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Multistable Switching Series in Chaotic Nets

Robert A. M. Gregson

Abstract: Examples of conscious and interpretable responses that have two or more forms alternating to the same stimuli have been known for centuries, and methods of describing how such situations arise have evolved in biological science. When switches between transient, perceptual or cognitive responses can occur and are mixed serially within time series exhibiting local terminal stability, then patterns arise where psychological data series are too brief to analyse empirically, and neurophysiological data and mathematical simulation are necessary. Modelling such conditions can be approached by using one modified Markov matrix, which we illustrate if we allow some singularities to exist in the dynamics. As soon as networks cease to be homogeneous and have a number of attractors present and operate with different local structures, then one or more response patterns may potentially exist at the same time. The patterns may be addressed within the behavioural dynamics by incorporating in turn very short transients, that can be voluntary or involuntary, in sensory and cognitive data. Related software work for modelling, employing hierarchical Dirichlet structures projected into hidden Markov matrices is noted.

Key Words: Multistability systems, Time Series, Switching, Chaos, Networks.

Correspondence address: Robert A. M. Gregson, Department of Psychology, Australian National University, Canberra, ACT 0200, Australia.
e-mail: ramgdd@bigpond.com

"Do you see yonder cloud that's almost in shape of a camel?...
Very like a whale?"

Shakespeare, W. (1600) *Hamlet, Act III, Scene II*, lines 393-398.

Our topic has a long and evolving history. At least four hundred years ago we knew that the picture of vague sensations that are alternatively switched from one remembered and associated image to another is something easily created by suggestions. Clouds provide us with images of fractal boundaries such that their size in any physical dimension or distances cannot be simply estimated by the observer, and vary over time so that similarities between images can change for observers. Yet there is a difficulty in entertaining two images at the same moment, it is rare for the observer to see at one instant a mixture of a camel and a whale. Within neural networks are memories that store simultaneously, for rapid recall, records of diverse patterns that in multistability become sensations that might apparently alternate. So we have to attempt to explain what have been known to us for a long time, things switch almost instantaneously, and some things cannot be simultaneous. About 200 years ago in the beginnings of experimental psychology we looked in more detail at such switchings. Based on modern neurophysiological data, from the last half century, we became able to record events in our neural bases that are there at the same time and take time to change, involving vast numbers of connected elements within networks.

The developments of theories about nonlinear dynamics have increased our interest in what happens when information is stored and processed in networks, the networks can be employed as explanations of mental brain activity, and in particular we can simulate networks where the elements within them are variable in their connectivity and are discontinuously fluctuating. Neural networks can have some properties that resemble perceptual and memory data. We have shifts between different subsystems, the changes in those subsystems can be very abrupt and the jumps are so brief that they cannot yet be reliably explored by neurophysiological recording such as EEG. The jumps can sometimes be treated as bifurcations, and at a bifurcation there is a singularity. External sensory observations may or may not be reversible in the changes associated with the perceived magnitudes of sensations, and they can persist for relatively transient epochs and therein be stable, yet at the same time their dynamics appear to be chaotic in their detailed sequences (Zumdieck, Timme, Geisel & Wolf, 2004, Liu, Cao & Huang, 2010).

We are specifically concerned here with concepts, sensations and perceptions that exhibit multistability, and can make transitions between them *almost* instantaneously. It is necessary to note the qualification of *almost* because there are limitations on speeds of synchronization (Timme, Wolf & Geisel, 2004). Particularly we need to distinguish simple descriptions of visual human experiences, in the form of linear stationary time series models, from the processes in neural networks that underlie those sensations and may be fractal or chaotic. Three levels used to describe processes we designate as macro, meso and micro theories, a distinction that is not original or unique to us, though it is used in by others in different contexts, not necessarily about multistability. Multistability can be distinguished as having three levels of precision:

Macrostability Behavioural response outputs, using continuous time series that only encode terse jumps between symbolic (not quantitative) encoding of locally stable states. Data, expressed in ordinary language, from overt perceptual or sensory experiences, are sufficient.

Mesostability Psychophysical data that distinguish magnitudes above and below threshold levels in both noise and output response signals. Psychological and physiological processes coexist. Chaotic subseries arise.

