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Pareto Improvements under Matching Mechanisms in a Public Good Economy

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Abstract

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Keywords

Public goods, Matching mechanisms, Pareto improvements, Aggregative games, Interiority

JEL Classification

C78, H41, H77, Q54

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Pareto Improvements under Matching Mechanisms in a Public Good Economy

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1. Introduction

It is well known that public goods are generally underprovided in the context of voluntary contribution. A large literature explores possible solutions to this issue among which matching mechanisms have gained considerable attention. The mechanisms, first suggested by Guttman (1978, 1987), work as a two-stage game. At the first stage, each agent announces a matching rate indicating by how much the agent would subsidize public good contributions of all other agents. For example, should one announce a matching rate of 0.1, the agent would provide 0.1 units of the public good as a matching contribution if another agent provides one unit of the public good. At the second stage, all agents decide independently how much of the public good they would provide. The idea is still to stick to the non-cooperative mode of public good provision but meanwhile to subsidize individual public good contributions and hence to lower the effective price of the public good. In that seminal work, Guttman shows that with quasi-linear preferences the sub-game perfect equilibrium in such a two-stage game of two identical players is fully efficient. Danziger and Schnytzer (1991) generalize the result to more general preferences and to any number of agents. This approach has been refined and applied in various ways[1].

The existing literature focuses on the Pareto optimal equilibrium under matching mechanisms. Nevertheless, reaching the Pareto optimal equilibrium is very ambitious in practice for several reasons. First, full knowledge about individual preferences is indispensable to attain efficient allocations but the information is difficult to obtain. Deaton (1987) and Myles (1995) compare informational requirements in policy reform and both show that optimal policy requires global knowledge of relevant parameters while Pareto-improving reform only needs limited information of the current position[2]. Moreover, agents are often more uncertain about responses of larger changes from the current position, and thus gradually Pareto-improving reform is more desirable. This information problem becomes much more severe at the international level because it is always difficult, if not impossible, to have information of the aggregate preference of one country.

Second, a very important but implicit assumption of matching mechanisms is the credibility of commitments that players are able to match contributions of other players at the second stage. These commitments should not be taken for granted in the absence of a central government (Boadway, Song and Tremblay, 2007, 2011). This paper will show that the Pareto optimal equilibrium requires larger matching rates and hence more ambitious commitments than Pareto-improving equilibria. The fact that no supranational authority can force sovereign countries to implement their commitments makes public good provision extremely hard at the international level. In the last two decades the negotiation experiences of climate protection which is a global public good sufficiently suggest that it is more realistic and feasible to focus on Pareto-improving rather than optimal equilibria in practice.

Third, even if the required information is complete and the commitments are credible, this paper will show that the sub-game perfect equilibrium of the matching game may not be Pareto-improving in some situations compared to the initial Nash equilibrium without matching, i.e., one player is worse

[1] See Boadway, Pestieau and Wildasin (1989), Althammer and Buchholz (1993), Varian (1994a, 1994b), Andreoni and Bergstrom (1996), Falkinger (1996), Kirchsteiger and Puppe (1997), Buchholz, Cornes and Rübbelke (2011, 2012). Falkinger, Hackl and Pruckner (1996), Boadway, Song and Tremblay (2007, 2011) apply matching mechanisms in global environmental protection.

[2] Deaton (1987) also argues that even Pareto-improving reform is an ambitious requirement relative to data and techniques actually available. It's less challenging nowadays with improved databases and statistical techniques in the last several decades.

off than without matching. Cornes and Sandler (2000) argue that policies that can increase public good supply and improve everyone's well-being have desirable normative properties and are more interesting than policies that just augment public good provision. In the presence of transboundary or even global public goods, voluntary participation of agents in a matching scheme requires that each agent should not be worse off with matching because there is no central government with enforcement power at the international level.

Therefore, this paper focuses on Pareto-improving moves from the initial equilibrium, which has been little examined under matching mechanisms. As it is difficult to reach the Pareto optimal equilibrium, players may negotiate less ambitious matching schemes given local information of their preferences. Then questions come up: (1) Under what conditions do there exist matching schemes to generate Pareto-improving outcomes? (2) What could those matching schemes be? (3) If a matching scheme generates Pareto-improving outcomes, do players have incentives to take free rides? (4) If one player is too poor to provide any public good contribution, is it possible that the poor player offers matching to induce the rich to provide a larger public good contribution resulting in Pareto-improving outcomes? Or is it likely that the rich offers matching to induce the poor to provide a positive public good contribution resulting in Pareto-improving outcomes?

In most situations such as climate protection, agents differ from one another in ways that play a significant role in outcomes. The agent heterogeneity in this paper comes from two aspects: incomes (or wealth) and preferences. In climate negotiations, for example, some countries are rich while some others are poor, which would lead to different decisions of how to allocate their incomes between private consumption and climate protection. Even countries at a similar income level may attach different weights of value to climate protection relative to private consumption. Although countries may vary in the unit cost of public good production and population sizes also matter, these are left for further research.

This paper first shows that providing matching contributions is better off while accepting matching contributions is worse off at interior equilibria. In this sense, this matching mechanism avoids free riding and each player has incentives to provide matching contributions. Furthermore, the paper identifies the interiority zone within which any income distribution would reach an interior equilibrium, and then proves that given any income distribution within the interiority zone there always exist small matching schemes to generate Pareto-improving outcomes. This indicates that, if income distribution is within the interiority zone, two players can always implement some small matching schemes to make them both better off. This finding is useful for cooperation, particularly in the context without complete information of global preferences or at the international level without a central government which can force sovereign countries to implement their commitments. On the contrary, at corner equilibria providing matching contributions is worse off while accepting matching contributions is better off. Thus, if income distribution is beyond the interiority zone, there are no Pareto-improving matching equilibria.

The paper illustrates that although the sub-game perfect equilibrium achieves the Pareto optimal equilibrium, it may not generate Pareto-improving equilibrium, which is overlooked in the literature. The reason one player may be worse off in a game of voluntary participation is that the underlying assumption of the matching game is that players share common knowledge that players agree to participate in the game and players match others' contributions and accept others' matching.

The remainder of this paper is organized as follows. Section 2 provides the model and introduces the aggregative game approach to characterize equilibria. Section 3 investigates existence of Pareto-improving matching equilibria at interior solutions followed by the case of corner solutions. Section 4 extends the analysis to general preferences and Section 5 discusses participation in the game. Section 6 concludes and discusses possible extensions.

2. The framework

2.1 The model

Consider the standard pure public good economy (See Bergstrom, Blume and Varian, 1986; Cornes and Sandler, 1996). There is one private good, one pure public good, and two players with the Cobb-Douglas utility function $u_i(x_i, G) = x_i^{\alpha_i} G, i = 1, 2$, where x_i is the private good consumption of player i and G is the total public good provision. Assume that two players have complete information of their preferences[3].

