Time-varying Models for Macroeconomic Forecasts

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Except where otherwise indicated, this thesis is my own original work.

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I dedicate this dissertation to my family.

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Abstract

This thesis consists of three studies focusing on ways to detect and model time variation among macroeconomic variables. In these three studies, errors with autoregressive moving averages (ARMA), model averaging, and stochastic volatility (SV) are used to investigate the uncertainty and the instability of macroeconomic dynamics. In particular, I expand upon both univariate (autoregressive; AR) and multivariate (vector autoregressive; VAR) time series models.

Chapter 1 provides a general introduction to the research interest of this thesis. Next, Chapter 2 introduces an ARMA component with SV into the unobserved component model. A transformation to a stacked matrix form of the model is conducted for posterior fast simulation. The proposed model is then used to study macroeconomic time series in the United States (US). The proposed new model provides good full-sample simulation for the majority of the macroeconomic variables, and can improve both the point and the interval forecasting performance of these variables across different horizons.

In Chapter 3, I use real-time macroeconomic variables and both time-varying and equal weights with time-varying parameter models to forecast inflation in the US. Three time-varying coefficient models with three specifications of their error terms are studied. The alternative error-term assumptions are errors with a Gaussian distribution, errors with SV, and errors with moving average SV. Both point forecasts and density forecasts suggest that adding variables and allowing time-varying lag length choice can significantly improve forecasting performance. The forecasting performance of the time-varying and equal weights model combination methods show that adding SV can improve density forecasts but not point forecasts.

Finally, in Chapter 4, I employ a time-varying parameter VAR with SV (**TVP-VAR-SV**) to analyze the dynamics of renewable electricity generation (REG), gross domestic product (GDP) growth, and CO_2 emissions. **TVP-VAR-SV** and other restricted variants are employed for forecasting REG with data from the US. The empirical results suggest that **TVP-VAR-SV** is suitable for studying the relationship between REG, GDP, and CO_2. The forecasting results suggest that VARs with a time-varying volatility specification can perform much better than those without SV, while allowing for time-varying coefficients does not improve forecasting performance.

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Chapter 1

Introduction

In macroeconomic empirical research, the movement of economic variables attracts strong interest from researchers. To reveal the evolution of these macrovariable time series, both univariate and multivariate frameworks with many time-varying specifications have been considered in studies. The present thesis contributes to the literature by modeling these time series with time-varying features. Specifically, the correlation of error terms, model averaging, time-varying parameters, and stochastic volatility (SV) are highlighted in the three main chapters. After evaluating the in-sample fitness of the proposed models, the results of out-of-sample forecasts are reported in the "Application" section. The empirical parameter simulation and modeling evaluation in this thesis are conducted by Bayesian econometrics. Both a block-banded sparse matrix and a precision-based algorithm are used for the rapid and efficient simulation of parameters.

Chapter 2 introduces SV with autoregressive moving average (ARMA; SV-ARMA) errors in the univariate unobserved components (UC) model. Another contribution of this thesis is that it is the first study to run forecasts on United States (US) macrovariables with SV-ARMA error terms. Other competing models include UC models with or without error correlation and SV assumptions; moreover, some simple but hard to beat univariate models, such as random walk (RW) and AR models, are also considered. In this chapter, both point and interval forecasting results are presented, and the analysis of forecasting performance is based on 22 quarterly macroeconomic time series with 13 modeling specifications allocated in four groups.

Chapter 3 focuses on inflation forecasts. Inflation is a core macroeconomic indicator and has received considerable attention from both central bankers and macroeconomic researchers. In this chapter, model averaging time-varying weight and equal weight strategies are considered for either point forecasts or density forecasts with eight inflation predictors. The modeling competition is conducted using UC models with SV and SVMA errors, with dynamic model averaging and selection for these highly competitive models.

Chapter 4 employs a time-varying parameter vector autoregressive (TVP-VAR) model with SV to investigate the relationship between the renewables, output, and CO_2 emissions. In fact, TVP-VARs are already widely accepted and applied to macroeconomic studies to investigate the dynamic interactions between variables. The empirical results indicate that the SV specification shows a better fitness to the data than the homoscedastic variance models in the full-sample application. The forecasting results suggest that the specification of SV can substantially improve the forecasting performance in comparison with constant variances, whereas specification of the time-varying parameters is not helpful.

Chapter 5 presents the general conclusions of the full thesis and future research prospects.

Chapter 2

Forecasting Macroeconomic Series by Models with ARMA-SV Errors

2.1 Introduction

In empirical macroeconomics, researchers are generally interested in testing classical economic theories or exploring empirical relationships among macroeconomic variables. To empirically model such variables, practitioners tend to rely on multivariate time series techniques such as VARs and vector error correction models (VECMs). While historically they have been popular, recent literature has shown that multivariate systems may only work well for in-sample fitness or out-of-sample forecasts in some episodes (Stock and Watson, 2007), and the number of variables in the information set may change substantially over time when better estimation results are being pursued (Chan et al., 2012). Under such circumstances, revealing the evolution of these time series through univariate models could be a better strategy.

The instability of coefficients in univariate models has been widely acknowledged and has been explored in a variety of ways. Examples include shifts in local means, structure breaks, and more recently, time-varying coefficients (e.g., Koop and Potter, 2007; Stock and Watson, 1996; Chan, Koop, Leon-Gonzalez, and Strachan, 2012). For example, Stock and Watson (2007) find that the underlying trend of the time series shows stochastic changes over time, and the standard deviation (SD) also varies over time. This indicates that a constant variance assumption is not sufficient for all situations. In this sense, UC with SV could cover the concerns regarding both the time-varying underlying trend and the stochastic transitory disturbance. In order to achieve better estimation results, researchers explore different SV setups, such as an unknown degree of freedom of Student's t-distribution and a jump component in the error term (Chib et al., 2002), SV models with leverage (Omori et al., 2007), and moving-average error with SV specification (Chan, 2013), which models the errors with serial correlation.

In macroeconomic empirical studies, it is not surprising to note that significant serial dependence is found among the observations. Under this circumstance, it is reasonable to assume that the error terms follow a serially dependent process rather than an independent distribution. Many applications show that a properly assumed ARMA error process can be better accommodated with the same structure of variable evolution as those without it. For example, Tsay (1984) studied the least squares estimation with both stationary and nonstationary ARMA errors, while Chib and Greenberg (1994) and Wu and Wang (2012) discuss linear regression models with ARMA errors under a Bayesian framework. However, to our knowledge, none of the existing work focuses on a UC model with ARMA errors or even ARMA errors with an SV component.

Regarding the empirical parameter simulation using Bayesian methods, Chib and Greenberg (1994) develop a practical Markov chain Monte Carlo (MCMC) method that works well in low dimensions when resorting to the Kalman filter. However, it can be very time consuming when the estimation applies to much higher dimension models, especially the state space model with time-varying parameters. Chan and Jeliazkov (2009) solve this problem by introducing a transparent precision-based algorithm for a state space model with a constant covariance matrix, and it presents rapid and efficient properties due to their block-banded and sparse matrix algorithms. Later, Chan (2013) presents his work on state space models with a moving average error, which builds on this algorithm, and it shows that the MCMC converges well. However, for models with the ARMA component, McCausland et al. (2011) believe that the estimation could be less efficient when using precision-based algorithms than it is with the Kalman filter, as it is difficult to find an expression where the covariance matrix with stacked innovation terms could avoid full rank.

To conquer these difficulties, we introduce an approach to working on a univariate UC model with ARMA-SV (abbreviated as **UC-ARMA** in this thesis) evolution using a precision-based algorithm. The developed algorithm in this chapter can still present rapid and efficient properties as achieved by Chan (2013).

This is the first study to present forecasting exercises on US macroeconomic time series by **UC-ARMA** specification. Other competing models include UC models with or without error correlation and SV assumptions; moreover, some simple univariate models, such as RW and AR models (Stock and Watson, 2007) are also considered.

In the literature, there are only a few forecasting applications for macroeconomic time series, and most studies consider multivariate specifications. For example, Swanson and White (1997) compare the forecasting performance of linear and nonlinear models with a variety of fixed and flexible specifications using nine macroeconomic variables, Athanasopoulos and Vahid (2008) explore five multivariate models including both VAR and VAR moving average (VARMA) for macroeconomic forecasting. With respect to the univariate models, Bauwens et al. (2015) investigate the forecasting performance of two groups of structural break models as well as AR models for 23 quarterly macroeconomic series, while Marcellino et al. (2006) use 170 monthly time series to compare iterated forecast and direct forecast results for an autoregressive model. In this chapter, we provide both point and interval forecasting results and analyze the forecasting performance of 22 quarterly macroeconomic time series in the US in four groups—in total, 13 specifications.

In the next section, we present the framework of the **UC-ARMA** model, followed by the analytical likelihood function as well as the posterior analysis and simulation methods for the parameters. In Section 2.3, we discuss the application of the US macroeconomic time series with full-sample estimation of the key parameters. Section 2.4 introduces the forecast methods and compares the forecasting results among the competing models. The final section presents the concluding remarks.

2.2 UC Models with ARMA Errors and SV

Consider a general state space modeling framework by generating the observation y_t at time t from two parts, a UC term τ_t, and an error term ε_t^y, which is expressed in terms of an ARMA(p,q) process with SV:

$$y_t = \tau_t + \varepsilon_t^y, \tag{2.1}$$

$$\tau_t = \tau_{t-1} + \varepsilon_t^\tau, \quad \tau_1 \sim \mathcal{N}(0, \sigma_{0\tau}^2), \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2), \tag{2.2}$$

$$\varepsilon_t^y = \phi_1 \varepsilon_{t-1}^y + \cdots + \phi_p \varepsilon_{t-p}^y + u_t + \psi_1 u_{t-1} + \cdots + \psi_q u_{t-q}, \quad u_t \sim \mathcal{N}(0, \mathrm{e}^{h_t}), \tag{2.3}$$

$$h_t = h_{t-1} + \varepsilon_t^h, \quad h_1 \sim \mathcal{N}(0, \sigma_{0h}^2), \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2), \tag{2.4}$$

where $t = 1, \cdots, T$ and we assume that the error terms ε_t^h, ε_t^τ and u_t are all independent from each other for all the observations. Equation (2.1) is referred to as the measurement equation or observation equation, which is composed of the unobserved states τ and the error terms ε^y, whereas Equation (2.2) is the state or transition equation indicating the evolution of the states. Here, τ is assumed to follow an RW. The initial value τ_1 is generated from a Gaussian distribution whose variance $\sigma_{0\tau}^2$ is given in advance, and ε_t^τ is the smooth parameter whose variance σ_τ^2 is also known.

Equation (2.3) can be rewritten in terms of a polynomial with the lag operator L as:

$$\phi(L)\varepsilon_t = \psi(L)u_t,$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, and $\psi(L) = 1 + \psi_1 L + \cdots + \psi_q L^q$. We assume that all roots of $\phi(L)$ stay outside the unit circle for stationarity of the ARMA process, and all roots of $\psi(L)$ fall outside the unit circle for invertibility of the process (see Chib and Greenberg (1994)) for identification purposes.

The SV parameter h_t enters this specification as an instantaneous volatility of the model, and itself follows an RW evolution with h_1 drawn from a stationary Gaussian distribution.

2.2.1 Estimation

We perform the estimation by exploring the Bayesian paradigm using Gibbs sampling and the Metropolis-Hasting algorithm, which are powerful data-based MCMC methods for simulating the joint distributions of interest with suitable convergent speed. Another matter needs to be considered here: the serially dependent series of the error terms when they have an ARMA structure. When resorting to a conventional simulation Kalman filter, the original data need to be transformed to independence (Chib and Greenberg, 1994); however, as in Chan (2013), our direct approach using precision-based algorithms does not need to use this transformation, and the MA component also contains serial dependence for the time series.

2.2.1.1 Observations Likelihood Function

We first investigate the likelihood function of our model. A likelihood function can provide rich information on the data and describe the precise manner of specified parameters. For estimation purposes, we also provide the log-likelihood function below.

Since the likelihood function $L(\theta \mid \mathbf{y})$ is defined by the joint distribution $f(\mathbf{y} \mid \theta)$ given observations $\mathbf{y} = (y_1, \cdots, y_T)'$, and it is $L(\theta \mid \mathbf{y}) = f(\mathbf{y} \mid \theta)$, we first stack Equation (2.3) into matrices and vectors:

$$\mathbf{H}_\phi \boldsymbol{\varepsilon}^y = \mathbf{H}_\psi \mathbf{u}, \quad \mathbf{u} \sim \mathcal{N}(0, \boldsymbol{\Omega}_\mathbf{u})$$

then,

$$\boldsymbol{\varepsilon}^y = \mathbf{H}_\phi^{-1} \mathbf{H}_\psi \mathbf{u},$$

put it into Equation (2.1),

$$\mathbf{y} = \boldsymbol{\tau} + \mathbf{H}_\phi^{-1} \mathbf{H}_\psi \mathbf{u}, \tag{2.5}$$

where

$$\mathbf{H}_\phi = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -\phi_1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ -\phi_p & \cdots & -\phi_1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & -\phi_p & \cdots & -\phi_1 & 1 \end{pmatrix}, \quad \mathbf{H}_\psi = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \psi_1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \psi_q & \cdots & \psi_1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & \psi_q & \cdots & \psi_1 & 1 \end{pmatrix}.$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix}, \quad \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_T \end{bmatrix}, \quad \boldsymbol{\varepsilon}^y = \begin{bmatrix} \varepsilon_1^y \\ \vdots \\ \varepsilon_T^y \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_T \end{bmatrix},$$

and

$$\boldsymbol{\Omega}_u = \begin{pmatrix} \mathrm{e}^{h_1} & & \mathcal{O} \\ & \ddots & \\ \mathcal{O} & & \mathrm{e}^{h_t} \end{pmatrix}.$$

Given $\boldsymbol{\Omega}_y = \mathbf{H}_\phi^{-1}\,\mathbf{H}_\psi\,\boldsymbol{\Omega}_u\,(\mathbf{H}_\phi^{-1}\,\mathbf{H}_\psi)'$, the conditional joint probability density function of $\mathbf{y}$ is:

$$(\mathbf{y} \,|\, \phi, \psi, \boldsymbol{\tau}, \mathbf{h}) \sim \mathcal{N}(\boldsymbol{\tau}, \boldsymbol{\Omega}_y),$$

where $\mathbf{h} = (h_1, \ldots, h_T)'$. It is worth noting that $\mathbf{H}_\phi$ and $\mathbf{H}_\psi$ are $T \times T$ banded matrices. The values of p and q are normally much smaller than the number of observations T in empirical studies, so it is useful to implement banded or sparse matrix algorithms for precise estimation and rapid computation. Although $\mathbf{H}_\phi$ and $\mathbf{H}_\psi$ are both lower triangular banded matrices and $\boldsymbol{\Omega}_u$ is a diagonal matrix, the product $\boldsymbol{\Omega}_y$ is no longer a sparse matrix, due to $\mathbf{H}_\phi^{-1}$ introduced in the multiplication. The transformation for obtaining sparse matrices is discussed in the next section. The log-likelihood function is followed as:

$$\ell(\boldsymbol{\theta} \,|\, \mathbf{y}) = \log p(\mathbf{y} \,|\, \phi, \psi, \boldsymbol{\tau}, \mathbf{h}) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T} h_t - \frac{1}{2}(\mathbf{y}-\boldsymbol{\tau})'\boldsymbol{\Omega}_y^{-1}(\mathbf{y}-\boldsymbol{\tau}) \quad (2.6)$$

To calculate the log-likelihood function, we refer to the Cholesky decomposition and forward (backward) substitution introduced by Chan (2013), as it takes computer arithmetic into account for faster algorithms when implementing them in certain software packages (e.g., MATLAB).We first calculate the Cholesky decomposition of $\boldsymbol{\Omega}_y$:

$$\mathbf{C}_y = \text{chol}(\boldsymbol{\Omega}_y),$$

then by forward substitution and backward substitution:

$$A = \boldsymbol{\Omega}_y^{-1}(\mathbf{y} - \boldsymbol{\tau}) = \mathbf{C}_y' \backslash (\mathbf{C}_y \backslash (\mathbf{y} - \boldsymbol{\tau})),$$

which is equal to $A = \mathbf{C}_y^{-1'}(\mathbf{C}_y^{-1}(\mathbf{y} - \boldsymbol{\tau})) = \boldsymbol{\Omega}_y^{-1}(\mathbf{y} - \boldsymbol{\tau})$, and followed by:

$$B = (\mathbf{y} - \boldsymbol{\tau})' A.$$

Thus, the log-likelihood function (2.6) can be efficiently evaluated.

2.2.1.2 Posterior Analysis and Simulation

We refer to a Bayesian approach to study the property of the parameters in the proposed specifications. Given the information on the observations $\mathbf{y}$ and the prior distribution of the parameters $p(\boldsymbol{\theta})$, the likelihood $p(\mathbf{y} \mid \boldsymbol{\theta})$ can be obtained by (2.6) and then the posterior density function $p(\boldsymbol{\theta} \mid \mathbf{y})$ can be simulated according to Bayes rule (see Koop (2003)). Formally, our expression for the posterior is:

$$p(\boldsymbol{\theta} \mid \mathbf{y}) = \frac{p(\boldsymbol{\theta}, \mathbf{y})}{p(\mathbf{y})} = \frac{p(\mathbf{y} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})}{p(\mathbf{y})} \tag{2.7}$$

Since we are only interested in the performance of $\boldsymbol{\theta}$, the terms that do not involve $\boldsymbol{\theta}$ can be ignored. Here, we ignore the term $p(\mathbf{y})$, and (2.7) can be written as:

$$p(\boldsymbol{\theta} \mid \mathbf{y}) \propto p(\mathbf{y} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})$$

Before introducing the posterior analysis for MCMC sampler simulation, we first set the initial value of τ_t as $\tau_1 \sim \mathcal{N}(\tau_0, \sigma^2_{0\tau})$ and h_t as $h_1 \sim \mathcal{N}(h_0, \sigma^2_{0h})$, where $\tau_0, h_0, \sigma^2_{0\tau}$ and σ^2_{0h} are some known constants. Considering the variances of macroeconomic variables and literature (e.g., Stock and Watson, 2007; Chan, 2013), we initialize UC models with SV by setting $\tau_0 = 0, h_0 = 0, \sigma^2_{0\tau} = 5$, and $\sigma^2_{0h} = 5$.

The priors for $\phi, \psi, \sigma^2_\tau$, and σ^2_h are assumed to be independent of each other, and are:

$$\sigma^2_\tau \sim \mathcal{IG}(\nu_\tau, S_\tau), \quad \sigma^2_h \sim \mathcal{IG}(\nu_h, S_h), \quad \phi \sim \mathcal{N}(\phi_0, V_\phi), \quad \psi \sim \mathcal{N}(\psi_0, V_\psi).$$

Note that the priors of σ^2_τ and σ^2_h follow inverse-gamma distributions, which means that the priors are natural conjugates and they have the same functional form as the likelihood function. The conjugate prior has two advantages. One is that the posterior has the same distribution form as the prior and likelihood function, and thus further analytical discussion is clearer and simpler, and it can be easily used in posterior analysis and simulation. The other advantage is that the conjugate prior can reduce the computational demand substantially, because when MCMC methods are used, other priors may require a heavy computational burden (Koop and Korobilis, 2009). The priors of ϕ and ψ are multivariate normal distributions. Here, we assume that they have a low-dimension structure, which provides ARMA error structure to the state space model, but still retains simplicity.

We sample the posteriors in the following sequence cyclically:

1. $p(\boldsymbol{\tau} \,|\, \mathbf{y}, \mathbf{h}, \phi, \psi, \sigma_\tau^2)$;
2. $p(\mathbf{h} \,|\, \mathbf{y}, \boldsymbol{\tau}, \phi, \psi, \sigma_h^2)$;
3. $p(\psi, \sigma_h^2, \sigma_\tau^2 \,|\, \mathbf{y}, \boldsymbol{\tau}, \phi, \mathbf{h}) = p(\psi \,|\, \mathbf{y}, \boldsymbol{\tau}, \phi, \mathbf{h})\, p(\sigma_h^2 \,|\, \mathbf{h})\, p(\sigma_\tau^2 \,|\, \boldsymbol{\tau})$;
4. $p(\phi \,|\, \mathbf{y}, \boldsymbol{\tau}, \psi, \mathbf{h})$.

Sampling for $\boldsymbol{\tau}$

To investigate how to draw samplers efficiently from $p(\boldsymbol{\tau} \,|\, \mathbf{y}, \mathbf{h}, \phi, \psi, \sigma_\tau^2)$, we first propose that:

$$\mathbf{H}_\phi \mathbf{H}_\psi = \mathbf{H}_\psi \mathbf{H}_\phi \quad \& \quad \mathbf{H}_\phi^{-1} \mathbf{H}_\psi = \mathbf{H}_\psi \mathbf{H}_\phi^{-1},$$

The proof is given in Appendix 2.B. Thus, (2.5) becomes:

$$\mathbf{y} = \boldsymbol{\tau} + \mathbf{H}_\psi \mathbf{H}_\phi^{-1} \mathbf{u}. \tag{2.8}$$

Then we pre-multiply (2.8) both sides by $\mathbf{H}_\psi^{-1}$, which becomes:

$$\widetilde{\mathbf{y}} = \widetilde{\boldsymbol{\tau}} + \mathbf{H}_\phi^{-1} \mathbf{u},$$

where $\widetilde{\mathbf{y}} = \mathbf{H}_\psi^{-1} \mathbf{y}$ and $\widetilde{\boldsymbol{\tau}} = \mathbf{H}_\psi^{-1} \boldsymbol{\tau}$, so that $\boldsymbol{\tau} = \mathbf{H}_\psi \widetilde{\boldsymbol{\tau}}$, which means that once we obtain draws of $\widetilde{\boldsymbol{\tau}}$, the estimations of $\boldsymbol{\tau}$ can be obtained by pre-multiplying $\widetilde{\boldsymbol{\tau}}$ by $\mathbf{H}_\psi$. The log posterior density for $\widetilde{\boldsymbol{\tau}}$ is:

$$\log p(\widetilde{\boldsymbol{\tau}} \,|\, \widetilde{\mathbf{y}}, \mathbf{h}, \phi, \psi, \sigma_\tau^2) \propto \log p(\widetilde{\boldsymbol{\tau}} \,|\, \sigma_\tau^2) + \log p(\widetilde{\mathbf{y}} \,|\, \widetilde{\boldsymbol{\tau}}, \mathbf{h}, \phi, \psi), \tag{2.9}$$

where $p(\widetilde{\boldsymbol{\tau}} \,|\, \sigma_\tau^2)$ is the prior for $\widetilde{\boldsymbol{\tau}}$ and $p(\widetilde{\mathbf{y}} \,|\, \widetilde{\boldsymbol{\tau}}, \mathbf{h}, \phi, \psi)$ is the likelihood for $\widetilde{\mathbf{y}}$. Similar to the derivation for $\mathbf{y}$, the log-likelihood for $\widetilde{\mathbf{y}}$ is obtained by:

$$\log p(\widetilde{\mathbf{y}}\,|\,\widetilde{\boldsymbol{\tau}},\mathbf{h},\phi,\psi) \propto -\frac{1}{2}\sum_{t=1}^{T} h_t - \frac{1}{2}(\widetilde{\mathbf{y}}-\widetilde{\boldsymbol{\tau}})'\mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi(\widetilde{\mathbf{y}}-\widetilde{\boldsymbol{\tau}}), \tag{2.10}$$

Compared with (2.6), (2.10) can be calculated much faster due to the sparse structure of the resulting $\mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi$ matrix. As $\boldsymbol{\Omega}_u^{-1}$ is a diagonal matrix, and both $\mathbf{H}_\phi'$ and $\mathbf{H}_\phi$ are banded matrices, their multiplication is still a banded matrix. For $t = 1, \cdots, T$, (2.2) can be stacked as:

$$\mathbf{H}\boldsymbol{\tau} = \boldsymbol{\varepsilon}_{\boldsymbol{\tau}}, \quad \boldsymbol{\varepsilon}_{\boldsymbol{\tau}} \sim \mathcal{N}(0, \boldsymbol{\Omega}_{\varepsilon_\tau}),$$

where $\mathbf{H}$ is the first difference matrix

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & -1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & -1 & 1 \end{pmatrix},$$

and $\boldsymbol{\Omega}_{\varepsilon_\tau} = \mathrm{diag}(\sigma_{0\tau}^2, \sigma_\tau, \cdots, \sigma_\tau)$. So that:

$$\boldsymbol{\tau} = \mathbf{H}^{-1}\boldsymbol{\varepsilon}_\tau, \tag{2.11}$$

where $\boldsymbol{\tau} \sim \mathcal{N}(0, \boldsymbol{\Omega}_\tau)$ and $\boldsymbol{\Omega}_\tau^{-1} = \mathbf{H}'\boldsymbol{\Omega}_{\varepsilon_\tau}^{-1}\mathbf{H}$. Now we pre-multiply $\mathbf{H}_\psi^{-1}$ on both sides of (2.11), so that $\widetilde{\boldsymbol{\tau}} \sim \mathcal{N}(0, \boldsymbol{\Omega}_{\widetilde{\tau}})$ and $\boldsymbol{\Omega}_{\widetilde{\tau}}^{-1} = \mathbf{H}_\psi'\boldsymbol{\Omega}_\tau^{-1}\mathbf{H}_\psi$. It is easy to see that $\mathbf{H}_\psi'\boldsymbol{\Omega}_\tau^{-1}\mathbf{H}_\psi$ also has a sparse structure like $\mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi$. Noting that $|\mathbf{H}| = |\mathbf{H}_\psi| = 1$ and $|\boldsymbol{\Omega}_\tau| = \sigma_{0\tau}^2(\sigma_\tau^2)^{T-1}$. Finally, we have the log prior for $\widetilde{\tau}$ as:

$$\log p(\widetilde{\boldsymbol{\tau}}\,|\,\sigma_\tau^2) \propto -\frac{T-1}{2}\log\sigma_\tau^2 - \frac{1}{2}\widetilde{\boldsymbol{\tau}}'\mathbf{H}_\psi'\boldsymbol{\Omega}_\tau^{-1}\mathbf{H}_\psi\widetilde{\boldsymbol{\tau}}, \tag{2.12}$$

Then, putting (2.10) and (2.12) into (2.9), we have:

$$\begin{aligned} \log p(\widetilde{\boldsymbol{\tau}}\,|\,\widetilde{\mathbf{y}},\mathbf{h},\phi,\psi,\sigma_\tau^2) &\propto -\frac{1}{2}\widetilde{\boldsymbol{\tau}}'\mathbf{H}_\psi'\boldsymbol{\Omega}_\tau^{-1}\mathbf{H}_\psi\widetilde{\boldsymbol{\tau}} - \frac{1}{2}(\widetilde{\mathbf{y}}-\widetilde{\boldsymbol{\tau}})'\mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi(\widetilde{\mathbf{y}}-\widetilde{\boldsymbol{\tau}}) \\ &\propto -\frac{1}{2}(\widetilde{\boldsymbol{\tau}}'(\mathbf{H}_\psi'\boldsymbol{\Omega}_\tau^{-1}\mathbf{H}_\psi + \mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi)\widetilde{\boldsymbol{\tau}} - 2\widetilde{\boldsymbol{\tau}}'\mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi\widetilde{\mathbf{y}}) \\ &\propto -\frac{1}{2}(\widetilde{\boldsymbol{\tau}}-\widehat{\boldsymbol{\tau}})'\mathbf{D}_{\widetilde{\tau}}^{-1}(\widetilde{\boldsymbol{\tau}}-\widehat{\boldsymbol{\tau}}), \end{aligned}$$

where $\mathbf{D}_{\widetilde{\tau}} = (\mathbf{H}_\psi'\boldsymbol{\Omega}_\tau^{-1}\mathbf{H}_\psi + \mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi)^{-1}$ is a sparse matrix and $\widehat{\boldsymbol{\tau}} = \mathbf{D}_{\widetilde{\tau}}\mathbf{H}_\phi'\boldsymbol{\Omega}_u^{-1}\mathbf{H}_\phi\widetilde{\mathbf{y}}$.

Thus:

$$(\widetilde{\boldsymbol{\tau}} \,|\, \widetilde{\mathbf{y}}, \mathbf{h}, \phi, \psi, \sigma_\tau^2) \sim \mathcal{N}(\widehat{\boldsymbol{\tau}}, \mathbf{D}_{\widetilde{\tau}}).$$

Similar to the approach discussed before, we can use the Cholesky decomposition $\mathbf{C}_{\widetilde{\tau}}$ for $\mathbf{D}_{\widetilde{\tau}}^{-1}$ firstly, then $\widehat{\boldsymbol{\tau}}$ can be calculated rapidly by the precision-based algorithm. By implementing the forward and backward substitution, we have:

$$\widehat{\boldsymbol{\tau}} = \mathbf{C}'_{\widetilde{\tau}} \backslash (\mathbf{C}_{\widetilde{\tau}} \backslash (\mathbf{H}'_\phi \boldsymbol{\Omega}_u^{-1} \mathbf{H}_\phi \widetilde{\mathbf{y}})),$$

so that the draws of $\widetilde{\boldsymbol{\tau}}$ can be obtained by:

$$\widetilde{\boldsymbol{\tau}} = \widehat{\boldsymbol{\tau}} + \mathbf{C}'_{\widetilde{\tau}} \backslash \mathbf{R}, \quad \mathbf{R} \sim \mathcal{N}(0, \mathbf{I}),$$

where $\mathbf{R}$ is an RW vector that follows an independent standard normal distribution.

Finally, we can return $\boldsymbol{\tau}$ by $\boldsymbol{\tau} = \mathbf{H}_\psi \widetilde{\boldsymbol{\tau}}$.

Sampling for h

The MCMC sampling for $\mathbf{h}$ uses a mixture of a normal distribution, which is designed for the SV component in a log form for any time series model. This improved MCMC algorithm was first introduced by Kim et al. (1998) and has been proven to be an efficient approximation for SV using just seven normal distributions. To adopt this method, we transform (2.8) into the following form:

$$\begin{aligned} \mathbf{y}^* &= \log(\mathbf{H}_\psi^{-1} \mathbf{H}_\phi (\mathbf{y} - \boldsymbol{\tau})) \\ &= \log(\mathbf{e}^{\mathbf{h}} \cdot \boldsymbol{\varepsilon}_{y^*}^2) \\ &= \mathbf{h} + \log \boldsymbol{\varepsilon}_{y^*}^2 \end{aligned}$$

In the empirical coding, we have:

$$\mathbf{y}^* = \log(\mathbf{H}_\psi^{-1} \mathbf{H}_\phi (\mathbf{y} - \boldsymbol{\tau}) + \mathbf{c})$$

where $\mathbf{c}$ is the offset element in case the estimation of $\boldsymbol{\varepsilon}_{y^*}^2$ is too small. We follow Kim et al. (1998) and set $\mathbf{c} = 0.001$. Then:

$$\log \boldsymbol{\varepsilon}_{ty^*}^2 \approx \sum_{t=1}^{7} p_i f_N(x_j \mu_i, \sigma_i^2),$$

where p_i, μ_i and σ_i^2 are all known in advance and are given in Kim et al. (1998). The seven Gaussian distributions $\mathbf{S_t} \in 1, 2, \cdots, 7$ are drawn in the probability $\mathbb{P}(\mathbf{S}_t = j) = p_j$, and the covariance matrix of y^* is just $\boldsymbol{\Omega}_{y^*} = \mathrm{diag}(\sigma_{S_1}^2, \sigma_{S_2}^2, \cdots, \sigma_{S_T}^2)$. If the simulation of $\mathbf{y}^*$ has been obtained, the posteriors $(\mathbf{h} \,|\, \mathbf{y}, \boldsymbol{\tau}, \phi, \psi, \sigma_h^2) \sim \mathcal{N}(\widehat{\mathbf{h}}, \mathbf{D}_h)$ can be computed by the same forward-backward smoothing method as before, which is also based on a precision-based sampler. That is:

$$\mathbf{D}_h^{-1} = \mathbf{H}'\boldsymbol{\Omega}_h^{-1}\mathbf{H} + \boldsymbol{\Omega}_{y^*}^{-1}, \quad \widehat{\mathbf{h}} = \mathbf{D}_h(\boldsymbol{\Sigma}_{y^*}^{-1}(\mathbf{y}^* - \boldsymbol{\mu})),$$

where $\boldsymbol{\Omega}_h = \text{diag}(\sigma_{0h}^2, \sigma_h^2, \cdot, \sigma_h^2)$ comes from (3.2.9).

Sampling for σ_h^2 and σ_τ^2

We assume that both σ_h^2 and σ_τ^2 are conditionally independent and the derivations of their posteriors can follow the standard method discussed in Koop (2003). Thus, their posteriors can be obtained after a simple transformation. The simulations for posteriors σ_h^2 and σ_τ^2 can use the standard variance result for linear regression models in Koop (2003). Given a conjugate inverse-gamma prior $\sigma_\tau^2 \sim \mathcal{IG}(\nu_\tau, S_\tau)$, we can receive an inverse-gamma posterior for $(\sigma_\tau^2 \,|\, \boldsymbol{\tau})$:

$$\begin{aligned} p(\sigma_\tau^2 \,|\, \tau) &\propto p(\boldsymbol{\tau} \,|\, \sigma_\tau^2) + p(\sigma_\tau^2) \\ &= (\sigma_\tau^2)^{-\frac{T}{2}} \exp\left(-\frac{1}{2\sigma_\tau^2}\sum_{t=2}^{T}(\tau_t - \tau_{t-1})^2\right) \cdot (\sigma_\tau^2)^{-(\nu_0-1)} \exp(-\frac{S_\tau}{\sigma_\tau^2}) \\ &\propto (\sigma_\tau^2)^{-((\frac{T}{2}+\nu_0)-1)} \exp\left(-\frac{1}{\sigma_\tau^2}(\sum_{t=2}^{T}(\tau_t - \tau_{t-1})^2/2 + S_\tau)\right). \end{aligned}$$

That is:

$$(\sigma_\tau^2 \,|\, \boldsymbol{\tau}) \sim \mathcal{IG}\left(T/2 + \nu_\tau, \quad \sum_{t=2}^{T}(\boldsymbol{\tau}_t - \boldsymbol{\tau}_{t-1})^2/2 + S_\tau\right).$$

Similarly, the posterior density of σ_h^2 can be derived as:

$$(\sigma_h^2 \,|\, \mathbf{h}) \sim \mathcal{IG}\left(T/2 + \nu_h, \quad \sum_{t=2}^{T}(\mathbf{h}_t - \mathbf{h}_{t-1})^2/2 + S_h\right).$$

Sampling for ψ and ϕ

Unlike $\mathbf{y}, \boldsymbol{\tau}, \sigma_h^2$ and σ_τ^2, which all follow affine normal or inverse-gamma standard family distributions, the distributions of parameters ψ and ϕ are unknown and require suitable candidate generation densities for sampling. The candidate density sampling for ψ and ϕ here is adopted using the acceptance-rejection algorithm (see Kroese et al. (2011)).