Microstability Representing internal network dynamics with a combined use of EEGs and fMRI series in cellular neural networks. Chaotic components of multivariate internal process series arise. Stability can arise in irregular networks when the dynamics are complex (Jahnke, Memmesheimer & Timme, 2008).

The term micro-meso-macro is also used by Liljenström and Svedin (2005) in a related but different sense, where micro can be used to refer to quantum dynamics within brain cells. They focussed on the level "in between"--the meso level, through a number of cases from different domains of science, including physical and social sciences, economics and technology. There are facets regarding meso-level issues that are similar between domains, but also that the domains differ in various ways. This was particularly exemplified by the differences in perspectives from which the natural and social sciences deal with scaling issues. I am not here concerned with physical activity within single neurophysiological cells, as distinct from cellular automata on aggregates of neurocells (Grush & Churchland, 1995).

This three level approach was also employed (Mountcastle & Werner, 1964) quite early in neuroanatomy to distinguish stimulus-response relations, Weber functions, and information transmission. There are more differences between multistability that arise in these three levels, for example a very simple form is created by competent musicians who can switch at choice between two different languages, alternating when singing a song. Involuntary shifts are modelled by some researchers using an analogue of sand heap slipping or neuronal ‘avalanches’(Kitzbichler et al, 2009). The most complex shifts are exemplified in activity where “the brain is spontaneously at a continuously changing (in space and time) intermediate state between two extremes, one of excessively cortical integration and the other of complete segregation”, such happens in variable levels of coarseness in networks, even during sleep (Expert et al, 2010).

What is employed in the three models labelled here is symbolic dynamics grounded in multivariate time series, and if the models can exist in combination then we have a time series M mapping into a complex of levels m, which if each transits in a time $\odot$ coded as 0,1 they can have seven forms (0,0,0 is omitted),

$$\text{ts}\, M \subset ma \odot me \odot mi$$

that distinguish three patterns of possible multistability interacted at mixed levels..

Observed behavioural multistability.

Our sensations of physical events can be experienced as so stable that they are communicated and are reported as existing independently of the observer. Some of them are stored with limited completeness and rate, and expressed as the subjective confidence in their recall (Rabinovich, Huerta, Varona & Afraimovich,2008). Much of the new mathematical modelling of networks was not primarily developed to represent psychological examples. Here we want to focus on looking at them when they have abrupt jumps, in discontinuities and switching between at least two subsystems (Freeman & Holmes, 2005). *Metastability* means here the ability to jump between states. *Multistability* means the existence of a multiplicity of states. Some previous forms have been reviewed by Gregson (2011), and this paper is a sequel, extending theories by using Markov matrices. Using state-space discrete-time dynamics but without Markov matrices had been importantly employed by van Leeuwen, Steyvers & Nooter (1997) to introduce chaotic dynamics supporting stability switching. Some of the jumps, and some of the subsystems internally may be chaotic. We want to suggest that such phenomena are intrinsic to human activity, and replacing such accounts with stationary linear time series are intrinsically ineffectual. The distinction between voluntary and involuntary switches between transiently stable systems clearly arises in perceptual or cognitive processes, and when the generation of networks in the brain involves complex multistability the observer may be able to select and control a desired state, even in the presence of a high noise level (Gadaleta & Dangelmayr, 2001), if the dynamics are chaotic and learning can occur.

The default option in Metastability

What is called the *resting* state of the brain, one in which there is no explicit external observable stimuli or responses, is in fact very active, but in a different way in terms of its mental dynamics, more detail is needed on switching from one cognitive state to another via transient apparent inactivity. There is in resting activity resembling an Ising lattice, a network that is fractal with a transition between order and disorder (Fraiman, Balenzuela, Foss & Chialvo, 2009). This pattern is a *default mode* (Fair et al, 2008) incorporating a collection of metastable states. As soon as any cognitive task progresses then the default mode is shut down, it is immediately replaced by a pattern that can persist for some stable time series. The switches discussed here are activities replacing the metastability in the default mode by Macrostability, but as a jump across a threshold of correlated networks. The correlations are mixed positive and negative, and usually in balance except in some brain pathologies detected in fMRI data (Wink et al, 2008). So we might tentatively predict that a multistability skip will tersely pass through a metastable default mode, reappearing as a macrostable state supporting recognizable cognitive functioning.

