

INTRODUCTION

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In recent years there has been a large amount of theoretical work on nuclei near closed shells. In particular, the doubly closed shell system O^{16} and its neighbouring isotope O^{17} have been the subject of much discussion, due in part to the relative simplicity expected in the structure of these nuclei. The theoretical interest shown in the nuclei has provoked numerous experimental studies, providing a sound basis for comparison with theoretical predictions. However, a number of anomalies remain regarding the detailed knowledge of the nuclear level structures. Many apparently conflicting experimental results have been obtained for the structure of O^{16} in the region of 13 MeV excitation, while little is known about the energy levels of O^{17} above ~ 9 MeV excitation.

It has been the purpose of the work presented in this thesis to examine these deficiencies by studying the elastic scattering of alpha particles from C^{12} and C^{13} respectively, and further, by examining the (α, n) reaction in the case of O^{17} .

In the course of the above studies, some work was also performed on the fluctuations in energy loss of low energy protons in passage through matter. In particular, an extensive study was made of Symon's theory to account for straggling phenomena, over the full range of applicability of the theory.

COMPOUND NUCLEUS STUDIES IN OXYGEN

by

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PREFACE

This thesis describes a series of experiments which were performed using the 12 MeV Tandem Van de Graaff and 2 MeV Van de Graaff accelerators in the Department of Nuclear Physics at the Australian National University.

The $C^{12}(\alpha, \alpha_0)C^{12}$ studies on the energy level structure of the compound nucleus system O^{16} in the region of 13 MeV excitation, reported in Chapter 1, were initiated by Dr T.R. Ophel as a continuation of previous investigations in this laboratory on the O^{16} nucleus. The experimental work was shared equally between Dr T.R. Ophel, Mr J.M. Morris and myself, and the analysis performed under supervision of Dr T.R. Ophel by Mr J.M. Morris and myself.

The work of Chapter 2 on Symon's theory of energy loss distributions originated from a suggestion by Dr Ophel as to a possible means of investigating an experimental discrepancy in the $C^{12}(\alpha, \alpha_0)C^{12}$ work. Most of the experimental work and all of the analysis were performed by myself.

The measurements of Chapter 3 on the level structure of O^{17} using the $C^{13}(\alpha, \alpha_0)C^{13}$ and $C^{13}(\alpha, n)O^{16}$ reactions were suggested by Professor J.R. Risser, and the initial experimental work was carried out by Professor Risser, Mr Morris and myself. Further experimental work and all of the analysis were carried out by Mr Morris and myself following Professor Risser's return to Rice University. The author again participated actively in all aspects of the investigations.

The chapters are not presented in chronological order, the work of Chapter 3 having been performed first.

Some of the work reported here has been presented in the following papers:

Kerr, G.W., Morris, J.M. and Risser, J.R. - 'Energy Levels of O^{17} from $C^{13}(\alpha, \alpha_0)C^{13}$ and $C^{13}(\alpha, n)O^{16}$ ' ;

Morris, J.M., Kerr, G.W. and Ophel, T.R. - 'Energy Levels of O^{16} in the Vicinity of 13 MeV - III' .

Both papers are presently in press.

I am pleased to take this opportunity of thanking Dr Ophel for his excellent supervision during the major part of my scholarship, and also Mr Morris (now Dr Morris) for a valuable partnership during most of my three years at the A.N.U. I would further like to thank the staff members of the department for their much appreciated assistance on many occasions, especially Dr H.J. Hay for the use of his parameter optimisation program, FITALL, which was an essential part of much of the work presented in this thesis. I must also express my gratitude to Professor E.W. Titterton for his encouragement and support of these investigations, and to the A.N.U. for the award of a research scholarship.

No part of this thesis has been submitted for a degree at any other University.

George W. Kerr.

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CHAPTER 1

A STUDY OF LEVELS OF THE COMPOUND NUCLEUS SYSTEM O^{16} IN THE REGION OF 13 MeV

1.1 INTRODUCTION

The available experimental information on the natural parity states of the compound nucleus system O^{16} in the region of 13 MeV excitation is often contradictory in the many channels open at this energy, and has been the subject of much discussion.

Bittner and Moffat (Bi 54), from phase shift analysis of $C^{12}(\alpha, \alpha_0)C^{12}$ excitation functions between alpha particles bombarding energies from 3.8 to 7.6 MeV, observed a 1^- state in O^{16} of CM width 173 KeV at 7.04 MeV ($E_X = 12.44$ MeV), and noted that the $\ell = 0$ and $\ell = 2$ phases were increasing, implying the existence of a 0^+ state and a 2^+ state at higher excitation. Jones et al (Jo 62), in similar measurements up to 5 MeV bombarding energy, found that the s-wave phase was decreasing, and the discrepancy between the two sets of measurements has since been attributed to multiple solutions in the analysis of Bittner and Moffat. Clark et al (Cl 67) in this laboratory, have recently confirmed the results of Jones et al, and have shown that the s-wave phase continues to decrease up to 6.5 MeV. Ferguson and McCallum (Fe 61), also in studies of elastic alpha particle scattering from C^{12} in the region $7\text{MeV} \leq E_\alpha \leq 11\text{ MeV}$ confirmed a 1^- state at about 12.44 MeV excitation, but at a bombarding energy of 7.065 MeV and of CM width only 80 KeV. They observed several further anomalies at higher energies, and in particular were able to assign 3^- to the state at $E_\alpha = 8.15\text{ MeV}$ ($E_X = 13.26\text{ MeV}$). However, due

mainly to experimental difficulties, they were unable to fit the data in the region of $E_{\alpha} \sim 7.9$ MeV ($E_X \sim 13.1$ MeV) and were able only to indicate that a 1^- and a 2^+ of approximately the same width and energy most nearly accounted for the data. In studies of the $C^{12}(\alpha, \alpha_1)C^{12*}$ reaction, they observed the same resonances as in the elastic scattering except for the 1^- level at $E_X = 12.44$ MeV which did not decay via the inelastic channel.

Mitchell et al (Mi 64a) reported resonances corresponding to O^{16} excitations of 13.13 and 13.27 MeV in studies of the $C^{12}(\alpha, \alpha'\gamma)C^{12}$ reaction, and found that the angular distributions were 'consistent' with 3^- assignments for both states. However, they pointed out that the $T = 1$ (1^-) analogue of the 392 KeV state in N^{16} was expected to occur at about 13.1 MeV excitation in O^{16} , that all previous experimenters had obtained a J^{π} of 1^- in the region, and that suitably mixed 1^- and 2^+ states as suggested by Ferguson and McCallum could account for their results.

Larson and Spear (La 61) measured the ground state gamma radiation from the $C^{12}(\alpha, \gamma)O^{16}$ capture reaction and found resonances at $E_{\alpha} = 7.08$, 7.89 and 8.11 MeV which they associated with the 1^- level at $E_X = 12.44$ MeV, a 1^- level at 13.10 MeV and with the 3^- level at 13.26 MeV respectively. However, in concurrent studies of the $C^{12}(\alpha, \alpha'\gamma)C^{12}$ reaction they corroborated the results of Mitchell et al, observing resonances at $E_{\alpha} = 7.93$ MeV and 8.14 MeV. No explanation was offered regarding the 50 KeV bombarding energy difference between the resonances observed at ~ 7.9 MeV in the two reaction channels. In a more recent paper (La 64), the same authors (with a revised energy scale) quote Barker as pointing out from relative yield considerations that the level in O^{16} corresponding to the 7.96 MeV state is almost certainly not the 1^- observed in other reactions at $E_X \sim 13.10$ MeV. [The information acted on by Barker derived

originally from Mitchell and Ophel (Mi 64b) who pointed out that inclusion of a 3^- state at $E_\alpha = 7.96$ MeV ($E_X = 13.13$ MeV) would explain much of the experimental data in the 13.1 MeV region.]

In the same paper, Larson and Spear demonstrated that asymmetries about 90° existed in the ground state gamma ray angular distributions of the $C^{12}(\alpha, \gamma)O^{16}$ reaction in the region of $E_\alpha \sim 7.9$ MeV, confirming preliminary results of Tanner and Mitchell (Ta 63), and giving strong evidence for a 2^+ state of approximately the same width and excitation as the 1^- state.

Hebbard (He 60) has given a description of the $N^{15}(p, \alpha_0)C^{12}$ reaction in a comprehensive analysis of the available data from threshold to 1200 KeV, in terms of the constructive interference of the two 1^- states at 338 KeV ($E_X = 12.44$ MeV) and 1010 KeV ($E_X = 13.10$ MeV) respectively, and has also postulated the existence of a 0^+ state at ~ 500 KeV and a 2^+ state at ~ 1000 KeV to account for interference terms in the ground state α -particle angular distributions. Hebbard also observed the 12.44 MeV gamma radiation from the $N^{15}(p, \gamma)O^{16}$ capture reaction, confirming the 1^- assignment. The ground state radiation from the same reaction has been observed at the 1010 KeV resonance by Kraus (Kr 54) and found to be isotropic, implying s-wave formation of the state in accord with the 1^- assignment. Bailey (Ba 63) has shown that no evidence exists for interference of the 1^- level at 1010 KeV with a state of positive parity which, in conjunction with Hebbard's analysis, implies a very small proton width for the 2^+ level believed to exist near 1000 KeV proton energy. This is to be contrasted with the substantial evidence found for the 2^+ state by Larson and Spear through the $C^{12}(\alpha, \gamma)O^{16}$ reaction.

Studies of the $N^{15}(p, \alpha' \gamma)C^{12}$ and $N^{15}(p, \alpha_1)C^{12*}$ reactions by Schardt et al (Sc 52) show no resonance at 338 KeV and a weak resonance at ~ 1050 KeV. The upper resonance has been associated

with the 1^- level at $E_X = 13.10$ MeV (He 60, Mi 64), and the displacement to higher energies was attributed to penetrability effects on the low energy inelastic alpha particles. However, it is not clear that the resonance is not associated with the 3^- state proposed by Mitchell and Ophel at 13.13 MeV. Measurements of the same reactions at proton energies in the region of 1210 KeV have at various times given assignments to the level at $E_X = 13.26$ MeV of 3^- (Ha 57), 3^- or 4^+ (Ha 57, Ba 57), 4^+ (Ha 57, Kr 57) and 3^- (Ba 59).

Investigations of the elastic scattering of protons from N^{15} have confirmed the J^π assignments of 1^- and 3^- for the 1010 KeV and 1210 KeV resonances (Ha 57, Ba 59), and Hagedorn (Ha 57) has claimed that the 1010 KeV anomaly was sufficiently well fitted to exclude the possibility of more than one state being involved.

Finally, in the two most recent papers covering the 13 MeV region, Mitchell and Ophel (Mi 64, Mi 65) have summarised the published data and presented a self-consistent level scheme for the region together with conclusive experimental evidence, which explains most of the contradictory results previously obtained by the many experimenters, as discussed above. They measured excitation functions and angular distributions in a series of reactions $C^{12}(\alpha, \gamma)O^{16}$, $C^{12}(\alpha, \alpha' \gamma)C^{12}$, $C^{12}(\alpha, \gamma \gamma)O^{16}$, $C^{12}(\alpha, p)N^{15}$ and $C^{12}(\alpha, \alpha_1)C^{12}$, and confirmed the J^π assignments of 1^- at $E_X = 13.10$ MeV, 3^- at 13.13 MeV and 2^+ at approximately the same energy, and 3^- at 13.26 MeV.

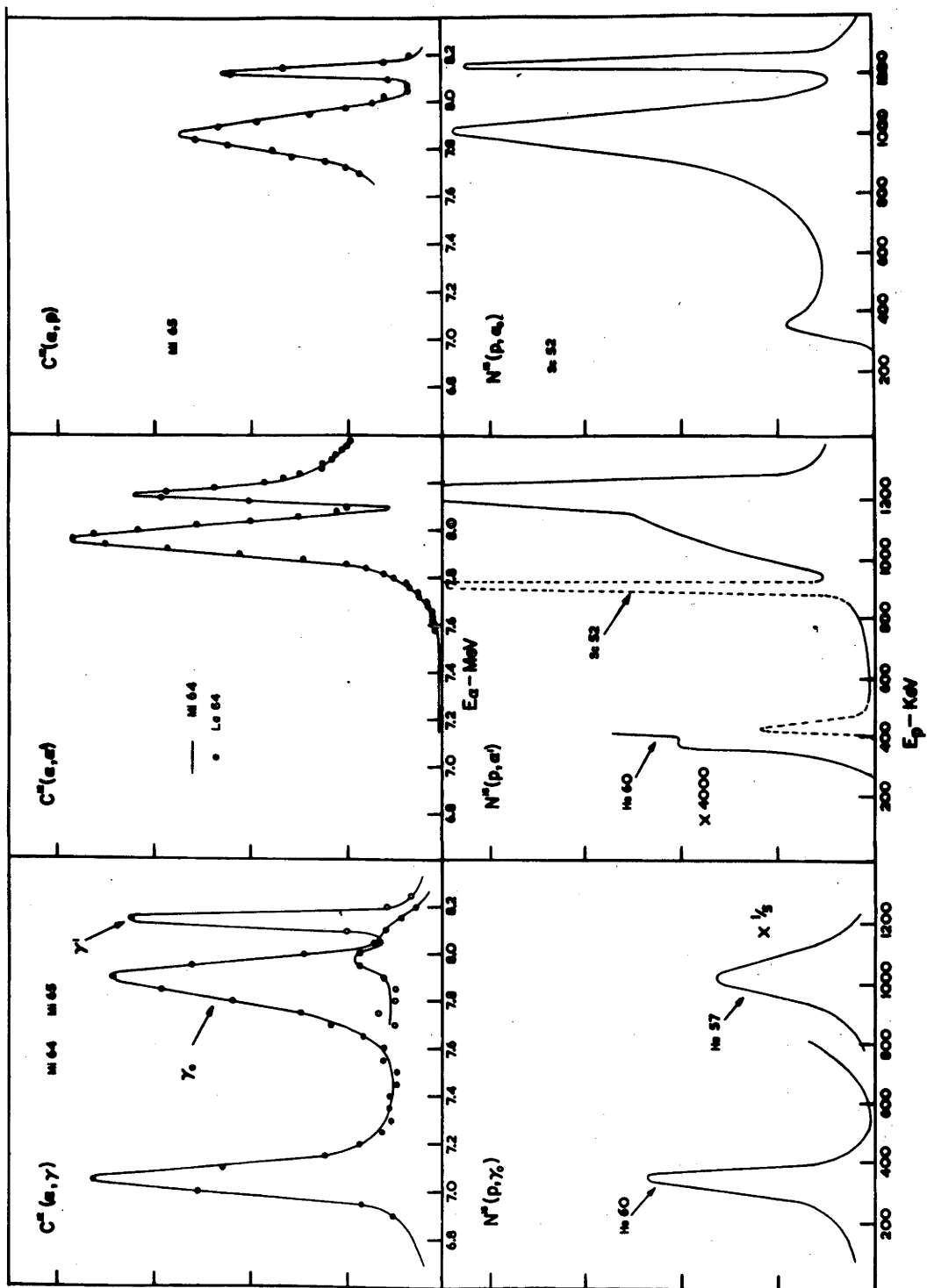
Table 1.1 presents a compilation of the results of the many experiments which have been performed at ~ 13 MeV in O^{16} , and fig 1.1 presents a comprehensive diagram of the excitation functions measured in the various open channels.

TABLE 1.1

I = interference			AD = angular distribution(s)		EF = excitation function(s)		PSA = phase shift analysis	
E_X (MeV)	E_α (MeV)	E_p (MeV)	REACTION	J^π	E_{CM} (KeV)	REF.	COMMENTS	
12.44	7.04	7.065	338 $N^{15}(p, \alpha_0)C^{12}$	0^+ ($I, 1^-$)	173	Co 53	α_0 AD	
			338 $N^{15}(p, \alpha_0)C^{12}$	1^- ($I, J^\pi \approx 2$)		Ha 57a, Ba 59	α_0 AD	
			338 $N^{15}(p, \alpha_0)C^{12}$	1^-		He 60	comprehensive study of published data	
			338 $N^{15}(p, \alpha_0)C^{12}$	1^-		He 60	γ_0 AD ; 0^+ excluded	
			$C^{12}(\alpha, \alpha_0)C^{12}$	1^-		Bi 54	PSA	
			$C^{12}(\alpha, \alpha_0)C^{12}$	1^-		Fe 61	PSA	
			$C^{12}(\alpha, \gamma_0)O^{16}$	1^-		La 64	γ_0 AD, EF	
			$C^{12}(\alpha, \gamma_0)O^{16}$	1^-		Mi 64	γ_0 AD, EF	
			$\sim 500 N^{15}(p, \alpha_0)C^{12}$	0^+		He 60	I terms in α_0 AD	
			$\sim 1000 N^{15}(p, \alpha_0)C^{12}$	2^+		He 60	I terms in α_0 AD	
~ 13.1	7.04	7.065	1010 $N^{15}(p, \alpha_0)C^{12}$	1^-	140	He 60	comprehensive study of published data	
			1050 $N^{15}(p, \alpha_0)C^{12}$	1^-		Ha 57a	α_0 AD	
			$\sim 1050 N^{15}(p, \alpha_0)C^{12}$	1^-		Sc 52	α' EF ; weak	
			1010 $N^{15}(p, \gamma_0)O^{16}$	1^-		Kr 54	γ_0 AD	
			1010 $N^{15}(p, \gamma_0)O^{16}$	1^-		Ba 63	γ_0 AD ; no evidence of I, J^π	
			1050 $N^{15}(p, p_0)N^{15}$	1^-		Ha 57	p_0 AD ; no further states present!	
			1010 $N^{15}(p, p_0)N^{15}$	1^-		Ba 59	p_0 AD	
			$C^{12}(\alpha, \alpha_0)C^{12}$	$I, 0^+$ and 2^+		Bi 54	PSA	
			$C^{12}(\alpha, \alpha_0)C^{12}$	$I, 1^-$ and 2^+		Fe 61, Fe 65	PSA ; poor fits obtained	
			$C^{12}(\alpha, \gamma_0)O^{16}$	$I, 1^-$ and 2^+		La 64	γ_0 AD ; EF	
> 7.6	7.89	7.95	$C^{12}(\alpha, \gamma_0)O^{16}$	$I, 1^-$ and 2^+	140(1^-) ; 250(2^+)	Mi 64, Mi 65	γ_0 AD ; EF	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^- or $I, 1^-$ and 2^+		Mi 64a	4.43γ AD, EF ; α' AD, EF	
			$C^{12}(\alpha, \gamma_0)O^{16}$	(3^-)		La 64	4.43γ EF ; not $I, 1^-$ and 2^+	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		Mi 64	4.43γ AD, EF ; not $I, 1^-$ and 2^+	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		Mi 65	α_1 AD, EF ; definitely not $I, 1^-$ and 2^+	
			$C^{12}(\alpha, \gamma_0)O^{16}$	2^+		Mi 65	I terms in α_1 AD	
			$C^{12}(\alpha, p_0)N^{15}$	1^-		Mi 65	p_0 EF	
			1210 $N^{15}(p, \alpha_0)C^{12}$	3^-		Ha 57a, Ba 59	α_0 AD	
			1210 $N^{15}(p, \alpha_0)C^{12}$	3^-		Ha 57	$\alpha - \gamma$ correlation	
			1210 $N^{15}(p, \alpha_0)C^{12}$	3^- or 4^+		Ha 57, Ba 57	4.43γ AD	
~ 13.26	8.15	8.14	1210 $N^{15}(p, \alpha_0)C^{12}$	4^+	26	Ha 57, Kr 57	α_1 AD	
			1210 $N^{15}(p, \alpha_0)C^{12}$	3^-		Ba 59	α_1 AD	
			1210 $N^{15}(p, p_0)N^{15}$	3^-		Ha 57, Ba 59	p_0 AD	
			$C^{12}(\alpha, \alpha_0)C^{12}$	3^-		Fe 61	PSA	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		La 64	γ_0 AD, EF	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		Mi 64a	4.43γ AD, EF ; α_1 AD	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		Mi 64, La 64	4.43γ AD, EF	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		Mi 65	α_1 AD, EF	
			$C^{12}(\alpha, p_0)N^{15}$	3^-		Mi 65	p_0 EF	
			$C^{12}(\alpha, \gamma_0)O^{16}$	3^-		Mi 65	γ EF	

Fig. 1.1

EXCITATION FUNCTIONS OF THE SEVERAL INELASTIC $N^{15} + p$ AND $C^{12} + \alpha$ CHANNELS FOR THE RANGE OF EXCITATION IN O^{16} BETWEEN 12.2 AND 13.4 MeV . THE PROTON AND ALPHA ENERGIES HAVE BEEN ADJUSTED SO THAT THE CURVES MAY BE DIRECTLY COMPARED WITH REGARD TO O^{16} EXCITATION ENERGY. THE $N^{15}(p,\gamma)$ DATA HAS HAD THE ENERGY-DEPENDENT FACTORS (PENETRABILITY) REMOVED. THE DOTTED CURVES REPRESENT UNNATURAL PARITY STATES (WHICH ARE NOT EXCITED IN THE $C^{12}(\alpha,\alpha_o)C^{12}$ REACTION).



In summary, almost all of the data obtained to the present time is consistent with the following level scheme:

- (i) a 1^- level at 12.44 MeV of large α_0 width and small or negligible proton and inelastic alpha particle widths
- (ii) a 1^- level at 13.10 MeV of appreciable α_0 and p widths, but with small α_1 width
- (iii) a 3^- level at 13.13 MeV of appreciable α_0 and α_1 widths, but with small p width
- (iv) a 2^+ level at ~ 13.1 MeV of large α_0 width and small or negligible p and α_1 widths
- (v) a 3^- level at 13.26 MeV with appreciable width in each of the three particle channels α_0 , α_1 and p.

Fig 1.2 and table 1.2, present the level scheme of O^{16} in the region of 13 MeV, after Mitchell and Ophel.

The major exception to the above description of O^{16} in the region of 13 MeV rests in the alpha particle elastic scattering data which have never been measured or fitted well. The object of the present experiment has been to repeat the alpha particle scattering measurements of Ferguson and McCallum in order to test the above scheme in the elastic channel, and to deduce more quantitative level parameters for the 2^+ at ~ 13.1 MeV and for the 3^- at ~ 13.13 MeV.

It is worth noting that elastic alpha particle scattering from C^{12} is the only experiment in which all five states presented in the summary above would be expected to resonate appreciably, since the only partial width large in all of the states together is the elastic alpha particle width.

TABLE 1.2

E_{α} (MeV)	E_p (KeV)	O^{16} (MeV)	J^{π}	Γ_{cm} (KeV)
7.09	338	12.44	1^+	94
7.90	1010	13.10	1^-	140
7.95	not observed	13.13	3^-	100
~7.98	~1000	~13.15	2^+	~250
8.14	1210	13.25	3^-	225

The table lists the parameters of the levels in the 13 MeV region of O^{16} , after Mitchell and Ophel (M1 65).

1.2 EXPERIMENTAL DETAILS

1.2.1 BEAM

The A.N.U. 2 MeV Van de Graff injector and the 12 MeV Van de Graaff tandem accelerator facilities provided a beam of doubly charged alpha particles ($0.1 - 0.5 \mu\text{A}$) which was magnetically analysed and focussed through collimators onto the target.

1.2.2 TARGETS

Ferguson (Fe 65) ascribed much of the difficulty experienced in analysing the data obtained by Ferguson and McCallum (Fe 61) in the present region to experimental problems associated with the thin self-supporting carbon-foil targets used. In particular, it was suggested that the difficulties were due to uncertainty in the absolute cross section scale resulting from use of foils, and that the problem was accentuated by non-trivial inaccuracies in the angular distribution data (which showed very rapid variations with angle), resulting from buckling of the carbon foils under bombardment by the beam. McCallum (Mc 65) has pointed out that discrepancies in the angular scale up to 1° could occur from foil buckling.

In view of the above, the initial intention in the present experiment was to use a gas target. However, preliminary measurements made with foils to ascertain the details of the region indicated that, by taking adequate care, foils could in fact be used successfully to obtain an internally self-consistent set of angular distributions and excitation functions, requiring, in principle, one measurement to relate the data to an absolute cross section scale. Since foils have several advantages over gas targets,

such as better energy resolution and a greater measurable angular range, it was decided to use foils for the measurements.

The foils were generally of 20-30 $\mu\text{gm}/\text{cm}^2$ thickness, made by evaporating natural carbon onto glass slides coated with RBS 25 detergent, and floating the carbon layer off with distilled water. A very thin gold layer was evaporated onto the foils to enable checking of beam integration by means of Rutherford scattering.

1.2.3 SCATTERING CHAMBER

Details of the chamber (figs 1.3 and 1.4) are discussed by Ohlsen and Young (Oh 64).

The detectors generally subtended a solid angle of $\sim 10^{-3}$ sterad. at the target, defined by rectangular apertures mounted in the counter blocks. The detector angles could be set to a nominal accuracy of better than 0.05° by means of vernier scales attached to the rotating lids. The beam entered the chamber through a narrow collimation system optically aligned with the chamber centre.

In view of the problems experienced by Ferguson and McCallum, checks were made of the positional accuracy of the beam-target-detector system. In particular, scattering of the alpha particle beam from a thin gold layer on the targets was used to check left-right symmetry of the system, about the beam axis. Agreement to within statistics ($< 1\%$) was found for the yields on either side of the chamber. The fractional discrepancy in the Rutherford scattering cross section for small angular inaccuracies is given by

$$\frac{\Delta\sigma_R}{\sigma_R} = 2 \cot\left(\frac{\theta}{2}\right) \cdot \Delta\theta \quad (1.1)$$

so that a 1% error at 25° , the most forward angle measured, represents an error in the nominal angle θ , of $\sim 0.05^\circ$.

Fig. 1.3 HORIZONTAL CROSS-SECTION OF THE 51 cm SCATTERING CHAMBER. THE NUMBERED PARTS ARE:

- (1) Rotating Lid
- (2) Counter Angle Vernier Scale
- (3) Beam Collimating System
- (4) Target
- (5) Faraday Cup Housing
- (6) Solid State Counter Mounting Block
- (7) Counter Collimating Slits
- (8) Solid State Counter

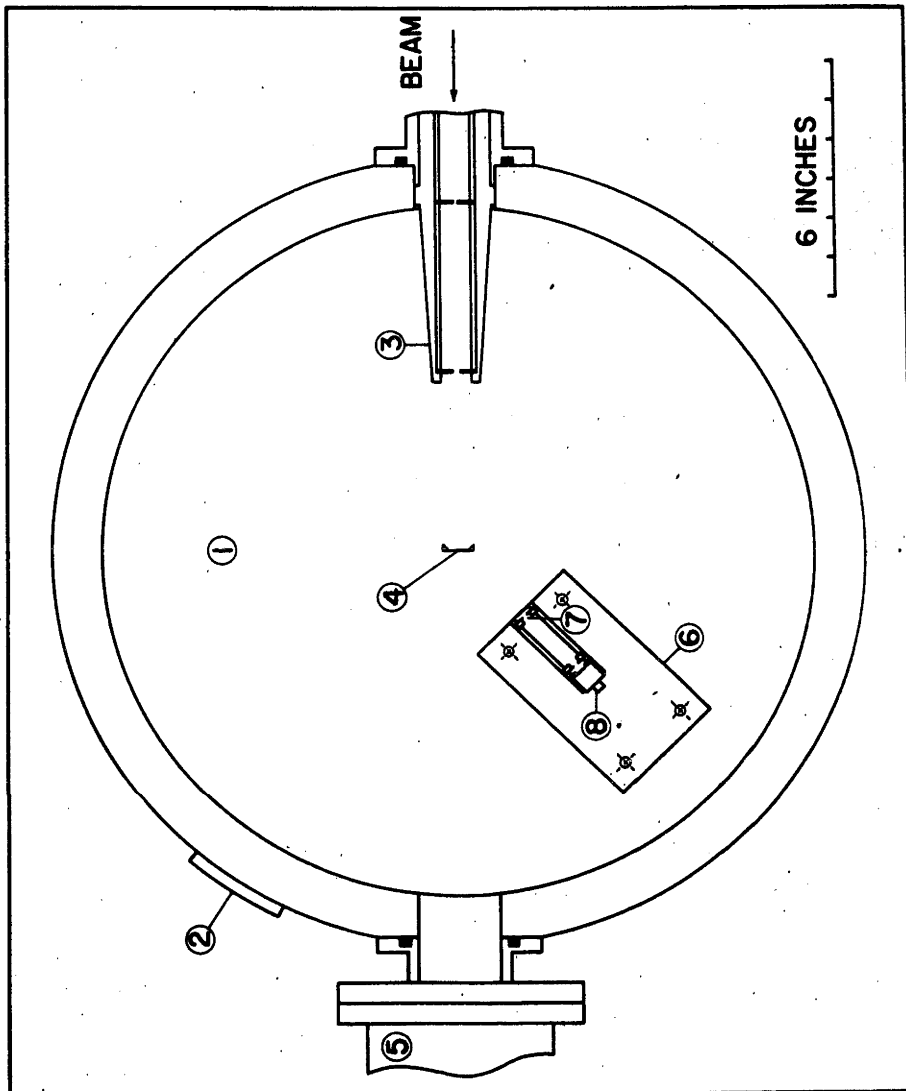
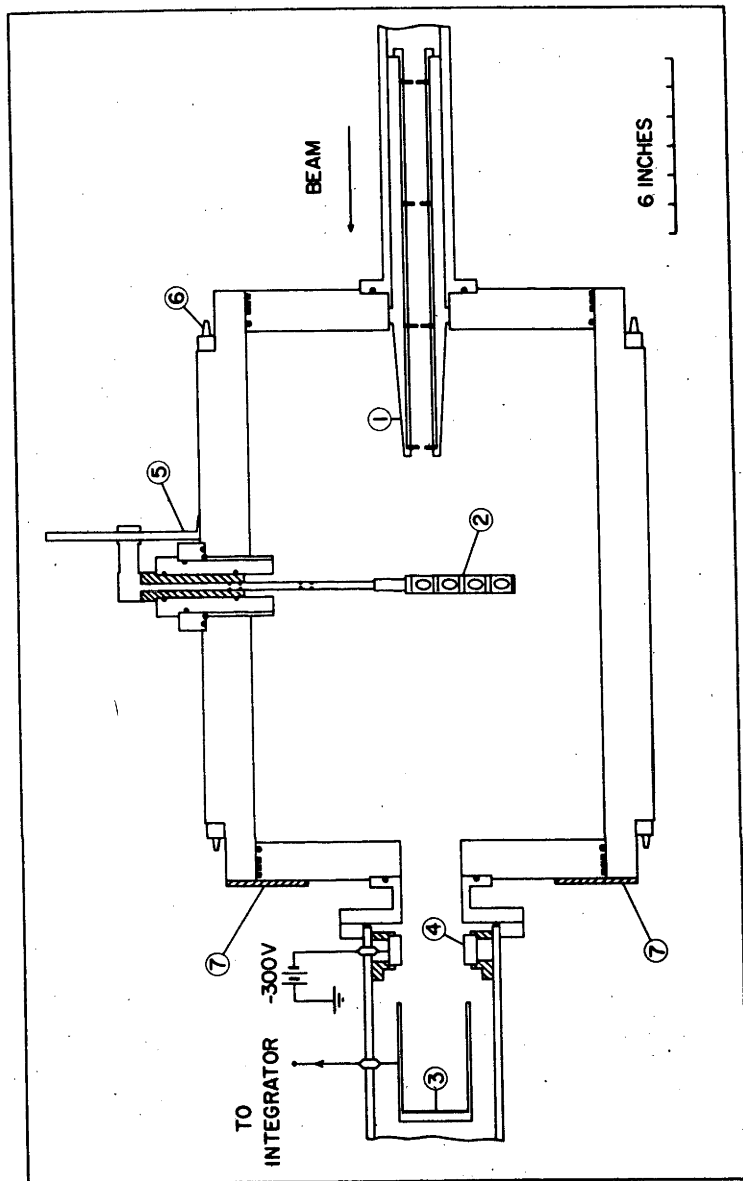


Fig. 1.4 VERTICAL CROSS-SECTION OF THE 51 cm SCATTERING CHAMBER. THE NUMBERED PARTS ARE:

- (1) Beam Collimating System
- (2) Target Ladder
- (3) Faraday Cup
- (4) Suppression Magnet
- (5) Pointer giving Angular Position of the Target
- (6) Gears for Rotating Upper and Lower Lids
- (7) Counter Angle Vernier Scales



(Note that the measurements at backward angles furnished a measure of the relative accuracy of the detector slit systems, due to the insensitivity of Rutherford scattering with angle at backward angles). Furthermore, the absolute angular scale was tested at the same time by comparing the yield as a function of angle with the calculated Rutherford scattering cross section. The measurements indicated that alpha particle scattering from gold at the present energies was pure coulomb over the full angular range. A typical measurement is shown in fig 1.5. Fig 1.6 indicates that errors in the angular scale were less than 0.1° in the angle sensitive region forward of 50° . Similar results were obtained when the targets (45° to the beam) were rotated about 90° and 180° , indicating that target distortion effects were small.

1.2.4 DETECTORS

Silicon surface barrier detectors of 500 ohm.cm resistivity coupled to ORTEC 103-203 amplifier systems were used for the data collection.

1.2.5 DEAD-TIME MEASUREMENT

'Busy' pulses from the RIDL 400 channel and RCL 512 channel pulse height analysers used for the data collection were used to gate fixed frequency oscillators, and comparison of the gated output of the oscillators and the ungated output, provided a measure of the dead time. Dead times were kept less than 5% by adjustment of the beam current, to reduce errors in the correction factors resulting from beam variations.

Fig. 1.5 THE MEASURED ANGULAR DISTRIBUTION OF ELASTIC ALPHA PARTICLE SCATTERING FROM GOLD. EACH DATUM POINT WAS RECORDED FOR NOMINALLY EQUAL INTEGRATED CHARGE AND HAS NOT BEEN NORMALIZED TO THE MONITOR YIELD. THE EXPERIMENTAL ERRORS ARE SMALLER THAN THE DATA POINTS. THE SOLID CURVE IS THE CALCULATED RUTHERFORD SCATTERING DISTRIBUTION.

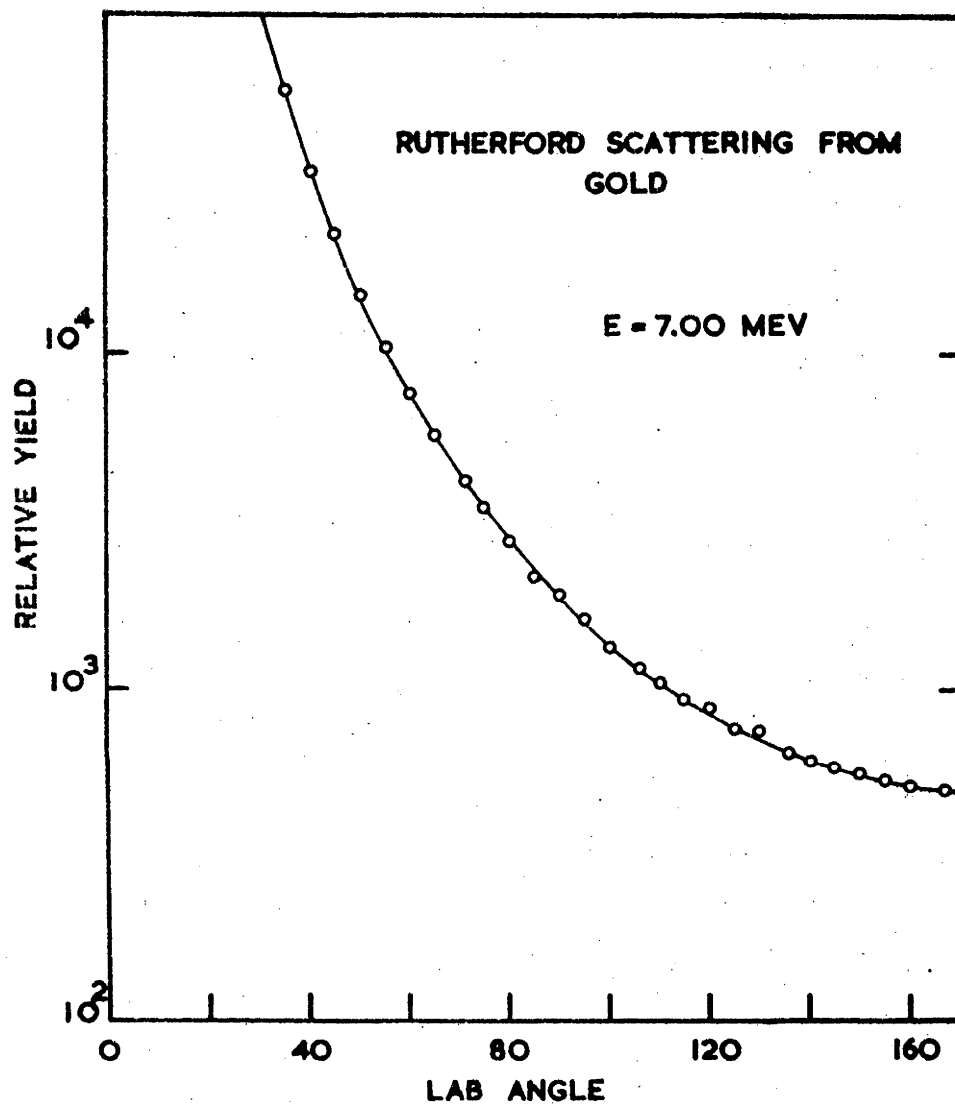
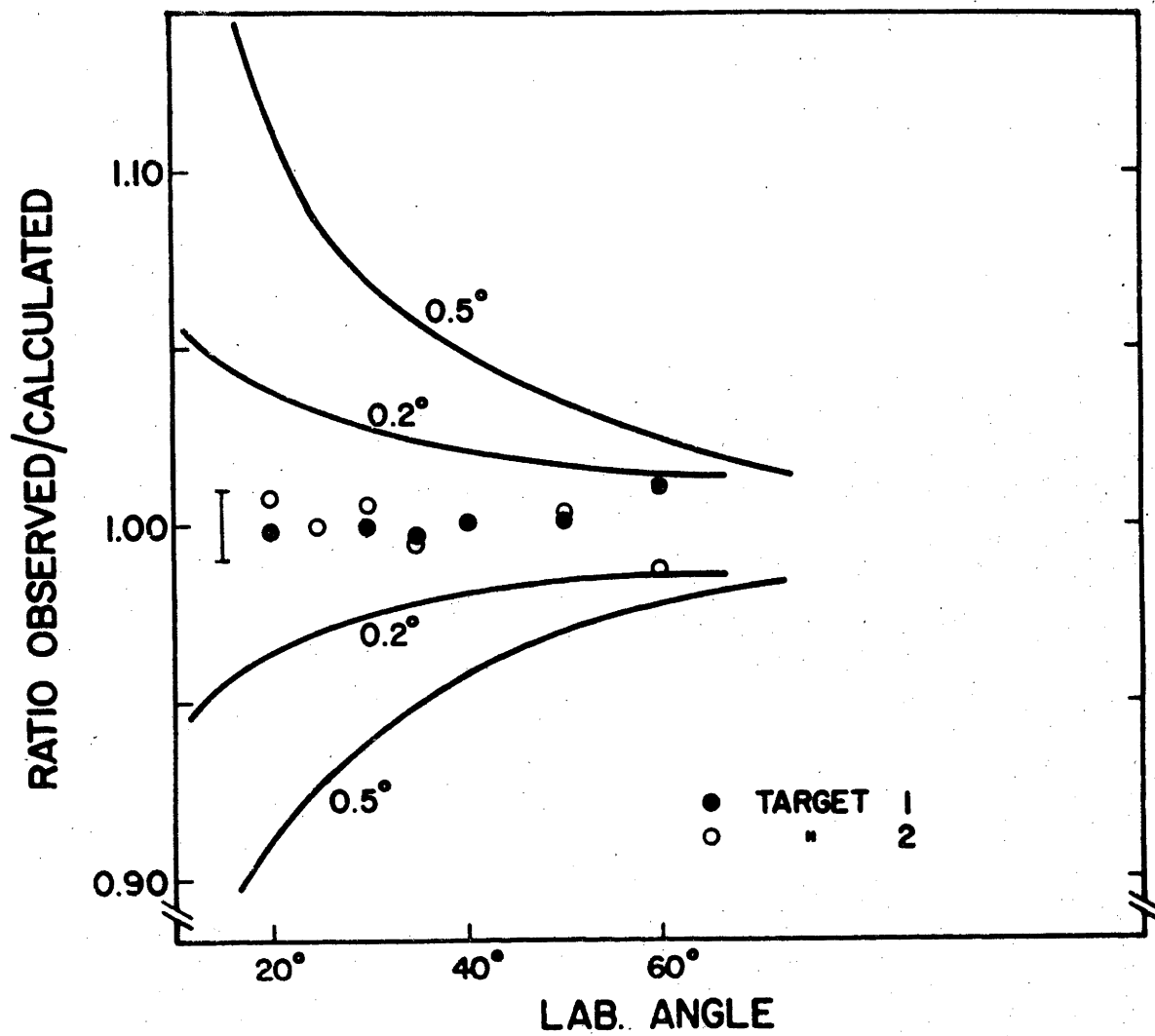


Fig. 1.6 THE RATIO OF THE OBSERVED-TO-CALCULATED YIELD OF ALPHA PARTICLES SCATTERED FROM GOLD AS A FUNCTION OF ANGLE FOR TWO DIFFERENT TARGETS. THE ERROR BAR REPRESENTS THE MAXIMUM UNCERTAINTY IN THE MEASURED POINTS. THE SOLID CURVES INDICATE RATIOS CALCULATED ASSUMING THAT THE TRUE ANGULAR SCALE DIFFERED FROM THE NOMINAL SCALE BY $\pm 0.2^\circ$ AND $\pm 0.5^\circ$.



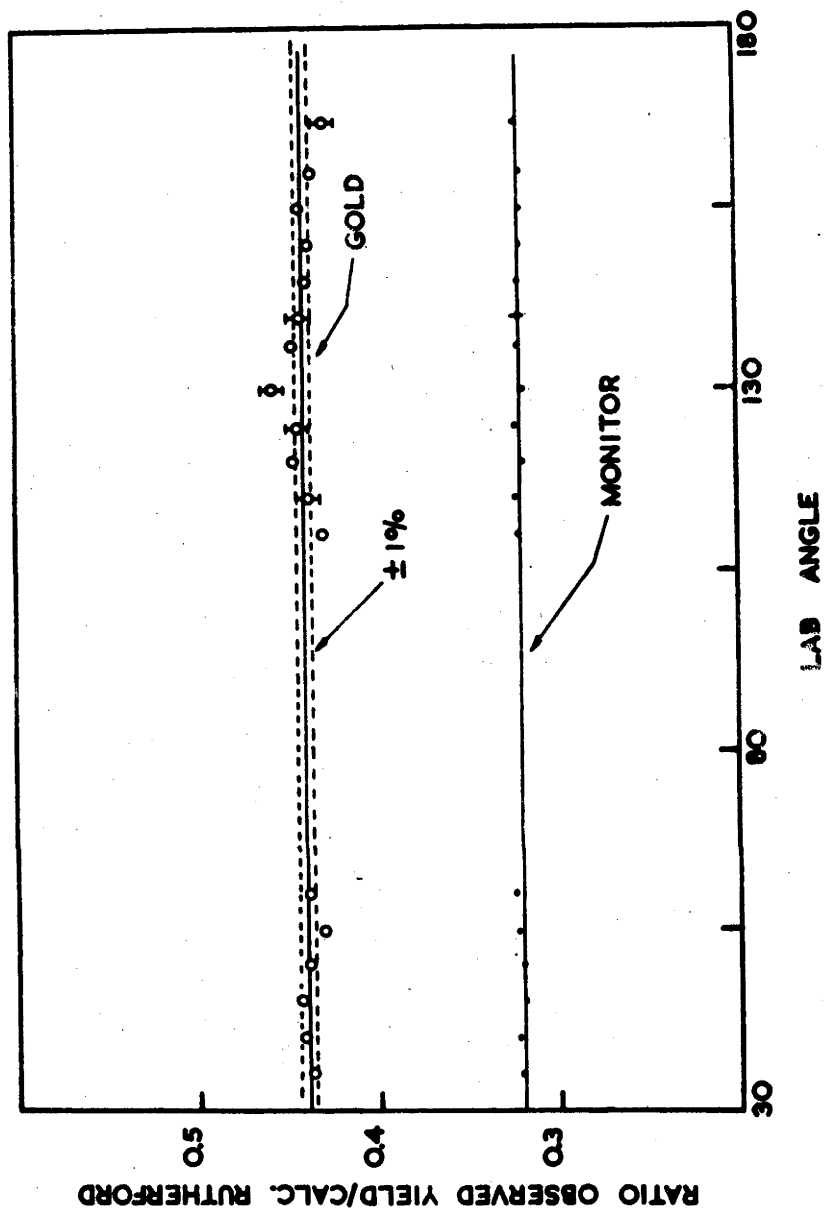
1.2.6 BEAM CURRENT INTEGRATION

Charge was collected in an electrically and magnetically suppressed Faraday cup (see fig 1.4) and fed directly to a charge integrator. The integrator was checked periodically with a constant current source and electronic timer, and found to have long term stability of better than 1%. Further confirmation of the stability resulted from the consistency of the angular distribution monitor yield, and comparison of the gold scattered yield with Rutherford scattering. Fig 1.7 presents the results from a typical angular distribution measurement. The forward angle gold data show the angular scale to be correct, while the data at backward angles indicate an integrator stability of $\sim 1\%$, as does the constancy of the carbon scattered yield for the fixed monitor counter.

1.2.7 BUILD UP

In general, build up of carbon on the targets was found not to be a significant problem during the course of the measurements. Build up was tested for in the excitation functions by repeating each measured excitation function in coarse energy steps over the range, and was always found to be insignificant. Fig 1.8, although not strictly an example of the above, provides an excellent illustration of the point. Shown are the results of measuring two excitation functions at the angle 136.06° , with the same target. (The dotted curve was measured in finer energy steps than the crosses). Note that this figure also presents good evidence for the integrator stability, as the data for the two excitation functions were collected for different integrated charge (an appropriate factor being applied for normalisation to equal integrated charge). During the angular distribution measurements, one detector remained fixed as a monitor, so that any systematic variation in yield from this

Fig. 1.7 THE RATIO OF THE OBSERVED-TO-CALCULATED
SCATTERING FROM GOLD AS A FUNCTION OF ANGLE
TAKEN FROM A TYPICAL ANGULAR DISTRIBUTION
MEASUREMENT. ALSO SHOWN IS THE YIELD OF THE
CARBON SCATTERED ALPHA PARTICLES, RECORDED
AT THE FIXED MONITOR COUNTER. THE ERROR BAR
ON THE MONITOR YIELD REPRESENTS $\pm 5\%$ VARIATION.



