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**Economic Solubility of the Agency Problem with
State-Contingent Production**

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1. Economic solubility of the agency problem with state-contingent production

The theory of principal-agent relationships has been used to analyze a wide range of economic problems. The most common formulation of the problem is that due to Grossman and Hart (1983). Although Grossman and Hart demonstrated the existence of a solution to the mathematical problem of deriving the optimal contract, there has been relatively little analysis of the conditions under which the agency problem is soluble in an economic sense. A situation in which the principal can do no better than to guarantee the agent a constant payment in return for zero effort is one in which the agency problem is economically insoluble, since the cost of inducing the agent to put forth positive effort exceeds the benefits of any feasible contract.

Haubrich (1994) presents a number of examples in which the optimal contract requires zero effort, so that the agency problem is economically insoluble. Quiggin (2001) derives necessary and sufficient conditions for economic solubility of the Grossman-Hart formulation of the agency-cost problem and shows that economically insoluble agency problems are likely to arise for plausible parametrizations of principal-agent relationships.

An alternative model of the moral hazard model is proposed by Quiggin and Chambers (1998). Whereas Grossman and Hart (1983) and most other treatments of moral hazard since Mirrlees (1974) have adopted a parametrized distribution formulation of the problem, Quiggin and Chambers (1995) return to the more economically intuitive state-contingent model. The difficulties observed with this model by Mirrlees are overcome by the use of a more flexible representation of production under uncertainty (Chambers and Quiggin 2000). The central idea is that producers choose not only an effort vector but a state-contingent output vector as well. Thus, for any effort bundle, a large set of state-contingent output vectors may be feasible instead of the single state-contingent output vector the traditional model imposes.

The object of the present note is to show that, under plausible conditions, the state-contingent formulation of the agency problem will be economically soluble.

2. Model

There are two states of nature, and production of a single output is uncertain (state-contingent). For a fixed vector of inputs, $\mathbf{x} \in \mathfrak{R}_+^n$, the agent's state-contingent output set is illustrated by the state-contingent production possibility frontier in Figure 1. Production in state 1, z_1 , is measured along the horizontal axis while state-2 production, z_2 , is measured vertically. The set of feasible state-contingent outputs, for given $\mathbf{x}$, consists of all output combinations on or below the production possibility frontier (free disposability of output). As drawn, the output set is convex, reflecting the assumption that for fixed input $\mathbf{x}$, state-1 production can be transformed into state-2 production only at increasing cost.

Thus, the technology operates much the same as a joint product model without uncertainty: The agent's ex post preferences are additive in returns and the vector of inputs committed to production, i.e.,

$$w(y, \mathbf{x}) = u(y) - g(\mathbf{x}),$$

where u is a differentiable, concave, strictly increasing von Neumann-Morgenstern utility function, y is the return to the agent, and g is a strictly convex and increasing function. Chambers and Quiggin (2000) show that for g of this form and a state-contingent production set like that in Figure 1, there exists a nondecreasing and convex effort-cost function $C(z_1, z_2)$ which represents the minimum of $g(\mathbf{x})$ consistent with $\mathbf{x}$ producing (z_1, z_2) . We assume that C is convex, increasing in both arguments and that $C(0, \mathbf{0}^n) = 0$, where $\mathbf{0}^n \in \mathfrak{R}^n$ is the input vector with all elements equal to zero.

The agent's maximum expected utility given state-contingent payments y_1 and y_2 and consistent with producing the state-contingent output vector (z_1, z_2) is

$$E[w(y, \mathbf{x})] = \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2),$$

where $\pi_i > 0$ is the probability of state i and E is the expectations operator.

