table 100,000 males at birth are expected to live collectively 6512707 years, then the average individual can expect, at birth, to live 65.1 years. Similarly, expectations of life at other ages are obtained by dividing T_x by the corresponding l_x.

The survival rates were calculated from the relations:

$S(b) = 5\ L_0 / 5\ l_0$

$S(x) = 5L_{x+5}\ /\ 5L_x$ for x = 0, 5, 10............70, and

$S(75) = T(80)\ /_{T(75)}$.

Abridged Life Table for Singapore, 1956-58 (Females)

Year of Age	Mortality Rate of 10,000 Born Alive: Number dying per 1,000 alive at beginning of age	Number alive at beginning of age interval	Number dying during age interval	Stationary Population: In the Age interval	In this and all subsequent age intervals	Survival rates	Average Remaining Lifetime: Average number of years of life remaining at beginning of age interval
x to x+4	1,000 $_{n}q_{x}$	l_{x}	$_{n}d_{x}$	$_{n}L_{x}$	T_{x}	S_{x}	$\mathring{e}_{x}$
0	37.61	10,000	376	9,735	666,102	.95564*	66.6
1	8.21	9,624	79	9,581	656,367		68.2
2	4.81	9,545	46	9,522	646,786		67.8
3	3.00	9,499	29	9,484	637,264		67.1
4	2.02	9,470	19	9,460	627,780	.98598**	66.3
5 - 9	5.98	9,451	57	47,112	618,320	.99525	65.4
10 - 14	3.49	9,394	33	46,888	571,208	.99599	60.8
15 - 19	4.49	9,361	42	46,700	524,320	.99469	56.0
20 - 24	6.13	9,319	57	46,452	477,620	.99296	51.2
25 - 29	8.02	9,262	74	46,125	431,168	.99068	46.6
30 - 34	10.64	9,188	98	45,695	385,043	.98676	41.9
35 - 39	15.82	9,090	144	45,090	339,348	.98059	37.3
40 - 44	23.03	8,946	206	44,215	294,258	.97248	32.9
45 - 49	32.13	8,740	281	42,998	250,043	.96063	28.6
50 - 54	46.78	8,459	396	41,305	207,045	.94141	24.5
55 - 59	70.90	8,063	572	38,885	165,740	.91264	20.6
60 - 64	104.98	7,491	787	35,488	126,855	.87072	16.9
65 - 69	156.38	6,704	1048	30,900	91,367	.80890	13.6
70 - 74	232.32	5,656	1314	24,995	60,467	.72334***	10.7
75 - 79	334.51	4,342	1452	18,080	35,472	.49030	8.2
80 - 84	477.50	2,890	1380	11,000	17,392		6.0
85+	1,000.00	1,510	1510	6,392	6,392		4.2

* S(Birth) = L(0-4)/$_{5}l_{o}$, **S(0-4) = L(5-9)/L(0-4), *** S(75+0) = T(80)/T(75)

Source: Saw Swee-Hock, Singapore Population Transition, 1970:p.179.

Abridged Life Table for Singapore, 1956-58 (Males)

Year of age	Mortality Rate of 10,000 Born Alive: Number dying per 1,000 alive at beginning of age	Number alive at beginning of age interval	Number dying during age interval	Stationary Population: In the Age interval	In this and all subsequent age intervals	Survival rates	Average Remaining Lifetime: Average number of years of life remaining at beginning of age interval
x to x+4	$1000\ {}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	S_x	$\overset{o}{e}_x$
0	47.51	10,000	475	9648	605278	.94630*	60.5
1	8.12	9,525	77	9482	595630		62.5
2	4.69	9,448	44	9426	586148		62.0
3	2.83	9,404	27	9390	576722		61.3
4	1.24	9,377	12	9371	567332	.98643**	60.5
5 - 9	6.38	9,365	60	46675	557961	.99422	59.6
10 - 14	5.19	9,305	48	46405	511286	.99440	54.9
15 - 19	6.03	9,257	56	46145	464881	.99313	50.2
20 - 24	7.72	9,201	71	45828	418736	.99153	45.5
25 - 29	9.16	9,130	84	45440	372908	.98904	40.8
30 - 34	12.72	9,046	115	44942	327468	.98488	36.2
35 - 39	19.85	8,931	177	44712	282526	.96368	31.6
40 - 44	31.21	8,754	273	43088	237814	.95892	27.2
45 - 49	51.30	8,481	435	41318	194726	.93603	23.0
50 - 54	77.29	8,046	622	38675	153408	.89743	19.1
55 - 59	129.92	7,424	965	34708	114733	.84159	15.4
60 - 64	190.99	6,459	1234	29210	80025	.76282	12.4
65 - 69	294.21	5,225	1537	22282	50815	.66771	9.7
70 - 74	386.32	3,688	1425	14878	28533	.56809	7.7
75 - 79	505.74	2,263	1145	8452	13655	.38103***	6.0
80 - 84	641.97	1,118	718	3795	5203		4.6
85+	1000.00	400	400	1408	1408		3.5

* S(Birth) = $L_{0-4}/5l_o$, ** S(0-4) = L_{5-9}/L_{0-4}, *** S(75+) = T(80)/T(75).