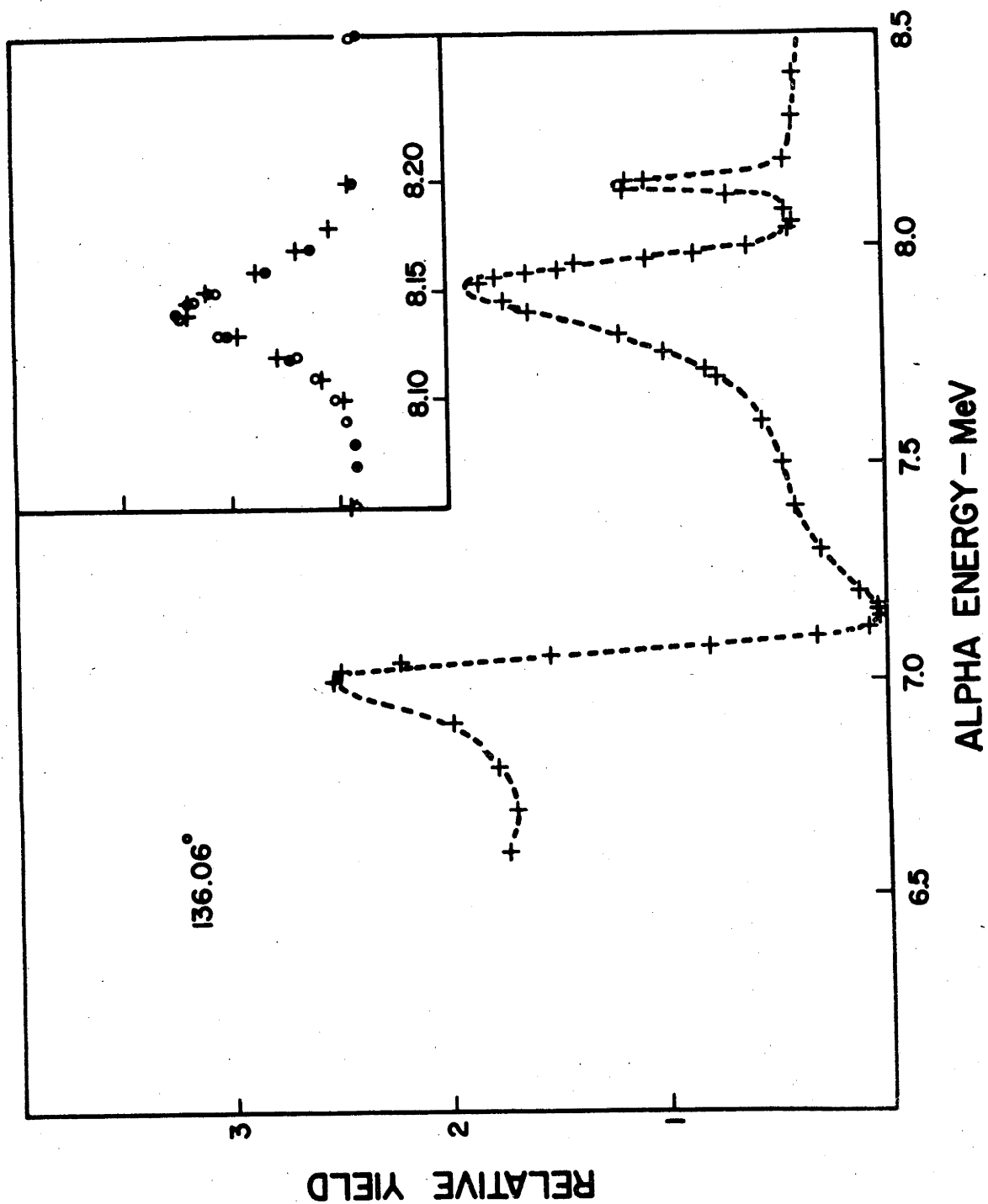
monitor counter furnished a measure of the build up. Although build up was found generally to be small during most runs, target increments as large as 10% were observed on occasion. The reason for the occasional occurrence of significant build up was not clear, although there appeared to be some correlation with the quality of the chamber vacuum.

1.3 DIFFERENTIAL CROSS SECTION MEASUREMENTS

Twenty-three angular distributions and six excitation functions were measured in the range of alpha particle bombarding energies from 6.6 to 8.5 MeV. The angular distributions were generally recorded in 5° steps from 20° to 165° lab and at the excitation function angles, and the excitation functions measured at intervals ranging between 5 KeV in regions of rapid yield variation with energy and 100 KeV in regions of slow variation, for laboratory angles of 136.06° , 124.89° , 106.6° and 71.55° , which are the zeroes of P_4 , P_3 , P_2 and all odd Legendre polynomials respectively in the CM system. Excitation functions were also measured at 167° - the most backward laboratory angle attainable, and 104.5° for comparison with the data of Bittner and Moffat.

As indicated in the preceding section, one detector was fixed at a suitable angle as monitor during measurement of the angular distributions, although not necessarily at the same angle for each energy. At angles forward of 60° lab., the elastic alpha group from the 1% C^{13} in the target merged with the C^{12} group. This was not considered a serious problem, as C^{13} appears to exhibit only weak broad resonances in this energy region. A small quantity of O^{16} was found to be present on all of the targets. The O^{16} group was not clearly resolved from the C^{12} group at angles forward of 40° , leading to a further increasing uncertainty as a function of decreasing angle.

Fig. 1.8 $C^{12}(\alpha, \alpha_0)C^{12}$ EXCITATION FUNCTION RECORDED AT 136.06° . THE DOTTED CURVE SHOWS THE EXCITATION FUNCTION MEASURED IN FINE ENERGY INCREMENTS (10 - 20 KeV). CROSSES SHOW A REPEATED MEASUREMENT OF THE YIELD, MADE IN COARSE ENERGY STEPS WITH THE SAME TARGET. THE INTEGRATOR SETTINGS WERE DIFFERENT FOR THE TWO RUNS, AND AN APPROPRIATE FACTOR HAS BEEN APPLIED FOR NORMALISATION TO EQUAL INTEGRATED CHARGE. THE INSET SHOWS A PORTION OF THE DATA ACROSS THE NARROW RESONANCE AT $E_\alpha = 8.15 \text{ MeV}$ FOR SEVERAL DIFFERENT RUNS.



For the excitation function measurements, one detector was maintained at 136.06° and the excitation functions at each of the five other angles determined in turn with a second detector so that the yields could be directly related. Furthermore, the relative energy scales of all the excitation functions could be established to within 3 KeV by use of the rapid yield variation with energy of the 136.06° measurements - in particular, of the narrow anomaly at $E_\alpha \sim 8.15$ MeV. Fig 1.8 shows two measurements at 136.06° , and the inset shows data from several different runs over the 8.15 MeV anomaly.

It is worth noting that the characteristics of the beam handling system of the A.N.U. tandem accelerator are such that, while energy reproducibility of better than 3 KeV can be obtained under reasonably constant accelerator conditions (e.g. during the course of a continuous two or three day run), larger energy uncertainties of 5 - 10 KeV have been observed under different 'set-up' conditions. The energy shifts have been attributed to small angular differences in beam trajectory through the 90° analysing magnet. These problems were more severe for some of the later measurements which were performed in a newly constructed target area. In this case, the beam had to pass between the poles of a degaussed 90° magnet before being analysed, and incorrect degauss conditions of this magnet could result in larger than usual variations in beam trajectory through the analysing magnet. Problems associated with small energy shifts are discussed in the next section.

1.3.1 RELATIVE NORMALISATION

Severe difficulties were encountered in the initial stages of data reduction. In particular, it was found that a consistent set of normalising factors could not be readily determined to relate the angular distributions and the excitation functions. The problems

arose from several sources, mainly the rapid variation of elastic yield with energy and angle (eg. see figs 1.13 and 1.14) coupled with small energy uncertainties as discussed in the previous section, together with further problems introduced by the selection of different monitor angles for the various angular distributions and the fact that not all of the distributions included the angles at which the excitation functions were recorded.

It was felt that it should be possible to correct errors in the angular distribution energies with reasonable certainty, by taking advantage of the strong constraints imposed on the positions of the angular distributions relative to the excitation functions, through the rapid and often opposing yield variations in the region at the various excitation function angles (cf. fig 1.14).

To this end, a FORTRAN program was developed for use on the A.N.U. IBM 360/50 computer, which allowed simultaneous free variation of the angular distribution energies, together with the normalising factors associated with the angular distributions and excitation functions, to minimise the value of a function, χ^2 , defined as the weighted sum of the squared deviations of the angular distribution yield from the excitation function yield at the corresponding energy and angle, summed over all 'cross-over' points of angular distributions with excitation functions. It was further decided to incorporate the possibility of varying the excitation function angles from the nominal angle, and to allow for possible target buckle in the beam direction which could lead to systematic errors of angle in the angular distributions.

The basis of the program lay in a general, non-linear least squares fitting routine, FITALL, developed by Hay and Barker (Ha 66) which minimises χ^2 by means of two procedures:

- (i) descent-cycle: the parameters are varied one at a time according to internally calculated step sizes, and the program then moves iteratively to the minima of quadratics fitted through the χ^2 curves for the various parameters, until χ^2 changes slowly for each parameter.
- (ii) contour-cycle: when the descent cycle has moved into a region of parameter space in which χ^2 decreases slowly for each parameter, χ^2 contours of the region are examined by changing the parameter values singly and in pairs, and a multi-dimensional quadratic is then fitted through the χ^2 surface. Following this, the parameters are changed to move to the stationary point of the fitted quadratic surface.

The cycles are repeated in an order determined by the program until satisfactory convergence is achieved.

Since the weights associated with the deviations are intimately connected with the shape of the χ^2 surface, some effort was expended in the selection of a suitable system of weighting. The procedure finally employed was to weight each 'cross-over' point with the normal statistical error associated with the value, together with the inverse of the slopes of the yield curves in both angle and energy, according to the form

$$W = \frac{\sqrt{N}}{\cos(\arctan(S(E))) \times \cos(\arctan(S(\theta)))} , \quad (1.2)$$

where the scales of the slopes, $S(E)$ and $S(\theta)$, were determined according to accuracy considerations of the energy and angle settings.

'Cross-over' values which had to be determined by interpolation were calculated from quadratics fitted through adjacent measured values on the appropriate angular distribution or excitation function.

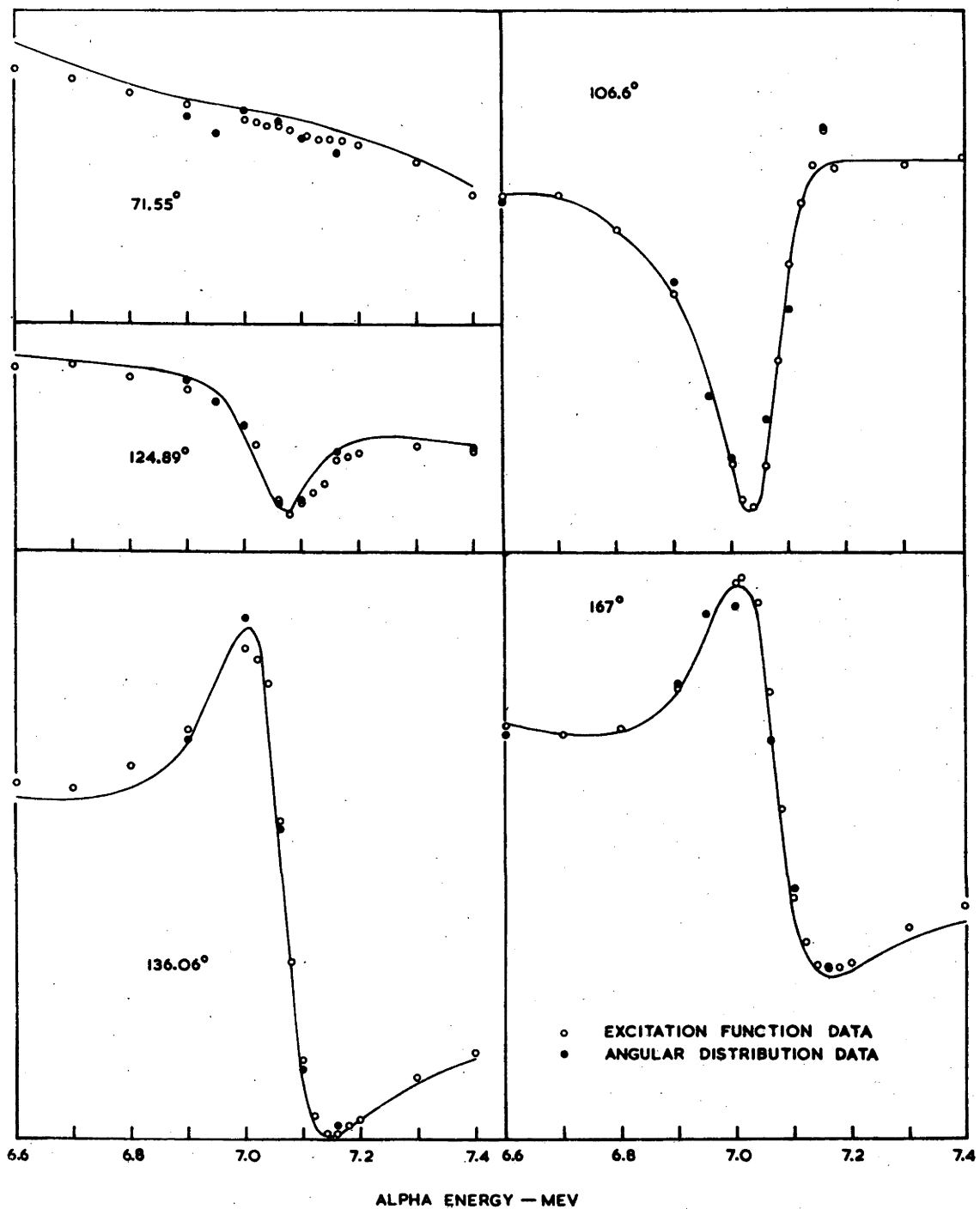
Acceptable normalisation of the data was achieved with energy shifts in the angular distributions generally $< 5 \text{ KeV}$, although several shifts were as large as 15 KeV . Angle shifts in the excitation functions were found to be insignificant ($< 0.05^\circ$) as were shifts of the target position in the chamber ($< 1 \text{ mm}$ from centre).

In view of the nature of the program, it was felt important to test consistency of the results in as many ways as possible.

Some of the tests which were applied are indicated below:

- (i) the excitation functions, which already had well determined experimental normalisations, were entered into the program with arbitrary factors, as were the angular distributions, and the results for the excitation functions were in each case within one or two percent of the experimental determinations.
- (ii) where several angular distributions were obtained in one run with the same target, related normalisations and the same energy shifts would be expected. This was found to be so without exception. In particular, the 15 KeV energy shifts mentioned above were associated with three angular distributions performed during the same run in the newly constructed target area where larger than usual energy uncertainties were not unexpected, as discussed in section 1.3.
- (iii) toward the latter stages of the experiment, when the problems of normalisation were better understood, it was decided to repeat carefully some of the measurements

Fig. 1.9 REPEAT MEASUREMENTS IN THE LOW ENERGY REGION OF THE $C^{12}(\alpha, \alpha_0)C^{12}$ EXCITATION FUNCTIONS AND ANGULAR DISTRIBUTIONS (OPEN AND CLOSED CIRCLES), RECORDED OVER TWO RUNS AND NORMALISED EXPERIMENTALLY, COMPARED WITH A PORTION OF THE TOTAL EXCITATION FUNCTION DATA (SOLID CURVES), NORMALISED AS DESCRIBED IN THE TEXT. THE 167° EXCITATION FUNCTION SCALE HAS BEEN REDUCED BY A FACTOR FIVE.



in the more simple low energy region. It was found that these data, collected over two runs, could be normalised together satisfactorily by hand. On development of the normalising program, the angular distributions from these measurements, whose relative normalising factors were thus known, were entered into the program with arbitrary factors together with the complete original set of data over the full energy region. The factors calculated by the program were found to be within several percent of the known experimental values. Fig 1.9 shows the normalised excitation functions of the original data (solid curves) over the low energy range, together with the remeasured experimental points.

It was felt that the results of the above three checks provided good confirmation of the validity of results from the program.

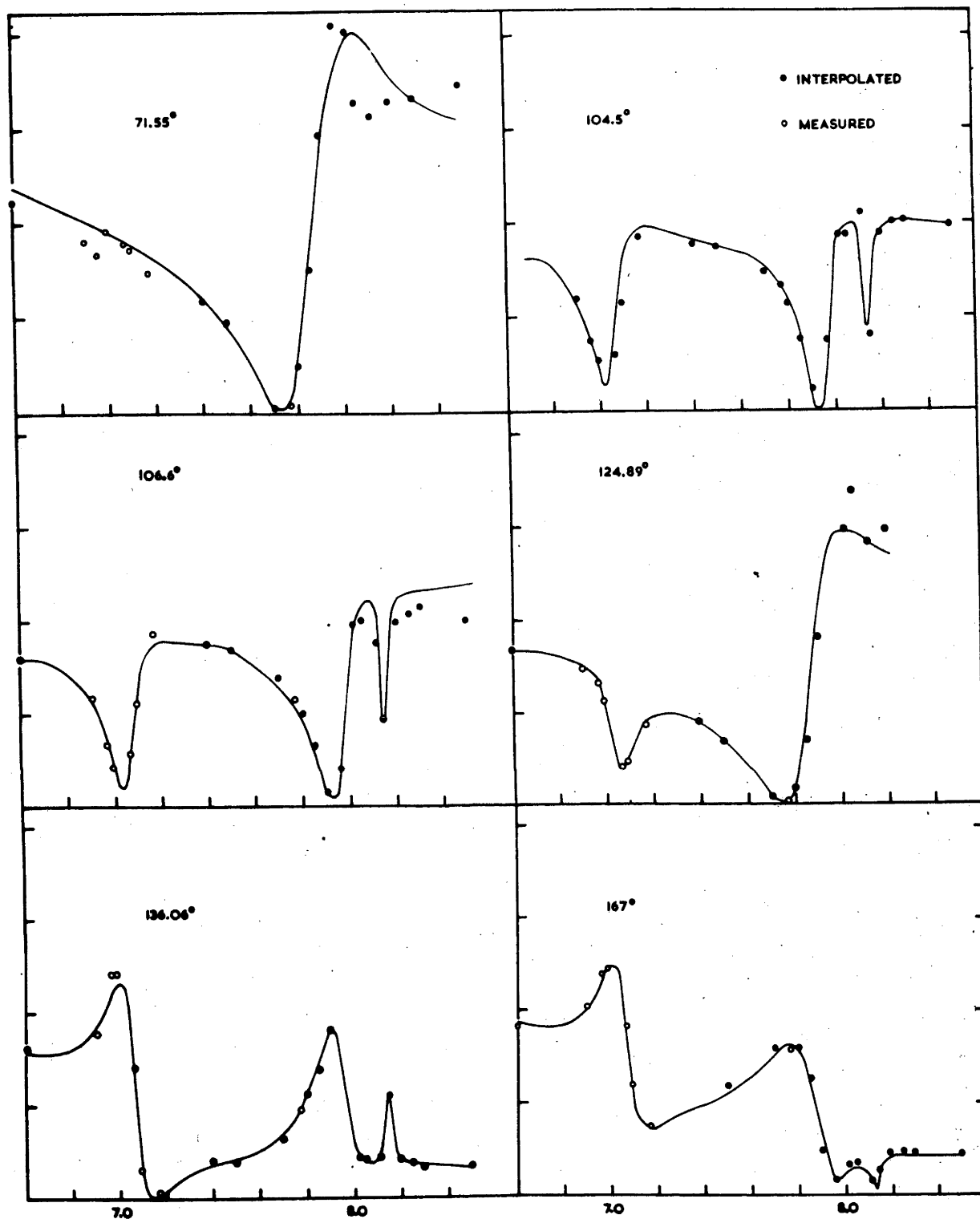
Fig 1.10 shows the excitation functions compared with the measured and interpolated 'cross-overs' of the angular distributions over the full energy range, after normalisation.

1.3.2 CROSS SECTION DETERMINATION

Three methods were used to determine absolute cross sections:

- (i) The present data were related to the absolute measurements of Jones et al (Jo 62) by comparison of yields at alpha particle energies of 4.0 and 6.6 MeV at angles of 71.55° , 106.60° and 136.06° which were common to both sets of data.

Fig. 1.10 COMPARISON OF THE EXCITATION FUNCTION DATA AND ANGULAR DISTRIBUTION DATA AFTER NORMALIZATION TO A COMMON RELATIVE YIELD SCALE. WHERE DISTRIBUTION DATA WERE NOT RECORDED AT THE EXCITATION FUNCTION ANGLES THE YIELD, INTERPOLATED AS DESCRIBED IN THE TEXT, IS SHOWN. THE 167° EXCITATION FUNCTION SCALE HAS BEEN REDUCED BY A FACTOR FIVE.



INCIDENT ALPHA ENERGY - MEV

- (ii) Gas-target measurements using natural CO_2 were made at alpha particle energies of 6.6, 7.4 and 8.4 MeV and angles of 60° , 71.55° and 90° employing a gas cell with thin nickel entrance and exit windows and a mylar window for the reaction products. The gas pressure (22 mm of mercury) and temperature were monitored continuously. Beam integration was achieved as discussed in section 1.2.6 .
- (iii) The proton energy equivalents of several of the carbon foils used in the $\text{C}^{12}(\alpha, \alpha_0)$ measurements were determined for an incident energy of 894 KeV , using a target modulation technique (cf Chap 2) to measure the displacement of the 874 KeV $\text{F}^{19}(\text{p}, \alpha\gamma)\text{O}^{16}$ resonance. Tables of stopping power allowed conversion of the energy equivalents into units of carbon atoms/ cm^2 , so that absolute normalisation of the $\text{C}^{12}(\alpha, \alpha_0)$ data could again be obtained.

The values obtained from methods (i) and (ii) agreed to within 10% , whereas the results of method (iii), using tables of stopping powers tabulated by Williamson and Boujot (Wi 66), yielded values ~25% larger. Sources of error in the last measurement are discussed in chapter 2. The cross sections from the gas-target measurements were used in the analysis.

1.4 ANALYSIS

1.4.1 GENERAL

The elastic scattering cross section for charged spinless particles on a spinless target may be expressed as follows

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{k^2} \left| A_c + \left(\frac{i}{2}\right) \sum_{\ell=0}^{\infty} (2\ell + 1) [e^{i\alpha_{\ell}} - U_{\ell}] P_{\ell}(\cos \theta) \right|^2, \quad (1.3)$$

where,

A_c is the usual coulomb scattering amplitude

$$\alpha_{\ell} = 2 \sum_{s=1}^{\ell} \tan^{-1} \left(\frac{\eta}{s} \right), \quad \text{the coulomb phase shift} \quad (\alpha_0 = 0)$$

$$\eta = \frac{Z_1 Z_2 e^2}{k v}$$

k is the wave number.

The U-matrix, or collision matrix, relates the scattered wave to the incoming wave. When elastic scattering only is possible, the U-matrix is diagonal so that U may be expressed in the form

$$U_{\ell} = e^{i\Psi_{\ell}}, \quad (1.4)$$

where Ψ_{ℓ} is a real phase from unitarity.

If inelastic processes are also possible, the U-matrix has non-zero off-diagonal elements. In this case, the diagonal

elements describing the elastic scattering may be expressed as

$$U_\ell = A_\ell e^{i\Psi_\ell}, \quad (1.5)$$

where, again from unitarity, $0 \leq A_\ell \leq 1$ and Ψ_ℓ is again a real phase. A_ℓ is a measure of the absorption of the incoming partial wave of angular momentum ℓ into channels other than elastic scattering.

It is useful to express Ψ in the form

$$\Psi_\ell = \alpha_\ell + 2\Delta_\ell, \quad (1.6)$$

where α_ℓ is defined above.

The expression for the scattering cross section (equation (1.3)) reduces to:

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{k^2} \left| A_c + \frac{i}{2} \sum_{\ell=0}^{\infty} (2\ell + 1) e^{i\alpha_\ell} [1 - A_\ell e^{2i\Delta_\ell}] P_\ell(\cos \theta) \right|^2, \quad (1.7)$$

the expression employed in the data analysis.

So far, no reference has been made to particular models of the scattering problem, and the expressions presented have been deduced wholly from the laws of quantum mechanics. The phase shift extraction is thus to be regarded as a simple parameterisation of the data in terms of the real phase shifts Δ_ℓ and absorption coefficients A_ℓ , and interpretation of the resulting parameters treated as a separate problem.

Level parameters were extracted from the phase shifts on the assumption that the scattering process was described by the compound nucleus mechanism, using the R-matrix theory presented by Lane and Thomas (La 58).

1.4.2 ONE-LEVEL APPROXIMATION

(The suffix l is omitted in the following discussion.)

Defining the absorption parameter

$$a = \frac{\Gamma_c}{\Gamma} , \quad (1.8)$$

where Γ_c and Γ are the elastic partial width and total width respectively of the level at the resonance energy (it follows that $0 \leq a \leq 1$), the diagonal elements of the U-matrix may be written, (M1 62).

$$U = e^{i(\alpha+2\varphi)} [1 + a(e^{2i\beta} - 1)] , \quad (1.9)$$

where α is defined above, the resonant phase β is defined according to

$$\tan \beta = \frac{\Gamma/2}{E_0 - E} , \quad (1.10)$$

and the hard sphere phase φ is given by

$$\varphi = - \tan^{-1} \frac{F_l}{G_l} . \quad (1.11)$$

From equations (1.5, (1.6) and (1.7), the following simple expressions for A and Δ may be calculated:

$$\Delta = \Delta^0 + \varphi, \quad \text{where} \quad \tan 2\Delta^0 = \frac{a \sin 2\beta}{(1-a) + a \cos 2\beta}, \quad (1.12)$$

$$A^2 = a^2 + 2a(1-a)\cos 2\beta + (1-a)^2. \quad (1.13)$$

The behaviour of the phase shift Δ^0 and the absorption coefficient A , calculated from (1.12) and (1.13), for a reaction proceeding through an isolated resonance and for several values of the parameter a , are shown in fig 1.11. (The energy dependence of a due to penetrability has been neglected in fig 1.11 for simplicity.)

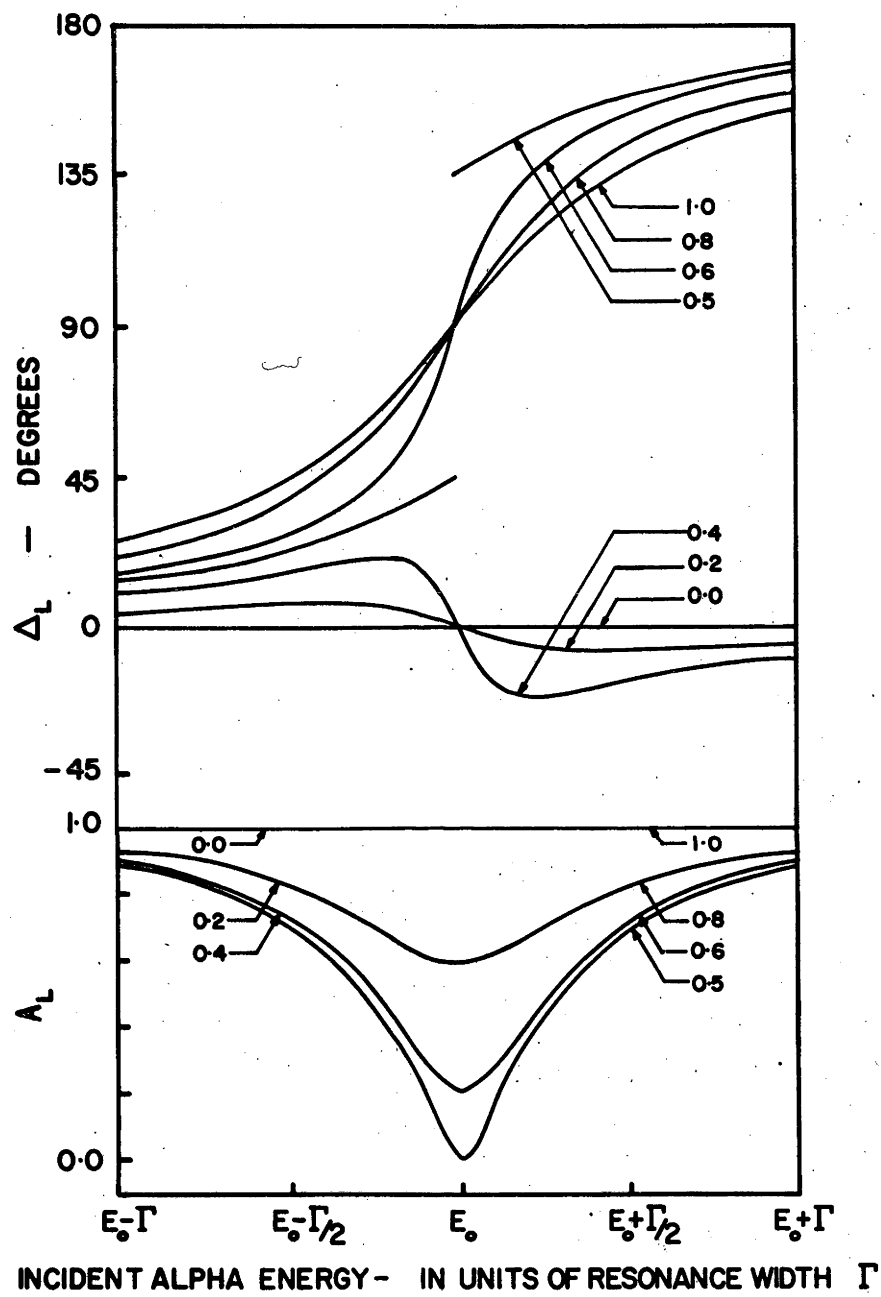
It is of interest to note that whereas Δ^0 has essentially the same shape as the resonant phase, β , for all $a > 0.5$, the shape for $a < 0.5$ is 'anomalous', in that Δ^0 no longer passes through 90° at the resonant energy, but turns over and passes through 0° instead. Similar behaviour has been reported by Duval et al (Du 66).

The resonant phase β is of course independent of a and always increases monotonically through 180° . It is clear from fig 1.11 that an erroneous result will obtain if the width of a level which decays by particle emission via more than one channel is determined in the conventional manner of taking the energy difference between the 45° and 135° phase angles of Δ^0 (assuming $a > 0.5$).

The 90° discontinuity in Δ^0 at $E = E_0$ with $a = 0.5$ poses no problem physically since A vanishes under these conditions and the cross section thus remains continuous through the resonance energy.

From equations (1.10) and (1.13) it is straightforward to show that A^2 has a simple Breit-Wigner form,

Fig. 1.11 THE CALCULATED BEHAVIOUR OF THE REAL PHASE SHIFT Δ AND ABSORPTION COEFFICIENT A , FOR AN ISOLATED LEVEL OF RESONANT ENERGY E_0 AND WIDTH Γ , SHOWN AS A FUNCTION OF THE INCIDENT ENERGY. CURVES ARE SHOWN FOR VALUES OF $a = \Gamma_{\alpha_0} / \Gamma$, 1.0, 0.8, 0.6, 0.5, 0.4, 0.2 AND 0.0. THE ENERGY DEPENDENCE OF THE LEVEL WIDTH AND LEVEL SHIFT, AND THE HARD SPHERE COMPONENT OF THE PHASE SHIFT, HAVE BEEN NEGLECTED.



$$A^2 = 1 - a(1 - a) \cdot \frac{\Gamma^2}{(E - E_0)^2 + (\Gamma/2)^2} , \quad (1.14)$$

justifying use of equation (1.14) in section (1.5.2). Equation (1.14) demonstrates (as do equation (1.13) and fig 1.11) that the absorption parameter, a , is not uniquely determined by the value of A at $E = E_0$, but has two possible solutions. The correct solution is determined by consideration of the behaviour of Δ^0 , the value $a > 0.5$ being appropriate if Δ^0 passes through 90° at the resonant energy, whilst 'anomalous' behaviour of Δ^0 specifies the value which is less than 0.5.

1.4.3 TWO-LEVEL APPROXIMATION

In the following section, the two-channel case only is considered.

The following definitions are useful -

$\Gamma_i(E_i)$ = total width of level i at the resonant energy E_i

$$a_i = \frac{\Gamma_{i1}(E_i)}{\Gamma_i(E_i)} = \text{constant}$$

Γ_{ij} = partial width for channel j of level i , where $j = 1$ is the elastic channel.

In terms of the above definitions it is straightforward to show that

$$\Gamma_{i1} = a_i \cdot \frac{P_1(E)}{P_1(E_i)} \cdot \Gamma_i(E_i) \quad (1.15)$$

and
$$\Gamma_{i2} = (1 - a_i) \cdot \frac{P_2(E)}{P_2(E_i)} \Gamma_i(E_i) ,$$

where $P_j(E)$ is the penetrability factor appropriate to channel j at energy E . The above expressions (equation 1.15)) can then be used for the partial widths in the two-level, two-channel expression for the diagonal elements of the U -matrix (M1 62), in which level shifts have been neglected,

$$U = \left| \begin{array}{c} 1 + \frac{\Gamma_{12} - \Gamma_{11}}{E - E_1} + \frac{\Gamma_{22} - \Gamma_{21}}{E - E_2} - \frac{(\sqrt{\Gamma_{21}\Gamma_{12}} - \sqrt{\Gamma_{11}\Gamma_{22}})^2}{(E - E_1)(E - E_2)} \\ 1 + \frac{\Gamma_{12} + \Gamma_{11}}{E - E_1} + \frac{\Gamma_{22} + \Gamma_{21}}{E - E_2} + \frac{(\sqrt{\Gamma_{21}\Gamma_{12}} - \sqrt{\Gamma_{11}\Gamma_{22}})^2}{(E - E_1)(E - E_2)} \end{array} \right| \cdot e^{i(\alpha+2\varphi)} \quad (1.16)$$

Equating the above expression (equation 1.16) to the general expression for U (equation (1.5)), it is again possible to obtain expressions for A and Δ^0 similar to, if more complicated than, those previously derived in the one-level approximation. The same general results hold for the shape of Δ^0 as a function of a , at each level, as held in the one-level approximation, but A^2 is in general no longer a true Breit-Wigner as before, being skewed to an extent dependent on the width and absorption parameter, a , of the adjacent level. The Breit-Wigner form of A^2 equation (1.14) used in the fitting procedure is thus not strictly valid if two or more levels of the same spin and parity are present, each with more than one channel open (cf e.g. fig 1.17). The shape is further modified by penetrability effects.

1.5 PHASE SHIFT EXTRACTION

The phase shifts and absorption coefficients enter non-linearly into the cross section expression (equation (1.7)), and iterative search techniques must therefore be employed in obtaining values which best reproduce the experimental data. The parameter optimisation program FITALL (see section 1.3.1) was used to this end.

Two complementary methods were employed in the parameterisation of the data in terms of the phase shifts Δ_ℓ and absorption coefficients A_ℓ :

1.5.1 METHOD I - ANGULAR DISTRIBUTIONS

The approach taken was conventional in that the fitting program varied the phase shifts and absorption coefficients (hereinafter referred to as PSAC) freely from given initial values, to minimise the function

$$\chi^2 = \sum_{N=1}^{ND} \left[\frac{\sigma_{TH}(\theta_N, E) - \sigma_{EXP}(\theta_N, E)}{ERR(\theta_N, E)} \right]^2, \quad 1.17$$

for each angular distribution independently.

In the above expression,

$\sigma_{EXP}(\theta_N, E)$ is the measured differential cross section at the N^{th} angle, θ_N , in the angular distribution at energy E ,

$ERR(\theta_N, E)$ is the error associated with the appropriate experimental value above. (Errors were calculated in the manner indicated in section equation (1.2).)

$\sigma_{TH}(\theta_N, E)$ is calculated from the cross section expression (equation (1.7)).

In programs of the type under discussion, the final parameter values obtained may depend on the starting values, as 'non-physical' minima can exist in χ^2 space along with the 'true' physical minimum; i.e., multiple solutions are possible in terms of the simple χ^2 criterion, and the fitting program can easily become 'trapped' in the associated 'minima' if the starting values are unrealistic. Furthermore, there is no a priori reason

for assuming that, of several different solutions obtained, the solution with the smallest value of χ^2 is necessarily the 'correct' one. One must couple χ^2 considerations with the physical requirement of energy continuity of the PSAC through neighbouring angular distributions.

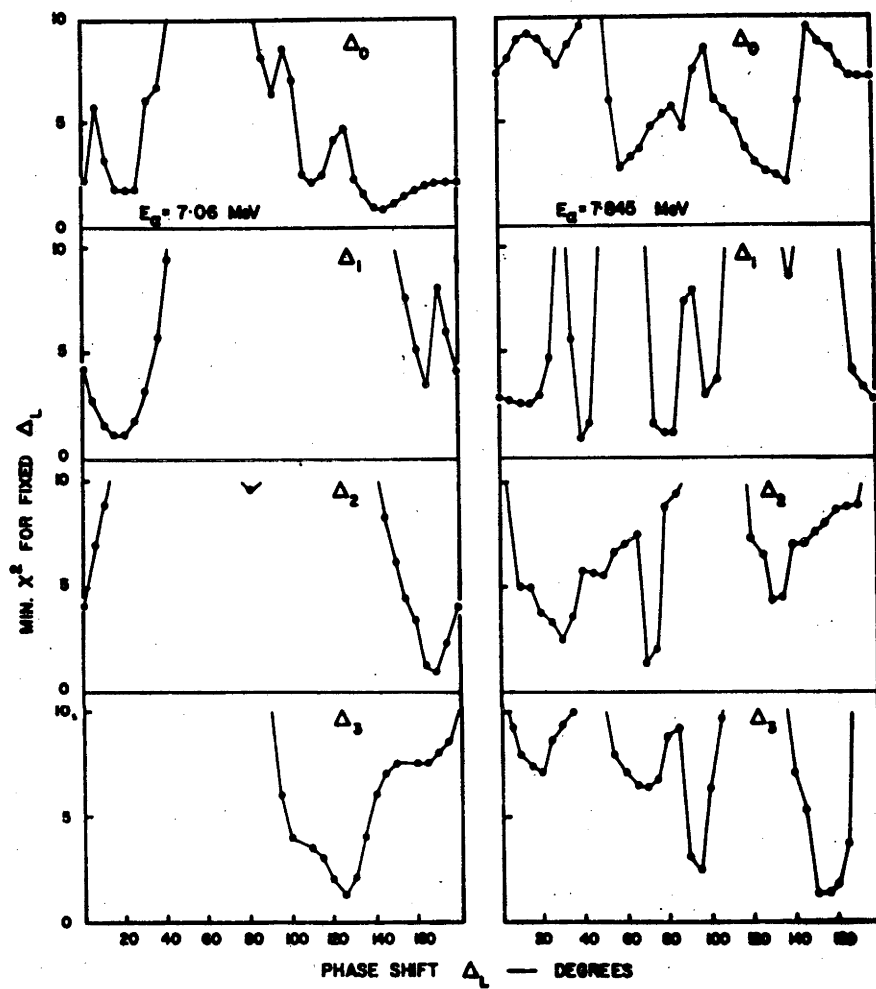
Several procedures were followed in attempting to locate solutions of equivalent χ^2 and hence deduce the 'correct' solution, of which the main are described below:

- (i) various sets of starting values were chosen for the parameters;
- (ii) phase shifts were stepped singly and in pairs through 180° in 5° and 10° steps, simultaneously minimising χ^2 by varying some or all of the remaining parameters;
- (iii) sequential runs were made through all of the angular distributions in order of both increasing and decreasing energy, using the final parameter values from one angular distribution as starting values for the adjacent distribution.

In practice it was found that multiple solutions generally obtained only in complex regions where more than one partial wave was resonating in both elastic and non-elastic channels. eg. cf. fig 1.12 which shows typical differences between the behaviour of the $\chi^2_{\min}$ surfaces as a function of one parameter (as in (ii) above) for angular distributions in two different regions of the compound nucleus.

The highest angular momentum value considered in the program was $\ell = 6$, and since in fact neither the $\ell = 5$ nor the $\ell = 6$ phase was found to vary significantly over the region,

Fig. 1.12 CONTOURS OF THE MINIMUM χ^2 FOR FIXED VALUES OF
THE PHASE SHIFT PARAMETER Δ_0 , Δ_1 , Δ_2 AND Δ_3
FOR THE 7.06 AND 7.845 MeV ANGULAR DISTRIBUTIONS.



Δ_5 and Δ_6 were set to the calculated hard sphere values in the final stages of fitting.

An investigation into the effect of varying the cross section by $\pm 20\%$ showed that, in general, only the s-wave phase was affected significantly, changing by as much as 50° ($\sim 20^\circ$) in some regions, to compensate.

1.5.2 METHOD II - EXCITATION FUNCTIONS

Although the criterion of smooth energy dependence of the PSAC, from the angular distribution analyses, was adhered to as much as practical, the final PSAC exhibited some scatter as evidenced by fig 1.15. Furthermore, values derived from smooth curves fitted through the PSAC resulted in calculated excitation functions which were in poor agreement with the data, especially over the regions containing resonances. However, in initial attempts to improve the fits by manual manipulation of the shapes of the curves, it became evident that the excitation functions were very sensitive to small changes in the PSAC, and it was decided to take advantage of this fact by attempting a non-linear least-squares analysis of the excitation functions in a manner analogous to that used for the angular distributions.

The method chosen involved varying the shapes of the smooth curves mentioned above through use of the fitting program FITALL so as to obtain best fits to the excitation functions by minimisation of the function

$$\chi^2 = \sum_{N=1}^{NE} \sum_{M=1}^{ND(N)} \frac{[\sigma_{TH}(\theta_N, E_M) - \sigma_{EXP}(\theta_N, E_M)]^2}{\sigma_{EXP}(\theta_N, E_M)}, \quad (1.18)$$

where,

NE = number of excitation functions

ND(N) = number of data points in the N^{th} excitation function
at angle θ_N

$\sigma_{\text{TH}}(\theta_N, E_M)$ = calculated cross section at energy E_M ,
angle θ_N

$\sigma_{\text{EXP}}(\theta_N, E_M)$ = measured cross section at energy E_M ,
angle θ_N .

Note that this method automatically ensures smoothness of the PSAC over the region of fitting.

The sequence of the program was as follows:

- (i) linear least squares fits to the phase shifts resulting from Method I were obtained for each partial wave with a quasi-Fourier series of the form,

$$\Delta_{\ell}(E) = B_{\ell} + C_{\ell} \cdot E + \sum_{N=1}^{\text{NS}(\ell)} D_{\ell}^{(N)} \sin\left[N\pi \frac{(E - E_1)}{(E_2 - E_1)}\right], \quad (1.19)$$

where,

$\Delta_{\ell}(E)$ is the value of the phase shift of angular momentum ℓ at energy E ;

B_{ℓ} , C_{ℓ} and $D_{\ell}^{(N)}$ are $(\text{NS}(\ell) + 2)$ free parameters for each partial wave ℓ ;

$\text{NS}(\ell)$ is the number of Fourier series terms allowed in the fit for the phase of angular momentum ℓ ;

E_1 is the lower bound of fit;

E_2 is the upper bound of fit.

In practice, the number of terms included in the series was limited to $NS(\ell) \leq 4$ by appropriate choice of boundaries E_1 and E_2 , and $NS(\ell)$ was set to 0 or 1 in regions where the phase was effectively linear as a function of energy.

- (ii) in regions where the angular distribution results indicated absorption into non-elastic channels for some partial wave ℓ (i.e., A_ℓ consistently less than unity over the region of a resonance) the absorption coefficients, A_ℓ , of equation (1.7) were entered into the program in the form given by equation (1.14), initial values of the parameters Γ_ℓ , a_ℓ and E_{o_ℓ} being obtained from the angular distribution results.

Although use of the Breit-Wigner form of equation (1.14) limited the generality of the fitting procedure, this form is applicable in the single level case (for reasonably narrow levels), and the convenience of the expression together with the small number of parameters required to describe it and the fact that the cross section was found not to be critically dependent on A_ℓ , led to its use in the program. The effect of this restriction on the shape of A_ℓ is discussed later.

- (iii) the fitting section then attempted to minimise χ^2 as defined by equation (1.18), by varying the parameters of the quasi-Fourier series (equation (1.19)), and the parameters Γ , a and E_o of equation (1.14), for chosen ℓ values. In other words, the program minimised χ^2 by a non-linear variation of the shapes of the phases, and of the absorption coefficients if appropriate, over the region of fit.

In practice, the 2 MeV region over which the data extended was divided into smaller regions of from 200 - 600 KeV , which were fitted independently. These (arbitrary) divisions were introduced to simplify the phase shapes entered into the program, thereby reducing the number of parameters necessary to describe the shapes in each region.

It is to be noted that, if the phases are well determined by the data and the data are internally consistent, there should be continuity across the region boundaries, thus providing a good consistency test of the results.

1.6 EXPERIMENTAL RESULTS

The excitation functions are compared with measured and interpolated data from the angular distributions in fig 1.10. The complete angular distribution and excitation function data are shown in figs 1.13 and 1.14, together with fits calculated from the phases and absorption parameters (fig 1.15) resulting from METHOD II above.

The p- and f-wave phases (fig 1.15) each contain regions of 'anomalous' behaviour, indicating the presence in O^{16*} of 1^- and 3^- states at energies of 13.10 and 13.25 MeV respectively, whose elastic widths have less than half the magnitude of the respective total widths (cf section 1.4.2). The presence of three further states is indicated by the more typical 'resonant' behaviour of the p- , d- and f-wave phases also apparent in fig 1.15. These states are identified as 1^- , 2^+ and 3^- at energies of 12.44, 13.01 and 13.13 MeV respectively.

Shown in figs 1.16 and 1.17 are two-level two-channel R-matrix fits to the p- and f-wave phases. The asymmetry (section 1.4.3) of the theoretical shape of the absorption coefficient

Fig. 1.13 MEASURED $C^{12}(\alpha, \alpha_0)C^{12}$ ANGULAR DISTRIBUTIONS
RECORDED FOR THE INCIDENT ALPHA PARTICLE
ENERGIES SHOWN. EACH DISTRIBUTION IS DISPLAYED
ON THE SAME CROSS-SECTION SCALE AND DISPLACED
BY AN ARBITRARY AMOUNT. THE HORIZONTAL BARS
INDICATE THE 0.1 mb LEVEL FOR INDIVIDUAL
DISTRIBUTIONS. THE SOLID CURVES ARE DISTRIBUTIONS
CALCULATED FROM THE PHASE SHIFTS DERIVED FROM THE
 $C^{12}(\alpha, \alpha_0)C^{12}$ EXCITATION FUNCTION DATA.

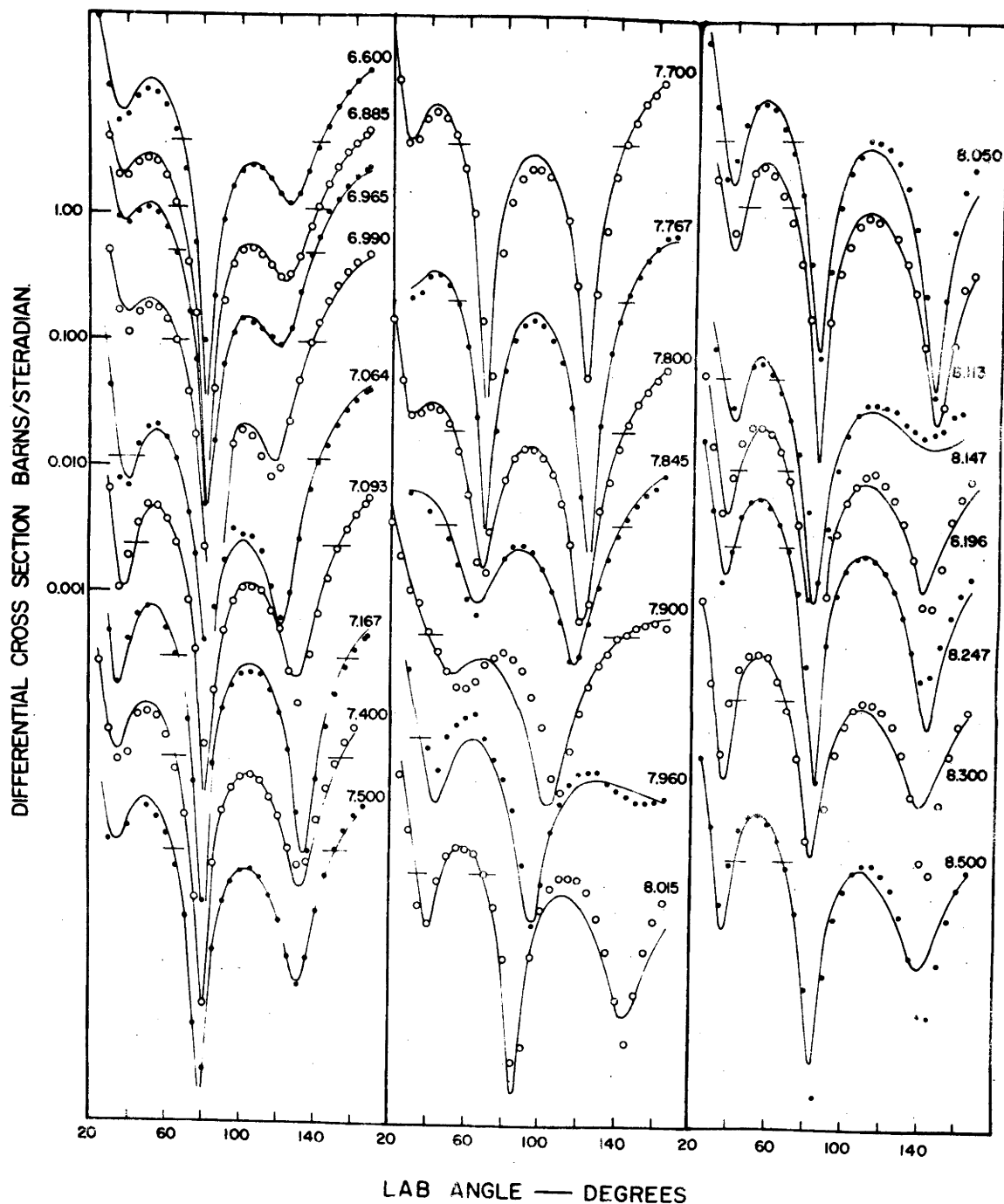
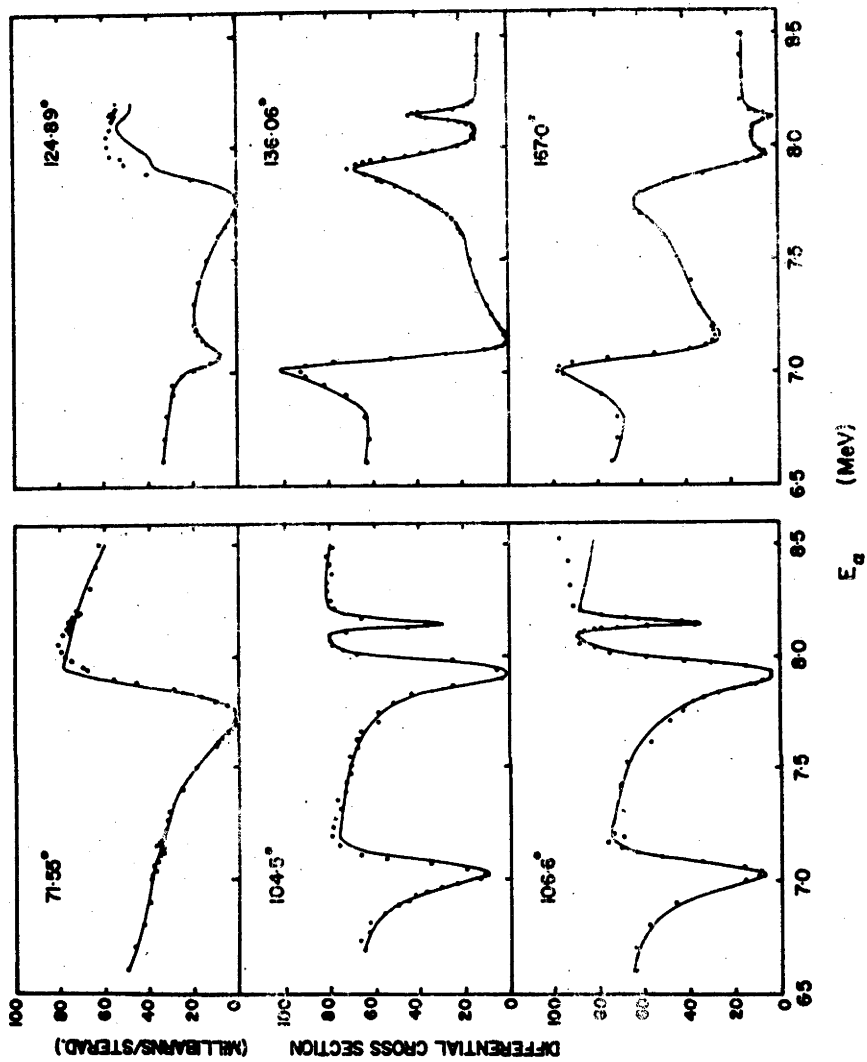


Fig. 1.14 MEASURED CENTRE-OF-MASS DIFFERENTIAL CROSS-
SECTIONS FOR THE $C^{12}(\alpha, \alpha_0)C^{12}$ REACTION RECORDED
AT THE ANGLES INDICATED. THE SOLID CURVES WERE
CALCULATED FROM THE PHASE SHIFTS DISCUSSED IN
THE TEXT. THE ORDINATE FOR THE EXCITATION
FUNCTION AT 167° IS REDUCED BY A FACTOR OF
FIVE.



