Essays on Non-Gaussian Time Series Analysis

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For my family and friends

Declaration

I hereby declare that the work in this thesis is my own work except where otherwise stated. Part of this thesis is based on joint research with my supervisor Dr Kin-Yip Ho.

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Abstract

This thesis is a compilation of essays on the extension of financial econometric techniques to various fields of financial and non-financial risk management—namely, longevity risk, disaster risk and food security risk.

First, longevity risk is quantified by proposing a mortality forecasting methodology based on a modified survival function and nonparametric residual-based bootstrapping. The parameters of the survival function are estimated through time and are modelled with a time series model structure. The estimated model is used to generate forecasts of parameter values and life expectancy. Confidence intervals are generated by residual-based bootstrapping through an autoregressive sieve based on the estimated model. The methodology is applied to life tables of male and female subjects from the United States, Australia and Japan, and compared with the Lee–Carter model in terms of forecasting life expectancy. From the results for the three countries, the proposed survival function has better long-term forecasting performance than does the Lee–Carter model.

Second, a proposed methodology for estimating disaster risk is devised using bootstrapped multivariate extreme value theory methods. A disaster risk measure called storm-at-risk is created. The risk measure can be estimated through semiparametric and nonparametric approaches and is applied to weather extremes data generated by typhoons that enter the western North Pacific basin. Robustness checks on the performance of the approaches are conducted. The semiparametric approach performs better than the nonparametric approach in longer periods, but not in smaller periods.

Third, food security risk is quantified by proposing risk measures for hierarchical agricultural time series data, which are generated for national and sub-national levels. The risk measures are created by a combination of forecast reconciliation methods for hierarchical time series data and residual-based bootstrapping methods. The methodology is applied to Philippine rice production time series data that are collected from the regions and are aggregated to the macro-regional and national levels.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Risk is present in every human activity. Our interaction with the environment and society exposes us to dangers. These events may have miniscule probabilities of occurrence, yet their impacts can be large, widespread and persistent. Therefore, it is vital that we gain understanding and insights on these risks so that human activities can be managed and the impacts of these dangers can be reduced if not eliminated.

Statistical methods pave the way to developing an understanding of risk. However, research on the estimation and accounting of risk in various fields differs in terms of methodology. Risk in finance has been pursued in many works of literature (Artzner et al. 1999; Jorion 2006; McNeil, Frey & Embrechts 2005; Tsay 2005), with stylised facts over the nature of the problem cemented in the field (Tsay 2005). Conversely, there have been open questions in pursuing risk estimation and accounting in the fields of demography, particularly in accounting for longevity risk (Crawford, de Haan & Runchey 2008); disasters from natural hazards (World Meteorological Organization 1999), particularly typhoons (Okazaki, Watabe & Ishihara 2005; Yonson, Gaillard & Noy 2016); and food security management (Jones et al. 2013; Scaramozzino 2006).

People in the Organisation for Economic Co-operation and Development (OECD) countries (OECD 2011) and in East Asia (National Institute of Ageing 2011) are living longer because of improvements in health care and in access to such services. However, this exposes people to longevity risk, defined as the exposure to dangers related to increased longevity. Although it may seem positive at first glance, living longer is associated with practical financial considerations. For individuals, this means the risk of a person running out of retirement benefits and savings or outliving family or other informal sources of support (Stone & Légaré 2012). The risk manifests for pension fund managers as depleted cash reserves because they are ill prepared for the increasing number of beneficiaries and for being in contract for longer periods (Modu 2009). Government agencies that provide benefits for their elderly citizens, such as medical care, pensions and tax breaks, are also at risk with longevity as underestimation of the costs leads to budget deficits (Stone & Légaré 2012).

In these situations, longevity risks are realised by inaccurate estimates of life expectancy and rates of mortality (Crawford, de Haan, and Runchey 2008).

To properly account for or reduce the impact of longevity risk, there is an open field of research in mortality modelling. This can be traced back to the Gompertz–Makeham parametric mortality model (Gompertz 1825; Makeham 1860), which had reasonably good fit in modelling adult mortality. Flexible and dynamic models of mortality for the purpose of forecasting life expectancy have been proposed. An example of these models is the model by McNown and Rogers (1989), which describes a parametric model for age-specific survival probability and generates forecasts using time series analysis on the parameters. Another is by Lee and Carter (1992), which proposes a decomposition of the central mortality rate into age-specifc and time-specific components; in addition, forecasts of mortality rates are generated by time series analysis methods on the time-specific component. Forecasts for life expectancy in the United States and G7 countries have been generated using the Lee–Carter (LC) model (Tuljapurkar, Li & Boe 2000). More recently, Wong and Tsui (2015) proposed the CH survival function, which decomposes the survival probability into two components, the youth-to-adulthood component and the old-to-oldest component, and fared well in fitting the US population, compared with the LC model and the Bongaarts (2005) shifting logistic model.

In the first essay of the thesis, we propose a modified CH (MCH) survival function in which probabilities are decomposed to the same youth-to-adulthood and old-to-oldest components, but the number of parameters is reduced from six to five. These parameters are estimated using the nonlinear least squares method. The parameters are then modelled using time series analysis, such as the univariate and vector autoregressive models, and interval forecasts are generated through residual-based bootstrapping. We show in the essay that the MCH function performs well in long-term forecasts over the LC model for the US, Australian and Japanese populations.

Climate change has brought more intense natural hazards such as floods, cyclones, heat waves and droughts with increasing frequency. In particular, much of temperate-climate Asia, which covers China, the Korean peninsula, Taiwan and Japan, will experience an increase in weather hazards, and tropical Asia, which covers much of South and Southeast Asia, is exposed to risks of more intense cyclones, which can cause displacement of populations in low-lying areas (Mirza 2003). Thus, Southeast Asias growing economies have a higher likelihood of suffering from the effects of climate change, compared with the rest of the world, if no policies to mitigate the risk are established (Asian Development Bank 2009). Laos, Malaysia, Myanmar, the Philippines, Thailand and Vietnam are countries in the region that have cyclonic storms as their dominant disaster risk (ASEAN Disaster Risk Management Initiative 2010), and in Japan, typhoons that occurred from 1970 to 2004 caused the highest insurance losses in the countrys record (Okazaki, Watabe & Ishihara 2005).

The intensities of typhoon characteristics were revealed as important determinants in the impacts of cyclone disasters in the Philippines (Yonson, Gaillard & Noy 2016). Therefore, weather and climate extremes are highlighted as major areas of concern, and estimating and predicting weather extremes has been selected as one of the World Climate Research Programme Grand Challenges (Sillmann et al. 2017), emphasising the importance of the problem for academics. However, even before the posing of the challenge, modelling of extreme weather events has been conducted. There has been extensive research on modelling extreme or maximum wind speed of hurricanes in at-risk regions of the United States. Walshaw (2000) modelled extreme wind speeds in Boston and in the Key West area by using the generalised extreme value (GEV) distribution (Fisher & Tippet 1928; Gnedenko 1943) with a Bayesian approach, and concluded that standard models give misleading results in the regions of their scope. In the Gulf Coast, Florida, and the East Coast regions of the United States, Jagger and Elsner (2006) devised climatology models on extreme hurricane winds by using the generalised Pareto distribution (Balkema & de Haan 1974; Pickands 1975) with both maximum likelihood and Bayesian approaches, and generated estimates on maximum wind speed levels for certain return periods. Wind hazard conditions on the Chinese coast were modelled using the polynomial family of probability distributions—namely, the uniform, trapezoidal and quadratic distributions—to generate return values for 50-year and 100-year periods (Li & Hong 2015). Annual maximum wind conditions in the western North Pacific region were estimated using the Gumbel distribution, a special case of the GEV distribution, and there was fair agreement between the estimated and observed 48-year wind maxima for the extreme winds data from the Philippines (Ott 2006). Estimations of extreme wind and pressure events for storms were conducted by Economou, Stephenson and Ferro (2014) for the northern Atlantic basin by using point process extreme value models with the North Atlantic Oscillation index (NAO) as a significant covariate in the model, and a negative relationship was revealed between minimum pressure levels and the NAO.

The second essay of the thesis proposes an estimation procedure for the threshold estimation of extreme wind and pressure characteristics, similar in concept to the value-at-risk (Jorion 2006), which we call the storm-at-risk. The proposed risk measure is estimated using the multivariate extreme value distribution (Pickands 1981) with bootstrapped confidence interval forecasts. We demonstrate the methodology on typhoon data for the western North Pacific basin, which includes the high-at-risk regions of East and Southeast Asia. For lower coverage probabilities and lower return periods, the nonparametric storm-at-risk curves provide more robust results within desired coverage probabilities. For higher coverage probabilities, the semiparametric approach provides more robust results within the desired coverage.

Food security, as defined by the World Food Summit (1996, p. 4, par. 1, clause 2), 'exists when all people, at all times, have physical and economic access to sufficient, safe and nutritious food that meets their dietary needs and food preferences for an active and healthy life'. The definition underlines four key dimensions of food security: availability; access; utilisation; and stability (Food and Agricultural Organization 2016). The key contributing factors to food security in developing countries are agricultural productivity, foreign exchange earnings and population growth. By increasing production, increasing food imports and reducing population growth, developing countries can achieve food security (Shapouri & Rosen 1999). If left unchecked, food insecurities and emergencies threaten long-term development in developing countries and increase the risk of communities of these countries to future disasters (Haile & Bydekerke 2012). By developing risk assessment and analysis systems, risk mapping, and monitoring and early warning systems, governments in developing countries can manage and reduce the impact of food insecurity (Asian Development Bank 2013; Haile & Bydekerke 2012).

Measuring food security risk is an open research question. A compendium of food security metrics has been provided by Jones et al. (2013). They described the metrics by their purposes—namely, (1) to provide national estimates of food supply, (2) to inform global monitoring and early warning systems, (3) to assess household food access and acquisition and (4) to measure food consumption and utilisation. Scaramozzino (2006) proposed a value-at-risk approach to measuring vulnerability to food security. The methodology introduced a financial risk management style to mitigating and addressing food security risk, considered of particular importance in the area of early warning systems (Haile & Bydekerke 2012).

With the ideas of using value-at-risk methodologies in food insecurity, providing a measurement to be used for early warning systems, and an estimation system that can provide national and sub-national estimates, we propose a food security risk estimation method called food-at-risk in the third essay. It adapts the forecast reconciliation methodology of Hyndman et al. (2011) and Hyndman, Lee and Wang (2016) to produce consistent and agreeable time series forecasts for national and sub-national estimates. The reconciliation method is bootstrapped to produce the desired risk measure for a defined probability of risk. We apply the estimation technique on Philippine regional rice production data.

1.1 Significance

The significance of the proposed methodologies is in their uses in providing information and understanding of the risks that each methodology addresses. In proposing the MCH function, adequately forecasted life expectancy can make pension fund institutions more solvent by having adequate cash reserves to continue servicing clients (Modu 2009). Robust life expectancy forecasts in the long term can also help government agencies that serve the elderly to manage their resources (Stone & Légaré 2012) and properly estimate cash inflows and outflows of the services. These concerns can be met by the proposed model because it possesses better long-term forecasting ability.

For disaster risks from extreme weather events, the understanding and prediction of these events has been posed as a global challenge to all researchers in the field (Sillmann et al. 2017). Proper estimation of extreme weather conditions is also vital for insurance companies that give financial support in cases of disasters due to such natural hazards (Okazaki, Watabe & Ishihara 2005). Estimates from the proposed methodology in the second essay can guide disaster risk managers and policymakers for appropriate plans of action in case of severe cyclonic storms.

Measuring food security risks is vital to facilitate growth in developing countries and combat extreme poverty (Asian Development Bank 2013; Food and Agricultural Organization 2016; Haile & Bydekerke 2012). Estimation of food security risk requires a multi-level approach that provides understanding on national and local levels. The food-at-risk information system proposed by the third essay addresses the concerns of food insecurity by examining the level of food supply at both national and sub-national levels.

1.2 Outline

The thesis is assembled as follows. Chapter 1 covers the introduction to the theme of the thesis and a discussion of the significance of the proposed methods. Longevity risk is addressed by the proposed survival model called the MCH function in the first essay presented in the second chapter. The second essay discusses the storm-at-risk curves methodology and is presented in the third chapter. A discussion of the food-at-risk methodology is presented in the fourth chapter of the thesis. Finally, the fifth chapter contains the conclusion and summary of the thesis with a discussion on future work that will be pursued.

Chapter 2

Longevity Risk: Forecasting Life Expectancy

2.1 Introduction

Improvements in healthcare services and increased access to these services over the past 50 years have resulted in populations living longer than previously anticipated. The United Nations Department of Economic and Social Affairs, Population Division (2012) reported that a growing number of populations around the world have had positive trends in their life expectancies at birth, from an average of approximately 48 years of age in 1950–1955 to 68 years in 2005–2010. For member countries in the OECD, life expectancy at birth in 2008 was 79.3 years, with an average difference of life expectancy over all OECD countries of 6 years from 1983 to 2008 (OECD 2011). Such trends reflect the general improvement in the quality of life in the world with lower mortality rates.

With growing elderly populations, an increase in costs and demand for aged healthcare, retirement plans, and pensions provided by financial institutions is observed or can be anticipated, along with increased utilization of government healthcare services for senior citizens. A rough estimate on a five-year improvement on longevity for retirement-age individuals in the United States would mean an increase on the present value of benefits by 10% to 15%. Insurance portfolios with many old individuals and few active employees that contribute to revenues are heavily affected by changes in mortality (Gutterman, et al 2002). Demand for full-time physicians in the United States is expected to rise from 778,200 physicians in 2013 to between 865,000 to 911,400 physicians in 2025 that should be fulfilled by the government and private sectors (IHS Inc. 2015). An increase of 50% in the proportion of Americans that will receive healthcare through the Medicare program between 2000 and 2050 is expected. The cost of Medicare expected to increase from 2.2% of GDP in 2000 to 6% of GDP in 2050. Total cost of all available services for the elderly, which includes Medicare, Social Security, and Medicaid long-term care is expected to increase from 6.8% of GDP in 2000 to 13.2% of GDP by 2050 (Wiener and Tilly 2002). Demand for caregivers in the

United States may increase from 5% of the non-elderly adult population in 2010 to a low scenario of 7% or a high scenario of 11% in 2050. (Congressional Budget Office 2013). As these examples demonstrate, it is imperative for these institutions to assess and prepare for the risks associated with increased cash outflows due to the growing elderly population.

One type of risk associated with the growing elderly population is known as longevity risk, which arises because of unexpectedly high life expectancies leading to higher payout and other cost ratios for private financial institutions (Modu 2009) and government agencies (Stone & Légaré 2012). An example of impending longevity risk to governments is the report of the Board of Trustees, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds (2015) of the United States, which states that the balances among their income rates, excluding interest income and cost rates, have been in the negative range since 2009, and may continue to be in the negative up to the year 2090, in the three scenarios of low-, intermediate- and high-cost situations. The negative balance is due to the baby-boom generation moving to the retirement and senior cohort in 2016–2035 and the declining death rates forecast in 2050–2089. To address such risk, institutions initially prepare models to forecast future mortality tables and to design policies or risk transfer products. These models enable them to not only create improved products and services in response to growing demand from the elderly population but also reduce their exposure through the process of reinsurance. Therefore, the development of accurate mortality forecasting models is vital for these institutions.

Given these concerns, current mortality models have shortcomings in terms of forecasting accuracy. Before 1992, mortality forecasting was constructed through structural econometric modelling of socioeconomic and demographic factors (e.g., Land 1986; Olshansky 1988) or deterministic cohort-component forecasts (e.g., Alho 1990; Guralnik, Yanagishita & Schneider 1988). However, these methodologies produced unsatisfactory forecasts for longevity (Giacometti et al. 2012) until the methodology of Lee and Carter (1992), using a statistical time series forecasting approach, was introduced into a demographical model of mortality. This model, however, had flaws documented in the literature. Its strict assumptions on age-specific components have been empirically disproven, and the model was extended for this criterion (Li, Lee & Gerland 2013). The model has performed very well in forecasting life expectancies for lower and middle starting ages but has been found poor for older ages, where longevity risk is more imminent (Wong & Tsui 2015). Leng and Peng (2016) concluded that the LC model and its extensions in the literature cannot describe the true dynamics of the mortality index, which may lead to questionable forecasts and projections on mortality. With these considerations, we propose a model that addresses these issues with improved forecasting ability.

By synthesising the existing statistical methodologies of forecasting mortality, we show that our proposed procedure can significantly enhance the power of forecasting future mortality rates. The first step is the estimation of a survival function from mortality tables within each year. The survival function, the MCH function, is based on a simplified form of the Wong and Tsui (2015) CH function, which considers two components of survivability: young-to-old and old-to-oldest components. Changing trends in the oldest cohort, which are different from those in the younger cohorts, is the consideration of the CH function. The MCH function has a reduced number of parameters and pragmatic parameter constraints, which improves longevity estimates and the interpretability of the function parameters. A mix of univariate and multivariate time series analysis through autoregressive models is performed to generate longevity forecasts. Autoregressive models take into account the autocorrelation of each MCH parameter, and the vector form of the model the cross-correlation between parameters within each MCH component, which improves forecasting ability by reducing estimation errors. To augment longevity forecasts, residual-based multivariate bootstrapping is used in generating confidence intervals. To demonstrate the methodology, the US, Australian and Japanese male and female life tables from 1950 to 2010 were used. Results on confidence intervals and forecasted life expectancies are shown. Robustness checks through cross-validation forecast error statistics and the Diebold–Mariano (DM) test (Diebold & Mariano 1995) for forecasting comparisons with the LC model are shown below. We have concluded that the proposed procedure performs favourably over the LC model in terms of out-of-sample forecasting.

The remainder of the chapter is constructed as follows. The second part describes the mathematical background of the LC and Wong–Tsui models. Our proposed methodology is discussed in detail in the third part. A discussion on the steps of demonstrating the method with the US, Australian and Japanese annual life tables is outlined in the fourth part, with discussion of the results in the fifth part. We provide our conclusions on the methodology in the sixth part of the chapter.

2.2 Background Literature

Lee and Carter (1992) proposed a methodology for modelling mortality rates and forecasting life expectancy by using the following decomposition model; denoting $\mathbf{m}_{x,t}$ as the matrix of central mortality rates, it is assumed that:

$$\log\left(\mathbf{m}_{x,t}\right) = \mathbf{a}_x + \mathbf{b}_x\mathbf{k}_t + \epsilon_{x,t} \tag{2.1}$$

where $\mathbf{a}_x$ is the main component of age in mortality, $\mathbf{b}_x$ is the interaction factor of age to and independent of time, and $\mathbf{k}_t$ is defined as the mortality index of time. They are estimated by singular value decomposition, and $\mathbf{k}_t$ is reestimated to conform with the relationship:

$$D(t) = \sum \left[N(x,t)\exp\{\mathbf{a}_x + \mathbf{b}_x\mathbf{k}_t\}\right] \tag{2.2}$$

where $D(t)$ is the observed number of deaths in time t, and $N(x,t)$ is the population distribution for age x in a given time t. To make forecasts, it is assumed that $\mathbf{a}_x$ and $\mathbf{b}_x$ will not change in time and $\mathbf{k}_t$ is fitted with an econometric model. According to Lee and Carter (1992), the model is estimated with a random walk with drift and an exogenous variable, flu, which accounts for the 1918 influenza outbreak:

$$\mathbf{k}_t = \mathbf{k}_{t-1} - 0.365 + 5.24\text{flu} + e_t \tag{2.3}$$

From the econometric model of $\mathbf{k}_t$ and the structural model of $\log(\mathbf{m}_{x,t})$, forecasted life tables and life expectancies are generated.

However, the original LC model is too restrictive in its assumptions, such as $\mathbf{b}_x$ being constant in time (Li, Lee & Gerland 2013), and is not suitable for inference (Leng & Peng 2016); thus, it has not performed well in forecasting for older ages over time (Wong & Tsui 2015), which warrants alternative models for forecasting longevity.

Wong and Tsui (2015) proposed a methodology for forecasting life expectancy by using a new survival function and combining the function with autoregressive models to facilitate forecasting. The CH survival function $S_{CH}(x)$ of Wong and Tsui (2015) is specified as follows:

$$S_{CH}(x) = \alpha_1 \exp\{-\exp\{(x/\beta_1)^{\gamma_1}\}\} + \alpha_2 \exp\{-\cosh\{(x/\beta_2)^{\gamma_2}\}\} \tag{2.4}$$

where α_1 and α_2 act as weights, β_1 and β_2 act as scaling parameters and give information on typical ages that distinguish each component, and γ_1 and γ_2 describe the shape of the two components on how fast the survival probabilities descend to zero as the age x increases. The parameter ranges are as follows: $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2 > 0$. The first addend corresponds to the "youth-to-adulthood" component whilst the second is the "old-to-oldest-old" component.

In the methodology by Wong and Tsui (2015) for each life table in year $t = 1, 2, \ldots, T$, the parameters are estimated by nonlinear least squares, creating the parameter series $\{\alpha_{1,t}\}_{t=1}^T$, $\{\beta_{1,t}\}_{t=1}^T$, $\{\gamma_{1,t}\}_{t=1}^T$, $\{\alpha_{2,t}\}_{t=1}^T$, $\{\beta_{2,t}\}_{t=1}^T$, and $\{\gamma_{2,t}\}_{t=1}^T$. Each parameter series is modeled individually by rate of change differencing and univariate autoregressive models (Box, Jenkins & Reinsel 1994), such as the AR(1) model as shown below:

$$\frac{\Delta y_t}{y_t} = \mu + \phi_1 \left(\frac{\Delta y_{t-1}}{y_{t-1}} - \mu \right) + \epsilon_t, \quad \epsilon_t \sim \left(0, \sigma^2\right) \tag{2.5}$$

From the estimtes of the model above, forecasts on parameter values are generated and life expectancy estimates $\tilde{e}_x$ at age x are generated by actuarial methods, in which for any survival function $S(x)$:

$$\tilde{e}_x = \sum_{k=0}^{\infty} {}_k p_x + \frac{1}{2}, \quad {}_k p_x = \frac{S(x+k)}{S(x)} \tag{2.6}$$

Based on the results from in-sample and out-of-sample forecasts, the Wong and Tsui (2015) methodology outperforms that of Lee and Carter (1992).

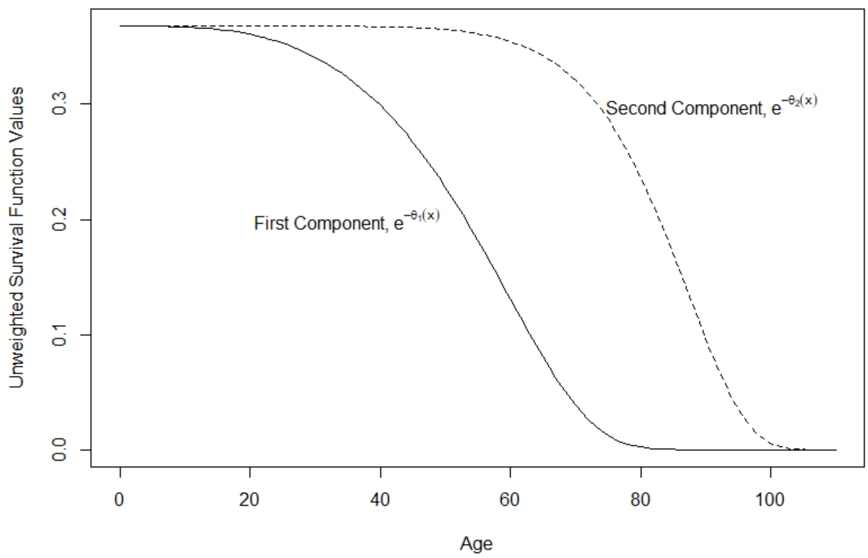
2.3 Proposed Methodology

We notice certain parameter relationships and structures in the Wong and Tsui (2015) model whereby the model can be further simplified to enhance the estimation process. They are:

$$\alpha_1 + \alpha_2 = e, \alpha_1 \leq \alpha_2, \beta_1 \leq \beta_2, \gamma_1 \leq \gamma_2 \tag{2.7}$$

When evaluated at zero, $S_{CH}(0) = (\alpha_1 + \alpha_2)e^{-1}$. To force the value to 1, $\alpha_1 e^{-1} = \alpha$ and $\alpha_2 e^{-1} = 1 - \alpha$, thus making equation (2.8) below. In the paper of Wong and Tsui (2015), $\alpha_1 \leq \alpha_2$ for any year, emphasising that survivability is domimated by the old-to-oldest-old component over the youth-to-adulthood component. For the new parameter, $\alpha \leq 1 - \alpha$, thus $\alpha \leq 0.5$. The relationships for $\beta_1, \beta_2, \gamma_1$ and γ_2 are based on the features of the components in the original CH function results. The relationship is shown in Figure 2.1.

Figure 2.1: Unweighted CH Components Using Wong and Tsui's (2015) Results for US Females for the Year 2000



The parameter β_1 is related to the centre of the first component, and β_2 the second. As the first is linked to youth-to-adulthood, compared with old-to-oldest-old for the second, the centre of the first is less than or equal to the centre of the second; thus, $\beta_1 \leq \beta_2$. The parameters γ_1 and γ_2 are polynomial powers with respect to the speed of reaching zero survival probability. With links to age periods of an individuals life, the speed of the first is less than or equal to the speed of the second; thus, $\gamma_1 \leq \gamma_2$.

We propose the MCH survival function, which has a reduced number of parameters and maintains appropriate properties as a survival function:

$$S_{MCH}(x) = \alpha \exp\{-\exp\{(x/\beta_1)^{\gamma_1}\} + 1\} + (1-\alpha)\exp\{-\cosh\{(x/\beta_2)^{\gamma_2}\} + 1\}. \tag{2.8}$$

The MCH survival function has the following parameter space:

$$\alpha \leq 0.50, 0 < \beta_1 \leq \beta_2, 0 < \gamma_1 \leq \gamma_2 \tag{2.9}$$

To create life expectancy forecasts, let $l_x(t)$ be the number of lives age x at year $t = 1, 2, \ldots, T$. The parameters $\{\alpha_t\}_{t=1}^T, \{\beta_{1,t}\}_{t=1}^T$, $\{\gamma_{1,t}\}_{t=1}^T, \{\beta_{2,t}\}_{t=1}^T, \{\gamma_{2,t}\}_{t=1}^T$ are estimated by minimising the sum of square errors (SSE) for all years t, where x_{max} is the maximum age:

$$SSE(t) = \sum_{x=0}^{x_{max}} \left[l_x(t) - 100,000 S_{MCH,t}(x)\right]^2 \tag{2.10}$$

A p_α-order autoregressive [AR(p_α)] model is fitted to the adjusted parameter series $\{f_1(\alpha_t)\}_{t=1}^T$, while a p_i-order vector AR [VAR(p_i)] model (Lütkepohl 2006; Sims 1980) is fitted on the adjusted parameter vector series $\{\theta_{\mathbf{t}}^{(\mathbf{1})}\}_{t=1}^T = \{[f_2(\beta_{1,t}), f_3(\gamma_{1,t})]'\}_{t=1}^T$ and $\{\theta_{\mathbf{t}}^{(\mathbf{2})}\}_{t=1}^T = \{[f_4(\beta_{2,t}), f_5(\gamma_{2,t})]'\}_{t=1}^T$:

$$\theta_{\mathbf{t}}^{(\mathbf{i})} = \sum_{j=1}^{p_i} \mathbf{\Phi}_{\mathbf{j}}^{(\mathbf{i})} \theta_{\mathbf{t-j}}^{(\mathbf{i})} + \epsilon_{\mathbf{t}}^{(\mathbf{i})}; \quad \epsilon_{\mathbf{t}}^{(\mathbf{i})} \sim \left(\mathbf{0}, \mathbf{\Sigma}^{(\mathbf{i})}\right), i = 1, 2 \tag{2.11}$$

A life table is generated on the basis of $S_{MCH}(x)$ by using the in-sample predictions $\{\mu_{\mathbf{t}}\}_{t=1}^T = \{\mu_{\mathbf{t}}^\alpha, \mu_{\mathbf{t}}^{(\mathbf{1})'}, \mu_{\mathbf{t}}^{(\mathbf{2})'}\}_{t=1}^T$ and k-periods forecasts $\{\mu_{\mathbf{t}}\}_{t=T+1}^{T+k}$ of the AR(p_α) and VAR(p_i) models for the parameters with their formulas shown below:

$$\mu_t^\alpha = \sum_{i=1}^{p_\alpha} \hat{\mathbf{\Phi}}_i^\alpha f_1(\alpha_{t-i}); \mu_t^{(i)} = \sum_{j=1}^{p_i} \hat{\mathbf{\Phi}}_j^{(i)} \theta_{t-j}^{(i)}, i = 1, 2 \tag{2.12}$$

The life expectancy $\{\tilde{e}_{x,t}\}_{t=1}^{T+k}$ for age x at year t is derived from the estimated life table by using equation (2.6).

