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Measuring financial synergies in cross-border M&A transactions using diffusion processes

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Abstract: In a cross-border M&A framework, the question and measurement of financial synergy can be important in the analysis of the transaction and consideration needs to be given to whether the specific cross-border financial risks outweigh operational synergies. This paper develops a diffusion model to explore a set of optimal capital structures of the acquiring firm, target firm and merged firm. Differential taxes, bankruptcy costs, interest rate risk and foreign exchange risk are considerations of the model. The model results in a measure of pure financial synergy. Further, capital structure can impact on the structure of the offer and the model allows for the determination of an optimal stock exchange ratio. Worked examples reveal that the cross-border M&A of two symmetric firms can result in negative financial synergy, whereas the merger of two asymmetric firms can lead to positive financial synergy.

Keywords: cross-border M&A; financial synergies; mergers; stock exchange ratio; capital structure.

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1 Introduction

The liberalisation of restrictions to international capital flows and the trend towards an integrated world economy have led to increased activity in the global market for cross-border mergers and acquisitions. Such M&A activity was especially prevalent during the latter half of the 1990s which saw major icon brands and companies change ownership and structure. Many of these mergers and acquisitions have been expansionary in which firms have sought scale and access to global markets. Examples include Deutsche Telekom (Germany) which acquired One2One (USA) for a cost of US$13.629 billion in 1999; Deutsche Bank (Germany) which acquired Bankers Trust New York Corp. (USA) with a US$9.082 billion offer in 1998; Texas Utilities Co. (USA) which acquired Energy Group PLC (UK) for US$10.947 billion in 1998 and Daimler-Benz (Germany) which acquired Chrysler (USA) for US$40.467 billion in 1998.

There is a considerable body of literature on M&A activity generally, however research on international aspects of M&As, while growing, has not been as voluminous. Much of the existing research on international M&As has been conducted within the entry mode stream, wherein the M&A is considered as either a strategic alternative to an alliance or a greenfield investment.[1] Focarelli and Pozzolo (2001) explore the pattern of cross-border M&As in the banking industry relative to the non-financial sector and investigate which factors make it more likely that a bank will expand its activities abroad. Rossi and Volpin (2002) study the determinants of M&As around the world during the 1990s by focusing on differences in laws and enforcement across countries. They find that the volume of M&A activity and the premium paid are significantly greater in countries with better investor protection.

In addition to the risks associated with a standard M&A transaction, an international M&A transaction involves further considerations. In addition to the business and management risks that partly depend on cultural differences and political environments, there are additional financial matters that need to be considered. These include interest rate differentials and exchange rate factors. These risks can be substantial, and if not considered properly they have the potential to outweigh gains from the transaction arising elsewhere. This study focuses on these issues and constructs a dynamic diffusion model to analyse a cross-border M&A.

More specifically, in any M&A transaction, there are two main types of synergy, namely operating synergy and financial synergy. Operating synergy generally comes in three forms from revenue enhancements, cost reductions and productivity gains. Revenue enhancements may result from enlarged market share or expanded product lines. Cost-reducing synergies are common sources of gain, and often revolve around achieving lower labour costs or economies of scale. Productivity gains may result form technological or process improvements that can be leveraged over the broader base of the combined entity. Gupta and Gerchak (2001) construct a dynamic-programming real-option model to quantify operating synergies. They show how size and flexibility of capacity, and larger or less variable demand patterns interact to produce different levels of operational synergy in a typical M&A.

In contrast, financial synergy refers to the impact of the transaction on the costs of capital to the firm. A positive effect may be obtained if the firms' cash flow streams are not perfectly correlated. Thus, if the acquisition or merger lowers the volatility of the cash flows, suppliers of capital may consider the firm less risky. Higgins and Schall (1975) explain this result as the debt coinsurance effect wherein the benefits accrue to debtholders at the expense of shareholders. Another financial synergy may arise from an enhanced debt capacity of the combined firm, and tax savings. Leland (2005) considers multiple activities with imperfectly correlated cash flows and zero operational synergies and provides some closed-form measures of financial synergies of combining activities. In the presence of corporate taxes and bankruptcy costs, mergers can realise positive financial synergies due to reduced risk and the potential for greater leverage, but at the cost of losing separate capital structure choices and separate limited liability.

In a cross-border M&A, foreign exchange risk becomes a further consideration, as the comparative strength of the foreign currency can impact on cross-border debt coinsurance. Further, differential interest rates and differential tax and bankruptcy regulations may affect (expected) cross-border synergies. Thus, a model that incorporates these factors such that the pure cross-border financial synergy can be measured is a valuable contribution to both theory and practice.

A related issue is the optimal stock exchange ratio when stock is used as the compensation in a cross-border M&A transaction. The model of Larson and Gonedes (1969) is one of the first developments to consider the optimal stock exchange ratio. However, this model is static and does not consider the capital structure of the firms either pre or post-merger. A more realistic model would incorporate the stochastic features of interest rates.

This paper develops a diffusion model to quantify the pure financial synergy of a cross-border M&A transaction and provides for an optimal stock exchange ratio. The model first constructs the optimal capital structures of the acquiring firm, target firm and merged firm, where the interest rate and foreign exchange risks are taken into consideration. Based on the model, we provide the closed-form formulas to measure both the pure financial synergy and the optimal stock exchange ratio. Numerical examples show that the cross-border M&A of two symmetric firms can result in a negative financial synergy, whereas that of two asymmetric firms can lead to a positive financial synergy.

The remainder of this paper is organised as follows. Section 2 describes the framework for the model including background assumptions, while the model derivation is provided in Section 3. Section 4 provides some numerical examples and Section 5 concludes this paper.

2 The framework

Leland (1994) first utilises a diffusion approach to analysing the firm's optimal capital structure with infinite maturity debt. Leland and Toft (1996) further develop the model to allow for finite maturity debt via a stationary debt structure. Recently, Leland (2005) has developed a simple diffusion model which has direct applications to mergers and spin-offs, and provides a rationale for structured finance techniques such as asset securitisation and project finance. Ju and Ou-Yang (2005) provide a model where the optimal capital structure and debt maturity of a firm are jointly determined in the stochastic interest rate environment of Vasicek (1977).

