Analysing Electoral Systems Through Five Simple Questions:
A Journey of Discovery from Local Government in Australia's Northern Territory to the World of Electoral Science

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Introduction

The argument of this paper is that current common categorizations of electoral systems in political science are insufficiently analytic and deductive. They do not allow us to fully understand how, and why, different electoral systems contribute to certain patterns in electoral outcomes. By asking five simple questions about electoral systems, this paper develops alternative groupings of electoral systems which can be discussed more clearly and analytically. The five questions are:

1) What is the rule for winning?

2) How many representatives are to be elected?

3) How are votes marked?

4) How are votes counted? and

5) How many votes do electors have?

The paper begins by arguing that existing inductive, historical discussions of electoral systems in political science tend to jumble answers to questions 1 and 4 and to neglect question 5. It then recounts a journey of discovery in which I first encountered a particular electoral system in local government in Australia's Northern Territory in 2008, against which I had an instant intuitive adverse reaction. It was in attempting to understand and substantiate that reaction that my dissatisfaction with existing inductive historical discussions in electoral science became apparent and this more deductive analysis of electoral systems through these five simple questions began to emerge. The particular circumstances of this journey of discovery will, I hope, ground this discussion empirically without in any way compromising its analytic generality. For it is to clear deductive analytic generality that the paper ultimately aspires.

Existing Categorisations and Discussions

In their 1997 analysis of the world's national electoral systems, Reynolds and Reilly talk of three 'families' of systems: plurality/ majority systems, semi-proportional systems and proportional representation systems (Reynolds and Reilly 1997: pp18, see Figure 1). The

labeling of these families appears, at first, to be in response to the first of our five simple questions about electoral systems: what is the rule for winning? The terms 'plurality' and 'majority' identify two answers to that question. Candidates win either by having more votes than other candidates (plurality) or by having a majority of votes. Just why these *two* rules for winning are grouped together in *one* electoral system family is not entirely clear, but that issue can be left for later. Here I want to note that in opening their discussion of this first electoral system family, Reynolds and Reilly actually address *all five* of our simple questions in a rather jumbled way. They begin by focusing on question 2: how many representatives are to be elected? But they answer that question in a very equivocal way.

> The distinguishing feature of plurality-majority systems is that they almost always use single member electorates (Reynolds and Reilly 1997: 18).

Within two sentences, having explained the rule for winning in plurality systems, Reynolds and Reilly's equivocation on question 2 becomes complete and their analysis shifts to question 5, as well as back to question 1.

> When this (plurality) system is used in multi-member districts it becomes the Block Vote. Voters have as many votes as there are seats to be filled, and the highest polling candidates fill the positions regardless of the percentage of the vote they actually achieve (Reynolds and Reilly 1997: 18).

This last statement sets up a contrast with majority systems in which candidates do have to 'achieve' a particular 'percentage of the vote' in order to be elected, answering question 1. This then shifts the discussion, implicitly, to questions 3 and 4: how are the votes marked and counted? Majority systems, they say, make use of:

> voters' second preferences to produce a majority winner if one does not emerge from the first round of voting (Reynolds and Reilly 1997: 18-19).

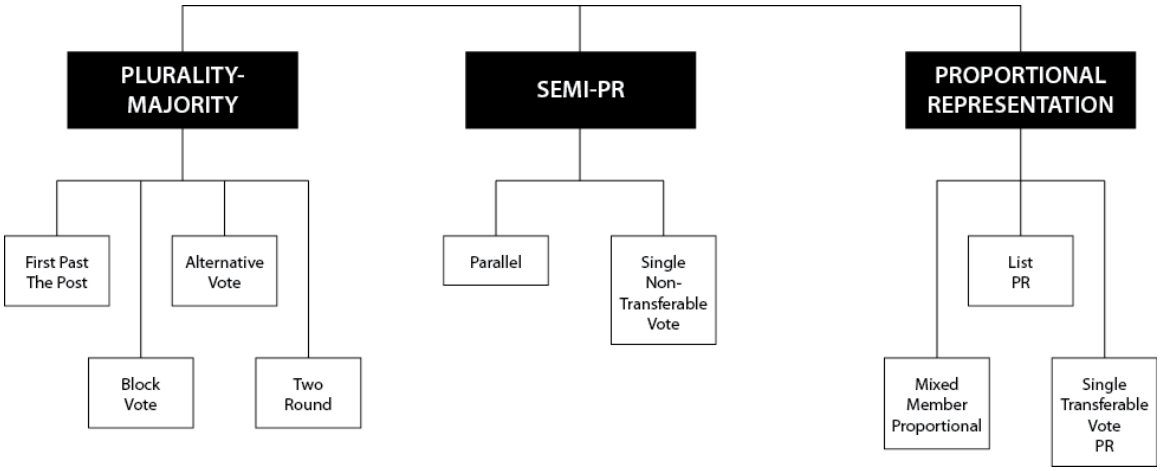
In attempting to be brief and general, Reynolds and Reilly's discussion is rather too compressed and does not achieve the clarity required for a full analytic understanding of even this first family of electoral systems. However even more is asked, or assumed, of the reader when they attempt to introduce their other two electoral system families.

In discussing semi-proportional systems, Reynolds and Reilly simply begin by listing common systems by name. Then in discussing proportional representation systems they begin as follows:

> The rationale underpinning all proportional representation (PR) systems is to consciously reduce the disparity between a party's share of the national vote and its share of the parliamentary seats; if a major party wins 40% of the votes, it should win approximately 40% of the seats, and a minor party with 10% of the votes should also gain 10% of the parliamentary seats (Reilly and Fraser 1997: 19).

This statement, like the labeling of these latter two electoral system families, does not address *any* of our five simple questions for analyzing electoral systems. Instead, there is a switch to the discussion of *outcomes* of electoral systems in terms of proportionality between seats won in assemblies and votes cast, viewed primarily as votes for parties rather than individual candidates. This is a *huge* analytic jump, which also implies or assumes, as background knowledge, that the plurality/ majority family of electoral systems might not be very good at achieving such proportional outcomes.

Figure 1. Electoral System Families, following Reynolds and Reilly 1997.



An earlier critical survey of the 'underdevelopment' of electoral systems research by Lijphart distinguished between:
-research which tries to explain the 'causes of different electoral systems' historically across nations, and
-research which aims 'to discover the consequences of the different aspects of electoral rules'.
It was the latter, Lijphart argued in the 1980s, that was 'the more important task' and the more 'underdeveloped' (Lijphart 1985: 5-7). Clearly, it is to this latter style of research that I am attempting to contribute here.

Lijphart went on to identify the three most important 'elements of electoral systems' that had been identified and studied as: 'electoral formula', 'district magnitude' and 'ballot structure', before adding another twelve minor variables of his own which he argued also

needed to be considered (Lijphart 1985:7). 'District magnitude' corresponds to our question 2: how many representatives are to be elected? While 'ballot structure' corresponds to our question 3: how are votes marked? 'Electoral formula' appears to correspond to question 1 and in discussing it Lijphart identified the same three groupings of electoral systems as Reynolds and Reilly over a decade later. But interestingly, Lijphart referred to plurality/ majority, semi-proportional and proportional representation as 'methods', as much as systems, which possibly also alludes to our question 4: how are votes counted?