The blocked AR(1)–VAR(1) modeling approach was chosen to make forecasts of MCH parameter values. One may use higher orders for forecasting, but because of the parameters estimation on each year would produce short time series data, e.g., for the real data application, there would be 60 annual periods of data, higher orders would risk convergence issues, especially on bootstrapping later.

Residual-based bootstrapping (Paparoditis & Streitberg 1991) can be used to generate confidence intervals to augment life expectancy forecasts. Here are the following steps for the procedure:

- From the AR(p_α) and VAR(p_i) models in equations (2.5), (2.11) and (2.12), the residual vectors $\{\hat{\epsilon}_{\mathbf{t}}\}_{t=1}^{T}$ where $\hat{\epsilon}_{\mathbf{t}} = \theta_{\mathbf{t}} - \mu_{\mathbf{t}}$ are generated.

- Letting n_B be the number of bootstrap samples to be generated, the bootstrapping is looped for $b = 1, 2, \ldots, n_B$

  1. a bootstrap sample of residual vectors $\{\mathbf{e}_{t,b}\}_{t=1}^{T}$ is drawn from $\{\hat{\epsilon}_{\mathbf{t}}\}_{t=1}^{T}$ and get $\tilde{\theta}_{t,b} = \mu_{\mathbf{t}} + \mathbf{e}_{t,b}$.
  2. AR(p_α) and VAR(p_i) models are fitted to $\{\tilde{\theta}_{t,b}\}_{t=1}^{T}$.
  3. A life table is generated based on $S_{MCH}(x)$ by using the in-sample predictions $\{\tilde{\mu}_{t,b}\}_{t=1}^{T}$ and k-periods forecasts $\{\tilde{\mu}_{t,b}\}_{t=T+1}^{T+k}$ ofAR(p_α) and VAR(p_i) models for the parameters:

$$\mu_{t,b}^{\alpha} = \sum_{i=1}^{p_\alpha} \hat{\mathbf{\Phi}}_{i,b}^{\alpha} f_1\left(\alpha_{t-i}\right); \mu_{t,b}^{(i)} = \sum_{j=1}^{p_i} \hat{\mathbf{\Phi}}_{j,b}^{(i)} \theta_{t-j}^{(i)}, i = 1, 2 \tag{2.13}$$

  4. The life expectancy $\tilde{e}_{x,t,b}$ is estimated from the generated life table using equation (2.6).

- The $(1-\alpha)\,100\%$ confidence interval for $e_{x,t}$ for age x at time t, with $\tilde{e}_{x,t,(k)}$ meaning the kth smallest value of $\tilde{e}_{x,t,b}$, is:

$$\left(\tilde{e}_{x,t,\left(\left[n_B \frac{\alpha}{2}\right]\right)}, \tilde{e}_{x,t,\left(\left[n_B \frac{2-\alpha}{2}\right]\right)}\right) \tag{2.14}$$

We adjust the parameters using the following functions:

$$f_1\left(r_t\right) = \log\left(\frac{r_t}{1-r_t}\right) - \log\left(\frac{r_{t-1}}{1-r_{t-1}}\right) \tag{2.15}$$

$$f_q\left(r_t\right) = \log\left(r_t\right) - \log\left(r_{t-1}\right), q = 2, 3, 4, 5 \tag{2.16}$$

where the first function is called the dlogit function while the second function is called the dlog function.

2.4 Methodology Demonstration

The MCH methodology is demonstrated on the US, Australian and Japanese annual life tables data for males and females in 1950–2010. The data were downloaded on 2 January 2016 from the Human Mortality Database. Life expectancy is evaluated at ages 0 (at birth), 20, 40, 65 and 80, and forecast for 2011–2050, with 95% confidence intervals. The predictive performance of the MCH model is compared with that of the LC model, as set up in the demography package (Hyndman et al. 2014) in R and performed with 10 years of hold-out data (2001–2010). The in-sample period for estimation and predictive performance is 1950–2005. The predictive performance statistics for comparisons are the mean absolute error (MAE) and the two-sided one-step-ahead DM test:

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |actual_i - predicted_i| \tag{2.17}$$

$$DM = \frac{\bar{d}}{\sqrt{\frac{\hat{\sigma}_d^2}{n}}} \sim N(0,1) \quad as \quad n \to \infty \tag{2.18}$$

$$d_i = \left(error^2_{i,model1}\right) - \left(error^2_{i,model2}\right), i = 1, 2, \ldots, n$$
$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i; \quad \hat{\sigma}_d^2 = \frac{1}{n}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2$$

2.5 Discussion of Results

2.5.1 US Life Tables

The following graphs show the descriptive statistics and table of results of the demonstrations on US data. From Figure 2.2, the graph of life expectancy values for US males in 1950–2010, there is an upward trend in life expectancy over all ages. The trend, however, becomes flatter as we solve for the life expectancy from birth up to age 80.

Figure 2.2: Life Expectancy of US Males, by Age, 1950–2010

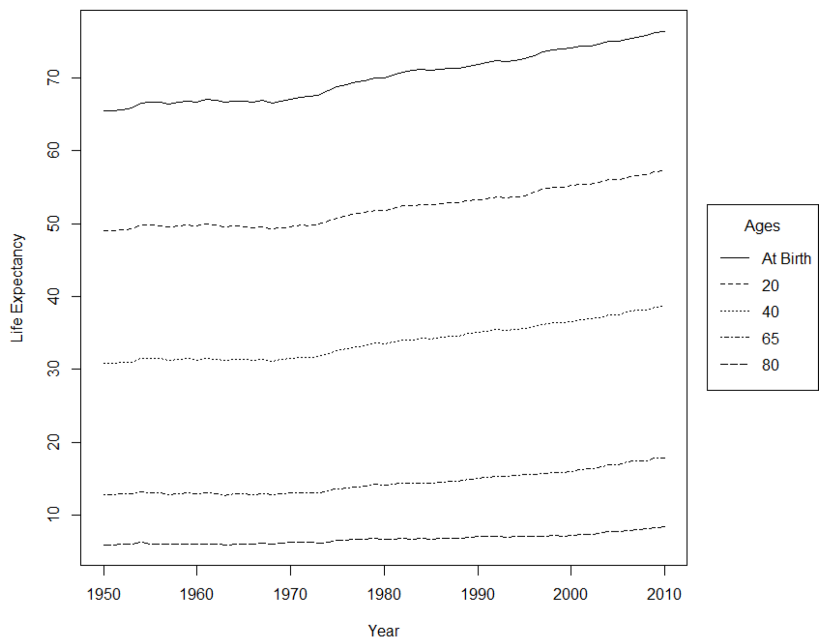


Figure 2.3: Parameter Estimates of the MCH Function for US Males

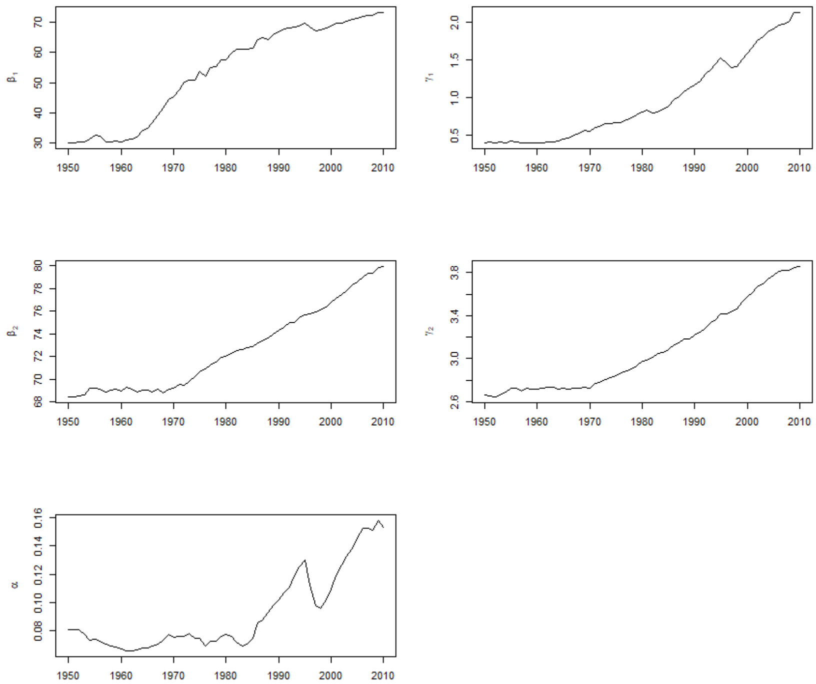


Figure 2.3 is the graph of the parameter estimates of the proposed methodology over the covered period. From the estimated parameter series of the MCH function for US males, the beta and gamma parameters generally have an upward trend starting at 1960 for the first component, while it starts at 1970 for the second component. The alpha parameter for males is relatively flat from 1950 to the mid-1980s before developing an increasing trend. For the parameters of the youth-to-adulthood component and alpha, there is a decline in 1995–1998. After the period, the parameters show an upward trend. The upward trend in alpha signifies that, for the MCH function, the survivability of US males tends to have bigger weight on the youth-to-adulthood component than the old-to-oldest component. The numerical results can be found in Table 2.1. It shows the R-square of the fit of the proposed survival function for each life table of the years. The R-square is always above 99.98%. Other summary statistics of the parameters are shown in Table 2.2.

Table 2.1: Parameter Estimates of the MCH Function for US Males, 1950–2010

Year	α	β_1	γ_1	β_2	γ_2	R-square
1950	0.08095	30.11515	0.40597	68.41750	2.65831	99.9913%
1951	0.08103	29.93896	0.40632	68.42830	2.65243	99.9913%
1952	0.08068	30.34482	0.40610	68.52568	2.64008	99.9918%
1953	0.07751	30.34034	0.40948	68.63079	2.66238	99.9924%
1954	0.07346	31.22199	0.40086	69.20069	2.68720	99.9938%
1955	0.07389	32.74988	0.41938	69.21561	2.72225	99.9935%
1956	0.07239	32.39476	0.41693	69.17866	2.72373	99.9934%
1957	0.07048	30.48612	0.40228	68.84197	2.70013	99.9938%
1958	0.06931	30.49449	0.39826	69.05374	2.72126	99.9941%
1959	0.06885	30.71491	0.40347	69.17052	2.71678	99.9942%
1960	0.06737	30.50508	0.39402	68.95348	2.71379	99.9946%
1961	0.06557	30.97055	0.40319	69.27922	2.72768	99.9943%
1962	0.06551	31.30714	0.41157	69.11689	2.73227	99.9942%
1963	0.06638	32.00199	0.41426	68.88007	2.73504	99.9942%
1964	0.06802	34.17906	0.43169	69.07630	2.71714	99.9940%
1965	0.06778	35.03456	0.44516	69.02151	2.72528	99.9939%
1966	0.06907	36.80348	0.46590	68.90725	2.71691	99.9938%
1967	0.07036	39.43931	0.49888	69.13815	2.72655	99.9937%
1968	0.07355	41.67378	0.52163	68.82065	2.72551	99.9933%
1969	0.07724	44.53032	0.56013	69.10719	2.73634	99.9930%
1970	0.07511	45.44515	0.55294	69.22324	2.72842	99.9934%
1971	0.07576	47.54643	0.60456	69.55554	2.76717	99.9937%
1972	0.07599	50.30757	0.62171	69.50683	2.78079	99.9940%
1973	0.07791	50.74621	0.65488	69.78869	2.80689	99.9940%
1974	0.07446	50.95305	0.65381	70.24518	2.82478	99.9944%
1975	0.07446	53.86511	0.67012	70.68670	2.84371	99.9944%
1976	0.06924	52.10698	0.67292	70.92032	2.86460	99.9952%
1977	0.07268	55.17276	0.70741	71.29319	2.88723	99.9952%
1978	0.07249	55.34002	0.73464	71.50546	2.90888	99.9953%
1979	0.07577	57.59404	0.77553	71.95174	2.93341	99.9953%
1980	0.07737	57.69412	0.81587	71.99930	2.97635	99.9954%
1981	0.07583	59.83110	0.83509	72.28147	2.98844	99.9958%
1982	0.07182	61.14238	0.80319	72.52052	3.01096	99.9960%
1983	0.06912	61.25181	0.80906	72.59878	3.04672	99.9963%
1984	0.07057	61.13745	0.85033	72.80760	3.05938	99.9964%
1985	0.07439	61.39307	0.89244	72.89652	3.08677	99.9961%
1986	0.08551	64.34461	0.98015	73.22052	3.12722	99.9958%
1987	0.08748	64.89376	1.01540	73.42686	3.14253	99.9957%
1988	0.09290	64.38272	1.08298	73.63627	3.18058	99.9955%
1989	0.09767	65.74466	1.12399	74.00277	3.18590	99.9952%
1990	0.10221	66.94121	1.16918	74.33765	3.21839	99.9954%
1991	0.10672	67.67891	1.22399	74.61203	3.24841	99.9955%
1992	0.11049	68.05616	1.31155	74.97004	3.27886	99.9956%
1993	0.11890	68.49314	1.37511	75.02808	3.33222	99.9954%
1994	0.12531	68.89534	1.45257	75.39440	3.36609	99.9952%
1995	0.12990	69.59608	1.52239	75.65471	3.41353	99.9955%
1996	0.11206	68.55962	1.46137	75.74452	3.41415	99.9956%
1997	0.09784	67.11171	1.40237	75.93831	3.43579	99.9956%
1998	0.09587	67.52334	1.41170	76.14474	3.46258	99.9955%
1999	0.10167	68.26278	1.50436	76.38578	3.52459	99.9949%
2000	0.10924	68.74757	1.59542	76.77923	3.57414	99.9944%
2001	0.11877	69.62737	1.67230	77.14243	3.61613	99.9943%
2002	0.12659	69.73075	1.75944	77.45423	3.66942	99.9929%
2003	0.13273	70.28062	1.81182	77.78716	3.69407	99.9920%
2004	0.13812	71.11015	1.86885	78.35483	3.73799	99.9913%
2005	0.14560	71.23329	1.91736	78.57172	3.77952	99.9903%
2006	0.15237	71.93645	1.95606	79.02906	3.81209	99.9903%
2007	0.15220	72.22768	1.97260	79.30786	3.82040	99.9895%
2008	0.15089	72.33843	2.01870	79.37364	3.82377	99.9893%
2009	0.15795	73.10333	2.12437	79.85772	3.84719	99.9891%
2010	0.15314	73.28042	2.13050	79.95006	3.85749	99.9899%

Table 2.2: Summary Statistics of Parameter Estimates for US Males

	α	β_1	γ_1	β_2	γ_2
Mean	0.09221	53.62089	0.96942	72.63688	3.08555
Maximum	0.15795	73.28043	2.13050	79.95006	3.85749
Minimum	0.06551	29.93897	0.394021	68.41750	2.64008
Standard Deviation	0.02773	15.87067	0.55511	3.66533	0.39635
Skewness	1.09587	-0.38014	0.68823	0.53176	0.66347
Excess Kurtosis	-0.09444	-1.48379	-0.86931	-1.05322	-0.95367

Table 2.3: Time Series Model Results for Parameter Estimates of the MCH Function for US Males

Terms	Equations				
	dlogit(α)	dlog(β_1)	dlog(γ_1)	dlog(β_2)	dlog(γ_2)
constant	0	0.011175*	0.019413***	0.0019562***	0.003796***
(se)	- -	0.004336	0.005389	0.0005534	0.001084
dlogit(α).l1	0.4948***				
(se)	0.1106				
dlog(β_1).l1		0.205536	0.014588		
(se)		0.148681	0.18478		
dlog(γ_1).l1		0.032339	0.301317*		
(se)		0.117308	0.14579		
dlog(β_2).l1				-0.0916912	0.601523*
(se)				0.148069	0.29011
dlog(γ_2).l1				0.1471083*	0.155718
(se)				0.0713006	0.139699

Signifincance codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
se: standard error; l1: lag of order 1

Table 2.3 above shows the results of the time series models used in estimating the in-sample predictions for the data from 1950 to 2010 and forecasts from 2011 to 2050.

Figure 2.4: Forecasted Life Expectancy for US Males from the LC and the MCH Model with 95% Confidence Bands, by Age

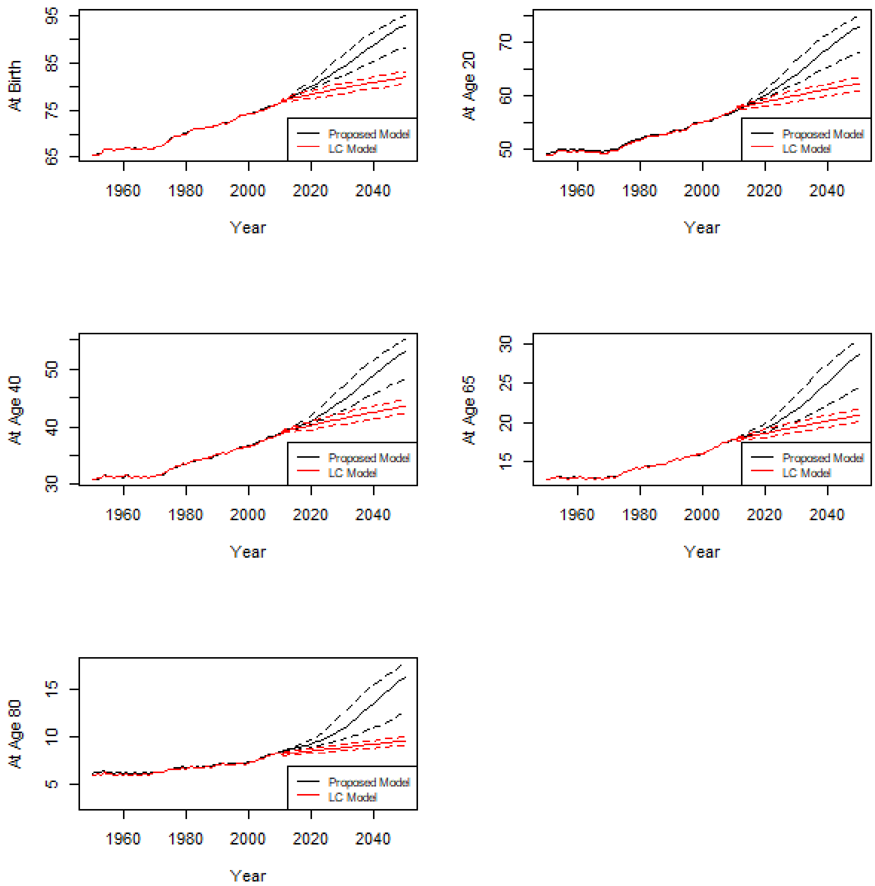


Figure 2.4 shows graphs of estimates and forecasting results by the proposed methodology and the LC model for each age for US males. The LC model in red lines has narrower intervals over the

MCH model indicated in black lines. The proposed methodology tends to have higher forecasted values, compared with the LC model, over all ages. We show the confidence interval estimates from the MCH model in Tables 2.4 and 2.5 .

Table 2.4: Estimates and Confidence Limits of Life Expectancy at Birth, Age 20 and Age 40 as Predicted by the MCH Function for US Males, 2011–2050

	$\tilde{e}_0$			$\tilde{e}_{20}$			$\tilde{e}_{40}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	76.82	76.72	77.00	57.38	57.28	57.53	38.93	38.84	39.04
2012	77.15	77.02	77.42	57.66	57.53	57.89	39.12	39.01	39.30
2013	77.48	77.30	77.85	57.93	57.76	58.26	39.33	39.18	39.58
2014	77.81	77.58	78.30	58.22	58.00	58.65	39.54	39.35	39.86
2015	78.15	77.86	78.75	58.51	58.23	59.05	39.76	39.52	40.17
2016	78.49	78.14	79.19	58.81	58.48	59.47	39.99	39.69	40.50
2017	78.84	78.43	79.67	59.12	58.73	59.89	40.22	39.86	40.86
2018	79.20	78.72	80.14	59.45	58.98	60.33	40.47	40.05	41.20
2019	79.56	79.01	80.62	59.78	59.24	60.78	40.73	40.24	41.57
2020	79.93	79.31	81.10	60.12	59.51	61.24	41.00	40.44	41.95
2021	80.31	79.61	81.60	60.47	59.78	61.72	41.29	40.64	42.36
2022	80.69	79.89	82.13	60.83	60.06	62.21	41.59	40.85	42.76
2023	81.09	80.19	82.68	61.20	60.33	62.74	41.89	41.07	43.21
2024	81.49	80.49	83.24	61.59	60.61	63.29	42.22	41.30	43.68
2025	81.89	80.79	83.79	61.98	60.90	63.83	42.55	41.55	44.18
2026	82.31	81.09	84.37	62.38	61.19	64.40	42.90	41.80	44.68
2027	82.74	81.40	84.93	62.80	61.48	64.96	43.26	42.04	45.20
2028	83.17	81.70	85.51	63.22	61.77	65.52	43.64	42.28	45.72
2029	83.61	82.01	86.09	63.65	62.07	66.10	44.02	42.54	46.25
2030	84.06	82.31	86.66	64.09	62.36	66.66	44.42	42.79	46.79
2031	84.51	82.62	87.22	64.54	62.66	67.23	44.83	43.05	47.33
2032	84.98	82.93	87.77	65.00	62.96	67.78	45.25	43.32	47.86
2033	85.44	83.24	88.28	65.46	63.28	68.28	45.68	43.59	48.36
2034	85.92	83.54	88.78	65.93	63.58	68.78	46.12	43.87	48.85
2035	86.39	83.85	89.29	66.40	63.87	69.29	46.57	44.16	49.35
2036	86.87	84.15	89.81	66.88	64.17	69.81	47.02	44.44	49.85
2037	87.35	84.45	90.26	67.35	64.46	70.26	47.48	44.69	50.30
2038	87.82	84.78	90.68	67.83	64.79	70.68	47.93	44.99	50.72
2039	88.30	85.11	91.11	68.30	65.12	71.11	48.39	45.28	51.14
2040	88.77	85.42	91.52	68.77	65.43	71.52	48.85	45.57	51.54
2041	89.23	85.73	91.94	69.24	65.74	71.94	49.30	45.85	51.96
2042	89.69	86.03	92.32	69.69	66.04	72.32	49.75	46.15	52.33
2043	90.14	86.33	92.67	70.14	66.34	72.67	50.18	46.45	52.68
2044	90.58	86.64	93.03	70.58	66.64	73.03	50.61	46.74	53.04
2045	91.00	86.90	93.38	71.00	66.90	73.38	51.03	47.01	53.39
2046	91.42	87.17	93.72	71.42	67.17	73.72	51.44	47.26	53.73
2047	91.82	87.44	94.07	71.82	67.45	74.07	51.84	47.50	54.08
2048	92.20	87.70	94.40	72.20	67.70	74.40	52.22	47.75	54.41
2049	92.58	87.95	94.72	72.58	67.95	74.72	52.59	47.99	54.73
2050	92.94	88.21	95.04	72.94	68.21	75.04	52.95	48.24	55.04

Table 2.5: Estimates and Confidence Limits of Life Expectancy at Ages 65 and 80 as Predicted by the MCH Function for US Males, 2011–2050

	$\tilde{e}_{65}$			$\tilde{e}_{80}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	17.95	17.88	18.05	8.51	8.45	8.57
2012	18.07	17.98	18.20	8.58	8.51	8.66
2013	18.20	18.08	18.37	8.65	8.55	8.77
2014	18.33	18.17	18.55	8.73	8.60	8.88
2015	18.47	18.27	18.74	8.81	8.65	9.00
2016	18.61	18.37	18.95	8.90	8.70	9.12
2017	18.76	18.47	19.16	8.98	8.75	9.25
2018	18.91	18.57	19.38	9.08	8.81	9.40
2019	19.08	18.67	19.63	9.18	8.86	9.56
2020	19.25	18.80	19.88	9.28	8.92	9.72
2021	19.43	18.92	20.14	9.39	8.99	9.90
2022	19.62	19.05	20.42	9.51	9.06	10.09
2023	19.82	19.18	20.73	9.64	9.15	10.31
2024	20.03	19.30	21.06	9.78	9.22	10.56
2025	20.25	19.44	21.41	9.92	9.30	10.83
2026	20.49	19.59	21.78	10.08	9.37	11.11
2027	20.74	19.75	22.17	10.25	9.47	11.40
2028	21.00	19.89	22.56	10.44	9.56	11.72
2029	21.28	20.04	22.98	10.63	9.65	12.04
2030	21.57	20.20	23.41	10.84	9.75	12.36
2031	21.88	20.39	23.85	11.06	9.85	12.70
2032	22.19	20.59	24.30	11.30	9.97	13.04
2033	22.52	20.77	24.75	11.54	10.07	13.39
2034	22.86	20.94	25.15	11.80	10.18	13.70
2035	23.21	21.13	25.55	12.07	10.28	14.01
2036	23.57	21.32	25.96	12.35	10.39	14.34
2037	23.94	21.51	26.35	12.63	10.52	14.66
2038	24.31	21.70	26.71	12.92	10.64	14.93
2039	24.69	21.91	27.06	13.22	10.78	15.20
2040	25.07	22.13	27.41	13.51	10.92	15.49
2041	25.45	22.35	27.76	13.81	11.07	15.75
2042	25.82	22.56	28.09	14.10	11.22	15.98
2043	26.20	22.79	28.38	14.39	11.36	16.21
2044	26.57	23.02	28.71	14.68	11.51	16.43
2045	26.93	23.26	29.03	14.96	11.66	16.64
2046	27.29	23.48	29.34	15.24	11.84	16.85
2047	27.63	23.69	29.65	15.50	12.01	17.09
2048	27.97	23.89	29.93	15.76	12.18	17.28
2049	28.30	24.09	30.22	16.01	12.34	17.48
2050	28.62	24.29	30.50	16.25	12.50	17.68

Figure 2.5: Life Expectancy of US Females, by Age, 1950–2010

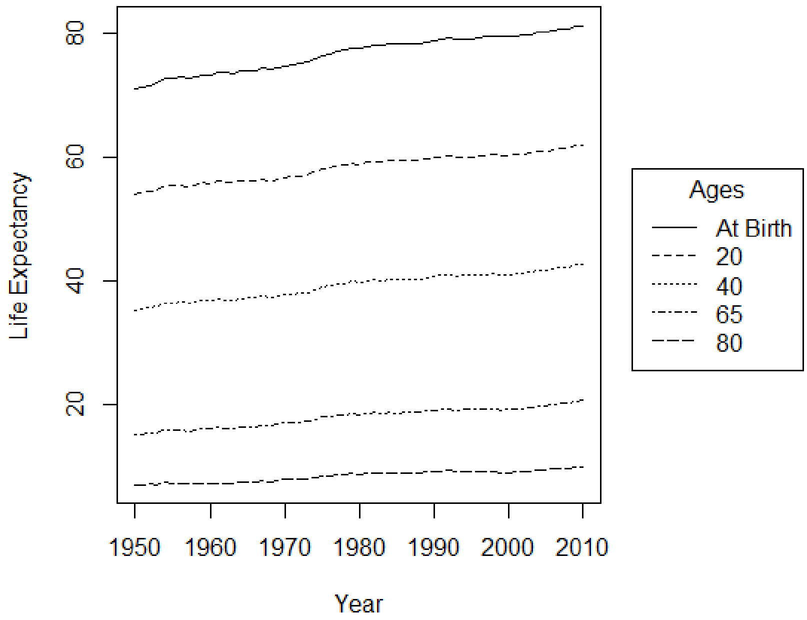


Figure 2.5 shows the life expectancy values for US females from 1950 to 2011 based on the life table data. For life expectancies at birth, age 20 and age 40, there are greater increases from 1950 to 2010, compared with ages 65 and 80.

