Erratum: Spatial sound intensity vectors in spherical harmonic domain [J. Acoust. Soc. Am. 145(2), EL149-EL155 (2019)]

Huanyu Zuo,[a)] **Prasanga N. Samarasinghe, and Thushara D. Abhayapala**

Research School of Engineering, College of Engineering and Computer Science, The Australian National University, Canberra, ACT2601, Australia

huanyu.zuo@anu.edu.au, prasanga.samarasinghe@anu.edu.au, thushara.abhayapala@anu.edu.au

Glenn Dickins

Dolby Laboratories, Sydney, NSW2060, Australia

glenn.dickins@dolby.com

[a)] Author to whom correspondence should be addressed.

This erratum concerns the expression of acoustic particle velocity (Eq.(4)) in the original paper. We have mistakenly omitted a coordinate transformation term for the expression. The expression in the original paper is correct for the direction r. However, for the direction θ and ϕ, the correct equations should be as follows:

$$V_\theta(\boldsymbol{x}, k) = \frac{i}{k\rho_0 c} \frac{\partial P(\boldsymbol{x}, k)}{\partial \theta} \frac{1}{r}, \tag{4a}$$

$$V_\phi(\boldsymbol{x}, k) = \frac{i}{k\rho_0 c} \frac{\partial P(\boldsymbol{x}, k)}{\partial \phi} \frac{1}{r \sin\theta}. \tag{4b}$$

Accordingly, the correct versions of Eq.(5b), (5c), (9b), (9c) are expressed, respectively, as

$$V_\theta(\boldsymbol{x}, k) = \frac{i}{k\rho_0 c} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_{nm}(k) \frac{j_n(kr)}{r} A_{nm} P'_{nm}(\cos\theta) e^{im\phi}, \tag{5b}$$

$$V_\phi(\boldsymbol{x}, k) = \frac{i}{k\rho_0 c} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} im\alpha_{nm}(k) \frac{j_n(kr)}{r} A_{nm} \hat{P}_{nm}(\cos\theta) e^{im\phi}, \tag{5c}$$

$$I_\theta(\boldsymbol{x}, k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \sum_{n'=0}^{\infty} \sum_{m'=-n'}^{n'} A_{n'm'} \frac{T_{nmn\prime m\prime}(k, r)}{r} Y^*_{nm}(\theta, \phi) P'_{n\prime m\prime}(\cos\theta) e^{im'\phi}, \tag{9b}$$

$$I_\phi(\boldsymbol{x}, k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \sum_{n'=0}^{\infty} \sum_{m'=-n'}^{n'} im' A_{n'm'} \frac{T_{nmn\prime m\prime}(k, r)}{r} Y^*_{nm}(\theta, \phi) \hat{P}_{n\prime m\prime}(\cos\theta) e^{im'\phi}, \tag{9c}$$

where

$$\hat{P}_{nm}(\cos\theta) = \begin{cases} P'_{nm}(1), & \text{if } \theta = 0 \\ -P'_{nm}(-1), & \text{if } \theta = \pi \\ \frac{P_{nm}(\cos\theta)}{\sin\theta}, & \text{otherwise.} \end{cases}$$

Therefore, the expressions of intensity coefficients for the θ (Eq.(14)) and ϕ (Eq.(10b)) direction in the original paper should be replaced, respectively, with

$$S_{pq}^{(\theta)}(k,r) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'} A_{nm}A_{n'm'}A_{pq}\mathcal{P}_{nmn'm'pq}\mathcal{E}_{mm'q}\frac{T_{nmn'm'}(k,r)}{r}, \tag{14}$$

$$S_{pq}^{(\phi)}(k,r) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'} im'A_{nm}A_{n'm'}A_{pq}\hat{\mathcal{P}}_{nmn'm'pq}\mathcal{E}_{mm'q}\frac{T_{nmn'm'}(k,r)}{r}, \tag{10b}$$

where

$$\begin{aligned}\hat{\mathcal{P}}_{nmn'm'pq} =& H(n,m)H(n',m')H(p,q)\mathcal{G}(\frac{m+m'+q+1}{2}, \frac{4-\delta_{m+n}-\delta_{m'+n'}-\delta_{p+q}}{2}; \\ & \frac{1+m-n-\delta_{m+n}}{2}, \frac{1+m'-n'-\delta_{m'+n'}}{2}, \frac{1+q-p-\delta_{p+q}}{2}; \frac{2+m+n}{2} \\ & -\frac{\delta_{m+n}}{2}, \frac{2+m'+n'-\delta_{m'+n'}}{2}, \frac{2+p+q-\delta_{p+q}}{2}; m+1, m'+1, q+1).\end{aligned}$$