My fundamental contention is that much common existing inductive historical discussion of electoral systems moves far too quickly to discussing *outcomes*, in terms of proportionality between seats won in assemblies and votes cast, and that it does not sufficiently address the internal elements of electoral systems reflected in our five simple questions. When occasionally such discussion does address these questions, it tends to jumble answers to questions 1 and 4 and to pay insufficient attention to question 5. These fourth and fifth questions address basic elements of electoral systems which need to be considered early in our analysis. How votes are counted (q4) is a different matter from the rule for winning (q1). It is also crucially important to think about the number of votes electors have (q5).

Let me now recount how I came to this analysis by observing local government elections in Australia's least populous, sub-national jurisdiction, the Northern Territory.

An Australian Journey of Discovery

The Northern Territory is a vast, but sparsely populated area of Australia which had, until 2008, a *permissive* system of local government only covering about 10 per cent of its land area. About 60 little local governments had developed, covering four main urban areas and over 50 smaller more remote settlements, or groups of settlements. In a major reform which moved towards *mandatory* local government in 2008, these over 50 remote local governments were restructured into just eight much larger shires covering almost all the land area of the Northern Territory.[i] The electoral system given to these new shires by the Northern Territory Government was that already used in the urban municipalities. It

allowed for both single- and multi-member districts, which could exist side-by-side, and it relied on ordinal, or preferential, vote marking with numbers; as do most electoral systems in Australia. The aspect of this system against which I intuitively reacted was its vote counting method in multi-member districts, which was referred to in the labeling of the system as 'Exhaustive Preferential'. I have since learned that, in the larger world of electoral system analysis, this system has also been known as 'Preferential Block Majority' and 'Multiple Majority Preferential' voting. Both these labelings are better than 'Exhaustive Preferential', but the last is perhaps best at giving an indication of how the system operates.[ii]

Basically this vote counting system runs elections for multi-member districts by running repeated single member elections with a majority winning rule. After the initial tally of No.1 votes using ordinal vote marking, the candidate with the least votes is eliminated and their votes are transferred, or distributed, to the candidates marked No.2 on those ballot papers. This elimination and vote transfer process is repeated until there are just two candidates left in the election, one of whom has more and one less than 50 per cent of the votes. This 'tally, eliminate and transfer' vote counting method is very familiar to Australians as the way in which many elections for single members in electoral districts are conducted under the Alternative Vote, or preferential voting as we more commonly call it. So the idea of repeating this procedure several times over to elect multiple members in an electoral district seems, perhaps, to be a reasonable common-sense approach. However, this understandably attractive common-sense approach will have some very undesirable consequences for electoral outcomes which, to me, could be anticipated intuitively and which observation readily verified.

I observed one district in one of the new shires in which there were four members to be elected. Candidates were all standing as individuals without party affiliations, as is often the case in Australian local government. They did however have affiliations of other sorts that were known to electors, such as to towns or other places within the electoral district. In this election there were 9 candidates with a residential affiliation to the largest town in the district and then one candidate from each of two smaller towns. In the initial tally of No.1 votes, the candidates from the two smaller towns did quite well, each capturing

about 20 per cent of votes. The nine candidates from the larger town shared the other 60 per cent of votes, some with around 15 per cent of primary votes and some with as few as two or three per cent. In the first 'tally, eliminate and transfer' count, it was understandably one of the candidates from the larger town with around 15 per cent of the primary vote who went on to win as they benefitted from the elimination of candidates with low numbers of votes from the large town and the transfer of those votes using the ordinal ballot markings.

Once this election for the first position was complete, a totally new count of the votes was begun in which the first elected person, let's call them A, was no longer a candidate. In this new election, A's No.1 votes were re-allocated to those marked No.2 on the ballot papers. This is not the 'distribution of preferences', as Australians refer to the process of vote transfer in the elimination process on the way down to a two person race. Rather it is simply the beginning of a new election in which A is no longer a candidate and A's votes are 're-allocated'. In this second single member election, another candidate from the large town who began with about 15 per cent of the initial votes came through to win the second position, which also possibly seemed reasonable. However, in the third and fourth elections it was candidates from the large town who had started with only two or three per cent of the No.1 votes who came through to win and this clearly seemed far less reasonable.

What was happening in these repeated single member elections was that the 're-allocated' votes of the just-elected candidate were bringing up through the field another candidate who had *much the same voter support base*. The large town won all four positions and the two candidates from the small towns never progressed much from their original 20 per cent share of the votes. Indeed I started to say that if there had been more positions available, *all nine* candidates from the large town would probably have been elected before either of the small town candidates, as people were voting largely by town affiliation. My intuitive reaction was that, although it had some understandable common-sense appeal this was, in logic, a *mistaken* vote counting system which led to highly disproportional and concentrated representational outcomes.

As I began asserting that this vote counting system was a mistake, I began to learn some Australian political history of which I was embarrassingly ignorant. Known at the time as Preferential Block Majority, this was the vote counting system that had been used in the Australian Senate from 1919 until 1946.[iii] In that situation, where party affiliations were significant, the system had also been shown to lead to highly disproportional and concentrated representational outcomes. In those years, in normal half Senate elections, there were three Senators to be elected from single, state-wide districts. In 60 elections (ten elections in each of Australia's six states), 55 had resulted in the party which won the first Senate position also winning the second and third positions (see Lijphart 1997, Reilly and Maley 2000). I was astounded not by these results, but by how long Australians had abided this electoral system against which I had an instant intuitive adverse reaction. What was political science saying about this vote counting system?

Before I recount my journey of discovery into the world of electoral science, I will add that I have since discovered that this electoral system has been used a number of other times in Australian electoral history and in each case it has ultimately been recognized as producing a disproportional concentration of representation, rather than representational spread. It was used in elections for the South Australian Legislative Council from 1930 to 1973, where the same party would almost always win both first and second seats in five two-member districts (Jaensch 1977: 55). It was used in Victorian local government after a major amalgamation and reform process in the early 1990s, but was abandoned in 2003 when it became evident that it could lead to major disproportional concentrations of representation both by party and by location, such as between large and small towns (see Burdess and O'Toole 2004, author reference 2009). I have also since learned that this system still exists as an option in New South Wales local government and that a small number of councils continue to choose it, producing narrow virtual one-party local governments. So if, as I contend, this vote counting system is a *mistake,* then it is a quite common, repeated and persistent one in Australian electoral practice – instances of which I am still slowly discovering.

