Macroeconomic impacts: uncertainty shocks and bubbles

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Abstract

This dissertation includes three working papers showing my research on the macroeconomic impact of uncertainty and bubbles. The first two papers look at uncertainty shocks in a dynamic stochastic general equilibrium framework, while the last one focuses on economic bubbles and collateral constraints in an overlapping-generations model.

The first paper provides empirical contributions to the uncertainty shock literature by estimating a dynamic stochastic general equilibrium (DSGE) model for the Australian economy. This study aims at distinguishing the impact of uncertainty shocks from structural shocks in driving Australian business-cycle fluctuations from 1981 to 2015. Principally, the model includes a wide range of structural shocks incorporated with time-varying volatility components to quantify the effect of uncertainty shocks on the output variations. Overall, the findings suggest that time-varying volatilities have contributed substantially to the Australian business fluctuations, while major driver factors are attributed to systematical shocks. Domestic policy shocks account for the largest fraction of the GDP variations, while international spillover effects have a negligible fraction in the output movements.

The second paper enriches the empirical research on uncertainty shocks with a multisector model for a small, open economy. With a more refined classification of sources of uncertainty shocks, and separate identification of mining sector and resource-price shocks, the model emphasises the importance of the resource sector in the Australian economy during the mining boom period from 1991 to 2013. The results show that foreign shocks are the most crucial drivers, accounting for more than 50% of the economic variations, while demand and productivity shocks have minimal impacts. Additionally, the shocks are further decomposed into their mean and volatility components. Consequently, time-varying uncertainty shocks are more important than systematic shocks to the fluctuations, with great importance attributes to foreign shocks.

The third paper focuses on rational bubbles and collateral constraints in an overlapping-generations model with heterogeneous beliefs. The leverage equilibrium and asset-pricing theories are employed to explore the role of leverage in creating and popping bubbles. The study shares similar findings with the rational bubble literature that bubbles cannot exist in a frictionless endowment economy. Other noticeable results emphasise the existing conditions of bubble equilibrium in a fixed-bubble supply environment. First, bubbly equilibrium can endure in high-income countries with highly leveraged financial markets. However, high collateralisation might trigger deleveraging behavior from nervous lenders, causing bubbles to collapse. Second, bubbly equilibrium is prone to exist in low-income countries at any level of collateralisation. The dynamic analysis of bubbly equilibrium with growing bubble supply is more complex and will be tackled outside of this dissertation in the future.

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Nomenclature

ABS	Australian Bureau of Statistics
AR	auto-regression
CES	constant elasticity of substitution
COV	covariance
CPI	consumer price index
DFD	domestic final demand
DSGE	dynamic stochastic general equilibrium
EDF	expected discount factor
ES	elasticity of substitution
FEVD	forecast error variance decomposition
FP	fiscal policy
G	gamma distribution
GDP	gross domestic production
GFC	Global Financial Crisis
GST	good and service tax
GVA	gross value added
H-F	home-foreign
i.i.d	independent and identically distributed
IG	inverse gamma distribution
LR	long-run

MCMC	Markov Chain Monte Carlo
MP	monetary policy
N	normal distribution
NPISHs	non-profit institution serving households
OLG	overlapping-generations
RBA	Reserve Bank of Australia
REE	rational expectation equilibrium
ROW	rest of the world
SV	stochastic volatility
TFP	total factor productivity
TN	truncated normal distribution
U.S	United States of America
UIP	uncovered interest parity
VAR	vector auto-regression

1. Introduction

Economic researchers use a DSGE model to study business cycles, conduct scenario analysis and provide policy advice. Economic agents are specified in the DSGE models. Typically, these include households, firms, government and a foreign economy. These agents interact and behave optimally in response to random states in a competitive equilibrium.

Economic researchers first postulate stochastic shocks to the model to study macroeconomic fluctuations. The shocks might be productivity shocks, preference shocks, policy shocks, shocks from international economies, and commodity price shocks, which will drive the model.

In the first modern DSGE model, Kydland and Prescott [1982] studied the source of macroeconomic fluctuations in the U.S. Later, a New Keynesian DSGE model was developed, considering nominal and real rigidities, for example, sticky prices or wages as in Woodford [2003]. In the model, the volatilities of stochastic shocks are assumed to be invariant over time. However, these assumptions have been relaxed to consider time-varying variances in the innovations of the stochastic shocks. These relaxations originated from the realisation of time-varying variance components in aggregate time series. The dynamic effect of changing volatilities and the systematic shocks will drive the model [Fernández-Villaverde and Rubio-Ramírez, 2010].

This thesis consists of three working papers showing my learning journey on macroeconomics and uncertainty during my doctoral program. The first two papers look at uncertainty shocks in a DSGE framework, while the third paper focuses on economic bubbles and collateral constraints in an overlapping-generations model.

Chapter 2 (paper 1) provides empirical contributions to the uncertainty shock literature by estimating a DSGE model for the Australian economy. In this chapter, the model was populated with four primary economic agents: representative households, firms, government, and a foreign economy. There are nine structural shocks in the model, accounting for shocks to domestic and international economic conditions. A stochastic process is incorporated in the variances of each structural shock to capture uncertainty shocks. The model is calibrated to match the Australian data during the period from 1982 to 2015. The impact of mean shocks and uncertainty shocks to the aggregate fluctuations are explicitly distinguished. Overall, the findings can be compared to several studies for other small, open economies as in Cross et al. [2018] and Gómez-González et al. [2013]. In short, this chapter has empirically contributed to the existing literature that has estimated DSGE models of Australian economy. Notably, time-varying volatility shocks have contributed substantially to the Australian economic fluctuations.

The shocks account for more than 20% of the variations in which uncertainty in labour tax and cost-push shocks are the most important. However, my results share a similarity with the estimated Canadian model in Cross et al. [2018] that the systematic shocks are primary factors driving aggregate movements. The Australian domestic monetary and fiscal policy shocks are the largest components, accounting for nearly 50% of its gross domestic product (GDP) fluctuations. Moreover, the foreign shocks account for a negligible fraction (less than 10%) in the Australia output movements.

Chapter 3 (paper 2) continues the research journey by extending the model, providing more structure for the production sector to include non-traded, non-resource tradable, resource, import, and final goods and services sectors. This extension was motivated by the fact that a strong resource sector is widely considered a distinct feature of the Australian economy over the last thirty years. The mining boom has favoured a commodity-exporting country like Australia, with mining exports and investments shooting up and average living standards and wages increasing. Despite a great benefit from Australian dollar appreciation to the import sector, export industries, such as manufacturing and agriculture worse off [Downes et al., 2014]. The model explicitly examines the impact of uncertainty shocks on the Australian economy during the mining boom. Remarkably, the results confirm that foreign shocks are the most crucial drivers accounting for more than 50% of the Australian economic variations, while demand and productivity shocks have minimal impacts. After further decomposition of the shocks, the time-varying uncertainty components account for more than 60% of the Australian output variations. The foreign-related shocks, including risk-premium, resource-prices, foreign policies, and economic condition shocks, are the most considerable factors contributing about 40% in the uncertainty shocks decomposition.

Chapter 4 (paper 3) focuses on rational bubbles and collateral constraints in an overlapping-generations model with heterogeneous beliefs. Economic agents with different beliefs on bubble values will use different trading strategies to maximise their utility functions. The bubbles are fueled by credit resulting in a premium value of collateral assets. The study shows that a median buyer will determine the existence of bubbles in an economy. A bubbleless economy is identical to an autarky economy when no one believes in bubbles. A bubbly equilibrium can endure in high-income countries with highly leveraged financial markets. However, high collateralisation might trigger deleveraging behavior from nervous lenders, causing bubbles to collapse. Besides, bubbly equilibrium is also prone to exist in low-income countries at any level of collateralisation.

Chapter 5 concludes my thesis with the remarkable findings and future work.

2. Uncertainty shocks in markets and policies: an Australian case study[1]

Abstract. This chapter studies the driving factors of Australian business cycles by developing a DSGE model with various structural shocks and stochastic volatilities. There are nine structural shocks in the model, accounting for shocks to domestic and international economic conditions. The time-varying volatilities are incorporated into the structural innovations to explicitly distinguish the contributions of the shocks and their volatilities to the Australian economic variations. The finding shows that time-varying volatilities have contributed substantially to the Australian business fluctuations, while major driver factors are attributed to systematical shocks. Domestic policy shocks account for the most considerable fraction of the GDP variations, while international spillover effects have a negligible fraction in the output movements.

2.1. Introduction

New Keynesian DSGE models are widely used by macroeconomists and central bank for policy analysis. The model shows enormous advantages in empirical and theoretical research on the dynamic effects of shocks on business-cycle fluctuations. Researchers have traditionally modeled shocks to aggregate variables as stochastic processes with a homoscedasticity assumption [Christiano et al., 2011].

Starting from data observations during the Great Moderation in the U.S. between 1984 and 2007, research confirms that aggregate time series have robust time-varying variance components. This has led to attempts to relax the homoscedasticity assumption on shocks. Particularly, they explicitly model time-varying volatilities of the shocks of the stochastic processes, or the so-called uncertainty shocks [Born and Pfeifer, 2014, Fernández-Villaverde et al., 2015, Bloom, 2017, Baker et al., 2016].

Since different shocks will bring different impacts to an economy, policymakers have a strong interest in the source of shocks. One dimension is the origin of shocks: for instance, whether

[1] I would like to thank Jamie Cross, Timothy Kam, and Aubrey Poon for initial discussion and sharing the source codes. I also benefited from comments and feedback from seminar participants at the ANU Macroeconomics Group (AMG), Research School of Economics weekly seminars (Macroeconomics).

the shocks are from foreign economic and policy conditions, or from domestic conditions. Another dimension is the type of shocks: in other words, whether the shocks themselves drive aggregate fluctuations as imperfectly predictable factors (e.g. mean shocks) or by uncertainty in the distributions of the shocks (e.g. uncertainty shocks) [Fernández-Villaverde and Rubio-Ramírez, 2010, Cross et al., 2018, Fernández-Villaverde and Guerrón-Quintana, 2020]. Macroeconomic interventions responding to stabilise the business cycles can also build up uncertainty [Ludvigson et al., 2015]. Unclear responses, bad timing and hyperactivity can boost uncertainty [Baker et al., 2016]. Financial instabilities also appear to be correlated with uncertainty in financial markets [Alessandri and Mumtaz, 2014, Nodari, 2014]. Besides, uncertainty can affect domestic economies through international spillovers that bring the negative implications of unemployment and asset prices through international financial markets and trades [Castelnuovo et al., 2017, Davis, 2016].

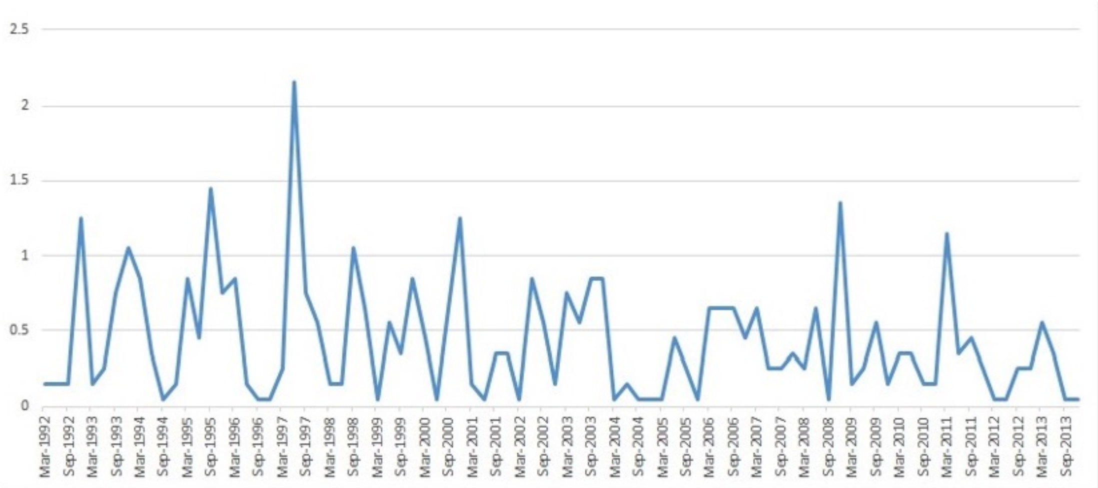
The presence of time-varying variance is found in Australian real GDP growth between 1992 and 2013. Figure 2.1.1 shows the absolute deviations of the Australian real GDP growth with respect to their mean. In this figure, we notice that volatilities fluctuated overtime. There are noticeable spikes, which coincided with the significant turbulent periods in Australia, namely the Asian Financial Crisis in 1997–1998, the goods and services taxes introduction in 2000, and the Global Financial Crisis (GFC) in 2008–2009. In contrast, there are tranquil periods when the volatilities were at low levels. Additionally, the table in figure 2.1.1 shows that the standard deviation of real GDP growth falls by nearly 40 per cent after 2004.Q1. The volatilities of other Australian variables have also been shown by the other studies, for example, volatility in term of trade as in Andrews and Rees [2009] and commodity prices in Tran [2019].

The evidence of time-varying variance in Australian output growth has motivated the work in this chapter. In particular, this chapter aims to gain insights into the sources of business-cycle fluctuations in a small, open economy like Australia. To what extent, are Australian business-cycle fluctuations driven by external and internal shocks? How important are stochastic volatility shocks to the aggregate variations?

To answer the questions, I borrowed the structural Bayesian DSGE model from Cross et al. [2018] with nine structural shocks, namely domestic monetary and fiscal policy, technology growth, cost-push, investment, international spill-over factors. The model also explicitly distinguishes the impact of uncertainty shocks from the structural shocks. More precisely, a stochastic process is incorporated in variances of all nine structural shocks to account for uncertainty shocks. I then calibrated the model to match Australian data during the period from 1981 to 2015. The effects of each shock were quantified. I further decomposed the total effects of each structural shock to show the contributions of mean and uncertainty shocks to the aggregate fluctuations.

The results can be compared to the recent studies on uncertainty shocks for small, open economies (e.g. Canada, New Zealand). This paper empirically contributes to the existing

Figure 2.1.1.: Australian real GDP growth 1992–2013



	Mean	Standard deviation
Q1.1992—Q4.1993	0.85	0.59
Pre 2004.Q1	0.96	0.67
Post 2004.Q1	0.71	0.42
Post 2004.Q1/Pre 2004.Q1	0.74	0.63

Source: ABS 5026.001

literature that has estimated DSGE models of the Australian economy. More specifically, its findings show that time-varying volatilities have contributed more than 20% to business fluctuations in Australia. Uncertainty in labour tax and cost-push shocks are the most important factors to output disturbances among the stochastic volatilities. However, it shares a similarity with the estimated Canadian model in Cross et al. [2018] that the systematic shocks are the primary factors driving aggregate movements. Domestic monetary and fiscal policies account for the largest components, accounting for nearly 50% of GDP fluctuations. Moreover, the foreign shocks account for a negligible fraction (less than 10%) in domestic output movements.

Literature review

This study was initially motivated by Fernández-Villaverde and Rubio-Ramírez [2007] and Justiniano and Primiceri [2008], who incorporated stochastic volatilities in structural shocks to improve model fitnesses. They developed a DSGE model with time-varying volatilities and proposed a computation procedure for computing and estimating the model using the Bayesian method and non-linear filtering theory. Their empirical application is a medium-scale DSGE model for a closed economy, which confirmed the importance of both changes in volatility and policy changes in understanding the U.S. monetary history. The other application is a small, open economy DSGE model for the Argentinian economy with time-varying volatilities incorporated in real interest rates. The model confirms that stochastic volatilities play an important role in driving output, consumption, investment, and other economic conditions, leading to aggregate fluctuations in the emerging country. Compared with them, my paper complements other works studying the macroeconomic effects of time-varying uncertainty. I consider a broader set of internal and external shocks with stochastic volatility components and quantify the effects of each shock on aggregate fluctuations in Australia.

My work also contributes to the existing literature that has estimated a DSGE model for the Australian economy. Buncic and Melecky [2008], Nimark [2009] developed and estimated New Keynesian small-scale models to study the dynamic effects of external shocks in the Australian economy. Buncic and Melecky [2008] found that domestic shocks are the most influential to the Australian business cycles, while foreign demand shocks also are important in some periods. Nimark [2009] also strengthened the role of foreign block shocks to domestic variables. Jääskelä and Nimark [2011] constructed a medium-scale DSGE model with a wide range of shocks and frictions but featured a less detailed production sector for the Australian economy. They found that both foreign and domestic shocks are influential to Australian business cycles.

The research in this chapter is closer to Cross et al. [2018], who developed and estimated a DSGE model for a small, open economy with a broad set of domestic and foreign shocks, incorporating stochastic volatility to the structural shocks for the Canadian economy. Their finding shows that domestic policy shocks account for a substantial proportion of the Canadian output fluctuations, while international effects seem comparably small. Also, they found that time-varying volatilities are insignificant in driving the output variations while the major drivers are the unanticipated policy shifts. Compared with their research, my empirical contributions are the decompositions of the source of business cycles using an estimated DSGE model for Australia. The results can be compared with other studies for small, open economies [Castelnuovo et al., 2017, Garcıa Cicco et al., 2013, Gómez-González et al., 2013]. In that sense, this paper empirically contributes to the existing literature that has estimated DSGE models of the Australian economy.

The remainder of this chapter is structured as follows: Section 2.2 describes the model and equilibrium conditions. Section 2.3 shows exogenous stochastic shocks, while section 2.4 describes the Bayesian estimation method and shows the data and calibrations. The results and discussion are presented in section 2.5, following by a conclusion in section 2.6.

2.2. A DSGE model

This section describes a DSGE model for a small, open economy with incomplete markets. There are representative households, firms, government, and rest of the world (ROW) in the model. The households derive their utility from consuming homogeneous goods and disutility from providing labour services to firms. They smooth their consumption by buying bonds denominated in home and foreign currencies and accumulating capital stocks. The firms use a Cobb–Douglas production technology to produce differentiated goods sold to the domestic government, domestic and foreign households. According to the conventional Taylor rules, monetary authorities adjust the nominal interest rates to stabilise inflation and aggregate output. Fiscal authorities balance their budget constraints by tax revenue from capital and labour income taxes.

The model has some features that match the Australian data, such as price stickiness, convex price adjustment costs, and convex capital adjustment costs. The convex price adjustment costs are what firms need to pay when they change their prices, while the price stickiness controls how responsive the prices are. These features guarantee that monetary policies influence both real activities and prices. The convex capital adjustment costs are applied to match volatilities in investment data, as in Fernández-Villaverde et al. [2015].

The dynamics of the model are driven by domestic economic and policy shocks and shocks to international economies. These shocks include monetary policy shocks, fiscal policy shocks, investment shocks, technology shocks, cost-push shocks, and shocks to foreign output, inflation, and interest rates.

2.2.1. A representative household

A continuum of identical households consume goods, invest domestically, hold domestic and foreign money bonds, supply labour and rent capital that is subject to labour and capital tax and capital depreciation, and receive a dividend from firm ownership. They have to decide on their optimal plan to maximise their lifetime utility functions:

$$E_0 \left\{ \sum_{t=0}^{\infty} \delta_t U(C_t, N_t) \right\}. \tag{2.1}$$

Their decisions are also subject to budget constraints:

$$P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} + \frac{S_t B^*_{t+1}}{1 + r^*_t} \leq (1 - \tau_{W,t}) W_t N_t + (1 - \tau_{K,t}) R_{k,t} K_t + P_t \tau_{k,t} \xi K_t + B_t + S_t B^*_t + P_t \Theta_t, \tag{2.2}$$

where P_t represents the domestic consumer price index (CPI). Domestic and foreign money bonds in home and foreign currency are respectively denoted as B_t and B^*_t, while domestic and foreign nominal interest rates are r_t and r^*_t, respectively. The domestic and foreign gross nominal interest rates are $R_t = (1 + r_t)$ and $R^*_{t+1} = (1 + r^*_t)$. The nominal exchange rate as the domestic currency per unit of foreign currency is S_t. Domestic investment is I_t. Constant elasticity of substitution (CES) composite index of home and foreign consumption goods is C_t. The marginal labour income tax rate is $\tau_{w,t}$, W_t is the hourly nominal wage rate; and N_t the number of working hours. The marginal capital income tax rate is $\tau_{K,t}$ with K_t representing the capital level and $R_{K,t}$ represents the capital rental rate. The capital depreciation rate denoted $\xi \in (0, 1)$. The total number of dividend payments from ownership of firms $i \in [0, 1]$ is defined as $\Theta \equiv \int_{[0,1]} \Theta(i) di$.

The endogenous discount factor δ_t is modeled in equation (2.3) following Uzawa [1968] to ensure a unique non-stochastic steady-state in incomplete markets with international borrowing and lending:

$$\delta_t := \begin{cases} \beta(C^a_{t-1}/A_{t-1})\delta_{t-1} & t > 0 \\ 1 & t = 0 \end{cases}, \tag{2.3}$$

$$\beta(C^a_t/A_t) = \frac{\tilde{\beta}}{1 + \zeta[ln(C^a_t/A_t) - \vartheta]}, \tag{2.4}$$

where A_t is the total factor productivity (TFP) or technology. The average consumption across households C^a_t is detrended by dividing to A_t. Equation (2.4) describes the parametric form of endogenous discount factors, following Ferrero et al. [2008]. In this equation, $\tilde{\beta} \in (0, 1)$ and two positive parameters ζ and ϑ are chosen to ensure the unique existence condition of a non-stochastic steady-state equilibrium and minimise the effect of the endogenous discount factor on the model dynamics.

The utility function of an individual household is assumed to be additive and separable in the following form:

$$U(C_t, N_t) := \frac{C_t^{1-\rho}}{1-\rho} + \nu(\tilde{G}_t) - \psi(A_t^{1-\rho})\frac{N_t^{1+\varphi}}{1+\varphi}. \tag{2.5}$$

A positive parameter $\rho > 0$ represents inter-temporal elasticity of substitution. The utility of a representative household also increases with the stationary level of government spending $\tilde{G}_t$, with $\nu(.)$ an increasing, concave, and abounded above function. The level of technology A_t is included in the household utility function to assure model balanced-growth path [Fernández-Villaverde et al., 2015]. A positive parameter $\psi > 0$ is a scale parameter while a positive parameter $\varphi > 0$ denotes the inverse of Frisch elasticity of labour supply.

Following Fernández-Villaverde et al. [2015], a capital accumulation is assumed to include the convex cost function for a capital adjustment:

$$K_{t+1} = (1-\xi)K_t + \mu_t I_t \left[1 - \mathcal{D}\left(\frac{I_t}{I_{t-1}}\right)\right], \tag{2.6}$$

$$\mathcal{D}\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - exp(g_A)\right). \tag{2.7}$$

Equation (2.6) represents the law of motion for the capital accumulation at a capital depreciation rate ξ. An investment shock μ_t is introduced here to capture an investment efficiency. It follows a mean-reverting process with a time-varying variance component to capture the uncertainty level in the future paths of investments. More detail is provided in the next section. $\mathcal{D}(.)$ is a convex cost function for the capital adjustment described in equation (2.7). A positive parameter κ is the capital adjustment cost parameter, g_A is the growth rate of TFP along a balanced-growth path so that $\mathcal{D}(g_A) = \mathcal{D}'(g_A) = 0$; $\mathcal{D}''(g_A) = \kappa$.

The first-order conditions for the representative household problem are presented below:

$$C_t: \quad \lambda_t = \frac{1}{P_t C_t^{\rho}}, \tag{2.8}$$

$$N_t: \quad A_t^{1-\rho}\psi N_t^{\varphi} C_t^{\rho} = (1-\tau_{W,t})\frac{W_t}{P_t}, \tag{2.9}$$

$$B_{t+1}: \quad E_t\lambda_{t+1} = \frac{\lambda_t}{R_t}, \tag{2.10}$$

$$B_{t+1}^*: \quad E_t\lambda_{t+1} = \frac{\lambda_t S_t}{(1+r_t^*)\, S_{t+1}}, \tag{2.11}$$

$$C_{t+1}: \quad E_t C_{t+1}^{-\rho}\beta(C_t^a/A_t) = E_t\left(\frac{P_{t+1}}{P_t}\right)\frac{C_t^{\rho}}{R_t}, \tag{2.12}$$

$$\begin{aligned} I_t: \quad \lambda_t P_t = \tilde{q}_t & \left(\left[1-\mathcal{D}\left(\frac{I_t}{I_{t-1}}; g_A\right)\right] - \mu_t I_t \mathcal{D}'\left(\frac{I_t}{I_{t-1}}; g_A\right)\right) \\ & -\delta_t E_t\left\{\tilde{q}_{t+1}\mu_{t+1} I_{t+1}\mathcal{D}'\left(\frac{I_{t+1}}{I_t}; g_A\right)\right\}, \end{aligned} \tag{2.13}$$

$$K_{t+1}: \quad \tilde{q}_t = \delta_t E_t\left\{\lambda_{t+1}\left(\left(1-\tau_{K,t+1}\right)R_{K,t+1} + P_{t+1}\tau_{K,t+1}\xi\right) + \tilde{q}_{t+1}(1-\xi)\right\}, \tag{2.14}$$

where λ_t denotes the Lagrange multiplier representing a marginal utility. The nominal Tobin's-q is $q_{N_t t}$. The Lagrange multiplier $\tilde{q}_t$ expresses a nominal Tobin's-q in terms of marginal utility $\tilde{q} = \lambda_t q_{N_t}$.

After some manipulations I get:

$$C_t^{-\rho} = R_t E_t\left\{\beta(C_t^a/A_t)\frac{P_t}{P_{t+1}}C_{t+1}^{-\rho}\right\}, \tag{2.15}$$

$$C_t^{-\rho} = (1+r_t^*)E_t\left\{\beta(C_t^a/A_t)\frac{P_t^* S_{t+1}}{P_{t+1}^* S_t}C_{t+1}^{-\rho}\right\}. \tag{2.16}$$

Each household chooses its optimal consumption to minimise its expenditure based on given prices. This results in optimal demand functions for home and foreign goods and a composite consumer price index:

$$\begin{aligned} C_{H,t} &= (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \\ C_{F,t} &= \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \\ P_t &= \left[(1-\gamma)\, P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{1/(1-\eta)}, \end{aligned} \tag{2.17}$$

where $\eta > 0$ is the elasticity of substitution between home and foreign (H-F) goods.

2.2.2. Firms

The production side of the economy is modeled following Gali and Monacelli [2005], which consists of a continuum of firms producing differentiated goods. The demand side includes a domestic economy, particularly government and households, and an international market. The demand functions are:

$$Y_{H,t+s}(i) = \left(\frac{P_{H,t+s}(i)}{P_{H,t+s}}\right)^{-\epsilon_{H,t}} Y_{H,t+s}, \tag{2.18}$$

$$Y_{H,t+s} := C_{H,t+s} + I_{H,t+s} + G_{H,t+s} + C^*_{H,t+s}. \tag{2.19}$$

$Y_{H,t+s}(i)$ and $Y_{H,t+s}$ are the demand functions for an individual firm i and aggregate output, respectively. $\frac{P_{H,t+s}(i)}{P_{H,t+s}}$ is the relative prices between domestic producer price index for goods i and composite producer price index. $\epsilon_{H,t}$ is the elasticity of substitution between the differentiated goods. The right-hand side of equation (2.19) shows the domestic aggregate output will be sold to domestic households for their consumption and investment, to a home government for its expenditure, and to foreign households for their consumption. In this model, I assume that the home government only consumes home-produced goods, for simplicity.

Each firm i possesses its own Cobb–Douglas production technology and requires labour $N_t(i)$ and capital $K_t(i)$ inputs to produce goods i:

$$Y_{H,t}(i) = [A_t N_t(i)]^{1-\alpha}[K_t(i)]^{\alpha}. \tag{2.20}$$

Parameter $\alpha \in (0,1)$ governs the input share of capital, while $(1-\alpha)$ is the input share of labour. A_t is a labour-augmenting productivity term.

Each firm i chooses its optimal labour and capital inputs to solve its cost minimisation problem. It implies an unique capital-to-labour ratio and marginal cost of production for all differentiated firms:

$$MC_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{W_t^{1-\alpha}R_{K,t}^{\alpha}}{A_t^{1-\alpha}}, \tag{2.21}$$

$$\frac{K_t(i)}{N_t(i)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_{K,t}}. \tag{2.22}$$

In equation (2.21), MC_t represents the nominal marginal cost of production. This is the Lagrange multiplier on the firm's technology constraint.

Each firm i participates in monopolistic competition, choosing the prices to charge for their goods i. In this model, I assume a convex price adjustment cost on an average cost function for each firm as in Rotemberg [1982]:

$$AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i)\right) := \frac{\omega}{2}\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-\Pi}(i)} - \Pi\right)^2 \times Y_{H,t+s}(i). \tag{2.23}$$

Equation (2.23) describes the average cost function of firm i. The parameter ω controls how responsive the prices are. For instance, the larger ω is, the more sticky prices are, and firms are reluctant to change their prices. Π is the gross long-run CPI inflation rate, or the inflation target rate in monetary policies. Each firm will choose a price $P_{H,t+s}(i)$ to maximise its profit subject to the constraints in equations (2.19), (2.20) and (2.21):

$$\begin{aligned}\Theta_t(i) = \max_{\{P_{H,t+s}(i)\}_{s\in N}} \Bigg\{ E_t \sum_{s=0}^{\infty} \digamma_t \Bigg[&\frac{P_{H,t+s}(i)}{P_{H,t+s}} Y_{H,t+s}(i) \\ &- \frac{W_{t+s}}{P_{H,t+s}} N_{t+s}(i) - AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i)\right)\Bigg]\Bigg\},\end{aligned}$$

with $\digamma_t = \delta_{t+s}\frac{U_c(C_{t+s},N_{t+s})}{U_c(C_t,N_t)}$ denoting the stochastic discount factor as Uzawa [1968].

After some manipulations, the optimal pricing strategy for firm i implies that, in the real term, the current marginal revenue for firm i with its price variation is equal to the sum of three terms.

$$\begin{aligned}(1-\epsilon_{H,t})\frac{Y_{H,t}(i)}{P_{H,t}} &= \frac{MC_t}{P_{H,t}}\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} + \frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)}{\partial P_{H,t}(i)} \\ &+ \beta(C_t^a/A_t)E_t\left\{\frac{U_c(C_{t+s},N_{t+s})}{U_c(C_t,N_t)}\frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)}{\partial P_{H,t}(i)}\right\},\end{aligned} \tag{2.24}$$

$$\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} = -\epsilon_{H,t}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_{H,t}-1}\left(\frac{Y_{H,t}}{P_{H,t}}\right),$$

$$\begin{aligned}\frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)}{\partial P_{H,t}(i)} &= \frac{\omega}{2}AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)\frac{1}{Y_{H,t}(i)}\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} \\ &- \omega\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right)\frac{Y_{H,t}(i)}{P_{H,t-1}(i)}.\end{aligned}$$

The first term on the right-hand side of equation (2.24) is the real marginal cost of production associated with the marginal variation of labour inputs. The other two terms in the right-hand side of the equation indicate current and expected future marginal impacts of pricing variation on profit through the sticky price terms.

2.2.3. Market-clearing conditions

Economic agents include households and firms, will make their optimal choices subject to the assumptions and market-clearing conditions. This model has four markets: domestic labour and capital markets, differentiated goods and aggregate goods markets.

Labour and capital are assumed to be immobile across countries. Then the domestic labour market must be clear in a competitive equilibrium, which implies the labour supply in equation (2.9) must be equal to the labour demand in equation (2.21):

$$A_t^{1-\rho}\psi N_t^{\varphi}C_t^{\rho} = (1-\tau_{W,t})mc_{H,t}A_t p_{H,t}, \tag{2.25}$$

with $mc_{H,t} = \frac{MC_t}{P_{H,t}}$ being the real marginal cost of production, and $p_{H,t} = \frac{P_{H,t}}{P_t}$ being the home final goods price index to the CPI index.

The capital market must also be clear in a competitive equilibrium, implying the capital supply in equations (2.13), (2.14) must be equal to the capital demand in equations (2.21), (2.22).

All differentiated goods markets must be clear in a competitive equilibrium. From equations (2.19) and (2.32), goods market-clearing conditions imply the two following equalities:

$$\begin{aligned}\left[1-\frac{\omega}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}-\Pi\right)^2 \times Y_{H,t}(i)\right] &= C_{H,t}(i)+I_{H,t}(i)+G_{H,t}(i)+C^*_{H,t}(i) \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_{H,t}} Y_{H,t}.\end{aligned} \tag{2.26}$$

The composite demand for goods i is given by the home households and government, and the foreigners:

$$Y_{H,t} = \left[(1-\gamma)\,p_{H,t}^{\eta}\,(C_{H,t}+I_{H,t}+G_{H,t}) + \gamma\left(\frac{P_{H,t}(i)}{S_t P^*_{F,t}}\right)^{-\epsilon_{F,t}}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t\right].$$

The aggregate home production index is given by:

$$Y_t = \left[\int_0^1 Y_{H,t}^{(\epsilon_{H,t}-1)/\epsilon_{H,t}}(i)d(i)\right]^{\epsilon_{H,t}/(\epsilon_{H,t}-1)}.$$

The aggregate goods market-clearing condition is given by:

$$\left[1-(\Pi_{H,t}-\Pi)^2\right]Y_t = Y_{H,t} \equiv (p_{H,t})^{-\eta}\left[(1-\gamma)\,(C_t+G_t)+\gamma Q_t^{\eta}C^*_t\right]. \tag{2.27}$$

2.2.4. Government and policy shocks

In the model, I follow the standard assumptions for monetary and fiscal policies. In particular, I assume that monetary authorities follow the conventional Taylor rule, which empirically proves to fit central bank behavior in reality:

$$\frac{R_t}{R} = \frac{R_{t-1}^{\phi_R}}{R}\frac{\Pi_t^{(1-\phi_R)\phi_\Pi}}{\Pi}\frac{Y_t^{(1-\phi_R)\phi_Y}}{YA_t}exp\{\sigma_{R,t}\varepsilon_{R,t}\}, \tag{2.28}$$

where R, Π, Y are a gross long-term interest rate, a CPI inflation, and an output, respectively. $\Pi_{H,t} := \frac{P_{H,t}}{P_{H,t-1}}$ represents the CPI inflation in home country; $\phi_R \in [0,1)$ denotes an interest rate smoothing parameter in monetary policy (MP), while $\phi_\Pi > 0$ is a CPI inflation parameter showing how MP responds to the CPI inflation. $\phi_Y > 0$ is a contemporaneous output showing how MP responds to the output gap. $\varepsilon_{R,t} \sim \mathcal{N}(0,1)$ is a structural shock capturing non-systematic MP shocks, interpreted as policy mistakes, while $\sigma_{R,t}$ captures the time-varying uncertainty of MP.

Following Fernández-Villaverde et al. [2015], I assume that fiscal authorities do not keep debt stock. They always balance their budget constraint according to:

$$G_t = \tau_{W,t}\frac{W_tN_t}{P_{H,t}} + \tau_{K,t}\frac{R_{K,t}K_t}{P_{H,t}}. \tag{2.29}$$

The labour and capital income taxes follow the mean-reverting processes. This implies that the tax rate increases with GDP growth representing demand management by policymakers.

$$\tau_{i,t} - \tau_i = \alpha_i(\tau_{i,t-1} - \tau_i) + \phi_{i,Y}(\frac{Y_t}{Y_{t-1}} - 1) + exp\{\sigma_{\tau_{i,t}}\}\varepsilon_{\tau_{i,t}}, \tag{2.30}$$

where α_K is a capital smoothing parameter in fiscal policy (FP). $\phi_{K,Y}$ represents a capital output feedback in FP, α_W is a labour smoothing parameter in FP, $\phi_{W,Y}$ represents a labour output feedback in FP; $\varepsilon_{\tau_{w,t}}$ and $\varepsilon_{\tau_{K,t}}$ are structural shocks capturing non-systematic FP while $\sigma_{\tau_{W,t}}$ and $\sigma_{\tau_{K,t}}$ are time-varying variances in FP shocks [2].