In our model, cross-border M&As are concerned with exchange rate risk, and therefore, we construct a dynamic model which takes five kinds of risk factors into consideration, including the risks of unlevered asset values of both acquiring and target firms, the interest rate risks faced by both acquiring and target firms and the exchange rate risk. Without loss of generality, the model is initially built in a domestic and foreign risk-neutral framework, although the risk-neutral assumption is not necessary.[2] The assumptions of the model are explained below.

The domestic firm is taken as the acquiring firm which intends to target a foreign firm in a stock exchange transaction.

Assumption 1: *The dynamics of the unleveraged asset value of the domestic acquiring firm under domestic risk neutrality are given as:*

$$\frac{dV_d(t)}{V_d(t)} = r_d(t)dt + {}_{V_d} dW_{V_d}^Q(t), \text{ given } V_d(0) \equiv V_d \tag{1}$$

where $r_d(t)$ is the instantaneous domestic risk-free short rate, σ_{V_d} is the constant volatility of the rate of return of the unlevered asset of the domestic firm, and $W_{V_d}^Q$ is a Wiener process under domestic risk-neutral filtered probability space.

The domestic risk-free interest rate is stochastic and is represented by the domestic zero-coupon bond price.

Assumption 2: *The dynamics of the price of the domestic zero-coupon bond matured at T under domestic risk neutrality are given as:*

$$\frac{dB_d(t,T)}{B_d(t,T)} = r_d(t)dt + \sigma_{B_d} dW_{B_d}^Q(t), \text{ given } B_d(0,T) \tag{2}$$

where σ_{B_d} is the constant volatility of the rate of return of the domestic zero-coupon bond, and $W_{B_d}^Q$ is another Wiener process under domestic risk-neutral filtered probability space.

Equation (2) shows that the domestic zero-coupon bond value is log-normal distributed, implying the domestic interest rate is normal distributed. Varying σ_{B_d} would result in different short rate models, such as Vasicek (1977).[3] Note that the maturity of the bond could vary. For notational simplicity, we assume all the rates of returns of the zero-coupon bonds have the same volatility and are perfectly correlated.

Assumption 3: *The dynamics of the exchange rate (in units of foreign currency) under domestic risk neutrality are given as:*

$$\frac{dX(t)}{X(t)} = \left(r_d(t) - r_f(t)\right)dt + \sigma_X dW_X^Q(t), \text{ given } X(0) \equiv X \tag{3}$$

where $r_f(t)$ is the instantaneous foreign risk-free short rate, σ_X is the constant volatility of the rate of return of the exchange rate, and W_X^Q is Wiener process under domestic risk-neutral filtered probability space.

The target firm is assumed to be foreign.

Assumption 4: *The dynamics of the unleveraged asset value of the foreign target firm under foreign risk-neutrality are given as follows:*

$$\frac{dV_f(t)}{V_f(t)} = r_f(t)dt + \sigma_{V_f} dW_{V_f}^{Q_f}(t), \text{ given } V_f(0) \equiv V_f \tag{4}$$

where σ_{V_f} is the constant volatility of the rate of return of the unleveraged asset of the foreign firm, and $W_{V_f}^{Q_f}$ is a Wiener process under foreign risk-neutral filtered probability space.

Assumption 5: *The dynamics of the price of the foreign zero-coupon bond matured at T under foreign risk neutrality are given as:*

$$\frac{dB_f(t,T)}{B_f(t,T)} = r_f(t)dt + \sigma_{B_f} dW_{B_f}^{Q_f}(t), \text{ given } B_f(0,T) \tag{5}$$

where σ_{B_f} is the constant volatility of the rate of return of the foreign zero-coupon bond matured at T, and $W_{B_f}^{Q_f}$ is a Wiener process under foreign risk-neutral filtered probability space.[4]

Following Assumptions 1–5, there are five kinds of risk factors in the framework which are mutually correlated. The instantaneous correlation coefficients are assumed to be constant, and are defined by: $\rho_{ij}dt = E^Q\left[dW_i^Q(t)dW_j^Q(t)\right]$ where $i,j = V_d, B_d, X, V_f$ and B_f.

Assumption 6: *Assume there would be zero operating synergy, that is, $V_m(t) = V_d(t) + V_f(t)X(t)$, for all $t \geq 0$, where V_m is the unlevered asset value of the merger firm in units of domestic currency, and the initial time zero denotes the time that the merger is determined.*

The unlevered asset value of the firm can be viewed as the after-tax operational value of the firm. Consequently, zero operating synergy would imply that $V_m(t) = V_d(t) + V_f(t)X(t)$, for all $t \geq 0$. Using this result we can then focus solely on the financial synergy and evaluate the pure financial synergy of a cross-border M&A.

From Assumptions 1, 3 and 4, one can see that the unlevered asset value of the merged firm is not lognormally distributed. The following assumption is therefore used to keep the model tractable.

Assumption 7: *Assume the dynamics of the unleveraged asset value of the acquiring firm under domestic risk-neutrality are as follows:*

$$\frac{dV_m(t)}{V_m(t)} = w_d \frac{dV_d(t)}{V_d(t)} + w_f \frac{d\left(V_f(t)X(t)\right)}{V_f(t)X(t)}, \text{ given } V_m(0) \equiv V_m = V_d + V_f X \tag{6}$$

where $w_d = V_d / V_m$ *and* $w_f = V_f X / V_m$ *are two known constants.*

From Assumptions 1, 3, 4 and 7, we can rewrite Equation (6) as

$$\frac{\mathrm{d}V_m(t)}{V_m(t)} = r_d(t)\mathrm{d}t + \sigma_{V_m} \mathrm{d}W_{V_m}^Q(t) \tag{7}$$

where

$$\sigma_{V_m} = \sqrt{\begin{aligned}&\left(w_d\sigma_{V_d}\right)^2 + \left(w_f\sigma_{V_f}\right)^2 + \left(w_f\sigma_X\right)^2 + 2w_d\sigma_{V_d} w_f\sigma_{V_f}\rho_{V_dV_f}\\ &+2w_d\sigma_{V_d} w_f\sigma_X\rho_{V_dX} + 2w_f\sigma_{V_f} w_f\sigma_X\rho_{V_fX}\end{aligned}}$$