Player i has an initial income of w_i units of the private good whose price is normalized to one. The total income is $W = w_1 + w_2$, and their income ratio is $k = w_1 / w_2$. The public good is produced from the private good. Player i's contribution to the public good, under a given matching scheme, consists of a direct flat contribution y_i, and of an indirect matching contribution that player i makes by matching the flat contribution of the other player. The flat contribution is non-negative, i.e., $y_i \geq 0$. The matching rate that player 1 offers to match player 2's flat contribution is μ_1 and that of player 2 is $\mu_2 (\mu_1 \geq 0, \mu_2 \geq 0)$, which composes a matching scheme $m(\mu_1, \mu_2)$. The total public good provision is $G = (1 + \mu_2) y_1 + (1 + \mu_1) y_2$.

Assume that two players have the same linear production technology of the public good. By choosing units the public good price (or unit cost) is normalized to one. Therefore, the budget constraints are respectively $x_1 + y_1 + \mu_1 y_2 = w_1$ and $x_2 + y_2 + \mu_2 y_1 = w_2$. The private marginal rate of transformation between the private good and the public good is $\pi_i = 1 + \mu_j (j = 1, 2, j \neq i)$. The effective public good price that player i has to pay for an additional unit of the public good is $p_i = 1/\pi_i$.

There are two states of the economy: one is the initial Nash equilibrium without matching (hereafter 'the initial equilibrium') and the other is the equilibrium under matching (hereafter 'the matching equilibrium'). The initial equilibrium is a special case of the matching equilibrium when $\mu_1 = \mu_2 = 0$. Given any matching scheme, preferences and incomes, a matching equilibrium is defined as follows.

[3] Players may not have complete information of global preferences but they may have complete information of local preferences by observing the current position. As this paper considers less ambitious matching schemes, the matching equilibrium is not far away from the initial equilibrium and therefore it is reasonable to assume that players have complete information of preferences in the neighbourhood of the current position.

Definition 1 A pairwise (y_1^*, y_2^*) is a matching equilibrium in flat contributions if, for player $i = 1, 2$, the flat contribution y_i^* maximizes $u_i(x_i, G) = u_i(w_i - (y_i + \mu_i y_j^*), (1+\mu_j)y_i + (1+\mu_i)y_j^*)$ where $j = 1, 2,\ j \neq i$.

The following definition distinguishes between interior equilibria and corner equilibria.

Definition 2 (i) An interior equilibrium is an equilibrium where each player chooses a positive flat contribution, i.e., $y_i > 0, i = 1, 2$; (ii) A corner equilibrium is an equilibrium where at least one player chooses a zero flat contribution, i.e., $y_i = 0, i = 1$ or 2. Player i is thus said to be at the corner.

From the Samuelson rule, an interior matching equilibrium is Pareto optimal if and only if $\sum_{i=1}^{2} p_i = 1$, i.e., $\mu_1 \mu_2 = 1$.

2.2 Aggregative game approach

This paper applies the aggregative game approach developed by Cornes and Hartley (2003, 2007) to characterize interior matching equilibria. Let $e_i(G, p_i)$ denote player i's income expansion path which is a function of the total public good provision, on which player i's effective public good price is p_i. At an interior matching equilibrium, the following two conditions must be satisfied:

$$x_i = e_i(G, p_i)$$

$$G + \sum_{i=1}^{2} e_i(G, p_i) = W$$

The first condition holds because, when any player chooses a positive flat contribution, the marginal rate of substitution between the private good and the public good must be equal to the private marginal rate of transformation between the two goods, so that the choice is on the income expansion path. The second condition is the aggregate budget constraint.

Immediately, an interior initial equilibrium (with a superscript N for each variable) must satisfy the following conditions:

$$x_i^N = e_i(G^N, 1)$$

$$G^N + \sum_{i=1}^{2} e_i(G^N, 1) = W$$

Unfortunately, interior matching equilibria only emerge for specific initial income distributions. Interiority of equilibria is even much harder to get with matching than without (Buchholz, Cornes and Rübbelke, 2011). However, corner solutions in voluntary public good provision games are important (See Bergstrom, Blume and Varian, 1986; Itaya, de Meza and Myles, 1997; Cornes and Sandler, 2000). This paper takes corner equilibria into consideration. For discussion of Pareto-improving equilibria, the following definition is useful.

Definition 3 An equilibrium under a matching scheme $m(\mu_1, \mu_2)$ is Pareto-improving if, for player $i = 1, 2$, the utility under the matching scheme, $u_i(x_i, G)$, is higher than without matching, i.e., $u_i(x_i, G) > u_i\left(x_i^N, G^N\right)$.

3. Pareto-improving equilibria

This section investigates conditions of Pareto-improving moves from the initial equilibrium. The initial equilibrium is either an interior or a corner equilibrium. The section first considers interior equilibria and identifies the interiority zone, and then links the interiority zone to Pareto-improving equilibria, followed by the case of corner equilibria. As the condition of the Pareto optimal equilibrium is $\mu_1\mu_2 = 1$, this paper looks at situations in which $\mu_1 \geq 0, \mu_2 \geq 0, \mu_1\mu_2 < 1$.

3.1 Interiority and neutrality

If the matching equilibrium is an interior solution, two players would be on their income expansion paths. Thus,

$$(3.1) \qquad \frac{\partial u_i / \partial x_i}{\partial u_i / \partial G} = 1 + \mu_j \Rightarrow x_i = \frac{\alpha_i G}{1 + \mu_j} \; (i, \; j = 1, 2, i \neq j)$$

Combining the aggregate budget constraint yields the equilibrium as

$$(3.2) \qquad G = \frac{W}{\frac{\alpha_1}{1+\mu_2} + \frac{\alpha_2}{1+\mu_1} + 1}, \; x_1 = \frac{\alpha_1}{1+\mu_2} \frac{W}{\frac{\alpha_1}{1+\mu_2} + \frac{\alpha_2}{1+\mu_1} + 1}, \; x_2 = \frac{\alpha_2}{1+\mu_1} \frac{W}{\frac{\alpha_1}{1+\mu_2} + \frac{\alpha_2}{1+\mu_1} + 1}$$

The initial equilibrium is immediately obtained by setting $\mu_1 = \mu_2 = 0$ and the flat contributions are

$$y_1^N = w_1 - \frac{\alpha_1 W}{\alpha_1 + \alpha_2 + 1}, \; y_2^N = w_2 - \frac{\alpha_2 W}{\alpha_1 + \alpha_2 + 1}$$

An interior equilibrium requires $y_1^N > 0, y_2^N > 0$ which is solved as

$$\frac{\alpha_1}{\alpha_2 + 1} < k < \frac{\alpha_1 + 1}{\alpha_2}$$

The above range is hereafter referred to as 'the interiority zone'. Any income distribution within the interiority zone would generate an interior equilibrium. The total public good provision and the private good consumption of each player depend on the total income rather than the individual income. Put it differently, they are unaffected by income heterogeneity or income redistribution among contributors, which is Warr neutrality (See Warr, 1982, 1983; Bergstrom, Blume and Varian, 1986; Cornes and Sandler, 1996). This is because two players have the same unit cost of the public good production. If two players have different unit costs of the public good production, the private good consumption and the public good provision both depend on the income distribution. The neutrality breaks down.