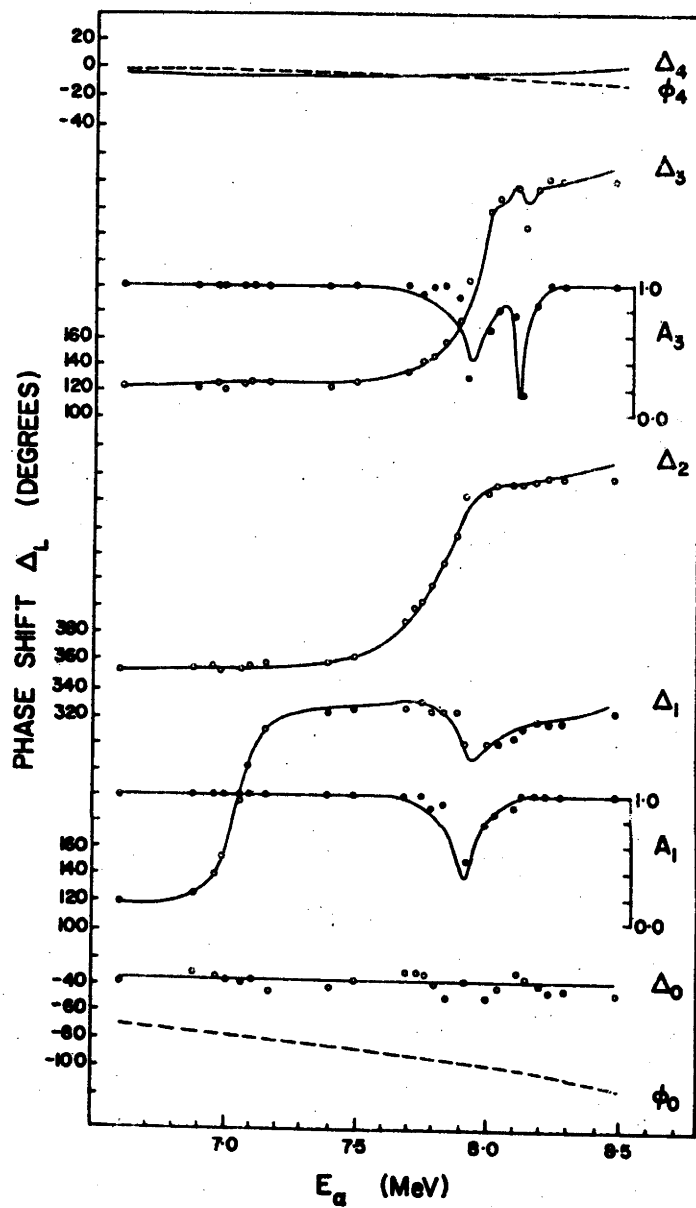
A_ℓ is clearly seen in the fit to the upper 3^- level, fig 1.17. This departure from Breit-Wigner form, together with some interdependence of the phases and absorption parameters in fitting the data, can be expected to account for some part of the small discrepancies observed between the predicted phase shift shapes and those obtained experimentally.

The upper 1^- state in the region decays via proton emission to the N^{15} ground state. Angular momentum and parity conservation allow both s- and d- wave emission, but s-wave decay was assumed to be the dominant mode from penetrability considerations. In the two-level two-channel fits to the f-wave phase, fig 1.17, the reaction channel for decay of the two 3^- levels was taken as inelastic p-wave scattering of alpha particles to the first excited state of C^{12} (Mi65). Proton decay to the ground state of N^{15} also takes place from the higher energy 3^- state, but inclusion of this third decay channel in the fits to the phase shifts did not alter the elastic parameters significantly, and the simpler two-channel approximation was used in the extraction of these parameters. A one-level one-channel fit to the d-wave elastic phase is also shown in fig 1.16.

In each of the above cases, the addition of a constant background phase, $\Delta\phi_\ell$, was found necessary. During the course of this part of the analysis, no attempt was made to test the overall dependence of the phases on channel radius. The values used throughout the analysis were 5.3 fm in the elastic and inelastic alpha channels, and 3.8 fm in the $N^{15} + p$ channel, calculated from the formula

$$r_c = 1.1 (A_1^{1/3} + A_2^{1/3}) . \quad (1.20)$$

Fig. 1.15 PHASE SHIFTS (Δ_ℓ) AND ABSORPTION COEFFICIENTS (A_ℓ) EXTRACTED FROM THE $C^{12}(\alpha, \alpha_0)C^{12}$ EXCITATION FUNCTION DATA (solid curves) AND FROM THE ANGULAR DISTRIBUTIONS (open and solid data points). ABSORPTION COEFFICIENTS FOR $\ell = 0$, $\ell = 2$ AND $\ell = 4$ DID NOT DIFFER FROM UNITY AND ARE NOT SHOWN. THE HARD SPHERE PHASE SHIFTS ϕ_0 AND ϕ_4 (CALCULATED FROM $\phi_\ell = \tan^{-1} F_\ell / G_\ell$ FOR A CHANNEL RADIUS OF $5.4f$) ARE INCLUDED FOR COMPARISON (dotted curves).



The final parameter values extracted from the PSAC of the levels are shown in table 1.3. Several of the parameters have large errors quoted, due mainly to interactions between the parameters in such a way as to give acceptable fits to the PSAC for ranges of parameter values. eg a_ℓ and Γ_ℓ are two such interacting parameters.

1.7 DISCUSSION

The elastic scattering reaction has proven to be a powerful technique for deducing the spins of the natural parity states of the compound nucleus system O^{16} in the region of 13 MeV. The results of the present experiment give complete verification of the level scheme proposed by Mitchell and Ophel, and accurate values have been deduced for the parameters of the 3^- and 2^+ levels discussed in the introduction.

It is worth noting that the elastic s-wave phase shift has a much slower energy variation than predicted by the calculated hard-sphere s-wave phase, fig 1.15. Clark (Cl 67) has pointed out on the basis of similar observations at incident alpha energies below 6.5 MeV, that such behaviour may be explained by the existence of a very broad 0^+ state ($\Gamma_{CM} \sim 5.5$ MeV) at an excitation energy in O^{16} of ~ 14.5 MeV.

1.7.1 1^- LEVELS

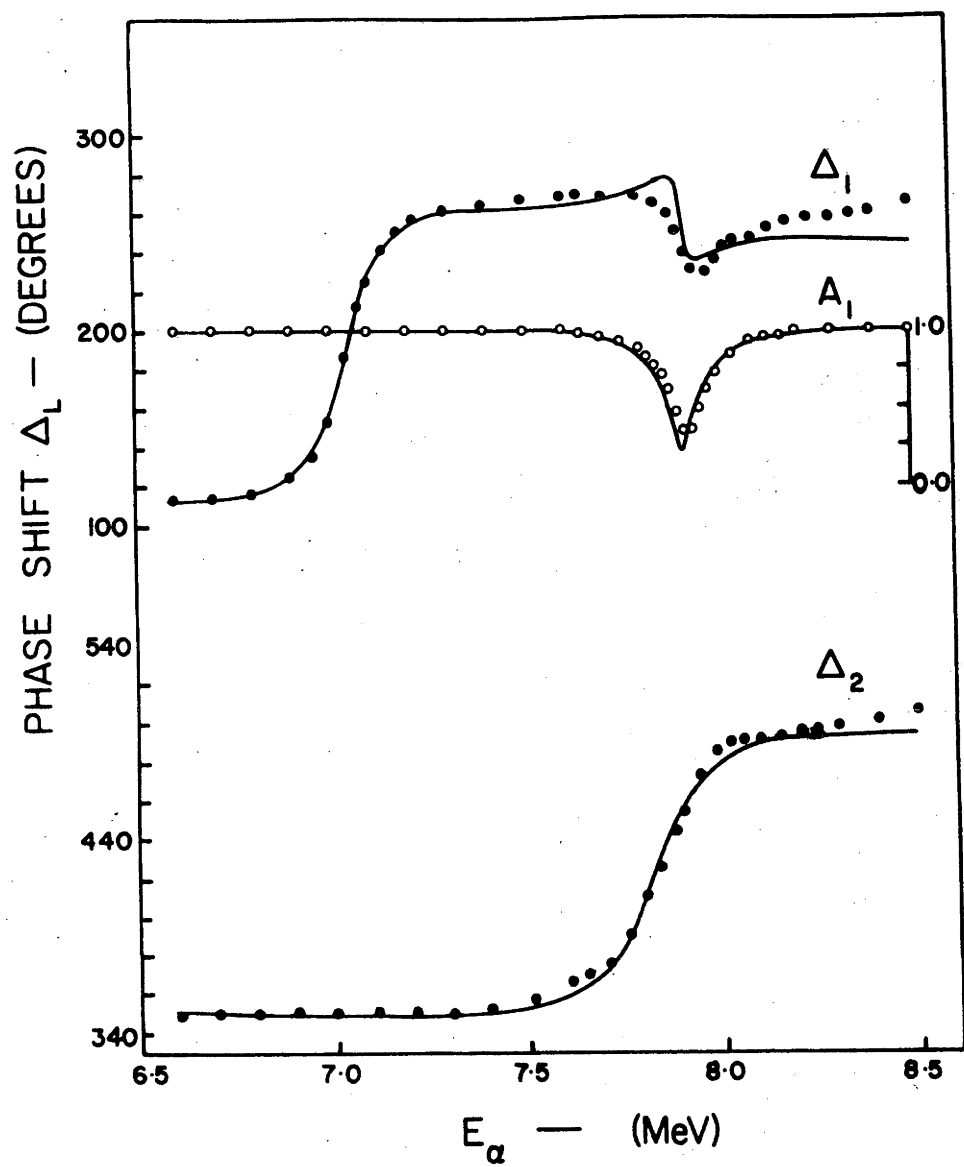
Both of the 1^- states present in the region, at $E_\alpha = 7.045$ MeV and at $E_\alpha = 7.915$ MeV respectively, are well known and their properties have been discussed by several authors. The absorption parameter value, $\underline{a} > 0.95$, obtained for the lower level (table 1.3)

TABLE 1.3

E_{α} (MeV)	J^{π}	$\Gamma_{\text{lab.}}$ (KeV)	$a. = \frac{\Gamma_{\alpha_0}}{\Gamma}$	$\Delta\varphi$ (degrees)
7.045 ± 0.005	1^{-}	130 ± 10	> 0.95	-20
7.820 ± 0.010	2^{+}	200 ± 15	> 0.95	+29
7.915 ± 0.010	1^{-}	150 ± 20	0.40 ± 0.05	-20
7.960 ± 0.010	3^{-}	170 ± 15	0.70 ± 0.05	-40
8.130 ± 0.015	3^{-}	35 ± 10	0.35 ± 0.05	-40

The table lists the values of the level parameters obtained in the present experiment.

Fig. 1.16 CALCULATED FITS TO THE MEASURED $l = 1$ AND
 $l = 2$ PHASE SHIFTS ASSUMING THE LEVEL
PARAMETERS LISTED IN TABLE 1.3.



is consistent with the non-appearance of this level in the $C^{12}(\alpha, \alpha_1)C^{12*}$ and $C^{12}(\alpha, p)N^{15}$ reactions. The width determined for the upper level, $\Gamma_{CM} = 113$ KeV, is narrower than the 140 KeV quoted by previous experimenters, and the ratio of elastic width to total width, $\underline{a} = 0.4$, is somewhat larger than the 30% given by Hebbard (He 60), (table 1.4), from two-level multi-channel fits to the total $N^{15}(p, \alpha_0)C^{12}$ cross section. However, as mentioned in section 1.6, $\underline{a}$ and Γ are interdependent to some extent in the fitting procedures of the present experiment, and the results quoted by Mitchell and Ophel and by Hebbard (table 1.4) are therefore probably more accurate for this level.

1.7.2 2^+ LEVEL

As in the case of the 1^- level at $E_\alpha = 7.045$ MeV, the 2^+ level at $E_\alpha = 7.82$ MeV was found to decay almost entirely via the elastic channel ($\underline{a} > 0.95$) in agreement with all other available experimental information. In particular, Hebbard, in his analysis of the $N^{15} + p$ data, noted that the 2^+ level postulated by Bittner and Moffat to account for the behaviour of the d-wave phase in the elastic scattering of alpha particles from C^{12} , was evident only in interference terms of the $N^{15}(p, \alpha_0)C^{12}$ angular distributions, and concluded that the state would have a large α_0 and a small p width. The presence of the level is apparent also in interference terms of $C^{12}(\alpha, \gamma_0)O^{16}$ and $C^{12}(\alpha, \alpha_1)C^{12*}$ angular distributions (La 64, Mi 64) and arguments similar to Hebbard's above, lead to the conclusion that the state should also have small inelastic alpha particle partial width.

Since absorption into channels other than elastic is small, the effect of the absorption parameter $\underline{a}$ on the level width due to correlations in the fitting, is negligible, and the value given in

TABLE 1.4

The Level Parameters of the 12.44, 13.01, 13.08, 13.12 and 13.25 MeV Levels of O^{16}

J^π	Present Work					Previous Work					Γ_0/Γ	T	Ref.
	$E_{EX}^{O^{16}}$ (MeV)	$\Gamma_{total}^{O^{16}}$ (KeV)	$\Gamma_{\sigma}^{O^{16}}$ (KeV)	$a = \Gamma_{\sigma}/\Gamma$	$E_{EX}^{O^{16}}$ (MeV)	Γ_{total} (KeV)	Γ_{σ} (KeV)	Γ_{α_1} (KeV)	Γ_p (KeV)	Γ_{γ} (eV)			
1^-	12.436 ± 0.005	98 ± 8	98 ± 8	1.00	12.44	94	93	0.025	1.1	12.8	1.0	0	(a)
2^+	13.01 ± 0.010	150 ± 11	150 ± 11	1.00	13.15 ± 0.1	250	250	small	0	0.6 ± 0.3	1.0		(b)
1^-	13.08 ± 0.010	113 ± 15	45 ± 18	0.40 ± 0.05	13.10 ± 0.010	140	40	1	100	88	0.29	1	(a)
3^-	13.12 ± 0.010	128 ± 11	90 ± 14	0.70 ± 0.05	13.13 ± 0.010	100	80(c)	20(c)	1	-	0.80	0(d)	(b)
3^-	13.25 ± 0.015	28 ± 8	9 ± 4	0.35 ± 0.05	13.25 ± 0.010	22.5	10.5	7.5	4.5	-	0.44	1(d)	(a)

(a) Hebbard (He 60).

(b) Mitchell and Ophel (MI 64, MI 65).

(c) Estimated from relative (σ, σ') yield at $E_{\alpha} = 7.96$ and 8.14 MeV - see text.

(d) See text.

Fig. 1.17 CALCULATED FITS TO THE MEASURED $\ell = 3$ PHASE SHIFTS AND ABSORPTION COEFFICIENTS FOR THE LEVEL PARAMETERS LISTED IN TABLE 1.3. TWO DIFFERENT VALUES OF Γ_{α_0}/Γ FOR THE UPPER (8.13 MeV) 3^- LEVEL HAVE BEEN USED:

(a) $\Gamma_{\alpha_0}/\Gamma = 0.35$, and

(b) $\Gamma_{\alpha_0}/\Gamma = 0.25$.

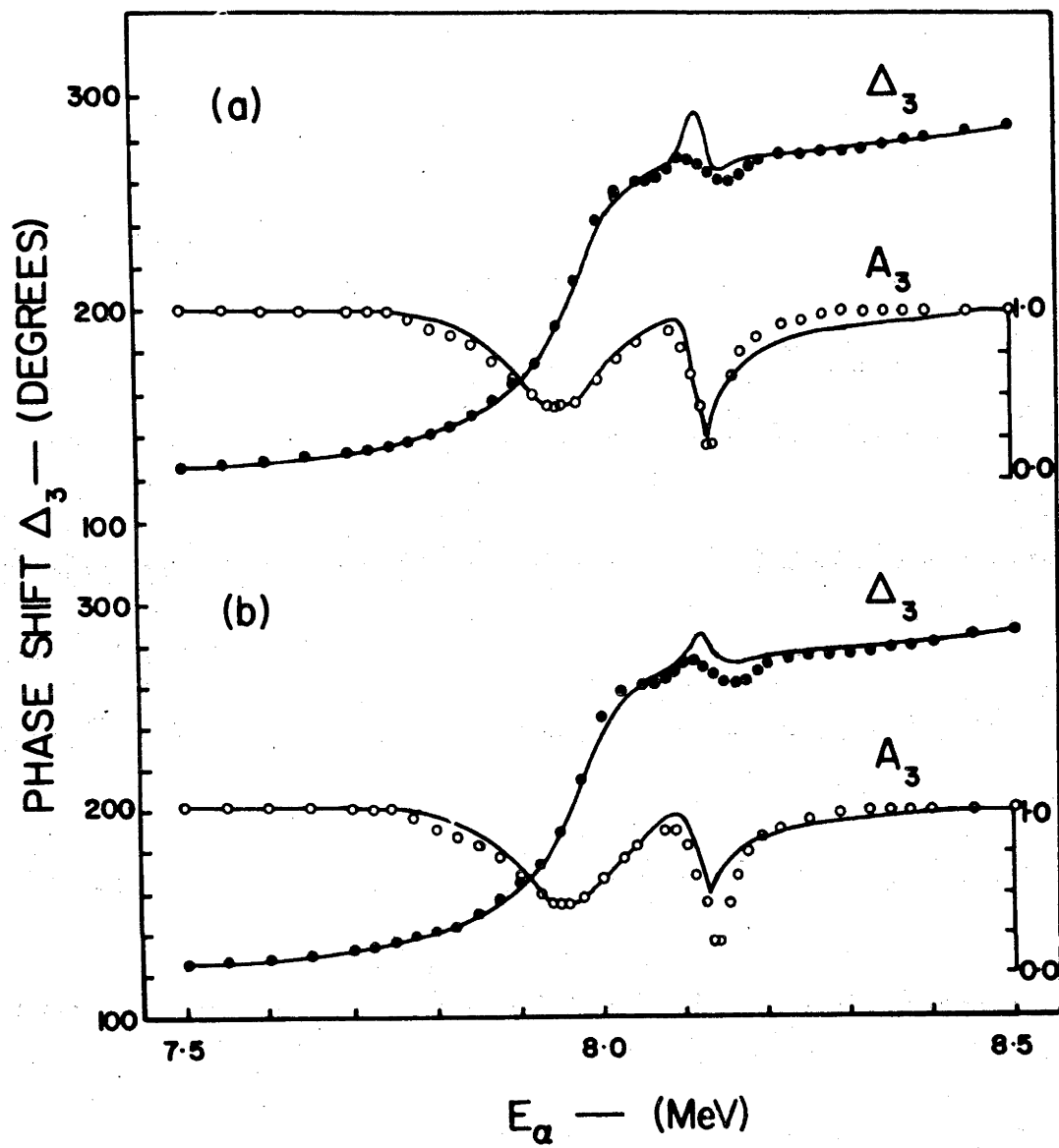


table 1.3, $\Gamma_{CM} = 150 \text{ KeV}$, is considered more accurate than that given from previous work on the $N^{15}(p, \alpha_0)$, $C^{12}(\alpha, \gamma_0)$ and $C^{12}(\alpha, \alpha')$ reactions.

1.7.3 3^- LEVELS

The 3^- level proposed by Mitchell and Ophel (Mi 64, Mi 65) is verified in the present work, and the 3^- assignment to the $E_\alpha = 8.13 \text{ MeV}$ is confirmed. In contrast to the 1^- level at 7.915 MeV which decays via proton emission to the ground state of N^{15} in the non-elastic channel, the absorption channel of the 3^- level at 7.96 MeV consists of inelastic alpha decay to the first excited state of C^{12} . It is this very difference in the absorption channel decay modes of the two levels which has given rise to much of the confusion in the 13 MeV region of O^{16} excitation.

The width ($\Gamma_{CM} = 26 \text{ KeV}$) given for the upper 3^- level compares well with previous results. Hebbard found the ratio of elastic width to total width for this level to be 0.45 as compared to the value 0.35 given in table 1.4; however, the interdependence of a and Γ can account for this difference, and accordingly the value quoted by Hebbard is again considered more accurate.

1.8 CONSISTENCY

Several consistency tests may be applied to the values of the 3^- level parameters presented in table 1.3. The 3^- state at $E_X = 13.26 \text{ MeV}$ appears to be the fourth member of the $T = 1$ analogue quadruplet corresponding to the ground state and first three excited states of N^{16} . The other 3 members of the quadruplet are identified as the 0^- level at $E_X = 12.79 \text{ MeV}$, the 2^- level at $E_X = 12.97 \text{ MeV}$ (neither of which show in the present experiment

due to their unnatural parity), and the 1^- level at $E_X = 13.10$ MeV (Aj 59). The reason for selecting the upper 3^- level as $T = 1$ lies partly in the strong cascade gamma radiation from this state compared to that from the neighbouring 3^- state at $E_X = 13.13$ MeV (Mi 65). (Gamma ray selection rules inhibit $T = 0$, E1 transitions in self-conjugate nuclei).

Some further evidence will be discussed later. Since the state at 13.26 MeV participates strongly in the $C^{12}(\alpha, \alpha_0)C^{12}$ reaction, which only populate $T = 0$ states, there must be isobaric spin mixing of nearby $T = 0$ states into this state, and vice versa. The 3^- level at 13.13 MeV is likely to contribute most of the $T = 0$ impurity, in which case the ratio of the reduced widths for α_0 break up of the two levels should equal the ratio of the reduced widths for α_1 break up, and also the inverse ratio of the widths for E_1 gamma ray transitions from the levels. From the parameters of table 1.4, the reduced widths for α_0 and α_1 break up of the level at $E_\alpha = 7.96$ MeV are calculated to be 66 KeV and 1300 KeV respectively. Since no attempt has been made in the present experiment to separate the absorption width of the level at $E_\alpha = 8.13$ MeV into partial widths for inelastic α and for p decay, the values given by Hebbard for these parameters (He 60) (table 1.4) are used. The reduced widths for elastic and inelastic α decay of this level are then 5.5 KeV and 110 KeV respectively, so that the ratios of the α_0 and α_1 reduced widths for the two levels are then 12 : 1 and 13 : 1 respectively, providing strong confirmation of the arguments presented above. The fact that the upper 3^- has an alpha particle reduced width some twelve times smaller than the lower 3^- level provides the further evidence that the upper level contains most of the $T = 1$ state.

The inverse relationship for electric dipole γ decay of the levels is more difficult to verify since no data exist which

allow a quantitative determination of the partial widths involved. However, qualitative estimates of these widths may be made by consideration of the relative yields of the 6-7 MeV cascade gamma radiation from the resonances at $E_{\alpha} \sim 7.96$ MeV and $E_{\alpha} = 8.13$ MeV respectively, using the $C^{12}(\alpha, \gamma\gamma)O^{16}$ excitation function data of Mitchell and Ophel (Mi 65), measured at 45° . Assuming the transitions pass through intermediate 2^{+} states, and assuming isotropy of the angular distributions involved, the inverse ratio is calculated as $\sim 10 : 1$ (making allowance for the underlying background).

Similar arguments regarding the reduced width ratios have been followed by Hebbard in his discussion of the 1^{-} states.

As a final test on the parameters of the 3^{-} states, the ratio of the peaks of the total 4.43 MeV gamma ray yields from the $C^{12}(\alpha, \alpha'\gamma)C^{12}$ reaction can be calculated and compared with experiment. On the basis of the single level approximation, the ratio R is given by

$$R = \left[\frac{\Gamma_{\alpha_0} \Gamma_{\alpha_1}}{\Gamma^2} \right]_{13.12} : \left[\frac{\Gamma_{\alpha_0} \Gamma_{\alpha_1}}{\Gamma^2} \right]_{13.25}, \quad (1.21)$$

resulting in the value, $R = 1.31$, from the parameters of table 1.4. Experimental measurements of the yield ratio at 90° (Mi 64) and 45° (La 64) give values of 1.24 and 1.27 respectively, comparing favourably with the calculated value.

CHAPTER 2

A STUDY ON THE APPLICABILITY OF SYMON'S THEORY OF ENERGY LOSS DISTRIBUTIONS

2.1 INTRODUCTION

The work presented in this chapter developed initially through an attempt to explain the discrepancies mentioned in chapter 1, section 1.3.2, but the subject proved to be sufficiently interesting to merit study beyond the scope of the original project.

The discrepancies discussed in section 1.3.2 consisted simply in the fact that three independent methods of determining an absolute cross section for the $C^{12}(\alpha, \alpha_0)$ work resulted in two significantly different answers. In particular, the target thicknesses determined from the energy loss of 890 KeV protons in passage through the targets (thin, self-supporting carbon foils), resulted in a cross section some 25% higher than that measured with a gas cell.

In essence, each target thickness was determined by measuring the shift in energy of the leading edge of the yield curve from the narrow $F^{19}(p, \alpha\gamma)O^{16}$ resonance at 874 KeV when the carbon foil was inserted in the beam path. The energy thickness was converted into units of atoms/cm² by use of the calculated stopping power tables published by Williamson, Boujot and Picard (Wi 66).

Several points may be raised regarding the procedure used to calculate the foil thickness from the energy loss data:

- (i) there is no guarantee that the corrections (Wa 52) applied to the Bethe formula by Williamson et al result in sufficiently accurate stopping powers for the present problem
- (ii) the validity of the assumption that the energy equivalent of the foil is the difference between the energies of the mid-yield points

of the leading edges of the distributions, before and after insertion of the foil in the beam path, warrants examination. It might be argued that a more meaningful quantity would be the energy difference between the leading edges of the actual beam spreads for the respective yields (*i.e.*, after extracting the Breit-Wigner dependence of the resonance from the yield curves). In fact, it is not clear that even this quantity is more correct than the energy losses as measured by considering either the peak energy or the mean energy displacements of the beam spreads.

(iii) an independent method for extracting the foil thickness from the data exists in the theory of energy loss distributions presented by Symon (Sy 48). Strictly speaking the theory should only be applicable when the particle velocity is much larger than that of the atomic electrons in the absorbing material. The $1s$ electrons in carbon have a mean velocity of $\sim 0.04 c$, which corresponds to a proton energy of $\sim 1 \text{ MeV}$. Symon states that the lower limit of validity is $\sim 10 \text{ MeV}$.

This chapter is concerned mainly with an experimental study of the degree of applicability of Symon's theory below the theoretical energy limit of validity, although it should be emphasized that there are two aspects of the problem, *viz.* the energy loss distribution, and the actual energy loss defined from the mean of the distribution. While both are implicitly related by Symon, an experimental study could demonstrate the correctness of only one under some circumstances.

2.2 FUNDAMENTAL EQUATIONS

The yield from a target can be represented by a triple integral of the form

$$Y(E_b, T) \sim \int_{E=0}^{\infty} G(E_b, E) \times \int_{E_i=0}^{\infty} \int_{x=0}^T \omega(E, E_i, x) \times \sigma(E_i) dE_i dE dx, \quad (2.1)$$

where,

$Y(E_b, T)$ is the yield from a target of thickness T bombarded with a beam of energy E_b ,

$G(E_b, E)$ characterises the incoming beam spread, and is the probability that a particle in the beam of nominal energy E_b , has an energy between E and $E + dE$,

$\omega(E, E_i, x)$ is the probability that a particle with incoming energy E will have an energy between E_i and $E_i + dE_i$ when it is at depth x ,

$\sigma(E_i)$ is the nuclear reaction cross section at energy E_i .

Evaluation of the quantities G and σ poses no problem since G can in general be measured, or at least estimated to a good approximation, and σ can normally be represented analytically. However calculation of the form of ω , which is the result of complicated statistical processes whose detailed behaviour is not easily obtained, is much more difficult.

A simple derivation of the integro-differential equation defining ω has been given by Rossi (Ro 52):

$\omega(E, E_i, x)$ may be redefined as the fractional number of particles of initial energy E which reach the depth x with an energy between E_i and $E_i + dE_i$. As the particles traverse an additional thickness dx , the number of particles in the energy interval dE_i at E_i changes because of the following two phenomena:

(i) some of the particles that have an energy between E_i and $E_i + dE_i$ at x , undergo a collision in dx and are thereby

removed from this energy interval

(ii) some of the particles that arrive at depth x with an energy $E_i + \xi$, greater than E_i , undergo in dx a collision that brings them into the energy interval dE_i at E_i .

It is useful to introduce the differential collision probability,

$$\Phi(E_i, \xi) d\xi dx$$

defined as the probability that a charged particle of energy E_i , traversing a thickness dx , will transfer energy between ξ and $\xi + d\xi$ to an atomic electron.

Then, from (i) and (ii) above, it follows that

$$\begin{aligned} \omega(E, E_i, x + dx) - \omega(E, E_i, x) &= -\omega(E, E_i, x) dx \times \\ &\int_0^\infty \Phi(E_i, \xi) d\xi + dx \int_0^\infty \omega(E, E_i + \xi, x) \Phi(E_i + \xi, \xi) d\xi . \end{aligned} \quad (2.2)$$

$\Phi(E_i, \xi) = 0$ for $\xi > E_m$, where E_m is the maximum energy transferable to an atomic electron.

$$E_m \sim 2m_e c^2 \cdot \frac{\beta^2}{1 - \beta^2} , \quad (2.3)$$

where

$$\beta = \frac{v}{c}$$

m_e = electron mass.

$$\omega(E, E_i, x) = 0 \text{ for } E_i > E .$$

Equation (2.2) may be rewritten,

$$\frac{\partial \omega(E, E_i, x)}{\partial x} = \int_0^\infty [\omega(E, E_i + \xi, x) \Phi(E_i + \xi, \xi) - \omega(E, E_i, x) \Phi(E_i, \xi)] d\xi . \quad (2.4)$$

A form for Φ has been calculated by Bhabha (Bh 38), and by Massey and Corben (Ma 39) for particles of mass m , spin $\frac{1}{2}$, in terms of the silhouette area C of the electrons in one gram of target material,

$$\Phi(E_i, \xi) d\xi = \frac{2Cm_e c^2}{\beta^2} \cdot \frac{d\xi}{(\xi)^2} \left[1 - \beta^2 \cdot \frac{\xi}{E_{m'}} + \frac{1}{2} \left(\frac{\xi}{E_i + mc^2} \right)^2 \right] , \quad (2.5)$$

where m is the mass of the incident particle of velocity v

$$C = \pi N \frac{Z}{A} r_e^2$$

N is Avogadro's number

$$r_e = \frac{e^2}{m_e c^2} , \text{ the classical electron radius.}$$

The last term in equation (2.5) is generally small enough to be ignored.

2.2.1 SOLUTIONS

In the following discussion, the parameter G defined below is useful:

$$G = \frac{2C m_e c^2 x}{\beta^2 E_m'} \quad , \quad (2.6) \quad x \text{ measured in } \text{gm/cm}^2$$

$$= Cx \cdot \frac{(1 - \beta^2)}{\beta^4} \quad .$$

Equation (2.4) was first solved by Bethe (Be 30) on the assumption that G was large (≥ 10 , say) which is effectively the case of a thick target, and ω was shown to have a symmetric Gaussian form.

Landau (La 44) obtained a solution for $G < 0.05$ by the method of the Laplace transformation, applicable to the case of moderately thin targets, which is decidedly asymmetric.

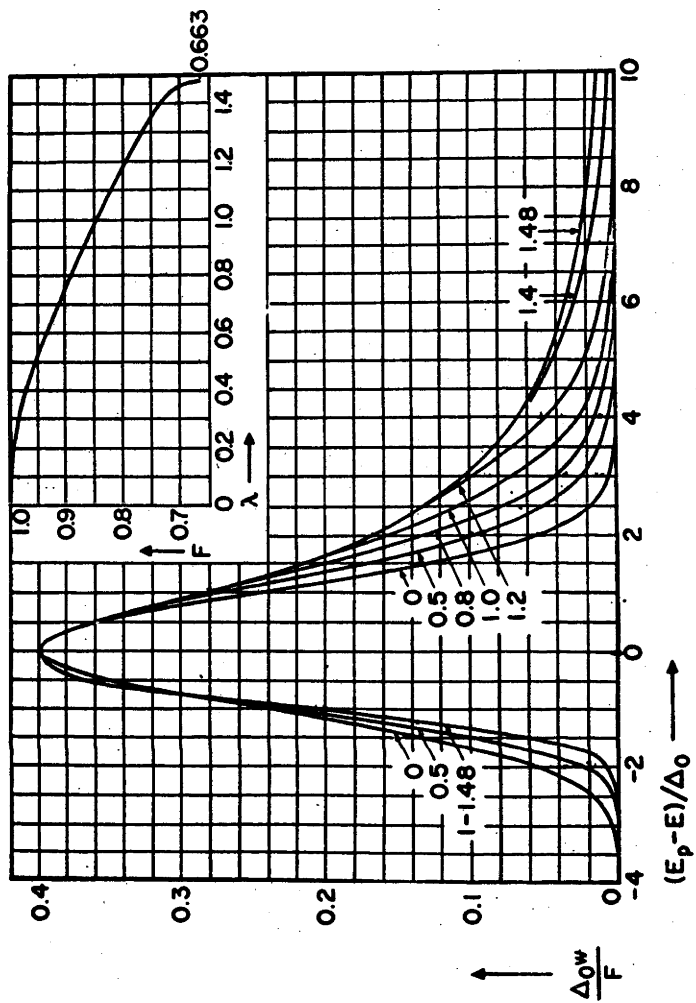
2.2.2 SYMON'S THEORY

Symon solved equation (2.4) in the intermediate region, $0.01 \leq G \leq 10.0$, without any of the restrictive mathematical assumptions invoked by Bethe and by Landau. [It should be pointed out that all three treatments implicitly assume a particle velocity much greater than that of the atomic electrons in their use of equation (2.5) for the form of the collision probability, ϕ . At lower energies, an increasing number of the collisions are adiabatic, resulting in an overestimation of the value as given by equation (2.5)].

Symon demonstrated that cases in the intermediate region could be suitably represented by a single parameter set of curves^{*} $\phi_\lambda(\Delta)$ (Fig 2.1) of varying skewness, providing a smooth transition between the Bethe and Landau curves. These curves, in conjunction with a set of 3 weighted parameters, approximate the energy loss distribution function $\omega(E, E_1, x)$ to good accuracy.

* Rosenzweig (Ro 59) disputes this fact in the low energy region. He claims that the approximations made by Symon to arrive at a one parameter set of curves become untenable at low energies.

Fig. 2.1 THE SET OF CURVES, $\varphi_{\lambda}(\Delta)$, PRESENTED BY SYMON DETERMINE THE VALUE OF THE FUNCTION ω IN TERMS OF THE ABSCISSA $\Delta = (E_p - E_1/\Delta_0)$, FOR GIVEN V VALUES OF THE SKEWNESS PARAMETER λ . CURVES ARE PRESENTED FOR A RANGE OF VALUES OF λ BETWEEN 0 AND 1.48. THE VALUE OF ω IS OBTAINED FOR GIVEN Δ AND λ BY MULTIPLYING THE ORDINATE BY THE NORMALISING FACTOR F (PRESENTED AS A FUNCTION OF λ IN THE INSET), AND DIVIDING BY THE TERM Δ_0 . THE PARAMETERS Δ , Δ_0 AND λ ARE OBTAINED AS DESCRIBED IN THE TEXT.



The weighted parameters are:

(i) the parameter j (Fig 2.2) which locates the peak of the curve by determining the most probable energy loss according to

$$E - E_p = G \cdot E_m \left[\log_e \left(\frac{2m_e c^2 G E_m'}{I^2(z)} \cdot \frac{\beta^2}{1 - \beta^2} \right) - \beta^2 + j \right], \quad (2.7)$$

where $I(z)$ is the average ionisation potential. The parameter, j has the form

$$j = \log_e \left[\frac{E_m' \cdot \beta^2}{2m_e c^2 \cdot Cx} \right] - \beta^2 - \frac{\beta^2}{2m_e c^2 \cdot Cx} \cdot (E_p - \bar{E}), \quad (2.8)$$

where $\bar{E}$ is the mean energy of the energy loss distribution.

(ii) the r.m.s. fluctuation b (Fig 2.3) which is a scale factor for the width of curve, according to

$$\Delta_0 = G \cdot E_m' \cdot b, \quad (2.9)$$

and (iii) the skewness λ (Fig 2.4), which determines the appropriate choice of curve from the set ϕ_λ .

The three parameters j , b , and λ are functions of G and β only. The family of curves $\phi_\lambda(\Delta)$ defines the energy loss distribution in terms of the dimensionless quantity Δ .

$$\Delta = (E_p - E_i) / \Delta_0 \quad (2.10)$$

which is the difference between the actual energy loss $E - E_i$ and the most probable energy loss $E - E_p$, measured in units of Δ_0 .

Fig. 2.2

THE SET OF CURVES j — THE PARAMETER IN SYMON'S THEORY WHICH LOCATES THE PEAK OF THE ENERGY LOSS DISTRIBUTIONS IN TERMS OF THE MEAN ENERGY LOSS, AS CALCULATED FROM BETHE'S FORMULA. THE ABSCISSA, G , IS OBTAINED AS DESCRIBED IN THE TEXT. THE CURVES ARE FOR VALUES OF $\beta^2 = 0.0$, 0.4 , 0.7 AND 1.0 FROM THE TOP.

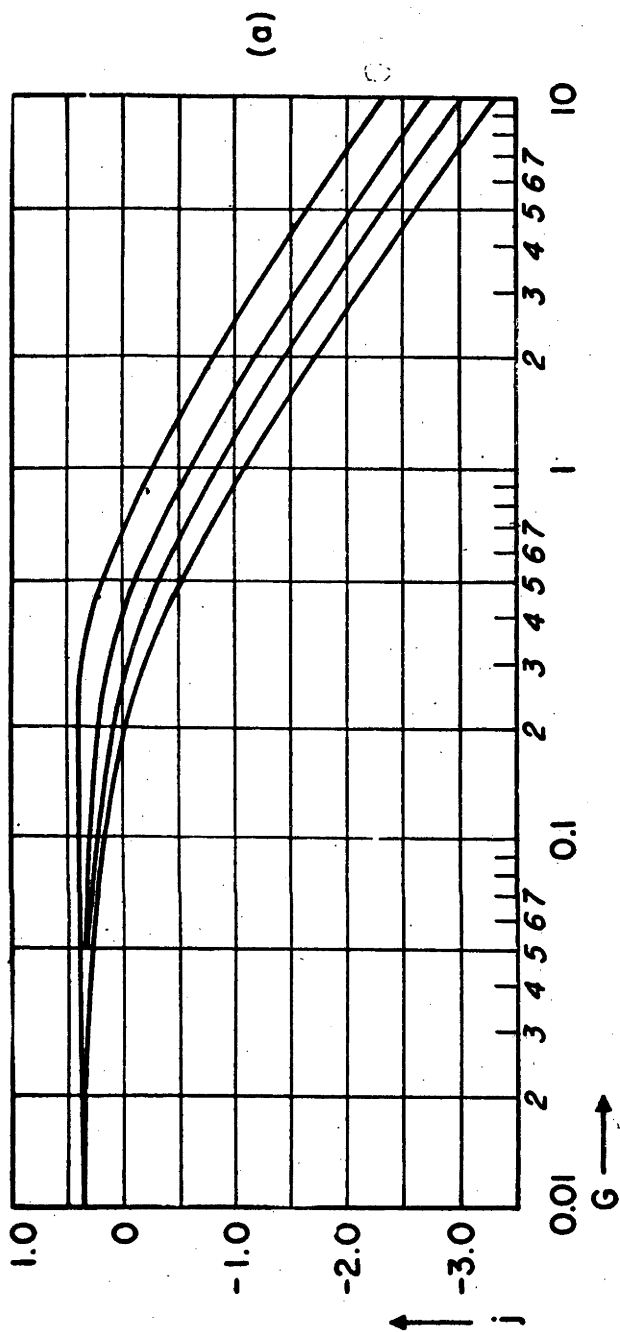


Fig. 2.3

THE SET OF CURVES b — A SCALE FACTOR WHICH DETERMINES THE WIDTH OF SYMON'S ENERGY LOSS DISTRIBUTIONS (see text). THE CURVES ARE FOR VALUES OF $\beta^2 = 0.0$, 0.4 , 0.7 , AND 1.0 FROM THE TOP.

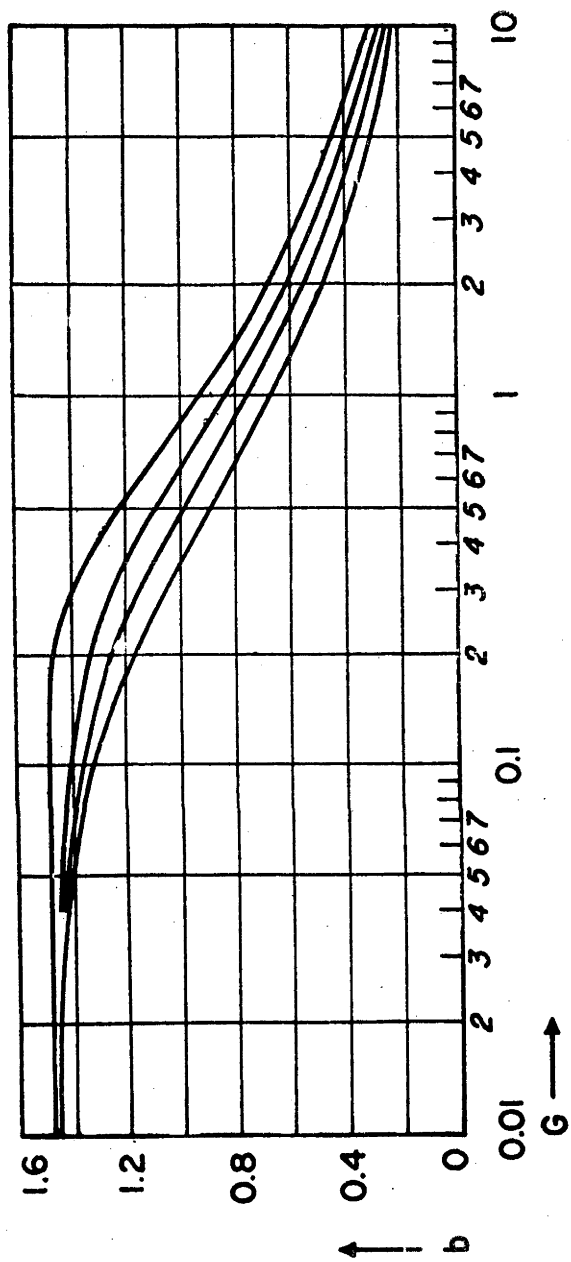
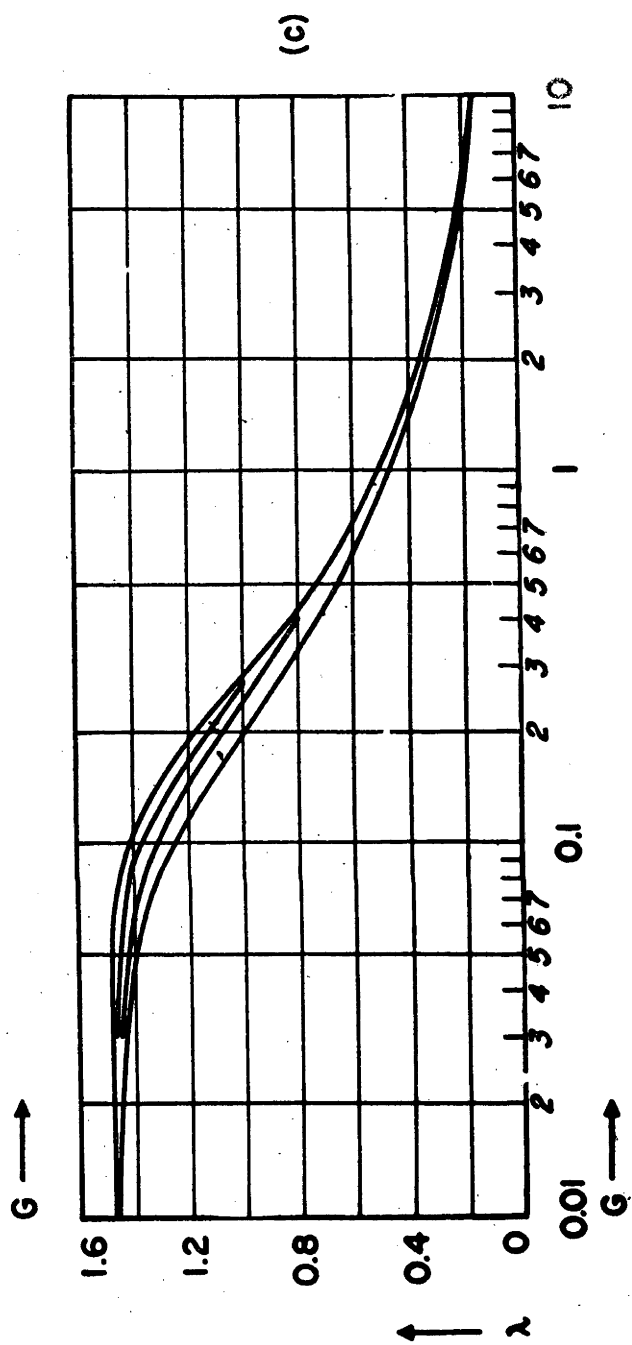


Fig. 2.4

THE SET OF CURVES λ — A SKEWNESS FACTOR WHICH DETERMINES THE APPROPRIATE CHOICE OF CURVE FROM THE SET ϕ_λ . AS BEFORE, THE CURVES ARE FOR VALUES OF $\beta^2 = 0.0, 0.4, 0.7$ AND 1.0 FROM THE TOP.



