

Delay times and rates for Type Ia supernovae and thermonuclear explosions from double-detonation sub-Chandrasekhar mass models

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ABSTRACT

We present theoretical delay times and rates of thermonuclear explosions that are thought to produce Type Ia supernovae (SNe Ia), including the double-detonation sub-Chandrasekhar mass model, using the population synthesis binary evolution code *STARTRACK*. If detonations of sub-Chandrasekhar mass carbon–oxygen white dwarfs following a detonation in an accumulated layer of helium on the white dwarf’s surface (‘double-detonation’ models) are able to produce thermonuclear explosions which are characteristically similar to those of SNe Ia, then these sub-Chandrasekhar mass explosions may account for at least some substantial fraction of the observed SN Ia rate. Regardless of whether all double-detonations look like ‘normal’ SNe Ia, in any case the explosions are expected to be bright and thus potentially detectable. Additionally, we find that the delay time distribution of double-detonation sub-Chandrasekhar mass SNe Ia can be divided into two distinct formation channels: the ‘prompt’ helium–star channel with delay times < 500 Myr (13 per cent of all sub-Chandras), and the ‘delayed’ double white dwarf channel, with delay times $\gtrsim 800$ Myr spanning up to a Hubble time (87 per cent). These findings coincide with recent observationally derived delay time distributions which have revealed that a large number of SNe Ia are prompt with delay times < 500 Myr, while a significant fraction also have delay times spanning ~ 1 Gyr to a Hubble time.

Key words: binaries: close – stars: evolution – supernovae: general – white dwarfs.

1 INTRODUCTION

The exact nature of the stars that produce Type Ia supernovae (SNe Ia) – which are believed to be thermonuclear explosions of carbon–oxygen (CO) white dwarfs (WDs) close to the Chandrasekhar mass limit – remains unknown (e.g. Branch et al. 1995). The most widely favoured SN Ia progenitor scenarios involve the double degenerate scenario (DDS; Iben & Tutukov 1984; Webbink 1984), and the single degenerate scenario (SDS; Whelan & Iben 1973). In the DDS, the merger of two CO WDs with a total mass exceeding the Chandrasekhar mass limit, $M_{\text{Ch}} \sim 1.4 M_{\odot}$, can lead to explosive carbon burning which causes a SN Ia explosion. In the SDS, a CO WD accretes from a hydrogen-rich stellar companion via stable Roche lobe overflow (RLOF) and undergoes hydrogen

burning on the surface, enabling the WD to accumulate mass towards M_{Ch} until carbon is ignited explosively in the centre of the WD leading to a SN Ia. However, in the stable RLOF configuration the companion does not have to be a hydrogen-rich main sequence (MS) or giant-like star, but can be a non- or semidegenerate helium-burning star, or a (degenerate) helium WD (e.g. Iben et al. 1987; Yoon & Langer 2003; Solheim & Yungelson 2005). Like the SDS, the WD explodes once it approaches M_{Ch} (this helium-rich donor scenario will hereafter be referred to as HeRS).

Recently, Rüter et al. (2009, hereafter Paper I) carried out a population synthesis study showing rates and delay times – time from birth of a progenitor system in a short burst of star formation to SN – for three formation channels of SNe Ia: DDS, SDS and HeRS. But it is also worth exploring in detail: what is the likelihood that *sub-Chandrasekhar mass* WDs contribute to the population of explosive (SN Ia-like) events? We point out that in some scenarios, a layer of accumulated helium on an accreting, sub-Chandrasekhar mass WD’s surface may undergo several shell flashes (e.g. leading up to a ‘Ia’ as discussed in Bildsten et al. 2007), and while these explosions can be bright none of them result in a SN Ia. However, it is

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possible that the detonation in the helium layer causes the underlying CO WD to detonate as well, resulting in an (final) explosion that does look like a SN Ia. Here, we extend our study of progenitors and focus our analysis and discussion on these thermonuclear ‘double-detonation’ events involving helium-rich donors, hereafter referred to as the sub- M_{Ch} model, in which a CO WD accretes from a helium-rich companion filling its Roche lobe and explodes as a SN Ia before reaching the M_{Ch} limit.

The sub- M_{Ch} model has thus far been regarded as an unlikely model for SNe Ia owing to the fact that most synthetic light curves and spectra of these objects from previous studies did not match those observed for SNe Ia. However, it has recently been argued that the model might be capable of producing a better match to observation, depending on details regarding the manner in which the accreted helium burns (e.g. Fink et al. 2010; Kromer et al. 2010). In either case, the explosion mechanism is expected to produce events that are bright and should be detectable. Thus, quantification of their predicted rates and delay times is an important step for testing our population synthesis models, and for determining what fraction of SNe Ia could conceivably be associated with this channel.

Since these calculations are based on the work that was performed for Paper I, the reader is referred to that paper for a more detailed description of the DDS, SDS and HeRS scenario. The layout of this paper is as follows. In Section 2 we summarize some background information on SN Ia progenitors from the literature. In Section 3 we discuss the population synthesis modelling. In Section 4 we present delay time distributions (DTDs) and rates as a function of stellar mass as well as distributions showing the exploding CO WD core mass. In Section 5 we close with a discussion of these findings and possible implications/predictions for SN progenitors and their host stellar populations.

2 BACKGROUND

2.1 Recent observations of SNe Ia delay times

The idea that SN Ia progenitors belong to at least two distinct populations (Scannapieco & Bildsten 2005; Mannucci, Della Valle & Panagia 2006; Pritchett, Howell & Sullivan 2008) has been gaining ground. A picture is emerging which supports populations of both quickly evolving (prompt) progenitors with short delay times less than ~ 500 Myr, as well as more slowly evolving progenitors with (sometimes rather) long delay times spanning up to a Hubble time (but see also Greggio 2010).

The DTD is a useful tool in determining the age of the progenitor stellar population, which places strong constraints on the different proposed progenitor scenarios. There are a growing number of observationally derived DTDs presented in the literature from various groups (see Section 1 of Maoz & Badenes 2010, for an overview of these previous studies). Totani et al. (2008) derived the DTD from a large population of old galaxies which only probed delay times $\gtrsim 100$ Myr, and they found that the DTD follows a relatively smooth power-law distribution (t^{-1}) from ~ 0.1 to 8 Gyr (see also Horiuchi & Beacom 2010). Probing younger stellar populations, Maoz & Badenes (2010) were able to determine that a substantial fraction of SNe Ia are prompt.¹ Among these prompt SNe Ia

(35–330 Myr delay times in that study) the SN Ia rate² is ~ 0.09 – 0.40 SNUm, compatible with the results of Li et al. (2011a), whereas delayed SNe Ia in that study (330 Myr–14 Gyr delay times) had an overall smaller rate: < 0.0024 SNUm. This study confirmed that roughly half of SNe Ia occur with delay times $\lesssim 330$ Myr, thus giving strong support for a prompt component of the DTD. Maoz et al. (2011) reconstructed the star formation histories for a sample of LOSS SN host galaxies and found strong evidence for both a prompt Ia component with delay times < 420 Myr and a delayed component with long delay times (> 2.4 Gyr). Brandt et al. (2010) used SN light curves and spectra from host galaxies of 101 SNe Ia with $z < 0.3$ to construct the DTD, and arrived at a similar conclusion: that roughly half of SNe Ia occur with delay times < 400 Myr, while roughly the other half have long (> 2.4 Gyr) delay times. Further, they find that the short delay time events are more luminous with slowly declining light curves, and are associated with young stellar populations, whereas the SNe with long delay times are typically fast-declining, sub-luminous events.

As one can see, the aforementioned recent studies indicate that SNe Ia are observed to occur over a range of long delay times (with less events as time goes on), with a substantial fraction also occurring at very early times.

2.2 Two progenitor scenarios: DDS and SDS

For some time, population synthesis calculations (Iben & Tutukov 1984; Yungelson et al. 1994; Yungelson & Tutukov 1997; Yungelson & Livio 1998; Nelemans et al. 2001a; Ruitter, Belczynski & Fryer 2009) have predicted that the number of merging CO WDs with a total mass exceeding M_{Ch} (DDS) is sufficient to match, and thus possibly account for, the rate of SNe Ia (0.4 ± 0.2 per century for the Galaxy, Cappellaro, Evans & Turatto 1999). At the same time, the theoretically predicted SN Ia rate from the SDS channel is usually unable to explain the observed rates of SNe Ia (see also Gilfanov & Bogdán 2010). There are very few SNe Ia that show any hint of hydrogen lines in their spectra; if the progenitor involved a hydrogen-rich companion, in particular a giant donor, one may expect $H\alpha$ to be detectable in the nebular spectra more frequently (Leonard 2007; Hayden et al. 2010).