Figure 2.6: Parameter Estimates of the MCH Function for US Females

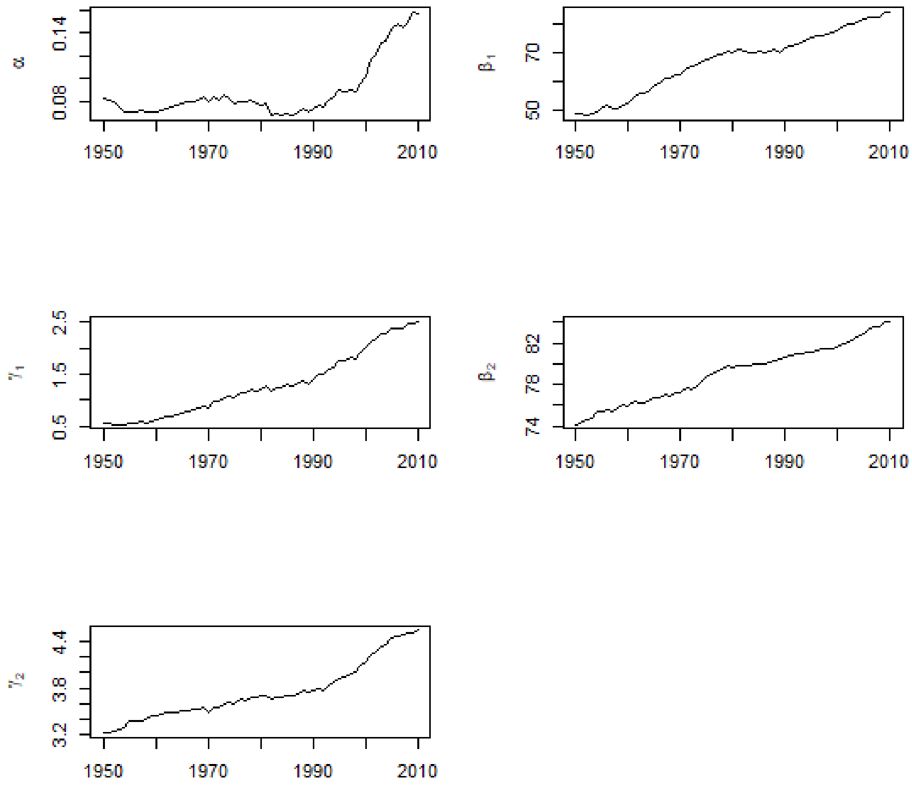


Figure 2.6 is a graph of the parameter estimates for US females for the coverage period. In contrast to US males, US females do not show a decline; each series has an upward trend. The alpha parameter is relatively flat from 1950 to 1990, before having a positive trend from the latter year. This may mean that the youth-to-adulthood component became more important for the survivability of US females by 1990. We show the parameter results in Table 2.6 and the summary statistics in Table 2.7. They show that the R-square maintains a value of at least 99.97%.

Table 2.6: Table of Parameter Estimates of the MCH Function for US Females, 1950 to 2010

Year	α	β_1	γ_1	β_2	γ_2	R-square
1950	0.08189	48.67920	0.55321	74.15176	3.22102	99.9784%
1951	0.08043	48.27816	0.54550	74.31886	3.23815	99.9771%
1952	0.07920	47.88114	0.53399	74.58214	3.24694	99.9789%
1953	0.07440	48.25102	0.53128	74.77087	3.27897	99.9799%
1954	0.07050	49.17792	0.52630	75.36778	3.30127	99.9815%
1955	0.07104	50.76786	0.55049	75.48524	3.37506	99.9811%
1956	0.07032	51.29027	0.55830	75.58754	3.38602	99.9810%
1957	0.07150	50.29155	0.57071	75.45447	3.37633	99.9804%
1958	0.07010	50.22395	0.55866	75.68344	3.41271	99.9798%
1959	0.07051	51.65296	0.58318	75.98405	3.44124	99.9797%
1960	0.07092	52.06526	0.61022	75.98184	3.44122	99.9786%
1961	0.07160	54.38907	0.63590	76.34274	3.47330	99.9775%
1962	0.07407	55.62843	0.67478	76.29007	3.49355	99.9758%
1963	0.07514	55.51552	0.69395	76.24484	3.49379	99.9747%
1964	0.07681	56.72273	0.71750	76.58941	3.49125	99.9733%
1965	0.07832	58.33628	0.75540	76.70482	3.50982	99.9726%
1966	0.07892	58.98006	0.77627	76.73143	3.51960	99.9741%
1967	0.07979	60.66701	0.82167	77.04581	3.52761	99.9742%
1968	0.08181	60.83825	0.86049	76.84493	3.52367	99.9736%
1969	0.08367	62.10180	0.88411	77.18176	3.54742	99.9748%
1970	0.07995	62.07443	0.86107	77.26970	3.48598	99.9771%
1971	0.08418	64.32276	0.97047	77.62210	3.55474	99.9772%
1972	0.08128	64.88285	0.98108	77.60740	3.54972	99.9808%
1973	0.08500	65.92812	1.05277	77.90969	3.60356	99.9783%
1974	0.08225	66.72579	1.06072	78.32620	3.62504	99.9789%
1975	0.07802	67.43809	1.03327	78.75834	3.60050	99.9807%
1976	0.08022	67.95235	1.12829	79.02873	3.66349	99.9791%
1977	0.07967	68.97214	1.15175	79.34762	3.64862	99.9808%
1978	0.08058	69.33938	1.19652	79.50536	3.68089	99.9808%
1979	0.07801	70.47728	1.17953	79.81389	3.68825	99.9820%
1980	0.07629	69.64567	1.19702	79.58017	3.69560	99.9827%
1981	0.07746	71.00797	1.26510	79.85069	3.69934	99.9825%
1982	0.06809	70.31424	1.15823	79.86322	3.65226	99.9852%
1983	0.06890	70.05545	1.23786	79.87698	3.68262	99.9846%
1984	0.06753	69.81607	1.23159	79.92710	3.67813	99.9856%
1985	0.06943	70.17602	1.29379	79.96550	3.71042	99.9851%
1986	0.06814	69.80634	1.27893	80.02796	3.69927	99.9867%
1987	0.07136	70.27538	1.34449	80.20092	3.73185	99.9860%
1988	0.07433	70.81702	1.37676	80.23119	3.76949	99.9853%
1989	0.07032	70.10697	1.30555	80.41992	3.74591	99.9866%
1990	0.07304	71.70889	1.40884	80.67979	3.76176	99.9870%
1991	0.07603	72.28957	1.48708	80.85339	3.78368	99.9873%
1992	0.07463	72.91817	1.49993	80.98068	3.77934	99.9886%
1993	0.08035	73.13215	1.58260	80.88050	3.84321	99.9884%
1994	0.08471	74.51214	1.63834	81.04775	3.87435	99.9892%
1995	0.09031	75.15524	1.74627	81.16258	3.92104	99.9893%
1996	0.08868	75.69606	1.75452	81.22504	3.94099	99.9899%
1997	0.08997	75.94906	1.81942	81.39663	3.98712	99.9895%
1998	0.08793	76.24137	1.79068	81.39981	4.01005	99.9898%
1999	0.09600	76.93670	1.92223	81.46115	4.09125	99.9898%
2000	0.10182	77.60422	2.00722	81.62740	4.13560	99.9900%
2001	0.11662	79.47767	2.12930	81.91480	4.21593	99.9902%
2002	0.12188	79.73437	2.17725	82.10375	4.27344	99.9901%
2003	0.13119	80.06556	2.28186	82.40594	4.33596	99.9895%
2004	0.13298	81.20804	2.26849	82.76715	4.34646	99.9894%
2005	0.14382	81.70747	2.36454	82.96676	4.43979	99.9892%
2006	0.14809	82.20330	2.38924	83.32211	4.46455	99.9891%
2007	0.14525	82.37390	2.36431	83.53216	4.47097	99.9892%
2008	0.14927	82.29292	2.46103	83.61380	4.50819	99.9896%
2009	0.15870	83.99502	2.48258	83.99502	4.50301	99.9906%
2010	0.15678	84.10306	2.52350	84.10310	4.53919	99.9913%

Table 2.7: Summary Statistics of Parameter Estimates for US Females

	α	β_1	γ_1	β_2	γ_2
Mean	0.08820	66.90455	1.28436	79.11337	3.74902
Maximum	0.15870	84.10306	2.52350	84.10310	4.53919
Minimum	0.06753	47.88114	0.52630	74.15176	3.22102
Standard Deviation	0.02491	10.85392	0.61419	2.75112	0.35749
Skewness	1.76578	-0.31109	0.57652	-0.05410	0.83534
Excess Kurtosis	1.88193	-1.01046	-0.78920	-1.06648	-0.16928

Comparing the differences between the sexes for the US population from tables 2.1and 2.6, the females tend to have higher values in β_1, γ_1, β_2, and γ_2. The β parameters are associated with the typical ages between the two components, youth-to-adulthood and old-to-oldest-age. With females having higher values in the β parameters, this implies that females typically live longer than males, which agrees with typically observed life expectancy values as seen in Figures 2.2 and 2.5.

The γ parameters for females are always higher than the males. These are attributed to the speed in which zero survival probability is reached for each of the two components of the MCH function. Higher γ values mean that once the typical age of has been achieved by the individual, the probability of surviving more years declines faster than having lower γ values. For the difference between the sexes in the US, this means that once females reach the typical age of their component in which they belong as described by the β parameter, their chance of survival will decline faster than when males in the same situation with respect to their typical age.

With the α parameters between the sexes, the US males had an incline between 1984 to 1995, but dropped from 1996 to 1998 to be in the same levels as the female population. As this parameter is interpreted as the contribution of the youth-to-adulthood component to overall longevity, this meant that from the period of 1984 to 1995, the youth-to-adulthood related activities grew more important for the longevity of males. This may be attributed to the changing demographic for US males in the period. The years 1996 to 1998 were an adjustment period for the US male population to be in line with females with respect to the importance of youth-to-adulthood component. From 1998 onwards, the patterns between males and females in terms of the α parameters were similar which meant that from the start of the new millennium, the importance of the components in terms of survivability have become similar between the sexes.

Table 2.8 shows the results of time series model estimation used to generate the estimates and forecasts of the life expectancy for US females in the prescribed periods.

Table 2.8: Time Series Model Results for Parameters of the MCH Function for US Females

Terms	Equations				
	dlogit(alpha)	dlog(beta1)	dlog(gamma1)	dlog(beta2)	dlog(gamma2)
constant	0.0121*	0.00938***	0.030895***	0.0022543***	0.006617***
(se)	0.0069	0.002115	0.005802	0.0004257	0.001597
dlogit(alpha).l1	0.1163				
(se)	0.1279				
dlog(beta1).l1		-0.011813	0.613764		
(se)		0.150873	0.413896		
dlog(gamma1).l1		0.005402	-0.416918**		
(se)		0.051363	0.140906		
dlog(beta2).l1				-0.1242139	0.106631
(se)				0.1338445	0.502054
dlog(gamma2).l1				0.0183699	-0.196858
(se)				0.0352951	0.132393

Significance codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Figure 2.7 shows graphs of estimates and forecasting results by the proposed methodology and the LC model for each age for US females. They have the same behaviour as the estimates and forecasts for US males. The life expectancy forecasts and confidence limits are shown in Tables 2.9 and 2.10.

Figure 2.7: Life Expectancy Estimates and Forecasts with 95% Confidence Bands for the MCH Function and LC Model for US Females, by Age, 1950–2050

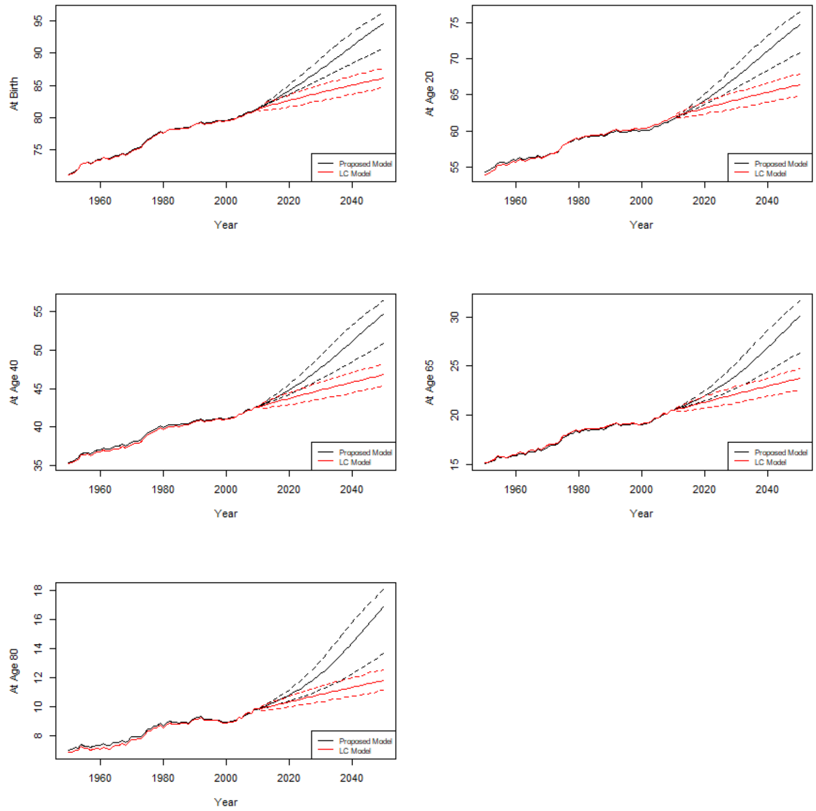


Table 2.9: Estimates and Confidence Limits of Life Expectancy at Birth, Age 20 and Age 40 as Predicted by the MCH Function for US Females, 2011–2050

	$\tilde{e}_0$			$\tilde{e}_{20}$			$\tilde{e}_{40}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	76.82	76.72	77.00	57.38	57.28	57.53	38.93	38.84	39.04
2012	77.15	77.02	77.42	57.66	57.53	57.89	39.12	39.01	39.30
2013	77.48	77.30	77.85	57.93	57.76	58.26	39.33	39.18	39.58
2014	77.81	77.58	78.30	58.22	58.00	58.65	39.54	39.35	39.86
2015	78.15	77.86	78.75	58.51	58.23	59.05	39.76	39.52	40.17
2016	78.49	78.14	79.19	58.81	58.48	59.47	39.99	39.69	40.50
2017	78.84	78.43	79.67	59.12	58.73	59.89	40.22	39.86	40.86
2018	79.20	78.72	80.14	59.45	58.98	60.33	40.47	40.05	41.20
2019	79.56	79.01	80.62	59.78	59.24	60.78	40.73	40.24	41.57
2020	79.93	79.31	81.10	60.12	59.51	61.24	41.00	40.44	41.95
2021	80.31	79.61	81.60	60.47	59.78	61.72	41.29	40.64	42.36
2022	80.69	79.89	82.13	60.83	60.06	62.21	41.59	40.85	42.76
2023	81.09	80.19	82.68	61.20	60.33	62.74	41.89	41.07	43.21
2024	81.49	80.49	83.24	61.59	60.61	63.29	42.22	41.30	43.68
2025	81.89	80.79	83.79	61.98	60.90	63.83	42.55	41.55	44.18
2026	82.31	81.09	84.37	62.38	61.19	64.40	42.90	41.80	44.68
2027	82.74	81.40	84.93	62.80	61.48	64.96	43.26	42.04	45.20
2028	83.17	81.70	85.51	63.22	61.77	65.52	43.64	42.28	45.72
2029	83.61	82.01	86.09	63.65	62.07	66.10	44.02	42.54	46.25
2030	84.06	82.31	86.66	64.09	62.36	66.66	44.42	42.79	46.79
2031	84.51	82.62	87.22	64.54	62.66	67.23	44.83	43.05	47.33
2032	84.98	82.93	87.77	65.00	62.96	67.78	45.25	43.32	47.86
2033	85.44	83.24	88.28	65.46	63.28	68.28	45.68	43.59	48.36
2034	85.92	83.54	88.78	65.93	63.58	68.78	46.12	43.87	48.85
2035	86.39	83.85	89.29	66.40	63.87	69.29	46.57	44.16	49.35
2036	86.87	84.15	89.81	66.88	64.17	69.81	47.02	44.44	49.85
2037	87.35	84.45	90.26	67.35	64.46	70.26	47.48	44.69	50.30
2038	87.82	84.78	90.68	67.83	64.79	70.68	47.93	44.99	50.72
2039	88.30	85.11	91.11	68.30	65.12	71.11	48.39	45.28	51.14
2040	88.77	85.42	91.52	68.77	65.43	71.52	48.85	45.57	51.54
2041	89.23	85.73	91.94	69.24	65.74	71.94	49.30	45.85	51.96
2042	89.69	86.03	92.32	69.69	66.04	72.32	49.75	46.15	52.33
2043	90.14	86.33	92.67	70.14	66.34	72.67	50.18	46.45	52.68
2044	90.58	86.64	93.03	70.58	66.64	73.03	50.61	46.74	53.04
2045	91.00	86.90	93.38	71.00	66.90	73.38	51.03	47.01	53.39
2046	91.42	87.17	93.72	71.42	67.17	73.72	51.44	47.26	53.73
2047	91.82	87.44	94.07	71.82	67.45	74.07	51.84	47.50	54.08
2048	92.20	87.70	94.40	72.20	67.70	74.40	52.22	47.75	54.41
2049	92.58	87.95	94.72	72.58	67.95	74.72	52.59	47.99	54.73
2050	92.94	88.21	95.04	72.94	68.21	75.04	52.95	48.24	55.04

Table 2.10: Estimates and Confidence Limits of Life Expectancy at Ages 65 and 80 as Predicted by the MCH Function for US Females, 2011–2050

Years	$\tilde{e}_{65}$ Estimate	Lower CI	Upper CI	$\tilde{e}_{80}$ Estimate	Lower CI	Upper CI
2011	17.95	17.88	18.05	8.51	8.45	8.57
2012	18.07	17.98	18.20	8.58	8.51	8.66
2013	18.20	18.08	18.37	8.65	8.55	8.77
2014	18.33	18.17	18.55	8.73	8.60	8.88
2015	18.47	18.27	18.74	8.81	8.65	9.00
2016	18.61	18.37	18.95	8.90	8.70	9.12
2017	18.76	18.47	19.16	8.98	8.75	9.25
2018	18.91	18.57	19.38	9.08	8.81	9.40
2019	19.08	18.67	19.63	9.18	8.86	9.56
2020	19.25	18.80	19.88	9.28	8.92	9.72
2021	19.43	18.92	20.14	9.39	8.99	9.90
2022	19.62	19.05	20.42	9.51	9.06	10.09
2023	19.82	19.18	20.73	9.64	9.15	10.31
2024	20.03	19.30	21.06	9.78	9.22	10.56
2025	20.25	19.44	21.41	9.92	9.30	10.83
2026	20.49	19.59	21.78	10.08	9.37	11.11
2027	20.74	19.75	22.17	10.25	9.47	11.40
2028	21.00	19.89	22.56	10.44	9.56	11.72
2029	21.28	20.04	22.98	10.63	9.65	12.04
2030	21.57	20.20	23.41	10.84	9.75	12.36
2031	21.88	20.39	23.85	11.06	9.85	12.70
2032	22.19	20.59	24.30	11.30	9.97	13.04
2033	22.52	20.77	24.75	11.54	10.07	13.39
2034	22.86	20.94	25.15	11.80	10.18	13.70
2035	23.21	21.13	25.55	12.07	10.28	14.01
2036	23.57	21.32	25.96	12.35	10.39	14.34
2037	23.94	21.51	26.35	12.63	10.52	14.66
2038	24.31	21.70	26.71	12.92	10.64	14.93
2039	24.69	21.91	27.06	13.22	10.78	15.20
2040	25.07	22.13	27.41	13.51	10.92	15.49
2041	25.45	22.35	27.76	13.81	11.07	15.75
2042	25.82	22.56	28.09	14.10	11.22	15.98
2043	26.20	22.79	28.38	14.39	11.36	16.21
2044	26.57	23.02	28.71	14.68	11.51	16.43
2045	26.93	23.26	29.03	14.96	11.66	16.64
2046	27.29	23.48	29.34	15.24	11.84	16.85
2047	27.63	23.69	29.65	15.50	12.01	17.09
2048	27.97	23.89	29.93	15.76	12.18	17.28
2049	28.30	24.09	30.22	16.01	12.34	17.48
2050	28.62	24.29	30.50	16.25	12.50	17.68

Tables 2.11 and 2.12 show the out-of-sample statistics and DM test results for forecasting given the prescribed forecast periods with the LC model and the MCH function for males and females. The tables show that the proposed methodology has better out-of-sample performance in most cases, particularly at ages 0 and 80, for both males and females. In these tables, the MCH function performs well over the LC model for all ages for both the MAE with the MCH function having lower values and the DM test indicating that the LC has larger errors than the MCH. In-sample results are shown in Tables 2.13 and 2.14. The LC model has better fit in-sample, but this may be a sign of overfitting the data.

Table 2.11: Out-of-Sample Statistics for 10-Year Forecasts, US Males

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At Birth	0.60132	0.20479	3.08845	0.01296
Age 20	0.69088	0.15566	3.55640	0.00615
Age 40	0.56586	0.29007	3.18325	0.01113
Age 65	0.78118	0.66125	2.80297	0.02061
Age 80	0.36113	0.29847	2.28909	0.04785

Table 2.12: Out-of-Sample Statistics for 10-Year Forecasts, US Females

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At Birth	0.64287	0.34319	4.04047	0.00293
Age 20	0.51267	0.13107	3.71202	0.00483
Age 40	0.52044	0.09748	3.70022	0.00492
Age 65	0.35938	0.28267	2.66376	0.02589
Age 80	0.17370	0.14452	2.32826	0.04488

Table 2.13: In-Sample Results, US Males

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At Birth	0.0071	0.0431	-10.7167	0.0000
Age 20	0.0007	0.2307	-11.8438	0.0000
Age 40	0.0006	0.0528	-9.6181	0.0000
Age 65	0.0008	0.0228	-8.7927	0.0000
Age 80	0.0018	0.1072	-5.6686	0.0000

Table 2.14: In-Sample Results, US Females

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At Birth	0.0048	0.0950	-23.5130	0.0000
Age 20	0.0038	0.1814	-8.1725	0.0000
Age 40	0.0038	0.2161	-12.9889	0.0000
Age 65	0.0044	0.1015	-12.6139	0.0000
Age 80	0.0069	0.1429	-10.5038	0.0000

2.5.2 Australian Life Tables

In this part, we will discuss the results for the Australian male and female life tables and forecasts generated by the LC and MCH functions. Figure 2.8 shows the life expectancy values based on the life tables for Australian males. On graphs for people from ages 0 or at birth to 40, there is a noticeable flat line of life expectancy up until 1970, from which the trend goes upward.

Figure 2.8: Life Expectancy of Australian Males, by Age, 1950–2010

The parameter series of the MCH function for Australian males are plotted in Figure 2.9 and the corresponding parameter series outputs are in Table 2.15. For all of the years, the MCH function fits the life tables with R-square always above 99.99%. The period 1989–1999 has low values for α and β_1. The spikes that can be seen in the plots of α and β_1 are found in the year 1996. After 1999, the trend goes back to the general pattern that is observed before 1989. We only account this with possible demographic changes during 1989 and 1999 with the lifestyles of the youth-to-adulthood segment of the population, as α_1 discuss the contribution of this component on the survivability and β_1 describes the typical age of this component. For the parameters γ_1, β_2, and γ_2, there is a flat pattern until 1970, from which the parameters start an incline. This phenomenon is similar to the observed life expectancy values for the younger ages. Table 2.16 shows the summary statistics for the parameter series of Australian males.