Source: Saw Swee-Hock, Singapore Population in Transition, 1970: p.178.

Abridged Life Table for Singapore, 1969-71 (Males)

Year of age	Mortality Rate of 10,000 Born Alive: Number dying per 1,000 alive at beginning of age	Number alive at beginning of age interval	Number dying during age interval	Stationary Population: In the Age interval	In this and all subsequent age intervals	Survival rates	Average Remaining Lifetime: Average number of years of life remaining at beginning of age interval
x to x+4	$1{,}000\ {}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	S_x	e^o_x
0	22.50	10,000	225	9806	659489	.97536*	65.9
1	2.37	9,775	23	9762	649683		66.5
2	1.50	9,752	15	9744	639921		65.6
3	1.00	9,737	10	9732	630177		64.7
4	0.74	9,727	7	9724	620445	.99543**	63.8
5 - 9	2.29	9,720	22	48545	610721	.99759	62.8
10 - 14	2.59	9,698	25	48428	562176	.99591	58.0
15 - 19	5.58	9,673	54	48230	513748	.99374	53.1
20 - 24	6.92	9,619	67	47928	465518	.99263	48.4
25 - 29	7.72	9,552	74	47575	417590	.99264	43.7
30 - 34	6.97	9,478	66	47225	370015	.98979	39.0
35 - 39	13.51	9,412	127	46743	322790	.98250	34.3
40 - 44	21.53	9,285	200	45925	276047	.97154	29.7
45 - 49	35.50	9,085	323	44618	230122	.95092	25.3
50 - 54	63.09	8,762	553	42428	185504	.92144	21.2
55 - 59	95.02	8,209	780	39095	143076	.87441	17.4
60 - 64	159.44	7,429	1184	34185	103981	.80497	14.0
65 - 59	237.52	6,245	1483	27518	68786	.71608	11.2
70 - 74	344.90	4,762	1642	19705	42278	.62015***	8.9
75 - 79	433.19	3,120	1352	12220	22573	.45865	7.2
80+	1,000.00	1,768	1768	10353	10353		5.9

* S(Birth) = $L(0\text{-}4)/{}_5l_0$, ** S(0-4) = L(5-9)/L(0-4), *** S(75+) = T(80)/T(75).

Abridged Life Table for Singapore, 1969-71 (Females)

Year of age	Mortality Rate of 10,000 Born Alive: Number dying per 1,000 alive at beginning of age	Number alive at beginning of age of interval	Number dying during age interval	Stationary Population: In the Age interval	In this and all subsequent age intervals	Survival rates	Average Remaining Lifetime: Average number of years of life remaining at beginning of age interval
x to x+4	1,000 $_nq_x$	l_x	$_nd_x$	$_nL_x$	T_x	S_x	e^o_x
0	17.74	10,000	177	9853	722968	.98050	72.3
1	2.20	9,823	21	9811	713115		72.6
2	1.19	9,892	12	9796	703304		71.8
3	0.92	9,790	9	9788	693508		70.8
4	0.77	9,781	8	9777	683720	.9955 **	69.9
5 - 9	2.44	9.773	24	48805	673943	.99795	69.0
10 - 14	1.60	9,749	16	48705	625138	.99791	64.1
15 - 19	2.59	9,733	25	48603	576433	.99706	59.2
20 - 24	3.29	9,708	32	48460	527830	.99604	54.4
25 - 29	4.69	9,676	45	48268	479370	.99472	49.5
30 - 34	5.93	9,631	57	48013	431102	.99281	44.8
35 - 39	8.46	9,574	81	47668	383089	.98878	40.0
40 - 44	14.01	9,493	133	47133	335421	.98264	35.3
45 - 49	20.79	9,360	194	46315	288288	.97323	30.8
50 - 54	32.94	9,166	302	45075	241973	.95692	26.4
55 - 59	53.63	8,864	475	43133	196898	.92929	22.2
60 - 64	86.43	8,389	725	40083	153765	.89115	18.3
65 - 69	130.88	7,644	1000	35720	113682	.83351	14.9
70 - 74	207.56	6,644	1379	29773	77962	.75347 ***	11.7
75 - 79	295.79	5,265	1557	22433	48188	.55630	9.2
80+	1,000.00	3,708	3708	26807	26807		7.2

* S(Birth) - L(0-4)/$_5l_0$, ** S(0-4) - L(5-9)/ L(0-4), *** S(75+) - T(80) / T(75)

APPENDIX C

ESTIMATING NUPTIALITY FROM THE STANDARD TABLES OF FIRST-MARRIAGE FREQUENCIES

The standard schedule of first-marriage frequencies is presented in Table 2.3 and 4 of Age Patterns of Marriage by Ansley J. Coale (Coale, 1971:193-214). They are described as marriage frequencies, proportions ever-married and person-years lived ever-married. The ages serving as the basis of the tabulation run from zero to 39.9 years. The origin is not at birth, and hence 'age' is not ordinary chronological age. The origin is the earliest age of marriage, and 'age' is chronological age less this value.