The DDS is an attractive model for SNe Ia, given the theoretically predicted occurrence rate as well as the fact that CO WD mergers are systems that are essentially devoid of hydrogen. The main argument against the DDS is that detailed WD merger calculations between two CO WDs with a total mass $> M_{\text{Ch}}$ indicate that the merging process, while it can potentially lead to a thermonuclear explosion if the correct conditions are satisfied (Yoon, Podsiadlowski & Rosswog 2007), is more likely to result in collapse and form a neutron star; an accretion induced collapse (AIC, Miyaji et al. 1980; Saio & Nomoto 1985; Nomoto & Kondo 1991, and references therein). Based on modern collapse calculations (Fryer et al. 1999; Dessart et al. 2007; Abdikamalov et al. 2010), Fryer et al. (2009) found that these AICs produced outbursts that were ~ 1 to 3 magnitudes dimmer than typical SNe Ia, arguing that AICs could only explain a few abnormal Ia explosions.

¹ A complimentary result was also determined by Maoz, Sharon & Gal-Yam (2010a), who found that in galaxy clusters the DTD is well fitted by a power law of $t^{-1.2}$ for delay times > 400 Myr.

² The supernova rate in the local Universe as a function of Hubble type was recently presented in Li et al. (2011a) who found the SN Ia rate to be constant across galaxy Hubble type, with a value of 0.136 ± 0.018 SNUm ($1 \text{ SNUm} = \text{SNe Ia per century per } 10^{10} M_{\odot}$).

In a merger of two CO WDs, once the larger (less massive) WD fills its Roche lobe, it is likely to be disrupted and rapidly accreted by the companion. This process can be quite violent, and might under the right conditions lead to a SN Ia explosion (Piersanti et al. 2003). For example, in the DDS case involving the merger of two WDs with a mass ratio close to unity and WD masses $\sim 0.9 M_{\odot}$ (Pakmor et al. 2010a), critical conditions for the successful initiation of a detonation (Seitenzahl et al. 2009) can be obtained. The Pakmor et al. (2010a) study found that these DDS systems can both in number and in observational characteristics account for the population of subluminous 1991bg-like SNe Ia. However, the 1991bg-like systems only account for a small fraction of SNe Ia (Li et al. 2011b).

For lower mass ratios (see Pakmor et al. 2010; 2011) it is unlikely that the merger would lead to a SN Ia as the achieved densities are not high enough to enable a detonation to occur. In such WD mergers, high accretion rates on to the relatively ‘cold’ primary WD can lead to carbon burning off-centre, where the densities are too low, and carbon does not burn explosively. The primary CO WD will in turn burn carbon and evolve into an oxygen–neon–magnesium (ONeMg) WD (Nomoto & Kondo 1991). As the ONeMg WD increases in mass, density and temperature conditions become more favourable for electron captures, which in turn remove electron degeneracy pressure from the undisturbed WD. As the WD approaches M_{Ch} , the central densities continue to increase and the WD collapses to become a neutron star before a thermonuclear explosion can take place.

Despite this, in population synthesis calculations it is typically assumed that all mergers of CO–CO WDs produce a SN Ia provided that the total mass exceeds M_{Ch} . If some (or many) of these CO–CO mergers result in AIC then the observed SN Ia rates cannot be fully explained by the DDS model. Thus, if it is true that the majority of the DDS systems cannot produce events that look like SNe Ia, and there are not enough SDS or HeRS events, then a significant fraction of SNe Ia remain to be accounted for.

2.3 The helium donor formation channels

2.3.1 Chandrasekhar mass explosions

We delineate between the SDS and HeRS since the latter can involve either one degenerate star where the donor is helium burning, or two, where the donor is a helium-rich WD. Double WDs will have very close orbits (orbital periods < 70 min) as is the case for typical AM CVn binaries (see Nelemans et al. 2001b, 2010, for a discussion on AM CVn stars). In Paper I the rates and delay times from three formation channels involving exploding WDs with masses $\geq M_{\text{Ch}}$ (DDS, SDS and HeRS) were investigated. Both helium-donor channels were referred to as the ‘AM CVn channel’ in Paper I. In this paper we adopt the acronym HeRS for all SN Ia progenitors in which the *Chandrasekhar mass WD* explodes once it has accreted sufficient mass in stable RLOF from a helium-rich companion, whether the donor is degenerate or non-degenerate. This scenario includes AM CVn binaries as well as WDs accreting from all helium-burning stars.

SN Ia rates of the HeRS scenario leading to SNe Ia have been previously investigated by different groups: Solheim & Yungelson (2005); Ruiter et al. (2009); Wang et al. (2009a, b); Meng & Yang (2010), some of whom considered only the helium-burning star channel. In the majority of studies it was found that the HeRS is un-

able to account for the rates of SNe Ia.³ In many cases, theoretically motivated studies of the HeRS channel produce SNe Ia with short delay times (\lesssim a few hundred Myr), and are not able to account for a large number of systems at long delay times.

2.3.2 Sub-Chandrasekhar mass explosions

Thermonuclear explosions may occur in systems with a sub- M_{Ch} (probably CO) WD accreting via stable RLOF from a helium-rich companion (Iben & Tutukov 1991; Tutukov & Yungelson 1996; Yungelson & Livio 2000). It has been shown that at certain (low) accretion rates on to the WD helium flashes on the WD surface are inhibited (Kawai, Saio & Nomoto 1987; Ivanova & Taam 2004), and the WD can steadily and efficiently build up a massive layer of helium on the surface. In such a massive degenerate helium shell (e.g. $\sim 0.1 M_{\odot}$ of helium, Taam 1980), a flash may likely evolve as a violent detonation, which may also trigger a detonation of the CO core, and thus a thermonuclear explosion of the complete star (e.g. Livne 1990; Woosley & Weaver 1994; Livne & Arnett 1995).⁴ Tutukov & Yungelson (1996) found that the rate of sub- M_{Ch} SNe Ia from the non-degenerate helium star channel might be high enough to account for the Galactic rate if these explosions are comparable in luminosity to normal SNe Ia, though these types of events were found to have somewhat short delay times and cannot account for the number of SNe Ia in old stellar populations.

Sub- M_{Ch} models of SNe Ia are appealing for a number of reasons. Population synthesis calculations have already shown that the DTD for the double-detonation sub- M_{Ch} explosion model (sometimes referred to as edge-lit detonation, ELD; see Yungelson & Livio 2000, and references therein) spans a wide range at early times. Yungelson & Livio (2000) investigated the double-detonation scenario for helium star and hydrogen-rich donors, and found a corresponding delay time of ~ 30 Myr to 1.3 Gyr for the helium star donor systems. Also, population synthesis modelling has indicated that the number of potential progenitors for the double-detonation sub- M_{Ch} SNe Ia involving helium-rich donors alone may be large enough to account for the observed rates (e.g. Tout et al. 2001; Regos et al. 2003; Paper I). Additionally, simplistic studies of pure detonations of sub- M_{Ch} WDs (in the absence of any overlying helium shell) indicate that the synthetic light curves and spectra from sub- M_{Ch} explosions may be able to reproduce a surprising number of the observed properties of SNe Ia (Sim et al. 2010), and even variations within the class that could be associated with differences in the mass of the exploding WD.

³ Wang et al. (2009a) found Galactic SN Ia rates which were higher than the other studies: $\sim 10^{-3} \text{ yr}^{-1}$. However this rate is likely somewhat optimistic because they consider a rather large range of orbital periods at the moment of RLOF onset between the WD and the helium star (up to > 100 d for the most massive WD accretors).

⁴ It is possible that the companion can be hydrogen-rich where hydrogen burns steadily on the surface of the WD, building up a helium-rich layer on top of the CO WD which can detonate (Kenyon et al. 1993; Piersanti et al. 1999; Yungelson & Livio 2000). Such a double-detonation scenario for hydrogen-rich donors was investigated by Yungelson et al. (1995), who have shown that sub-Chandrasekhar mass WDs accreting hydrogen in symbiotic binaries may be capable of producing up to $\sim 1/3$ of all SNe Ia, provided that accreting WDs with masses as low as $0.6 M_{\odot}$ are able to successfully undergo double-detonations. However in this work, we only consider helium-rich donors as possible companions for double-detonation sub- M_{Ch} SN Ia progenitors.