Figure 2.9: Parameter Estimates of the MCH Function for Australian Males, 1950–2010

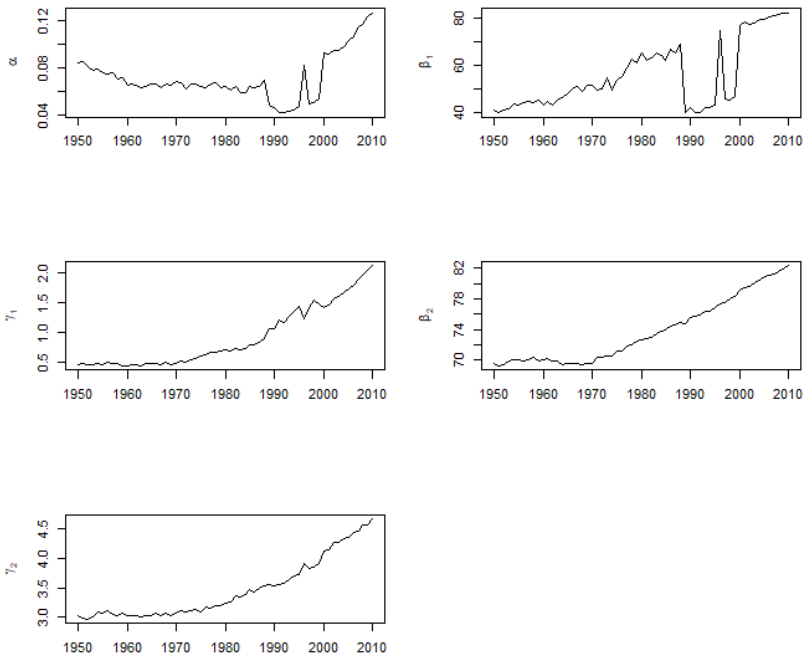


Table 2.15: Parameter Estimates of the MCH Function for Australian Males, 1950–2010

Year	α	β_1	γ_1	β_2	γ_2	R-square
1950	0.08377	40.85934	0.46919	69.60545	3.02463	99.9936%
1951	0.08547	39.69914	0.48683	69.24027	2.98542	99.9928%
1952	0.08062	40.78786	0.46297	69.33248	2.97389	99.9930%
1953	0.07825	41.20562	0.46528	69.87313	3.01393	99.9939%
1954	0.07908	43.65442	0.48218	69.99065	3.08638	99.9926%
1955	0.07567	43.00148	0.46606	69.99699	3.06122	99.9933%
1956	0.07470	44.20673	0.50326	69.87640	3.11789	99.9938%
1957	0.07539	44.50152	0.47940	70.02817	3.04722	99.9919%
1958	0.07017	44.04826	0.47914	70.30296	3.03530	99.9931%
1959	0.07179	45.16570	0.45451	69.83648	3.06383	99.9919%
1960	0.06477	42.85592	0.44631	70.05500	3.02815	99.9934%
1961	0.06651	44.60019	0.47281	70.19173	3.03202	99.9941%
1962	0.06548	42.70025	0.46888	69.95607	3.02876	99.9944%
1963	0.06296	44.94645	0.45036	69.83920	3.01639	99.9938%
1964	0.06429	46.04942	0.47860	69.43126	3.02589	99.9940%
1965	0.06605	47.91016	0.49234	69.64211	3.02123	99.9927%
1966	0.06681	50.05208	0.49451	69.55416	3.06573	99.9922%
1967	0.06363	50.72772	0.45958	69.62994	3.02092	99.9923%
1968	0.06631	48.76988	0.51291	69.44739	3.07805	99.9936%
1969	0.06580	51.48865	0.46655	69.64034	3.02821	99.9915%
1970	0.06889	51.31171	0.48552	69.48107	3.08145	99.9919%
1971	0.06751	49.23454	0.52439	70.31413	3.10432	99.9938%
1972	0.06258	49.78005	0.50299	70.31890	3.09137	99.9944%
1973	0.06600	54.56338	0.54046	70.53768	3.11838	99.9941%
1974	0.06588	49.46046	0.57706	70.46636	3.12916	99.9953%
1975	0.06381	53.43672	0.61858	71.13256	3.09153	99.9961%
1976	0.06361	54.97252	0.63141	71.15556	3.17614	99.9961%
1977	0.06600	58.67602	0.67220	71.80408	3.16377	99.9958%
1978	0.06733	62.76508	0.67936	72.01334	3.20707	99.9948%
1979	0.06339	61.11757	0.70282	72.41942	3.20010	99.9958%
1980	0.06380	65.11682	0.72446	72.57851	3.25021	99.9955%
1981	0.06084	62.08510	0.69558	72.80879	3.27138	99.9959%
1982	0.06435	63.32691	0.74224	72.94172	3.35660	99.9958%
1983	0.05867	65.41690	0.71931	73.52216	3.33778	99.9961%
1984	0.05936	64.09566	0.74540	73.72738	3.37932	99.9964%
1985	0.06422	61.96667	0.80573	74.00781	3.46810	99.9967%
1986	0.06266	66.58688	0.80214	74.42822	3.43743	99.9965%
1987	0.06394	65.35464	0.84422	74.53224	3.49654	99.9968%
1988	0.06946	69.03111	0.90629	74.86074	3.54427	99.9960%
1989	0.04826	39.88100	1.07435	74.76571	3.55894	99.9959%
1990	0.04683	41.87399	1.07619	75.50622	3.53237	99.9962%
1991	0.04212	39.59258	1.20513	75.81676	3.54848	99.9947%
1992	0.04197	39.61505	1.17138	75.85270	3.57802	99.9964%
1993	0.04334	41.89701	1.27739	76.41503	3.64097	99.9960%
1994	0.04440	41.87273	1.36325	76.36836	3.70099	99.9972%
1995	0.04709	43.06492	1.43539	76.95197	3.73267	99.9967%
1996	0.08171	74.60848	1.22606	77.33319	3.90657	99.9975%
1997	0.04972	45.81245	1.41819	77.53470	3.82235	99.9973%
1998	0.05111	44.87426	1.53633	77.95358	3.85484	99.9974%
1999	0.05330	46.64760	1.48470	78.37873	3.92235	99.9971%
2000	0.09235	76.63155	1.41813	79.08349	4.12473	99.9981%
2001	0.09160	78.26971	1.45255	79.42007	4.15304	99.9972%
2002	0.09416	77.44740	1.56180	79.67053	4.27410	99.9978%
2003	0.09454	77.94119	1.60435	80.05699	4.26314	99.9974%
2004	0.09771	79.27241	1.65819	80.39058	4.32616	99.9972%
2005	0.10266	79.63271	1.72799	80.92033	4.34713	99.9967%
2006	0.10579	80.36787	1.79854	81.18265	4.44585	99.9965%
2007	0.11416	81.24258	1.90230	81.24258	4.46092	99.9961%
2008	0.11683	81.53917	1.95371	81.53917	4.57605	99.9965%
2009	0.12297	81.94236	2.02933	81.94236	4.57208	99.9964%
2010	0.12618	82.34719	2.11478	82.34720	4.67085	99.9950%

Table 2.16: Summary Statistics of Parameter Estimates for Australian Males

	α	β_1	γ_1	β_2	γ_2
Mean	0.07145	55.76891	0.91642	73.75727	3.47004
Maximum	0.12618	82.34719	2.11478	82.34720	4.67085
Minimum	0.04197	39.59258	0.44631	69.24027	2.97389
Standard Deviation	0.01944	14.20426	0.50356	4.19430	0.50598
Skewness	1.02616	0.65598	0.86607	0.63821	0.99798
Excess Kurtosis	0.92969	-1.00826	-0.58626	-0.98270	-0.27475

Table 2.17: Time Series Model Results for Parameters of the MCH Function for Australian Males

Terms	Equations				
	dlogit(α)	dlog(β_1)	dlog(γ_1)	dlog(β_2)	dlog(γ_2)
constant	0.0075	0.01829	0.032320***	0.002633***	0.004905*
(se)	0.0147	0.02117	0.008726	0.000616	0.002152
dlogit(α).l1	-0.2383 .				
(se)	0.1242				
dlog(β_1).l1		-0.28802 .	-0.020354		
(se)		0.16063	0.066215		
dlog(γ_1).l1		-0.10168	-0.289484 .		
(se)		0.39035	0.160907		
dlog(β_2).l1				-0.091994	1.718478***
(se)				0.122042	0.426327
dlog(γ_2).l1				0.079976 *	-0.29573 *
(se)				0.032127	0.112229

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Table 2.17 shows the time series model estimation results for the parameters of the MCH function for Australian males. These estimates are used to generate forecasts and for bootstrapping to generate the confidence bands for life expectancy.

Figure 2.10: Life Expectancy Estimates and Forecasts with 95% Confidence Bands for the MCH Function and LC Models for Australian Males, by Age, 1950–2050

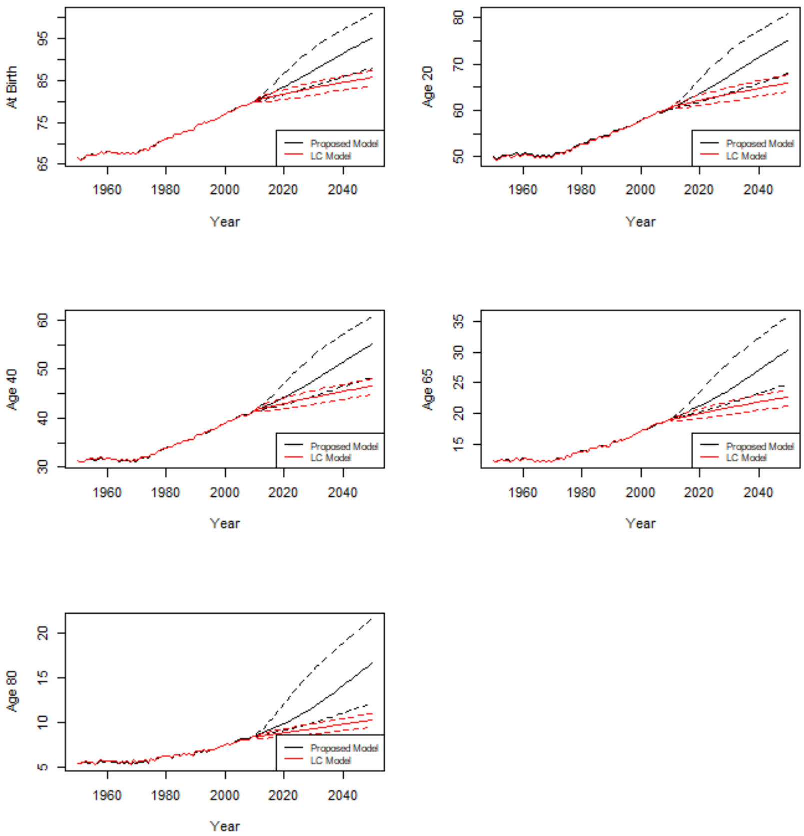


Figure 2.10 shows the forecasts results based on the LC model and the MCH function. The

bands of the two models often intersect in all ages. The bands of the MCH model are wider than those of the LC model in the plots. It can be inferred that, in cases where the bands intersect, the forecast quality of the two models is similar. Tables 2.18 and 2.19 show the MCH forecasts with 95% confidence limits for all ages under consideration.

Table 2.18: Estimates and Confidence Limits of Life Expectancy at Birth, Age 20, and Age 40 as Predicted by the MCH Function for Australian Males, 2011–2050

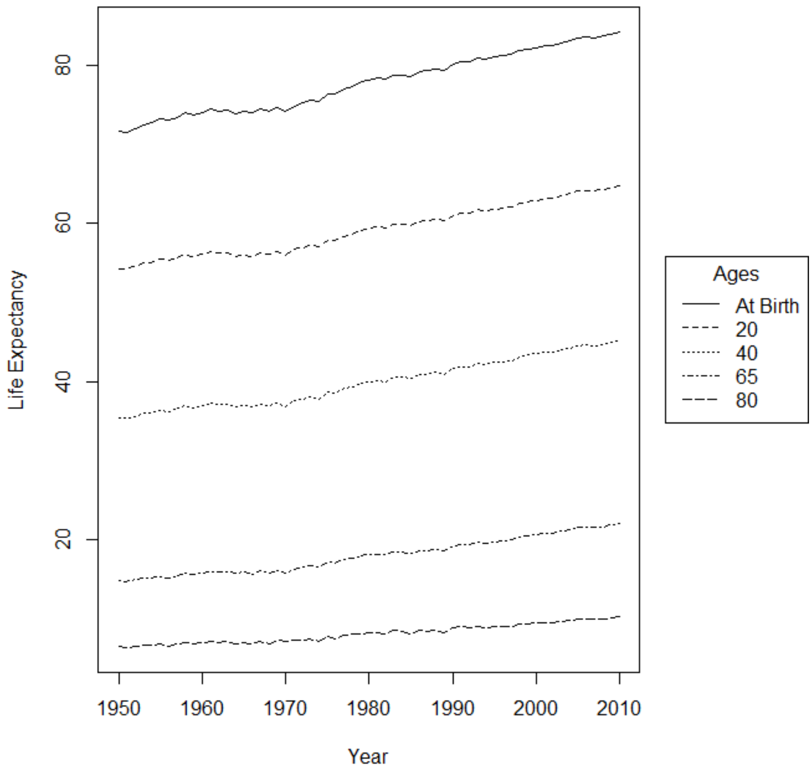
	$\tilde{e}_0$			$\tilde{e}_{20}$			$\tilde{e}_{40}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	80.34	79.92	80.74	60.73	60.33	61.09	41.77	41.43	42.05
2012	80.67	80.14	81.19	61.02	60.51	61.53	42.00	41.59	42.44
2013	81.01	80.34	81.82	61.33	60.68	62.12	42.27	41.73	42.94
2014	81.35	80.53	82.44	61.63	60.85	62.71	42.53	41.88	43.44
2015	81.69	80.74	83.09	61.95	61.02	63.33	42.80	42.04	44.03
2016	82.04	80.92	83.77	62.27	61.19	63.99	43.07	42.19	44.62
2017	82.39	81.10	84.47	62.60	61.36	64.63	43.35	42.33	45.20
2018	82.75	81.30	85.15	62.93	61.53	65.31	43.64	42.48	45.81
2019	83.11	81.50	85.85	63.27	61.69	66.02	43.93	42.64	46.46
2020	83.47	81.68	86.54	63.62	61.86	66.71	44.23	42.78	47.10
2021	83.84	81.86	87.23	63.97	62.03	67.40	44.54	42.94	47.74
2022	84.21	82.04	87.91	64.32	62.20	68.06	44.86	43.11	48.38
2023	84.59	82.22	88.58	64.69	62.40	68.71	45.18	43.27	48.99
2024	84.97	82.40	89.23	65.05	62.57	69.31	45.51	43.45	49.56
2025	85.35	82.61	89.84	65.43	62.75	69.89	45.84	43.62	50.11
2026	85.74	82.81	90.43	65.80	62.95	70.47	46.18	43.80	50.65
2027	86.13	83.02	90.99	66.19	63.14	71.03	46.53	43.97	51.17
2028	86.53	83.24	91.54	66.57	63.32	71.57	46.89	44.14	51.70
2029	86.92	83.44	92.10	66.96	63.52	72.12	47.25	44.33	52.23
2030	87.32	83.63	92.68	67.36	63.72	72.69	47.61	44.51	52.80
2031	87.73	83.83	93.23	67.75	63.92	73.24	47.98	44.70	53.30
2032	88.13	84.04	93.74	68.15	64.13	73.75	48.36	44.89	53.80
2033	88.54	84.26	94.20	68.55	64.33	74.21	48.73	45.07	54.26
2034	88.94	84.47	94.66	68.96	64.54	74.66	49.12	45.26	54.69
2035	89.35	84.69	95.09	69.36	64.75	75.09	49.50	45.48	55.11
2036	89.75	84.91	95.50	69.76	64.97	75.50	49.89	45.67	55.52
2037	90.16	85.13	95.92	70.17	65.18	75.92	50.27	45.87	55.93
2038	90.56	85.35	96.32	70.57	65.40	76.32	50.66	46.06	56.33
2039	90.96	85.56	96.71	70.97	65.59	76.71	51.05	46.26	56.72
2040	91.36	85.77	97.11	71.37	65.78	77.11	51.43	46.45	57.11
2041	91.76	85.98	97.49	71.76	66.00	77.49	51.82	46.64	57.50
2042	92.15	86.19	97.87	72.15	66.20	77.87	52.20	46.83	57.88
2043	92.53	86.40	98.25	72.54	66.46	78.26	52.58	47.02	58.26
2044	92.92	86.60	98.63	72.92	66.66	78.63	52.95	47.21	58.63
2045	93.29	86.79	99.02	73.29	66.85	79.02	53.32	47.40	59.02
2046	93.66	87.00	99.40	73.66	67.06	79.40	53.69	47.60	59.41
2047	94.03	87.22	99.76	74.03	67.29	79.76	54.05	47.79	59.76
2048	94.39	87.45	100.12	74.39	67.52	80.12	54.40	47.99	60.12
2049	94.74	87.67	100.46	74.74	67.71	80.46	54.75	48.18	60.46
2050	95.08	87.88	100.80	75.09	67.93	80.80	55.10	48.37	60.80

Table 2.19: Estimates and Confidence Limits of Life Expectancy at Ages 65 and 80 as Predicted by the MCH Function for Australian Males, 2011–2050

	$\tilde{e}_{65}$			$\tilde{e}_{80}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	19.31	19.08	19.48	8.71	8.52	8.88
2012	19.48	19.22	19.78	8.80	8.59	9.10
2013	19.69	19.33	20.16	8.94	8.65	9.39
2014	19.88	19.45	20.57	9.06	8.72	9.71
2015	20.09	19.58	21.02	9.19	8.80	10.12
2016	20.29	19.71	21.49	9.33	8.87	10.49
2017	20.51	19.84	22.01	9.47	8.94	10.91
2018	20.73	19.96	22.50	9.61	9.01	11.34
2019	20.95	20.08	23.01	9.76	9.07	11.77
2020	21.18	20.21	23.54	9.91	9.15	12.17
2021	21.42	20.33	24.08	10.07	9.23	12.56
2022	21.66	20.47	24.61	10.24	9.31	12.94
2023	21.91	20.61	25.13	10.41	9.39	13.33
2024	22.17	20.76	25.61	10.59	9.48	13.73
2025	22.43	20.90	26.08	10.77	9.57	14.11
2026	22.70	21.04	26.54	10.96	9.67	14.49
2027	22.98	21.18	27.00	11.15	9.77	14.87
2028	23.26	21.33	27.47	11.35	9.86	15.24
2029	23.55	21.49	27.95	11.56	9.95	15.57
2030	23.84	21.65	28.40	11.78	10.04	15.91
2031	24.14	21.79	28.86	11.99	10.14	16.24
2032	24.45	21.95	29.28	12.22	10.24	16.56
2033	24.76	22.10	29.68	12.45	10.34	16.87
2034	25.08	22.25	30.07	12.68	10.44	17.16
2035	25.40	22.42	30.45	12.92	10.54	17.45
2036	25.73	22.58	30.83	13.17	10.65	17.74
2037	26.06	22.73	31.21	13.41	10.76	18.03
2038	26.39	22.88	31.58	13.66	10.88	18.31
2039	26.72	23.04	31.96	13.91	10.98	18.60
2040	27.06	23.21	32.33	14.17	11.08	18.88
2041	27.39	23.37	32.70	14.42	11.19	19.16
2042	27.73	23.54	33.07	14.68	11.31	19.44
2043	28.06	23.71	33.43	14.94	11.42	19.71
2044	28.39	23.88	33.79	15.19	11.54	19.99
2045	28.73	24.05	34.14	15.45	11.66	20.26
2046	29.06	24.22	34.51	15.70	11.78	20.54
2047	29.38	24.39	34.87	15.96	11.91	20.81
2048	29.71	24.57	35.21	16.21	12.05	21.09
2049	30.03	24.74	35.55	16.46	12.18	21.36
2050	30.34	24.93	35.89	16.71	12.31	21.63

For Australian females, figure 2.11 shows the life expectancy values based on the life tables. With the plots, there are consistent positive trends for all ages, although the trend weakens as the life expectancy is computed for older ages.

Figure 2.11: Life Expectancy of Australian Females, by Age, 1950–2010



In figure 2.12, the parameter series of the MCH function for Australian females are plotted. The corresponding parameter series outputs are in Table 2.20. The minimum R-square for the fit of the MCH model for each year was 99.97%. The parameter values for Australian females are more volatile compared to the US male and female and Australian male parameter results. These estimates for the Australian females were the most stable that can be found given some initial values necessary for the optimisation algorithm that has been programmed. Much of the volatility can be found only in the α parameter. This may mean volatility in the importance of the youth-to-adulthood component in determining survivability. The parameter α does not have any trend from 1960 until 1990, from which it increases to a peak in 2000 then drops back in 2006 to levels similar to 1990. There may have been demographic changes during 1990 to 2006 that made youth-to-adulthood more relevant to survivability but after 2006 conditions have reverted similar to 1990. For the β_1 parameter, positive trend can be observed from 1950 until the mid-2000s. For the other three parameters, trends are consistently positive. The summary statistics are shown in Table 2.21 for the parameter series of Australian females.

Figure 2.12: Parameter Estimates of the MCH Function for Australian Females, 1950–2010

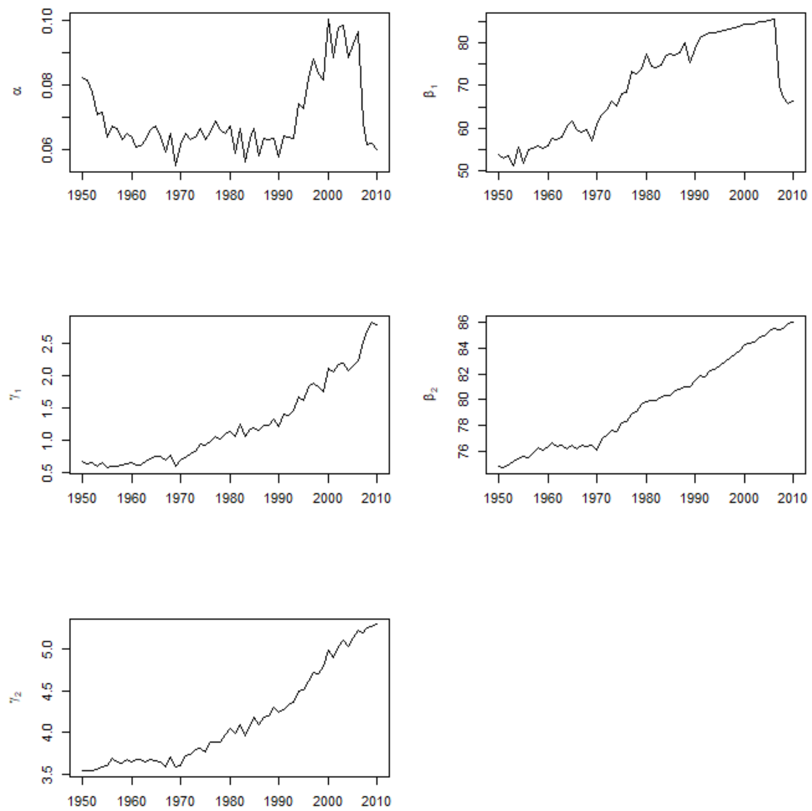


Table 2.20: Parameter Estimates of the MCH Function for Australian Females, 1950–2010

Year	α	β_1	γ_1	β_2	γ_2	R-square
1950	0.08226	53.95936	0.66169	74.79286	3.54318	99.9797%
1951	0.08156	52.97611	0.62623	74.69611	3.55088	99.9787%
1952	0.07764	53.42839	0.64949	74.94309	3.54344	99.9783%
1953	0.07095	51.26657	0.58888	75.27075	3.55728	99.9825%
1954	0.07174	55.47657	0.64302	75.38371	3.59744	99.9820%
1955	0.06384	51.71949	0.55910	75.64506	3.60357	99.9867%
1956	0.06728	55.12711	0.59306	75.52900	3.69511	99.9849%
1957	0.06655	55.27363	0.59669	75.89250	3.65366	99.9865%
1958	0.06292	55.94358	0.60852	76.26972	3.63276	99.9855%
1959	0.06516	55.17584	0.63534	76.11886	3.68173	99.9837%
1960	0.06403	55.97067	0.63930	76.33998	3.64806	99.9856%
1961	0.06072	57.54582	0.60524	76.64823	3.68629	99.9880%
1962	0.06101	57.40554	0.61340	76.41872	3.67745	99.9865%
1963	0.06313	57.86698	0.67350	76.47427	3.64496	99.9855%
1964	0.06620	60.64880	0.69891	76.17556	3.68574	99.9851%
1965	0.06739	61.76133	0.74642	76.42886	3.65860	99.9851%
1966	0.06392	59.74178	0.74161	76.15853	3.65501	99.9866%
1967	0.05940	58.97729	0.69384	76.48451	3.59707	99.9860%
1968	0.06511	59.65830	0.75596	76.40914	3.70912	99.9849%
1969	0.05508	57.16161	0.59243	76.50385	3.58365	99.9893%
1970	0.06152	60.89633	0.68195	76.13789	3.59857	99.9884%
1971	0.06498	63.17147	0.72559	76.98631	3.72551	99.9852%
1972	0.06307	64.17110	0.78400	77.27829	3.73389	99.9859%
1973	0.06370	66.35462	0.82756	77.63598	3.79198	99.9842%
1974	0.06639	65.28882	0.93692	77.51535	3.81256	99.9831%
1975	0.06327	67.98742	0.91297	78.22745	3.76936	99.9870%
1976	0.06523	68.37583	0.98196	78.28836	3.88617	99.9854%
1977	0.06896	73.18120	1.05583	78.87228	3.89354	99.9864%
1978	0.06619	72.63614	1.01673	79.09611	3.88361	99.9874%
1979	0.06518	73.94032	1.09058	79.67859	3.97849	99.9884%
1980	0.06747	77.53904	1.12242	79.86840	4.04600	99.9875%
1981	0.05875	74.36565	1.04547	79.92056	3.99312	99.9908%
1982	0.06652	74.26977	1.24052	79.96470	4.08908	99.9876%
1983	0.05600	74.82311	1.04474	80.19743	3.95845	99.9906%
1984	0.06340	77.20047	1.17932	80.35006	4.07461	99.9906%
1985	0.06647	77.32048	1.18884	80.29732	4.18061	99.9888%
1986	0.05807	76.97398	1.15035	80.68594	4.09646	99.9929%
1987	0.06338	77.57791	1.23778	80.85133	4.19008	99.9921%
1988	0.06307	80.09296	1.23681	81.01108	4.19530	99.9926%
1989	0.06337	75.22327	1.33249	80.96115	4.30346	99.9921%
1990	0.05787	78.51269	1.21338	81.49819	4.24004	99.9937%
1991	0.06419	81.24945	1.40548	81.84047	4.27785	99.9945%
1992	0.06386	81.81465	1.39338	81.81664	4.33153	99.9937%
1993	0.06341	82.27809	1.46051	82.27809	4.36325	99.9945%
1994	0.07424	82.31653	1.67944	82.31653	4.49785	99.9940%
1995	0.07286	82.63155	1.62043	82.63526	4.51416	99.9943%
1996	0.08184	82.89174	1.83295	82.89174	4.61973	99.9932%
1997	0.08806	83.17082	1.87879	83.17082	4.72419	99.9925%
1998	0.08336	83.54964	1.83475	83.54964	4.70994	99.9941%
1999	0.08146	83.76661	1.75529	83.76661	4.79176	99.9930%
2000	0.10046	84.27638	2.11415	84.27638	4.98412	99.9922%
2001	0.08840	84.33558	2.06178	84.33558	4.90255	99.9936%
2002	0.09788	84.44715	2.18462	84.44715	5.03217	99.9918%
2003	0.09853	84.84880	2.20823	84.84880	5.10851	99.9932%
2004	0.08859	84.93008	2.08974	84.93009	5.03281	99.9935%
2005	0.09276	85.32567	2.16081	85.32568	5.14448	99.9911%
2006	0.09682	85.49404	2.23922	85.49405	5.22516	99.9927%
2007	0.06859	69.93747	2.53565	85.47414	5.19852	99.9922%
2008	0.06169	67.21274	2.69679	85.52546	5.26271	99.9927%
2009	0.06214	65.81256	2.84245	85.90237	5.26482	99.9920%
2010	0.05999	66.40364	2.80271	86.02026	5.29391	99.9923%

Table 2.21: Summary Statistics of Parameter Estimates for Australian Females

	α	β_1	γ_1	β_2	γ_2
Mean	0.06980	69.73247	1.24141	79.75003	4.15288
Maximum	0.10046	85.49404	2.84245	86.02026	5.29391
Minimum	0.05508	51.26657	0.55910	74.69611	3.54318
Standard Deviation	0.01145	11.30244	0.64573	3.55029	0.56650
Skewness	1.32770	-0.08989	0.93083	0.28851	0.77305
Excess Kurtosis	0.79878	-1.48599	-0.17050	-1.28217	-0.77442

Table 2.22: Time Series Model Results for Parameters of the MCH Function for Australian Females

Terms	Equations				
	dlogit(α)	dlog(β_1)	dlog(γ_1)	dlog(β_2)	dlog(γ_2)
constant	-0.0055	0.006475	0.037574 ***	0.0027499 ***	0.006142 *
(se)	0.0093	0.005395	0.009884	0.0005016	0.00254
dlogit(α).l1	-0.3060 *				
(se)	0.1216				
dlog(β_1).l1		-0.072478	-0.103093		
(se)		0.131573	0.241031		
dlog(γ_1).l1		-0.097211	-0.478693 ***		
(se)		0.063908	0.160907		
dlog(β_2).l1				-0.2850106 *	1.449231 *
(se)				0.1218055	0.61669
dlog(γ_2).l1				0.0464229 *	-0.413400 ***
(se)				0.0227981	0.115425

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

The time series results in Table 2.22 are used to generate forecasts and for bootstrapping to generate the confidence bands for life expectancy.