Note that the proof for $S_{pq}^{(\phi)}(k,r)$ here is similar to the proof for $S_{pq}^{(\theta)}(k,r)$ in the original paper, which is correct. Without the Wigner 3-j symbols in the expression of $S_{pq}^{(\phi)}(k,r)$, the active order is not $2N$ any more. However, the truncation error also falls to an acceptable value as the truncation order increases. Figure 2 from the original paper must also be replaced with the figure given here. Note that the performance of sound intensity in the ϕ direction, similar to the θ direction, is slightly worse than that in the r direction as well due to the truncation error.

Acknowledgments

The authors very much appreciate Byeongho Jo of Korea Advanced Institute of Science and Technology for discovering this problem.

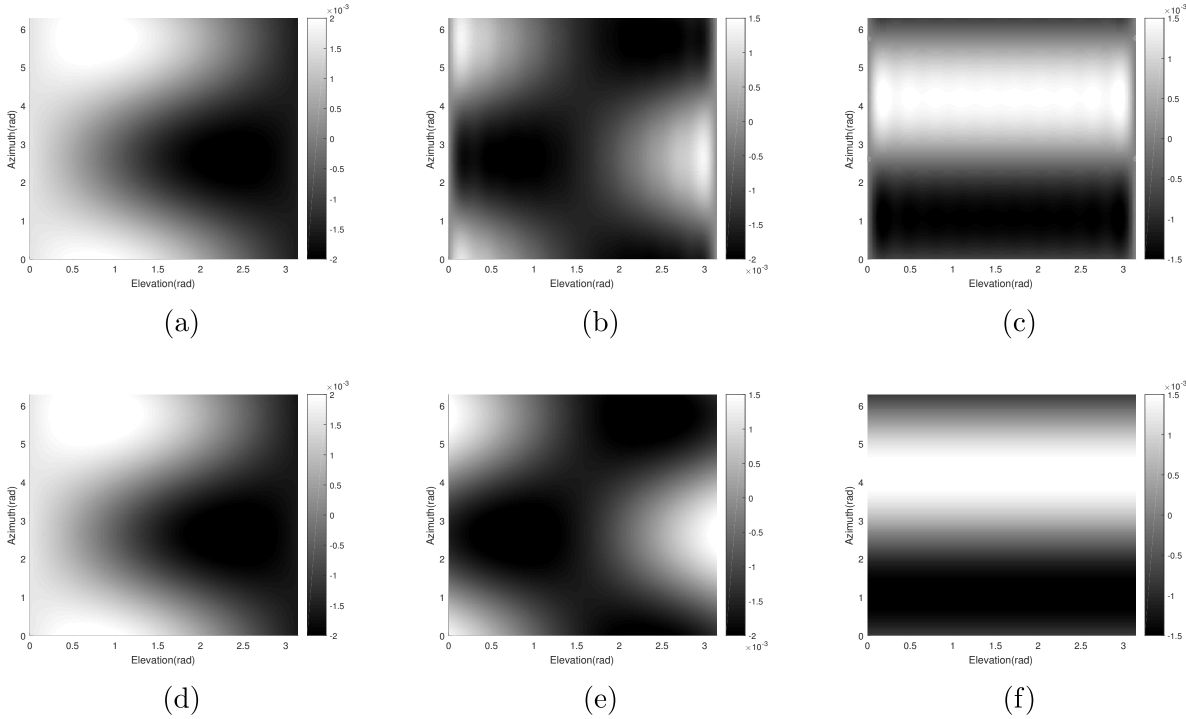

Fig. 2. Sound intensity on a sphere with radius of 0.05 m, generated by a plane wave from $(3\pi/4, 5\pi/6)$, with frequency 600 Hz. (a-c) Sound intensity in r, θ and ϕ direction, separately, calculated using the proposed theory, (d-f) sound intensity in r, θ and ϕ direction, separately, obtained from point by point measurement.