Simple Voluntary and Involuntary Multistability

It is illustrative to begin with simple perceptual examples, where the role of internal oscillations is not detected or tracked, first with examples that show the experienced multistability is under control of the observer and reversible or static, the switches when they occur are almost instantaneous. The number of alternative multistable patterns is two or more, it can be perceptual or social (Collins, Pecher, Zeelenberg & Coulson, 2011). The next case of switching we may illustrate is involuntary once it begins to occur, and becomes in time almost of regular periodicity. Golland, Golland, Bentin & Malach (2008) shows that we have two global systems and the distinctions in usage of extrinsic and extrinsic processing of sensation correspond, and are reflected in fMRI data. That resembles the earlier discussion of Attneave (1971) that was cited by Feudel (2008) and again by Gregson (2011). Then both real and simulated data on internal oscillations are brought into network theory, and chronicity and phase locking are involved (Laurent et al, 2010).

Many of us must have seen a picture, drawn in flat perspective, of a staircase, as reminiscent of the work of the Dutch artist Escher. It was first created in 1854 by Schröder's staircase, following observations by Necker's lattice in 1832. It has been reproduced many times, and related familiar images used in perception were clearly recently displayed by Pitts, Nerger & Davis (2007) and by Ehm, Bach & Kornmeier (2011).
We may see it in three different ways, as a zig-zag picture running diagonally across the page, or as a view of one as we are walking up or down the staircase and looking down on the steps, or as standing on the floor below and looking up at the underside of the staircase. When we first see it, perhaps as a child, it may just be a pattern. Once it is interpreted for us as viewing the solid zig-zag surface it becomes alive as a view either from above or below, and we may alternate in how we see it. The interest that is being studied here is the almost instantaneous transit between above and below the two postures. In the staircase picture we can revert to the total flat view, but in multistability processes there is no third representation, if we are actually seeing a physical staircase, that is sometimes built spirally within a lighthouse, then seeing it as flat is an illusion. There are now manyexperienced examples of abrupt and reversible processes, the modern interest here turns to cases in mathematical networks in neural processes, that can display nonlinear dynamics.

Wet-ware within the brain and nervous system exists to process information, to collect, filter, store, process, integrate, transform, control and project information about data to its own organism, and to other organisms with compatible structures. We are both individual and social animals, sharing environments both organic and inorganic. It doing all this we have serious limitations in capacity and structure, and the price of transforming data that is compatible with our environments is that we generate added error and nonsense in surprising levels. The development of computer storage and structure means that we now can and do interact and augment our behaviour with systems that have vastly more data and less of some types of errors.

Cassac expresses a central issue we focus on in this review, when he notes "non-linearity may induce effective paths among units that are not directly connected to the graph of interactions. This type of behaviour is not specific to weakly-coupled neural networks,.., we shall exhibit a similar behaviour for a chaotic Neural Network." (Cassac & Samuelides, 2007, p. 42, Cassac, 2010). An exploration of stage transitions in nonlinear dynamics that also involves chaos has been anticipated by Molenaar, Newell, Hartelman & Raijmakers (1999), and a review of relevant sources in non-stationary time series which surveys processes like our problem, but without an emphasis on abrupt shifts, is pertinent (Molenaar, Sinclair, Rovine, Ram & Corneal, 2009).

We noted that multistability describes a situation where there are two or more of such units and they jump from one to another, with each while functioning retaining its recognizable properties, including its oscillations and bifurcations within its micro dynamics. The network as a whole has to be structurally unstable in order to exhibit real nonlinear effects and to be relevant for representing brain dynamics. Such unstability is described in some various forms of what is called synaptic plasticity, an idea that was early used by Hebb (1949) in modelling learning, back at that time chaotic dynamics and nonlinearity were not central ideas and multistability (Attneave, 1971) was not linked to neural plasticity in mathematical models, particularly in time series analysis (Gregson, 2006).