To simplify notation, all the diffusion terms (the return volatilities) of Equations (1)–(5) and (7) are assumed to be constant. It is worth noting that those volatilities could be time-dependent and deterministic. Only the non-stochastic nature of the diffusion terms are required to derive valuation formulas in closed form.[5]

Assumption 8: *Assume both the domestic and foreign firms have a debt/liability, respectively, with principal value P_i, discrete constant coupon flows C_{i,t_j}, and maturity T_i, where $i = d, f$ and $j = 1, \ldots, n_i$. Acquisitions by merger result in combinations of the assets and liabilities of acquired and acquiring firms.*

Assumption 8 is the simplification of the debt structure of Leland and Toft (1996), which is also used to keep the model tractable. The discrete coupon payment setting coincides with practice since the firm usually pays coupons on a semi-annual or annual basis. In what follows, we will derive the optimal capital structure models for the domestic firm, foreign firm and a merged firm.

3 Derivation of the model

The stochastic nature of the interest rate complicates the model. To deal with stochastic interest rates, the change of measure (numeraire) technique is utilised to derive valuation formulas in closed form.

3.1 Preliminary development

Before we present the model, some preliminary discussion is required. First, define the first passage time $\tau_i^B \equiv \inf\{0 < t \le T_i : V_i(t) \le B_i(t,T)V_i^B\}$, where $i = d, f,$ and m, and V_i^B is a constant. In other words, τ_i^B is the first time where the unlevered asset value $V_i(t)$ goes down and touches the stochastic default barrier $B_i(t,T)V_i^B$. That is, τ_i^B is the stochastic time that the firm i decides to announce bankruptcy. Equivalently, τ_i^B could be viewed as the first time that the forward unlevered asset value $V_i(t,T)/B_i(t,T)$ hits V_i^B. Note that until now, V_i^B is an exogenously given constant, but it would be endogenously determined by the shareholders later. According to the technique of changing measure, there is a well-known result which plays an important role in our model, provided in the following.

Lemma 1: *Using* $B_d(t,T)$ (or $B_f(t,T)$) *as the numeraire, all the forward asset values in units of domestic (foreign) currency would be martingale processes under domestic (foreign) forward risk-neutral probability measure.*

Corollary 1: *According to Assumptions 1–7 and Lemma 1, together with Itô's Lemma, the processes that we need to derive the formulas are summarised in the following.*

$$d\ln\left(\frac{V_d(t)}{B_d(t,T)}\right) = -0.5\sigma^2_{V_d/B_d}\,dt + \sigma_{V_d/B_d}\,dW^{B_d}_{V_d/B_d}(t) \tag{8}$$

$$d\ln\left(\frac{V_f(t)}{B_f(t,T)}\right) = -0.5\sigma^2_{V_f/B_f}\,dt + \sigma_{V_f/B_f}\,dW^{B_f}_{V_f/B_f}(t) \tag{9}$$

$$d\ln\left(\frac{V_m(t)}{B_d(t,T)}\right) = -0.5\sigma^2_{V_m/B_d}\,dt + \sigma_{V_m/B_d}\,dW^{B_d}_{V_m/B_d}(t) \tag{10}$$

and

$$d\ln\left(\frac{V_m(t)}{B_d(t,T)}\right) = \left(\mu_{V_m/B_d} - 0.5\sigma^2_{V_m/B_d}\right)dt + \sigma_{V_m/B_d}\,dW^{B_f}_{V_m/B_d}(t) \tag{11}$$

where

$$\sigma_{V_d/B_d} = \sqrt{\sigma^2_{V_d} + \sigma^2_{B_d} - 2\sigma_{V_d}\sigma_{B_d}\rho_{V_dB_d}}$$

$$\sigma_{V_f/B_f} = \sqrt{\sigma^2_{V_f} + \sigma^2_{B_f} - 2\sigma_{V_f}\sigma_{B_f}\rho_{V_fB_f}}$$

$$\sigma_{V_m/B_d} = \sqrt{\sigma^2_{V_m} + \sigma^2_{B_d} - 2\sigma_{B_d}\left(w_d\sigma_{V_d}\rho_{V_dB_d} + w_f\sigma_{V_f}\rho_{V_fB_d} + w_f\sigma_X\rho_{XB_d}\right)}$$

and

$$\begin{aligned}\mu_{V_m/B_d} &= \sigma_X\left(w_d\sigma_{V_d}\rho_{V_dX} + w_f\sigma_{V_f}\rho_{V_fX} - (1-w_f)\sigma_X - \sigma_{B_f}\rho_{XB_f}\right)\\ &\quad + \sigma_{B_f}\left(w_d\sigma_{V_d}\rho_{V_dB_f} + w_f\sigma_{V_f}\rho_{V_fB_f} - (1-w_f)\sigma_X\rho_{XB_f} - \sigma_{B_f}\right)\\ &\quad - \sigma_{B_d}\left(w_d\sigma_{V_d}\rho_{V_dB_d} + w_f\sigma_{V_f}\rho_{V_fB_d} - (1-w_f)\sigma_X\rho_{XB_d} - \sigma_{B_f}\rho_{B_dB_f}\right)\end{aligned}$$

It is worth explaining the relation between Equations (10) and (11). Equation (10) demonstrates the forward unlevered asset value of the merged firm in units of domestic currency under a domestic forward risk-neutral probability measure, which is a martingale by Lemma 1. Alternatively, Equation (11) shows that under a foreign forward risk-neutral probability measure, the drit term accounts for the risk-adjusted premium of the forward unlevered asset domestic value of the merged firm from the perspective of foreign investors.

In our framework, the distribution of the first passage of time τ_i^B under domestic and foreign forward martingale measures is important. The distribution of the stopping time of a standard Brownian motion with drit and diffusion can be found by Harrison (1985), and the result is summarised below.