2.2.5. Competitive equilibrium conditions

Asset-pricing condition

[2] Note that the average tax rates in chapter 2 are effective, not the official taxes rates. As in Australia, income tax rates are progressive. As a result, the aggregate effective tax rates are often volatile.

Perfect competition in a final goods market leads to zero profit. Equating the Euler equations in (2.15), (2.16) implies an uncovered interest parity (UIP) condition:

$$R_t E_t \left\{ \beta(C_t^a/A_t) \frac{P_t}{P_{t+1}} C_{t+1}^{-\rho} \right\} = E_t \left\{ \beta(C_t^a/A_t) \frac{P_t^* S_{t+1}}{P_{t+1}^* S_t} C_{t+1}^{-\rho} R_{t+1}^* \right\}. \tag{2.31}$$

Phillips curve

For simplicity, this work only considers a symmetric equilibrium. The firms' optimal pricing condition implies a Phillips curve functional equation:

$$\begin{aligned} \Pi_{H,t}(\Pi_{H,t} - \Pi) - \frac{\epsilon_{H,t}}{2}(\Pi_{H,t} - \Pi)^2 = & \\ & \beta(C_t^a/A_t) E_t \left\{ \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \Pi_{H,t+1}(\Pi_{H,t+1} - \Pi) \frac{Y_{H,t+1}}{Y_{H,t}} \right\} \\ & + \frac{\epsilon_{H,t}}{\omega} \left[mc_{H,t} - \frac{\epsilon_{H,t} - 1}{\epsilon_{H,t}} \right]. \end{aligned} \tag{2.32}$$

Other identities

The ratio of the home final good price index to the CPI index is derived:

$$p_{H,t} := \frac{P_{H,t}}{P_t} = \left[\frac{1 - \gamma(Q_t)^{1-\eta}}{1 - \gamma} \right]^{1/(1-\eta)}.$$

Other useful identities are:

$$\Pi_t = \frac{\Pi_{H,t}}{p_{H,t}/p_{H,t-1}} = \Pi_{H,t} \times \left[\frac{1 - \gamma(Q_{t-1})^{1-\eta}}{1 - \gamma(Q_t)^{1-\eta}} \right]^{1/(1-\eta)},$$

$$Y_{H,t} = [A_t N_t]^{1-\alpha} [K_t]^{\alpha}.$$

Recursive competitive equilibrium

Given the monetary and fiscal policies in equations (2.28) and (2.30), a recursive competitive equilibrium is a system [3] $s_t(B_t, B_t^*, K_t, A_t, Z_t^*, \Sigma_t^{\mathbf{1/2}}) \rightarrow (C, I, N, K, G, Y_H, mc_H)(s_t)$ and pricing functions $s_t(B_t, B_t^*, K_t, A_t, Z_t^*, \Sigma_t^{\mathbf{1/2}}) \rightarrow (\Pi_H, p_H, \Pi, Q)(s_t)$ such that:

1. households maximise their lifetime utility functions subject to their budget constraints and capital accumulation processes: (2.9), (2.15), (2.16), (2.13), (2.14);

[3] The definition of Z_t^* and $\Sigma_t^{\mathbf{1/2}}$ will be discussed in the next section.

2. firms minimise their costs subject to the production functions and maximise their profits subject to the demand functions: (2.21), (2.22), (2.32);

3. markets clear each period: (2.13), (2.14), (2.21), (2.22), (2.25); and

4. governments balance their budgets: (2.29).

2.3. Exogenous stochastic shocks

2.3.1. Domestic policy shocks

In this model, I include two structural shocks in the monetary and fiscal policies, respectively in equations (2.28) and (2.30).

2.3.2. Other domestic shocks

I also include domestic technology (A_t), cost-push (ϵ_H), investment shocks (μ), that follow a mean-reverting process:

$$ln(i_t) = (1 - \rho_i)i + \rho_i ln(i_t) + \sigma_{i,t}\epsilon_{i,t}, \tag{2.1}$$

where $\epsilon_{i,t} \sim N(0,1)$; $\rho_i \in (0,1)$ is a smoothing coefficient, $i \in \{g_A, \epsilon_H, \mu\}$. $g_A := \frac{A_t}{A_{t-1}}$ being a technology growth rate; ϵ_H is a markup rate. The innovations have time-varying variance to reflect uncertainty about future growth of domestic technology, markups, and investment over time.

2.3.3. Foreign sectors

With the standard assumptions for a small, open economy setting, a foreign block is modeled as a large, closed economy where total output equals total consumption in global markets. In this model, I assume the rest of the world (ROW) follows a recursive first-order VAR-SV[4] process, which can be presented as a generic linear regression model:

$$Z_t^* = X_t^* \beta_z + w_t, \tag{2.2}$$

$$W_t^* = \begin{bmatrix} 0 & 0 & 0 \\ -\Delta ln(Y_t^*) & 0 & 0 \\ & -\Delta ln(Y_t^*) & -\pi_t^* \end{bmatrix}, \tag{2.3}$$

where $w_t \sim N(0, D_t)$; $Z_t^* := [\Delta ln(Y_t^*), \pi_t^*, r_t^*]'$ includes the demeaned real GDP growth rate, inflation rate, and nominal interest rate of the foreign economy; $\beta_z = [\beta_1, \beta_2]$ is a vector of the

[4]Vector Autoregression - Stochastic Volatility

foreign variables including VAR coefficients β_1 and free elements β_2; $X_t^* = [Z_{t-1}^*, W_t^*]$. Using this model, I can obtain prior means for the ROW variables using the maximum likelihood estimates for β_z. I assume truncated and unrestricted Gaussian distributions with an interval (-1,1) for the priors for AR coefficients and covariance terms in the foreign model.

2.3.4. Uncertainty shocks

Let $\boldsymbol{u_t}$ be an 9x1 vector collecting all policy and economic shocks above, or structural shocks $\boldsymbol{u_t} = \{u_{i,t}\}_{i\in I}$ with $I := \{A, R, \tau_W, \tau_K, \epsilon_H, \mu, Y^*, \pi^*, r^*\}$:

$$\boldsymbol{u_t} = \boldsymbol{\Sigma_t^{1/2}} \boldsymbol{\epsilon_t}, \tag{2.4}$$

where $\boldsymbol{\epsilon_t} \sim N(0, I_9)$ and I_9 is an identity matrix, $\boldsymbol{\epsilon_t} = \{\epsilon_{i,t}\}_{i\in I}$ is a Gaussian shock to the variables or the mean shocks. $\boldsymbol{\Sigma_t^{1/2}}$ is a diagonal matrix Σ_t, called a stochastic volatility matrix:

$$\Sigma_t = \begin{bmatrix} F_t & 0_{(3\times 6)} \\ 0_{(6\times 3)} & D_t \end{bmatrix}, \tag{2.5}$$

$$F_t = \begin{bmatrix} \sigma^2_{Y^*,t} & 0 & 0 \\ 0 & \sigma^2_{\pi^*,t} & 0 \\ 0 & 0 & \sigma^2_{r^*,t} \end{bmatrix}, \tag{2.6}$$

$$D_t = \begin{bmatrix} \sigma^2_{A,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{R,t} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\tau_{W},t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\tau_{K},t} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\epsilon_H,t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\mu,t} \end{bmatrix}. \tag{2.7}$$

Each element of the matrix Σ_t follows a stochastic process:

$$\log \sigma_{i,t} = \log \sigma_{i,t-1} + \upsilon_{i,t}, \tag{2.8}$$

where $\upsilon_{i,t} \sim N(0, \omega_i^2)$, for $i \in I := \{A,\, R,\, \tau_w,\, \tau_K,\, \epsilon_H,\, \mu,\, Y^*,\, \pi^*,\, r^*\}$. The uncertainty shocks $\upsilon_{i,t}$ with variance parameters ω_i, describe a permanent shock to the volatility of distribution of each $u_{i,t} \in \boldsymbol{u_t}$.

Algorithm 2.1 A four-step Metropolis within Gibbs algorithm

1. Draw parameter vector $\boldsymbol{\theta}^g$ at iteration g by applying a random walk Metropolis-Hastings algorithm following Schorfheide [2000], given $\boldsymbol{\omega}^{\boldsymbol{g}}$ and $\{\boldsymbol{\Sigma}^{\boldsymbol{g}}\}_{t=1}^{T}$

2. Simulate the structural shocks $\{\boldsymbol{u_t}^{g+1}\}_{t=1}^{T}$ for iteration $(g+1)$ using efficient disturbance smoother by Durbin and Koopman [2002]

3. Draw stochastic volatility terms $\{\boldsymbol{\Sigma}^{\boldsymbol{g+1}}\}_{t=1}^{T}$ for iteration $(g+1)$ by precision sampling algorithm by Chan et al. [2014].

4. Draw variance of stochastic volatilities $\boldsymbol{\omega}^{g+1}$ for iteration $(g+1)$ by sampling from inverse Gamma distribution.

The state vector s_t will be defined as $s_t = (B_t, B_t^*, K_t, A_t, Z_t^*, \Sigma_t^{\mathbf{1/2}})$.

2.4. Bayesian estimation

2.4.1. The model solution

The model is normalised and transformed into a stationary version, which has a deterministic steady-state equilibrium. The model is solved using the first-order perturbation method following Justiniano and Primiceri [2008]. The standard algorithm is used to find a stable rational expectation equilibrium (REE) solution. It served as a conditionally linear and Gaussian state–space system, including a state transition equation and a measurement equation as follows:

- a state transition equation: $\boldsymbol{x}_{t+1} = \boldsymbol{A}(\boldsymbol{\theta}^{\boldsymbol{g}})\boldsymbol{x_t} + \boldsymbol{B}(\boldsymbol{\theta}^{\boldsymbol{g}})\boldsymbol{u_t}$,
- a measurement equation: $\boldsymbol{y}_t^o = \boldsymbol{H}^o \boldsymbol{y}_t$,

with $\boldsymbol{x_t}$ is a vector of all endogenous variables; $\boldsymbol{y}_t^o$ is a vector of observables; $A(\theta^g)$, $B(\theta^g)$ are parameter matrices; $\boldsymbol{H}^o$ is a linear observation relationship.

To obtain the posterior joint distribution of model structural parameters $(\boldsymbol{\theta}, \boldsymbol{\omega})$ and stochastic volatility matrices Σ_t, I follow a four-step Metropolis within Gibbs algorithm as in Algorithm 2.1. More detail is shown in Appendix A.1.

2.4.2. Data and calibration

2.4.2.1. Data

To estimate the model, I use six Australian time series for the domestic economy and three U.S. time series for the ROW block. The data have corresponded to the vector of observable variables in the state-space representation. The domestic time series includes a per capita real GDP growth rate, average labour and capital income tax rates[5], a demeaned gross nominal interest rate, a demeaned gross inflation rate, and a per capita real investment growth rate. The ROW time series are a demeaned per capita real GDP growth rate, a demeaned nominal interest rate and a demeaned CPI inflation rate.

The data are quarterly. The sample includes 138 quarters, which spans from 1981:Q3 to 2015:Q3. The choice of starting date reflects the turning point of the Australian economy by economic liberalisation and financial deregulation under the Hawke government in the early 1980s.

The domestic data was mainly collected from the Australian Bureau of Statistics (ABS), while foreign data were taken from the Federal Reserve Bank of St. Louis [6].

2.4.2.2. Calibration

Following Justiniano and Preston [2010], I set $\epsilon_H = 8$ to fix the steady-state markup at 14%[7]. The discount factor is 0.99, while the Inverse Frisch labour supply elasticity is 1.27[8]. Growth rate in TFP are set to 2% annum or $g_A = 0.005$ according to Fernández-Villaverde et al. [2015]. The consumption scaling factor $\vartheta = 10^{-6}$ is parameterised to limit the effect of the endogenous discount factor in the model dynamic and ensures the unique existence of non-stochastic steady-state equilibrium. The capital share in the production function $\alpha = 0.24$ is set to match government expenditure and investment share of output in the Australian data. This is summarised in table 2.4.1.

Table 2.4.2 shows a comparison of the long-run properties of the model and the data over the sample period. Specifically, based on GDP, the U.S. economy (as a representative for ROW) is 14.65 times larger than the Australian economy. The share of consumption expenditure is 56.9% of the Australian GDP. The gross saving share is 21.3%, while the government consumption share accounts for 18.8%. Foreign long-run variables are the mean of quarterly series in the U.S., while tax rates are the mean of labour and capital tax series calculated following Born and Pfeifer [2014] and Fernández-Villaverde et al. [2015]. The long-run labour

[5] Labour and capital income tax rates are calculated following Born and Pfeifer [2014] and Fernández-Villaverde et al. [2015]. More detail in Appendix A.2.2

[6] Appendix A.2 includes a full list of data sources, descriptions and constructions.

[7] The steady-state markup is given by $\frac{\epsilon_H}{\epsilon_H - 1} - 1$.

[8] Note that the inverse Frisch labour supply elasticity is assumed to have a moderately loose Gamma prior with large tails given the diversity from macro and micro research.

Table 2.4.1.: Calibration of model parameters

Parameters	Value	Description
ϑ	10^{-6}	Consumption scaling factor
φ	1.27	Inverse Frisch labour supply elasticity
β	0.99	Discount factor
g_A	0.005	Quarterly growth rate in steady-state
ϵ_H	8	Inverse markup
α	0.24	Fixed to match government expenditure and investment share of output in Australia

working hours is 0.3333, which is equivalent to eight hours working per day. Overall, this suggests that the model can capture many critical features of the Australian data.

Some parameters are pinned down by steady-state relation, which is described in table 2.4.3.

2.4.2.3. Prior distributions

The majority of structural parameters are bounded by their prior distributions and are estimated. The prior distributions will be chosen in line with the literature and economic theory [Justiniano and Preston, 2010]. For instance, smoothing parameters are bounded in the interval $(0, 1)$, so I assume the prior belongs to a beta distribution family. A positive parameter prior is assumed to have a gamma distribution, and the other is assumed to have a normal distribution. The priors for auto-regression (AR) coefficients and covariance terms in the foreign VAR-SV model follows normal distributions obtained from maximum likelihood estimates for the coefficients. The prior distributions are reported in the third and fourth column to the right in table 2.4.4 and 2.4.5.

Following Cross et al. [2018], the prior distributions for stochastic volatility terms $\{\omega_i\}$ with $i \in I := \{A,\ R,\ \tau_w,\ \tau_K,\ \epsilon_H,\ \mu,\ Y^*,\ \pi^*,\ r^*\}$ are assumed to follow a normal inverse Gamma distribution with mean and standard deviation, as reported in table 2.4.6.

2.5. Results and discussion

2.5.1. Structural parameter estimates

The posterior means and standard variations of all structural parameters are reported in the last two columns in table 2.4.4. The posterior densities were generated by the Metropolis–Hasting algorithm using 10^6 Monte Carlo Markov Chain draws. The starting points are from the means of the prior distributions and the first 50,000 draws are discarded as a burn-in period, and I retain one in 50 subsequent draws.

Table 2.4.2.: Long-run variables from data

Parameters	Value	Description	Resource
$systar$	14.65	Relative size of ROW to home economy	World bank data base GDP 1981-2017
sc	0.569	Consumption share of output	World bank data base1981-2017 for households and NPISHs final consumption expenditure (% of GDP)
si	0.213	Gross savings share of output	World bank data base1981-2017 for gross savings (% of GDP)
sg	0.188	Government final consumption share of output	World bank data base1981-2017 for general government final consumption expenditure (% of GDP)
$\bar{i}^*$	0.0127	LR nominal interest rate in foreign country	Mean of quarterly nominal interest in US from 1981-2017
$\bar{\pi}^*$	0.0067	LR inflation in foreign country	Mean of quarterly CPI inflation in US from 1981-2017
$\bar{R}$	1.0185	LR real gross interest rate in home country	Mean of quarterly gross interest rate in Australia from 1981-2017
$\bar{N}$	0.33333	8 hour working per day	
τ_W	0.1876	Average labour tax rate	Australia tax data 1981-2017
τ_K	0.3052	Average capital tax rate	Australia tax data 1981-2017

Table 2.4.3.: Parameters are pinned down using steady-state relations

Parameters	Value	Resource
$\bar{R}^{*}$	$\frac{1+\bar{i}^{*}}{1+\bar{\pi}^{*}}$	Long-run foreign real gross interest rate
$\bar{\pi}$	$\frac{\bar{R}}{\bar{R}^{*}}$	Long-run inflation in home country
$\bar{K}$	$\frac{\bar{N}}{\bar{R}/\alpha}$	Long-run capital in home country
$\bar{Y}$	$\bar{N}^{1-\alpha}\bar{K}^{\alpha}$	Long-run home production
$\bar{Y^{*}}$	$systar * \bar{Y}$	Long-run foreign production
$\bar{C}$	$sc * \bar{Y}$	Long-run home consumption
$\bar{mc}$	$1 - \frac{1}{\epsilon_H} = 0.875$	Long-run marginal cost of production
ζ	$\frac{\left(\beta\frac{\bar{R}}{\bar{\pi}}\right)exp(g_A)^{-\rho}-1}{log(\bar{C})-\vartheta}$	Expected Discounted Factor (EDF) parameter
$\bar{\delta}$	$\frac{\beta}{1+\zeta(log(\bar{C})-\vartheta)}$	EDF parameter in steady-state
ξ	$\frac{\alpha\bar{Y}\bar{mc}}{\bar{K}} + \frac{1-\frac{1}{\bar{\delta}exp(g_A)}}{1-\tau_K}$	Capital depreciation rate
$\bar{I}$	$\bar{K}\left[exp(g_A) - 1 + \xi\right]$	Long-run Investment
si	$\frac{\bar{I}}{\bar{Y}}$	Match with the data
sg	$((1-\alpha)\tau_W + \alpha\tau_K)\,\bar{mc}$	Match with the data
γ	$\frac{1-sc-si-sg}{systar-sc-si-sg}$	Openness, foreign good share in home consumption

Table 2.4.4.: Prior and posterior distribution with mean and standard deviation

Parameters	Description	Family[a]	Prior Mean	Prior Std.	Post. Mean[b]	Post. Std.[b]
ρ	Inter-temporal ES	G	1	0.71	0.87	0.53
η	Elasticity H-F goods	G	0.9	0.52	0.80	0.44
ω	Price stickiness	N	60	2.24	59.99	2.30
ϕ_R	MP, smoothing	B	0.60	0.10	0.61	0.10
ϕ_π	MP, inflation	G	1	0.87	0.91	0.68
ϕ_Y	MP, output	N	0.25	0.13	0.27	0.12
ϕ_W	FP, output	G	0.25	0.22	0.27	0.22
α_W	FP, smoothing	B	0.60	0.10	0.60	0.10
ρ_A	TFP, smoothing	B	0.60	0.10	0.60	0.11
κ	Capital adjustment stickiness	N	60	2.24	60.02	2.30
ϕ_K	FP, output	G	0.25	0.22	0.26	0.22
α_K	FP, smoothing	B	0.60	0.10	0.59	0.10
ρ_{ϵ_H}	Elasticity H-F goods, smoothing	B	0.60	0.10	0.61	0.09
ρ_μ	Investment, smoothing	B	0.60	0.10	0.62	0.10

[a] *B is for Beta, G for Gamma, N for Normal distributions.*

[b] *Posterior moments are generated from a thinned sample of 10^6 MCMC draws. As conventional, it cuts out half of sample as burn-in then saves 1 in 50 draws.*

Table 2.4.5.: Prior and posterior distribution with mean and standard deviation (continued)

Parameters	Description	Family[a]	Prior Mean	Prior Std.	Post. Mean[b]	Post. Std.[b]
$\rho(Y^*, Y^*)$	VAR-SV,AR	TN	0.39	0.10	0.41	0.10
$\rho(Y^*, \pi^*)$	VAR-SV,AR	TN	-0.15	0.10	-0.14	0.10
$\rho(Y^*, i^*)$	VAR-SV,AR	TN	0.04	0.10	0.05	0.10
$\rho(\pi^*, Y^*)$	VAR-SV,AR	TN	0.02	0.10	0.03	0.09
$\rho(\pi^*, \pi^*)$	VAR-SV,AR	TN	0.72	0.10	0.71	0.10
$\rho(\pi^*, i^*)$	VAR-SV,AR	TN	0.05	0.10	0.07	0.10
$\rho(i^*, Y^*)$	VAR-SV,AR	TN	0.06	0.10	0.08	0.11
$\rho(i^*, \pi^*)$	VAR-SV,AR	TN	0.28	0.10	0.26	0.10
$\rho(i^*, i^*)$	VAR-SV,AR	TN	0.88	0.10	0.86	0.08
$\sigma(\pi^*, Y^*)$	VAR-SV,COV	N	-0.03	0.10	-0.04	0.10
$\sigma(i^*, Y^*)$	VAR-SV,COV	N	-0.10	0.10	-0.10	0.10
$\sigma(i^*, \pi^*)$	VAR-SV,COV	N	-1.79	0.10	-1.78	0.09

[a] *B is for Beta, G for Gamma, N for Normal distributions.*

[b] *Posterior moments are generated from a thinned sample of 10^6 MCMC draws. As conventional, it cuts out half of sample as burn-in then saves 1 in 50 draws.*

Table 2.4.6.: Prior distribution for stochastic volatility terms

Parameter	Family [a]	Mean μ_h	Standard Deviation σ_h
ω_i	IG	0.1	0.01

[a] IG stands for an inverse Gamma distribution

Firstly, the posterior distributions are tested to be an ergodic distribution of the Markov chain with convergence diagnosis and mean test shown in Appendix A.4.

All coefficient estimates have more concentrated distributions compared to their prior distributions. For most of the structural parameters, posterior standard deviations are smaller than in the prior distributions, suggesting that information is quite informative. The inter-temporal elasticity of substitution and the elasticity of home and foreign goods are relatively below unity, but this is consistent with other studies [Cross et al., 2018, Justiniano and Preston, 2010].

The policy parameters are in line with the literature. The monetary response to inflation is relatively low, while it is the opposite for output response. The fiscal policy is modelled as mean-reverting processing. The smoothing parameters are similar to each other. The feedback effects of fiscal taxes on output fluctuation is a little high, the level of correlation in labour rate was slightly higher than in capital tax rate. This associated with the tax reform periods in Australia over the last 20-30 years. In 1998 government released its comprehensive "A new Tax system plan" and in 2000 a GST was introduced. In 2010, the comprehensive study on Australian Future Tax system was released to include recommendations focusing on four efficient tax bases: personal and business income taxes, private consumption expenditure and economic rents from land and natural resources.

The AR(1) smoothing coefficient of the exogenous shocks are moderately persistent, around 0.60 for $\rho_A, \rho_{\epsilon_H}, \rho_\mu$. The estimated standard deviation for the technological shock is the highest level of the three, but just 0.11, while the lowest is 0.09 for the cost-push shock.

For the ROW block, the estimates are just slightly different from their prior distributions. The differences might be because the foreign economy is exogenous, so a small economy cannot affect it. However, the international capital flow might connect the two economies, as presenting in the UIP equation in (2.31).

The noticeable point is that the posterior densities have a reasonably different shape to their prior densities for several parameters. More specifically, the capital stationary and auto-regressive elasticity coefficients change their posterior densities from skewed right to left, indicating more low-value observations. On the other hand, other AR coefficients in the foreign VAR-SV model change their skewness in the opposite direction toward the right for a higher value. The posterior densities of the foreign VAR-SV model have a wider spread than their prior distribution, implying higher variability of the data and the uncertainty that data brings in.

2.5.2. Uncertainty shocks

In this section, I focus on the possible impact of uncertainty shocks on the domestic economy. Firstly, I look at the behavior of the time-varying component in each structural shocks through the estimated stochastic volatilities (SVs). Figure 2.5.1 displays the posterior mean of the estimated SVs $\sigma_{i,t}$ with $i := \{A, R, \tau_W, \tau_K, \epsilon_H, \mu, Y^*, \pi^*, r^*\}$ for the whole sample period.

Regarding the shocks from overseas, the Great Moderation in the United States might be one reason to keep the uncertainty stable at low levels from the mid-1980s to mid-2000s across the three foreign SVs in output, inflation, and interest rate shocks. The volatilities were at a higher level in output shocks during the Global Financial Crisis (GFC) in 2008, which has caused the recession in the U.S. and adverse effects on other countries.

Regarding the domestic economy, the Australian economy experienced a significant downward trend from the 1980s to 2015. However, there are several spikes which coincided with the undeniable crisis in Australian history, namely the Black Monday event in 1987; the Asian Financial crisis in 1997, the introduction of the good and service tax (GST) in 2000, the mining boom in the 2000s and the GFC in 2008.

Uncertainty shock estimates in the domestic monetary policies fluctuates during the sample period. A noticeable point is the adoption of inflation targeting monetary policy framework in 1993 following New Zealand, Canada, and Sweden. The new regime led to a spike in the gross domestic interest rate volatility in the mid-1990s. This was followed by a stable decade in volatility, with a significant drop in the early 2000s, before increasing during the GFC in 2008.

Labour and capital tax uncertainty estimates show a similar pattern. High spikes seen before 2000 might indicate high uncertainty associated with the introduction of GST.

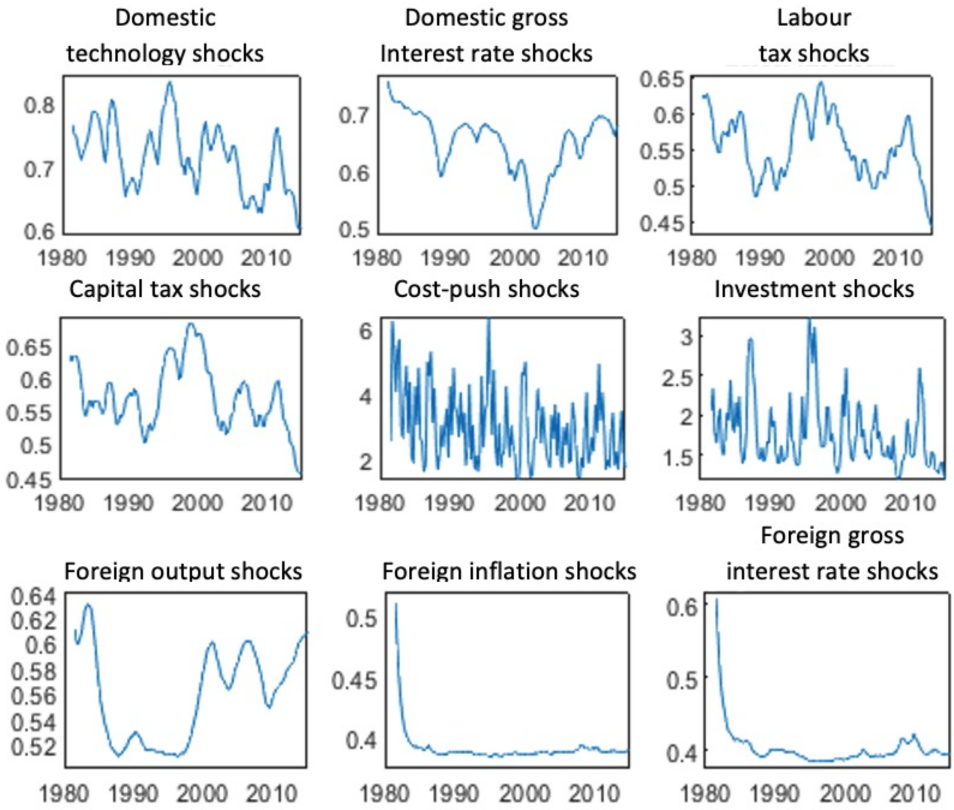
The SVs estimates in cost-push, and investment shocks highly fluctuate. Obviously, there are spikes in both series during the historical crisis in the 1980s, 1990s and 2000s. This seems to be consistent with the Canadian case studied by Cross et al. [2018].

2.5.3. Volatility shock accounting

This section will answer the research questions of the driving factor of the Australian business cycles in the last three decades. I will decompose and quantify the effect of time-varying components that account for uncertainty shocks in contributing to business-cycle fluctuations.

The long-run variance decomposition of the Australian real domestic GDP was analysed, over 40 quarters. The decompositions are constructed following Justiniano and Primiceri [2008] to analyse the impact of each shock on the variability of observable variables. With this method,

Figure 2.5.1.: Stochastic volatility shocks



the variance decompositions are changing over time because the variances of endogenous observable variables are time-varying.

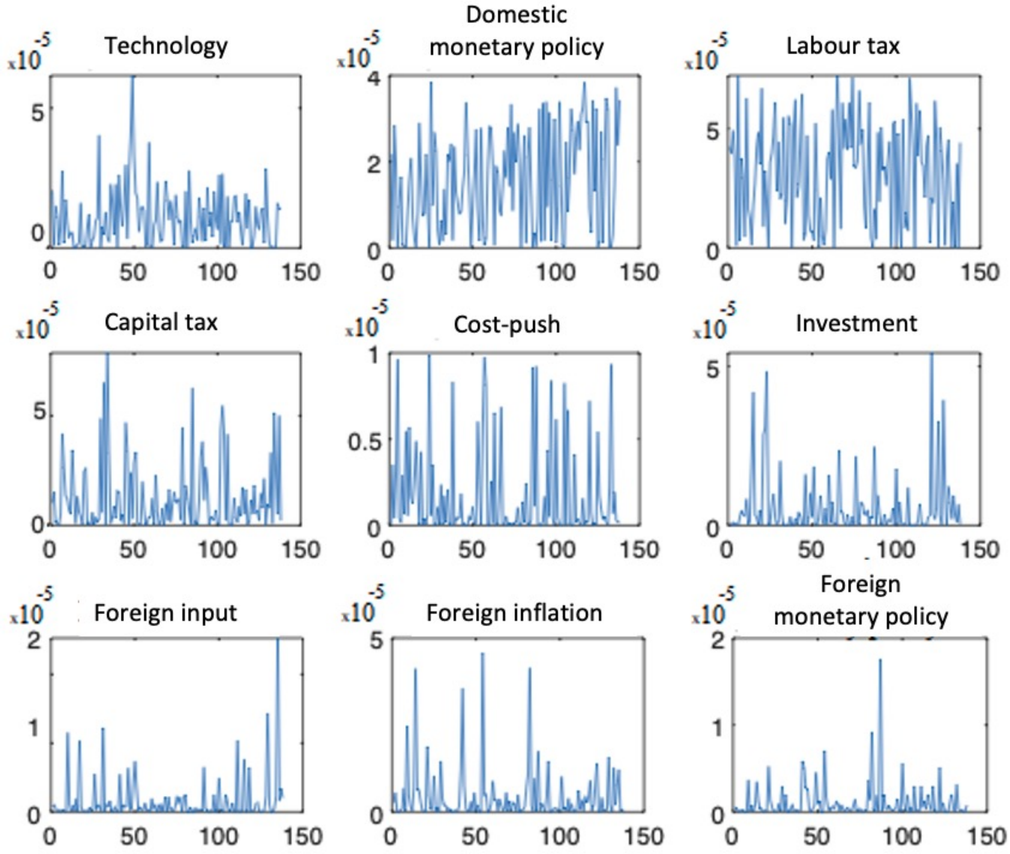
Combined effects

Figure 2.5.2 shows the forecast error variance decomposition (FEVD) to the GDP growth attributed to each structural shock. It represents the average share of each structural shock to the GDP growth fluctuations. The largest contribution is from labour tax shock (almost 30%), while most domestic shocks are nearly equally contributing to the GDP growth fluctuation, roughly 15%. Cost-push shock is more influential in some periods (especially in 1987), while investment shock overall is less relevant (about 5%). The ROW block contributes minimally to the GDP growth variations (less or equal to 5%) with higher impacts arising in the 2000s with the GFC. Similar to Justiniano and Preston [2010], international spillovers are also found less important. The findings here share similarities with the studies by Justiniano and Preston [2010] and Cross et al. [2018] where domestic policies contributed to output fluctuations substantially.

Absolute proportion of uncertainty component in the composite shocks

I then further decomposed the structural shocks u_t to distinguish and quantify the contributions of the i.i.d innovations ϵ_t and uncertainty components σ_t.

Figure 2.5.2.: Variance decomposition for real GDP growth (40 quarters ahead)



The following ratio calculates the absolution proportion of uncertainty share over composite structural shocks:

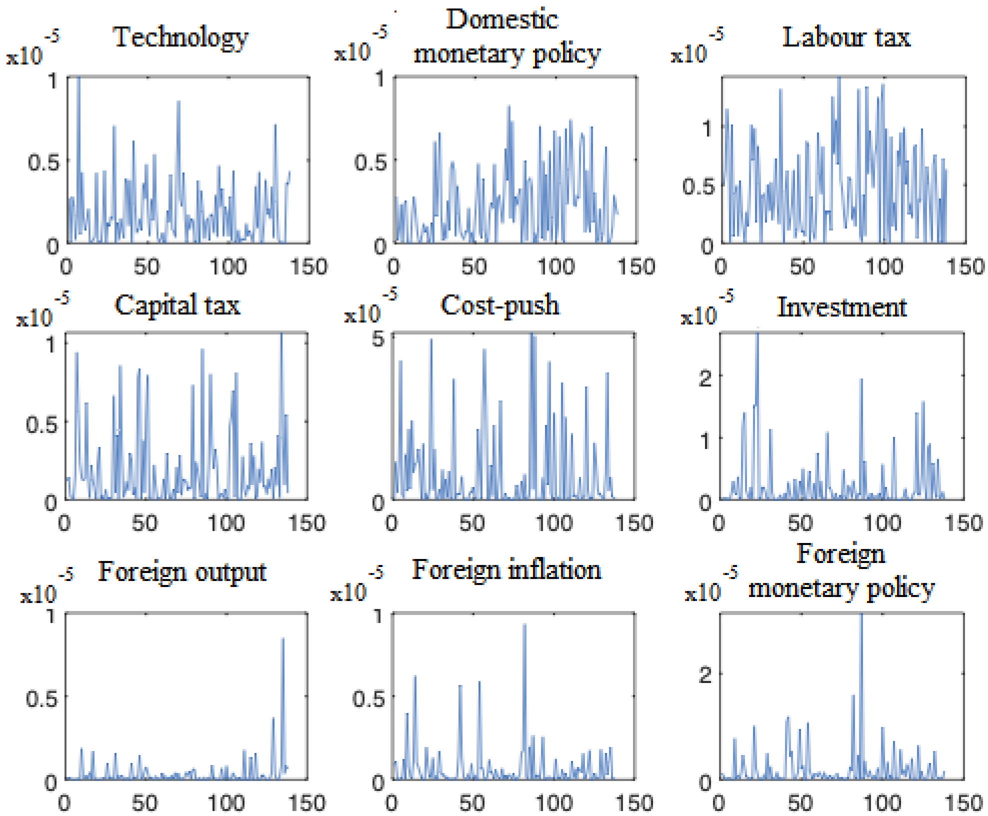
$$\xi_{i,t} = \frac{|log(\sigma^2_{i,t})|}{|log(u^2_{i,t})|}. \tag{2.1}$$

Figure 2.5.3 shows the evolution of absolute uncertainty proportions. Uncertainty share fluctuated around 20% for most structural shocks. More importantly, it appears primarily driven by the cost-push and investment shocks (almost 50%) while remained relatively stable for the ROW block at roughly 20%. For the technology shock, uncertainty was more important in some periods, namely early 2000 (internet dotcom) and in 2010 (GFC). For the domestic monetary and fiscal policies, uncertainty shares seem stable around 20%.

Variance decomposition breakdown: uncertainty effect to GDP growth

Multiplying the absolute uncertainty proportion with variance decomposition, I get the share of GDP growth fluctuations attributed to the uncertainty shocks and i.i.d components over the sample period. Table 2.5.1 shows the average GDP FEVD due to uncertainty, i.i.d, and composite shocks for the nine structural shocks. The last column represents the FEVD of structural shocks, corresponding to figure 2.5.1. In contrast, the second column shows the uncertainty share to FEVD, which corresponds to figure 2.5.3. In general, i.i.d innovations contribute nearly 80% to the fluctuations, while the SV components only account for about

Figure 2.5.3.: Uncertainty share evolution



20%. The SV components are more important in some cases, for instance, the cost-push and investment shocks during the mining boom period in the 2000s. The labour tax shock is the largest driver contributing to the GDP growth fluctuations, mainly caused by i.i.d innovations (83.43%). For the cost-push shock, the composite shocks account for 16.7% GDP growth fluctuations, while uncertainty and i.i.d components are nearly equally contributed (43% and 57%, respectively). The investment shock only accounts for 5% of the GDP fluctuations, but uncertainty accounts for 35%.