For the moving average term ψ, we stack (2.1) and (2.3) into matrix form:

$$\mathbf{H}_\phi(\mathbf{y} - \boldsymbol{\tau}) = \mathbf{H}_\psi\mathbf{u}. \tag{2.13}$$

Remember that the prior $\psi \sim \mathcal{N}(\psi_0, V_\psi)$ is a multivariate normal distribution, and the log-likelihood of the posterior is:

$$\begin{aligned}\log p(\psi \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h}) &\propto \log p(\mathbf{y} \,|\, \psi, \boldsymbol{\tau}, \mathbf{h}) + \log p(\psi) \\ &\propto \log p(\psi) - \frac{1}{2}(\mathbf{H}_\phi(\mathbf{y} - \boldsymbol{\tau}))'(\mathbf{H}'_\psi \boldsymbol{\Omega}_u \mathbf{H}_\psi)^{-1}\mathbf{H}_\phi(\mathbf{y} - \boldsymbol{\tau}).\end{aligned}$$

When ψ is high dimensional, adaptive MCMC samplers can be implemented instead of the high-dimensional numerical maximization (Andrieu and Thoms, 2008). Then, we use the Metropolis-Hastings algorithm detailed in Chib and Greenberg (1995) to sample ψ, which is widely used to simulate the distribution of multivariates. The proposal density for ψ is multi-normal distribution $q(\psi)$, and the updated ψ^c is accepted with the probability:

$$\min\{1, \quad \frac{p(\psi^c \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h})}{p(\psi \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h})} \cdot \frac{q(\psi)}{q(\psi^c)}\}.$$

In the subsequent sections, ψ is a scalar, which can be evaluated numerically by a MATLAB built-in function by searching for ψ within (-1,1).

For sampling ϕ, we first derive a suitable candidate generation density, so that the Metropolis-Hastings algorithm can be implemented with a high success rate for accepting the candidate draws, and the sampling of ϕ can be much faster.

The proposal density for ϕ is a truncated normal distribution. We let $\mathbf{z} = \mathbf{y} - \boldsymbol{\tau}$ and change (2.13) into:

$$\mathbf{H}_\phi \mathbf{z} = \mathbf{H}_\psi \mathbf{u},$$

then move $\mathbf{H}_\psi$ to another side and rearrange it as:

$$\mathbf{z} = \mathbf{X}_z \phi + \mathbf{H}_\psi \mathbf{u}$$

where $\mathbf{X}_\mathbf{z} = (\mathbf{z}_1, \cdots, \mathbf{z}_{T-1})'$. So ϕ is equivalent to the coefficients of a standard linear regression model. Given the truncated normal prior $\phi \sim \mathcal{N}(\phi_0, \mathbf{V}_\phi)\mathbb{1}(\phi \in \mathbf{A}_\phi)$, the posterior density of ϕ is just:

$$(\phi \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h}) \sim \mathcal{N}(\widehat{\phi}, \mathbf{D}_\phi)\mathbb{1}(\phi \in \mathbf{A}_\phi),$$

where $\mathbf{D}_\phi^{-1} = \mathbf{V}_\phi^{-1} + \mathbf{X}'_z(\mathbf{H}_\psi \boldsymbol{\Omega}_u \mathbf{H}'_\psi)^{-1}\mathbf{X}_z$ and $\widehat{\phi} = \mathbf{D}_\phi \cdot (\mathbf{V}_\phi^{-1}\phi_0 + \mathbf{X}_z(\mathbf{H}_\psi \boldsymbol{\Omega}_u \mathbf{H}'_\psi)^{-1}\mathbf{z})$ (see Koop (2003)), $\mathbf{A}_\phi$ are a set that satisfies the stationarity restriction to the roots of autoregressive polynomial, which all lie outside the unit circle.

2.3 Application to the US Macroeconomic Series

Here we investigate four groups of univariate models among our **UC-ARMA** model, including RW, which is used as the benchmark, UC models, and two autoregressive models. For the selection of time series, we refer to the US quarterly macroeconomic series used in Bauwens et al. (2015). These time series are among the most important nominal and real activity indicators studied by macroeconomists.

2.3.1 Competing Models

Recent studies show that allowing SV in the standard variances can provide better model fitness and forecasting performance than models with constant standard variance (e.g., Clark and Doh, 2014; Chan, 2013), and therefore it is considered an empirically significant component for macroeconomic time series. We include SV in the error term as a default component in our models, so that all the specifications are with SV unless the models are marked specifically as "NoSV". The models with "NoSV" in their names are models defined as those that have constant variances other than SV.

On the other hand, the benchmark model is a standard RW model with fixed variance. The reason we include RW here is that RW is still a competitive model among both univariate and multivariate models, and it is often adopted as the benchmark in literature (e.g., Atkeson and Ohanian, 2001; Stock and Watson, 2007; Stella and Stock, 2013).

We also consider autoregressive models as competitive models. In order to focus on the studied specification and keep the discussion compact, we fix the lag order for these models with two lags (AR2) and four lags (AR4), with and without ARMA error or SV for comparison (Marcellino et al. (2006) present a careful discussion of the AR model lag order choice) instead of referring to a data-dependent lag order choice using the Akaike information criterion (AIC) or Bayes information criterion (BIC). With this setting, both short lags (AR2) and long lags (AR4) are covered for comparison.

1. the RW model:
$$y_t = y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2).$$

2. the UC model:
$$\begin{aligned} y_t &= \tau_t + \varepsilon_t^y, \\ \tau_t &= \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2), \end{aligned}$$

with only SV error (**UC**):

$$\begin{aligned}
&\varepsilon_t^y \sim \mathcal{N}(0, \mathrm{e}^{ht}),\\
&h_t = h_{t-1} + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2),
\end{aligned}$$

with MA-SV error (**UC-MA**):

$$\begin{aligned}
&\varepsilon_t^y = u_t + \psi_1 u_{t-1} + \cdots + \psi_q u_{t-q}, \quad u_t \sim \mathcal{N}(0, \mathrm{e}^{h_t}),\\
&h_t = h_{t-1} + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2),
\end{aligned}$$

with ARMA-SV error (**UC-ARMA**):

$$\begin{aligned}
&\varepsilon_t^y = \phi_1 \varepsilon_{t-1}^y + \cdots + \phi_p \varepsilon_{t-p}^y + u_t + \psi_1 u_{t-1} + \cdots + \psi_q u_{t-q}, \quad u_t \sim \mathcal{N}(0, \mathrm{e}^{h_t}),\\
&h_t = h_{t-1} + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2).
\end{aligned}$$

with ARMA error but without SV (**UC-ARMA-NoSV**):

$$\varepsilon_t^y = \phi_1 \varepsilon_{t-1}^y + \cdots + \phi_p \varepsilon_{t-p}^y + u_t + \psi_1 u_{t-1} + \cdots + \psi_q u_{t-q}, \quad u_t \sim \mathcal{N}(0, \sigma_y^2).$$

3. the autoregressive (2) model:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t^y,$$

where ε_t^y has the same four specifications as the UC model group; we refer to them as **AR, ARMA, AR-ARMA and AR-ARMANoSV** to be consistent with the above group.

4. the autoregressive (4) model:

$$y_t = \phi_0 + \phi_1 y_{t-1} + + \cdots + \phi_4 y_{t-4} + \varepsilon_t^y,$$

which has a similar model assumption to that of Model Group 3 but with lag length 4, so we name them **AR4, AR4-MA, AR4-ARMA and AR4-ARMANoSV**.

As presented above, the **UC** model has SV in the observation equation. However, for simplicity, we do not expand models with SV in the transition equation as in Stock and Watson (2007). For Groups 3 and 4, we assume that the AR process is stationary, and all roots of the characteristic polynomial related to the AR coefficients in the estimation are outside the unit circle.

2.3.2 Data and Priors

We conduct the estimation for all the competing models in empirical macroeconomics using 22 quarterly series (detailed in Table 2.1) in the US. For each time series, the data set is composed of 218 discrete time observations from the third quarter of 1958 to the last quarter of 2012, and the first four quarters data are separated as initial known lags. As indicated in the table, we leave the series, which is already measured as rates, in its original quantity but transform the others to growth rates using the first difference of logarithms. We do not transform the series using other methods (e.g., second difference of logarithms) because only univariate models are considered here.

Table 2.1: Variables used in model comparison.

No.	Acronym	T	Definition
1	**GDPC96**	R	Real Gross Domestic Product, 3 Decimal
2	**CPIAUCSL**	R	Consumer Price Index for All Urban Consumers: All Items
3	**FEDFUNDS**	O	Effective Federal Funds Rate
4	**BORROW**	R	Total Borrowing of Depository Institutions from Federal Reserve
5	**SP500**	R	S&P 500 Stock Price Index
6	**M2SL**	R	M2 Money Stock
7	**PINCOME**	R	Personal Income
8	**PCECC96**	R	Real Personal Consumption Expenditures
9	**INDPRO**	R	Industrial Production Index
10	**UNRATE**	O	Civilian Unemployment Rate
11	**HOUST**	R	Housing Starts: Total New Privately Owned Housing Units Start
12	**PPIFCG**	R	Producer Price Index: Finished Consumer Goods
13	**PCECTPI**	R	Personal Consumption Expenditures: Chain-Type Price Index
14	**AHEMAN**	R	Average Hourly Earning Of Production: Nonsupervisory Employee
15	**M1SL**	R	M1 Money Stock
16	**OILPRICE**	R	Spot Oil Price: West Texas Intermediate
17	**GS10**	O	10-Year Treasury Constant Maturity Rate
18	**GPDIC96**	R	Real Gross Private Domestic Investment, 3 Decimal
19	**PAYEMS**	R	All Employees: Total Nonfarm
20	**PMI**	R	Purchasing Managers Index
21	**NAPMNOI**	R	ISM Manufacturing: New Orders Index
22	**OPHPBS**	R	Business Sector: Output Per Hour of All Persons

The third column refers to the transformation methods: O = original series, R = growth rate after the first difference of logged variables. The sample period (for both O and R transformation methods) was 1958Q3 to 2012Q4. Data were obtained from the St. Louis FRED database (http://research.stlouisfed.org).

The full sample estimation results are reported in Appendix 2.A.

2.4 Forecasting Results

2.4.1 Forecast Evaluation Methods

We divide the data into three sub-samples for a pseudo out-of-sample forecast. The first part (1958Q3 to 1959Q2) is the separated initial four observations that consider the adopted four lags in the autoregressive models so that all models contain the same estimation starting point (1959Q3). The second part (1959Q3 to 1974Q4) is the estimation sample, consisting of 62 observations in each macroeconomic variable. The third part (1975Q1 to 2012Q4) is the hold-out sample, which contains 148 observations.

2.4.1.1 Recursive and Direct Forecasts

The parameters are first estimated from the first data part (say it is $\mathbf{y}_{1:T_0+t-1}$) using MCMC simulation, and they are used to generate the forecasts ($\widehat{y}_{T_0+t+k-1}$, where k is k-step-ahead) to be compared with the real data $\mathbf{y}_{T_0+t+k-1}$ in the third part. Later on, we expand the observation window by one more data point $\mathbf{y}_{1:T_0+t+1-1}$, update the parameters for the next-step discrete time point and forecast again until we consume the complete data set. In the end, the entire forecast series is obtained by a recursive computation. For each forecasting loop, the parameter simulation is still based on 50,000 draws with a burn-in period of 5,000.

The expanding windows make the estimation sample part larger, and more information is used for the newer forecasts. Here we do not use a rolling window, which keeps the number of estimation samples the same for every forecast, as it does not like expanding windows that can include more known information with time marching. The expanding windows can also examine whether the forecasting performance of a candidate model can have less influence from structural breaks in the data.

A direct forecast is conducted in the present chapter rather than an iterated forecast, since UC group models do not have an iterated formula such as AR group models do. We provide five horizons forecast results, that is, one-, four-, eight-, 12-, and 16-step-ahead forecasts for each of these 22 macroeconomic variables using these 13 specifications. This means that the forecasting results cover from one quarter to four years, so that both shorter horizon and longer horizon performance are investigated using the proposed specifications.

2.4.1.2 Criteria for Model Selection

To evaluate the quantity of the forecasting performance, two metrics are used: the mean square forecast error (MSFE) and the log-predictive-likelihood (LPL). The MSFE can be used to evaluate the point forecast performance, where a smaller value indicates a better performance, while the predictive likelihood is used to evaluate the density forecast performance, where a larger predictive likelihood value implies a better interval forecast performance. The MSFE is used widely as a criterion for model selections. Similar to the variant of an error term, it is a measurement of the magnitude of the forecasting error (Tsurumi and Wago, 1991). To calculate the MSFE, we first evaluate the forecast $\widehat{y}_{T_0+t+k-1}$ by averaging all the posterior means $\mathbb{E}(y_{T0+t+k-1} \,|\, \mathbf{y}_{1:T_0+t})$ when it is time T_0+t; then the forecasting error is just $\mathbf{e}^2_{T_0+t+k-1} = \mathbf{y}^0_{T_0+t+k-1} - \mathbb{E}(y_{T0+t+k-1} \,|\, \mathbf{y}_{1:T_0+t})$, where $\mathbf{k}$ denotes a k-step-ahead forecast. The next step is to calculate the mean of the forecasting error. Thus, the MSFE is defined as:

$$\text{MSFE} = \frac{1}{T-T_0-k+1} \sum_{t=1}^{T-T_0-k+1} \mathbf{e}^2_{T_0+t+k-1}$$

As mentioned in the previous section, an RW model is used as the benchmark. We use the MSFE values of other models to divide that of the RW model and obtain the relative MSFE (RelMSFE). Thus, the forecast performance standardizes by setting the performance of the RW to 1.00. The predictive likelihood $p(\widehat{y}_{T_0+t+k-1} = \mathbf{y}_{T_0+t+k-1} \,|\, \mathbf{y}_{1:T_0+t})$ is used to evaluate the density forecast performance $p(\widehat{y}_{T_0+t+k-1} \,|\, \mathbf{y}_{1:T_0+t})$, which is the predictive density of $\widehat{y}_{T_0+t+k-1}$ evaluated at the observed value $\mathbf{y}_{T_0+t+k-1}$. There is a close connection between the predictive likelihood and the marginal likelihood. There is more detailed discussion of the log-predictive likelihood in Geweke and Amisano (2010). The estimated parameters are conditional on the observed data $\mathbf{y}_{T_0+t+k-1}$ producing a larger predictive likelihood value when the observed data fall into the interval of the posterior predictive distribution with higher probability (Hinkley, 1979). The summarized LPL is used here to evaluate the density forecast when considering easier computation; that is:

$$\text{LPL} = \sum_{t=1}^{T-T_0-k+1} \log p(\widehat{y}_{T_0+t+k-1} = y_{T_0+t+k-1} \,|\, \mathbf{y}_{1:T_0+t})$$

Since there is no prediction density available for RW, the relative LPL (RelLPL) is computed using the UC model as the benchmark, and the RelLPL is then obtained using the value of the other models LPL minus that of the UC model. It can be seen that the RelLPL of UC are all zero and a larger value indicates better prediction density forecasting for all models except RW.

2.4.2 MSFE Forecast Results

The MSFE and RelMSFE forecast results are tabulated in Appendix 2.B. Here, we summarize the specifications with the best performance over the 22 macroeconomic variables according to different errors and different model groups, respectively. The following two tables present the winning times for each specification across short to long horizons, and their winning percentages in the whole data sets are given below the winning times. Although the results of the MSFE and RelMSFE are kept to four decimal places, there is still one case where two models, **AR2** and **AR4-MA**, tied for the best performance in the $k = 16$ horizon for the No. 22 variable forecasting exercise, so the total number of winning models is 23 and the outperforming percentages are also calculated by dividing by 23 other than 22.

Table 2.2: The number and percentage of the best models with different error specifications based on MSFE for 22 macroeconomic variable forecasting.

	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
RW	2	1	0	0	0
	9.1%	4.5%	0%	0%	0%
with only SV error	4	1	6	5	5
	18.2%	4.5%	27.3%	22.7%	21.7%
with MA error	5	7	2	3	4
	22.7%	31.8%	9.1%	13.6%	17.4%
with ARMA error	9	9	7	6	8
	40.9%	40.9%	31.8%	27.3%	34.8%
with ARMA NoSV error	2	4	7	8	6
	9.1%	18.2%	31.8%	36.4%	26.1%
Total	22	22	22	22	23

Table 2.2 shows *ex ante* forecasting evidence that models (UC, AR2, and AR4) with ARMA errors yield the best performance for point forecasting prediction in almost all horizons except Horizon $k = 12$. For example, in the one-step-ahead forecast, models with ARMA error have the smallest MSFE for nine variables in comparison with the other models, and this takes up to 40.9% in all 22 variables.

The results also suggest that models with MA error sometimes provide significantly better forecasting results in short horizons (1 and 4), while models with ARMA NoSV error can give the best predictions in intermediate to long horizons (8 and 12). As might be expected, even the benchmark RW can win twice in Horizon 1. However, none of these models can present steady good performance over all horizons like the models with ARMA error. In other words, the results in Table 2.2 support that the proposed ARMA error term is a robust forecasting device for point forecasts.

We interpret the results in Table 2.3 as important evidence that the UC model is a powerful tool for improving forecasting performance compared with the autoregressive models and the RW model. In Table 2.3, we can see that the UC group models dominate other models in all horizons except Horizon 4. More specifically, half the variables in longer horizons (Horizons 12 and 16) prefer UC models, and UC models generate more accurate forecasts than other models for over one-third of variables for a short horizon (Horizon 1) and an intermediate horizon (Horizon 8).

Table 2.3: The number and percentage of the best models with different model group specifications based on MSFE for 22 macroeconomic variable forecasting.

	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
RW group	2	1	0	0	0
	9.1%	4.5%	0%	0%	0%
UC group	8	7	9	11	11
	36.4%	31.8%	40.9%	50.0%	50.0%
AR2 group	7	5	7	4	5
	31.8%	22.7%	31.8%	18.2%	22.7%
AR4 group	5	9	6	7	7
	22.7%	40.9%	27.3%	31.8%	31.8%
Total	22	22	22	22	23

Table 2.4: The number of the best models based on MSFE for 22 macroeconomic variable forecasting.

	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
RW	2	1	0	0	0
UC	0	1	3	1	2
UC-MA	4	3	0	2	3
UC-ARMA	3	2	3	2	1
UC-ARMANoSV	1	1	3	6	5
AR2	2	0	2	2	2
AR2-MA	1	2	1	0	0
AR2-ARMA	3	1	2	1	3
AR2-ARMANoSV	1	2	2	1	0
AR4	2	0	1	2	1
AR4-MA	0	2	1	1	1
AR4-ARMA	3	6	2	3	4
AR4-ARMANoSV	0	1	2	1	1
Total	22	22	22	22	23

For the two group autoregressive models, the short lag group (AR2) seems to have better performance over shorter horizons, while the long lag group (AR4) shows better relative performance in longer horizons. It seems that the added lags in AR models can capture the underlying persistence of the variables in longer horizons but occupy too much SV in short horizons.

Table 2.4 presents the detailed forecasting performance for each specification. It is not surprising that no specification can dominate the others for all variables in all horizons. On one hand, the results in the preceding table suggest that the proposed **UC-ARMA** can produces better performance than other specifications for some variables, such as real gross domestic product in shorter horizons (horizon one and four), personal income in all horizons except horizon sixteen, real personal consumption expenditures in horizon eight, 10-year treasury constant maturity rate in longer horizons (horizon eight to sixteen), and all employees: total nonfarm in horizon one. On the other hand, **UC-ARMANoSV** is selected as the best model for many variables in horizon eight to sixteen, which indicates that UC-ARMA models without SV specification can still work well in longer horizons for some variables. Meanwhile, **AR4-ARMA** is also a highly competitive specifications across forecasting horizons.

2.4.3 LPL Forecast Results

Turning to the analysis of the LPL forecast results in the preceding tables, we can see that the forecasting performance of all models is slightly different from the MSFE results.

Table 2.5: The number and percentage of the best models with different error specifications based on RelLPL for 22 macroeconomic variable forecasting.

	$k=1$	$k=4$	$k=8$	$k=12$	$k=16$
with only SV error	5	4	5	6	8
	22.7%	18.2%	22.7%	27.3%	36.4%
with MA error	9	5	3	1	2
	40.9%	22.7%	13.6%	4.5%	9.1%
with ARMA error	6	7	7	6	5
	27.3%	31.8%	31.8%	27.3%	22.7%
with ARMA NoSV error	2	6	7	9	7
	9.1%	27.3%	31.8%	40.9%	31.8%
Total	22	22	22	22	22

Table 2.5 shows that models with ARMA error can produce fairly stable and above

average numbers of best forecasting models in all horizons. The relative performance of models with only SV and ARMANoSV often improves considerably with longer forecast horizons. These results indicate that a flexible error innovation process may be good at predicting the short-term evolution of a variable, while long horizon forecasts require less dynamic specifications in error terms.

Table 2.6: The number and percentage of the best models with different model group specifications based on RelLPL for 22 macroeconomic variable forecasting.

	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
UC group	4	5	6	6	9
	18.2%	22.7%	27.3%	27.3%	40.9%
AR2 group	8	11	7	7	4
	36.4%	50.0%	31.8%	31.8%	18.2%
AR4 group	10	6	9	9	9
	45.5%	27.3%	40.9%	40.9%	40.9%
Total	22	22	22	22	22

Table 2.7: The number of the best model for each specification based on LPL for 22 macroeconomic variable forecasting.

	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
UC	1	0	4	4	6
UC-MA	2	1	1	0	1
UC-ARMA	1	3	1	1	1
UC-ARMANoSV	0	1	0	1	1
AR2	1	3	0	0	0
AR2-MA	3	2	0	1	0
AR2-ARMA	3	2	3	4	3
AR2-ARMANoSV	1	4	4	2	1
AR4	3	1	1	2	2
AR4-MA	4	2	2	0	1
AR4-ARMA	2	2	3	1	1
AR4-ARMANoSV	1	1	3	6	5
Total	22	22	22	22	22

In reference to the model group comparison in Table 2.6, the UC group performs less well and there is weak evidence that it outperforms in the long horizons. However, this time the AR4 group shows a relatively better performance in both the short and long horizons.

The results in Table 2.7 indicate that each model has its own advantages in forecasting variables with different characteristics. Although the **UC-ARMA** model does not show a significant forecasting performance in interval forecasts, it has its advantages in all the horizons. It provides the best forecasts for real gross domestic product in horizon four to twelve, personal income in horizon one and four, and real personal consumption expenditures in horizon four and sixteen. It suggests that **UC-ARMA** can forecast some real activity variables well, and it could be helpful in predict the business cycle for central banks. The ARMA error dynamic introduced in **AR2-ARMA** also achieves a substantial forecasting improvement in all the specifications.

The results in the above MSFE and LPL tables suggest that no model or specification can dominate the others, and the best performing model varies for different variables and forecast horizons. This holds in particular for forecasting metric changes from point forecasts to interval forecasts. When introducing the ARMA error term into the UC model, the answer as to whether this model can produce out-of-sample forecasts that are better than those from other univariate models seems to be mixed. Further, the byproducts **AR2-ARMA** and **AR4-ARMA** are also highly competitive models.

2.5 Concluding Remarks and Future Research

In this chapter, we have extended the UC model by introducing an ARMA component with SV evolution into the error term. By transforming the stacked matrix form of the model, the computation time is significantly reduced, and the serial dependence induced by the ARMA component is resolved by an efficient precision-based algorithm developed especially for this model.

This innovation in the error term was tested using 22 macroeconomic variables in the US data, and the results show that this new component is necessary in estimation and forecasting exercises for many variables, although not all of them. The point forecasting performance of **UC-ARMA** displays an improvement in forecast accuracy above the average number of winning models, while the interval forecast results of **UC-ARMA** show that **UC-ARMA** can produce scores in all forecasting horizons. Future research could investigate the time series suitable for ARMA-SV in a multivariate model setting.

Appendix 2.A Full-Sample Estimation Results

Our implementation of an MCMC algorithm and simulation of parameters is straightforward under a fast precision-based algorithm. The initial values of variables are set to zero and the prior of parameters are given a mean of zero with large variances. In total, 50,000 draws are taken and the first 5,000 draws are discarded, so the next 45,000 draws are retained for computing the posterior properties of the interesting parameters.

Posterior Modes of ϕ and ψ

Following Chan (2013) we set the moving average order in the MA-SV model variants to be one. For consistency, we also set each of the specifications with ARMA(p, q) errors to be ARMA(1,1). The results for ϕ and ψ of models UC, AR2, and AR4 with ARMA errors, as well as the **UC-MA** model, are summarized in Table 2.8.

Table 2.8: The modes of ϕ and ψ in models with **ARMA-SV** errors and the mode of ψ in the **UC-MA** model.

No.	Acronym	**UC-ARMA**		**UC-MA**	**AR2-ARMA**		**AR4-ARMA**	
		ϕ	ψ	ψ	ϕ	ψ	ϕ	ψ
1	**GDPC96**	0.60	-0.17	0.04	0.26	0.03	0.40	0.12
2	**CPIAUCSL**	0.92	-0.17	0.16	0.83	0.38	0.87	-0.18
3	**FEDFUNDS**	0.94	0.35	0.99	0.97	0.06	0.96	0.77
4	**BORROW**	0.99	-0.92	0.05	0.88	-0.99	-0.30	-0.99
5	**SP500**	0.20	0.20	0.12	-0.34	-0.26	0.31	-0.30
6	**M2SL**	0.53	0.24	0.41	0.67	0.27	0.95	0.28
7	**PINCOME**	0.27	-0.19	0.02	0.86	0.07	0.92	-0.05
8	**PCECC96**	0.43	-0.37	-0.02	0.27	-0.15	0.76	-0.14
9	**INDPRO**	0.71	0.20	0.38	0.33	-0.31	0.49	0.07
10	**UNRATE**	0.97	0.99	0.99	0.85	0.14	0.91	0.77
11	**HOUST**	0.86	-0.33	-0.03	-0.46	-0.99	0.36	0.96
12	**PPIFCG**	0.35	0.25	0.16	-0.48	0.15	0.59	-0.11
13	**PCECTPI**	0.89	-0.16	0.11	0.88	0.12	0.76	0.03
14	**AHEMAN**	0.97	-0.52	-0.15	0.85	-0.48	0.94	-0.05
15	**M1SL**	0.58	-0.55	0.32	-0.21	-0.05	0.97	-0.17
16	**OILPRICE**	-0.35	-0.18	0.02	0.99	-0.38	-0.53	-0.51
17	**GS10**	0.89	0.10	0.99	-0.29	0.14	0.95	0.66
18	**GPDIC96**	0.99	-0.86	-0.04	0.64	0.98	0.39	0.00
19	**PAYEMS**	0.75	-0.09	0.30	0.56	-0.07	0.80	-0.29
20	**PMI**	-0.99	0.99	0.08	0.56	-0.98	0.72	-0.99
21	**NAPMNOI**	0.99	-0.93	-0.09	0.55	-0.98	0.48	-0.99
22	**OPHPBS**	0.74	-0.85	-0.20	-0.04	-0.22	0.47	-0.14

The key parameters of interest are the ARMA error coefficients ϕ and ψ in the **UC-ARMA** model, and the results in Table 2.8 suggest that most posterior modes of ϕ and ψ which are the AR and the MA components are away from zero, except $|\psi| < 0.1$ in the No. 19 variable. There are six posterior modes of ϕ and ψ in **AR2-ARMA**, and five in **AR4-ARMA** concentrating around zero with absolute values smaller than 0.1. While **UC-ARMA** model seems to be more favor of ϕ and ψ, as there is only one posterior mode of ϕ and ψ with absolute value smaller than 0.1. There are also some absolute values of ϕ and ψ closing to 1, indicating highly persistent error terms. It suggests that the truncated normal distribution of ARMA(1,1) may not a suitable assumption for all variables. Some variables need more suitable specifications.

It is worth noting that the posterior means of ϕ and ψ are sensitive to the specifications for some variables. The simulated results vary between models, not just in absolute values but also in signs, and even the two autoregressive models have quite different estimations from each other. The results indicate that autoregressive models could be mis-specified for macroeconomic variable studies.

Another interesting point is that ψ in **UC-ARMA**, **AR2-ARMA**, and **AR4-ARMA** have 14, 12, and 12 negative values respectively, whereas the number of negative ψ in **UC-MA** is only 6, which indicates that the appearance of error lag ϕ captures some positive autocorrelations for the error term ε_y, and induces more negative autocorrelations in the SV part than that of the **UC-MA** model with only MA lag.

Marginal Density for ϕ and ψ

The marginal probability of a parameter tells a story about the properties of the concerned parameter by marginalizing over other parameters in the model. The marginal probabilities $p_\phi(\phi \,|\, \mathbf{y})$ and $p_\psi(\psi \,|\, \mathbf{y})$ here can be marginalized out by summing over the posterior draws of other parameters; that is:

$$p_\phi(\phi \,|\, \mathbf{y}) = \int p_{\phi \,|\, h}(\phi \,|\, \mathbf{y}, \mathbf{h}) p(\mathbf{h}) dh = \mathbb{E}_h(p_{\phi \,|\, \mathbf{h}}(\phi \,|\, \mathbf{y}, \mathbf{h})),$$

$$p_\psi(\psi \,|\, \mathbf{y}) = \int p_{\psi \,|\, h}(\psi \,|\, \mathbf{y}, \mathbf{h}) p(\mathbf{h}) dh = \mathbb{E}_h(p_{\psi \,|\, \mathbf{h}}(\psi \,|\, \mathbf{y}, \mathbf{h})).$$

In practice, we first compute the conditional density $p_{\phi \,|\, h}(\phi \,|\, \mathbf{y}, \mathbf{h})$ and $p_{\psi \,|\, h}(\psi \,|\, \mathbf{y}, \mathbf{h})$ in their truncated area [-1,1] and obtain the Monte Carlo average for ϕ and ψ by summarizing the 45,000 posterior draws for $\mathbf{h}$. Then the times of the draws are scattered on the grid and normalized as the rate of recurrence, so that the area under the curve in total is 1. Figure 2.1 displays the marginal probability estimated for ϕ and ψ under **UC-ARMA** models:

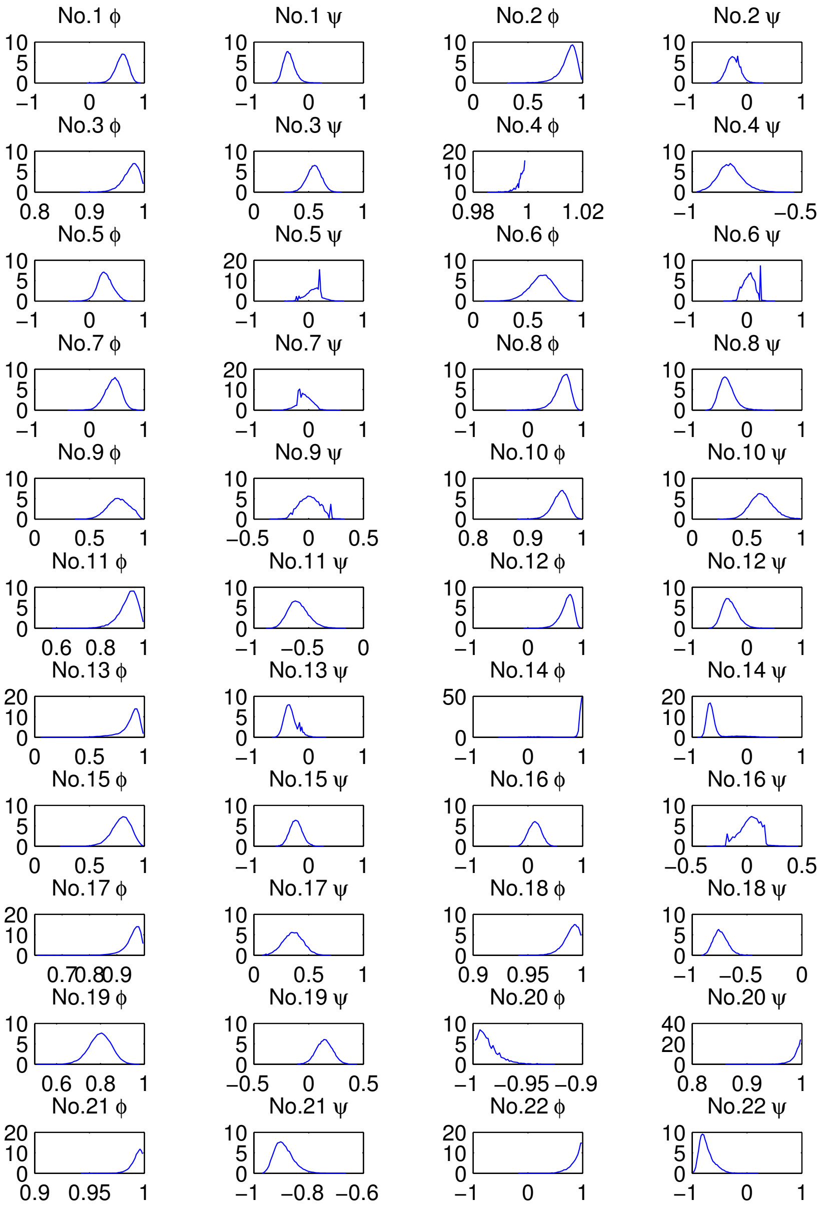

Figure 2.1: Marginal probability estimates for ϕ and ψ under **UC-ARMA** models.

It is not surprising to find that ϕ and ψ related to different macroeconomic variables have a variety of marginal distributions. Most of them have normal or truncated normal distributions or approximate truncated normal distributions, while some may have their own particular densities, such as ϕ in No. 4 and No. 14, and ψ in No. 5, 6, 7, and 16. Moreover, some truncated normal distributions may keep the major part of the distribution in the [-1,1] area, while others just keep the tail parts (ϕ in No. 14 and 22, ψ in No. 20) or a small part (ϕ in No. 17, 18, and 21).

On the one hand, the results of ϕ in Figure 2.1 also present strong evidence that the AR components are all significantly away from zero. On the other hand, some results of ψ (No. 6, 7 and 9) have a substantial mass around zero, which indicates that ψ in these data seems to have a small value and the evidence of the MA component is not supported by the empirical results. However, since the models with AR and MA components are both nested in the model with the ARMA component, the forecasting exercises presented in the next section are still based on the ARMA(1,1) specification.