The Sim et al. (2010) work, however, neglects the issue of how the outer helium layer will affect the observables. Several previous studies have calculated detailed synthetic light curves and spectra of sub- M_{Ch} double-detonation models (e.g. Hoefflich & Khokhlov 1996; Nugent et al. 1997). These concluded that such explosions would likely not lead to events with observational properties characteristic of normal SNe Ia. In general, the light curves were found to rise and fall too rapidly compared to ‘normal’ SNe Ia while their spectra were too blue to match subluminous SNe Ia and lacked sufficiently strong features of intermediate mass elements, such as Si and S. Importantly, most of these discrepancies with observation can be traced to the presence of the products of helium burning (^{56}Ni and other iron-group elements) in the outer regions of the ejecta, and those studies mainly considered systems in which a relatively massive ($\sim 0.2 M_{\odot}$) helium layer had accumulated on the WD ($\sim 0.6 M_{\odot}$) surface. More recently, Bildsten et al. (2007) and Shen & Bildsten (2009) have shown that conditions suitable for detonation in the WD might be reached for somewhat lower helium shell masses than considered in most previous studies: perhaps as low as $0.05 M_{\odot}$ for a CO WD (core) mass of $1.0 M_{\odot}$ (in general the more massive the CO WD, the less massive the accumulated helium layer needs to be for a detonation). Fink et al. (2010) have shown that, even for such low helium shell masses, detonation of the helium will robustly lead to an explosion of the underlying WD. With a significantly lower He shell mass (and thus fewer iron-group elements in the outer ejecta), this may open the door for double-detonation sub- M_{Ch} models whose spectra and light curves are in better agreement with observed SNe Ia. This has been investigated by Kromer et al. (2010) who computed synthetic observables for the Fink et al. (2010) simulations. They showed that even very low-mass ($0.05 M_{\odot}$) helium shells affect the observable display and can lead to spectroscopic signatures that are not characteristic of observed SNe Ia. However, Kromer et al. (2010) also highlighted that the results are highly sensitive to the details of the nucleosynthesis that occur during burning of the helium shell. Modifications to the burning – as might be achieved by considering a composition other than pure helium – could allow the model predictions to achieve much better agreement with observation.

Taken together, the body of theoretical work strongly suggests that the sub- M_{Ch} double-detonation scenario is physically realistic. Depending on the details of the accumulated helium layer and its burning products, the explosion may closely resemble observed ‘normal’ SNe Ia or it might be highly spectroscopically peculiar – but regardless of this, it certainly can be bright enough to be readily observable – for a CO WD of around $1.0 M_{\odot}$ the luminosity produced following detonation is expected to be close to that of a normal SN Ia (Shigeyama et al. 1992; Sim et al. 2010). Given that potential progenitors are also expected to be common, we are therefore compelled to further investigate this progenitor scenario. If these explosions can produce events that resemble ‘normal’ SNe Ia then it is of interest to quantify the fraction of observed SNe Ia that might be accounted for via this channel. Alternatively, if these explosions are realized in nature but are spectroscopically peculiar, it is important to estimate their predicted rate and consider whether the apparent lack of observational detections is a major concern for the established theory; the lack of such events may challenge our understanding of either (or both) the explosion physics or the progenitor binary evolution. For this scenario it is of particular interest to consider the DTD predicted for this class of explosion and to investigate any correlations between the properties of the exploding system (particularly the mass of the primary WD, which likely determines the brightness

of the explosion) and the age of the stellar population in which it resides.

3 MODEL

It has been shown that a WD accumulating helium-rich material may be capable of exploding as a SN Ia if the correct conditions are satisfied, even if the WD is below M_{Ch} (Taam 1980; Iben et al. 1987; Iben & Tutukov 1991; Woosley & Weaver 1994; Livne & Arnett 1995; Ivanova & Taam 2004). In Belczynski, Bulik & Ruitter (2005), sub- M_{Ch} models were calculated, though DTDs for different formation channels were not discussed separately, and rates were not presented in that work. In Paper I, rates and delay times were presented only for $\gtrsim M_{\text{Ch}}$ WD mass models where as here, we additionally include sub- M_{Ch} SNe Ia progenitors in our study.

All sub- M_{Ch} SN progenitors in our calculations involve a CO WD accreting via RLOF from a helium-rich companion. As in Paper I, if the donor is a WD then it can be either a helium WD or a hybrid WD; a WD with a CO core and a helium-rich mantle (e.g. Tutukov & Yungelson 1996, and references therein). Hybrid WDs are formed through binary evolution when a red giant is stripped of its envelope through binary interactions. In cases where the stripped helium core does not reach the helium-burning phase, a helium WD is formed.

In the following sections we compute and discuss rates and delay times for the aforementioned SN Ia evolutionary models that have been proposed as the most promising formation channels for SNe Ia:

- (i) the DDS,
- (ii) the SDS,
- (iii) the HeRS,
- (iv) the double-detonation sub- M_{Ch} scenario involving helium-rich donors.

It is important to keep in mind that, while evolution of close binaries remains an active field of research and discovery, no concrete constraints currently exist for the evolution of mass-transferring binaries, nor for the common envelope (CE) phase. The CE phase is certainly one of the most poorly understood phenomena in close binary evolution, and a theoretical picture of CE evolution is not yet available. Since there are a limited (though growing) number of observations available to guide our choice of parameters, we present results for three different CE realizations which most effectively bracket the uncertainties. For the growth of CO WDs during stable RLOF, as was done in Paper I we present the results from our population synthesis model using a detailed WD accretion scheme, which was constructed by adopting various input physics from the literature.

We use the `STARTRACK` population synthesis binary evolution code (Belczynski et al. 2008) to evolve our stellar populations employing Monte Carlo methods. The code has undergone many revisions since the first code description (Belczynski, Kalogera & Bulik 2002). Many of the updates concerning accretion on to WDs can be found in Belczynski et al. (2005, 2008), though since then we have incorporated an updated prescription for accretion of hydrogen on WDs by including calculations from Nomoto et al. (2007) in addition to the existing Prialnik & Kovetz (1995) prescription. The initial distributions for binary orbital parameters (orbital periods, mass ratios, etc.) are the same as described in Paper I.

In Paper I, it was assumed that the ejection of the envelope of the mass-losing star during a CE phase came at the expense of removing the orbital energy of the binary, as dictated by the well-known

‘energy-balance’ (or ‘ α -formalism’) equation (Webbink 1984), with α_{CE} representing the efficiency with which the binary orbital energy can unbind the CE, and λ is a parametrization of the structure of the donor star (de Kool 1990); both α_{CE} and λ are fairly uncertain.

For Models 1 and 2 from Paper I, $\alpha_{\text{CE}} \times \lambda$ values of 1 and 0.5 were adopted, respectively. The major difference was that Model 1 (more efficient removal of the CE) resulted in an overall higher number of SNe. In the current paper, we keep all model parameters the same as in Model 1 of Paper I for one model; we refer to this model as Model A1 (standard model). However, in order to explore the sensitivity of the physical mechanism of CE ejection, which is still not understood, we have run two additional sets of models. There has been some recent observational (Zorotovic et al. 2010) as well as theoretical (De Marco et al. 2011) evidence that the value for α_{CE} lies between 0.2 and 0.3. Additionally, for low-mass stars a value of 0.5 is often adopted for λ (van der Sluys, Verbunt & Pols 2006). Thus to best bracket our uncertainties for the energy balance prescription of CE evolution, in a second model we employ a very low CE ejection efficiency: $\alpha_{\text{CE}} = 0.25$ and $\lambda = 0.5$ yielding $\alpha_{\text{CE}} \times \lambda = 0.125$. We refer to this model as Model A.125. In a third model, we assume a different parametrization for the treatment of the CE phase. We employ the ‘ γ ’ prescription for CE evolution (Nelemans et al. 2000) every time a CE event is encountered in the code. Although the combination ‘ α - γ ’ prescription is considered to be the preferred prescription by some groups, the two formalisms stress different physics (one focusing on energy conservation, one on angular momentum balance), and we do not mix the formalisms in the current paper. Henceforth we refer to this model as Model G1.5. For Model G1.5, all physical parameters are identical to Models A1 and A.125, except that when unstable mass transfer is encountered and a CE ensues, the orbital separation of the binary changes not as a consequence of removing gravitational binding energy from the orbit, but linearly as a function of mass loss (and hence angular momentum loss), parametrized by the factor γ :

$$\frac{a_f}{a_i} = \left(1 - \gamma \frac{M_{\text{ej}}}{M_{\text{tot},i}}\right)^2 \frac{M_{\text{tot},f}}{M_{\text{tot},i}} \left(\frac{M_{\text{don},i} M_{\text{com}}}{M_{\text{don},f} M_{\text{com}}}\right)^2, \quad (1)$$

where $M_{\text{don},i}$ is the initial mass of the (giant) donor star just prior to the CE, M_{ej} is the ejected mass (assumed to be the mass of the giant’s envelope), M_{com} is the mass of the companion (assumed to be unchanged during the CE), $M_{\text{don},f}$ is the final mass of the donor once the envelope has been ejected, a_i is the initial orbital separation, a_f is the final orbital separation, and $M_{\text{tot},i}$ and $M_{\text{tot},f}$ represent the total mass of the binary before and after CE, respectively. Following Nelemans et al. (2000), we have chosen $\gamma = 1.5$.⁵

3.1 Sub-Chandrasekhar mass model: assumptions

We adopt the prescription of Ivanova & Taam (2004), applied to accretion from helium-rich companions only, to determine when a particular binary undergoes a sub- M_{Ch} SN Ia (Belczynski et al. 2008, see section 5.7.2 for equations). In short, we consider three different accretion rate regimes for accumulation of helium-rich material on *all* CO WDs, adopting the input physics for helium accretion on to WDs of Kato & Hachisu (1999, 2004). Clearly, our results are sensitive to the adopted helium accumulation efficiency. It may also be possible that for the less common hybrid donors,

the star’s helium-rich envelope becomes stripped in RLOF before a helium shell detonation on the CO WD occurs. This would have an effect on the purity of the helium-rich shell, though we do not investigate the consequences here. We assume that mass is exchanged conservatively in all cases where the transferred material would be CO-rich.