The ordinate, when multiplied by a normalising factor $F(\lambda)$ (Fig 2.1), and by the term dE/Δ_0 , gives the probability that a particle of incident energy E undergoes an energy loss between $E - E_1$ and $E - E_1 + dE_1$ in traversing the thickness x .

2.3 PREVIOUS WORK

Bondelid and Butler (Bo 63) have performed some very careful and accurate work on the shapes of the yield curves from narrow (p,γ) resonances, mainly at the 991 KeV $Al^{27}(p,\gamma)Si^{28}$ resonance. They measured the yields from a comprehensive series of Aluminium targets ranging from very thin to thick, with varying amounts of contamination on the surfaces. They showed that Symon's theory was able to account well for the shapes of the measured yield curves. In particular, they were able to show that several unexpected phenomena observed by them were explained on the basis of Symon's theory.

The main observations were:

- (a) the peaks of thin-target resonance yield curves were not shifted from resonance energy by as much as half the target thickness;
- (b) the midpoint of the rise of thick-target yield curves was shifted to bombarding energies below the resonance energy, instead of being symmetrical about that energy as was generally thought to be the case;
- (c) a hump, or 'overshoot', on the thick target yield curve was evident above the plateau.

Palmer et al (Pa 63), also in studies of the $Al^{27}(p,\gamma)Si^{28}$ resonance at 991 KeV, obtained essentially the same results as Bondelid and Butler analysing their data using Monte Carlo techniques based on an exponential distribution of path lengths derived from a characteristic mean collision path length λ , and on the $1/Q^2$

energy loss distribution for an energy loss Q derived randomly in terms of assumed minimum and maximum possible energy losses to the atomic electrons.

They claim furthermore that the work of Bondelid and Butler cannot have been properly analysed, since Symon's theory is stated as **being inapplicable** at the low energies used, and for the target thicknesses employed. However, it should be pointed out that Bondelid and Butler have not relied very heavily on the detailed behaviour of Symon's theory. The nature of their experiment (in which the energy straggling occurs in the actual target material) is such that the theory is only being used to apply first order modifications to the shapes of the yield curves. In fact, any theory which used the statistical nature of the energy loss process as its basis would explain the basic results of Bondelid and Butler.

Ophel and Morris (Op 65) present an energy loss distribution using the 991 KeV resonance, but in which the energy loss occurs almost wholly in a carbon foil placed in front of a thin aluminium target. The fit they present appears to show that the low energy tail is not well fitted by Symon's theory.

Vavilov (Va 57) has presented a theory equivalent to Symon's, but which is mathematically more rigorous. Furthermore, it is straightforward to modify the theory to take account of electron binding energy effects such as proposed by Walske, in contrast to the theory presented by Symon.

Experimental data analysed in terms of Vavilov's theory appear to be rather scarce, but in the one case known to the author (Go 57) Symon's theory can be seen to fit the data more accurately.

2.4 EXPERIMENTAL METHOD I

The method of the following section is essentially the same as that of the foil thickness measurement discussed in chapter 1 and at the beginning of this chapter.

A technique involving voltage modulation of the target was employed to determine the shift and broadening of a series of narrow (p, γ) resonances when the foil whose thickness was to be measured was inserted in the beam path.

The method (Mo 65) involves applying a triangular voltage waveform with a period of several seconds and an amplitude of several tens of kilovolts to a well insulated target. The proton beam was set at an energy some kilovolts above the resonance of interest so that the modulated voltage on the target swept the effective energy of the beam incident on the target back and forth through the resonance. The pulse output from a detector placed near the target was used to trigger a linear gate whose output, consisting of pulses directly proportional to the instantaneous voltage on the target, was fed directly into a PHA to provide a high resolution excitation function of the reaction over the range of voltage modulation.

In regard to the present series of measurements, the device is effectively a spectrometer which can measure the energy loss distributions of the foils with a resolution equal to the quadrature sum of the beam width and the resonance width. Instrumental contributions can be shown to have negligible effects on the resolution.

The triangular waveform was obtained from a 100 KV power supply whose control circuit reference voltage was derived from a motor driven linear potentiometer with a six second period. The input voltage to the potentiometer could be selected (by means of a six position switch) to obtain outputs of 3 , 6 , 12 , 25 , 50 and 100 KV .

Fig. 2.5 SCHEMATIC DIAGRAM OF THE 100 KV TARGET
MODULATION EQUIPMENT.

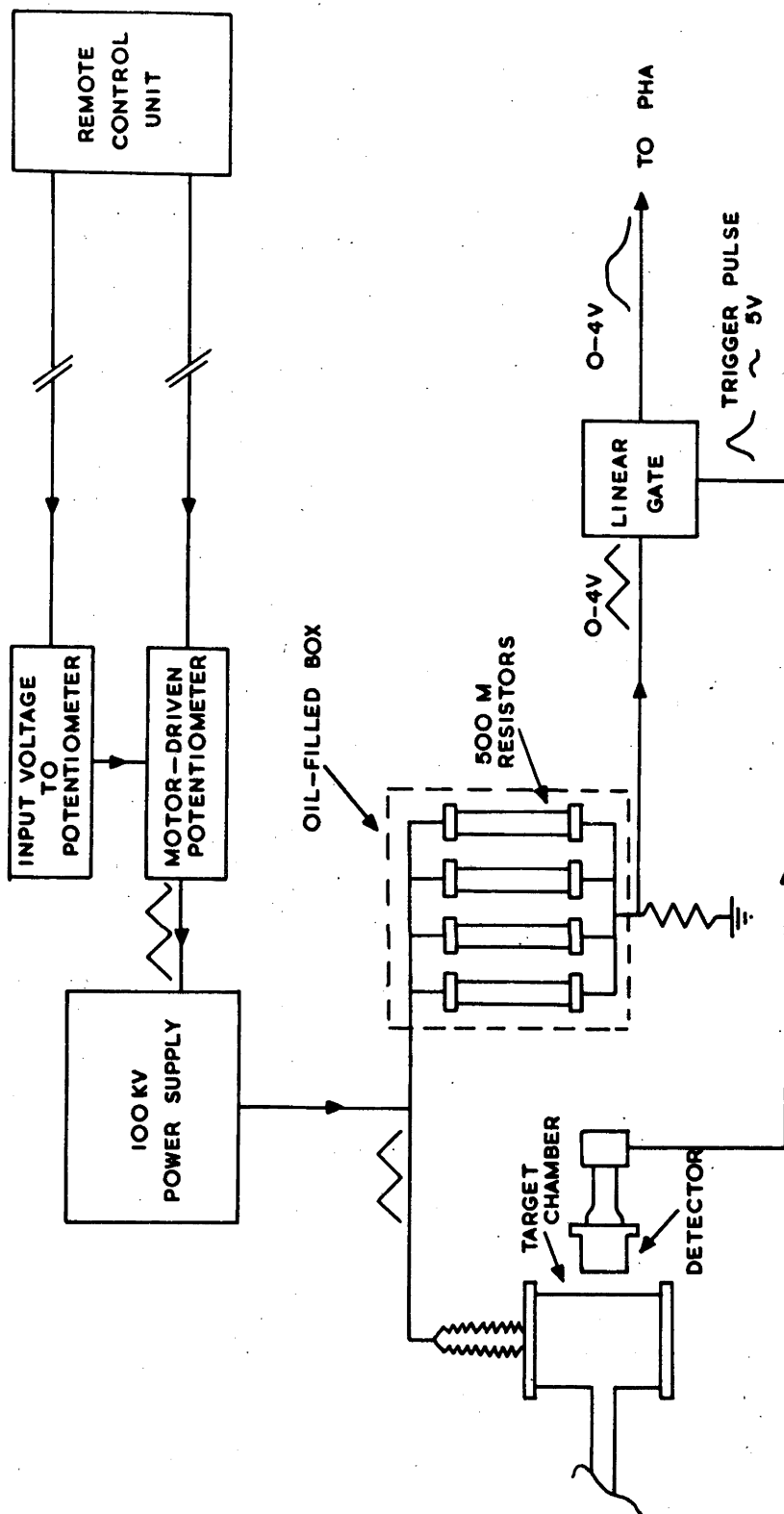
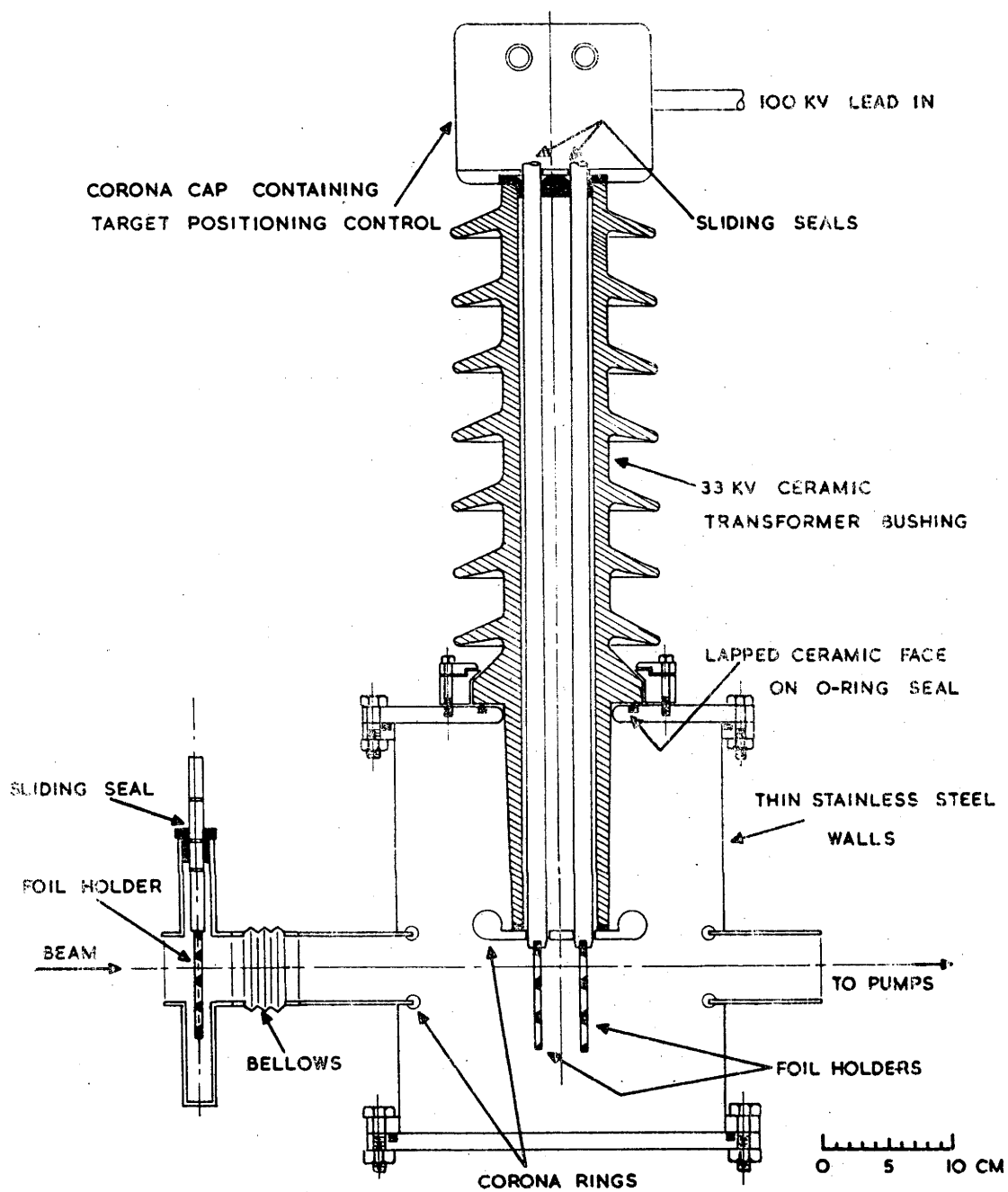


Fig. 2.6 DIAGRAM OF THE TARGET CHAMBER IN WHICH THE
ENERGY LOSS DISTRIBUTION MEASUREMENTS WERE
PERFORMED. ALTHOUGH THE SWEEP VOLTAGE WAS
ONLY 25 KV IN THE MEASUREMENTS DESCRIBED,
THE CHAMBER IS ABLE TO HOLD SWEEP VOLTAGES
UP TO 100 KV .



Manual control of the potentiometer setting was available for calibration purposes. The output of the supply was connected in parallel to the target holder (insulated by means of a large ribbed porcelain insulator) and to a voltage divider made up of four paralleled 500 M Ω resistors earthed through a resistance of several thousand ohms; the exact value was selected by means of a switch ganged to the voltage selector. (Figs. (2.5) and (2.6) describe the apparatus used).

2.4.1 TARGETS

Three types of target were used for the (p,γ) measurements:

- (i) Fluorine, using several of the narrow $F^{19}(p,\alpha\gamma)O^{16}$ resonances between 340 KeV and 1.348 MeV .
- (ii) Aluminium, using the 991 KeV resonance in $Al^{27}(p,\gamma)Si^{28}$.
- (iii) Carbon-13 , using the $C^{13}(p,\gamma)$ resonance at 1.747 MeV .

The fluorine and aluminium targets were made by r.f. heating of lithium fluoride and aluminium onto carefully cleaned copper backings in a good, contaminant-free vacuum system. Confirmation of the cleanliness of the system obtained from the observation of little yield of 6 MeV gamma rays from the aluminium target in the region of the strong fluorine resonance at a proton energy of 874 KeV .

The carbon-13 targets were made under somewhat less ideal conditions by cracking methyl iodide (55% C^{13}) onto heated tantalum strip. (cf chapter 3, section 3.2.2).

The energy widths of the targets used in the measurements were much less than the beam width (as opposed to the carbon foils, which generally had much greater energy widths).

2.4.2 BEAM

The A.N.U. 2MeV van de Graff accelerator provided a beam of protons continuously variable energy from 300 KeV to 2 MeV , focussed at the ion source and magnetically analysed by a 20° deflecting magnet. The accelerator was energy stabilised by a triode controlled corona discharge feedback system, taking error signals from narrow image slits placed along the beam path following the 20° magnet. High beam currents were readily available, so that both object and image slit systems could be set with very narrow gaps to obtain good beam energy resolution and stability. Beam currents on target were typically 3-5 μ A . The beam resolution was ~ 1.8 KeV and the stability was generally ~ 0.15 KeV .

2.5 ANALYSIS

Each measurement of a foil consisted of two parts - one with the foil in the path of the beam, and one with the foil out.

The analysis was performed in two stages, corresponding to the 'foil in' and 'foil out' measurements respectively. In the first stage, the initial beam energy distribution was calculated from the 'foil out' measurement, using the known resonance width. The second stage then used this result, together with Symon's ω function, to calculate the 'foil in' distribution, the foil thickness being a free parameter.

Before discussing the details of the analysis, it is convenient to modify the subscripting of equation (2.1). In particular, instead of considering the yield from a target of thickness T at beam energy E_b , *i.e.*, $Y(E_b, T)$, it is more useful to use the yield in channel c of the analyser, when the target voltage is $V(c)$, and the beam energy is E_b . The notation $Y(c, T)$ is sufficient.

Furthermore, the meaning of the subscript T has now altered. In equation (2.1), T represented the thickness of the target in which the reaction occurred; with the notation $Y(c,T)$, T is to be taken as the thickness of the foil in the path of the beam. The thickness of the actual target is unimportant provided it has an energy width appreciably less than the beam width, which was the case during the course of these measurements.

2.5.1 STAGE I (FOIL OUT)

Since the energy thickness of the target can be taken as much less than the beam spread, the integrals over E_i and x in equation (2.1) may be removed with little error, so that the triple integral reduces to the single integral over E ,

$$Y(c,0) = \int_{E=0}^{\infty} G(E_b, E) \sigma(E - V(c)) dE. \quad (2.11)$$

All pulses from the linear gate analysed into channel c of the analyser result from nuclear events occurring in the target when the potential on the target is $V(c)$; hence the term $E - V(c)$ in the argument of σ . Calibration of V as a function of c is discussed in section 2.6.1.

The usual Breit-Wigner form for the energy dependence of the reaction was chosen, *i.e.*,

$$\sigma(E) = \frac{\Gamma^2}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \quad (2.12)$$

$G(E_b, E)$ was assumed to be a skew-gaussian of the form

$$G(E_b, E) = H \cdot \exp(-0.653 X^2) , \quad (2.13)$$

$$\text{where} \quad X = \frac{E_b - E}{2\Gamma_G(1 \pm s)} \quad (2.14)$$

and where, Γ_G is the characteristic width of the gaussian

s is the associated skewness; choice of sign depending on the sign of $E_b - E$

H is a parameter which takes into account all neglected multiplicative constants.

The parameter s is defined to be

$$s = \frac{U - L}{U + L} = \frac{U - L}{\Gamma_G} \quad (2.15)$$

where U and L are the half widths of the gaussian on the positive and negative sides respectively of E_b .

Justification of the skew gaussian form for G rests basically in the fact that it was found to work well. e.g., see fig 2.9.

This stage of the analysis reduces to the problem of finding the best values of the parameters H , Γ_G , s and E_b in order to fit the 'foil out' distribution. Optimisation of these parameters in terms of a suitably defined 'chi-squared' is of course a non-linear problem, and the previously discussed fitting program FITALL was employed (cf chapter 1, section 1.3.1).

It was necessary to treat E_b as a parameter in the fitting procedure since in general neither the beam energy nor the resonance energy was known to an accuracy greater than ~ 0.2 KeV.

The presence of some background had to be taken into account in a few cases.

The integral over E was evaluated over the restricted range $E_b - 3\Gamma_G$ to $E_b + 3\Gamma_G$ using Simpson's rule.

2.5.2 STAGE II (FOIL IN)

The following section discusses analysis of the 'foil in' distribution, employing Symon's theory to take account of the effect of the foil on the beam energy distribution.

The yield from the target is described by a double integral of the form,

$$Y(c,T) = \int_{E=0}^{\infty} G(E_b, E) \int_{E_1=0}^{\infty} \omega(E, E_1, T) \sigma(E_1 - V(c)) dE_1 dE . \quad (2.16)$$

It is of course not necessary to integrate over the foil thickness T , since the function $\omega(E, E_1, T)$ describes exactly the distribution of energies E_1 leaving the foil for given incident energy E , and this is just the distribution incident on the target in which the reaction occurs.

As in the previous section, the integral over E extended from $E_b - 3\Gamma_G$ to $E_b + 3\Gamma_G$. The integral over E_1 extended from $E - E_\ell$ to E , where E_ℓ was determined such that $\omega(E, E_\ell, T) \sim 0$. For values $E_1 > E$, ω is necessarily zero.

The integrations were again performed using Simpson's rule.

The values of the gaussian parameters resulting from STAGE I were used for the function G (excluding the background term).

It is worth emphasising that the problem of analysis is essentially

a one parameter problem, since the foil thickness T has to account for all three of the energy loss phenomena observed, viz, the shift of the resonance position, the broadening of the distribution and finally the increased asymmetry or straggling. [It is not strictly correct to say that the analysis is a one parameter problem since differences in yields of the 'foil out' and associated 'foil in' distribution due to lack of beam integration required that the parameter H be left free (cf previous section). Furthermore, the presence of some background had again generally to be taken into account.]

Because of the lengthy time involved in calculating a 'foil in' distribution, the value of T was adjusted manually rather than by the fitting program. The background and the height H were also adjusted externally.

2.5.2.1 SECOND FIT TO 'FOIL IN' DISTRIBUTION

It was felt that one further step in the analysis could be of some interest. This entailed refitting the 'foil in' distributions in exactly the same manner as the 'foil out' distributions.

i.e., using the method of STAGE I. Thus one arrives at two sets of skew gaussian parameters describing respectively the actual beam distributions before and after the foil is inserted, enabling straightforward calculation of the displacements of the peaks, the mid yield points of the leading edges, and the mean energies of the appropriate beam distributions. From these energy displacements, one can calculate the respective foil thicknesses from tables of stopping power. Conversely, if Symon's theory is able to provide a unique foil thickness from the data, one can calculate stopping powers.

2.6 EXPERIMENTAL DETAILS

2.6.1 CALIBRATION OF INSTRUMENTS

Several instrumental effects can cause distortion of the spectrum obtained with the pulse height analyser. Because of the accuracy required in the measurements discussed here, it was important to determine the magnitudes of such distortions and correct the data accordingly if necessary.

The effects can be classified in three main categories:

- (i) non-linearity of the triangular voltage waveform
- (ii) non-linear response of the linear gate
- (iii) non-linearity of the analyser

The simplest method of testing the existence of one or more of these effects consists of using the pulse output from a detector placed near a source to trigger the gate. The detector signals are uncorrelated with the voltage waveform, so that the spectrum of pulses from the linear gate should be a flat distribution. The spectrum from such a test is shown in fig 2.7, indicating that a small amount of non-linearity exists in the system.

The gate output was found to be linear when a series of accurately known D.C. voltage levels over its full range was applied to the input. Although the analyser was already known to be slightly non-linear from measurements with a 20 c.c. Ge-Li gamma ray detector of ~ 6 KeV resolution, the magnitude of the effect was too small to account for the distortion of the 'randoms spectrum' of fig 2.7. Thus it was concluded that the major factor in the spectrum distortion was non-linearity of the triangular waveform. Correction for the distortion was made by normalising the data to the inverse of the 'randoms' spectrum.

Fig. 2.7 'RANDOMS' SPECTRUM OBTAINED AS DESCRIBED IN THE
TEXT. DEVIATIONS FROM THE EXPECTED FLAT
DISTRIBUTION, AT THE LOW AND HIGH VOLTAGE
ENDS OF THE SPECTRUM, ARE DUE TO NON-
LINEARITIES IN THE TRIANGULAR VOLTAGE WAVEFORM.

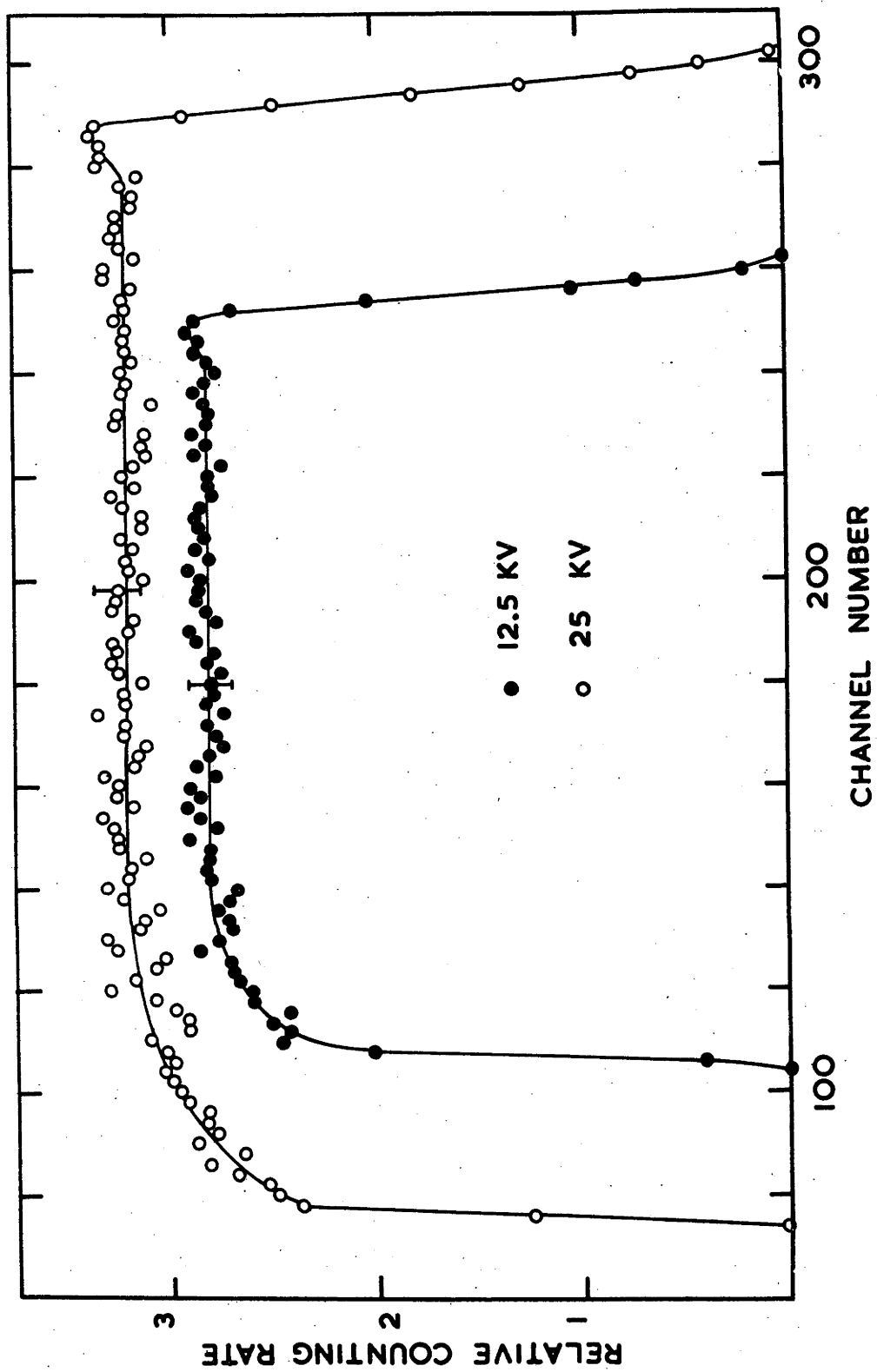
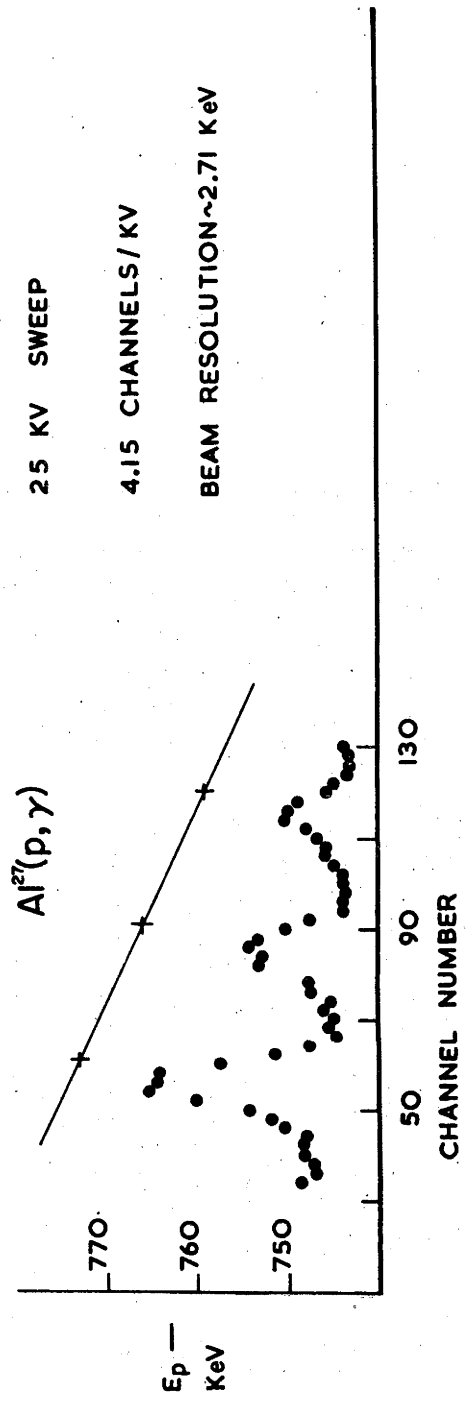


Fig. 2.8

THE CALIBRATION CURVE AS DETERMINED BY MANUAL
ADJUSTMENT OF THE VOLTAGE MODULATING EQUIPMENT
(straight line), IS COMPARED WITH CALIBRATION POINTS
OBTAINED FROM A 25 KV SWEEP MEASUREMENT ON THE
THREE NARROW RESONANCES IN THE $\text{Al}^{27}(\text{p},\gamma)\text{Si}^{28}$
REACTION NEAR 770 KeV (crosses).



25 KV SWEEP

4.15 CHANNELS/KV

BEAM RESOLUTION ~2.71 KeV

1 Voltage vs channel number calibration of the system was performed by manually adjusting the output of the supply in 2 KV steps, and reading the output voltage on a sensitive moving coil galvanometer.

The response of the meter was checked by recording an excitation function over the three narrow $\text{Al}^{27}(\text{p},\gamma)$ resonances near 770 KeV , and agreement with the calibration curve was within the small experimental error (See fig 2.8).

2.6.2 TARGET THICKNESS

As mentioned in section 2.5.2, analysis is much simpler if the target in which the reaction occurs can be treated as having essentially zero thickness compared to the beam width. Choice of target was made as follows:

A series of targets of regularly decreasing thickness was made (cf section 2.4.1) by setting the copper backings at differing distances in the bell-jar from the target material before the evaporation. Each member of the series was bombarded in turn by 484 KeV protons, and the yield from a window over the full energy peak in the γ -ray spectrum measured in a scalar for the same integrated charge. Since the resonance width is $\sim 0.9 \text{ KeV}$ and the beam width is $\sim 1.8 \text{ KeV}$, the yield should be approximately constant for targets of thickness $\gtrsim 3 \text{ KeV}$ at that proton energy, and should fall off for thinner targets. Using this method, a target of energy width $\sim 0.35 \text{ K V}$ was chosen for the measurements.

Similar methods were applied to the Al^{27} and C^{13} targets.

2.6.3 ENERGY LOSS MEASUREMENTS

Four foils were used in all of the measurements. Before and after each 'foil in' measurement, runs were performed with the foil out

to check energy stability. The gain setting, fixed for all measurements of the series, was ~ 3 channels/KeV, and if any 'foil out' distribution showed a shift of more than one channel, the preceding 'foil in' data were discarded, and the measurement repeated.

The gain setting of 3 channels/KeV was a compromise between resolution and time taken to analyse the data, the latter being almost a linear function of the number of channels in the spectrum. All of the analysis was performed on the A.N.U. 360/50 I.B.M. computer and the average time taken on the central processor was approximately one hour per foil per resonance, for a complete analysis. Since several measurements were repeated, total time on the computer was considerable.

2.6.4 RESONANCES USED

The resonances used in the series of measurements were as follows:

(a) $F^{19}(p,\alpha\gamma)O^{16}$

(i) 340 KeV (ii) 484 KeV (iii) 874 KeV (iv) 1.348 KeV

(b) $Al^{27}(p,\gamma)Si^{28}$

(i) 991 KeV

(c) $C^{13}(p,\gamma)N^{14}$

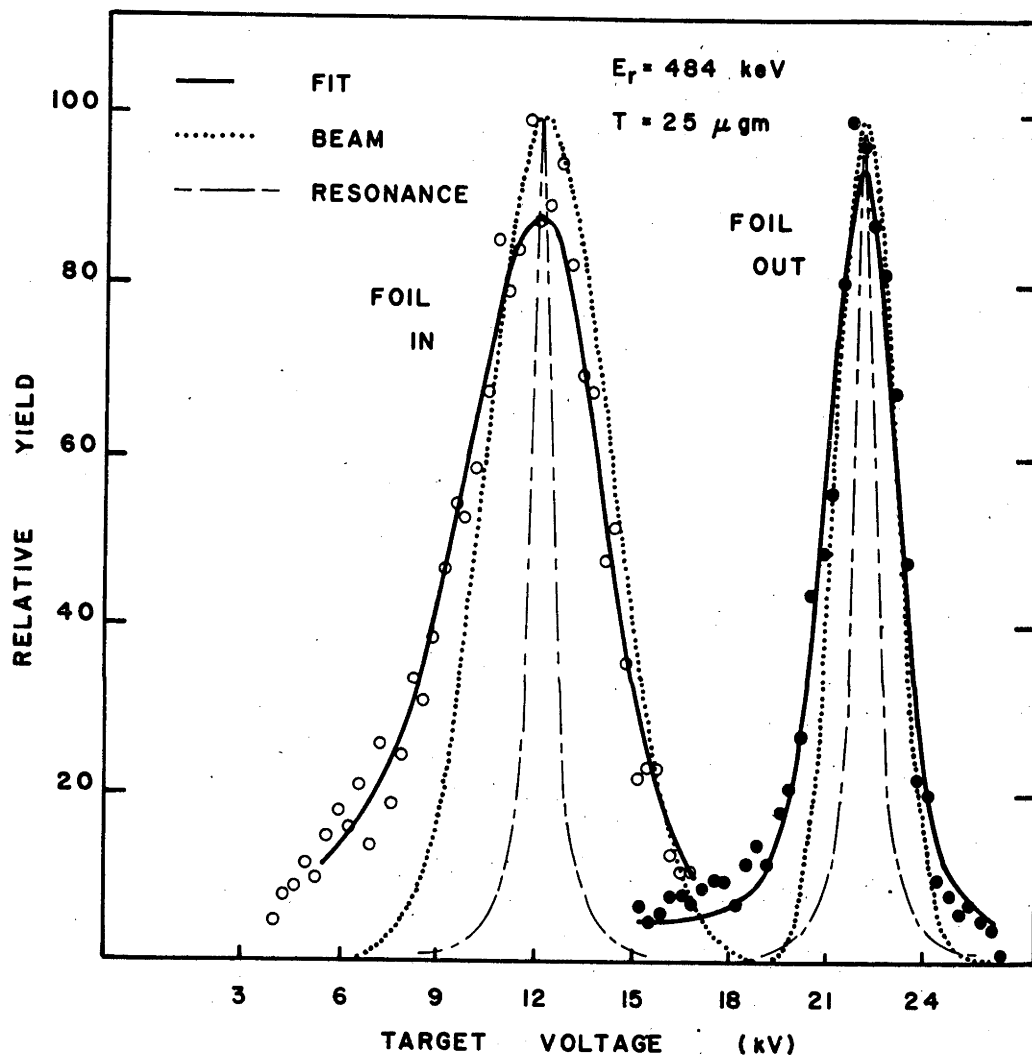
(i) 1.347 MeV .

2.6.5 BUILD UP

Since the average time spent by each foil in the beam was approximately 10-15 minutes per resonance, (giving a total time of < 2 hours) build up of carbon was not expected to be a serious

Fig. 2.9

YIELD CURVES FROM THE $F^{19}(p,\alpha\gamma)O^{16}$ REACTION AT THE 484 KeV RESONANCE, WITH AND WITHOUT A CARBON FOIL IN THE BEAM PATH, AND WITH A BEAM ENERGY OF ~ 506 KeV. THE FIGURE INDICATES THE QUALITY OF THE 'SKEW-GAUSSIAN BREIT-WIGNER' FITS TYPICALLY OBTAINED TO THE 'FOIL IN' AND 'FOIL OUT' DISTRIBUTIONS. THE PEAK OF THE BREIT-WIGNER SHAPE (broken curves) GIVES THE PRECISE LOCATION OF THE RESONANCE ENERGY (NOT NECESSARILY AT THE PEAK YIELD POSITION) . NOTE THAT, WHILE THE YIELD CURVES ARE SKEWED TOWARDS LOWER TARGET VOLTAGE (HIGHER EFFECTIVE BEAM ENERGY ON TARGET), THE BEAM ENERGY DISTRIBUTIONS (dotted curves) ARE SKEWED TOWARDS HIGHER VOLTAGE (lower beam energy). THE PEAKS OF THE BEAM ENERGY DISTRIBUTIONS ARE ARBITRARILY SHOWN AT THE RESONANCE ENERGY IN EACH CASE. THE YIELD CURVES WERE OBTAINED BY INTEGRATING THE BEAM DISTRIBUTIONS SHOWN TOGETHER WITH THE B-W DISTRIBUTION, MOVING THE PEAK OF THE BEAM DISTRIBUTIONS SUCCESSIVELY THROUGH EACH CHANNEL.



problem. However, to mitigate the possibility of any small systematic effects, the measurements were performed out of energy sequence (as indicated in fig 2.12).

2.6.6 BEAM CONDITIONS

The stability of the beam energy was found to be better than ~ 200 eV over long periods, based on the magnitudes of energy shifts in the 'foil out' measurements. The beam energy resolution was ~ 1.8 KeV, based on the results of fitting the 'foil out' distributions at the various resonance energies with the energy width of the beam as a free parameter (cf section 2.5.1).

2.7 EXPERIMENTAL RESULTS

The fits to the energy loss distributions obtained through Symon's theory are shown for one foil in fig 2.10. The fits to the 'foil out' data are not shown in the figures, but were generally of high quality, as could be expected (eg. see fig 2.9). The numerical results of the measurements are presented in table 2.1.

It was felt that the best way to present such a large quantity of related numbers would be to normalise and average them in some reasonable way. The foil thicknesses used to obtain the fits were normalised to foil D, using one factor for a given foil over the energy range, and the normalised values were then averaged. Finding some physically meaningful way to average the actual energy shifts obtained from the simple 'skew-gaussian + Breit Wigner' fits to the distributions (cf section 2.5.2.1) presents more of a problem:

Fig 2.11 presents the relationship between the most probable energy loss $E_0 - E_p$ as given by Symon (equation 2.7, using a foil thickness of $30 \mu\text{gm}/\text{cm}^2$), and the mean energy loss $E_0 - \bar{E}$

TABLE 2.1

Energy (MeV)	Foil	T_S ($\frac{\mu\text{gm}}{\text{cm}^2}$)	Δp (KeV)	ΔL (KeV)	ΔM (KeV)	Stopping* power ($\frac{\text{KeV} \cdot \text{cm}^2}{\text{mg}}$)	$T_{\Delta p}$ ($\frac{\mu\text{gm}}{\text{cm}^2}$)	$T_{\Delta L}$ ($\frac{\mu\text{gm}}{\text{cm}^2}$)	$T_{\Delta M}$ ($\frac{\mu\text{gm}}{\text{cm}^2}$)
0.360	A	15.5	7.8	7.6	7.8	445	17.5	17.1	17.5
	B	16.0	8.1	8.0	8.1		18.2	18.0	18.2
	C	23.0	11.6	11.6	11.5		26.1	26.1	26.0
	D	22.0	10.8	10.7	10.7		24.3	24.0	24.0
0.506	A	17.5	7.0	6.6	7.4	368	19.0	17.9	20.1
	B	18.0	7.4	7.0	7.5		20.1	19.0	20.4
	C	25.0	10.3	9.9	10.7		28.0	26.9	29.1
	D	24.0	9.9	9.2	9.9		26.9	25.0	26.9
0.894	A	19.5	5.6	5.5	5.6	256	21.9	21.5	21.9
	B	21.0	5.6	5.6	5.6		21.9	21.9	21.9
	C	28.5	7.9	7.8	7.9		30.9	30.5	30.9
	D	26.5	7.4	7.8	7.4		28.9	30.5	28.9
1.007	A	20.0	4.9	4.3	5.0	235	20.9	18.3	21.3
	B	21.0	5.2	4.9	5.3		22.1	20.9	22.6
	C	28.5	7.1	6.5	7.0		30.2	27.7	29.8
	D	26.5	6.7	6.0	6.9		28.5	25.5	29.4
1.368	A	24.0	4.8	4.9	4.8	189	25.4	25.9	25.4
	B	22.0	4.2	4.2	4.1		22.2	22.2	21.7
	C	30.0	6.1	6.0	6.0		32.3	31.7	31.7
	D	26.0	5.5	5.3	5.2		29.1	28.0	27.5
1.767	D	31.0	4.7	4.4	4.9	156	30.1	28.2	31.4
	E	33.0	5.4	4.9	5.6		34.6	31.4	35.9

* As tabulated by Boujot and Williamson

T_S is the foil thickness determined from Symon's theory.

Δp , ΔL , ΔM are respectively the 'peak' energy displacements, the 'leading edge' energy displacements, and the 'mean' energy displacements of the actual beam energy distributions.

$T_{\Delta p}$, $T_{\Delta L}$, $T_{\Delta M}$ are foil thicknesses as calculated from the respective beam energy displacements, using the tabulated stopping powers.

from Bethe's formula (Be 30). As expected, these values diverge increasingly with energy. It is clear that any average over the sets of energy losses in table 2.1 should include a factor relating the mean energy loss to the most probable energy loss (or 'peak' energy loss).

Since no obvious method presents itself for relating the 'leading edge' energy loss to the most probable or to the mean energy loss, and since these values clearly start to differ from each of the other quantities at higher energies, it was decided to exclude the 'leading edge' energy losses from any averaging.

Table 2.2 presents the values of the resultant quantities, appropriately normalised and averaged from table 2.1 according to the procedure indicated above, and figs. 2.12 and 2.13 give a graphical description. Fig 2.11 indicates why the foil thicknesses, as calculated from Williamson's stopping powers, should be greater than those resulting from application of Symon's theory (see fig 2.12). The difference lies in the fact that Symon used Bethe's original formula for the mean energy loss, whereas Williamson modified the formula to take account of the decrease in stopping power at lower energies due to atomic electron binding energy effects.

Fig 2.14 shows a 'fit' to the $^{484}\text{K}\text{eV}$ resonance using $T = 33 \mu\text{gm}/\text{cm}^2$ for comparison with the appropriate fit in fig 2.10.

Errors were calculated on the basis of ± 1 channel, coupled with a subjective addition regarding the quality of the various fits involved.

Fig 2.15 (and table 2.3) presents the results of determining foil thicknesses by simple leading edge displacement of the yield curves (using Williamson's tables to convert from KeV to $\mu\text{gm}/\text{cm}^2$)

Fig. 2.11

THE FIGURE PRESENTS THE RELATIONSHIP BETWEEN THE 'MEAN ENERGY LOSS STOPPING-POWER' AS CALCULATED FROM BETHE'S ORIGINAL FORMULA FOR $\frac{dE}{dx}$ (solid curve), AND THE 'PEAK ENERGY LOSS STOPPING-POWER' AS CALCULATED FROM A FORMULA GIVEN BY SYMON (see text) BASED ON BETHE'S FORMULA (broken curve), AND USING A FOIL THICKNESS OF $30 \mu\text{gm}/\text{cm}^2$. THE ENERGY LOSS THROUGH THE FOIL DECREASES WITH INCREASING ENERGY, RESULTING IN INCREASED STRAGGLING OF THE ENERGY LOSS DISTRIBUTION, WHICH RESULTS IN AN INCREASING DISPARITY BETWEEN THE 'MEAN' AND 'PEAK' STOPPING POWERS, AS INDICATED. ALSO SHOWN IS THE STOPPING POWER CURVE OBTAINED FROM WILLIAMSON'S TABLES, WHICH TAKES SOME ACCOUNT OF ELECTRON BINDING ENERGY EFFECTS (dotted curve).

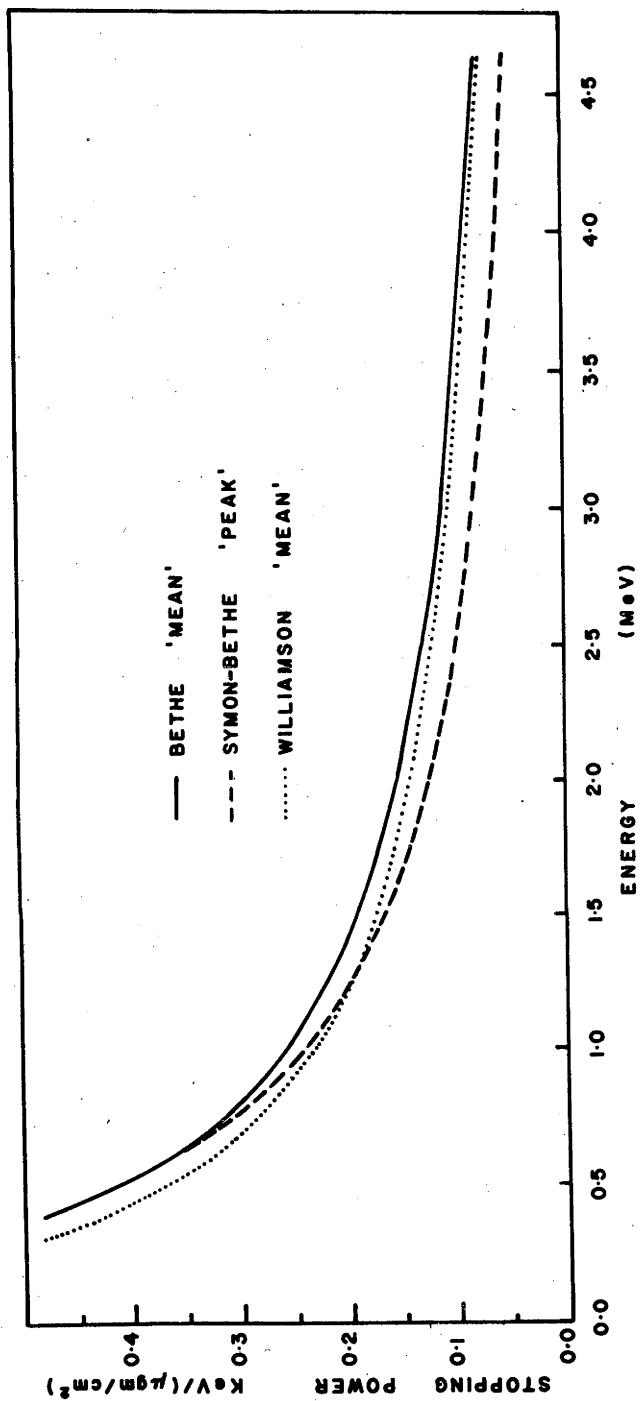


Fig. 2.12 AVERAGED RESULTS OF FOIL THICKNESSES AS DETERMINED BY FITTING THE ENERGY LOSS DISTRIBUTIONS WITH SYMON'S THEORY, AS A FUNCTION OF ENERGY. THE AVERAGE WAS PERFORMED OVER THE FOUR FOILS A , B , C , D AT EACH ENERGY (see text). THE NUMBERS, 1 - 6 , INDICATE THE TIME ORDER IN WHICH THE MEASUREMENTS WERE PERFORMED, AND INDICATE THAT BUILD UP OF MATERIAL ON THE FOILS WAS NOT A SIGNIFICANT FACTOR IN THE MEASUREMENTS. THE CURVE WAS DRAWN BY EYE THROUGH THE DATA POINTS. IT MUST BE EMPHASISED THAT THE FOIL THICKNESS UNITS ARE $\mu\text{gm}/\text{cm}^2$, COMPLETELY INDEPENDENT OF ENERGY, SO THAT FOR AN IDEAL THEORY THE POINTS SHOULD LIE ON A HORIZONTAL LINE.