To conclude, under the model construction and interpretation, the output variations in the Australian economy in the last 30 years was mainly due to the domestic monetary and fiscal policy shocks. The cost-push shock was also crucial in some period, especially in 1987. The ROW blocks were not causing the business cycles. The fluctuations were mainly due to the policy on its own (good or bad policies), but uncertainty shocks have also contributed substantially to drive the output variations. Compared with the other studies for OECD countries by Cross et al. [2018], they share similar findings. The mean shocks are the main drivers of GDP growth fluctuations. In contrast, my study finds that uncertainty was much more important in Australia than in Canada (over 20% in Australia compared to 3% in Canada) during 1981–2015.

Table 2.5.1.: Driving factors of real GDP growth variance decomposition

Shocks	SV component	i.i.d component	Total
Technology	0.015	0.091	0.106
Domestic monetary policy	0.023	0.129	0.152
Labour tax	0.050	0.252	0.303
Capital tax	0.021	0.111	0.132
Cosh push	0.072	0.095	0.167
Investment	0.019	0.035	0.053
Foreign Output	0.003	0.015	0.018
Foreign Inflation	0.007	0.048	0.056
Foreign Monetary policy	0.002	0.012	0.014
Total	0.212	0.788	1

2.6. Conclusion

Throughout the study the primary purpose is to understand how important the mean and uncertainty shocks in driving the business cycles of a small, open economy like Australia. The model followed a similar structure to the one proposed by Cross et al. [2018]. The study uses the Bayesian techniques with six domestic and three foreign macroeconomics variables to estimate the model with nine structural shocks and stochastic volatilities to decompose and quantify the underlying drivers for real GDP fluctuations in Australia. This also accounts for international economic and policy spillover effects.

The Australian data was collected from ABS and other trusted sites to derive the findings. The finding shows that time-varying volatilities have contributed substantially to the business fluctuations in Australia (more than 20%) in which the labour tax and cost-push are the most considerable factor contributing to output disturbances among stochastic volatilities. However, the major driver factor is attributed to systematical shocks in which domestic policies are accounted for the greatest fraction (nearly 50%) of the GDP fluctuations. The international spillovers account for a negligible fraction (less than 10%) in the output movements. The labour tax shock is the most influential factor driving the business cycles (30%).

3. Uncertainty shocks in a multisector model for small, open economies: an Australian case study[1]

Abstract. This paper enriches the empirical research on the macroeconomic impacts of uncertainty shocks with a multisector DSGE model for a small, open economy. The model considers 17 structural shocks in six categories, incorporated with time-varying volatility components to quantify uncertainty shock effects on output variations. With a more refined classification of sources of uncertainty shocks and separate identification of mining sector and resource-price shocks, the model emphasises the importance of the resource sector in the Australian economy during the mining boom period from 1991 to 2013. The results show that foreign shocks are the most crucial drivers accounting for more than 50% of the Australian economic variations, while demand and productivity shocks have minimal impacts. Additionally, the shocks are further decomposed into their mean and volatility components. Consequently, the time-varying uncertainty shocks are found to be more important than the systematic shocks to the fluctuations, with great importance attributed to foreign shocks.

3.1. Introduction

The Australian economy has changed sharply over the past 30 years, as reflected by shifts in contribution to gross value added. The industry with the largest share of current price gross valued added now is the mining industry. This is due to the mining boom of the early 2000s which brought a massive investment in technology, manufacturing inputs, employment and imports. It has thus significantly increased the Australian standard of living. However, the adverse impact of the resource shocks caused by an immense appreciation of the Australian dollar has discouraged other industries. Therefore, a multisector model is one of the most crucial features of the Australian economy during the last three decades [Downes et al., 2014].

[1] I would like to thank Jamie Cross, Timothy Kam, Aubrey Poon, Daniel M Rees, Penelope Smith, and Jamie Hall for initial discussion and sharing the source codes. I also benefited from comments and feedback from seminar participants at the ANU Macroeconomics Group (AMG), Research School of Economics weekly seminars (Macroeconomics).

Table 3.1.1.: Changes in volatility of Australian aggregate variables

	Mean			Standard deviation		
	Inflation	Government spending	Resource price	Inflation	Government spending	Resource price
Q1.1992 - Q4.2013	0.64	0.71	0.00	0.56	1.08	0.03
Pre 2004.Q1	0.60	0.53	0.00	0.63	1.36	0.02
Post 2004.Q1	0.69	0.92	0.01	0.46	0.50	0.03
Post 2004.Q1/Pre 2004.Q1	**1.15**	**1.73**	**12.90**	**0.74**	**0.37**	**2.03**

Source: ABS 5206.005, ABS 6401.001, ABS 6401.002, RBA I2 - Index of Commodity Prices

Recent work by Rees et al. [2016] has developed a DSGE model with five production sectors to identify the source of Australian business-cycle fluctuations, but the model fails to produce a reasonable estimate in an appearance of uncertainty. More specifically, their model works well with the reasonably stable data but hardly captures the more volatile data such as resource prices and exchange rate series. They over-predicts GDP growth during the global financial crisis due to errors in the non-tradeable sector.

Using an example of resource prices, as a small, open economy, Australia is a price taker for resource goods. Resource prices are determined by global supply and demand and are highly volatile. Table 3.1.1 presents means and standard deviations for inflation, government spending, and resource-price series. As can be seen from the table, the volatility of resource prices in Australia was 12.9 times higher after 2004 than prior to 2004, indicating a robust time-varying feature of the series.

In chapter 2, I have found that the time-varying volatilities have contributed substantially to the business-cycle fluctuations in Australia. Even though the model considers a wide range of structural shocks, including foreign factors that can spill over, it fails to capture a necessary feature of a small resource-exporting economy like Australia. As a result, the findings reflect a negligible fraction of output movements attributed to international spill-over factors.

The evidence of time-varying variance in Australian output growth and resource price motivates the extensions in this chapter. More specifically, this research aims at answering these two questions: How much do domestic and international shocks contribute to the Australian aggregate variations during the mining boom? How important are stochastic volatility shocks in driving the business cycles?

Seeking answers to address these questions, I develop and estimate a DSGE model with a more refined production sector, emphasising the importance of the mining industry. The model includes time-varying volatilities in the innovations of 17 structural shocks in six categories. I explicitly distinguishes the effects of uncertainty shocks from the structural shocks on economic fluctuations within a multisector framework that fits the Australian economy. To the best of my knowledge, this multisector, small, open economy model contributes to the uncertainty shocks literature that has estimated the Australian economy at first [2].

The results show that foreign shocks, particularly risk-premium shocks, were the most crucial drivers accounting for more than 50% of the Australian economic variations during the mining boom. In contrast, the demand and productivity shocks had minimal impacts. However, after the shocks are further decomposed into their mean and volatility components, the time-varying uncertainty shocks were more important than the systematic shocks to the fluctuations, with great importance attributed to foreign shocks.

Literature review

The study was following Fernández-Villaverde and Rubio-Ramírez [2007] and Justiniano and Primiceri [2008] to import stochastic volatility structures in DSGE models. The time-varying volatility was estimated using their proposed computational procedure based on the Bayesian method and non-linear filtering theory. They applied the method to a medium-scale DSGE model for a closed economy to understand the importance of volatility and policy changes in the U.S monetary history. In addition, they also looked at a small open economy DSGE model, particularly for Argentina. The time-varying volatility was incorporated in real interest rates to understand its impacts on aggregate economic variables such as output, consumption, investment, and other economic conditions. This contributes to understanding the sources of aggregate fluctuations in emerging countries. Unlike them, this chapter complements other works studying the macroeconomic effects of time-varying uncertainty with a broader set of shocks. The set includes internal and external shocks with stochastic volatility components and quantify the effects of each shock on aggregate fluctuations in Australia. All shocks are assumed to have stochastic volatility components so that the model can indicate the most important factors in driving the business cycle without presuming uncertainty in a certain sector.

Next, the research complements other literature that develops the DSGE model to study the macroeconomic effects of uncertainty and time-varying volatility [Basu and Bundick, 2017, Carriero et al., 2018, Jurado et al., 2015, Garcıa Cicco et al., 2013, Pfeifer et al., 2012]. In terms of similarities, their research takes a structural approach in modeling DSGE models in which uncertainty shocks have explicit economic interpretations. However, my research considers a broader range of shocks and explicitly distinguishes the macroeconomic effect of uncertainty shocks from the structural shocks. The model considers 17 structural shocks in six categories: productivity, demand, supply, monetary, commodity, and foreign shocks. To the best of my knowledge, this structural model is the first study on a small, open economy modeling interactions between different sectors, to contribute to the literature of uncertainty shocks.

[2]The use of DSGE model for macroeconomic scenario analysis has been discussed widely in the literature and chapter 1 of this thesis. VAR models were used as empirical evidence to motivate this research.

Furthermore, this study builds on the literature of macroeconomic consequences of domestic and international spill-over shocks. Justiniano and Preston [2010] suggested that around half of two year-ahead Canadian output growth volatility is explained by the first-moment U.S. shocks. Besides, Cross et al. [2018] found that time-varying uncertainty has a negligible effect on Canadian business-cycle fluctuations. Gómez-González et al. [2013] found that the direct effects of terms-of-trade volatility shocks on output, consumption and investment are generally small for the Australian economy. Nevertheless, when interacting with the level shocks, the volatility shocks account for around one-quarter of the total impact of terms of trade shocks on macroeconomic outcomes. Rees et al. [2016] developed a DSGE model with five production sectors to identify the source of Australian business-cycle fluctuations but the model fails to produce a good estimate in an appearance of uncertainty. Compare with this research, I studied and documented the significant impact of uncertainty shocks in driving business cycles in Australia.

There is more direct empirical evidence where uncertainty shocks are exogenous for the Australian business cycles. Specifically, Tran [2019] studied the commodity price uncertainty index for Australia from 1994 to 2017. He found that high uncertainty often associated with aggregate fluctuations, such as iron ores market fluctuation in 2005, Global Financial Crisis in 2008 and China's slowdown in 2014. Compared with the Economic Uncertainty index by Moore [2017] and CBOE volatility index, the commodity uncertainty index weakly correlated with domestic and financial uncertainty in Australia. The commodity uncertainty index has a higher correlation with other uncertainty indexes, for instance, Global Economic Policy Uncertainty Index by Baker et al. [2016] and Jurado et al. [2015].

My work employed the multisector framework from Rees et al. [2016], who developed a DSGE model with five production sectors to identify the source of Australian business-cycle fluctuations. However, their model fails to produce a reasonable estimate in an appearance of uncertainty. Compared with theirs, my research emphasises the time-varying variance feature in Australian output growth and resource prices. The model includes 17 structural shocks with stochastic volatilities incorporated to estimate the impact of uncertainty shocks in driving the business cycles. Thus, the model by Rees et al. [2016] will be used as a benchmark model where the stochastic volatility structures are not imposed.

More recently, Tran [2019] studied commodity uncertainty in a multisector DSGE framework. He found that the commodity uncertainty shocks negatively impact aggregate variables, namely output, consumption, investment, and trade, via a precautionary saving mechanism. Compared with his paper, my model considers uncertainty shocks in a broader range, including domestic, international, and policy shocks, to provide a more comprehensive opinion on the source of the Australian aggregate fluctuations, notably through the mining boom.

The remainder of this paper is structured as follows: Section 3.2 describes the model and equilibrium conditions. Section 3.3 shows exogenous stochastic shocks, while section 3.4

describes the Bayesian estimation method and shows the data and calibrations. Results and a discussion are presented in section 3.5, followed by a conclusion in section 3.6.

3.2. A multi-production DSGE model

This section describes a multisector DSGE model used to study the macroeconomic effects of domestic and international spill-over shocks on real economic activity. The model extends the DSGE model for Australia from Rees et al. [2016] by incorporating time-varying components to the structural shocks faced by economic agents. Figure 3.2.1 provides a visual summary of the populated economy with economic agents and their interactions.

3.2.1. A representative household

A representative household chooses to work H_t and consume C_t to maximise its lifetime utility:

$$U(C_t, H_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_{c,t} ln(C_t - hC_{t-1}) - A_L \frac{H_t^{1+\eta}}{1+\eta} \right\}, \tag{3.1}$$

where β is an inter-temporal discount rate; h controls a habit persistence; η is an inverse elasticity of labour supply to wages; A_L is a normalised constant to ensure the model's steady-state matching with sample data; $\xi_{c,t}$ governs preference shocks to consumption following a stationary auto-regressive process. The choice of a log utility in consumption and linear marginal substitution between consumption and leisure ensures the presence of a balanced-growth path; this also represents the common fact that households are shifting their preference.

The investment $I_{j,t}$ induces a law of motion for the sector-specific capital accumulation $K_{j,t}$ as follows:

$$K_{j,t+1} = (1-\delta)K_{j,t} + \Upsilon_t I_{j,t} \left[1 - \frac{\Phi_K}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1 \right)^2 \right], \tag{3.2}$$

where $j = \{n, m, z\}$ indicating non-traded (n), traded (m) and resource (z) sectors, respectively; δ is a common capital depreciation rate across sectors; Φ_K is quadratic adjustment costs parameters; Υ_t is an investment shock following a stationary auto-regressive process.

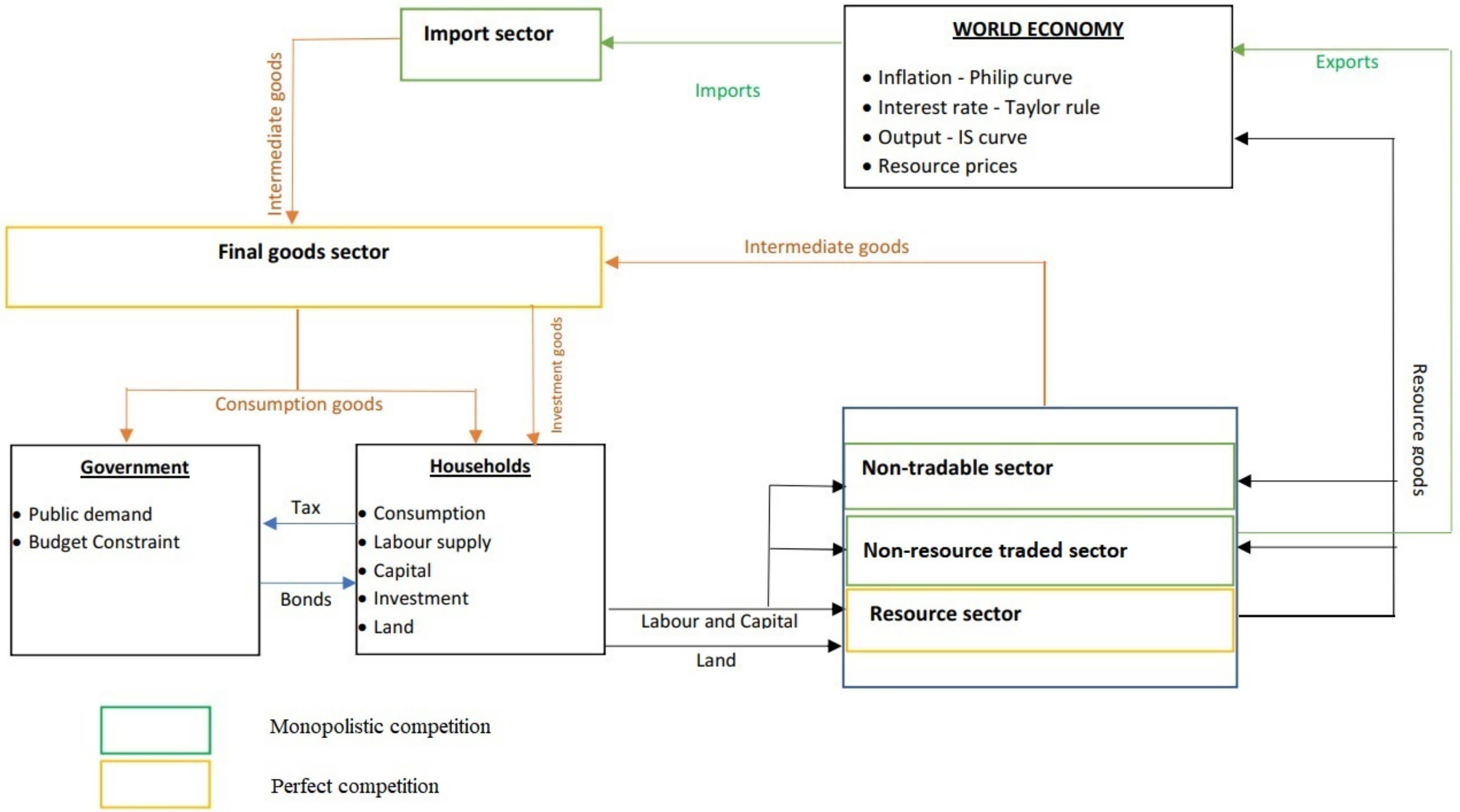
The labour supply index is a composite of labour supply in non-traded (n), traded (m) and resource (z) sectors as follows:

$$H_t = \left[H_{n,t}^{1+\sigma} + H_{m,t}^{1+\sigma} + H_{z,t}^{1+\sigma} \right]^{1/(1+\sigma)}, \tag{3.3}$$

where a positive parameter $\sigma \geq 0$ is an elasticity of labour substitution across sectors.

A household trades its domestic bonds B_{t-1} and foreign bonds B_{t-1}^* subject to a nominal exchange rate S_t. Bonds are risk-free and mature after one period. The domestic and foreign bond prices are $1/R_t$ and $S_t/(R_t^* v_t)$,and are proportional to gross nominal interest rates in

Figure 3.2.1.: Overview of the multiple sector economy



domestic and foreign economies, R_t and R_t^*, respectively. A variable υ_t indicates a country-specific risk-premium that positively related to a foreign debt and a risk-premium shock Ψ_t following a stationary auto-regressive process:

$$\upsilon_t = exp\left[-\chi_{rp}\left(\frac{S_{t-1}B_{t-1}^*}{P_{t-1}Y_{t-1}}\right) + \Psi_t\right], \tag{3.4}$$

where χ_{rp} is a foreign debt risk-premium parameter. A household can trade its domestic and foreign bonds in international financial markets subject to the domestic and foreign interest and exchange rates. The household also provides labour, own equity of domestic firms, rent land endowment L for firms in production and resource sectors [3]. In exchange, it receives wages, profit from equity ownership, rental income and income from asset holdings. The household then uses its total income and asset holdings to purchase new bonds, consumption or investment goods and pay taxes T_t:

$$P_tC_t+P_tI_t+\frac{B_t}{R_t}+\frac{S_tB_t^*}{R_t^*\upsilon_t} \leq \sum_{j=n,m,z}(W_{j,t}H_{j,t}+R_{j,t}K_{j,t})+R_{L,t}L+B_{t-1}+S_tB_{t-1}^*+\Gamma_t-T_t, \tag{3.5}$$

where P_t is a domestic final goods price; $j = \{n,\ m,\ z\}$ indicates non-traded (n), traded (m) and resource (z) sectors; $I_t = \sum_{j=n,m,z} I_{j,t}$ is an aggregate investment, equivalent to a total of investment in each sector j; $\Gamma_t = \sum_{j=n,m,z}\int_0^1 \Gamma_{j,t}(i)di$ is an aggregate profit, equivalent to a total of profit from all firms in each sector j; $W_{j,t}$ and $R_{j,t}$ are wage and capital return rate in sector j; $R_{L,t}$ is a land rental rate; T_t is a lump sum transfer to government.

3.2.2. Non-traded sector

The non-traded sector consists of a continuum of firms in an imperfect competition using capital, labour and resource goods as inputs to produce differentiated goods. They sell their output to a retailer with technology to transform intermediate goods into homogeneous goods sold to the final goods producer. The transformation technology in the non-traded sector follows the constant elasticity of substitution (CES) function:

$$Y_{n,t} = \left[\int_0^1 Y_{n,t}(i)^{\frac{\theta^n-1}{\theta^n}}di\right]^{\frac{\theta^n}{\theta^n-1}}, \tag{3.6}$$

where $Y_{n,t}$ is an aggregate output in non-traded sector; $Y_{n,t}(i)$ is an output of non-traded firm i; θ^n is an elasticity of substitution between differentiated goods.

[3] For the long-term model, the model can allow land to be time-varying based on the amount of land usage in production as land needs time to switch between active and inactive positions. However, in this paper, I only focus on the short-term business cycle fluctuations, so that land assumes to be fixed.

The demand function for each firm's output is given by:

$$Y_{n,t}(i) = \left(P_{n,t}(i)/P_{n,t}\right)^{-\theta^n} Y_{n,t}, \tag{3.7}$$

where $P_{n,t}(i)$ is price for a differentiated goods produced by firm i; $P_{n,t}$ is price for a composite goods in non-traded sector.

The production function for each non-traded firm i:

$$Y_{n,t}(i) \leq a_{n,t} \left(\mu_t H_{n,t}(i)\right)^{\alpha_n} K_{n,t}(i)^{\gamma_n} Z_{n,t}(i)^{1-\alpha_n-\gamma_n}, \tag{3.8}$$

where $H_{n,t}(i)$, $K_{n,t}(i)$, $Z_{n,t}(i)$ are hours worked, capital and resource inputs used by non-traded firm i; α_n, γ_n and $1-\alpha_n-\gamma_n$ are Cobb–Douglas parameters governing the input shares of labour, capital and resource in producing goods in the non-traded sector; $a_{n,t}$ is a sector-specific technology shock in non-traded sector following a stationary auto-regressive process, while μ_t is a permanent productivity shock common across sectors.

The prices are assumed to be sticky following Rotemberg [1982] with a quadratic cost of adjusting price settings. In each period, firms can change their prices subject to the adjustment cost. Thus, all firms face the same problem then choose the same solution for their profit maximisation problem. The indexation controls by $\chi \in [0,1]$ where $\chi = 0$ when all firms choose to change their prices, while $\chi = 1$ when no firms change their prices. Under monopolistic competition, given demand functions, firms choose prices and inputs to maximise their real profits. The profit is equal to real revenue from selling output at monopolistic prices minus total production cost, which are real marginal cost and average cost, or the following:

$$\Gamma_{n,t}(i) = \frac{P_{n,t}(i)Y_{n,t}(i)}{P_t} - \frac{MC_{n,t}Y_{n,t}(i)}{P_t} - \frac{\tau_{\pi^n}}{2}\left[\frac{P_{n,t}(i)}{\Pi_{n,t-1}^{\chi}\Pi^{1-\chi}P_{n,t-1}(i)} - 1\right]^2 \frac{P_{n,t}Y_{n,t}}{P_t}, \tag{3.9}$$

where $\Gamma_{n,t}(i)$ is profit of firm i in sector n; τ_{π^n} is a positive parameter measuring the cost of price adjustment in units of non-traded output; the term in square brackets describes the total price-adjustment cost.

$MC_{n,t}$ is a nominal marginal cost which is identical for all firms in non-traded sector:

$$MC_{n,t} = \frac{e_{\pi_n,t}}{a_{n,t}}\left(\frac{W_{n,t}}{\alpha_n}\right)^{\alpha_n}\left(\frac{R_{n,t}}{\gamma_n}\right)^{\gamma_n}\left(\frac{P_{Z,t}}{1-\alpha_n-\gamma_n}\right)^{1-\alpha_n-\gamma_n}, \tag{3.10}$$

where $e_{\pi_n,t}$ is a positive markup shock that changes the nominal marginal cost of non-traded production for reasons other than changes in wage, capital return rate and resource price, which is assumed to be a white noise process.

3.2.3. Non-resource traded sector

The non-resource traded sector consists of a continuum of imperfect competition firms using capital, labour and resource goods as inputs to produce differentiated goods.

The firms in this sector will sell a fraction of their output in the domestic market while the rest of the output was exported to overseas markets. They sell directly to the retailers or exporters who have transformation technology to turn differentiated products into a homogeneous good for selling to the final goods producers or exporting to foreign markets.

The transformation technology in non-resource traded sector follows a CES function:

$$Y^j_{m,t} = \left[\int_0^1 Y^j_{m,t}(i)^{\frac{\theta^m - 1}{\theta^m}} di \right]^{\frac{\theta^m}{\theta^m - 1}}, \tag{3.11}$$

where $j \in \{d, x\}$ indicates domestic and export markets, respectively; $Y^j_{m,t}$ is an aggregate output in the non-resource traded sector for market j; $Y^j_{m,t}(i)$ is an output of non-resource traded firm i for market j; θ^m is an elasticity of substitution between differentiate goods.

The demand function for each firm's output for each market j is given by:

$$Y^d_{m,t}(i) = \left(P_{m,t}(i)/P_{m,t}\right)^{-\theta^m} Y^d_{m,t}, \tag{3.12}$$

$$Y^x_{m,t}(i) = \left(P^*_{m,t}(i)/P^*_{m,t}\right)^{-\theta^m} Y^x_{m,t}, \tag{3.13}$$

where $P_{m,t}(i)$, $P^*_{m,t}(i)$ are prices for a differentiated good produced by firm i in domestic and overseas markets; $P_{m,t}$, $P^*_{m,t}$ are prices for a composite non-resource traded products selling to final goods producers and overseas market.

The production function for each non-resource traded firm i is:

$$Y_{m,t}(i) \leq a_{m,t} \left(\mu_t H_{m,t}(i)\right)^{\alpha_m} K_{m,t}(i)^{\gamma_m} Z_{m,t}(i)^{1-\alpha_m-\gamma_m}, \tag{3.14}$$

where $H_{m,t}(i)$, $K_{m,t}(i)$, $Z_{m,t}(i)$ are hours worked, capital and resource inputs used by non-resource traded firm i; α_m, γ_m and $1 - \alpha_m - \gamma_m$ are Cobb–Douglas parameters governing the input shares of labour, capital and resources in producing goods in the non-resource traded sector; $a_{m,t}$ is a sector-specific technology shock in the non-resource traded sector following a stationary auto-regressive process, while μ_t is a permanent productivity shock common across sectors.

The prices are assumed to be sticky following Rotemberg [1982] with a quadratic cost of adjusting price settings. In each period, firms can change their prices subject to the adjustment cost. Thus, all firms face the same problem then choose the same solution for their profit maximisation problem. The indexation controls by $\chi \in [0,1]$ where $\chi = 0$ when all firms choose to change their prices, while $\chi = 1$ when no firms change their prices.

Under monopolistic competition, given demand functions, firms choose prices and inputs to maximise their real profits. Their real profits are equal to the real revenue from selling output at monopolistic prices minus total production cost. Given that the total production cost is the real marginal cost plus average cost, the real profits are derived as follows:

$$\begin{aligned}\Gamma_{m,t}(i) = & \frac{P_{m,t}(i)Y^d_{m,t}(i)}{P_t} + \frac{S_t P^*_{m,t}(i)Y^x_{m,t}(i)}{P_t} \\ & - \frac{MC^d_{m,t}Y^d_{m,t}(i)}{P_t} - \frac{MC^x_{m,t}Y^x_{m,t}(i)}{P_t} \\ & - \frac{\tau_{\pi^m}}{2}\left[\frac{P_{m,t}(i)}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}P_{m,t-1}(i)} - 1\right]^2 \frac{P_{m,t}Y^d_{m,t}}{P_t} \\ & - \frac{\tau_{\pi^{m*}}}{2}\left[\frac{P^*_{m,t}(i)}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}P^*_{m,t-1}(i)} - 1\right]^2 \frac{S_t P^*_{m,t}Y^x_{m,t}}{P_t},\end{aligned} \tag{3.15}$$

where $\Gamma_{m,t}(i)$ is a profit of firm i in sector m; τ_{π^m}, $\tau_{\pi^{m*}}$ are positive parameters measuring the cost of price adjustment in units of non-resource traded output; the terms in square brackets describe the total price adjustment cost in each market j.

$MC^j_{m,t}$ is a nominal marginal cost which is identical for all firms in the non-resource traded sector:

$$MC^j_{m,t} = \frac{e^j_{\pi_{m,t}}}{a_{m,t}}\left(\frac{W_{m,t}}{\alpha_m}\right)^{\alpha_m}\left(\frac{R_{m,t}}{\gamma_m}\right)^{\gamma_m}\left(\frac{P_{Z,t}}{1-\alpha_m-\gamma_m}\right)^{1-\alpha_m-\gamma_m}, \tag{3.16}$$

where $e^j_{\pi_{m,t}}$ is a positive markup shock that changes the nominal marginal cost of non-resource traded production in each market $j \in \{d, x\}$ for reasons other than changes in wage, capital return rate, and resource price, which is assumed to be a white noise process.

Since I consider a small, open economy, the foreign demand for non-resource traded product is given as follows:

$$Y^x_{m,t} = \omega^*_m \left(P^*_{m,t}/P^*_t\right)^{\zeta^*} Y^*_t, \tag{3.17}$$

with $Y^x_{m,t}$ is a quantity of non-resource traded products demand to export; P^*_t and Y^*_t are the price and GDP levels in foreign economies; ω^*_m is a share of non-resource traded goods exported to the overseas market; ζ^* is an elasticity of substitution between domestic and foreign goods in the overseas market.

3.2.4. Resource sector

The resource sector consists of a single firm that produces homogeneous output from labour, capital and land inputs under perfect competition with given prices. The production function is as follows:

$$Y_{z,t} = a_{z,t}\left(\mu_t H_{z,t}\right)^{\alpha_z} K^{\gamma_z}_{z,t}\left(\mu_t L\right)^{1-\alpha_z-\gamma_z}, \tag{3.18}$$

where $H_{z,t}$, $K_{z,t}$, L are hours worked, capital and land inputs used by resource firm; α_z, γ_z and $1-\alpha_z-\gamma_z$ are Cobb–Douglas parameters governing the input shares of labour, capital, and land in producing goods in the resource sector; $a_{z,t}$ is a sector-specific technology shock in the resource sector following a stationary auto-regressive process, while μ_t is a permanent productivity shock common across sectors.

The resource firm chooses labour and capital inputs to maximise its profit each period with a given price $P_{z,t}$ in domestic currency:

$$\Gamma_{z,t} = P_{z,t}Y_{z,t} - W_{z,t}H_{z,t} - R_{z,t}K_{z,t} - R_{L,t}L. \tag{3.19}$$

The resource price $P^*_{z,t}$ is determined by foreign demand and unrelated to the domestic economy. In the long-run, I assume domestic and foreign prices are identical to the law of one price. However, the model considers a short-term delay for the domestic price changes to adapt to the foreign price movement in the short-run. This assumption is backup by two facts: first, a large fraction of the Australian resource goods exported are priced periodically with contracts; second, resource firms hedge their overseas currency exposures. Following Rees et al. [2016], I choose the sticky price parameters that attribute equal weights to the foreign and domestic prices. Then, the domestic currency resource prices are derived as:

$$P_{z,t} = \left(S_t P^*_{z,t}\right)^{1/2} \left(P_{z,t-1}\right)^{1/2}. \tag{3.20}$$

3.2.5. Import sector

The import sector consists of firms using their production technology to aggregate a continuum of imported varieties to homogeneous products sold to the final goods producers.

The production technology in the import sector follows the CES function:

$$Y_{f,t} = \left[\int_0^1 Y_{f,t}(i)^{\frac{\theta^f-1}{\theta^f}} di\right]^{\frac{\theta^f}{\theta^f-1}}, \tag{3.21}$$

where $Y_{f,t}$ is an aggregate output in the import sector; $Y_{f,t}(i)$ is an imported varieties from firm i; θ^f is an elasticity of substitution between varieties. The demand function for each variety is given by:

$$Y_{f,t}(i) = \left(P_{f,t}(i)/P_{f,t}\right)^{-\theta^f} Y_{f,t}, \tag{3.22}$$

where $P_{f,t}(i)$ is prices for variety imported by firm i selling in the domestic market; $P_{j,t}$ is prices for a composite imported product sold to the final goods producer.

Import firms also face a quadratic cost of adjusting price function following Rotemberg [1982]. In each period, firms can change their prices subject to the adjustment cost. Thus, all firms face the same problem, then choose the same solution for their profit maximisation problem. The indexation controls by $\chi \in [0,1]$ where $\chi = 0$ when all firms choose to change their prices, while $\chi = 1$ when no firms change their prices. They choose prices to maximise their profit as follows:

$$\Gamma_{f,t}(i) = \frac{P_{f,t}(i)Y_{f,t}(i)}{P_t} - \frac{MC_{f,t}Y_{f,t}(i)}{P_t} - \frac{\tau_{\pi^f}}{2}\left[\frac{P_{f,t}(i)}{\Pi_{f,t-1}^{\chi}\Pi^{1-\chi}P_{f,t-1}(i)} - 1\right]^2 \frac{P_{f,t}Y_{f,t}}{P_t}. \quad (3.23)$$

$MC_{n,t}(i)$ is a nominal marginal cost of firm i:

$$MC_{f,t}(i) = e_{\pi_{f,t}} \frac{S_t P_{f,t}^*}{P_{f,t}}, \quad (3.24)$$

where $P_{f,t}^*$ is an importing price in foreign currency; $e_{\pi_{f,t}}$ is a positive markup shock that change the nominal marginal cost for reasons other than domestic and overseas economic conditions, which is assumed to be a white noise process.

3.2.6. Final goods sector

The final goods sector consists of competitive firms that combine composite outputs from all intermediate good producers in non-traded, non-resource traded, and import sectors to produce a single final goods. The production function is given as follows:

$$DFD_t = \left[\omega_n^{1/\zeta} Y_{n,t}^{\frac{\zeta-1}{\zeta}} + \omega_m^{1/\zeta} Y_{m,t}^{d\frac{\zeta-1}{\zeta}} + \omega_f^{1/\zeta} Y_{f,t}^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \quad (3.25)$$

where ω_n, ω_m, ω_f govern shares of non-traded, non-resource tradeable and import sector in the final domestic goods: $\omega_n + \omega_m + \omega_f = 1$; DFD_t is a domestic final demand at time t.

Firms in this sector face perfect competition, maximise their profits, with given intermediate prices and final good price. The final goods price index is:

$$P_t = \left[\omega_n P_{n,t}^{1-\zeta} + \omega_m P_{m,t}^{1-\zeta} + \omega_f P_{f,t}^{1-\zeta}\right]^{\frac{1}{1-\zeta}}. \quad (3.26)$$

3.2.7. Monetary authority

The central bank sets nominal interest rates following a conventional Taylor rule that responds to the inflation, output growth and real exchange rate as follows:

$$ln\left(\frac{R_t}{R}\right) = \rho_r ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left[\phi_\pi ln\left(\frac{\Pi_t}{\Pi}\right) + \phi_y ln\left(\frac{Y_t^{va}}{Y^{va}}\right) + \phi_{\Delta_y} ln\left(\frac{Y_t^{va}}{Y_{t-1}^{va}}\right) + \phi_q \frac{Q_t}{Q_{t-1}}\right] + e_{r,t}, \quad (3.27)$$

where Π_t is a CPI inflation rate which is equal to $\Pi_t = \frac{P_t}{P_{t-1}}$ and Π is a CPI inflation rate target; Y_t^{va} is a deviation of real GDP from long-run trend; ρ_r controls a smoothing degree of interest rate; ϕ_π, ϕ_y, ϕ_{Δ_y}, ϕ_s control the responsiveness of interest rate to the current value of CPI inflation, output level, output growth rate and real exchange rate; $Q_t = S_t \frac{P_t^*}{P_t}$ is a real exchange rate which is equal to the nominal exchange rate multiplied by the relative price level between foreign and home countries; $e_{r,t}$ is a monetary shock, which is assumed to be a white noise process.

