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PT-Symmetry of Coupled Fiber Lasers

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ABSTRACT

In this work, we propose a concept of a coupled fiber laser exhibiting PT-symmetry properties. We consider a system operated via Raman gain. The scheme comprises two identical fiber loops (ring cavities) connected by means of two fiber couplers with variable phase shift between them. We show that by changing the phase shift one can switch between generation regimes, realizing either PT-symmetric or PT-broken solution. Furthermore, the paper investigates some peculiarities of the system such as power oscillations and the role of nonlinear phase shift in fiber rings.

Keywords: fibre laser, PT symmetry.

1. INTRODUCTION

The concept of parity-time symmetry in optics attracts great deal of attention. Recently, a parity-time symmetry-breaking microring laser has been experimentally demonstrated[1,2]. The latter paper considered two coupled microrings, one ring with losses and another one with gain. A single-mode operation in an initially multi-mode system was achieved through stronger mode discrimination close to a PT-symmetry breaking transition. Further on, a single transverse mode operation in a system of coupled microring lasers was demonstrated near the exceptional point[3]. In a pair of coupled microdisk quantum cascade lasers, the reversal of generated power dependence was identified in the vicinity of exceptional points[4]. Further on, a realization of the PT symmetry based mode-locking has been theoretically proposed[5], and PT fiber cavity laser was developed[6].

In this work, we propose a concept of a coupled fiber lasers exhibiting PT-symmetry properties. We consider a system that comprises two fiber loops and operates via Raman gain. We propose and investigate mathematical model of such system. In particular, we show that by changing the phase shift one can switch between generation regimes, realizing either PT-symmetric or PT-broken solution. We also investigate some peculiarities of the system under consideration such as power oscillations, stability of system eigenmodes and the role of nonlinear phase shift in fiber rings.

2. LASER LAYOUT

The scheme of PT-symmetric fiber system is shown in Fig. 1. The scheme comprises an infinite chain of segments with gain and loss inside each of them. Sequential segments are connected with each other by means of pair of 50/50 fiber couplers with phase shift between them. A single period of such system is shown in Fig. 1. Let's note that the scheme shown in Fig. 1 coincides with itself after the mirror reflection and reversal of propagation direction of the radiation, indicating PT-symmetry of the whole system. (Here P stands for parity operator which reflects space coordinate, whereas T is time reflection, i.e. changing t to -t.)

For practical implementation in the form of a fiber loop system, we can map a chain period shown in Fig. 1 into a circle by connecting fiber ends on the right to left-side fiber ends. Moreover, the system can be further shortened almost twice by means of cross-connection of fiber ends as shown in Fig. 2. Let's stress that the systems shown in Fig. 1 and Fig. 2 are equivalent from the point of view of radiation that travels into optical fibers. Indeed, in the case of Fig. 2 the light passes the same sequence of optical element as in the case of infinite fiber chain shown in Fig. 1.

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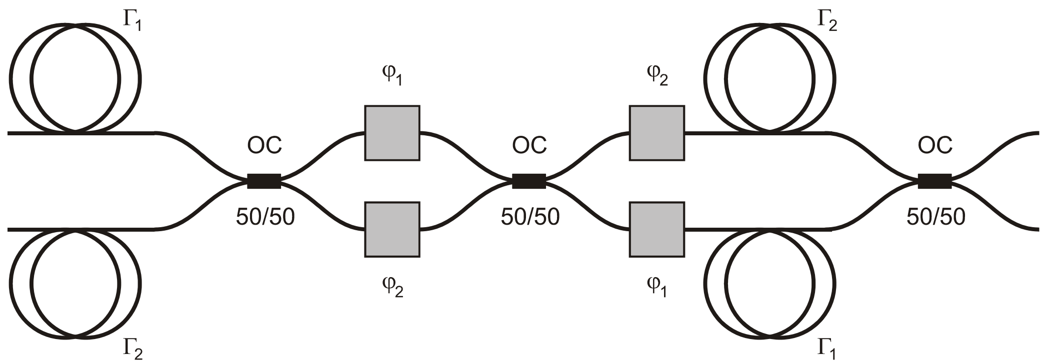

Fig. 1. Single period of an infinite PT-symmetrical optical circuit. Γ_1 and Γ_2 – optical gain and loss, φ_1 and φ_2 – variable phase shifters, OC – 50/50 optical couplers.