Let (x_s) be age on the standard scale, $g_s(x_s)$ be standard first-marriage frequency at age x_s, $G_s(x_s)$ the standard proportion ever-married at age x_s, and $Z_s(x_s)$ the average number of person-years lived ever-married at age x_s in a cohort not subject to mortality.

$$G_s(x_s') = g_s(x_s)dx_s \quad \text{and} \quad Z_s(x_s') = G_s(x_s)dx_s.$$

Now suppose there is a cohort subject to a schedule of first-marriage frequencies of the standard form, with an origin a_o, a proportion ultimately ever-married C (rather than 1.0), and a horizontal scale k times the standard. Thus the distance from the origin to age a is k times x_s, or $a-a_o=kx_s$; hence

$$x_s = \frac{a - a_o}{k}.$$

The proportion ever-married at age a is

$$G(a) = C.g_s\frac{(a-a_o)}{k}$$

The first-marriage frequency at age a is $g(a) = \frac{C}{k}G_s\frac{(a-a_o)}{k}$.

The average number of person-years lived ever-married at age a is

$$Z(a) = C \cdot kZ_s \frac{(a - a_o)}{k}.$$

For determining the values of a_o, k and C, we shall use the recommended procedure in Age Patterns of Marriage (Coale, 1971:214). The designated parameters R_1, R_2 and R_3 can be calculated as follows:

$$R_1 = \frac{\text{Proportion ever-married 15-19}}{\text{Proportion ever-married 20-24}}$$

$$R_2 = \frac{\text{Proportion ever-married 20-24}}{\text{Proportion ever-married 25-29}}$$

$$R_3 = \frac{\text{Proportion ever-married 25-29}}{\text{Proportion ever-married 30-34}}$$

k and a_o can be estimated by locating (through interpolation in Table IA of Age Patterns of Marriage (Coale, 1971:210-211) the values that would yield the observed R_1, and R_2, or R_2 and R_3.
If $R_1 > (1-R_3)$ then estimate from R_2 and R_1, and
if $R_1 < (1-R_3)$ then estimate from R_2 and R_3.

Since our a_o for Singapore 1970 proportion ever-married falls between 15-19, the value of k and a_o will be estimated from R_2 and R_3.

Age Group	Proportion Ever-Married (Singapore) 1970	
15-19	0.048	R_1 = 0.136
20-24	0.354	R_2 = 0.457
25-29	0.774	R_3 = 0.856
30-34	0.904	$(1-R_3)$ = 0.144

From the table mentioned above,

$a_o = 2.5$ | $a_o = 3.0$
$R_3 = 0.856$ | $R_3 = 0.856$
$R_2 = 0.462$ | $R_2 = 0.430$
$k = 0.59$ | $k = 0.55$

Again, we can interpolate these values for required value of R_2 and R_3.

$a_o = 2.6$

$R_3 = 0.856$

$R_2 = 0.457$

$k = 0.58$

Since the first age group is 15-19, the final estimate of a_o is 17.6.

The value of C (proportion ultimately ever married) is estimated by determining the person-years lived ever-married at the beginning and end of the third five-year age interval (25 and 30 if marriages begin between 15 and 20) in the transformed standard schedule with the estimated values of k and a_o. Specifically, one determines

$$kZ_s \frac{(25-a_o)}{k} \quad \text{and } kZ_s \frac{(30-a_o)}{k} .$$

The difference between these two, divided by 5.0 is the proportion ever-married in a cohort subject to a curve characterized by the calculated values of a_o and k, and with an ultimate proportion ever-married of 1.0. The required estimate of C is, then,

$$C = \frac{\text{(proportion ever-married 25-30)}}{k/5\ (Z_s(30-a_o)/k) - Z_s(25-a_o)/k))}$$

$$= \frac{0.7940}{0.8199} = 0.968$$ ($Z_s(x_s)$ values are interpolated from standard schedule (Coale, 1971:202).