3.2.8. Government

Government revenue is raised from issuing bonds to borrow from the economy through open market operations. The bonds are assumed to be at interest rate R_t. Another source of government revenue is lump-sum transfers. The government uses its revenue to pay for its expenditure on goods and services. The government keeps a balanced budget as follows:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}, \tag{3.28}$$

where G_t is a public demand. It follows an exogenous process:

$$ln\left[\frac{G_t}{\mu_t}\right] = \rho_g ln\left[\frac{G_{t-1}}{\mu_{t-1}}\right] + e_{g,t}, \tag{3.29}$$

where $e_{g,t}$ is a public demand shock, which is assumed to be a white noise process.

3.2.9. Foreign economy

As in Rees et al. 2016, the foreign block is modeled as a closed economy presented by the four following equations: IS curve, Phillip curve, Taylor rule, and real resource prices. All variables are in log deviations from their steady-state, denoted with a "hat".

First, the foreign IS curve represents a relationship between foreign output and interest rate with the presence of demand shock:

$$\hat{y}_t^* = E_t\{\hat{y}_{t+1}^*\} - \left(\hat{r}_t^* - E_t\{\hat{\pi}_{t+1}^*\}\right) - E_t\{\hat{\xi}_{y^*,t+1}\} + \hat{\xi}_{y^*,t}, \tag{3.30}$$

where y_t^*, r_t^*, π_t^* are the foreign output, interest rate and CPI inflation rate, respectively; $\xi_{y^*,t}$ is a foreign demand shock following a stationary auto-regressive process.

Second, the foreign Phillip curve describes a relationship between inflation and expected output gap:

$$\hat{\pi}_t^* = \beta E_t\{\hat{\pi}_{t+1}^*\} + \frac{\kappa^*}{100}\hat{y}_t^* + e_{\pi^*,t}, \tag{3.31}$$

where $e_{\pi^*,t}$ is a cost-push shock following a stationary auto-regressive process.

Third, the Taylor rule governs the monetary regulation of foreign central bank to decide foreign interest rate:

$$\hat{r}_t^* = \rho_{r^*}\hat{r}_{t-1}^* + (1-\rho_{r^*})(\phi_{\pi^*}\hat{\pi}_t^* + \phi_{y^*}\hat{y}_t^*) + \phi_{\Delta_{y^*}}(\hat{y}_t^* - \hat{y}_{t-1}^*) + e_{r^*,t}, \tag{3.32}$$

where $e_{r^*,t}$ is a monetary shock, which is assumed to be a white noise process.

Last, the real resource price in foreign currency is affected by foreign demand shocks and resource-specific price shocks:

$$\hat{p}_{z,t}^* = \rho_{p_{z^*}}\hat{p}_{z,t-1}^* + \phi_{zy,t}\hat{\xi}_{y^*,t} + e_{p_z^*,t}, \tag{3.33}$$

with $\hat{p}_{z,t}^* = log\left[P_{z,t}^*/P_t^*\right]$ is a relative price of resource goods in foreign currency; $e_{p_z^*,t}$ is a resource-price shock, which is assumed to be a white noise process.

3.2.10. Market-clearing conditions

First, the market-clearing condition in the domestic final good markets is:

$$DFD_t = C_t + I_t + G_t. \tag{3.34}$$

Second, the market-clearing condition in the non-resource traded sector implies that all outputs in this sector must be sold in the either domestic or overseas markets:

$$Y_{m,t} = Y_{m,t}^d + Y_{m,t}^x. \tag{3.35}$$

Third, the market-clearing condition in the resource sector implies that all resource outputs must be exported or sold to non-traded and non-resource traded sectors to use as inputs for their productions:

$$Y_{z,t} = Z_{x,t} + Z_{n,t} + Z_{m,t}. \tag{3.36}$$

Fourth, the nominal net exports NX_t is given by the total exports from resource and non-resource traded sectors minus total import from import sector:

$$NX_t = P_{z,t}Z_{x,t} + S_tP_{m,t}^*Y_{m,t}^x - S_tP_{f,t}^*Y_{f,t}. \tag{3.37}$$

Fifth, the current account equation describes the evolution of net foreign debt as:

$$\frac{S_tB_t^*}{R_t^*v_t} = S_tB_{t-1}^* + NX_t. \tag{3.38}$$

Sixth, the measurement of GDP should avoid double counting the resource outputs since they are also used as inputs in other production sectors:

$$Y_{m,t}^{va} = Y_{m,t} - (P_z/P_m)\, Z_{m,t}, \tag{3.39}$$

$$Y_{n,t}^{va} = Y_{n,t} - (P_z/P_n)\, Z_{n,t}, \tag{3.40}$$

where $Y_{m,t}^{va}$, $Y_{n,t}^{va}$ are the gross value added in non-resource traded and non-traded sectors; P_m, P_n, P_z are composite prices in non-resource traded, non-traded, resource sectors fixed at the steady-state values.

The real GDP is calculated to abstract from price changes using fixed prices at the steady-state values:

$$Y_t = \frac{P_n}{P} Y_{n,t}^{va} + \frac{P_m}{P} Y_{m,t}^{va} + \frac{P_z}{P} Y_{z,t}, \tag{3.41}$$

where P is a domestic price level fixed at the steady-state value.

Seventh, the market-clearing condition in the investment good markets is:

$$I_t = \sum_{j=n,m,z} I_{j,t}. \tag{3.42}$$

Eighth, the market-clearing condition in the labour markets is:

$$H_t = \left[H_{n,t}^{1+\sigma} + H_{m,t}^{1+\sigma} + H_{z,t}^{1+\sigma} \right]^{1/(1+\sigma)}.$$

3.2.11. Competitive equilibrium conditions

Asset-pricing condition

The UIP condition requires zero profitable arbitrage in international financial markets, implying equality between the two Euler equations in the household's problem:

$$E_t \left[\frac{Q_{t+1}}{Q_t} \frac{\Pi_{t+1}}{\Pi_{t+1}^*} \right] R_t^* v_t = R_t. \tag{3.43}$$

The model tackles the exchange rate indirectly through the interest rate parity condition equation. Lubik and Schorfheide [2007] and Kam et al. [2009] showed that the optimal policy rules for central bank respond to exchange rate movements even they do not explicitly stabilise exchange rates. Thus, even though RBA does not target the exchange rate, but their optimal Taylor rules will have on stabilising impact on the exchange rate through the inflation channel. Then RBA indirectly targets the real exchange rate.

Given government policy rules in equations (3.27) and (3.29), a recursive competitive equilibrium is a system of decision functions and pricing kernels such that:

1. households maximise their lifetime utility functions, subject to the budget constraints and capital accumulation;
2. intermediate firms in non-resource sectors minimise their costs subject to the production functions and maximise their profits, subject to thedemand functions;
3. intermediate firms in resource sector maximise their profits, subject to production functions;
4. a final goods firm maximise its profit, subject to production functions and the price index;
5. markets must be clear each period: (3.3), (3.34), (3.35), (3.36), (3.37), (3.38), (3.41), (3.42); and
6. governments balance their budgets: (3.28).

3.3. Exogenous stochastic shocks

3.3.1. Structural shocks

The law of motion for labour-augmenting domestic permanent technology follows the process:

$$ln(\mu_t) - ln(\mu_{t-1}) = ln(\mu) + \sigma_\mu \epsilon_{\mu,t}, \tag{3.1}$$

where $\epsilon_{\mu,t} \sim N(0,1)$ and $ln(\mu)$ is the productivity growth rate in long-run.

The sector-specific technology shocks follow the AR(1) process:

$$\hat{a}_{j,t} = \rho_{a_j} \hat{a}_{j,t-1} + \sigma_{a_j,t} \epsilon_{a_j,t}, \tag{3.2}$$

where $\epsilon_{a_j,t} \sim N(0,1)$; $j \in \{n, m, z\}$, and all variables with the hats in log deviations from their steady-state.

The household efficient investment shock follows the AR(1) process:

$$\hat{\Upsilon}_t = \rho_\Upsilon \hat{\Upsilon}_{t-1} + \sigma_{\Upsilon,t} \epsilon_{\Upsilon,t}, \tag{3.3}$$

where $\epsilon_{\Upsilon,t} \sim N(0,1)$.

The household preference shock follows the AR(1) process:

$$\hat{\xi}_{ct} = \rho_{\xi_c} \hat{\xi}_{ct-1} + \sigma_{\xi_c,t} \epsilon_{\xi_c,t}, \tag{3.4}$$

where $\epsilon_{\xi_c,t} \sim N(0,1)$.

The sector-specific markup shocks to marginal cost in sector $j \in \{n, m, m^*, f\}$:

$$\hat{\pi}_{j,t} = \frac{\kappa_j}{100}\hat{mc}_{j,t} + \beta E_t\{\hat{\pi}_{j,t+1}\} + \sigma_{\pi_j,t}\epsilon_{\pi_j,t}, \tag{3.5}$$

where $\epsilon_{\pi_j,t} \sim N(0,1)$ and $\kappa_j = \frac{100\left(\theta^j - 1\right)}{\tau_j}$.

The domestic fiscal policy follows the AR(1) process with public demand shocks:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \sigma_{g,t}\epsilon_{g,t}, \tag{3.6}$$

where $\epsilon_{g,t} \sim N(0,1)$.

The domestic monetary policy follows the Taylor rule with interest rate shocks:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}^{va}\right) + \phi_{\Delta_y}(\hat{y}_t^{va} - \hat{y}_{t-1}^{va}) + \phi_q(\hat{q}_t - \hat{q}_{t-1}) + \sigma_{r,t}\epsilon_{r,t}, \tag{3.7}$$

where $\epsilon_{r,t} \sim N(0,1)$.

The risk-premium shock can cause a change in the nominal exchange rate that is unrelated to economic conditions, that makes it a pure exchange rate shock:

$$\hat{\Psi}_t = \rho_\Psi \hat{\Psi}_{t-1} + \sigma_{\Psi,t}\epsilon_{\Psi,t}, \tag{3.8}$$

where $\epsilon_{\Psi,t} \sim N(0,1)$.

The foreign resource-price shock:

$$\hat{p}_{z,t}^* = \rho_{p_{z^*}} \hat{p}_{z,t-1}^* + \rho_{zy}\hat{\xi}_{y^*,t} + \sigma_{p_z^*,t}\epsilon_{p_z^*,t}, \tag{3.9}$$

where $\epsilon_{p_z^*,t} \sim N(0,1)$.

The foreign demand shocks follow the AR(1) process:

$$\hat{\xi}_{y^*,t} = \rho_{\xi_{y^*}} \hat{\xi}_{y^*,t-1} + \sigma_{\xi_{y^*},t}\epsilon_{\xi_{y^*},t}, \tag{3.10}$$

where $\epsilon_{\xi_{y^*},t} \sim N(0,1)$.

The foreign monetary policy shock:

$$\hat{r}_t^* = \rho_{r^*} \hat{r}_{t-1}^* + (1-\rho_{r^*})\left(\phi_{\pi^*} \hat{\pi}_t^* + \phi_{y^*} \hat{y}_t^*\right) + \phi_{\Delta_{y^*}}(\hat{y}_t^* - \hat{y}_{t-1}^*) + \sigma_{r^*,t}\epsilon_{r^*,t}, \tag{3.11}$$

where $\epsilon_{r^*,t} \sim N(0,1)$.

The foreign cost-push shock (Phillip curve) follows the AR(1) process:

$$\hat{e}_{\pi^*,t} = \rho_{e_{\pi^*}} \hat{e}_{\pi^*,t-1} + \sigma_{e_{\pi^*},t}\epsilon_{e_{\pi^*},t}, \tag{3.12}$$

where $\epsilon_{e_{\pi^*},t} \sim N(0,1)$.

3.3.2. Uncertainty shocks

Denote $\boldsymbol{u_t}$ an 17x1 vector collecting all policy and economic shocks above, or structural shocks $\boldsymbol{u_t} = \{u_{i,t}\}_{i \in I}$ with $I := \{\mu, \Upsilon, a_n, a_m, a_z, \xi_c, \pi_n.\pi_m, \pi_{m^*}, \pi_f, g, r, \Psi, p_z^*, \xi_y^*, r^*, e_{\pi^*}\}$:

$$\boldsymbol{u_t} = \boldsymbol{\Sigma_t^{1/2}} \boldsymbol{\epsilon_t}, \tag{3.13}$$

where $\boldsymbol{\epsilon_t} \sim N(0, I_{17})$ with I_{17} is an identity matrix; $\boldsymbol{\epsilon_t} = \{\epsilon_{i,t}\}_{i \in I}$ is a Gaussian shock to the variables or the mean shocks; $\boldsymbol{\Sigma_t^{1/2}}$ is a diagonal matrix Σ_t, called the stochastic volatility matrix[4]:

$$\Sigma_t = \begin{bmatrix} F_t & 0_{(5\times4)} & 0_{(5\times8)} \\ 0_{(5\times4)} & Te_t & 0_{(4\times8)} \\ 0_{(5\times8)} & 0_{(4\times8)} & D_t \end{bmatrix}, \tag{3.14}$$

$$F_t = \begin{bmatrix} \sigma^2_{\Psi,t} & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{p_z^*,t} & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\xi_y^*,t} & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{r^*,t} & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{e_{\pi^*},t} \end{bmatrix}, \tag{3.15}$$

$$Te_t = \begin{bmatrix} \sigma^2_{\mu,t} & 0 & 0 & 0 \\ 0 & \sigma^2_{a_n,t} & 0 & 0 \\ 0 & 0 & \sigma^2_{a_m,t} & 0 \\ 0 & 0 & 0 & \sigma^2_{a_z,t} \end{bmatrix}, \tag{3.16}$$

[4] It is possible to allow correlation among the shocks to improve the model fitness to actual data. However, this study aims at the identification of sources of uncertainty shocks and economic interpretations. Allowing such correlations might make problems complicated.

$$D_t = \begin{bmatrix} \sigma^2_{\Upsilon,t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\xi_c,t} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\pi_n,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\pi_m,t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\pi_{m^*},t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\pi_f,t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{g,t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{r,t}0 \end{bmatrix}. \tag{3.17}$$

Each element of the matrix Σ_t follows a stochastic process:

$$\log \sigma_{i,t} = \log \sigma_{i,t-1} + \upsilon_{i,t}, \tag{3.18}$$

where $i \in I := \{\mu, \Upsilon, a_n, a_m, a_z, \xi_c, \pi_n.\pi_m, \pi_{m^*}, \pi_f, g, r, \Psi, p_z^*, \xi_y^*, r^*, e_{\pi^*}\}$, $\upsilon_{i,t} \sim N(0, \omega_i^2)$ is called uncertainty shocks which describe a permanent shock to the volatility of distribution of each $u_{i,t} \in \boldsymbol{u_t}$. Uncertainty shocks in this thesis are modelled as unobserved variables in a structural model, which are part of the equilibrium solutions. This is a key feature that distinguishes the model from the ones where uncertainty shocks are exogenous. As an accounting exercising, this study aims to distinguish the impact of mean shocks and uncertainty shocks. The proportion of uncertainty shocks to drive the business cycles is different across the shocks. I develop well-defined notions of domestic versus foreign economic and policy shocks with explicit economic interpretations for time-varying uncertainty. Allowing uncertainty shocks to appeare in all variables is to avoid presuming uncertainty shocks in certain sectors.

3.4. Bayesian estimation

3.4.1. The model solution

The model solution is similar to previous chapter using the first-order perturbation method following Justiniano and Primiceri [2008] to find a stable REE solution. The solution is a conditionally linear and Gaussian state-space system. The posterior joint distribution of model structural parameters $(\boldsymbol{\theta}, \boldsymbol{\omega})$ and stochastic volatility matrices Σ_t will be obtained by a four-step Metropolis within Gibbs algorithm as in Algorithm 2.1.

3.4.2. Data and calibration

3.4.2.1. Data

The model uses quarterly data for 88 quarters from 1992:Q1 to 2013:Q4. The sample period in this chapter is a subset of the study in chapter 2 to explicitly look at the impact of shocks during the mining boom period in Australia. The observable series that are used in this chapter include the growth rates of Australian GDP, consumption, investment, public demand, resource exports, and non-resource exports; growth rates of value-added output in the Australian non-traded, non-resource traded, and resource sectors; Australian CPI inflation and non-traded inflation rate; Australian cash rate; the growth of resource prices; the foreign GDP growth rate, inflation, and cash rate.

The data are quarterly, seasonally adjusted and demeaned. They are all in real terms, sourced from ABS and Reserve Bank of Australia (RBA).

The foreign GDP is calculated by the trade-weighted average GDP growth of Australia's three major trading partners. Foreign inflation is average inflation among G7 economies. The foreign cash rate is a simple average of G3 (US, Japan, Euro).

More information about the data source and construction are shown in Appendix B.3.

3.4.2.2. Calibration

The model is calibrated to match the Australian data from 1992 to 2013 [5]. Some structural parameters can be directly identified and externally calibrated.

The steady-state inflation rate is 1.0062. The household discount factor is 0.9996 to give an annual interest rate of 6%. The inverse of Frisch labour supply elasticity and the elasticity of labour switching across sectors are both set to 2, following Horvath [2000].

The parameters governing the share of intermediate goods production sectors in the domestic final demand are chosen to match the Australian data. The intersectoral elasticity and the substitution elasticity between home and foreign goods overseas are equal to 0.8 Justiniano and Primiceri [2008].

The substitution elasticity between differentiated goods in non-traded, non-resource traded, and import sectors are set to 6, or the average markup is equivalent to 20 per cent. The quarterly depreciation rate of capital is 1.75 per cent, which closely matches data in the Australian national accounts. The government spending share and foreign asset are calibrated following the average share of public demand in nominal GDP and average trade deficit in the Australian data. Other sector-specific parameters are calibrated using data on factor income for each sector according to Rees et al. [2016].

The foreign block is modeled as a closed economy recursive from the rest of the model. It can thus be estimated separately. Estimating big models with time-varying volatilities is a particular challenge. While, the theoretical techniques are available, in practice, they usually suffer from numerical problems and are time-consuming. For this reason, I focus on estimating the structural shocks in the home country. Fortunately, the parametrisation of the foreign economy were estimated in the study of Rees et al. [2016] and I will take it as given in this model.

[5] A summary table shown in Appendix B.3.

The calibrations are shown in table 3.4.1 and 3.4.2.

Table 3.4.3 shows a comparison of the long-run properties of the model and the data over the sample period. The long-run property results from my model are relatively similar to the one in Rees et al. [2016] when shutting down the stochastic volatility processes. This suggests that the models can capture many critical features of the Australian data.

3.4.2.3. Prior distributions

The prior means and standard deviations of the estimated parameters are presented in the third and fourth columns to the right of table 3.4.4. The choice of informative prior distributions follows common values in the literature. The prior distributions for AR(1) coefficients are chosen to have beta distributions to ensure they do not go to zero or one.

3.5. Results and discussion

3.5.1. Structural parameter estimates

The posterior medians and standard deviations of the estimated parameters are shown in the last two columns to the right of table 3.4.4. Firstly, the posterior distributions are tested to be an ergodic distribution of the Markov chain with convergence diagnosis and mean test shown in Appendix B.4.

All coefficient estimates have more concentrated distributions compared to their prior distributions. The posterior standard deviations are smaller than in their prior distributions for most coefficients, suggesting that information is quite informative.

Sectoral price-stickiness coefficients imply that the New Keynesian Phillip curves are very flat, which share a similar conclusion with the stylised facts and literature [Roberts, 2004, Beaudry and Doyle, 2000, Rees et al., 2016]. It reflects that the inflation is less responsive to the output fluctuations in all production sectors. More specifically, the non-traded sector exhibited the highest degree of price rigidity, followed by the import sector and domestic non-resource traded sector. Prices in the foreign non-resource traded sector are the most flexible. An increased openness from globalisation and high competition are the two popular explanations for changes in the price-setting behavior in production sectors [Kulish et al., 2020].

The habit parameter has a posterior median of 0.68, indicating a large degree of consumption inertia. It is similar to Jääskelä and Nimark [2011] and Rees et al. [2016].

The investment adjustment coefficient has a posterior mean of 4.87 with a small standard deviation. The mean is comparatively small compared with that found by Jääskelä and Nimark [2011], suggesting a lower cost of adjusting capital stock in the capital accumulation process. This is plausibly because of the multisector feature allowing investment to respond differently across sectors.

Table 3.4.1.: Parameter calibrations

Parameter	Value	Description
μ	1.008	Steady-state total factor productivity (TFP) growth rate
Π	1.0062	Steady-state inflation rate
δ	0.0175	Quarterly depreciation rate
b^*	25	Steady-state foreign asset to GDP
g	0.672	Public demand share in expenditure
r	1.015	steady-state domestic gross nominal interest rate
r^*	1.013	steady-state foreign gross nominal interest rate
β	0.9996	Household discount factor
ζ	0.8	Elasticity of substitution between intermediate sectors in domestic final demand (DFD)
ζ^*	0.8	Elasticity of substitution between home and foreign goods in overseas
η	2	Inverse of Frisch labour supply elasticity
σ	2	Elasticity of substitution of labour supply across sectors
ω_n	0.60	Share of non-traded goods in DFD
a_n	2.2	Steady-state TFP growth rate in non-traded sector
α_n	0.7	Labour share in non-traded goods production
γ_n	0.24	Capital share in non-traded goods production
θ^n	6	Elasticity of substitution between differentiate non-traded goods
ω_m	0.12	Share of non-resource traded goods in DFD
a_m	1.4	Steady-state TFP growth rate in non-resource traded sector
α_m	0.61	Labour share in non-resource traded goods production
γ_m	0.31	Capital share in non-resource traded goods production
θ^m	6	Elasticity of substitution between differentiate non-resource traded goods
ω_m^*	1.17	Share of exported non-resource traded goods in overseas market

Table 3.4.2.: Parameter calibrations (continued)

Parameter	Value	Description
a_z	1	Steady-state TFP growth rate in resource sector
α_z	0.2	Labour share in resource goods production
γ_z	0.25	Capital share in resource goods production
p_z^*	2.5	Steady-state foreign-denominated resource price
θ^f	6	Elasticity of substitution between differentiate imported goods
ω_f	0.28	Share of imported goods in DFD
ρ_{ξ^*}	0.95	Foreign demand shock/ smoothing
$\rho_{e_{\pi^*}}$	0.32	Foreign cost-push shock/ smoothing
ρ_{r^*}	0.93	Foreign monetary policy/ smoothing
ϕ_{π^*}	1.48	Foreign Taylor rule response to inflation
ϕ_{y^*}	0.21	Foreign Taylor rule response to output
$\phi_{\Delta y^*}$	0.14	Foreign Taylor rule response to output growth
ρ_{zy}	0.21	Resource price response to foreign output shocks
$\rho_{p_{z^*}}$	0.94	Foreign resource-price shock/ smoothing
κ^*	3.53	Foreign Phillips curve–price stickiness

Table 3.4.3.: Long-run properties of the model

Target	Data	Model
Expenditure (per cent of GDP)		
Household consumption	0.54	0.56
Private investment	0.26	0.23
Public demand	0.21	0.22
Exports	0.19	0.19
Imports	0.20	0.20
Production (per cent of GVA)		
Non-traded	0.65	0.64
Other traded	0.24	0.24
Resource	0.11	0.12
Trade (per cent of exports)		
Resource export	0.43	0.44
Other exports	0.57	0.56
Investment demand (per cent of private investment)		
Non-traded	0.62	0.59
Other traded	0.28	0.28
Resource	0.10	0.13

Table 3.4.4.: Prior and posterior distributions of estimated parameters

Parameters	Description[a]	Shape[b]	Prior		Posterior[c]	
			Mean	Std	Median	Std
ρ_r	MP; smoothing	B	0.75	0.10	0.69	0.06
ρ_g	FP; smoothing	B	0.50	0.15	0.53	0.06
ρ_{ξ_c}	Household preference; smoothing	B	0.50	0.15	0.43	0.15
ρ_Υ	Efficient investment; smoothing	B	0.50	0.15	0.55	0.06
ρ_Ψ	Risk-premium; smoothing	B	0.50	0.15	0.65	0.08
ρ_{a_n}	Sectoral TFP; smoothing	B	0.50	0.15	0.53	0.05
ρ_{a_m}	Sectoral TFP; smoothing	B	0.50	0.15	0.45	0.05
ρ_{a_z}	Sectoral TFP; smoothing	B	0.50	0.15	0.39	0.06
h	Household habit formation	B	0.50	0.15	0.67	0.09
τ_{π_n}	Sectoral price-stickiness	G	50	30	1.18	0.19
τ_{π_f}	Sectoral price-stickiness	G	50	30	0.32	0.27
τ_{π_m}	Sectoral price-stickiness	G	50	30	0.34	0.19
$\tau_{\pi_{m^*}}$	Sectoral price-stickiness	G	50	30	0.85	0.29
χ	Foreign debt risk-premium	B	0.30	0.15	0.09	0.05
Φ	Investment cost adjustment	G	4.00	1.00	3.38	0.57
ϕ_π	MP; inflation	N	0.75	0.10	1.33	0.03
ϕ_y	MP; output	N	0.13	0.05	0.09	0.05
ϕ_{Δ_y}	MP; output growth	N	0.00	0.03	0.02	0.02
ϕ_q	MP; exchange rate	N	0.00	0.05	0.02	0.06

[a] *MP and FP stands for monetary policy and fiscal policy rules. TFP stands for total factor productivity.*
[b] *B is for Beta, G for Gamma, N for Normal distributions.*
[c] *Posterior moments are generated from a thinned sample of* 10^6 *MCMC draws. As conventional, it cuts out half of sample as burn-in then saves 1 in 50 draws.*

The Taylor rules coefficients are similar to the results from Lubik and Schorfheide [2007], Kam et al. [2009], Rees et al. [2016]. It shows a more robust response to inflation than the output gap, while negligible impacts on output and exchange rate movements.

The coefficients for the AR processes are around 0.5, suggesting a moderate persistence in shocks.

3.5.2. Uncertainty shocks

In this section, I focus on the possible impact of uncertainty shocks on the economy. Initially, I look at the behavior of the time-varying component in each structural shocks through the estimated stochastic volatilities (SVs). Figure 3.5.1 shows the evolution of posterior means of the SVs in the period from 1991–2013.

Clearly, the time-varying variances of the shocks to risk-premium, resource-specific prices, and foreign inflation seem co-moving. Prominently, the highest spikes in the three series are around 2003 when the mining boom was about to start. During that period, the Australian economy witnessed a jump in resource prices, underlined by a robust demand from China's industrialisation. Inflation was prevalent among the Australian major trading partners. It was also well documented that the exchange rate was highly appreciated, profiting the import sector while discouraging commodity-exporting industries.

The evolution of markup shock SVs in the import and commodity-exporting sectors share a similar story. Besides, this period marked increased uncertainty in resource technology shocks with numerous discoveries of new resources and technology. A high level of uncertainty to the markup shocks in the domestic tradable sector presumably relates to the demand for manufacturing inputs from the mining sector [Downes et al., 2014].

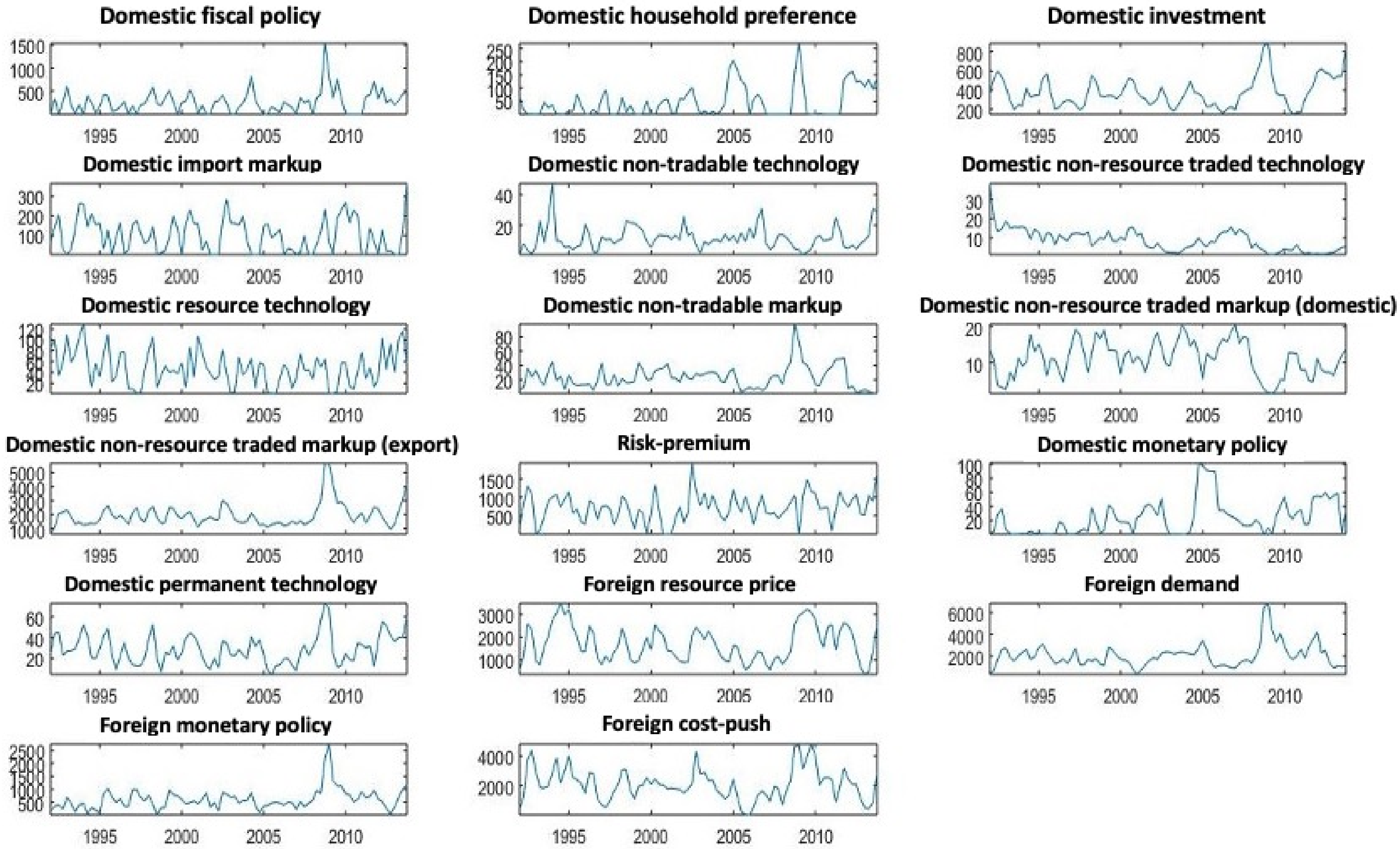
Another notable period was during the GFC from 2007–2009. The severe recession in the U.S. hit the Australian economy, causing a high degree of uncertainty in foreign demand, monetary policy, inflation, resource-specific prices, risk-premium, consumer preference, investment, sectoral markup, and domestic policy shocks.

Overall, the SVs in resource technology and risk-premium shocks are the most fluctuated among others. This is reasonably supported by a narrative around the Australian mining boom from empirical and historical research [Downes et al., 2014, Tran, 2019]. The volatilities of investment shocks are often larger and earlier than consumer preference shock volatilities. This finding is similar to Bloom [2017].

3.5.3. Volatility shock accounting

This section addresses the research question of what are the driving factors of the Australian business cycles in the last three decades. I decompose and quantify the effect of mean and uncertainty shocks on the aggregate fluctuations. I analyse the variance decomposition of

Figure 3.5.1.: Evolution of stochastic volatilities of the shocks



the Australian real GDP in the long-run (40 quarters). The variance decompositions are constructed to analyse the impact of each shock to the variability of observable variables, following Justiniano and Primiceri [2008]. With this method, the variance decompositions change over time because the variances of endogenous observable variables are time-varying.

Combined effects. Following Rees et al. [2016], I grouped the structural shocks into six categories to analyse. These groups are: i) productivity group, including permanent and sectoral technology shocks and investment shocks; 2) demand group, including fiscal policy and consumer preference shocks; 3) supply group, including sectoral markup shocks; 4) monetary policy shocks; 5) commodity group refer to resource-price shocks; and 6) world group, including risk-premium, monetary policy, cost-push and demand shocks from overseas.

Figure 3.5.2 presents the evolution of variance shares of GDP growth in Australia from 1991 to 2013 attributed to each shock category. Generally, the foreign shocks substantially account for the domestic GDP fluctuations (almost 60%), while the domestic supply and resource prices shocks are the second and third largest drivers of the economic variations. The demand, productivity, and domestic monetary shocks similarly contribute small portions to the output variations during this sample period.

Stochastic volatility. In this part, I decompose each structural shock $u_{i,t}$ into two parts: i) mean shocks $\epsilon_{i,t}$ which are Gaussian shocks to the variables; ii) uncertainty shocks $\sigma^2_{i,t}$ arising from the variation of distributions of shocks. Following Cross et al. [2018], I define the (absolute) proportion of stochastic volatility component in the structural shocks as:

$$SV_{i,t} = \frac{|log(\sigma^2_{i,t})|}{|log(u^2_{i,t})|}.$$

The larger $SV_{i,t}$ is the immense contribution of uncertainty component in accounting for structural shock i. The complement indicates the contribution of mean shocks to the changes in structural shocks.

Variance decomposition. In this part, I computed the variance decomposition of the GDP fluctuations attributed to each shock component. It is done by multiplying the total variation with the stochastic volatility shares. Table 3.5.1 shows the proportion of i.i.d. and SV components to the total shocks in driving the Australian output variations.

Figure 3.5.2.: Historical decompositions of combined shocks in absolute terms by category

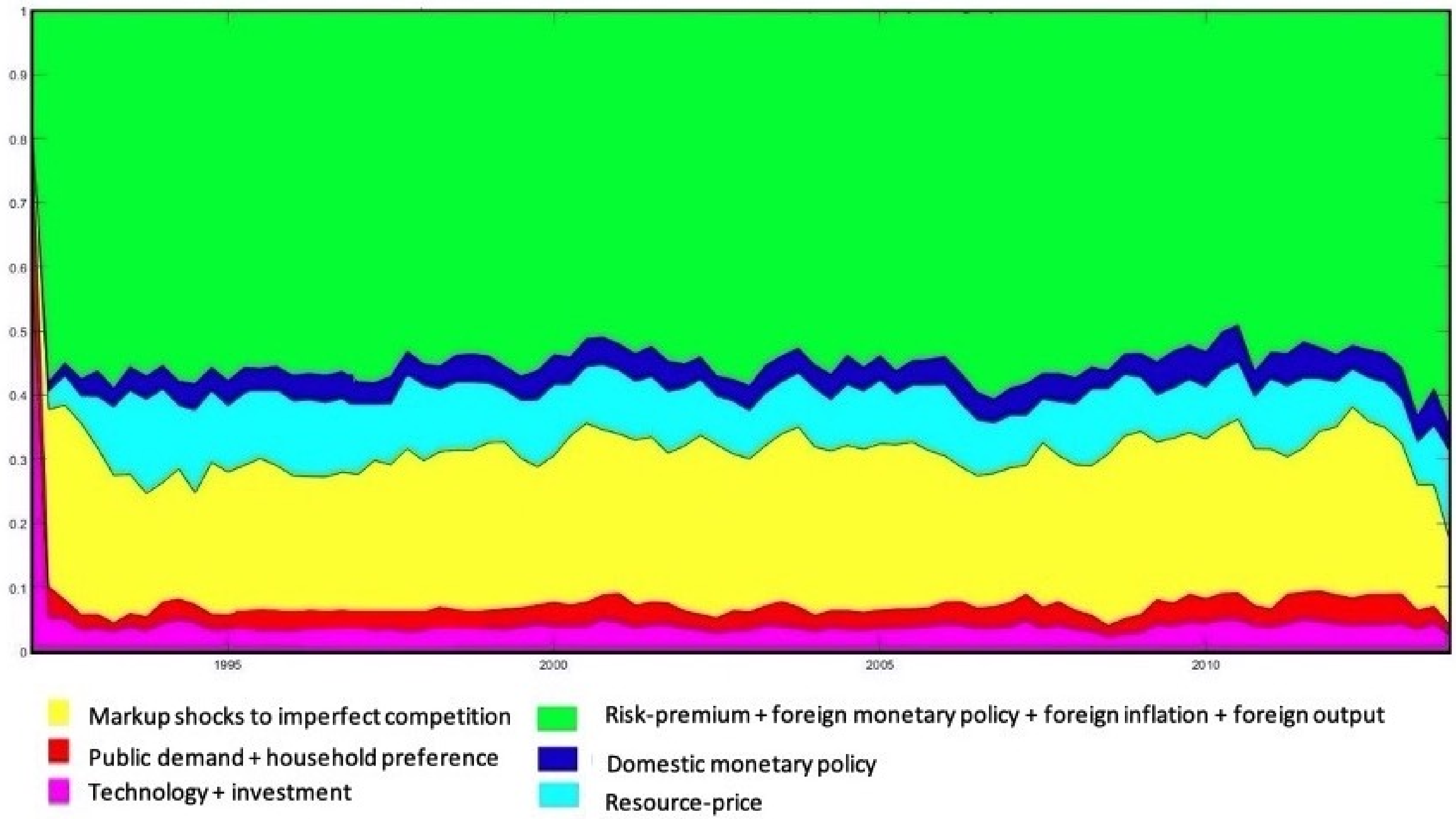


Table 3.5.1.: Variance decompositions

Group of shocks	iid	SV	Total
Productivity	0.02	0.02	0.04
Demand	0.01	0.02	0.03
Supply	0.11	0.12	0.23
Resource price	0.04	0.07	0.11
Monetary policy	0.01	0.02	0.03
World	0.23	0.33	0.56
Total	0.42	0.58	1.00

Uncertainty shocks contribute almost 60% of the economic variations. Among the shocks, the foreign uncertainty shocks account for the most significant proportion of the variations, roughly over 50% of the total uncertainty shock GDP decomposition, or about 30% of the output variations. Foreign monetary policy uncertainty shock has the lowest impact. In contrast, the risk-premium, foreign demand, and cost-push uncertainty shocks contribute an equal share to the economic fluctuations. They are also consistently highly volatile [6].