At high accretion rates ($\sim 10^{-6} M_{\odot} \text{ yr}^{-1}$, and for all cases where the WD accretor mass is $< 0.7 M_{\odot}$), helium burning is stable and thus mass accumulation on the WD is fully efficient ($\eta_{\text{acu}} = 1$). At somewhat lower accretion rates helium burning is unstable and the binary enters a helium-flash cycle, thus accumulation is possible but is not fully efficient ($0 < \eta_{\text{acu}} < 1$). In both of these aforementioned accretion regimes, the CO WD is allowed to accrete (and burn) helium, and its total mass may reach M_{Ch} and explode as a SN Ia through the HeRS channel. However, for low accretion rates ($\sim 10^{-8} M_{\odot} \text{ yr}^{-1}$), compressional heating at the base of the accreted helium layer plays no significant role, and a layer of unburned helium can be accumulated on the WD surface. Following Ivanova & Taam (2004), we assume that if such a CO WD accumulating helium enters this ‘low’ accretion rate regime and accumulates $0.1 M_{\odot}$ of helium on its surface, a detonation is initiated at the base of the helium shell layer. Consequently, a detonation in the core of the CO WD is presumed to follow, and we assume that a sub- M_{Ch} SN Ia takes place. Only accreting WDs with a *total mass* $\geq 0.9 M_{\odot}$ are considered to lead to potential sub- M_{Ch} SNe Ia in this work, since lower mass cores may not detonate, and if they do they are unlikely to produce enough radioactive nickel and hence will not be visible as SNe Ia (e.g. Sim et al. 2010, table 1). Thus in all future discussions we refer to sub- M_{Ch} systems whose total WD mass (CO core + helium shell) is at least $0.9 M_{\odot}$ at the time of SN Ia unless otherwise noted; for our population synthesis model, this intrinsically implies that all exploding sub- M_{Ch} SNe Ia have CO WD ‘core’ masses $> 0.8 M_{\odot}$. Helium-rich WDs are simply not massive enough, and we assume that ONeMg WDs do not make SNe Ia.

4 RESULTS

Here we present the DTD and rates for all of our SN Ia models, as well as CO WD core masses for our sub- M_{Ch} models. Our results are discussed in the following subsections, but here we give a brief outline of our findings.

We have investigated the DDS, SDS, HeRS and the sub- M_{Ch} scenario for three different CE realizations. Within the framework of our adopted models, we find that only two SN Ia formation scenarios are capable of matching the observed SNe Ia rates: the DDS and the sub- M_{Ch} channels. The most favourable model in terms of matching observational rates is model A1 ($\alpha \times \lambda = 1$). For models A1 and G1.5 ($\gamma = 1.5$), the adopted sub- M_{Ch} scenario is dominant at nearly all epochs $\lesssim 5$ Gyr; however, the sub- M_{Ch} channel rate is too low for our low-efficiency CE model (A.125). For Model A.125, no single progenitor, nor an admixture of all of the progenitors combined are able to account for the observed rates of SNe Ia.

4.1 Delay times

In Fig. 1 we show the DTD of the four aforementioned progenitor channels for Models A1, A.125 and Model G1.5. We note that the bumpiness in the smoothed plot is due to Monte Carlo noise. For our DTD normalization of all models, we have assumed a binary fraction across the entire initial stellar mass function of 50 per cent

⁵ We note here that the γ CE equation in Belczynski et al. (2008, equation 55) is missing an exponent, though the CE evolution is properly carried out in the `STARTRACK` code with the correct equation.

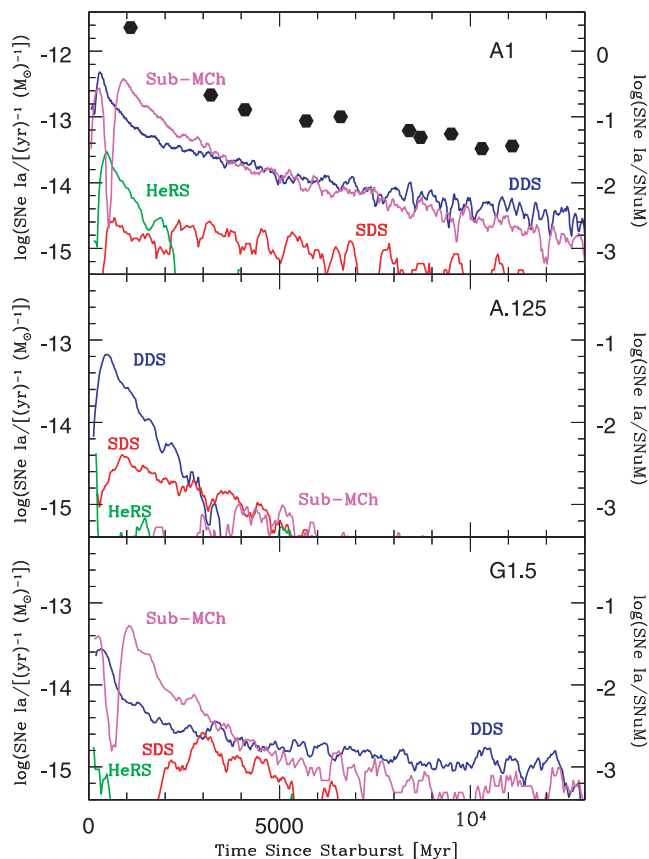


Figure 1. Lines represent the DTD for SNe Ia. Top panel: Model A1. Middle panel: Model A.125. Bottom panel: Model G1.5. The number of SNe Ia yr^{-1} per unit stellar mass born in stars (at starburst $t = 0$, 50 per cent binarity) is shown for the DDS (blue), SDS (red), HeRS (green) and sub- M_{Ch} (magenta) channels. The sub- M_{Ch} SN Ia DTD clearly shows two distinct populations for Models A1 and G1.5: the helium star channel (spike at delay times $\lesssim 500$ Myr) and the WD channel (from ~ 800 Myr to a Hubble time). The helium star channel, however, is absent in Model A.125. In the top panel we additionally show the DTD [SNuM] compiled by Maoz et al. (2010, table 1), which is fitted relatively well by a power law $\sim t^{-1.2}$. The data points showing the observed DTD are computed using a different normalization technique (see text), and thus we show the points for comparison of the DTD shapes and not the absolute numbers. We note that assuming a different binary fraction or initial mass function would change the level of our normalization.

(2/3 of stars are in binaries), and we show the DTD normalized to stellar mass (SNuM and $\text{SNe Ia yr}^{-1} M_{\odot}^{-1}$). The mass represents the mass in formed stars, which includes mass which has potentially been expelled from stars in SNe or thermal pulses for example. In Section 4.2, we give the delay times in tabular form.