Figure 2.13: Life Expectancy Estimates and Forecasts with 95% Confidence Bands for the MCH Function and LC Models for Australian Females, by Age, 1950–2050

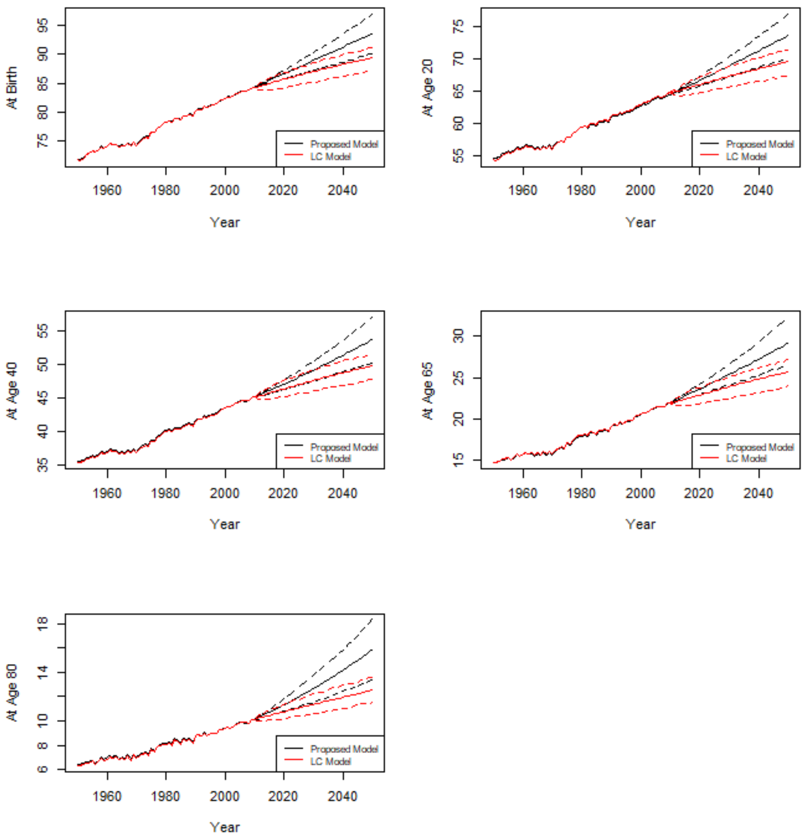


Figure 2.13 shows the forecasts results based on the Lee-Carter and the MCH function for Australian females. Similar to the results with the Australian males, the bands of the two models often intersect in all ages. However, the bands of the MCH model are of similar width as the LC

model in the plots. It can be inferred that the forecast quality of the two models are similar for the Australian females as well. Tables 2.23 and 2.24 show the MCH forecasts with 95% confidence limits for all ages under consideration.

Table 2.23: Estimates and Confidence Limits of Life Expectancy at Birth, Age 20 and Age 40 as Predicted by the MCH Function for Australian Females, 2011–2050

	$\tilde{e}_0$			$\tilde{e}_{20}$			$\tilde{e}_{40}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	84.57	84.40	84.71	64.69	64.54	64.84	45.31	45.18	45.48
2012	84.78	84.59	84.94	64.90	64.70	65.06	45.50	45.32	45.67
2013	85.02	84.73	85.23	65.12	64.84	65.33	45.69	45.43	45.91
2014	85.24	84.88	85.50	65.33	64.98	65.61	45.88	45.54	46.16
2015	85.47	85.02	85.80	65.55	65.11	65.89	46.07	45.65	46.42
2016	85.70	85.17	86.09	65.77	65.25	66.17	46.26	45.76	46.67
2017	85.93	85.32	86.39	65.99	65.39	66.45	46.46	45.88	46.93
2018	86.16	85.46	86.68	66.21	65.53	66.73	46.65	46.00	47.19
2019	86.39	85.61	86.97	66.44	65.66	67.02	46.85	46.11	47.45
2020	86.61	85.75	87.26	66.66	65.80	67.31	47.05	46.23	47.72
2021	86.84	85.90	87.56	66.88	65.94	67.60	47.25	46.35	47.99
2022	87.07	86.04	87.87	67.11	66.08	67.89	47.45	46.47	48.26
2023	87.30	86.19	88.19	67.33	66.23	68.20	47.66	46.60	48.52
2024	87.54	86.34	88.51	67.56	66.37	68.52	47.86	46.72	48.79
2025	87.77	86.49	88.81	67.79	66.51	68.82	48.07	46.84	49.06
2026	88.00	86.63	89.11	68.02	66.65	69.12	48.28	46.97	49.34
2027	88.23	86.78	89.41	68.25	66.80	69.43	48.49	47.09	49.64
2028	88.46	86.93	89.73	68.48	66.94	69.73	48.70	47.23	49.95
2029	88.69	87.07	90.04	68.71	67.09	70.06	48.91	47.37	50.24
2030	88.93	87.23	90.35	68.94	67.24	70.36	49.12	47.50	50.53
2031	89.16	87.38	90.65	69.17	67.39	70.67	49.34	47.62	50.81
2032	89.39	87.52	90.97	69.40	67.53	70.98	49.56	47.75	51.11
2033	89.63	87.67	91.28	69.63	67.67	71.29	49.78	47.89	51.40
2034	89.86	87.81	91.59	69.86	67.82	71.60	49.99	48.04	51.69
2035	90.09	87.96	91.91	70.10	67.96	71.92	50.22	48.19	51.99
2036	90.33	88.10	92.23	70.33	68.11	72.23	50.44	48.33	52.30
2037	90.56	88.25	92.55	70.57	68.25	72.55	50.66	48.48	52.62
2038	90.80	88.39	92.89	70.80	68.40	72.89	50.89	48.63	52.94
2039	91.03	88.54	93.22	71.03	68.54	73.22	51.11	48.76	53.27
2040	91.27	88.69	93.54	71.27	68.69	73.54	51.34	48.88	53.61
2041	91.51	88.83	93.87	71.51	68.83	73.87	51.57	49.01	53.91
2042	91.74	88.98	94.20	71.74	68.98	74.20	51.80	49.14	54.24
2043	91.98	89.12	94.53	71.98	69.12	74.53	52.03	49.27	54.57
2044	92.22	89.27	94.88	72.22	69.27	74.88	52.26	49.40	54.90
2045	92.45	89.42	95.23	72.45	69.42	75.23	52.49	49.54	55.24
2046	92.69	89.56	95.58	72.69	69.56	75.58	52.72	49.67	55.58
2047	92.93	89.71	95.92	72.93	69.71	75.92	52.96	49.80	55.92
2048	93.17	89.86	96.26	73.17	69.86	76.26	53.19	49.93	56.27
2049	93.41	90.01	96.61	73.41	70.01	76.61	53.43	50.07	56.68
2050	93.64	90.15	96.96	73.64	70.15	76.96	53.66	50.20	57.01

Table 2.24: Estimates and Confidence Limits of Life Expectancy at Ages 65 and 80 as Predicted by the MCH Function for Australian Females, 2011–2050

	$\tilde{e}_{65}$			$\tilde{e}_{80}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	22.10	21.98	22.26	10.26	10.15	10.39
2012	22.26	22.11	22.41	10.36	10.24	10.49
2013	22.43	22.22	22.63	10.48	10.30	10.64
2014	22.59	22.33	22.84	10.59	10.37	10.80
2015	22.76	22.43	23.06	10.71	10.43	10.97
2016	22.93	22.54	23.29	10.82	10.50	11.13
2017	23.10	22.65	23.51	10.94	10.57	11.29
2018	23.27	22.76	23.74	11.06	10.64	11.47
2019	23.44	22.86	23.97	11.19	10.71	11.64
2020	23.62	22.97	24.21	11.31	10.78	11.81
2021	23.79	23.09	24.44	11.44	10.85	11.99
2022	23.96	23.19	24.68	11.56	10.93	12.18
2023	24.14	23.30	24.92	11.69	11.00	12.36
2024	24.31	23.41	25.15	11.83	11.07	12.55
2025	24.49	23.52	25.39	11.96	11.14	12.75
2026	24.67	23.63	25.63	12.09	11.22	12.94
2027	24.84	23.75	25.86	12.23	11.30	13.14
2028	25.02	23.87	26.11	12.37	11.38	13.34
2029	25.20	23.99	26.36	12.51	11.46	13.54
2030	25.38	24.10	26.62	12.65	11.55	13.74
2031	25.56	24.22	26.87	12.80	11.63	13.94
2032	25.74	24.34	27.13	12.94	11.71	14.15
2033	25.93	24.46	27.39	13.09	11.79	14.35
2034	26.11	24.60	27.66	13.24	11.88	14.56
2035	26.29	24.69	27.92	13.39	11.96	14.77
2036	26.48	24.79	28.20	13.54	12.06	14.98
2037	26.67	24.89	28.48	13.70	12.15	15.19
2038	26.85	25.02	28.74	13.86	12.24	15.41
2039	27.04	25.16	29.03	14.01	12.33	15.62
2040	27.23	25.28	29.31	14.17	12.42	15.84
2041	27.42	25.41	29.61	14.33	12.51	16.08
2042	27.61	25.53	29.95	14.50	12.60	16.34
2043	27.81	25.67	30.26	14.66	12.69	16.56
2044	28.00	25.81	30.55	14.83	12.81	16.80
2045	28.19	25.93	30.83	14.99	12.91	17.08
2046	28.39	26.05	31.12	15.16	13.01	17.35
2047	28.59	26.17	31.43	15.33	13.11	17.59
2048	28.79	26.30	31.75	15.51	13.20	17.86
2049	28.99	26.44	32.04	15.68	13.30	18.10
2050	29.19	26.59	32.36	15.85	13.40	18.34

Tables 2.25 and 2.26 show the out-of-sample statistics and DM test results for forecasting, given the prescribed forecast periods with the LC model and the MCH function for males and females. For Australian males, the MCH function is the best-performing model over the LC model in the 10-year forecast, as can be concluded in both the MAE and DM test. The MCH model is better than the LC model in terms of the MAE for all ages except in expectancies for age 80. DM tests reveal that the MCH is the better model for forecasting for ages 20, 40 and 65. In-sample results are shown in Tables 2.27 and 2.28. Consistent with results from the US populations, the LC model has better fit in-sample, but this may again be a sign of overfitting.

Table 2.25: Out-of-Sample Statistics for 10-Year Forecasts, Australian Males

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At birth	0.62074	0.42631	2.83899	0.01944
Age 20	0.71884	0.34706	3.96654	0.00327
Age 40	0.59171	0.15300	4.03419	0.00295
Age 65	0.76250	0.39268	4.15062	0.00248
Age 80	0.34288	0.11355	3.62156	0.00556

Table 2.26: Out-of-Sample Statistics for 10-Year Forecasts, Australian Females

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At birth	0.35703	0.23530	1.22076	0.25319
Age 20	0.37140	0.18986	3.83924	0.00397
Age 40	0.39349	0.23707	1.97519	0.07967
Age 65	0.36024	0.11717	4.29905	0.00199
Age 80	0.10140	0.14839	-1.47060	0.17548

Table 2.27: In-Sample Results, Australian Males

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At birth	0.00659	0.02091	-11.11789	0.00000
Age 20	0.00046	0.20329	-12.24674	0.00000
Age 40	0.00025	0.07600	-10.38637	0.00000
Age 65	0.00009	0.04031	-6.33417	0.00000
Age 80	0.00006	0.06667	-5.89744	0.00000

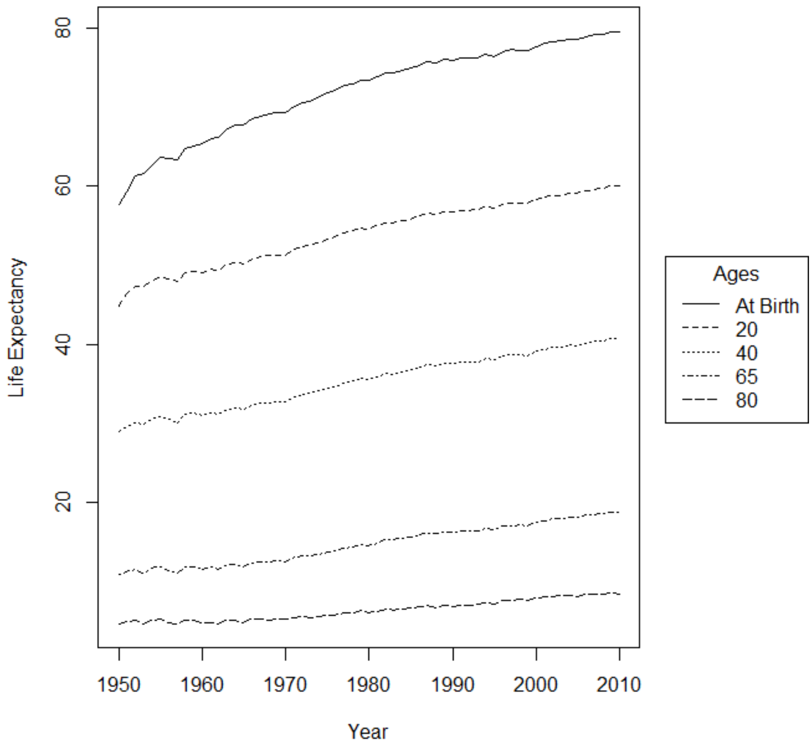
Table 2.28: In-Sample Results, Australian Females

Age	MAE		DM Test	
	LC	MCH	Statistic	P-Value
At birth	0.00496	0.06726	-25.08206	0.00000
Age 20	0.00062	0.14096	-8.32092	0.00000
Age 40	0.00025	0.18272	-15.00841	0.00000
Age 65	0.00012	0.08165	-10.63623	0.00000
Age 80	0.00006	0.08142	-10.71089	0.00000

2.5.3 Japanese Life Tables

In this part, we will discuss the results for the Japanese male and female life tables and forecasts generated by the LC and MCH functions. Figure 2.14 shows the life expectancy values based on the life tables for Japanese males. For all ages, there is an upward trend in life expectancy.

Figure 2.14: Life Expectancy of Japanese Males, by Age, 1950–2010



The parameter series of the MCH function for Japanese males are plotted in Figure 2.15, and the corresponding parameter series outputs are in Table 2.29. For all of the years, the MCH function fits the life tables with R-square always above 99.92%. The α of Japanese males has a negative trend from 1950 to 1970, and thereafter a fluctuating pattern. This means that the youth-to-adulthood component becomes less important than old-to-oldest in contributing to survivability. This is heavily related to the changes happening in Japan during the recovery period after the Second World War in which the country was demilitarised and young people do not have to serve in a military. For β_1, there is a positive trend from 1950 until 2003, after which the parameter drops to 63 years. Because this is a recent change, we find no other possible explanation yet in literature. The three other parameters have a positive trend for the years covered. Table 2.30 shows the summary statistics for the parameter series of Japanese males.

Figure 2.15: Parameter Estimates of the MCH Function for Japanese Males, 1950–2010

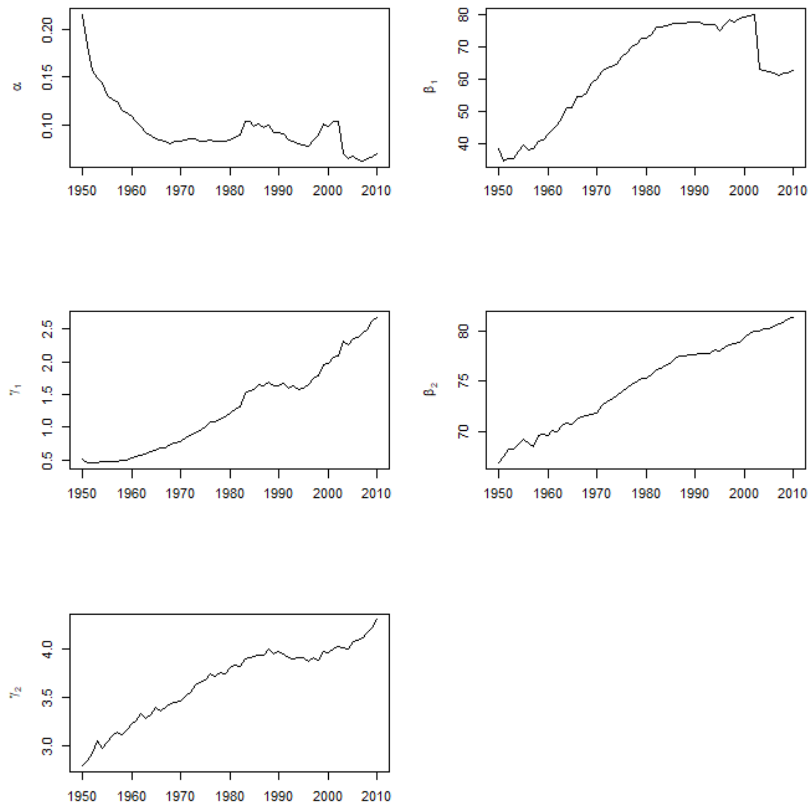


Table 2.29: Parameter Estimates of the MCH Function for Japanese Males, 1950–2010

Year	α	β_1	γ_1	β_2	γ_2	R-square
1950	0.21549	38.57509	0.52353	66.79142	2.79608	99.9251%
1951	0.18371	34.59698	0.45122	67.45875	2.84810	99.9523%
1952	0.15674	35.47897	0.44928	68.16717	2.92288	99.9687%
1953	0.14961	35.15594	0.45079	68.21528	3.05589	99.9704%
1954	0.14355	37.43371	0.47293	68.68896	2.97228	99.9729%
1955	0.13156	39.58168	0.48158	69.15668	3.04225	99.9779%
1956	0.12739	38.15562	0.47409	68.87804	3.10056	99.9780%
1957	0.12420	38.25818	0.47533	68.51652	3.14196	99.9795%
1958	0.11568	40.51858	0.48725	69.54812	3.11328	99.9821%
1959	0.11294	41.27394	0.49586	69.74509	3.16810	99.9833%
1960	0.10829	43.19349	0.52672	69.64072	3.23313	99.9844%
1961	0.10420	44.17410	0.54964	70.03409	3.26368	99.9851%
1962	0.09836	45.82406	0.57088	69.97825	3.33958	99.9853%
1963	0.09139	48.18742	0.58375	70.53426	3.28065	99.9881%
1964	0.08926	51.04898	0.62491	70.76181	3.32672	99.9894%
1965	0.08556	51.16884	0.64510	70.64005	3.39503	99.9898%
1966	0.08404	54.77156	0.69415	71.17525	3.35619	99.9915%
1967	0.08290	54.52610	0.68816	71.48076	3.40364	99.9909%
1968	0.08013	55.69208	0.73449	71.61480	3.43883	99.9919%
1969	0.08318	58.95695	0.76426	71.72265	3.45186	99.9919%
1970	0.08253	60.01200	0.78027	71.76261	3.46778	99.9919%
1971	0.08448	62.21052	0.84092	72.58238	3.52023	99.9922%
1972	0.08594	63.54172	0.88272	72.97604	3.54992	99.9921%
1973	0.08593	63.70738	0.92041	73.21454	3.62726	99.9916%
1974	0.08223	64.57284	0.95926	73.52631	3.65222	99.9920%
1975	0.08327	66.86221	1.01253	73.94658	3.68853	99.9920%
1976	0.08463	68.18724	1.08662	74.29408	3.74428	99.9918%
1977	0.08294	70.11805	1.09283	74.69657	3.71996	99.9917%
1978	0.08292	70.79283	1.11527	74.96431	3.75419	99.9919%
1979	0.08319	72.71511	1.15991	75.33241	3.74905	99.9917%
1980	0.08455	72.73993	1.22062	75.29440	3.81330	99.9917%
1981	0.08643	74.05922	1.27715	75.69091	3.83108	99.9914%
1982	0.08879	76.08941	1.31737	76.08941	3.82457	99.9911%
1983	0.10319	76.30817	1.51823	76.30817	3.90669	99.9903%
1984	0.10321	76.62933	1.56797	76.62933	3.91359	99.9897%
1985	0.09867	76.80577	1.57102	76.80577	3.93080	99.9907%
1986	0.10153	77.20815	1.64708	77.20815	3.93950	99.9907%
1987	0.09734	77.48660	1.63629	77.48660	3.94216	99.9906%
1988	0.09936	77.45730	1.69170	77.45730	4.00897	99.9913%
1989	0.09174	77.67252	1.63237	77.67252	3.95364	99.9923%
1990	0.09230	77.67176	1.64400	77.67176	3.98083	99.9919%
1991	0.09062	77.80809	1.67950	77.80810	3.95841	99.9925%
1992	0.08455	76.78158	1.59808	77.72450	3.91545	99.9926%
1993	0.08282	77.08109	1.62840	77.77137	3.90035	99.9932%
1994	0.08042	76.81540	1.58535	78.10229	3.90861	99.9935%
1995	0.07915	75.18190	1.59869	77.96159	3.91899	99.9944%
1996	0.07815	77.08933	1.65341	78.41920	3.87015	99.9953%
1997	0.08380	78.61243	1.75214	78.65088	3.92035	99.9955%
1998	0.08916	77.81530	1.79292	78.77233	3.89393	99.9957%
1999	0.10074	78.86500	1.95393	78.86500	3.98293	99.9958%
2000	0.09821	79.32649	1.96860	79.32649	3.96444	99.9958%
2001	0.10361	79.74632	2.06925	79.74632	4.00899	99.9962%
2002	0.10305	79.98065	2.08869	79.98065	4.02859	99.9962%
2003	0.06950	63.21254	2.32297	80.01229	4.01601	99.9960%
2004	0.06471	62.59584	2.26241	80.18428	4.00382	99.9962%
2005	0.06747	62.48650	2.34753	80.19030	4.08339	99.9957%
2006	0.06358	61.78938	2.37404	80.53078	4.08790	99.9958%
2007	0.06211	61.27078	2.44817	80.69388	4.11314	99.9955%
2008	0.06441	62.10658	2.48020	80.87773	4.17624	99.9955%
2009	0.06623	61.80345	2.61725	81.27079	4.21736	99.9949%
2010	0.07017	62.84384	2.68273	81.37873	4.30524	99.9943%

Table 2.30: Summary Statistics of Parameter Estimates for Japanese Males

	α	β_1	γ_1	β_2	γ_2
Mean	0.09600	62.66611	1.28890	74.79715	3.66301
Maximum	0.21549	79.98065	2.68273	81.37873	4.30524
Minimum	0.06211	34.59698	0.44928	66.79142	2.79608
Standard Deviation	0.02780	14.74658	0.66998	4.27492	0.38559
Skewness	2.17228	-0.57037	0.37101	-0.19953	-0.59809
Excess Kurtosis	6.13036	-1.02210	-1.01937	-1.29056	-0.75483

Table 2.31: Time Series Model Results for Parameters of the MCH Function for Japanese Males

Terms	Equations				
	dlogit(α)	dlog(β_1)	dlog(γ_1)	dlog(β_2)	dlog(γ_2)
constant	-0.0221	0.008174	0.029263 ***	0.0032316 ***	0.001424
(se)	0.0138	0.006256	0.005196	0.0007295	0.00219
dlogit(α).l1	0.2770 *				
(se)	0.1297				
dlog(β_1).l1		0.081849	0.191361 .		
(se)		0.12589	0.104565		
dlog(γ_1).l1		0.047256	-0.021176		
(se)		0.130532	0.108421		
dlog(β_2).l1				-0.082838	1.902190 ***
(se)				0.1324745	0.397797
dlog(γ_2).l1				0.0320808	-0.107248
(se)				0.0376874	0.113168

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

The estimates are used to generate forecasts and for bootstrapping to generate the confidence bands for life expectancy shown in Table 2.31, which has the time series model estimation results for the parameters of the MCH function for Japanese males.

Figure 2.16: Life Expectancy Estimates and Forecasts with 95% Confidence Bands for the MCH Function and Lee-Carter Models for Japanese Males, by Age, 1950–2050

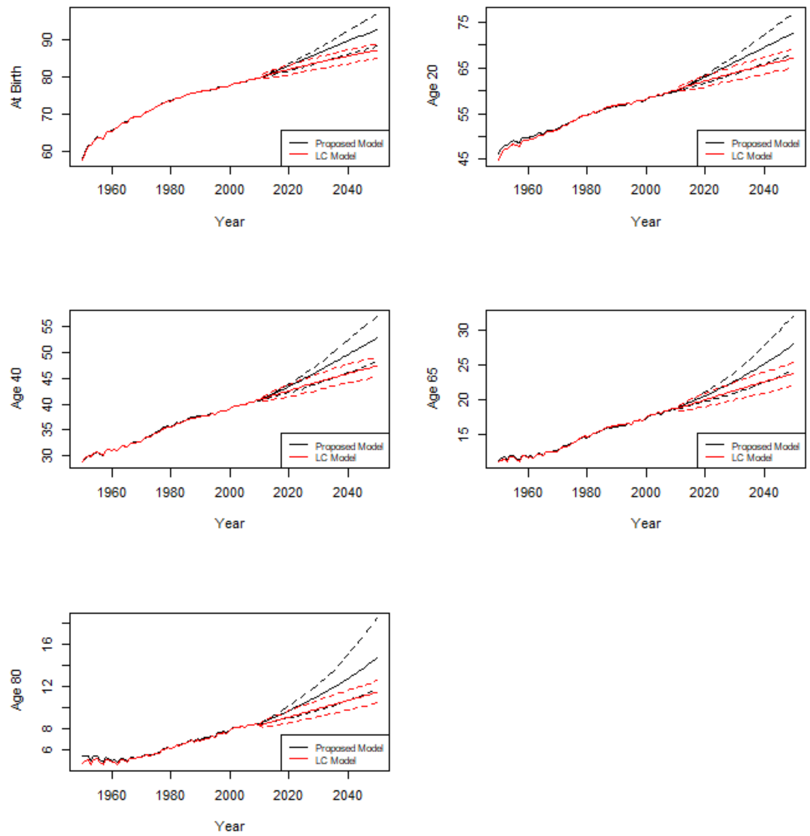


Figure 2.16 shows the forecasts results based on the LC model and the MCH function. The

bands of the two models often intersect in all ages. The bands of the MCH model are wider than those of the LC model in the plots. It can be inferred that, in cases where the bands intersect, the forecast quality of the two models is similar. Tables 2.32 and 2.33 show the MCH forecasts with 95% confidence limits for all ages under consideration.