An Electoral Science Journey of Discovery

In an attempt to show how this Multiple Majority Preferential vote counting system is a mistake, I began to contrast it with alternatives for electing multiple members to districts using single votes: such as the Single Non Transferable Vote (SNTV) (a single vote used to elect multiple members under a plurality counting rule) and Single Transferable Vote Proportional Representation (STVPR). Where this Multiple Majority Preferential system differed most markedly from these, it seemed to me, was in effectively giving voters as many votes as there were people to be elected. With each repeated single member election under this system, voters were being given *another* vote. The consequence was that a similar majority to the one that dominated the first election could re-emerge to dominate the second and subsequent elections as well. What seemed important here was to recognize that in moving from electing one to multiple members per district it is not desirable to give electors multiple votes. All this does is risk reproducing the *weakness* of single member elections many times over. Rather what is important is to find ways of counting electors' single votes in ways which gradually spread elected representatives to *different* voter bases. SNTV, as a single vote, multi-member, plurality system, spreads representation in a very simple unsophisticated way, while STVPR does so in a slightly more sophisticated way, informed by basic mathematics. But both, importantly, stay with one vote per elector as they move to multi-member districts, in contrast to Multiple Majority Preferential.

Was this insight about single and multiple votes in elections for multi-member districts made apparent in the general discussions of electoral systems that I referred to at the outset?

In their 1997 survey of the world's national electoral systems, Reynolds and Reilly made two passing references to the Australian Senate experience of 1919 to 1946 which noted that the 'Alternative Vote' system when applied to 'multi-member districts' does not 'work well' because it leads to 'highly disproportional', 'lop-sided and unrepresentative results' (Reynolds and Reilly 1997: 10, 39). While these references identified the problem with this system, they did not clearly relate it, analytically and deductively, to

this being a multi-vote system. They simply relied on induction from historical experience.

Elsewhere in their 1997 monograph, Reynolds and Reilly also discussed the Block Vote system, identifying it as a multi-vote, multi-member adaptation of First Past The Post voting. They noted that some 10 countries still used the Block Vote for national elections but that some had recently moved away from it because of a tendency to 'exaggerate' the 'disproportionality' of FPTP (Reynolds and Reilly 1997: 36). But beyond this Reynolds and Reilly had little clear analytic discussion of the difference between multi-vote and single vote systems.

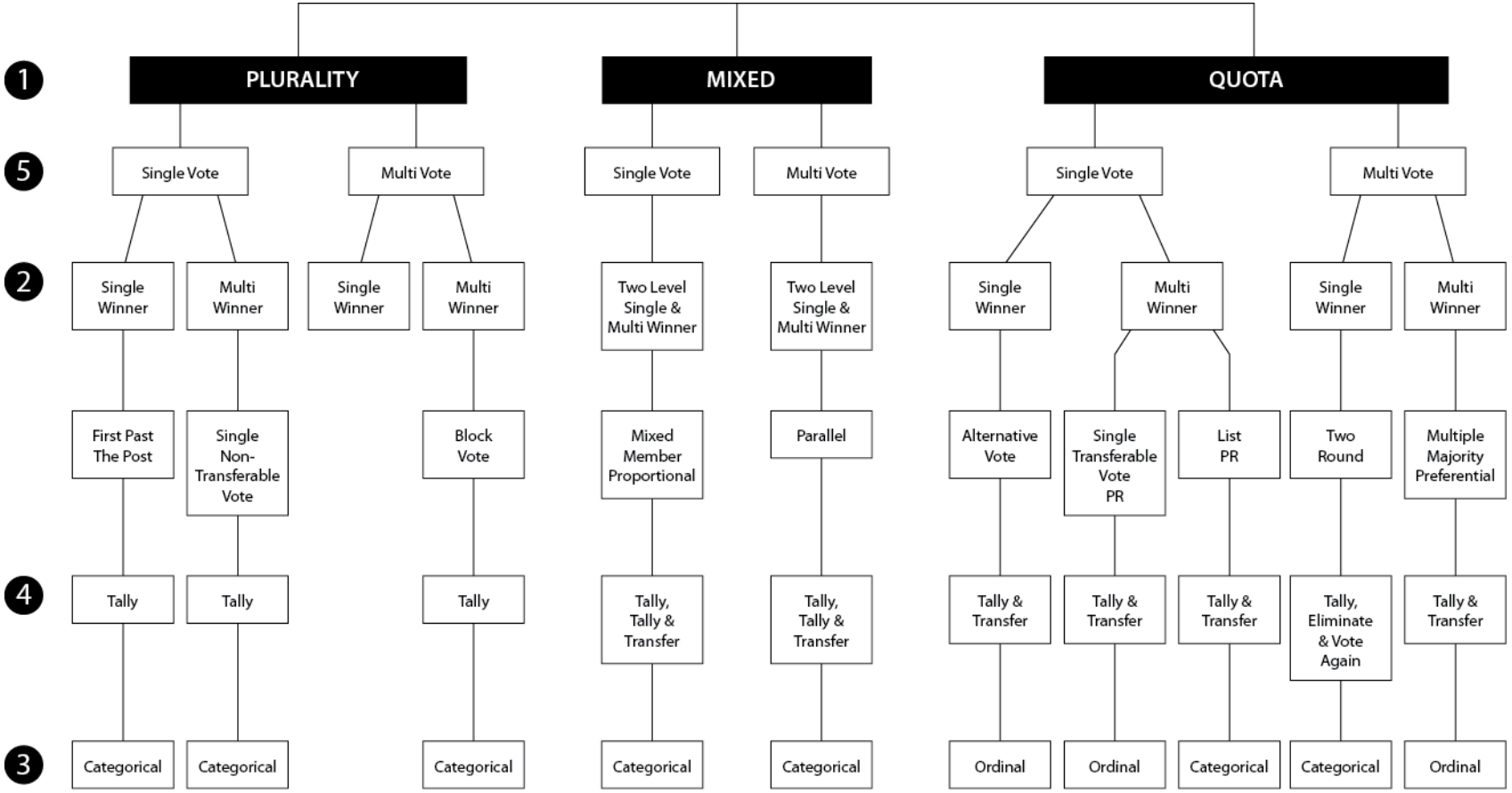
Lijphart's 1985 critical survey of electoral systems research did not discuss individual systems in detail, but a later article extensively analysed the Australian Senate experience from 1919 to 1946 as part of a larger debate with Horowitz about whether Alternative Vote systems lead to greater 'proportionality' of electoral outcomes than plurality systems (Lijphart 1997). Lijphart argued convincingly against the Alternative Vote systematically achieving such greater representational spread, particularly if used in multi-member districts. But again he did so on the basis of historical induction, rather than on the basis of clear deductive analytic reasoning which identified the multi-vote, rather than single vote nature of this system when used in multi-member districts.

It was at about this point in my investigations that I felt there were quite serious analytic deficiencies in the study of electoral systems and I began playing around with my five simple questions. Question 5 seemed to be given insufficient attention in this inductive historical style of research which focuses on the world-wide distribution and performance of these three families of electoral systems. What would happen to these families if we *systematically* asked of each electoral system *all five* of these simple questions?