Appendix 2.B A proof for $\mathbf{H}_\psi^{-1}\mathbf{H}_\phi = \mathbf{H}_\phi\mathbf{H}_\psi^{-1}$

Proposal: Suppose $\mathbf{H}_\phi$ and $\mathbf{H}_\psi$ are the following matrices, then: $\mathbf{H}_\psi^{-1}\mathbf{H}_\phi = \mathbf{H}_\phi\mathbf{H}_\psi^{-1}$.

$$\mathbf{H}_\phi = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -\phi_1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ -\phi_p & \cdots & -\phi_1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & -\phi_p & \cdots & -\phi_1 & 1 \end{pmatrix}, \quad \mathbf{H}_\psi = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \psi_1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \psi_q & \cdots & \psi_1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & \psi_q & \cdots & \psi_1 & 1 \end{pmatrix}.$$

Proof: Let L_i be a $T \times T$ matrix that has only the nonzero elements 1 on the i-th lower diagonal. In particular, $L_0 = I$ (identity matrix) and $L_{T+j} = 0$ for $j \geqslant 1$,

$$L_i = \begin{pmatrix} 0_0 & 0 & 0 & 0 & \cdots & & 0 \\ \vdots & 0 & 0 & 0 & \cdots & & 0 \\ 0_{i-1} & \ddots & \ddots & \ddots & & & \vdots \\ 1_i & \ddots & \ddots & 0 & \cdots & & 0 \\ 0_{i+1} & \ddots & 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{pmatrix}.$$

It is easy to check that $L_iL_j = L_{i+j} = L_jL_i$, when $i, j \geqslant 0$ and $i + j \leqslant T$. Then we can write $\mathbf{H}_\phi$ and $\mathbf{H}_\psi$ as:

$$\mathbf{H}_\phi = I - \sum_{i=1}^{p} \phi_i L_i, \quad \mathbf{H}_\psi = I - \sum_{j=1}^{q} \psi_j L_j.$$

So that:

$$\begin{aligned}
\mathbf{H}_\phi\mathbf{H}_\psi &= (I - \sum_{i=1}^{p}\phi_i L_i)(I - \sum_{j=1}^{q}\psi_j L_j) \\
&= I - \sum_{i=1}^{p}\phi_i L_i - \sum_{j=1}^{q}\psi_j L_j + \sum_{i=1}^{p}\sum_{j=1}^{q}\phi_i\psi_j L_i L_j \\
&= I - \sum_{j=1}^{q}\psi_j L_j - \sum_{i=1}^{p}\phi_i L_i + \sum_{j=1}^{q}\sum_{i=1}^{p}\psi_j\phi_i L_j L_i \\
&= (I - \sum_{j=1}^{q}\psi_j L_j)(I - \sum_{i=1}^{p}\phi_i L_i) \\
&= \mathbf{H}_\psi\mathbf{H}_\phi.
\end{aligned}$$

Then we have:

$$\mathbf{H}_\psi^{-1}(\mathbf{H}_\phi\mathbf{H}_\psi)\mathbf{H}_\psi^{-1} = \mathbf{H}_\psi^{-1}(\mathbf{H}_\psi\mathbf{H}_\phi)\mathbf{H}_\psi^{-1}.$$

Finally,

$$\mathbf{H}_\psi^{-1}\mathbf{H}_\phi = \mathbf{H}_\phi\mathbf{H}_\psi^{-1}.$$

Appendix 2.C Forecasting Results

		MSFE for Point Forecasting					LPL for Density Forecasting				
		h1	h4	h8	h12	h16	h1	h4	h8	h12	h16
	RW	12.718	18.316	23.017	24.596	19.909					
	UC	8.9716	9.5263	9.7616	9.9815	9.7403	-324.69	-338.25	-341.82	-342.46	-334.84
	UC_MA	8.2319	9.5159	9.7418	9.9706	9.7916	-318.24	-333.05	-337.6	-341.24	-336.26
	UC_ARMA	7.6148	9.2405	9.6173	9.8324	9.4932	-313.83	-328.21	-333.68	-339.39	-336.13
	UC_ARMANoSV	7.6934	9.602	9.6095	9.7795	9.4628	-329.99	-343.99	-345.59	-347.14	-343.96
	AR2	7.7939	9.7007	9.9513	10.103	10.056	-313.17	-333.09	-337.18	-340.77	-337.08
MACRO 1	AR2_MA	7.7867	9.6921	10.021	10.117	10.06	-312.67	-333.82	-338.43	-342.49	-339.12
	AR2_ARMA	7.7155	10.2	11.042	11.729	11.599	-315.88	-341.49	-347.42	-352.34	-349.42
	AR2_ARMANoSV	7.7762	10.132	10.734	11.07	10.804	-319.94	-339.54	-342.09	-343.89	-341.31
	AR4	8.0013	9.7337	10.029	10.142	10.062	-313.66	-334.1	-337.74	-341.87	-338
	AR4_MA	7.9659	9.7228	10.093	10.154	10.052	-313.67	-333.8	-337.94	-341.84	-338
	AR4_ARMA	7.9767	9.9241	10.316	10.965	10.647	-319.11	-340.24	-348.23	-353.71	-351.21
	AR4_ARMANoSV	7.9422	9.8822	10.561	11.126	10.801	-322.48	-337.33	-340.36	-343.99	-341.53
	RW	5.565	9.0055	11.813	14.044	15.398					
	UC	5.705	7.1572	9.1106	10.256	10.556	-273.06	-301.95	-320.32	-320.83	-325.86
	UC_MA	4.6959	6.9824	8.4487	9.2928	9.5417	-270.26	-300.23	-314.61	-314.67	-317.26
	UC_ARMA	4.6579	7.0136	7.9388	8.4739	8.3658	-270.67	-302.8	-315.95	-313.28	-311.51
	UC_ARMANoSV	4.8221	6.3749	8.0518	8.7681	8.7357	-311.71	-318.53	-333.54	-339.91	-337.97
	AR2	5.031	6.5897	8.6648	10.054	10.363	-274.66	-304.71	-324.6	-330.53	-333.08
MACRO 2	AR2_MA	4.9078	6.4778	8.6403	9.9225	10.237	-273.44	-308.07	-330.15	-336.65	-338.25
	AR2_ARMA	4.5804	5.7866	7.4403	8.1655	7.9824	-267.74	-287.02	-297.61	-301.32	-299.21
	AR2_ARMANoSV	4.6176	5.8337	7.556	8.4439	8.1911	-292.51	-289.91	-302.38	-309.23	-309
	AR4	4.7023	6.4394	8.6489	9.8004	10.257	-268.26	-303.73	-323.88	-327.1	-330.16
	AR4_MA	4.6496	6.4089	8.6974	9.908	10.378	-268.11	-303.01	-322.59	-326.23	-329.68
	AR4_ARMA	4.5407	6.0157	7.6478	8.2365	8.2137	-267.2	-293.84	-306.94	-306.5	-306.21
	AR4_ARMANoSV	4.6188	6.073	7.8926	8.7918	9.3795	-283.95	-309.46	-317.82	-313.93	-316.86
	RW	0.88466	4.3436	10.399	15.612	18.904					
	UC	1.5377	4.7	10.093	15.113	17.335	-146.14	-301.35	-394.09	-419.86	-407.54
	UC_MA	1.9269	7.2364	11.053	13.848	14.508	-149.33	-348.48	-441.43	-462.3	-437.16
	UC_ARMA	0.94427	4.6619	9.5606	12.967	14.521	-103.96	-337.53	-444.2	-472.68	-472.2
	UC_ARMANoSV	1.0234	4.7069	9.6363	12.587	14.102	-197.63	-305.16	-366.4	-402	-407.93
	AR2	0.99399	4.5933	9.989	14.065	16.052	-90.445	-281.57	-373.89	-405.11	-414.08
MACRO 3	AR2_MA	0.99614	4.4631	9.6312	13.599	15.615	-92.606	-294.51	-388.65	-419.75	-424.18
	AR2_ARMA	0.92645	6.5561	12.097	15.557	16.979	-93.605	-302.02	-366.94	-390.22	-396.05
	AR2_ARMANoSV	0.96561	6.2554	10.621	13.358	14.951	-146.3	-310.09	-365.68	-384.27	-392.47
	AR4	1.0176	4.2308	9.7096	13.868	15.763	-93.097	-284.5	-376.87	-408.44	-416.31
	AR4_MA	1.0179	4.2248	9.609	13.638	15.548	-92.697	-284.85	-375.47	-406.48	-415.26
	AR4_ARMA	1.0169	4.4057	9.5694	13.236	15.272	-105.51	-282.96	-362.37	-397.78	-404.8
	AR4_ARMANoSV	1.2017	4.1724	9.1098	12.904	15.258	-155.74	-282.25	-349.16	-383.52	-390.47
	RW	1.86E+05	1.08E+05	1.28E+05	1.53E+05	1.65E+05					
	UC	1.06E+05	1.06E+05	1.06E+05	1.06E+05	1.06E+05	-960	-965.61	-966.11	-966.2	-958.04
	UC_MA	1.11E+05	1.06E+05	1.06E+05	1.06E+05	1.06E+05	-962.71	-966.51	-966.34	-967.64	-960.29
	UC_ARMA	1.16E+05	1.06E+05	1.22E+05	1.86E+05	5.13E+05	-983.78	-1014.3	-1042.2	-1062.1	-1069.9
	UC_ARMANoSV	1.11E+05	1.13E+05	1.12E+05	1.12E+05	1.13E+05	-983.92	-980.28	-981.19	-981.81	-975.81
	AR2	96618	98816	1.06E+05	1.06E+05	1.06E+05	-955.51	-966.99	-968.07	-968.67	-959.86
MACRO 4	AR2_MA	97260	99643	1.06E+05	1.06E+05	1.06E+05	-956.23	-967.15	-967.73	-968.33	-959.77
	AR2_ARMA	95940	97743	1.05E+05	1.05E+05	1.06E+05	-954.93	-966.87	-969.15	-970.11	-961.62
	AR2_ARMANoSV	89893	1.35E+05	1.47E+05	1.84E+05	1.86E+05	-949.61	-983.62	-986.58	-989.48	-982.91
	AR4	88264	86824	1.01E+05	1.04E+05	1.06E+05	-949.53	-961.16	-966.98	-970.69	-961.5
	AR4_MA	88905	86893	1.00E+05	1.04E+05	1.06E+05	-949.97	-961.01	-966.39	-970.48	-961.52
	AR4_ARMA	88472	86675	99727	1.03E+05	1.05E+05	-949.54	-961.48	-967.87	-971.1	-962.17
	AR4_ARMANoSV	88769	77851	1.03E+05	1.13E+05	1.17E+05	-983.92	-946.16	-961.49	-969.48	-963.77
	RW	893.86	1317.6	1392.5	1355.5	1350.6					
	UC	703.63	710.06	710.73	712.05	715.92	-626.62	-634.36	-634.59	-631.09	-620.73
	UC_MA	643.76	711.41	713.7	714.08	718.56	-620.39	-634.04	-635.67	-632.66	-623.09
	UC_ARMA	644.23	732.59	736.56	735.89	738.77	-621.4	-637.41	-644.38	-647.08	-642.06
	UC_ARMANoSV	684.52	875.28	832.43	843.99	827.04	-633.23	-652.2	-660.08	-656.58	-664.67
	AR2	643.29	699.3	696.59	696.81	701.48	-619.58	-631.44	-633.2	-629.51	-620.64
MACRO 5	AR2_MA	644.17	698.9	696.28	696.67	700.76	-619.56	-631.93	-633.33	-630.21	-621.52
	AR2_ARMA	646.29	704.4	695.6	700.55	705.01	-621.32	-636.12	-638.35	-636.46	-628.46
	AR2_ARMANoSV	645.18	701.09	700.8	702.75	707.15	-640.75	-632.93	-632.81	-633.04	-628.71
	AR4	647.46	705.76	697.61	697.46	701.41	-621.15	-632.39	-633.63	-629.6	-620.75
	AR4_MA	646.87	706.36	697.21	696.99	700.74	-620.92	-632.57	-633.57	-629.65	-620.71
	AR4_ARMA	658.11	735.97	704.59	690.07	698.56	-626.07	-636.95	-638.78	-636.44	-628.61
	AR4_ARMANoSV	667.31	724.43	751.78	733.37	724.05	-636.85	-643.74	-634.36	-627.54	-620.34

	RW	9.2632	14.843	16.49	17.474	19.209					
	UC	8.7605	10.478	12.353	13.597	15.038	-334.03	-360.11	-374.21	-379.63	-381.9
	UC_MA	7.6086	10.607	11.975	12.987	14.102	-325.58	-357.24	-366.78	-373.8	-370.4
	UC_ARMA	8.0666	10.334	11.621	12.471	13.468	-325.79	-354.63	-362.66	-369.86	-365.9
	UC_ARMANoSV	8.7603	11.899	12.206	13.957	14.756	-349.3	-360.43	-362.53	-376.44	-375.38
	AR2	8.3751	11.779	12.684	12.96	13.265	-329.1	-359.44	-367.64	-370.39	-364.52
MACRO 6	AR2_MA	8.3629	11.553	12.516	12.855	13.18	-328.69	-357.89	-366.22	-369.5	-363.21
	AR2_ARMA	8.4908	22.82	19.263	18.009	16.834	-328.28	-402.1	-404.34	-409.91	-411.14
	AR2_ARMANoSV	8.5507	28.154	23.972	22.847	20.906	-332.3	-410.15	-414.63	-416.33	-412.09
	AR4	8.1946	11.498	12.477	12.863	13.289	-327.69	-357.45	-366.15	-369.91	-364.34
	AR4_MA	8.2499	11.542	12.427	12.794	13.211	-327.63	-358.06	-366.17	-369.97	-364.51
	AR4_ARMA	8.1464	10.037	10.425	10.784	11.41	-329.92	-359.5	-372.27	-380.87	-384.29
	AR4_ARMANoSV	8.0362	10.143	10.383	11.225	12.14	-324.05	-339.56	-347.01	-350.73	-354.47
	RW	16.013	22.195	25.59	26.938	24.663					
	UC	13.78	14.738	15.256	15.768	15.923	-358.41	-372.77	-378.34	-379.85	-374.41
	UC_MA	12.903	14.637	15.108	15.574	15.771	-355.09	-372.71	-378.33	-379.27	-373.73
	UC_ARMA	12.26	14.551	15.058	15.555	15.802	-352.38	-370.63	-377.2	-378.02	-372.86
	UC_ARMANoSV	12.602	15.618	15.803	16.305	16.383	-372.62	-394.61	-394.16	-396.52	-394.94
	AR2	12.414	15.701	16.853	17.343	17.474	-352.41	-370.97	-379.91	-383.82	-381.31
MACRO 7	AR2_MA	12.407	15.688	16.784	17.374	17.579	-352.56	-371.16	-380.79	-385.27	-382.98
	AR2_ARMA	12.628	16.647	19.699	21.548	23.592	-355.7	-384.06	-395.77	-398.27	-397.6
	AR2_ARMANoSV	12.818	17.535	20.738	23.269	26.778	-362.3	-383.96	-388.25	-393.17	-390.65
	AR4	12.65	16.051	16.638	17.356	18.149	-353.84	-372.09	-379.08	-385.1	-384.87
	AR4_MA	12.555	16.005	16.668	17.371	18.113	-353.6	-371.71	-378.96	-384.97	-384.68
	AR4_ARMA	12.992	16.448	16.297	16.134	16.332	-357.86	-377.01	-381.35	-382.56	-381.82
	AR4_ARMANoSV	13.081	17.021	16.829	16.227	16.31	-365.65	-377.78	-374.76	-369.05	-366.16
	RW	8.2403	10.171	12.874	14.022	13.528					
	UC	5.9035	6.5744	7.1589	7.2857	7.2322	-300.87	-319.16	-327.98	-324.34	-315.67
	UC_MA	5.6762	6.5503	7.0954	7.2458	7.2331	-297.56	-317.08	-326.2	-324.12	-316.06
	UC_ARMA	5.4375	6.4532	7.0573	7.2167	7.216	-294.13	-313.51	-323.39	-322.83	-315.61
	UC_ARMANoSV	5.3639	6.4974	7.0905	7.1843	7.1046	-303.94	-316.78	-322.27	-323.01	-320.15
	AR2	5.5123	6.6967	7.2087	7.313	7.3786	-294.41	-317.02	-326.73	-324.25	-318.2
MACRO 8	AR2_MA	5.4303	6.5841	7.1727	7.296	7.3785	-293.37	-315.4	-325.44	-323.71	-318.27
	AR2_ARMA	5.3539	7.347	8.2648	8.7517	9.3043	-296.73	-329.57	-338.82	-336.71	-333.19
	AR2_ARMANoSV	5.4418	7.3089	7.83	8.0965	8.289	-297.13	-316.47	-320.7	-320.53	-318.13
	AR4	5.3973	6.7158	7.1734	7.3146	7.3812	-290.67	-318.21	-327.5	-323.96	-317.15
	AR4_MA	5.4302	6.7443	7.192	7.3243	7.3893	-291.03	-318.13	-327.93	-324.02	-317.2
	AR4_ARMA	5.1656	6.4425	8.0606	8.7022	9.3037	-296.06	-323.94	-336.93	-339.67	-338.31
	AR4_ARMANoSV	5.3157	6.5874	8.2662	8.9339	9.4817	-295.03	-314.94	-324.75	-328.1	-327.42
	RW	24.61	62.327	70.524	72.133	66.993					
	UC	31.972	33.673	33.965	34.564	34.405	-407.35	-439.75	-437.96	-429.57	-420.3
	UC_MA	20.307	33.635	34.026	34.417	34.314	-376.53	-429.24	-431.14	-422.53	-415.04
	UC_ARMA	20.638	49.234	50.714	53.522	55.608	-375.84	-457.28	-475.39	-480.67	-485.7
	UC_ARMANoSV	20.779	38.272	39.371	35.656	36.671	-394.4	-439.62	-444.26	-448.46	-451.73
	AR2	19.415	35.158	35.245	35.745	35.782	-371.25	-425.23	-427.22	-419.81	-413.14
MACRO 9	AR2_MA	18.941	34.872	34.938	35.373	35.396	-370.72	-425.42	-427.01	-419.45	-412.61
	AR2_ARMA	18.371	34.987	37.237	37.977	38.162	-373.08	-426.02	-430.32	-425.03	-421.99
	AR2_ARMANoSV	18.701	33.865	43.269	41.303	40.127	-378.2	-422.47	-425.55	-426.3	-424.62
	AR4	18.671	34.917	34.918	35.433	35.437	-370.93	-424.02	-425.6	-418.74	-412.47
	AR4_MA	18.739	35.061	34.886	35.319	35.301	-371.02	-425.35	-426.36	-419.23	-412.97
	AR4_ARMA	19.271	36.631	36.165	37.253	37.94	-376.9	-429.72	-431.66	-426.2	-422.82
	AR4_ARMANoSV	18.594	35.309	35.417	35.474	36.183	-377.38	-418.83	-418.84	-419.66	-418.09
	RW	0.10588	1.111	2.8718	4.2685	5.0208					
	UC	0.12741	1.1825	2.9066	4.2739	5.019	-47.411	-257.67	-360.47	-390.09	-350.53
	UC_MA	0.18452	1.3908	3.0456	4.3003	4.9375	-46.923	-257.86	-376.93	-397.28	-386.15
	UC_ARMA	0.081121	1.2271	2.9545	4.2919	5.0987	-3.5403	-244.02	-378.96	-428.43	-434.59
	UC_ARMANoSV	0.074858	1.0162	2.5059	3.5109	3.9952	-16.768	-208.85	-288.24	-309.29	-312.53
	AR2	0.062938	0.9442	2.7521	4.1194	4.9818	4.9359	-173.08	-268.41	-313.84	-328.27
MACRO 10	AR2_MA	0.0643	0.93872	2.6828	4.0264	4.8724	3.6297	-174.05	-269.8	-311.74	-325.52
	AR2_ARMA	0.074955	2.2848	5.5848	7.6918	9.1508	-0.72958	-240.92	-333.92	-366.39	-372.92
	AR2_ARMANoSV	0.075733	2.4175	5.2363	7.4581	8.7127	-6.7529	-280.89	-416.3	-399.75	-380.15
	AR4	0.062803	0.92192	2.691	4.0507	4.9166	5.3615	-172.54	-267.05	-310.28	-323.76
	AR4_MA	0.06361	0.9191	2.718	4.112	5.0007	5.3505	-170.72	-266.36	-309.25	-323.57
	AR4_ARMA	0.085699	1.2563	2.9255	4.338	5.2097	-7.113	-175.79	-251.97	-286.67	-311.09
	AR4_ARMANoSV	0.10303	1.2317	2.571	3.5904	4.0854	-20.684	-184.76	-254.09	-285.03	-301.65
	RW	1651.9	2216	2295.9	2632.5	2823.8					
	UC	1140.3	1144.1	1148.3	1151.4	1154.4	-648.9	-654.98	-663.24	-669.93	-662.6

	UC_MA	1048	1143.4	1146.9	1149.2	1153.1	-645.14	-652.74	-661.51	-668.29	-662.41
	UC_ARMA	1091.5	1592.9	1404	1497.7	1494	-654.32	-679.34	-693.42	-703.5	-702.48
	UC_ARMANoSV	1203	1549.3	1717.9	1737.1	1966.7	-665.45	-682.3	-696.96	-705.93	-711
	AR2	1062.7	1156	1157.1	1162.8	1171	-646.06	-654.65	-662.75	-669.7	-663.88
MACRO 11	AR2_MA	1062.3	1159.5	1156.5	1163.1	1170.6	-646.18	-655.05	-662.59	-669.47	-663.95
	AR2_ARMA	1065.2	1164.9	1145.3	1154.6	1166.8	-646.37	-654.79	-661.03	-667.77	-662.55
	AR2_ARMANoSV	1217.4	1802.7	1920.9	1921.4	2073.3	-654.62	-688.11	-695.45	-697.03	-693.27
	AR4	1096.9	1186.1	1158.5	1157.6	1164.2	-648.75	-656.71	-662.35	-668.3	-662.64
	AR4_MA	1095.7	1188.9	1161.8	1156.7	1165.3	-648.72	-656.81	-662.43	-668.28	-662.73
	AR4_ARMA	1076.6	1150	1144.9	1150.4	1165.2	-648.54	-655.75	-662.56	-668.33	-663.04
	AR4_ARMANoSV	1147	1495.2	1719.9	1866	1953.8	-654.54	-671.2	-685.62	-690.08	-685.73
	RW	33.798	57.205	56.715	58.316	61.94					
	UC	28.814	30.986	32.213	33.303	33.849	-403.45	-417.32	-420.21	-414.31	-412.43
	UC_MA	24.901	30.183	31.301	32.131	32.453	-392.92	-415.41	-418.86	-412.83	-408.68
	UC_ARMA	26.385	35.864	31.432	31.058	30.476	-390.73	-417.94	-419.39	-411.57	-405.09
	UC_ARMANoSV	26.336	32.457	31.165	31.316	31.036	-420.6	-432.94	-423.07	-423.45	-419.78
	AR2	27.138	31.144	30.929	31.402	30.818	-392.3	-413.27	-417.75	-412.83	-406.81
MACRO 12	AR2_MA	27.169	32.319	31.924	32.106	31.701	-391.98	-419.6	-427.92	-424.79	-419.35
	AR2_ARMA	25.442	29.502	29.77	30.083	29.775	-388.21	-408.62	-410.74	-407.56	-401.86
	AR2_ARMANoSV	25.32	29.361	29.46	29.997	30.14	-423.52	-428.3	-416.73	-419	-417.88
	AR4	26.384	32.065	32.226	32.414	32.117	-389.66	-415.09	-419.81	-413.16	-407.72
	AR4_MA	26.346	32.075	32.287	32.477	32.176	-389.93	-415.24	-419.97	-413.2	-407.81
	AR4_ARMA	25.858	30.453	30.685	30.33	30.162	-388.34	-411.09	-413.91	-408.43	-403.47
	AR4_ARMANoSV	26.522	30.511	30.904	30.983	31.191	-410.58	-436.87	-439.01	-429.93	-430.91
	RW	2.498	4.2252	5.5956	6.7976	7.6029					
	UC	2.5488	3.5183	4.7071	5.6013	6.1751	-227.28	-260.73	-284.67	-289.15	-296.44
	UC_MA	2.2038	3.5741	4.589	5.3558	5.8439	-225.87	-261.51	-282.79	-286.32	-291.46
	UC_ARMA	2.0746	3.4867	3.935	4.2901	4.4985	-222.91	-257.69	-275.51	-272.5	-272.74
	UC_ARMANoSV	2.1296	3.1444	3.9855	4.4503	4.7991	-245.52	-270.23	-285.33	-294.17	-298.55
	AR2	2.1901	3.219	4.1707	4.7784	5.183	-225.18	-258.74	-281.2	-286.14	-290.69
MACRO 13	AR2_MA	2.1583	3.138	4.1501	4.7397	5.1374	-224.66	-260.1	-283.92	-289.75	-293.43
	AR2_ARMA	2.0677	2.8885	3.869	4.311	4.4385	-223.1	-248.26	-262.43	-265.93	-265.76
	AR2_ARMANoSV	2.0973	2.8225	3.8578	4.3146	4.6315	-241.46	-245.87	-264.22	-266.52	-272.14
	AR4	2.0995	3.0908	4.1852	4.7624	5.2386	-221.7	-257.85	-279.6	-282.74	-287.95
	AR4_MA	2.0922	3.0847	4.1795	4.7392	5.1906	-221.98	-257.53	-279.05	-282.3	-287.53
	AR4_ARMA	2.1157	3.0572	3.9028	4.2598	4.3957	-222.27	-251.56	-267.63	-266.81	-268.18
	AR4_ARMANoSV	2.1456	3.0725	3.8915	4.2397	4.5748	-226.45	-258.81	-271.62	-266.89	-272.56
	RW	2.5307	3.4325	4.7711	4.8809	6.0411					
	UC	2.287	3.1213	4.1553	5.2068	6.2602	-231.91	-252.67	-267.91	-285.06	-296.42
	UC_MA	2.3564	3.1828	4.2202	5.249	6.3263	-232.3	-251.41	-267.15	-283.83	-295.67
	UC_ARMA	2.2246	3.3362	4.3418	5.1002	6.2362	-230.2	-265.98	-285.77	-300.94	-314.72
	UC_ARMANoSV	2.1355	2.8707	3.7721	4.6742	5.886	-243.35	-263.93	-284.2	-301.88	-322.11
	AR2	1.7684	2.7779	3.721	4.2863	4.9633	-230.38	-258.06	-275.31	-287.42	-293.77
MACRO 14	AR2_MA	1.735	2.649	3.5242	4.1538	5.0063	-228.06	-258.14	-277.61	-291.69	-299.38
	AR2_ARMA	1.7451	3.7285	5.2625	5.8258	7.0404	-232.12	-284.27	-302.78	-307.84	-309.6
	AR2_ARMANoSV	1.8314	9.2359	9.8202	10.42	11.665	-231.66	-347.72	-369.22	-366.42	-365.67
	AR4	1.826	2.6294	3.4568	3.9483	4.8619	-227.85	-249.26	-264.11	-278.06	-286.31
	AR4_MA	1.8029	2.6277	3.462	3.9682	4.8806	-227.53	-249.63	-264.73	-278.93	-287.09
	AR4_ARMA	1.7882	2.8342	4.5258	5.842	7.5578	-232.6	-261.78	-286.15	-302.04	-308.91
	AR4_ARMANoSV	1.8009	2.9702	4.832	6.4194	8.4414	-231.71	-260.79	-286.42	-302.82	-313.07
	RW	23.865	48.594	55.012	68.597	75.557					
	UC	32.522	39.616	44.54	45.93	46.776	-422.2	-446.93	-450.33	-463.18	-458.36
	UC_MA	27.777	41.784	43.862	44.478	43.798	-412.21	-450.7	-451.28	-459.99	-459.63
	UC_ARMA	22.022	37.393	40.855	42.364	42.661	-397.66	-446.16	-449.25	-459.16	-460.03
	UC_ARMANoSV	23.101	38.174	38.446	41.581	42.414	-433.66	-476.78	-489.57	-502.08	-510.02
	AR2	21.267	35.441	38.411	39.404	39.598	-394.77	-438.81	-439.3	-444.37	-444
MACRO 15	AR2_MA	21.363	35.429	38.382	39.436	39.646	-395.06	-437.78	-438.93	-443.82	-443.46
	AR2_ARMA	21.437	41.252	43.112	43.458	43.536	-394.74	-442.05	-442.98	-447.12	-446.79
	AR2_ARMANoSV	21.884	39.83	40.381	41.366	41.525	-428.07	-462.24	-449.53	-447.28	-446.11
	AR4	21.779	35.496	38.371	39.504	39.711	-396.66	-438.57	-439.13	-445.01	-443.47
	AR4_MA	21.775	35.513	38.331	39.424	39.642	-396.56	-438.39	-438.74	-444.67	-443.38
	AR4_ARMA	21.853	37.606	44	48.709	49.761	-394.77	-438.86	-448.09	-457.41	-456.47
	AR4_ARMANoSV	22.365	38.108	44.892	49.228	48.918	-429.43	-490.01	-500.09	-498.66	-486.88
	RW	4691.1	6765.9	6540.3	6272.4	6366.9					
	UC	3333.4	3332.1	3331.5	3331.4	3200.6	-735.07	-749.76	-741.1	-735.3	-721.67
	UC_MA	3271.9	3958.9	3.52E+10	4.27E+15	9.94E+20	-757.52	-805.76	-833.98	-856.29	-866.9
	UC_ARMA	3289.3	8.73E+06	4.09E+16	4.25E+19	2.52E+25	-759.24	-806.41	-836.8	-859.6	-869.72

	UC_ARMANoSV	3193.5	3404.8	3516.1	3291.6	3217.1	-741.54	-747.52	-744.49	-743.54	-737.44
	AR2	3346.3	3.21E+07	1.47E+17	3.42E+23	1.44E+30	-786.01	-849.39	-888.55	-914.05	-924.7
MACRO 16	AR2_MA	3346.3	2.07E+12	5.24E+23	1.88E+34	2.30E+38	-786.24	-838.87	-881.72	-908.15	-920.31
	AR2_ARMA	3345	3.06E+16	3.49E+22	1.28E+32	2.57E+41	-783.46	-839.52	-878.5	-906.26	-918.31
	AR2_ARMANoSV	3200.2	3374	3342.3	3339.8	3205.1	-756.95	-749.05	-744.51	-740.02	-735.94
	AR4	3345.6	1.62E+08	6.23E+17	2.36E+25	4.63E+36	-784.38	-846.77	-885.01	-910.47	-921.46
	AR4_MA	3345.7	1.73E+07	1.66E+23	4.28E+26	2.03E+32	-784.21	-846.54	-884.77	-910.27	-920.77
	AR4_ARMA	3336.3	5.27E+15	2.52E+21	3.80E+28	6.51E+32	-777.96	-833.74	-871.12	-897.41	-908.47
	AR4_ARMANoSV	3229.5	3396.8	3351	3339.6	3206.5	-753.96	-763.95	-749.13	-744.92	-740.3
	RW	0.29869	1.6849	3.1751	4.2937	6.1046					
	UC	0.7714	2.0254	3.1914	4.5783	6.2929	-136.48	-264.22	-359.86	-407.42	-413.51
	UC_MA	0.63997	2.2584	3.2669	4.5289	5.8662	-125.59	-260.22	-332.55	-376.54	-386.11
	UC_ARMA	0.31372	1.8465	3.1143	3.9817	5.2167	-101.95	-254.86	-315.14	-339.37	-357.84
	UC_ARMANoSV	0.37631	1.8404	3.2863	4.4039	6.1597	-151.15	-322.13	-531.87	-724.4	-901.54
	AR2	0.30808	1.7138	3.1753	4.2014	5.8454	-103.72	-220.68	-275.65	-304.96	-319
MACRO 17	AR2_MA	0.30372	1.7151	3.1721	4.2128	5.8793	-101.88	-223.32	-276.75	-305.34	-320.75
	AR2_ARMA	0.30903	5.2773	13.541	15.159	14.646	-102.2	-273.03	-318.04	-332.91	-337.7
	AR2_ARMANoSV	0.32565	4.497	9.6252	11.744	12.26	-101.67	-281.67	-328.56	-340.98	-344.65
	AR4	0.30555	1.7645	3.2688	4.1922	5.7556	-101.89	-222.45	-278.01	-306.87	-321.62
	AR4_MA	0.30631	1.7513	3.2366	4.1891	5.7763	-101.63	-221.57	-277.21	-306.19	-320.86
	AR4_ARMA	0.47851	2.3258	4.3624	6.0551	7.7323	-113	-224.95	-256.9	-275.34	-292.88
	AR4_ARMANoSV	0.4585	1.932	3.5496	4.7449	6.2305	-105.74	-233.1	-254.92	-269.17	-286.53
	RW	365.94	610.79	622.14	652.95	582.08					
	UC	283.8	286.63	287.35	288.26	286.36	-561.06	-571.58	-574.92	-572.35	-566.41
	UC_MA	270.24	286.77	287.62	288.76	286.73	-558.06	-571.28	-575.91	-572.68	-568.58
	UC_ARMA	294.39	595.79	563.54	569.99	518.75	-568.91	-623.39	-646.33	-656.31	-663.48
	UC_ARMANoSV	296.76	388.38	376.59	416.47	402.08	-572.46	-590.01	-590.53	-597.11	-591.9
	AR2	274.89	291.78	289.39	291.17	289.34	-557.9	-570.81	-575.58	-572.84	-568.6
MACRO 18	AR2_MA	273.04	289.47	290.21	289.97	289.17	-557.83	-577.72	-586.02	-586.18	-583.41
	AR2_ARMA	271.91	294.17	297.84	298.07	289.62	-559.1	-574.28	-579.46	-578.68	-573.81
	AR2_ARMANoSV	293.12	316.47	387.95	351.31	365.19	-560.69	-580.02	-585.67	-587.47	-584.98
	AR4	284.29	291.96	290.83	291.54	289.84	-559.18	-571.34	-576.22	-573.43	-569.2
	AR4_MA	279.06	292.11	291.05	291	288.78	-559.14	-571.43	-576.05	-573.26	-568.84
	AR4_ARMA	295.67	311.74	288.23	293.07	289.93	-563.1	-575.41	-580.2	-580.42	-576.52
	AR4_ARMANoSV	309.81	329.35	317.79	307.85	315.22	-564.99	-576.74	-582.45	-584.7	-583.58
	RW	1.2541	5.4993	9.1232	10.676	9.9516					
	UC	2.1877	5.6837	8.0437	8.6576	7.3337	-199.55	-296.84	-334.5	-339.74	-325.58
	UC_MA	1.5594	5.0075	6.6917	7.0648	5.9753	-208.72	-307.09	-338.61	-335.64	-317.9
	UC_ARMA	1.0593	4.0768	5.0796	5.2249	5.0765	-181.14	-292.95	-328.96	-323.18	-309.09
	UC_ARMANoSV	1.0812	4.0728	4.9226	4.9533	4.7289	-199.04	-285.48	-298.89	-299.05	-293.02
	AR2	1.0996	4.321	5.4153	5.62	5.6592	-181.31	-278.81	-312.48	-314.26	-308.53
MACRO 19	AR2_MA	1.0918	4.2906	5.4096	5.6213	5.6461	-181.57	-278.02	-312.97	-314.22	-308.7
	AR2_ARMA	1.0615	4.1602	5.8464	6.4417	6.7843	-181.54	-281.62	-324.54	-329.39	-324.42
	AR2_ARMANoSV	1.0921	4.2227	5.5204	5.7706	5.8498	-193.9	-287.63	-321.94	-325.84	-324.39
	AR4	1.0785	4.5259	5.6065	5.6386	5.6303	-180.17	-279.04	-314.15	-313.79	-307.51
	AR4_MA	1.0726	4.4901	5.5677	5.6193	5.622	-180.04	-278.32	-312.4	-312.55	-306.82
	AR4_ARMA	1.0753	4.3473	6.0286	6.8212	7.2539	-183.92	-290.76	-327.94	-329.95	-324.08
	AR4_ARMANoSV	1.1026	4.4318	5.9337	6.478	6.763	-193.41	-324.74	-361.84	-358.83	-363.5
	RW	1691.3	2606	2256.1	2168.4	2308					
	UC	1080.5	1083.9	1077.5	1080.9	1076.3	-644.29	-657.11	-652.67	-649.16	-638.56
	UC_MA	1046.5	1083.7	1077	1081.4	1075.9	-635.17	-656.8	-653.71	-652.72	-642.01
	UC_ARMA	1273	2599.6	2055.8	3683.9	12180	-647.18	-719.04	-716.94	-739.67	-745.25
	UC_ARMANoSV	1354.2	1361.6	1245.1	1414.6	1477.1	-669.87	-669.31	-665.32	-672.2	-670.04
	AR2	1053.5	1097.1	1075.7	1086.1	1074.5	-635.02	-656.16	-653.28	-653.87	-643.05
MACRO 20	AR2_MA	1007.6	1068.4	1073.8	1085.7	1073.8	-630.71	-663	-662.64	-661.88	-652.01
	AR2_ARMA	996.41	1135.2	1083.9	1079.4	1072.9	-629.72	-670.59	-669.54	-669.51	-659.99
	AR2_ARMANoSV	1314.7	2659.7	2213.7	2195	2260.5	-657.95	-731.95	-733.47	-741.89	-743.38
	AR4	956.46	996.81	1099.8	1075.4	1076.1	-629.92	-650	-654.86	-651.68	-642.3
	AR4_MA	957.62	995.88	1099.8	1078.5	1076.7	-629.61	-650.01	-654.77	-651.93	-642.48
	AR4_ARMA	946.64	994.75	1098.7	1082.6	1078.3	-628.19	-648.87	-655.12	-653.79	-645.42
	AR4_ARMANoSV	1475.5	1308.4	1424.4	1254.9	1180.7	-671.36	-674.98	-687.78	-692.2	-689.93
	RW	4663.6	5409.5	5083	5004.8	5161.2					
	UC	2458.2	2460.7	2454.2	2458.8	2466	-700.69	-718.01	-717.53	-712.89	-697.98
	UC_MA	2495.9	2460.3	2453.9	2459.2	2465.7	-694.17	-714.43	-718.53	-717.45	-702.86
	UC_ARMA	2705.2	5376.4	4543.9	4787.7	17549	-697.83	-760.36	-774.9	-790.42	-791.09
	UC_ARMANoSV	2911.8	2687.3	2613	2711.3	2744.3	-727.22	-724.13	-720.67	-724.29	-718.72
	AR2	2476.6	2477.3	2453.5	2464.4	2464.4	-692.3	-713.02	-717.41	-717.95	-704.54

