

**INFERENCE FOR DISCRETE TIME STOCHASTIC PROCESSES
USING AGGREGATED SURVEY DATA**

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DECLARATION OF AUTHENTICITY

This thesis is my own work and all sources used have been acknowledged.

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CONTENTS

Declaration of Authenticity	ii
Acknowledgements	iii
Summary	1
1 Inference for discrete time finite state space stochastic processes using aggregate data	3
1.0 Introduction	3
1.1 Model based inference	5
1.1.1 Time series modelling of survey data	6
1.1.2 Linear modelling of data with autocorrelated responses	11
1.1.3 Estimating a continuous time Markov chain	15
1.2 Non-parametric inference for finite Markov chains	18
1.3 Stochastic interpolation	22
1.4 Measures of occupation time	24
1.4.1 Model based estimators and the Sullivan method	24
1.4.2 Disability adjusted life expectancy	28
2 Using partial odds to model discrete time categorical data	33
2.0 Introduction	33
2.1 Notation and preliminary results	35
2.2 The asymptotic distribution of the log partial odds	39
2.3 Linear regression modelling of the log partial odds	49
2.3.1 Estimation of the one-step transition probabilities	50
2.3.2 Hypothesis testing	54
2.4 An example: voting intentions	58
3 Estimation of a Markov chain by interpolation	66
3.0 Introduction	66
3.1 Weighted least squares interpolation	67
3.2 Recurrence relations for the derivatives of the least squares loss function	70
3.2.1 Differentials of the loss function with respect to the log partial odds	70

3.2.2	Derivatives of the log partial odds	71
3.2.3	Recursion relationships for the transition probability derivatives	73
3.2.4	Derivatives of the one –step transition probabilities	76
3.2.5	Competing risks	78
3.3	Example 3.1: Australian male mortality	80
3.3.1	Data and methods	80
3.3.2	Results	83
3.4	Example 3.2: Finnish work life expectancies	88
3.4.1	Data and parameter estimation	88
3.4.2	Work life expectancies: results and discussion	101
3.5	Example 3.3: French period mortality data for the oldest ages	110
3.5.1	Data and parameter estimation	110
3.5.2	Results	115
4	Estimating marginal probabilities using cross-sectional survey data	123
4.0	Introduction	123
4.1	Assumptions and main results	124
4.2	Regression modelling of the log odds	135
4.3	Marginal occupation times	139
4.4	Example 4.1: Australian surveys of disability and ageing	141
4.4.1	Data and methods	141
4.4.2	Health expectancies: results and discussion	146
4.5	Example 4.2: Finnish marginal work life expectancies	152
4.5.1	Methods	152
4.5.2	Marginal work life expectancies: results and discussion	158
5	The use of superimposed sample data in inference for discrete time stochastic processes	166
5.0	Introduction	166
5.1	Rotation group sampling	167
5.2	Asymptotic theory for superimposed sample data	170
5.3	Example 5.1: Transitions in labour force status of Australian males	185
5.3.1	The CURF data	185

5.3.2 Estimates of transition probabilities	190
5.3.3 Occupation times	198
Appendix 1 Datasets	202
A1.1 Data for Section 3.4	202
A1.2 Data for Section 4.4	205
A1.3 Data for Section 4.5	213
A1.4 Data for Section 5.3	215
Appendix 2 S-Plus code	220
A2.0 Introduction	220
A2.1 Programs for Section 3.4	221
A2.2 Programs for Section 4.5	233
A2.3 Programs for Section 5.3	245
Bibliography	253

SUMMARY

We consider a longitudinal system in which transitions between the states are governed by a discrete time finite state space stochastic process X . Our aim, using aggregated sample survey data of the form typically collected by official statistical agencies, is to undertake model based inference for the underlying process X . We will develop inferential techniques for continuing sample surveys of two distinct types. First, longitudinal surveys in which the same individuals are sampled in each cycle of the survey. Second, cross-sectional surveys which sample the same population in successive cycles but with no attempt to track particular individuals from one cycle to the next. Some of the basic results have appeared in Davis et al (2001) and Davis et al (2002).

Longitudinal surveys provide data in the form of transition frequencies between the states of X . In Chapter Two we develop a method for modelling and estimating the one-step transition probabilities in the case where X is a non-homogeneous Markov chain and transition frequencies are observed at unit time intervals. However, due to their expense, longitudinal surveys are typically conducted at widely, and sometimes irregularly, spaced time points. That is, the observable frequencies pertain to multi-step transitions. Continuing to assume the Markov property for X , in Chapter Three, we show that these multi-step transition frequencies can be stochastically interpolated to provide accurate estimates of the one-step transition probabilities of the underlying process. These estimates for a unit time increment can be used to calculate estimates of expected future occupation time, conditional on an individual's state at initial point of observation, in the different states of X .

For reasons of cost, most statistical collections run by official agencies are cross-sectional sample surveys. The data observed from an on-going survey of this type are marginal frequencies in the states of X at a sequence of time points. In Chapter Four we develop a model based technique for estimating the marginal probabilities of X using data of this form. Note that, in contrast to the longitudinal case, the Markov assumption does not simplify inference based on marginal frequencies. The marginal probability estimates enable estimation of future occupation times (in each of the states of X) for an individual of unspecified initial state. However, in the applications of the technique that we discuss (see Sections 4.4 and 4.5) the estimated occupation times will be conditional on both gender and initial age of individuals.

The longitudinal data envisaged in Chapter Two is that obtained from the surveillance of the same sample in each cycle of an on-going survey. In practice, to preserve data quality it is necessary to control respondent burden using sample rotation. This is usually achieved using a mechanism known as *rotation group sampling*. In Chapter Five we consider the particular form of rotation group sampling used by the Australian Bureau of Statistics in their Monthly Labour Force Survey (from which official estimates of labour force participation rates are produced). We show that our approach to estimating the one-step transition probabilities of X from transition frequencies observed at incremental time intervals, developed in Chapter Two, can be modified to deal with data collected under this sample rotation scheme. Furthermore, we show that valid inference is possible even when the Markov property does not hold for the underlying process.