In what follows we consider the scheme shown in Fig. 2 that comprises two fiber loops, one with gain Γ_1, another one with loss (optical attenuator) Γ_2, connected to each other by means of a pair of optical couplers with phase shift between them. Varying phase shifts φ_1, φ_2, one can change coupling ratio between two fiber loops. Thus, when $\delta\varphi = \varphi_1 - \varphi_2 = 0$ the loops are decoupled. In contrast, when $\delta\varphi = \pi$, the loops are fully coupled so that 100% of radiation exiting the first loop enters the second one and vice versa. As we'll see below, when the coupling between loops is strong enough, PT-symmetric lasing can be achieved, providing symmetrical power distribution between two fiber loops. In contrast, when coupling is weak, PT-broken lasing regime is realized, with the power inside amplifying fiber loop higher then the power inside the attenuator loop.

Let's note that the proposed PT-symmetric fiber laser shown in Fig. 2 is pretty simple and can be easily assembled from conventional optical fiber components. Actually, the basic element of such a scheme comprise. In practical implementation, however, one should take care of possible de-phasing processes from various sources including noise transfer from partially incoherent pump wave to the signal (generation) wave[7] and formation of patterns via internal correlations[8].

In the following, we study in more details intriguing lasing properties of this scheme.

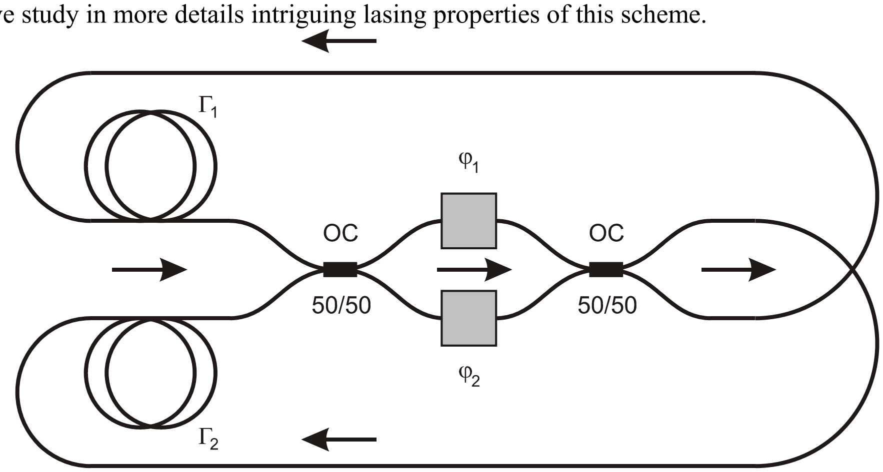

Fig. 2. Proposed scheme of PT-symmetrical fiber laser. Γ_1 and Γ_2 – optical gain and loss, φ_1 and φ_2 – variable phase shifters, OC – 50/50 optical couplers. The arrows indicate light propagation directions. Amplifying optical fiber Γ_1 requires optical pump module (not shown in this figure.)

3. MATHEMATICAL MODEL

In order to describe lasing properties of the novel fiber laser system under consideration, we use a transfer matrix method. Evolution of radiation amplitudes vector (u_1, u_2), where u_1 and u_2 are radiation amplitudes inside gain and loss fiber loops, is described by means of 2x2 matrix, defined as a product of 2x2 matrices of separate optical fiber elements shown in Fig. 2. The correspondence of the optical elements and matrices is as follows:

$$A_{OC} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix},\quad A_{\varphi} = \begin{pmatrix} \exp(i\varphi_1) & 1 \\ 1 & \exp(i\varphi_2) \end{pmatrix},\quad A_{\Gamma} = \begin{pmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{pmatrix},\quad A_{CC} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{1}$$