Attneave's first usage of multistability was specifically in the examination of vision, in describing what happens over time as the observer tries to report, in stability, the perception when viewing normally the projecting plane image of a wire-line diagram of the 12 edges of an empty cube, slowly rotating before a blank white field. This study involves the transformations of two-dimensional images into three-dimensional percepts, and three-dimensional images into two-dimensional percepts, all having a short stable period and abrupt transformations back and forth. There exists a number of contextual variables, as reviewed by Dodwell and Caelli (1984), which affect the content and stability of transformations, and the very problem of using dimensionally fixed integer assumptions in modelling dynamics was questioned by Marr (1982) who wrote of two-and-a-half dimensionality in vision with depth and distance cues.

How extending the question of multistability to the analyses of senses other than vision requires us to come back to the neurophysiology of networks that are not based solely in vision, and hence not locally based in those specialised parts of the brain. A related example, of fluctuations in the sequential perception of auditory sounds of short ambiguous words, is known (Ladefoged & Broadbent, 1960). Obviously we can also construct sequences, as in music, that have deliberately structured repeated multistable patterns (Geake & Gregson, 1999).

Since the first offered definitions of multistability were produced, our understanding of chaotic dynamics has become more extensive, and is expressed pertinently by Timme, Wolf and Giesel (2003, p.378) when they write

"In general, it is hard to leave a stable attractor after convergence, e. g., the complement of a task. With an unstable attractor, however, a small perturbation is sufficient to leave the attractor and to switch towards another one."

Gayler and Levy (2011) note that localist connectionist nets, in what here would be the microstructures underlying the mesostructure states that are representable in hidden Markov dynamics, cannot make switches sufficiently rapidly to be physiologically plausible. Therefore the nets have instead to be distributed based on symbolic architectures, which are those coherent mesostructures that respond to instabilities. It follows that experimental investigation of unstable dynamics requires exploration of a diversity of perturbations rather that seeking stable stimulus conditions. We need to explore what sorts of unstability are created in psychological circumstances that might represent real sensory, neural or social operators, and unstabilities are encoded as transitions through time series data, to which we now turn.

Time Series representations

The dynamic situations being explored here were first formalised in statistical models that did not draw on neural network theory. Some time series models with jumps in levels were covered usefully by Gössel (1982) and Tong (1983). We consider time series where the output or response series are locally stationary and locally autocorrelated, but can make abrupt jumps between two or more processes U, V, W,…, in all the possible bidirectional pairs such as U ↔ V. The length of a local series is variable, the duration of the jumps is taken as inaccessible and instantaneous in time, the jumps are in space and can be measured in metric relative distances. Such processes are used in cusp catastrophes by Guastello, Koopmans & Pincus (2009, p.25) and in threshold autoregressive series by Tong (1983).

Psychophysics, here put within the mesostability level, can be built with time series of complex cubic dynamics (Gregson, 1996), and hence to some but not all levels of inputs the process responds with bifurcation and fractal activity. The macro perceptual example that Attneave used to illustrate multistability involves an input that is ambiguous, and hence a potential basis for jumping between structures. So the psychophysics can have multiple ambiguities fed into its dynamics that are themselves instable and have multiple series, grounded in the networks where they are stored in microinstability. We need some or all of the interactions that are symbolised by the possible $ma \odot me \odot mi$ combinations; the empirical problem is to find out which are operating in real data.

New methods of more tractable ways for scaling time series that lead into symbolic dynamics include permutation entropy (Bandt & Pompe, 2002), and can represent non-linear transformations between the three *m* levels we have distinguished. In high-dimensional time series it is possible to use symbolic dynamics to convert ordinal patterns into pattern matrices (Keller & Wittfeld, 2004).

State Transition Probability Matrices locally accelerated.

The core of a Markov chain is the matrix of transitions between states symbolised here as (STP,M). The actual probability distribution of clock times within a persisting state to transitions between states are not built in, but can be derived by setting the value of the increment δt and the probability of persisting in one state (Gregson,1983). Only one distinction is being used here, between transitions between variable slow switching and almost instantaneous transitions. Let us consider only two main states, the argument can be extended if required to such situations as the children's game using paper-stone-scissors. The states here are S_A, S_ε and S_B, the transition probabilities $p(.)$ in STM are by definition fixed;

$$
\text{STM:}\quad t\quad
\begin{array}{c|ccc|c}
 & & t+\delta t & & \\
 & S_A & S_\varepsilon & S_{B.} & \\
S_A & p(a\rightarrow a) & p(a\rightarrow\varepsilon) & 0 & \\
S_\varepsilon & 0.5 & 0 & 0.5 & [M1] \\
S_{B.} & 0 & p(b\rightarrow\varepsilon) & p(b\rightarrow b) & \\
\text{Duration:} & \text{variable} & \text{instantaneous} & \text{variable} &
\end{array}
$$