Lemma 2: *Let* $d\ln\tilde{B}(t) = adt + bdB(t)$ *and* $t = inf\left(t > 0 : \tilde{B}(t) \le c\right)$, *where a, b and c are three positive constants, and B is a standard Brownian motion, and then*

$$F\left(t;a,b,c\right) \equiv \Pr\left(\tau > t\right) = N\left(\frac{\ln\left(\tilde{B}(0)/c\right)+at}{b\sqrt{t}}\right) - \left(\frac{\tilde{B}(0)}{c}\right)^{-2a/b^2} N\left(\frac{-\ln\left(\tilde{B}(0)/c\right)+at}{b\sqrt{t}}\right)$$

where $N(\cdot)$ *is the cumulative normal distribution.*

Corollary 2: *By Corollary 1 and Lemma 2, the distributions of the first-passage times* $\tau_d^B = inf(0 < t \le T_d : V_d(t)/B_d(t,T) \le V_d^B)$ $\tau_f^B = inf\left(0 < t \le T_f : V_f(t)/B_f(t,T) \le V_f^B\right)$ $\tau_m^B = inf\left(0 < t \le T_d : V_m(t)/B_d(t,T) \le V_m^B\right)$ $\tau_m^B = \inf\left(0 < t \le T_d : V_m(t)/B_d(t,T) \le V_m^B\right)$ *with respect to the processes of Equations (8)–(11) can be derived.*

Due to stochastic interest rates, the risk-neutral pricing method is changed to the forward risk-neutral pricing method. The pricing methodology is summarised by the following.

Lemma 3 (forward risk-neutral pricing): *Any domestic (foreign) contingent claim* Y_d *(*Y_f*) under stochastic interest rates can be priced by changes of probability measure as below.*[6]

$$Y_d(0) = E^Q\left[exp\left(-\int_0^T r(s)ds\right)Y_d(T)\right] = B_d(0,T)E^{B_d}\left[Y_d(T)\right]$$

and

$$XY_f(0) = XE^{Q_f}\left[exp\left(-\int_0^T r_f(s)\mathrm{d}s\right)Y_f(T)\right] = XB_f(0,T)E^{B_f}\left[Y_f(T)\right]$$
$$= B_d(0,T)E^{B_d}\left[X(T)Y_f(T)\right]$$

3.2 Optimal capital structure

To derive the optimal capital structures for the (domestic) acquiring firm, (foreign) target firm and the merged firm, we first price their outstanding debt. For the acquiring firm, by Lemma 3, the forward risk-neutral pricing method, the debt value can be expressed as:

$$\begin{aligned} D_d\left(V_d^B\right) = & \mathrm{E}^{B_d}\left[B_d\left(0,T_d\right)1_{\{\tau_d^B>T_d\}}P_d\right]+\sum_{i=1}^{n_d}\mathrm{E}^{B_d}\left[B_d\left(0,t_{d,i}\right)1_{\{\tau_d^B>t_{d,i}\}}C_{d,i}\right] \\ & +\mathrm{E}^{B_d}\left[B_d\left(0,\tau_d^B\right)\left(1-\alpha_d\right)V_d^B\left(\tau_d^B\right)1_{\{\tau_d^B\leq T_d\}}\right] \end{aligned} \qquad (12)$$

The first term of Equation (12) shows the expected present value of the par value in the absence of bankruptcy and before the maturity. The second term demonstrates the expected present value of the cumulative coupon payments in the absence of default. The last term is expected present value of the firm's scrap value if the firm files for bankruptcy, where $(1-\alpha_d)V_d^B(\tau_d^B)$ is the scrap value at the time of default and α_d, a constant, represents proportional bankruptcy costs.

The total firm value of the acquiring firm can be expressed as follows.

$$\begin{aligned} v_d\left(V_d^B\right) = & V_d+\sum_{i=1}^{n_d}\mathrm{E}^{B_d}\left[B_d\left(0,t_{d,i}\right)1_{\{\tau_d^B>t_{d,i}\}}\delta_d C_{d,i}\right] \\ & -\mathrm{E}^{B_d}\left[B_d\left(0,\tau_d^B\right)\alpha_d V_d^B\left(\tau_d^B\right)1_{\{\tau_d^B\leq T_d\}}\right] \end{aligned} \qquad (13)$$

From Equation (13), the total firm value is equal to the initial unlevered asset value plus the expected present value of the cumulative tax benefits, minus the expected present value of the bankruptcy costs, where δ_d is the domestic corporate tax rate. Following the accounting identity, the equity value of the domestic firm is equal to the total firm value minus the debt value. The explicit expression of the debt value, total firm value and equity value are provided by the following.

Proposition 1: *The debt value, total firm value and equity value of the domestic firm are given as:*

$$\begin{aligned} D_d\left(V_d^B\right) = & B_d\left(0,T_d\right)\left(P_d-\left(1-\alpha_d\right)V_d^B\right)F\left(T_d;-0.5\sigma_{V_d/B_d}^2,\sigma_{V_d/B_d},V_d^B\right) \\ & +\sum_{i=1}^{n_d}B_d\left(0,t_{d,i}\right)C_{d,i}F\left(t_{d,i};-0.5\sigma_{V_d/B_d}^2,\sigma_{V_d/B_d},V_d^B\right)+B_d\left(0,T_d\right)\left(1-\alpha_d\right)V_d^B \end{aligned}$$

$$\begin{aligned} v_d\left(V_d^B\right) = & V_d+\sum_{i=1}^{n_d}B_d\left(0,t_{d,i}\right)\delta_d C_{d,i}\ F\left(t_{d,i};-0.5\sigma_{V_d/B_d}^2,\sigma_{V_d/B_d},V_d^B\right) \\ & -B_d(0,T_d)\alpha_d V_d^B\left(1-F\left(T_d;-0.5\sigma_{V_d/B_d}^2,\sigma_{V_d/B_d},V_d^B\right)\right) \end{aligned}$$

$$\begin{aligned} v_d\left(V_d^B\right) = & V_d+\sum_{i=1}^{n_d}B_d\left(0,t_{d,i}\right)\delta_d C_{d,i}\ F\left(t_{d,i};-0.5\sigma_{V_d/B_d}^2,\sigma_{V_d/B_d},V_d^B\right) \\ & -B_d(0,T_d)\alpha_d V_d^B\left(1-F\left(T_d;-0.5\sigma_{V_d/B_d}^2,\sigma_{V_d/B_d},V_d^B\right)\right) \end{aligned}$$

$$E_d\left(V_d^B\right)=v_d\left(V_d^B\right)-D_d\left(V_d^B\right)$$

Proof: By Corollary 1 and Lemma 2.