MACRO 21	AR2_MA	2343.9	2432.7	2454.7	2463.2	2467.4	-687.28	-714.75	-721.04	-721.58	-708.35
	AR2_ARMA	2327.7	2491.8	2467	2460.7	2469.7	-686.73	-722.6	-726.23	-726.48	-714.61
	AR2_ARMANoSV	2992.2	2912.3	3115.9	2861.3	3213.7	-709.18	-712.21	-715.75	-715.47	-711.05
	AR4	2397.1	2409.3	2511.7	2455.9	2465	-690.19	-710.73	-718.62	-716.55	-703.59
	AR4_MA	2358	2382.8	2500.6	2454.8	2463.7	-688.15	-709.56	-718.3	-717.07	-703.9
	AR4_ARMA	2349.3	2376.3	2510.9	2459.2	2463.6	-687.57	-708	-717.17	-715.81	-703.87
	AR4_ARMANoSV	2556.5	2502.6	2543.1	2457.1	2442.1	-723.56	-716.82	-718.04	-712.2	-704.32
MACRO 22	RW	15.696	16.959	17.511	19.541	14.305					
	UC	7.9133	8.0258	8.1257	8.4283	8.1923	-331.12	-333.41	-336.59	-338.3	-334.41
	UC_MA	7.948	8.0189	8.1245	8.4394	8.1891	-332.27	-334.55	-338.82	-340.81	-336.88
	UC_ARMA	8.3371	12.282	11.426	12.011	9.8228	-335.8	-374.65	-383.73	-391.19	-388.08
	UC_ARMANoSV	7.9782	8.0534	8.1445	8.3872	8.1989	-333.24	-334.08	-335.04	-336.61	-333.45
	AR2	7.8652	8.0214	8.0306	8.1434	8.0662	-331.26	-333.13	-336.93	-338.26	-336.59
	AR2_MA	7.8542	8.0353	8.0571	8.1449	8.0714	-331.26	-336.07	-339.87	-341.23	-339.85
	AR2_ARMA	7.8491	8.1364	8.2688	8.5015	8.3057	-332.13	-336.6	-340.72	-342.75	-341.44
	AR2_ARMANoSV	7.784	7.9947	8.1764	8.2014	8.0827	-328.42	-329.61	-331.12	-330.72	-327.95
	AR4	7.9948	8.0501	8.0949	8.1642	8.0773	-332.17	-333.7	-337.57	-338.66	-337.36
	AR4_MA	7.98	8.0359	8.1016	8.1602	8.0664	-332.05	-333.5	-337.68	-338.68	-337.27
	AR4_ARMA	7.9873	8.2407	8.3637	8.7482	8.4707	-334	-341.11	-347.44	-350.57	-349.7
	AR4_ARMANoSV	7.9361	8.164	8.339	8.5775	8.4006	-329.59	-330.34	-332.64	-333.86	-331.43

		Relative MSFE for Point Forecasting					Relative LPL for Density Forecasting				
		h1	h4	h8	h12	h16	h1	h4	h8	h12	h16
MACRO 1	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.7054	0.5201	0.4241	0.4058	0.4892	0	0	0	0	0
	UC_MA	0.6473	0.5195	0.4232	0.4054	0.4918	6.45	5.2	4.22	1.22	-1.42
	UC_ARMA	0.5987	0.5045	0.4178	0.3998	0.4768	10.86	10.04	8.14	3.07	-1.29
	UC_ARMANoSV	0.6049	0.5242	0.4175	0.3976	0.4753	-5.3	-5.74	-3.77	-4.68	-9.12
	AR2	0.6128	0.5296	0.4324	0.4108	0.5051	11.52	5.16	4.64	1.69	-2.24
	AR2_MA	0.6123	0.5292	0.4354	0.4113	0.5053	12.02	4.43	3.39	-0.03	-4.28
	AR2_ARMA	0.6067	0.5569	0.4797	0.4769	0.5826	8.81	-3.24	-5.6	-9.88	-14.58
	AR2_ARMANoSV	0.6114	0.5532	0.4664	0.4501	0.5427	4.75	-1.29	-0.27	-1.43	-6.47
	AR4	0.6291	0.5314	0.4357	0.4123	0.5054	11.03	4.15	4.08	0.59	-3.16
	AR4_MA	0.6264	0.5308	0.4385	0.4128	0.5049	11.02	4.45	3.88	0.62	-3.16
	AR4_ARMA	0.6272	0.5418	0.4482	0.4458	0.5348	5.58	-1.99	-6.41	-11.25	-16.37
	AR4_ARMANoSV	0.6245	0.5395	0.4588	0.4524	0.5425	2.21	0.92	1.46	-1.53	-6.69
MACRO 2	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.0252	0.7948	0.7712	0.7303	0.6855	0	0	0	0	0
	UC_MA	0.8438	0.7754	0.7152	0.6617	0.6197	2.8	1.72	5.71	6.16	8.6
	UC_ARMA	0.8370	0.7788	0.6720	0.6034	0.5433	2.39	-0.85	4.37	7.55	14.35
	UC_ARMANoSV	0.8665	0.7079	0.6816	0.6243	0.5673	-38.65	-16.58	-13.22	-19.08	-12.11
	AR2	0.9040	0.7317	0.7335	0.7159	0.6730	-1.6	-2.76	-4.28	-9.7	-7.22
	AR2_MA	0.8819	0.7193	0.7314	0.7065	0.6648	-0.38	-6.12	-9.83	-15.82	-12.39
	AR2_ARMA	0.8231	0.6426	0.6298	0.5814	0.5184	5.32	14.93	22.71	19.51	26.65
	AR2_ARMANoSV	0.8298	0.6478	0.6396	0.6013	0.5320	-19.45	12.04	17.94	11.6	16.86
	AR4	0.8450	0.7151	0.7322	0.6978	0.6661	4.8	-1.78	-3.56	-6.27	-4.3
	AR4_MA	0.8355	0.7117	0.7363	0.7055	0.6740	4.95	-1.06	-2.27	-5.4	-3.82
	AR4_ARMA	0.8159	0.6680	0.6474	0.5865	0.5334	5.86	8.11	13.38	14.33	19.65
	AR4_ARMANoSV	0.8300	0.6744	0.6681	0.6260	0.6091	-10.89	-7.51	2.5	6.9	9
MACRO 3	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.7382	1.0821	0.9706	0.9680	0.9170	0	0	0	0	0
	UC_MA	2.1781	1.6660	1.0629	0.8870	0.7675	-3.19	-47.13	-47.34	-42.44	-29.62
	UC_ARMA	1.0674	1.0733	0.9194	0.8306	0.7681	42.18	-36.18	-50.11	-52.82	-64.66
	UC_ARMANoSV	1.1568	1.0836	0.9267	0.8062	0.7460	-51.49	-3.81	27.69	17.86	-0.39
	AR2	1.1236	1.0575	0.9606	0.9009	0.8491	55.695	19.78	20.2	14.75	-6.54
	AR2_MA	1.1260	1.0275	0.9262	0.8711	0.8260	53.534	6.84	5.44	0.11	-16.64
	AR2_ARMA	1.0472	1.5094	1.1633	0.9965	0.8982	52.535	-0.67	27.15	29.64	11.49
	AR2_ARMANoSV	1.0915	1.4401	1.0213	0.8556	0.7909	-0.16	-8.74	28.41	35.59	15.07
	AR4	1.1503	0.9740	0.9337	0.8883	0.8338	53.043	16.85	17.22	11.42	-8.77
	AR4_MA	1.1506	0.9727	0.9240	0.8736	0.8225	53.443	16.5	18.62	13.38	-7.72
	AR4_ARMA	1.1495	1.0143	0.9202	0.8478	0.8079	40.63	18.39	31.72	22.08	2.74
	AR4_ARMANoSV	1.3584	0.9606	0.8760	0.8265	0.8071	-9.6	19.1	44.93	36.34	17.07
MACRO 4	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.5675	0.9773	0.8246	0.6899	0.6444	0	0	0.0000	0.0000	0.0000
	UC_MA	0.5968	0.9815	0.8281	0.6928	0.6424	-2.71	-0.9	-0.23	-1.44	-2.25
	UC_ARMA	0.62366	0.98148	0.95313	1.2157	3.1091	-23.78	-48.69	-76.09	-95.9	-111.86
	UC_ARMANoSV	0.5968	1.0463	0.8750	0.7320	0.6849	-23.92	-14.67	-15.08	-15.61	-17.77
	AR2	0.5195	0.9150	0.8256	0.6908	0.6447	4.49	-1.38	-1.96	-2.47	-1.82
	AR2_MA	0.5229	0.9226	0.8256	0.6907	0.6443	3.77	-1.54	-1.62	-2.13	-1.73
	AR2_ARMA	0.5158	0.9050	0.8175	0.6888	0.6446	5.07	-1.26	-3.04	-3.91	-3.58
	AR2_ARMANoSV	0.4833	1.2500	1.1484	1.2026	1.1273	10.39	-18.01	-20.47	-23.28	-24.87

	AR4	0.4745	0.8039	0.7917	0.6829	0.6430	10.4700	4.45	-0.87	-4.49	-3.46
	AR4_MA	0.4780	0.8046	0.7851	0.6795	0.6416	10.03	4.6000	-0.28	-4.28	-3.48
	AR4_ARMA	0.4757	0.8026	0.7791	0.6732	0.6364	10.46	4.13	-1.76	-4.9	-4.13
	AR4_ARMANoSV	0.47725	0.72084	0.80469	0.73856	0.70909	-23.92	19.45	4.62	-3.28	-5.73
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.7872	0.5389	0.5104	0.5253	0.5301	0	0	0	0	0
	UC_MA	0.7202	0.5399	0.5125	0.5268	0.5320	6.23	0.32	-1.08	-1.57	-2.36
	UC_ARMA	0.7207	0.5560	0.5290	0.5429	0.5470	5.22	-3.05	-9.79	-15.99	-21.33
	UC_ARMANoSV	0.7658	0.6643	0.5978	0.6226	0.6124	-6.61	-17.84	-25.49	-25.49	-43.94
	AR2	0.7197	0.5307	0.5002	0.5141	0.5194	7.04	2.92	1.39	1.58	0.09
MACRO 5	AR2_MA	0.7207	0.5304	0.5000	0.5140	0.5189	7.06	2.43	1.26	0.88	-0.79
	AR2_ARMA	0.7230	0.5346	0.4995	0.5168	0.5220	5.3	-1.76	-3.76	-5.37	-7.73
	AR2_ARMANoSV	0.7218	0.5321	0.5033	0.5184	0.5236	-14.13	1.43	1.78	-1.95	-7.98
	AR4	0.7243	0.5356	0.5010	0.5145	0.5193	5.47	1.97	0.96	1.49	-0.02
	AR4_MA	0.7237	0.5361	0.5007	0.5142	0.5188	5.7	1.79	1.02	1.44	0.02
	AR4_ARMA	0.7363	0.5586	0.5060	0.5091	0.5172	0.55	-2.59	-4.19	-5.35	-7.88
	AR4_ARMANoSV	0.7466	0.5498	0.5399	0.5410	0.5361	-10.23	-9.38	0.23	3.55	0.39
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.9457	0.7059	0.7491	0.7781	0.7829	0	0	0	0	0
	UC_MA	0.8214	0.7146	0.7262	0.7432	0.7341	8.45	2.87	7.43	5.83	11.5
	UC_ARMA	0.8708	0.6962	0.7047	0.7137	0.7011	8.24	5.48	11.55	9.77	16
	UC_ARMANoSV	0.9457	0.8017	0.7402	0.7987	0.7682	-15.27	-0.32	11.68	3.19	6.52
	AR2	0.9041	0.7936	0.7692	0.7417	0.6906	4.93	0.67	6.57	9.24	17.38
MACRO 6	AR2_MA	0.9028	0.7784	0.7590	0.7357	0.6861	5.34	2.22	7.99	10.13	18.69
	AR2_ARMA	0.9166	1.5374	1.1682	1.0306	0.8764	5.75	-41.99	-30.13	-30.28	-29.24
	AR2_ARMANoSV	0.9231	1.8968	1.4537	1.3075	1.0883	1.73	-50.04	-40.42	-36.7	-30.19
	AR4	0.8846	0.7746	0.7566	0.7361	0.6918	6.34	2.66	8.06	9.72	17.56
	AR4_MA	0.8906	0.7776	0.7536	0.7322	0.6878	6.4	2.05	8.04	9.66	17.39
	AR4_ARMA	0.8794	0.6762	0.6322	0.6172	0.5940	4.11	0.61	1.94	-1.24	-2.39
	AR4_ARMANoSV	0.8675	0.6834	0.6297	0.6424	0.6320	9.98	20.55	27.2	28.9	27.43
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.8606	0.6640	0.5962	0.5853	0.6456	0	0	0	0	0
	UC_MA	0.8058	0.6595	0.5904	0.5781	0.6395	3.32	0.06	0.01	0.58	0.68
	UC_ARMA	0.7656	0.6556	0.5884	0.5774	0.6407	6.03	2.14	1.14	1.83	1.55
	UC_ARMANoSV	0.7870	0.7037	0.6176	0.6053	0.6643	-14.21	-21.84	-15.82	-16.67	-20.53
	AR2	0.7753	0.7074	0.6586	0.6438	0.7085	6	1.8	-1.57	-3.97	-6.9
MACRO 7	AR2_MA	0.7748	0.7068	0.6559	0.6450	0.7128	5.85	1.61	-2.45	-5.42	-8.57
	AR2_ARMA	0.7886	0.7500	0.7698	0.7999	0.9566	2.71	-11.29	-17.43	-18.42	-23.19
	AR2_ARMANoSV	0.8005	0.7900	0.8104	0.8638	1.0858	-3.89	-11.19	-9.91	-13.32	-16.24
	AR4	0.7900	0.7232	0.6502	0.6443	0.7359	4.57	0.68	-0.74	-5.25	-10.46
	AR4_MA	0.7841	0.7211	0.6514	0.6449	0.7344	4.81	1.06	-0.62	-5.12	-10.27
	AR4_ARMA	0.8113	0.7411	0.6369	0.5989	0.6622	0.55	-4.24	-3.01	-2.71	-7.41
	AR4_ARMANoSV	0.8169	0.7669	0.6576	0.6024	0.6613	-7.24	-5.01	3.58	10.8	8.25
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.7164	0.6464	0.5561	0.5196	0.5346	0	0	0	0	0
	UC_MA	0.6888	0.6440	0.5511	0.5168	0.5347	3.31	2.08	1.78	0.22	-0.39
	UC_ARMA	0.6599	0.6345	0.5482	0.5147	0.5334	6.74	5.65	4.59	1.51	0.06
	UC_ARMANoSV	0.6509	0.6388	0.5508	0.5124	0.5252	-3.07	2.38	5.71	1.33	-4.48
	AR2	0.6689	0.6584	0.5599	0.5215	0.5454	6.46	2.14	1.25	0.09	-2.53
MACRO 8	AR2_MA	0.6590	0.6473	0.5572	0.5203	0.5454	7.5	3.76	2.54	0.63	-2.6

	AR2_ARMA	0.6497	0.7224	0.6420	0.6241	0.6878	4.14	-10.41	-10.84	-12.37	-17.52
	AR2_ARMANoSV	0.6604	0.7186	0.6082	0.5774	0.6127	3.74	2.69	7.28	3.81	-2.46
	AR4	0.6550	0.6603	0.5572	0.5217	0.5456	10.2	0.95	0.48	0.38	-1.48
	AR4_MA	0.6590	0.6631	0.5587	0.5223	0.5462	9.84	1.03	0.05	0.32	-1.53
	AR4_ARMA	0.6269	0.6334	0.6261	0.6206	0.6877	4.81	-4.78	-8.95	-15.33	-22.64
	AR4_ARMANoSV	0.6451	0.6477	0.6421	0.6371	0.7009	5.84	4.22	3.23	-3.76	-11.75
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.2991	0.5403	0.4816	0.4792	0.5136	0	0	0	0	0
	UC_MA	0.8252	0.5397	0.4825	0.4771	0.5122	30.82	10.51	6.82	7.04	5.26
	UC_ARMA	0.8386	0.7899	0.7191	0.7420	0.8301	31.51	-17.53	-37.43	-51.1	-65.4
	UC_ARMANoSV	0.8443	0.6141	0.5583	0.4943	0.5474	12.95	0.13	-6.3	-18.89	-31.43
	AR2	0.7889	0.5641	0.4998	0.4955	0.5341	36.1	14.52	10.74	9.76	7.16
MACRO 9	AR2_MA	0.7697	0.5595	0.4954	0.4904	0.5284	36.63	14.33	10.95	10.12	7.69
	AR2_ARMA	0.7465	0.5614	0.5280	0.5265	0.5696	34.27	13.73	7.64	4.54	-1.69
	AR2_ARMANoSV	0.7599	0.5433	0.6135	0.5726	0.5990	29.15	17.28	12.41	3.27	-4.32
	AR4	0.7587	0.5602	0.4951	0.4912	0.5290	36.42	15.73	12.36	10.83	7.83
	AR4_MA	0.7614	0.5625	0.4947	0.4896	0.5269	36.33	14.4	11.6	10.34	7.33
	AR4_ARMA	0.7831	0.5877	0.5128	0.5165	0.5663	30.45	10.03	6.3	3.37	-2.52
	AR4_ARMANoSV	0.7556	0.5665	0.5022	0.4918	0.5401	29.97	20.92	19.12	9.91	2.21
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.2033	1.0644	1.0121	1.0013	0.9996	0	0	0	0	0
	UC_MA	1.7427	1.2518	1.0605	1.0074	0.9834	0.488	-0.19	-16.46	-7.19	-35.62
	UC_ARMA	0.7662	1.1045	1.0288	1.0055	1.0155	43.871	13.65	-18.49	-38.34	-84.06
	UC_ARMANoSV	0.7070	0.9147	0.8726	0.8225	0.7957	30.643	48.82	72.23	80.8	38
	AR2	0.5944	0.8499	0.9583	0.9651	0.9922	52.347	84.59	92.06	76.25	22.26
MACRO 10	AR2_MA	0.6073	0.8449	0.9342	0.9433	0.9704	51.041	83.62	90.67	78.35	25.01
	AR2_ARMA	0.7079	2.0565	1.9447	1.8020	1.8226	46.681	16.75	26.55	23.7	-22.39
	AR2_ARMANoSV	0.7153	2.1760	1.8234	1.7472	1.7353	40.658	-23.22	-55.83	-9.66	-29.62
	AR4	0.5932	0.8298	0.9370	0.9490	0.9793	52.773	85.13	93.42	79.81	26.77
	AR4_MA	0.6008	0.8273	0.9464	0.9633	0.9960	52.761	86.95	94.11	80.84	26.96
	AR4_ARMA	0.8094	1.1308	1.0187	1.0163	1.0376	40.298	81.88	108.5	103.42	39.44
	AR4_ARMANoSV	0.9731	1.1086	0.8953	0.8411	0.8137	26.727	72.91	106.38	105.06	48.88
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.6903	0.5163	0.5002	0.4374	0.4088	0	0	0	0	0
	UC_MA	0.6344	0.5160	0.4995	0.4365	0.4084	3.76	2.24	1.73	1.64	0.19
	UC_ARMA	0.6608	0.7188	0.6115	0.5689	0.5291	-5.42	-24.36	-30.18	-33.57	-39.88
	UC_ARMANoSV	0.7283	0.6991	0.7483	0.6599	0.6965	-16.55	-27.32	-33.72	-36	-48.4
	AR2	0.6433	0.5217	0.5040	0.4417	0.4147	2.84	0.33	0.49	0.23	-1.28
MACRO 11	AR2_MA	0.6431	0.5232	0.5037	0.4418	0.4146	2.72	-0.07	0.65	0.46	-1.35
	AR2_ARMA	0.6448	0.5257	0.4989	0.4386	0.4132	2.53	0.19	2.21	2.16	0.05
	AR2_ARMANoSV	0.7370	0.8135	0.8367	0.7299	0.7342	-5.72	-33.13	-32.21	-27.1	-30.67
	AR4	0.6640	0.5352	0.5046	0.4397	0.4123	0.15	-1.73	0.89	1.63	-0.04
	AR4_MA	0.6633	0.5365	0.5060	0.4394	0.4127	0.18	-1.83	0.81	1.65	-0.13
	AR4_ARMA	0.6517	0.5190	0.4987	0.4370	0.4126	0.36	-0.77	0.68	1.6	-0.44
	AR4_ARMANoSV	0.6944	0.6747	0.7491	0.7088	0.6919	-5.64	-16.22	-22.38	-20.15	-23.13
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.8525	0.5417	0.5680	0.5711	0.5465	0	0	0	0	0
	UC_MA	0.7368	0.5276	0.5519	0.5510	0.5239	10.53	1.91	1.35	1.48	3.75
	UC_ARMA	0.7807	0.6269	0.5542	0.5326	0.4920	12.72	-0.62	0.82	2.74	7.34
	UC_ARMANoSV	0.7792	0.5674	0.5495	0.5370	0.5011	-17.15	-15.62	-2.86	-9.14	-7.35

	AR2	0.8030	0.5444	0.5453	0.5385	0.4976	11.15	4.05	2.46	1.48	5.62
MACRO 12	AR2_MA	0.8039	0.5650	0.5629	0.5506	0.5118	11.47	-2.28	-7.71	-10.48	-6.92
	AR2_ARMA	0.7528	0.5157	0.5249	0.5159	0.4807	15.24	8.7	9.47	6.75	10.57
	AR2_ARMANoSV	0.7492	0.5133	0.5194	0.5144	0.4866	-20.07	-10.98	3.48	-4.69	-5.45
	AR4	0.7806	0.5605	0.5682	0.5558	0.5185	13.79	2.23	0.4	1.15	4.71
	AR4_MA	0.7795	0.5607	0.5693	0.5569	0.5195	13.52	2.08	0.24	1.11	4.62
	AR4_ARMA	0.7651	0.5324	0.5410	0.5201	0.4870	15.11	6.23	6.3	5.88	8.96
	AR4_ARMANoSV	0.7847	0.5334	0.5449	0.5313	0.5036	-7.13	-19.55	-18.8	-15.62	-18.48
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.0203	0.8327	0.8412	0.8240	0.8122	0	0	0	0	0
	UC_MA	0.8822	0.8459	0.8201	0.7879	0.7686	1.41	-0.78	1.88	2.83	4.98
	UC_ARMA	0.8305	0.8252	0.7032	0.6311	0.5917	4.37	3.04	9.16	16.65	23.7
	UC_ARMANoSV	0.8525	0.7442	0.7123	0.6547	0.6312	-18.24	-9.5	-0.66	-5.02	-2.11
	AR2	0.8767	0.7619	0.7454	0.7030	0.6817	2.1	1.99	3.47	3.01	5.75
MACRO 13	AR2_MA	0.8640	0.7427	0.7417	0.6973	0.6757	2.62	0.63	0.75	-0.6	3.01
	AR2_ARMA	0.8277	0.6836	0.6914	0.6342	0.5838	4.18	12.47	22.24	23.22	30.68
	AR2_ARMANoSV	0.8396	0.6680	0.6894	0.6347	0.6092	-14.18	14.86	20.45	22.63	24.3
	AR4	0.8405	0.7315	0.7479	0.7006	0.6890	5.58	2.88	5.07	6.41	8.49
	AR4_MA	0.8376	0.7301	0.7469	0.6972	0.6827	5.3	3.2	5.62	6.85	8.91
	AR4_ARMA	0.8470	0.7236	0.6975	0.6267	0.5782	5.01	9.17	17.04	22.34	28.26
	AR4_ARMANoSV	0.8589	0.7272	0.6955	0.6237	0.6017	0.83	1.92	13.05	22.26	23.88
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.9037	0.9093	0.8709	1.0668	1.0363	0	0	0	0	0
	UC_MA	0.9311	0.9273	0.8845	1.0754	1.0472	-0.39	1.26	0.76	1.23	0.75
	UC_ARMA	0.8791	0.9719	0.9100	1.0449	1.0323	1.71	-13.31	-17.86	-15.88	-18.3
	UC_ARMANoSV	0.8438	0.8363	0.7906	0.9577	0.9743	-11.44	-11.26	-16.29	-16.82	-25.69
	AR2	0.6988	0.8093	0.7799	0.8782	0.8216	1.53	-5.39	-7.4	-2.36	2.65
MACRO 14	AR2_MA	0.6856	0.7717	0.7387	0.8510	0.8287	3.85	-5.47	-9.7	-6.63	-2.96
	AR2_ARMA	0.6896	1.0862	1.1030	1.1936	1.1654	-0.21	-31.6	-34.87	-22.78	-13.18
	AR2_ARMANoSV	0.7237	2.6907	2.0583	2.1349	1.9309	0.25	-95.05	-101.31	-81.36	-69.25
	AR4	0.7215	0.7660	0.7245	0.8089	0.8048	4.06	3.41	3.8	7	10.11
	AR4_MA	0.7124	0.7655	0.7256	0.8130	0.8079	4.38	3.04	3.18	6.13	9.33
	AR4_ARMA	0.7066	0.8257	0.9486	1.1969	1.2511	-0.69	-9.11	-18.24	-16.98	-12.49
	AR4_ARMANoSV	0.7116	0.8653	1.0128	1.3152	1.3973	0.2	-8.12	-18.51	-17.76	-16.65
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.3627	0.8152	0.8096	0.6696	0.6191	0	0	0	0	0
	UC_MA	1.1639	0.8599	0.7973	0.6484	0.5797	9.99	-3.77	-0.95	3.19	-1.27
	UC_ARMA	0.9228	0.7695	0.7427	0.6176	0.5646	24.54	0.77	1.08	4.02	-1.67
	UC_ARMANoSV	0.9680	0.7856	0.6989	0.6062	0.5614	-11.46	-29.85	-39.24	-38.9	-51.66
	AR2	0.8911	0.7293	0.6982	0.5744	0.5241	27.43	8.12	11.03	18.81	14.36
MACRO 15	AR2_MA	0.8952	0.7291	0.6977	0.5749	0.5247	27.14	9.15	11.4	19.36	14.9
	AR2_ARMA	0.8983	0.8489	0.7837	0.6335	0.5762	27.46	4.88	7.35	16.06	11.57
	AR2_ARMANoSV	0.9170	0.8197	0.7340	0.6030	0.5496	-5.87	-15.31	0.8	15.9	12.25
	AR4	0.9126	0.7305	0.6975	0.5759	0.5256	25.54	8.36	11.2	18.17	14.89
	AR4_MA	0.9124	0.7308	0.6968	0.5747	0.5247	25.64	8.54	11.59	18.51	14.98
	AR4_ARMA	0.9157	0.7739	0.7998	0.7101	0.6586	27.43	8.07	2.24	5.77	1.89
	AR4_ARMANoSV	0.9372	0.7842	0.8160	0.7176	0.6474	-7.23	-43.08	-49.76	-35.48	-28.52
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.7106	0.4925	0.5094	0.5311	0.5027	0	0	0	0	0
	UC_MA	0.6975	5.851E-01	5.382E+06	6.808E+11	1.561E+17	-22.45	-56	-92.88	-120.99	-145.23

	UC_ARMA	0.7012	1.290E+03	6.254E+12	6.776E+15	3.958E+21	-24.17	-56.65	-95.7	-124.3	-148.05
	UC_ARMANoSV	0.6808	5.032E-01	5.376E-01	5.248E-01	5.053E-01	-6.47	2.24	-3.39	-8.24	-15.77
	AR2	0.7133	4.747E+03	2.248E+13	5.451E+19	2.259E+26	-50.94	-99.63	-147.45	-178.75	-203.03
MACRO 16	AR2_MA	0.7133	3.061E+08	8.015E+19	2.995E+30	3.609E+34	-51.17	-89.11	-140.62	-172.85	-198.64
	AR2_ARMA	0.7131	4.518E+12	5.337E+18	2.039E+28	4.040E+37	-48.39	-89.76	-137.4	-170.96	-196.64
	AR2_ARMANoSV	0.6822	4.987E-01	5.110E-01	5.325E-01	5.034E-01	-21.88	0.71	-3.41	-4.72	-14.27
	AR4	0.7132	2.392E+04	9.523E+13	3.764E+21	7.274E+32	-49.31	-97.01	-143.91	-175.17	-199.79
	AR4_MA	0.7132	2.556E+03	2.537E+19	6.820E+22	3.181E+28	-49.14	-96.78	-143.67	-174.97	-199.1
	AR4_ARMA	0.7112	7.789E+11	3.853E+17	6.058E+24	1.023E+29	-42.89	-83.98	-130.02	-162.11	-186.8
	AR4_ARMANoSV	0.6884	0.5021	0.5124	0.5324	0.5036	-18.89	-14.19	-8.03	-9.62	-18.63
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	2.5826	1.2021	1.0051	1.0663	1.0308	0	0	0	0	0
	UC_MA	2.1426	1.3404	1.0289	1.0548	0.9610	10.89	4	27.31	30.88	27.4
	UC_ARMA	1.0503	1.0959	0.9809	0.9273	0.8546	34.53	9.36	44.72	68.05	55.67
	UC_ARMANoSV	1.2599	1.0923	1.0350	1.0257	1.0090	-14.67	-57.91	-172.01	-316.98	-488.03
	AR2	1.0314	1.0172	1.0001	0.9785	0.9575	32.76	43.54	84.21	102.46	94.51
MACRO 17	AR2_MA	1.0168	1.0179	0.9991	0.9812	0.9631	34.6	40.9	83.11	102.08	92.76
	AR2_ARMA	1.0346	3.1321	4.2647	3.5305	2.3992	34.28	-8.81	41.82	74.51	75.81
	AR2_ARMANoSV	1.0903	2.6690	3.0315	2.7352	2.0083	34.81	-17.45	31.3	66.44	68.86
	AR4	1.0230	1.0472	1.0295	0.9764	0.9428	34.59	41.77	81.85	100.55	91.89
	AR4_MA	1.0255	1.0394	1.0194	0.9756	0.9462	34.85	42.65	82.65	101.23	92.65
	AR4_ARMA	1.6020	1.3804	1.3739	1.4102	1.2666	23.48	39.27	102.96	132.08	120.63
	AR4_ARMANoSV	1.5350	1.1467	1.1179	1.1051	1.0206	30.74	31.12	104.94	138.25	126.98
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.7755	0.4693	0.4619	0.4415	0.4920	0	0	0	0	0
	UC_MA	0.7385	0.4695	0.4623	0.4422	0.4926	3	0.3	-0.99	-0.33	-2.17
	UC_ARMA	0.8045	0.9754	0.9058	0.8730	0.8912	-7.85	-51.81	-71.41	-83.96	-97.07
	UC_ARMANoSV	0.8110	0.6359	0.6053	0.6378	0.6908	-11.4	-18.43	-15.61	-24.76	-25.49
	AR2	0.7512	0.4777	0.4652	0.4459	0.4971	3.16	0.77	-0.66	-0.49	-2.19
MACRO 18	AR2_MA	0.7461	0.4739	0.4665	0.4441	0.4968	3.23	-6.14	-11.1	-13.83	-17
	AR2_ARMA	0.7431	0.4816	0.4787	0.4565	0.4976	1.96	-2.7	-4.54	-6.33	-7.4
	AR2_ARMANoSV	0.8010	0.5181	0.6236	0.5380	0.6274	0.37	-8.44	-10.75	-15.12	-18.57
	AR4	0.7769	0.4780	0.4675	0.4465	0.4979	1.88	0.24	-1.3	-1.08	-2.79
	AR4_MA	0.7626	0.4783	0.4678	0.4457	0.4961	1.92	0.15	-1.13	-0.91	-2.43
	AR4_ARMA	0.8080	0.5104	0.4633	0.4488	0.4981	-2.04	-3.83	-5.28	-8.07	-10.11
	AR4_ARMANoSV	0.8466	0.5392	0.5108	0.4715	0.5415	-3.93	-5.16	-7.53	-12.35	-17.17
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	1.7444	1.0335	0.8817	0.8109	0.7369	0	0	0	0	0
	UC_MA	1.2434	0.9106	0.7335	0.6618	0.6004	-9.17	-10.25	-4.11	4.1	7.68
	UC_ARMA	0.8447	0.7413	0.5568	0.4894	0.5101	18.41	3.89	5.54	16.56	16.49
	UC_ARMANoSV	0.8621	0.7406	0.5396	0.4640	0.4752	0.51	11.36	35.61	40.69	32.56
	AR2	0.8768	0.7857	0.5936	0.5264	0.5687	18.24	18.03	22.02	25.48	17.05
MACRO 19	AR2_MA	0.8706	0.7802	0.5930	0.5265	0.5674	17.98	18.82	21.53	25.52	16.88
	AR2_ARMA	0.8464	0.7565	0.6408	0.6034	0.6817	18.01	15.22	9.96	10.35	1.16
	AR2_ARMANoSV	0.8708	0.7679	0.6051	0.5405	0.5878	5.65	9.21	12.56	13.9	1.19
	AR4	0.8600	0.8230	0.6145	0.5282	0.5658	19.38	17.8	20.35	25.95	18.07
	AR4_MA	0.8553	0.8165	0.6103	0.5264	0.5649	19.51	18.52	22.1	27.19	18.76
	AR4_ARMA	0.8574	0.7905	0.6608	0.6389	0.7289	15.63	6.08	6.56	9.79	1.5
	AR4_ARMANoSV	0.8792	0.8059	0.6504	0.6068	0.6796	6.14	-27.9	-27.34	-19.09	-37.92
	RW	1.0000	1.0000	1.0000	1.0000	1.0000					