Table 2.32: Estimates and Confidence Limits of Life Expectancy at Birth, Age 20, and Age 40 as Predicted by the MCH Function for Japanese Males, 2011–2050

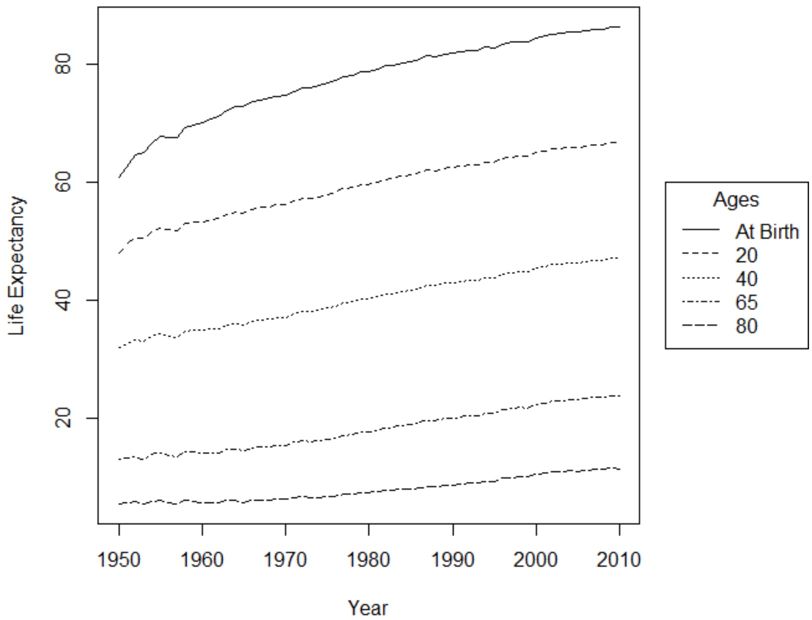
	$\tilde{e}_0$			$\tilde{e}_{20}$			$\tilde{e}_{40}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	79.95	79.75	80.03	60.14	59.94	60.22	40.99	40.81	41.06
2012	80.26	79.97	80.38	60.42	60.14	60.54	41.22	40.97	41.33
2013	80.60	80.17	80.75	60.74	60.32	60.89	41.48	41.13	41.62
2014	80.94	80.37	81.11	61.06	60.50	61.23	41.74	41.27	41.92
2015	81.28	80.57	81.49	61.38	60.69	61.59	42.00	41.42	42.22
2016	81.62	80.77	81.88	61.70	60.87	61.96	42.27	41.57	42.54
2017	81.95	80.97	82.26	62.02	61.07	62.33	42.54	41.72	42.86
2018	82.29	81.17	82.65	62.35	61.26	62.70	42.82	41.87	43.19
2019	82.62	81.38	83.04	62.67	61.44	63.09	43.10	42.03	43.52
2020	82.95	81.58	83.43	63.00	61.63	63.47	43.38	42.20	43.87
2021	83.29	81.78	83.84	63.32	61.83	63.87	43.66	42.37	44.22
2022	83.62	81.99	84.26	63.65	62.04	64.28	43.95	42.53	44.58
2023	83.95	82.20	84.67	63.97	62.24	64.69	44.24	42.70	44.95
2024	84.28	82.41	85.08	64.30	62.44	65.10	44.54	42.87	45.33
2025	84.61	82.62	85.50	64.62	62.64	65.51	44.83	43.04	45.72
2026	84.94	82.83	85.92	64.95	62.85	65.93	45.13	43.22	46.11
2027	85.27	83.05	86.34	65.27	63.06	66.35	45.43	43.41	46.49
2028	85.59	83.26	86.74	65.60	63.27	66.75	45.74	43.60	46.90
2029	85.92	83.47	87.16	65.92	63.48	67.17	46.04	43.80	47.32
2030	86.24	83.69	87.60	66.25	63.69	67.60	46.35	43.99	47.71
2031	86.57	83.90	88.03	66.57	63.91	68.03	46.66	44.17	48.10
2032	86.89	84.11	88.46	66.90	64.12	68.46	46.97	44.36	48.54
2033	87.22	84.33	88.93	67.22	64.33	68.93	47.28	44.56	48.99
2034	87.54	84.56	89.40	67.54	64.56	69.40	47.60	44.76	49.45
2035	87.87	84.79	89.89	67.87	64.79	69.89	47.91	44.96	49.91
2036	88.19	85.01	90.38	68.19	65.01	70.38	48.23	45.17	50.39
2037	88.51	85.23	90.88	68.51	65.23	70.88	48.54	45.37	50.89
2038	88.83	85.44	91.39	68.83	65.44	71.39	48.86	45.59	51.40
2039	89.16	85.67	91.85	69.16	65.67	71.85	49.18	45.78	51.86
2040	89.48	85.89	92.32	69.48	65.90	72.32	49.49	46.00	52.33
2041	89.80	86.12	92.77	69.80	66.13	72.77	49.81	46.23	52.78
2042	90.12	86.34	93.22	70.12	66.34	73.22	50.13	46.45	53.23
2043	90.44	86.56	93.74	70.44	66.56	73.74	50.45	46.69	53.75
2044	90.76	86.78	94.18	70.76	66.78	74.18	50.77	46.90	54.19
2045	91.08	86.99	94.66	71.08	67.00	74.66	51.09	47.11	54.66
2046	91.40	87.21	95.14	71.40	67.22	75.14	51.41	47.33	55.14
2047	91.72	87.43	95.62	71.72	67.43	75.62	51.73	47.55	55.62
2048	92.04	87.65	96.03	72.04	67.65	76.03	52.05	47.77	56.04
2049	92.36	87.87	96.46	72.36	67.87	76.46	52.37	47.99	56.47
2050	92.68	88.09	96.88	72.68	68.09	76.88	52.68	48.20	56.88

Table 2.33: Estimates and Confidence Limits of Life Expectancy at Ages 65 and 80 as Predicted by the MCH Function for Japanese Males, 2011–2050

		$\tilde{e}_{65}$			$\tilde{e}_{80}$	
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	18.89	18.71	18.95	8.63	8.46	8.66
2012	19.05	18.84	19.15	8.72	8.53	8.80
2013	19.24	18.96	19.39	8.84	8.59	8.96
2014	19.42	19.08	19.63	8.96	8.65	9.13
2015	19.61	19.19	19.88	9.08	8.72	9.30
2016	19.80	19.30	20.12	9.20	8.78	9.48
2017	19.99	19.42	20.37	9.33	8.84	9.66
2018	20.18	19.54	20.62	9.46	8.92	9.85
2019	20.38	19.65	20.87	9.58	8.99	10.03
2020	20.58	19.77	21.13	9.72	9.05	10.23
2021	20.77	19.88	21.39	9.85	9.12	10.42
2022	20.98	20.00	21.65	9.98	9.18	10.61
2023	21.18	20.12	21.92	10.12	9.25	10.80
2024	21.39	20.23	22.18	10.26	9.32	11.00
2025	21.60	20.35	22.47	10.40	9.40	11.20
2026	21.81	20.47	22.78	10.55	9.48	11.41
2027	22.02	20.60	23.08	10.69	9.56	11.62
2028	22.24	20.74	23.38	10.84	9.63	11.83
2029	22.47	20.88	23.69	10.98	9.71	12.05
2030	22.69	21.00	24.03	11.13	9.78	12.27
2031	22.92	21.13	24.36	11.29	9.86	12.51
2032	23.16	21.28	24.69	11.44	9.96	12.77
2033	23.40	21.41	25.04	11.60	10.03	13.00
2034	23.64	21.54	25.40	11.75	10.12	13.24
2035	23.89	21.70	25.77	11.91	10.20	13.49
2036	24.14	21.87	26.16	12.08	10.29	13.79
2037	24.39	22.01	26.57	12.24	10.38	14.10
2038	24.65	22.18	26.99	12.41	10.48	14.39
2039	24.91	22.34	27.44	12.59	10.58	14.72
2040	25.18	22.50	27.87	12.77	10.68	15.04
2041	25.45	22.65	28.28	12.95	10.78	15.41
2042	25.72	22.81	28.69	13.13	10.92	15.71
2043	26.00	22.97	29.13	13.32	11.02	16.05
2044	26.28	23.15	29.53	13.52	11.10	16.38
2045	26.56	23.36	29.97	13.72	11.23	16.73
2046	26.85	23.56	30.44	13.92	11.33	17.09
2047	27.14	23.72	30.90	14.13	11.45	17.44
2048	27.43	23.90	31.33	14.35	11.57	17.81
2049	27.72	24.16	31.73	14.57	11.68	18.15
2050	28.02	24.39	32.09	14.79	11.83	18.46

For Japanese females, figure 2.17 shows the life expectancy values based on the life tables. With the plots, there are consistent positive trends for all ages, although the trend weakens as the life expectancy is computed for older ages.

Figure 2.17: Life Expectancy of Japanese Females, by Age, 1950–2010



In Figure 2.18, the parameter series of the MCH function for Japanese females are plotted. The corresponding parameter series outputs are in Table 2.34. The minimum R-square for the fit of the MCH model for each year was 99.90%. The summary statistics are shown in Table 2.35 for the parameter series of Japanese females. Of interest, are the α and β_1 parameters. There is a negative trend for the α parameter from 1950 to 1970, which is attributed to the changing demographic conditions for young people after the Second World War. It is flat from 1970 until 1987, during which it shows values of approximately 0.49. The hypothesis is that it may be attributable to the impending stagnation of the Japanese economy which has resulted to demographic shifts of youth-to-adulthood component with respect to importance of survivability. There is one spike in 1996 in the parameter. For β_1, there is a positive trend from 1950 to 1986, until a shift in 1987 starting with a value of 58.49 years. We hypothesise that this is a demographic change before the stagnation of the Japanese economy during the 1990s. There is also a spike for β_1 in 1996. We have not found any possible explanation for the spike in this year. The other three parameters continue with a positive trend.

Figure 2.18: Parameter Estimates of the MCH Function for Japanese Females, 1950–2010

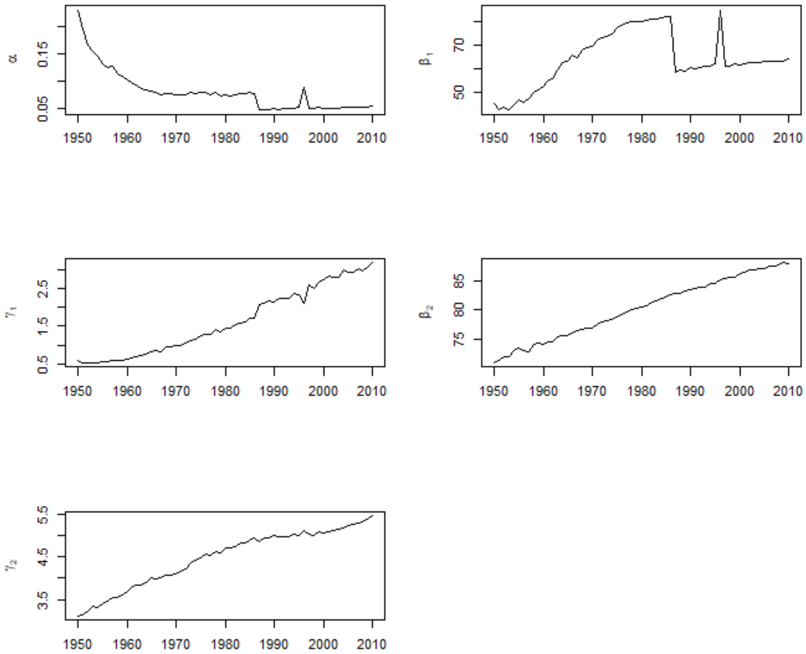


Table 2.34: Parameter Estimates of the MCH Function for Japanese Females, 1950–2010

Year	α	β_1	γ_1	β_2	γ_2	R-square
1950	0.22941	45.14258	0.60404	71.07496	3.11843	99.9078%
1951	0.19540	42.32700	0.53608	71.53936	3.15078	99.9291%
1952	0.16569	43.31979	0.54077	72.17216	3.21342	99.9478%
1953	0.15397	42.57639	0.52555	72.10370	3.33528	99.9516%
1954	0.14415	44.38190	0.54117	73.09203	3.31784	99.9582%
1955	0.13089	46.90405	0.55985	73.48823	3.38608	99.9644%
1956	0.12489	45.67742	0.54719	73.18660	3.46368	99.9659%
1957	0.12570	47.46283	0.57816	73.05246	3.53895	99.9652%
1958	0.11373	49.73151	0.57468	74.14545	3.53955	99.9702%
1959	0.10959	50.85660	0.59710	74.35410	3.59631	99.9717%
1960	0.10254	52.51506	0.62382	74.31565	3.69706	99.9742%
1961	0.09759	54.69413	0.66003	74.69040	3.77204	99.9753%
1962	0.09277	56.04459	0.67890	74.71203	3.84496	99.9759%
1963	0.08619	59.51500	0.71502	75.41622	3.84214	99.9794%
1964	0.08387	62.49584	0.76960	75.66044	3.89970	99.9808%
1965	0.08263	63.44918	0.82823	75.62041	4.00073	99.9815%
1966	0.07928	65.73305	0.86238	76.19119	3.97625	99.9855%
1967	0.07574	64.93712	0.83599	76.43196	4.00303	99.9856%
1968	0.07765	68.19006	0.93939	76.65172	4.07435	99.9865%
1969	0.07624	68.94757	0.95297	76.88738	4.06432	99.9879%
1970	0.07492	69.67219	0.97385	76.85555	4.10491	99.9886%
1971	0.07420	72.25394	0.98259	77.58028	4.15847	99.9880%
1972	0.07434	73.10204	1.04058	77.99166	4.19988	99.9891%
1973	0.07856	74.07119	1.12497	78.15981	4.35718	99.9877%
1974	0.07663	75.16480	1.15737	78.38598	4.41127	99.9887%
1975	0.07942	77.57392	1.24750	78.84119	4.48010	99.9885%
1976	0.07859	78.80699	1.29269	79.17368	4.55698	99.9891%
1977	0.07406	79.61427	1.26856	79.61427	4.53456	99.9893%
1978	0.07906	80.03613	1.40514	80.03613	4.62604	99.9900%
1979	0.07256	80.41455	1.34832	80.41455	4.59917	99.9892%
1980	0.07468	80.43746	1.43813	80.43747	4.69572	99.9905%
1981	0.07339	80.79836	1.44994	80.79836	4.71986	99.9897%
1982	0.07581	81.35631	1.54886	81.35631	4.73782	99.9891%
1983	0.07736	81.49515	1.58411	81.49515	4.81587	99.9901%
1984	0.07678	81.89461	1.61494	81.89461	4.83603	99.9901%
1985	0.07894	82.17910	1.71092	82.17910	4.88317	99.9900%
1986	0.07765	82.58462	1.71135	82.58462	4.92637	99.9892%
1987	0.04684	58.49697	2.06259	82.90244	4.84744	99.9866%
1988	0.04839	59.27251	2.09668	82.85125	4.93954	99.9869%
1989	0.04772	59.09350	2.16559	83.30124	4.92600	99.9868%
1990	0.04927	60.63757	2.15343	83.39695	5.00077	99.9882%
1991	0.04854	60.11128	2.23712	83.69301	4.98202	99.9876%
1992	0.04925	60.60219	2.23992	83.82693	4.97806	99.9878%
1993	0.04871	60.88126	2.23586	83.95542	4.97202	99.9886%
1994	0.04884	60.85598	2.37018	84.42032	5.02780	99.9885%
1995	0.05260	62.03333	2.33060	84.41272	5.01152	99.9891%
1996	0.08777	85.18326	2.10911	85.18326	5.10859	99.9906%
1997	0.04979	61.12447	2.58498	85.28492	5.03427	99.9891%
1998	0.05007	61.12635	2.51464	85.50204	5.00457	99.9895%
1999	0.05177	61.99869	2.67914	85.51330	5.07323	99.9906%
2000	0.05008	61.64081	2.74162	86.08501	5.05284	99.9908%
2001	0.05055	62.27369	2.81408	86.41798	5.09331	99.9913%
2002	0.05040	62.53686	2.80939	86.72125	5.10142	99.9908%
2003	0.05078	62.68927	2.79492	86.84914	5.15070	99.9909%
2004	0.05186	62.76452	2.97917	87.12789	5.17460	99.9913%
2005	0.05265	63.22782	2.93223	87.07403	5.22479	99.9920%
2006	0.05161	63.12175	2.91521	87.35252	5.24945	99.9917%
2007	0.05167	63.21398	3.02104	87.53386	5.29077	99.9915%
2008	0.05149	63.29514	2.95901	87.61913	5.32718	99.9919%
2009	0.05154	63.45037	3.05221	88.00591	5.38938	99.9919%
2010	0.05358	64.05089	3.17903	87.96732	5.46221	99.9918%

Table 2.35: Summary Statistics of Parameter Estimates for Japanese Females

	α	β_1	γ_1	β_2	γ_2
Mean	0.08017	64.09901	1.61227	80.25546	4.47378
Maximum	0.22941	85.18326	3.17903	88.00591	5.46221
Minimum	0.04684	42.32700	0.52555	71.07496	3.11843
Standard Deviation	0.03720	11.60997	0.87332	5.11616	0.67006
Skewness	1.98252	-0.00350	0.31305	-0.11150	-0.52730
Excess Kurtosis	4.61949	-0.66833	-1.35323	-1.27540	-1.00112

Table 2.36: Time Series Model Results for Parameters of the MCH Function for Japanese Females

Terms	Equations				
	dlogit(α)	dlog(β_1)	dlog(γ_1)	dlog(β_2)	dlog(γ_2)
constant	-0.0273 *	0.007955	0.035332 ***	0.0037330 ***	0.007339 *
(se)	0.0138	0.012052	0.007232	0.0008433	0.002772
dlogit(α).l1	-0.2411 .				
(se)	0.1262				
dlog(β_1).l1		-0.262313 .	0.133982		
(se)		0.152027	0.091229		
dlog(γ_1).l1		0.021116	-0.216165		
(se)		0.226821	0.136112		
dlog(β_2).l1				-0.2104506	0.709445
(se)				0.1348852	0.443328
dlog(γ_2).l1				0.0574622	-0.062825
(se)				0.0418252	0.137467

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

The time series results in Table 2.36 are used to generate forecasts and for bootstrapping to generate the confidence bands for life expectancy.

Figure 2.19: Life Expectancy Estimates and Forecasts with 95% Confidence Bands for the MCH Function and Lee-Carter Models for Japanese Females, by Age, 1950–2050

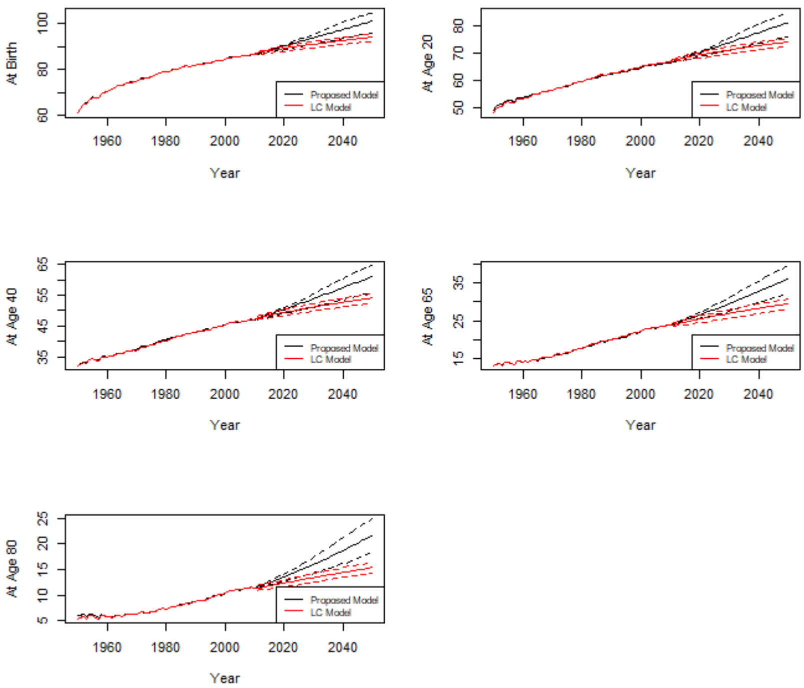


Figure 2.19 shows the forecasts results based on the LC model and the MCH function for Japanese females. Similar to the results for Japanese males, the bands of the two models often intersect in all ages. The bands of the MCH model are wider than those of the LC model in the plots. Tables 2.37 and 2.38 show the MCH forecasts with 95% confidence limits for all ages under consideration.

Table 2.37: Estimates and Confidence Limits of Life Expectancy at Birth, Age 20, and Age 40 as Predicted by the MCH Function for Japanese Females, 2011–2050

	$\tilde{e}_0$			$\tilde{e}_{20}$			$\tilde{e}_{40}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	86.83	86.50	86.83	66.91	66.58	66.91	47.45	47.14	47.45
2012	87.16	86.75	87.19	67.23	66.83	67.26	47.72	47.37	47.78
2013	87.55	86.98	87.60	67.61	67.05	67.66	48.06	47.57	48.14
2014	87.91	87.21	88.00	67.96	67.27	68.06	48.37	47.78	48.51
2015	88.28	87.45	88.41	68.32	67.50	68.46	48.70	47.98	48.88
2016	88.65	87.69	88.84	68.69	67.73	68.88	49.03	48.18	49.28
2017	89.02	87.92	89.28	69.05	67.96	69.31	49.36	48.38	49.67
2018	89.39	88.16	89.72	69.41	68.19	69.75	49.69	48.59	50.08
2019	89.75	88.39	90.16	69.77	68.42	70.18	50.02	48.80	50.48
2020	90.12	88.63	90.59	70.14	68.65	70.61	50.36	49.01	50.88
2021	90.49	88.86	91.02	70.50	68.88	71.04	50.70	49.23	51.29
2022	90.85	89.10	91.47	70.86	69.11	71.49	51.04	49.44	51.70
2023	91.22	89.33	91.95	71.22	69.35	71.96	51.38	49.65	52.14
2024	91.58	89.56	92.39	71.59	69.58	72.39	51.72	49.88	52.56
2025	91.95	89.79	92.85	71.95	69.80	72.85	52.07	50.09	53.00
2026	92.31	90.03	93.33	72.31	70.03	73.33	52.42	50.31	53.45
2027	92.68	90.26	93.80	72.68	70.27	73.80	52.77	50.52	53.91
2028	93.04	90.49	94.31	73.04	70.50	74.31	53.12	50.74	54.39
2029	93.40	90.73	94.86	73.41	70.74	74.87	53.47	50.95	54.92
2030	93.77	90.98	95.41	73.77	70.98	75.41	53.83	51.17	55.47
2031	94.13	91.22	95.95	74.13	71.22	75.96	54.18	51.41	55.98
2032	94.50	91.46	96.46	74.50	71.46	76.46	54.54	51.64	56.48
2033	94.86	91.70	96.97	74.86	71.70	76.97	54.89	51.87	56.99
2034	95.22	91.93	97.48	75.23	71.94	77.48	55.25	52.11	57.50
2035	95.59	92.17	98.00	75.59	72.17	78.00	55.61	52.34	58.01
2036	95.95	92.41	98.51	75.95	72.41	78.51	55.97	52.58	58.52
2037	96.32	92.66	99.00	76.32	72.66	79.00	56.33	52.81	59.01
2038	96.68	92.91	99.46	76.68	72.91	79.46	56.69	53.05	59.47
2039	97.05	93.16	99.93	77.05	73.16	79.93	57.06	53.29	59.93
2040	97.41	93.42	100.42	77.41	73.42	80.42	57.42	53.53	60.42
2041	97.77	93.67	100.92	77.77	73.67	80.92	57.78	53.78	60.92
2042	98.14	93.92	101.41	78.14	73.92	81.41	58.14	54.03	61.42
2043	98.50	94.18	101.85	78.50	74.18	81.85	58.50	54.28	61.86
2044	98.86	94.43	102.25	78.86	74.43	82.25	58.87	54.53	62.26
2045	99.22	94.68	102.65	79.22	74.68	82.65	59.23	54.79	62.65
2046	99.58	94.93	103.05	79.58	74.93	83.05	59.59	55.04	63.05
2047	99.94	95.18	103.44	79.94	75.18	83.44	59.94	55.31	63.44
2048	100.30	95.43	103.82	80.30	75.43	83.82	60.30	55.57	63.82
2049	100.66	95.69	104.20	80.66	75.69	84.20	60.66	55.85	64.20
2050	101.01	95.95	104.57	81.01	75.95	84.57	61.01	56.12	64.57

Table 2.38: Estimates and Confidence Limits of Life Expectancy at Ages 65 and 80 as Predicted by the MCH Function for Japanese Females, 2011–2050

	$\tilde{e}_{65}$			$\tilde{e}_{80}$		
Years	Estimate	Lower CI	Upper CI	Estimate	Lower CI	Upper CI
2011	24.09	23.84	24.13	11.69	11.48	11.73
2012	24.31	24.04	24.43	11.85	11.62	11.95
2013	24.60	24.24	24.76	12.06	11.75	12.20
2014	24.86	24.42	25.10	12.25	11.88	12.46
2015	25.14	24.61	25.45	12.45	12.01	12.71
2016	25.41	24.80	25.79	12.66	12.14	12.98
2017	25.69	24.99	26.15	12.87	12.28	13.25
2018	25.97	25.17	26.50	13.08	12.41	13.54
2019	26.25	25.36	26.86	13.30	12.55	13.84
2020	26.53	25.55	27.21	13.52	12.69	14.13
2021	26.81	25.73	27.58	13.74	12.84	14.43
2022	27.10	25.92	27.95	13.97	12.99	14.73
2023	27.39	26.11	28.32	14.20	13.14	15.04
2024	27.68	26.30	28.69	14.44	13.29	15.35
2025	27.97	26.50	29.07	14.68	13.44	15.67
2026	28.27	26.69	29.46	14.92	13.60	15.99
2027	28.57	26.90	29.84	15.17	13.76	16.32
2028	28.87	27.11	30.24	15.42	13.92	16.67
2029	29.17	27.33	30.68	15.68	14.08	17.01
2030	29.48	27.56	31.12	15.93	14.25	17.38
2031	29.79	27.77	31.59	16.19	14.43	17.77
2032	30.10	28.00	32.02	16.46	14.60	18.18
2033	30.41	28.21	32.47	16.73	14.78	18.55
2034	30.73	28.42	32.95	17.00	14.95	18.91
2035	31.05	28.66	33.43	17.27	15.13	19.30
2036	31.37	28.88	33.89	17.55	15.31	19.68
2037	31.69	29.11	34.31	17.83	15.51	20.06
2038	32.02	29.33	34.73	18.11	15.69	20.44
2039	32.35	29.55	35.14	18.39	15.89	20.82
2040	32.68	29.78	35.56	18.68	16.09	21.19
2041	33.01	29.99	36.01	18.97	16.29	21.56
2042	33.35	30.25	36.49	19.25	16.49	21.94
2043	33.69	30.53	36.91	19.55	16.69	22.31
2044	34.03	30.80	37.31	19.84	16.90	22.72
2045	34.37	31.05	37.70	20.13	17.10	23.13
2046	34.71	31.30	38.09	20.43	17.32	23.50
2047	35.05	31.55	38.48	20.72	17.56	23.84
2048	35.40	31.82	38.85	21.02	17.84	24.17
2049	35.74	32.06	39.22	21.32	18.08	24.51
2050	36.08	32.30	39.58	21.61	18.30	24.88

Tables 2.39 and 2.40 show the out-of-sample statistics and Diebold–Mariano test results for forecasting, given the prescribed forecast periods with the LC model and the MCH function for males and females. Results from Japan are mixed for males and females. The MAE of the MCH for Japanese males is lower for life expectancies at birth, age 20 and age 80, but the DM test concludes similar forecasting power for the LC and MCH. For Japanese females, the MCH model has lower MAE than the LC model for all ages except age 65. The DM test concludes that the MCH model has better forecasting ability for life expectancies at birth, age 20 and age 40. In-sample results are shown in Tables 2.41 and 2.42. Consistent with results from the US and Australian populations, the LC model has better fit in-sample, but again overfitting may be present.

Table 2.39: Out-of-Sample Statistics for 10-Year Forecasts, Japanese Males

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At birth	0.80289	0.74185	-0.49874	0.62992
Age 20	0.68054	0.57719	0.18197	0.85964
Age 40	0.40415	0.42767	-0.88465	0.39936
Age 65	0.08324	0.28548	-2.68171	0.02514
Age 80	0.26527	0.20934	0.81254	0.43745

Table 2.40: Out-of-Sample Statistics for 10-Year Forecasts, Japanese Females

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At birth	1.39886	0.64645	10.43369	0.00000
Age 20	1.14723	0.57424	5.34545	0.00047
Age 40	0.87351	0.47568	4.76764	0.00102
Age 65	0.19581	0.38845	-2.07317	0.06801
Age 80	0.35985	0.31179	-0.00700	0.99457

Table 2.41: In-Sample Results, Japanese Males

Age	MAE		DM Test (LC-MCH)	
	LC	MCH	Statistic	P-Value
At birth	0.00691	0.06992	-12.59271	0.00000
Age 20	0.00060	0.28746	-4.05150	0.00018
Age 40	0.00030	0.09324	-7.74054	0.00000
Age 65	0.00008	0.08486	-4.30786	0.00008
Age 80	0.00007	0.11544	-2.71567	0.00906

Table 2.42: In-Sample Results, Japanese Females

Age	MAE		DM Test	
	LC	MCH	Statistic	P-Value
At birth	0.00542	0.07636	-8.43138	0.00000
Age 20	0.00075	0.30200	-4.13054	0.00014
Age 40	0.00038	0.13437	-10.39194	0.00000
Age 65	0.00012	0.08792	-9.17589	0.00000
Age 80	0.00007	0.11765	-3.06734	0.00348

2.6 Conclusion and Summary

As financial institutions and government agencies face increasing demand for elderly healthcare services because of increasing life expectancy and a growing elderly population, they should prepare for increasing longevity risks by having appropriate models that account for the changing population and mortality dynamics.