The principal cannot observe either the agent's effort or the state of nature that actually occurs, and must therefore design the payment schedule to induce the agent to report the state truthfully. The principal's problem is:

Choose (z_1, z_2, y_1, y_2) to maximize

$$W^p = \pi_1(z_1 - y_1) + \pi_2(z_2 - y_2),$$

subject to the constraints that

$$\pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq \bar{u} \quad (T.1)$$

$$\begin{aligned} \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) &\geq \pi_1 u(y_1) + \pi_2 u(y_1) - C(z_1, z_1) \\ &= u(y_1) - C(z_1, z_1) \quad (T.2) \end{aligned}$$

$$\begin{aligned} \pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) &\geq \pi_1 u(y_2) + \pi_2 u(y_2) - C(z_2, z_2) \\ &= u(y_2) - C(z_2, z_2) \quad (T.3) \end{aligned}$$

and

$$\pi_1 u(y_1) + \pi_2 u(y_2) - C(z_1, z_2) \geq \pi_1 u(y_2) + \pi_2 u(y_1) - C(z_2, z_1) \quad (T.4)$$

T.1 is the agent's voluntary participation constraint, saying that the contract offered by the principal must yield the agent his reservation utility, which we denote by $\bar{u}$. T.2 through T.4 are the constraints arising from the requirements for truthful reporting of the states.

Quiggin and Chambers (2000) show that for any given (z_1, z_2) , with $z_2 \geq z_1$, the required payments are given by

$$y_i = (u_i(\mathbf{z})),$$

where

$$h(u_i) = u^{-1}(y_i)$$

is a strictly increasing convex function and

$$u_1(z) = \bar{u} + C(z_1, z_1) = \bar{u} + C(z_1, z_2) - \pi_2 \delta \quad (2.1)$$

$$u_2(z) = \bar{u} + C(z_1, z_2) + (1 - \pi_2)\delta \quad (2.2)$$

where

$$\delta = \{C(z_1, z_2) - C(z_1, z_1)\}/\pi_2$$

measures the agent's gain from shirking by producing (z_1, z_1) instead of (z_1, z_2) , divided by the probability of being punished by receiving the low payment y_1 in state 2.

2.1. Conditions for Solubility

We will define the moral hazard problem to be economically soluble if the principal can make non-negative profit for some $\mathbf{z} \neq \mathbf{0}$. Thus, the problem of solubility is equivalent to whether there exists $\mathbf{z} \neq \mathbf{0}$ with

$$\pi_1 z_1 + \pi_2 z_2 - \pi_1 h(u_1(\mathbf{z})) - \pi_2 h(u_2(\mathbf{z})) \geq 0$$

A trivially obvious necessary condition is that there exist $\mathbf{z} \neq \mathbf{0}$ such that

$$\pi_1 z_1 + \pi_2 z_2 > h(\bar{u} + C(\mathbf{z})) \quad (2.3)$$

Otherwise, production is unprofitable even in the first best, and there is no moral hazard problem to solve. We shall assume that holds. We then derive:

Proposition 2.1. *Let*

$$u^* = \max\{\pi_1 u(z_1) + \pi_2 u(z_2) - C(z_1, z_2)\},$$

and let $\mathbf{z}^$ be the associated output vector. Then if $\mathbf{z}^* \neq \mathbf{0}$ and $\bar{u} \leq u^*$, the moral hazard problem is soluble.*

Proof: Suppose the agent contracts to produce $\mathbf{z}^*$. Then it is necessary to show that

$$\pi_1 h(u_1(\mathbf{z}^*)) + \pi_2 h(u_2(\mathbf{z}^*)) \leq \pi_1 z_1^* + \pi_2 z_2^*$$

By convexity of h and concavity of u

$$\pi_1 h(u_1) + \pi_2 h(u_2) \geq h(\pi_1 u_1 + \pi_2 u_2)$$

and

$$u(\pi_1 z_1 + \pi_2 z_2) \geq \pi_1 u(z_1) + \pi_2 u(z_2),$$

so it is sufficient to prove

$$\pi_1 u_1 + \pi_2 u_2 \leq \pi_1 u(z_1) + \pi_2 u(z_2)$$

From 2.1 and 2.2 ,

$$\begin{aligned} \bar{u} + C(\mathbf{z}^*) &= \pi_1 u_1 + \pi_2 u_2 \\ &\leq \pi_1 u(z_1) + \pi_2 u(z_2), \\ &= u^* + C(\mathbf{z}^*) \end{aligned}$$

and the result is proved. ■

Corollary 2.2. *If for some $z > 0$, $h(\bar{u} + C(z, z)) \leq z$, the moral hazard problem is economically soluble.*