MACRO 20	UC	0.6389	0.4159	0.4776	0.4985	0.4663	0	0	0	0	0
	UC_MA	0.6188	0.4159	0.4774	0.4987	0.4662	9.12	0.31	-1.04	-3.56	-3.45
	UC_ARMA	0.7527	0.9975	0.9112	1.6989	5.2773	-2.89	-61.93	-64.27	-90.51	-106.69
	UC_ARMANoSV	0.8007	0.5225	0.5519	0.6524	0.6400	-25.58	-12.2	-12.65	-23.04	-31.48
	AR2	0.6229	0.4210	0.4768	0.5009	0.4656	9.27	0.95	-0.61	-4.71	-4.49
	AR2_MA	0.5958	0.4100	0.4760	0.5007	0.4653	13.58	-5.89	-9.97	-12.72	-13.45
	AR2_ARMA	0.5891	0.4356	0.4804	0.4978	0.4649	14.57	-13.48	-16.87	-20.35	-21.43
	AR2_ARMANoSV	0.7773	1.0206	0.9812	1.0123	0.9794	-13.66	-74.84	-80.8	-92.73	-104.82
	AR4	0.5655	0.3825	0.4875	0.4959	0.4663	14.37	7.11	-2.19	-2.52	-3.74
	AR4_MA	0.5662	0.3822	0.4875	0.4974	0.4665	14.68	7.1	-2.1	-2.77	-3.92
	AR4_ARMA	0.5597	0.3817	0.4870	0.4993	0.4672	16.1	8.24	-2.45	-4.63	-6.86
	AR4_ARMANoSV	0.8724	0.5021	0.6314	0.5787	0.5116	-27.07	-17.87	-35.11	-43.04	-51.37
MACRO 21	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.5271	0.4549	0.4828	0.4913	0.4778	0	0	0	0	0
	UC_MA	0.5352	0.4548	0.4828	0.4914	0.4777	6.52	3.58	-1	-4.56	-4.88
	UC_ARMA	0.5801	0.9939	0.8939	0.9566	3.4002	2.86	-42.35	-57.37	-77.53	-93.11
	UC_ARMANoSV	0.6244	0.4968	0.5141	0.5417	0.5317	-26.53	-6.12	-3.14	-11.4	-20.74
	AR2	0.5311	0.4580	0.4827	0.4924	0.4775	8.39	4.99	0.12	-5.06	-6.56
	AR2_MA	0.5026	0.4497	0.4829	0.4922	0.4781	13.41	3.26	-3.51	-8.69	-10.37
	AR2_ARMA	0.4991	0.4606	0.4853	0.4917	0.4785	13.96	-4.59	-8.7	-13.59	-16.63
	AR2_ARMANoSV	0.6416	0.5384	0.6130	0.5717	0.6227	-8.49	5.8	1.78	-2.58	-13.07
	AR4	0.5140	0.4454	0.4941	0.4907	0.4776	10.5	7.28	-1.09	-3.66	-5.61
	AR4_MA	0.5056	0.4405	0.4920	0.4905	0.4774	12.54	8.45	-0.77	-4.18	-5.92
	AR4_ARMA	0.5038	0.4393	0.4940	0.4914	0.4773	13.12	10.01	0.36	-2.92	-5.89
	AR4_ARMANoSV	0.5482	0.4626	0.5003	0.4910	0.4732	-22.87	1.19	-0.51	0.69	-6.34
MACRO 22	RW	1.0000	1.0000	1.0000	1.0000	1.0000					
	UC	0.5042	0.4733	0.4640	0.4313	0.5727	0	0	0	0	0
	UC_MA	0.5064	0.4728	0.4640	0.4319	0.5725	-1.15	-1.14	-2.23	-2.51	-2.47
	UC_ARMA	0.5312	0.7242	0.6525	0.6147	0.6867	-4.68	-41.24	-47.14	-52.89	-53.67
	UC_ARMANoSV	0.5083	0.4749	0.4651	0.4292	0.5732	-2.12	-0.67	1.55	1.69	0.96
	AR2	0.5011	0.4730	0.4586	0.4167	0.5639	-0.14	0.28	-0.34	0.04	-2.18
	AR2_MA	0.5004	0.4738	0.4601	0.4168	0.5642	-0.14	-2.66	-3.28	-2.93	-5.44
	AR2_ARMA	0.5001	0.4798	0.4722	0.4351	0.5806	-1.01	-3.19	-4.13	-4.45	-7.03
	AR2_ARMANoSV	0.4959	0.4714	0.4669	0.4197	0.5650	2.700	3.800	5.470	7.580	6.460
	AR4	0.5094	0.4747	0.4623	0.4178	0.5647	-1.05	-0.29	-0.98	-0.36	-2.95
	AR4_MA	0.5084	0.4738	0.4627	0.4176	0.5639	-0.93	-0.09	-1.09	-0.38	-2.86
	AR4_ARMA	0.5089	0.4859	0.4776	0.4477	0.5922	-2.88	-7.7	-10.85	-12.27	-15.29
	AR4_ARMANoSV	0.5056	0.4814	0.4762	0.4390	0.5873	1.53	3.07	3.95	4.44	2.98

Chapter 3

Real-Time Inflation Forecast Combination for Time-Varying Coefficient Models

3.1 Introduction

Inflation is a core macroeconomic indicator that is closely monitored by both central bankers and macroeconomic researchers for a range of reasons. For example, forecasting future inflation accurately is vital for the conduct of monetary and fiscal policies. Many studies have investigated the time series properties of inflation, and there is now consensus in the literature that the underlying trend and the volatility of inflation have changed considerably over time, although there is no agreement on the best way to model the dynamic of inflation (e.g., Stock and Watson, 2007; Cogley and Sbordone, 2008; Koop and Korobilis, 2012; Chan, 2013).

In their influential paper, Stock and Watson (2007) show that it is hard to outperform a flexible model with stochastic volatility using only inflation data. Following the seminal work of Clark (2011) and Clark and Ravazzolo (2015), it is generally accepted that stochastic volatility is indispensable for producing accurate inflation forecasts. Chan (2013) later introduced moving average errors to flexible models similar to those in Stock and Watson (2007). He showed that allowing for moving average errors improves upon the forecasts by univariate models using only inflation data. More generally, stochastic volatility is widely accepted to be indispensable for modeling macroeconomic data (e.g., Primiceri, 2005; D'Agostino, Giannone, and Gomez, 2013; Clark and Ravazzolo, 2015).

Economic theory suggests that inflation should be affected by a range of macroeconomic variables. In fact, the Phillips curve represents the well-known empirical relationship between the unemployment rate and inflation, which was first noted by Friedman (Friedman, 1968). However, Phillips curve models, such as those considered in Stock and Watson (2007), do not forecast inflation well. One reason could be that these models employ constant coefficients and homoscedastic errors. It is plausible that the forecasting performance of Phillips curves with constant coefficients can be improved by allowing time variation. Several time-varying parameter (**TVP**) multivariate models have been considered, such as the new Keynesian Phillips curve considered by Cogley and Sbordone (2008) and the state-dependent Phillips curve studied by Stella and Stock (2013). In the present chapter, our featured models not only allow for time-varying parameters, stochastic volatility, and moving average errors, but also consider other macroeconomic variables as explanatory variables in addition to the unemployment rate.

There is a growing body of literature that considers time-varying parameter Phillips curve models for forecasting inflation. For example, Koop and Korobilis (2012) introduced dynamic model averaging (**DMA**) and dynamic model selection (**DMS**), which uses a forgetting factor strategy to update time-varying coefficients and averaging models with a set of explanatory variables and different lag lengths. Chan et al. (2012) use a time-varying dimension (**TVD**) approach to allow the model dimension to change over time, which addresses the concern of parameter-rich and over-fitting in **TVP** models by choosing a more parsimonious representation automatically. Groen et al. (2013) used Bayesian model averaging to study structural breaks in the regression parameters and error variance. They concluded that structural breaks in the error variance can provide better forecasting performance, especially after 1984. However, the computational burden needs to be considered when the number of lags is greater than two with multiple explanatory variables. For instance, eight explanatory variables with three lags could produce more than 400 million candidate models. Since quarterly data are widely used for inflation forecasts, most models use four lags (e.g., Cogley and Sargent, 2005; Stock and Watson, 2007; Clark and Ravazzolo, 2015).

Motivated by the forecasting results from **DMA** and **DMS** that show high weights are given to parsimonious models or parsimonious models that rarely have more than two predictors selected, we employ only one explanatory variable in each component model with certain lags, and average the component models in the next step. Since combining forecasts based on a single variable reduces the number of models significantly, the lag length can increase to four without a heavy computational burden.

We also investigate the temporal relationship between inflation and other explanatory variables. In particular, we consider models with contemporaneous predictors in the estimation part and different lags of predictors in the forecasting part. In the present study, we use real-time data in the forecasting exercise instead of heavily revised data. There has already been much work studying real-time macroeconomic variable forecasting, such as forecasts using Bayesian vector autoregressive models (Clark, 2011), forecasts of inflation and the output gap (Garratt et al., 2011), UK monetary aggregates (Garratt et al., 2009) and inflation forecasts by Bayesian model averaging (Groen et al., 2013). The present study follows these pioneer researchers and employs both Bayesian estimation and real-time data for the study of inflation.

Point and density forecasts from a wide variety of models are combined using both time-varying weights and equal weights strategies, as it remains unresolved whether or not equal weights can produce better forecasts (e.g., Stock and Watson, 2004; Clark and McCracken, 2009; Jore, Mitchell, and Vahey, 2010). Our forecasting results suggest that compared with the traditional specification of the unemployment rate Phillips curve, models that include other macroeconomic variables with proper error specifications can outperform univariate models **DMA** and **DMS** in both point forecasts and density forecasts.

The remainder of the chapter proceeds as follows. Section 3.2 describes the specifications of time-varying coefficients models and the component models for inflation forecast combination. Section 3.3 provides a brief introduction to the real-time data and presents a full-sample estimation using US inflation data. Section 3.4 discusses the forecasting results of the model combination for point and density inflation forecasts, univariate models, dynamic model averaging, and dynamic modeling selection. In Section 3.5 we conclude.

3.2 Component Models

We consider three broad classes of time-varying coefficient models with different specifications of error terms: (i) models with constant variance (**TVC**); (ii) models with stochastic volatility (**TVC-SV**); and (iii) models with moving average stochastic volatility (**TVC-SVMA**). Within each class of model, we consider eight specifications, each with a different measure of economic activities for full-sample estimation. Finally, seven specifications are used for forecasting inflation. In addition, for each inflation predictor, various lag structures are considered, such as from one to four single lags, and up to four more lags.

By combining all lag structures with the seven predictors, this model averaging approach can be conducted for **TVC**, **TVC-SV** and **TVC-SVMA**, respectively. In the following subsections, first, the specifications of **TVC**, **TVC-SV** and **TVC-SVMA** are described, and then the eight inflation predictor candidates are described.

3.2.1 Time-Varying Coefficient Models

3.2.1.1 Constant Variance

A generic **TVC** model can be described as a generalized Phillips curve with time-varying coefficients:

$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{n} \beta_{2+j,t} x_{t-j} + \varepsilon_t^y, \quad \varepsilon_t^y \sim \mathcal{N}(0, \sigma_y^2), \tag{3.2.1}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\varepsilon}_t^{\beta}, \quad \boldsymbol{\varepsilon}_t^{\beta} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}), \tag{3.2.2}$$

$$\boldsymbol{\beta}_t = \begin{pmatrix} \beta_{1,t} & \beta_{2,t} & \cdots & \beta_{n,t} \end{pmatrix}', \mathbf{Q}_0 = \begin{pmatrix} \sigma_{0\beta_1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{0\beta_n}^2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \sigma_{\beta_1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\beta_n}^2 \end{pmatrix},$$

where k is the forecasting horizon. For the full sample estimation in Section 3.3.2, we set $k = 0$. Note that the subscripts in the specification represent the order of the coefficients and time points (e.g., $\boldsymbol{\beta}_{1,t}$ to $\boldsymbol{\beta}_{n,t}$), while the superscripts indicate the underlying relationship between variables (e.g., $\mathbf{y}$ and $\boldsymbol{\varepsilon}^y$). x_t is a vector of covariates that may include lagged values of inflation or inflation predictors. The model above incorporates both a time-varying intercept and regression coefficients.

In Equation (3.2.2), the intercept and coefficients are assumed to follow independent random walks (e.g., Clark, 2011; Clark and Ravazzolo, 2015). By allowing the coefficients to evolve gradually over time, this specification accommodates a slowly changing relationship between inflation and the explanatory variables. Atkeson and Ohanian (2001) criticize the ability of the Phillips curve models to forecast inflation as compared with random walk forecasts. However, the Phillips curve models they adopt all have constant coefficients, and they do not perform as well as random walk naive forecasts in some historical periods. It can be expected that a time-varying Phillips curve performs better than one with constant coefficients.

The covariance matrices $\mathbf{Q}_0$ and $\mathbf{Q}$ of $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_t$ respectively are assumed to be diagonal matrices, where $\mathbf{Q}_0$ and $\boldsymbol{\beta}_0$ are initial values of $\mathbf{Q}$ and $\boldsymbol{\beta}_t$, respectively. It indicates that $\boldsymbol{\beta}_{1,t} \cdots \boldsymbol{\beta}_{n,t}$ have individual independent white noise disturbances with zero means and variances $\varepsilon_t^{\beta_1} \cdots \varepsilon_t^{\beta_n}$, respectively. Both ε_t^y and $\boldsymbol{\varepsilon}_t^{\beta}$ have constant variances.

3.2.1.2 Stochastic Volatility

Next, we extend (3.2.1)-(3.2.2) to allow for stochastic volatility (e.g., Groen, Paap, and Ravazzolo, 2013):

$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{n}\beta_{2+j,t}x_{t-j} + \varepsilon_t^y, \quad \varepsilon_t^y \sim \mathcal{N}(0, \mathrm{e}^{h_t}), \tag{3.2.3}$$

$$h_t = h_{t-1} + \boldsymbol{\varepsilon}_t^h, \quad \boldsymbol{\varepsilon}_t^h \sim \mathcal{N}(0, \sigma_h^2), \tag{3.2.4}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\varepsilon}_t^\beta, \quad \boldsymbol{\varepsilon}_t^\beta \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}), \tag{3.2.5}$$

where $\boldsymbol{\beta}_t$ and $\boldsymbol{\varepsilon}_t^\beta$ are the same as those in **TVC**, but the variance of ε_t^y is time-varying. The variance of ε_t^y is controlled by log stochastic volatility h_t, which changes over time. Equation (3.2.4) presents the evolution of the stochastic volatility parameter, which is an instantaneous volatility component of the model. The log-volatilities in Equation (3.2.4) are initialized by $h_1 \sim \mathcal{N}(0, \sigma_{0h}^2)$ with variance given in advance, and h_t follows random walk innovation.

3.2.1.3 Moving Average Stochastic Volatility

We further extend the SV model (3.2.3)-(3.2.5) using the framework in Chan (2013). The **TVC-SVMA** model is given below:

$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{n}\beta_{2+j,t}x_{t-j} + \varepsilon_t^y, \tag{3.2.6}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\varepsilon}_t^\beta, \quad \boldsymbol{\beta}_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_0), \quad \boldsymbol{\varepsilon}_t^\beta \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}), \tag{3.2.7}$$

$$\varepsilon_t^y = \omega_t + \psi_1\omega_{t-1} + \cdots + \psi_q\omega_{t-q}, \quad \omega_t \sim \mathcal{N}(0, \mathrm{e}^{h_t}), \tag{3.2.8}$$

$$h_t = h_{t-1} + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2), \tag{3.2.9}$$

where $\boldsymbol{\beta}_t$ and $\boldsymbol{\varepsilon}_t^\beta$ are the same as those in **TVC**. Equation (3.2.8) presents the moving average error feature of the specification. It can be rewritten as a polynomial of the lag operator L:

$$\varepsilon_t = \psi(L)u_t,$$

where $\psi(L) = 1 + \psi_1 L + \cdots + \psi_q L^q$. For identification, we assume that all roots of $\psi(L)$ are outside the unit circle. We assume $q = 1$ for simplicity.

3.2.2 Inflation Predictors

We consider a total of eight inflation predictors for each time-varying coefficient model (**TVC**, **TVC-SV**, and **TVC-SVMA**). As mentioned, we use real-time data.

However, data availability limits the number of variables used. Motivated by the predictive performance of inflation predictors in Groen et al. (2013), we select eight variables that include both real economy activities and a nominal variable (M2). The real-time inflation predictors are from the Real-Time Data Set for Macroeconomists (RTDSM) database at the Federal Reserve Bank of Philadelphia. The inflation predictors are:

1. Real unemployment rate (UR),
2. Real capacity utilization rate in manufacturing (CUR),
3. Housing starts (HSTS),
4. Real imports of goods and services (IMP),
5. M2 growth rate (M2),
6. Real durable consumption growth (RCON),
7. Real residential investment (RINV),
8. Real output growth (ROUT).

Among these eight predictors, there are seven real economy activity predictors, and one nominal predictor, M2.

3.2.3 Lag Structure

For each inflation predictor, we consider different forms of lag structure: coincident, and as a leading inflation predictor (one quarter ahead, half-year ahead, three quarters ahead, and one year ahead). We also include different numbers of lags (see Stock and Watson (1999)). Explicitly, the list of models is as follows:

$$y_{t+k} = \beta_{1,t} + \beta_{2,t}x_t + \varepsilon_t^y, \tag{3.2.10}$$
$$y_{t+k} = \beta_{1,t} + \beta_{2,t}x_{t-1} + \varepsilon_t^y, \tag{3.2.11}$$
$$y_{t+k} = \beta_{1,t} + \beta_{2,t}x_{t-2} + \varepsilon_t^y, \tag{3.2.12}$$
$$y_{t+k} = \beta_{1,t} + \beta_{2,t}x_{t-3} + \varepsilon_t^y, \tag{3.2.13}$$
$$y_{t+k} = \beta_{1,t} + \beta_{2,t}x_{t-4} + \varepsilon_t^y, \tag{3.2.14}$$
$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{1}\beta_{2+j,t}x_{t-j} + \varepsilon_t^y, \tag{3.2.15}$$
$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{2}\beta_{2+j,t}x_{t-j} + \varepsilon_t^y, \tag{3.2.16}$$
$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{3}\beta_{2+j,t}x_{t-j} + \varepsilon_t^y, \tag{3.2.17}$$
$$y_{t+k} = \beta_{1,t} + \Sigma_{j=0}^{4}\beta_{2+j,t}x_{t-j} + \varepsilon_t^y, \tag{3.2.18}$$

where k is the same as that in Section 3.2.1.1. In total, there are nine component models for each predictor. The time-varying coefficients in each component model and time-varying weights of component models make it possible to update the information set at any time.

3.2.4 The Priors

Each component model is estimated using Bayesian methods that incorporate parameter uncertainty. The priors of the parameter initial values are set as follows. We assume that the priors of the intercept and coefficient initial values are normal: $\beta_{1,1} \sim \mathcal{N}(\beta_{1,0}, V_{\beta_1})$, $\beta_{2,1} \sim \mathcal{N}(\beta_{2,0}, V_{\beta_2}), \cdots, \beta_{j,1} \sim \mathcal{N}(\beta_{j,0}, V_{\beta_j}), \cdots$, and $\beta_{n,1} \sim \mathcal{N}(\beta_{n,0}, V_{\beta_n})$ for $j = 3, \cdots, n-1$. We set $\beta_{1,0} = 5, \beta_{2,0} = -0.2, \beta_{j,0} = -0.1, V_{\beta_1} = 2, V_{\beta_2} = 0.2$ and $V_{\beta_j} = 0.1$. The prior means of $\beta_{j,1}$ are set to small values, which reflects the belief that the covariates are weakly informative about the inflation initial conditions.

Next, the variances $\sigma^2_{\beta_1}, \sigma^2_{\beta_2}, \cdots, \sigma^2_{\beta_n}$ and the elements of the covariance matrix Q for $\boldsymbol{\beta}$ are assumed to have independent inverse-gamma priors: $\sigma^2_{\beta_1} \sim \mathcal{IG}(\nu_{\beta_1}, S_{\beta_1})$, $\sigma^2_{\beta_2} \sim \mathcal{IG}(\nu_{\beta_2}, S_{\beta_2})$, and $\sigma^2_{\beta_j} \sim \mathcal{IG}(\nu_{\beta_j}, S_{\beta_j})$. In order to have vague priors for the variances, we choose large prior variances. Specifically, we set small values for the degree of freedom parameters: $\nu_{\beta_1} = \nu_{\beta_2} = \nu_{\beta_j} = 10$. We then set the scale parameters $S_{\beta_1} = 2$,$S_{\beta_2} = 0.2, S_{\beta_j} = 0.1$, so that the prior means are $E(\sigma^2_{\beta_1}) = 0.22, E(\sigma^2_{\beta_2}) = 0.02$, and $E(\sigma^2_{\beta_j}) = 0.01$. The prior means indicate that the parameters transit in the desired smoothness from one state to another.

The prior of the stochastic volatility parameter initial value h_1 is also supposed to be normal: $h_1 \sim \mathcal{N}(h_0, V_h)$ and $\sigma^2_h \sim \mathcal{IG}(\nu_{\sigma_h}, S_{\sigma_h})$, where $h_0 = 0, V_h = 0.05, \nu_h = 10$ and $S_h = 0.45$, so that the prior mean of $\sigma^2_h = 0.05$.

Finally, the prior of the MA (1) coefficient ψ is assumed to be a truncated normal prior following that of Chan (2013): $\psi \sim \mathcal{N}(\psi_0, V_\psi)\mathbb{1}(|\psi| < 1)$, where $\psi_0 = 0.9$ and $V_\psi = 1$.

A detailed description of the Bayesian estimation methods is provided in Appendix 3.A.

3.3 Full Sample Estimation

Before presenting the forecasting results, in this section we first describe the data and then the posterior results. For the full sample study, we present only the empirical results of the **TVC-SVMA** model and the posterior estimates of eight potential inflation predictors in a coincident form. The model with a coincident inflation predictor is one of the specifications suggested in Stock and Watson (1999). For the full sample estimation, we use the vintage published in 2014Q2, which spans from 1960Q2 to 2014Q1. For the forecasting exercise in the next section, real-time data are used. First, a brief introduction to real-time data in this study is provided.

3.3.1 Real-Time Data

Real-time data are related to vintage changing and can be heavily-revised and updated over time. Specifically, a data vintage only contains the time series that a policymaker can observe at that date, thus all the model estimations and forecasts at that moment are conducted by the data set of that vintage, which is much closer to reality and is believed as a more reasonable way to test the robust of the forecasting models (Croushore and Stark, 2001). For example, the vintage 2000Q2 refers to the time series available in 2000Q2, and the time series of vintage 2000Q2 is from 1959Q1 to 2000Q1 in our examples which is the first available estimator for 2000Q1. Thus, the pseudo out of sample forecasts in the last section use the latest revised data set referring to the final vintage data.

We use a comprehensive real-time data set compiled by the Federal Reserve Bank of Philadelphia, as discussed in Croushore and Stark (2001, 2003). Since macroeconomic data are typically heavily revised, it is vital to use real-time data as opposed to using the last vintage if we want to simulate the experience of forecasters in real time. We use the second available estimates as actuals, because Corradi et al. (2009) provide tests showing that the second revision error is concentrated around zero better than the first revision error, which is normally distributed. For example, considering that we use vintage 2000Q2 data to make one-step-ahead forecasts for 2000Q2, the second available estimate for 2000Q2 is just in the time series 1959Q1 to 2000Q3 in vintage 2000Q4.

The real-time data that we use for forecasting exercises is taken from the Federal Reserve Bank of Philadelphias RTDSM. The starting point of modeling estimation is 1960Q2, and the time period of 1959Q2 to 1960Q1 is trimmed as lags. We use the personal consumption expenditures (PCE) deflator as the measure of US inflation, rather than the customer price index (CPI) since the first available vintage of CPI is vintage 1994Q3, which is too late for comparing with PCE, for which we can adopt a vintage as early as 1985Q3. Thus, the data from 1960Q2 to 1985Q2 in vintage 1985Q3 is considered the first evaluation time period, and the last vintage is 2014Q2, so the forecasting exercises finish when the forecasting results of 2014Q1 are obtained.

Specifically, all these inflation activity predictors are measured by their percentage quarterly changes. If there are only monthly time series available, quarterly averages will be made. To illustrate, the quarterly inflation rate y_t is calculated as the first difference of the logged inflation deflator:

$$y_t = 400 \times \log(PCE_t/PCE_{t-1}).$$

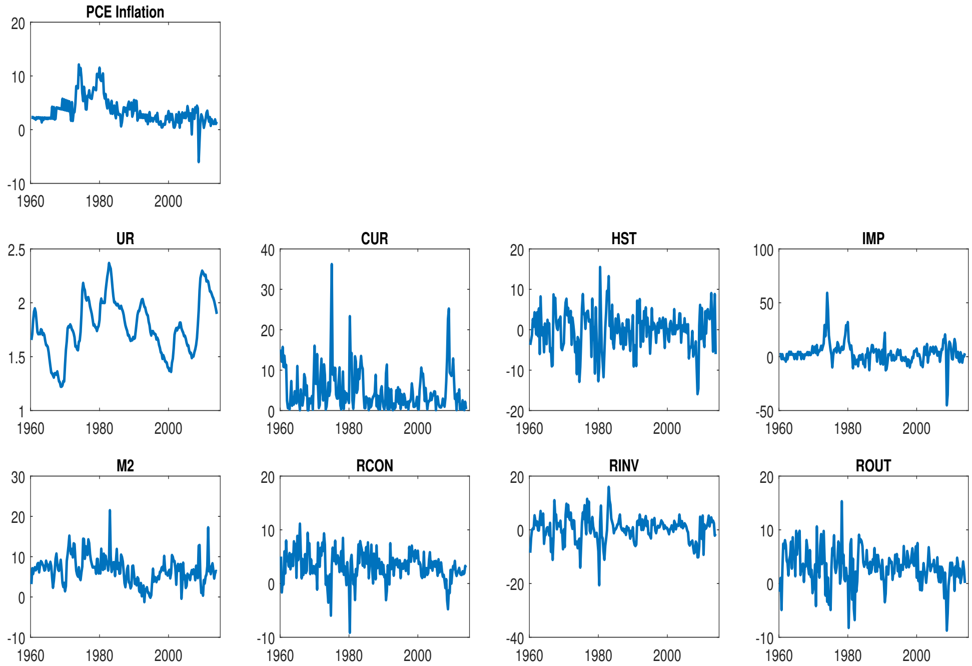

Figure 3.1: PCE inflation and inflation predictors.

The quarterly UR is the number of unemployed as a percentage of the labor force, and it is available in the RTDSM at the Federal Reserve Bank of Philadelphia. The original data vintages of the real UR are used for both estimation and forecasting studies. In order to obtain real capacity utilization rates in manufacturing, the original monthly data are first transformed into quarterly average data, and then the first differences are taken on the logarithm to measure quarterly changes. Housing starts are also monthly vintages, while they are transformed by taking the second difference logarithm. The monthly M2 growth rate vintages are transformed in the same way as the real capacity utilization rate in manufacturing. However, vintages 1981Q1 and 1981Q2 of M2 are incomplete, so we replace them with vintage 1981Q3. For real imports of goods and services, the original quarterly data vintages are available for imports, so we just take the natural logarithm of the raw data to construct the quarterly frequency of import price inflation. For real durable consumption growth, the quarterly data vintages are transformed by the first difference of logarithm. For real RINV and real output growth, the original quarterly data vintages are all available, so we follow the same transformation as for real imports of goods and services to construct the growth rates. The last vintages of the time series after transformation are plotted in Figure 3.1.

3.3.2 Full Sample Empirical Results

We present the empirical results of the full sample estimation for eight **TVC-SVMA** specifications with PCE inflation and different inflation predictors. The full sample results are based on the data in the 2014Q2 vintage. The posterior means and percentiles are all based on 25,000 draws with 5,000 burn-in draws obtained using the MCMC algorithm discussed in Section 3.A.

All the graphs in Figure 3.2 present the posterior means, the 5th and the 95th percentiles of the time-varying parameters $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$ and $\exp(\mathbf{h}/2)$. The graphs also plot the posterior density of the moving average parameter ψ for **TVC-SVMA** with inflation predictors based on Equation (3.2.10), where $k = 0$.

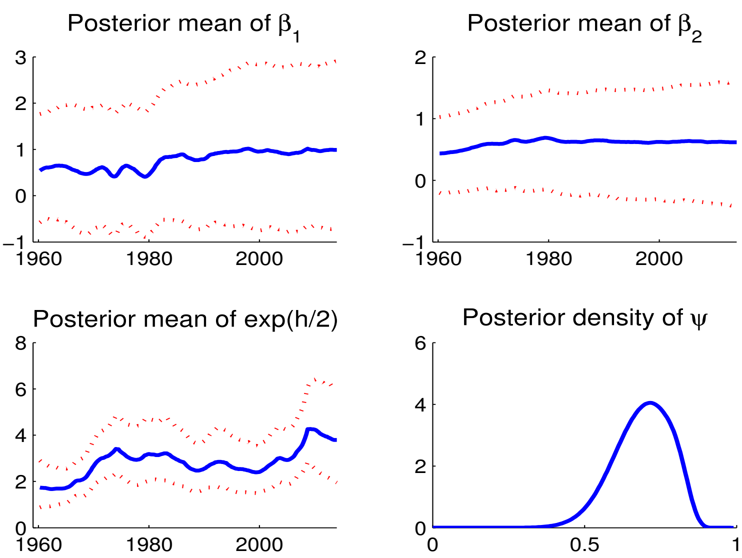

Figure 3.2: Posterior estimates of **TVC-SVMA** where the predictor is the UR.

Figure 3.2 presents the posterior results of parameters with UR as the inflation predictor. Apparently, the posterior mean of the coefficient β_2 is positive in the sample period, which is counterintuitive, as Phillips curves generally describe a negative relationship between inflation and the UR. Moreover, the 5th and the 95th percentile credible intervals of both the intercept and the coefficient always include zero. Therefore, we do not use UR as the inflation predictor in forecasting exercises and introduce other variables as inflation predictors.

Figure 3.3 to Figure 3.9 show that all the posterior means of the parameter $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$ and $\exp(\mathbf{h}/2)$ exhibit time variation, especially the log volatility $\mathbf{h}$. This is in line with the literature that finds inflation volatility changes substantially over time (e.g., Primiceri, 2005; Stock and Watson, 2007; Chan, 2013). Most of the mass of the posterior densities of the moving average parameter ψ in all models are far away from zero, indicating that the moving average errors in all specifications play an important role in describing the dynamics of inflation.

The posterior means of these time-varying parameters correspond to the empirical results in the previous literature studying time-varying inflation specifications (e.g., Cogley and Sargent, 2005; Primiceri, 2005; Groen, Paap, and Ravazzolo, 2013). The credible intervals, being far from zero, indicate that both intercept and coefficient are significant in the specified **TVC-SVMA**. Since we take the natural logarithm of UR, its new vintage values present more stable and less volatile properties. Furthermore, the positive values of coefficients indicate that these seven inflation predictors all have a positive correlation with inflation. Almost all the coefficients reach their highest peak around 1979, and some of them have a second peak around 2007. Although all eight coefficients exhibit time-varying properties under **TVC-SVMA**, the degrees of time-varying properties they present are not the same. For example, the intercept of CUR and M2 and $\boldsymbol{\beta}_2$ of HSTS and M2 are flatter than other variables, which implies that one or two coefficients between inflation and these variables are relatively stable with time.

The estimates of the standard deviation $\exp(\mathbf{h}/2)$ exhibit substantial time variation, which indicates the importance of including stochastic volatility in modeling inflation. The figures show that $\exp(\mathbf{h}/2)$ can successfully capture the surge in inflation volatility in the 1970s and early 1980s, as well as the financial crisis in 2007. Although the specifications here allow for time-varying coefficients, the estimates of $\exp(\mathbf{h}/2)$ are still similar to those reported in Chan (2013). However, the variance ranges across the examined time period are different between the inflation predictors. For example, the largest variance value of IMP is over 15, while that of M2 is below 8. Almost all the variance results suggest that there are two peak values captured before 1980 and one peak value around 2007, while the results of CUR, HSTS, IMP, and RINV suggest that there is a smaller peak around 1990.

The lower right graph in each figure is the posterior density of the moving average parameter ψ. None of the densities are concentrated around zero, which indicates that the moving average parameter is necessary for **TVC-SVMA**. In addition, the values of

ψ suggest that error terms are positively autocorrelated. This strong evidence suggests that all moving average parameters ψ of the examined variables are significant in **TVC-SVMA**.

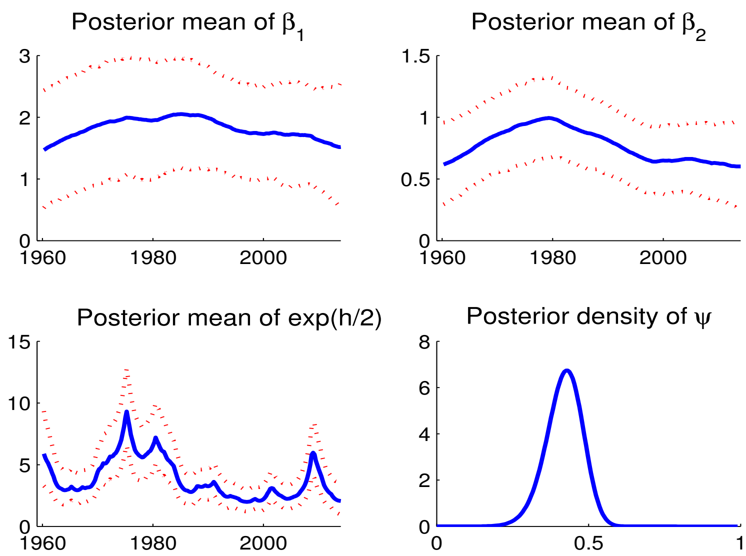

Figure 3.3: Posterior estimates of **TVC-SVMA** where the predictor is CUR.