Distributions would model the probability duration of the major states S_A and $S_{B,}$ as the t to t+ δt steps are repeatedly incremented over long series (Vissen, Raijmakers and Molenaar, 2000). The S_ε are fixed in duration as a value ε, and var(ε^2)→0 as set in STM. The STM matrix line at tS_ε is set so that the transition has no information about its past, in that sense it is like a bifurcation that does not locally predict a single outcome. If some stochastic prediction is implied, then the values 0.5, 0, 0.5 can be written as α, 0, 1-α, $0 < \alpha < 1$ .Thus the STM is like a Markov with a variation so that one transition is in three states but the rest is in two states. Instead of, as usual, predicting from the STM to long time series we examine tδ time series to see if the empirical data are like, for example,
$A_1,\ldots,A_n,\varepsilon,B_1,\ldots,B_m$ or $A_1,\ldots,A_n,\varepsilon,A_1,\ldots,A_m$ or $A_1,..,A_n,\varepsilon,B_1,..,B_m\ \varepsilon,A_1,..,A_n$.
There is no need or justification a priori to postulate any stable trigonometric component periodicity such as sin(S,δt) which has been done in some modelling. In some visual cases of switching over a long time some approximate periodicity arises due to a reactive inhibition to sensory responses.
There are other complications that would have to be represented at micro and not a meso level in creating the STM Markov [M1] for the transient state S_ε because the EEG and ERP processes, in the networks creating the states associated with the perception of bistable figures, are active is a short period of EEG gamma modulation (Keil, Muller, Ray, Gruber & Elbert, 1999, Lachaux *et al*, 2005), preceding the transitions, for at least 200 ms, and short phases of P300 occipital ERP of about 300 ms. Kornmeier and Bach (2007) and Ehm, Bach and Konmeier (2011) describe these pre-shifting processes, so state transition is extended in time at a micro level but overtly almost instantaneous, as we depicted in [M1] at the meso and hence the macro levels. Another instability that can affect the point at which a S_ε transition happens is related to the phase of gamma EEG oscillating activity (VanRullen, Busch, Drewes. & Dubois, 2011).

Mixing Markov Samples

At one time I had to analysis some survey data on brand loyalties for the consumption of things like tea, tobacco and soft drinks. One might think is this hardly an example of profound work on multistability, but it turned out that we can readily have a population that suddenly moves in consumption preferences, but over a brief time may be described by a mixture of two Markov chains on two competing brands *a, b,* which have the two simultaneous social groups with probabilities *p, q* and in 2 x 2 matrices of the consumption *a,b* transition probabilities. In it *a* is brand loyal for the *p* group and *b* is loyal for the *q* group initially before a marketing campaign starts, the remaining respondents are nearly indifferent, and the transition is from time *t* to *t+1.*

$$M(p,a) = \begin{array}{c|cc} & \multicolumn{2}{c}{t+1} \\ & a & b \\ a_t & .9, & .1 \\ b_t & .4 & .6 \end{array} \quad \text{and} \quad M(q,b) = \begin{array}{c|cc} & \multicolumn{2}{c}{t+1} \\ & a & b \\ a_t & .5 & .5 \\ b_t & .1 & .9 \end{array} \qquad [M2]$$

and if one erroneously treats the total social group (almost certainly falsely) as stationary, homogeneous and defined sufficiently by one Markov then we have a pooled Markov matrix [M3] that becomes (with then re-scaling afterwards to make the rows each add to 1)

$$M(p+q) = \begin{array}{cc} .9p + .4q & .1p + .6q \\ .5p + .1q & .5p + .9q \end{array} \qquad [M3]$$

The values of p, q, with $q = 1\text{-}p$ are unknown because the surveyor forgot to ask about their prior probabilities of a,b. What is missing is another Markov transition chain of the two social groups p,q. When we have one sort of Markov nested inside another then we have something called *hierarchical*, that resembles state-space equations. One ends up in my example with only slight spurious probabilities of brand loyalties in one model. We could for example define another problem with similar structure by replacing the notional problem with two populations one of which p is liable to infection a and the other is resistant with a drug b. There are related network theories about changes in religious affiliation (Bainbridge, 2006). For realism one must add noise as oscillation and shifts in resistance over time. If we erroneously mix statistical data into an heterogenous stationary sample it can become a platykurtic pdf with non-Gaussian β_1 and β_2 values, that might be used to reveal where something is being misidentified.