Proposition 1 Given $u_i\left(x_i, G\right) = x_i^{\alpha_i} G, \; i = 1, 2$ without matching,

(1) Any income distribution within the interiority zone $\frac{\alpha_1}{\alpha_2+1} < k < \frac{\alpha_1+1}{\alpha_2}$ would generate an interior equilibrium.
(2) If two players have the same unit cost of the public good production, the neutrality holds; otherwise, the neutrality breaks down.

3.2 Interior equilibria

Given an interior initial equilibrium, income distribution must be within the interiority zone. If the matching equilibrium is also an interior solution, the following conditions must hold:

$$(3.3) \qquad y_1 > 0 \Rightarrow \mu_2 > \frac{\alpha_1(1+\mu_1)}{k(1+\alpha_2)-\mu_1} - 1$$

$$(3.4) \qquad y_2 > 0 \Rightarrow \mu_2 < \frac{\frac{1}{k}(1+\alpha_1)(1+\mu_1)-\alpha_2}{1+\mu_1+\alpha_2}$$

If the matching equilibrium is Pareto-improving, the following conditions must be satisfied:

$$(3.5) \qquad u_1 > u_1^N \Rightarrow 1+\mu_1 > \frac{\alpha_2(1+\mu_2)}{(\alpha_1+\alpha_2+1)(1+\mu_2)^{\frac{1}{\alpha_1+1}} - \alpha_1 - 1 - \mu_2}$$

$$(3.6) \qquad u_2 > u_2^N \Rightarrow 1+\mu_2 > \frac{\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1)^{\frac{1}{\alpha_2+1}} - \alpha_2 - 1 - \mu_1}$$

Based on (3.3)-(3.6), the following proposition links the interiority zone to the existence of Pareto-improving equilibria.

Proposition 2 Given $u_i(x_i, G) = x_i^{\alpha_i} G,\ i = 1, 2$ and any income distribution within the interiority zone $\frac{\alpha_1}{\alpha_2+1} < k < \frac{\alpha_1+1}{\alpha_2}$, there always exist small matching rates $(\mu_1\mu_2 < 1)$ to generate Pareto-improving equilibria.

Proof: See Appendix A.

This proposition indicates that, if income distribution is within the interiority zone, two players can always implement some small matching schemes to make them both better off. This finding is very useful for cooperation, particularly in the context without complete information of global preferences or at the international level without a central government which can force sovereign countries to implement their commitments.

The intuition of generating Pareto-improving matching equilibrium is as follows. The allocative function of matching mechanisms is achieved by distorting the relative price between the public good and the private good through reciprocal subsidization of the flat contribution. By slightly distorting the relative price, players are induced to provide larger contributions and, to some extent, the externality of the public good is corrected. The economy becomes more efficient and players are better off.

Fig.1 provides a graphic explanation in a numerical example with $\alpha_1 = 1, \alpha_2 = 2$. Each panel presents the space of (μ_1, μ_2) given different income ratios. The two axes denote μ_1 and μ_2 respectively, and thus the origin represents the initial equilibrium. The dashed curve Y_2 represents $y_2 = 0$, the dotted curve Y_1 is $y_1 = 0$ and the hyperbola is $\mu_1\mu_2 = 1$. The three curves intersect at E. The area enclosed by Y_2 and Y_1 together with the two axes represents matching schemes which generate interior equilibria (hereafter 'the interiority area'). The upper bound of the lens-shaped area denotes the indifference curve of player 1 and the lower bound denotes the indifference curve of player 2 at their respective utility level of the initial equilibrium. Thus, the lens-shaped area represents matching schemes which lead to Pareto improvements. The overlapping (shaded) area between the lens-shaped area and the interiority area represents the matching schemes which generate Pareto-improving equilibria. When income distribution is within the interiority zone $1/3 < k < 1$, there is always an overlapping area representing Pareto-improving equilibria. The Pareto-improving matching schemes can be quite flexible in the overlapping area. They can be either fixed on the focal point of the same matching rate, or somehow adjusted for the income ratio and the weights of value between the two goods. Fig.1 (a) and (c) present two boundary cases.

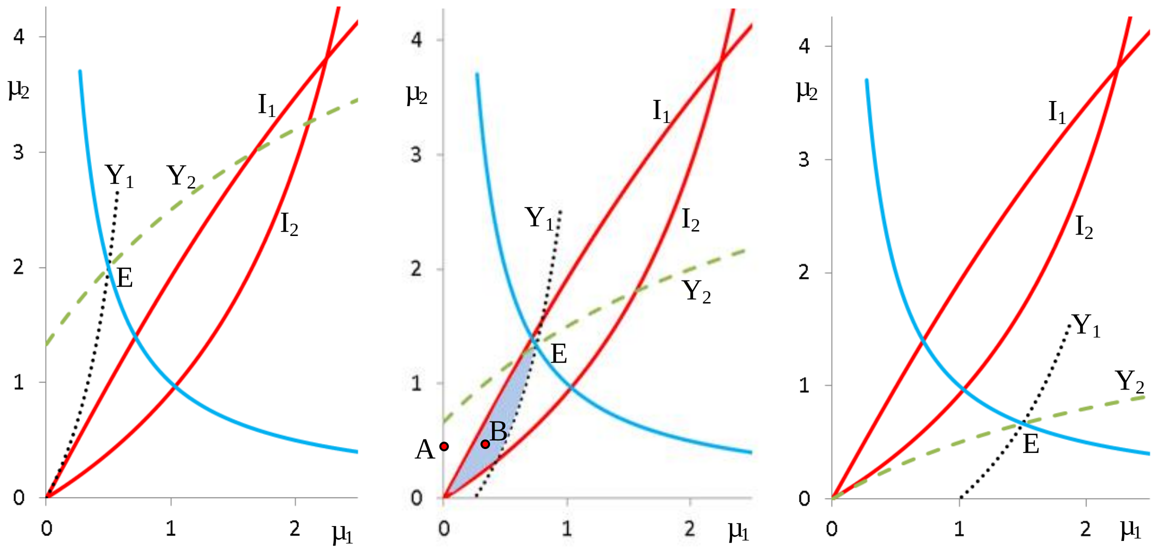


Fig.1 (a) $k = 1/3$ Fig.1 (b) $k = 0.5$ Fig.1 (c) $k = 1.0$

It is due to free riding that public goods are often underprovided in the context of voluntary contribution. Do players also have incentives to take free rides in the matching game? The following proposition shows how players respond to the own matching rate and the opponent's matching rate.