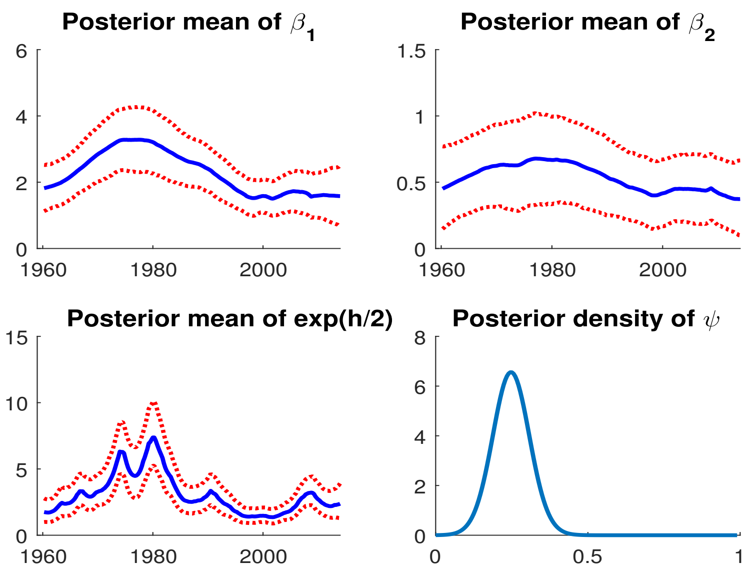

Figure 3.4: Posterior estimates of **TVC-SVMA** where the predictor is HSTS.

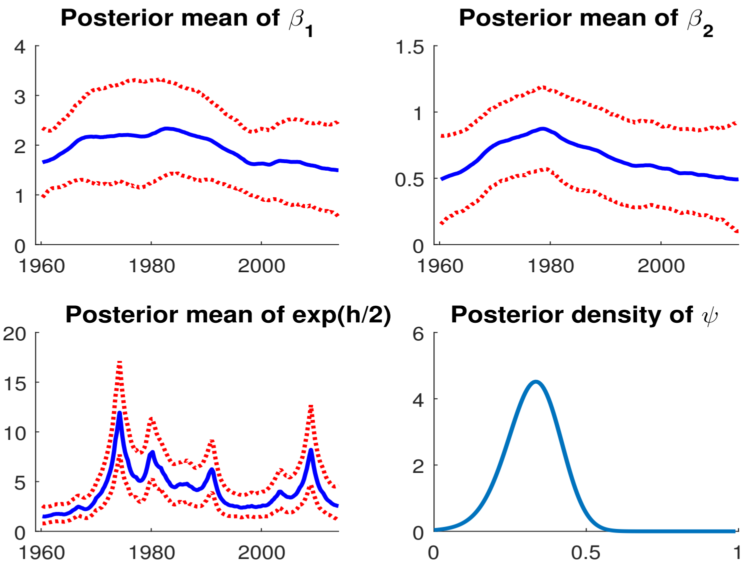

Figure 3.5: Posterior estimates of **TVC-SVMA** where the predictor is IMP.

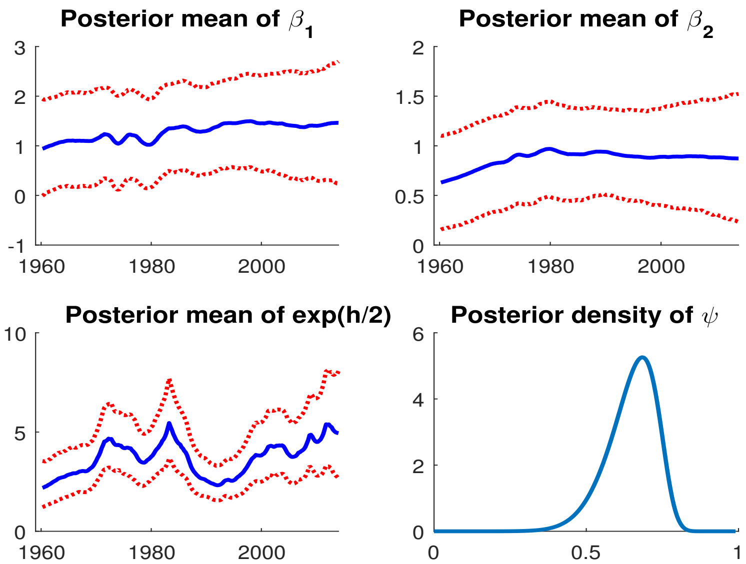

Figure 3.6: Posterior estimates of **TVC-SVMA** where the predictor is M2.

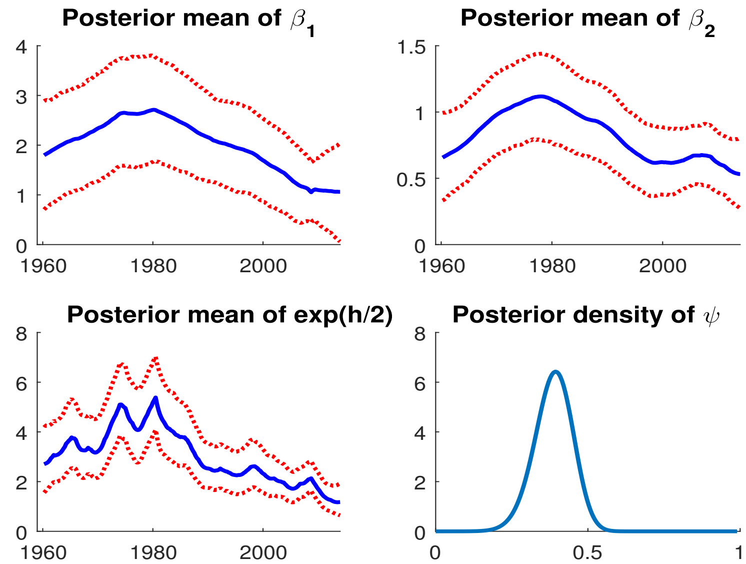

Figure 3.7: Posterior estimates of **TVC-SVMA** where the predictor is RCON.

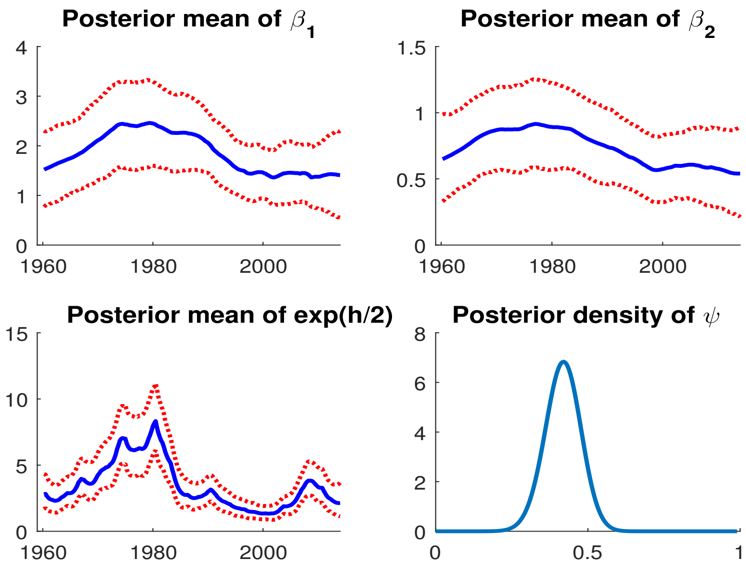

Figure 3.8: Posterior estimates of **TVC-SVMA** where the predictor is RINV.

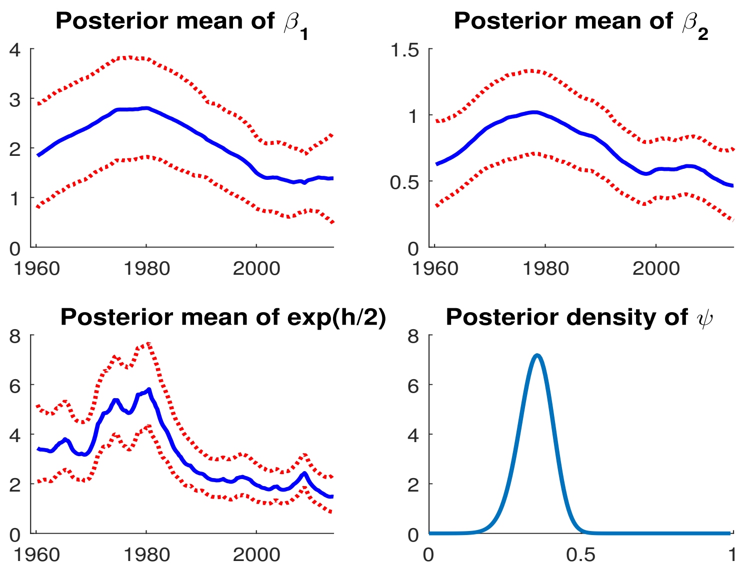

Figure 3.9: Posterior estimates of **TVC-SVMA** where the predictor is ROUT.

According to the full sample results, all seven predictors give sensible posterior estimation results and present various relationships with inflation, and the results strongly support the inclusion of time-varying parameters, stochastic volatility, and the moving average component in the error terms. They are used for model combination. Since there are seven variables and nine component models, the total number of component models for each specification is 63.

3.4 Real-Time Forecasts of US Inflation

The forecasting exercises are carried out using real-time data from vintage 1985Q3 to 2014Q2. In particular, for the first set of forecasts, the parameters are estimated using data from 1960Q2 to 1985Q2, and the forecasting period runs from 1985Q3 to 2014Q1. Then, the estimation window is a rolling window that keeps the same window length for calculating time-varying weight. The rolling window is conducted by adding the next data in and deleting the oldest data since 1985Q3 (Jore et al., 2010). The forecasting horizons are one quarter, one year, two years and four years ($k = 1, 4, 8$ and 16). In the 1-, 4-, 8- and 16-step-ahead forecasts, $y_{t+1}, \cdots, y_{t+16}$ are calculated by Equation (3.2.10) to (3.2.18).

We also compute forecast combinations by weighting the forecasts from different models. Specifically, recursive weights based on historical forecasting performance are used for model averaging forecasts (e.g., Jore, Mitchell, and Vahey, 2010; Garratt, Mitchell, Vahey, and Wakerly, 2011), so that the model averaging weights are evaluated repeatedly by the following vintages.

Model averaging based on three time-varying coefficient models with specification on error terms are considered as competing models in the following forecasting exercises sections. Various lag forms for the seven macroeconomic variables are the component models for combining forecasts as discussed before (see Equation (3.2.10) to (3.2.18)). Instead of using Bayesian model selection criteria to determine the optimal forecast model, such as the Bayesian information criterion or deviance information criterion, model averaging does not need to select a single model, which may not forecast well all the time, but gives more weights to models with better forecasting performance.

3.4.1 List of Competing Models

As mentioned, we consider three classes of models: **TVC**, **TVC-SV**, and **TVC-SVMA**. In the inflation forecasting literature, stochastic volatility is found to be an important component in inflation forecasting models, and we would like to determine whether or not the moving average stochastic volatility can improve forecasting results.

Other models are also considered, including an unobserved components model with stochastic volatility (**UC-SV**), an unobserved components model with moving average stochastic volatility (**UC-SVMA**), a dynamic model averaging (**DMA**), and dynamic model selection (**DMS**). Specifically, both **UC-SV** (Stock and Watson, 2007) and **UC-SVMA** (Chan, 2013) are well-performing univariate models for inflation forecasts in the literature. **DMA** and **DMS** (Koop and Korobilis, 2012) with a forgetting-factors strategy achieve substantial forecasting improvements, while time-varying volatility is implemented through exponentially weighted moving average estimates. The competing models are:

1. Unobserved components model with stochastic volatility model (**UC-SV**):

$$
\begin{aligned}
&y_t = \tau_t + \varepsilon_t^y, && \varepsilon_t^y \sim \mathcal{N}(0, \mathrm{e}^{h_t^y}),\\
&\tau_t = \tau_{t-1} + \varepsilon_t^\tau, && \varepsilon_t^\tau \sim \mathcal{N}(0, \mathrm{e}^{h_t^\tau}),\\
&h_t^y = h_{t-1}^y + \varepsilon_t^{h^y}, && h_1^y \sim \mathcal{N}(0, \sigma_{0h^y}^2), \quad \varepsilon_t^{h^y} \sim \mathcal{N}(0, \sigma_{h^y}^2),\\
&h_t^\tau = h_{t-1}^\tau + \varepsilon_t^{h^\tau}, && h_1^\tau \sim \mathcal{N}(0, \sigma_{0h^\tau}^2), \quad \varepsilon_t^{h^\tau} \sim \mathcal{N}(0, \sigma_{h^\tau}^2).
\end{aligned}
$$

2. Unobserved components model with moving average stochastic volatility (**UC-**

SVMA):

$$
\begin{aligned}
&y_t = \tau_t + \varepsilon_t^y, \qquad \varepsilon_t^y \sim \mathcal{N}(0, \mathrm{e}^{ht}),\\
&\tau_t = \tau_{t-1} + \varepsilon_t^\tau, \qquad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2),\\
&\varepsilon_t^y = u_t + \psi_1 u_{t-1} + \cdots + \psi_q u_{t-q}, \quad u_t \sim \mathcal{N}(0, \mathrm{e}^{h_t}),\\
&h_t = h_{t-1} + \varepsilon_t^h, \quad h_1 \sim \mathcal{N}(0, \sigma_{0h}^2), \quad \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2).
\end{aligned}
$$

3. Time-varying coefficient model without stochastic volatility (**TVC**), and $\beta_{2,t}x_t$ can be replaced according to Equation (3.2.10) to (3.2.18):

$$
\begin{aligned}
&y_{t+k} = \beta_{1,t} + \beta_{2,t}x_t + \varepsilon_t^y, \quad \varepsilon_t^y \sim \mathcal{N}(0, \sigma_y^2),\\
&\beta_t = \beta_{t-1} + \varepsilon_t^\beta, \qquad \beta_1 \sim \mathcal{N}(\mathbf{0}, Q_0), \quad \varepsilon_t^\beta \sim \mathcal{N}(\mathbf{0}, Q).
\end{aligned}
$$

4. Time-varying coefficient model with stochastic volatility (**TVC-SV**), and $\beta_{2,t}x_t$ can be replaced according to Equation (3.2.10) to (3.2.18):

$$
\begin{aligned}
&y_{t+k} = \beta_{1,t} + \beta_{2,t}x_t + \varepsilon_t^y, \quad \varepsilon_t^y \sim \mathcal{N}(0, \mathrm{e}^{h_t})\\
&\beta_t = \beta_{t-1} + \varepsilon_t^\beta, \quad \beta_1 \sim \mathcal{N}(\mathbf{0}, Q_0), \varepsilon_t^\beta \sim \mathcal{N}(\mathbf{0}, Q),\\
&h_t = h_{t-1} + \varepsilon_t^h, \quad h_1 \sim \mathcal{N}(0, \sigma_{0h}^2), \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2).
\end{aligned}
$$

5. Time-varying coefficient model with moving average stochastic volatility (**TVC-SVMA**) with lag of ψ equal to one for simplicity just as in Chan (2013), and $\beta_{2,t}x_t$ can be replaced according to Equation (3.2.10) to (3.2.18):

$$
\begin{aligned}
&y_{t+k} = \beta_{1,t} + \beta_{2,t}x_t + \varepsilon_t^y,\\
&\beta_t = \beta_{t-1} + \varepsilon_t^\beta, \quad \beta_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_0), \varepsilon_t^\beta \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}),\\
&\varepsilon_t^y = \omega_t + \psi\omega_{t-1}, \quad \omega_t \sim \mathcal{N}(0, \mathrm{e}^{h_t}),\\
&h_t = h_{t-1} + \varepsilon_t^h, \quad h_1 \sim \mathcal{N}(0, \sigma_{0h}^2), \varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2),
\end{aligned}
$$

6. Dynamic model averaging (**DMA**) has the same model structure as **TVC**, but all the candidate inflation predictors are used as explanatory variables. Model averaging is conducted by seven predictors with Equation (3.2.10) to (3.2.18). In the present study, zero and one lag are considered for inflation, and up to two lags are considered for the explanatory variables due to the computational burdens discussed above.

7. Dynamic model selection (**DMS**) uses the same strategy as **DMA**, but only a

single model is selected at each time point.

3.4.2 Forecasting Metrics

In this section, we discuss the metrics for evaluating the forecasts. Both the mean absolute forecast errors (MAFE) and the average of log predictive likelihoods (ALPL) are employed to compare forecasts from different models.

Specifically, MAFE uses absolute values of the difference between the actual values and the forecasts to evaluate the forecasting performance. It measures point forecasts accuracy. The rolling window MAFE is defined as:

$$MAFE_{k,i} = \frac{1}{T - T_0 - k + 1} \sum_{t=1}^{T-T_0-k+1} |\widehat{y}_{T_0+t+k-1,i} - y^0_{T_0+t+k-1}|,$$

where $\widehat{y}_{T_0+t+k-1,i}$ is the k-step ahead point forecast for model i, and $y^0_{T_0+t+k-1}$ is the realized inflation at time $T_0 + t + k - 1$.

For density forecasts, ALPL can give an average evaluation of the quality of the predictive density, and it is used to evaluate the forecasting performance of density forecasts. The rolling window average of log predictive likelihood is defined as:

$$ALPL_{k,i} = \frac{1}{T - T_0 - k + 1} \sum_{t=1}^{T-T_0-k+1} \log p(\widehat{y}^0_{T_0+t+k-1,i} = y_{T_0+t+k-1} \,|\, \mathbf{y}_{1:T_0+t}).$$

It can be seen as summarizing out-of-sample forecasting prediction performance in the form of predictive likelihood (Geweke, 1999), and it can describe the density forecasting performance of competitive models. Therefore, ALPL is very attractive to both policymakers and forecasters. In practice, a bigger value of ALPL implies a better density forecast.

3.4.3 Forecast Combination

For point forecasts, both equal weights and time-varying weights are considered. Similar to the combination weights of mean square forecast error in Stock and Watson (2004), we use a window of the previous 40 periods to calculate MAFE so that for each forecast horizon k and for each type of specifications (**TVC**, **TVC-SV**, and **TVC-SVMA**), the MAFE at time $T_0 + t$is: $\text{MAFE}_{T_0+t,T_0+t+k,i} = \sum_{\tau=T_0+t-40}^{T_0+t-1} |y_\tau - \widehat{y}_{\tau,i}|$, where i stands for a specific inflation predictor or one model in model combination. Then the time-varying weights, based on the inverse absolute forecast errors for point forecast model averaging, are calculated by:

$$\widehat{w}^{\text{MAFE}}_{T_0+t,T_0+t+k,i} = \frac{1}{\text{MAFE}_{T_0+t,T_0+t+k,i}} / \sum_{j=1}^{N} (1/\text{MAFE}_{T_0+t,T_0+t+k,i}), \tag{3.4.1}$$

where N is the total number of inflation predictors. The weights $w^{\text{MAFE}}_{T_0+t,T_0+t+k,i}$ are all non-negative and can be summed to unity. This may vary with recursive forecasts in the entire forecasting evaluation time period.

Thus the forecast combination of each type of time-varying coefficient specifications (**TVC**, **TVC-SV** and **TVC-SVMA**) for k-step ahead forecasts is:

$$\widehat{y}^{\text{comb-MAFE}}_{T_0+t,T_0+t+k} = \sum_{i=1}^{N} (\widehat{w}^{\text{MAFE}}_{T_0+t,T_0+t+k,i} \cdot \widehat{y}_{T_0+t,T_0+t+k,i}).$$

For density forecast combinations, in addition to using equal weights, we consider an approach that is based on forecasting performance. Specifically, the density combination weights use the products of predictive likelihoods (PPL) with a rolling window of 40 periods (Chan et al., 2012). That is, given information up to time $T_0 + t$, the weight $\widehat{w}^{\text{ALPL}}_{T_0+t,T_0+t+k,i}$ for each model i is proportional to products of $\prod_{\tau=T_0+t-k-40}^{T_0+t-k} P(y_{\tau+k} = y^0_{\tau+k} | y_{1:T_0+t})$. The weights are then normalized so that they sum to 1. As ALPL is used in forecasting evaluation, the present study uses linear combinations based on the log predictive likelihood of each candidate model. Then the forecast combination of each type of time-varying coefficient specifications (**TVC**, **TVC-SV** and **TVC-SVMA**) for the k-step ahead forecasts is:

$$\widehat{y}^{\text{comb-ALPL}}_{T_0+t,T_0+t+k} = \sum_{i=1}^{N} (\widehat{w}^{\text{ALPL}}_{T_0+t,T_0+t+k,i} \cdot \widehat{y}_{T_0+t,T_0+t+k,i}).$$

3.4.4 Forecasting Results

Both point and density forecasting results of PCE deflator inflation are summarized in Table 3.1. For easy comparison, the relative MAFE and ALPL scores to the **UC-SV** benchmark are reported. The relative MAFE is the ratio of MAFE of a given model to that of **UC-SV**. Thus, a value less than one indicates better forecasting performance than the benchmark. The relative ALPL is the difference of ALPL of a given model to that of **UC-SV**. Hence a positive value indicates better forecast performance. The forecasting results are listed in four blocks, relating to unobserved components models, time-varying coefficient models with CUR coincident, model combinations, and dynamic model averaging and selection.

Table 3.1: Real-time forecasts for PCE inflation.

	MAFE				**ALPL**			
	$k=1$	$k=4$	$k=8$	$k=16$	$k=1$	$k=4$	$k=8$	$k=16$
UC-SV	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
UC-SVMA	1.14	1.11	1.23	1.02	-0.10	-0.07	-0.23	0.00
CUR Coincident								
TVC	1.06	1.15	1.11	1.08	1.67	2.37	2.67	3.02
TVC-SV	1.05	1.10	1.08	1.04	1.62	2.30	2.60	2.92
TVC-SVMA	1.02	1.11	1.08	1.05	1.53	2.34	2.65	2.98
Time-varying Weights								
TVC	1.00	0.95	0.94	0.89	2.12	2.77	3.05	3.50
TVC-SV	1.00	0.97	0.98	0.93	3.54	3.74	3.81	4.14
TVC-SVMA	0.99	0.97	0.99	0.94	3.54	3.71	3.85	4.12
Equal Weights								
TVC	1.01	0.95	0.93	0.89	2.10	2.44	2.74	2.96
TVC-SV	1.01	0.97	0.98	0.93	2.39	2.84	3.14	3.47
TVC-SVMA	1.00	0.97	0.98	0.93	2.37	2.86	3.16	3.50
DMA (1,0)	1.43	1.56	1.47	1.20	-0.42	-0.13	-0.29	-0.84
DMS (1,0)	1.60	1.59	1.55	1.33	-0.08	0.08	-0.07	-0.65
DMA (1,1)	1.89	1.54	1.42	1.43	-0.19	-0.12	-0.28	-0.73
DMS (1,1)	2.05	1.61	1.53	1.66	0.08	0.22	0.09	-0.41
DMA (2,0)	2.04	1.60	1.55	1.24	-0.09	-0.01	-0.20	-0.77
DMS (2,0)	2.13	1.68	1.60	1.38	0.10	0.20	0.03	-0.56
DMA (2,1)	2.07	1.63	1.54	1.41	-0.04	0.01	-0.15	-0.62
DMS (2,1)	2.18	1.67	1.66	1.67	0.22	0.32	0.18	-0.33

In Table 3.1, the real-time forecasting results suggest that the forecast combinations based on time-varying coefficient models can outperform univariate models and a single inflation predictor (e.g. CUR) for both point forecasts and density forecasts. In particular, using additional predictors and considering different lag structures improve forecast performance relative to univariate models (**UC-SV** and **UC-SVMA**) and time-varying coefficient models with one inflation predictor. It supports the conclusion of Koop and Korobilis (2012) that allowing for changes of models over time is more important than any other model specifications for inflation forecasts.

The results of both the point forecasts and the density forecasts suggest that the specification of **TVC** has the best forecasting performance for both time-varying weights and equal weights. For all three specifications (**TVC**, **TVC-SV**, and **TVC-**

SVMA), equal weights and time-varying weights have similar forecasting performance. These results are in line with the findings that equal weights may not have worse forecasting performance than time-varying weights (e.g., Stock and Watson, 2004; Clark and McCracken, 2009; Jore, Mitchell, and Vahey, 2010).

Comparing the forecasting performance of **TVC-SV** and **TVC-SVMA** with that of **TVC**, it appears that adding SV improves density forecasts but not point forecasts. For point forecasts, the performance of models with stochastic volatility worsens in longer horizons. However, for density forecasts, the results indicate that the specifications with stochastic volatility tend to forecast better.

Although the specifications of both **TVC-SV** and **TVC-SVMA** have competitive forecasting performance in time-varying averaging and equal weight blocks, their performance is not significantly different from each other. It suggests that when forecast combination is allowed, the specification of moving average does not significantly assist in inflation forecasts. For forecasting performance improvement, model combination with various covariates is more important than SV or SVMA.

In the **DMA** and **DMS** block, no single specification consistently outperforms the others in either point or density forecasts. Time-varying coefficient models with one inflation predictor can have better forecasting performance than those of **DMA** and **DMS**, while model averaging can keep improving the forecasting performance with introducing more inflation predictors. For point forecasts, **DMA(1,0)** with one lag of inflation and no lags of inflation predictors provides the optimal forecasts in all forecasting horizons, while the density forecasting results suggest that **DMS (2,1)** has the best forecasting performance results in all horizons. The forecasting results of **DMA** and **DMS** also suggest that forecasts with more lags of inflation are not helpful for shorter horizon forecasts, while forecasts with lags of both inflation and inflation predictors are not good for longer horizon forecasts. However, some **DMA** and **DMS** specifications are better than the unobserved components models, which tend to perform worse than combination forecasts.

3.4.5 Weights of Inflation Predictors Grouped by Lag Forms

In this section, we present the time-varying weights computed for forecast combinations in the previous sections. For one-step-ahead point forecasts, the weights of 63 models with **TVC** specification are grouped by nine lag forms (see Equation 3.2.10-3.2.18) and are plotted in Figure 3.10. Similar plots for other forecast horizons are given in Appendix 3.A.

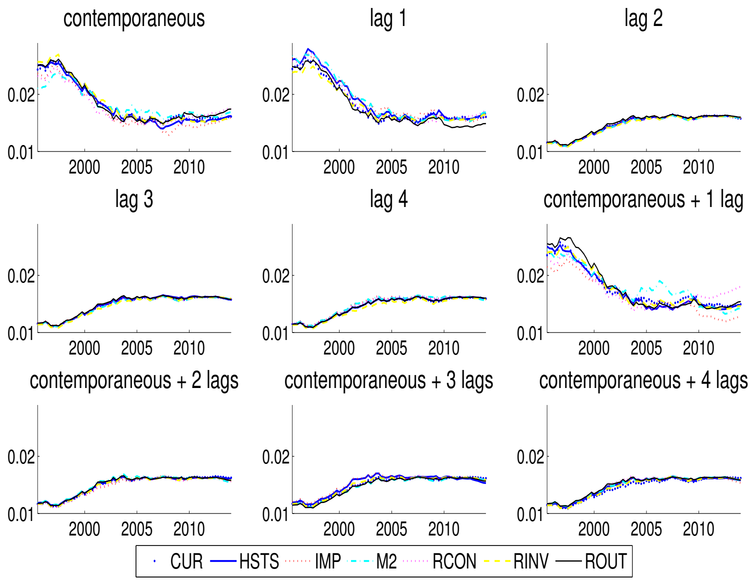

Figure 3.10: Weights of component models for **TVC**, forecasting horizon one.

The results in Figure 3.10 show that models with contemporaneous and one lag tend to have more weights than models with more lags before 2000, but these weights decline after 2000. In contrast, the weights of other lag forms keep growing and the weights are similar among lag forms at the end of the sample. It indicates that the choice of lag length is more important than that of inflation predictor, and the optimal choice of lag length is time-varying.

Table 3.2: Correlation coefficients between inflation predictors.

	CUR	HSTS	IMP	M2	RCON	RINV	ROUT
CUR	1.00	-0.07	-0.11	0.20	-0.23	-0.14	-0.28
HSTS		1.00	-0.08	0.26	0.38	0.66	0.31
IMP			1.00	-0.12	-0.17	-0.17	-0.02
M2				1.00	0.19	0.30	0.08
RCON					1.00	0.54	0.67
RINV						1.00	0.53
ROUT							1.00

On the other hand, when model averaging is introduced, the numbers of inflation predictors and specifications are not small. The results in each sub-figure suggest that no single inflation predictor can forecast significantly better than any other predictor, which is consistent with literature (e.g., Clark and McCracken, 2009; Jore, Mitchell, and Vahey, 2010). Figure 3.1 presents the inflation predictors with stationary transformation, which suggests that the time series are not cointegrated. Table 3.2 reports the correlation coefficients between inflation predictors, and most correlation coefficients are below 0.5. It indicated that the inflation predictors are not highly correlated with each other. Apparently, it is hard to find one particular inflation predictor that outperforms the others, which highlights the importance of forecast combinations.

3.5 Concluding Remarks

In this chapter, forecast combinations using both time-varying and equal weights of 63 component models have been investigated. Three specifications of time-varying coefficient models were employed for PCE inflation forecasts in the US. In the real-time forecasting exercise, forecast combinations tend to improve univariate models with only inflation, which suggests that adding variables can significantly improve forecasting performance. Our results show that the specification of **TVC** has the best forecasting performance in point forecasts. The analysis of weights among inflation predictors and lag lengths for **TVC** further suggests that a time-varying lag length choice is more important than the choice of inflation predictors.

For density forecasts, both **TVC-SV** and **TVC-SVMA** have better forecasting performances, which suggests that adding stochastic volatility is helpful for density forecasts. The importance of allowing for stochastic volatility in forecasting better combinations can improve density forecasting performance.

Both point forecasts and density forecasts suggest that equal weights and time-varying weights have similar forecasting performance. Moreover, the moving average does not seem to improve forecast performance when forecast combinations are used.

Appendix 3.A Bayesian Estimation Method: MCMC Algorithm

The model is estimated by Bayesian methods using the priors specified in Section 3.2.4. Specifically, we derive a Markov chain Monte Carlo (MCMC) algorithm to sample from the joint posterior distribution. In particular, Gibbs sampling, the Metropolis-Hastings algorithm, and precision-based algorithm are employed for simulation.

Here, we take the estimation of parameters in **TVC-SVMA** as an example. There are five blocks in the MCMC algorithm. We draw $\boldsymbol{\beta}$, stochastic volatility parameter $\mathbf{h}$, the covariance matrix $\mathbf{Q}$ of $\boldsymbol{\beta}$, and the moving average coefficient ψ sequentially by conditioning on the inflation and inflation predictors:

1. $p(\boldsymbol{\beta} \,|\, \mathbf{y}, \mathbf{u}, \psi, \mathbf{h}, \mathbf{Q})$;
2. $p(\mathbf{h} \,|\, \mathbf{y}, \mathbf{u}, \psi, \sigma_h^2)$;
3. $p(\mathbf{Q} \,|\, \mathbf{y}, \mathbf{u}, \boldsymbol{\beta})$;
4. $p(\psi \,|\, \mathbf{y}, \mathbf{u}, \boldsymbol{\beta}, \mathbf{h})$;
5. $p(\sigma_h^2 \,|\, \mathbf{h})$.

In the first step, we draw vectors of the intercept and coefficients $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_j$ together by rearranging them into one vector $\boldsymbol{\beta} = (\beta_{1,1}, \beta_{2,1}, \beta_{j,1}, \ldots, \beta_{1,T}, \beta_{2,T}, \beta_{j,T})'$. Equation (3.2.6) and (3.2.8) can be stacked over t

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}^y, \tag{3.A.1}$$

$$\boldsymbol{\varepsilon}^y = \mathbf{H}_\psi \mathbf{u}. \tag{3.A.2}$$

Hence, we have

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{H}_\psi \mathbf{u}. \tag{3.A.3}$$

This is a standard normal model. We assume normal conjugate priors for the variables and the intercept and coefficients $\boldsymbol{\beta}$, so that the posterior is also normal. In fact, we have:

$$(\boldsymbol{\beta} \,|\, \mathbf{y}, \mathbf{u}, \psi, \mathbf{h}, \mathbf{Q}) \sim \mathcal{N}(\widehat{\boldsymbol{\beta}}, \mathbf{D}_{\boldsymbol{\beta}}).$$

The **TVC-SVMA** model has a similar model structure as the state space model used by Chan (2013); we can use the precision sampler (Chan and Jeliazkov, 2009) to draw the posterior of $\boldsymbol{\beta}$ efficiently.

The second step draws the stochastic volatility $\mathbf{h}$ conditioning on the observed variables, and other parameters. This is done using the auxiliary mixture sampler of Kim et al. (1998). This method provides an accessible and efficient approximation of a nonlinear stochastic volatility model using a mixture of linear Gaussian state space models.

The third step draws the covariance matrix of $\boldsymbol{\beta}$. Stacking (3.2.7) over t, we have:

$$\mathbf{H}\boldsymbol{\beta} = \boldsymbol{\varepsilon}^{\beta}, \quad \boldsymbol{\varepsilon}^{\beta} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}), \tag{3.A.4}$$

where

$$\boldsymbol{\Omega} = diag(\mathbf{Q}_0, \mathbf{Q}, \cdots, \mathbf{Q}).$$

As $\mathbf{Q}$ is assumed to be diagonal, $\boldsymbol{\Omega}$ is also diagonal. Thus, σ^2_β in step 3 and σ^2_h in step 5 are both conditionally independent. Both of them have conjugate inverse-gamma priors, and their posteriors can be estimated by the standard method discussed in Koop (2003).

Below, we present the details of each step for deriving the conditional posterior distributions of parameters in the proposed model.

Step 1: Sampling for the Coefficient Parameter $\boldsymbol{\beta}$

To derive the conditional posterior distribution $p(\boldsymbol{\beta} \,|\, \mathbf{y}, \mathbf{u}, \psi, \mathbf{h}, \mathbf{Q})$, we first have:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{H}_\psi \mathbf{u},$$

$$\boldsymbol{\beta} = \mathbf{H}^{-1}\boldsymbol{\varepsilon}_{\boldsymbol{\beta}},$$

where

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & -1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & -1 & 1 \end{pmatrix}, \quad \mathbf{H}_\psi = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \psi_1 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \psi_q & \cdots & \psi_1 & 1 & \cdots & 0 \\ \vdots & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & \psi_q & \cdots & \psi_1 & 1 \end{pmatrix}.$$

Then we pre-multiply both sides of the above two equations by $\mathbf{H}_\psi^{-1}$:

$$\widetilde{\mathbf{y}} = \mathbf{X}\widetilde{\boldsymbol{\beta}} + \mathbf{u},$$

$$\widetilde{\boldsymbol{\beta}} = \mathbf{H}_{\psi}^{-1}\mathbf{H}^{-1}\boldsymbol{\varepsilon}_{\beta},$$

where $\widetilde{\mathbf{y}} = \mathbf{H}_{\psi}^{-1}\mathbf{y}$ and $\widetilde{\boldsymbol{\tau}} = \mathbf{H}_{\psi}^{-1}\boldsymbol{\tau}$.