The supply uncertainty shocks are the second-largest drivers, accounting for nearly 25% of the total uncertainty shock GDP decomposition, or 12% of the output variations. The impact was driven mainly by the markup shocks to the commodity-exporting marginal cost. The resource price uncertainty shock is the third-largest driver, involving about 7% of the aggregate variations. The effect of resource-specific price shocks was amplified by the foreign uncertainties, especially risk-premium shocks. It shares a similar story with Bean [1987] that the UK's oil booms during the 1970s led to an exchange rate appreciation, followed by a massive impact on business-cycle fluctuations. The negative impact of commodity uncertainty shocks on the Australian economic fluctuations was confirmed in a recent study by Tran [2019]. The impacts of productivity, demand, and monetary uncertainty shocks are relatively equal at a small level.

Mean shocks contribute over 40% to the economic variations. The foreign shocks are still the most important among the systematic shocks, accounting for over 50% of total mean shock GDP decomposition, or 23% of the output variations. The supply shocks are the second-highest drivers, contributing about 25% to the total uncertainty shock GDP decomposition, or 11% of the output variations. The resource-specific price shocks are the third-largest contributor, accounting for 4% of the aggregate fluctuations.

To conclude, under my model construction and interpretation, the Australian output variations from 1992–2013 were mainly driven by foreign shocks, particularly the risk-premium, demand,

[6] This is important to note that the foreign block in chapter 2 only included the U.S as the rest of the world while in chapter 3 the model includes three trading partners (U.S, China, Japan) with China is the biggest trading partner during the mining boom decades. This is to explain the differences regarding foreign impacts between chapter 2 and chapter 3.

and cost-push shocks. These shifts substantially came from uncertainty in the distribution over time (i.e. 'world uncertainty') while a smaller impact was explained by the mean shocks (i.e. 'good or bad conditions').

3.6. Conclusion

The primary purpose of this chapter is to understand the importance of mean and uncertainty shocks in driving the business cycles in a multisector, small, open economy. A small, open economy DSGE model was extended to include five production sectors, incorporated stochastic volatilities into 17 structural shocks. The model is estimated using the Bayesian method to quantify and decompose the impact of mean and uncertainty shocks on the output variations. The findings consist of three aspects. First, the stochastic volatilities are found to be counter-cyclical. They tends to peak during economic recessions and vice versa. Second, the uncertainty shocks appeared to be the main driver of business fluctuation in Australia, accounting for about 60%. This is a similar conclusion to Moore [2017] for a 'net importer' of uncertainty like Australia. Third, among all shock categories, the foreign shocks significantly impact the output variations in Australia. This was mainly driven by risk-premium, demand, and cost-push shocks.

4. Bubbles and collateral constraints

Abstract. This chapter focuses on rational bubbles and collateral constraints in an overlapping-generations model with heterogeneous beliefs. The leverage equilibrium and asset-pricing theories are employed to explore the role of leverage in creating and popping bubbles. The study shares similar findings with the rational bubble literature that bubbles cannot exist in a frictionless endowment economy. Other noticeable results emphasise the existing conditions of bubble equilibrium in a fixed-bubble supply environment. First, bubbly equilibrium can endure in high-income countries with highly leveraged financial markets. However, high collateralisation might trigger deleveraging behavior from nervous lenders, causing bubbles to collapse. Second, bubbly equilibrium is prone to exist in low-income countries at any level of collateralisation. The dynamic analysis of bubbly equilibrium with growing bubble supply is more complex and will be tackled outside of this dissertation in the future.

4.1. Introduction

It has been well documented that prior to the global financial crisis (GFC) investment leverage increased significantly in the U.S. during the security and housing market booms, getting as high as 98% for mortgage security and 97% for home loans. Following the crash, the ability of borrowers to leverage was dramatically curtailed by new government regulations and by nervous lenders who wanted more collateral for their loans. Leverage has a significant impact on asset pricing, contributing to economic bubbles since a class of buyers, who value certain assets more than the rest of the economy, are willing to pay more with the hope of reselling at higher prices. Increasing leverage drives up prices while deleveraging is a major reason why security and housing market booms subsequently burst. The leverage cycle is defined as a period of high leverage, followed by low leverage [Geanakoplos, 2010].

The impact of increasing and decreasing leverage is a key element in this chapter. A simple model of bubbly assets with heterogeneous agents has been developed to explore the role of leverage in creating and popping bubbles. Following the literature of rational bubbles, I construct a simple overlapping generations (OLG) model in which economic agents are different in their beliefs in the growth path of bubbly assets. With beliefs that are more vigorous than the general public, a particular class of people become natural buyers. This optimistic group of agents inclines to borrow from the public, which drives up the bubble prices. Loans are given in incomplete markets where borrowers cannot commit to repay in the future, then requiring collateral. A fraction of their expected wealth is pledged as collateral, which limits the borrowing capacity of borrowers.

The main feature of the model is a new endogenous notion of optimism and how that interacts with collateral constraints in characterising bubbly and bubbleless equilibrium by employing the theory of leverage equilibrium and asset-pricing from Geanakoplos [2010].

Our findings share a similar conclusion with Tirole [1985] that bubbles cannot exist in frictionless endowment economies. Moreover, this points out two conditions in which bubbly equilibrium exists. First, bubbly equilibrium can endure in high-income countries with highly leveraged financial markets. However, high collateralisation might trigger deleveraging behavior from nervous lenders, causing bubbles to collapse. This finding is consistent with the narratives along with the leveraged security and housing bubbles in the U.S. in 2006–2008. Second, bubbly equilibrium is prone to exist in low-income countries at any level of collateralisation. Indeed, lenders will become less risk-averse when their incomes are low since their risk of losing is small. Natural buyers thus have more access to loans, which then drive up the bubble prices. This is supported by several historical bubble periods in East Asian economies where easy credit and financial deregulation ended in crisis.

Literature review

This chapter complements the literature on macroeconomics of rational bubbles Samuelson [1958], Tirole [1985], Diamond [1965]. It has been well documented that bubbles are impossible in a standard Arrow–Debreu economy since the arbitrage conditions will eliminate any mispricing causing bubbles. Research shows that bubbles can emerge in a dynamically efficient economy with financial frictions [Kocherlakota, 1992, Woodford, 2003, Kiyotaki and Moore, 1997, Bernanke and Gertler, 1990]. The financial frictions are typically a borrowing constraint when an economic agent chooses optimal decisions with limited commitment and imperfect enforcement. The constraint limits his ability to borrow, depresses the interest rate in equilibrium, and makes a good condition for bubbles to emerge. Compared with the early literature, my work also uses the overlapping-generation framework with an incomplete financial market to explore the existing condition of rational bubbles. Nevertheless, my model focuses on pledgeability and collateral conditions in creating a bubbly environment.

In the conventional models, bubbles are deterministic and unlike real-world bubbles in the sense that those modeled bubbles never burst and are very predictable. Weil [1987] extended Tirole [1985] to study the conditions for stochastic bubbles to emerge with a constant and exogenous collapsing probability. Compared with their research, this chapter also looks at the existence of stochastic bubbles in an overlapping-generations model, but attempt to endogenize the probability of bubble collapsing based on the theory of equilibrium leverage and asset-pricing by Geanakoplos [2010].

Furthermore, my model relates to the existing research on the rational bubbles by Martin and Ventura [2016], Miao and Wang [2011], Bengui and Phan [2018], who looked at the collateral value generated from bubbly assets. However, unlike us, they do not allow the probability of bubble collapse to be endogenous. In my model, the agents are populated with heterogeneous beliefs. Following Geanakoplos [2010], I provide an endogenous mechanism in which the cut-off distinction between natural buyers and the general public is not presumed. It then decides if the bubble exists or collapses.

This work borrows insights from the literature of financial frictions by Kiyotaki and Moore [1997], Aiyagari [1994], Bernanke and Gertler [1990]. They consider limited enforcement and collateral constraint environment, generating a feedback loop between credit and asset prices.

Finally, my findings contribute to the literature on leverage bubbles. It supports empirical research to confirm the link between bubble asset prices and leverage. Geanakoplos [2010] shows that leverage variations have an enormous impact on asset prices. He evidenced the leverage cycle throughout global crisis history, such as the financial derivative crisis in 1994, Black Monday in 1987, and GFC in 2008. Increasing leverage drove asset prices up before dropping down to pop the bubbles.

The rest of the chapter is presented as follows. Section 4.2 describes the environment, including the type of agents, assumptions, and markets, while section 4.3 shows the optimal behavior of each type of agent and characterises the competitive equilibrium. Section 4.4 analyses the model's solutions in a frictionless and friction economy. Section 4.5 concludes and presents future research plans.

4.2. Environment

4.2.1. Agents and assumptions

This chapter considers a closed economy populated with a continuum of equal-sized and overlapping-generations of young-age and old-age agents. Time is discrete and starts in period $t = 0$ until infinite. There is a single perishable consumption good. Young-age agents born in period t are given an identical y endowment unit of consumption goods. They can choose either buy bubble b_t, borrow d_t^H or lend d_t^L to others depending on their beliefs in the future growth path of bubbles. In the next period $t + 1$, the agents become old and consume all of their accumulated wealth $c_{o,t+1}$ and leave no bequests to other generations.

Each agent maximises its lifetime expected utility as follows:

$$E_t \left[U(c_{y,t}, c_{o,t+1}) \right] = u(c_{y,t}) + \beta E_t(c_{o,t+1} | \pi_t^*, s_t). \tag{4.1}$$

Equation (4.1) takes a quasi-linear form, which is increasing and concave in consumption. Discount factor $\beta > 0$. $c_{y,t}$ and $c_{o,t+1}$ are non-negative and respectively denote the young-age and old-age consumption. The utility function of the young-age agents is represented with $u(.)$, which takes the following form $u(c_t, n_t) = ln(c_{y,t})$.

Following the theory of equilibrium leverage in Geanakoplos [2010], the model emphasises the belief heterogeneity between young-age agents. Each young-age agent i is born with a certain belief in bubbles $\pi^i \in (0, 1)$. The higher the belief π^i is, the more optimistic he is and the more willing to buy bubbles he has. Agents with firmer beliefs will be natural buyers who want to hold more bubbles than the general public. The literature suggests many reasons underlie this type of behavior. For instance, natural buyers might be less risk-averse, or they might have access to different hedging techniques, production technology, or particular information than the general public. However, for simplicity, I use a similar assumption with Geanakoplos [2010] in which natural buyers are more optimistic by nature. The natural buyers will hold the bubbles and drive their prices up. If their access to borrowing is reduced, there will be a subsequent crash.

The equilibrium leverage theory provides an endogenous mechanism in which the cut-off distinction between natural buyers and the general public is not presumed. At each period t, there is a median buyer whose belief is π_t^*. The median buyer is the one who thinks the probability of bubble boom is π_t^* and bubble burst is $1 - \pi_t^*$. In each period, an agent i will choose to be a natural buyer of a bubbly asset if his belief is greater than the median buyer's belief; or a lender otherwise. With this setting, the median buyer's belief will be endogenous. Having a continuous unit mass of young-age agents helps to avoid a rigid categorisation of agents. Therefore, at each period t, there always exists a fraction of $(1 - \pi_t^*)$ of the young-age agents who are optimistic, while a fraction of π_t^* are pessimistic.

The old-age agents who are natural buyers in the previous period are assumed to be the only sellers of bubbles. Since the model does not allow for a bequest, the old-age generation will consume all of their accumulated wealth.

4.2.2. Bubbly asset and credit market

There are two markets that agents can trade on, particularly the bubbly asset market and the credit market.

Bubbly asset market

The bubbly asset market is modeled following the conventional rational bubble literature. The standard assumptions are that bubbles are traded in a competitive market in a fixed-unit supply. The asset that I called 'bubbles' is durable and perfectly divisible, and it pays no dividend but might be traded at a positive price in equilibrium. Bubbles are often interpreted as the difference between market prices and fundamental values.

Bubbles are typically fragile because they are supported by the belief of reselling at higher prices in the future. In this model, I employ the assumption from Weil [1987] to let bubbles collapse permanently with a probability. More specifically, let $s_t P_t$ denote a bubble price in time t. P_t is a bubble price at time t conditional on the persistent bubbles. A binary random variable s_t takes the value of one indicating persistent bubble, while zero if the bubble collapses [1].

I incorporate this setting into the theory of leverage equilibrium. The median buyer's belief π_t^* will determine the frangibility of the bubbles every period. The natural buyers buy into bubbles because they believe that they will profit from reselling at a higher price. When π_t^* becomes higher, it incurs a higher collapsing risk since it might be harder to find the potential buyers who are more optimistic. In other words,

$$Pr(s_{t+1} = 0|s_t = 1) = \pi_t^*, \tag{4.2}$$
$$Pr(s_{t+1} = 0|s_t = 0) = 1. \tag{4.3}$$

Equation (4.2) shows that the probability of a current persistent bubble to collapse in the next period is determined by the current median buyer's belief. Furthermore, a collapsed bubble is not expected to reemerge as in equation (4.3). Therefore, the median buyer's hypothesis allows us to endogenise the bubble fragility [2].

Credit market

This model considers an incomplete financial market where borrowers have limited commitment to paying their loans. Hence, all loans need to be collateralised. A natural buyer seeks a loan to buy a bubbly asset when they are young. A one-period and non-contingent debt is lent at the gross interest rate R_t at time t. The borrower needs to pledge a fraction φ of his asset and future income as collateral for the loan. He faces a borrowing constraint that limits his lending capacity:

$$d_t^H R_t \leq \phi E_t s_{t+1} P_{t+1} b_t. \tag{4.4}$$

The left-hand side of equation (4.4) represents the repayment for a loan d_t^H at the gross interest rate R_t. The right-hand side indicates a fraction ϕ of expected value of bubbly asset b_t at an expected price P_{t+1}, that has been pledged as a collateral for the loan d_t^H conditional on the existence of bubbles.

[1] The assumption that bubbles cannot re-emerge is made following Weil [1987] to model the fragility of bubbles as the coordination in expectation across agents. I am happy to explore the possibility to relax the assumption with nonzero fundamental value for a future project. For this paper, I do not consider the evolution of bubble and fundamental value and assume zero fundamental value as well for simplicity.

[2] The bubbles in this chapter are modelled as an asset that can boom and burst depending on the belief of economic agents. Bubble prices have been only determined by the market supply and demand. The evolution of bubbles has not been studied in this thesis. This would possibly be a good extension when we allow the asset price of the bubbly asset to follow a sub-martingale property as in the empirical literature on bubbles [Blanchard and Watson, 1982, Phillips et al., 2011, 2015].

The economic intuition behind this is the natural buyer with a higher expectation on a bubbly asset borrows to buy into bubbles since his expected return rate on investing in bubbles is higher than the gross lending interest rate.

The endowment income follows an AR(1) process:

$$y_t = \xi y_{t-1} + \epsilon_t; \epsilon_t \sim N(0, \sigma_\epsilon^2).$$

Suppose that the total bubble supply is growing at a fixed rate g. As the model assumes no bequest, the optimistic agents will sell all their bubbly assets at their old age. That implies the following exogenous bubble supply growth path:

$$(1 - \pi_t^*)b_t = (1 + g)(1 - \pi_{t-1}^*)b_{t-1}.$$

4.3. Optimal behavior

Each agent will choose his optimal behavior subject to his lifetime budget constraints and credit frictions in equilibrium. This section describes the optimal behavior of optimistic and pessimistic agents.

4.3.1. Optimistic agents

An agent is given an identical endowment income at his young age, a y unit of consumption goods. He is also granted a certain belief in bubble growth, which determines their trading behavior. The optimistic agent is a natural buyer, whose belief is higher than the median buyer's belief. Given bubble and consumption goods prices, the optimistic agent chooses his consumption bundle $c_{y,t}^H$, bubble holdings position b_t, and debt level d_t^H to maximise his lifetime utility function:

$$\max_{c_{y,t}^H, b_t, d_t^H} \left\{ u\left(c_{y,t}^H\right) + \beta E_t \left[u\left(c_{o,t+1}^H | (\pi_t^*, s_t)\right) \right] \right\}. \tag{4.1}$$

He also faces budget constraints in the young and old age:

$$c_{y,t}^H + P_t b_t = y + d_t^H, \tag{4.2}$$

$$c_{o,t+1}^H = s_{t+1} P_{t+1} b_t - s_{t+1} R_t d_t^H. \tag{4.3}$$

Equation (4.2) shows a budget constraint for a young-age optimistic agent. He chooses to consume $c_{y,t}^H$ and hold b_t bubbles at price P_t. It is financed by giving up of y endowment income units and taking a debt d_t^H.

Equation (4.3) presents a budget constraint for the agent at his old age. After the bubble state and price are realised, the agent sells all his bubble holding to repay the loan and consume

$c^H_{o,t+1}$. Hence, the loan repayment depends on the state of the world s_{t+1}. If bubbles burst, he probably defaults with no consumption. Otherwise, he likely consumes a positive amount and repays the loan at the agreed lending rate.

Note that I implicitly assume that optimistic agents do not lend in the model. Without loss of generality, an optimistic agent never wants to lend in equilibrium under this parametrisation.

Since the optimistic agent participates in an incomplete market where commitment does not exist without enforcement, he faces a borrowing constraint, representing a collateral condition on his loan:

$$d^H_t R_t \leq \phi E_t s_{t+1} P_{t+1} b_t. \tag{4.4}$$

Equation (4.4) simply says that the debt must be backed by the expected value of bubbles.

In this environment, short-selling is not allowed. Therefore, an agent must face no short-selling constraint:

$$b_t \geq 0. \tag{4.5}$$

Let $\lambda^H_{j,t}$ with $j = \{1, 2, 3, 4\}$ be the Lagrange multipliers in the four constraints in equations (4.2), (4.3), (4.4), (4.5), respectively. The first-order conditions of a young-age optimistic agent are shown below.

The first-order condition with respect to $c^H_{y,t}$ implies that the marginal utility of consumption at his young age is equal to the shadow value λ^H_{1t}. In other words, an additional fund at his young age increases his marginal utility of consumption:

$$\lambda^H_{1t} = u'(c^H_{y,t}) = \frac{1}{c^H_{y,t}}. \tag{4.6}$$

The first-order condition with respect to b_t implies that the shadow value of the no short-selling constraint is equal to the differences between the marginal cost of buying bubbles and the marginal benefit of holding them. The marginal benefit consists of two components: (i) the discounted expected price of bubbles in the next period; (ii) the collateral value, which is an additional value of bubbles generating from using bubbles as credit collateral:

$$\lambda^H_{4t} = P_t \lambda^H_{1t} - \beta E_t s_{t+1} P_{t+1} - \phi \lambda^H_{3t} E_t P_{t+1} s_{t+1}. \tag{4.7}$$

The first-order condition with respect to d^H_t implies that the shadow value of the borrowing constraint is equal to the differences between the marginal benefit of borrowing and the discounted expected cost of loan repayment:

$$\lambda^H_{3t} R_t = \lambda^H_{1t} - \beta R_t E_t s_{t+1}. \tag{4.8}$$

The Kuhn—Tucker conditions for the two inequality constraints in equations (4.4) and (4.5) are:

$$E_t s_{t+1} P_{t+1} b_t - d_t^H R_t \geq 0 \;\&\; \lambda_{3t}^H \geq 0,$$

$$(\phi_t E_t s_{t+1} P_{t+1} b_t - d_t^H R_t)\lambda_{3t}^H = 0, \tag{4.9}$$

$$b_t \geq 0 \;\&\; \lambda_{4t}^H \geq 0,$$

$$b_t \lambda_{4t}^H = 0. \tag{4.10}$$

4.3.2. Pessimistic agents

The pessimistic agent is a natural lender, whose belief is lower than the median buyer's belief. A pessimistic agent is also given identical endowment income, a y unit of consumption goods. Given bubble and consumption goods prices, the pessimistic agent chooses his consumption bundle $c_{y,t}^L$ and lending position d_t^L to maximise his lifetime utility function:

$$\max_{c_{y,t}^L, d_t^L} \left\{ u\left(c_{y,t}^L\right) + \beta E_t \left[u\left(c_{o,t+1}^L | (\pi_t^*, s_t)\right)\right]\right\}. \tag{4.11}$$

He also faces budget constraints in the young and old age:

$$c_{y,t}^L + d_t^L = y, \tag{4.12}$$

$$c_{o,t+1}^L = s_{t+1} R_t d_t^L. \tag{4.13}$$

Equation (4.12) shows a budget constraint for a young-age pessimistic agent. He chooses to consume $c_{y,t}^L$ and issue debt d_t^L at the gross return rate of R_t. It is financed by giving up y endowment income units.

Equation (4.13) presents a budget constraint for the agent at his old age. After the bubble state and price are realised, the agent receive the loan repayment and consume. The consumption at his old age depends on the state of the world $t+1$. If bubbles burst, the loan probably defaults and he has no consumption. Otherwise, he likely consumes a positive amount and receives loan repayment at the agreed lending rate. Similarly, I implicitly assume that pessimistic agents do not borrow under this parametrisation.

Likewise, a pessimistic agent must face no short-selling constraint:

$$d_t^L \geq 0. \tag{4.14}$$

Let $\lambda_{j,t}^L$ with $j = \{1, 2, 3\}$ be the Lagrange multipliers in the three constraints (4.12), (4.13), (4.14), respectively. The first-order conditions are shown below.

The first-order condition with respect to $c_{y,t}^L$ implies that the marginal utility of consumption at his young age is equal to the shadow value λ_{1t}^L. In other words, an additional fund at his young age increases his marginal utility of consumption:

$$\lambda_{1t}^{L} = u'(c_{y,t}^{L}) = \frac{1}{c_{y,t}^{L}}. \tag{4.15}$$

The first-order condition with respect to d_t^L implies that the shadow value of the borrowing constraint is equal to the differences between the marginal cost of lending and the discounted expected benefit of loan repayment:

$$\lambda_{3t}^{L} = \lambda_{1t}^{L} - \beta E_t s_{t+1} R_t. \tag{4.16}$$

The Kuhn–Tucker conditions for the inequality constraints (4.14) is:

$$d_t^L \geq 0 \; \& \; \lambda_{3t}^{L} \geq 0,$$

$$d_t^L \lambda_{3t}^{L} = 0. \tag{4.17}$$

4.3.3. Competitive equilibrium

In equilibrium, given a bubble initial price $P_0 \geq 0$ and a stochastic process of the binary random variables $\{s_t\}_{t=0}^{\infty}$ that determines bubble persistence, a stationary competitive equilibrium consists of seven portfolio choices c_y^H, c_y^L, c_o^H, c_o^L, b, d^H, d^L, cutoff belief π^*, bubble price P, and interest rate R, that satisfy the following conditions:

1. bubble demand: (4.7),
2. bubble market clears every period:

$$(1 - \pi_t^*) b_t = 1, \tag{4.18}$$

3. loan demand: (4.8),
4. loan supply: (4.16),
5. credit market-clearing condition:

$$\pi_t^* d_t^L = (1 - \pi_t^*) d_t^H, \tag{4.19}$$

6. non-arbitrage condition:

$$R_t = \frac{E_t s_{t+1} P_{t+1}}{P_t}, \tag{4.20}$$

7. resource constraint for optimistic: (4.2), (4.3),
8. resource constraint for pessimistic: (4.12), (4.13),

9. borrowing constraint: (4.4).

4.4. Solutions

Under the endowment parameterisation with a fixed-bubble supply, I only focus on the steady-state where all variables are time invariant. The bubble transition path results in three types of solutions depending on the initial state of bubble prices.

Suppose an equilibrium bubble price is P. At the beginning $t = 0$, an initial bubble price is lower than the equilibrium bubble price; for instance, $P_0 < P$, the bubble price probably converts to a bubbleless equilibrium where the bubble has no value, $P = 0$. In this case, since there is no bubble in equilibrium, no borrowing and lending activities happen.

In another case, when the bubble price starts greater than the equilibrium bubble price, $P_0 > P$, it grows to infinity.

The last case is when the initial bubble price is at the equilibrium price, $P_0 = P$, it maintains at bubble equilibrium.

This chapter considers the three scenarios: a first-best allocation where no credit frictions; a bubbleless equilibrium where bubbles have no value; and a bubbly equilibrium where bubbles have a positive value.

4.4.1. The first-best scenario with no credit frictions

Consider a complete market where people fully commit to repaying the loan. It means that the borrowing constraint is unnecessary or $\lambda_{3t}^H = 0$. Therefore, the interest rate is simply a discounted marginal utility for borrowers, which also is equal to the discounted marginal utility for lenders. Equation (4.7) and (4.16) thus imply the first-best condition as:

$$\lambda_{1,t}^H = \lambda_{1,t}^L - \lambda_{3,t}^L. \tag{4.1}$$

The optimistic optimisation problem suggests that the rate of return from holding bubbles is:

$$\frac{(1-\pi_t^*)P_{t+1}}{P_t} = \frac{\lambda_{1t}^H}{\beta} - \frac{\lambda_{4t}^H}{\beta P_t}. \tag{4.2}$$

Equation (4.1) and (4.2) imply that the bubble return rate is less or equal to the interest rate. In other words, considering the collapsing risk, a high level of interest rate would not benefit the optimist from his borrowing strategy.

If $\lambda_{4t}^H > 0$, the complementary slackness conditions in equation (4.10) imply that $b_t = 0$ or no one buys bubbles. If no one believes that their investment will grow in the future, a bubble will not develop. As a result, no bubble is traded, and no borrowing or lending activities happen. Thus, the economy is equivalent to the autarky economy, where agents only consume what they have, with no trade and resource reallocations.

If $\lambda_{4t}^H = 0$, the complementary slackness conditions in equation (4.10) implies that bubble stocks can be non-negative or $b_t \geq 0$. Equation (4.7) implies that the rate of return from bubbles is:

$$\lambda_{1,t}^H P_t = \beta E_t s_{t+1} P_{t+1}$$
$$\Leftrightarrow \frac{\lambda_{1,t}^H}{\beta} = \frac{E_t s_{t+1} P_{t+1}}{P_t}. \tag{4.3}$$

Together with the non-arbitrage condition in equation (4.20), this indicates that the rate of return on bubble trade is equal to the interest rate. Therefore, there are no incentive the optimists to borrow:

$$\frac{\lambda_{1,t}^H}{\beta} = R_t. \tag{4.4}$$

Combining equation (4.3) with equation (4.1), gives:

$$\frac{\lambda_{1,t}^L}{\beta} \geq \frac{\lambda_{1,t}^L - \lambda_{3,t}^L}{\beta} = R_t. \tag{4.5}$$

Equation (4.14) leads to two following cases.

If $\lambda_{3,t}^L = 0$, equation (4.5) becomes an equality, $\frac{\lambda_{1,t}^L}{\beta} = R_t$. It signifies that the bubble return rate is equal to the interest rate. Thus the pessimistic agents are all indifferent between lending and holding a bubble.

If $\lambda_{3,t}^L > 0$, equation (4.5) will be an inequality, $\frac{\lambda_{1,t}^L}{\beta} > R_t$. It clearly shows that the bubble return rate is greater than the interest rate. Hence, pessimistic agents have incentives to borrow to buy bubbles, which contradicts the no short selling condition requiring $d_t^L \geq 0$. Consequently, no lend/borrow occurs; the economy becomes autarky.

Remark

The first-best scenario shares a similar conclusion with the rational bubble literature that bubbles cannot exist in a frictionless endowment economy [Tirole, 1985].

4.4.2. Bubble equilibrium characterisation

There will be two types of equilibrium: a bubbleless equilibrium in which there are no bubbles and the bubble market price is zero; and a bubbly equilibrium in which bubbles are traded with a positive market price. The bubbly equilibrium is supported by the strong belief in bubbles from optimists with firm beliefs. The bubbleless equilibrium is backed by a pessimistic belief. If no one believes in a bubble, it cannot exist.

The first-best scenario confirms that bubbles only exist when $\lambda^H_{4.t} = 0$ and $b_t \geq 0$. Consequently, there are two cases: first, a bubbleless equilibrium with $b_t = 0$ and $P_t = 0$ and second, a bubbly equilibrium where $b_t > 0$ and $P_t > 0$.

4.4.2.1. Bubbleless equilibrium

A bubbleless equilibrium is defined as a situation when a bubble has no value in the equilibrium or $P_t = 0$ at all t. Plugging $P_t = 0$ in equation (4.7) gives:

$$0 = (\beta + \phi\lambda^H_{3t})E_t s_{t+1} P_{t+1}. \tag{4.6}$$

Mathematically, a bubble expected value is zero or $E_t s_{t+1} = 0$. The intuition is that if no one believes in a bubble, the bubble never exists. Bubbles are not traded because it has no value. Therefore, credits are unnecessary. Consequently, the bubbleless economy is identical to an autarky economy.

4.4.2.2. Bubblely equilibrium

A bubbly equilibrium is when a bubble has a positive value in the equilibrium or $P_t > 0$ for all t. It means that bubbles are held by optimistic agents or $b_t > 0$. According to the complementary slackness conditions for the no short selling constraint, $\lambda^H_{4t} = 0$. Then equation (4.10) becomes:

$$P_t \lambda^H_{1t} = \beta E_t s_{t+1} P_{t+1}(1 + \frac{\phi\lambda^H_{3t}}{\beta}). \tag{4.7}$$

The left-hand side of equation (4.7) is the cost of purchasing one unit bubble, while the right-hand side shows the marginal discounted expected benefit from holding the bubble in the next period. The benefit of holding bubbles has two components: (i) a value from reselling the bubble in the next period at the price P_{t+1}; and (ii) holding the bubble help agents generate additional wealth. Specifically, an additional dollar of the bubble price increases the borrowing capacity in the credit constraint. This additional credit allows optimists to invest more in bubbles. Thus this additional benefit is called a collateral value. The optimistic shadow wealth on the credit constraint will be positive.

If someone believes in bubbles, the bubbles will have a positive value. Bubbles can be self-fulfilling so that the economy can reach either a bubbleless or a bubbly equilibrium. If bubbles start at the initial price P_0, which is smaller than the equilibrium price, the prices probably convert to a bubbleless equilibrium where the bubble has no value. In another case, when the bubble price starts greater than the equilibrium bubble price, there will be no equilibrium since bubbles grow to infinity. The last case is when the initial bubble price is at the equilibrium price, the bubbles will then sustain, and bubbly equilibrium is possible.

The transversality condition fails because of the additional term generated by the borrowing constraint, which guarantees a bubbly equilibrium. This is relatively standard in the literature of rational bubbles. There will be no bubble without borrowing constraints [Kocherlakota, 2009, Tirole, 1982].

Some algebra manipulations of equation (4.7) are shown below:

$$\begin{aligned} P_t\lambda_{1t}^H &= E_t s_{t+1} P_{t+1}(\beta + \phi\lambda_{3t}^H) \\ \iff \frac{1}{c_{y_t,t}^H} &= \frac{E_t s_{t+1} P_{t+1}}{P_t}(\beta + \phi\lambda_{3t}^H) \\ \iff \frac{1}{c_{y_t,t}^H} &= R_t(\beta + \phi\lambda_{3t}^H) \\ \iff c_{y_t,t}^H &= \frac{1}{R_t(\beta + \phi\lambda_{3t}^H)}. \end{aligned} \tag{4.8}$$

The demand side in equation (4.7) is:

$$\begin{aligned} \lambda_{3t}^H R_t &= \lambda_{1t}^H - \beta R_t E_t s_{t+1} \\ \iff \frac{1}{c_{y_t,t}^H} &= R_t[\lambda_{3t}^H + \beta E_t s_{t+1}] \\ \iff c_{y_t,t}^H &= \frac{1}{R_t[\lambda_{3t}^H + \beta E_t s_{t+1}]}. \end{aligned} \tag{4.9}$$

Combining equations (4.8) and (4.9):

$$\begin{aligned} \frac{1}{R_t(\beta + \phi\lambda_{3t}^H)} &= \frac{1}{R_t[\lambda_{3t}^H + \beta E_t s_{t+1}]} \\ \iff \lambda_{3,t}^H &= \frac{\beta(1 - E_t s_{t+1})}{1-\phi} > 0. \end{aligned} \tag{4.10}$$

Equation (4.10) implies that the shadow value is negatively related to the collapsing probability. When the probability is low, the shadow value on credit is increasing. Moreover, the shadow value is positively related to pledgeability ϕ. Specifically, when a financial market is fully collateralised, the shadow value becomes unidentified as the pledgeability is one. Agents are indifferent in their belief in bubbles. Full collateralisation means that lenders are optimistic enough to give loans without any collateral asset. Hence, it rules out the bubbles. In contrast, lenders never give loans to borrowers when the pledgeability is zero. This case will be identical to the first-best scenario.

Now, let's look at the case when $\pi^* \in (0,1)$ and $\phi \in (0,1)$, $\lambda_3^H > 0$. From pessimist's optimisation problem in equation (4.16), the young-age consumption is:

$$\begin{aligned} \lambda_{3t}^L &= \lambda_{1t}^L - \beta R_t E_t s_{t+1} \\ \iff c_{y_t,t}^L &= \frac{1}{\beta R_t E_t s_{t+1}}. \end{aligned} \tag{4.11}$$

Compared with equation (4.9), it shows that $c_y^H \leq c_y^L$ or optimistic agents tend to consume no larger than pessimistic ones at their young age. The young-age agents consume identically when $\lambda_3^H = 0$ and the economy only exists one type of agents and no bubble trading.

From the young-age pessimistic budget constraint in equation (4.12), the optimal loan is:

$$d_t^L = y_t - \frac{1}{\beta R_t E_t s_{t+1}}. \tag{4.12}$$

Credit market-clearing condition implies:

$$d_t^H \pi_t^* d_t^L = (1 - \pi_t^*) d_t^H = \frac{\pi^*}{1 - \pi^*}(y_t - \frac{1}{\beta R_t E_t s_{t+1}}). \tag{4.13}$$

Since the shadow value on credit constraint is strictly positive, then by complementary slackness condition in equation (4.17), the borrowing constraint in equation (4.4) should be always binding:

$$\begin{aligned}
&\phi E_t s_{t+1} P_{t+1} b_t - d_t^H R_t = 0 \\
&\iff \phi E_t s_{t+1} P_{t+1} b_t = d_t^H \frac{E_t s_{t+1} P_{t+1}}{P_t} \\
&\iff P_t b_t = \frac{d_t^H}{\phi}.
\end{aligned} \tag{4.14}$$

The intuition is that optimistic agents pledge a fraction of ϕ bubble value to buy bubbles.