Proposition 3 Given $u_i(x_i, G) = x_i^{\alpha_i} G,\ i = 1, 2$ with a matching scheme (μ_1, μ_2), at interior matching equilibria,

(1) The matching contribution is increasing in the own matching rate and the flat contribution is decreasing in the own matching rate to a larger extent resulting that the individual public good provision is decreasing in the own matching rate;
(2) The matching contribution is decreasing in the opponent's matching rate and the flat contribution is increasing in the opponent's matching rate to a larger extent resulting that the individual public good provision is increasing in the opponent's matching rate;
(3) The total public good provision is increasing in both matching rates;

(4) The private good consumption is increasing in the own matching rate and decreasing in the opponent’s matching rate;
(5) The utility is increasing in the own matching rate and decreasing in the opponent’s matching rate.

This matching game changes players’ behaviour - They decrease their flat contributions and increase their matching contributions to induce the opponent to provide a larger public good provision. The proposition shows that providing matching contributions is better off while accepting matching contributions is worse off. Therefore, once players participate in the matching game, each has incentives to provide a matching contribution and to increase their own matching rate. In this sense, this matching mechanism avoids free riding. Buchholz et. al (2014) consider a special case of a unilateral matching and provide a graphical explanation of this matching paradox. In Fig.1 (b), given a unilateral matching scheme, for example, only player 2 provides a matching contribution, i.e., $\mu_1 = 0, \mu_2 = \hat{\mu}_2 > 0$ represented by A on the vertical axis, by reading the diagram, the matching equilibrium is outside the lens-shaped area and player 1 is worse off while player 2 is better off. In this special case, the above proposition immediately reads as

Proposition 4 Given $u_i(x_i, G) = x_i^{\alpha_i} G,\ i = 1, 2$ at an interior matching equilibrium with a unilateral matching scheme,

(1) The matching player increases the matching contribution and decreases the flat contribution to a larger extent resulting in a lower individual public good provision;
(2) The matched player increases the flat contribution resulting in a higher individual public good provision;
(3) The total public good provision is increased;
(4) The matching player increases the private good consumption while the matched decreases the private good consumption;
(5) The matching player is better off and the matched is worse off.

Now consider a bilateral matching scheme $\mu_1 = \hat{\mu}_1 > 0, \mu_2 = \hat{\mu}_2 > 0$ represented by B within the shaded area in Fig. 1(b). To have a better understanding of the bilateral matching scheme, it can be decomposed into two stages: At the first stage, $\mu_1 = 0, \mu_2 = \hat{\mu}_2$; at the second stage $\mu_1 = \hat{\mu}_1, \mu_2 = \hat{\mu}_2$. After player 2 sets a positive matching rate at the first stage, player 2 is better off with the utility u_2^A and player 1 is worse off with the utility u_1^A, i.e., $u_2^A > u_2^N, u_1^A < u_1^N$. Given player 2’s matching rate, player 1 sets a positive matching rate at the second stage. Player 1 is better off with the utility u_1^B and player 2 is worse off with the utility u_2^B, i.e., $u_1^B > u_1^A, u_2^B < u_2^A$. In this special matching scheme, player 2 is better off by providing a matching contribution and worse off by being matched to a smaller extent resulting in a higher utility, and so is player 1.

3.3 Corner equilibria

If income distribution is beyond the interiority zone, the poor player does not provide any public good at the initial equilibrium. The poor player may not necessarily have a lower income than the rich. If $k \leq \frac{\alpha_1}{\alpha_2 + 1}$, player 1 is referred to as the poor player and player 2 as the rich player. Is it possible that the poor player offers matching to induce the rich to provide a larger public good contribution resulting in Pareto-improving outcomes? Or is it likely that the rich offers matching to induce the poor to provide a positive public good contribution resulting in Pareto-improving outcomes? The answers are both NO. These two cases are considered respectively. If the initial equilibrium is at a corner, the income ratio must satisfy $k \le \frac{\alpha_1}{\alpha_2 + 1}$ or $k \ge \frac{\alpha_1 + 1}{\alpha_2}$. Due to symmetry, consider the former case only.

Case I: Corner matching equilibria

Suppose that the poor player does not provide a flat contribution but offers a matching contribution to the rich. Therefore, the rich player does not provide a matching contribution. Without loss of generality, $\mu_2 = 0$. The poor player offers a matching contribution to the rich, i.e., $\mu_1 > 0, \mu_2 = 0$.

Proposition 5 Given $u_i(x_i, G) = x_i^{\alpha_i} G,\ i = 1, 2$, if the poor player at the corner offers matching to the rich,

(1) The poor player provides a positive matching contribution;
(2) The rich player does not change the flat contribution;
(3) The total public good provision is increased;
(4) The poor player decreases the private good consumption while the rich does not change the private good consumption;
(5) The poor player is worse off and the rich is better off;
(6) The utility of the poor player is decreasing in the matching rate.

The poor player at the initial equilibrium does not provide any flat contribution, and thus cannot take advantage of the initial flat contribution to offer matching. Instead, the player has to reduce the private consumption. In fact, even if the poor player provides a matching contribution, the rich player does not change the flat contribution of the public good. This is because, when the rich is matched, the substitution effect moves the rich player from the private good to the public good but the public good is increased by the poor player. The matching contribution exactly offsets the substitution effect leaving unchanged the flat contribution of the rich. The poor player provides a matching contribution but does not induce a larger contribution from the rich, and thus becomes worse off. Therefore, at corner matching equilibria, there are no Pareto-improving outcomes.

Case II: Interior matching equilibria

Suppose that the rich player offers a matching contribution to induce the poor to provide a positive flat contribution, which implies an interior solution. The matching scheme $\mu_1 > 0, \mu_2 > 0$ can be decomposed into two stages: At the first stage, $\mu_1 > 0, \mu_2 = 0$; at the second stage, $\mu_1 > 0, \mu_2 > 0$. From Proposition 5, player 1 at the corner is worse off with $\mu_1 > 0, \mu_2 = 0$ compared to no matching. Furthermore, from Proposition 3, at interior matching equilibria, the utility of player 1 is decreasing in the opponent's matching rate, so player 1 is worse off with $\mu_1 > 0, \mu_2 > 0$ than with $\mu_1 > 0, \mu_2 = 0$. Put it differently, the poor is worse off by matching and also by being matched - The player at the corner suffers 'double losses'. Unambiguously, at interior matching equilibria, there are no Pareto-improving outcomes.

The two cases above show that there are no Pareto-improving matching equilibria if income distribution is beyond the interiority zone. The intuition is as follows: If income distribution is too large, the rich player would provide a relatively large flat contribution to the public good. As the poor player has a very small income, the marginal rate of substitution between the private good and the public good would be very large. If the poor gives up some of the private good to provide a positive contribution (either matching or flat), the player requires a large increase in the total provision of the public good to compensate for the forgone private good, but the rich player would not provide that much.