Thus, the log posterior density for $\widetilde{\boldsymbol{\beta}}$ is:

$$\log p(\widetilde{\boldsymbol{\beta}} \mid \widetilde{\mathbf{y}}, \mathbf{h}, \psi, \sigma_{\beta}^2) \propto \log p(\widetilde{\boldsymbol{\beta}} \mid \sigma_{\beta}^2) + \log p(\widetilde{\mathbf{y}} \mid \widetilde{\boldsymbol{\beta}}, \mathbf{h}, \psi), \tag{3.A.5}$$

where $p(\widetilde{\boldsymbol{\beta}} \mid \sigma_{\beta}^2)$ is the prior for $\widetilde{\boldsymbol{\beta}}$ and $p(\widetilde{\mathbf{y}} \mid \widetilde{\boldsymbol{\beta}}, \mathbf{h}, \psi)$ is the likelihood for $\widetilde{\mathbf{y}}$. So that the log-likelihood of $\widetilde{\mathbf{y}}$ and the prior of $\widetilde{\boldsymbol{\beta}}$ are:

$$\log p(\widetilde{\mathbf{y}} \mid \widetilde{\boldsymbol{\beta}}, \mathbf{h}, \phi, \psi) \propto -\frac{1}{2}\sum_{t=1}^{T} h_t - \frac{1}{2}(\widetilde{\mathbf{y}} - \mathbf{X}\widetilde{\boldsymbol{\beta}})'\boldsymbol{\Omega}_u^{-1}(\widetilde{\mathbf{y}} - \mathbf{X}\widetilde{\boldsymbol{\beta}}), \tag{3.A.6}$$

$$\log p(\widetilde{\boldsymbol{\beta}} \mid \sigma_{\tau}^2) \propto -\frac{T-1}{2}\log \sigma_{\beta}^2 - \frac{1}{2}\widetilde{\boldsymbol{\beta}}'\mathbf{X}'\mathbf{H}_{\psi}'\mathbf{H}'\mathbf{Q}^{-1}\mathbf{H}\mathbf{H}_{\psi}\mathbf{X}\widetilde{\boldsymbol{\beta}}, \tag{3.A.7}$$

where $\boldsymbol{\Omega}_u = diag(\mathrm{e}^{h_1}, \cdots, \mathrm{e}^{h_t})$.

Putting (3.A.7) and (3.A.6) into (3.A.5), we have:

$$\begin{aligned}
\log p(\widetilde{\boldsymbol{\beta}} \mid \widetilde{\mathbf{y}}, \mathbf{h}, \phi, \psi, \sigma_{\tau}^2) &\propto -\frac{1}{2}\widetilde{\boldsymbol{\beta}}'\mathbf{H}_{\psi}'\mathbf{H}'\mathbf{Q}^{-1}\mathbf{H}\mathbf{H}_{\psi}\widetilde{\boldsymbol{\beta}} - \frac{1}{2}(\widetilde{\mathbf{y}} - \mathbf{X}\widetilde{\boldsymbol{\beta}})'\boldsymbol{\Omega}_u^{-1}(\widetilde{\mathbf{y}} - \mathbf{X}\widetilde{\boldsymbol{\beta}}) \\
&\propto -\frac{1}{2}(\widetilde{\boldsymbol{\beta}}'(\mathbf{H}_{\psi}'\mathbf{H}'\mathbf{Q}^{-1}\mathbf{H}\mathbf{H}_{\psi} \ + \mathbf{X}'\boldsymbol{\Omega}_u^{-1}\mathbf{X})\widetilde{\boldsymbol{\beta}} - 2\mathbf{X}'\widetilde{\boldsymbol{\beta}}'\boldsymbol{\Omega}_u^{-1}\widetilde{\mathbf{y}}) \\
&\propto -\frac{1}{2}(\widetilde{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}})'\mathbf{D}_{\widetilde{\beta}}^{-1}(\widetilde{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}}),
\end{aligned}$$

where $\mathbf{D}_{\widetilde{\beta}} = (\mathbf{H}_{\psi}'\mathbf{H}'\mathbf{Q}^{-1}\mathbf{H}\mathbf{H}_{\psi} + \mathbf{X}'\boldsymbol{\Omega}_u^{-1}\mathbf{X})^{-1}$ is a sparse matrix and $\widehat{\boldsymbol{\beta}} = \mathbf{D}_{\widetilde{\beta}}\mathbf{X}'\boldsymbol{\Omega}_u^{-1}\widetilde{\mathbf{y}}$. So that:

$$(\widetilde{\boldsymbol{\beta}} \mid \widetilde{\mathbf{y}}, \mathbf{h}, \phi, \psi, \sigma_{\tau}^2) \sim \mathcal{N}(\widehat{\boldsymbol{\beta}}, \mathbf{D}_{\widetilde{\beta}}).$$

By Cholesky decomposition and forward and backward substitution (details in Chan (2013)), we can finally sample $\boldsymbol{\beta}$ by $\boldsymbol{\beta} = \mathbf{H}_{\psi}\widetilde{\boldsymbol{\beta}}$.

Step 2: Sampling for h

We follow Kim et al. (1998) to sample the SV component $\mathbf{h}$, which uses an auxiliary mixture of seven normal distributions to draw $\mathbf{h}$ efficiently. In practice, we adopt the algorithm used in Chan (2013), which is a precision-based sampler, instead of the forward-backward smoothing algorithm used in Kim et al. (1998). To apply this method, we first define:

$$\mathbf{y}^* = \mathbf{H}_{\psi}^{-1}(\mathbf{y} - \beta\mathbf{X})),$$

So that $\mathbf{y}^* = \mathbf{u}, \mathbf{u} \sim \mathcal{N}(\mathbf{0}, \mathbf{S}_y)$, where $\mathbf{S}_y = \text{diag}(e^{h_1}, \cdots, e^{h_T})$. The detailed sampling procedure can also be obtained from Koop and Korobilis (2009).

Step 3: Sampling for σ_h^2 and $\mathbf{Q}$

We assume that σ_h^2 and the diagonal elements of $\mathbf{Q}$ ($\sigma_{\beta_1}^2, \cdots, \sigma_{\beta_k}^2$) are conditionally independent, so that the derivations of their posteriors can follow the standard method discussed in Koop (2003). Thus, their posteriors can be obtained after a simple transformation. Given a conjugate inverse-gamma prior $\sigma_h^2 \sim \mathcal{IG}(\nu_h, S_h)$, we can derive an inverse-gamma posterior for $(\sigma_h^2 \,|\, \mathbf{h})$,

$$\begin{aligned}
p(\sigma_h^2 \,|\, \mathbf{h}) &\propto p(\mathbf{h} \,|\, \sigma_h^2) + p(\sigma_h^2) \\
&= (\sigma_h^2)^{-\frac{T}{2}} \exp\left(-\frac{1}{2\sigma_h^2}\sum_{t=2}^{T}(h_t - h_{t-1})^2\right) \cdot (\sigma_h^2)^{-(\nu_{0h}-1)} \exp(-\frac{S_h}{\sigma_h^2}) \\
&\propto (\sigma_h^2)^{-((\frac{T}{2}+\nu_0)-1)} \exp\left(-\frac{1}{\sigma_h^2}(\sum_{t=2}^{T}(h_t - h_{t-1})^2/2 + S_h)\right).
\end{aligned}$$

That is:

$$(\sigma_h^2 \,|\, \mathbf{h}) \sim \mathcal{IG}\left(T/2 + \nu_h, \quad \sum_{t=2}^{T}(h_t - h_{t-1})^2/2 + S_h\right).$$

Similarly, the posterior density of each diagonal element in $\mathbf{Q}$ can be derived as above, and we can stack them into matrix form in empirical simulation:

$$(\mathbf{Q} \,|\, \boldsymbol{\beta}) \sim \mathcal{IG}\left(T/2 + \nu_\beta, \quad \sum_{t=2}^{T}(\mathbf{H}\boldsymbol{\beta})^2/2 + S_\beta\right).$$

Step 4: Sampling for ψ

Note that given $\mathbf{y}, \boldsymbol{\beta}$ and $\mathbf{h}, \psi$ is conditionally independent from σ_h^2 and $\mathbf{Q}$; thus, we can draw σ_h^2, $\mathbf{Q}$ and ψ sequentially in the simulation. We first stack (3.2.6) and (3.2.8) into matrix form:

$$\mathbf{H}_\phi(\mathbf{y} - \boldsymbol{\tau}) = \mathbf{H}_\psi \mathbf{u}. \tag{3.A.8}$$

Then, the log-likelihood of posterior $(\psi \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h})$ is:

$$
\begin{aligned}
\log p(\psi \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h}) &\propto \log p(\mathbf{y} \,|\, \psi, \boldsymbol{\tau}, \mathbf{h}) + \log p(\psi) \\
&\propto \log p(\psi) - \frac{1}{2}(\mathbf{H}_{\phi}(\mathbf{y} - \boldsymbol{\tau}))'(\mathbf{H}_{\psi}'\boldsymbol{\Omega}_u\mathbf{H}_{\psi})^{-1}\mathbf{H}_{\phi}(\mathbf{y} - \boldsymbol{\tau}).
\end{aligned}
$$

Unlike $\mathbf{y}, \boldsymbol{\beta}, \sigma_h^2$ or σ_τ^2, the distributions of moving average parameters ψ are unknown, and ψ is typically low dimensional in empirical economic studies (Chan, 2013), so it can be evaluated numerically by maximizing $\log p(\psi|\mathbf{y}, \boldsymbol{\beta}, \mathbf{h})$ to obtain the mode and the negative Hessian evaluated at the mode. Then we use the Metropolis-Hastings algorithm detailed in Chan (2013) to sample ψ, which is widely used to simulate the posterior of a multivariate model. The proposed density for ψ is multi-normal distribution $q(\psi)$, and the updated ψ^c is accepted with the probability:

$$
\min\{1, \quad \frac{p(\psi^c \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h})}{p(\psi \,|\, \mathbf{y}, \boldsymbol{\tau}, \mathbf{h})} \cdot \frac{q(\psi)}{q(\psi^c)}\}.
$$

Appendix 3.B Figures of Weights for Each Inflation Predictor and Lag Form

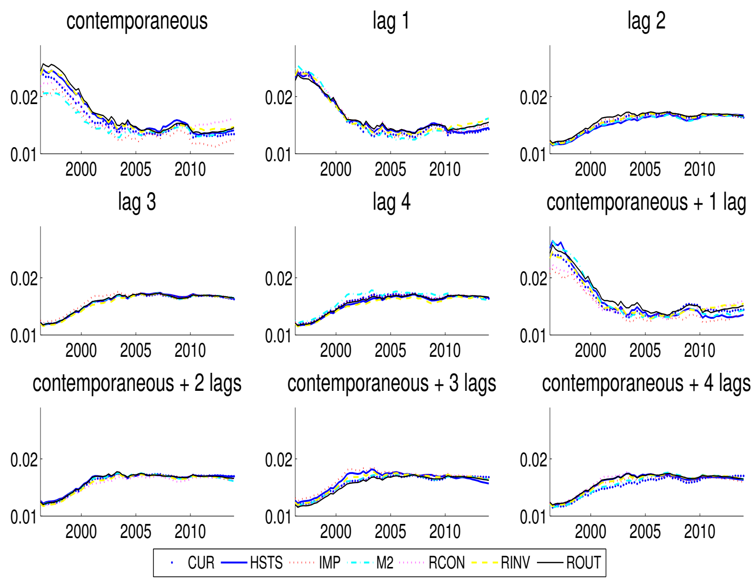

Figure 3.11: Weights of component models for **TVC**, forecasting horizon four.

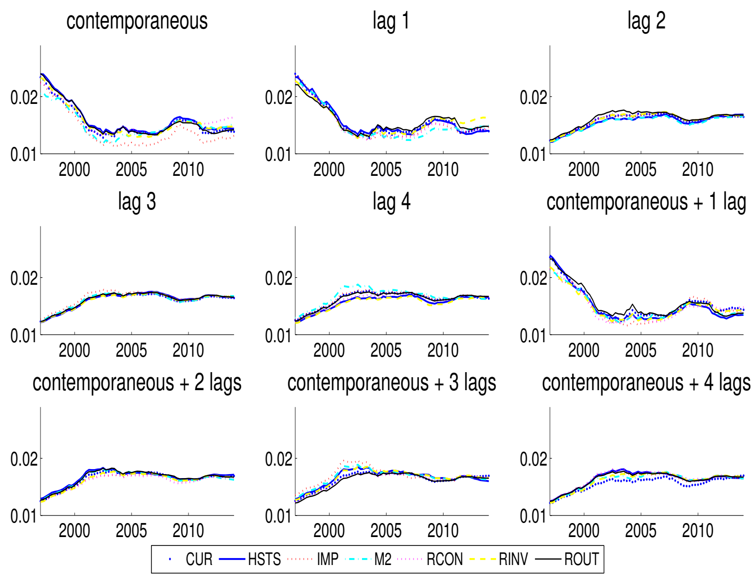

Figure 3.12: Weights of component models for **TVC**, forecasting horizon eight.

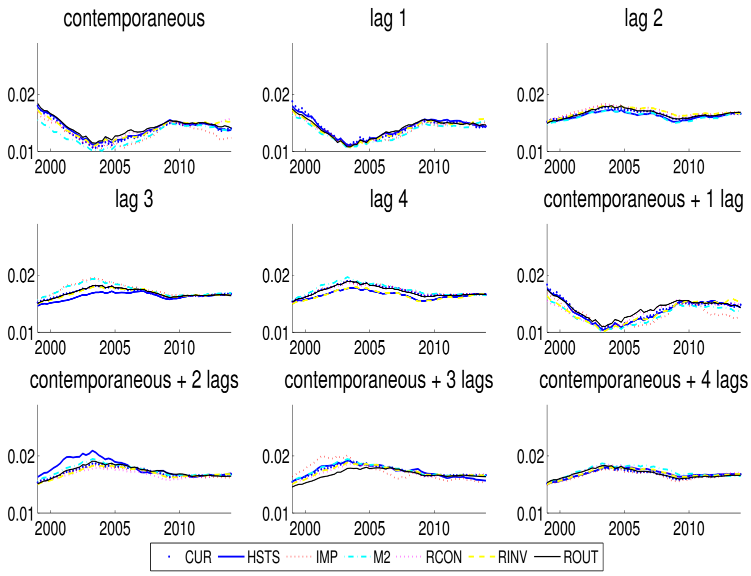

Figure 3.13: Weights of component models for **TVC**, forecasting horizon sixteen.

Chapter 4

The Importance of Stochastic Volatility in Renewable Energy Forecasts

4.1 Introduction

It is widely accepted that traditional energy sources, such as coal and crude oil, cause serious environmental problems. For instance, it is well known that the consumption of these resources releases greenhouse gases that contribute to climate change (e.g., Soytas and Sari, 2009; Apergis and Payne, 2010). In order to limit the emission of CO_2, particularly in industrialized countries, the Kyoto Protocol was instituted in 1997. In the early 1990s, the concept of the environmental Kuznets curve arose, and critical studies followed, suggesting that other frontier models need to be considered for the emissions and economic growth relationship (Grossman and Krueger, 1991; Stern, 2004). More recently, researchers and governments have begun to consider the interaction of the combustion of fossil fuels, the quality of the natural environment, especially the atmosphere, and economic growth (e.g., Stern, 2004; Halicioglu, 2009; Soytas and Sari, 2009).

Renewable energy sources have attracted increasing attention, as energy security is another factor that needs to be considered by governments. In addition, the promotion of renewable energy can not only reduce the emission of greenhouse gases but also support sustainable economic development. Given this context, techniques that can generate environmentally friendly renewable energy continue to receive considerable government support and attract generous industrial investment. As a result of these favorable government policies, rising crude oil prices, and market investment, renewable energy is now a rapidly growing sector of the energy market. Thus, the relationship between renewable energy and economic development has attracted increasing attention. There is a growing body of literature showing that renewable energy makes a significant contribution to economic growth and can increase gross domestic product (GDP) in the long run (e.g., Payne, 2010; Sari and Soytas, 2004; Apergis and Payne, 2010; Menegaki, 2011; Tugcu et al., 2012; Pao and Fu, 2013).

Regarding GDP, and CO_2 these two of the most closed variables to renewable energy generation (REG), the vector autoregressive (VAR) method is a useful technique to study the interactions between all these three variables in a model (Silva et al., 2012) and do forecasts for REG. In practice, a time-varying parameter vector autoregression model can be adopted to study REG via a time-varying interaction with GDP and CO_2. In fact, there is increasing acceptance of the view that models with a constant variance are not flexible enough to capture the changeable behaviors of macroeconomic variables (e.g., Primiceri, 2005; Nakajima et al., 2011; Baumeister and Peersman, 2013; Clark and Ravazzolo, 2015; Cross and Nguyen, 2017). In this chapter, we examine both time-varying parameter and stochastic volatility (SV) specifications in VAR models, and the forecasting results suggest that VARs with SV can provide better forecasts for REG than those with constant variances.

In the empirical section, we first present the evidence on time-varying parameters and SV. The reason for allowing for time-varying parameters is that although there are studies that apply structural break methods to model changes in the economy (such as Lee and Chang (2007), who use panel VARs for energy consumption and real GDP, and Ohler and Fetters (2014), who use an error correction model to examine the relationship between renewable energy uptake and GDP growth), they cannot precisely predict the movement among variables in a gradually updated macroeconomic environment. There are also a number of studies using constant parameter VAR methodology to investigate energy-related problems (research on exploring the relationships of energy consumption, real GDP, CO_2, and crude oil prices (see Lee and Chang, 2007; Narayan et al., 2008; Soytas and Sari, 2009; McPhail, 2011)), and studies on renewable energy employing constant parameter VAR methods (e.g., Silva et al., 2012; Onafowora and Owoye, 2015). On the other hand, there are studies forecasting renewable energy, but they are mainly on hourly and daily generation or price, so more frequent time series are used (Foley et al., 2012; Ordiano et al., 2017). This study differs from these by adopting quarterly data, and the forecasting exercise is conducted from one quarter up to four years.

The rest of the chapter is organized as follows. In Section 4.2, VAR models with different specifications are described and a Bayesian estimation strategy is discussed. In Section 4.3, the preliminaries to the empirical study are presented. In Section 4.4, the empirical evidence on SV and the results of forecasting are discussed. The last section presents the conclusions.

4.2 VAR Models

In this section, two types of VAR specifications are considered: time-varying parameter and SV. The most general model, **TVP-VAR-SV**, is discussed in this section.

4.2.1 TVP-VAR-SV

The time-varying parameter VAR can take a reduced form. Let $\mathbf{y}_t = (y_{1t}, \cdots, y_{nt})'$ denote an $n \times 1$ vector of endogenous variables in **TVP-VAR-SV**. Both intercepts and coefficients are assumed to be time-varying. Let $\mathbf{B}_{1t}, \cdots, \mathbf{B}_{pt}$ denote $n \times n$ time-varying coefficient matrices, and let $\boldsymbol{\beta}_{0t}$ denote the time-varying intercepts.

$$\mathbf{y}_t = \boldsymbol{\beta}_{0t} + \mathbf{B}_{1t}\mathbf{y}_{t-1} + \cdots + \mathbf{B}_{pt}\mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_{yt}, \quad \boldsymbol{\varepsilon}_{yt} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{yt}), \tag{4.2.1}$$

Following Primiceri (2005), the covariance matrix can be factorized using a Cholesky decomposition. We specify a lower-triangular matrix A for the simultaneous response to external shocks, so that the identification process follows an order of variables given in matrix A. The SV is reflected in a diagonal matrix Ω_{yt} incorporated by the standard deviations (SD) of shocks:

$$\mathbf{A}_t \Sigma_{yt} \mathbf{A}_t' = \Omega_{yt}\Omega_{yt}',$$

where the parameters of $\mathbf{A}_t$ and $\boldsymbol{\Sigma}_t$ are all time-varying, and:

$$\mathbf{A}_t = \begin{pmatrix} 1 & & \mathcal{O} & \\ \alpha_{1t} & \ddots & & \\ \vdots & \ddots & \ddots & \\ \alpha_{n,1t} & \cdots & \alpha_{n,n-1t} & 1 \end{pmatrix}, \quad \Omega_{yt} = \begin{pmatrix} \mathrm{e}^{\frac{1}{2}h_{1t}} & & \mathcal{O} \\ & \ddots & \\ \mathcal{O} & & \mathrm{e}^{\frac{1}{2}h_{nt}} \end{pmatrix}.$$

$h_{1t}, \cdots, h_{nt}$ above are SV parameters. Both $\mathbf{A}_t$ and Ω_{yt} are used to decompose the variance covariance matrix in (4.2.1). This decomposition is considered to be an efficient way to estimate covariance matrices in time-varying parameter VAR models (e.g., Primiceri, 2005; Nakajima et al., 2011).

Thus (4.2.1) becomes:

$$\mathbf{y}_t = \boldsymbol{\beta}_{0t} + \mathbf{B}_{1t}\mathbf{y}_{t-1} + ... + \mathbf{B}_{pt}\mathbf{y}_{t-p} + \mathbf{A}_t^{-1}\boldsymbol{\Omega}_{yt}\boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, I_n), \tag{4.2.2}$$

where I_n stands for a $n \times n$ identity matrix.

The parameter components of $\boldsymbol{\beta}_{0t}$ and $\mathbf{B}_t$ are stacked into a vector $\boldsymbol{\beta}_t$ and the free elements of $\mathbf{A}_t$ can be stacked into a vector $\boldsymbol{\alpha}_t$. It is worth noting that the total number of elements of $\boldsymbol{\beta}_{0t}, \mathbf{B}_{1t}, \cdots, \mathbf{B}_{pt}$ are $n + n^2k$, $\boldsymbol{\alpha}_t$ is $((n-1) \times n)/2$ and $\mathbf{h}_t$ is n, respectively. These vectors evolve according to independent random walks:

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\varepsilon}_{\beta_t}, \quad \beta_1 \sim \mathcal{N}(0, \sigma^2_{0\beta}), \boldsymbol{\varepsilon}_{\beta_t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\beta), \tag{4.2.3}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\varepsilon}_{\alpha_t}, \quad \alpha_1 \sim \mathcal{N}(0, \sigma^2_{0\alpha}), \boldsymbol{\varepsilon}_{\alpha_t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\alpha), \tag{4.2.4}$$

$$\mathbf{h}_t = \mathbf{h}_{t-1} + \boldsymbol{\varepsilon}_{h_t}, \quad h_1 \sim \mathcal{N}(0, \sigma^2_{0h}), \boldsymbol{\varepsilon}_{h_t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_h), \tag{4.2.5}$$

where

$$\boldsymbol{\Sigma}_\beta = diag(\sigma^2_{\beta_1}, \cdots, \sigma^2_{\beta_{n+n^2}}), \tag{4.2.6}$$

$$\boldsymbol{\Sigma}_\alpha = diag(\sigma^2_{\alpha_1}, \cdots, \sigma^2_{\alpha_{((n-1)\times n)/2}}), \tag{4.2.7}$$

$$\boldsymbol{\Sigma}_h = diag(\sigma^2_{h_1}, \cdots, h^2_n). \tag{4.2.8}$$

It is common for macroeconomic quarterly time series to be considered as a random walk process, and for financial daily time series to be depicted with first-order autoregressive innovations. Although there is a potential for a random walk process to generate explosive and non-stationary behaviors, limited lengths of macroeconomic time series and carefully pre-set priors can prevent some undesired changes and provide reasonable estimations, as discussed in Primiceri (2005). The estimation of parameters uses a Bayesian estimation method. The details of parameter simulation are given in Appendix 4.A.

Based on whether or not these two specifications contain time-varying parameters or SV, four models are obtained: constant parameter VAR with constant variance (**C-VAR-C**), constant parameter VAR with SV (**C-VAR-SV**), time-varying parameter VAR with constant variance (**TVP-VAR-C**), and time-varying parameter VAR with SV (**TVP-VAR-SV**). Each of these models is applied to the study with the three variables of REG, GDP, and CO_2 emissions. The other specifications are specified either on constant parameters or constant variance compared with **TVP-VAR-SV** to verify the time-varying properties in parameters and variances.

4.3 Preliminary Empirical Study

In this section, we first describe the three macroeconomic time series we used in the analysis, namely, REG, real GDP growth, and CO_2 emission. Then, the identification scheme is discussed to construct the structural shocks, followed by a brief outline of the priors and initial values. Finally, the selection of lag lengths is discussed based on two Bayesian model comparison metrics, namely, the average log-predictive-likelihood (LPL) and the continuous ranked probability score (CRPS).

4.3.1 Data

The multivariate framework uses three aggregate macroeconomic variables. The growth rate of REG and carbon dioxide emissions are both sourced from the US Energy Information Administrations online database. According to the classification of the US Energy Information Administration, REG covers electricity generated from hydroelectric power, wood, waste, geothermal, solar, and wind sources. The real GDP growth rate is obtained from the St. Louis FRED database (https://fred.stlouisfed.org/). Data on REG and CO_2 emissions, particularly from electric power sector publications (https://www.eia.gov/renewable/data.php) are available for research and social communities. The sample period starts from 1973Q1 and ends in 2014Q4. The sample is relatively short because both REG and CO_2 are not available until 1973. Since both REG and CO_2 emissions are monthly data, seasonally adjusted quarterly data are obtained by implementing the X-13 seasonal adjustment method on the monthly data. GDP is seasonally adjusted quarterly data, so a further smoothing method is not needed. All of these three variables (REG, GDP, and CO_2) are detrended by the first difference and converted to growth rates, so the time period is from 1973Q2 to 2014Q4.

Figures 4.1, 4.2 and 4.3 plot the REG growth rate, the real GDP growth rate, and CO_2 emission growth rate, respectively.

Figure 4.1 shows the variations of REG across the examined time period. REG generally increases, but occasionally contracts, from the time of the oil crisis in 1973; there is also an increase in 1978. Aguilar et al. (2011) claim that the renewable electricity mandates passed in 1978 reduced renewable energy costs and drove the investment in renewable power facilities. In 1989, there is an unusual spike. It could be a sharp growth of REG that year or a change of the way in which data were collected. The electricity generation from conventional hydroelectric power does not increase significantly, but other forms of REG grow dramatically. For example, the monthly electricity generation from wood energy increases from 77 to 2,408 thousand megawatt-hours (1 megawatt-hour = 1,000 kilowatt hours) from December 1988 to January 1989, 54 to 519 for waste, 849 to 1,279 for geothermal, 0.06 to 7 for solar, and 0.05 to 146 for wind. In the following years, there are also various policies corresponding to the fluctuations in REG. The related policy instruments include state-level financial incentive projects from 1990; tax credits and grants for renewable electricity from 1992; renewable energy government bonds established in 2005; tax credits for residential renewable energy in 2006; green power purchase requirements from 2007; rural grants since 2008; and the renewable energy grant program created in 2009. There are also regulations promoting the development of the renewable energy industry, which have contributed to the uptake of renewables, such as the National Energy Act (1978), the Energy Policy Act (2005), and the Energy Independence and Security Act (2007). In fact, renewable energy policies play a crucial role in REG sector development (Zhao et al., 2013). Although there are substantial environmental and social benefits from the renewable energy sector, the high fixed cost and long-term return are still major barriers for renewable energy projects (Verbruggen et al., 2010). Hence, renewable energy policies such as feed-in tariffs and investment incentives from governments play a decisive role in reducing the costs of REG and penetrating the energy market (Beck and Martinot, 2004). Several renewable energy policies are examined and identified as contributing to the use of renewables (e.g., Aguilar et al., 2011; Zhao et al., 2013). However, there is also knowledge-intensive renewable technology innovation, which can attract effective investments to renewable industrial (Ragwitz et al., 2009). Apparently, these external variances have various effects on the renewable industrial sector (e.g., Lehr et al., 2008; Hillebrand et al., 2006). The different scales of effects from these instruments cannot be ignored in REG forecasts, and the various effects can be viewed as stochastic volatility of the VAR models.

It can be seen in Figure 4.2 that there are dramatic fluctuations in the GDP growth rate before 1985. After 1985, the Great Moderation period begins and continues until 2007. This is followed by the Global Financial Crisis, and the GDP growth rate drops sharply.

In Figure 4.3, the CO_2 emission growth rate exhibits more substantial fluctuations than the other two variables. On one hand, CO_2 emissions increase with rapid economic development. On the other hand, CO_2 emissions can be significantly decreased by emission reduction policies or the adoption of air purification technology.

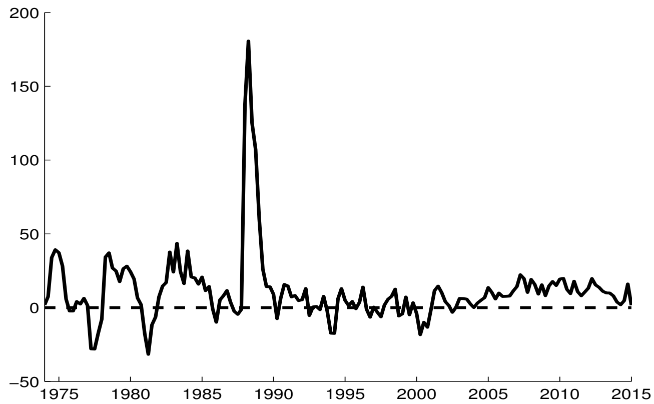

Figure 4.1: Plot of REG growth rate.

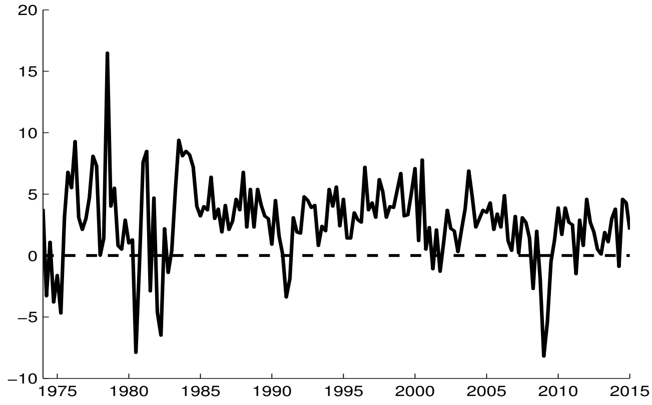

Figure 4.2: Plot of annualized real GDP growth rate.

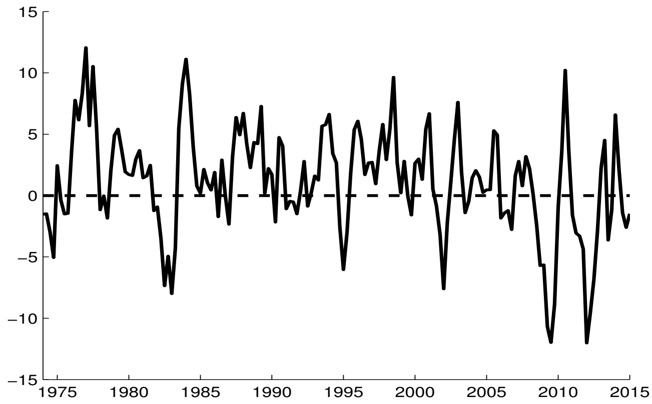

Figure 4.3: Plot of CO_2 emission growth rate.

4.3.2 Ordering of Variables in the VARs

For the variable order of the VARs, we follow Silva et al. (2012) and Tiwari (2011), and order the variables as follows: REG, GDP, and CO_2. These restrictions are mainly based on the assumption that there are some storage levels in hydro systems (Amundsen and Bergman, 2002), so that the shocks from REG can affect CO_2 emissions and have some relatively very small impacts on GDP. Meanwhile, the growth hypothesis (unidirectional causality running from renewable energy to economic growth) is adopted for the causal relationship between REG and GDP, which is based on the results of the Toda-Yamamoto causality test (Payne, 2011). The emission of CO_2 from energy consumption is one by-product of economic growth (e.g., Amundsen and Bergman, 2002; Stern, 2004; Silva et al., 2012), and this is another reason that it is ordered behind GDP in VAR models. Moreover, it is assumed that the first variable, REG, has impacts on both GDP and CO_2, but does not receive any contemporaneous impacts from them. The second variable, GDP, receives impacts from REG and does not have any effect on REG, whereas it affects CO_2 but does not receive impacts from CO_2. The third variable, CO_2, does not have any impact on either REG or GDP but receives impacts from them.

4.3.3 The Choice of Renewable Electricity Time Series

In the empirical study section, we use the generation rather than the consumption of renewable electricity. This covers the concern that some portion of the electricity consumption is imported instead of being generated domestically. It is also consistent with another variable, GDP growth rate, which is an index of goods and services produced within a country. Although there are quite a few studies focusing on renewables consumption, the relationship of energy generation and GDP has not been studied extensively (e.g., Silva et al., 2012; Yoo and Kim, 2006).

We could frame an empirical study to consider the share of REG in total electricity generation. However, it does not increase much, and its movement with other variables is not significant. Menyah and Wolde-Rufael (2010) also suggest that the share of renewables in total energy consumption increased very little in the US in their study periods. The main reason for the above is that renewable generation increases with rapid economic growth over time, but so does total electricity generation. Moreover, when the renewable share is treated as an endogenous variable correlated with the GDP growth rate and the CO_2 emission growth rate in VARs, the economic significance of the variable can be vague.

The priors and initial values of parameters are discussed in Appendix 4.B.

4.3.4 VAR Specification Selection and Lag Length

Bayesian estimation is used for parameter prediction. The sum of the MLL from $t = 1$ up to $t = T$ and the CRPS are used for Bayesian model selection. Both are adopted for full-sample fitness comparisons of lag length selection in the present study. The traditional standard criteria for model comparison can also be used for the lag selection of VARs, such as the Akaike information criterion, Bayesian information criterion, and Hannan-Quinn information criterion, but the marginal likelihood needs to be calculated first to compare the models and then the lag selection of VARs explored second. Because **TVP-VAR-SV** has high dimensional time-varying parameters, its covariance matrix is not a constant, whereas it is in **TVP-VAR-C**.

There are studies discussing modeling comparison attempts of time-varying parameter VARs without SV, such as Ljung-Box statistics (Santis, 2007) and the deviance information criterion through observed data (Chan and Grant, 2016a), and modeling comparison with constant parameters and SV (Schwarz information criterion implemented by an adaptive importance sampling (Chan and Grant, 2016b)). However, the computation of the marginal likelihood can be quite a difficult task in practice. Furthermore, the integrated likelihood needs to be calculated, which does not have a closed form (Chan and Eisenstat, 2018). Under these circumstances, some researchers suggest that lag length can be set to one or two directly for modeling the inherent dynamics of the economy (e.g., Nicolini, 2007; Santis, 2007; Cao, 2012). In fact, the inclusion of just one more lag length can increase the number of time-varying parameters dramatically and potentially lead to pathological simulation results. The results of MLL and CRPS below also suggest that fewer lag lengths can provide better model fitness.