where A_{OC} stands for optical coupler matrix, A_{φ} describes phase rotator, A_{Γ} is matrix of optical amplification and losses, A_{CC} stands for matrix of optical cross-connection. Since the most conventional way is to write vectors to the right of matrices so that the last matrix in the product transforms the vector first and the first matrix in the product acts the last, the matrices of aforementioned optical elements should be multiplied in the reverse order. It yields the matrix A of entire optical system

$$A = A_{CC} \times A_{OC} \times A_{\varphi} \times A_{OC} \times A_{\Gamma}\,. \tag{2}$$

The matrix A (2) describes cavity round-trip evolution of optical field amplitudes (u_1, u_2) at the beginning of gain/loss segments of fiber cavity shown in Fig. 2. Cross over coupling facilitates PT symmetry with static elements, whereas active modulation was required is previous realizations of mesh lattices[9].

Let's examine the properties of the proposed PT-symmetric laser system under different operating regimes.

4. LINEAR AMPLIFICATION

Despite the fact that lasing is a substantially nonlinear optical phenomenon which occurs at the balance of optical gain and losses, it's always useful to start examination with the case of a linear system. Such approximation may be valid for real laser systems at the beginning of a lasing process. Thus for the sake of simplicity, we consider all coefficients in Eq. (1) to be constant, *i.e.* independent on intra-cavity field amplitudes (u_1, u_2), including amplification coefficient Γ_1 as well. This means that we consider a linear optical system that transforms optical field amplitudes (u_1, u_2) always in the same way, irrespective of the amplitudes (u_1, u_2). In order to understand how such system acts, we find the linear eigenmodes as:

$$A\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \lambda\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \tag{3}$$

Then, we find two eigenvectors ($u_1^{(1)}$, $u_2^{(1)}$) and ($u_1^{(2)}$, $u_2^{(2)}$) that do not change after a full laser cavity round-trip, but are only multiplied by corresponding eigenvalues $\lambda^{(1)}$ and $\lambda^{(2)}$. We solve the eigenvalue problem according to Eq. (3) in the case $\Gamma_1\,\Gamma_2 = 1$, *i.e.* when optical gain Γ_1 compensates exactly total optical losses Γ_2. As a result we find that following cases for eigenvalues can be realized depending on the ratio of other parameters such as differential phase shift $\delta\varphi = \varphi_1 - \varphi_2$:

1. If coupling ratio between two fiber loops is strong enough, optical gain compensates losses, meaning that the optical power does not change after cavity round-trip and both $|\lambda^{(1)}| = 1$ and $|\lambda^{(2)}| = 1$. Two eigenvalues $\lambda^{(1)}$ and $\lambda^{(2)}$ differ from each other only in complex arguments, i.e. different phase shifts are acquired by two eigenmodes.

2. If coupling ratio between two fiber loops is weak, two eigenvalues differ in absolute values: $|\lambda^{(1)}| > 1$ and $|\lambda^{(2)}| < 1$. This means that the first eigenmode ($u_1^{(1)}$, $u_2^{(1)}$) exhibits exponential growth, whereas another one ($u_1^{(2)}$, $u_2^{(2)}$) decays exponentially with time (i.e. with number of laser cavity round-trips).

Let's stress that linear amplification model admits solution $|\lambda^{(1)}| > 1$ with unlimited power growth. Therefore, such model can't be directly applied to real physical systems. However, based on the findings discussed above, one can expect existence of two different lasing regimes. The first regime is PT-symmetric lasing with equal powers at the middle of gain and loss fiber segments with possible power oscillations due to interference of eigenmodes ($u_1^{(1)}$, $u_2^{(1)}$) and ($u_1^{(2)}$, $u_2^{(2)}$). The second regime is PT-broken with only one mode ($u_1^{(1)}$, $u_2^{(1)}$) allowed, whereas the second eigenmode ($u_1^{(2)}$, $u_2^{(2)}$) vanishes rapidly and thus can't contribute to stationary lasing. The first (PT-symmetric) case is expected to be realized at relatively large differential phase shift $\delta\varphi = \varphi_1$-$\varphi_2$, whereas the second one (PT-broken regime) should be found at relatively small $\delta\varphi$.