Consider a special case in which the rich player offers matching when the poor is at the corner.

Proposition 6 Given $u_i(x_i, G) = x_i^{\alpha_i} G,\ i = 1, 2$, if the poor player at the corner is matched by the rich at the matching rate $\mu_2 > 0$,

Case I: If $\frac{\alpha_1}{(1+\mu_2)(\alpha_2+1)} < k < \frac{\alpha_1}{\alpha_2+1}$,

(1) The rich player increases the matching contribution and decreases the flat contribution to a larger extent resulting in a lower individual public good provision;
(2) The poor player increases the flat contribution resulting in a positive provision;
(3) The total public good provision is increased;
(4) The rich player increases the private good consumption while the poor decreases the private good consumption;
(5) The rich player is better off and the poor is worse off;
(6) The utility of the rich player is increasing in the matching rate.

Case II: If $k \le \frac{\alpha_1}{(1+\mu_2)(\alpha_2+1)}$,

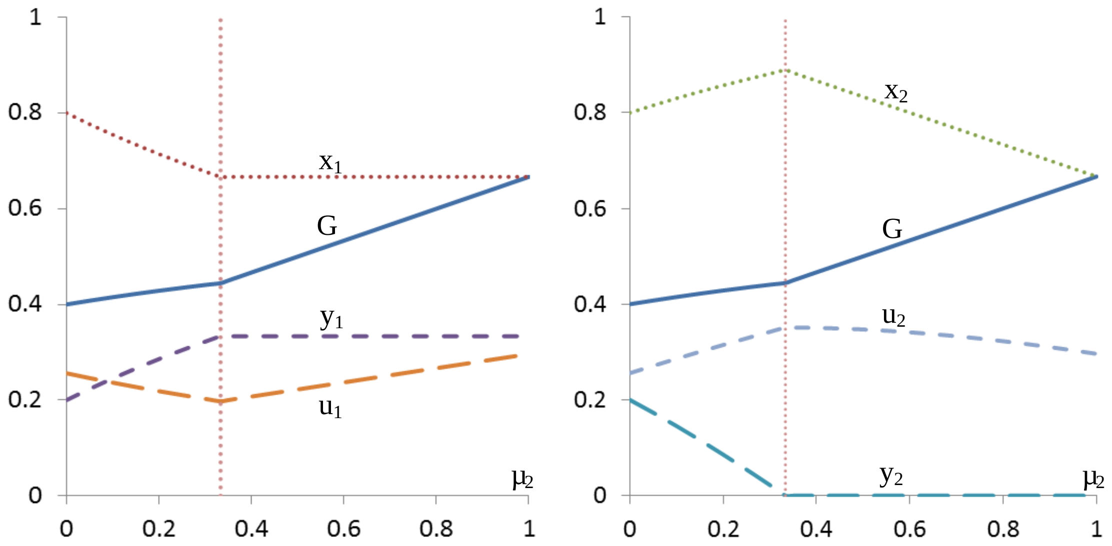
(1) The poor player is still at the corner without any public good provision;
(2) Both players are at the initial equilibrium with nothing changed.

Again this shows that providing matching contributions is better off while accepting matching contributions is worse off. This is because although the poor player is at the corner the rich player can take advantage of the initial flat contribution to offer matching, which is consistent with the findings in the case of interior equilibria.

Combining Propositions 3-6, it can be concluded that if one player provides a positive contribution at the initial equilibrium, the player would be better off through providing a matching contribution. This is because the player can reduce the flat contribution while providing a matching contribution. If the player does not provide any public good at the initial equilibrium, the player would be worse off through matching. The next section shows that there is a kink if one player transitions from an interior equilibrium to a corner.

3.4 From interior equilibria to corner equilibria

Suppose it is an interior equilibrium at the initial state and now player 2 gradually increases the matching rate from zero. Section 3.2 shows that player 2 increases the matching contribution while decreasing the flat contribution. With the matching rate increasing, player 2's flat contribution continues to decline until it reaches zero. By solving $y_2 = 0$, the critical matching rate is $\bar{\mu}_2 = \dfrac{1+\alpha_1-\alpha_2 k}{(1+\alpha_2)k}$. If $\mu_2 > \bar{\mu}_2$, player 2 is at a corner. Section 3.3 indicates that player 2 would be worse off if the matching rate continues to increase. Fig. 2 presents how private good consumption, public good provision and utility change with an increasing matching rate given $\alpha_1 = \alpha_2 = 2, w_1 = w_2 = 1$. The vertical dotted line represents the critical matching rate from interior equilibria to corner equilibria, i.e., $\bar{\mu}_2 = 1/3$ in this example.



(a) The matched player - Player 1 (b) The matching player - Player 2

Fig. 2 Changes from interior equilibria to corner equilibria

Fig.2 connects interior equilibria and corner equilibria, and well illustrates Propositions 3 and 4. At an interior equilibrium, the matching player is able to decrease the flat contribution while increasing the matching contribution to induce a larger flat contribution of the other player. However, the matching player cannot further decrease the flat contribution when it reaches zero with an increasing matching rate. The player has to reduce the private good consumption to provide the matching contribution resulting in lower utility. Either at an interior equilibrium or at a corner equilibrium, the total public good provision is increased.

3.5 Optimal equilibria

Boadway, Song and Tremblay (2007) prove that the sub-game perfect equilibrium of this matching game is optimal and the matching rates satisfy $\mu_1\mu_2 = 1$. Given the Cobb-Douglas utility function, at the sub-game perfect equilibrium (See Appendix B),

$$\mu_1 = \frac{k(1+\alpha_2)}{1+\alpha_1}, \mu_2 = \frac{1+\alpha_1}{k(1+\alpha_2)}$$

Thus, the intersection E in Fig. 1 represents the matching scheme at the sub-game perfect equilibrium. Clearly, the intersection may be outside the lens-shaped area given some income ratios. This indicates that the sub-game perfect equilibrium may not generate Pareto-improving equilibrium although it achieves the Pareto optimal equilibrium.

4. General preferences

This section extends previous analysis from the Cobb-Douglas preference to general preferences which satisfy the following standard assumptions: The utility function is continuous and differentiable, strictly increasing in both variables and strictly quasi-concave; Both goods are strictly normal, and indifference curves asymptote to the two axes (hereafter 'the standard utility function').

Proposition 7 Given any standard utility function without matching, there exists an interiority zone within which any income distribution would generate an interior equilibrium.