We propose the MCH model, which considers different components of mortality and is adaptable to the changes in longevity through time by using autoregressive models. Confidence intervals for forecasted life expectancies are generated through bootstrapping techniques. The model is demonstrated on US, Australian and Japanese life tables data. In the estimation of the parameters, α and β_1 tend to be more volatile than the three other parameters in the MCH model. We see this as the youth-to-adulthood component of survivability being more volatile and dynamic than old-to-oldest component. It may mean that this aspect of survivability is more sensitive to demographic changes than the old-to-oldest. In terms of its long-term forecasts, the MCH model tends to give higher life expectancies, compared with the LC model. This is attributed with the emphasis of the MCH model on describing two components of longevity: youth-to-adulthood and old-to-oldest. In the estimation for the three countries, often the old-to-oldest has a higher share as seen in the values attained among the three countries and between the sexes. This leads to the observed higher life expectancies as the survivability of the old-to-oldest ages are emphasised. This has led to better forecasts seen from the MCH over the LC model in most of the ages and sexes of the countries under demonstration in terms of the MAE and the DM test.

We note the difference in forecasting performance among the countries under study, which emphasises that there can be no one model to describe all populations; thus, researchers should trial more than one family of models to search for the optimal fit for their specific situation.

Chapter 3

Disaster Risk: Storm-at-Risk Using Extreme Value Theory

3.1 Introduction

With climate change, the return periods of storms are decreasing, implying increased frequency through time. In terms of economics, storms tend to have a negative effect on growth, with the agricultural sector most heavily affected (World Bank and United Nations 2010). Disaster risk has also affected the insurance industry in the United Kingdom, particularly with floods, storms and drought, with the rate of change in these risks rapidly increasing (Dlugolecki 2008).

Disasters are considered extreme events, and their occurrences are more frequent than expected if they are assumed to follow a normal distribution. More often, disasters are a cause for or caused by a cascade of other disasters. Storms are often the main factor in these cascades, leading to other disasters such as dam overflows, floods, death, loss of livelihood and homelessness. Therefore, disaster risk mitigation and management are imperative for susceptible countries to prepare for such disasters (Helbing, Ammoser & Khnert 2006).

The focus of this research is on hurricanes as disasters, and the goal of estimating the characteristics of the worst possible storms. The number of storms entering the Philippine jurisdiction, part of the western North Pacific basin, may not have significant trends, but the impact has been increasing in terms of economic losses and damages (Cinco et al. 2016). The proposed methodology is applied with typhoon data from the western Northern Pacific basin in understanding the phenomenon for disaster risk management and climate change adaptation.

The flow of this chapter is as follows. An introduction to disaster risk is discussed in the first part, followed by a discussion on extreme value theory methods in the second part. The proposed methodology called storm-at-risk is outlined in the third part. We apply the proposed risk measure to the western North Pacific basin cyclone data and perform robustness checks on different levels of return periods and risk probabilities in the fourth part. Finally, a summary and some concluding remarks are expressed in the fifth part.

3.2 Extreme Value Theory Methods

We use extreme value statistics to generate the proposed methodology. Within the theory of extreme values, we define a random variable variable Z having the GEV distribution $G(z) \sim GEV(\mu, \sigma, \xi)$ if and only if (Fisher & Tippet 1928; Gnedenko 1943):

$$G(z) = exp\left\{-\left[1+\xi\frac{z-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\right\} \tag{3.1}$$

$$\text{for } 1+\xi\frac{z-\mu}{\sigma} \geq 0, \quad \sigma > 0, \mu \in \mathbb{R}, \xi \in \mathbb{R}$$

The random vector $(Z_1, Z_2)'$ with marginals G_1 and G_2 has the bivariate exteme value distribution (BEVD) if and only if (Berlaint et al. 2004; Pickands 1981):

$$G(z_1, z_2) = \exp\left\{\log\left[G_1(z_1)G_2(z_2)\right] A\left[\frac{\log\left[G_2(z_2)\right]}{\log\left[G_1(z_1)G_2(z_2)\right]}\right]\right\} \tag{3.2}$$

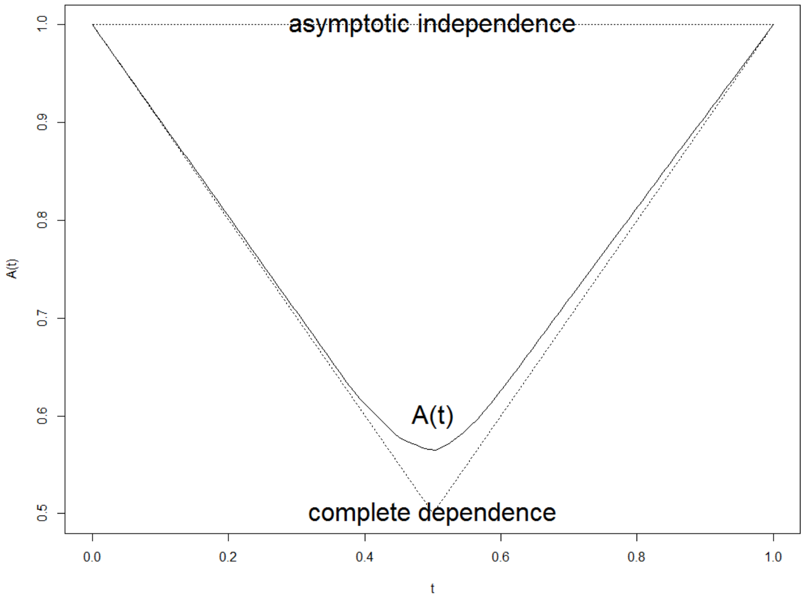
The function $A(\bullet)$ is the Pickands (or tail) dependence function, which describes the dependence between the two random variables. The Pickands function has the following properties: $\max\{t, 1-t\} \leq A(t) \leq 1$ for $t \in [0,1]$ and $A(\bullet)$ is convex. An example of a Pickands function is shown in Figure 3.1. When $A(t) = 1$ for all t, then there is asymptotic independence. When $A(t) = \max\{t, 1-t\}$, the two random variables have complete dependence. the Pickands dependence function is also used for extracting the measure for asymptotic independence χ defined as:

$$\chi = \lim_{u \uparrow 1} P\left[G_2(Z_2) > u | G_1(Z_1) > u\right] \tag{3.3}$$

In terms of the Pickands dependence function, χ has the formula:

$$\chi = 2 - 2A\left(1/2\right) \tag{3.4}$$

Figure 3.1: Example of the Pickands Dependence Function



There are three approaches (Pickands 1981; Berlaint 2004; Stephenson 2002) for estimating the BEVD:

1. nonparametric approach, no fitting of the GEV distribution on the marginal data and estimating $A(\bullet)$ nonparametrically,

2. semiparametric approach, fitting the GEV distribution on the marginal but estimating $A(\bullet)$ nonparametrically, and

3. parametric approach, fiiting a GEV distribution and the dependence between variables is modeled by a copula, specifically extreme value copulas.

Only the first two approaches were used for purposes of proposing a methodology. We have forgone the estimation through parametric means because prevailing parametric models available in statistical software do not fit well with the estimated dependence function of the data.

Given an estimator for the marginals $\hat{G}_1$ and $\hat{G}_2$ for the bivariate data $\{(z_{1,i}, z_{2,i})\}_{i=1}^n$, let $x_i = -\log \hat{G}_1(z_{1,i})$ and $y_i = -\log \hat{G}_2(z_{2,i})$. the Pickands estimator for $A(\bullet)$ is (Pickands 1981):

$$\hat{A}_n^P(t) = \left\{ \frac{1}{n} \sum_{i=1}^{n} \min\left(\frac{x_i}{1-t}, \frac{y_i}{t} \right) \right\}^{-1}, \quad t \in [0,1]. \tag{3.5}$$

The estimator $\hat{A}_n^P(t)$ is not a valid dependence function, so the estimator that will be used is based on the adjustment by Hall and Tajvidi (2000), where $\bar{x}$ and $\bar{y}$ are corresponding marginal sample means:

$$\hat{A}_n^{HT}(t) = \left\{ \frac{1}{n} \sum_{i=1}^{n} \min\left(\frac{x_i/\bar{x}}{1-t}, \frac{y_i/\bar{y}}{t} \right) \right\}^{-1}, \quad t \in [0,1]. \tag{3.6}$$

Another problem is that $\hat{A}_n^{HT}(t)$ is not necessarily convex, so the convex minorant is finally used, i.e., The largest convex function $\hat{A}_n^{HTconv}(t)$ in $[0,1]$ bounded by $\hat{A}_n^{HT}(t)$. The choice of setting up the convex minorant in estimation is available in the R package "evd" (Stephenson 2002) as is automated. We explain how the convex minorant is achieved by the following approach (Hall and Tajvidi 2000). Let $0 = t_0 < t_1 < ... < t_{m-1} < t_m = 1$ be a set of candidate points in $[0,1]$ and $s > 0$ is a smoothing parameter, and P^3 is a set of polynomial smoothing spline functions of degree 3. The convex minorant function is specified below:

$$\hat{A}_n^{HTconv}(t) = \arg\inf_{f_s(t) \in P^3} \left[\sum_{i=1}^{m} \left\{ \hat{A}_n^{HT}(t_j) - f_s(t_j) \right\}^2 + s \int_0^1 f_s^{''}(t)dt \right] \tag{3.7}$$

The choice of m has been declared by Hall and Tajvidi (2000) as not relevant so one may choose to have higher values. The choice of s is made by cross-validation of which $\int E(f_s - f)^2$ is minimised.

Based on estimates of the marginal distributions and the dependence function $\hat{A}_n^{HTconv}(t)$, the estimated joint cumulative distribution $\hat{G}(z_1, z_2)$ is (Berlaint 2004):

$$\hat{G}(z_1, z_2) = \exp\left\{ \log\left[\hat{G}_1(z_1)\hat{G}_2(z_2) \right] \hat{A}_n^{HTconv} \left[\frac{\log\left[\hat{G}_2(z_2)\right]}{\log\left[\hat{G}_1(z_1)\hat{G}_2(z_2)\right]} \right] \right\} \tag{3.8}$$

From the estimated joint distribution, other statistics can be generated, such as the $100p\%$ cumulative quantile curve $Q\left(\hat{G}, p\right)$, defined as follows:

$$Q\left(\hat{G}, p\right) = \left\{ (y_1, y_2) : \hat{G}(y_1, y_2) = p \right\} \tag{3.9}$$

and the $100p\%$ survival quantile curve $Q\left(\hat{G}, p\right)$,defined as follows:

$$Q_S\left(\hat{G}, p\right) = \left\{ (y_1, y_2) : P\left[Z_1 > y_1, Z_2 > y_2 | \hat{G} \right] = p \right\}. \tag{3.10}$$

The survival quantile curves are used later for the proposed methodology of estimating disaster risk of typhoons.

The proposed methodology would incorporate bootstrapping in the estimation process of the multivariate extreme value. Bootstrapped extreme value theory is performed for estimating the parameters of a conditional extreme value model (Heffernan & Tawn 2004) and the estimation of the stable tail dependence function, $l(\bullet)$ which simplifies to the Pickands dependence function for bivariate extreme value theory (Peng & Qi 2008).

3.3 Proposed Methodology

The proposed methodologies are an extension of the bootstrapped bivariate extreme value theory (Peng & Qi 2008) where $100p\%$ survival quantile curves are estimated and bootstrap confidence curves are created. These proposed survival curves for hurricanes would be called storm-at-risk curves. These proposed survival curves for hurricanes are called storm-at-risk curves. These curves are extensions in concept of the financial risk measure known as value-at-risk (Jorion 2006), defined as the maximum level of loss in holding a financial asset given a risk probability that such level is exceeded. The bootstrapping methodology provides confidence bands for the curves. The proposed methodologies may be estimated nonparametrically or semiparametrically; both methods are demonstrated. The advantages and disadvantages of the methods are discussed.

The steps in creating these curves are as follows:

- let $\{(z_{1,i}, z_{2,i})\}_{i=1}^{n}$ be the bivariate data.
- generate the direct estimates for $\hat{A}_n^{HTconv}(t), t \in [0,1]$, $\hat{\chi} = 2 - 2\hat{A}_n^{HTconv}(1/2)$, and $Q_S\left(\hat{G}, p\right)$ from $\{(z_{1,i}, z_{2,i})\}_{i=1}^{n}$ as in equations (3.6) with adjustment to convexity in equation (3.7), (3.4), and (3.10) respectively.
- suppose n_B is the desired number of boostrap samples. For $j = 1, 2, \ldots, n_B$
    1. let $\{(\tilde{z}_{1,i,j}, \tilde{z}_{2,i,j})\}_{i=1}^{n}$ be the bootstrap sample generated from $\{(z_{1,i}, z_{2,i})\}_{i=1}^{n}$
    2. generate the estimates for $\tilde{\hat{A}}_{n,j}^{HT}(t), t \in [0,1]$, $\tilde{\hat{\chi}}_j = 2 - 2\tilde{\hat{A}}_{n,j}^{HTconv}(1/2)$, and $Q_S\left(\tilde{\hat{G}}_j, p\right)$ from $\{(\tilde{z}_{1,i,j}, \tilde{z}_{2,i,j})\}_{i=1}^{n}$, where

$$Q_S\left(\tilde{\hat{G}}_j, p\right) = \left\{(y_{1,j}, y_{2,j}) : P\left[Z_1 > y_1, Z_2 > y_2 | \tilde{\hat{G}}_j\right] = p\right\} \tag{3.11}$$

For the purpose of inference, confidence intervals can be produced from the bootstrap results. The $(1-\alpha)\,100\%$ confidence interval for the Pickands function (Peng and Qi 2008): for every $t \in [0,1]$

$$\left(\tilde{\hat{A}}^{HTconv}_{n,\left(\left[n_B \frac{\alpha}{2}\right]\right)}(t), \tilde{\hat{A}}^{HTconv}_{n,\left(\left[n_B \frac{2-\alpha}{2}\right]\right)}(t)\right). \tag{3.12}$$

For the measure of asymptotic dependence, the $(1-\alpha)\,100\%$ confidence interval is:

$$\left(\tilde{\hat{\chi}}_{\left(\left[n_B \frac{\alpha}{2}\right]\right)}, \tilde{\hat{\chi}}_{\left(\left[n_B \frac{2-\alpha}{2}\right]\right)}\right) \tag{3.13}$$

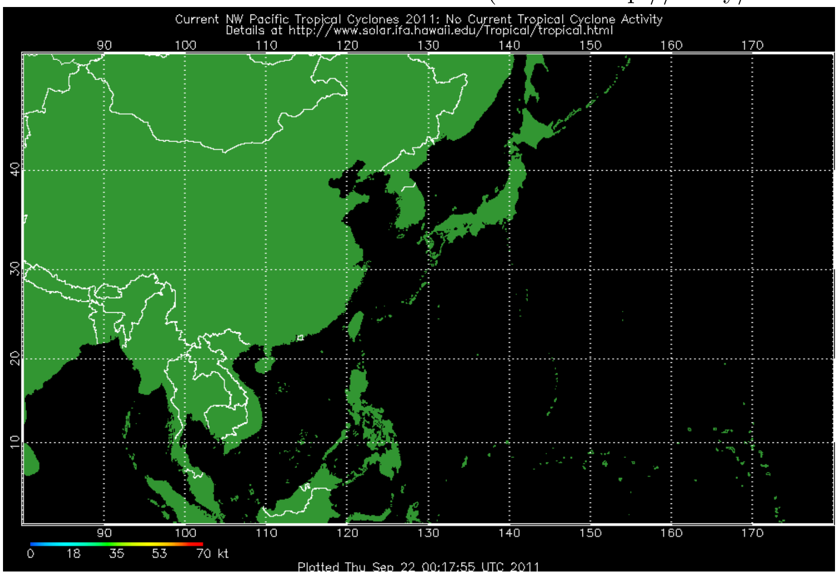
The $(1-\alpha)\,100\%$ confidence band for the $100p\%$ storm-at-risk curve is:

$$\left(Q_S\left(\tilde{\hat{G}}_{\left(\left[n_B \frac{\alpha}{2}\right]\right)}, p\right), Q_S\left(\tilde{\hat{G}}_{\left(\left[n_B \frac{2-\alpha}{2}\right]\right)}, p\right)\right) \tag{3.14}$$

3.4 Data Application: Tropical Systems in the Western North Pacific Basin

A tropical cyclone (TC) is a cyclone occurring in the western North Pacific basin, defined as the region of the Pacific Ocean between 0N and 60N, and between 100E and 180E. (World Meteorological Organization 2015). An image of the western North Pacific basin is shown in Figure 3.2.

Figure 3.2: The Western North Pacific Basin (Source: http://bit.ly/2BGrDWV)



The hurricane data for the demonstration is from the International Best Track Archive for Climate Stewardship (IBTrACs) database (Knapp et al. 2010), for the years 1881 to 2014. The span of time for the demonstration is from 1977 to 2014, a period of 37 years. This span of data is most reliable with respect to identifying hurricanes that have entered the western North Pacific basin, the only storms considered in the demonstration. From each TC, two variables were gathered: (1) wind speed in knots and (2) barometric pressure in millibars measured every 6 hours. For practical purposes, the negative of barometric pressure is used in estimation, as low barometric pressure is an indicator of storm generation. The component-wise maxima of wind speed and negative pressure are gathered from each TC. The data are refined further by using only data with positive wind speed; TCs with zero wind speed are removed. Overall, 942 individual TCs are included in the demonstration data.

The following outputs are displayed for both nonparametric and semiparametric approaches: (1) the Pickands dependence function, with 95% confidence intervals; (2) the estimated , with 95% confidence intervals; (3) the estimated 5% storm-at-risk curve, with 95% confidence bands; (4) the once-in-10-years (0.3928%) storm-at-risk curve, with 95% confidence bands; and (5) the once-in-100-years (0.03928%) storm-at-risk curve, with 95% confidence bands.

Robustness checks on the proposed approaches are conducted. The hold-out period spans from 2005 to 2014, with 227 storms in the 10-year span. The following risk probabilities and return periods are selected for the checks: 10% and 5% probabilities, and 5-year, 7-year, 10-year, 15-year and 20-year return periods. The proportion of observations lying outside the curve is the statistic used for the robustness check, as the closest or lower the proportion to the desired risk probability, the better.

Figure 3.3: Histogram of Wind Speed in Knots

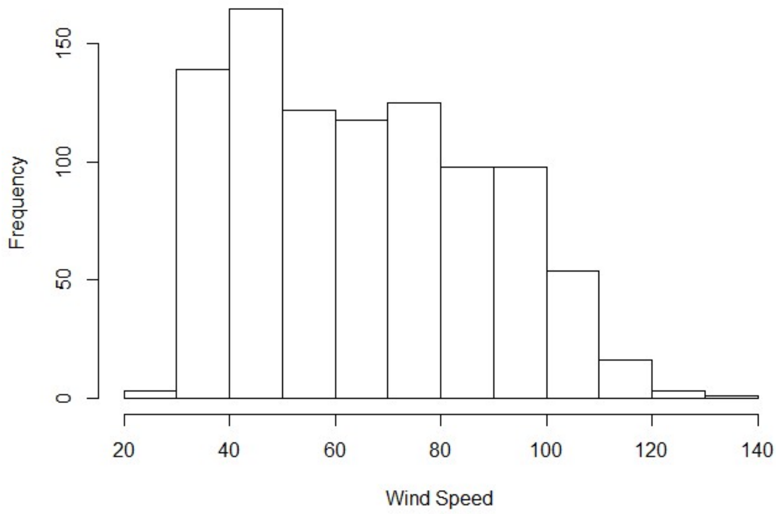


Figure 3.3 shows the histogram of the component-wise maximum wind speed in the life of each storm. It is skewed to the right, with most being less than 100 knots.

Figure 3.4: Histogram of Negative Barometric Pressure in Millibars

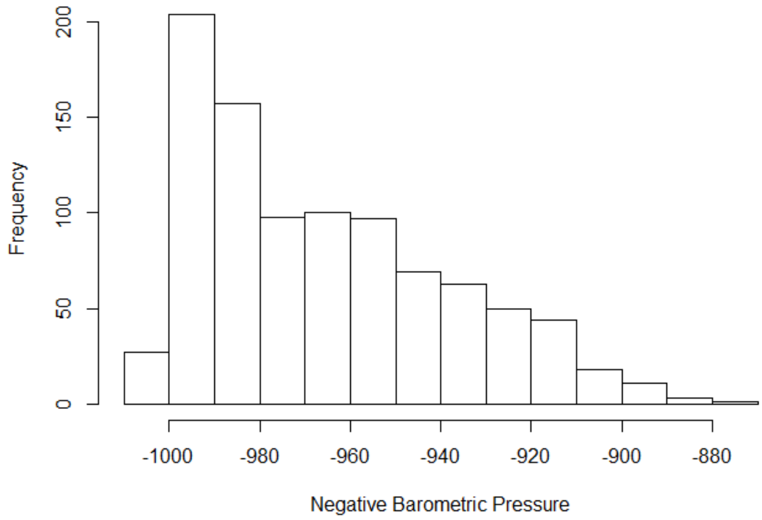


Figure 3.4 shows the histogram of the component-wise maximum negative barometric pressure in millibars in the life of each storm. It is skewed to the right, most frequently in the −1000 to −990 millibars.

Table 3.1: Summary Statistics for the Componentwise Maxima of Tropical Cyclones

	Wind Speed	Negative Barometric Pressure
Mean	68.16	-963.40
Standard Deviation	23.00512	27.98223
Skewness	0.33846	0.66403
Excess Excess Kurtosis	2.10558	2.52472
Minimum	25	-1006
1st Quartile	50	-985
Median	65	-970
3rd Quartile	85	-945
Maximum	140	-870

Table 3.1 shows the descriptive statistics of the component-wise maxima data. With regard to the shape of their distributions, both variables exhibit positive skewness and heavy tail features.

Figure 3.5: Scatterplot of Componentwise Maxima

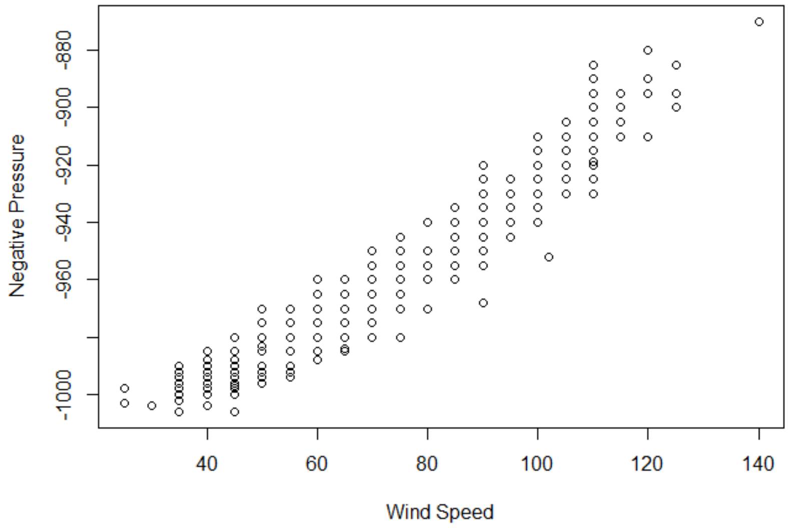


Figure 3.5 shows the scatterplot of the component-wise maxima for each storm. There is a strong positive relationship between extreme wind speed and extreme negative barometric pressure of a cyclone.

Figure 3.6: The Estimated Pickands Dependence Function by Approach

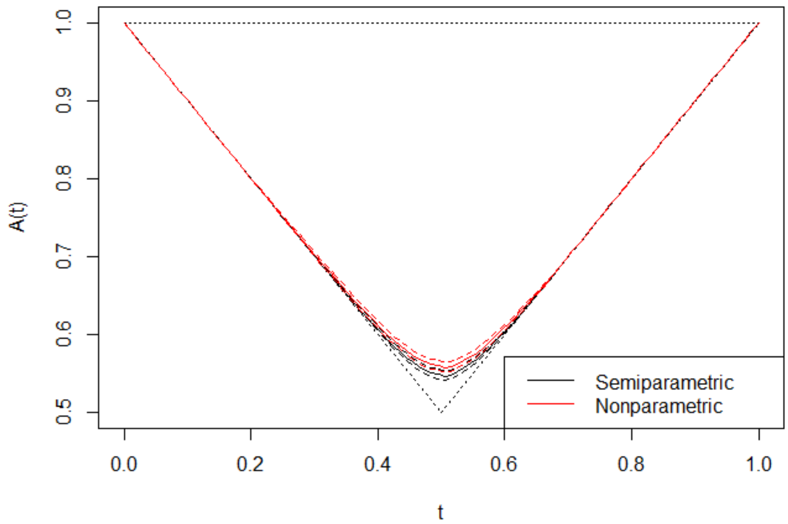


Figure 3.6 shows the estimates for the Pickands dependence function for the component-wise maxima of the two variables, estimated nonparametrically and semiparametrically. The graph indicates strong dependence as it is farther from $A(t) = 1$ and closer to $A(t) = \max\{t, 1 - t\}$. There is an

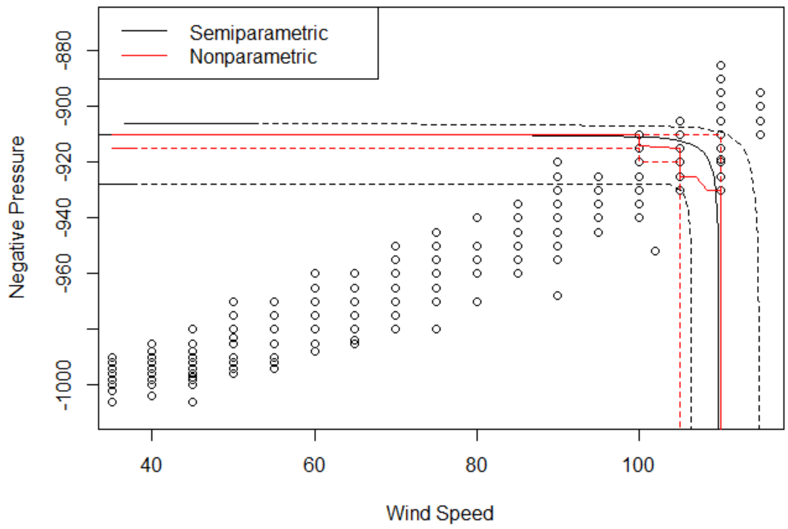
overlap between the semiparametric and nonparametric estimates, indicating a fit of the semiparametric method.

Table 3.2: Confidence Intervals for χ

	Estimate	Lower 95%	Upper 95%
Semiparametric	0.9232114	0.9063379	0.9375825
Nonparametric	0.8920272	0.8788396	0.9052175

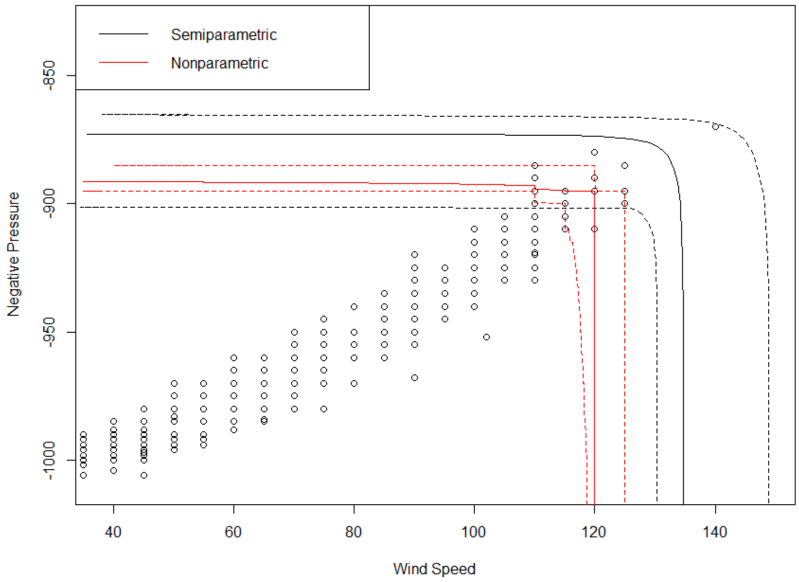
The estimates for χ and 95% confidence intervals for each method are shown in table 3.2. Both indicate high dependence in the componentwise maxima. The confidence intervals overlap, indicating again a fit of the semiparametric method.