Figure 2 is my attempt to systematically answer all five questions for the electoral systems discussed by Reynolds and Reilly in their 1997 survey of the world's national electoral systems. I start in the first line with question 1: what is the rule for winning? While the first answer to that question is 'Plurality', I do not pair this answer with

majority. Indeed I do not use the word majority in this line of Table 2, nor do I use the words proportional representation which, as noted previously, refer to desired outcomes rather than a rule for winning. Rather I use the word 'Quota' as the other category of rule for winning, alongside 'Plurality', with some 'Mixed' systems in the middle. The basic idea here is that some electoral systems set a quota of votes needed to win when the *number* of votes is known, but independent of the allocation of those votes to candidates. Other systems, by way of contrast, just start allocating votes to candidates and see who gets the most. The latter are Plurality systems, while the former are Quota systems. Majority systems, it can be seen from Figure 2, move out of the Plurality family and into the Quota family, alongside various named Proportional Representation systems. Majority systems effectively set a quota of 50%+1 to win, once the number of votes is known, independent of the allocation of votes to particular candidates.[iv]

Figure 2. Revised Electoral System Families, systematically answering 5 questions



The second line of Figure 2 asks question 5: how many votes do electors have? The answers here are simply either single or multiple votes, though in empirical instances the number of multiple votes will also need to be clarified. The third line of Figure 2 asks question 2: how many representatives are to be elected? Again the answers are basically either single or multiple, though again in empirical instances precise numbers of 'winners' would need to be specified. There are also some interesting instances of 'Mixed' systems in which both single and multiple representatives are elected at two different district levels.

There could be much further discussion of these answers to questions 1, 5 and 2 in Figure 2, but at this stage I want to list, in the fourth row of Figure 2, the various named electoral systems which Reynolds and Reilly discussed in their 1997 survey of the world's national electoral systems. The arrangement of these named electoral systems in Figure 2, both into three primary families and then into smaller groups within them, is *significantly different* from Reynolds and Reilly's Figure 1. Not only have majority systems moved right to the Quota family, but two systems have also moved left. The Single Non Transferable Vote system has moved from Reynolds and Reilly's middle group, which they called Semi-Proportional, to the Plurality family on the left. Also the Mixed Member Proportional system, which Reynolds and Reilly classified as a Proportional Representation system on the right of Figure 1, has moved to the Mixed systems in the middle of Figure 2 rather than being in the Quota systems on the right. There is thus a *significant re-arrangement* of electoral system families between Figures 1 and 2, as a result of systematically asking questions 1, 5 and 2.

Another difference in Figure 2 is that I have explicitly identified and located Multiple Majority Preferential as a distinct electoral system, on the multi-winner, multi-vote Quota branch of our electoral systems taxonomic tree. Reynolds and Reilly did not give Multiple Majority Preferential the status of a distinct electoral system in Figure 1, referring to it in their text as just 'the Alternative Vote in multi-member districts' (Reynolds and Reilly 1997: 10). But as can be seen from Figure 2, the Alternative Vote and Multiple Majority Preferential systems are on quite different branches of the electoral

system tree separated by answers to not one, but *two* of our five simple questions. The single-winner, single-vote Alternative Vote appears, on this taxonomy, to be more closely related to Single Transferable Vote Proportional Representation than to Multiple Majority Preferential. I will return to this point shortly. However first let us look at the answers to questions 3 and 4 in the bottom two rows of Figure 2.

The answer to how votes are counted (q4) in Plurality systems, is basically by a simple 'tally'. In Quota systems the predominant vote counting method is 'tally and transfer'. Transfers can occur for two reasons. The first reason is because at certain points in the count no candidate is achieving a quota, so candidates with low numbers of votes are eliminated and their votes transferred to others. The second reason is because candidates have achieved over a quota of votes and their surplus votes need to be allocated to other candidates. These are *very different processes* which can be referred to as 'preference distribution' and 'surplus allocation', but they are both *vote transfers*. The other vote counting method I identify among Quota systems is 'tally, eliminate and vote again', which describes the procedure in Two Round systems[v]. Mixed systems, understandably, mix these vote counting methods of 'tally' and 'tally and transfer'.

The last line of Figure 2 simply identifies whether systems have categorical or ordinal vote marking. Plurality systems tend to have categorical vote marking. Quota systems can to some extent be run using categorical vote marking if combined with party lists or repeated rounds of votes. But Quota systems which allow voting for individual candidates in single round elections rely on ordinal vote marking. Existing national Mixed systems in 1997, as I understand them from Reynolds and Reilly's descriptions, tended to involve categorical vote marking, but could in principle also involve ordinal vote marking (see Figure 2).

There is clearly much more wide-ranging discussion that could arise from Figure 2. Indeed my hope is that Figure 2 will provoke a major re-assessment of the conventional families of electoral systems used in recent political science and provoke great discussion. For the remainder of this essay, however, I will focus my use of Figure 2 on helping me make the case that Multiple Majority Preferential is an unacceptable voting

system which should be condemned by political science as a mathematical mistake and as undemocratic. In the process I will also be suggesting that the Block Voting can be subject to the same analysis. This paper, therefore, involves a fairly significant challenge to what political science has in the past accepted as some reasonable, democratic electoral systems.

Let me now return to the point that in Figure 2, the single-winner, single-vote Alternative Vote seems more closely related to Single Transferable Vote Proportional Representation than to Multiple Majority Preferential. This is because the former are both single vote systems and the latter is a multi-vote system. Single- and multi-vote systems in multi-member districts, I argue, respond *very differently* to what I have referred to above as the *weakness* of single member elections. This weakness is simply that only one candidate can win. Some electors vote for that winning candidate, but many do not. There is thus an *inevitable major disproportionality* in single member elections between votes cast and positions won. Single vote, multi-member systems reflect some awareness of this weakness and spread subsequent elected positions to other voter bases, either simply or in a more mathematically sophisticated fashion. But multi-vote, multi-member systems seem unaware of or inattentive to this weakness of single member elections and as a consequence risk repeating the disproportionality between votes cast and positions won as they repeat (or aggregate) such elections in order to elect multiple members. This can be seen most easily by a simple example.

A Simple Example to Count in Different Ways

Table 1 is an array of eleven votes cast for four candidates using ordinal vote marking. Although this is hypothetical array of votes, it is I would argue, quite realistic. It contains considerable variety among voter preferences, but also discernable voting groups, reminiscent of party alignment, or town alignment in local government. Voters 1-5 are a clear voting group favouring a single candidate, while voters 6-11 are a slightly larger voting group supporting three other candidates.

Table 2 is the procedure for counting these eleven votes under the Multiple Majority Preferential system which I argue is mistaken. In a standard Alternative Vote procedure, C2 wins the election for the first position even though C1 has 5 primary votes. This is because, following the eliminations of C3 and C4 and the transfer of their 3 votes to add to C2's 3 primary votes, C2 has a majority of 6 votes. If there is a second position available in the district, Multiple Majority Preferential leaves this counting in place, declares C2 elected and starts a new count of the 11 votes in which C2 is not a candidate. In this count electors are effectively being given a *second vote* after C2 is elected and C2's votes are 're-allocated' to those marked No2 on the ballot papers where C2 is marked No1. In this second count, in the second panel of Table 2, C1 starts with 5, C3 with 4 and C4 with 2 initial votes. After the elimination of C4 and transfer of those 2 votes using the ordinal preferences, C3 is elected with 6 votes. If there is a third position available in the district, again all this counting is left in place and electors are effectively given a *third* vote after C2 and C3 are declared elected. In this third election, C4 is 're-allocated' the votes that were previously with C2 and C3 and wins with 6 votes.