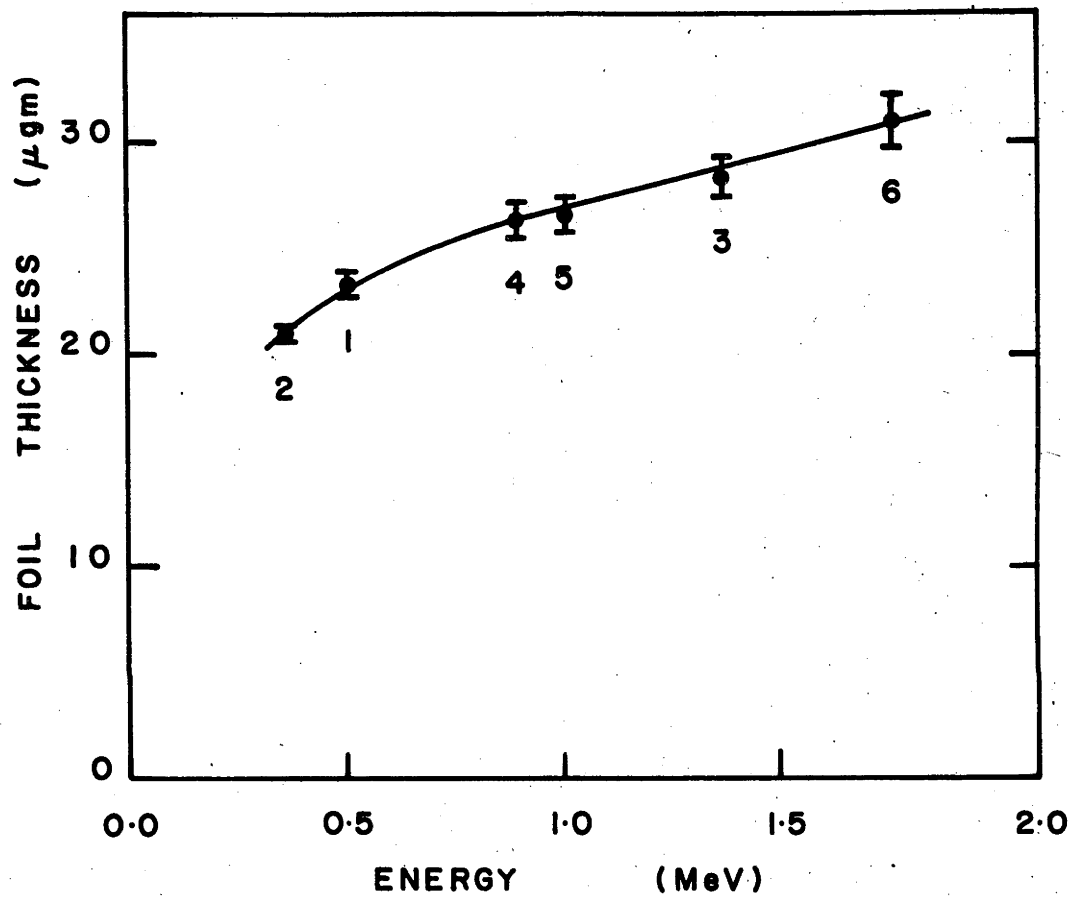


Fig. 2.13 AVERAGED RESULTS OF FOIL THICKNESSES AS DETERMINED FROM THE ENERGY LOSS OF THE BEAM ON PASSAGE THROUGH THE FOILS, AND USING WILLIAMSON'S TABLES OF STOPPING POWER. THE AVERAGE WAS PERFORMED OVER THE FOUR FOILS A , B , C D FOR THE 'PEAK' AND 'MEAN' ENERGY LOSSES, USING THE 'PEAK : MEAN' RATIO, R (as determined from fig 2.11), TO CONVERT 'PEAK' ENERGY LOSS INTO 'MEAN' ENERGY LOSS BEFORE AVERAGING (see text). THE BROKEN CURVE IS THE CURVE OF FIG 2.12 — THE VARIATION OF FOIL THICKNESS WITH ENERGY OBTAINED FROM FITTING THE DATA WITH SYMON'S THEORY. THE SOLID CURVE IS A PREDICTED VARIATION OF FOIL THICKNESS WITH ENERGY, AND WAS OBTAINED FROM THE BROKEN CURVE USING THE RATIO OF THE BETHE 'MEAN' STOPPING POWER TO THE WILLIAMSON'S 'MEAN' STOPPING POWER AS DEPICTED IN FIG 2.11 (cf text).

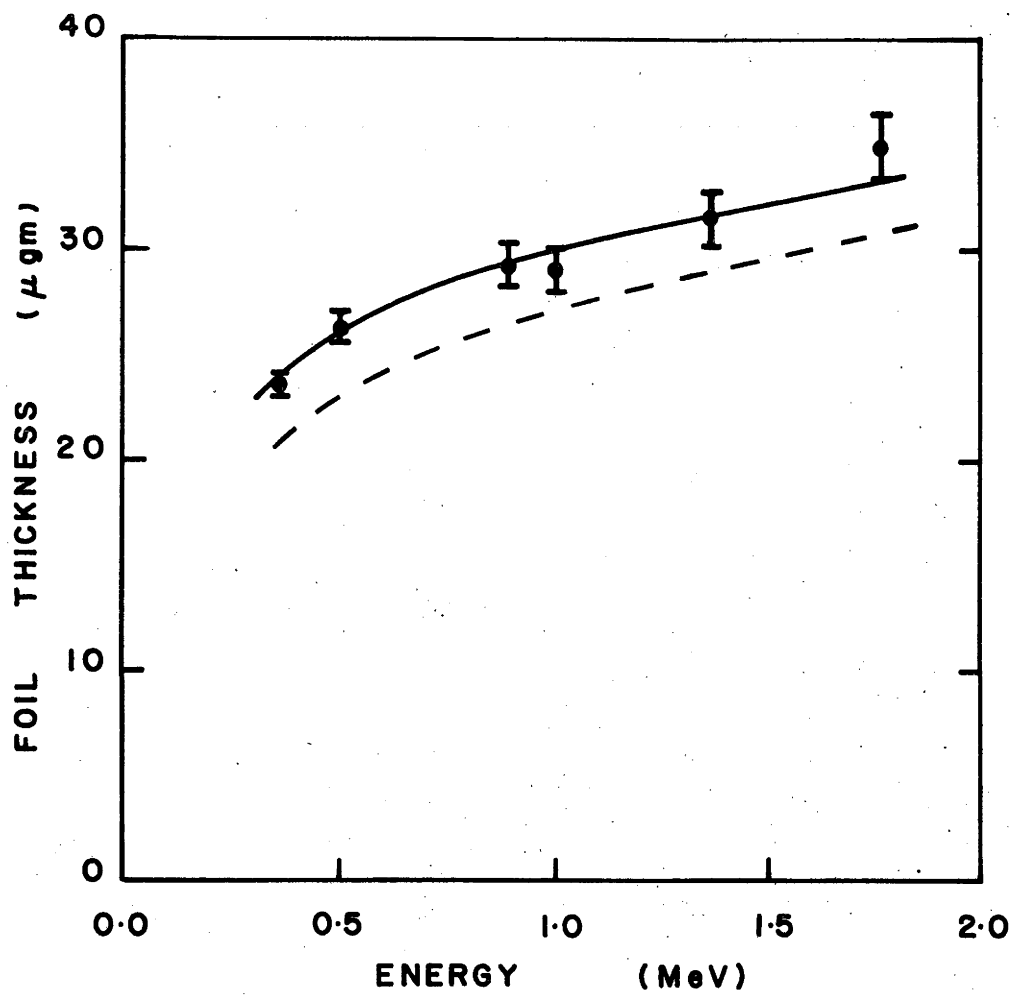


TABLE 2.2

ENERGY (MeV)	T_S ($\mu\text{gm}/\text{cm}^2$)	$\Delta M^{(A)}$ (KeV)	$\Delta p^{(A)}$ (KeV)	MEAN: PEAK (R)	$\Delta p^{(A)}$ (KeV)	XR	AVERAGE $\Delta M^{(A)}, \Delta p^{(A)}$	XR	$T_{\Delta M}^{(A)}$ ($\mu\text{gm}/\text{cm}^2$)	$T_{\Delta p}^{(A)}$ ($\mu\text{gm}/\text{cm}^2$)	AVERAGE $(T_{\Delta M}^{(A)}, T_{\Delta p}^{(A)})$
0.360	21.0	10.5	10.5	1.0	10.5		10.5		23.5	23.7	23.6
0.506	23.3	9.8	9.5	1.01	9.6		9.7		26.6	25.8	26.3
0.894	26.3	7.3	7.3	1.05	7.7		7.5		28.5	28.5	29.2
1.007	26.5	6.7	6.6	1.06	7.0		6.8		28.4	28.0	29.0
1.368	28.3	5.3	5.4	1.12	6.0		5.6		29.5	30.1	31.6
1.767	30.9	5.1	4.8	1.20	5.8		5.4		32.4	31.1	34.9

T_S represents the average foil thickness obtained by normalising to foil D (as described in text), the thicknesses used in fitting Symon's theory to the energy loss data, and averaging the resultant ΔM values.

$\Delta M^{(A)}$ and $\Delta p^{(A)}$ are the averaged values of the 'mean' and 'peak' beam energy-shifts respectively. As in the calculation of T_S , the shifts were normalised to foil D before averaging.

R, the ratio of the 'mean' energy shift to the 'peak' energy shift, is obtained from fig 2.11 (see text).

The column under the heading $\Delta p^{(A)} \times R$ gives 'peak' energy shifts corrected for the 'mean': 'peak' ratio, R. Ideally the values should equal the corresponding values in the $\Delta M^{(A)}$ column.

$T_{\Delta M}^{(A)}$ and $T_{\Delta p}^{(A)}$ were obtained from the energy shifts $\Delta M^{(A)}$ and $\Delta p^{(A)}$ by use of Williamson's tables.

Fig. 2.14 THE FIGURE SHOWS A 'FIT' TO AN ENERGY LOSS DISTRIBUTION
AT THE 484 KeV FLUORINE RESONANCE USING A FOIL
THICKNESS OF $T = 33 \mu\text{gm}/\text{cm}^2$, FOR COMPARISON WITH
THE APPROPRIATE FIT IN FIGURE 2.10 ($T = 25 \mu\text{gm}/\text{cm}^2$).

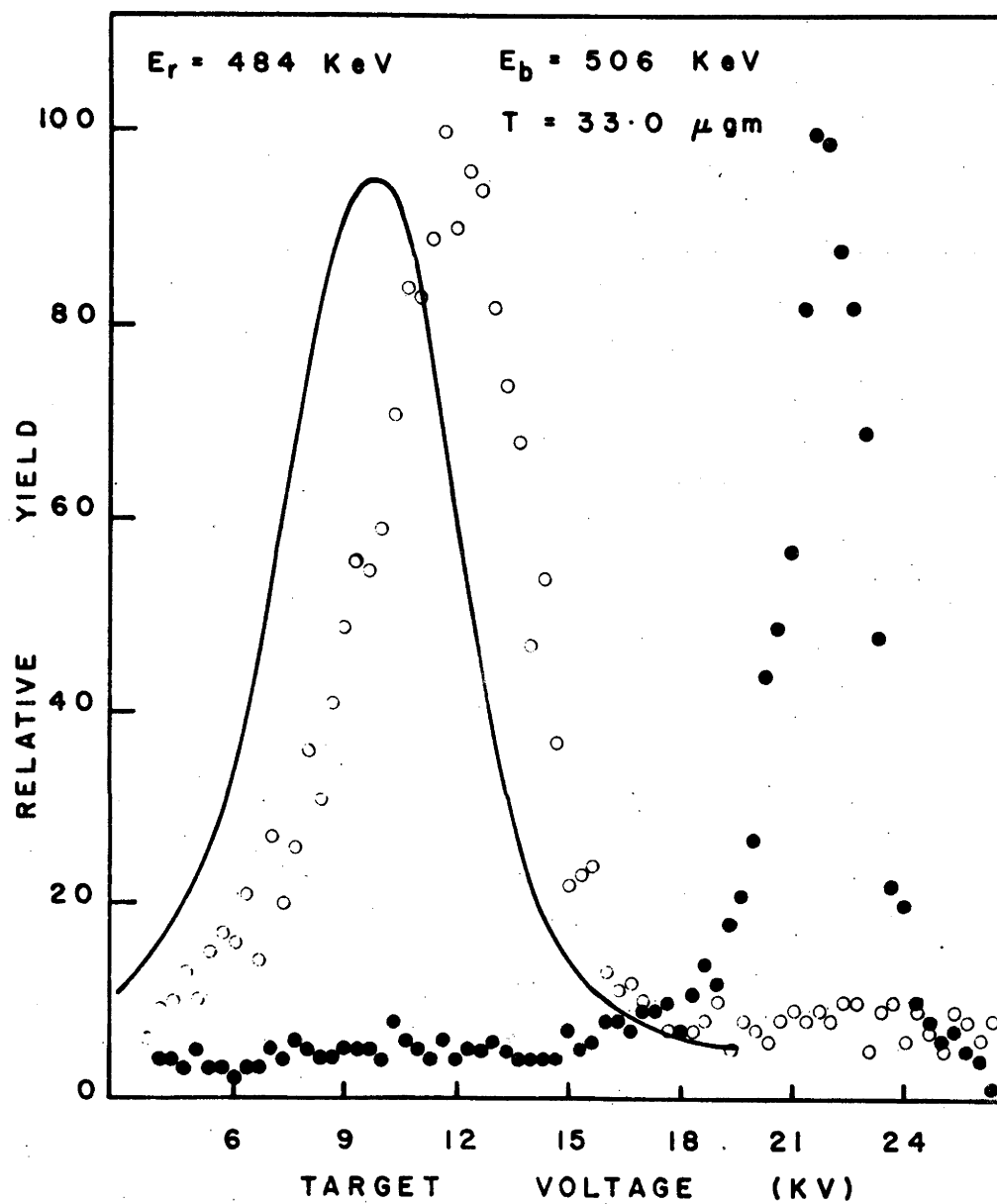


Fig. 2.15 AVERAGED RESULTS OF FOIL THICKNESSES AS DETERMINED FROM THE 'LEADING EDGE' ENERGY LOSS, AND USING WILLIAMSON'S TABLES. THE SOLID CURVE WAS CALCULATED AS INDICATED IN THE DESCRIPTION OF FIG 2.13. THE POINTS ARE OBSERVED TO SHOW MUCH MORE SCATTER THAN IN THE TWO PREVIOUSLY DESCRIBED METHODS OF OBTAINING FOIL THICKNESS.

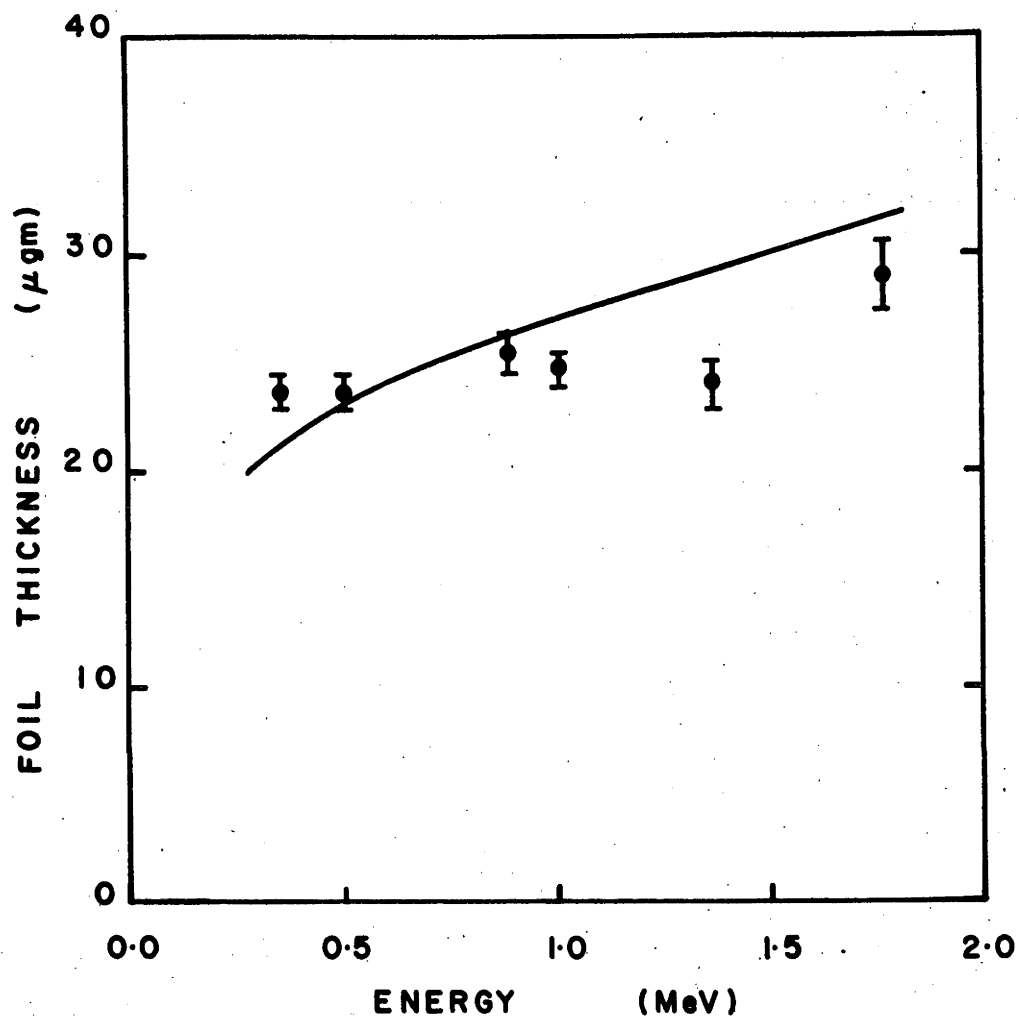


TABLE 2.3.

ENERGY (MeV)	FOIL	ΔL (KeV)	AVERAGE ΔL	S (KeV/(mg/cm ²))	$T_{\Delta L}$ ($\mu\text{gm/cm}^2$)
0.360	A	7.8	10.7	445	24.3
	B	8.0			
	C	12.0			
	D	11.2			
0.506	A	6.0	8.5	368	23.2
	B	6.2			
	C	10.0			
	D	8.9			
0.894	A	4.8	6.5	256	25.2
	B	4.9			
	C	7.3			
	D	6.7			
1.007	A	4.4	5.8	235	24.6
	B	4.6			
	C	6.1			
	D	5.9			
1.368	A	3.8	4.5	189	23.8
	B	3.3			
	C	5.2			
	D	4.2			
1.767	D	4.2	4.5	156	28.9
	E	5.2			

ΔL is the displacement of the mid-yield points of the leading edges of the yield curves.

The average was performed on values normalised to foil D, as before.

The stopping power, S, was obtained from Williamson's tables and $T_{\Delta L}$ was calculated from S and the averaged ΔL .

2.8 DISCUSSION

Several points are apparent at this stage.

1. Symon's theory is able to account well for the broadening and asymmetry of the proton beam energy distribution when the beam is passed through a thin carbon foil, even at the low energies of the present investigation. Since straggling is not expected to be important for the foil thickness used below ~ 1 MeV ($G \gtrsim 0.5$, $\Rightarrow \lambda \lesssim 0.75$ and cf. fig 2.1), no serious attempt was made to investigate this effect. For the measurements above ~ 1 MeV, background obscures most of the straggling (fig 2.10)
2. Equally consistent results are obtained for foil thickness, whether measured by converting the mean energy loss, obtained from fitting gaussians (section 2.5.2.1) to the 'foil in' and 'foil out' distributions, through use of Williamson's tables, or by fitting the distributions through Symon's theory with the foil thickness as a free parameter. (Allowing for the different expressions used for the stopping cross section.)

Furthermore, determination of foil thicknesses by the method which gave rise to this experiment (i.e., leading edge displacement of the yield curves), resulted in much less consistent values (see fig 2.15) than foil thicknesses determined from the mean or peak displacements of the actual beam distributions. This is not surprising on the basis of the variation of the shape of Symon's distribution function w with energy loss (fig 2.1). The mid-yield point of the leading edge does not have a constant (or even easily determinable) relation to the peak or mean yield abscissae. The differing widths of the various resonances may also be expected to contribute some kind of effect on the measurements.

3. The existence of a distinct increase in foil thickness with energy, determined by both Symon's theory and Williamson's tables, indicates that the expressions used for the stopping cross section are overestimated in each case by an amount which decreases with increasing energy, a result not unexpected, particularly in Symon's case. The fact that the Williamson curve is higher than the Symon curve at low energies has been commented upon in the previous section.

4. It is clear from fig 2.12 that a continuation of the measurements to higher energies is strongly indicated in order to see if the trend continues upward, which would be rather surprising, or to see if the curve flattens out at some energy. The latter would be a much more reasonable result, since expressions for the stopping cross section are expected to correspond more closely to reality as the binding energy effects decrease.

Since high yield, narrow (p,r) resonances cease to exist at higher energies, other methods were considered.

One possible method would have been to look at shifts of the (p,n) thresholds using the voltage modulation technique with a modified target chamber. However, such measurements would be of limited value due to the sparseness of suitable thresholds.

At about this time, a high resolution double focussing spectrometer instrumented with a flat response 5 cm position sensitive detector (p.s.d.) became available in the laboratory, and it was decided to try and make use of this device to perform the higher energy measurements.

2.9 EXPERIMENTAL METHOD II

The method chosen involved scattering the beam from a very thin gold foil (2 K ϵ V at $\sim$ 1 MeV proton energy) at the centre of the target chamber, and using the scattered beam at an angle of 10° as the incident beam for the carbon foil. The carbon foil was placed approximately 5 mm behind the gold foil along the beam direction. See fig 2.17.

The p.s.d. was placed in the focal plane of the magnet. This method has several advantages over the target modulation technique, viz.

- (a) the measurements are performed rapidly due to the high count rate, reducing problems of energy drift.
- (b) a wide range of energies can be covered, and the actual energies can be chosen to suit the experimenter, since there is no reliance on the existence of narrow nuclear anomalies in (p, γ) or other reactions.
- (c) the measurements can be performed with any type of beam, in contrast to part 1 type methods.
- (d) it is no longer necessary to fold in a Breit-Wigner form in the analysis of the distributions.
- (e) there is no problem as regards a finite target thickness distorting results.

This latter point however introduces several unfavourable aspects of the spectrometer in regard to the measurements:

- (a') the energy resolution of the instrument is not a negligible factor, in contrast to the voltage modulating equipment.
i.e., a δ -function beam energy input appears on the p.s.d. as a finite spread of energies. [In fact, although the apparatus of

METHOD I has little effect on the distributions, the resonance width can be regarded as an equivalent factor to the magnet-p.s.d. response, limiting the final resolution]. Furthermore, the resolution deteriorates with increasing energy, which is particularly unfortunate since the energy shifts from the foils are decreasing with energy. The resultant decrease in accuracy of the measurements could have been obviated by using thicker foils; however it was felt desirable to continue with the same foils, especially in regard to measuring the energy straggling expected to occur at the higher energies.

(b') Aberrations in the magnetic field and the response of the p.s.d. further distort the actual beam distributions.

The measurements were made with the magnet entrance slits set as narrow as possible, and with tight collimation of the incoming beam, to try and offset these effects to some extent.'

2.9.1. BEAM

The proton beam obtained from the A.N.U. 12 MeV tandem van de Graaff, was magnetically analysed by a 90° magnet and stabilised by a corona feedback system from slits placed behind the 90° magnet and with a gap of ~ 50 thou'. The energy was determined in the usual way by measuring the resonant frequency of protons in a probe placed in the 90° magnet. Recalibration of the frequency - energy curve is achieved periodically by means of the (p,n) thresholds.

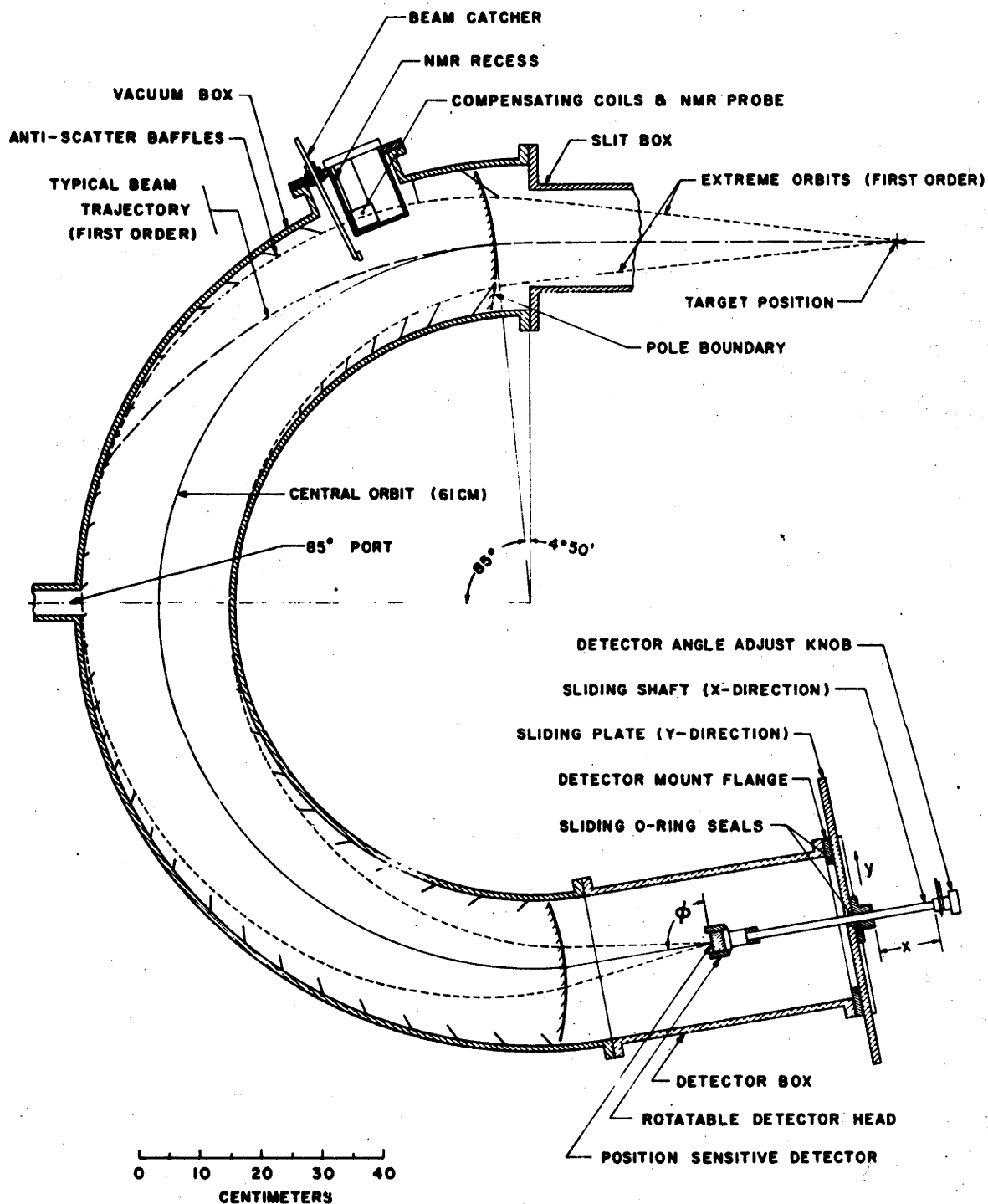
2.9.2 THE SPECTROMETER

Fig 2.16 is a schematic representation of the vacuum box of the 61 cm, 180° double-focussing magnetic particle spectrometer in use with the A.N.U. tandem accelerator.

The magnet was constructed by Spectromagnetic Industries, and

Fig. 2.16

A SECTION OF THE DOUBLE-FOCUSSING SPECTROMETER USED IN THE ENERGY LOSS MEASUREMENTS. THE SECTION SHOWS THE VACUUM BOX, PARTICLE ORBITS, THE NMR RECESS AND THE MOUNTING FOR THE POSITION SENSITIVE DETECTOR.



is similar to one constructed by them to specifications of the California Institute of Technology. It weighs 12 tons and sits on a carriage which rotates about a vertical axis through the target. The acceptance solid angle of the spectrometer can be varied by means of slits inserted between the target and the entrance to the vacuum box. The slits consist of four independent tantalum vanes which may be adjusted to define a rectangle of continuously variable size and location. During the course of the present measurements, the slits were set to define an aperture of ~ 40 thou square, located about the central orbit. These settings defined an entrance solid angle of $\lesssim 0.01$ m.sr .

Fig 2.17 shows the foil arrangements in the target chamber. Details of calibration of the spectrometer are given in ref E1 68.

2.9.3 THE DETECTOR

The 5 cm position sensitive detector was supplied by Nuclear Diodes. In essence, the detector supplies two signals for each incident particle, one of which, the energy signal is proportional to the energy of the incident particle, while the other, the position signal, is dependent on the position along the major axis of the detector at which the particle is incident.

The amplitude of the position pulse is defined by,

$$E_p = \frac{Y}{X + Y} \cdot E$$

where the symbols are explained in fig 2.18.

A negative bias is applied to a contact at the detector front surface. An incident particle causes electron-hole pairs to be formed in the depletion layer and all of the holes are collected at

Fig. 2.17 SIMPLIFIED VERTICAL SLICE THROUGH THE TARGET CHAMBER USED IN THE ENERGY LOSS DISTRIBUTION MEASUREMENTS IN THE DOUBLE-FOCUSSING SPECTROMETER. THE SPECTROMETER, RIGIDLY COUPLED TO THE CHAMBER, WAS INCLINED AT AN ANGLE OF 10° TO THE VERTICAL PLANE THROUGH THE BEAM. THE SWIVEL POINT OF CHAMBER AND SPECTROMETER IS INDICATED BY THE 'AXIS OF ROTATION' IN THE FIGURE. THE 'STRAIGHT THROUGH' BEAM WAS PREVENTED FROM REACHING THE DETECTOR BY MEANS OF A RECTANGULAR SLIT SYSTEM IN THE SPECTROMETER (see text).

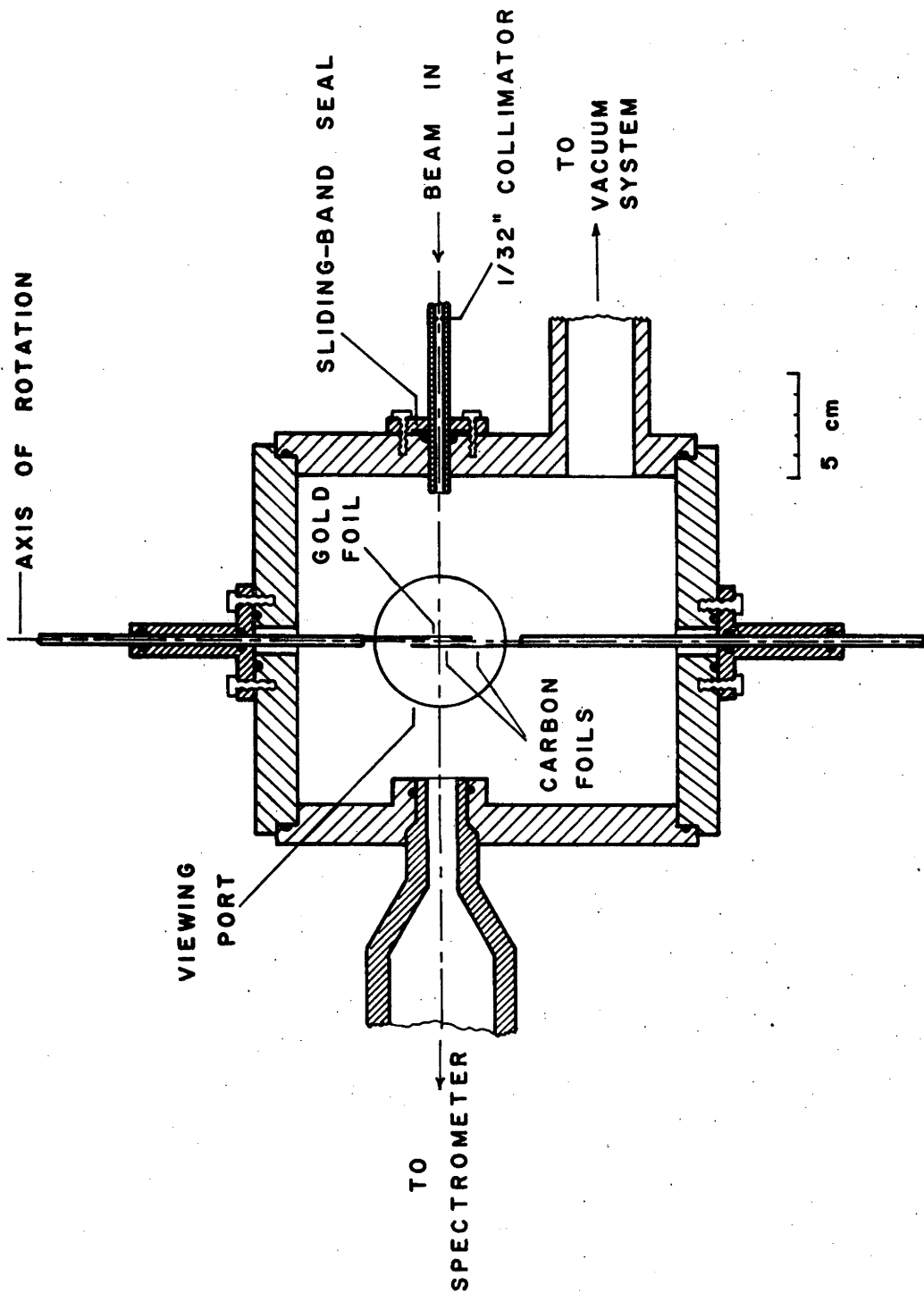


Fig. 2.18 SCHEMATIC DIAGRAM OF THE POSITION SENSITIVE DETECTOR.

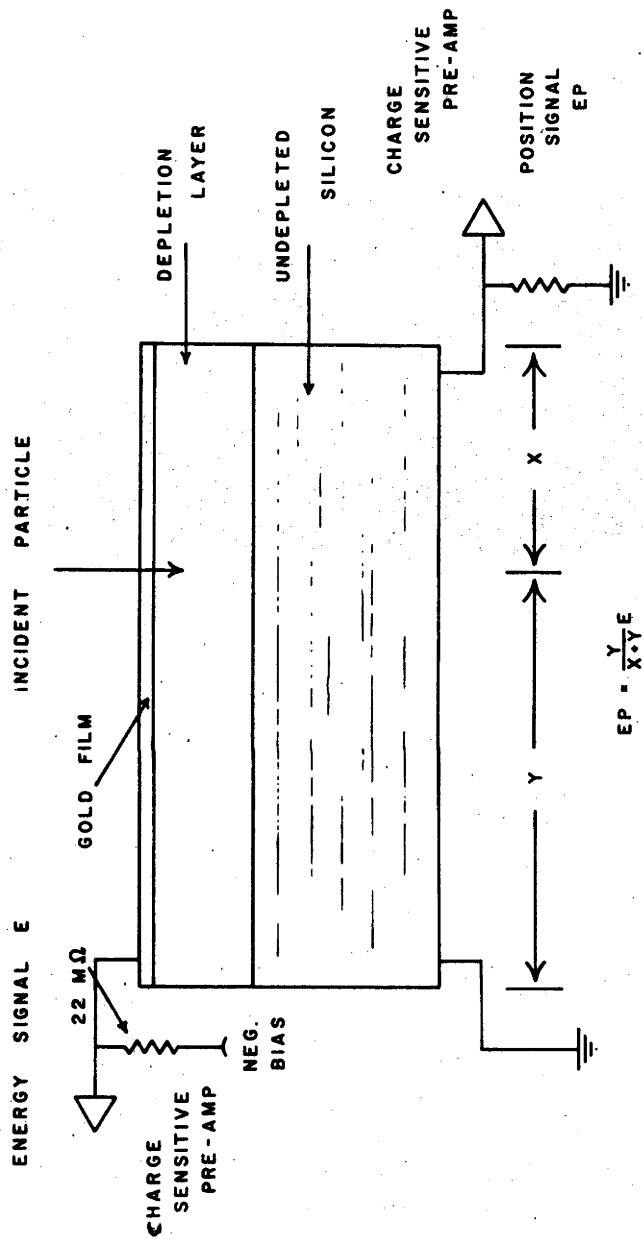
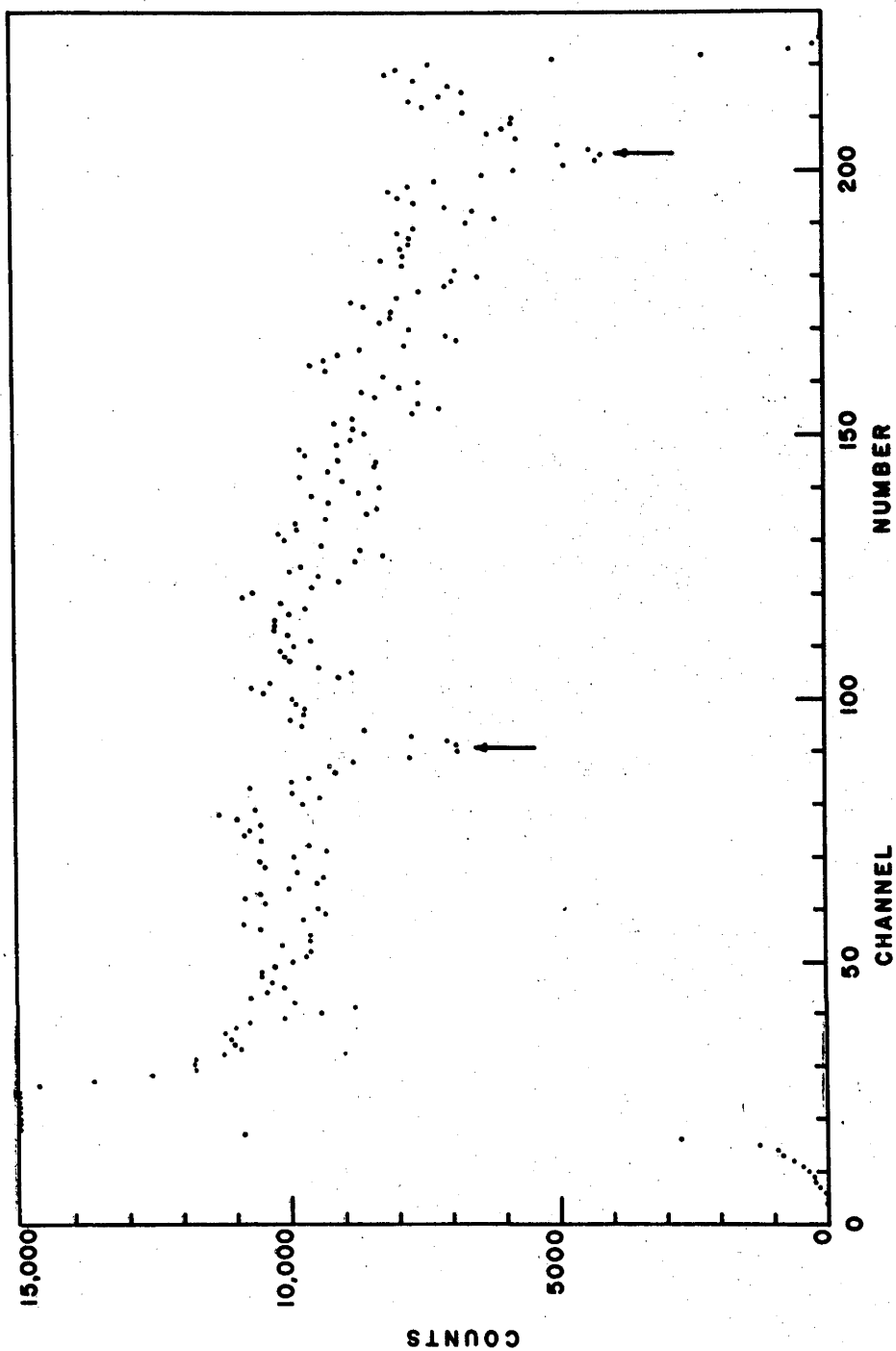


Fig. 2.19 RESPONSE OF THE 5cm POSITION-SENSITIVE DETECTOR TO IRRADIATION OF ITS SURFACE BY 8.78 MeV α -PARTICLES. THE DOWNWARD SLOPE TO THE 'HIGH-ENERGY' END IS DUE TO A NON-PARALLEL SLIT SYSTEM. (THE SLITS WERE USED TO MASK THE EDGES OF THE DETECTOR ALONG THE MAJOR AXIS). THE ARROWS INDICATE THE EFFECT OF TWO THIN WIRES (1 mm dia) USED TO SUPPORT THE DETECTOR FOR THE RESPONSE CURVE MEASUREMENT. IT IS OF INTEREST THAT, WHEREAS THE POINTS SHOW SOME DEGREE OF SCATTER OVER THE 5 cm, THE DIPS IN THE RESPONSE CURVE YIELD DUE TO THE SUPPORTING WIRES ARE REMARKABLY WELL DEFINED.



the front contact giving a positive pulse of height E proportional to the total energy the particle loses in the depletion layer.

Electrons migrate to two electrical contacts at the back of the detector, one at each end, through the resistive undepleted layer, which acts as a voltage divider. One of these contacts is grounded, as shown in fig 2.18, and at the other a negative signal appears which is a fraction $\frac{Y}{X+Y}$ of the energy signal E .

The response of the detector to uniform irradiation of its surface by 8.78 MeV α particles is shown in fig 2.19.

2.10 ANALYSIS

Bearing in mind the factors presented in section 2.9, several approaches are possible in analysis of the data:

(i) Effects of resolution and aberrations can be ignored and the p.s.d. spectrum taken as an accurate representation of the incoming beam energy distribution. This simplifies and greatly speeds the rate of analysis since the integral in equation (2.11) vanishes, and the double integral of equation (2.17) reduces to a single integral. Due to the high premium on computer time, this factor assumes some importance.

(ii) The energy distribution of the beam scattered at 10 degrees from the gold foil can be assumed to be a δ -function, and the p.s.d. spectrum treated as the response of the system (cf. the Breit-Wigner shape in the voltage modulation measurements, as mentioned above).

This method also has the advantage of faster analysis because of the reduction in the number of integrations. Furthermore, it is probably more realistic than (i) above because of the thinness of the gold foil (2.0 KeV at 1 MeV proton energy) and the high resolution of the tandem (~ 1 KeV).

(iii) The gold foil can be measured as described in METHOD I of this chapter at, say, the 991 KeV resonance in $Al^{27}(p,\gamma)Si^{28}$ and an appropriate 'beam' distribution calculated at each energy of the present set of measurements, using Symon's theory. This distribution, together with an assumed energy distribution of the actual tandem beam could be folded into the 'foil out' p.s.d. spectrum, and the true response of the magnet thus calculated. The effect of the carbon foil can be extracted accurately from the 'foil in' distribution. This method has the advantage of greater accuracy, but the disadvantage of long analysis time.

Without further comment at this stage, analysis in terms of method (ii) will now be discussed:

2.10.1 STAGE I ('FOIL OUT')

In comparison with equation (2.11), and a δ -function incoming beam energy distribution at E_b ,

$$Y(c,0) = R(E_b,E), \quad \text{where } E = E(c) \quad (2.17)$$

R is the magnet system response function, and is assumed to be gaussian with a skewness factor, s , to account for slit scattering, energy loss in the gold foil, etc.

2.10.2 STAGE II ('FOIL IN')

In comparison with equation (2.16), and the beam δ -function at E_b ,

$$Y(c,T) = \int_{E_\lambda}^{E_b} \omega(E_b,E_i,T) \times R(E_i,E) dE_i, \quad (2.18)$$

where E_ℓ is chosen as in section 2.5.2, and where $E = E(c)$ as above.

On closer examination of the form of these equations, and of the function ω , it becomes evident that if ω can be treated as independent of the absolute value of E_b (within the limits of a small incoming beam spread, say) then method (i) and method (ii) give identical answers. Furthermore, although not so obvious mathematically, since method (iii) is intermediate between the extreme cases of (i) and (ii), method (iii) must also reduce to the same form under the approximation. Inspection of the way in which E_b enters into ω (viz. as the quantity β) shows that the required approximation is valid at these bombarding energies. Analysis of the data in terms of method (ii) was performed, optimising the usual parameters as before with the program FITALL.

The 'foil in' data were finally refitted in the manner described at the end of the analysis section in METHOD I (Section 2.5.2.1) in order to extract the actual energy displacements of the 'beam', as before. The results in this case must be expected to be less accurate than in METHOD I because of the lack of knowledge of the incoming 'beam' distribution.

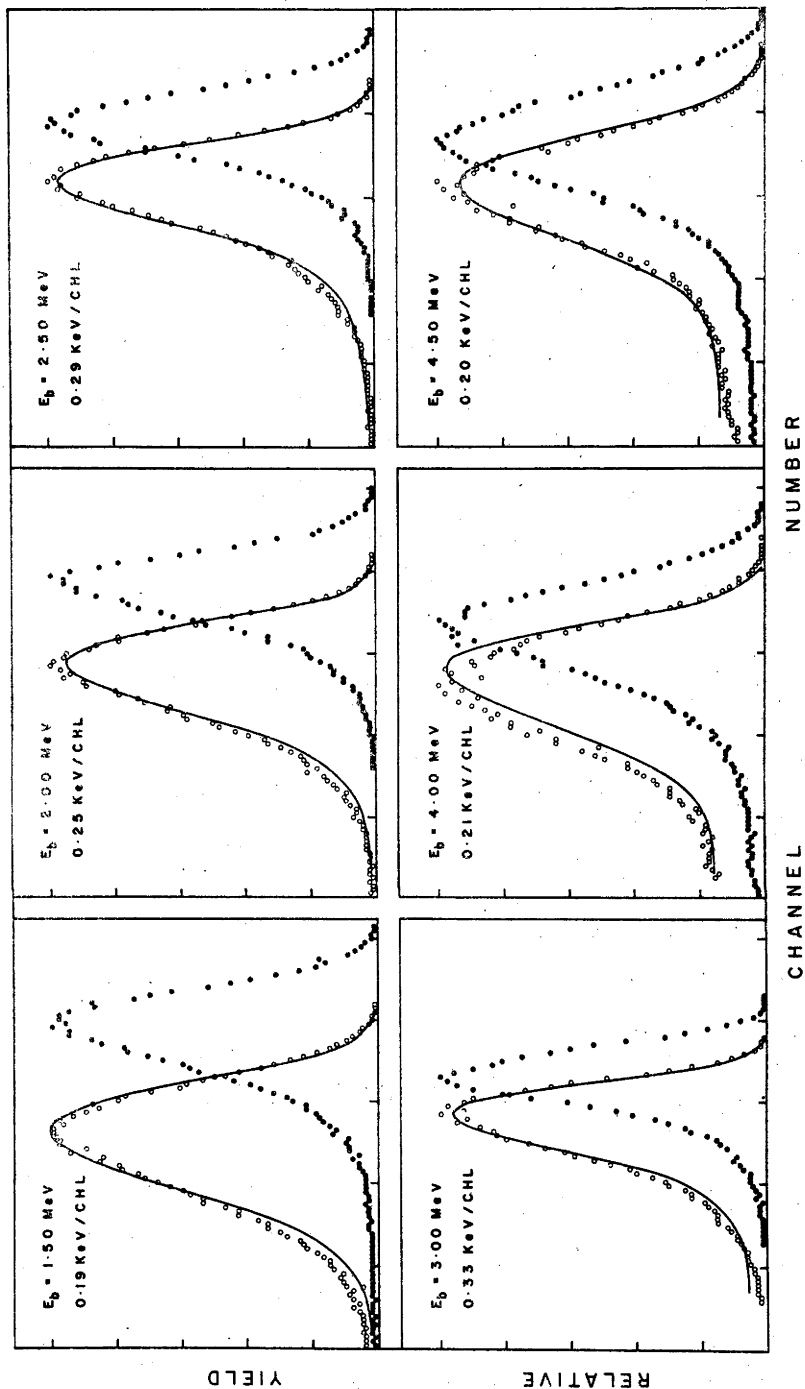
2.11 EXPERIMENTAL DETAILS

Two foils were used during the series of measurements, one of which was foil D from METHOD I (the other foils had broken).

The distributions were measured in energy steps of 500 KeV, from 1.5 MeV to 4.5 MeV.

Energy calibration of the PHA was achieved at each energy by setting the tandem energy 15-20 KeV below the energy used in the energy loss measurements, and measuring the resultant displacement of 'foil out' peak.

Fig. 2.20 FITS TO THE ENERGY LOSS DISTRIBUTIONS (solid curves) F
ONE FOIL, USING SYMON'S THEORY AS DESCRIBED IN THE TEX
THE 'FOIL OUT' DISTRIBUTIONS ARE GIVEN BY THE FULL
CIRCLES AND THE 'FOIL IN' DISTRIBUTIONS BY THE OPEN
CIRCLES. THE SKEW-GAUSSIAN FITS TO THE 'FOIL OUT'
DISTRIBUTIONS ARE NOT SHOWN. THE RELATIVELY POOR
QUALITY OF THE FIT AT $E_b = 4.00 \text{ MeV}$ HAS BEEN
ATTRIBUTED TO A GAIN DRIFT DURING THE RUN — A DRIFT
OF $< 0.5 \text{ KeV}$ (i.e. LESS THAN 1 PART IN 8000) WOULD
ACCOUNT FOR THE DISCREPANCY.



As in the voltage modulation measurements, 'foil out' distributions were measured before and after each 'foil in' measurement, and the series was discarded if energy shifts of one or more channels were observed. The gain was generally ~ 4 channels/KeV. No attempt was made to ensure fixed gain throughout the course of the measurements, mainly because of the fact that the energy range subtended by the p.s.d. is a function of the magnetic field of the spectrometer.

Ortec 109 FET pre-amplifiers coupled to Ortec 410 amplifiers were used to obtain good electronic stability.

It should be mentioned that no precautions were taken as regards carbon build up on the foils because of the very small beam intensities and short duration of the foils in the beam.

2.12 EXPERIMENTAL RESULTS

The fits to the energy loss distributions are shown in fig 2.20, and the details are presented in table 2.4. Averaging was performed over the values as described in METHOD I (Section 2.7), making the same corrections for the ratio of 'peak' to 'mean' displacement, R (fig 2.11), and excluding the leading edge displacement. The averaged quantities are presented in table 2.5. [Note that at the higher energies the values presented for the 'mean' displacement are probably underestimated. This is due to the fact that the mean energy of the 'foil in' distribution is calculated on the basis of a skew gaussian fit to the distribution, and such a function cannot take account of straggling. (cf fig 2.21).]