From the young-age optimistic budget constraint in equation (4.2), we get:

$$\begin{aligned}
&c_{yt,t}^H + P_t b_t = y_t + d_t^H \\
&\iff c_{yt,t}^H = y_t + d_t^H - P_t b_t \\
&\iff c_{yt,t}^H = y_t + d_t^H - \frac{d_t^H}{\phi} \\
&\iff c_{yt,t}^H = y_t + \frac{\pi_t^*}{1 - \pi_t^*}(y_t - \frac{1}{\beta R_t E_t s_{t+1}})(1 - \frac{1}{\phi}) \\
&\iff c_{y,t}^H = y[1 + \frac{\pi_t^*(1 - \frac{1}{\phi})}{1 - \pi_t^*}] - \frac{\pi_t^*(1 - \frac{1}{\phi})}{1 - \pi_t^*} \frac{1}{\beta E_t s_{t+1} R_t} \\
&\iff c_{y,t}^H = \frac{1}{1 - \pi_t^*}[y(1 - \frac{\pi_t^*}{\phi}) - \frac{\pi_t^*(1 - \frac{1}{\phi})}{\beta E_t s_{t+1} R_t}].
\end{aligned} \tag{4.15}$$

Combining with equation (4.15), λ_{3t}^H and $E_t s_{t+1} = (1 - \pi_t^*)$, we get:

$$y[1 + \frac{\pi_t^*(1 - \frac{1}{\phi})}{1 - \pi_t^*}] - \frac{\pi_t^*(1 - \frac{1}{\phi})}{1 - \pi_t^*} \frac{1}{\beta(1 - \pi_t^*) R_t} = \frac{1}{R_t(\beta + \phi \lambda_{3t}^H)}$$

$$\iff y(1-\frac{\pi^*}{\phi})\frac{1}{1-\pi_t^*} - \frac{\pi^*(1-\frac{1}{\phi})}{\beta(1-\pi^*)^2 R_t} = \frac{1}{R_t(\beta+\phi\frac{\beta\pi_t^*}{1-\phi})}$$

$$\iff \beta y(\phi-\pi_t^*)(1-\pi_t^*)^2(1-\phi+\phi\pi_t^*)\frac{P_{t+1}(s_{t+1}=1)}{P_t}+\pi^*(\phi-1)^2-2(\phi^2-\phi)\pi_t^{*2} = (\phi-\phi^2)(1-2\pi_t^*)$$

$$\iff \frac{P_{t+1}(s_{t+1}=1)}{P_t}[(1-\phi)\beta y\phi - \beta y\pi_t^*\{1+\phi-3\phi^2\} + \beta y\pi_t^{*2}\{2-2\phi-3\phi^2\}$$
$$-\beta y\pi_t^{*3}\{1-3\phi-\phi^2\} - \beta y\pi_t^{*4}\phi] - 2(\phi^2-\phi)\pi_t^{*2} - (\phi-\phi^2) - \pi_t^*(\phi^2-1) = 0. \tag{4.16}$$

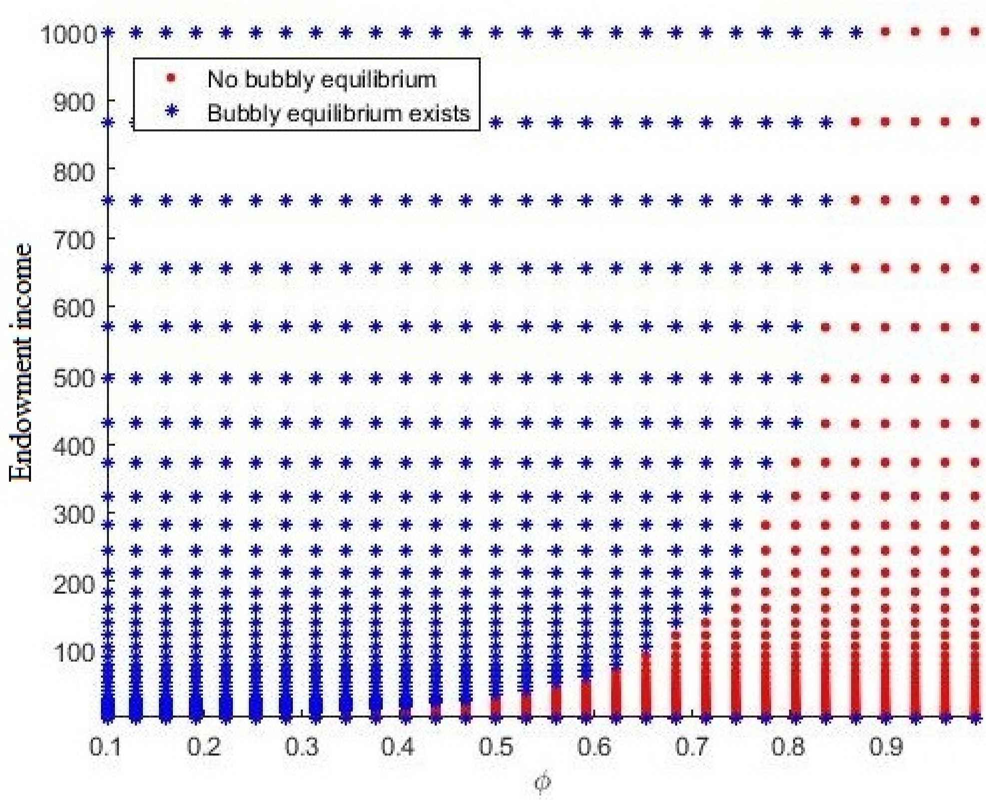
Existence of bubbly equilibrium in a fixed-bubble supply case

When bubble supply is fixed, I only consider the steady-state case where variables are time invariant. The cut-off belief is a solution to the quadratic equation (4.16) with $\frac{E_t s_{t+1}P_{t+1}}{P_t} = 1$:

$$-\beta y\phi\pi_t^{*4} - \beta y(1-3\phi-\phi^2)\pi_t^{*3} + \{\beta y(2-2\phi-3\phi^2) - 2(\phi^2-\phi)\}\pi_t^{*2}$$
$$-\{\beta y(1+\phi-3\phi^2) + (\phi^2-1)\}\pi_t^* + [(1-\phi)\beta y\phi] - (\phi-\phi^2) = 0. \tag{4.17}$$

With $\phi \in (0,1)$ and endowment income unit y varies from 1 to 1000, figure 4.4.1 shows the region where equation (4.16) has feasible solutions π_t^*. Obviously, a bubbly equilibrium exists on the left side of the graph. It indicates that the bubbly equilibrium is possible, corresponding with a high-income level and not too large collateralisation. Indeed, too high collateralisation causes a significant risk for lenders, restricting their lending. However, when the income is too low, lenders become riskier to finance bubbles since the risk of loss is small. Consequently, the bubbly equilibrium still exist along the bottom line at all levels of collateralisation ϕ.

Figure 4.4.1.: Bubble existence conditions



4.5. Conclusion

This chapter develops a simple overlapping-generations model in which agents have heterogeneous beliefs in bubbly assets. The model employs the theory of equilibrium leverage and asset-pricing from Geanakoplos [2010] to endogenise the bubble collapsing probability in an incomplete financial market environment. The economic agents have limited commitment in their loan repayment, requiring collateral to secure the loan. The collateral generates premium values, which drives up asset prices up. Deleveraging behavior is thus perhaps the main reason causing the security and house booms to burst. The more optimistic the median buyers are, the more people willing to lend, but the harder to find people with a higher optimistic expectation to resell.

There is a cut-off at the equilibrium where the median buyer's belief results in two different cases. The first case is a bubbleless equilibrium economy where no one believes in higher bubble prices. The bubbles become useless, and this bubbleless economy is identical to autarky and frictionless economy. The second case is a bubbly equilibrium economy where bubbles have a positive value in equilibrium. In this scenario, the cut-off belief is feasible in relation to the endowment income and collateral ratio conditions. Particularly, a bubbly equilibrium is more likely in more developed countries with reliable income and a highly leveraged financial market. However, an extremely high collateral ratio might trigger deleveraging behavior from nervous lenders, reducing natural buyers' lending capacity, causing prices to decline and bubbles to collapse. This finding is consistent with the narratives along with the leveraged security and housing bubbles in the U.S. Another possibility is that a bubbly equilibrium is likely to exist in developing countries with low income at any level of collateralsation. Indeed, lenders will become less risk-averse when their incomes are low since their loss risks are small. Natural buyers thus have more access to loans, which drives up the bubble prices. This is supported by other bubble periods in East Asian economies where easy credit and financial deregulation ended in crisis.

5. Conclusion

This thesis consists of three research papers that study the macroeconomic impacts of uncertainty shocks and bubbles. Empirically, the papers contribute to the research on a small, open economy, particularly a commodity-exporting economy like Australia, and complement the literature on uncertainty shocks and rational bubbles.

Chapter 2 started the journey by replicating a DSGE small, open economy model based on studies by Justiniano and Primiceri [2008], Justiniano and Preston [2010], Cross et al. [2018]. The aim is to explore the macroeconomic impact of uncertainty shocks in driving business-cycle fluctuations in Australia from 1981 to 2015. The model was populated with four basic economic agents: representative households, firms, government, and foreign economy. Nine structural shocks account for domestic and international shocks. The model explicitly distinguishes the impact of uncertainty shocks from the structural shocks on economic variations. I contribute to the existing literature that has estimated DSGE models of the Australian economy. Notably, I show that time-varying volatility shocks have contributed substantially to the Australian economic fluctuations. The shocks account for more than 20% of the variations in which uncertainty in labour tax and cost-push shocks are the most important. However, my results share a similarity with the estimated Canadian model in Cross et al. [2018] that the systematic shocks are the primary factors driving aggregate movements. The Australian domestic monetary and fiscal policy shocks are the largest components, accounting for nearly 50% of GDP fluctuations. Moreover, the foreign shocks account for a negligible fraction (less than 10%) in domestic output movements.

What distinguishes Australia from other small, open economies is a strong mining sector that dominates the export sector. The mining boom has favoured a commodity-exporting country like Australia, with mining exports and investments shooting up and average living standards and wages increasing. Despite a great benefit from Australian dollar appreciation to the import sector, export industries, such as manufacturing and agriculture worse off [Downes et al., 2014]. Thus, in chapter 3, I continue my research journey by extending the model, providing more structure for the production sector to include non-traded, non-resource tradable, resource, import, and final goods and services sectors. The model emphasises the importance of the resource sector in a small, commodity-exporting economy like Australia. I use Australian data from 1992 to 2013 to explicitly examine the impact of uncertainty shocks on the Australian economy during the mining boom. The results show that foreign shocks are the most crucial drivers accounting for more than 50% of the Australian economic variations, while demand and productivity shocks have minimal impacts. After further decomposition of the shocks, the time-varying uncertainty components account for more than 60% of the Australian output variations. The foreign-related shocks, including risk-premium, resource-prices, foreign policies, and economic condition shocks, are the largest factors contributing about 40% in the uncertainty shocks decomposition.

Chapter 4 focuses on rational bubbles and collateral constraints in an overlapping-generations model with heterogeneous beliefs. Economic agents with different beliefs on bubble values will use different trading strategies to maximise their utility function. The model employed the theory of equilibrium leverage and asset-pricing in Geanakoplos [2010] to endogenise the collapsing probability of bubbles in an incomplete financial market environment. The study shows that a median buyer will determine the existence of bubbles in an economy. A bubbleless economy is identical to an autarky economy when no one believes in bubbles. A bubbly equilibrium can endure in high-income countries with highly leveraged financial markets. However, high collateralisation might trigger deleveraging behavior from nervous lenders, causing bubbles to collapse. Bubbly equilibrium is also prone to exist in low-income countries at any level of collateralisation.

In short, this thesis demonstrates the role of uncertainty in explaining macroeconomic fluctuations through different channels. Specifically, the critical challenge remains whether uncertainty shocks drive business-cycle fluctuation or aggregate fluctuations drive uncertainty shocks [Van Nieuwerburgh and Veldkamp, 2006]. There is room for further research on the endogeneity of uncertainty shocks where the structural shocks can create variations in second-moment shocks to capture uncertainty [Atkinson et al., 2020]. In terms of the rational bubble study, the model can be extended to characterise a dynamic bubbly equilibrium. This research will be tackled outside of this thesis in future research.

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A. Appendix A: Uncertainty shocks in markets and policies: an Australian case study.

A.1. Bayesian estimation algorithm

1. Transform the model to a stationary model: a stationalised variable is the ratio of the original level and domestic TFP.
2. Calibrate model parameters.
3. Solving the linear rational expectation system by the QZ algorithm. The solution can be written in this form: $x_{t+1} = A_\theta x_t + B_\theta \tilde{u}_t$ where x_t is vector of model variables, and the coefficient matrices A_θ, B_θ are a function of structural parameters θ. This equation is called a state transition equation. This step can be done with Dynare.
4. Setting up a state-space framework for Bayesian estimation as follows:

    a) state transition equation: $x_{t+1} = A_\theta x_t + B_\theta \tilde{u}_t$. We can extract matrix A_θ, B_θ after Dynare program,

    b) measurement equation: $y_t^o = H^o y_t$. This equation is to match the data with the state-space representation. We use the pair (i, j):

        i. where i is variable order in data, $i \in [1, 9]$ representing: domestic Real GDP growth, domestic average labour income tax, domestic average capital income tax, domestic gross nominal interest rate, demeaned domestic gross inflation rate, domestic real investment growth, demeaned foreign real GDP growth, demeaned foreign nominal interest rate, demeaned foreign CPI inflation rate, respectively,

        ii. where j is the variable order in the state-space representation, $j \in [1, 26]$ representing the structural parameters,

iii. there might be measurement errors which are caused by the data collecting process. It follows the white noise process and uncorrelated to other variables and lags. These errors capture the gap between theoretical variables and observable data. Adding measurement errors is necessary to avoid a stochastic singularity.

c) Random-walk plus noise SV model:

$$\tilde{u}_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \tag{1.1}$$

$$h_{i,t} = h_{i,t-1} + v_{i,t}, \tag{1.2}$$

where $h_{i,t} = log\sigma_{i,t}$, $\varepsilon_{i,t} \sim N(0,1)$ is white noise, and $v_{i,t} \sim N(0, \omega_i^2)$ and $i = 1...6$ are structural shocks. Denote θ^g, H^g, ω^g are last saved draws, we are estimating in iteration $g+1$.

5. Draw structural shocks $\tilde{u}^{g+1}$ following Durbin and Koopman 2002 by efficient disturbance simulation smoother.

6. Draw SV H^{g+1} following Chan et al. 2014 by a precision sampling.

7. Draw variance of SV ω^{g+1} by sampling from inverse Gamma distribution with $\omega_i^2 \sim IG(v_{\omega_i^2}, S_{\omega_i^2})$:

$$(\omega_i^{g+1}|\tilde{u}_i^{g+1}, h_i^{g+1}) \sim IG\left(v_{\omega_i^2} + \frac{T-1}{2}, S_{\omega_i^2} + \sum_{t=2}^{T}(h_t - h_{t-1})^2\right). \tag{1.3}$$

8. Draw DSGE parameters, resolve the model return candidate parameters θ and candidate model defined by matrices $A_\theta$and B_θ:

   a) draw candidate DSGE parameters $\hat{\theta}$ by random walk transition equation,

   b) resolve the model and check the results to make sure the model is solvable, uniquely determined, stable, no indeterminacy, consistent,

   c) updating the matrices $\hat{A}_{\hat{\theta}}$ and $\hat{B}_{\hat{\theta}}$ to get the candidate model.

9. Using Metropolis-Hasting to decide updating parameters:

   a) evaluate model likelihood with Kalman filter for old $\mathcal{L}(Y|\theta^g, H^g)$ and candidate models $\mathcal{L}(Y|\theta^c, H^c)$,

b) compute prior density for structural parameters by Schorfheide 2000 for old $p(\theta^g)$ and candidate models $p(\theta^c)$,

c) the accepting probability is given by:

$$\alpha = min\left\{1, \frac{\mathcal{L}(Y|\theta^c, H^c)p(\theta^c)}{\mathcal{L}(Y|\theta^g, H^g)p(\theta^g)}\right\}. \tag{1.4}$$

A.2. Data sources and definitions

A.2.1. Data sources

GDP: quarterly percentage change of Australian Gross Domestic Product (GDP) in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.01 'Australian National Accounts: National Income, Expenditure and Product'.

Inflation: quarterly percentage change of Australian implicit price deflator, seasonally adjusted. Source: ABS Cat No 5206.05 'Australian National Accounts: National Income, Expenditure and Product'.

Population: quarterly population estimates by state/territory, sex and age. Source: ABS Cat No 3101.04 'National, state and territory population'.

Investment: quarterly gross fixed capital formation in constant prices, seasonally adjusted. Source: Federal Reserve Bank of St. Louis - NAEXKP04AUQ189S.

Nominal interest rate: monthly immediate rates for Australia, seasonally adjusted. Source: Federal Reserve Bank of St. Louis - IRSTCI01AUM156N.

Personal income tax, taxes on corporate income: quarterly data, seasonally adjusted. Source: ABS Cat No 5206.17 'Australian National Accounts: National Income, Expenditure and Product'.

Wages and salaries, proprietor's income, corporate profit, compensation of employees: quarterly data, seasonally adjusted. Source: ABS Cat No 5206.07 'Australian National Accounts: National Income, Expenditure and Product'.

Rental income, net interest income and other payments, government social insurance: quarterly data, seasonally adjusted. Source: ABS Cat No 5206.20 'Australian National Accounts: National Income, Expenditure and Product'.

Property taxes: quarterly data, seasonally adjusted. Source: ABS Cat No 5206.22 'Australian National Accounts: National Income, Expenditure and Product'.

Real GDP per capita: quarterly real GDP per capita for the United States, seasonally adjusted. Source: Federal Reserve Bank of St. Louis - A939RX0Q048SBEA.

GDP deflator: quarterly GDP implicit price deflator for the United States, seasonally adjusted. Source: Federal Reserve Bank of St. Louis - GDPCTPI.

Nominal interest rate: monthly immediate rates for the United States, seasonally adjusted. Source: Federal Reserve Bank of St. Louis - FEDFUNDS.

A.2.2. Domestic data construction

Domestic real GDP per capita growth rate is calculated by taking nominal GDP divided by GDP deflator and then divided by population before taking thelog-difference of the result.

Per capita investment is calculated by gross fixed capital formation divided by population; then taking the log-difference of the result. Gross inflation is calculated by taking the log of GDP deflator difference. The gross nominal interest rate is calculated by dividing by 400 and transforming to gross rate and taking a natural log, finally subtracting the implied long-run mean.

Labour and capital income tax rates are calculated following Born and Pfeifer [2014] and Fernández-Villaverde et al. [2015]. Firstly, the average personal income tax (PIT) rate τ_p is calculated as:

$$\tau_p = \frac{PIT}{WS + PRI/2 + CI}. \tag{1.1}$$

where PIT denotes personal income tax, WS is wage and salaries, PRI is proprietor's income, CI is capital income which is the sum of rental income RI, corporate profit CP, net interest income and other payments NI. Having PIT rate τ_p, I proceed to calculate average labour income tax rate τ_N and average capital income tax rate τ_K as:

$$\tau_N = \frac{\tau_p(WS + PRI/2) + CSI}{CE + PRI/2}, \tag{1.2}$$

$$\tau_K = \frac{\tau_p CI + CT + PT}{CI + PT}, \tag{1.3}$$

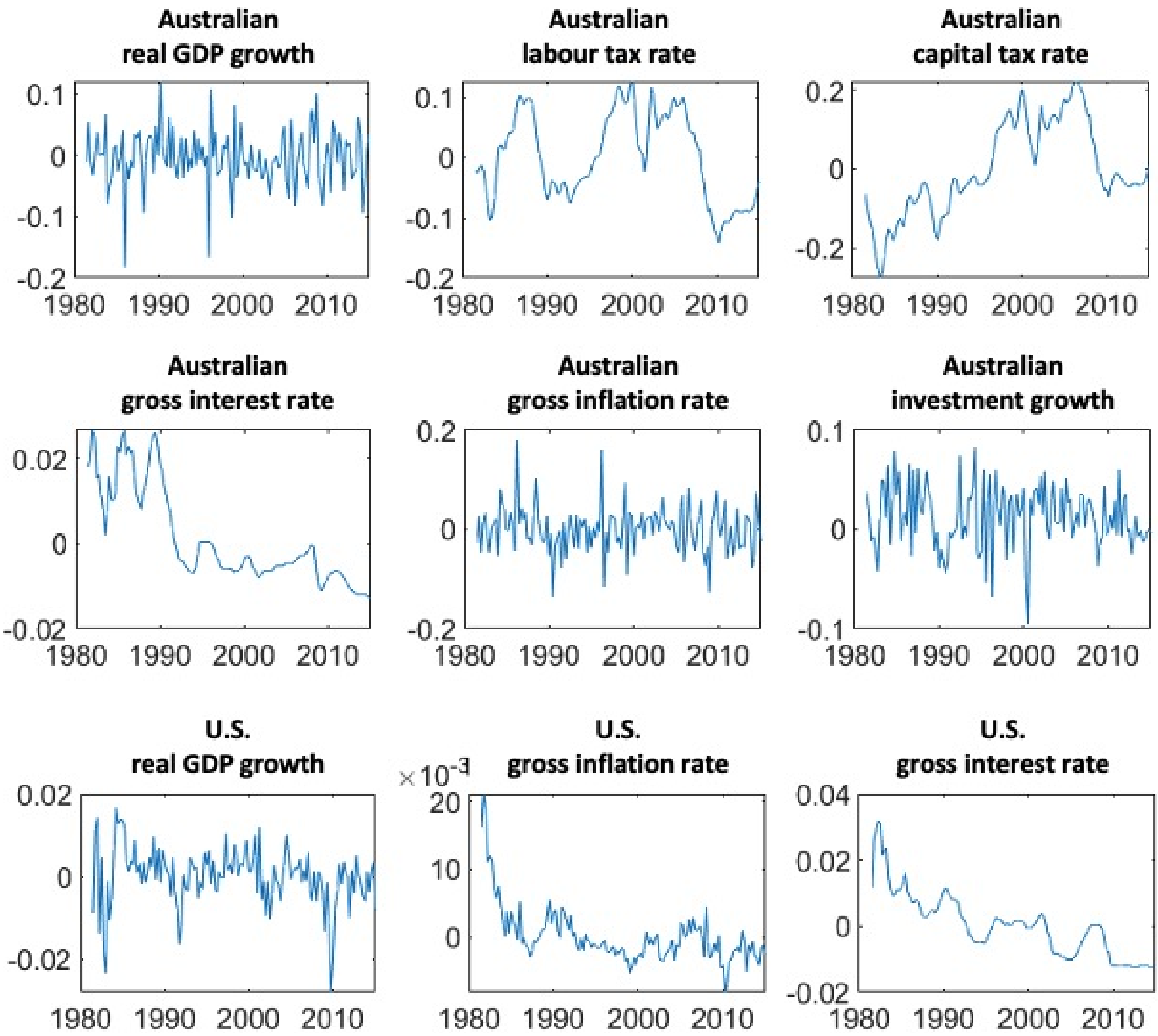
where CSI denotes government social insurance, CE is compensation of employees, CT denotes taxes on corporate income, PT is property taxes. Taking log-difference and subtracting the implied long-run mean of the average labour and capital income tax rate τ_N and τ_K for model estimation.

In addition, the consumption share of output and government expenditure share of output is sourced from World Bank 1981—2015 while the relative size of home country production to the ROW is calculated from GDP data from World Bank 1981–2015.

A.2.3. Foreign data

I use the U.S. as a representative for ROW. I use three series of U.S. quarterly data (real GDP, GDP deflator, and nominal interest rate), which are sourced from the FRED database. Real GDP growth and CPI inflation rate are calculated respectively by log-difference of real GDP and GDP deflator series. Gross nominal interest rate is converted to quarterly rate by geometric mean and dividing by 400, then transforming to gross rate. Before entering the model, the three series are demeaned by subtracting the implied long-run mean.

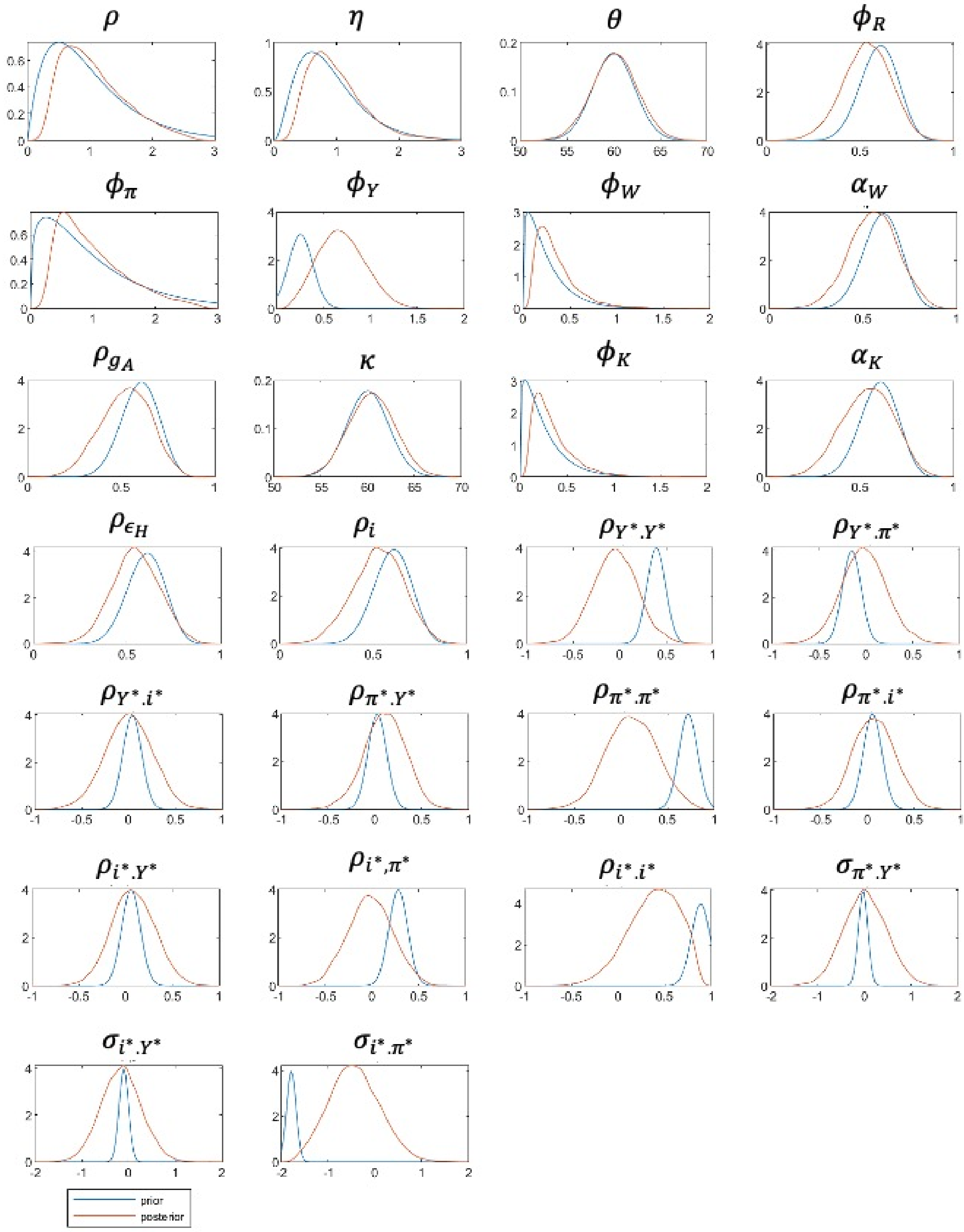
Figure A.2.1.: Australian data overview 1981–2015



Source: ABS, Federal Reserve Bank of St. Louis.

A.3. Prior and posterior distribution

Figure A.3.1.: Prior densities versus posterior densities



A.4. Convergence diagnosis

Figure A.4.1.: Inefficiency factors for estimated parameters

.