5. SATURABLE GAIN

In order to make the mathematical model discussed above applicable to real laser systems, one should take into consideration gain saturation which plays a key role in real lasing. Indeed, when intra-cavity power is small, the laser amplifies it rapidly, but when the power becomes too high, amplification saturates, resulting in power stabilization (instead of unrealistic power growth within the linear model discussed above). We consider gain saturation according to a simple phenomenological law:

$$\Gamma_1 = \frac{\Gamma_0}{1+|u_1|^2}, \quad (4)$$

where Γ_0 is unsaturated gain coefficient, $|u_1|^2$ is the power at the beginning of gain fiber segment of the cavity, see the laser scheme in Fig. 2. It should be stressed that saturated amplification coefficient Γ_1 according to Eq. (4) depends on intra-cavity power $|u_1|^2$, meaning that the matrix A defined according to Eq. (2) becomes nonlinear, *i.e.* it depends now on the solution vector (u_1, u_2).

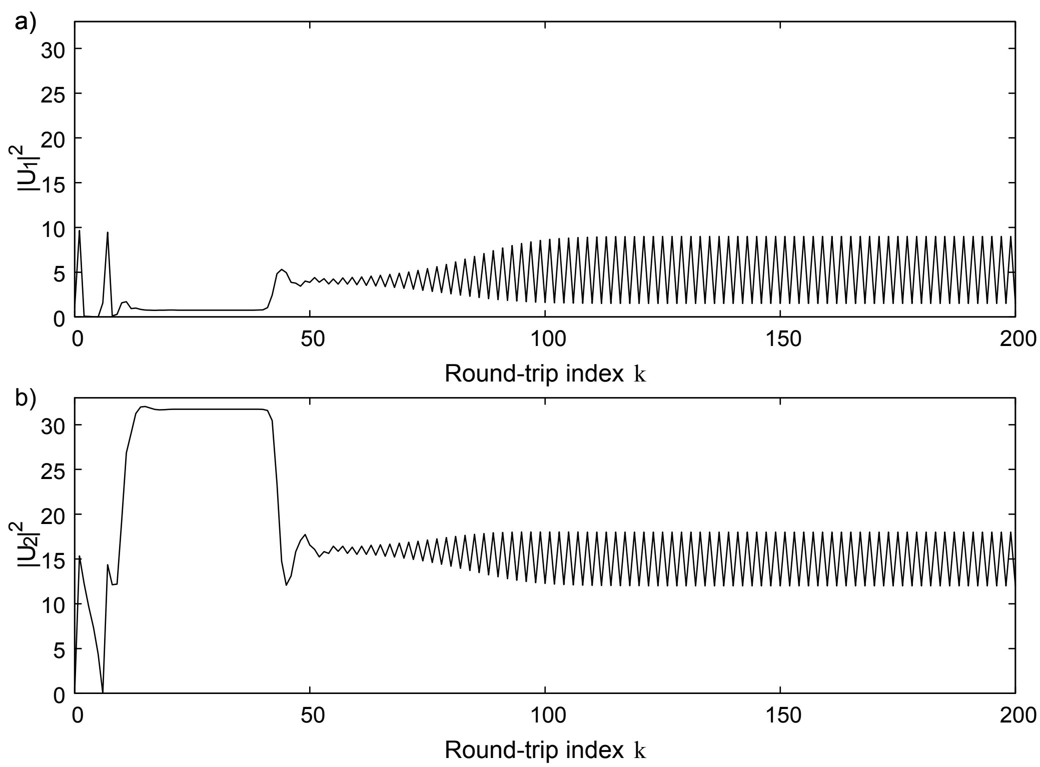

Fig. 3. Temporal power dynamics in (a) amplification segment and (b) loss fiber loop.