Similarly, the debt value, total firm value and equity value of the foreign target firm, in units of foreign currency, can be expressed as follows.

Proposition 2: *The debt value, total firm value and equity value of the foreign target firm are given as:*

$$D_f\left(V_f^B\right)=B_f\left(0,T_f\right)\left(P_f-(1-\alpha_f)V_f^B\right)F\left(T_f;-0.5\sigma_{V_f/B_f}^2,\sigma_{V_f/B_f},V_f^B\right)$$
$$+\sum_{i=1}^{n_f}B_f\left(0,t_{f,i}\right)C_{f,i}F\left(t_{f,i};-0.5\sigma_{V_f/B_f}^2,\sigma_{V_f/B_f},V_f^B\right)+B_f\left(0,T_f\right)(1-\alpha_f)V_f^B$$

$$v_f\left(V_f^B\right)=V_f+\sum_{i=1}^{n_f}B_f\left(0,t_{f,i}\right)\delta_f C_{f,i}\ F\left(t_{f,i};-0.5\sigma_{V_f/B_f}^2,\sigma_{V_f/B_f},V_f^B\right)$$
$$-B_f(0,T_f)\alpha V_f^B\left(1-F\left(T_f;-0.5\sigma_{V_f/B_f}^2,\sigma_{V_f/B_f},V_f^B\right)\right)$$

$E_f(V_f^B)=v_f(V_f^B)-D_f(V_f^B)$, *where* δ_f *is the foreign corporate tax rate.*

Proof: By Corollary 1 and Lemma 2.

To assist with simplicity and without loss of generality, we further assume $T_f<T_d$. Further, the proportional bankruptcy of the target firm is a value-weighted combination of the individual bankruptcy costs, that is, $\alpha_m=w_d\alpha_d+w_f\alpha_f$.

Thus, the combined debt value of the merged firm in units of domestic currency can be expressed as:

$$D_m\left(V_m^B\right)=\mathrm{E}^{B_d}\left[B_d\left(0,T_d\right)1_{\{\tau_m^B>T_d\}}P_d\right]+X\mathrm{E}^{B_f}\left[B_f\left(0,T_f\right)1_{\{\tau_m^B>T_f\}}P_f\right]$$
$$+\sum_{i=1}^{n_d}\mathrm{E}^{B_d}\left[B_d\left(0,t_{d,i}\right)1_{\{\tau_m^B>t_{d,i}\}}C_{d,i}\right]+X\sum_{i=1}^{n_f}\mathrm{E}^{B_f}\left[B_f\left(0,t_{f,i}\right)1_{\{\tau_m^B>t_{f,i}\}}C_{f,i}\right] \quad (14)$$
$$+\mathrm{E}^{B_d}\left[B_d\left(0,\tau_m^B\right)\left(1-\alpha_m\right)V_m^B\left(\tau_m^B\right)1_{\{\tau_d^B\le T_d\}}\right]$$

From Equation (14), the first two terms are the expected present value of the par values of the domestic and foreign firms in the absence of bankruptcy and before maturity. The next two terms represent the expected present values of the cumulative coupon payments with respect to the two firms again in the absence of bankruptcy. The last term denotes the expected present value of the merged firm's value in bankruptcy. It is worth emphasising that the distributions of the stopping time τ_m^B under domestic and foreign forward risk-neutral probability measure are different.

Following the same logic as above, the total firm value of the merged firm can be expressed as:

$$v_m\left(V_m^B\right)=V_m+\sum_{i=1}^{n_d}\mathrm{E}^{B_d}\left[B_d\left(0,t_{d,i}\right)1_{\{\tau_m^B>t_{d,i}\}}\delta_d C_{d,i}\right]+X$$
$$\sum_{i=1}^{n_f}\mathrm{E}^{B_f}\left[B_f\left(0,t_{f,i}\right)1_{\{\tau_m^B>t_{f,i}\}}\delta_f C_{f,i}\right]-\mathrm{E}^{B_d}\left[B_d\left(0,\tau_m^B\right)\alpha_m V_m^B\left(\tau_m^B\right)1_{\{\tau_d^B\le T_d\}}\right] \quad (15)$$

The explicit formulas of the merger firm's debt value, total firm value and equity value are provided in the following.

Proposition 3: *The debt value, total firm value and equity value of the merger firm are given as:*

$$
\begin{aligned}
D_m\left(V_m^B\right) &= B_d\left(0,T_d\right)P_dF\left(T_d;-0.5\sigma^2_{V_m/B_d},\sigma_{V_m/B_d},V_m^B\right) \\
&+XB_f\left(0,T_f\right)P_fF\left(T_f;\left(\mu_{V_m/B_d}-0.5\sigma^2_{V_m/B_d}\right),\sigma_{V_m/B_d},V_m^B\right) \\
&+\sum_{i=1}^{n_d}B_d(0,t_{d,i})C_{d,i}\cdot F\left(T_d;-0.5\sigma^2_{V_m/B_d},\sigma_{V_m/B_d},V_m^B\right) \\
&+X\sum_{i=1}^{n_f}B_f(0,t_{f,i})C_{f,i}F\left(t_{f,i};\left(\mu_{V_m/B_d}-0.5\sigma^2_{V_m/B_d}\right),\sigma_{V_m/B_d},V_m^B\right) \\
&+B_d\left(0,T_d\right)\left(1-\alpha_m\right)V_m^B\left(1-F\left(T_d;-0.5\sigma^2_{V_m/B_d},\sigma_{V_m/B_d},V_m^B\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
V_m\left(V_m^B\right) &= V_m+\sum_{i=1}^{n_d}B_d\left(0,t_{d,i}\right)\delta_dC_{d,i}F\left(T_d;-0.5\sigma^2_{V_m/B_d},\sigma_{V_m/B_d},V_m^B\right) \\
&+X\sum_{i=1}^{n_f}B_f\left(0,t_{f,i}\right)\delta_fC_{f,i}F\left(t_{f,i};\left(\mu_{V_m/B_d}-0.5\sigma^2_{V_m/B_d}\right),\sigma_{V_m/B_d},V_m^B\right) \\
&-B_d(0,T_d)\alpha_mV_m^B\left(1-F\left(T_d;-0.5\sigma^2_{V_m/B_d},\sigma_{V_m/B_d},V_m^B\right)\right)
\end{aligned}
$$