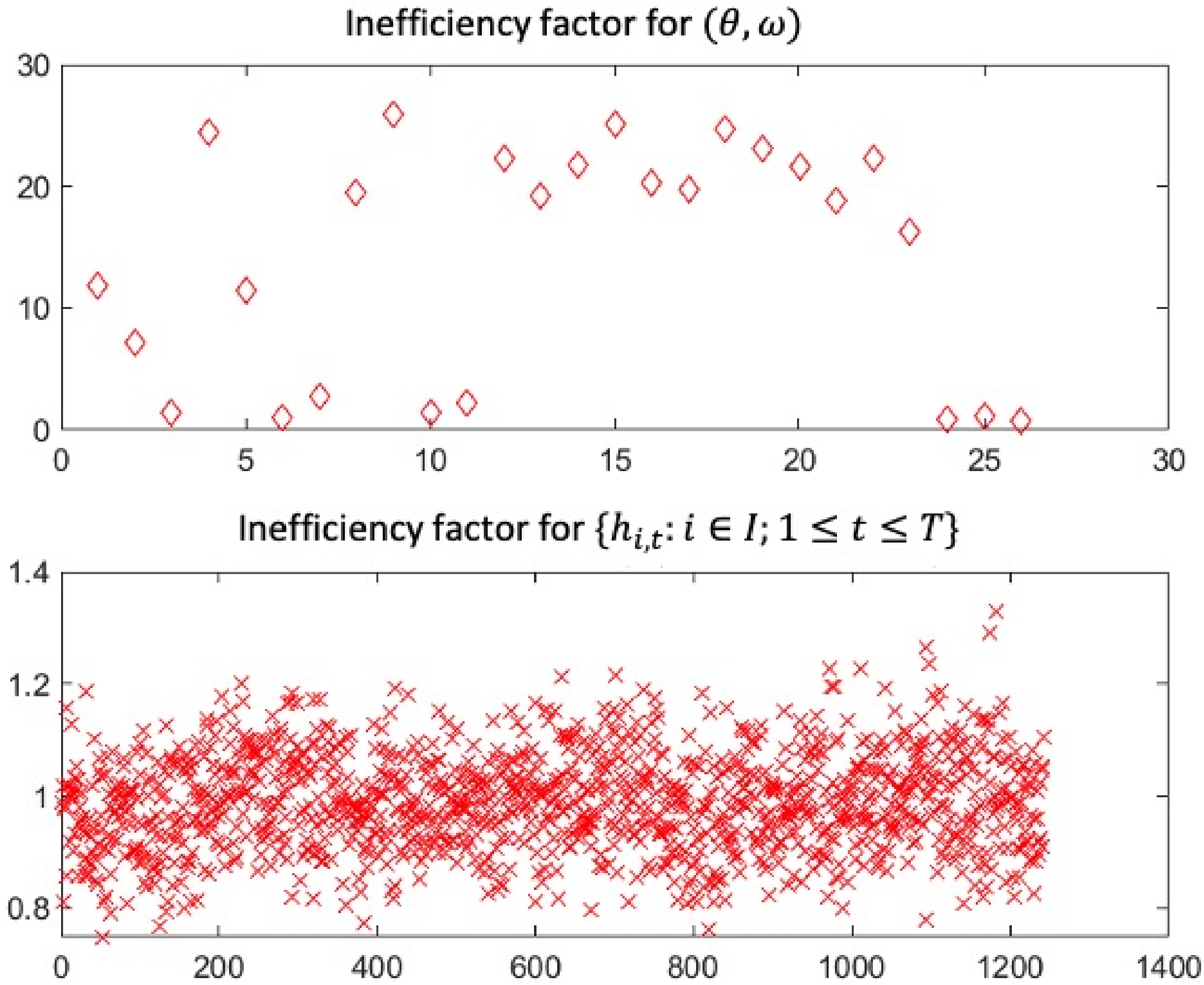


Figure A.4.2.: Plot of Markov Chain mean of posteriors for estimated parameters

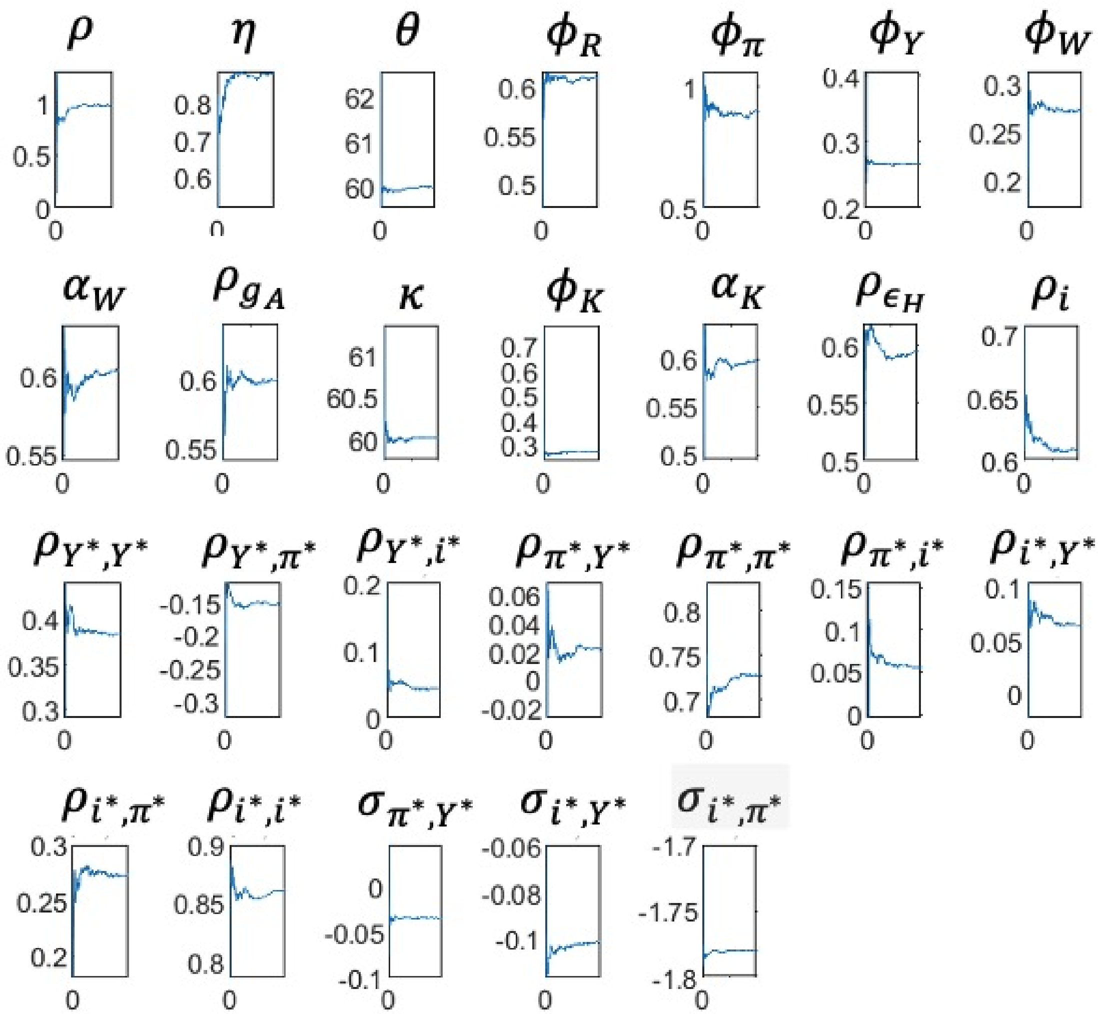
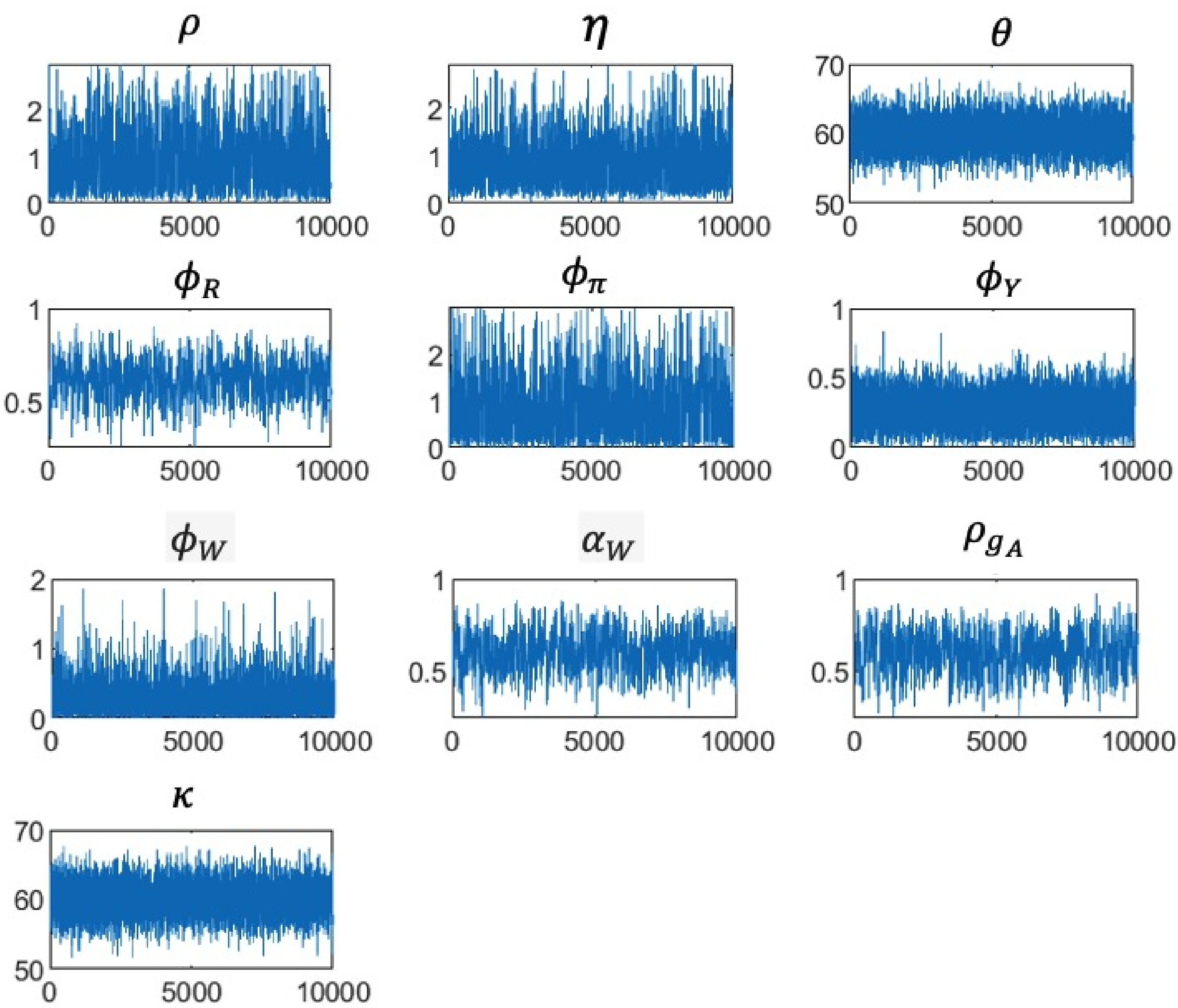
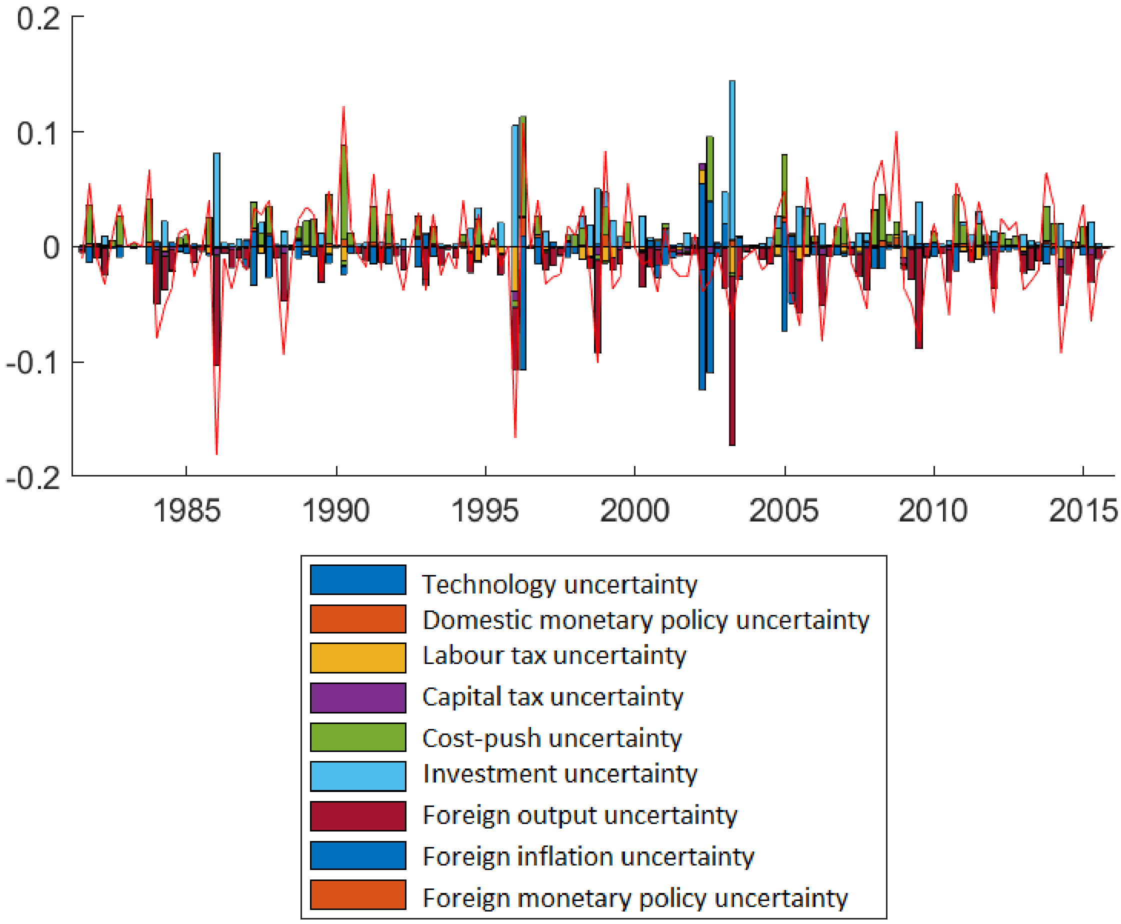


Figure A.4.3.: Trace plots for estimated parameters



A.5. Historical decomposition

Figure A.5.1.: Historical decomposition for each shock (combined)



B. Appendix B: Uncertainty shocks in a multisector model for small, open economies: an Australian case study.

B.1. Derivation of equilibrium conditions

B.1.1. A representative household

The Lagrangian function associated with the household's problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_{c,t} ln(C_t - hC_{t-1}) - A_L \frac{\left[H_{n,t}^{1+\sigma} + H_{m,t}^{1+\sigma} + H_{z,t}^{1+\sigma}\right]^{\frac{1+\eta}{1+\sigma}}}{1+\eta} \right. \tag{2.1}$$

$$+\lambda_t \left[\sum_{j=n,m,z} (W_{j,t}H_{j,t} + R_{j,t}K_{j,t}) + R_{L,t}L + B_{t-1} + S_t B_{t-1}^* \right.$$

$$\left. +\Gamma_t - T_t - P_t C_t - P_t I_t - \frac{B_t}{R_t} - \frac{S_t B_t^*}{R_t^* \upsilon_t} \right]$$

$$\left. + \lambda_t^j \left[(1-\delta)K_{j,t} + \Upsilon_t I_{j,t} \left[1 - \frac{\Phi_K}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1 \right)^2 \right] - K_{j,t+1} \right] \right\},$$

where $j = \{n, m, z\}$, λ_t is the Lagrangian multiplier associated with the budget constraints; $\lambda_t^j$is the Lagrangian multiplier associated with the capital accumulation.

The first-order conditions (FOCs) with respect to:

- $\frac{\partial \mathcal{L}}{\partial C_t}$:

$$\xi_{c,t} \frac{1}{C_t - hC_{t-1}} - \lambda_t P_t - h\beta E_t \xi_{c,t+1} \frac{1}{C_{t+1} - hC_t} = 0. \tag{2.2}$$

Let $\Lambda_t = \lambda_t P_t$, then:

$$\xi_{c,t}\frac{1}{C_t - hC_{t-1}} - h\beta E_t \xi_{c,t+1}\frac{1}{C_{t+1} - hC_t} = \Lambda_t. \tag{2.3}$$

- $\frac{\partial \mathcal{L}}{\partial B_t}$:

$$\beta E_t \lambda_{t+1} - \frac{\lambda_t}{R_t} = 0. \tag{2.4}$$

or

$$\beta E_t \Lambda_{t+1}\frac{P_t}{P_{t+1}} - \frac{\Lambda_t}{R_t} = 0. \tag{2.5}$$

- $\frac{\partial \mathcal{L}}{\partial B_t^*}$:

$$\beta E_t \lambda_{t+1} S_{t+1} - \lambda_t \frac{S_t}{R_t^* \upsilon_t} = 0; \tag{2.6}$$

$$\beta E_t \Lambda_{t+1}\frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}} - \Lambda_t \frac{1}{R_t^* \upsilon_t} = 0. \tag{2.7}$$

- $\frac{\partial \mathcal{L}}{\partial H_{j,t}}$:

$$-A_L \frac{1+\eta}{1+\sigma}\frac{\left[H_{n,t}^{1+\sigma} + H_{m,t}^{1+\sigma} + H_{z,t}^{1+\sigma}\right]^{\frac{1+\eta}{1+\sigma}-1}}{1+\eta}\left(1+\sigma\right) H_{j,t}^{1+\sigma-1} + \lambda_t W_{j,t} = 0. \tag{2.8}$$

After some manipulations:

$$\begin{aligned} -A_L \left[H_{n,t}^{1+\sigma} + H_{m,t}^{1+\sigma} + H_{z,t}^{1+\sigma}\right]^{\frac{\eta-\sigma}{1+\sigma}} H_{j,t}^{\sigma} + \lambda_t W_{j,t} = 0 \\ \Longleftrightarrow -A_L H_{j,t}^{\sigma} H_t^{\eta-\sigma} + \lambda_t W_{j,t} = 0. \end{aligned} \tag{2.9}$$

or

$$-A_L H_{j,t}^{\sigma} H_t^{\eta-\sigma} + \frac{\Lambda_t}{P_t} W_{j,t} = 0. \tag{2.10}$$

- $\frac{\partial \mathcal{L}}{\partial K_{j,t+1}}$:

$$-\lambda_t^j + \beta E_t\left[\lambda_{t+1} R_{j,t+1} + \lambda_{t+1}^j (1-\sigma)\right] = 0. \tag{2.11}$$

Let $Q_{j,t} = \frac{\lambda_t^j}{\Lambda_t}$ is Tobin's Q for each sector j, then

$$-Q_{j,t}\Lambda_t + \beta E_t\left[\frac{\Lambda_{t+1}}{P_{t+1}} R_{j,t+1} + Q_{j,t+1}\Lambda_{t+1}(1-\sigma)\right] = 0, \tag{2.12}$$

or

$$-Q_{j,t} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{R_{j,t+1}}{P_{t+1}} + Q_{j,t+1}(1-\sigma)\right] = 0. \tag{2.13}$$

- $\frac{\partial \mathcal{L}}{\partial I_{j,t}}$:

$$-\Lambda_t + \lambda_t^j \Upsilon_t \left[1 - \frac{\Phi_K}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1 \right)^2 - \Phi_K \frac{I_{j,t}}{I_{j,t-1}} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1 \right) \right] \\ + \beta E_t \lambda_{t+1}^j \Upsilon_{t+1} \Phi_K \frac{I_{j,t+1}^2}{I_{j,t}^2} \left(\frac{I_{j,t+1}}{I_{j,t}} - 1 \right) = 0, \tag{2.14}$$

or

$$-1 + Q_{j,t} \Upsilon_t \left[1 - \frac{\Phi_K}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1 \right)^2 - \Phi_K \frac{I_{j,t}}{I_{j,t-1}} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1 \right) \right] \\ + \beta E_t Q_{j,t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \Upsilon_{t+1} \Phi_K \frac{I_{j,t+1}^2}{I_{j,t}^2} \left(\frac{I_{j,t+1}}{I_{j,t}} - 1 \right) = 0. \tag{2.15}$$

If there is no adjustment cost, the last equation becomes $Q_{j,t} = 1/\Upsilon_t$ meaning that Tobin's Q is just the replacement cost of capital.

B.1.2. Non-traded sector

The non-traded firms solve a two-stages problem: first, they choose labour, capital and resource inputs to minimise production cost at given input prices; second, they choose prices to maximise the profit.

First, the cost minimisation problem for each firm i in sector n in a perfectly competitive factor input market is as follows:

$$\min_{H_{n,t}(i), K_{n,t}(i), Z_{n,t}(i)} W_{n,t} H_{n,t}(i) + R_{n,t} K_{n,t}(i) + P_{z,t} Z_{n,t}(i), \tag{2.16}$$

subject to production function:

$$Y_{n,t}(i) = a_{n,t} \left(\mu_t H_{n,t}(i) \right)^{\alpha_n} K_{n,t}(i)^{\gamma_n} Z_{n,t}(i)^{1-\alpha_n-\gamma_n}. \tag{2.17}$$

FOCs with respect to $H_{n,t}(i), K_{n,t}(i), Z_{n,t}(i)$ with $\lambda_{n,t}$ is the Lagrangian multiplier:

$$W_{n,t} = \lambda_{n,t} a_{n,t} \alpha_n \left(\mu_t H_{n,t}(i) \right)^{\alpha_n - 1} K_{n,t}(i)^{\gamma_n} Z_{n,t}(i)^{1-\alpha_n-\gamma_n}, \tag{2.18}$$

$$R_{n,t} = \lambda_{n,t} a_{n,t} \left(\mu_t H_{n,t}(i) \right)^{\alpha_n} \gamma_n K_{n,t}(i)^{\gamma_n - 1} Z_{n,t}(i)^{1-\alpha_n-\gamma_n}, \tag{2.19}$$

$$P_{z,t} = \lambda_{n,t} a_{n,t} \left(\mu_t H_{n,t}(i) \right)^{\alpha_n} K_{n,t}(i)^{\gamma_n} \left(1 - \alpha_n - \gamma_n \right) Z_{n,t}(i)^{-\alpha_n-\gamma_n}. \tag{2.20}$$

This implies:

$$\frac{R_{n,t}}{P_{z,t}} = \frac{\gamma_n Z_{n,t}(i)}{(1-\alpha_n-\gamma_n)\, K_{n,t}(i)}; \frac{W_{n,t}}{P_{z,t}} = \frac{\alpha_n Z_{n,t}(i)}{(1-\alpha_n-\gamma_n)\, \mu_t H_{n,t}(i)}. \tag{2.21}$$

The firm real cost is:

$$\frac{1}{P_{n,t}} \left(W_{n,t} H_{n,t}(i) + R_{n,t} K_{n,t}(i) + P_{z,t} Z_{n,t}(i)\right) \tag{2.22}$$

$$\iff \frac{1}{P_{n,t}} \left(\frac{\alpha_n P_{z,t} Z_{n,t}(i)}{(1-\alpha_n-\gamma_n)\, \mu_t} + \frac{\gamma_n P_{z,t} Z_{n,t}(i)}{(1-\alpha_n-\gamma_n)} + P_{z,t} Z_{n,t}(i) \right)$$

$$\iff \frac{1}{P_{n,t}} P_{z,t} Z_{n,t}(i) \left(\frac{\alpha_n + \mu_t \gamma_n + (1-\alpha_n-\gamma_n)\, \mu_t}{(1-\alpha_n-\gamma_n)\, \mu_t} \right)$$

$$\iff \frac{1}{P_{n,t}} P_{z,t} Z_{n,t}(i) \left(\frac{\alpha_n + \mu_t - \mu_t \alpha_n}{(1-\alpha_n-\gamma_n)\, \mu_t} \right). \tag{2.23}$$

A real marginal cost MC_t is calculated by setting labour supply, capital and resource inputs to the level required to produce one unit of non-traded goods:

$$a_{n,t} \left(\mu_t H_{n,t}(i)\right)^{\alpha_n} K_{n,t}(i)^{\gamma_n} Z_{n,t}(i)^{1-\alpha_n-\gamma_n} = 1 \tag{2.24}$$

$$\iff a_{n,t} \left(\frac{\alpha_n P_{z,t} Z_{n,t}(i)}{(1-\alpha_n-\gamma_n)\, W_{n,t}} \right)^{\alpha_n} \left(\frac{\gamma_n P_{z,t} Z_{n,t}(i)}{(1-\alpha_n-\gamma_n)\, R_{n,t}} \right)^{\gamma_n} Z_{n,t}(i)^{1-\alpha_n-\gamma_n} = 1$$

$$\iff Z_{n,t}(i) = \frac{1}{a_{n,t}} \left(\frac{(1-\alpha_n-\gamma_n)\, W_{n,t}}{\alpha_n P_{z,t}} \right)^{\alpha_n} \left(\frac{(1-\alpha_n-\gamma_n)\, R_{n,t}}{\gamma_n P_{z,t}} \right)^{\gamma_n}. \tag{2.25}$$

Plug this into the firm real cost, the nominal marginal cost $mc_{n,t}$ is:

$$\begin{aligned} MC_{n,t} &= P_{z,t} Z_{n,t}(i) \left(\frac{\alpha_n + \mu_t - \mu_t \alpha_n}{(1-\alpha_n-\gamma_n)\, \mu_t} \right) \\ &= \frac{1}{a_{n,t}} \left(\frac{W_{n,t}}{\alpha_n} \right)^{\alpha_n} \left(\frac{R_{n,t}}{\gamma_n} \right)^{\gamma_n} \left(\frac{P_{Z,t}}{1-\alpha_n-\gamma_n} \right)^{1-\alpha_n-\gamma_n} \\ &= \frac{\epsilon_{\pi_n,t}}{a_{n,t}} \left(\frac{W_{n,t}}{\alpha_n} \right)^{\alpha_n} \left(\frac{R_{n,t}}{\gamma_n} \right)^{\gamma_n} \left(\frac{P_{Z,t}}{1-\alpha_n-\gamma_n} \right)^{1-\alpha_n-\gamma_n}. \end{aligned} \tag{2.26}$$

Assuming a positive markup shock changes the nominal marginal cost of non-traded production for reasons unrelated to changing wage, capital return rate, and resource price, denote this shock as $\epsilon_{\pi_n,t}$. Notice that the nominal marginal cost is not dependent on i, implying that all non traded firms face the same technology shocks and rental input prices.

Second, non-traded firms solve their profit maximisation problem as follows:

$$\max_{P_{n,t}(i)} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{P_{n,t}(i)Y_{n,t}(i)}{P_t} - \frac{MC_{n,t}Y_{n,t}(i)}{P_t} - \frac{\tau_{\pi^n}}{2} \left[\frac{P_{n,t}(i)}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}P_{n,t-1}(i)} - 1 \right]^2 \frac{P_{n,t}Y_{n,t}}{P_t} \right\}, \tag{2.27}$$

subject to the demand function:

$$Y_{n,t}(i) = \left(P_{n,t}(i)/P_{n,t}\right)^{-\theta^n} Y_{n,t}. \tag{2.28}$$

The FOCs is:

$$\begin{aligned}(1-\theta^n)\left(\frac{P_{n,t}(i)}{P_{n,t}}\right)^{-\theta_n} \frac{Y_{n,t}}{P_t} + \theta_n MC_{n,t} \left(\frac{P_{n,t}(i)}{P_{n,t}}\right)^{-\theta_n} \left(\frac{P_{n,t}(i)}{P_{n,t}}\right)^{-1} \frac{Y_{n,t}}{P_t} \\ -\tau_{\pi^n}\frac{Y_{n,t}}{P_t}\left[\frac{P_{n,t}(i)}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}P_{n,t-1}(i)} - 1\right] \frac{P_{n,t}}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}P_{n,t-1}(i)} \\ +\beta\tau_{\pi^n}E_t\left\{\left[\frac{P_{n,t+1}(i)}{\Pi^{\chi}_{n,t}\Pi^{1-\chi}P_{n,t}(i)} - 1\right]\frac{P_{n,t+1}(i)}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}P_{n,t}(i)}\frac{P_{n,t+1}Y_{n,t+1}}{P_{t+1}P_{n,t}(i)}\right\} = 0.\end{aligned} \tag{2.29}$$

I only consider symmetric equilibrium in this model, then all firms choose to set the same price. Together with $P_{n,t}(i) = P_{n,t}$, and the inflation definition with a friction $\Pi_{n,t} = \frac{P_{n,t}}{P_{n,t-1}}$, equation (2.29) becomes:

$$\begin{aligned}(1-\theta^n) + \theta^n MC_{n,t} - \tau_{\pi^n}\left(\frac{\Pi_{n,t}}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}} - 1\right)\frac{\Pi_{n,t}}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}} \\ +\beta\tau_{\pi^n}E_t\left\{\frac{Y_{n,t+1}}{Y_{n,t}}\frac{P_t}{P_{t+1}}\left(\frac{\Pi_{n,t}}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}} - 1\right)\frac{\Pi^2_{n,t+1}}{\Pi^{\chi}_{n,t-1}\Pi^{1-\chi}}\right\} = 0.\end{aligned} \tag{2.30}$$

B.1.3. Non-resource traded sector

The non-resource traded firms also solve a two-stage problem: first, they choose labour, capital and resource inputs to minimise production cost at given input prices; second, they choose price to maximise the profit.

First, the cost minimisation problem for each firm i in sector n is:

$$\min_{H_{m,t}(i),K_{m,t}(i),Z_{m,t}(i)} W_{m,t}H_{m,t}(i) + R_{m,t}K_{m,t}(i) + P_{z,t}Z_{m,t}(i), \tag{2.31}$$

subject to production function:

$$Y_{m,t}(i){=}a_{m,t}\left(\mu_t H_{m,t}(i)\right)^{\alpha_m} K_{m,t}(i)^{\gamma_m} Z_{m,t}(i)^{1-\alpha_m-\gamma_m}. \tag{2.32}$$

FOCs with respect to $H_{m,t}(i), K_{m,t}(i), Z_{m,t}(i)$:

$$W_{m,t} = MC_{m,t}a_{m,t}\alpha_m \left(\mu_t H_{m,t}(i)\right)^{\alpha_m - 1} K_{m,t}(i)^{\gamma_m} Z_{m,t}(i)^{1-\alpha_m-\gamma_m}, \tag{2.33}$$

$$R_{m,t} = MC_{m,t}a_{m,t} \left(\mu_t H_{m,t}(i)\right)^{\alpha_m} \gamma_m K_{m,t}(i)^{\gamma_m - 1} Z_{m,t}(i)^{1-\alpha_m-\gamma_m}, \tag{2.34}$$

$$P_{z,t} = MC_{m,t}a_{m,t} \left(\mu_t H_{m,t}(i)\right)^{\alpha_m} K_{m,t}(i)^{\gamma_m} \left(1-\alpha_m-\gamma_m\right) Z_{m,t}(i)^{-\alpha_m-\gamma_m}. \tag{2.35}$$

This implies:

$$\frac{R_{m,t}}{P_{z,t}} = \frac{\gamma_m Z_{m,t}(i)}{\left(1-\alpha_m-\gamma_m\right) K_{m,t}(i)}; \frac{W_{m,t}}{P_{z,t}} = \frac{\alpha_m Z_{m,t}(i)}{\left(1-\alpha_m-\gamma_m\right) \mu_t H_{m,t}(i)}. \tag{2.36}$$

The firm real cost is:

$$\frac{1}{P_{m,t}} \left(W_{m,t}H_{m,t}(i) + R_{m,t}K_{m,t}(i) + P_{z,t}Z_{m,t}(i)\right) \tag{2.37}$$

$$\Longleftrightarrow \frac{1}{P_{m,t}} \left(\frac{\alpha_m P_{z,t}Z_{m,t}(i)}{\left(1-\alpha_m-\gamma_m\right)\mu_t} + \frac{\gamma_m P_{z,t}Z_{m,t}(i)}{\left(1-\alpha_m-\gamma_m\right)} + P_{z,t}Z_{m,t}(i)\right)$$

$$\Longleftrightarrow \frac{1}{P_{m,t}} P_{z,t}Z_{m,t}(i) \left(\frac{\alpha_m + \mu_t\gamma_m + \left(1-\alpha_m-\gamma_m\right)\mu_t}{\left(1-\alpha_m-\gamma_m\right)\mu_t}\right)$$

$$\Longleftrightarrow \frac{1}{P_{m,t}} P_{z,t}Z_{m,t}(i) \left(\frac{\alpha_m + \mu_t - \mu_t\alpha_m}{\left(1-\alpha_m-\gamma_m\right)\mu_t}\right). \tag{2.38}$$

A real marginal cost MC_t is calculated by setting labour supply, capital and resource inputs to the level required to produce one unit of non-resource traded goods:

$$a_{m,t} \left(\mu_t H_{m,t}(i)\right)^{\alpha_m} K_{m,t}(i)^{\gamma_m} Z_{m,t}(i)^{1-\alpha_m-\gamma_m} = 1 \tag{2.39}$$

$$\Longleftrightarrow a_{m,t} \left(\frac{\alpha_m P_{z,t}Z_{m,t}(i)}{\left(1-\alpha_m-\gamma_m\right)W_{m,t}}\right)^{\alpha_m} \left(\frac{\gamma_n P_{z,t}Z_{m,t}(i)}{\left(1-\alpha_m-\gamma_m\right)R_{m,t}}\right)^{\gamma_m} Z_{m,t}(i)^{1-\alpha_m-\gamma_m} = 1$$

$$\Longleftrightarrow Z_{m,t}(i) = \frac{1}{a_{m,t}} \left(\frac{\left(1-\alpha_m-\gamma_m\right)W_{m,t}}{\alpha_m P_{z,t}}\right)^{\alpha_m} \left(\frac{\left(1-\alpha_m-\gamma_m\right)R_{m,t}}{\gamma_m P_{z,t}}\right)^{\gamma_m}. \tag{2.40}$$

Plug this into the firm real cost, the nominal marginal cost $mc_{m,t}$ is

$$\begin{aligned} MC^{j}_{m,t} &= P_{z,t} Z_{m,t}(i) \left(\frac{\alpha_m + \mu_t - \mu_t \alpha_m}{(1-\alpha_m-\gamma_m)\,\mu_t} \right) \\ &= \frac{1}{a_{m,t}} \left(\frac{W_{m,t}}{\alpha_m} \right)^{\alpha_m} \left(\frac{R_{m,t}}{\gamma_m} \right)^{\gamma_m} \left(\frac{P_{Z,t}}{1-\alpha_m-\gamma_m} \right)^{1-\alpha_m-\gamma_m} \\ &= \frac{\epsilon^{j}_{\pi_m,t}}{a_{m,t}} \left(\frac{W_{m,t}}{\alpha_m} \right)^{\alpha_m} \left(\frac{R_{m,t}}{\gamma_m} \right)^{\gamma_m} \left(\frac{P_{Z,t}}{1-\alpha_m-\gamma_m} \right)^{1-\alpha_m-\gamma_m} . \end{aligned} \tag{2.41}$$

Assuming a positive markup shock that changes the nominal marginal cost of non-resource traded production in each market $j \in \{d, x\}$ for reasons unrelated to changing wage, capital return rate, and resource price, denote this shock as $\epsilon^{j}_{\pi_m,t}$. Notice that the nominal marginal cost in each market j is not dependent on i, implying that all non-resource traded firms face the same technology shocks and rental input prices.

Second, non-resource traded firms solve their profit maximisation problem as:

$$\begin{aligned} \max_{P_{m,t}(i)} E_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{P_{m,t}(i) Y^d_{m,t}(i)}{P_t} + \frac{S_t P^*_{m,t}(i) Y^x_{m,t}(i)}{P_t} \\ & - \frac{MC^d_{m,t} Y^d_{m,t}(i)}{P_t} - \frac{MC^x_{m,t} Y^x_{m,t}(i)}{P_t} \\ & - \frac{\tau_{\pi^m}}{2} \left[\frac{P_{m,t}(i)}{\Pi^{\chi}_{m,t-1} \Pi^{1-\chi} P_{m,t-1}(i)} - 1 \right]^2 \frac{P_{m,t} Y^d_{m,t}}{P_t} \\ & - \frac{\tau_{\pi^{m*}}}{2} \left[\frac{P^*_{m,t}(i)}{\Pi^{*\chi}_{m,t-1} \Pi^{1-\chi} P^*_{m,t-1}(i)} - 1 \right]^2 \frac{S_t P^*_{m,t} Y^x_{m,t}}{P_t} \Bigg\}, \end{aligned} \tag{2.42}$$

subject to the demand functions:

$$Y^d_{m,t}(i) = \left(P_{m,t}(i)/P_{m,t} \right)^{-\theta^m} Y^d_{m,t}, \tag{2.43}$$

$$Y^x_{m,t}(i) = \left(P^*_{m,t}(i)/P^*_{m,t} \right)^{-\theta^m} Y^x_{m,t}. \tag{2.44}$$

The FOCs is:

$$(1-\theta^m)\left(\frac{P_{m,t}(i)}{P_{m,t}}\right)^{-\theta_m}\frac{Y^d_{m,t}}{P_t}+(1-\theta^m)\left(\frac{P^*_{m,t}(i)}{P^*_{m,t}}\right)^{-\theta_m}\frac{S_tY^x_{m,t}}{P_t} \tag{2.45}$$

$$+\theta_m MC^d_{m,t}\left(\frac{P_{m,t}(i)}{P_{m,t}}\right)^{-\theta_m}\left(\frac{P_{m,t}(i)}{P_{m,t}}\right)^{-1}\frac{Y^d_{m,t}}{P_t} \tag{2.46}$$

$$+\theta_m MC^x_{m,t}\left(\frac{P^*_{m,t}(i)}{P^*_{m,t}}\right)^{-\theta_m}\left(\frac{P^*_{m,t}(i)}{P^*_{m,t}}\right)^{-1}\frac{S_tY^x_{m,t}}{P_t}$$

$$-\tau_{\pi^m}\frac{Y^d_{m,t}}{P_t}\left[\frac{P_{m,t}(i)}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}P_{m,t-1}(i)}-1\right]\frac{P_{m,t}}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}P_{m,t-1}(i)}$$

$$-\tau_{\pi^{m*}}\frac{S_tY^x_{m,t}}{P_t}\left[\frac{P^*_{m,t}(i)}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}P^*_{m,t-1}(i)}-1\right]\frac{P^*_{m,t}}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}P^*_{m,t-1}(i)}$$

$$+\beta\tau_{\pi^m}E_t\left\{\left[\frac{P_{m,t+1}(i)}{\Pi^{\chi}_{m,t}\Pi^{1-\chi}P_{m,t}(i)}-1\right]\frac{P_{m,t+1}(i)}{\Pi^{\chi}_{m,t}\Pi^{1-\chi}P_{m,t}(i)}\frac{P_{m,t+1}Y^d_{m,t+1}}{P_{t+1}P_{m,t}(i)}\right\}$$

$$+\beta\tau_{\pi^{m*}}E_t\left\{\left[\frac{P^*_{m,t+1}(i)}{\Pi^{*\chi}_{m,t}\Pi^{1-\chi}P^*_{m,t}(i)}-1\right]\frac{P^*_{m,t+1}(i)}{\Pi^{*\chi}_{m,t}\Pi^{1-\chi}P^*_{m,t}(i)}\frac{P^*_{m,t+1}Y^x_{m,t+1}S_t}{P_{t+1}P^*_{m,t}(i)}\right\}=0.$$

Since I only consider symmetric equilibrium where all firms choose to set the same price in each market j, with $P^j_{m,t}(i)=P^j_{m,t}$, equation (2.46) becomes:

$$(1-\theta^m)\left(Y^d_{m,t}+S_tY^x_{m,t}\right)+\theta^m\left(MC^d_{m,t}Y^d_{m,t}+MC^x_{m,t}S_tY^x_{m,t}\right) \tag{2.47}$$

$$-\tau_{\pi^m}Y^d_{m,t}\left(\frac{\Pi_{m,t}}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}}-1\right)\frac{\Pi_{m,t}}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}}-\tau_{\pi^{m*}}S_tY^x_{m,t}\left(\frac{\Pi^*_{m,t}}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}}-1\right)\frac{\Pi^*_{m,t}}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}}$$

$$+\beta\tau_{\pi^m}E_t\left\{Y^d_{m,t+1}\frac{P_t}{P_{t+1}}\left(\frac{\Pi_{m,t}}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}}-1\right)\frac{\Pi^2_{m,t+1}}{\Pi^{\chi}_{m,t-1}\Pi^{1-\chi}}\right\}$$

$$+\beta\tau_{\pi^{m*}}E_t\left\{Y^x_{m,t+1}\frac{P_t}{P_{t+1}}\left(\frac{\Pi^*_{m,t}}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}}-1\right)\frac{\Pi^{*2}_{m,t+1}}{\Pi^{*\chi}_{m,t-1}\Pi^{1-\chi}}\right\}=0.$$

B.1.4. Resource sector

Firms in the resource sector solve their profit maximisation problem to choose labour and capital inputs given prices:

$$\max_{H_{z,t},K_{z,t}} P_{z,t}Y_{z,t}-W_{z,t}H_{z,t}-R_{z,t}K_{z,t}-R_{L,t}L, \tag{2.48}$$

subject to

$$Y_{z,t}{=}a_{z,t}\left(\mu_tH_{z,t}\right)^{\alpha_z}K^{\gamma_z}_{z,t}\left(\mu_tL\right)^{1-\alpha_z-\gamma_z}. \tag{2.49}$$

The FOCs are:

$$W_{z,t} = a_{z,t}\alpha_z P_{z,t}\mu_t \left(\mu_t H_{z,t}\right)^{\alpha_z - 1} K_{z,t}^{\gamma_z} \left(\mu_t L\right)^{1-\alpha_z-\gamma_z}, \tag{2.50}$$

$$R_{z,t} = a_{z,t}\gamma_z P_{z,t} \left(\mu_t H_{z,t}\right)^{\alpha_z} K_{z,t}^{\gamma_z - 1} \left(\mu_t L\right)^{1-\alpha_z-\gamma_z}, \tag{2.51}$$

or:

$$\frac{W_{z,t}}{R_{z,t}} = \frac{\alpha_z K_{z,t}}{\gamma_z H_{z,t}}. \tag{2.52}$$

B.1.5. Import sector

The marginal cost MC_t is calculated by dividing the importing price to selling price with a markup shock $\epsilon_{\pi_{f,t}}$:

$$MC_{f,t}(i) = \epsilon_{\pi_{f,t}} \frac{S_t P_{f,t}^*}{P_{f,t}}. \tag{2.53}$$

The import firms solve their maximisation problem to choose their selling price to optimise the profit:

$$\max_{P_{f,t}(i)} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{P_{f,t}(i) Y_{f,t}(i)}{P_t} - \frac{MC_{f,t} Y_{f,t}(i)}{P_t} - \frac{\tau_{\pi^f}}{2} \left[\frac{P_{f,t}(i)}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi} P_{f,t-1}(i)} - 1 \right]^2 \frac{P_{f,t} Y_{f,t}}{P_t} \right\}, \tag{2.54}$$

subject to the demand function:

$$Y_{f,t}(i) = \left(P_{f,t}(i)/P_{f,t}\right)^{-\theta^f} Y_{f,t}. \tag{2.55}$$

The FOCs is:

$$\begin{aligned} &(1-\theta^f)\left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\theta_f} \frac{Y_{f,t}}{P_t} + \theta_f MC_{f,t} \left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\theta_f} \left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-1} \frac{Y_{f,t}}{P_t} \\ &-\tau_{\pi^f} \frac{Y_{f,t}}{P_t} \left[\frac{P_{f,t}(i)}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi} P_{f,t-1}(i)} - 1 \right] \frac{P_{f,t}}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi} P_{f,t-1}(i)} \\ &+\beta \tau_{\pi^f} E_t \left\{ \left[\frac{P_{f,t+1}(i)}{\Pi_{f,t}^{\chi} \Pi^{1-\chi} P_{f,t}(i)} - 1 \right] \frac{P_{f,t+1}(i)}{\Pi_{f,t}^{\chi} \Pi^{1-\chi} P_{f,t}(i)} \frac{P_{f,t+1} Y_{f,t+1}}{P_{t+1} P_{f,t}(i)} \right\} = 0. \end{aligned} \tag{2.56}$$

Since I only consider symmetric equilibrium, then all firms choose to set the same price. Together with $P_{f,t}(i) = P_{f,t}$, and the inflation definition with a friction $\Pi_{f,t} = \frac{P_{f,t}}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi} P_{f,t-1}}$, equation (2.56) becomes:

$$\begin{aligned} &(1-\theta^f) + \theta^f MC_{f,t} - \tau_{\pi^f} \left(\frac{\Pi_{f,t}}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi}} - 1 \right) \frac{\Pi_{f,t}}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi}} \\ &+\beta \tau_{\pi^f} E_t \left\{ \frac{Y_{f,t+1}}{Y_{f,t}} \frac{P_t}{P_{t+1}} \left(\frac{\Pi_{f,t}}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi}} - 1 \right) \frac{\Pi_{f,t+1}^2}{\Pi_{f,t-1}^{\chi} \Pi^{1-\chi}} \right\} = 0. \end{aligned} \tag{2.57}$$

B.2. The log-linearised model

Since the common labour-augmenting technology process μ_t in each intermediate goods sector has a unit root to guarantee variables evolving along the stochastic growth path. Before solving the system, the variables first need to normalise by dividing to the trend level of technology $x_t = X_t/\mu_t$. The steady-state values are without a time subscript, while the hat symbol indicates the log deviations from steady-state values $\hat{x}_t = lnx_t - lnx$.

