Spatial Multizone Sound Field Reproduction Using Higher Order Loudspeakers

Debashish Chakraborty
U4610248

Supervised by
Dr. Akramus Salehin
Assoc. Prof. Thushara Abhayapala

November 2012

A thesis submitted in part fulfilment of the degree of
Bachelor of Engineering
Department of Engineering
Australian National University

This thesis is an account of research undertaken between February 2012 and November 2012 at the Research School of Engineering, College of Engineering and Computer Science, The Australian National University, Canberra, Australia.

It contains no material, which has been accepted for the award of any other degree or diploma in any university. To the best of the author's knowledge, it contains no material previously published or written by another person, except where due reference is made in the text.

Debashish Chakraborty
02 November 2012

ACKNOWLEDGEMENTS

I would like to thank Assoc. Prof. Thushara Abhayapala, for providing me with the opportunity to undertake this project. I would also like to thank Dr Akramus Salehin for constant guidance and support he has shown me throughout the year was instrumental in the development of this thesis and my enhanced understanding of research based engineering. For that I am truly grateful.

Thank you to Sandra Isabel Arcos-Holzinger and Arash Mahmoudzadeh for taking the time to proof-read this thesis. I would also like to extend a Thank You to all my fellow engineers. You have made this year, and the four before it, some of the most enjoyable of my life. I wish all of you the best of luck in your future endeavours.

I would like to thank my father, Purnendu Chakraborty, my mother, Anju Chakraborty, and my brother, Suvashish Chakraborty, for their support throughout my university career and more. Finally, I am grateful for the support and motivation provided by all my friends.

Abstract

Spatial multizone sound field reproduction aims to regenerate desired sound fields as multiple independent sound fields in isolated zones without unpremeditated interference within a zone from other zones. Such attempts over an extended region of open have proved to be quite arduous and intricate. At high frequencies, significantly large numbers of loudspeakers are required for the reproduction of the sound field over significant areas. This thesis is aimed at using higher order (up to Nth) loudspeakers, with polar responses up to $\cos(N\phi)$ and $\sin(N\phi)$ to resolve the high frequency reproduction issue.

The thesis includes the design of the system along with any equations derived to verify the performance of the system for arbitrary sound fields. The equations have been defined with respect to the frequency content of the sound fields and the order and number of sources used. Numerical simulations to verify the equations and corresponding results have also been included.

TABLE OF CONTENTS

Glossary of Terms

Symbol	Definition
2D	Two Dimensional
3D	Three Dimensional
$\mathbb{R}$	Real Numbers
$\mathbb{Z}$	Integers
c	Speed of Sound
w	Loudspeaker Weight
L	Number of Loudspeakers
r_L	Radius of Loudspeaker array
FT	Fourier Transform
IFT	Inverse Fourier Transform
DFT	Discrete Fourier Transform
DSP	Digital Signal Processing
f	Frequency
E	Normalised Expansion Error
$h_n^{(1)}(\cdot)$	Spherical Hankel Function of the First Kind
$h_n^{(2)}(\cdot)$	Spherical Hankel Function of the Second Kind
$J_n\ (\cdot)$	Ordinary Bessel Function of the First kind
$j_n\ (\cdot)$	Spherical Bessel Function of the First kind
$N_n\ (\cdot)$	Ordinary Bessel Function of the Second kind
$n_n\ (\cdot)$	Spherical Bessel Function of the Second kind
$P_{n\|m\|}\ (\cdot)$	Associated Legendre function
k	Wavenumber
$A_v^{(1)}(\cdot)$	Fourier Coefficient for the First Zone
$A_v^{(2)}(\cdot)$	Fourier Coefficient for the Second Zone

List of Figures

Chapter 1 Introduction

This chapter provides an introduction to the thesis topic. In Section 1.1, the background surrounding acoustic signal processing and sound field reproduction is introduced to provide context to the problem being addressed in this thesis. Section 1.2 describes the motivation for investigating the problem and then declares the aims of the thesis. In Section 1.3, a literature review of the topic is then included to establish a theoretical framework for the topic under discussion. Section 1.4 lists and briefs on the commercial opportunities from the solution of the sound field reproduction. Finally, Section 1.5 explains the outline and organisation of this thesis.