Proof: If the equilibrium is an interior solution, two players would be on their income expansion paths. The public good provision is then characterized by the aggregate budget constraint

$$G + e_1(G,1) + e_2(G,1) = W$$

If $\lim_{G\to 0} e_i(G,1) = 0$ and $\lim_{G\to\infty} e_i(G,1) = \infty$, the existence of a public good level $\hat{G}$ is implied by the Intermediate Value Theorem. Uniqueness is ensured by the strict monotonicity of the income expansion path. Given such $\hat{G}$, interiority requires

$$y_1 = \frac{k}{k+1}W - e_1(\hat{G},1) > 0, y_2 = \frac{1}{k+1}W - e_2(\hat{G},1) > 0$$

The interiority zone can be solved as

$$\frac{e_1(\hat{G},1)}{W - e_1(\hat{G},1)} < k < \frac{W - e_2(\hat{G},1)}{e_2(\hat{G},1)}$$

The lower bound is always smaller than the upper bound, so the interiority zone exists. QED

Proposition 8 Given two players with standard utility functions and any income distribution within the interiority zone, there always exist small matching rates to generate Pareto-improving equilibria.

The proof proceeds as follows: Lemma 1 proves the existence of individual public good prices so that the interior matching equilibrium strictly Pareto dominates the initial equilibrium, and Lemma 2 proves the individual public good prices can be realized by a matching scheme. Combining Lemma 1 and Lemma 2 yields the above proposition.

Lemma 1 Given W there exist infinitely many $P=(p_1, p_2)$ of individual public good prices so that the interior matching equilibrium $(x_1(P,W), x_2(P,W), G(P,W))$ strictly Pareto dominates the initial equilibrium.

Proof: Suppose that the initial equilibrium and matching equilibrium are both interior solutions. Let $h_i(G)$ denote player i 's indifference curve passing through $(x_i(P,W), G(P,W))$. Define $W(\tilde{G})=\sum_{i=1}^{2} h_i(\tilde{G})+\tilde{G}$ for any $\tilde{G}\geq G(P^N,W)$ where $P^N=(1,1)$ is the initial public good prices. Since $W'(G(P^N,W))=\sum_{i=1}^{2} h_i'(G(P^N,W))+1=-1$, it follows that $W(\tilde{G})<W$ for $\tilde{G}>G(P^N,W)$. The allocation $(h_1(\tilde{G}), h_2(\tilde{G}), \tilde{G})$ is the matching equilibrium given the public good prices $p_i(\tilde{G})=-h_i'(\tilde{G})$ and the total income $W(\tilde{G})$. Let $P(\tilde{G})=(p_1(\tilde{G}), p_2(\tilde{G}))$. At the matching equilibrium given $P(\tilde{G})$ and $W(\tilde{G})$, both players attain the same utility levels as in the initial equilibrium. If the total income is increased from $W(\tilde{G})$ to W while the public good prices are kept at $P(\tilde{G})$, both players move outwards along their expansion paths $e_i(G, p_i(\tilde{G}))$ resulting in higher utility levels. Therefore for all $\tilde{G}>G(P^N,W)$ the allocation given $P(\tilde{G})$ and W strictly Pareto dominates the initial equilibrium. QED

Lemma 2 Given an interior initial equilibrium, if the public good prices are sufficiently close to the initial prices, there exists a matching scheme realizing the public good prices.

Proof: Construct a matching scheme $\mu_i=1/p_j-1$ given $P=(p_1, p_2)$. If the matching equilibrium given P and W is realized by a matching scheme, the following system must have a positive solution in flat contributions (see Buchholz, Cornes and Rübbelke 2011).

$$w_i - x_i(P,W) = y_i + \mu_i(p_j) y_j(P,W)$$

In the matrix form this system of equations reads as

$$\begin{pmatrix} w_1 - x_1(P,W) \\ w_2 - x_2(P,W) \end{pmatrix} = \begin{pmatrix} 1 & \mu_1(p_2) \\ \mu_2(p_1) & 1 \end{pmatrix} \begin{pmatrix} y_1(P,W) \\ y_2(P,W) \end{pmatrix}$$

If p_i is close to one and thus μ_i is close to zero, the determinant of the first matrix on the RHS is close to one so that the system has a unique solution. For any P in a small neighbourhood of P^N, $y_i(P,W)$ must be positive because it is close to $y_i^N(P^N,W)$ which is assumed to be positive. QED

Consider the matching equilibrium given $P(\tilde{G})$ and W as constructed in Lemma 1 which strictly Pareto dominates the initial equilibrium for $\tilde{G} > G(P^N,W)$. If the public good level $\tilde{G}$ goes to $G(P^N,W)$, the public good price $p_i(\tilde{G})$ goes to one. It follows from Lemma 2 that for $\tilde{G}$ which is close to $G(P^N,W)$ there exists a matching scheme realizing the matching equilibrium.

In practice, the Cobb-Douglas utility function may be sufficiently useful. This paper focuses on small matching schemes and thus the matching equilibrium is not far away from the initial equilibrium, so the Cobb-Douglas function can be used to approximate any standard function in the local neighbourhood of the initial equilibrium. It is likely to observe the consumption bundle at the initial equilibrium as well as the unit cost of the public good production, and then to estimate the weights of value between the goods.

The following proposition proves that at interior equilibria providing matching contributions is better off while accepting matching contributions is worse off given any general preference.

Proposition 9 Given two players with any standard utility functions at interior matching equilibria, the utility is increasing in the own matching rate and decreasing in the opponent's matching rate.

Proof: At an interior matching equilibrium, the public good provision can be solved from the aggregate budget constraint

$$(4.1) \qquad G + e_1(G, 1+\mu_2) + e_2(G, 1+\mu_1) = W$$

Without loss of generality, differentiating utility with respect to μ_1,

$$(4.2) \qquad \begin{aligned} \frac{\partial u_1}{\partial \mu_1} &= \frac{\partial u_1(x_1, G)}{\partial \mu_1} = \frac{\partial u_1(e_1(G, 1+\mu_2), G)}{\partial \mu_1} = \frac{\partial u_1}{\partial x_1}\frac{\partial e_1}{\partial G}\frac{\partial G}{\partial \mu_1} + \frac{\partial u_1}{\partial G}\frac{\partial G}{\partial \mu_1} \\ \frac{\partial u_2}{\partial \mu_1} &= \frac{\partial u_2(x_2, G)}{\partial \mu_1} = \frac{\partial u_2(e_2(G, 1+\mu_1), G)}{\partial \mu_1} = \frac{\partial u_2}{\partial x_2}\left(\frac{\partial e_2}{\partial G}\frac{\partial G}{\partial \mu_1} + \frac{\partial e_2}{\partial \mu_1}\right) + \frac{\partial u_2}{\partial G}\frac{\partial G}{\partial \mu_1} \end{aligned}$$

At an interior matching equilibrium,

$$(4.3) \qquad \begin{aligned} &\frac{\partial u_1 / \partial x_1}{\partial u_1 / \partial G} = 1+\mu_2 \Rightarrow \frac{\partial u_1}{\partial x_1} = (1+\mu_2)\frac{\partial u_1}{\partial G} \\ &\frac{\partial u_2 / \partial x_2}{\partial u_2 / \partial G} = 1+\mu_1 \Rightarrow \frac{\partial u_2}{\partial x_2} = (1+\mu_1)\frac{\partial u_2}{\partial G} \end{aligned}$$