Normalised variables are:

$c_t = C_t/\mu_t; k_t = K_t/\mu_t; k_{n,t} = K_{n,t}/\mu_t; k_{m,t} = K_{m,t}/\mu_t; k_{z,t} = K_{z,t}/\mu_t;$

$i_t = I_t/\mu_t; i_{n,t} = I_{n,t}/\mu_t; i_{m,t} = I_{m,t}/\mu_t; i_{z,t} = I_{z,t}/\mu_t;$

$h_t = H_t/\mu_t; h_{n,t} = H_{n,t}/\mu_t; h_{m,t} = H_{m,t}/\mu_t; h_{z,t} = H_{z,t}/\mu_t;$

$q_t = Q_t/\mu_t; q_{n,t} = Q_{n,t}/\mu_t; q_{m,t} = Q_{m,t}/\mu_t; q_{z,t} = Q_{z,t}/\mu_t;$

$y_{n,t} = Y_{n,t}/\mu_t; y^d_{m,t} = Y^d_{m,t}/\mu_t; y^x_{m,t} = Y^x_{m,t}/\mu_t; y_{z,t} = Y_{z,t}/\mu_t.$

The log-linearised model is given by the system of equations as follows.

Household sector

1. FOC with respect to consumption:

$$(\mu - h)(\mu - \beta h)\hat{\Lambda}_t = (\mu - h)(\mu\xi_{c,t} - h\beta E_t\xi_{c,t+1}) - (\mu^2 + \beta h^2)\hat{c}_t + \mu h(\hat{c}_{t-1} + \beta E_t\hat{c}_{t+1} - \hat{\mu}_t + \beta E_t\hat{\mu}_{t+1}. \quad (2.1)$$

2. FOC with respect to labour:

$$\hat{w}_{j,t} = (\eta - \sigma)\hat{h}_t + \sigma\hat{h}_{j,t} - \hat{\Lambda}_t, \quad (2.2)$$

   with $j \in \{n, m, z\}$

3. FOC with respect to investment:

$$\hat{\lambda}^k_{j,t} = \Phi_K\mu^2\left[(1+\beta)\hat{i}_{j,t} - \hat{i}_{j,t-1} - \beta E_t\hat{i}_{j,t+1} + \hat{\mu}_t - \beta E_t\hat{\mu}_{t+1}\right] + \hat{\Lambda}_t - \hat{\Upsilon}_t, \quad (2.3)$$

   with $j \in \{n, m, z\}$

4. FOC with respect to capital induce the shadow price of installed capital:

$$\hat{\lambda}^k_{j,t} = \left[\frac{\mu - \beta(1-\sigma)}{\mu}\right] E_t\left[\hat{\Lambda}_{t+1} + \hat{r}_{j,t+1}\right] + \frac{\beta(1-\delta)}{\mu}E_t\hat{\lambda}^k_{j,t+1} - E_t\hat{\mu}_{t+1}, \quad (2.4)$$

   with $j \in \{n, m, z\}$

5. Law of motion for capital accumulation in each sector:

$$\hat{k}_{j,t+1} = \frac{1-\delta}{\mu}(\hat{k}_{j,t} - \hat{\mu}_t) + \frac{\mu - 1 + \delta}{\mu}(\hat{i}_{j,t} + \hat{\Upsilon}_t), \tag{2.5}$$

with $j \in \{n, m, z\}$

6. FOC domestic bonds:

$$\hat{r}_t = E_t(\hat{\Lambda}_{t+1} - \hat{\pi}_{t+1}) - \hat{\Lambda}_t. \tag{2.6}$$

7. Aggregate labour supply:

$$\hat{h}_t = \left[\frac{H_n}{H}\right]^{1+\sigma} \hat{h}_{n,t} + \left[\frac{H_m}{H}\right]^{1+\sigma} \hat{h}_{m,t} + \left[\frac{H_z}{H}\right]^{1+\sigma} \hat{h}_{z,t}. \tag{2.7}$$

8. Aggregate investment:

$$\hat{i}_t = \left[\frac{I_n}{I}\right] \hat{i}_{n,t} + \left[\frac{I_m}{I}\right] \hat{i}_{m,t} + \left[\frac{I_z}{I}\right] \hat{i}_{z,t}. \tag{2.8}$$

Production sectors

1. Phillips curves for non-traded, non-resource traded domestic and import sectors:

$$\hat{\pi}_{j,t} = \frac{1}{1+\chi\beta}\frac{\kappa_j}{100}\hat{mc}_{j,t} + \beta\frac{1}{1+\chi\beta}E_t\hat{\pi}_{t+1} + \frac{\chi}{1+\chi\beta}\hat{\pi}_{j,t-1} + \epsilon_{\pi_j,t}, \tag{2.9}$$

with $j \in \{n, m, f\}$; $\kappa_j = 100(\theta^j - 1)/\tau_{\pi_j}$.

2. Phillips curves for non-resource traded sector export overseas:

$$\hat{\pi}^*_{m,t} = \frac{1}{1+\chi\beta}\frac{\kappa_{m^*}}{100}(\hat{mc}_{m^*,t} + p_{m,t} - p_{m^*,t}) + \beta\frac{1}{1+\chi\beta}E_t\hat{\pi}_{t+1} + \frac{\chi}{1+\chi\beta}\hat{\pi}_{j,t-1} + \epsilon_{\pi_j,t}, \tag{2.10}$$

with $\kappa_{m^*} = 100(\theta^{m^*} - 1)/\tau_{\pi_{m^*}}$; $p_{m,t}, p_{m^*,t}$ are relative prices of non-resource traded good in domestic and foreign markets.

3. Marginal costs for non-traded and non-resource traded sectors:

$$\hat{mc}_{j,t} = \alpha_j\hat{w}_{j,t} + \gamma_j\hat{r}_{j,t} + (1-\alpha_j-\gamma_j)p_{z,t} - \hat{p}_{j,t} - \hat{a}_{j,t} \tag{2.11}$$

with $j \in \{n, m\}$;$p_{j,t} = ln[P_{j,t}/P_t]$ is relative price of good j with final good price.

4. Marginal costs for importing sector:

$$\hat{mc}_{f,t} = \hat{q}_t - \hat{p}_{f,t}. \tag{2.12}$$

Optimal choice for capital and labour in the demand side:

$$\hat{h}_{j,t} = \hat{k}_{j,t} + \hat{r}_{j,t} - \hat{w}_{j,t} - \hat{\mu}_t, \tag{2.13}$$

with $j \in \{n, m\}$.

5. Optimal choice for resource input for domestic intermediate production sectors:

$$\hat{z}_{j,t} = \hat{k}_{j,t} + \hat{r}_{j,t} - \hat{p}_{z,t} - \hat{\mu}_t, \tag{2.14}$$

with $j \in \{n, m\}$.

6. Optimal choice for labour input in resource sector:

$$\hat{h}_{z,t} = \hat{p}_{z,t} + \hat{y}_{z,t} - \hat{w}_{z,t}. \tag{2.15}$$

7. Optimal choice for capital input in resource sector:

$$\hat{k}_{z,t} = \hat{p}_{z,t} + \hat{y}_{z,t} - \hat{r}_{z,t} + \hat{\mu}_t. \tag{2.16}$$

8. Law of motion for price of resource good in domestic market:

$$\hat{p}_{z,t} = \frac{1}{2}(\hat{q}_t + \hat{p}^*_{z,t}) + \frac{1}{2}\hat{p}_{z,t-1}. \tag{2.17}$$

9. Foreign demand for non-resource traded goods:

$$\hat{y}^x_m = \zeta^*(\hat{p}^*_{m,t} - \hat{q}_t) + \hat{y}^*_t. \tag{2.18}$$

10. Domestic demand for domestic sectors:

$$\hat{y}^j_t = -\zeta\hat{p}_{j,t} + d\hat{f}d_t, \tag{2.19}$$

with $j \in \{n, m, f\}$.

Market-clearing conditions

1. In non-resource traded sector:

$$y_m\hat{y}_{m,t} = y^d_m\hat{y}^d_{m,t} + y^x_m\hat{y}^x_{m,t}. \tag{2.20}$$

2. In resource sector:

$$y_z\hat{y}_{z,t} = z_x\hat{z}_{x,t} + y^m_z\hat{y}^m_{z,t} + y^n_z\hat{y}^n_{z,t}. \tag{2.21}$$

3. In domestic final goods:

$$\hat{dfd}_t = \frac{c}{dfd}\hat{c}_t + \frac{g}{dfd}\hat{g}_t + \frac{i}{dfd}\hat{i}_t. \tag{2.22}$$

4. Nominal value added:

$$\hat{y}_t^{nva} = \frac{p_n y_n^{nva}}{y_n^{nva}}(\hat{p}_{n,t} + \hat{y}_{n,t}^{nva}) + \frac{p_m y_m^{nva}}{y_m^{nva}}(\hat{p}_{m,t} + \hat{y}_{m,t}^{nva}) + \frac{p_z y_z}{y_z}(\hat{p}_{z,t} + \hat{y}_{z,t}). \tag{2.23}$$

5. Real value added:

$$\hat{y}_t^{va} = \frac{p_n y_n^{nva}}{y^{nva}}\hat{y}_{n,t}^{va} + \frac{p_m y_m^{nva}}{y^{nva}}\hat{y}_{m,t}^{va} + \frac{p_z y_z}{y^{nva}}\hat{y}_{z,t}. \tag{2.24}$$

6. Balance of payment:

$$\begin{aligned} \frac{b_{t+1}^*}{\Psi r^*} &= \frac{b_t^*}{\pi^* \mu} + \frac{p_z z_x}{y^{nva}}(\hat{p}_{z,t} + \hat{z}_{x,t} - \hat{y}_t^{nva}) \\ &\quad + \frac{p_m y_m^x}{y^{nva}}(\hat{p}_{m,t}^* + \hat{y}_{m,t}^x - \hat{y}_t^{nva}) - \frac{q y_f}{y^{nva}}(\hat{q}_t + \hat{y}_{f,t} - \hat{y}_t^{nva}), \end{aligned} \tag{2.25}$$

   with $b_t^* = [(S_t B_t^*/P_t Y_t) - b^*]$ called net foreign asset to GDP ratio to ensure both positive and negative debt.

Government policies

1. Monetary policy:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}^{va}\right) + \phi_{\Delta_y}(\hat{y}_t^{va} - \hat{y}_{t-1}^{va}) + \phi_q(\hat{q}_t - \hat{q}_{t-1}) + \sigma_{r,t}\epsilon_{r,t}, \tag{2.26}$$

   with $\epsilon_{r,t} \sim N(0,1)$.

2. Fiscal policy:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \sigma_{g,t}\epsilon_{g,t}, \tag{2.27}$$

   with $\epsilon_{g,t} \sim N(0,1)$.

Other useful identities

1. Uncovered interest rate parity:

$$E_t(\hat{q}_{t+1}) - \hat{q}_t = E_t(\hat{\pi}_{t+1}^* - \hat{\pi}_{t+1}) + \hat{r}_t - \hat{r}_t^* - \hat{\upsilon}_t. \tag{2.28}$$

2. Risk-premium:

$$\hat{\upsilon}_t = -\chi_{rp}(b_t^*) - \hat{\Psi}_t. \tag{2.29}$$

3. Real exchange rate:
$$\hat{q}_t - \hat{q}_{t-1} + \hat{\pi}_t - \hat{\pi}_t^* = \Delta s_t. \tag{2.30}$$

4. Domestic CPI:
$$0 = \omega_j p_j^{1-\zeta} \hat{p}_{j,t}, \tag{2.31}$$

   with $j \in \{n, m, f\}$.

5. Relative price of good $j \in \{n, m, f\}$ evolution:

$$\hat{p}_{j,t} = \hat{p}_{j,t-1} + \hat{\pi}_{j,t} - \hat{\pi}_t. \tag{2.32}$$

Foreign Economy

1. Demand shock:

$$\hat{y}_t^* = E_t\{\hat{y}_{t+1}^*\} - \left(\hat{r}_t^* - E_t\{\hat{\pi}_{t+1}^*\}\right) - E_t\{\hat{\xi}_{y^*,t+1}\} + \hat{\xi}_{y^*,t}. \tag{2.33}$$

2. Phillips curve:
$$\hat{\pi}_t^* = \beta E_t\{\hat{\pi}_{t+1}^*\} + \frac{\kappa^*}{100}\hat{y}_t^* + e_{\pi^*,t}. \tag{2.34}$$

3. Monetary regulation:

$$\hat{r}_t^* = \rho_{r^*}\hat{r}_{t-1}^* + (1-\rho_{r^*})(\phi_{\pi^*}\hat{\pi}_t^* + \phi_{y^*}\hat{y}_t^*) + \phi_{\Delta_{y^*}}(\hat{y}_t^* - \hat{y}_{t-1}^*) + e_{r^*,t}. \tag{2.35}$$

4. Real resource price in foreign currency:

$$\hat{p}_{z,t}^* = \rho_{p_{z^*}}\hat{p}_{z,t-1}^* + \phi_{zy,t}\hat{\xi}_{y^*,t} + e_{p_z^*,t}. \tag{2.36}$$

B.3. Data sources and definitions

GDP: quarterly percentage change of Australian GDP in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.01 'Australian National Accounts: National Income, Expenditure and Product'.

Consumption: quarterly percentage change of household final consumption expenditure in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.02 'Australian National Accounts: National Income, Expenditure and Product'.

Investment: quarterly percentage change of private gross fixed capital formation in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.02 'Australian National Accounts: National Income, Expenditure and Product'.

Export, import: quarterly exports and imports of goods and services in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.02 'Australian National Accounts: National Income, Expenditure and Product'.

Non-traded sector includes electricity, gas, water and waste industry, construction industry, retail trade industry, information, media, and telecommunications industry, finance and insurance industry, real estate industry, professional services industry, administrative services industry, public administration industry, education industry, healthcare industry, arts and recreation industry, other services industry and ownership of dwellings.

Non-resource tradable sector includes agriculture, forestry and fishing industry, manufacturing industry, transport industry, wholesale trade industry, and accommodation and food services industry.

Resource sector includes mining sector.

Industry value added: quarterly percentage change of sectoral value added in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.06 'Australian National Accounts: National Income, Expenditure and Product'.

Public demand: quarterly percentage change of public demand, which consists of government final consumption expenditure and public gross fixed capital formation in chain volume measures, seasonally adjusted. Source: ABS Cat No 5206.0 'Australian National Accounts: National Income, Expenditure and Product'.

Inflation: quarterly percentage change of inflation. Source: RBA statistical table G1 Consumer Price Inflation.

Interest rate: quarterly average interbank overnight cash rate, seasonally adjusted. Source: RBA statistical table F1.1 Interest rates and yields money market.

Exchange rate: quarterly percentage change of average nominal exchange rate. Source: RBA statistical table F11 Exchange rates.

Resource prices: quarterly percentage change of non-rural commodity price index measured in Special Drawing Rights. Source: RBA statistical table I2 Commodity Prices.

Figure B.3.1.: Australian data

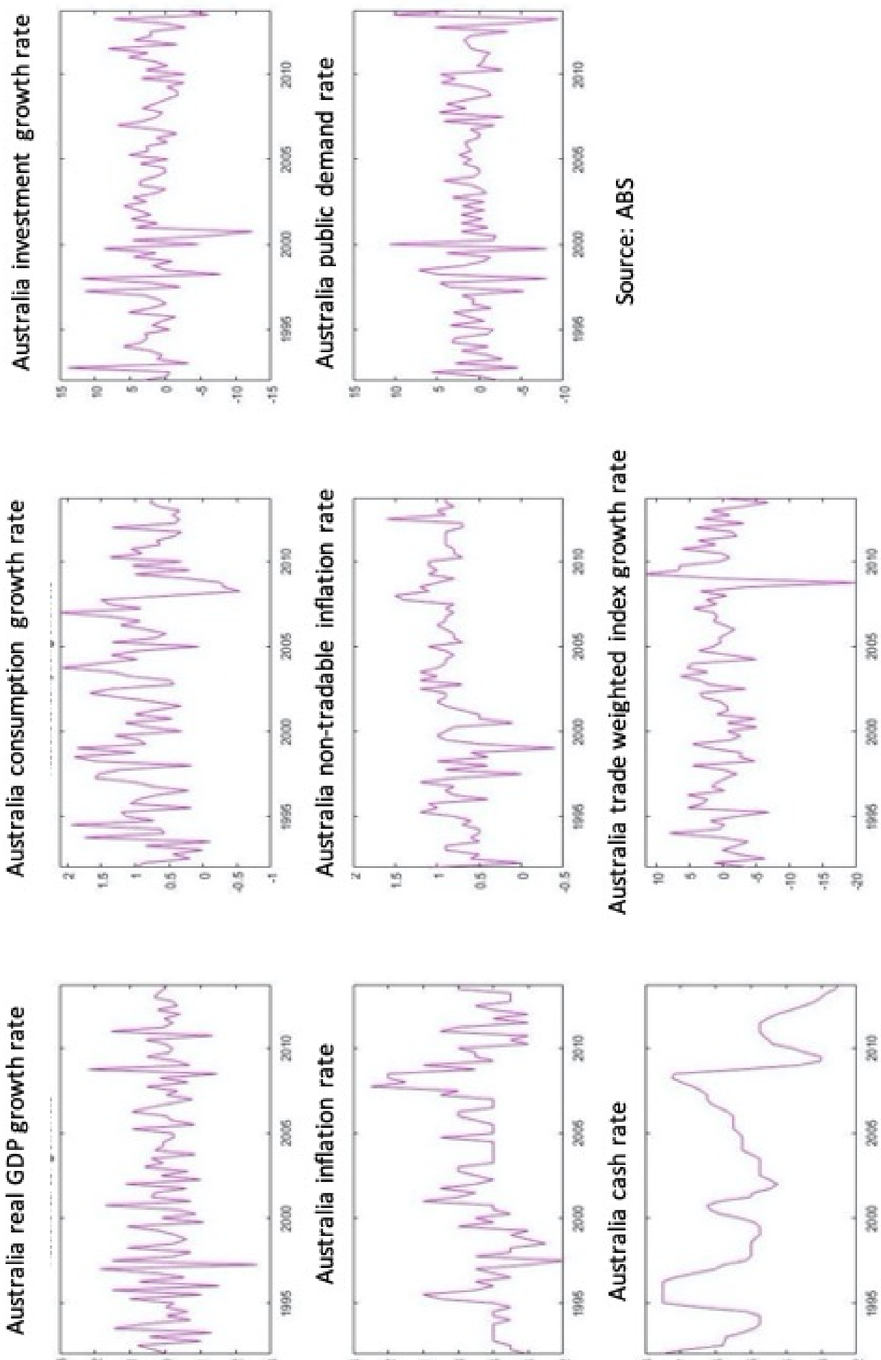
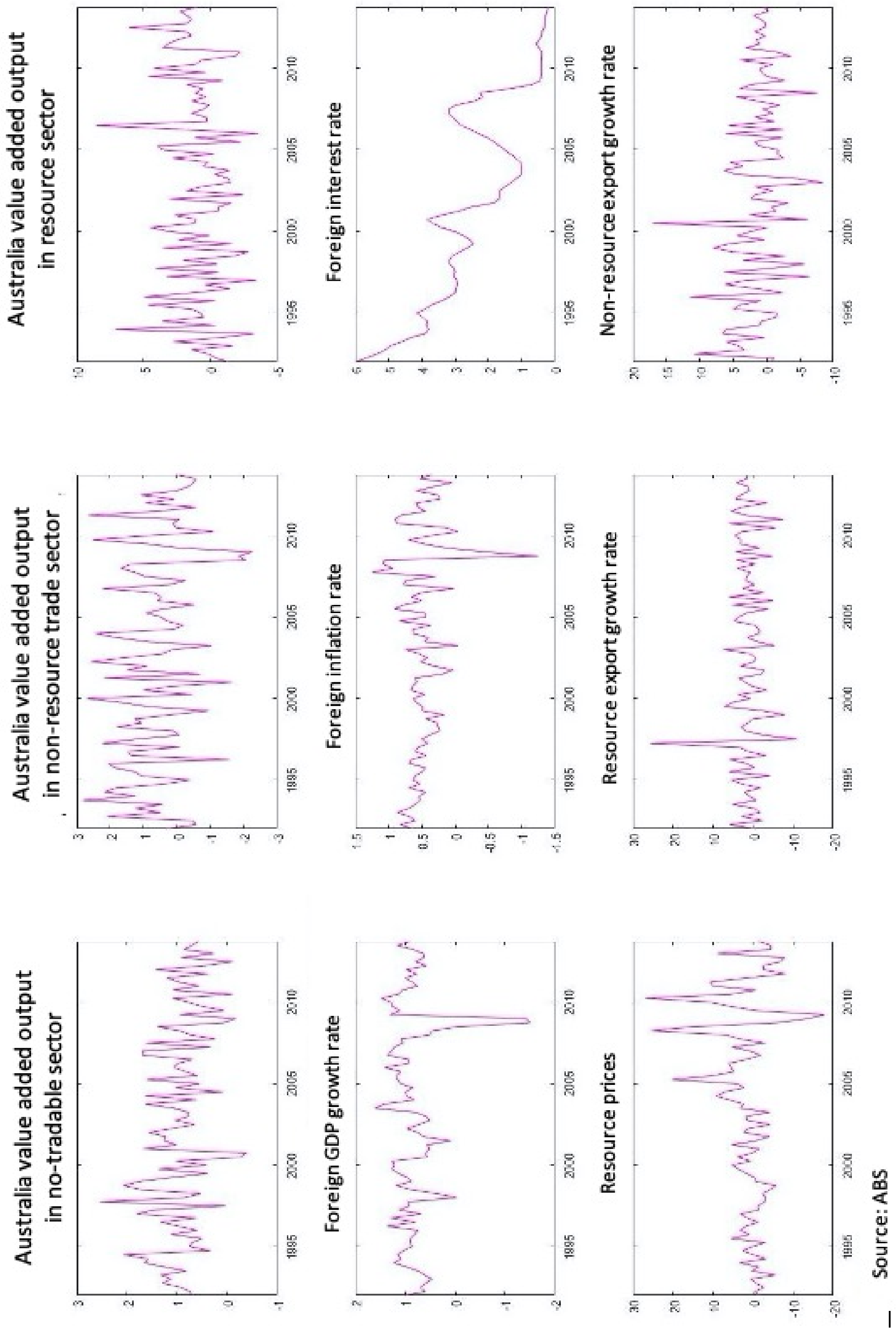


Figure B.3.2.: Australian data (continued)



B.4. Convergence diagnosis

Figure B.4.1.: Inefficiency factors for estimated parameters

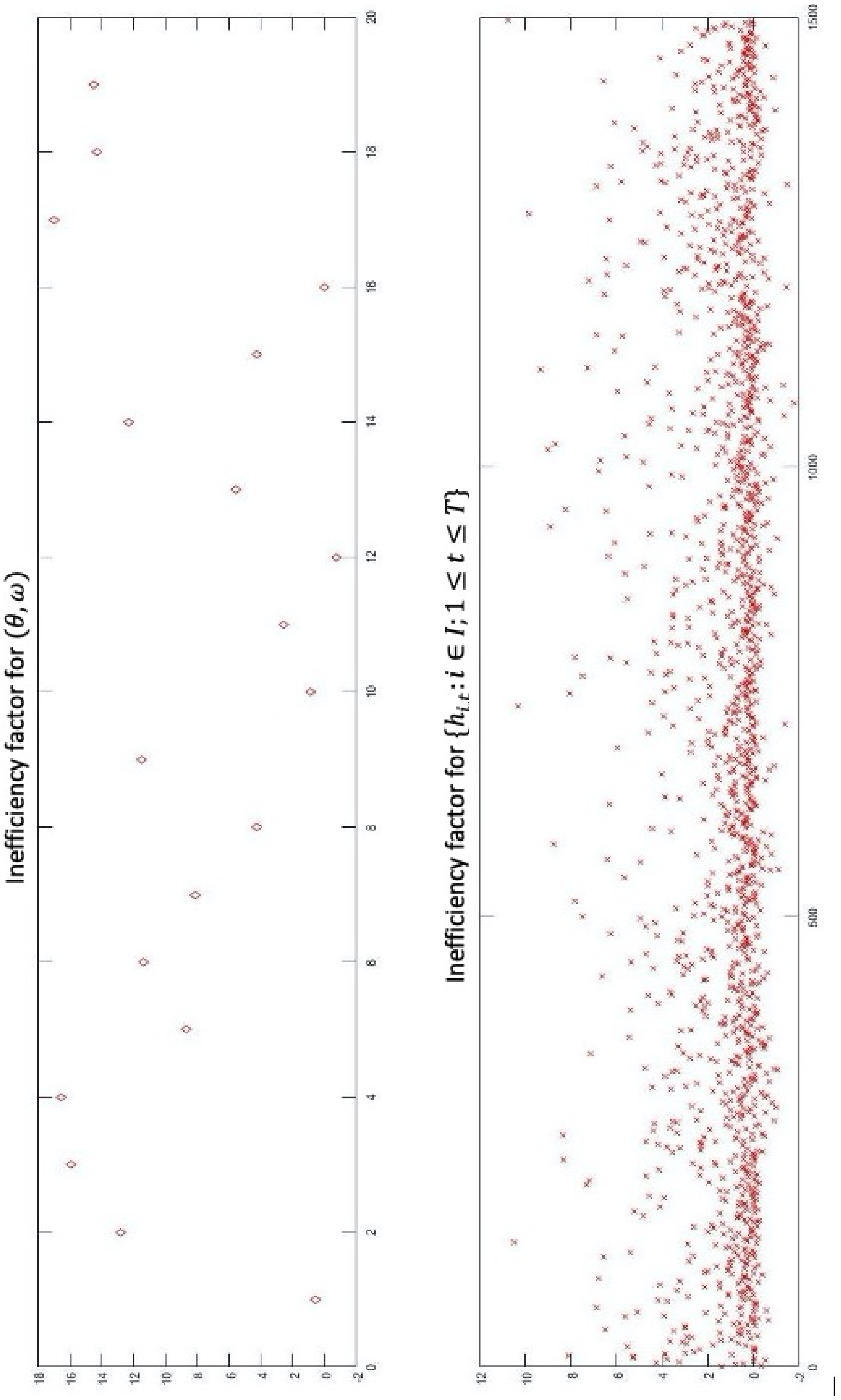


Figure B.4.2.: Plot of Markov Chain mean of posteriors for estimated parameters

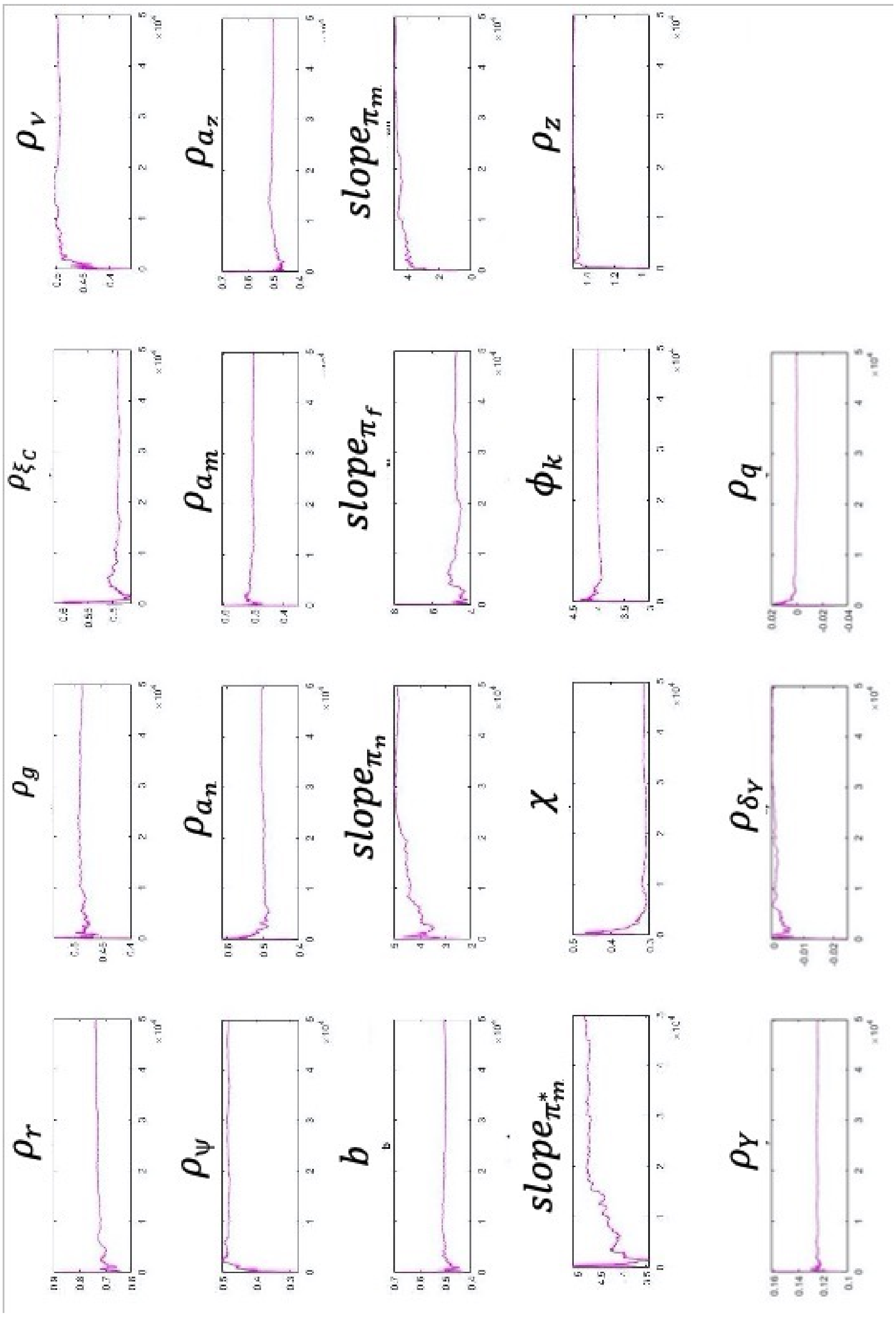
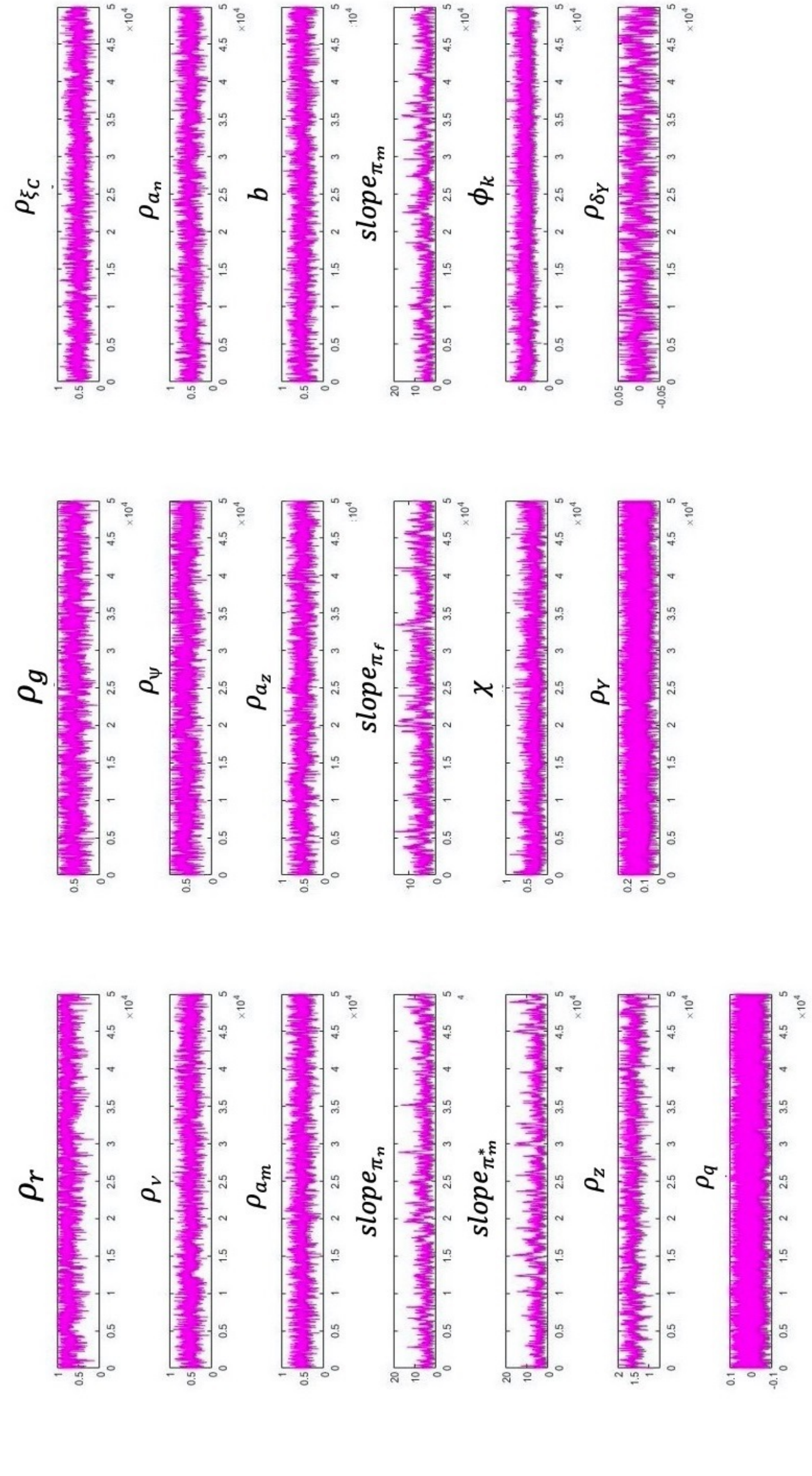


Figure B.4.3.: Trace plots for estimated parameters



B.5. Historical decomposition

Figure B.5.1.: Historical decomposition for each shock (combined)