Fig 2.22 gives a graphical description of the averaged foil thicknesses calculated from Symon's theory (henceforth known as T_s) from METHOD I and METHOD II combined. The curves through the points are drawn by eye, and indicate two possible interpretations which can be placed on the results.

Fig. 2.21 THE FIGURE ILLUSTRATES THE INABILITY OF SKEW-GAUSSIAN FUNCTION FITS TO TAKE ACCOUNT OF STRAGGLING IN 'FOIL IN' DISTRIBUTIONS. THIS RESULTS IN AN UNDERESTIMATION OF THE 'MEAN' ENERGY SHIFT, SINCE THE 'FOIL OUT' DISTRIBUTIONS, WHICH SHOW LITTLE STRAGGLING, ARE FITTED WELL BY SKEW-GAUSSIAN FUNCTIONS

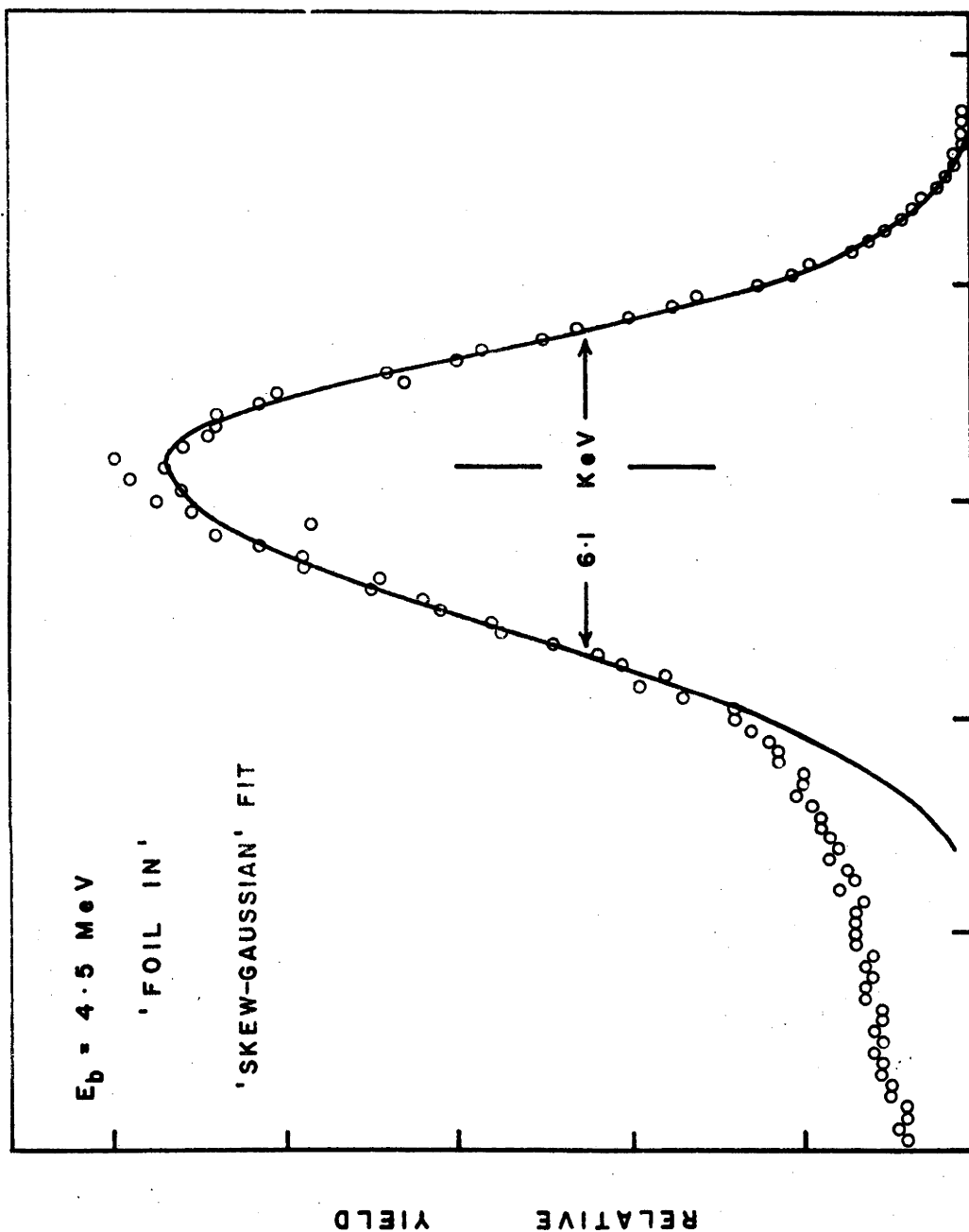


TABLE 2.4

ENERGY (MeV)	FOIL	T (SYMON) $\mu\text{gm}/\text{cm}^2$	ΔP (KeV)	ΔL (KeV)	ΔM (KeV)	S (KeV)/(mg/cm ²))	$T_{\Delta P}$ ($\mu\text{gm}/\text{cm}^2$)	$T_{\Delta L}$ ($\mu\text{gm}/\text{cm}^2$)	$T_{\Delta M}$ ($\mu\text{gm}/\text{cm}^2$)
1.50	D	29.0	5.5	4.8	5.8	175	31.4	27.4	33.2
	E	35.0	6.5	5.8	7.3		37.4	33.2	41.7
2.00	D	33.0	5.0	4.0	5.0	142	35.4	28.4	35.4
	E	39.0	5.6	5.1	6.3		39.8	36.2	44.7
2.50	D	33.0	3.8	3.6	4.4	120	31.6	30.0	37.6
	E	36.0	4.2	3.9	4.8		35.0	32.5	40.0
3.00	D	29.0	2.9	2.7	3.6	150	28.6	25.7	34.3
	E	30.0	2.9	2.6	3.6		28.6	24.8	28.2
3.50	D	27.0	2.1	1.7	2.6	92	22.8	18.5	28.2
	E	32.5	3.1	2.5	3.1		33.5	27.2	33.8
4.00	E	31.0	2.3	2.1	3.2	84	27.4	25.0	38.0
4.50	D	30.0	2.5	2.4	2.7	76	32.9	30.2	35.5
	E	33.0	2.5	2.3	2.7		32.9	30.2	35.5

The meanings of the column headings are explained in table 2.1.

Fig. 2.22 AVERAGED RESULTS OF FOIL THICKNESSES AS DETERMINED BY FITTING THE ENERGY LOSS DISTRIBUTIONS WITH SYMON'S THEORY. THE RESULTS FROM METHOD I (SEE FIG 2.12) AND METHOD II HAVE BEEN COMBINED IN THE FIGURE. THE CURVES HAVE BEEN DRAWN BY EYE AND ARE MEANT TO ILLUSTRATE TWO POSSIBLE INTERPRETATIONS OF THE DATA.

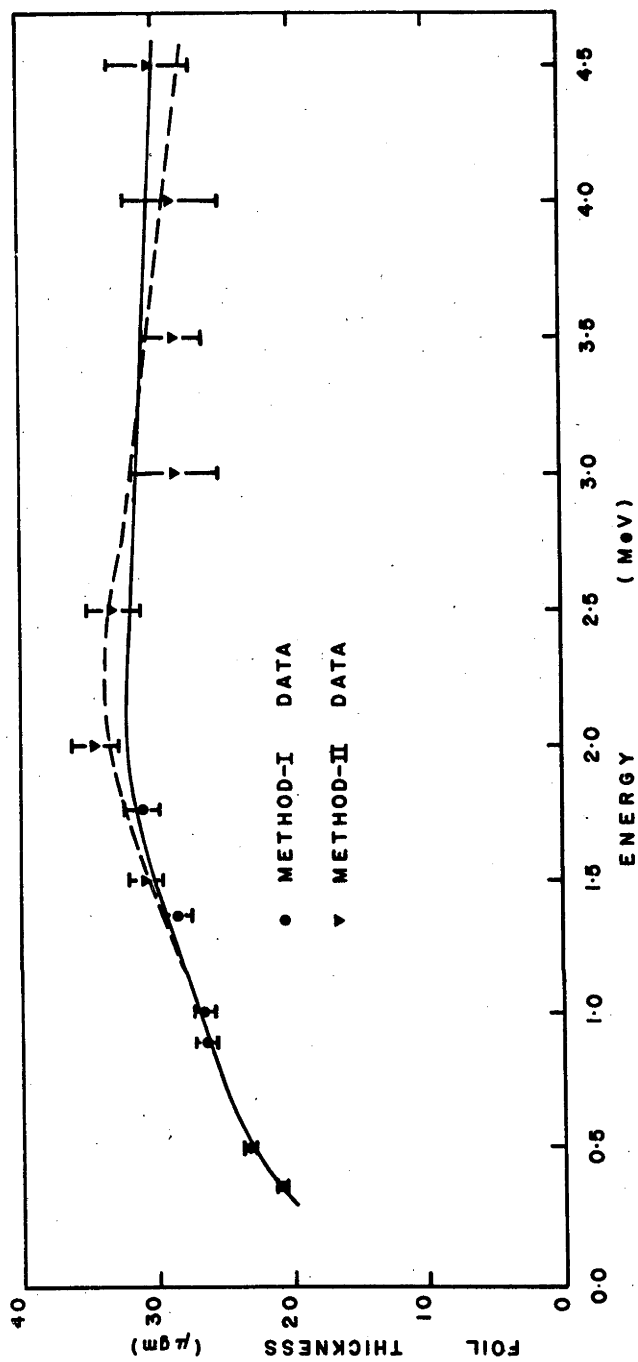


Fig. 2.23 THE DIAGRAMS SHOULD BE COMPARED WITH THE DIAGRAM OF FIG 2.13. THE POINTS REPRESENT THE AVERAGED RESULTS OF FOIL THICKNESSES AS DETERMINED FROM THE ENERGY LOSS OF THE BEAM ON PASSAGE THROUGH THE FOILS, AND USING WILLIAMSON'S TABLES OF STOPPING POWER.

- (a) The average was performed over the foils D and E , for the 'peak' and 'mean' energy losses using the 'peak : mean' conversion ratio R , (obtained from fig 2.11 - cf text).
- (b) The average was performed over the foils D and E , using only the 'mean' energy loss.

THE BROKEN AND SOLID CURVES WERE OBTAINED FROM THE CORRESPONDING CURVES OF FIG 2.22 USING THE 'WILLIAMSON: BETHE' STOPPING POWER RATIOS OBTAINED FROM FIG 2.11.

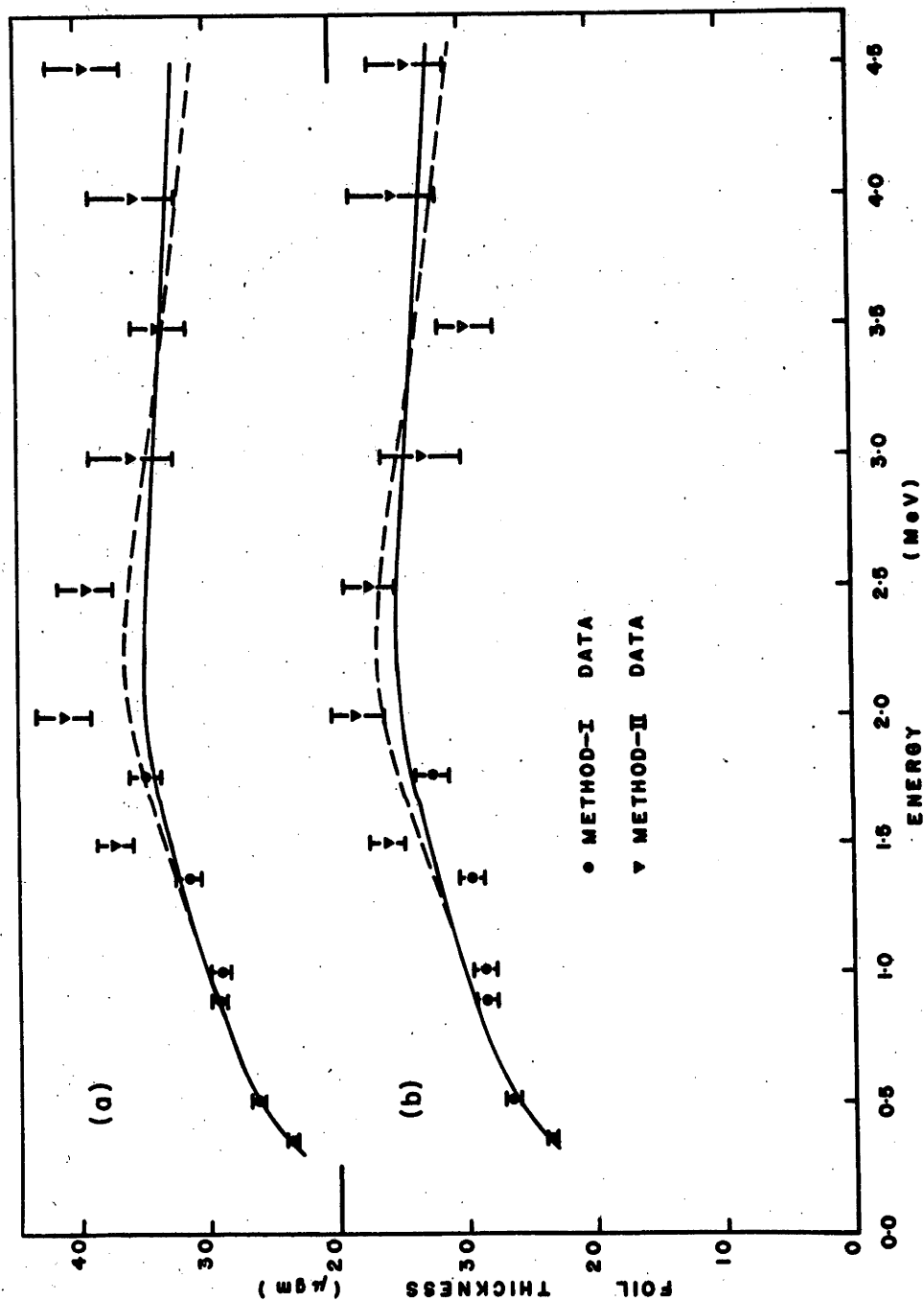


TABLE 2.5

ENERGY (MeV)	T_s ($\mu\text{gm}/\text{cm}^2$)	$\Delta M^{(A)}$ (KeV)	$\Delta P^{(A)}$ (KeV)	MEAN: PEAK (R)	$\Delta P^{(A)}_{XR}$ (KeV)	AVERAGE $\Delta M^{(A)}, \Delta P^{(A)}_{XR}$	$T_{\Delta M}^{(A)}$ (KeV)	$T_{\Delta P}^{(A)}$ (KeV)	AVERAGE $T_{\Delta M}^{(A)}, T_{\Delta P}^{(A)}$
1.5	30.8	6.3	5.8	1.17	6.8	6.6	36.0	33.2	37.4
2.0	34.6	5.5	5.1	1.24	6.3	5.9	38.5	36.2	41.5
2.5	33.2	4.5	3.9	1.30	5.1	4.8	37.4	32.1	39.5
3.0	28.5	3.5	2.8	1.41	3.9	3.7	33.1	27.6	36.0
3.5	28.6	2.8	2.5	1.42	3.5	3.2	29.8	27.0	33.9
4.0	28.8	3.0	2.1	1.43	3.0	3.0	35.3	25.5	35.7
4.5	30.3	2.6	2.4	1.44	3.4	3.0	34.2	31.7	39.4

The values presented in the table were obtained by averaging the results of table 2.4 in the manner described in the text and in table 2.2. The symbols have the same meaning as in table 2.2.

Errors were assigned in the same manner as before.

Apparent gain drifts during data collection are evident in one or two of the distributions. e.g., Fig 2.20, Symon's theory fits the data well at 3.0 MeV and 4.5 MeV, yet not at the intermediate energy of 4.0 MeV. Such behaviour is attributed to a gain drift or energy shift during the measurement. In fact, many of the measurements had to be repeated due to observed shifts in the 'foil out' distributions.

Fig 2.23a shows the foil thicknesses calculated from averaging the energy displacements as described (and using Williamson's tables), together with the expected curve(s) calculated from the T_s values of fig 2.22, (using the 'Williamson : Bethe' stopping power ratio fig 2.11).

Fig 2.23b presents similar data to fig 2.23a except that the 'peak' displacements have not been included in the averaging.

Finally, fig 2.24 (and table 2.6) presents a comparison of the stopping powers as calculated by Williamson, and those calculated from the present experiment on the basis of $T = 33 \mu\text{gm}/\text{cm}^2$ coupled with the 'mean' energy displacements.

2.13 DISCUSSION

Several points are again evident from the results:

1. Apart from one or two cases probably due to gain or energy drift during the run as described above, Symon's theory fits the energy loss distributions well. In particular, the straggling which shows strongly at the higher energies is reproduced remarkably well. e.g., 4 MeV and 4.5 MeV of fig 2.20.

Fig. 2.24 EXPERIMENTAL STOPPING POWER CURVE OBTAINED FROM THE
AVERAGED 'MEAN' ENERGY DISPLACEMENTS. THE 'TRUE'
FOIL THICKNESS WAS ASSUMED TO BE $33 \mu\text{gm}/\text{cm}^2$, ON
THE BASIS OF FIG 2.22 (cf text).

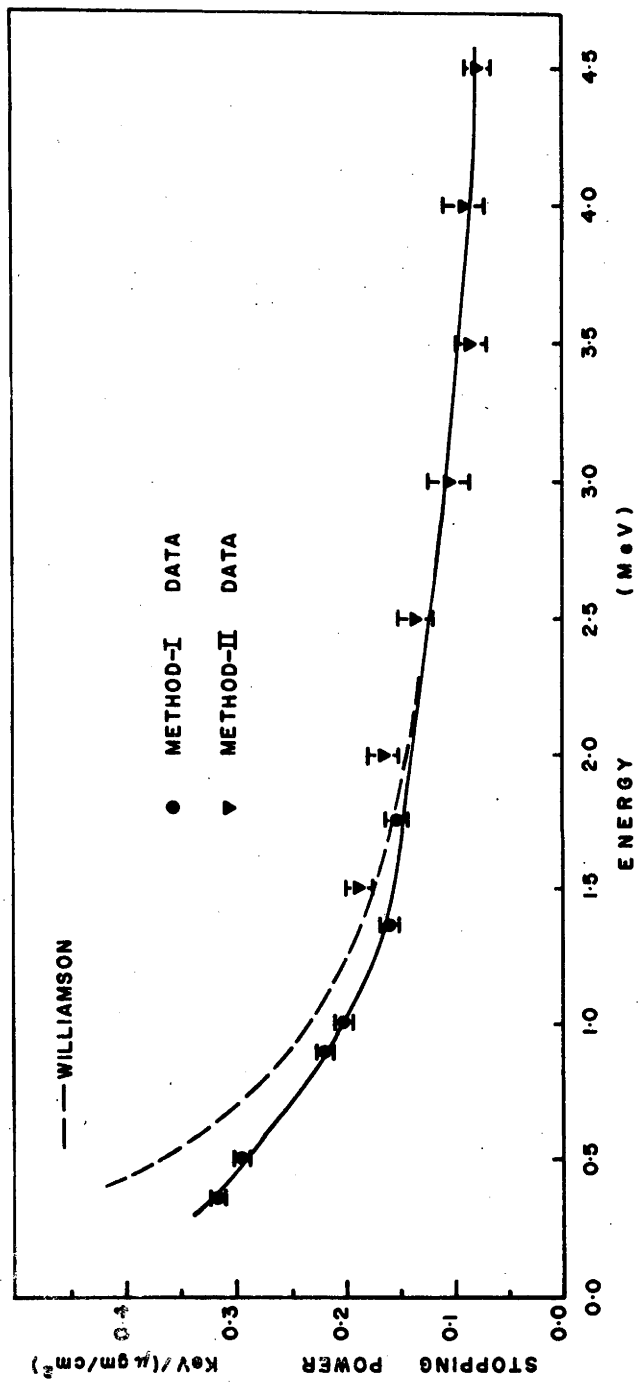


TABLE 2.6

ENERGY (MeV)	$\Delta M^{(A)}$ (KeV)	S (KeV/(mg/cm ²))
0.360	10.5	318
0.506	9.8	297
0.894	7.3	221
1.007	6.7	203
1.368	5.3	161
1.50	6.3	191
1.767	5.1	155
2.00	5.5	167
2.50	4.5	136
3.00	3.5	106
3.50	2.8	84
4.00	3.0	91
4.50	2.6	79

The values under the column heading $\Delta M^{(A)}$ are the averaged 'mean' beam-energy shifts from tables 2.2 and 2.5. The stopping powers, S, were calculated from the $\Delta M^{(A)}$ assuming a foil thickness of 33 $\mu\text{gm/cm}^2$ (cf text).

Note that no background was included in any of the fits.

2. Within the limits of the experimental accuracy, the curve T_s vs. E appears to flatten out as expected (solid curve, fig 2.22). However, the data do not exclude the possibility that the curve has a peak at about 2.3 MeV (dotted curve, fig 2.22). It is not obvious in terms of the decreasing effect of the binding energy of the atomic electrons how such a peak could occur. This possibly means that another effect is starting to increase in strength at the higher energies, where the foils are becoming very thin.

3. The curve of foil thickness as calculated from the averaged displacements and Williamson's tables does roughly what one would expect. (Fig 2.23). However there is a distinct discontinuity between the results from METHOD I which follow the expected curve, and the results from the spectrometer measurements which generally lie too high. As mentioned at the end of the analysis section (2.10.2), some sort of effect such as this is not unexpected due to the imprecise knowledge of the actual 'beam' in these measurements, as compared with METHOD I. This is confirmed to some extent by the fact that such a discontinuity is not apparent in the T_s curve (fig 2.22), in which knowledge of the incident beam distribution is not important (cf. section (2.10.2)).

A further curve is presented (fig 2.23b) which indicates the behaviour of the foil thickness as determined purely from the 'mean' displacements, excluding the contribution from the 'peak' displacements. These values appear to be more consistent than the values of fig 2.23, although the discontinuity is still evident to some extent. Several factors are expected to contribute to the difference between the two sets of values:

(i) the mean displacements are expected to be underestimated, increasingly with energy since the straggling is ignored in the skew gaussian fit to the 'foil in' data (cf. fig 2.21),

(ii) the results of the spectrometer measurements are expected to be in error due to lack of knowledge of the actual 'beam' distributions, as discussed above,

(iii) tables 2.2 and 2.5 indicate that multiplying the 'peak' displacement by the 'mean' : 'peak' ratio, R , generally results in values greater than the experimental 'mean' displacements, even at relatively low energies where the straggling is too small to affect the value calculated for the 'mean' displacement. This casts some doubt on the validity of the procedure of including the 'peak' displacements in the averaging as described.

4. The experimental stopping power curve (fig 2.24) was calculated on the assumption that the foil thickness was in fact $33 \mu\text{gm}/\text{cm}^2$. This value was obtained from fig 2.22, assuming that the region around 2-2.5 MeV had greater probability of giving the 'true' thickness than the other regions, for reasons which have been already discussed to some extent. There is no strong evidence at the moment, unfortunately, that $33 \mu\text{gm}/\text{cm}^2$ is indeed the 'true' value, and it may be that the experimental curve has to be multiplied by some normalising factor. However, one fact appears to be established beyond reasonable doubt, and that is that Williamson's curve is overestimated at the low energy end (since the foil thickness of fig 2.13 shows a marked increase with energy up to $\sim 2 \text{ MeV}$). Underestimation of the effect of the electron binding energies would give rise to such a result.

2.14 CONCLUSIONS

(a) The major conclusion is that Symon's theory accounts remarkably well for the broadening, skewness and straggling introduced to a beam of charged particles on passage through a thin foil, well below the lower energy limit at which the theory is expected to apply. This

has been shown only for protons on carbon, but may be expected to hold for protons on other light elements.

(b) At energies below ~ 2 MeV, the foil thickness used to produce the required shift of the distribution from Symon's theory is underestimated, but this can apparently be corrected for by application of a suitable factor.

(c) The method of calculating foil thickness by measuring the displacement of the leading edge of the yield curve of a resonance is not reliable, and even independently of Symon's theory, gives different answers to those obtained from calculating the mean energy shift of the actual 'beam' distribution. In the energy region where the beam distributions can be accurately evaluated (*i.e.*, METHOD I) the 'mean' energy displacements of the beam are consistent with the foil thicknesses obtained from Symon's theory (applying appropriate factors for the ratio of the unmodified Bethe stopping power used by Symon to the stopping power used by Williamson).

Note that the value obtained at the $874 \text{ KeV } F^{19}(p, \alpha)O^{16}$ resonance by the yield displacement method is $\sim 25 \text{ } \mu\text{gm/cm}^2$, as compared to the $33 \text{ } \mu\text{gm/cm}^2$ taken to be the actual foil thickness. This accounts completely for the 25% discrepancy in the cross section determination mentioned in the introduction to this chapter.

(d) Some work has been published which specifically evaluates corrections to Symon's theory in terms of binding energy effects. In particular, Rosenzweig (Ro 59) has presented a method for calculating correction factors to Symon's weighted parameters j , b and λ . The effects of these corrections on Symon's distribution function ω are as follows:

- (i) the most probable energy loss lies further from the mean energy loss

- (ii) the width of the curve is increased
- (iii) the skewness is decreased.

It is not clear that application of such corrections will improve the quality of the fits presented in fig 2.20, although it should be emphasized that no attempt has been made so far to refit any of the data using Rosenzweig's formulae. Morris and Ophel (Mo 65) claim that the corrections they applied at ~ 1 MeV actually worsen the fit compared to that obtained using the unmodified Symon theory.

(e) Gorodetzky et al (Go 67) have measured the stopping power of carbon for protons of energy between 400 KeV and 6 MeV by scattering protons from a gold foil (40-200 $\mu\text{gm}/\text{cm}^2$) on a carbon backing, and using the scattered protons at 90° as the incident beam on a carbon foil (100-150 $\mu\text{gm}/\text{cm}^2$) whose thickness was determined by weighing on a micro-balance. The distributions were measured on a surface barrier detector with a resolution of ~ 20 KeV, and the method of determining the shift produced by the foil was simply to subtract the mean energies of the 'foil out' and 'foil in' distributions. Gorodetzky's results agree very closely with the tabulated values in Williamson's tables. They have quoted maximum probable errors of $\pm 3.5\%$ over the complete energy range. This value includes probable errors in weighing the 100 μgm foils, uniformity considerations, possible energy shifts, and errors in determining the mean positions of the 'foil in' and 'foil out' distributions. In view of the fact that the present experiment was conducted with much higher resolution, better energy stability for the low energy measurements and with thinner foils, yet has greater probable errors over most of the range, the probable error assignments of Gorodetzky et al are thought to be optimistic.

2.15 FURTHER EXPERIMENTS

Although the work of this chapter has shown that Symon's theory is able to account satisfactorily for the energy loss distributions, it is felt that, owing to the importance of stopping power measurements, confirmation of the experimental stopping power curve presented herein is highly desirable. To this end, it is proposed to perform low energy Coulomb scattering measurements on a number of foils to determine absolute thicknesses, and to repeat the low energy measurements of METHOD I. Furthermore, it is proposed to repeat the higher energy measurements at closer energy spacings on the A.N.U. Beuchner mass spectrograph which is currently being modified for the purpose. The Beuchner has somewhat better resolution than the double focussing spectrometer, and energy stability which is better by an order of magnitude.

It is also proposed to modify the programs to include a section for analysis of the data in terms of Vavilov's theory, as it is felt that a direct comparison of the two theories over the complete range of the parameter G could prove interesting.

CHAPTER III
ENERGY LEVELS OF O^{17}

3.1 INTRODUCTION

Shell model calculations of nuclear energy levels are generally performed in mass regions near closed shells, and the nuclei adjacent to the O^{16} double closed shell system provide good examples.

In particular, mass-17 nuclei have recently been the subject of a number of theoretical studies, and a large body of experimental data has been obtained, enabling quantitative comparison with the theoretical predictions.

Lane and Flowers (La 57), assuming the O^{16} ground state to have a $(1s)^4 (1p)^{12}$ configuration, pointed out that the 1d and 2s shells would be expected to fill together on the basis of the oscillator potential, and suggested that the the three lowest positive parity states in O^{17} , viz the $\frac{5}{2}^+$ g.s., the 0.87 MeV $\frac{1}{2}^+$ and the 5.08 MeV $\frac{3}{2}^+$, would thus be single particle states with 1d $\frac{5}{2}$, 2s $\frac{1}{2}$ and 1d $\frac{3}{2}$ configurations, respectively. They noted that negative parity states in O^{17} appeared at lower excitations than expected on the basis of a simple description, and that such states could be explained on the basis of excited configurations formed either by core excitation or by exciting the last particle to the (2p,1f) shell.

Matthies et al (Ma 62) interpreted recent data on the lower levels of C^{12} and O^{16} as evidence for an α -particle model for the nuclei, and on this basis were able to predict with some success the negative parity states of O^{17} and F^{17} by taking into account the effect of nucleon - α -core binding.

Wildermuth and Carovillano (Wi 61), on the basis of an α - C^{13} cluster model, have predicted ~ 2 MeV wide optical model giant resonances in O^{17} for bombarding energies in the range 5 - 10 MeV .

Salisbury and Richards (Sa 62) measured a number of spins and parities of F^{17} levels, showing that the level structures of the O^{17} , F^{17} mirror nuclei were identical below 6.1 MeV. They suggested that, since the first three even parity states of F^{17} were probably single particle states, it was possible that higher levels could be explained on the basis of known excited states of O^{16} acting as parent 'single-particle cores' to the F^{17} states and demonstrated that the $\frac{1}{2}^+$ state at 6.56 MeV and the $\frac{3}{2}^+$ state at 7.48 MeV, each of appreciable nucleon reduced width, could be accounted for assuming as O^{16} parents, the 6.06 MeV (0^+) and 6.92 MeV (2^+) states, respectively.

They indicated that further even parity states in F^{17} could possibly be explained by promoting two p nucleons to s or d orbits, and also considered the possibility of a four-hole configuration with a quasi-alpha particle coupled to the $1d \frac{3}{2}$ orbital. They had little success in explaining the odd parity states assuming a Ne^{20} parent coupling Ne^{20} states to three p holes.

Barker (Ba 64) has pointed out that the simple model of O^{17} in which a $1d \frac{5}{2}$ or $2s \frac{1}{2}$ neutron is coupled to an inert O^{16} core cannot explain the non-vanishing of the O^{17} quadrupole moment and of the E2 decay of the O^{17} first excited state, and that it is necessary to assume contributions to the E2 matrix elements from the O^{16} core. The data have been explained in terms of a core distortion produced by a two-particle central interaction between the odd neutron and the core nucleons, involving a single 'collective' 2^+ state of O^{16} at an excitation of 12-18 MeV. However, it is not certain that such a state exists, and Barker has investigated the effect of considering the contributions to the E2 matrix elements from $T = 1$ excitations of the O^{16} core, and from recoil of the core.

Brown and Green (Br 65) list some of the information which shows that, even in the region of double closed shells, the description of nuclei in terms of the spherical shell model is by no means adequate. They have investigated a model in which there are low lying deformed states in addition to the usual shell model states, and show that this model can account for the observed rotational bands, large transition rates and the multiplicity of states (even parity) in O^{17} and O^{18} .

Quesne (Qu 66), following the success of the particle-hole model in describing the spectra of closed shell nuclei such as C^{12} and O^{16} , applied the model to nuclei with one nucleon outside closed shells, considering in particular the case of O^{17} . The model had some success in describing negative parity states, but resulted in poor agreement with the positive parity states.

Margolis and Takacsy (Ma 66) have similarly undertaken a series of calculations in order to see to what extent the low-lying negative parity states of the mass-17 nuclei can be described in terms of two-particle one-hole configurations. They report that the negative parity states in F^{17} and O^{17} up to ~ 6 MeV can be described adequately in terms of their model, but that levels above 6 MeV appear to arise mainly from higher configurations.

Most of the experimental work on unbound states in the O^{17} nucleus has involved either (n,α) or (α,n) studies on O^{16} and C^{13} respectively, together with some recent work measuring elastic scattering of alpha-particles from C^{13} and neutrons from O^{16} .

Walton et al (Wa 55) and Bonner et al (Bo 56) have measured the 0^0 and 90^0 neutron yields from the $C^{13}(\alpha,n)O^{16}$ reaction for alpha-particle bombarding energies in the ranges 1.5-4 MeV and 2-5 MeV, respectively. Both groups observed a series of narrow resonances, corresponding to states in O^{17} between 7 and 10 MeV excitation.

Becker and Barschall (Be 56) also measured the 0^0 and 90^0 neutron yields from the $C^{13}(\alpha,n)O^{16}$ reaction over the energy range $2 \text{ MeV} \leq E_{\alpha} \leq 4 \text{ MeV}$. At the same time they obtained the total neutron cross section of O^{16} , using the (α,n) neutrons, and showed the existence of strong correlations. Fossan et al (Fo 61) have also measured the total neutron cross section of O^{16} .

Rushbridge (Ru 56), Schiffer et al (Sc 57) and Walton et al (Wa 57) measured $C^{13}(\alpha,n)O^{16}$ angular distributions for excitation energies in O^{17} between 7 and 10 MeV, and determined spins and relative parities for a number of states in the region. Walton et al also observed the oxygen total neutron cross section and the $O^{16}(n,\alpha)C^{13}$ reaction, and compared the observed states in the (α,n) and (n,α) reactions using the principle of detailed balance.

Worley et al (Wo 60) measured the $O^{16}(n,\alpha)C^{13}$ reaction up to higher energies, using neutrons of energy from 5-9 MeV obtained from $D(d,n)He^3$, and compared a number of resonances in this reaction with resonances in the $C^{13}(\alpha,n)O^{16}$ reaction measured by Bonner et al.

Excitation functions of the 6-7 MeV gamma radiation from $C^{13}(\alpha,n\gamma)O^{16}$ have been measured by Spear and Larson (Sp 63) in the range of alpha-particle bombarding energies 5-10 MeV, and many resonances were observed corresponding to excited states of O^{17} in the region 10-14 MeV excitation.

Sekharn et al (Se 67) have recently determined the $C^{13}(\alpha,n)O^{16}$ total cross section in the region $2 \text{ MeV} \leq E_{\alpha} \leq 6 \text{ MeV}$, and measured total widths for a number of the observed levels.

The experiments discussed above are unable to determine parities of individual states, although relative parities have been determined in some cases from neutron angular distributions measured in regions of interfering states.

Barnes et al (Ba 65) have measured $C^{13}(\alpha, \alpha_0)C^{13}$ excitation functions at several angles, and the 0.0 degrees $C^{13}(\alpha, n)O^{16}$ excitation function, for alpha-particle bombarding energies from 2-3.5 MeV , together with a number of angular distributions.

Analysis of the data in terms of the single level dispersion theory was able to provide J^π assignments and reduced widths for a number of states in the 8-9 MeV region of O^{17} excitation.

Fowler and Johnson (Fo 65) and Lister and Sayres (Li 66) have measured angular distributions and excitation functions in the $O^{16}(n, n)O^{16}$ reaction, using 3-4 MeV neutrons, and phase shift analyses of the data enabled the determination of spins and parities of a number of states in the region ($E_X \sim 7-8$ MeV) .

Owing to the interest in the O^{17} nucleus, as indicated in the above discussion, it was felt worthwhile to extend measurements of the O^{17} level structure to the region of excitation above 9 MeV , where a number of reasonably isolated states were known to exist (Bo 56, Sc 57, Wo 60), with generally undetermined level parameters. To this end it was decided to complement the measurements of Barnes et al in the higher energy region.

In the present work, the $C^{13}(\alpha, \alpha_0)C^{13}$ differential cross sections were measured for energies between 3.5 and 6.5 MeV at CM angles of 54.7, 90.0, 149.4 and 169.9 degrees, and the $C^{13}(\alpha, n)O^{16}$ differential cross section at 0.0 degrees established over the same energy range, repeating some of the earlier work of Bonner et al. Angular distributions of the $C^{13}(\alpha, n)O^{16}$ reaction were obtained at a number of intermediate energies.

3.2 EXPERIMENTAL DETAILS

3.2.1 BEAM

The A.N.U. 2.0 MeV Van de Graaff injector and 12 MeV tandem Van de Graaff accelerator facilities provided a beam of doubly charged alpha particles which was magnetically analysed and focussed through collimators onto the targets.

3.2.2 TARGETS

The material used for the target preparation was methyl iodide enriched to 55% C^{13} . The accuracy of the manufacturer's assessment was confirmed by mass spectroscopic* and nuclear magnetic resonance analyses**.

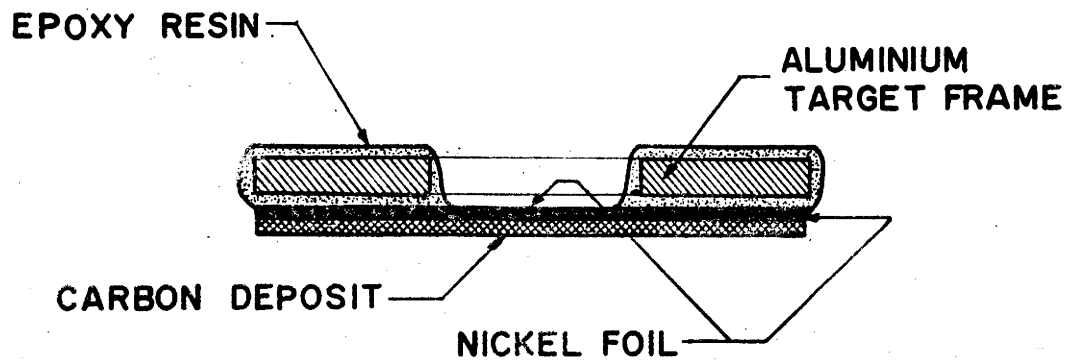
(i) Solid backed:

The targets used in the $C^{13}(\alpha,n)O^{16}$ measurements consisted of C^{13} enriched carbon, deposited on 0.001 inch thick tantalum backings. Preparation involved cracking the methyl iodide onto a tantalum strip, supported between large copper electrodes and heated to dull red by passage of current in an atmosphere (10-20 mm) of the vapour. Surface contaminants were removed from the tantalum by preheating it to white heat at 10^{-5} mm pressure for several minutes. Iodine deposited at the cool ends of the strip during the cracking procedure was removed by pumping the chamber for sometime afterwards. Vacuum grease was avoided so that carbon contamination, which would alter the enrichment

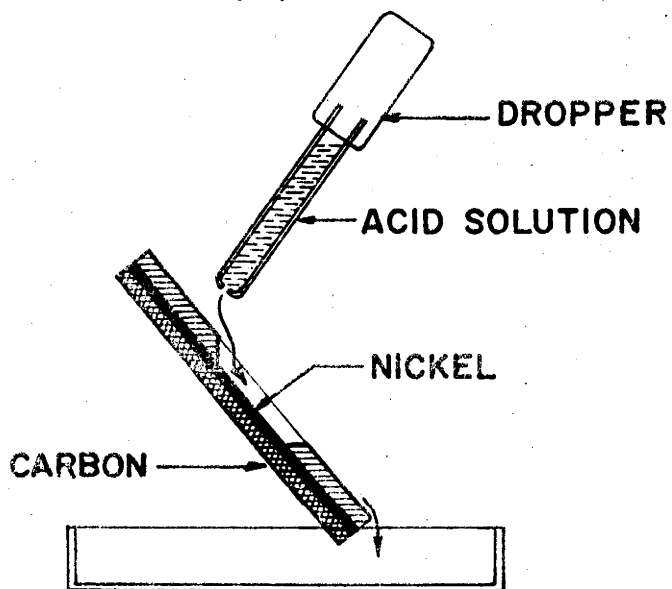
* The mass spectroscopic analysis was performed by Dr F.J. Bergenson of the C.S.I.R.O.

** The nuclear magnetic resonance analysis was performed by Dr J.J. Batterham of the Department of Medical Chemistry, A.N.U.

Fig. 3.1 METHOD OF MOUNTING (a), AND ETCHING (b),
NICKEL FOILS TO PREPARE ENRICHED SELF-SUPPORTING
 C^{13} FOILS. ALL SURFACES OF THE ALUMINIUM TARGET
FRAME WERE PROTECTED FROM THE ACID SOLUTION
BY A LAYER OF EPOXY RESIN.



(a)



(b)

ratio, was reduced. Furthermore, a liquid nitrogen cold-trap was interposed between the chamber and the pumping system to prevent contamination of the pumps by iodine and to reduce the amount of oil vapours reaching the chamber.

Target thicknesses were generally 20 - 40 $\mu\text{gm}/\text{cm}^2$ (initial estimates being obtained by weighing on a micro-balance), although thinner targets were made for use in resonance width determinations (see reactopm 3.4.3).

(ii) Self-supporting:

Carbon was deposited in the manner described above on the dull sides of thin nickel strips (0.00002 inch thick) supported back to back between the copper electrodes. The carbon-deposited nickel strips were subsequently attached to aluminium target frames (1.5 cm dia.) with epoxy resin and the nickel removed (fig 3.1) with a sulphuric-mitric acid mixture (2 parts H_2SO_4 , 2 parts HNO_3 , 1 part H_2O) (Ka 59). The carbon foils were washed thoroughly with dilute HNO_3 and distilled water. Residual impurities were found to be small as evidenced by inspection of the spectra of alpha particles elastically scattered from the foils (lower half of fig 3.2).

Target thicknesses were generally 20 - 30 $\mu\text{gm}/\text{cm}^2$, accurate values being derived from the known $\text{C}^{12}(\alpha, \alpha_0)\text{C}^{12}$ differential cross section (Jo 62).

3.2.3 TARGET CHAMBERS AND DETECTORS

(i) $\text{C}^{13}(\alpha, \alpha_0)\text{C}^{13}$;

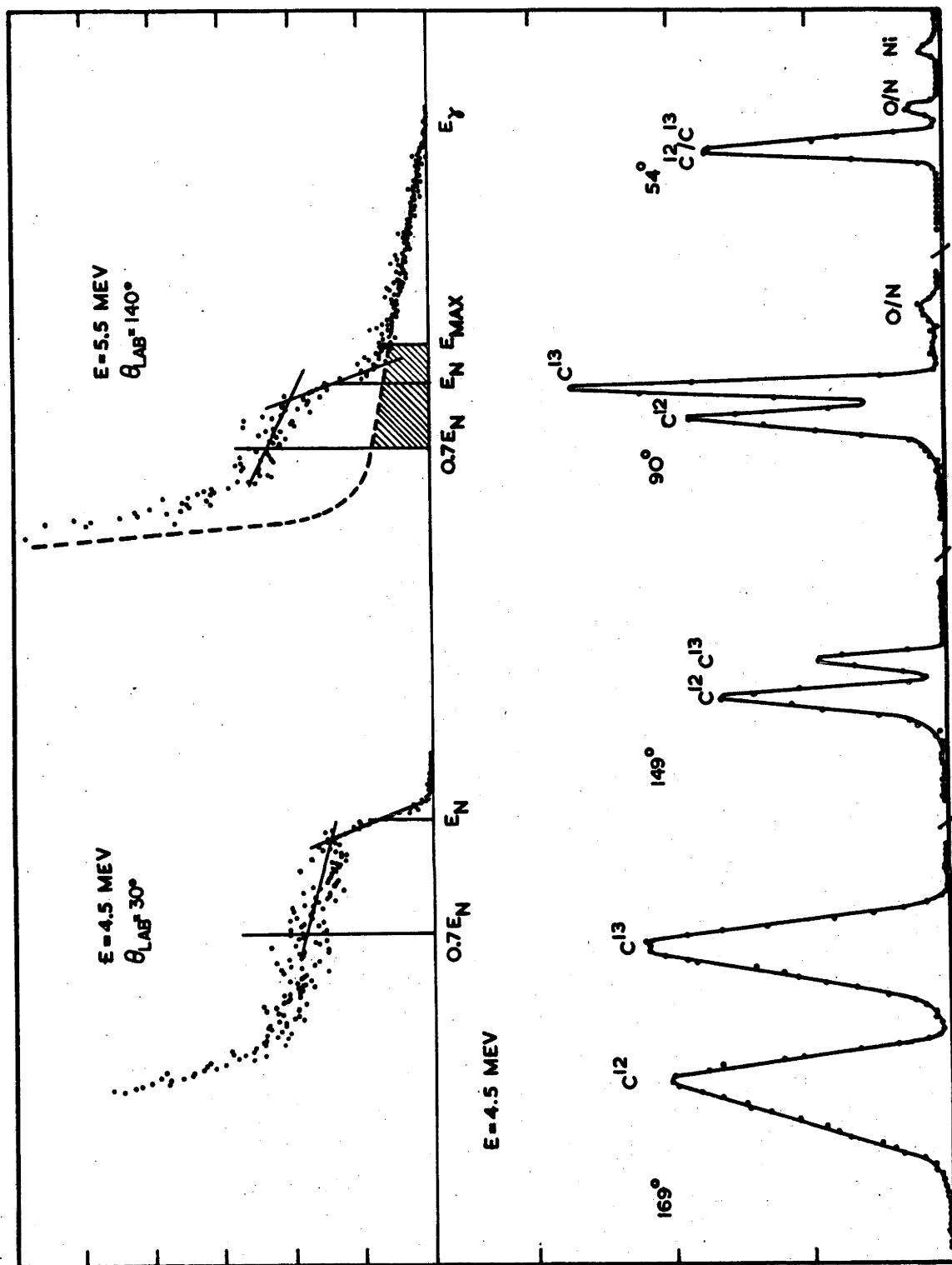
The 51 cm scattering chamber has been discussed in chapter 1, section 1.2.3. Two surface barrier detectors of 500 ohm.cm resistivity coupled to Ortec model 103 - 203 amplifier systems, operated in the RC mode with post-amplifier bias to retain optimum resolution, were employed for the measurements. Typical spectra

Fig. 3.2(a)

RECOIL PROTON SPECTRA AT $E_{\alpha} = 4.5 \text{ MeV}$ AND AT $E_{\alpha} = 5.5 \text{ MeV}$. PULSES WERE SUMMED FROM $0.7 E_N$ TO E_{MAX} . THE SHADED AREA IN THE 5.5 MeV SPECTRUM IS THE AREA SUBTRACTED FOR 6.14 MeV GAMMA RAY BACKGROUND, CALCULATED FROM THE MEASURED LINE SHAPE (broken curve) OF 6.14 MeV GAMMA RAYS IN THE $2.5 \text{ cm} \times 5.1 \text{ cm}$ PLASTIC SCINTILLATORS USED, AND THE SUM OF PULSES FROM E_{MAX} TO E_{γ} .

Fig. 3.2(b)

SPECTRA OF THE ELASTIC ALPHA PARTICLE GROUPS AT $E_{\alpha} = 4.5 \text{ MeV}$ FOR CENTRE OF MASS ANGLES 169.6° , 149.4° , 90° and 54.7° .



recorded at $E_{\alpha} = 4.5$ MeV are shown in the lower half of fig 3.2. The groups elastically scattered from C^{12} and C^{13} were essentially resolved at the two backward angles, and partially resolved at 90° CM. At 90° and 54.7° CM the contributions from C^{12} scattering were estimated from the $C^{12}(\alpha, \alpha_0)C^{12}$ cross section as calculated from the phase shifts of Jones et al (Jo 62).

(ii) $C^{13}(\alpha, n)O^{16}$:

The (α, n) measurements were made in a thin-walled brass chamber (fig 3.3), suspended so as to reduce the amount of scattering material in the vicinity of the chamber and detectors. A cold-trap consisting of a liquid-nitrogen cooled, insulated copper shield (Ba 61) surrounded the target and reduced carbon build up to a negligible amount. A negative potential was applied to the copper shield for electron suppression. Two $2.5 \text{ cm} \times 4 \text{ cm}$ plastic scintillators, mounted on a rotating table centred vertically below the chamber, were located in the plane of the beam 18 cm from the target and separated by 90° .

A relative measure of the yield at each angle and energy was obtained by summing pulses in the recoil proton spectrum above $0.7 E_p$ (MAX). The value 0.7 was chosen as a compromise between the rate of data collection and minimising the contributions of noise and γ -ray background, as well as avoiding the region of greatest non-linearity in the recoil proton spectra. Corrections for the energy dependence of the detection efficiency and for the non-linear response of the detectors, were applied. Data describing the detector response (fig 3.4) were obtained experimentally by consideration of the full energy recoil proton pulse height for a range of incident neutron energies (making use of the angle dependent neutron emission energy from the reaction, and obtaining the appropriate pulse height from the recoil proton spectrum end-point energy on the analyser (E_N of fig 3.2)).