Along with our theoretical DTDs, we show the observed (cosmic) DTD from the literature (Maoz et al. 2010). We wish only to compare the relative DTD shapes and not the absolute rates, since the normalization of our *STARTRACK* DTD differs substantially from the normalization techniques used in recovering the various observational DTDs. The difference between the (higher) rates of observed SNe Ia and the rates from population synthesis is visible, and the apparent discrepancy is not yet fully resolved. It has been suggested that binary population synthesis codes tend to underpredict the SN Ia rates compared to the rates inferred from recent observations, though one must keep in mind that many uncertainties are associated with the DTD recovery methods, i.e., extinction,

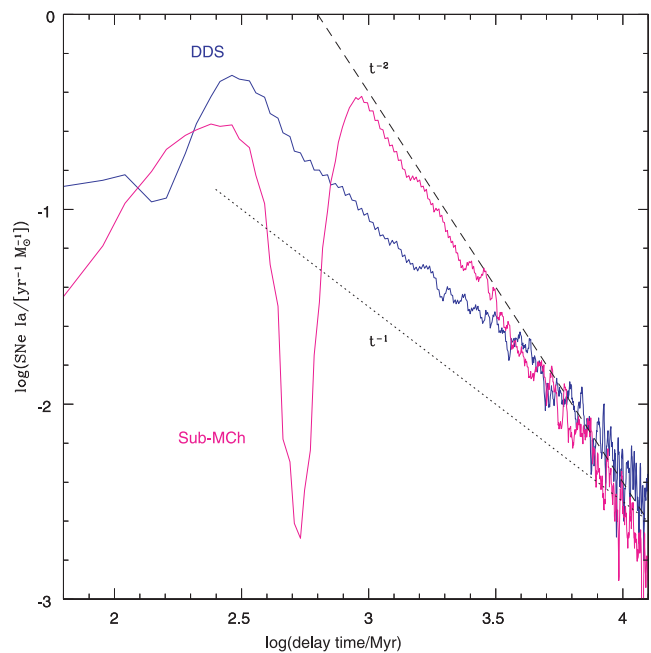


Figure 2. Delay time distribution for the DDS (blue) and sub- M_{Ch} (magenta) channels for model A1 (standard). We show two power laws alongside the DTDs: the DDS is relatively well fitted by a power law t^{-1} , where as the sub- M_{Ch} model closely follows a power law with t^{-2} beyond 1 Gyr, where all progenitors have helium-rich WD donors.

star formation history, and the use of spectral population synthesis codes which neglect the existence of binaries (see De Donder & Vanbeveren (2004), also Eldridge & Stanway (2009) have found that inclusion of massive binaries in spectral synthesis codes plays an important role in recovering accurate host galaxy properties).

Model A1. As was found in Paper I Model 1, the DDS distribution for Model A1 (top panel of Fig. 1) follows a power-law distribution with $\sim t^{-1}$ (see also Fig. 2), while the SDS distribution is somewhat flat with no events with delay times less than 460 Myr. The reason why the SDS does not harbour very prompt events is directly linked to the donor star's initial zero-age main sequence (ZAMS) mass. When the secondary ZAMS mass is $> 2.8 M_{\odot}$, the binary will enter a CE phase when the secondary fills its Roche lobe, rather than a stable RLOF phase. In such a case, the binary will not become an SDS SN Ia, though may under the right circumstances evolve to SN Ia from the HeRS channel. The SDS events at long delay times originate from progenitors with very low-mass MS donors, which take many Gyr to evolve to contact under the influence of magnetic braking. The HeRS DTD consists mostly of systems with relatively short (~ 100 Myr to 2 Gyr) delay times, with very few events at longer delay times. We refer the reader to Paper I for a description of these DTDs.

The sub- M_{Ch} systems can easily by eye be grouped into two classes: those prompt SNe which occur with delay times $\lesssim 500$ Myr, and those with delay times above ~ 800 Myr, with very little overlap. Not surprisingly, these two classes of SNe Ia stem from two very different formation scenarios. Those with short delay times consist of progenitors which involve a helium-burning star donor, whereas the rest mainly consists of helium WD donors (systems with hybrid WD donors span ~ 0.3 to 3 Gyr delay times). We find that progenitors with helium star, helium WD and hybrid WD donors comprise 13, 78 and 9 per cent of SNe Ia, respectively. We note that ~ 35 per cent of sub- M_{Ch} SNe Ia explode within 1 Gyr of star formation.

The prompt component accounts for 13 per cent of all sub- M_{Ch} SNe Ia that explode within 13 Gyr of star formation. Nearly all of these systems (96 per cent) have helium star donors, with the rest having hybrid WD donors. The delay time is governed by the MS lifetime of the donor star. The companions with ZAMS masses $\gtrsim 3 M_{\odot}$ evolve off of the MS within $\lesssim 400$ Myr. After the first CE, which leaves behind a CO primary WD and a MS secondary star, the secondary (e.g. on the Hertzsprung gap) will fill its Roche lobe and mass transfer is once again unstable leading to a second CE phase. The CE leaves the CO WD and newly formed naked helium star on a close orbit (~ 35 – 40 min). Within a few Myr, the orbit decreases to ~ 25 min, and the helium star fills its Roche lobe. However, initial mass transfer rates for the helium star channel are low enough to enable accumulation of the helium shell to commence immediately: typically such systems have initial mass transfer rates $\sim 2 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ (for a discussion on the evolution of low-mass helium stars in accreting binaries see Yungelson 2008).

The delayed component (delay times > 500 Myr) comprise the other ~ 87 per cent of the sub- M_{Ch} progenitors. Binaries with helium WD donors make up 90 per cent of the delayed component, while 10 per cent have hybrid donors. These binaries also evolve through two CE phases, as is expected for the evolution of AM CVn binaries. Similar to the DDS, the time-scale governing the DTD for the helium WD channel is largely set by the gravitational radiation time-scale (see also Tutukov & Yungelson 1996). However unlike the DDS, these WDs do not merge upon contact, but enter a stable phase of RLOF. Like the DDS DTD, the sub- M_{Ch} DTD follows a power law above 1 Gyr, however with a steeper functional form of t^{-2} (Fig. 2, see also Section 5.2).

Model A.125. In the middle panel of Fig. 1, we show the DTD for Model A.125. Contrary to the standard model, the DDS DTD is lacking progenitors at longer delay times since on average the time a progenitor spends as a detached double WD is decreased in this model (smaller orbital separation following the CE phase). For this model the SDS progenitors can have shorter delay times compared to the standard model due to the fact that the post-CE separations are overall smaller. Thus, a low CE efficiency model is more favourable for the production of SDS SNe Ia. The HeRS channel has some very prompt events (helium star channel), although the low CE efficiency serves to result in a merger during CE more frequently than in Model A1. The events at delay times ~ 1 to 2 Gyr belong to progenitors with helium WDs, while events at long delay times ($>$ a few Gyr) also involve helium WDs but belong to the evolved low-mass MS donor channel. The DTD of the sub- M_{Ch} progenitors looks drastically different from that of the standard model, and lacks a prompt component. This model is the only of the three which does not display a prominent division of the sub- M_{Ch} progenitor channels; in fact there are no sub- M_{Ch} SNe Ia originating from the helium star channel: those progenitors will encounter unstable RLOF too early in their evolution. With the adopted CE prescription, it is very difficult (or impossible) to produce helium star donor channel sub- M_{Ch} SNe Ia within our model framework, and thus there are no prompt events (the first SN Ia from the sub- M_{Ch} channel occurs at ~ 1.7 Gyr).

Model G1.5. In Fig. 1, bottom panel, we show the DTD for Model G1.5. Gravitational radiation plays a less significant role for the DDS since following the CE phase the binary orbit is still rather wide. Similar to the other two models, the DDS contributes the majority of its events at very early times followed by a decline. The SDS channel displays no prompt events, because the first CE event does not lead to a dramatic decrease in orbital separation. In general, the SNe Ia with short SDS delay times from Model A1

Table 1. Rates of SNe Ia (SNum, 50 per cent binarity) for the four progenitor formation scenarios considered in this work, following a starburst at $t = 0$. Models A1 (left), A.125 (middle) and G1.5 (right).