Jumping Oscillating Networks

Models that have evolved since the simpler examples noted above constitute richer structure and embrace older jumping examples as restricted special cases. The newer fuller complex examples of complex dynamics are particularly important for biological modelling, and they attempt to describe such cases as synchronized flashing of fireflies, or coherent oscillations observed in the human brain (Cross, Rogers, Lifshitz & Zumdieck, 2006). Sudden jumps in early language development are now of interest when considering intra-individual variability (van Dijk & Van Geert, 2007).

Instead of postulating two, or just a few more, alternative time series U,V that are locally stable, with one sort of reversible brief jumps, we have a large (uncounted) group of interacting dynamical components, and each on its own can exhibit complex dynamics. It is possible that a very complex network can exhibit both switching that repeatedly reverses, and points that are absorbing and terminal when they arise, with a low initial probability. That is in fact a legitimate model for slow learning. As the processes are not linear and not stationary it is necessary to explore models for very long series to find out if the switches are all reflecting or eventually terminal. It is easy to create Markov multi-state (>2) chains that will do that, as were explored in learning theory (Atkinson, 1964).

The findings in physical mathematics used here which mostly reflect oscillation series are often run to 1000 to 100,000 steps to obtain stability, and so have no certain confirmation of the possible evolution in some shorter biological series. The confusion of eventual switching and terminal absorption is similarly illustrated in the mixed Markov example [M2] given.

Stable and Unstable Trajectories

Winfree (1967) provided a coherent view of limit-cycling in oscillatory networks which is a precedent for what has become the active modelling of stable and mixed chaotic systems. After reviewing examples of mutual synchronized oscillation in self-entraining communities such as groups of fireflies or crickets, which have been reported since the 1930s, he looked finally at cases where two oscillators can exist transiently moving between one another. He observed (p. 36)

"If the loop (of the variables called the *influence* and *sensitivity* functions of the oscillators' periodicity) is so shaped that with all oscillators synchronous, the timing map has two maxima. So within limits, there is a continuum of ways to metastability partitioning the population between the two downslopes. This plasticity may camouflage a population of autonomous limit-cycling processes as a single conservative oscillator until an appropriately-timing perturbation reveals its composite character by transiently fragmenting the single rhythm into several of slightly divergent periods."

The individual organisms in each community were averaged in their pulse-coupling actions by methods such as Winfree used, but things become complicated unless the coupling is homogeneous and all-to-all. Timme and Wolf (2008, p. 1580) observe

"Biological neural networks using exact methods have, over the past decade, revealed numerous example of collective behaviours that obviously outside the standard repertoire of behaviours expected from a smoothly coupled dynamical system. These include several new and unexpected phenomena such as unstable attractors, stable chaos, and extreme sensitivity to network topology."

A single linear operator is insufficient, we need a large number of operators depending on the rank of the perturbation vector are needed to represent the dynamics of small perturbations. Pulse-like interactions play a profound role in network dynamical systems. We can have switching between irregular and synchronous dynamics (Timme and Wolf, p. 1585) as a function of the magnitude of random perturbations.

When there are a large number of operators instead of a single stability matrix, it is possible to study stability of the whole network by using the eigenvalues of the perturbations, within Perron-Frobenius matrices (Gregson, 2006, chapter 10).

The results of particular application in the cerebral cortex reveal a possibility of one type of multistability. Timme and Wolf (2008, p. 1596) "clearly demonstrate that states in which units fire in a temporally highly regular fashion and states with irregular asynchronous activity may be coexisting attractors of the same recurrent network." One thus can have systems such as populations of fireflies that interact by both sending and receiving pulses from one another. This begins to be interesting for describing human social interactions in conversations or perhaps for singing in a choir, or interactions between conflicting political groups.