Fig. 3.3 TARGET CHAMBER AND DETECTOR ARRANGEMENT
USED FOR THE $C^{13}(\alpha,n)O^{16}$ DISTRIBUTION
MEASUREMENTS. THE NUMBERED COMPONENTS ARE:

- (1) Chamber Wall
- (2) Target Support and Angle Scale
- (3) Cold Trap
- (4) Target
- (5) Rotating Counter
- (6) Liquid Nitrogen Container
- (7) Beam Entrance Tube
- (8) Collimators
- (9) Rotating Table
- (10) Fixed Monitor Counter

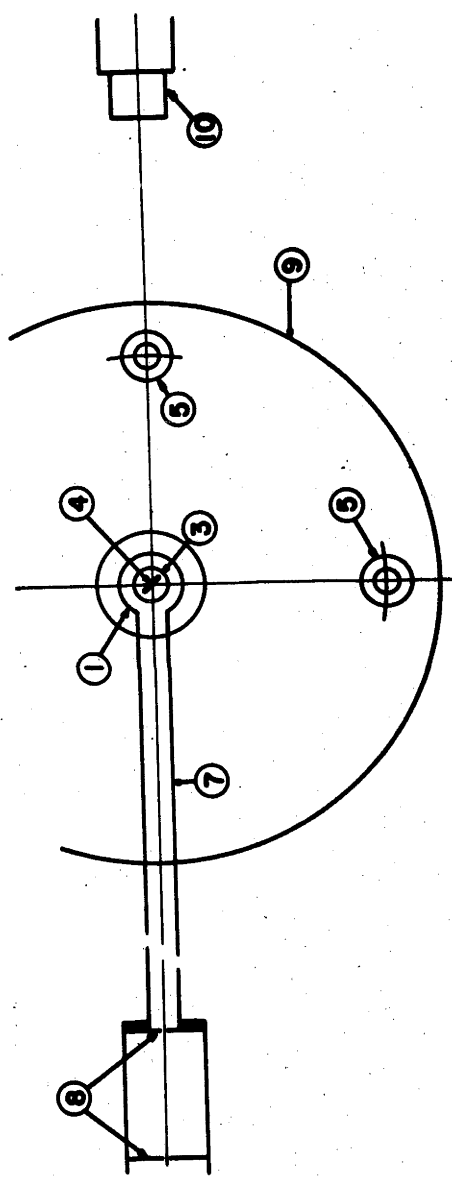
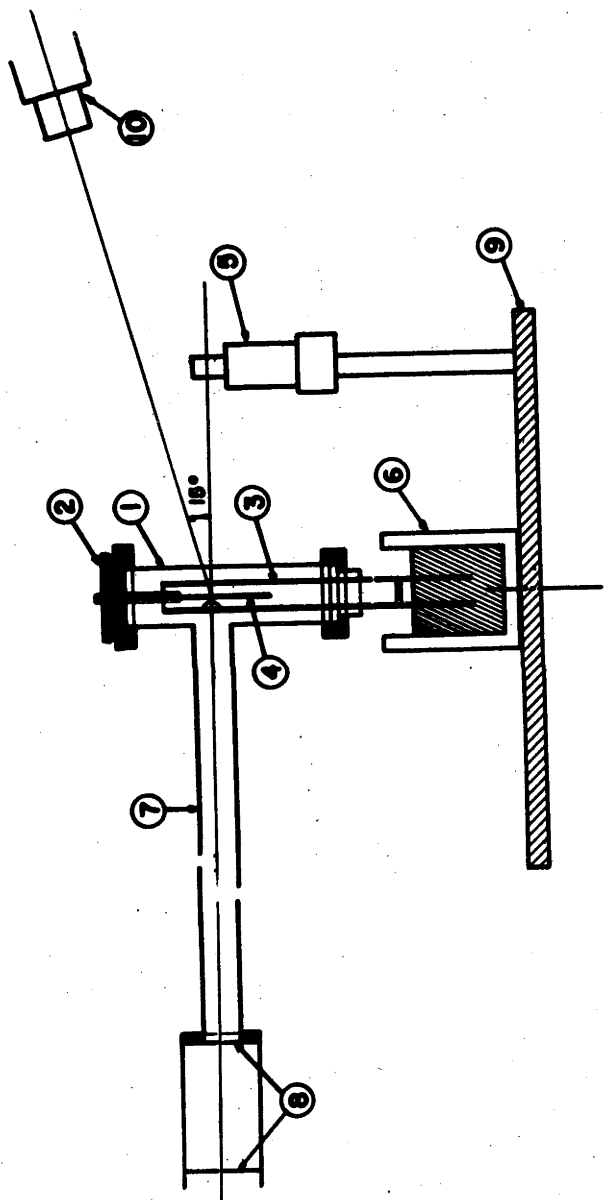
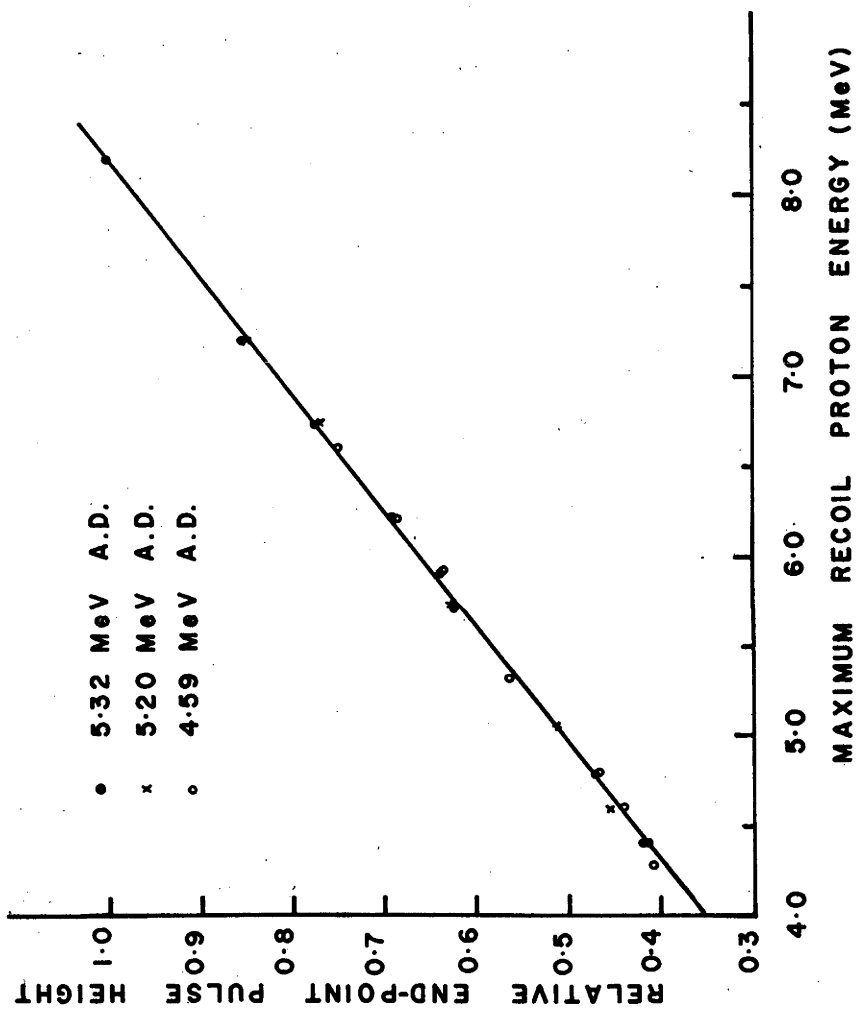


Fig. 3.4 RESPONSE OF THE PLASTIC SCINTILLATORS TO NEUTRONS.
THE RELATIVE END-POINT PULSE HEIGHT WAS CALCULATED
FROM THE ANALYSER CHANNEL NUMBER CORRESPONDING
TO THE POSITION E_N OF FIG 2. THE SOLID CURVE
IS A STRAIGHT LINE DRAWN BY EYE THROUGH THE
POINTS.



Above ~ 5 MeV bombarding energy, neutron decay to the 2nd excited state of O^{16} is energetically possible, resulting in a 6.14 MeV gamma ray background underlying, and extending beyond the ground state neutron yield. The $F^{19} (p, \alpha \gamma) O^{16}$ reaction was used to measure the line shape of this radiation in the plastic scintillators so that summation of the gamma ray yield above the recoil proton end point energy allowed subtraction of the background within the energy interval over which the recoil proton spectrum was summed.

Typical recoil proton spectra recorded at 4.5 MeV (30° lab), below the threshold for 6.14 MeV gamma-ray production, and at 5.5 MeV (140 lab) where the yield of gamma rays was large, are shown in the upper half of fig 2.2.

3.2.4 BUILD UP

As mentioned in the previous section, build up of carbon on the targets was negligible for the neutron measurements. However, this was not the case for the elastic scattering measurements where target thickness increments of typically 10%, though ranging between 0 - 25% on occasions, were observed. This is in contrast to the results discussed in chapter 1 for which build up was generally found to be small. It is believed that the different methods of preparation and structure of the carbon foils may have had some bearing on the rate at which carbon accumulated.

The amount of build up during each run was determined by remeasuring the first few points of the excitation functions. The data were corrected on the assumption that the deposit increased uniformly throughout the run.

3.2.5 DEAD TIME

Analyser dead times for both the neutron and elastic scattering measurements were kept below $\sim 5\%$, and the data were corrected according to the method discussed in chapter 1, section 1.2.5.

3.3 ANALYSIS

The data were analysed essentially according to the procedures described by Barnes et al (Ba 65). In contrast to the method discussed in chapter 1 in which the data were parametrised in terms of phase shifts independent of any particular nuclear model, the present method assumes a precise expression for the scattering matrix to calculate the cross sections, and reproduction of the data is attempted by assuming specific resonance parameter values (E_0, J^π, Γ) for the various anomalies in the scattering and reaction excitation functions.

For incident alpha particle energies below ~ 5 MeV, only two particle decay channels contribute appreciably to the reaction, viz. alpha particle emission to the ground state of C^{13} and neutron emission to the ground state of O^{16} . Inelastic scattering via the first excited state of C^{13} ($E_X = 3.09$ MeV) is energetically possible (for incident energies $E_\alpha \gtrsim 4$ MeV).

Thus the condition $\Gamma_\alpha + \Gamma_n = \Gamma$ can be considered valid in this energy region. (The radiative capture width is negligibly small compared with the particle decay widths). At bombarding energies greater than 5 MeV the condition becomes invalid due to neutron emission to excited states in O^{16} ($E_X = 6.06$ MeV, 6.14 MeV), and no detailed analysis was attempted for these energies - partly because of the complication of additional channels, but mainly because of the increasing number of overlapping resonances.

Although several overlapping resonances were evident in the region

below ~ 5 MeV , an analysis of the level structure was attempted in terms of the single level approximation to simplify the analysis and to reduce computation time.

3.3.1 ELASTIC SCATTERING

Cross-sections were calculated from the expression for the elastic scattering of spin-zero charged particles on spin- $\frac{1}{2}$ target (Be 61), using the single level form of the S-matrix given by equation 5.6 of Blatt and Biedenharn (Bl 52). The expression programmed is given below:

$$\frac{d\sigma(\theta)}{d\Omega} = |A|^2 + |B|^2 \quad (3.1)$$

where,

$$A = -\frac{\eta}{2k} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \exp[i\eta \log_e \operatorname{cosec}^2\left(\frac{\theta}{2}\right)]$$

$$+ \frac{i\sqrt{\pi}}{k} \sum_{\ell} (2\ell + 1)^{-\frac{1}{2}} [(2\ell + 1)e^{2i\alpha_{\ell}} + (\ell + 1)U_{\ell}^{+} + W_{\ell}^{-}] Y_{\ell}^0(\theta) ,$$

$$B = \frac{i\sqrt{\pi}}{k} \sum_{\ell} \left[\frac{\ell(\ell+1)}{2\ell+1}\right]^{\frac{1}{2}} [U_{\ell}^{+} - U_{\ell}^{-}] Y_{\ell}^1(\theta) ,$$

$$\text{and } U_{\ell}^{\pm} = e^{2i(\alpha_{\ell} + \phi_{\ell})} \left[1 + \sum_{J^{\pm}} \frac{\Gamma_{\ell}^{J^{\pm}}}{\Gamma_{J^{\pm}}} (1 - e^{2i\beta^{J^{\pm}}}) \right] \delta_{J, \ell + \frac{1}{2}}$$

The symbols used are defined in Appendix I.

Energy dependence of the level widths and level shifts are ignored.

In the single level approximation, it is expected that the hard sphere phases, ϕ_{ℓ} , may differ considerably from those calculated

from the expression $\phi_\ell = -\tan^{-1} F_\ell/G_\ell$, since these phase shifts are modified by contributions from other levels (La 58). Accordingly, the phases were entered into the program in a manner which enabled straightforward variation over the energy range, as

$$\phi_\ell = A_\ell + B_\ell \cdot E + C_\ell \cdot E^2, \quad (3.2)$$

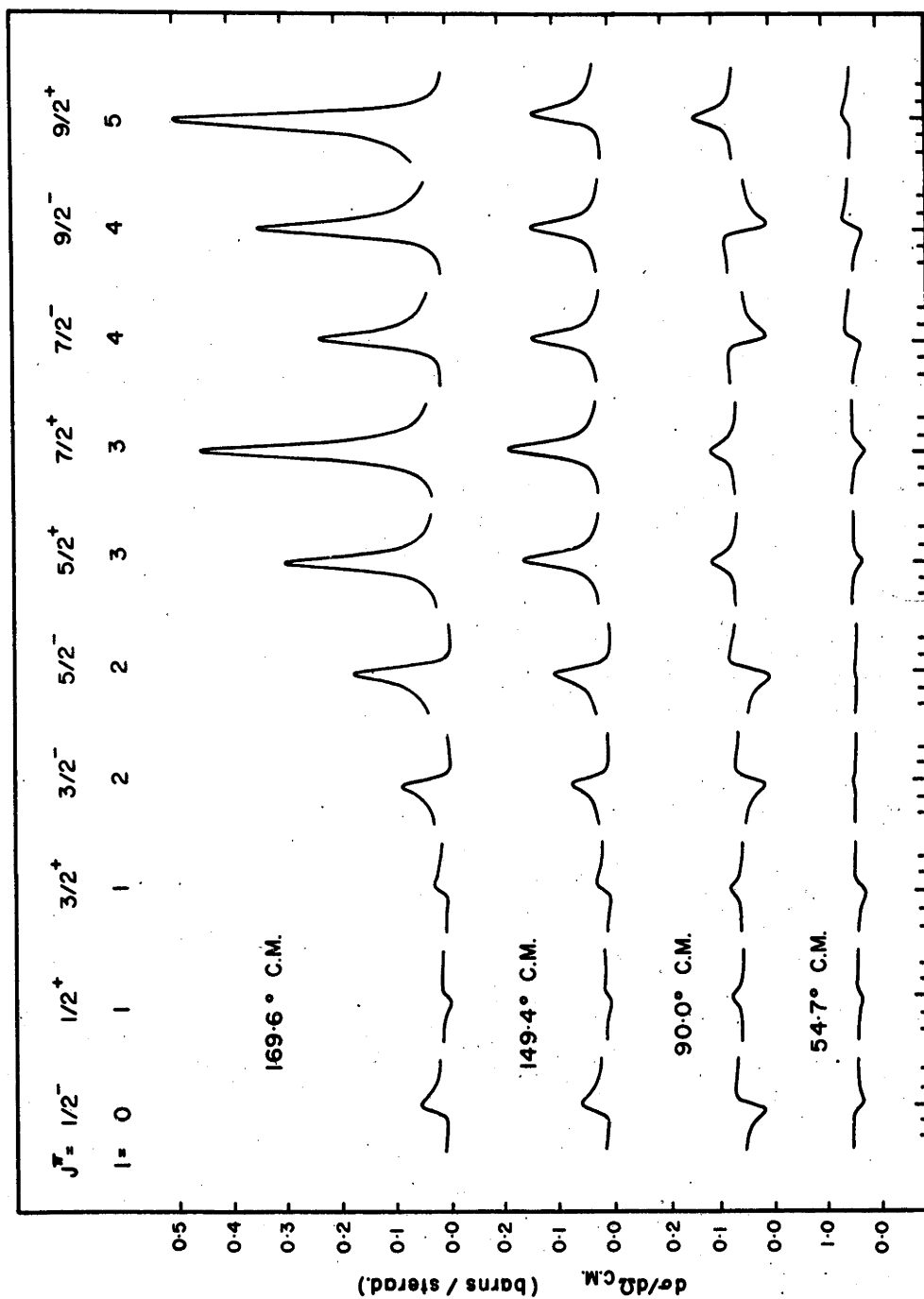
where initial values of the parameters A_ℓ , B_ℓ and C_ℓ were determined from the unmodified hard sphere behaviour with a channel radius of 5.7 fermi.

To reduce the number of J^π combinations tried in reproducing the cross section, a set of isolated resonance shapes for $\ell \leq 5$ was calculated (fig 3.5) from the cross section expression (equation (3.1)) to guide the choice of initial parameters. Inspection of the curves shows that the orbital angular momentum, ℓ , determines the shape of the anomaly at the various angles, thus establishing the parity and limiting the spin to $\ell \pm \frac{1}{2}$ (cf table 3.1). It is also evident from the figure that detailed examination of the peak yield ratios for a given anomaly at the various angles can differentiate between spin values for a given ℓ value. Extraction of spins in this way was found difficult in practice.

TABLE 3.1

$$\begin{array}{c}
 C^{13} + \alpha \xrightarrow{\ell, s = \frac{1}{2}} O^{17} \xrightarrow{\ell' (= \ell \pm 1), s = \frac{1}{2}} O^{16} + n \\
 \frac{1}{2}^- \quad 0^+ \qquad \qquad J = \ell \pm \frac{1}{2} \qquad \qquad 0^+ \quad \frac{1}{2}^+ \\
 \pi = (-)^{\ell+1} = (-)^{\ell'}
 \end{array}$$

Fig. 3.5 CALCULATED $C^{13}(\alpha, \alpha_0)C^{13}$ RESONANCE SHAPES FROM
ISOLATED STATES FORMED WITH $l \leq 5$.



3.3.2 $C^{13}(\alpha, n)O^{16}$

The expression used in calculating the reaction cross sections (Ka 60) was again based on single level dispersion theory, and was derived from equations (3.16), (4.5) and (4.6) of Blatt and Biedenharn (Bl 52):

$$\begin{aligned} \frac{d\sigma(\theta)}{d\Omega} = & \frac{1}{2k^2} \sum_{J_1 J_2 l_1 l_2 l_1' l_2'} Z(l_1 J_1 l_2 J_2; \frac{1}{2}v) Z(l_1' J_1 l_2' J_2; \frac{1}{2}v) P_Y(\cos \theta) \\ & \times \frac{\cos(\alpha_{l_1} + \varphi_{l_1} + \alpha_{l_1'} + \varphi_{l_1'} + \beta_1 - \alpha_{l_2} - \varphi_{l_2} - \alpha_{l_2'} - \varphi_{l_2'} - \beta_2)}{\{[(E_1 - E)/\frac{1}{2}\Gamma_1]^2 + 1\}^{\frac{1}{2}} \{[(E_2 - E)/\frac{1}{2}\Gamma_2]^2 + 1\}^{\frac{1}{2}}} \\ & \times \frac{(\Gamma_{l_1})^{\frac{1}{2}} (\Gamma_{l_1'})^{\frac{1}{2}} (\Gamma_{l_2})^{\frac{1}{2}} (\Gamma_{l_2'})^{\frac{1}{2}}}{\Gamma_1 \Gamma_2} . \end{aligned} \quad (3.3)$$

The two subscripts refer to interfering pairs of states of the compound nucleus, and the various symbols are defined in Appendix I .

It is to be noted that the relative signs of the square roots of the partial widths determine the signs of the interference terms in the above expression. For reasonably isolated resonances the interference terms are small and the effect of disregarding the signs is relatively small. However, in regions of strong interference the signs of the square root terms cannot be disregarded. Since there is no obvious theoretical method of determining the signs, the choice must be made from data fitting considerations.

3.3.3 ANGULAR DISTRIBUTIONS

As is evident from table 3.1, an isolated level of the compound nucleus characterised by angular momentum J and parity π can be formed by alpha particles with only one value of orbital angular momentum, l , and can decay by neutron emission with only one orbital angular momentum l' . From equation (3.3), the angular distribution will then be of the form

$$\frac{d\sigma(\theta)}{d\Omega} \sim \sum_v Z(lJlJ; \frac{1}{2}v) Z(l'Jl'J; \frac{1}{2}v) P_v(\cos \theta) . \quad (3.4)$$

Since the parities of incoming and outgoing channels are opposite, l' must be even if l is odd, and vice versa (table 3.1). Further, since Z has the same value for either of the two angular momentum possibilities ($J \pm \frac{1}{2}$), the angular distribution reduces to the simple form

$$\frac{d\sigma(\theta)}{d\Omega} \sim \sum_v Z(lJlJ; \frac{1}{2}v)^2 P_v(\cos \theta) \quad (3.5)$$

The distribution is the same for either parity, so that it is impossible to make a parity assignment at an isolated resonance on the basis of angular distributions. If more than one state of the compound nucleus is involved in the reaction at a given energy, the angular distribution will contain not only the sum of the angular distributions for the states separately, each multiplied by an appropriate weight factor from the Breit-Wigner energy dependence of the reaction cross section, but will also contain interference terms between pairs of states of the form

$$Z(l_1 J_1 l_2 J_2; \frac{1}{2}v) Z(l'_1 J_1 l'_2 J_2; \frac{1}{2}v) P_v(\cos \theta) .$$

Interfering states of different parity will produce odd Legendre polynomial terms, while states of the same parity will produce even terms. As a result, relative parities of neighbouring states can sometimes be determined from angular distributions.

Table 3.2 lists the coefficients of P_J for possible J values up to to $\frac{11}{2}$, including interference terms between pairs of J values.

It is to be noted that the signs of the partial width terms, as discussed above, will have marked effect on the angular distributions in regions of strong interference, reducing the value of the distributions in such regions.

3.4 EXPERIMENTAL RESULTS

3.4.1 $C^{13}(\alpha, \alpha_0)C^{13}$ MEASUREMENTS

The self-supporting carbon foils were bombarded with 0.1-0.5 μA of He_4^{++} over the energy range $3.5 \text{ MeV} \leq E_\alpha \leq 6.5 \text{ MeV}$ in steps of 10-20 KeV, and the scattered particles detected in two surface barrier detectors each subtending, by means of rectangular apertures, a solid angle of $\sim 10^{-3}$ sterad. at the target, at CM angles 54.7° , 90.0° , 149.4° and 169.6° . Fig 3.6 presents the excitation function data, together with fits as discussed in section 3.6.2. The energy scale corresponds to laboratory energy at the centre of the target.

During each of the excitation function measurements one counter was fixed at 169.6° to facilitate relative normalisation. The absolute cross section scale was determined by comparison with the $C^{12}(\alpha, \alpha_0)C^{12}$ cross section calculated from the phase shifts of Jones et al (Jo 62), and is estimated to be correct to within $\pm 10\%$ at the two backward angles where the C^{12} and C^{13} groups were essentially resolved. At the two forward angles the contribution from C^{12} had to be subtracted from the total yield, and the resultant absolute cross section scale is uncertain by $\sim 20\%$. However, since

TABLE 3.2
ANGULAR DISTRIBUTIONS FOR THE $C^{13}(O,n)O^{16}$ REACTIONS

Compound State Angular Momentum	Coefficients of P_V for the Expansion $\Sigma_V A_V P_V(\cos \theta)$				Compound State Angular Momentum	Coefficients of P_V for the Expansion $\Sigma_V A_V P_V(\cos \theta)$				
	A_0	A_2	A_4	A_6	A_8	A_3	A_1	A_5	A_7	A_9
Interference Between States of Same Parity										
$1/2^+$	2.00						$1/2 - 1/2$	-2.00		
$3/2^+$	4.00	4.00					$1/2 - 3/2$	4.00		
$5/2^+$	6.00	6.86	5.14			-6.00	$1/2 - 5/2$			
$7/2^+$	8.00	9.52	8.42	6.06		8.00	$1/2 - 7/2$			
$9/2^+$	10.00	12.12	11.33	9.70	6.85	-10.00	$1/2 - 9/2$			
$11/2^+$	12.00	14.69	14.10	12.83	10.08	12.00	$1/2 - 11/2$			
Interference Between States of Different Parity										
$1/2 - 3/2$		-4.00				-7.20	$3/2 - 3/2$	-0.80		
$1/2 - 5/2$		6.00				4.80	$3/2 - 5/2$	7.20		
$1/2 - 7/2$			-8.00			-2.67	$3/2 - 7/2$	-13.33		
$1/2 - 9/2$			10.00			13.33	$3/2 - 9/2$	6.67		
$1/2 - 11/2$				-12.00		-4.62	$3/2 - 11/2$	-19.39		
$3/2 - 5/2$		-1.71	-10.29			-3.20	$5/2 - 5/2$	-14.29		
$3/2 - 7/2$		10.29	5.71			8.00	$5/2 - 7/2$	5.71		
$3/2 - 9/2$			-3.64	-16.36		-1.82	$5/2 - 9/2$	-6.15	-22.03	
$3/2 - 11/2$			16.36	-7.64		18.18	$5/2 - 11/2$	10.77	7.05	
$5/2 - 7/2$		-1.14	-4.68	-18.18		-2.18	$7/2 - 7/2$	-6.59	-22.84	
$5/2 - 9/2$		14.29	9.35	6.36		10.91	$7/2 - 9/2$	9.23	6.53	
$5/2 - 11/2$		-2.52	-7.64	-25.85		-1.40	$7/2 - 11/2$	-4.31	-10.37	-31.93
$7/2 - 9/2$		-0.87	-3.24	-8.49	-27.41	-1.68	$9/2 - 9/2$	-4.62	-10.75	-32.65
$7/2 - 11/2$		1.82	12.59	10.18	7.05	13.71	$9/2 - 11/2$	12.31	10.37	7.26
$9/2 - 11/2$		-0.70	-2.52	-5.99	-12.99	-0.25	$11/2 - 11/2$	-3.62	-7.64	-15.55
					-37.81					-43.57

Fig. 3.6 EXPERIMENTAL $C^{13}(\alpha, \alpha_0)C^{13}$ CENTRE OF MASS DIFFERENTIAL CROSS-SECTIONS AT CENTRE OF MASS ANGLES 169.6° , 149.4° , 90° and 54.7° . THE CURVES ARE THE CALCULATED FITS WITH THE PARAMETERS LISTED IN TABLE 3.6. THE COEFFICIENTS OF THE QUADRATICS DESCRIBING THE 'HARD SPHERE' PHASES ϕ_1 (Ba 65) and ϕ_2 (PRESENT EXPERIMENT) ARE LISTED IN TABLE 3.5.

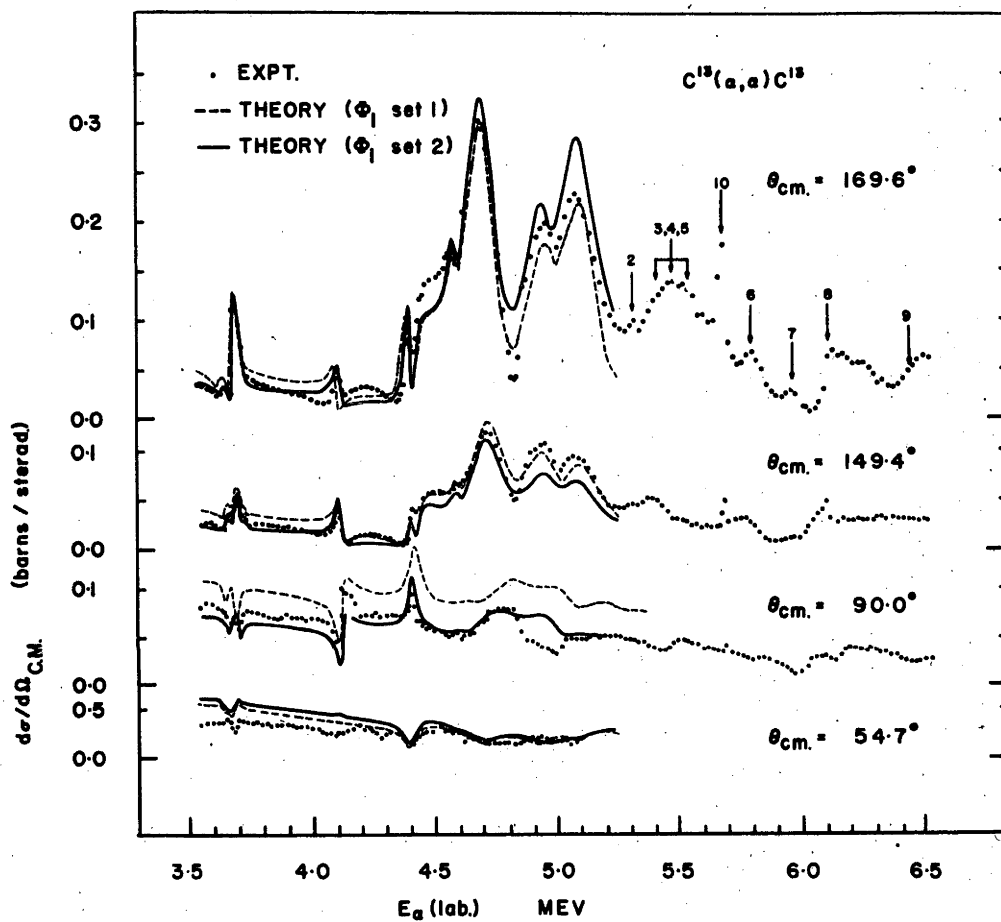
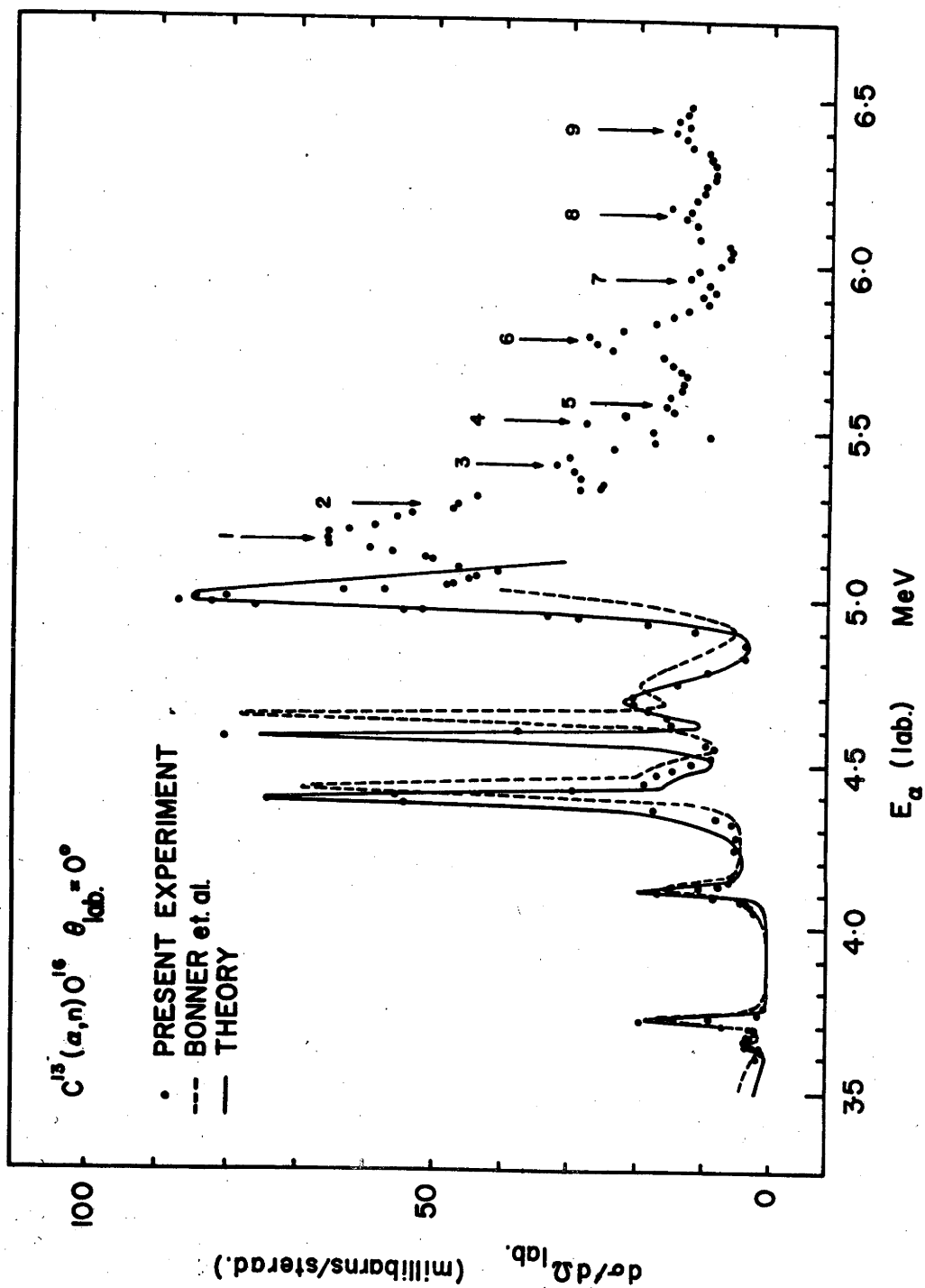


Fig. 3.7 EXPERIMENTAL $C^{18}(\alpha, n)O^{16}$ LABORATORY DIFFERENTIAL CROSS-SECTIONS AT 0° . THE SOLID CURVE IS THE CALCULATED FIT WITH THE PARAMETERS LISTED IN TABLE 3.6. THE DOTTED CURVE REPRESENTS THE DATA OF BONNER ET AL NORMALISED TO THE PRESENT CROSS SECTION SCALE.



the C^{12} yield varies slowly with energy over the region, the relative accuracy of the C^{13} cross section scale is estimated to be correct to within $\pm 5\%$ (except in the region 4.05 MeV to 4.25 MeV where the very large yield from the resonance corresponding to the 10.36 MeV, 4^+ state in O^{16} resulted in large uncertainties both in relative and absolute scales).

Charge integration was achieved as discussed in chapter 1, section 1.2.6, by use of an electrically and magnetically suppressed Faraday cup connected to a charge integrator known to be stable to $\sim 1\%$.

3.4.2 $C^{13}(\alpha, n)O^{16}$ MEASUREMENTS

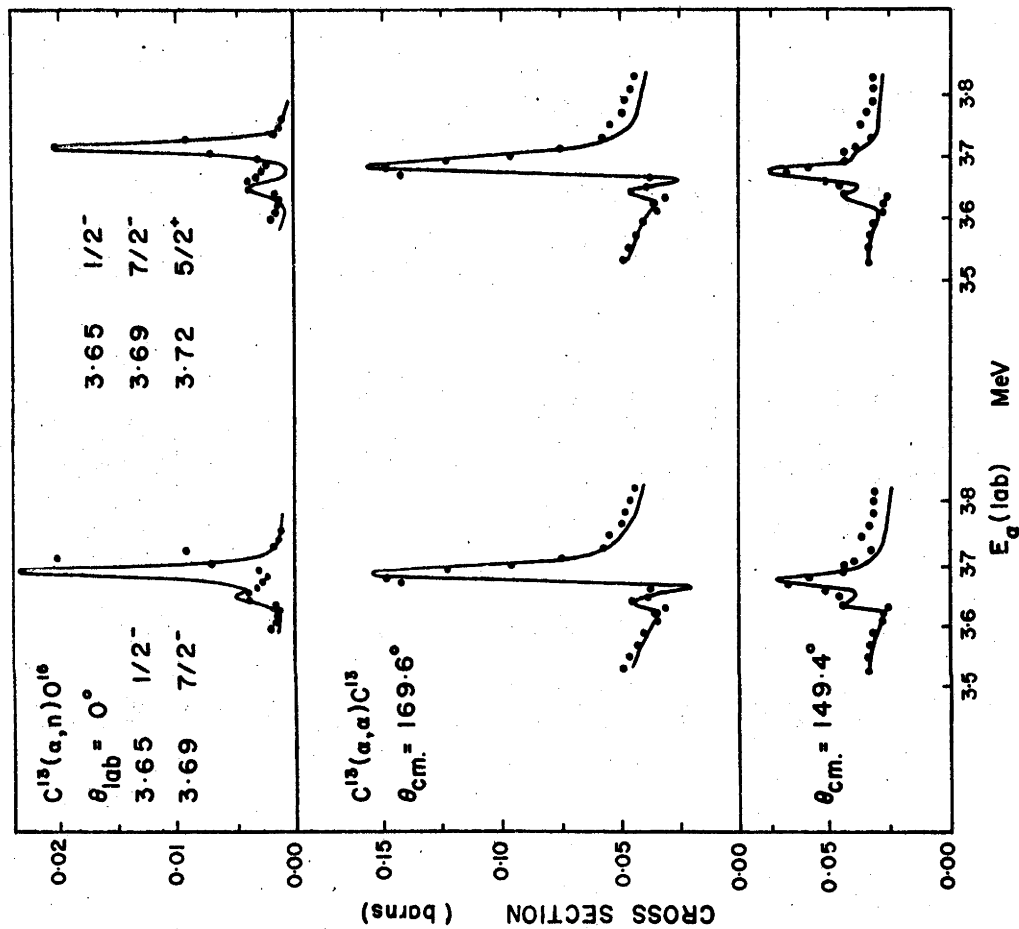
The O^0 excitation function was measured in 10-20 KeV steps over the energy range $3.5 \text{ MeV} \leq E_{\alpha} \leq 6.5 \text{ MeV}$, and the angular distributions obtained at intermediate energies in 5^0 - 10^0 steps over the angular range 0^0 - 160^0 lab.

Uncertainties in the relative accuracy of the measurements below 5 MeV are of the order of 5%. Above $\sim 5 \text{ MeV}$, in regions where the 6.14 MeV gamma ray background is large, uncertainties in the relative neutron yield are often as high as 20%.

(i) Excitation function:

The experimental laboratory differential cross section at O^0 is shown in fig 3.7. The energy scale corresponds to laboratory energy at the centre of the target. The solid curve is the calculated fit to the data, and the experimental data of Bonner et al (Bo 56), normalised to the present experimental cross section scale (cf section 3.4.4), are shown as a dotted curve. The present energy scale is believed accurate to within 10 KeV from calibration of the analysing magnet

Fig. 3.8 THE ELASTIC AND REACTION DATA BETWEEN $E_{\alpha} = 3.5$ MeV AND 3.8 MeV , COMPARED WITH CALCULATED CROSS-SECTIONS, ASSUMING RESONANCES AT 3.65 MeV AND 3.69 MeV , AND WITH A CALCULATION INCLUDING THE ADDITIONAL $5/2^{+}$ RESONANCE AT 3.72 MeV . THE 30 KeV ENERGY SHIFT BETWEEN THE PEAKS OF THE (α, α) and (α, n) DATA IN THE REGION OF 3.7 MeV IS CLEARLY VISIBLE.



with the thresholds:

$B^{11}(p,n)C^{11}$ (3.015 MeV, Ma 63.); $C^{13}(p,n)N^{13}$ (3.236 MeV, Ma 67);
and $Al^{27}(p,n)Si^{27}$ (5.800 MeV, Ma 67) .

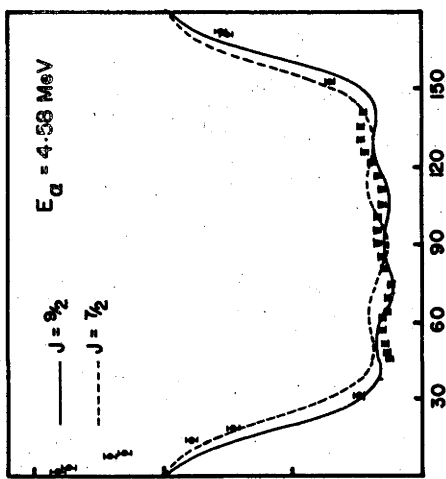
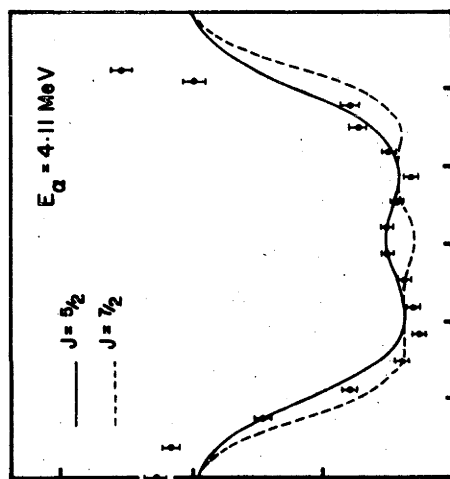
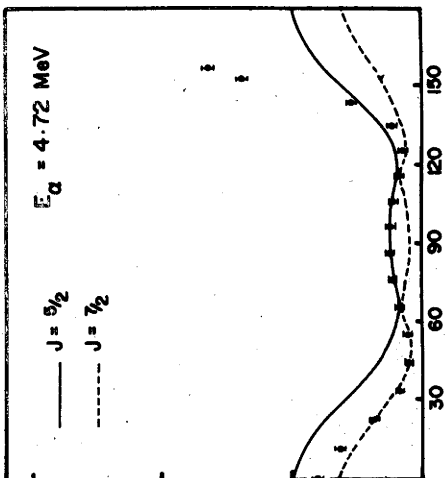
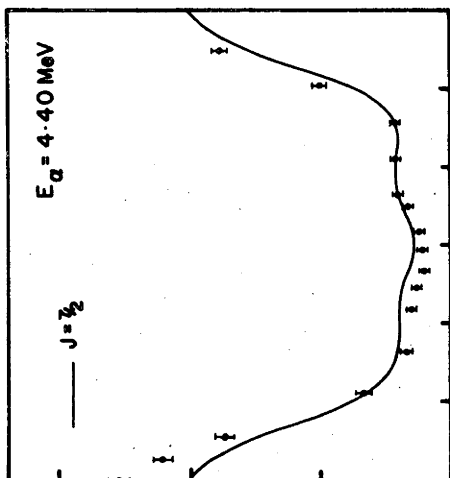
The energy scale of Bonner et al agrees with the present work up to about 4 MeV , but above this energy differences are observed, increasing to about 70 KeV at 4.8 MeV , which are probably due to saturation of the magnet employed in the earlier work.

Confirmation of the energy difference observed between the 3.69 MeV resonance in the $C^{13}(\alpha,\alpha_0)C^{13}$ excitation functions and the 3.72 MeV resonance in the $C^{13}(\alpha,n)O^{16}$ excitation function (cf fig 3.8) was established by simultaneous measurement of the two reaction yields over the region, using a self-supporting foil in the 51 cm scattering chamber. The energy difference between the resonances observed in the elastic and reaction data in the vicinity of 5 MeV (cf fig 3.6 and 3.7) was similarly checked and confirmed.

(ii) Angular distributions:

Although measurements over the angular range 0° - 172° lab. were mechanically possible, angular distributions were not measured beyond an angle of 160° lab. due to the possibility of neutron absorption and scattering in the beam pipe. The distributions were corrected for small errors in the nominal detector angle scales arising from displacement of the beam spot on the target relative to the central axis of the rotating table. The corrections were determined from left-right yield comparisons about the detector 'zero' angles. Each datum point on the angular distributions was normalised to the monitor yield as measured both by a BF_3 long counter and by a 7.5 cm x 7.5 cm plastic crystal, each of which was inclined at angle to the target-detector plane to avoid screening by the detectors.

Fig. 3.9 ANGULAR DISTRIBUTIONS OF NEUTRONS FROM THE
C¹³(α ,n)O¹⁶ REACTION AT SEVERAL RESONANCE
ENERGIES. THE CURVES INDICATE THE ANGULAR
DISTRIBUTIONS EXPECTED FROM THE INDICATED
SPINS OF THE COMPOUND SYSTEM WITHOUT
INTERFERENCE EFFECTS.



$\frac{d\sigma}{d\Omega}$ (arb. units)

$\theta_{c.m.}$ (DEGREES)

Legendre polynomial coefficients obtained from linear least squares fits to the angular distributions are presented in table 3.3.

The distributions were fitted according to the formula (cf section 3.3.3),

$$\sigma_{TH}(\theta) = a_0 \sum_{v=0}^{v_{MAX}} \frac{a_v}{a_0} P_v(\cos \theta) , \quad (3.6)$$

where v_{MAX} was effectively taken to be the lowest integer such that χ^2 , defined according to,

$$\chi^2 = \sum_{N=1}^{N_{DATA}} \left(\frac{\sigma_{EXP}(\theta_N) - \sigma_{TH}(\theta_N)}{ERROR(N)} \right)^2 / \begin{matrix} \text{degrees} \\ \text{of} \\ \text{freedom} \end{matrix} , \quad (3.7)$$

did not decrease significantly when v_{MAX} was increased by unity. Several of the angular distributions are shown in fig 3.9.

3.4.3 RESONANCE WIDTHS

Total widths for the narrow resonances were determined experimentally with a thin (1-2 KeV) backed target using the target modulation technique discussed in chapter 2. Fig 3.10 shows a composite of three spectra measured at different beam energies over the range $E_\alpha = 3.625$ to 3.745 MeV, with a 25 KV sweep voltage. [Note that although the sweep voltage was 25 KV, the energy span of the sweep was actually 50 KeV since the beam consisted of doubly charged alpha particles].

Symmetry of the 3.72 MeV resonance in fig 3.10 indicates that target thickness was not a significant factor in the width measurements. The widths and energies are presented in table 3.4.

Fig. 3.10 MEASUREMENT OF THE 3.65 MeV AND 3.72 MeV
RESONANCES OF $C^{13}(\alpha,n)O^{16}$ REACTION. THE
SPECTRUM IS A COMPOSITE OF THREE SPECTRA RECORDED
WITH INCIDENT BEAM ENERGIES OF 2.675 MeV , 3.715 MeV
AND 3.745 MeV AND WITH 25 KV MODULATION OF THE
TARGET.

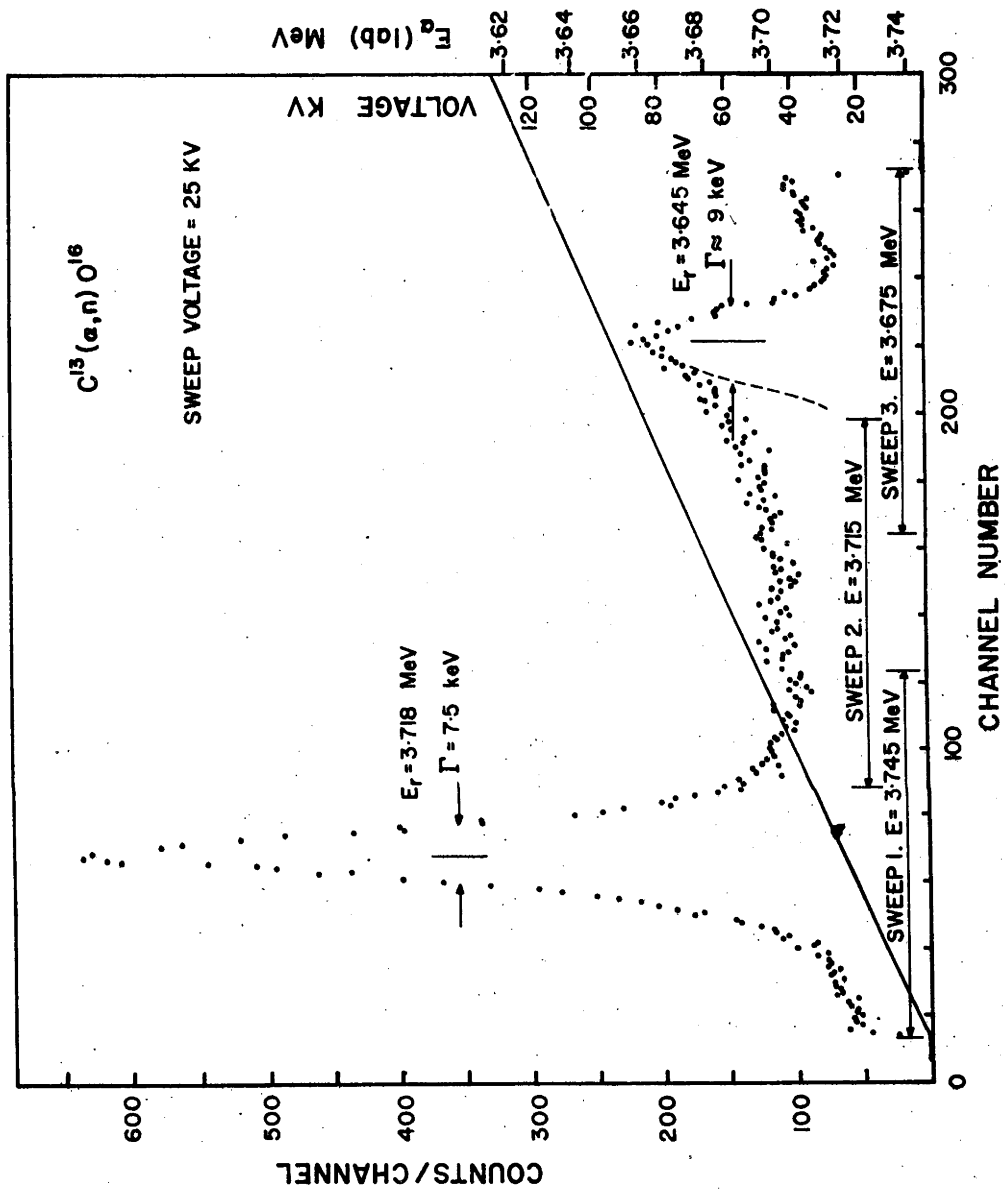


TABLE 3.4

THE ENERGIES AND WIDTHS OF NARROW RESONANCES

OBSERVED IN THE $C^{13}(\alpha, n)O^{16}$ REACTION

Using the target modulation technique with 25 kV sweep

Target thickness ~ 1 keV

E_{α} (MeV) (a)	Total Width Γ_{lab} (keV) Present Experiment	Total Width Γ_{lab} (keV) Bonner et al.
$3.645 \pm .005$	≤ 10 (b)	≤ 8
$3.718 \pm .005$	7.5 ± 1.0	≤ 5
$4.105 \pm .005$	19.5 ± 1.0	15
$4.387 \pm .005$	21.0 ± 1.0	25
$4.581 \pm .005$	15.5 ± 1.0	15
$4.981 \pm .005$	- (c)	65

(a) Energies listed are peak cross-section energies. Differences between these energies and energies E_0 listed in Table 3.6 are due to the fact that peaks in the differential cross-sections do not occur precisely at E_0 in cases of strong interference.