	A1	A.125	G1.5
DDS			
0.1 Gyr	2.0×10^{-1}	$< 10^{-4}$	2.0×10^{-2}
0.5 Gyr	1.6×10^{-1}	6.5×10^{-2}	2.2×10^{-2}
1 Gyr	8.0×10^{-2}	2.5×10^{-2}	5.3×10^{-3}
3 Gyr	2.5×10^{-2}	$\lesssim 10^{-4}$	$\sim 2 \times 10^{-3}$
5 Gyr	1.2×10^{-2}	0	$\sim 2 \times 10^{-3}$
10 Gyr	$\sim 5 \times 10^{-3}$	0	$\lesssim 10^{-3}$
SDS			
0.1 Gyr	0	10^{-3}	0
0.5 Gyr	$\sim 10^{-3}$	3.5×10^{-3}	0
1 Gyr	1.5×10^{-3}	5×10^{-3}	$\lesssim 10^{-3}$
3 Gyr	2.0×10^{-3}	$\lesssim 10^{-4}$	$\sim 2 \times 10^{-3}$
5 Gyr	$\sim 1 \times 10^{-3}$	$< 10^{-4}$	$\sim 10^{-4}$
10 Gyr	$\lesssim 10^{-3}$	~ 0	$\lesssim 10^{-4}$
HeRS			
0.1 Gyr	$\sim 3 \times 10^{-3}$	4×10^{-3}	$\lesssim 10^{-3}$
0.5 Gyr	2.2×10^{-2}	0	$\lesssim 10^{-3}$
1 Gyr	8.0×10^{-3}	$< 10^{-3}$	$< 10^{-4}$
3 Gyr	$< 10^{-3}$	$\lesssim 0$	$< 10^{-4}$
5 Gyr	$\lesssim 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
10 Gyr	~ 0	~ 0	$< 10^{-4}$
sub-M_{Ch}			
0.1 Gyr	$\sim 1 \times 10^{-1}$	0	$\lesssim 10^{-4}$
0.5 Gyr	$\sim 10^{-3}$	0	$\sim 10^{-3}$
1 Gyr	3.3×10^{-1}	$< 10^{-4}$	$\sim 7 \times 10^{-2}$
3 Gyr	4.0×10^{-2}	$< 10^{-4}$	$\sim 4 \times 10^{-3}$
5 Gyr	1.4×10^{-2}	$< 10^{-3}$	$\sim 2 \times 10^{-3}$
10 Gyr	$\sim 4 \times 10^{-3}$	~ 0	$\lesssim 10^{-4}$

will evolve into detached double CO WDs in Model G1.5, since the binary orbit is not small enough for mass transfer to begin once the secondary evolves off of the MS and fills its Roche lobe. The HeRS channel leads to SNe Ia with very short delay times, though there are events at long delay times but their frequency is for the most part too low to be seen on the figure. The sub- M_{Ch} DTD has the same general shape as Model A1: the prompt and the delayed components. We note, however, that SNe Ia with delay times less than 1 Gyr follow a different evolutionary sequence compared to the corresponding events of Model A1. In Model A1, the first mass exchange interaction occurs when the primary is an AGB star, where as for Model G1.5 the first mass exchange event (CE or stable RLOF) occurs when the primary is less evolved; a sub-giant or giant. This occurs since the semilata recta (and thus in general, the separations) of the G1.5 sub- M_{Ch} progenitors which explode in our models are smaller when mass transfer begins, as well as the fact that the primaries for this model are somewhat more massive than compared to the standard model and thus they evolve more quickly

4.2 Rates

In Table 1, we show the DTDs in tabular form (rates as a function of epoch) for our models. We estimate the Galactic SN Ia rate by convolving the DTD (in units of SNe/time) with a constant star formation history from 0–10 Gyr with a total mass born in stars of $6 \times 10^{10} M_{\odot}$; see Section 4 of Paper I. In this paper, we do not quantify the Galactic rate estimates explicitly since imposing a

particular star formation history serves to add sources of uncertainty to our DTD calculation; however, we give some numbers as a guide. In principle, all of the important information is already presented in the DTD plots and Table 1: it is possible to convolve the specific DTD with any star formation history of choice in order to achieve a particular SN Ia rate for a given stellar population.

Model A1. This model produces the highest number of SNe Ia out of our three models. Table 1 (left) is very similar to table 1 (elliptical column) in Paper I, though here for all tables we additionally include the rates of sub- M_{Ch} SNe Ia, as well as two additional epochs: 0.1 and 1 Gyr after star formation. Slight variations between the numbers in this study and table 1 of Paper I are due to a slight increase in volume of data, and thus a reduction in noise from low-number statistics. We find that the rate of our adopted sub- M_{Ch} SN Ia model exceeds all other progenitor channels between ~ 0.7 and 5 Gyr, and these systems are enough to account for the observed SN Ia rate, with a calculated Galactic rate of $\sim 2.6 \times 10^{-3}$ SN Ia yr^{-1} (including all systems with a total WD mass $\gtrsim 0.9 M_{\odot}$). For comparison, the DDS rate is $\sim 2 \times 10^{-3}$ SN Ia yr^{-1} . Both of these values are within the estimate from Cappellaro et al. (1999) of $4 \pm 2 \times 10^{-3}$ SN Ia yr^{-1} . As was determined in Paper I, the Model A1 DDS rates are able to (just) account for the observed Galactic rate of SNe Ia, whereas both the SDS and HeRS channels fall short by over an order of magnitude.

Model A.125. This model produces the least SNe Ia progenitors out of our three models. The DDS is significantly decreased in number (Table 1, middle), but is still the dominant channel at most times under a few Gyr. The SDS rate exceeds the DDS rate above 3 Gyr, though the overall rates are still too low for any progenitor in this model to account the observed SN Ia rates. The rates of the HeRS SNe Ia are too low; many binaries do not survive both CE events to become progenitors. Similarly, the sub- M_{Ch} progenitors are not easily formed in this model.

Model G1.5. The overall rates for this model (Table 1, right) are lower than found in the standard model, though not as low as found for Model A.125. In the DDS, since the binaries take a longer time to reach contact (e.g. it can easily be more than a Hubble time), the overall SN Ia rates are rather low compared to Model A1 with an estimated Galactic rate of $\sim 2 \times 10^{-4} \text{yr}^{-1}$, which is about a factor of 10 too low. The SDS channel produces very few events before 2 Gyr, and matches those of the DDS at ~ 3 Gyr, while the HeRS channel produces events with delay times < 1 Gyr and few events at later times. Even though the sub- M_{Ch} DTD exhibits the same general shape as found in the standard model, the rates are overall too low being roughly comparable to those of the DDS of this model (Galactic rate estimate $\sim 3 \times 10^{-4} \text{yr}^{-1}$).

4.3 CO core masses

In the sub- M_{Ch} scenario, the brightness is expected to be largely determined by the mass of the underlying CO WD. In Fig. 3, we show the mass of the CO WD ‘core’ (total WD mass minus the helium shell mass) at time of SN Ia. As mentioned previously, a detonation of a $\sim 0.7 M_{\odot}$ core WD would likely not look like a normal SN Ia. Since we currently lack a theoretical lower mass limit for which exploding CO core masses could potentially exhibit features which are characteristic of SNe Ia, for completeness we show the CO core mass at explosion for the entire mass spectrum for exploding sub- M_{Ch} cores. We draw a vertical line at $M_{\text{core}} = 0.8 M_{\odot}$, above which the systems are considered to be sub- M_{Ch} SNe Ia in our models.

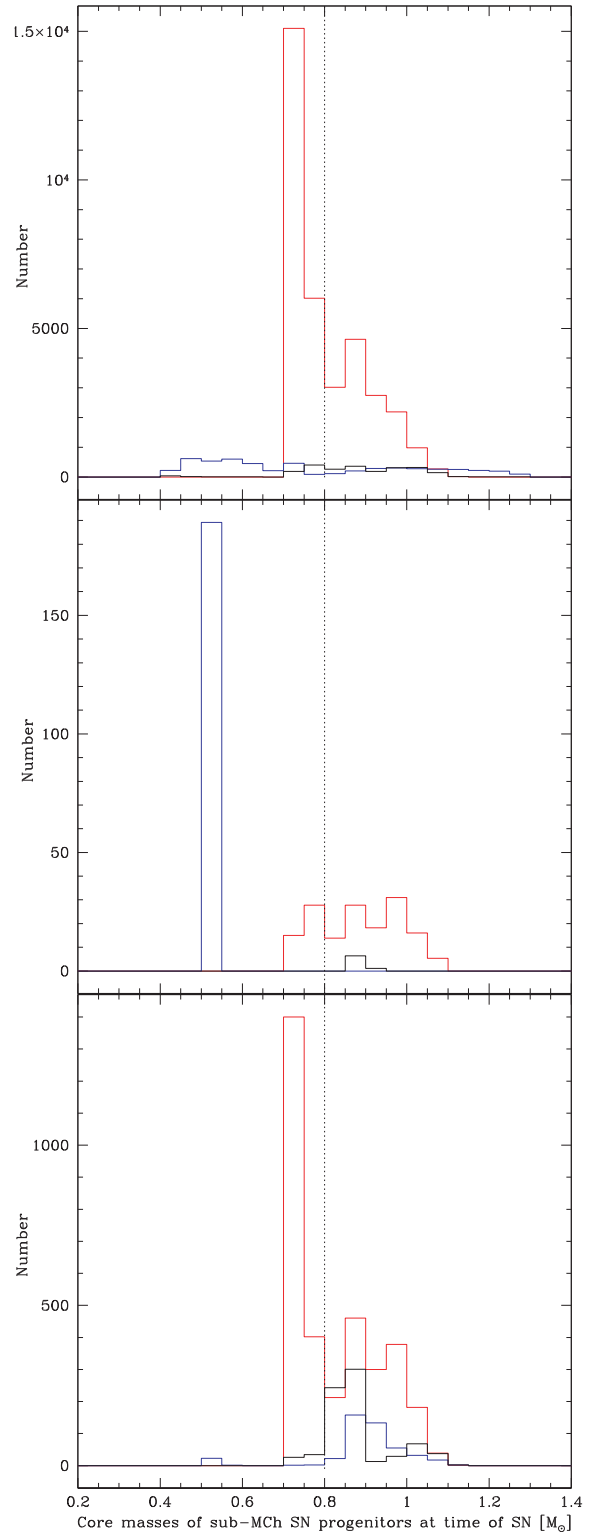


Figure 3. Distribution of STARTRACK CO WD core masses which managed to accumulate a $0.1 M_{\odot}$ shell of unburned helium. A double-detonation was assumed to follow in all cases. A vertical line is drawn at $M_{\text{CO core}} = 0.8 M_{\odot}$, above which all systems are assumed to lead to sub- M_{Ch} SN Ia in our three models. Top panel: Model A1. Middle panel: Model A.125. Bottom panel: Model G1.5. The helium star channel is outlined in blue, the He-WD channel is outlined in red and the hybrid WD channel is outlined in black. Note different scales on the y-axes.