Figure 3.7: Scatterplot of Component-Wise Maxima with 5% Storm-at-Risk Curves



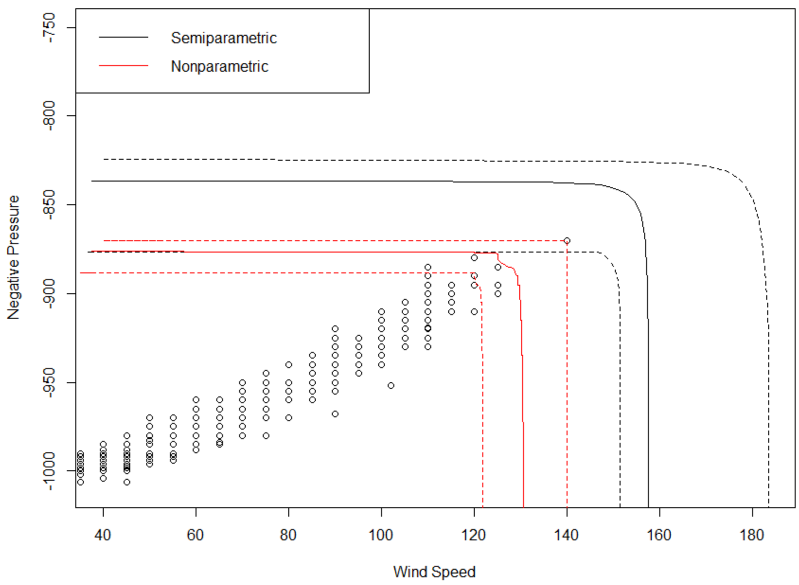
The 5% storm curves are displayed in Figure 3.7, with 95% confidence bands for the curves. The 5% curves indicate the features, in terms of pressure and wind speed, of the worst possible storms that may occur once every 1.333 years, or 1 year and 4 months. This is a small return period, but shows a demonstration of the method in terms of how many are classified as exceeding the curve.

Figure 3.8: Scatterplot of Componentwise Maxima with Once-in-10-Years Storm-at-Risk Curves



The once-in-10-years or 0.3928% storm-at-risk curves are displayed in Figure 3.8, with 95% confidence bands for the curves. The semiparametric approach has gone closest to the edge of the data with one storm being the worst, while the nonparametric method may indicate at most seven storms witihin the scope of the worst storms that occur once every 10 years. The formula to extract p in creating "once-in-k-years" storm-at-risk curves in data covering T years containing n_S storms is $p = \frac{T}{kn_S}$, thus $\frac{37}{10 \times 942} \times 100\% = 0.3928\%$ storm-at-risk curves.

Figure 3.9: Scatterplot of Component-Wise Maxima with Once-In-100-Years Storm-at-Risk Curves



The once-in-100-years or 0.03928% storm-at-risk curves are displayed in Figure 3.9, with 95% confidence bands for the curves. The semiparametric approach has gone beyond the range of the data, while the nonparametric method may indicate at most four storms, and already reaches the boundaries of the data, which only spanned 37 years. The nonparametric method cannot extrapolate for storms with larger return periods as it is restricted by the sample size. The semiparametric approach can extrapolate beyond the range of the data, but depends on the goodness of fit of the GEV marginal distributions.

Table 3.3: Robustness Results for Storm-at-Risk Curves by Return Period and Approach

Return Period (in Years)	Risk Probability	Proportion of Exceedance	
		Semiparametric	Nonparametric
0.378	0.10000	0.26432	0.11894
0.755	0.05000	0.14537	0.07048
5.000	0.00755	0.01762	0.00881
7.000	0.00539	0.01762	0.00881
10.000	0.00378	0.00881	0.00881
15.000	0.00252	0.00441	0.00881
20.000	0.00189	0.00441	0.00881

Table 3.3 shows the robustness results of the storm-at-risk curves at different return levels for each of the approaches. The best case for both models would be that the proportion matches the intended risk probability. We notice that for cases when the return period is less than 10 years, which is the span of years for the hold-out sample, the nonparametric is closest to the corresponding risk probability. However, for purposes of extrapolating beyond the span of the hold-out, the semiparametric is closest to the desired risk probability and the nonparametric is already stationary at the fixed level of the exceedance proportion. This means that for the estimation and forecasting of shorter return periods the nonparametric is better whilst for longer return periods the semiparametric case is best.

3.5 Conclusion and Summary

With climate change affecting the frequency and intensity of extreme weather events, it is of paramount importance that understanding and prediction of these events are a global challenge to climate researchers and academics. Weather hazards place developing countries at risk of disasters that will hamper their economic growth and the betterment of wellbeing of their populations.

A proposed measure of disaster risk with respect to typhoons called storm-at-risk is devised using multivariate extreme value theory. Confidence bands for the curves are generated using bootstrap methods. The methodology is demonstrated for the western North Pacific basin, a region of the world with both developed and developing economies but with higher typhoon risk due to frequent genesis of typhoons and cyclones. Robustness checks on the proposed methodology through cross-validation on a hold-out sample conclude that the nonparametric method is superior for shorter return periods while the semiparametric is best for extrapolation with longer return periods.

Chapter 4

Food Security Risk: Extensions of Forecast Reconciliation

4.1 Introduction

As described by Shapouri and Rosen (1999), one of the key contributing factors to food security is agricultural activity. If agricultural activity is deficient, countries either redistribute food supply from areas of plenty to regions of poverty or import food from other nations. If countries do not find approaches to mitigate or reduce the impact of food insecurities, they expose their populations to future threats (Haile & Bydekerke 2012). Countries should have risk assessment protocols and early warning systems to reduce the impact of food insecurity (Asian Development Bank 2013; Haile & Bydekerke 2012).

In measuring food security risks, the following purposes can be achieved: (1) providing national food supply estimates, (2) contributing information to global monitoring and early warning systems, (3) assessment of household food access and acquisition, and (4) measurement of food consumption and utilisation (Jones et al. 2013). Financial institutions have a specific measure as an early warning system—a financial risk management measure called value-at-risk (Jorion 2006). Scaramozzino (2006) proposed the measure as a statistic for measuring vulnerability to food insecurity. However, his methodology requires household-level monitoring of data for long periods of time, which is very expensive and data heavy. A value-at-risk approach that addresses the need for national level estimates and computes food security risks for sub-national levels would be most useful for developing countries, particularly as an early warning system (Haile & Bydekerke 2012).

In this chapter, we propose a measure of food security risk called food-at-risk, similar to the idea of Scaramozzino (2006) because it is based on the concept of value-at-risk (Jorion 2006). However, we integrate the needs of government agencies to produce consistent forecasts for both national and sub-national levels. To achieve this, the forecast reconciliation of hierarchical and grouped time series proposed by Hyndman et al. (2011) and Hyndman, Lee and Wang (2016) is used to generate consistent forecasts of food supply for the sub-national areas and aggregated data for the national level. We produce the food-at-risk estimates by a bootstrapped time series approach using the time series models from the reconciliation methodology. The food-at-risk measure is demonstrated on Philippine rice production volume data. Out-of-sample food-at-risk forecasts were generated in the demonstration due to short time series data.

The discussion of the chapter is as follows. An introduction of the topic is discussed in the first part. The forecast reconciliation technique is introduced in the second part. The food-at-risk methodology is explained in the third part of the chapter. The results on Philippine rice production volume data are discussed in the fourth part. Finally, we draw conclusions and present a summary of the chapter in the fifth part.

4.2 Forecast Reconciliation Techniques

Hyndman et al. (2011) and Hyndman, Lee and Wang (2016) proposed a methodology for reconciling forecasts of hierarchical and grouped time series data so that forecasts of individual time series data matched forecasts of aggregated series based on hierarchy or groups. To set up the methodology, let $\mathbf{y}_t = (y_{.,.,t}, y_{1,.,t}, \ldots, y_{.,1,t}, \ldots, y_{m,n,t})'$ be the vector of aggregated series and of grouped or hierarchical time series where $y_{.,.,t} = \sum_{i=1}^{m} \sum_{j=1}^{n} y_{i,j,t}$, $y_{i,.,t} = \sum_{j=1}^{n} y_{i,j,t}$, and $y_{.,j,t} = \sum_{i=1}^{m} y_{i,j,t}$ and $y_{i,j,t}$ are individual time series. Note that number n might be different for each of the m hierarchies or groups and that there can be more than 2 levels of indices than i, j. Let S be the matrix of zeroes and ones containing the information on how the groups generate the aggregate series. Suppose $\mathbf{b}_t$ be the vector of the bottom series $y_{i,j,t}$ at time t. Then, the relationship of the three can be summarised by

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t. \tag{4.1}$$

An example of equation (4.1) is shown. Let the bottom series be denoted by $\mathbf{b}_t = [y_{1,1,t}, y_{1,2,t}, y_{2,1,t}]'$ at time t, i.e., three bottom series. Assuming that the system is hierarchical, the first level of aggregation would be $y_{1,.,t} = y_{1,1,t} + y_{1,2,t}$ and $y_{2,.,t} = y_{2,1,t}$ and the second and final aggregation would be the overall sum series $y_{.,.,t} = y_{1,.,t} + y_{2,.,t}$. This would mean that $\mathbf{y}_t = [y_{.,.,t}, y_{1,.,t}, y_{2,.,t}, y_{1,1,t}, y_{1,2,t}, y_{2,1,t}]'$ at time t. To express the link between $\mathbf{b}_t$ and $\mathbf{y}_t$, S is expressed below to form an example of equation (4.1)

$$\mathbf{S} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

.

The methodology assumes a linear model for the desired reconciled h-step-ahead forecast of the bottom series, denoted by $\beta_h = E\left(\mathbf{b}_{T+h}|\mathbf{y}_1, \ldots, \mathbf{y}_T\right)$, given $\hat{\mathbf{y}}_h$ as known unreconciled h-step-ahead forecast of the top, middle and bottom series of the hierarchy:

$$\hat{\mathbf{y}}_h = \mathbf{S}\beta_h + \epsilon_h, \quad \epsilon_h \sim (\mathbf{0}, \mathbf{\Sigma}_h) \tag{4.2}$$

Equation (4.2) can be explained as follows. Using statistical time series modeling, the unreconciled forecasts $\hat{\mathbf{y}}_h$ and the S matrix that shows the structure that leads from the bottom series forecasts to forecasts of all hierarchy levels or groups of aggregation of the individual time series are inputs for reconciliation. The goal of the reconciliation is to estimate what would be the reconciled bottom series β_h that will create the reconciled forecasts for all hierarchies and groups of aggregation. The disturbance term ϵ_h describes the variance and covariance of the known unreconciled forecasts.

A system of solutions for estimating β_h and $\hat{\mathbf{y}}_h$ using generalised least squares would produce:

$$\hat{\beta}_h = \left(\mathbf{S}'\mathbf{\Sigma}_h^{-}\mathbf{S}\right)^{-1}\mathbf{S}'\mathbf{\Sigma}_h^{-}\mathbf{y}_t; \quad \mathbf{\Sigma}_h^{-} \text{ is a generalised inverse} \tag{4.3}$$

$$\tilde{\mathbf{y}}_h = \mathbf{S}\hat{\beta}_h = \mathbf{S}\left(\mathbf{S}'\mathbf{\Sigma}_h^{-}\mathbf{S}\right)^{-1}\mathbf{S}'\mathbf{\Sigma}_h^{-}\mathbf{y}_h \tag{4.4}$$

The matrix $\mathbf{\Sigma}_h^{-}$ may be replaced with any matrix $\mathbf{W_h}$ for weighted least squares.

Faster computations for these algorithms have been devised by Hyndman, Lee and Wang (2016). They involve an iterative approach for reconciling forecasts in terms of summing and matrix manipulations. This method is extended to create risk measures for food security for policy creation.

4.3 Proposed Methodology

A nonparametric approach to produce food security risk measures by residual-based bootstrapping the optimal forecast reconciliation methods of Hyndman, Lee and Wang (2016) for hierarchical and grouped agricultural data is proposed. The methodology is similar in concept to the value-at-risk discussed by Jorion (2006) and to the approach used by Scaramozzino (2006) on household survey data; however, the proposed methodology is used in grouped and hierarchical time series data for both national and sub-national levels, supplementing a countrys early warning system (Haile & Bydekerke 2012). The risk measure is called the food-at-risk (FaR).

With n_b as the number of bootstrap samples, the steps of the bootstrap approach are:

1. generate bootstraps of forecasts for the all series from each level of the hierarchy based on their selected time series models, $\{\hat{\mathbf{y}}_{t,i}\}_{i=1}^{n_b}$
2. generate reconciled forecasts for the hierarchy using equations (4.3) and (4.4): $\left\{\tilde{\hat{\mathbf{y}}}_{t,i}\right\}_{i=1}^{n_b}$
3. the $100p\%$ FaR is:
$$FaR(p) = \tilde{\hat{\mathbf{y}}}_{t,(n_b[1-p])} \tag{4.5}$$

4.4 Food Security Risk Assessment in the Philippines

The data used for the demonstration are the rice production volume data in metric tonnes gathered from CountrySTAT (Bureau of Agricultural Statistics 2012a). The data are hierarchical agricultural data, with 15 regional and three macro-regional (Luzon, Visayas and Mindanao) sublevels. Luzon covers seven regions: Ilocos; Cordillera Administrative Region (CAR); Cagayan Valley; Central Luzon; CALABARZON; MIMAROPA; and Bicol. The National Capital Region is in Luzon but does not have rice production data. The Visayas is divided into three regions: Western; Central; and Eastern. Mindanao is formed by the remaining regions: Zamboanga; Northern Mindanao; Davao; SOCCSKSARGEN; and the Autonomous Region of Muslim Mindanao (ARMM). A map of the Philippines is shown in Figure 4.1 .

Figure 4.1: Regional Map of the Philippines (Source: http://bit.ly/2mXpuTA)

Annual data are available from 1987 to 2015, and there are no missing values. The periods from 2011 to 2015 are hold-out samples for forecast evaluation. The forecast graphs for 5 years are presented. In the graphs, the black lines are the observed values, red lines are 5% food-at-risk and blue lines are the forecasted values.

Figure 4.2: The Observed, Forecasted, and 95% FaR Values for Rice Production Volume for the National and Macroregional Series

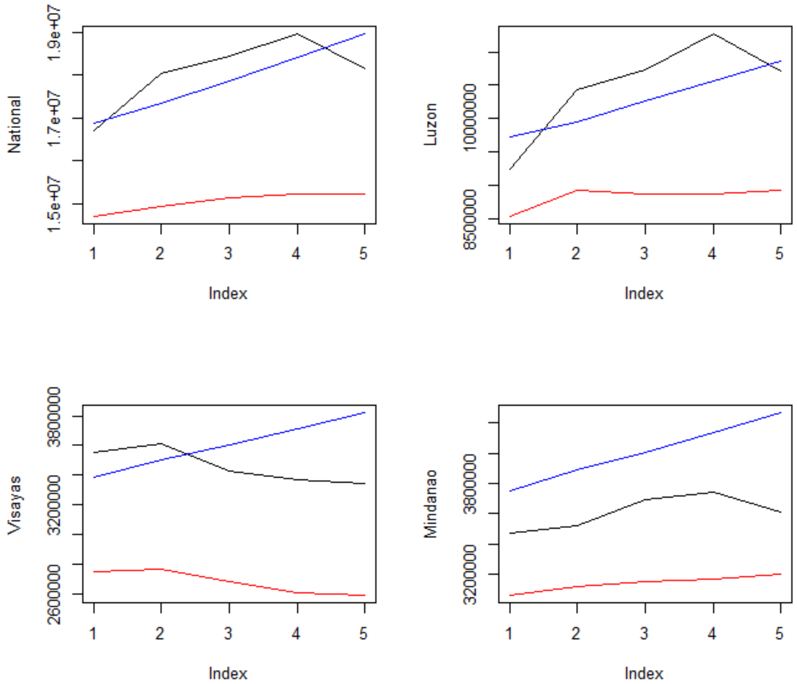


Figure 4.3: The Observed, Forecasted, and 95% FaR Values for Rice Production Volume for the CAR, Ilocos, Cagayan, and Central Luzon Regions

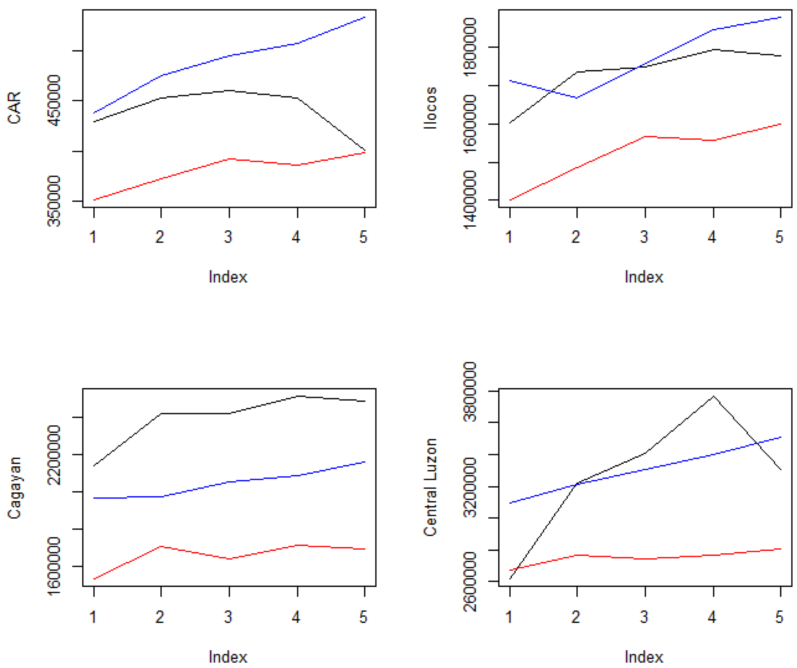


Figure 4.4: The Observed, Forecasted, and 95% FaR Values for Rice Production Volume for the CALABARZON, MIMAROPA, Bicol, and Western Visayas Regions

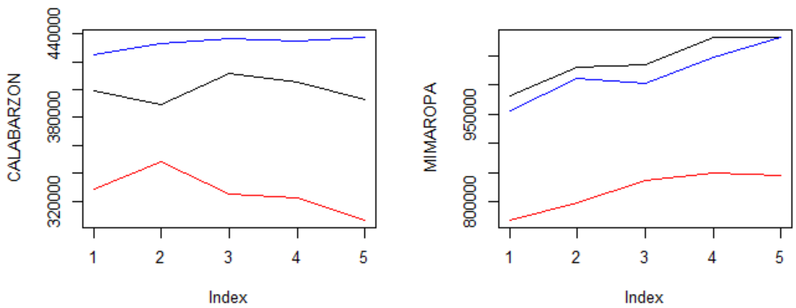


Figure 4.5: The Observed, Forecasted, and 95% FaR Values for Rice Production Volume for the Central Visayas, Eastern Visayas, Zamboanga, and Northern Mindanao Regions

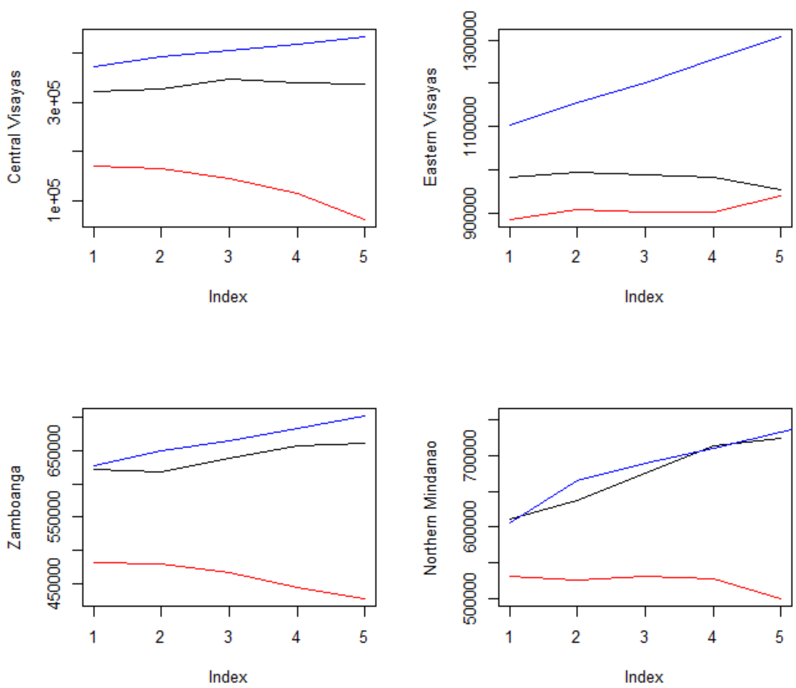
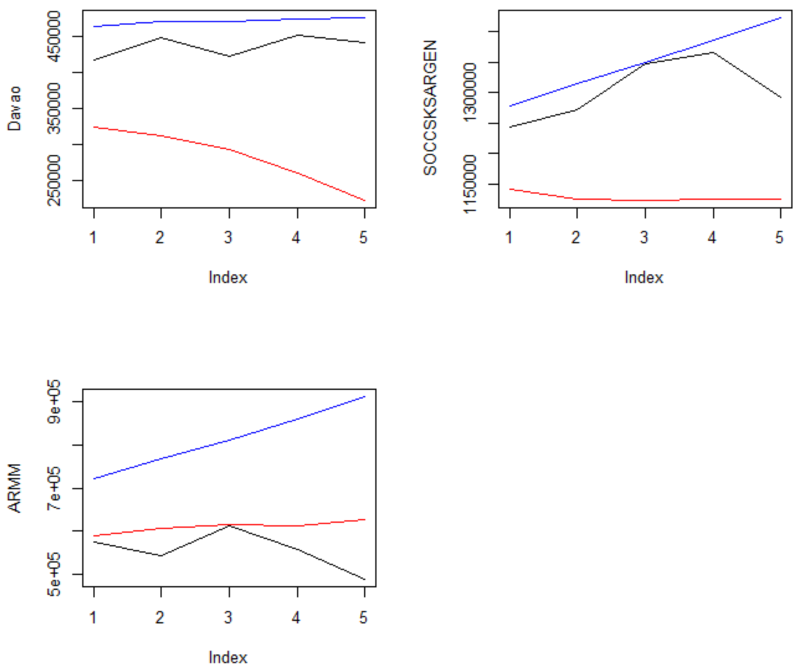


Figure 4.6: The Observed, Forecasted, and 95% FaR Values for Rice Production Volume for the Davao, SOCCSKSARGEN, and ARMM Regions



Figures 4.2 to 4.6 show the results for the 5% FaR.Nationally and in the macro-regions, production of palay is within expected levels and any decline in production does not pose a threat to food security. Regarding the individual regions, in Figure 31, the Central Luzon region breached its 5% FaR in period 1, which is the year 2011. It was able to recover its production from 2012 to 2015. The CAR plot indicates approaching lower-than-expected rice production, which may mean an impending food crisis in the future after 2015. Figure 33 shows an impending food crisis for the Eastern Visayas region. The ARMM region is consistently below or at its 5% FaR, which means that it is in a state of food crisis. The region is in an insurgency situation, which has affected food policy in the area (Food and Agriculture Organization 2015).

4.5 Summary

Food security is achieved when availability, access, utilisation and stability in food supply are secured. In developing countries, this is a difficult task if there are no systems in place to enable policymakers to understand and gather information regarding the food situation of their people. Therefore, information and early warning systems are needed to give governments and agencies the power to adapt to changing food situations to aid hungry populations.

In this essay, we have devised a financial risk management style in measuring food security risk through the food-at-risk methodology. It is constructed through the forecast reconciliation methods to create consistent aggregated forecasts for national estimates of food supply based on forecasts from regional levels. To create the food-at-risk for both the national and the sub-national levels, a time series bootstrapping methodology is integrated with the reconciliation methods. The methodology is applied to Philippine rice production volume data, which produces insights into the situation of the country and its regions.

The methodology can be adapted to measure various food situations, for not only rice but also other food items for which governments and agencies have a portfolio of food-at-risk measures to monitor the food situation in a holistic manner. This is a possible future direction of the current research.

Chapter 5

Conclusion

5.1 Summary

In this thesis, we aimed to measure risk manifested outside the field of finance through the use of non-Gaussian methodologies.

On the issue of longevity risk, governments and financial institutions can account for or reduce it by having an appropriate survival model to estimate mortality rates and life expectancy. In the second chapter, we proposed the MCH survival model, which is estimated through a nonlinear least squares approach for each year in the life table data. From the generated parameter time series, we created forecasts on life expectancy through residual-based bootstrapping. Our results have shown that the MCH outperforms the LC in long-term forecasts of up to 10 years, although this may vary by country.

Understanding extreme weather conditions is a global challenge taken up by climate researchers and academics because such conditions can hinder the growth of developing countries under risk and may cause severe disasters. To estimate extreme typhoon conditions, we devised the storm-at-risk curves in the third chapter, inspired by the value-at-risk methodology in financial risk management. These curves are created through multivariate extreme value theory as a means of estimating and extrapolating the maximum characteristics that storms can have given a predetermined return period or risk probability. To generate confidence bands on the curves, a bootstrapping approach was designed. Robustness checks were performed and we showed that, for low return periods or relatively high risk probabilities, the nonparametric approach is suitable but, for higher return periods or lower risk probabilities, the semiparametric approach is more suitable.

In the fourth chapter, we discussed issues of food security risk, as vulnerabilities of the risk may endanger poor and developing countries and, if left unchecked, may cause exposure to other human disasters. We proposed a FaR measure of food supply, inspired by value-at-risk methodology in finance, so that there is information and understanding on the scale of food security problems at which governments and agencies should take action. The FaR methodology involves the creation of agreements between forecasts of national and sub-national food supply levels through forecast reconciliation. The risk measure is then created by introducing a time series bootstrapping mechanism in the forecast reconciliation procedure. To demonstrate the insights that the procedure can give, we applied the FaR methodology to Philippine rice production data. We can make inferences on impending food crises that sub-national entities may have as they approach the measure and we can declare a food crisis when regions breach their FaR values.

5.2 Future Work

As risk can manifest in every human activity and in interaction of people with each other and their environment, the quest to gain understanding through statistical methodologies will always generate interesting research questions and novel means of estimation.

With regard to longevity risk, future work on the MCH function may determine which countries the model fits best in describing the demographic and mortality situation. As it is a survival model, research on its use for actuarial pricing and accounting of necessary cash reserves is another direction. As the MCH function is a mixture of two valid survival functions that describe two different components of mortality, generalisation of the MCH function is another research path.

The storm-at-risk methodology can be extended to include more storm characteristics given that ample data are available. We seek to introduce this measure to disaster risk management agencies as it may aid in information and understanding of extreme weather conditions. Country-specific applications of the methodology are research directions that can be pursued. Currently, the methodology is an unconditional measure, which entails frequent revisions of storm-at-risk estimates after a certain length of time. Future directions that we propose for storm-at-risk entail introducing a conditional model on the extreme value theory that takes into account the complex temporal behaviour of storms.

For the food-at-risk methodology, we seek to introduce the methodology into early warning systems of agricultural agencies, such as the one designed in the Philippines (Bureau of Agricultural Statistics 2012b; Yanson & Ramos 2010). A portfolio of food-at-risk estimates is the optimal utilisation of the methodology, particularly as a device for early warning systems for governments and agencies.

Overall, our future research directions are in devising methodologies for estimating risks in different aspects and fields of human activity. It is by understanding risk that people, governments, institutions and international bodies can prepare and devise policies and remedies to mitigate and reduce the impact of the realisation of risk.

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