The aggregate budget constraint (4.1) implies

(4.4) $$\frac{\partial G}{\partial \mu_1}+\frac{\partial e_1}{\partial G}\frac{\partial G}{\partial \mu_1}+\frac{\partial e_2}{\partial \mu_1}+\frac{\partial e_2}{\partial G}\frac{\partial G}{\partial \mu_1}=0 \Rightarrow \frac{\partial G}{\partial \mu_1}=-\frac{\partial e_2/\partial \mu_1}{1+\partial e_1/\partial G+\partial e_2/\partial G}$$

Substituting (4.3) and (4.4) into (4.2) yields

(4.5) $$\begin{aligned}\frac{\partial u_1}{\partial \mu_1}&=\frac{\partial u_1}{\partial x_1}\frac{\partial e_1}{\partial G}\frac{\partial G}{\partial \mu_1}+\frac{\partial u_1}{\partial G}\frac{\partial G}{\partial \mu_1}=-\frac{\partial u_1}{\partial G}\frac{\partial e_2/\partial \mu_1}{1+\partial e_1/\partial G+\partial e_2/\partial G}\left((1+\mu_2)\frac{\partial e_1}{\partial G}+1\right)\\ \frac{\partial u_2}{\partial \mu_1}&=\frac{\partial u_2}{\partial x_2}\left(\frac{\partial e_2}{\partial G}\frac{\partial G}{\partial \mu_1}+\frac{\partial e_2}{\partial \mu_1}\right)+\frac{\partial u_2}{\partial G}\frac{\partial G}{\partial \mu_1}=\frac{\partial u_2}{\partial G}\frac{\partial e_2}{\partial \mu_1}\frac{(1+\mu_1)*\partial e_1/\partial G+\mu_1}{1+\partial e_1/\partial G+\partial e_2/\partial G}\end{aligned}$$

As $\frac{\partial u_i}{\partial G}>0$ and $\frac{\partial e_2}{\partial \mu_1}<0$, $\frac{\partial u_1}{\partial \mu_1}>0, \frac{\partial u_2}{\partial \mu_1}<0$. QED

As in the case of the Cobb-Douglas utility function, the matched player is induced to increase the flat contribution resulting in a higher individual public good provision. The matching player increases the matching contribution but decreases the flat contribution to a larger extent resulting in a lower individual public good provision. Although the total public good provision is increased, the matched player decreases the private good consumption resulting in a lower utility level.

5. Participation in the game

Section 3.2 has shown that a unilateral matching scheme makes the matched player worse off. It seems that a unilateral matching scheme can be implemented by the matching player without a formal agreement with the matched player. Can the matched player reject the opponent's matching? Section 3.5 has shown that given some income distributions the sub-game perfect equilibrium is not Pareto-improving although it reaches the Pareto optimal equilibrium. Why is it possible that one player may be worse off in a game of voluntary participation with complete information? This is because the underlying assumption of the matching game is that players share common knowledge that players agree to participate in the game and players match others' contributions and accept others' matching. Suppose that the matched player does not accept matching contributions in the unilateral matching. If this is common knowledge, the matched player behaves as without matching and believes that the matching player's public good provision, denoted by z_1, is independent from the matched player's provision, and the matching player also believes that the matched player behaves as without matching. Then the budget constraints of two players are respectively $x_1+z_1=w_1, x_2+y_2=w_2$ and the total public good provision is $G=z_1+y_2$. The equilibrium immediately goes back to the initial equilibrium without matching.

Given complete information, players are able to expect outcomes of any matching scheme. If one player becomes worse off in a matching scheme, the player would not like to participate in the game. Therefore, it is indispensable to generate Pareto-improving outcomes for application of matching games in the context of voluntary participation.

6. Conclusions

Matching mechanisms have been proposed to improve public good provision in the context of voluntary contribution and ideally to reach Pareto optimal equilibria. Nevertheless, reaching Pareto optimal equilibria is very ambitious. This paper has focused on Pareto-improving equilibria under matching mechanisms with two heterogeneous players.

The paper has found that at interior equilibria providing matching contributions is better off while accepting matching contributions is worse off and hence each player has incentives to provide matching contributions. Second, given any income distribution within the interiority zone players can always implement small matching schemes to make them both better off. The matching schemes can be quite flexible in a certain range. This finding is useful for cooperation, particularly in the context without complete information of global preferences or at the international level without a central government. However, oppositely, at corner equilibria providing matching contributions is worse off while accepting matching contributions is better off. Thus, if income distribution is beyond the interiority zone, there are no Pareto-improving matching equilibria.

There are several interesting extensions based on this paper. One is to allow for productivity differentials of public goods and population sizes when countries play this matching game at the international level. The second one is to consider matching coalitions with multiple players. Another dimension of extension is to introduce incomplete information of preferences even in the neighbourhood of the initial equilibrium and then to investigate where the economy would reach through bargaining on matching schemes. The fourth one is to relax a pure public good to an impure one. In the climate example, fossil fuel combustion not only induces climate change but also generates air pollution. Climate protection is a global public good while air quality is mainly a private good in a country.

Appendix A

The proof proceeds as follows with the aid of Fig. A.

Step 1: Obtain the indifference curve at the utility level of the initial equilibrium. For player 1,

$$u_1(x_1, G) = u_1(x_1^N, G^N) \Rightarrow \mu_1 = \frac{\alpha_2(1+\mu_2)}{(\alpha_1+\alpha_2+1)(1+\mu_2)^{\frac{1}{\alpha_1+1}} - \alpha_1 - 1 - \mu_2} - 1$$

Differentiating μ_1 with respect to μ_2 yields

$$\frac{\partial \mu_1}{\partial \mu_2} = \frac{\alpha_1\alpha_2}{\alpha_1+1} \frac{(\alpha_1+\alpha_2+1)(1+\mu_2)^{\frac{1}{\alpha_1+1}} - \alpha_1 - 1}{\left((\alpha_1+\alpha_2+1)(1+\mu_2)^{\frac{1}{\alpha_1+1}} - \alpha_1 - 1 - \mu_2\right)^2} > 0$$

Thus, μ_1 is increasing in μ_2 and hence μ_2 is increasing in μ_1. The indifference curve of player 1, denoted by the solid curve I_1, is upward-sloping. Below I_1 represents a higher utility of player 1 than at the initial equilibrium.

Similarly, the indifference curve of player 2, denoted by the double-solid curve I_2, is upward-sloping. Above I_2 represents a higher utility of player 2 than at the initial equilibrium.