The core mass distributions look very different for all three models. In the top panel of Fig. 3, we show the core mass distribution for Model A1. The progenitors of binaries with low core masses ($<0.7 M_{\odot}$) go through a different evolutionary channel than those with higher core masses since they start out with smaller semilata recta and only evolve through one CE event. The cores associated with the helium star channel span both low and high masses, though for our adopted sub- M_{Ch} scenario they have slightly higher core masses on average compared to the WD channels. The hybrid WD channel shows a similarly flat distribution, which is not unexpected since many of these systems undergo an evolutionary sequence which is like that of a typical progenitor from the helium star channel. For the helium WD channel which comprises the majority, the masses decrease fairly steadily in number with increasing mass, since there are simply a larger number of less massive CO WD cores to start with. There is a clear lack of CO core masses below $\sim 0.7 M_{\odot}$. Typically these CO core progenitors will accrete (and burn) at least $0.1 M_{\odot}$ (often $\sim 0.2 M_{\odot}$) of helium at a high accretion rate before the phase of helium accumulation begins for the $0.1 M_{\odot}$ shell, and thus we find no CO cores from this channel with very low masses. However, there are a number of exploding cores with masses ~ 0.7 to $0.8 M_{\odot}$. One has to also consider the possibility that a low-mass ($<0.8 M_{\odot}$) CO core + helium shell may not reach sufficient conditions for a detonation to take place, which might explain why we would not see a large number of these events.

For Model A.125 (Fig. 3, middle panel), the separation between the helium star and double WD channels is quite distinct. The lowest mass CO cores belong to progenitors with helium star donors, and in our adopted sub- M_{Ch} model all of these binaries have CO core masses which are too low to qualify as SNe Ia. The ZAMS masses of these CO cores are small, ~ 1.8 to $2.1 M_{\odot}$, and these stars are unable to build a massive CO core before the first CE is encountered. Additionally, binaries which start their final RLOF phase when the secondary is a helium star have lower initial accretion rates ($\sim 10^{-8} M_{\odot} \text{ yr}^{-1}$), which allows the CO core to immediately accumulate (not burn) a shell of helium and produce a SN Ia without the CO WD having to grow in mass by an extra ~ 0.1 to $0.2 M_{\odot}$.

The distribution of CO core mass for the helium WD channel of Model G1.5 (Fig. 3, bottom panel) is very similar to that of the standard model. However the different evolutionary sequences allowed in this model enable the formation of more progenitors involving hybrid WD donors. The mass distributions for the helium star and hybrid WD channels peak between 0.85 and $0.9 M_{\odot}$ (total WD mass 0.95 – $1 M_{\odot}$), which is a noteworthy feature, especially if these systems are shown to contribute to the population of SNe Ia of ‘normal’ brightness (Sim et al. 2010).

5 DISCUSSION

Recent hydrodynamic explosion simulations of sub- M_{Ch} CO WDs (Fink et al. 2010) coupled with detailed nucleosynthesis and radiative transfer modelling (Kromer & Sim 2009) have revealed that sub- M_{Ch} mass SN Ia models exhibit features which are characteristically similar to those observed in SNe Ia (Kromer et al. 2010; Sim et al. 2010). Motivated by these new findings, as well as population synthesis rate estimates, we have investigated sub- M_{Ch} SN Ia formation channels and have calculated and presented the DTD and rates of their progenitors for three different parametrizations of the CE phase.

We find that only the sub- M_{Ch} progenitor channel is able to simultaneously

- (i) reproduce the observed rates for our standard model;
- (ii) provide an elegant explanation for the variety among SN Ia light curves (mass of exploding WD);
- (iii) naturally provide a system which is devoid of hydrogen;
- (iv) produce a DTD with distinct prompt ($\lesssim 500$ Myr) and delayed ($\gtrsim 500$ Myr) components, originating from two channels with very different evolutionary time-scales.

We think that this last point is one of the most interesting, considering the recent observational studies by different groups who have found evidence for such a DTD (Brandt et al. 2010; Maoz & Badenes 2010; Maoz et al. 2011).

5.1 Double WD mergers: implications

We note both the works of Guillochon et al. (2010), who investigated detonations in sub- M_{Ch} CO WDs undergoing rapid accretion during dynamically unstable mass transfer from a helium-rich WD companion, and van Kerkwijk, Chang & Justham (2010), who also considered mergers of WDs with a total mass below M_{Ch} as possible progenitors of SNe Ia. In this study we do not investigate sub- M_{Ch} mergers in detail though we briefly comment on them here. We find that the number of sub- M_{Ch} WD mergers in our standard model (considering all mergers where at least one WD is CO-rich, the other being CO and/or helium-rich) is nearly twice that of DDS mergers. While it is generally believed that a WD merger with a total mass below the Chandrasekhar mass limit would not lead to a SN Ia explosion, these mergers should produce other interesting objects; R Coronae Borealis stars are one example (Webbink 1984; Iben, Tutukov & Yungelson 1996; Clayton et al. 2007, see also Bogomazov & Tutukov 2009) and these types of merger events may be visible in upcoming transient surveys. If we make a constraint similar to that of van Kerkwijk et al. (2010) counting both sub- M_{Ch} and super- M_{Ch} WD mergers between CO WDs with near-equal masses, we find that the number of mergers drops to ~ 42 per cent of our standard model DDS rate, which is slightly too low to explain all SNe Ia.

In our models we have assumed the commonly adopted initial binary orbital configurations for population synthesis studies: i.e. initial separation flat in the logarithm (more binaries born on closer orbits relative to large orbits), and thus the ZAMS distribution of all semilata recta are the same for all three CE models. However, we find that for the low-CE efficiency case (Model A.125), DDS SNe Ia progenitors are only formed from systems with initial (ZAMS) orbital configurations which have rather large semilata recta compared to those for our standard model. In Model A.125, systems which would have made DDS SNe Ia in Model A1 merge too early, and never make double WDs. It was already mentioned in Hurley, Tout & Pols (2002) that the initial distribution of orbital separations in population synthesis studies should be distributed according to the (observed) distribution of semilata recta rather than semimajor axes or orbital periods alone. We note here that an initial distribution geared towards higher semilata recta than is canonically assumed would serve to augment the number of progenitors in models with low CE efficiency, making those DDS rates closer to those of observations.

While the predicted rates of the DDS for our models do not conflict with observations, these systems are theoretically expected to produce neutron stars via AIC. If this were the case, the AIC rate from the AIC-merger channel alone would be $\sim 10^{-3} \text{ yr}^{-1}$ for the Galaxy. We find the STARTRACK AIC rate from the ‘RLOF-AIC’ channel is a factor of 10 to 100 less: no more than 10^{-4} yr^{-1} for

our standard model. This rate is in agreement with the upper limit estimate derived from Solar system abundances of neutron-rich isotopes, which are expected to be produced in AICs (Fryer et al. 1999; Metzger, Piro & Quataert 2009). However, if (i) population synthesis estimates for the number of merging CO+CO WDs with a mass above M_{Ch} are correct and (ii) in most environments these mergers preferentially produce AICs and not SNe Ia, then this could potentially be in conflict with the predicted abundance of neutron-rich isotopes in the solar neighbourhood. However, modelling of AIC events, be it the ‘RLOF’ or ‘merger’ case, is still in its infancy, and many uncertainties remain (Dessart et al. 2006, 2007; Metzger et al. 2009, see also Darbha et al. 2010). If one can say for certain that AIC events formed from the merger of CO WDs produce very neutron-rich ejecta, then this provides a potentially strong constraint on the outcome of these mergers; namely that a non-negligible fraction of SNe Ia must be formed through the DDS channel. On the other hand, it is possible that population synthesis calculations overpredict the number of merging CO+CO WDs, which would also present an interesting problem for the binary evolution community, and may challenge the idea that the observed $\sim t^{-1}$ power-law DTD of SNe Ia originates from double WD mergers alone.