(b) Asymmetry in the observed resonance shape leads to uncertainty in width measurement.

(c) Only the resonance energy was determined, as the width is greater than the available sweep potential.

The results of the present measurements are in agreement with those of Bonner et al (Bo 56) except at the 3.72 MeV and 4.11 MeV resonances. No uncertainties are quoted for the widths given by Bonner et al, which were determined from the leading edges of the resonances observed with a 20 KeV thick target. The discrepancies are not easily explained since the present work yields widths narrower than or equal to the values of Bonner et al at two other resonances. Furthermore, remeasurement of the 3.72 MeV resonance using a thick target confirmed the results of the thin target measurement.

3.4.4 ABSOLUTE CROSS SECTION

Measurement of $C^{13}(\alpha,n)O^{16}$ absolute laboratory differential cross sections at 0° and at several energies was accomplished by comparison with the measured $D(d,n)He^3$ 0° differential cross section (Ma 63). The method involved bombarding deuterium and 55% C^{13} enriched carbon dioxide contained in a 1 cm x 2 cm gas cell with a thin (0.00002 inch thick) entrance window, with deuterons and alpha particles respectively. Appropriate deuteron energies were chosen to provide equivalent neutron energies to the (α,n) neutrons at the several comparison points. Gas temperature and pressure were monitored continuously, and mass spectroscopic analyses of both gases were obtained. The measurements were performed with the plastic scintillators discussed in section 3.2.3, and in the same geometrical configuration.

The absolute cross section scale obtained from the above measurements is in disagreement with the scale of Bonner et al (Bo 56) by a factor of ~ 4 , and is also a factor of ~ 2 higher than the scales of Becker and Barschall (Be 56) and of Barnes et al (Ba 65).

Bonner et al determined the cross-section by comparing the peak yield of the narrow 4.40 MeV resonance (4.42 MeV in their energy scale) with the yield from a calibrated Ra-Be neutron source.

The measurements were performed with a modified long counter 5 inches in length, which subtended a half angle of $\sim 25^\circ$ at the target. The number of C^{13} atoms per cm^2 was determined by weighing the target and assuming that the supplier's claimed ratio of C^{13}/C^{12} (2 : 1) was correct.

The absolute scale of Becker and Barschall was determined with a plastic scintillator calibrated from a long counter which was in turn calibrated with a Ra-Be source. They also determined the number of C^{13} atoms per cm^2 by weighing and by accepting the manufacturer's claimed 60% C^{13} content.

Barnes et al determined the cross section scale by use of a long counter calibrated with a standard Pu-Be neutron source. They obtained mass spectroscopic assays of the C^{13}/C^{12} ratio in the target material and determined the target thickness (self-supporting foils) from the measured $C^{12}(p,p_0)$ and $C^{13}(p,p_0)$ differential cross sections at a proton energy of 3 MeV and at a laboratory angle of 165° . The long counter subtended a half-angle of 14° at the target.

The present measurement is free of most of the sources of error inherent in the above measurements, but relies strongly on the measured $D(d,n)He^3$ absolute cross section scale. Probable errors associated with a smooth curve drawn through the various sets of available data in the energy region of interest are quoted by Brolley and Fowler (Ma 60) as $\pm 5\%$.

It is considered that the present cross section scale is more reliable than the values given from the previous experiments.

3.5 FITTING PROCEDURES

Because of the discrete nature of many of the parameters ($J, \pi, \pm \sqrt{\Gamma_{\alpha, n}}$) it was not possible to fit the data by means of the non-linear optimisation routine discussed in the previous chapters; instead, the parameters were varied manually. The problem was further complicated by the constraint that both reaction and scattering data had effectively to be fitted simultaneously.

The method used in fitting the resonance anomalies was essentially as follows:

The set of isolated resonance shapes (fig 3.5) was examined and compared with the behaviour of the various anomalies at the four angles of the elastic scattering excitation function data. In this way, the J^π associated with the anomalies could generally be narrowed down to two, or at most four possibilities. The total width, Γ , and resonance energy, E_0 , were deduced from the reaction data, the width being determined from the voltage modulation measurements for the narrower resonances, or along with the resonance energy from inspection of the 0° excitation function. In regions of strong interference, the peak yield of an anomaly in the excitation function, does not necessarily correspond to the resonance energy, so that the value of E_0 taken from the reaction data was treated simply as a first approximation to be improved upon in fitting the data. Fits to the 0° excitation function were obtained by manual adjustment of the level parameters associated with the various resonances, as indicated above. Using the relation $\Gamma_\alpha + \Gamma_n = \Gamma$ (section 3.3.2), the neutron and alpha particle partial widths were determined from the peak yields of the reaction cross section anomalies ($\sim \frac{\Gamma_\alpha \Gamma_n}{\Gamma^2}$, for isolated resonances). The parameters so obtained from fitting the reaction cross section data were used to calculate the elastic scattering cross sections at the appropriate angles. Optimisation of both sets of fits was achieved by further manual manipulation of the parameters.

Fitting the data in this way generally afforded unique determination of the J^π of the compound nucleus states. In some regions it was

necessary to accept minor compromises in both elastic and reaction fits to satisfy the ' $\Gamma_{\alpha} + \Gamma_n = \Gamma$ ' relation, but the final partial widths which gave satisfactory fits to both sets of data did not differ significantly from the values which gave the best fit to either set of data independently.

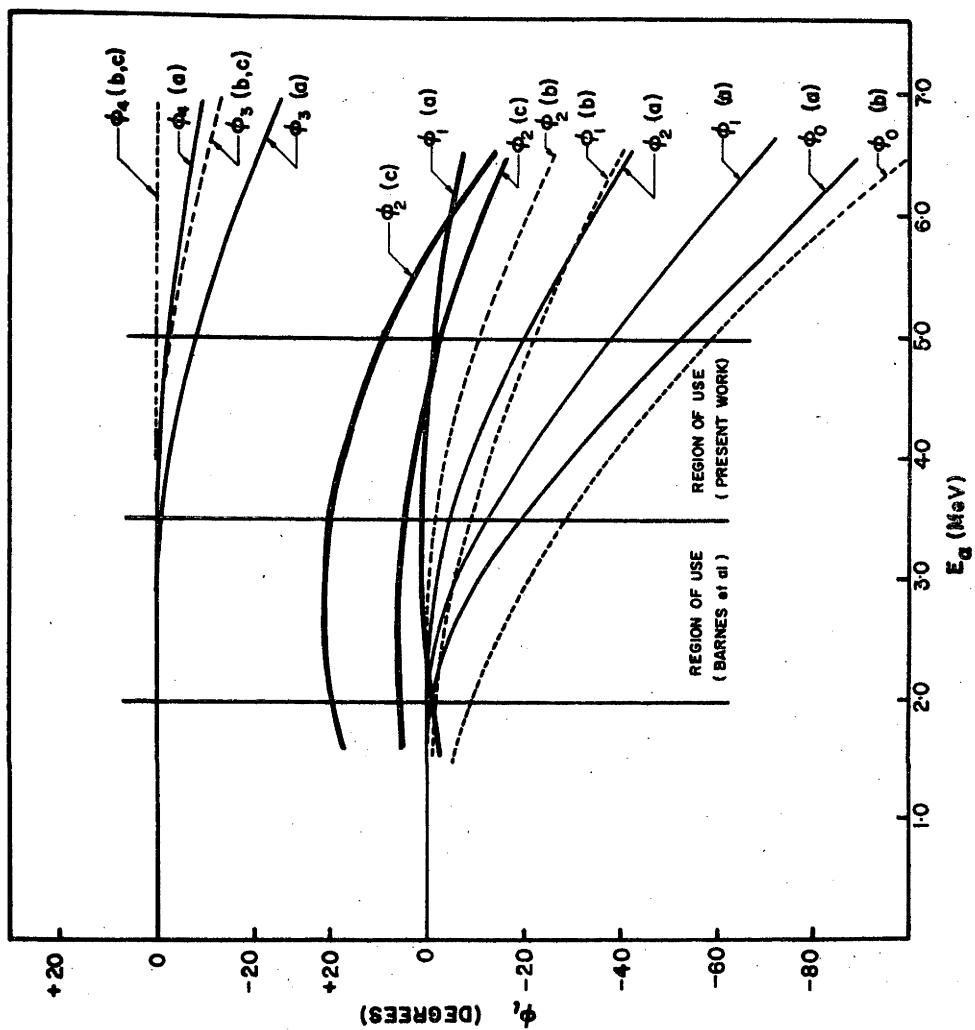
Since the targets, both self-supporting and backed, were thick compared to several of the resonances, the cross sections were calculated in small energy steps and averaged over target thickness. In fitting the neutron data, the calculated cross sections were also averaged over the solid angle subtended by the detector (half angle $\sim 4^{\circ}$)

Theoretical angular distributions and Legendre polynomial coefficients were calculated over the region of each resonance and compared with experiment as a check on the results and as a means of choosing between J^{π} assignments where the excitation function data were ambiguous. In the region above 5 MeV bombarding energy, where no detailed fitting was attempted, the angular distributions were used to infer possible spin assignments at some resonances.

As indicated previously, details of the reaction differential 0° cross sections, and angular distributions depended to some extent on the signs of the square roots of the partial widths. To determine the relative signs, a systematic, though not exhaustive investigation of the effect of sign changes was made in the latter stages of the analysis when the J^{π} of most resonances had been determined. The final relative signs are indicated in table 3.6.

During the initial fitting it was found that although the relative shapes of the elastic differential cross sections at the four angles could be reproduced reasonably well, the absolute magnitudes of the cross sections were in poor agreement with the data, particularly at 90° CM (cf dotted curves, fig 3.6). The problem was effectively independent of possible errors in J^{π} assignments, and appeared to be

Fig. 3.11 VALUES OF HARD SPHERE PHASES ϕ_ℓ USED IN THE ANALYSIS OF THE $C^{13}(\alpha, \alpha_0)C^{13}$ AND $C^{13}(\alpha, n)O^{16}$ DATA (heavy curves, set (c)), COMPARED WITH PHASES GIVEN BY BARNES et al., (Ba 65) (solid curves, set (a)), AND PHASES CALCULATED AS $\phi_\ell = \tan^{-1} F_\ell / G_\ell$ (broken curves, set (b)) .



due to a non-resonant 'nuclear background'. The hard sphere phases used at this stage were those of Barnes et al (Ba 65) extrapolated to higher energies according to the quadratic function discussed in section 3.3.1. [The table in reference Ba 65 is in error. The correct coefficients are as listed but with the quadratic and linear terms in ϕ_1 reversed and the quadratic term in ϕ_3 reduced by a factor 10 (Ri 66)] . Since attempts at varying the shapes and magnitudes of the phases by manual adjustment of the coefficients, resulted in some degree of success in improving the fit, further improvement was attempted using non-linear least squares techniques. The solid curves of fig 3.6 show the final fits obtained in this manner with the phases indicated in fig 3.11. The coefficients of the quadratic expressions are given in table 3.5.

Most of the improvement in the fit resulted from variation of ϕ_0 , and to some extent of ϕ_1 ; the higher l value phases had little effect.

Insertion of the phases, corresponding to best fits to the elastic data, into the reaction cross section expression did not affect the fit to the 0^0 excitation function appreciably. The 'hard sphere' phases of Barnes et al were used in the $0^{16} + n$ channel.

3.6 DISCUSSION

3.6.1 PARAMETERS

The values of the parameters used to calculate the elastic and reaction cross sections (figs 3.6 and 3.7) are listed in table 3.6 along with the excitation energies for the states of 0^{17} corresponding to the observed resonances, and the neutron and alpha-particle reduced widths expressed as a percentage of the Wigner limit $\theta^2 = \chi^2/2Ma^2$. Penetrabilities for the $C^{13} + \alpha$ and $0^{16} + n$ channels were calculated

TABLE 3.5

$$\text{COEFFICIENTS IN } \sigma_l = A_l E^2 + B_l E + C_l$$

USED IN CALCULATING THE ELASTIC CROSS-SECTIONS

E is in MeV. $\sigma_l = 0$ for $l > 3$

σ_l SET 1 (Broken Curve Fig. 1.5) From Barnes et al. (Ba 65)

l	A_l	B_l	C_l
0	-2.52	1.19	-2.08
1	-1.33	2.67	-1.93
2	-1.49	6.79	-7.67
3	-0.44	1.92	-2.09

σ_l SET 2 (Solid Curve Fig. 1.5)

l	A_l	B_l	C_l
0	-2.68	15.31	-0.49
1	-0.92	6.27	-9.81
2	-1.49	7.69	-3.89
3	-0.44	1.92	-2.09

TABLE 3.6

THE VALUES OF THE PARAMETERS USED IN THE FITS

TO THE ELASTIC AND REACTION DATA

$$(\Gamma_{\alpha} + \Gamma_n = \Gamma)$$

E_o (MeV)	Γ_{lab} (keV)	Γ_{α}/Γ	J^{π}	$E_x O^{17}$ (MeV)	% of Wigner Limit	
					θ_{α}^2	θ_n^2
3.64	8	0.45 *	$1/2^-$	9.14	0.18	0.03
3.69	4	≥ 0.98	$7/2^-$	9.18	5.08	0.00
3.72	7.5	0.20 *	$5/2^+$	9.20	0.39	0.06
4.11	19.5	0.85	$5/2^-$	9.50	1.24	0.07
4.40	21	0.70 *	$7/2^+$	9.72	1.79	0.60
4.42	80	0.90 *	$3/2^+$	9.74	2.81	0.08
4.58	15.5	0.18	$9/2^+$	9.86	5.16	1.22
4.70	140	0.78	$7/2^+$	9.95	10.75	2.72
4.94	180	0.85	$5/2^+$	10.14	12.55	0.25
4.98	60	0.15	$7/2^-$	10.17	2.03	0.97
5.08	160	0.60	$7/2^+$	10.24	7.15	4.83

* -ve sign used for $\sqrt{\Gamma_n}$

on the basis of channel radii 5.7 and 5.1 fermi, respectively.

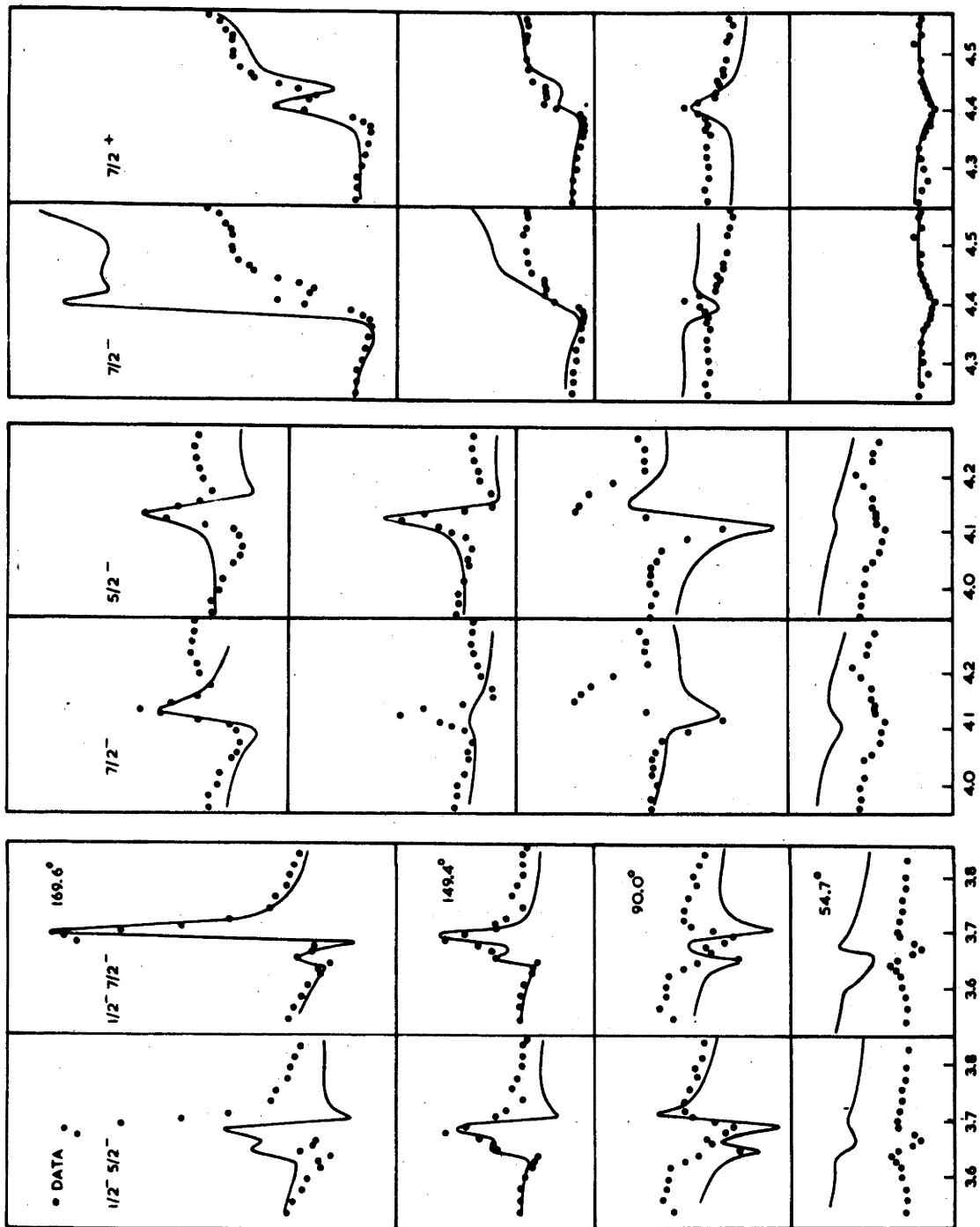
In general, the reduced widths are small compared to the Wigner limit, implying that the levels are not described in terms of single particle states at this excitation. However, in the region above 4.7 MeV bombarding energy, several of the levels have alpha-particle reduced widths smaller by less than one order of magnitude than the Wigner limit. This gives some indication of an ' α -particle — C^{13} core' configuration (cf section 3.6.5).

3.6.2 J^π ASSIGNMENTS

Both the elastic and reaction cross sections show two narrow resonances at 3.64 MeV and about 3.7 MeV. Comparison of the shape of the lower energy resonance in the elastic cross section with the calculated line shapes (fig 3.5) suggests a $1/2^-$ assignment which is consistent with the magnitude of the reaction cross section. The neutron angular distribution of Schiffer et al (Sc 57) suggests $J = 5/2$ for the 3.72 MeV resonance. Fits to the elastic cross section assuming $1/2^-$, $5/2^-$ and $1/2^-$, $7/2^-$ combinations for these resonances are shown in fig 3.12. A $5/2^-$ assignment for the 3.72 MeV resonance is seen to be in conflict with the experimental data while $7/2^-$ reproduces the data well, apart from a slight energy discrepancy. A careful check of the energies of the resonances observed in the elastic and reaction cross sections (as noted in section 3.4.2 and shown in fig 3.8) leads to the conclusion that three resonances contribute in this region. Fig 3.8 shows the elastic and reaction cross section fits with $1/2^-$ at 3.64 MeV, $7/2^-$ with small neutron width at 3.69 MeV and $5/2^+$ at 3.72 MeV. The fit is in good agreement with the data and is consistent with the $5/2$ assignment of Schiffer et al.

The angular distribution at the 3.64 MeV resonance shows strong interference, but is consistent with these assignments. In particular,

Fig. 3.12 THE ELASTIC DATA BETWEEN 3.5 AND 4.5 MeV
COMPARED WITH FITS FROM SEVERAL ASSIGNMENTS
AT THE 3.69 MeV , 4.11 MeV AND 4.40 MeV
RESONANCES.



ALPHA ENERGY — MEV

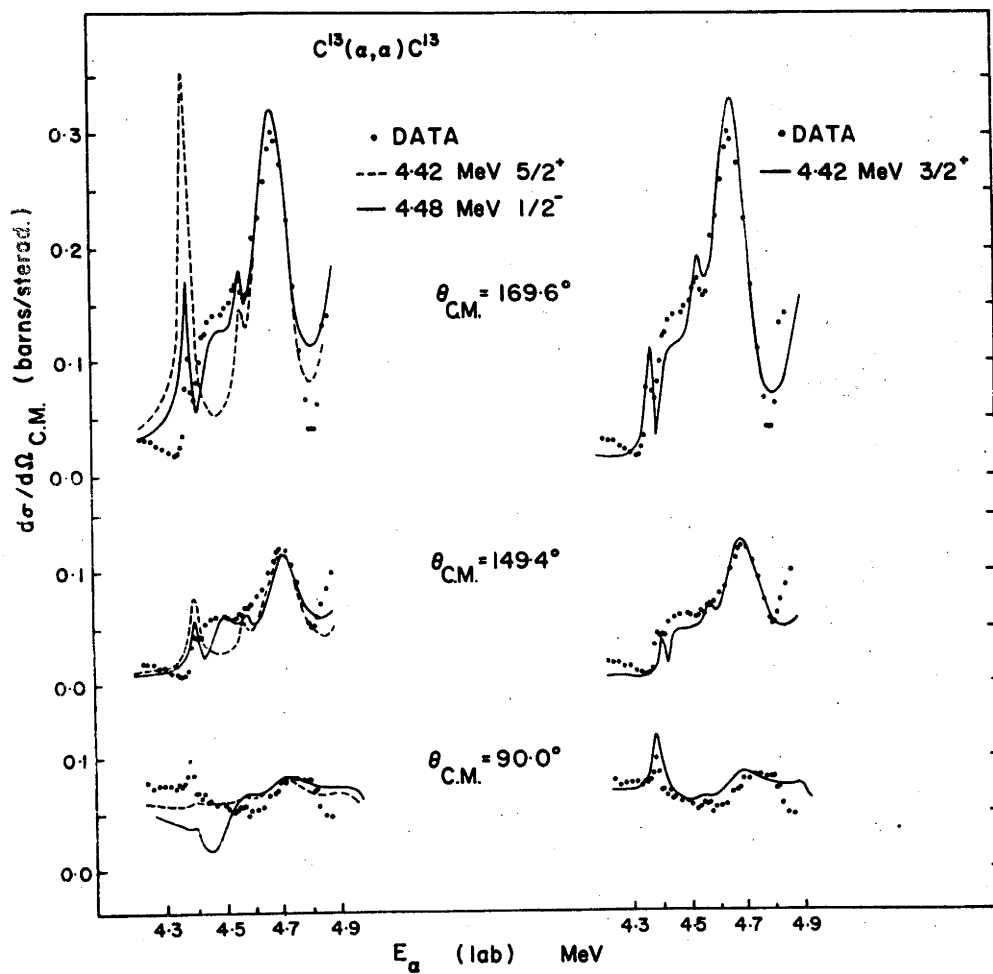
the presence of a P_6 term in the absence of a P_4 term suggests interference between $J = 1/2$ and $J = 7/2$ with $J = 3/2$ of opposite parity. A $3/2^+$ state of sufficient width to account for this interference is observed at ~ 3.3 MeV (Sc 57).

From analysis of the $C^{13}(\alpha, n)O^{16}$ angular distribution at 4.12 MeV, Schiffer et al suggest a probable spin of $7/2$ for the state at 9.50 MeV in O^{17} . A Legendre polynomial fit to the neutron angular distribution measured at 4.11 MeV in the present experiment gives coefficients differing from those of Schiffer et al. In particular, larger P_2 and P_4 coefficients were found with a smaller P_6 coefficient. Comparison with the theoretical distribution for $5/2$ and $7/2$ states (fig 3.9) indicates a probable spin of $5/2$ for this state. The dip observed in the 90° elastic cross section would require either a $5/2^-$ or $7/2^-$ assignment. Fig 3.12 shows the fits to the elastic data for these possibilities; clearly the $5/2^-$ assignment reproduces more closely.

The broad hump in the elastic data at 169.6° and 149.4° between 4.2 and 4.4 MeV was fitted well with a broad $J = 3/2$ resonance, which could explain the larger A_2 and A_4 . However as the elastic data in this region are uncertain due to the large resonance in the $C^{12}(\alpha, \alpha)C^{12}$ cross section, no such resonance was included in the fit. Further evidence for the $5/2$ assignment with a broad underlying $J = 3/2$ resonance is given by the neutron angular distribution at 4.18 MeV which contains predominantly P_0 , P_2 and P_4 terms and is well explained by a $J = 3/2$ state interfering strongly with a $J = 5/2$ state of like parity.

The neutron angular distribution taken on the 4.40 MeV resonance (fig 3.9) shows that $J = 7/2$ for the state in O^{17} corresponding to this resonance, in agreement with the assignment of Schiffer et al. Comparison of the elastic data with the calculated

Fig. 3.13 COMPARISON OF THE FITS FROM THE ASSIGNMENTS
5/2⁺ , 1/2⁻ AND 3/2⁺ AT THE 4.42 MeV
RESONANCE WITH THE $C^{13}(\alpha, \alpha_0)C^{13}$ CROSS-SECTIONS.
IN ALL CASES THE ASSIGNMENTS AT THE 4.40 MeV ,
4.58 MeV AND 4.72 MeV RESONANCES ARE AS LISTED
IN TABLE 3.6.



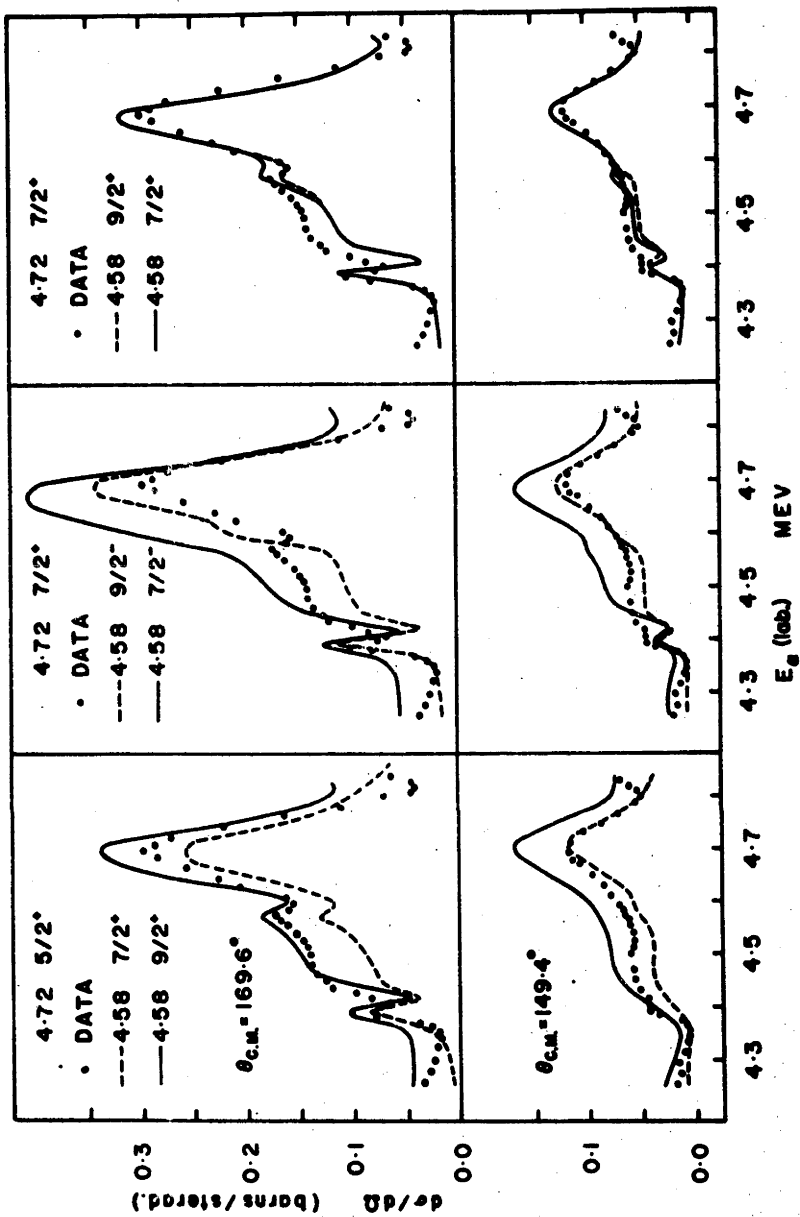
single level shapes (fig 3.5) shows that the state is formed with $l = 3$, suggesting a $J = 7/2$ state of positive parity. Fig 3.12 shows the fit to the elastic data for both $7/2^+$ and $7/2^-$ assignments. The $7/2^+$ assignment clearly reproduces the data more correctly.

The broad resonance observed as a shoulder on the high energy side of the 4.40 MeV resonance in both the elastic and reaction cross sections is best fitted with a $3/2^+$ state at 4.42 MeV. The ratio of the elastic cross sections at 169.6° and 149.4° indicates a spin of $1/2$, $3/2$ or $5/2$. The 90° elastic cross section shows no dip at this energy eliminating $1/2^-$, $3/2^-$ and $5/2^-$ assignments. The assignment $1/2^+$ results in a smaller calculated $C^{13}(\alpha, n)O^{16}$ cross section on the resonance than is observed, even with $\Gamma_\alpha = \Gamma_n = \Gamma/2$. Fig 3.13 shows the $C^{13}(\alpha, \alpha)C^{13}$ cross sections between 4.3 and 4.9 MeV, with several alternative assignments for the 4.42 MeV resonance. While agreement with the absolute elastic cross section is not particularly good, the $3/2^+$ assignment is clearly in closest agreement with the data. The large P_2 and P_4 coefficients observed in the neutron angular distribution at 4.45 MeV are in agreement with the assumption of a $J = 3/2$ state with strong interference from a $J = 7/2$ state of like parity.

The neutron angular distribution at the 4.58 MeV resonance indicates a probable spin of $9/2$, in agreement with the suggested assignment of Schiffer et al. Fits to the elastic and reaction cross sections with either a $9/2^+$ or $7/2^+$ assignment reproduce the data equally well (fig 3.14). However, the $9/2^+$ assignment is preferred on the basis of the experimental neutron angular distribution.

The angular distribution on the 4.72 MeV resonance, and the distribution measured at 4.75 MeV by Schiffer et al, are characterised by strong interference from states of like parity. Although

Fig. 3.14 COMPARISON OF THE FITS TO THE ELASTIC CROSS-
SECTIONS WITH $5/2^+$ AND $7/2^+$ ASSIGNMENTS AT
THE 4.72 MeV RESONANCES; IN BOTH CASES, ALL
OTHER ASSIGNMENTS ARE AS IN TABLE 3.6.

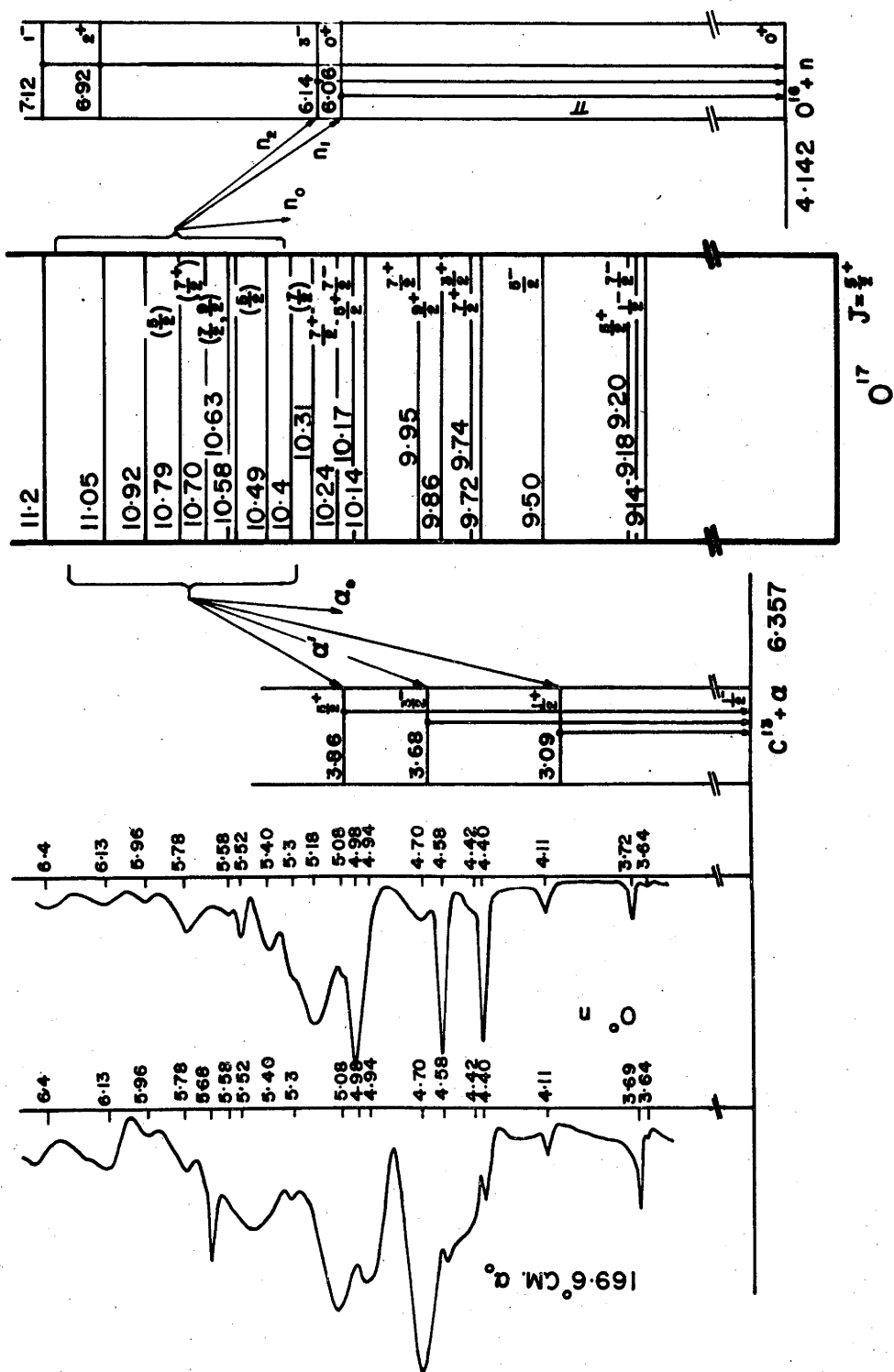


apparently predominantly due to a $J = 5/2$ state (fig 3.9) a $J = 7/2$ assignment would not be inconsistent with the data. Comparison with the calculated single level shapes (fig 3.5) for the elastic cross section at 90° indicates that the state is formed with $l = 3$. For a $5/2^+$ assignment the ratio of the elastic cross section at 169.6° to the cross section at 149.4° is too small (fig 3.14) while a $7/2^+$ assignment reproduces the data well.

Neutron angular distributions measured in the region 4.8 - 5.2 MeV all show evidence of very strong interference so that unique spin assignments cannot be made. However, an almost equal contribution from a $J = 5/2$ and a $J = 7/2$ state of opposite parity would explain the distribution measured on the large resonance at 4.98 MeV in the $0^\circ \text{ } ^{13}\text{C}(\alpha, n)^{16}\text{O}$ cross section. The elastic cross section in this region shows two broad resonances at about 4.94 MeV and 5.08 MeV. Comparison of the elastic and reaction data (cf section 3.4.2) leads to the conclusion that there are at least three resonances contributing to the cross section in this region, the two resonances at 4.94 MeV and 5.08 MeV having large alpha particle widths and the resonance at 4.98 MeV having a small alpha particle width. A number of resonance combinations were tried in this region and only the $5/2^+$, $7/2^-$, $7/2^+$ combination reproduces the data satisfactorily. The data indicate a number of additional broad resonances at higher energies, which have not been included in the calculation of the cross sections, and which most probably account for the discrepancies in the fits close to 5 MeV.

The dip in the 90° elastic cross section between 4.8 and 5 MeV can be reproduced by the inclusion of a broad underlying $1/2^-$ resonance at about 5 MeV; however, insufficient evidence is available to justify this assignment.

Fig. 3.15 ENERGY LEVEL DIAGRAM SHOWING THE STATES IN O^{17} OBSERVED IN THE $C^{13}(\alpha, \alpha_0)C^{13}$ ELASTIC SCATTERING AND THE $C^{13}(\alpha, n)O^{16}$ REACTIONS. THE O^0 NEUTRON YIELD AND 169.6^0 ELASTIC SCATTERING CROSS-SECTIONS ARE SHOWN TO AID IN IDENTIFYING THE RESONANCES REFERRED TO IN THE TEXT.



3.6.3 RESONANCES ABOVE 5.1 MeV

The resonances observed in this region (indicated by arrows on fig 3.6 and fig 3.7) are listed in table 3.7 together with the excitation energies, E_x , of the corresponding states of O^{17} . Possible spin assignments for some of the resonances, inferred from the neutron angular distributions, are shown. However, in all cases the distributions are characterised by strong interference from nearby resonances and assignments should be regarded as tentative. Previously reported states of O^{17} in the present experiment are also listed in table 3.7.

The single narrow resonance observed at 5.68 MeV in the $C^{13}(\alpha, \alpha_0)C^{13}$ cross section may be tentatively assigned spin $7/2^+$ by comparison of the observed resonance shape with the calculated shapes of fig 3.5. The relative magnitudes of the 90° , 149.4° and 169.6° cross sections favour $7/2^+$ over possible $5/2^+$ or $9/2^+$ assignments.

Fig 3.15 presents a level diagram of the O^{17} compound nucleus, with spins and parities of the states deduced in the present experiment.

3.6.4 'HARD SPHERE' PHASES

As noted in section 3.3.1, the s- and p- wave 'hard sphere' phases required to fit the non-resonant parts of the elastic excitation functions differed considerably from the values as calculated from single-level R-matrix theory. Such behaviour could indicate the existence of broad s- and p- wave states in O^{17} , formed by loose coupling of α -particles to a C^{13} core, near the region of excitation studied in the present experiment. However, it should be pointed out that the channel radius was not treated as a free parameter of the analysis. Variation

TABLE 3.7

SUMMARY OF RESONANCES OBSERVED VIA THE $C^{13}(\alpha, \alpha')C^{13}$ AND $C^{13}(\alpha, n)O^{16}$ REACTIONS

PRESENT EXPERIMENT					PREVIOUS WORK			
Resonance Number (a)	E_{α} (MeV)	$E_{EX} O^{17}$ (MeV)	$\Gamma_{c.m.}$ (keV)	J^{π}	$E_{EX} O^{17}$ (MeV)	$\Gamma_{c.m.}$ (keV)	J	Reference
(i) Resonances Observed via both $C^{13}(\alpha, n)$ and $C^{13}(\alpha, \alpha')$								
	3.64	9.14	6	$1/2^{-}$	9.14 9.15	4 ± 3 ≤ 6		Se 67 Bo 56
	3.72	9.20	5.5	$5/2^{+}$	9.18 9.19 9.20	≤ 17 4 ± 3 ≤ 40	$\geq 3/2$ 5/2	Fo 61 Se 67 Bo 56, Sc 57
	4.11	9.50	15	$5/2^{-}$	9.45 9.50 9.50	140 11 8 ± 3	$\geq 3/2$ 7/2	Fo 61 Bo 56, Sc 57 Se 67
	4.40	9.72	16	$7/2^{+}$	9.70 9.73 9.73	28 19 15 ± 3	$\geq 5/2$ 7/2	Fo 61 Bo 56, Sc 57 Se 67
	4.42	9.74	61	$3/2^{+}$	9.78 (9.78) 9.79	55 28	 $\geq 3/2$	Bo 56 Se 67 Fo 61
	4.58	9.86	12	$9/2^{+}$	9.87 9.88 9.89	25 25 ± 3 12	$\geq 1/2$ 9/2	Fo 61 Se 67 Bo 56, Sc 57
	4.70	9.95	107	$7/2^{+}$	9.98 (9.99) (10.05)	150 	(5/2, 7/2)	Bo 56, Sc 57 Se 67 Se 67
	4.94	10.14	138	$5/2^{+}$				
	4.98	10.17	46	$7/2^{-}$	10.16 10.20 10.21	38 50 50 ± 3	$\geq 3/2$	Fo 61 Bo 56 Se 67
	5.08	10.24	122	$7/2^{+}$				
1	5.18	10.23		(7/2)(b)	10.39			Se 67
2	5.3	10.4	≤ 20		10.423	14 ± 3		Sp 63
3	5.40	10.49	75 ± 30	(5/2)(b)	10.49			Se 67
4	5.52	10.58	45 ± 20	(7/2, 9/2)(b)	10.54 10.560 10.57	40 50 ± 20 	$\geq 3/2$	Fo 61 Sp 63 Ha 59, Wo 60
5	(5.58)	(10.63)						
6	5.78	10.79	75 ± 30	(5/2)(b)	10.770 10.80	85 ± 30		Sp 63 Ha 59, Wo 60
7	5.96	10.92	60 ± 20	$\geq 3/2(b)$	10.903 10.91 10.95	55 ± 20 70	$\geq 3/2$	Wo 60 Fo 61 Ha 59
8	6.13	11.05			11.02 11.027 11.11	45 ± 10		Wo 60 Sp 63 Ha 59
9	(~6.4)	(~11.2)			11.226 11.28	100 ± 30		Sp 63 Wo 60
(ii) Resonances Observed via $C^{13}(\alpha, \alpha')$ only								
	3.69	9.18	3	$7/2^{-}$				
10	5.68	10.70	≤ 25	(7/2 ⁺)(c)				

(a) Resonances above 5 MeV, not included in the detailed analysis, are numbered as shown on Figs. 1.5 and 1.6.

(b) Tentative assignments from $C^{13}(\alpha, n)O^{16}$ angular distribution data.

(c) J^{π} inferred from comparison of elastic yield with calculated level shapes.

of the channel radius will certainly alter the hard sphere phase shifts over the region, and it is possible that a channel radius could be found which would have the desired effect. A more comprehensive analysis than has been attempted here, would be required before any definite conclusions could be drawn.

3.6.5 PREVIOUS WORK

Most of the previous work on O^{17} has been discussed in section 3.1. Comparisons with some of this work up to 5 MeV bombarding energy have been effected in section 3.6.2 and, apart from the energy scale discrepancy already discussed (section 3.4.2), the present work is generally in good agreement with other measurements. Table 3.7 presents a summary of the results obtained in this experiment, together with previously obtained results. Difficulty was experienced in correlating some of the levels measured in the different reactions, especially at higher energies, and this is attributed to differences in the partial widths of the α , n and n' channels. e.g., the 3.69, 4.94, 5.08 and 5.68 MeV resonances observed in the $C^{13}(\alpha, \alpha_0)C^{13}$ reaction have not been observed in any previous experiments, all of which have been restricted to $O^{16} + n$ in either the entrance or exit channel at these excitations.

Some disagreement exists between the present work and results of Sekharn et al (Se 67) from total cross section measurements of the $C^{13}(\alpha, n)O^{16}$ reaction. In particular, the widths of the levels at 9.20 and 9.86 MeV excitation, as measured by Sekharn et al, are a factor of ~ 2 smaller than the present values (errors are quoted as $\sim 60\%$) with the corresponding alpha-particle, partial widths smaller by a factor of ~ 4 .

The total cross sections of the resonances in the $O^{16}(n,\alpha)C^{13}$ reaction, calculated from the reciprocity theorem, differ from the measured values of Davis et al (Da 63) by more than a factor of 2 .

The results of Sekharn et al are also in disagreement with those of Barnes et al (Ba 65) by similar factors for the 8.50 and 8.39 MeV states. 8.39 MeV . The total widths were determined from the leading edges of the thick target yields and the partial widths calculated from the total cross sections at the resonance peaks. The measurements were performed using a 4π , paraffin moderated $B^{10}F_3$ detector (Ma 60a) calibrated to 2.5 MeV neutron energy from the $Li^7(p,n)Be^7$ reaction, and at ~ 5 MeV using a standard Ra-Be neutron source.

A further discrepancy between the present measurements and those of Sekharn et al is noted for bombarding energies in the region of 4.8 MeV , where Sekharn et al propose two resonances on the basis of a small dip in the measured total cross section. The present measurements show no such dip, and only one resonance has been assumed. It is felt that the large resonance in the C^{12} total neutron cross section at the forward-angle neutron energies from the reaction (~ 6.3 MeV) may account for the discrepancy, as marked decreases ($\sim 10\%$) in long counter efficiencies in this region have been reported (Al 60). (The yields from the plastic scintillators used in the present measurements were corrected for variations in the C^{12} total neutron cross section - cf section 3.2.3).

In view of the obvious shortcomings of theoretical models of O^{17} at lower excitations, which limit the value of predictions in the present excitation region, no serious attempt has been made to account for individual states in the present measurements, on a theoretical basis. However, it is interesting to note that the large grouping of resonances in the region 4.5 - 6 MeV bombarding energy in the $C^{13}(\alpha,\alpha_0)C^{13}$ reaction (fig 3.6) lies just within the range predicted by Wildermuth

Carovillano for optical model giant resonances in O^{17} (cf section 3.1). Although the present work is not extensive enough to show systematics of the predicted nature, the region may be indicative of one such giant resonance state (cf section 3.6.1).

APPENDIX I

The symbols defined below are appropriate to equations (3.1) and (3.3) of chapter 3.

$k = \left(\frac{2ME}{\hbar^2}\right)^{\frac{1}{2}}$, is the wave number. (M and E are the reduced mass and energy of relative motion, respectively of the particles).

$\ell(\ell')$ is the orbital angular momentum of the incident (outgoing) particle. (Subscripts 1 and 2 refer to the angular momenta associated with states of spin J_1 or J_2).

$J^{\pm}$ represents the spin of states formed with orbital angular momenta $\ell = J + \frac{1}{2}$ (J_1 , say), or $\ell = J - \frac{1}{2}$ (J_2 , say).

U_{ℓ}^{+} and U_{ℓ}^{-} represent U_{ℓ}^J for collisions with total angular momentum $J = \ell + \frac{1}{2}$ and $J = \ell - \frac{1}{2}$, respectively.

$\phi_{\ell} = -\tan^{-1} \frac{F_{\ell}}{G_{\ell}}$, is the hard sphere phase shift.

$F_{\ell}(G_{\ell})$ is the regular (irregular) Coulomb function.

$\alpha_{\ell} = \sum_{s=1}^{\ell} \tan^{-1} \left(\frac{\eta}{s}\right)$, is the Coulomb phase shift ($\alpha_0 = 0$).

$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$, is the Coulomb field parameter for particles with charge Z_1 and Z_2 , and relative velocity v .

$\beta = \tan^{-1} \left(\frac{\Gamma/2}{E_0 - E}\right)$, is the resonant phase shift.

$\Gamma_{\ell}(\ell')$ is the partial width for the incident (outgoing) particle.

Γ is the total width of the state.

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