Step 2: Prove that I_1 is above I_2 near the origin, so the area enclosed by the two curves represents the Pareto-improving area. Denote

$$f(\mu_1,\mu_2)=\frac{\alpha_2(1+\mu_2)}{(\alpha_1+\alpha_2+1)(1+\mu_2)^{\frac{1}{\alpha_1+1}}-\alpha_1-1-\mu_2}-1-\mu_1$$

Immediately, any point on I_1 satisfies $f=0$. For any point (μ_1,μ_2) on I_2,

$$u_2(x_2,G)=u_2(x_2^N,G^N)\Rightarrow \mu_2=\frac{\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1)^{\frac{1}{\alpha_2+1}}-\alpha_2-1-\mu_1}-1$$

Substituting μ_2 into f yields

$$f=\frac{g}{(\alpha_1+\alpha_2+1)(1+\mu_2)^{\frac{1}{\alpha_1+1}}-\alpha_1-1-\mu_2}$$

Where

$$g=(\alpha_1+\alpha_2+1)\left(\frac{\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1)^{\frac{1}{\alpha_2+1}}-\alpha_2-1-\mu_1}\right)^{\frac{1}{\alpha_1+1}}-\frac{\alpha_1\alpha_2+\alpha_1(1+\mu_1)}{(\alpha_1+\alpha_2+1)(1+\mu_1)^{\frac{1}{\alpha_2+1}}-\alpha_2-1-\mu_1}-\alpha_1$$

μ_1 is small near the origin and hence the following approximations hold.

$$(1+\mu_1)^{\frac{1}{\alpha_2+1}}\approx 1+\frac{\mu_1}{\alpha_2+1},\ (1+\mu_1)^{\frac{1}{\alpha_1+1}}\approx 1+\frac{\mu_1}{\alpha_1+1},\ \left(1+\frac{\mu_1}{\alpha_2+1}\right)^{\frac{\alpha_1}{\alpha_1+1}}\approx 1+\frac{\mu_1}{\alpha_2+1}\frac{\alpha_1}{\alpha_1+1}$$

Substituting the above approximations into g yields

$$g\approx\frac{\alpha_1(\alpha_1+\alpha_2+1)\frac{\mu_1}{\alpha_1+1}\left(\frac{\alpha_2}{\alpha_2+1}+\frac{\mu_1}{\alpha_2+1}\frac{\alpha_1}{\alpha_1+1}\right)}{(\alpha_1+\alpha_2+1)\left(1+\frac{\mu_1}{\alpha_2+1}\right)-\alpha_2-1-\mu_1}>0$$

Therefore, $f>0$, i.e., I_1 is above I_2 near the origin.

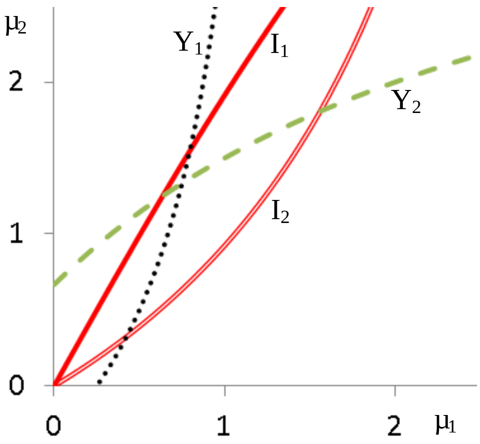


Fig.A

Step 3: Obtain the boundary condition of interior equilibria. For player 1,

$$y_1 = 0 \Rightarrow \mu_2 = \frac{\alpha_1(1+\mu_1)}{k(1+\alpha_2)-\mu_1} - 1$$

μ_2 is increasing in μ_1. This curve, denoted by the dotted curve Y_1, is upward-sloping. Above the curve is $y_1 > 0$. Similarly, the boundary condition of interior equilibria of player 2, denoted by the dashed curve Y_2, is upward-sloping. Below the curve is $y_2 > 0$.

Step 4: Prove that Y_2 is above Y_1 when $\mu_1\mu_2 < 1$, so the area enclosed by the two curves together with the two axes represents the matching schemes which generate interior solutions. Within the interiority zone $\frac{\alpha_1}{\alpha_2+1} \le k \le \frac{\alpha_1+1}{\alpha_2}$, (i) On Y_2, when $\mu_1 = 0$, $\mu_2 = \frac{\frac{1}{k}(1+\alpha_1)-\alpha_2}{1+\alpha_2} \ge 0$; (ii) On Y_1, when $\mu_1 = 0$, $\mu_2 = \frac{\alpha_1}{k(1+\alpha_2)} - 1 \le 0$. Therefore, Y_2 is above Y_1 within the interiority zone, which indicates that there is an interiority area near the origin.

Step 5: On Y_2, μ_2 is decreasing in k. Within the interiority zone, Y_2 is above I_1 near the origin. As k increases, Y_2 shifts downwards. When $k = \frac{\alpha_1+1}{\alpha_2}$, Y_2 intersects with I_1 at the origin. Similarly, on Y_1, μ_2 is decreasing in k. Within the interiority zone, Y_1 is below I_2 near the origin. As k decreases, Y_1 shifts leftwards. When $k = \frac{\alpha_1}{\alpha_2+1}$, Y_1 intersects with I_2 at the origin.

The Pareto-improving area near the origin is enclosed by the interiority area, so within the interiority zone there always exist small matching schemes to generate Pareto-improving equilibria. QED

Appendix B

The budget constraints of two players are

$$(B1) \qquad x_1 + y_1 + \mu_1 y_2 = w_1$$

$$(B2) \qquad x_2 + y_2 + \mu_2 y_1 = w_2$$

At the sub-game perfect equilibrium the matching rates satisfy $\mu_1\mu_2 = 1$. Multiply $(B2)$ by μ_1 and rewrite the budget constraints as follows:

$$(B3) \qquad y_1 + \mu_1 y_2 = w_1 - x_1$$

$$(B4) \qquad y_1 + \mu_1 y_2 = \mu_1(w_2 - x_2)$$

For equilibria to exist, the RHS of $(B3)$ and $(B4)$ must be equal.

$$(B5) \qquad w_1 - x_1 = \mu_1(w_2 - x_2)$$

At the interior equilibrium,

$$(B6) \qquad x_1 = \frac{\alpha_1}{1+\mu_2}\frac{W}{\frac{\alpha_1}{1+\mu_2}+\frac{\alpha_2}{1+\mu_1}+1},\ x_2 = \frac{\alpha_2}{1+\mu_1}\frac{W}{\frac{\alpha_1}{1+\mu_2}+\frac{\alpha_2}{1+\mu_1}+1}$$

By substituting $(B6)$ into $(B5)$, the matching rates at the sub-game perfect equilibrium are solved as $\mu_1 = \frac{k(1+\alpha_2)}{1+\alpha_1},\ \mu_2 = \frac{1+\alpha_1}{k(1+\alpha_2)}$. QED

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