5.2 Further remarks on delay times

The t^{-1} power-law shape found in the delay time study of Totani et al. (2008) implies that the majority of progenitors in elliptical-like galaxies originate from binaries for which the DTD is most strongly governed by the time-scale associated with gravitational wave radiation, thus these progenitors are likely to be DDS mergers (see Section 3 of Paper I). However, we point out that a change in the CE removal efficiency α_{ce} will have an effect on the amount of time between the last CE phase and final contact, thus affecting the shape (and perhaps to some degree the relevance) of the delayed DTD component. Similar to the DDS, our study has shown that the sub- M_{Ch} model DTD (e.g. Model A1) also exhibits a power law for delay times > 1 Gyr. This is not surprising, since the helium WD sub- M_{Ch} progenitors also spend an appreciable time as detached compact stars evolving to contact solely under the influence of gravitational radiation. However, the DTD of the sub- M_{Ch} channel falls off more steeply than the t^{-1} power-law fit of Totani et al. (2008) and the $t^{-1.1}$ to $t^{-1.3}$ power-law fits of Maoz et al. (2010), matching quite well to t^{-2} (Fig. 2). Thus, when comparing our results to observationally derived DTDs, the DDS channel matches more closely than the sub- M_{Ch} channel. However, a very recent study of Subaru/XMM-Newton Deep Survey (SXDS) SNe Ia indicates that the DTD may be well fitted by a power law of $t^{-1.5}$ (J. Okumura, private communication). It is of course possible that both DDS and sub- M_{Ch} progenitors contribute substantially to the SN Ia population, potentially yielding a DTD of functional form somewhere in between t^{-1} and t^{-2} above 1 Gyr, which would still be in agreement with the majority of recent observations.

5.3 Sub-Chandrasekhar SNe Ia connection to AM CVn stars and Ia events

Based on the observed local space density estimate of AM CVn binaries⁶ performed by Roelofs et al. (2007), Bildsten et al. (2007)

⁶ We would like to make the reader aware of the fact that population synthesis studies overpredict the number of AM CVn binaries in general compared to the observational results of Roelofs, Nelemans & Groot (2007) (e.g.

have calculated the occurrence rate of the final (explosive) helium flash from ‘.Ia’ systems in a typical E/S0 galaxy with a mass of $10^{11} M_{\odot}$ to be $(7-20) \times 10^{-5} \text{ yr}^{-1}$; i.e., 2–6 per cent of the SN Ia rate in E/S0 galaxies. .Ia events are expected to be about one tenth as bright as normal SNe Ia. Since our sub- M_{Ch} progenitors could also potentially lead to .Ia-like (and not SN Ia) explosions, we think it is useful to independently estimate the occurrence rate for such explosions in our standard (A1) model for similar (E/S0 galaxy) conditions. It was already found in Section 4.2 that the sub- M_{Ch} rate assuming a burst of star formation at $t = 0$ is $\sim 4 \times 10^{-3}$ at 10 Gyr. We find that among old stellar populations our double-detonation thermonuclear explosions will be roughly 30 times more frequent than the estimated .Ia explosion rate of Bildsten et al. (2007).

We also note that in the study of Bildsten et al. (2007) it was found that the ignition mass of the helium shell in .Ias varies as a function of the underlying CO core mass and the rate of accretion. However for our first investigation of double-detonation sub- M_{Ch} SNe we have used a more simplified model in which the ignition mass is always the same ($0.1 M_{\odot}$). The consequences of this on the resulting SN rate are not expected to be too drastic, as the time-scale for the helium accretion is relatively short compared to the evolutionary lifetime of the progenitors. This is particularly true for the helium WD donor case, which is the scenario most relevant for Bildsten et al. (2007).

Yoon & Langer (2004) found that rotation may pose a problem for the initiation of a detonation in accreted helium shells. They found that the spin-up of the WD due to the accretion and resulting dissipation due to differential rotation might cause helium flashes to occur for lower shell masses, which may lead to inhibition of a detonation in turn resulting in fewer sub- M_{Ch} SNe. However, Piro & Bildsten (2004) found that the accreted material will be brought into co-rotation with the WD already at low depths within the helium shell, and so as noted in Bildsten et al. (2007) rotation should not play a significant role in the heating of the helium shell and subsequent helium ignition.

5.4 The link between sub-Chandrasekhar SNe Ia and their progenitors

While it is useful to understand how the host galaxy environment influences the SN ejecta/observables, it is also fundamentally important to find a direct physical connection between the progenitor population and the observational characteristics of SNe Ia. For some time it has been known that brighter SNe Ia occur more frequently among young stellar populations (Hamuy et al. 1995). Could it be possible that sub- M_{Ch} SNe Ia arising from the (prompt) helium star channel are brighter than those from the double WD channel? This may be the case particularly considering Model A1, where the core mass of the exploding star for the helium star channel is on average slightly larger than for the double-WD channel (see Fig. 3), and thus is likely to produce more ^{56}Ni . We also note that for both Model A1 and Model G1.5, ~ 70 per cent of progenitors with delay times < 1 Gyr have CO WD masses $> 1.0 M_{\odot}$ (CO core masses $> 0.9 M_{\odot}$), while this fraction is only ~ 45 to 50 per cent for progenitors with delay times > 3 Gyr. However there is no *strong* trend in our models which indicates that more massive WDs explode among younger populations. The majority of the sub- M_{Ch} binaries are double WDs,

Nelemans et al. 2001a; Paper I). There may be several factors which conspire to cause the apparent difference, though the various possibilities are not explored in this study.

and the MS lifetime (ZAMS mass) of the primary star does not play a dominant role in setting the delay time.

Another point worth considering is that the helium star channel progenitors undergo two CE events on a relatively short time-scale compared to the time the stars spend as a post-MS detached binary. Thus these binary systems should be hotter and may be more readily detectable than their (colder, longer lived) double-WD counterparts. Since these helium star channel SNe in our models are expected to occur a few Myr after the last CE phase, the detection of such an explosion will probably not be inhibited by circumstellar matter from the companion. However, since these explosions involving helium stars are expected to be found among young stellar populations, they are likely to occur in regions of active star formation where their detection may be thwarted by the presence of dust and possibly circumstellar matter from nearby stellar systems. The binary progenitors of the helium WD channel on the other hand, although more abundant at most delay times, should be harder to detect as most of their evolutionary time is spent during the detached phase.

Thus far, we have only found (possibly) a weak correlation between the mass of the exploding WD and delay time, making it difficult to infer a connection between observed brightness (^{56}Ni synthesized in the explosion) and progenitor age. Nevertheless, if a connection between the age of the primary CO WD and the production of ^{56}Ni can be made in sub- M_{Ch} explosions such that dimmer SNe Ia occur among older populations, this would have very exciting consequences for our study.

5.5 Conclusion

Our standard model population synthesis indicates that there are potentially enough sub- M_{Ch} progenitors to account for the rates of SNe Ia. Nevertheless, much uncertainty still remains regarding the formation and evolution of close binary stars: mass transfer and accretion efficiencies, effects of rotation and magnetic fields, impact of metallicity on stellar winds and subsequent stellar and binary evolution, the CE phase, etc. Even given a large population of potential progenitors for sub- M_{Ch} explosions, there remain open questions about the explosion itself. Hydrodynamical studies have previously shown that sub- M_{Ch} WDs with an overlying helium shell can undergo a double-detonation which looks like a SN Ia, though the real answer as to what fraction of these systems lead to SNe Ia explosions depends on specific details. Most critically, under exactly which conditions does helium ignition occur, and how does the nucleosynthesis proceed?

The sub- M_{Ch} model is the first model which demonstrates a sufficient number of SNe Ia events to account for all, or at least some substantial fraction of, SNe Ia (Model A1), as well as *two distinct formation channels with their own characteristic DTD*: a prompt (<500 Myr) helium star channel originating from binaries with more massive secondaries, and a more delayed (>500 Myr) double WD channel originating from AM CVn-like progenitor binaries with lower mass. Whether some or all of the sub- M_{Ch} models explored in this work really lead to thermonuclear explosions that look like normal (or some subclass of) SNe Ia is still a topic which requires further study.

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