Apparent Jumps and Serial Switches

Jumps of responses to synchronized nonlinear oscillations has been explored by Cross, Rogers, Lifshitz and Zumdieck (2006). Such distributions can represent individual frequencies in a otherwise homogeneous social group. The jumps are marked, complicated and variable in form, and exhibit sometimes the hysteresis that one finds, for example, in a simple cusp catastrophe.
If jumps are regular in magnitude in a homogeneous stimulus series, and presented rapidly serially, then the response is perceived as continuous, as in a Johansson illusion (Gregson, 1976, 1998). Hence jumps can be displayed in a response process when the corresponding stimuli are nonlinear and heterogenous, switching abruptly between irregular and synchronous dynamics when perturbations of neural networks are imposed (Timme & Wolf, 2008). Even more complications arise in heterogenous networks (Zillmer, Livi, Politi & Torcini, 2006) and we can have long transients that are either chaotic or stochastic-like, "the evidence of two distinct phases is rather convincing" (Zimmer et al, p.1). The transient dynamics is characterized by a negative Lyapunov exponent, which is called 'stable chaos'. Our conclusion is that dynamical analyses that are explored by using the general linear model which is not multistable will fail to reveal many of the important variations in the evolution of biological processes that underlie human behaviour. Another type of changes between levels of complexity is found in EEG patterns, when epileptic changes can induce shifts between monofractal and multifractal patterns as observed through detrended fluctuation analyses (Dutta, 2010).

Statistical analysis of Markov matrices with a priori unknown cardinality

From the perspective of statistical methods for analysing experiments, assumptions about exchangeability and homoscedacity are usually assumed and underlie the familiar methods employed in multivariate statistics. Data can only be pooled over independent respondents if multistability is either absent or taken to be fully phase-coupled and homogeneous in variance, and stable in observed autoregression. There are approaches that work to relax such constraints.
New software for hidden Markov models, that are available and applicable to cases of switching nonlinear and heterogenous processes, is evolving. The use of Markov models as given in our main analysis [M1] is built on knowing or assuming in advance how many states exist between which jumps can occur. If a priori that cardinality is unknown, it has to be discovered by data analysis. Then initially more steps in the analysis have to be pursued. Work by Fox, Sudderth, Jordan and Willsky (2008,2009a, 2009b), give some available theory and computable software (Yau, Papaspiliopoulos, Roberts & Holmes, 2011, cited programs on page 44). Simple Markov STMs on their own are insufficient.
A number of models differing in structure and complexity is given by the authors listed, which include Hierarchical Dirichlet Processes (HDP). They were designed for topics (surprisingly) ranging from the social organization of Chinese restaurants with menus restricted within a chain of franchise and customer preferences (Teh, Jordan, Beal & Blei, 2006), to applications in nucleic acid genomes, and discriminating audio records of groups of speakers. Jump systems utilise sticky HDP-HMM models, where a priori the number of dynamical modes with varying state dimensions and autoregressive orders can be left partially unspecified. Fox et al (2009b) use an example of honey bee flight changes from *wiggle* to *turn right* dance.

The models note differences between very rapid switching and slow stable series, where constraints are imposed to make transitions much slower. Processes may be nested in some linear models and may be homogeneous or heterogeneous in their Markov states. One transition probability matrix with a single dominant sticky state [1], at cell 1,1, and substates [2 – 9] in it was drafted by Fox. If it had had two dominant states then it would be more clearly like the switching in psychophysics that interests us here. It is (Fig 10, p. 23) with each set of the missing cells, in a [2 to 9] row, summed to 0.1:

	1	2	3	………	9	
1	.6	.05	.05	………	.05	
2	.2	.7		…………….		[M4]
3	.2	….	.7	………		
.						
.				.		
9	.2	…………………			.7	

The difference between new hidden Markov models in framing analyses is that we have not got initial information about the precise number of states in the transition probability matrices, and have variable hierarchical processes with two or more Markov chains, with state persistence in sticky HDP-HMM constraining rapid dynamics transitions between states. This relates to our psychological modelling of multistability shifts. A very new explanatory paper on the use of HDP-HMM with psychological application is by Chow, Tang, Yuan, Sang & Zhu (2011**)** which again covers the relevant algebra cited by Fox et al (2009) above emphasising that it makes possible to advance without the structural assumptions in traditional linear stationary Gaussian factor analysis.