What is happening here is that in repeated single member elections with a majority winning rule, voters 6-11 are repeatedly electing the successful candidate and voters1-5, supporting C1, are repeatedly a losing minority, albeit a very substantial losing minority. C1 just can't win because even though he or she has a very strong base of support among five electors, a majority of 6 electors keep preferring other candidates. This multiple vote system is just reproducing the weakness of single member elections.

Table 1: Eleven Votes for Four Candidates

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11
C1	1	1	1	1	1	4	4	4	4	4	4
C2	2	3	4	4	4	1	1	1	2	3	2
C3	3	2	2	3	3	2	2	3	1	1	3
C4	4	4	3	2	2	3	3	2	3	2	1

Table 2: Multiple Majority Preferential Count

	Initial Votes	Votes after 1st elimination	Votes after 2nd elimination	Result for 1 Position
C1	5	5	5	
C2	3	4	6	C2 elected
C3	2	2	eliminated	
C4	1	eliminated		

	Initial Votes	Votes after 1st elimination	Result for 2 Positions
C1	5	5	
C2 elected	Re-allocated		
C3	4	6	C3 elected
C4	2	eliminated	

	Initial Votes	Result for 3 Positions
C1	5	
C2 elected	Re-allocated	
C3 elected	Re-allocated	
C4	6	C4 elected

Tables 3 and 4 set out two alternative vote counting procedures for these same 11 votes in which electors only have a single vote. Table 3 is a very simple Single Non Transferable Vote count for one, two or three positions available, in which ordinal vote marking is not actually being utilised in the counting procedure. Table 4 is the slightly more sophisticated Single Transferable Vote system, in which ordinal vote mark is being utilized to engage in the two different vote transfer procedures which I mention above and will explore further shortly. What is most notable in Tables 3 and 4 is that while the outcome differs when only one position is to be filled, with C1 being elected in Table 3 and C2 in Table 4, the outcomes converge when there are two or three positions to be filled. Both elect C1 and C2 when there are two positions to be filled and both elect C1,

C2 and C3 when there are three positions to be filled. This is because these are single vote systems which, after the first position is filled, look to *other* voter groups in order to fill subsequent positions and hence spread representation. Voters 1-5 and 6-11, as groups, each get a representative they quite like when there are two or more representative elected. This is in direct contrast to Multiple Majority Preferential, which keeps rewarding the same voter group (voters 6-11) over and over again.

Table 3: Single Non Transferable Vote Count

	Votes	Results
C1	5	Elected if 1 Position
C2	3	Elected if 2 Positions
C3	2	Elected if 3 Positions
C4	1	

In arguing for a change to the Northern Territory's local government vote counting system in multi-member districts, I would use Tables 3 and 4 as way of suggesting to the interested citizenry and officials that they needed to abandon the multi-vote, Exhaustive Preferential or Multiple Majority Preferential system in favour of a *single vote* system, whether simple or sophisticated. I would suggest that while the SNTV system does achieve representational spread across voting groups, Table 3 also shows us a possible weakness of this very simple system. If there are three positions available, does it seem fair that C1 is elected with 5 votes and C3 is elected with 2 votes? Many would think that this was not optimal and it is at this point in our discussions that I would suggest that the mathematically-inclined people who developed quota electoral systems had something to offer. These people had the idea that once you knew the number of votes in an election and the number of positions available, you could set a logical quota of votes for a candidate to win. That quota was thought, historically, to be the total number of votes (TV) divided by the number of positions available for representative members (M). However TV/M, I would explain, had turned out in practice to be a little high as a quota. If, for example, there were three positions available in a district and the quota was set at 1/3 of the total votes (TV/3) and two candidates had reached that quota, the remaining candidates would be competing for the last 1/3 of votes and any one of them would have trouble reaching it. A person called Droop, I would explain, had worked out somewhere

along the way that a slightly lower quota was better; TV/(M+1) +1 rounded down to the whole number below if this sum produced a fractional answer. Before losing the attention of the less mathematically inclined citizenry, I would ask them to imagine an election in which there were 3 positions available and the quota for election was set at ¼ of the votes plus one rounded down if not a whole number. Imagine that three candidates in an election achieved that number of votes, what would be the maximum number of votes that a fourth candidate could be left with? Even the less mathematically inclined citizenry could usually see that the fourth candidate could only have a little less than ¼ of the votes and hence would fall just short of achieving the required quota. The mathematical logic of this Droop quota, I would explain, is that it is the *lowest whole number* of votes which the required number of candidates can achieve but one more cannot. Thus it maximizes the likelihood that there will be *enough winners* to fill available positions, *but not too many*.

Table 4, I would explain, although it is like Table 2 in its first panel, is very different in its second and third panels. Unlike in Table 2, the three panels in Table 4 do not work together in an ongoing counting process. Rather they are separate counting processes depending on how many positions are available in the electoral district. In their top left cell the panels of Table 4 set the quota that is mathematically appropriate for the number of positions available. If there is one position available the quota is 11/(1+1) +1 or 6.5, which can be rounded down to 6. If there are two positions available the quota is 11/(2+1) +1 or 4.6666.., which can be rounded down to 4. If there are three positions available the quota is 11/(3+1) +1 or 3.75 which can be rounded down to 3. In each case the quota is the lowest whole number of votes that M candidates can win, but M+1 candidates cannot

The other thing to note about Table 4 is that the transfer of votes between candidates is occurring in the two very different ways I outlined earlier when discussing Figure 2 and question 4. There is transfer:
-by the ‘distribution of preferences’ when no candidate has a quota and the candidate with the least number of votes is eliminated, and/or

-by the ‘allocation of a surplus’ when a candidate has more than a quota of votes.
The first panel of Table 4 only needs to transfer votes in the first of these two ways, which is why it can look the same as the first panel of Table 2, except that it explicitly specifies the quota of 6 votes to win in this 11 vote election for a single available position.

The second and third panels of Table 4 usually require more explanation for the interested citizenry and officials. Perhaps political scientists do not require such explanation, but I will briefly give it anyway.

In the second panel of Table 4, where 2 positions are available and the quota is 4, C1 is immediately declared elected with 5 initial votes. But then the question arises as to how to give away C1’s surplus of one vote. The answer is that rather than give away any one vote, 1/5 of all five votes is given away. To do this we must return to Table 1 and look at the second preferences of the five votes for C1. One of those votes valued at 1/5 is transferred to C2 and two each are transferred to C3 and C4. Since no candidate has a quota after this allocation of C1’s surplus, the next step is to eliminate C4 and ‘distribute preferences’. C4 has 1 vote to be transferred at full value (vote11 to C2) and two votes to be transferred at 1/5 value (votes 4 &5 to C3). At this point C2 has more than a quota of votes and is declared elected.