$$
E_m\left(V_m^B\right)=V_m\left(V_m^B\right)-D_m\left(V_m^B\right)
$$

Proof: Again by Corollary 1 and Lemma 2.

Until now the bankruptcy triggers are exogenous and constant. Now, assume endogenously determined bankruptcy strategies, V_d^*, V_f^* and V_m^* by the following smooth-pasting conditions: $\partial E_d/\partial V_d|_{V_d=V_d^B=V_d^*}=0$, $\partial E_f/\partial V_f|_{V_f=V_f^B=V_f^*}=0$ and $\partial E_m/\partial V_m|_{V_m=V_m^B=V_m^*}=0$. The intuition behind these three conditions is that the shareholders of the respective firms decide their optimal bankruptcy triggers to maximise the equity value of each firm. By bringing these optimal bankruptcy strategies into the above pricing formulas, the optimal capital structures can be completely obtained.

Similar to Leland (2005), the pure financial synergy of a cross-border M&A can now be evaluated by the following measure:

$$
\text{FS}=\frac{v_m\left(V_m^*\right)-\left(v_d\left(V_d^*\right)+Xv_f\left(V_f^*\right)\right)}{v_d\left(V_d^*\right)+Xv_f\left(V_f^*\right)} \tag{16}
$$

This measure demonstrates the percentage change in total firm values between the merged firm and the sum of the values of the domestic acquiring firm and the foreign target firm. Note that the financial synergy can be decomposed into three components being the change in unlevered asset value, the change in value of the tax benefits and the change in value of the bankruptcy costs. The last two sources are directly related to changes in capital structure of the merged firm.

3.3 Optimal stock exchange ratio

A stock exchange is often used in M&A transactions, as is the case in cross-border transactions. Based on our diffusion model, the optimal stock exchange ratio can be expressed as:

$$ER = \frac{XE_f(V_f^*)/S_f}{E_d(V_d^*)/S_d} \tag{16}$$

where S_f and S_d respectively denote the total outstanding shares of the foreign and domestic firms at the time that the merger is determined.

The stock exchange ratio of Equation (16) reveals only the financial conditions of the acquiring and target firms, but does not take into account the financial synergy of the merger. To address this issue, we provide the modified optimal exchange ratio in (17):

$$\text{OER} = \text{ER}\left(1 + \frac{E_m\left(V_m^*\right) - \left(E_d\left(V_d^*\right) + XE_f\left(V_f^*\right)\right)}{E_d\left(V_d^*\right) + XE_f\left(V_f^*\right)}\right) \tag{17}$$

The additional term introduced in Equation (17) is the percentage change in the equity values between the merged firm and the sum of the domestic acquiring firm and foreign target firm. This term reflects the financial synergy of the transaction.

4 Worked examples

The model developed herein is flexible and could be applied to various scenarios. In this section, we will consider two cases. The first involves a cross-border M&A of two symmetric firms while the second concerns a cross-border M&A of two asymmetric firms. The comparative analysis focuses on the effects of all the correlation coefficients on the financial synergy, and the numerical results provide the optimal stock exchange ratio.

4.1 Case 1: symmetric firms

Consider two symmetric firms with the following parameters: $P_d = 100$, $P_f = 25$, $T_d = T_f = 10$, $C_{d,i} = 6, i = 1,\ldots,10$, $C_{f,i} = 1.5, i = 1,\ldots,10$, $V_d = 150$, $V_f = 37.5$, $\delta_d = \delta_f = 0.3$, $\alpha_d = \alpha_f = 0.5$, $S_d = S_f = 10$ and $\sigma_{V_d} = \sigma_{V_f} = 0.2$. The parameters of foreign exchange rate and both domestic and foreign zero-coupon bonds are: $X = 4$, $\sigma_X = 0.2$, $B_i(0,t_{i,1}) = 0.965, i = d,f$, $B_i(0,t_{i,2}) = 0.95, i = d,f$, $B_i(0,t_{i,3}) = 0.88$, $i = d,f$, $B_i(0,t_{i,4}) = 0.835, i = d,f$, $B_i(0,t_{i,5}) = 0.79, i = d,f$, $B_i(0,t_{i,6}) = 0.745$, $i = d,f$, $B_i(0,t_{i,7}) = 0.705, i = d,f$, $B_i(0,t_{i,8}) = 0.67, i = d,f$, $B_i(0,t_{i,9}) = 0.635$, $i = d,f$, $B_i(0,t_{i,10}) = 0.6, i = d,f$, $\rho_{ij} = 0.2, i,j = V_d, V_f, X, B_d,$ and B_f and $\sigma_{B_d} = \sigma_{B_f} = 0.1$.

Assume that the domestic acquiring firm and the foreign acquired firm have the same (in units of domestic currency) individual characteristics and are under the same term structures of the short rates. Following application of the model, the financial synergy is –0.141% and the optimal stock exchange ratio is 0.8246. This shows that the merger of two symmetric firms under the same environment is not beneficial in the grounds of financial synergy.