In order to investigate nonlinear dynamics within this model, we used the following parameters: differential phase shift $\delta\varphi = 1.8$, small signal gain coefficient $\Gamma_0 = 10$, loss coefficient $\Gamma_2 = 0.5$.

Using these parameters, we investigated temporal power dynamics using initial conditions ($u_{0;1}$, $u_{0;2}$) = (1, 0). In particular, we evaluated field amplitudes at successive passes according to the relation

$$A\begin{pmatrix} u_{k+1;1} \\ u_{k+1;2} \end{pmatrix} = \lambda \begin{pmatrix} u_{k;1} \\ u_{k;2} \end{pmatrix}. \quad (5)$$

Here the first index k denotes the number of round-trips made by radiation inside the cavity, whereas the second index (1 or 2) designates loop inside the cavity (either amplifier or loss segment of fiber laser shown in Fig. 2.) Simulation results (dependence of power $|u_{k;1}|^2$ on round-trip index k) are shown in Fig. 3.

It could be seen in Fig. 3 (a) that power at the beginning of amplification fiber loop (as well as complementary dynamics plotted for loss fiber loop, see Fig. 3 (b)) demonstrates complex dynamics. First of all, the power increase occurs (the lasing process starts within initially "cold" laser cavity.) After approximately 20 cavity round-trips the power stabilizes for a while. However this lasing point (eigenvector of matrix A) is not stable and the laser evolves away from it after 20 more cavity round-trips. After some transitional process that takes approximately 50 round-trips, the laser settles onto infinite oscillatory regime as can be seen in Fig. 3. Similar oscillations were expected *a priori* as a result of interference of two PT-symmetric laser eigenmodes. However, more detailed study allowed us to find out that the nature of these oscillations is different and comes from peculiarity of model (4) of gain saturation. Indeed, when the power $|u_1|^2$ becomes two high, amplifier introduces large losses instead of amplification. In particular, when $|u_1|^2 > 19$, optical losses in amplifier loop become larger than those of attenuator loop. It results in severe power drop at the next round-trip. Further on, the amplifier becomes unsaturated and the power grows again. This cycle repeats infinitely what can be readily seen in Fig. 3. Let's note that from physical point of view, amplifier can introduce optical losses instead of amplification in the case of insufficient pump power (or redundant signal power.) However it's not a typical case realized in fiber lasers in experiments. In order to make our results more precise, let's consider another model of gain saturation instead of Eq. (4):

$$\Gamma_1 = \exp\left(\frac{g_0}{1+|u_1|^2} - g_h\right), \tag{6}$$

where $g_h = 1$ dB is amplifier loss at high power limit, and (g_0-g_h) is amplification in opposite limit of low-level signal power. Let's note that within this model amplifier introduces relatively small optical losses when signal power becomes too high what prevents oscillation of sum intra-cavity power discussed above. Similar power dynamics within improved model of saturated gain Eq. (6) is shown in Fig. 4.