Estimated Proportion Ever Married from The Standard Tables

a	$\frac{a - a_o}{k} = x_s$	$Z_s(x_s)$	C.k. $Z_s(x_s)$	(Difference in column 4) divided by 5.0
(1)	(2)	(3)*	(4)	(5)
		Schedule I for:		
		$a_o = 17.6$, $k = .58$, $C = .968$		
20	4.14	0.0997	0.0560	.0112
25	12.76	3.3344	1.8721	.3632
30	21.38	10.4114	5.8454	.7947
35	30.00	18.6688	10.4814	.9272
40	38.62	27.2235	15.2844	.9606
45	47.24	35.8443	20.1244	.9680**
50	55.86	44.4649	24.9644	.9680**
		Schedule II for:		
		$a_o = 18$, $k = .78$, $C = .968$		
20	2.56	0.0209	0.0158	.0032
25	8.97	1.2467	0.9413	.1851
30	15.38	5.2410	3.9572	.6032
35	21.79	10.7898	8.1467	.8379
40	28.21	16.9172	12.7732	.9253
45	34.62	23.2375	17.5452	.9544
50	41.03	-	-	.9680**

* Interpolated from standard schedule (Coale, 1971:202). Column (4) yields estimated person-years ever married Column (5) estimated proportions ever married, for the age intervals 15-19, 20-24, etc.

** From C value.

APPENDIX D

ESTIMATION OF MARITAL FERTILITY

Louis Henry (Henry, 1961:81-91) found that there is a characteristic pattern of marital fertility in population in which there is little or no voluntary control of births. Later Ansley J. Coale (Coale, 1971:207) found that there is a typical age pattern of the departure of marital fertility from the pattern of natural fertility. He defined as follows:

Let n(a) be natural fertility (above age 20) and m(a) be marital fertility. There is a function v(a) expressing the typical pattern of the effect of the voluntary control of fertility, such that $m(a) = M.n(a)\ e^{m.v(a)}$, where M is the ratio of marital fertility at an early age (say 20-24) to fertility at that age in the schedule representing natural fertility, and m is a factor expressing the degree to which m(a) departs from the pattern of the Hutterites, 1921-30, one of the highest marital fertility schedules on reliable record.

Estimation of marital fertility for Singapore, 1970 based on:

$$m(a) = M, n(a)\ e^{m.v.(a)}$$

where $M = m(a)/n(a)$ at 20-24 = 0.722,

m(a) = Observed age-specific marital fertility rates (0.397,0.274,0.157,0.083,0.031,0.006) at 20-24 to 45-49 age groups,

n(a) = Hutterite age-specific fertility rates (0.550,0.502,0.447,0.406,0.222,0.061) at 20-24 to 45-49 age groups,

v(a) = General characteristic age-pattern of deviation from the natural fertility schedule (Coale, 1971: 207);
(0.000,-0.252,-0.623,-1.094,-1.519,-2.437),

m = 1.15 by using the formula; $\log(m(a)/M.n(a))/v(a)$, at 30-34 and 35-39 age groups, and taking the average (i.e. the average of the values that would yield the observed m(a) at 30-34 and 35-39).

Let us call the estimated age-specific fertility rate 'a' as $m_s(a)$; then the following can be easily computed:

$m_s(15\text{-}19)$ = 0.544 (as observed)

$m_s(20\text{-}24)$ = $(0.722)\ (0.550)\ (e^{0})$ = 0.3971

$m_s(25\text{-}29)$ = $(0.722\ (0.502)\ (e^{1.15(-0.252)})$ = 0.2713

$m_s(30\text{-}34)$ = $(0.722)\ (0.447)\ (e^{1.15(-0.623)})$ = 0.1771

$m_s(35\text{-}39)$ = $(0.722)\ (0.406)\ (e^{1.15(-1.094)})$ = 0.0833

$m_s(40\text{-}44)$ = $(0.722)\ (0.222)\ (e^{1.15(-1.519)})$ = 0.0279

$m_s(45\text{-}49)$ = $(0.722)\ (0.061)\ (e^{1.15(-2.437)})$ = 0.0027

Estimation of marital fertility for Japan, 1960 based on:

$m(a) = M \,.\, n(a)\ e^{m.v(a)}$

where M = m(a)/n(a) at 20-24 = 0.620.

m(a) = Observed age-specific marital fertility rates (0.338,0.235,0.091,0.027,0.006,0.001) at 20-24 to 45-49 age groups,

m = 1.91 by using the formula; $\log\ (m(a)/M.n(a))/v(a)$, at 30-34 and 35-39 age groups, and taking the average of those values, and n(a), v(a) are defined as previously.

Estimated age-specific marital fertility rates are as follows:

$m_s(15\text{-}19)$ = 0.319 (as observed)

$m_s(20\text{-}24)$ = $(0.62)\ (0.550)\ (e^{0})$ = 0.3410

$m_s(25\text{-}29)$ = $(0.62)\ (0.502)\ (e^{1.91(-0.252)})$ = 0.1926

$m_s(30\text{-}34)$ = $(0.62)\ (0.447)\ (e^{1.91(-0.623)})$ = 0.0843

$$m_s(35\text{-}39) = (0.62)\ (0.406)\ (e^{1.91(-1.904)}) = 0.0311$$

$$m_s(40\text{-}44) = (0.62)\ (0.222)\ (e^{1.91(-1.519)}) = 0.0076$$

$$m_s(45\text{-}49) = (0.62)\ (0.061)\ (e^{1.91(-2.437)}) = 0.0004$$

Note: Coale has corrected the values of n(a) and v(a) again in Population Index October 1975, Vol. 41, No. 4, p. 572. Values of n(a) and v(a) are as follows:

	20-24	25-29	30-34	35-39	40-44	45-49
n(a)	0.460	0.431	0.395	0.322	0.167	0.024
v(a)	0.000	-0.279	-0.677	-1.042	-1.414	-1.671

However, the values obtained for all standard marital fertility schedules are not affected by these changes.