Table 1 shows the effects of all the correlation coefficients on the financial synergy and optimal stock exchange ratio for these two symmetric firms. Note that the effects on the financial synergy are all negative. This is not surprising, since the debt-coinsurance effect of an M&A with two symmetric firms is minimal. Among the correlation coefficients, only $\rho_{V_dV_f}$ has a negative impact on the financial synergy, since the lower the $\rho_{V_dV_f}$, the greater the diversification effect. The other correlation coefficients all have a positive impact on financial synergy. From Table 1, ρ_{XB_d} is the most sensitive correlation coefficient, whereas ρ_{V_dX} and ρ_{V_fX} are the least sensitive (positive) correlation coefficients. In other words, the correlation between the foreign exchange rate and the domestic interest rate plays an important role.

4.2 *Case 2: asymmetric firms*

Consider two asymmetric firms with the following parameters: $P_d = 80$, $P_f = 5$, $T_d = 10$, $T_f = 5$, $C_{d,i} = 4.8, i = 1,...,10$, $C_{f,i} = 0.4, i = 1,...,5$, $V_d = 120$, $V_f = 8$, $X = 4$, $\delta_d = \delta_f = 0.3$, $\alpha_d = 0.4$, $\alpha_f = 0.6$, $S_d = 8$, $S_f = 2$, $\sigma_{V_f} = 0.4$ and $\sigma_{V_d} = 0.2$. The parameters of foreign exchange rate and both domestic and foreign zero-coupon bonds are: $X = 4$, $\sigma_X = 0.2$, $B_i(0,t_{i,1}) = 0.965, i = d,f$, $B_i(0,t_{i,2}) = 0.95, i = d,f$, $B_i(0,t_{i,3}) = 0.88, i = d,f$, $B_i(0,t_{i,4}) = 0.835, i = d,f$, $B_i(0,t_{i,5}) = 0.79, i = d,f$, $B_d(0,t_{d,6}) = 0.745$, $B_d(0,t_{d,7}) = 0.705$, $B_d(0,t_{d,8}) = 0.67$, $B_d(0,t_{d,9}) = 0.635$, $B_d(0,t_{d,10}) = 0.6$, $\rho_{ij} = 0.2, i,j = V_d, V_f, X, B_d$, and B_f and $\sigma_{B_d} = \sigma_{B_f} = 0.1$.

In this example, the domestic acquiring firm targets a foreign firm of smaller size (P_f, V_f and S_f), higher risk (σ_{V_f}, C_f/P_f and α_f) and shorter maturity (T_f). The foreign exchange rate, term structure of short rates and correlation coefficients remain unchanged from case 1 above.

The application of the diffusion model results in a measure of 0.012% and the optimal stock exchange ratio is 0.9895. This result demonstrates that the cross-border M&A of asymmetric firms results in a positive, albeit small, financial synergy.

Table 2 presents all of the correlation coefficients on the financial synergy and optimal stock exchange ratio for this case. Note that the effects on the financial synergy are all beneficial. Also note that because the unlevered asset's return volatility of the foreign acquired firm is much higher than that of the domestic acquiring firm, the diversification effect is very sensitive to the correlation coefficient between the foreign acquired firm and the foreign interest rate. In the example, $\rho_{V_dV_f}$ has a positive sign while $\rho_{V_d B_d}$ has a negative sign and these are negatively and positively correlated to the financial synergy, respectively. The other correlation coefficients play a minor role.

Table 1 Estimates of Financial Synergy (FS) and Optimal Stock Exchange Ratio (OER) from a case involving the cross-border M&A of two symmetric firms

ρ-Value	−0.75		−0.25		0.25		0.75	
	FS (%)	*OER*	*FS (%)*	*OER*		*FS (%)*	*OER*	*FS (%)*
$\rho_{V_d V_f}$	−0.132	0.8241	−0.135	0.8243	−0.142	0.8247	−0.154	0.8254
$\rho_{V_d B_d}$	−0.148	0.8206	−0.145	0.8240	−0.140	0.8246	−0.137	0.8245
$\rho_{V_d X}$	−0.150	0.8260	−0.142	0.8249	−0.141	0.8246	−0.141	0.8246
$\rho_{V_d B_f}$	−0.153	0.8256	−0.146	0.8250	−0.140	0.8246	−0.137	0.8244
$\rho_{V_f B_d}$	−0.152	0.8253	−0.145	0.8249	−0.140	0.8246	−0.137	0.8245
$\rho_{V_f X}$	−0.150	0.8260	−0.142	0.8249	−0.141	0.8246	−0.141	0.8246
$\rho_{V_f B_f}$	−0.149	0.8273	−0.145	0.8253	−0.140	0.8246	−0.137	0.8244
$\rho_{X B_d}$	−0.190	0.8282	−0.158	0.8258	−0.140	0.8246	−0.133	0.8242
$\rho_{X B_f}$	−0.132	0.8241	−0.133	0.8242	−0.142	0.8247	−0.166	0.8269
$\rho_{B_d B_f}$	−0.153	0.8256	−0.146	0.8250	−0.140	0.8246	−0.137	0.8244

Note: This table presents correlation coefficients on the financial synergy and optimal stock exchange ratio for case a involving a cross-border M&A of two symmetric firms. The financial synergy estimate is provided by

$$\mathrm{FS} = \frac{v_m\left(V_m^*\right) - \left(v_d\left(V_d^*\right) + Xv_f\left(V_f^*\right)\right)}{v_d\left(V_d^*\right) + Xv_f\left(V_f^*\right)}$$

while the optimal stock exchange ratio is provided by

$$\mathrm{OER} = \mathrm{ER}\left(1 + \frac{E_m\left(V_m^*\right) - \left(E_d\left(V_d^*\right) + XE_f\left(V_f^*\right)\right)}{E_d\left(V_d^*\right) + XE_f\left(V_f^*\right)}\right)$$

Parameter assumptions are: $P_d = 100$, $P_f = 25$, $T_d = T_f = 10$, $C_{d,i} = 6$, $i = 1,\ldots,10$, $C_{f,i} = 1.5, i = 1,\ldots,10$, $V_d = 150$, $V_f = 37.5$, $\delta_d = \delta_f = 0.3$, $\alpha_d = \alpha_f = 0.5$, $S_d = S_f = 10$, $\sigma_{V_d} = \sigma_{V_f} = 0.2$, $X = 4$, $\sigma_X = 0.2$, $B_i(0,t_{i,1}) = 0.965, i = d,f$, $B_i(0,t_{i,2}) = 0.95, i = d,f$, $B_i(0,t_{i,3}) = 0.88, i = d,f$, $B_i(0,t_{i,4}) = 0.835, i = d,f$, $B_i(0,t_{i,5}) = 0.79, i = d,f$, $B_i(0,t_{i,6}) = 0.745, i = d,f$, $B_i(0,t_{i,7}) = 0.705, i = d,f$, $B_i(0,t_{i,8}) = 0.67, i = d,f$, $B_i(0,t_{i,9}) = 0.635, i = d,f$, $B_i(0,t_{i,10}) = 0.6, i = d,f$ and $\sigma_{B_d} = \sigma_{B_f} = 0.1$.