Proof: By definition,

$$u^* \geq u(z) - C(z, z),$$

so

$$h(u^* + C(z, z)) \geq z.$$

Hence,

$$h(u + C(z, z)) \leq z \Rightarrow \bar{u} \leq u^*.$$

Corollary 2.3. : *Let $h(u) \leq 0$. Then the moral hazard problem is economically soluble.*

Proof: If $h(u) \leq 0$, condition 2.3 ensures

$$\pi_1 z_1 + \pi_2 z_2 > h(C(z))$$

or

$$(\pi_1 z_1 + \pi_2 z_2)/h(C(z)) > 1$$

for some $\mathbf{z} \neq \mathbf{0}$

Hence for some $\delta > 0$ and all $\lambda, 0 < \lambda \leq 1$,

$$u(\pi_1 \lambda z_1 + \pi_2 \lambda z_2) > (1 + \delta)C(\lambda z)$$

For sufficiently small λ , the fact that $u(0)$ is well-defined and therefore $u'(0)$ is finite implies

$$\lambda u(\pi_1 z_1) + \lambda u(\pi_2 z_2) > u(\pi_1 \lambda z_1 + \pi_2 \lambda z_2)/(1 + \delta) > C(\lambda z)$$

so $h(u^*) > 0, u^* > u$ and $\mathbf{z}^* \neq \mathbf{0}$.

Corollary 2.4. *There exist no $u, \bar{u}$ and C such that the moral hazard problem is economically insoluble but there exists a feasible contract with $\mathbf{z} = \mathbf{0}$.*

Proof: From Corollary 1.3, insolubility requires $h(\bar{u}) > 0$, and this implies that any contract with $\mathbf{z} = \mathbf{0}$ must be infeasible.

Proposition 1.1 indicates that if, for some output $\mathbf{z}$, the agent can earn at least reservation utility, while receiving the entire state-contingent output in both states, the moral hazard problem must be economically soluble. In particular, in many insurance contexts, the agent's reservation utility is defined to be the utility level he could achieve without contracting. In this case, $\bar{u} = u^*$ and Proposition 1.1 applies.

Corollary 1.2 shows that, whenever the agent can profitably organize production so as to eliminate risk, the moral hazard problem is economically soluble. The intuition is that for $\mathbf{z}$ near the bisector, the costs of moral hazard are small and a contract is feasible. The fact that the agent can reallocate resources between states, in this case increasing output in the bad state and reducing output in the good state, marks the central difference between the representation of production under uncertainty used here and that used in earlier attempts to apply the state-contingent approach to moral hazard problems.

Corollary 1.3 shows that the moral hazard problem is economically soluble whenever the principal is not required to pay a positive return unless the contract calls for the agent

to exert effort, that is, when there are no fixed costs associated with contracting. The intuition behind this result is straightforward. Condition assures us that production is profitable at some level. As long as there are no fixed costs associated with contracting, the convexity of the production technology assures us that there must be some net profit at smaller scales of production. As the scale of production becomes smaller, the cost associated with the moral hazard problem goes rapidly to zero.

Corollary 1.4 indicates that there can never be a situation where the optimal contract satisfying (T.1)-(T.4) has $\mathbf{z} = \mathbf{0}$.

In summary, whereas agency problems formulated in the Grossman–Hart framework are often economically insoluble, fairly weak conditions suffice to ensure that problems formulated in the state-contingent framework are economically soluble. This suggests that economic insolubility of the agency problem may be an artifact of the problem representation.

3. References

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