In the third panel of Table 4 there are three positions available and the quota is 3. Both C1 with 5 votes and C2 with 3 votes are declared elected after the initial tally of votes. But the question then arises as to how to give away C1’s surplus of two votes. Again all 5 votes are transferred, but at a value of 2/5 following the preferences of voters 1-5 expressed in Table 1. Votes 1-3 flow to C3 at 2/5 value and votes 4-5 to C4, after which C3 has more than a quota and is elected.

Table 4: Single Transferable Vote Count

1 Position Available Quota 6	Initial Votes	Votes after 1st elimination	Votes after 2nd elimination	Result
C1	5	5	5	
C2	3	4	6	C2 elected (6)
C3	2	2	eliminated	
C4	1	eliminated		

2 Positions Available Quota 4	Initial Votes	Allocate C1 Surplus at 1/5 value per vote	Votes after 1st elimination	Result
C1	5	C1 elected (4)	C1 elected (4)	C1 elected (4)
C2	3	3+(1/5X1)	3+1/5+1	C2 elected (4)
C3	2	2+(1/5X2)	2+2/5+2/5	
C4	1	1+(1/5X2)	eliminate	

3 Positions Available Quota 3	Initial Votes	Allocate C1 Surplus at 2/5 value per vote	Result
C1	5	C1 elected (3)	C1 elected (3)
C2	3	C2 elected (3)	C2 elected (3)
C3	2	2+(2/5X3)	C3 elected (3)
C4	1	1+(2/5X2)	

This example has helped me show the interested citizenry and officials of the Northern Territory how single vote systems in multi-member districts spread representation to different voter bases, while the multi-vote Multiple Majority Preferential system can unacceptably concentrate it. But there is more in this example than just that. It also suggests, through Tables 3 and 4, that the Single Non Transferable Vote and the Single Transferable Vote can each work in both single- and multi-member districts and that there is no need, analytically, to distinguish:

-in Plurality systems between First Past The Post in single member districts and the Single Non Transferable Vote in multi-member districts, or

-in Quota systems between the Alternative Vote in single member districts and the Single Transferable Vote in multi-member districts.

In attempting to make this point in relation to Quota systems, I have also developed Table 5, which simply sets out the mathematical logic of appropriate Droop quotas for the

election of different numbers of members per district, irrespective of the details of particular elections. If there are ten positions to be filled in a district, the appropriate quota is $1/11^{th}$ of the votes plus one, if there are five votes to be filled it is $1/6^{th}$ of the votes plus one, and so on. Although this is not a mathematical proof, the symmetry and logic of Table 5 perhaps suggests that if quotas were used in elections other than in this way, there could possibly be some strange outcomes. Imagine using the quota for ten positions available when there are in fact five. It is highly likely that would be too many winners. Or alternatively imagine using the quota for five positions available when there are in fact ten. There would be too few winners and people might look for ways to use the votes to elect more. This is in what Multiple Majority Preferential Voting does. It uses the quota appropriate for one position available when there are in fact more. Then, when it finds itself short of winners, it just recycles the votes and does another count!

The other major point of Table 5 is to show that the majority winning rule in single member elections is part of the logic of the Single Transferable Vote with Droop quotas, not a totally different electoral system called the Alternative Vote. This is something which some electoral system analysts have occasionally recognised, but then have not taken further because of the predominance in recent political science of the categorization on electoral system families of Figure 1.[vi] The Alternative Vote and the Single Transferable Voter seem in Figure 1 to be from different families. But with the help of Table 5 and Figure 2 they can be seen, mathematically and genealogically, to be in the same family; the single vote, Quota family. Why has recent political science not acknowledged this close relationship between the Alternative Vote and the Single Transferable Vote?

Table 5. STV Droop Quotas for number of positions available in a district

Positions	10	9	8	7	6	5	4	3	2	1
Quota	1/11+1	1/10+1	1/9+1	1/8+1	1/7+1	1/6+1	1/5+1	1/4+1	1/3+1	½+1

The Importance of Question 5 and the Problem of Multi-Vote Systems

Political science seems, when analyzing electoral systems, to have overlooked the importance of question 5: how many votes do electors have? Single and multi-vote

systems work very differently and lead to very different patterns of electoral outcomes. Multi vote systems in multi-member districts run the risk continuing the inevitable disproportional concentration of representation evident in single vote elections in single member districts. Single vote systems, by way of contrast, whether simple or mathematically sophisticated, tend to spread representation across different voter groups as they move up from one to larger numbers of members per district.

The problem of multi-vote electoral systems is so substantial, I now want to argue, that not only should Multiple Majority Preferential be regarded as unacceptable, undemocratic and mathematically mistaken, but so too should the Block Vote. This is clearly a much more live and significant issue for electoral system research as ten countries were still using the Block Vote as their national electoral systems at the time of Reynolds and Reilly's survey in the mid 1990s, whereas the Multiple Majority Preferential problem seems to be an Australian phenomenon increasingly restricted to local government. To make the case against the Block Vote I return to the 11 vote example used above.

Table 6 provides a Block Vote count of the 11 votes in Table 1 for one, two and three positions available. In doing so, like Table 3 in relation to the Single Non Transferable Vote, Table 6 does not actually use the ordinal vote marking evident in Table 1. If there are two positions available the Block Vote gives all electors two votes and so the count takes 1s and 2s as marks of equal value, like ticks or crosses, creating 22 votes in all. If there are three positions available it gives all electors three votes and takes 1s, 2s and 3s as marks of equal value, creating 33 votes in all. The outcomes of the Block Vote count in this hypothetical, but realistic example, are quite bizarre. While C1 wins if there is only one position available, C2 and C3 win if there are two positions available and C2, C3 and C4 win if there are three positions available.

Table 6: Block Vote Count

1 Position Available	Votes	Result
C1	5	C1 Elected
C2	3	
C3	2	
C4	1	

2 Positions Available	Votes	Result
C1	5	
C2	6	C2 elected
C3	6	C3 elected
C4	5	

3 Positions Available	Votes	Result
C1	5	
C2	8	C2 elected
C3	11	C3 elected
C4	9	C4 elected

Whereas Multiple Majority Preferential *repeats* single member elections in the Quota family of electoral systems using the same votes many times over, the Block Vote *aggregates* votes in multi–member elections in the Plurality family. By doing so it gradually gives larger voting groups more say and smaller voting groups less say. In our 11 vote example, voters 1-5, who prefer C1, have a winning say when there is only one position available because the larger voting groups, voters 6-11, are divided about which of the other three candidates they prefer. However when there are two or more positions available and voters 6-11 can equally support two or more of their preferred candidates, these six voters start to dominate the count. This is truly a bizarre winner take all system, which switches from one form of disproportionality of outcome to an opposing one. Either voters 1-5 elect the single representative, or voters 6-11 elect two or three representatives. But never is representation shared between these two voting groups.