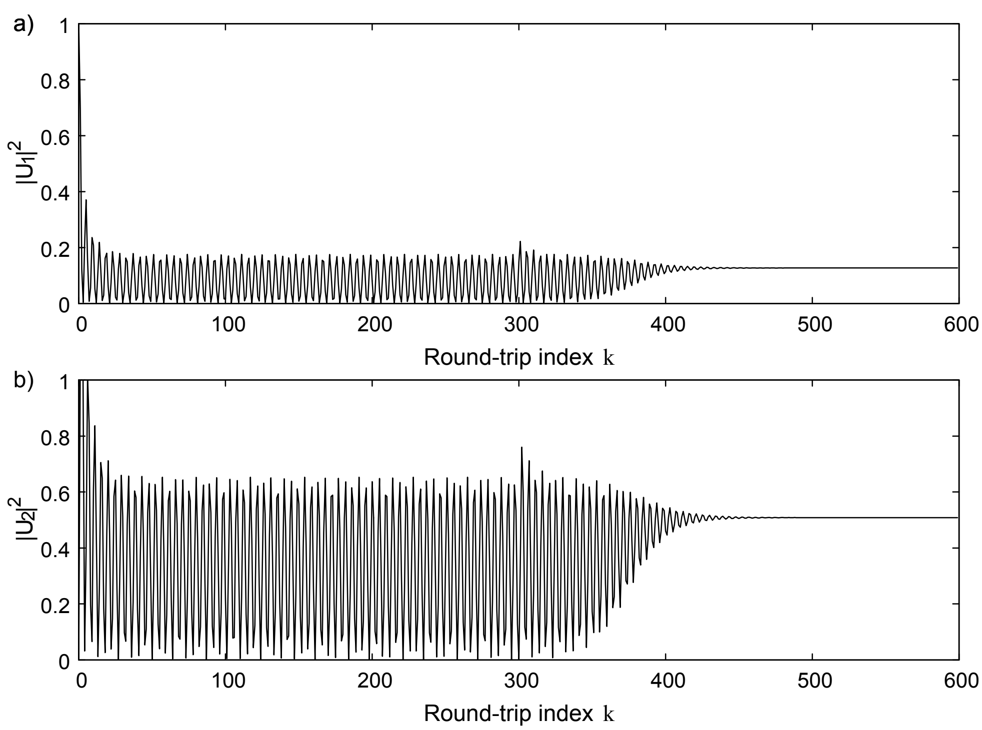

Fig. 4. Temporal power dynamics in (a) amplification segment and (b) loss fiber loop within improved model of saturated gain Eq. (6).

We see in Fig. 4 that as the lasing process starts, power in both fiber loops demonstrates oscillations during first 400 round-trips. These oscillations are due to the interference of two laser eigenmodes: at each moment laser state is a superposition of two eigenmodes that have different phase velocities, resulting in power oscillations. Finally oscillation amplitude decreases and power in both fiber loops becomes constant, meaning that the laser reaches a pure state (single eigenmode). Our investigation showed that both PT-symmetric eigenmodes are linearly stable so the whole laser dynamics is bi-stable. Thus the laser dynamics converges to a single pure state (eigenmode), either first or second, after a large number of round-trips (typically several hundreds, but in some case such intermediate oscillatory regime may last from dozens up to several thousands of round-trips.)

6. NONLINEAR PHASE SHIFT

Another important aspect of theoretical description of PT-symmetric fiber lasers is due to a nonlinear phase shift which is intrinsic to optical fiber systems. Thus let's consider additional phase shift in optical fiber amplifier caused by Kerr nonlinearity:

$$\Gamma_1 = \exp\left(\frac{g_0}{1+|u_1|^2} - g_h\right)\cdot \exp\left(i\gamma |u_1|^2\right), \tag{7}$$

where γ stands for nonlinear coefficient which is assumed to be small enough. Strictly speaking, once the nonlinear phase shift is introduced, the system ceases to be PT-symmetric. However, if coefficient γ is small enough, nonlinear phase shift can be considered as a small disturbance of PT-symmetric system. Our goal was to investigate the effect of nonlinear phase shift and to estimate appropriate level of nonlinearity in fiber laser, which still allows one to observe PT-transition phenomenon. To visualize the effect of the nonlinear phase shift, we plot the power ratio inside loss and gain segments, $P_{\text{loss}} / P_{\text{gain}}$, as a function of linear phase shift φ, see Fig. 5.