Table 2 Estimates of Financial Synergy (FS) and Optimal Stock Exchange Ratio (OER) from a case involving the cross-border M&A of two asymmetric firms

ρ *value*	−0.75		−0.25		0.25		0.75	
	FS (%)	*OER*	*FS (%)*	*OER*	*FS (%)*	*OER*	*FS (%)*	*OER*
$\rho_{V_dV_f}$	0.013	0.9894	0.013	0.9894	0.012	0.9895	0.007	0.9900
$\rho_{V_dB_d}$	0.003	0.9850	0.009	0.9888	0.012	0.9895	0.013	0.9894
ρ_{V_dX}	0.013	0.9894	0.012	0.9894	0.012	0.9895	0.011	0.9895
$\rho_{V_dB_f}$	0.011	0.9896	0.012	0.9895	0.012	0.9895	0.012	0.9894
$\rho_{V_fB_d}$	0.007	0.9900	0.010	0.9897	0.012	0.9895	0.013	0.9894
ρ_{V_fX}	0.012	0.9894	0.012	0.9895	0.012	0.9895	0.012	0.9895
$\rho_{V_fB_f}$	0.043	1.0086	0.024	0.9973	0.011	0.9887	0.002	0.9835
ρ_{XB_d}	0.009	0.9899	0.011	0.9896	0.012	0.9895	0.012	0.9894
ρ_{XB_f}	0.013	0.9894	0.012	0.9894	0.012	0.9895	0.011	0.9896
$\rho_{B_dB_f}$	0.011	0.9895	0.012	0.9895	0.012	0.9895	0.012	0.9895

Note: This table presents correlation coefficients on the financial synergy and optimal stock exchange ratio for case 2 involving a cross-border M&A of two asymmetric firms. The financial synergy estimate is provided by

$$\text{FS}=\frac{v_m\left(V_m^*\right)-\left(v_d\left(V_d^*\right)+Xv_f\left(V_f^*\right)\right)}{v_d\left(V_d^*\right)+Xv_f\left(V_f^*\right)}$$

while the optimal stock exchange ratio is provided by

$$\text{OER}=\text{ER}\left(1+\frac{E_m\left(V_m^*\right)-\left(E_d\left(V_d^*\right)+XE_f\left(V_f^*\right)\right)}{E_d\left(V_d^*\right)+XE_f\left(V_f^*\right)}\right)$$

Parameter assumptions are: $P_d=80$, $P_f=5$, $T_d=10$, $T_f=5$, $C_{d,i}=4.8, i=1,\ldots,10$, $C_i=0.4, i=1,\ldots,5$, $V_d=120$, $V_f=8$, $X=4$, $\delta_d=\delta_f=0.3$, $\alpha_d=0.4$, $\alpha_f=0.6$, $S_d=8$, $S_f=2$, $\sigma_{V_f}=0.4$, $\sigma_{V_d}=0.2$, $X=4$, $\sigma_X=0.2$, $B_i(0,t_{i,1})=0.965, i=d,f$, $B_i(0,t_{i,2})=0.95, i=d,f$, $B_i(0,t_{i,3})=0.88, i=d,f$, $B_i(0,t_{i,4})=0.835, i=d,f$, $B_i(0,t_{i,5})=0.79, i=d,f$, $B_d(0,t_{d,6})=0.745$, $B_d(0,t_{d,7})=0.705$, $B_d(0,t_{d,8})=0.67$, $B_d(0,t_{d,9})=0.635$, $B_d(0,t_{d,10})=0.6$ and $\sigma_{B_d}=\sigma_{B_f}=0.1$.

5 Conclusion

Cross-border M&A transactions represent a significant challenge. Any M&A transaction requires an analysis of the potential synergies which are generally considered as arising either through operating activities or financial activities. In addition to the international complexities of operating across national boundaries, the financial synergy associated with a cross-border M&A will be impacted by the additional risks associated with interest rate differentials, local tax and bankruptcy regulation and foreign exchange risk.

This paper provides a diffusion model to analyse a cross-border M&A. The model is general in that it allows for stochastic interest rates and foreign exchange rates. This paper reveals closed-form formulas to measure the pure financial synergy for a cross-border M&A transaction and provides for an optimal stock exchange ratio. Numerical examples show that the cross-border M&A of two symmetric firms may result in a negative financial synergy, whereas that of two asymmetric firms may lead to a positive financial synergy. This paper also explores the related issue of the optimal stock exchange ratio when stock is used as the compensation in the transaction. The model is capable of producing a measure of the optimal stock exchange ratio.

The model developed herein is powerful in that it permits a quantification of what is often a complex and subjective issue. The model has considerable potential in both theory and practice although it is recognised that in practice; there are a number of parameters that require initial estimation.

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Notes

[1]See Erramilli (1991) and Cho and Padmanabhan (1995).

[2]For simplicity, the market is assumed to be complete. See Musiela and Rutkowski (1997).

[3]See Musiela and Rutkowski (1997).

[4]Note that the maturity of the bond could vary. For notational simplicity, we assume rates of return on the zero-coupon bonds have the same volatility and are perfectly correlated.

[5]For time-dependent diffusion terms, see Liao and Huang (2005).

[6]See German et al. (1995).