APPENDIX E

ONE-SEX AND TWO-SEX MODELS FOR THE CONVERGENCE OF AGE-SEX DISTRIBUTION TO AN EQUILIBRIUM STATE

The following models are directly extracted from 'Convergence of Age-Sex Distributions and Population Change in the Presence of Migration' (Sivamurthy, 1970:97-103, 121-123), which provides the mathematical background for computing the duration of convergence. The basic idea of this mathematical formulation can be found in 'Introduction to the Mathematics of Population' (Keyfitz, 1968:41-62).

ONE-SEX MODEL

ONE-SEX MODEL WITH NO MIGRATION

The process through time which leads to the stable state could be studied by successively projecting the population over time and comparing the percentage age distribution at time t with that at time (t-1) or with the one obtained by applying the stable population equation:

$$p(x)' = b\ c^{-rx}\ s(x) \qquad \text{(E.1)}$$

where $p(x)$ is the proportion of the population at age x years, b and r the constant birth and growth rates and $s(x)$ the proportion surviving to age x years. The stable population is established when the difference between the two age distributions being compared, becomes smaller than a pre-assigned small quantity. The value of t, then, is the duration of the process of convergence.

Alternatively, we may decompose the sequence of births as the sum of a real exponential term and a series of relatively diminishing oscillatory terms and claim that

> ... the stable population is established no more than w years after the number of births last deviates by an arbitrarily small proportionate amount from (the) pure exponential sequence, where w is the highest age attained under the given mortality regime.[4]

We shall follow the procedure of population projections since it facilitates the inclusion of the migration component.

Let P(x,t) be the female (or male) population aged x years at time t. Then the population at time t can be obtained from that at time (t-1) by using the following equations:

$$P(0,t) = S(b) \sum_{\alpha}^{\beta} \frac{1}{2}\left[P(x,t-1) + P(x,t)\right] f(x) \qquad (E.2)$$

$$\text{and } P(x,t) = S(x-1)\; P(x-1,t-1), \text{ for } x = 1, 2, \ldots\ldots, w \qquad (E.3)$$

where the absence of t in the symbols for fertility and survival rates indicates that these are constant over time.

Making use of the matrix notation the above equations can be written as:

$$(P_t) = L(P_{t-1}) \qquad (E.4)$$

where (P_t) denotes a column vector giving the population by age which we shall call the population vector; and L is the matrix given below:

$$L = \begin{vmatrix} 0 & 0 & .0 & m(\alpha) & m(\alpha+1) & .\ m(\beta) & 0\ .\ 0 & 0 \\ S(0) & 0 & .0 & 0 & 0 & .\ 0 & 0\ .\ 0 & 0 \\ 0 & S(1) & .0 & 0 & 0 & .\ 0 & 0\ .\ 0 & 0 \\ . & . & .. & . & . & .\ . & .\ .\ . & . \\ 0 & 0 & .0 & 0 & 0 & .\ 0 & 0\ .\ S(w-1) & 0 \end{vmatrix}$$

$$\text{in which} \qquad m(x) = \frac{1}{2}\left[f(x) + S(x)\; f(x+1)\right] S(b)\ .. \qquad (E.5)$$

---

[4] A.J. Coale, 'Convergence of ..', <u>Journal of the American Statistical Association</u>, 1968, p.396.

The matrix representation has an advantage in studying the process of population growth since some of the aspects of population growth could be studied conveniently by the properties of the matrix L.

The repeated application of the equation (3.4) gives:

$$(P_t) = L^t \ (P_0) \qquad \text{(E.6)}$$

where (P_0) is the initial population vector.

We can partition the matrix L at the $(\beta+1)$th row and column, and write it as follows:

$$L = \begin{vmatrix} M & 0 \\ A & B \end{vmatrix},$$

then it can be shown that, in any power of L, the elements of the first row will not be affected by the elements in the second row and the resultant matrix will have the same form as L. Hence, the growth of the population at ages below β years (age β years is the oldest age at which reproduction occurs) at any time, is independent of the population at ages beyond β years[5]. But, in course of time, the populations at ages beyond β years become linear functions of the populations at ages below β years. The coefficients of the functions are determined entirely by the known fertility and mortality schedules. Therefore, the intrinsic growth rate obtained using the populations at ages below β years, must apply to the whole of the age range. In fact, it can be shown that the non-zero characteristic roots of the matrix L are the same as those of its sub-matrix M:

$$M = \begin{vmatrix} 0 & 0 & . & 0 & m(\alpha) & m(\alpha+1) & . & m(\beta-1) & m(\beta) \\ S(0) & 0 & . & 0 & 0 & 0 & . & 0 & 0 \\ 0 & S(1) & . & 0 & 0 & 0 & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & 0 & 0 & 0 & . & S(\beta-1) & 0 \end{vmatrix}$$

[5] N. Keyfitz, <u>Introduction to the Mathematics....</u>, 1968, p.37.