The Block Vote and Multiple Majority Preferential, I want to argue, are not acceptable democratic electoral systems at all. They are mathematically mistaken vote counting procedures which give multiple votes to electors when moving to multi-member districts. If Figure 2 is seen as having a normative as well as an analytic aspect, as categorizing

acceptable democratic electoral systems rather than just technically possible electoral procedures, then I argue that the Block Vote and Multiple Majority Preferential should be struck off.

Reworking Figure 2 After Questions 1 and 5

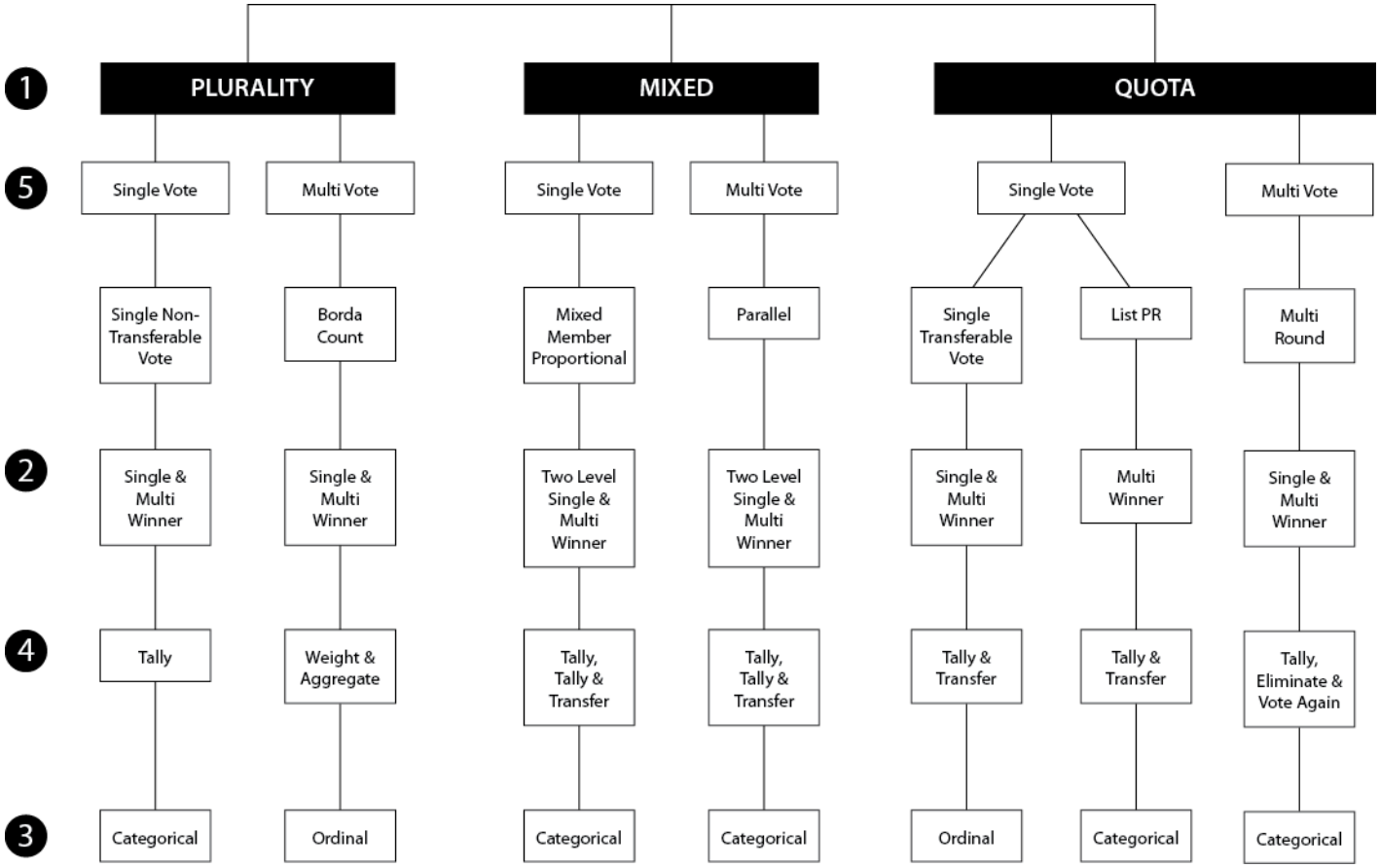
The previous two sections of this essay, while using the 11 vote example and Figure 2 to make an argument, also suggest some reworking of Figure 2 if it is seen as a taxonomy of acceptable democratic electoral systems. The last section suggested that both Multiple Majority Preferential and the Block Vote should be struck off as unacceptable and mistaken, while the previous section suggested that:

-within the Plurality family, First Past The Post and the Single Non Transferable Vote can be analytically collapsed into a single vote system which works in both single and multi-member districts, and

-within the Quota family, the Alternative Vote and Single Transferable Vote Proportional Representation can be analytically collapsed into a single vote system which works in both single and multi-member districts.

It now seems possible to me that acceptable democratic electoral systems might be able to be named after answering just two of our five simple questions: (q1)what is the rule for winning and (q5)how many votes do electors have? Figure 3 attempts this process, as a basis for further discussion. Within the Plurality family, Figure 3 simply names the single vote system the Single Non Transferable Vote, which covers but eliminates First Past The Post as a distinctly named system. Within the Quota family, Figure 3 simply names the single vote system the Single Transferable Vote, eliminating both the longer reference to Proportional Representation and the separately named Alternative Vote system.

Figure 3. Acceptable Democratic Electoral Systems arranged in Plurality, Quota and Mixed Families



There are also two other changes in Figure 3 on the multi-vote branches of the Plurality and Quota families in addition to the striking off of the Block Vote and Multiple Majority Preferential.

First, on the multi-vote branch of the Quota family I have changed the terminology from Two Round to Multi-Round system. This is because the procedure of 'tally, eliminate and vote again' until a predetermined quota is reached, which the French use for Presidential elections, could be applied several times over rather than just twice. It could also, with the appropriate quota, be applied to multiple as well as single winner elections.

Second, on the multi-vote branch of the Plurality family, which was empty after the Block Vote was struck off, I have now introduced another electoral system altogether known as the Borda Count. This is a system which Reynolds and Reilly did not name or place in Figure 1, but did discuss in their 1997 survey of the world's national electoral systems as it is used for parliamentary elections in tiny nation of Nauru (Reynolds and Reilly 1997: 39). When the multi-vote branch of the Plurality family of electoral systems was sitting empty, I was pushed to think about whether there was such an electoral system and it seemed to me that the Borda Count might fit the bill. Named after a French mathematician who devised it around 1770, this system uses ordinal vote markings in an interestingly different way which does not involve vote transfer. It gives electors multiple, but weighted votes which are indicated by the ordinal vote markings. Reynolds and Reilly (1997:39) suggest that in Nauru a No1 Vote is weighted at 1, a No2 vote at ½, a No 3 vote at 1/3, a No 4 votes at ¼, etc. These differentially weighted multiple votes are then *aggregated* in the counting process without any candidates ever being eliminated from the count. This is an interesting system which could, for example, in single member elections, strengthen the chances of a candidate who was the second choice of many electors.[vii] Whether the Borda Count can also work in multiple member elections, I leave as an open question. I wonder whether it might, in that circumstance, take us back towards the problems of the other multiple vote systems I have just argued against.