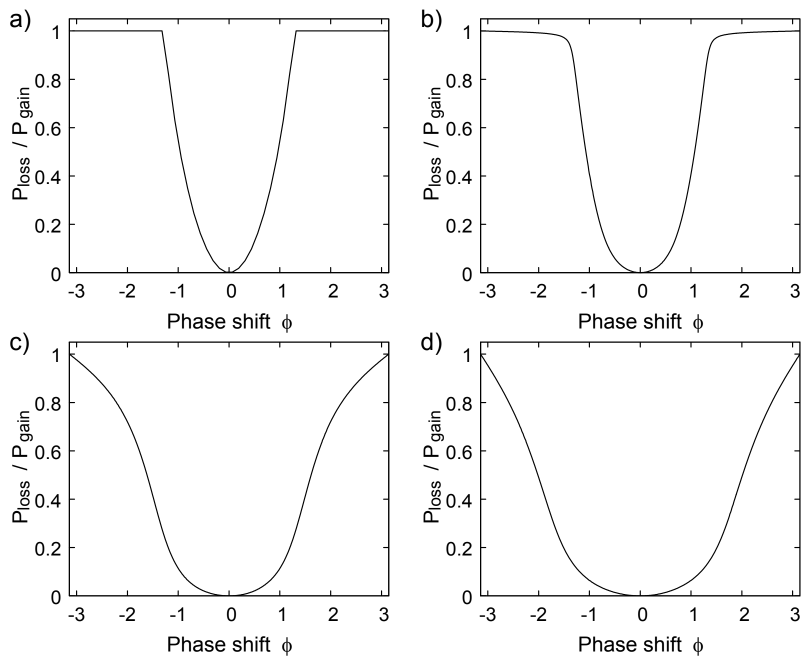

Fig. 5. Power ratio at the middle of loss / gain segments of fiber laser as a function of linear phase shift φ in (a) linear and (b), (c), (d) nonlinear ($\gamma = 0.1$) case at different levels of small-signal gain g_0: (b) 4.1 dB, (c) 7.55 dB, and (d) 11 dB.

We see from Fig. 5 (a) that in the case of purely linear phase shift there is a distinct boundary between the PT-symmetric and PT-broken regions. Specifically, relatively small phase shifts $|\varphi| < 1.2$ correspond to PT-broken states whith P_{loss} / $P_{\text{gain}} < 1$, where energy is concentrated in the amplifier segment. At larger linear phase shifts φ, the ratio P_{loss} / P_{gain} equals to unity, and such symmetric power distribution in fiber loops corresponds to PT-symmetric lasing mode. Thus, one can switch the laser between PT-symmetric and PT-broken lasing regimes by means of changing the phase shift φ between couplers and thus controlling the coupling ratio between amplification and loss segments of fiber laser.

In a nonlinear case close to lasing threshold [see Fig. 5 (b)] we observe a similar picture. The key difference between Figs. 5(a) and (b) is in fact that the latter has a smooth bound between PT-symmetric and PT-broken lasing regimes. As lasing power grows with small-signal gain g_0, the bound becomes more and more smooth, see Figs. 5(c) and (d). Let's stress that the ratio P_{loss} / P_{gain} is not equal to unity anymore, meaning that the system is not PT-symmetric in the strict sense. However, from the point of view of real experiments where lasing power is measured with typical accuracy of 5%, such system may look identical to "ideal" PT-symmetric system with no nonlinear phase shift in the amplifier.

We also see in Figs. 5(b-d) that the more is lasing power, the less is the area of quasi-PT-symmetry. The results of our simulations suggest that the nonlinear phase shift must be less than 0.15 in order to obtain a width of PT-symmetric area at the level of 20%.

7. CONCLUSION

In conclusion, we suggest a configuration of fiber loop laser which can be operated both in PT-symmetric and PT-broken regimes. Our analytical analysis and numerical modeling demonstrate that fiber laser can be switched between these regimes by changing a phase shift between fiber couplers. We identify power oscillations within the model and reveal that their nature is related to the oversaturation of an amplifier. Using improved phenomenological model of fiber amplifier, we demonstrate that PT-symmetric lasing in such system should be free of power oscillations, converging to a pure single spatial eigenmode after large enough time (typically several hundreds of cavity round-trips). We also outling the role of nonlinear phase shift and show that it should not exceed the level 0.15 in experiments in order to make PT-transition clearly visible.

Next possible steps are more rigorous study of the system by taking into account propagation effects within each fibre loop. Towards this, we will use a set of nonlinear Schrodinger equations describing successfully the interplay between pump wave and signal (generation) wave in long fiber lasers[10].

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