Below, the formulas of MLL and CRPS are described, followed by the corresponding metric values for VAR models with different lag lengths. The results are obtained by 55,000 draws with 5,000 burn-in based on the MCMC process discussed in the previous section.

Generally, the MLL can give an average quality evaluation of predictive densities for full-sample performance and provide insight on uncertainty. In practice, a larger value of MLL implies a better model fit, while a smaller value implies a worse model fit. It is defined as:

$$\mathrm{MLL}_{\mathrm{VAR}_i} = \frac{1}{T}\sum_{t=1}^{T} \log p(\widehat{y}_{(t+1,\mathrm{VAR}_i)} = y_{(t+1)} \mid y_{(1:t)}).$$

CRPS is calculated by measuring the difference between the cumulative distributions of the predicted and actual values. In contrast to MLL, CRPS prefers predictive densities with smaller distance and higher sharpness (e.g., Hersbach, 2000; Panagiotelis and Smith, 2008; Ravazzolo and Vahey, 2014). Formally, CRPS is defined as:

$$\begin{aligned}\mathrm{CRPS}_{\mathrm{VAR}_i} =& \frac{1}{T}\sum_{t=1}^{T} p(|\widehat{y}_{1,(t+1,\mathrm{VAR}_i)} - y_{(t+1)}| \\ & - 0.5 \times |\widehat{y}_{1,(t+1,\mathrm{VAR}_i)} - \widehat{y}_{2,(t+1,\mathrm{VAR}_i)}| \mid y_{(1:t)}),\end{aligned} \tag{4.3.1}$$

where $\widehat{y}_{1,(t+1)}$ and $\widehat{y}_{2,(t+1)}$ are two independent draws of a VAR model from the predictive density, y_{t+1} is the true value of y at time $t+1$, and each CRPS score is calculated by the observed data up to time t. Generally, the scores are positive. A smaller value of CRPS indicates that the predictive density has a better estimation regarding the true value, while a larger value of the score suggests worse predictive performance by the candidate model.

In Table 4.1, we present full-sample estimation results for both MLL and CRPS. Both MLL and CRPS results suggest that one lag is an optimal lag length for all four of these VAR models. Moreover, the results show that longer lags are not helpful for in-sample fit. Therefore, we use one lag for empirical analysis in the application part, and VARs with both one lag and two lags are used for the forecasting estimation.

Table 4.1: Sum of MLL and CRPS for the fitness of VARs.

	MLL			CRPS		
	REG	GDP	CO_2	REG	GDP	CO_2
TVP-VAR-SV						
lag 1	**-624.3**	**-404.0**	**-432.6**	**1104.2**	**309.1**	**337.1**
lag 2	-639.1	-420.1	-448.3	1150.6	377.3	355.7
lag 3	-644.9	-432.4	-455.7	1243.1	462.3	443.5
C-VAR-SV						
lag 1	**-608.0**	**-406.1**	**-415.2**	**1075.5**	**302.8**	**313.7**
lag 2	-608.9	-408.2	-415.5	1101.7	343.0	332.3
lag 3	-618.7	-417.6	-426.3	1173.4	422.0	397.3
TVP-VAR-C						
lag 1	**-693.4**	**-425.0**	**-430.1**	**1235.1**	**316.2**	**334.8**
lag 2	-824.8	-458.7	-463.5	1481.6	543.2	454.0
lag 3	-977.7	-467.2	-458.1	1417.0	548.8	482.4
C-VAR-C						
lag 1	**-608.0**	**-406.1**	**-415.7**	**1075.5**	**302.8**	**313.7**
lag 2	-608.9	-408.2	-417.5	1101.7	343.0	332.3
lag 3	-618.7	-417.6	-426.3	1173.4	422.0	397.3

4.4 Application of VARs with REG

The application of these four VAR models is conducted by full-sample quarterly time series. As mentioned in Section 4.3.2, the order of variables is REG, GDP, and CO_2, which is suggested in Silva et al. (2012) and Tiwari (2011). All of the posterior medians and credible intervals are obtained by 55,000 draws with 5,000 burn-in, the same as in the lag length selection step. Firstly, the SDs of innovations in the REG equation, the GDP equation, and the CO_2 emission equation are examined. The estimated results are compared between models with and without SV. Then the forecasting results of VARs and the benchmark are discussed.

4.4.1 Empirical Evidence of SV

Recently, a growing number of researchers have suggested that SV is a specification that cannot be ignored in macroeconomic time series modeling (e.g., Carriero et al., 2015; Chan and Eisenstat, 2018). In Figure 4.4, the evidence of the SVs of **C-VAR-SV** and **TVP-VAR-SV** is presented. The three subfigures in Figure 4.4 are estimations of the SD for the two VAR models with SV, respectively. They are calculated by $exp(h_t/2)$. Specifically, the SDs of **C-VAR-SV** are in dashed lines, while those of **TVP-VAR-SV** are in solid lines. The posterior medians and the 16th and 84th quantiles for the two VARs with constant variance (**C-VAR-C** and **TVP-VAR-C**) are reported in Table 4.2. The 16th and and 84th quantiles are used to present the credible interval of estimations in the VARs with constant parameters. We can check the credible interval of SV by inspecting the ranges from corresponding constant parameter VARs. We examine whether or not the values of $exp(h_t/2)$ are in the credible intervals of constant variance. In other words, if the curves of $exp(h_t/2)$ are out of the ranges, SV is significant in VARs. Apparently, all of the $exp(h_t/2)$ curves in these three equations are quite different from constant variances.

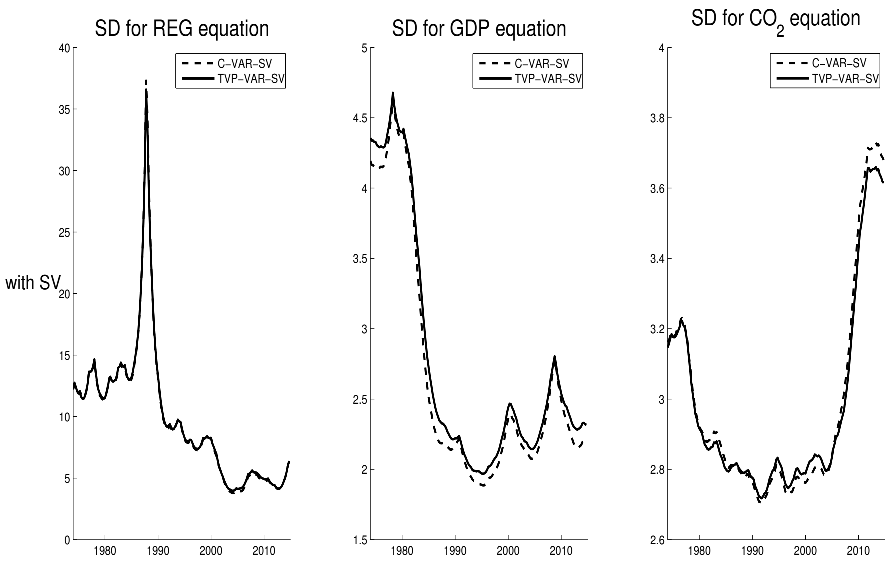

Figure 4.4: SD of posterior medians for **C-VAR-SV** and **TVP-VAR-SV**.

In Figure 4.4, the results suggest that more variation of the time series is allocated to time-varying parameters in **TVP-VAR-SV** for the SD curves of **TVP-VAR-SV** below **C-VAR-SV**. Relatively, the results suggest that more variation of the time series is allocated to the error terms in **TVP-VAR-SV** for the SD curves of **TVP-VAR-SV** above **C-VAR-SV**. For the REG equations, comparing the SD curves of **C-VAR-SV** with those of **TVP-VAR-SV**, the SD innovations are almost coincident with each other. It indicates that the SVs estimated from **C-VAR-SV** and **TVP-VAR-SV** are almost the same. For the GDP and CO_2 equations, these two SD innovations are slightly different from each other.

In the first subfigure of Figure 4.4, both the dashed line and the solid line are quite close to each other. The results show that the scales of SV in the renewable variable equation are much larger than that in the GDP equation. The highest SD value in the renewable variable equation reaches almost 40, in contrast to those in the GDP equation, which all remain below 5. Corresponding to the real world, the peak around 1989 implies an extremely high volatility related to the numerous renewable power facilities being completed and put into production after institution of renewable energy policies (see the Data section). After that, the amplitude of the SV for the renewable variable becomes smaller, but the implementation of a series of renewable energy acts in the US stimulates the REG growth rate, while the related volatility acts are quite moderate in recent years.

In the second subfigure of Figure 4.4, the SV of the GDP growth rate equation exhibits substantial variation across the estimated time periods, which is obviously out of the band of the 16th and 84th quantiles from **C-VAR-C**. Specifically, there is particularly high volatility in the 1970s and a sharp decline at the beginning of the Great Moderation. After the Great Moderation, the volatility of innovations rises again around 2000, corresponding to the early 2000s recession related to the 1997 Asian Financial Crisis, and a similar situation occurs during the Global Financial Crisis in 2007. Generally, the innovations of **C-VAR-SV** are always of larger magnitude than those of **TVP-VAR-SV**, which reflects that the SV of GDP in **C-VAR-SV** takes much more volatility than that allocated to **TVP-VAR-SV**.

In the third subfigure of Figure 4.4, the SD innovation in the CO_2 emission equation exhibits a similar volatility value range to that in the GDP equation, which implies that the CO_2 emission growth rate is a relatively stable time series compared with REG. Before 1980, the SD of the CO_2 equation is much higher than in other time periods. However, it drops sharply until 1990. There are also continuous large growth rates after 2005. All of the above imply that the SD of the CO_2 equation is not a constant value and SV is a specification that cannot be ignored.

Table 4.2: SD with 16th and 84th quantiles for **TVP-VAR-C** and **C-VAR-C**.

	REG			GDP			CO_2		
TVP-VAR	[14.45	15.24	16.13]	[2.77	2.93	3.11]	[2.81	2.97	3.15]
C-VAR	[14.27	15.06	15.92]	[2.81	2.97	3.14]	[2.78	2.93	3.10]

In Table 4.2, the posterior estimates of SD for **TVP-VAR-C** are quite close to those of **C-VAR-C** for models without SV. Specifically, the SDs of the REG and CO_2 equations of **TVP-VAR-C** are slightly higher than those of **C-VAR-C**. This may reflect that parts of the time series variation are treated as gradual innovations of time-varying parameters in the REG and CO_2 equations. In contrast, the SDs of the GDP equation of **TVP-VAR-C** are lower than those of **C-VAR-C**. This implies that a TVP-VAR model does not always increase the SDs.

Overall, a time-invariant volatility is not sensitive enough to capture the changing macroeconomic status, and the SV parameters appear to be a valuable addition to VAR models for macroeconomic studies.

4.5 Forecasting Results

In this section, a pseudo out-of-sample forecasting exercise for REG is conducted for VARs. We use autoregressive models (AR) with one lag AR(1) as a benchmark, and the forecasting results of AR with two and four lags are also reported. The forecasting performance of VARs with both one and two lags is presented in two blocks separately. The recursive forecasts of one-quarter-ahead, one-year-ahead, two-year-ahead, three-year-ahead and four-year-ahead ($k = 1, 4, 8, 12$ and 16) are evaluated. The forecasting starting time is 1995Q1.

4.5.1 Forecasting Metrics

Consistent with the full-sample lag selection, we use both the average LPL and the CRPS for model comparison. For easier comparison, average LPL and CRPS values of each model relative to the benchmark AR(1) are reported in Table 4.3.

The average LPL is defined as:

$$\begin{aligned}\text{Average LPL}_{\text{Model}_i} =& \frac{1}{T - T_0 - k + 1} \sum_{t=1}^{T-T_0-k+1} \\ & \log p(\widehat{y}_{(T_0+t+k-1,\text{Model}_i)} = y_{(T_0+t+k-1)} \,|\, y_{(1:T_0+t)}).\end{aligned} \tag{4.5.1}$$

Similar to the sum of the marginal-log-likelihood (MLL) (see Appendix 4.C), a larger value of LPL indicates better forecasting performance. The relative average LPL is the difference between the examined model and the benchmark. Thus, a positive score indicates better forecasting performance than the benchmark and vice versa.

The average CRPS is defined as:

$$\text{Average CRPS}_{\text{Model}_i} = \frac{1}{T - T_0 - k + 1} \sum_{t=1}^{T-T_0-k+1} p\big(|\widehat{y}_{1,(T_0+t+k-1,\text{Model}_i)} - y_{(T_0+t+k-1)}| - 0.5 \times |\widehat{y}_{1,(T_0+t+k-1,\text{Model}_i)} - \widehat{y}_{2,(T_0+t+k-1,\text{Model}_i)}| \,|\, y_{(1:T_0+t)}\big), \tag{4.5.2}$$

The relative average CRPS is the difference between the examined model and the benchmark, the same as the relative average LPL. However, in contrast to average LPL, a negative score represents a better forecasting performance, whereas a positive one indicates a worse forecasting performance than the benchmark.

Table 4.3: Relative average LPL and CRPS for REG forecasts from 1995.

	Relative Average LPL					Relative Average CRPS				
	k=1	k=4	k=8	k=12	k=16	k=1	k=4	k=8	k=12	k=16
AR(1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR(2)	0.03	-0.08	-0.03	-0.01	0.00	-0.02	0.58	0.25	0.00	-0.16
AR(4)	0.04	-0.08	-0.03	-0.01	0.00	-0.04	0.65	0.29	0.07	-0.10
One Lag										
TVP-VAR-SV	0.51	0.64	0.62	0.55	0.47	-1.11	-2.21	-2.41	-2.39	-1.81
C-VAR-SV	0.56	0.63	0.61	0.52	0.47	-1.16	-2.19	-2.46	-2.39	-2.24
TVP-VAR-C	-0.27	-0.07	-0.06	-0.06	-0.08	0.28	0.57	0.99	1.42	2.28
C-VAR-C	-0.02	-0.02	-0.02	-0.02	-0.02	0.16	0.23	0.19	0.16	0.17
Two Lags										
TVP-VAR-SV	0.34	0.54	0.55	0.51	0.47	-0.79	-1.54	-1.86	-1.81	-1.41
C-VAR-SV	0.47	0.52	0.56	0.53	0.53	-0.81	-1.54	-2.19	-2.42	-2.54
TVP-VAR-C	-0.36	-0.18	-0.15	-0.17	-0.20	0.50	2.04	2.13	3.11	4.92
C-VAR-C	0.01	-0.11	-0.03	-0.01	-0.01	0.31	1.29	0.37	0.05	-0.13

4.5.2 Relative Average LPL Results

The relative average LPL results for REG are reported in the left column of Table 4.3. Apparently, VARs allowing SV (**TVP-VAR-SV** and **C-VAR-SV**) have much better forecasting performance than the benchmark and other competing models at all forecast horizons. Specifically, **C-VAR-SV** with one lag performs the best at horizon 1 in forecasting REG, **TVP-VAR-SV** with one lag has the best forecasting performance at horizons 4, 8, and 12, and **C-VAR-SV** with two lags is the best model in forecasting REG at horizon 16. Generally, ARs with more lags have worse forecasting performance but do not produce rapidly deteriorating forecasts across horizons. Similarly, the forecasting performance of VARs with two lags is no better than those with one lag at shorter horizons in most cases, which indicates that longer lags are not helpful for multivariate models in shorter-term forecasts. It is worth noting that VARs with SV (**TVP-VAR-SV** and **C-VAR-SV**) have more accurate forecasts than those with constant variances (**TVP-VAR-C** and **C-VAR-C**). In other words, the specification of time-varying coefficients (**TVP-VAR-SV** and **TVP-VAR-C**) does not work better than models without it (**C-VAR-SV** and **C-VAR-C**) in forecasting exercises. Overall, the main forecasting improvement for REG is from the specification of time-varying variances but not from time-varying parameters, which is consistent with the literature studying models with time-varying parameters and SV for other macrovariables (Clark and Ravazzolo, 2015).

4.5.3 Cumulative Sum of LPL

To obtain the cumulative sum of LPL, the LPLs are first added up at each time point, then the sums of the LPLs are cumulated from the forecasting starting time point to the following time points. In Figure 4.5, one-quarter-ahead cumulative sums of LPL of VARs relative to **AR(1)** are plotted, and the forecasting performance of models at different time periods are presented.

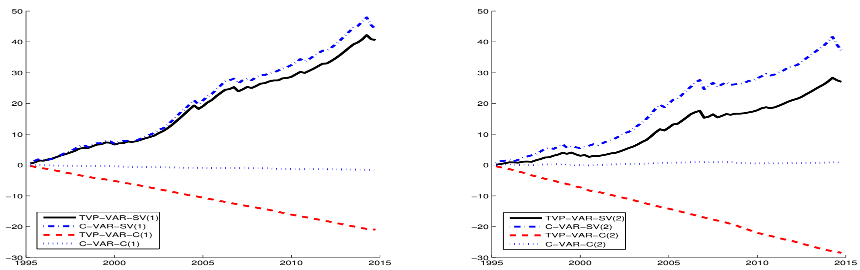

Figure 4.5: One-quarter-ahead cumulative sum of LPL for VAR models relative to AR(1) with one lag (left panel) and two lags (right panel).

In Figure 4.5, we can see that VARs with SV specification forecast much better than the benchmark and other VARs. Moreover, **TVP-VAR-SV** and **TVP-VAR-C** with one lag have better forecasting performance than those with two lags across the forecasting time period. It is also true for **TVP-VAR-C**. It can be seen from the figure that **TVP-VAR-SV** and **C-VAR-SV** produce similar cumulative sums of LPL for REG, and **C-VAR-SV** has a larger cumulative sum of LPL than **TVP-VAR-SV** in most cases. Moreover, the relative cumulative sums of LPL for **TVP-VAR-SV** and **C-VAR-SV** grow in time-varying speeds across the forecast time period, which is different from **C-VAR-C** and **TVP-VAR-C**. Although **TVP-VAR-C** nests **C-VAR-C**, **TVP-VAR-C** has worse forecasting performance than that of VARs without time-varying parameters or SV. Considering the number of time-varying parameters in **TVP-VAR-C**, the forecasts errors of **TVP-VAR-C** could be larger than those of **C-VAR-C** after all the parameters are updated for the forecasts.

4.5.4 Relative Average CRPS Results

The relative average CRPS results for REG are presented in the right column of Table 4.3. From the results of CRPS, we can see that both **TVP-VAR-SV** and **C-VAR-SV** with one lag and two lags have better forecasting performance than the benchmark at all forecast horizons. **C-VAR-SV** with two lags has the best forecasting performance at horizons 12 and 16, while other forecasting results are broadly similar to those in relative average LPL results. For VAR models, it is apparent that models with SV, either with time-varying parameters or constant coefficients, have a substantially improved forecasting performance than those without SV in all forecasting horizons. Apart from horizons 12 and 16, most forecasting results of **TVP-VAR-SV** and **C-VAR-SV** with two lags are no better than those with one lag, which is consistent with the findings in relative average LPL.

4.5.5 Cumulative Sum of CRPS

The cumulative sum of CRPS is obtained in a similar way to that of the cumulative sum of LPL. Differently from the cumulative sum of LPL, a smaller value of the cumulative sum of CRPS stands for a better forecasting performance, and vice versa. The results of the one-quarter-ahead cumulative sum of CRPS are presented in Figure 4.6 to show the relative forecasting performance of VARs across the forecast time period.

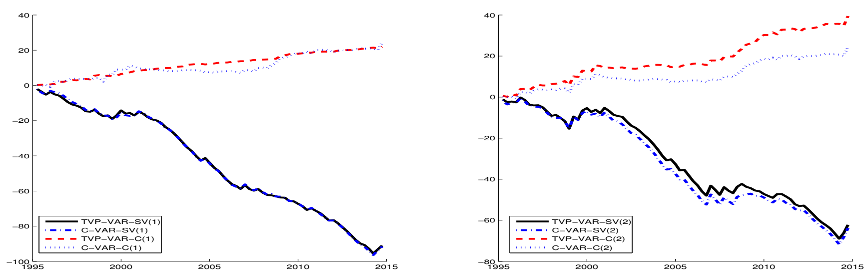

Figure 4.6: One-quarter-ahead cumulative sum of CRPS for VAR models relative to AR(1) with one lag (left panel) and two lags (right panel).

From Figure 4.6, we can see that VARs with time-varying variations have much better forecasting performance than those without it. VARs with constant variations forecast more poorly than the benchmark over all horizon 1 forecasts. **TVP-VAR-SV** and **C-VAR-SV** with one lag have a similar forecasting performance, and they produce more accurate forecasts than those with two lags. Differently from the cumulative sum of LPL, both **TVP-VAR-C** and **C-VAR-C** with one lag and two lags produce poorer forecasts than the benchmark. Moreover, their relative cumulative sums of CRPS are time-varying. In most time periods, **TVP-VAR-C** has worse forecasting performance than **C-VAR-C**, which indicates that time-varying parameter is not helpful for VARs in forecasts if the variations are constant.

4.5.6 Robustness of Forecasting Results

In this section, we conduct forecasting exercises for REG with two alternative forecast starting time points, 1990Q1 and 1985Q1. Both relative average LPL and CRPS scores are reported in Table 4.4 and Table 4.5.

Table 4.4: Relative average LPL and CRPS for REG forecasts from 1990.

	Relative Average LPL					Relative Average CRPS				
	k=1	k=4	k=8	k=12	k=16	k=1	k=4	k=8	k=12	k=16
AR(1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR(2)	0.03	-0.08	-0.02	0.00	0.00	-0.02	0.60	0.19	-0.02	-0.20
AR(4)	0.03	-0.08	-0.03	-0.01	-0.01	-0.04	0.66	0.29	0.05	-0.07
One Lag										
TVP-VAR-SV	0.45	0.57	0.58	0.54	0.47	-0.98	-1.90	-1.96	-2.06	-1.39
C-VAR-SV	0.49	0.57	0.56	0.52	0.48	-1.00	-1.94	-2.23	-2.33	-2.21
TVP-VAR-C	-0.27	-0.08	-0.07	-0.08	-0.10	0.25	-.64	1.15	1.72	2.59
C-VAR-C	-0.02	-0.02	-0.02	-0.02	-0.02	0.16	0.27	0.31	0.33	0.30
Two Lags										
TVP-VAR-SV	0.27	0.44	0.47	0.42	0.40	-0.66	-1.00	-1.39	-1.17	-0.54
C-VAR-SV	0.42	0.46	0.50	0.49	0.49	-0.74	-1.24	-1.96	-2.18	-2.32
TVP-VAR-C	-0.36	-0.18	-0.16	-0.18	-0.21	0.48	2.21	2.36	3.41	5.47
C-VAR-C	0.01	-0.11	-0.04	-0.02	-0.02	0.25	1.36	0.52	0.31	0.14

Broadly, sample sets with fewer parameter estimation periods and more forecasting time periods do not provide different forecasting results from Table 4.3. When forecasts start from 1985 and 1990, **TVP-VAR-SV** and **C-VAR-SV** with both one lag and two lags can have a better forecasting performance at all forecast horizons than the benchmark, and they consistently dominate VARs without SV. Moreover, **C-VAR-SV** does not always outperform **TVP-VAR-SV**, which is also true between **TVP-VAR-C** and **C-VAR-C**. This suggests that the specification of time-varying parameter does not improve the forecast accuracy, whereas allowing time-varying variation can improve forecasting performance significantly.

Table 4.5: Relative average LPL and CRPS for REG forecasts from 1985.

	Relative Average LPL					Relative Average CRPS				
	k=1	k=4	k=8	k=12	k=16	k=1	k=4	k=8	k=12	k=16
AR(1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR(2)	0.69	1.00	0.84	0.55	0.43	0.16	-4.73	-5.82	-5.76	-4.71
AR(4)	0.66	0.96	0.81	0.54	0.43	0.65	-3.44	-5.26	-6.20	-4.71
One Lag										
TVP-VAR-SV	0.56	0.90	0.82	0.50	0.29	-0.90	-2.35	-2.96	-3.74	-5.19
C-VAR-SV	0.58	0.88	0.78	0.48	0.37	-0.99	-2.47	-3.45	-4.54	-6.87
TVP-VAR-C	-0.07	0.24	0.30	0.03	-0.11	0.38	0.84	1.21	2.09	3.89
C-VAR-C	0.01	-0.08	0.13	0.02	-0.03	0.19	0.34	0.33	0.47	0.78
Two Lags										
TVP-VAR-SV	0.44	0.97	0.83	0.47	0.26	-0.78	-1.93	-3.07	-3.48	-3.33
C-VAR-SV	0.60	0.83	0.77	0.48	0.37	-0.94	-2.34	-3.80	-4.96	-7.34
TVP-VAR-C	-0.17	0.31	0.27	-0.02	-0.21	0.32	1.00	0.82	1.28	3.06
C-VAR-C	0.02	0.04	0.26	-0.05	-0.01	0.09	0.01	-1.37	-2.37	-4.38

4.6 Conclusion

In this study, VARs with time-varying parameters and SV (**TVP-VAR-SV**) were employed to study the interaction of REG, GDP growth, and CO_2 emissions. Full-sample estimation and pseudo out-of-sample forecasting comparisons of **TVP-VAR-SV** were made with AR models and three other VAR models: **C-VAR-C**, **C-VAR-SV** and **TVP-VAR-C**.

In the full-sample application of these models, the SV specification shows a better fitness to the data than the homoscedastic variance models. The results of pseudo out-of-sample forecasts also suggest that VARs with SV can substantially improve the forecasting performance compared with AR models and VARs with constant variances. However, the role of time-varying parameter specification is not obvious in REG forecasting exercises.

Appendix 4.A Bayesian Estimation

After specifying the TVP-VAR model, the next step is to estimate the time-varying parameters and SV. This is conducted by using Monte Carlo Markov chain (MCMC) methods. To do that, the TVP-VAR-SV model is written as a seemingly uncorrelated regression. Specifically, suppose X_t is a matrix of lagged endogenous variables, so that $\mathbf{X}_t = \mathbf{I}_p \otimes (\mathbf{1}, \mathbf{y}'_{t-1}, \ldots, \mathbf{y}'_{t-p})$ ($\otimes$ is the Kronecker product). Then the model can be written as:

$$\mathbf{y}_t = \mathbf{X}_t \boldsymbol{\beta}_t + \mathbf{A}_t^{-1} \boldsymbol{\Omega}_{y_t} \boldsymbol{\varepsilon}_t.$$

The posterior sampling of the parameters can be conducted by Metropolis-Hastings within the Gibbs sampling algorithm. In particular, the posterior draws of the parameters in **TVP-VAR-SV** can be obtained by sampling from the following MCMC algorithm:

1. $p(\boldsymbol{\beta} \,|\, \mathbf{y}, \boldsymbol{\alpha}, \mathbf{h}, \sigma_\beta^2, \boldsymbol{\beta}_0)$;
2. $p(\mathbf{h} \,|\, \mathbf{y}, \boldsymbol{\beta}, \sigma_h^2, \mathbf{h}_0)$;
3. $p(\boldsymbol{\alpha} \,|\, \mathbf{y}, \boldsymbol{\beta}, \mathbf{h}, \sigma_\alpha^2, \boldsymbol{\alpha}_0)$;
4. $p(\sigma_\beta^2, \sigma_h^2, \sigma_\alpha^2 \,|\, \boldsymbol{\beta}, \mathbf{h}, \boldsymbol{\alpha}) = p(\sigma_\beta^2 \,|\, \boldsymbol{\beta}) p(\sigma_h^2 \,|\, \mathbf{h}) p(\sigma_\alpha^2 \,|\, \boldsymbol{\alpha})$;
5. $p(\boldsymbol{\beta}_1, \mathbf{h}_1, \boldsymbol{\alpha}_1 \,|\, \boldsymbol{\beta}, \mathbf{h}, \boldsymbol{\alpha}, \sigma_\beta^2, \sigma_h^2, \sigma_\alpha^2) = p(\boldsymbol{\beta}_1 \,|\, \boldsymbol{\beta}, \sigma_\beta^2) p(\mathbf{h}_1 \,|\, \mathbf{h}, \sigma_h^2) p(\boldsymbol{\alpha}_1 \,|\, \boldsymbol{\alpha}, \sigma_\alpha^2)$;

In the above sequential sampling process, the simulation method of the parameter posteriors can be summarized as follows:

1. The posterior draws of $\boldsymbol{\beta}$ follow the textbook results for linear Gaussian regression (see Koop (2003)), as the state space form of the model can be written into a linear Gaussian form. A precision-based algorithm is employed for improving the simulation speed (see Chan and Jeliazkov (2009)).

2. We use the auxiliary mixture sampler approach introduced by Kim et al. (1998) to sample $\mathbf{h}$. It provides an efficient approximation of the SV and is widely used in macroeconomic SV studies.

3. The posterior draws of $\boldsymbol{\alpha}$ follow the simulation method used for linear Gaussian regression.

4. The posterior generation of $\sigma^2_\beta, \sigma^2_h$, and σ^2_α are straightforward with conjugate inverse-Gamma distribution priors according to the textbook results in Koop (2003).

5. The posterior densities of β_1, h_1, and α_1 can be directly obtained from the standard results of a linear Gaussian regression.

Appendix 4.B Priors and Initial Values

Considering the number of parameters and states, it is not surprising that the posteriors of parameters in the **TVP-VAR-SV** are quite sensitive to their corresponding priors. Thus, in order to avoid non-stationary estimations for **TVP-VAR-SV**, the hyperparameters of the priors need to be pre-set carefully. Meanwhile, we also treat the initial values of $\sigma_\beta^2, \sigma_h^2$, and σ_α^2 as parameters of the model. With the updated posterior variances of each parameter, the initial values of parameters β_1, h_1, and α_1 can also be updated after each loop.

Specifically, we assume that the priors of β_1, h_1, and α_1 are all non-informative. Meanwhile, all of these priors are independent of each other, so that:

$$p(\beta_1, h_1, \alpha_1, \sigma_\beta^2, \sigma_h^2, \sigma_\alpha^2) = p(\beta_1)p(h_1)p(\alpha_1)p(\sigma_\beta^2)p(\sigma_h^2)p(\sigma_\alpha^2).$$

$\boldsymbol{\beta}$ contains many more states and elements than do $\mathbf{h}$ and $\boldsymbol{\alpha}$, so tighter and alternative priors are considered for $\boldsymbol{\beta}$. The priors of variance of $\boldsymbol{\beta}$ are set to 0.001. For other parameters, the following independent priors are assumed as:

$$\begin{aligned} \beta_1 &\sim \mathcal{N}(\beta_0, V_{\beta_0}), \quad & \beta_h^2 &\sim \mathcal{IG}(\nu_\beta, S_{h\beta}), \\ h_1 &\sim \mathcal{N}(h_0, V_{h_0}), \quad & \sigma_h^2 &\sim \mathcal{IG}(\nu_h, S_h), \\ \alpha_1 &\sim \mathcal{N}(\alpha_0, V_{\alpha_0}), \quad & \sigma_\alpha^2 &\sim \mathcal{IG}(\nu_\alpha, S_\alpha). \end{aligned}$$

We set β_0, h_0, and α_0 to be 0, and let $V_{\beta_0} = V_{h_0} = V_{\alpha_0} = 10$. Then relative non-informative prior values are chosen for β_1, h_1, and α_1. Similarly, we choose relative non-informative values 5 for the shape parameters of σ_β^2, σ_h^2, and σ_α^2, and let $S_\beta = S_h = 0.01^2 \times (5-1)$ and $S_\alpha = 0.1^2 \times (5-1)$. Therefore, $\mathbb{E}\sigma_\beta^2 = \mathbb{E}\sigma_h^2 = 0.01^2$ and $\mathbb{E}\sigma_\alpha^2 = 0.1^2$. These hyperparameters are selected according to previous macroeconomic **TVP-VAR-SV** studies (e.g., Primiceri, 2005; Nakajima et al., 2011), and are designed to bring about a desired smoothing transition among states and non-explosive impulse responses.

For the macroeconomic variables used in our paper, the value of coefficient variances cannot be very large, as these variables are quarterly time series. Moreover, because the number of coefficients in a TVP-VAR model is quite large, a slightly larger variation of the coefficients can result in a pathology modeling. One important consequence is that the follow-up impulse responses can, therefore, be explosive, and the related analysis can be unreliable. If there is any related policy conclusion, it can be erroneous and far from reality. Thus, the hyperparameters of coefficient variance priors need to be pre-set properly, and we use coefficient variance priors with prior means equal to 0.01^2.

Chapter 5

Concluding Remarks and Future Research

5.1 Conclusion

The main findings of this thesis are that allowing time variations in models has empirical importance. It is important to determine the proper specifications for a specific macrovariable empirical study. In this thesis, ARMA specification in error terms, modeling average with time variations in models, and specifications of SV and time-varying parameters with VARs were examined in estimation and forecasting exercises in different macroeconomic topics. Chapter 2 reported a new fast algorithm for improving the computational efficiency of models with ARMA errors. The proposed algorithm is based on the stacked matrix form of the model and an efficient precision-based algorithm. The empirical results show that a UC model extended by ARMA errors with SV can have an improved forecasting performance in comparison with those without this extension. Both point and interval forecasting results confirm that the proposed **UC-ARMA** is helpful for improving forecast accuracy over the considered forecasting horizons.

Chapter 3 investigated the forecasting performance for inflation in the US from both model averaging and equal weight combination methods. In the real-time data forecasting exercises, candidate models combined by the model averaging combination method with specifications with both time-varying coefficients and SV always presented the best forecasting performance for both point and density forecasts. In fact, the modeling combination method is the most important aspect in improving forecast performance, since all three models with model averaging had the best forecasting performance when compared with those with other combination methods. With the model averaging combination method, SV is still an important specification for improving forecasting performance. However, the importance of moving average SV in time-varying coefficient models is not as significant as in the UC models studied in

Chapter 2.

Chapter 4 examined the specifications of both time-varying parameters and SV in VARs with REG, GDP growth, and CO2 emissions. In the full-sample application of these models, the SV specification showed a better fitness to the data than the homoscedastic variance models. The results of pseudo out-of-sample forecasts also suggest that VARs with SV can substantially improve the forecasting performance in comparison with VARs with constant variances. However, the role of time-varying parameter specification was not obvious in the REG forecasting exercises.

5.2 Future research

In future research, new studies need to consider the context of the empirical topics. Some other representative and practical macroeconomic variables can be studied for their real activity comovements by the proposed proper specifications. The prediction and forecasts of macroeconomic variables should be more connected with macroeconomic theories in the future studies. For instance, when doing inflation empirical exercises, the predictors of inflation could consider justification by literature before application.

In the present thesis, a linear approximation is used to estimate the time-varying parameter models. Future research can work on non-linear estimation with proper Bayesian econometric methods, such as approximate Bayesian computation and particle Markov chain Monte Carlo methods.

The time variation process in macroeconomic variables can be examined from an error term mixture or Markov mixture models. Other specifications of SV or non-Gaussian models can also be investigated in macroeconomic empirical studies and forecasting exercises. When doing multivariate analysis with the VAR models, time-varying impulse responses to a structure shock which are produced with a proposed error specification in VARs can be investigated.

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