There is clearly much for political science to debate in Figures 3 and 2 and in Table 5, if these are accepted as useful graphic presentations of electoral system families and mathematics. Asking five simple questions about electoral systems has taken me on a intellectual journey through electoral science that I had not anticipated when I had my intuitive adverse reaction to the Northern Territory's local government electoral system. I never, for a moment, thought that in order to analyse and substantiate that adverse reaction I would be challenging and recasting the categories through which political science views electoral systems. But that, interestingly, is where I have ended up.

Where to with proportionality?

In one last step, I want to return to the term proportionality, which might appear to have lost out in the course of this essay, having been dropped from the naming of electoral system families and of the Single Transferable Vote electoral system. This is because proportionality, or the lack of it, is an *outcome* of electoral systems, not a constituent element interrogated by our five simple questions. But proportionality is an important outcome which does flow from the constituent elements of electoral systems and which political scientists rightly worry about. So what, analytically, can we say about proportionality?

Above I have noted that, in single member electorates, proportionality between votes cast and positions won is inevitably very poor. Only one candidate can win. This disproportionality is, of course, *within* a single electoral district and it does not disappear when electoral systems move to two or three members per district with single vote systems. There are still inevitably significant numbers of voters within such districts who do not elect a candidate of their choice. Reynolds and Reilly (1997: 90) note that the electoral systems which produce *high* levels of proportionality are those that use a *large* 'district magnitude', or number of members per district. This is analytically true with one proviso which is at the heart of this paper: the systems must in moving to a large district magnitude stay with a single vote. If they move to multiple votes, like Multiple Majority Preferential or the Block Vote, multi-member systems run the risk of simply reproducing or even compounding the disproportionality of single member systems.

Reynolds and Reilly (1997:91) also note that the move to larger district magnitudes has often, quite reasonably, been resisted in favour of 'local' representation. With this persistence over the years of local representatives in districts with single or small numbers of representatives, many historical inductive political scientists have accepted disproportionality *within* districts and focused instead on proportionality *across* districts, between parties rather than individual candidates. Their investigations have become focused on the extent to which disproportionality towards one party in some electoral districts is counteracted by disproportionality away from that party in other districts. The larger question of interest becomes the extent to which there is proportionality between votes and positions won on a party basis across the whole jurisdiction. This is reasonable approach given the importance of parties in national politics. It has also, as Reynolds and Reilly (1997: 55, 74) note, led to some interesting experimentation with parallel and mixed two-level systems in recent years, in which disproportionality at individual district level is combined with or counteracted by efforts to reflect proportionality at a higher geographic level. It is also where Lijphart's more minor elements of electoral systems, like boundaries are drawn, start to have a significant effect (Lijphart 1985: 7-8).

All this attention to proportionality issues between parties *across* electoral districts at the national or jurisdictional level is good inductive historical political science. But it also needs to be underpinned by solid analytic attention to the basic elements of electoral systems and proportionality issues *within* electoral districts. This has been my concern with proportionality in this essay.

Recalling the Journey and Stretching the Family Metaphor

I began this essay with an adverse intuitive reaction to an electoral system I first encountered in local government in Australia's Northern Territory. In attempting to analyse and substantiate that reaction, I found myself asking five simple questions and, in the process, struggling with political science's accepted family categorization of electoral systems. Through systematically answering the five questions I was recasting those families. Four electoral systems which had long been regarded as in different families

were recast as two sets of close siblings, even identical twins which were genetically the same and could be analytically combined. First Past The Post disappeared, analytically, into the Single Non Transferable Vote, while the Alternative Vote was similarly combined into the Single Transferable Vote. Both were placed in happy new, democratic, single-vote families. But the electoral system against which I had the initial adverse reaction, Multiple Majority Preferential, was argued to be an illegitimate child from an unacceptable, undemocratic, multi-vote family. So too was one other system, the Block Vote, to which I introduced along the way. What a journey of discovery into family genealogies this essay has turned out to be! I am now too close to my newly discovered families of electoral systems and their members to know what's right and wrong. It is time for others to reflect on the re-arranged families of electoral systems and whether particular families and systems are acceptably democratic.

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[i] This useful language of permissive and mandatory local government is used in relation to other local government systems in the Australian states in Power, Wettenhall and Halligan eds 1981 from page 6.

[ii] In response to an early version of Author reference 2011, a representative of the Proportional Representation Society of Australia (Victoria – Tasmania) pointed out that their preferred terminology for this system is Multiple Majority Preferential voting. They also pointed out that the terminology 'Exhaustive Preferential' is used by some authorities on electoral systems to describe a multi-round electoral system in which one representative is elected through a series of repeated votes in which the candidate with the least number of votes is eliminated between each round of voting.

[iii] This naming of the electoral system for the Australian Senate from 1919 to 1946 was pointed out to me by a referee for Author reference 2011, for which I am grateful. The source for this naming identified was Parliament of the Commonwealth of Australia 1959: 23.

[iv] If this quota is not a whole number it can be rounded down to the whole number below. This happens when there are an odd number of votes and the quota effectively becomes the next whole number >50%.

[v] The Multi Round system which the Proportional Representation Society of Australia (Victoria- Tasmania) likes to refer to as Exhaustive Preferential could be added on this branch of the taxonomic tree. See note ii.

[vi] Wikipedia has an interesting website on Electoral Methods divided into single winner and multiple winner systems. Under the Instant Runoff Voting/ Alternative Vote entry within the former it notes, towards the end of its commentary:

> When the single transferable vote (STV) system is applied to a single-winner election it becomes IRV. For this reason IRV is sometimes considered to be merely a limited form of STV. However, IRV is usually excluded from discussions of STV, because STV was designed for multi-seat constituencies, redistributes votes from both the top (winners) and bottom (dropped candidates), and produces broadly proportional results (depending on the number of seats per constituency); none of which need apply to IRV

This commentary suggests that some people have in the past already made the link between the Alternative Vote and STVPR that I make in Figures 2&3, but have been unable or unwilling to follow that link to its logical conclusion in the face of electoral science's common families of electoral systems in Figure 1.
If my analysis is correct, Wikipedia will need to re-arrange their Electoral Methods website following the logic of Figure 3.

[vii] Reynolds and Reilly's suggested weighting of votes in the Nauru version of the Borda Count is almost but not quite exponential. Wikipedia suggests a linear weighting of votes in the Borda Count. Different weightings of votes could significantly change possible outcomes in Borda Count systems and the way we might judge them.