Discussion

The accounts from the early 19th century scientific journals, of switching between percepts of one image based on the same sharply defined physical patterns, are different from the one example of Hamlet switching between how he saw two images in a cloud. Both samples can described as having some multistability. But it is not clear if the switching is generated at the same rate, for the same reasons within the micro and meso activities. In both series the switches are like bifurcations, outside time, and can jump instantaneously to different apparent dimensionalities in the patterns perceived.

A number of examples of apparent multistability have been described, one possible difference between them is that some appear to be involuntary when arising in human perception, whereas others can be drawn on voluntarily and have stability and indeterminate sustained persistence. If multistability is generated by human neural networks then a continuum of basic oscillating activity is fundamental. Three multistable types of systems identified by Feudel (2008) appear to be reasonable candidates for representing network-based behaviours,; they include weakly dissipative systems, such as in cardiac activity, coupled map lattices fluctuating between ordered and turbulence patterns, and delayed feedback which can arise in examples in psychophysiology, or in social learning, in networks of groups. Obviously the scales in temporal evolution, for example between individual psychophysiology and changes in sociology, are apparently very diverse. But the different scales are not fixed in chronological time units but in the dynamical scales that can be expressed in fractal self-similarity. Using networks with internal oscillating to reproduce the capacities of perceptual discrimination appears to be necessary, unfortunately in order to do what seems to be easily performed by the human actor such network models may become complex and unsatisfying (Chen, Adhikara, Yoon & Kim, 2010).

There is a temptation to differentiate between series that are reversibly multistable and switch between two or more active states, and series that have dead series interrupted by transient active periods. An example would be a series of discontinuous headaches in brain sensations and hopefully longer periods with no pains. With evidence that human activity is continuously sustained during cognitive or sensory apparent inactivity, at microstability levels the difference is useless, it is simply a superficial labelling of overt human macro- or meso-stability self-description, categorizing activity as desirable or useful, or even perhaps misleading in generating response behaviours.

There are obviously unsolved problems in describing accurately the processes of multistability that build together micro-networks and their jumps in responses. There is some attempted explanation of how the rates of transitions between states can be so quick, almost instantaneous, “only a few milliseconds” (Beggs & Plenz, 2004), and distinguishing when changes are better described as from alternative processes, rearrangements or as replacements, such as “patterns… that include non-destructive interactions, in which the moving patterns modulate each other’s states, as well as destructive ones, in which one or both of the patterns are annihilated” (Gong & van Leeuwen, 2010, p.4) in the serial responses of cognition.

A parallel can be tentatively drawn with distinctions between classical and quantum theories in what is now called *model dependent realism* and *M-theory* (Hawking & Mlodinow, 2010) and distinctions drawn here between brain neural networks and nonlinear psychophysics in what we have here called micro- and meso-stability. The descriptions of how causality functions vary with the details and complexity of models used. The minimum presuppositions can be illustrated by examples such as Conway’s the Game of Life or Wolfram’s cellular automata, and using what is called an ‘effective theory’ (Hawking & Mlodinov, 2010, p.32) for describing certain known phenomena without describing in detail underlying processes which might resemble the minimum presuppositions employed. An effective theory would in our classification be macrostability.

We still have to involve processes that are employed almost universally in brain dynamics, of which self-organized criticality has emerged (Frigg, 2003). There are recurrent themes that apply to making sense of switching in sensory processes, non-equilibrium phase transitions (Kinouchi& Copelli, 2006) and unstable attractors in networks (Ashwin & Timme, 2005), and continuously changing between states of integration and segregation, between inhibition and excitation, between synchronization and desynchronization (Ernst, Pawelzik & Geisel, 1998: Expert et al., 2010), and varied delays in transient periods of stability and attention to inputs (Weissman, Warner & Woldorff, 2009). Stationary and linear statistical methods, and assumptions that the neurocortex can perform some algebraic differentiation and integration with the same operations that we use in constructing models to describe and predict consciousness, are implausible and have even been ridiculed (Dobosz & Duch, 2010). It appears to be safe to generalise and conclude that multistability can arise because neurophysiology is active in parallel but consciousness is mostly in series. The transition into consciousness involves a loss of information as the price of stability.

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