Hence it suffices, in this case, to study the sub-matrix M. This matrix is an irreducible non-negative square matrix which is primitive, i.e. when it is raised to some powers it becomes a matrix with all elements positive. It has therefore a real positive characteristic root, λ_1, which is unique and whose value is greater than the absolute value of any other root, and corresponding to λ_1, there exists a characteristic vector whose elements are positive. The matrix M will be primitive whenever there are at least two consecutive ages at which the fertility rates are strictly positive[6]. As far as human populations are concerned, this requirement appears to be always satisfied.

Let λ_2 be another characteristic root of M, whose absolute value is greater than that of any other characteristic root except λ_1. We assume that λ_2 is also distinct. In fact, in demographic applications, it is observed that all the characteristic roots are distinct[7].

Now, if we denote the population vector below the age years as K, then we have:

$$(K_t) = M^t \ (K_0) \qquad \text{(E.7)}$$

When t is sufficiently large, we assert that:

$$(K_t) = \lambda_1^t \ C_1 \quad (V_1) \qquad \text{(E.8)}$$

which implies that the other terms become relatively negligible. Here C_1 is called the stable equivalent and (V_1) the stable age distribution[8]. Since λ_2 is the root highest in absolute value next to λ_1 and is distinct, the ratio $\left[|\lambda_2|/|\lambda_1|\right]$ determines the

---

6 J.H. Pollard, 'On the Use of Direct Matrix Product ....', Biometrika, 1966.

7 N. Keyfitz, Introduction to the Mathematics ...., 1968, p.51.

8 N. Keyfitz, Introduction to the Mathematics ...., 1968, p.57.

value of t, the duration of convergence. If λ_2 is a complex root, then $|\lambda_2| = |\lambda_3|$ because λ_3 will be the complex conjugate of λ_2 and we can use any one of them to determine t. In fact, t can be calculated as follows:

$$\left[|\lambda_2| / |\lambda_1| \right]^t = \mathcal{E} \qquad (E.9)$$

where is a small quantity chosen arbitrarily to ensure the required approximation. Hence

$$t = \left[(\log \mathcal{E}) / \left\{ \log |\lambda_2| - \log |\lambda_1| \right\} \right) \qquad (E.10)$$

and we may take the highest integer contained in the expression plus one as the value of t. It can easily be seen that the value of t is always positive as it should be. Thus the examination of the duration of convergence reduces to the study of λ_1 and λ_2 of the characteristic roots of the projection matrix. λ_1 also gives the intrinsic growth rate.

For the convenience of interpretation, it is better to transform the s into r's by the transformation $\lambda = e^r$. Then r_1 corresponding to the real positive root λ_1 is the intrinsic rate of natural increase of the stable population and $r_2 = y + i\,z$ (or $r_2 = y - i\,z$), where y and z are real quantities, corresponding to the complex root λ_2 (or λ_3) determines the rate of dampening of the waves of the oscillatory terms and their wavelength which is given by $(2\pi/z)^9$. In terms of r_1 and r_2, equation (3.1) becomes:

$$t = \left[(\log_e \mathcal{E}) / (y - r_1) \right] \qquad (E.11)$$

in which y will be negative in practice, or if positive it must be less than r_1, because the dampening will not occur otherwise. Again, it will have to be greater than r_1 whenever r_1 is negative because λ_1 will not be greater than the absolute value of λ_2 otherwise. Hence t is always positive as it should be. It may also be inferred that as $(y - r_1)$ increases in magnitude the value of t decreases and vice versa.

TWO-SEX MODEL WITH NO MIGRATION

Using the subscripts m and f to distinguish the male and the female populations and assuming $f_f(x)$ to be the age-specific birth rate for females aged x years, taking births of both sexes and s to be the male proportion at birth, we can write the growth equations as follows:

$$P_f(0,t) = (1 - s)\ S_f(b) \sum_{\beta}^{\alpha} \frac{1}{2} \left[P_f(x,t-1) + S_f(x-1)\ P_f(x-1,t-1) \right.\ \ f_f(x)$$

$$P_f(x,t) = S_f(x-1)\ P_f(x-1,t-1), \quad \text{for } x = 1, 2, \ldots., w$$

$$P_m(0,t) = s\ S_m(b) \sum_{\beta}^{\alpha} \frac{1}{2} \left[P_f(x,t-1) + S_f(x-1)\ P_f(x-1,t-1) \right]\ \ f_f(x)$$

$$\text{and} \qquad P_m(x,t) = S_m(x-1)\ P_m(x-1,t-1), \text{ for } x = 1, 2, \ldots., w. \qquad \text{(E.12)}$$

If our population vector has, now, females by age as the first (w+1) elements and the males by age as the next (w+1) elements, the growth equations could be written as:

$$(P_t) = L'\ (P_{t-1}) \qquad \text{(E.13)}$$

where the matrix L' is

$$\begin{vmatrix}
0 & 0 & \cdot & 0 & m_f(\alpha) & \cdot & m_f(\beta) & 0 & \cdot & 0 & 0 & 0 & 0 & \cdot & 0 & 0 \\
S_f(0) & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & 0 & 0 & \cdot & 0 & 0 \\
0 & S_f(1) & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & 0 & 0 & \cdot & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & S_f(w-1) & 0 & 0 & 0 & \cdot & 0 & 0 \\
0 & 0 & \cdot & 0 & m_f'(\alpha) & \cdot & m_f'(\beta) & 0 & \cdot & 0 & 0 & 0 & 0 & \cdot & 0 & 0 \\
0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & S_m(0) & 0 & \cdot & 0 & 0 \\
0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & 0 & S_m(1) & \cdot & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & 0 & 0 & \cdot & S_m(w-1) & 0
\end{vmatrix}$$

in which $m_f(x) = \frac{1-s}{2} \left[f_f(x) + S_f(x)\ \ f_f(x+1) \right] S_f(b)$

and $\qquad m_f'(x) = \frac{s}{2} \left[f_f(x) + S_f(x)\ \ f_f(x+1) \right] \qquad S_m(b).$

Thus, in this case, the intrinsic growth rate and the duration of the convergence are determined by the characteristic roots of the matrix L'. We can partition this matrix into four parts by dividing at the (w+1)th row and at the (w+1)th column as shown above. Hence, at any time, the changes in the male population will not have any effect on the female population at any age. Further, it can easily be shown that in (w+1) years, the male populations at all ages and the female populations at ages beyond β years, become linear functions of the female populations at ages below β years. Thus the intrinsic growth rate and the duration of convergence, in this case, would be the same as for the female population at ages below β years provided that the duration of convergence is greater than (w+1) years. Hence, they depend on the characteristic roots of the matrix M, and could be calculated from the matrix roots.

APPENDIX F

PROGRAM LISTING: POWERS OF MATRIX M.

```
1:55 BAJOB   BATCON VERSION 13(1071)-2 RUNNING SP02 SEQUENCE 10497 IN STREAM 1 FOR KHI
1:55 BAFIL   INPUT FROM DSKB1:SP02.CTL[317,545]
1:55 BAFIL   OUTPUT TO  DSKB1:SP02.LOG[317,545]
1:55 BASUM   JOB PARAMETERS
             TIME:00:05:30   UNIQUE:YES      RESTART:NO

1:55 MONTR
1:55 MONTR    LOGIN 317/545
2:18 USER    JOB 9 COOMBS DEC10 603 VER 6 TTY46
2:34 USER    1402    29-JUN-78         THUR
2:41 MONTR
2:41 MONTR   ..TYPE YKX.FOR
2:53 USER            DOUBLE PRECISION X,R1
2:53 USER            INTEGER T,OUT
2:53 USER            INTEGER YK
2:53 USER            DIMENSION A(10,10),D(10),SCRAT(10,10)
2:53 USER            DIMENSION C(10,10)
2:53 USER            DIMENSION A64(10,10),F(10,10),F64(10,10),F65(10,10),F66(10,10)
2:53 USER    C
2:53 USER            COMMON /IO/ OUT,INP
2:53 USER    C
2:53 USER            DATA OUT,INP/6,10/
2:53 USER    C
2:53 USER            T=6
2:53 USER            READ(INP,99)((C(I,J),J=4,10),I=1,9)
2:53 USER    99      FORMAT(7F7.6)
2:53 USER    C
2:53 USER            READ(INP,222)(D(J),J=1,10)
2:53 USER    222     FORMAT(10F6.5)
2:53 USER            DO 333 YK=1,9
2:53 USER            DO 7766 I=1,10
2:53 USER            DO 7766 J=1,10
2:53 USER    7766    A(I,J)=0.0
2:53 USER            DO 339 J=1,9
2:53 USER    339     A(1,J)=((49025./20000.)*(C(YK,J)+(C(YK,J+1)*D(J))))
2:53 USER            A(1,10)=(49025./20000.)*(C(YK,10))
2:53 USER    C ** COMPLETE ARRAY A
2:53 USER    C
2:53 USER            DO 100 I=1,9
2:53 USER    100     A(I+1,I)=D(I)
2:53 USER    C
2:53 USER    C ** COMPUTE A**64
2:53 USER    C
2:53 USER            DO 110 I=1,10
2:53 USER            DO 110 J=1,10
2:53 USER    110     A64(I,J)=A(I,J)
2:53 USER    C
2:53 USER            DO 150 N=1,T
2:53 USER              CALL MULT( A64,A64,SCRAT,10 )
2:53 USER              DO 140 I=1,10
2:53 USER              DO 140 J=1,10
2:53 USER    140       A64(I,J)=SCRAT(I,J)
2:53 USER    150     CONTINUE
2:53 USER    C
2:53 USER    C ** NOW CALC E**65 SO WE HAVE ELEMENT (2,1)
2:53 USER    C
2:53 USER            CALL MULT( A64,A,SCRAT,10 )
2:53 USER    C
2:53 USER            R1=SCRAT(2,1)/A64(2,1)
```

```
53 USER    C
53 USER    C ** DISPLAY A,MAXN,A64,A65,R1
53 USER    C
53 USER          CALL DISPLA( 'A',1,A )
53 USER          WRITE(OUT,50) T
53 USER    50    FORMAT(' VALUE OF T IS ',I2)
53 USER          CALL DISPLA( 'A64',3,A64 )
53 USER          CALL DISPLA( 'A65',3,SCRAT )
53 USER          WRITE(OUT,52) R1
53 USER    52    FORMAT(' VALUE OF R1 IS',F12.8)
53 USER    C
53 USER    C ** CALC A64/R1**64
53 USER    C
53 USER          X=1.0/R1**64
:58 USER         DO 180 I=1,10
:58 USER         DO 180 J=1,10
:58 USER   180   SCRAT(I,J)=A64(I,J)*X
:58 USER   C
:58 USER   C ** NOW GET A-R1*A64/R1**64
:58 USER   C
:58 USER         DO 200 I=1,10
:58 USER         DO 200 J=1,10
:58 USER         F(I,J)=A(I,J)-SCRAT(I,J)*R1
:58 USER   200   F64(I,J)=F(I,J)
:58 USER   C
:58 USER   C ** DISPLAY THE ARRAYS
:58 USER   C
:58 USER         CALL DISPLA( 'A64/R1**64',10,SCRAT )
:58 USER         CALL DISPLA( 'F=A-R1*A64/R1**64',17,F )
:58 USER   C
:58 USER   C ** NEXT COMPUTE F**64
:58 USER   C
:58 USER   C
:58 USER         DO 220 N=1,T
:58 USER           CALL MULT( F,F,SCRAT,10 )
:58 USER           DO 210 I=1,10
:58 USER           DO 210 J=1,10
:58 USER   210     F(I,J)=SCRAT(I,J)
:58 USER   220   CONTINUE
:58 USER   C
:58 USER         CALL MULT( F,F64,F65,10 )
:58 USER         CALL MULT(F64,F65,F66,10 )
:58 USER   C
:58 USER   C ** DISPLAY FINAL
:58 USER   C
:58 USER         CALL DISPLA( 'F64',3,F )
:58 USER         CALL DISPLA( 'F65',3,F65 )
:58 USER         CALL DISPLA( 'F66',3,F66 )
:58 USER   333   CONTINUE
:58 USER   C
:58 USER         END
:58 USER

:58 USER         SUBROUTINE MULT( A,B,C,N )
:58 USER   C
:58 USER   C DO MATRIX MULT C=B*A
:58 USER   C
:58 USER         DIMENSION A(N,N),B(N,N),C(N,N)
:58 USER   C
:58 USER         DO 120 I=1,N
:58 USER         DO 120 J=1,N
:58 USER           S=0.0
:58 USER           DO 110 K=1,N
:58 USER   110     S=S+A(I,K)*B(K,J)
:58 USER   120   C(I,J)=S
:58 USER         RETURN
:58 USER         END
:58 USER

:58 USER         SUBROUTINE DISPLA( NAME,NCH,ARR )
:58 USER   C
:58 USER   C ** TO TYPE OUT ARRAY CONTENTS
:58 USER   C
:58 USER         DIMENSION ARR(10,10)
:58 USER         DIMENSION FMT(2)
:58 USER         INTEGER OUT
:58 USER         COMMON /IO/ OUT,INP
:58 USER   C
:58 USER         DATA FMT(1)/ '(/,1X' /
:58 USER   C
:58 USER         N=(NCH+4)/5
:58 USER         ENCODE(5,10,FMT(2)) N
:58 USER   10    FORMAT(1H,,I1,3HA5) )
:58 USER         WRITE(OUT,FMT) NAME
:58 USER         WRITE(OUT,50) ((ARR(I,J),J=1,10),I=1,10)
:58 USER   50    FORMAT(10(1X,E12.6))
:58 USER         END
:58 MONITR
```