1.1 ACOUSTICS AND SOUND FIELD

Acoustics deals with transmission of pressure fluctuations through media as the result of vibrational impetus imparted to the media. It comprises of the realm of sound transmission through solids and fluids. The mechanical transmission of sound waves is due to vibrational energy passing from molecule to molecule. Acoustic propagation contrasts from electromagnetic propagation (i.e., involving light, x-rays etc.) in that the acoustic transmission can occur only through matter, whether solid or fluid. Sound waves cannot promulgate through vacuum.

The study of acoustics dates back to 3000 B.C. when a Chinese philosopher tried to relate the pitch of sound with the five fundamental elements: air, water, fire, wind and earth. After a period of intellectual dormancy that characterized the Middle Ages, the evolution of acoustics recommenced, with major contributions by Galileo, Leonardo Da Vinci, Newton, Laplace, Fermat, Huygens, Euler, the Bernoulli family, etc. The derivation of ultrasonics [1] (i.e., the sound extending beyond human hearing at the high end of the frequency range), which essentially occurred with the discovery of the piezoelectric effect by the Curie brothers, provided the means of generating and detecting ultra-high frequency sounds.

1.2 MOTIVATION AND THESIS AIMS

The ability to manipulate sound field within a given region of space is a fundamental issue in acoustic signal processing. The multi-frequency behaviour of sound waves augments its complexity; this is not commonly perceived in other signal processing problems [3]. Reproduction of a given sound field is possible using an array of loudspeakers. These sound reproduction systems aim to reproduce an arbitrary desired sound field within a region of space. Spatial multizone sound field reproduction has a huge range of applications such as simultaneous car entertainment systems, personal loudspeaker systems in shared office space and surround sound systems in exhibition centers as discussed in Section 1.4. These applications have managed to attract a lot of attention and research opportunities. The primary objective of this process is to provide listeners with personal individual sound environment without any physical barriers or using headphones. At high frequencies, significantly large numbers of loudspeakers are required for the reproduction of the sound field over significant areas discussed in further details in Section 1.3. Production of a reverberant field (due to exterior field, reflection from room surfaces) corrupts the sound field within the array, which adds to the limitation of using large number of loudspeakers in rooms. This objective is explored and investigated throughout the course of this thesis using higher order loudspeakers.

1.3 Literature Review

Sound reproduction systems have developed in both quality and efficiency since 1877 when the preliminary artificial sound reproduction systems came into existence with the invention of telephone by Bell and Watson [2]. Recent times have seen a myriad of methods for sound reproduction. An outline of the historically successful and modern advancements in sound reproduction is provided below as a timeline.

In 1931, Alan Blumlein developed the first reproduction system that placed sounds in a plane. Dolby Surround technology, the leading tool in the majority of the modern commercial sound reproduction systems (discussed in 1.4), is a slight extension of Blumlein’s original work as reproduced sounds are contained with 2D space [3]. His work involved a stereo reproduction system using a combination of delays and level differences between the loudspeakers to create the effect.

The subsequent key advancement in sound reproduction was the step towards sound field reproduction. Pressure variations around the mean air pressure can be used to describe acoustic sound fields. These pressure variations are attempted to reproduce by sound field reproduction systems. One of the ways to achieve this is by holographic reproduction by reconstructing the sound field over a region of space. On the other hand, binaural or transaural reproduction recreates the sound field at the location of the ears [4].

One of the initial holographic systems was suggested by Grenzo in 1972 [5] which later developed into the ambisonics system [6]. This system became the pioneer to encode height information in the sound field reproduction method. The ambisonics system uses first order spherical harmonic representation to reproduce sound field. A second order spherical harmonic representation has also been considered in [7]. Ambisonics systems are based on the assumption that sources radiate plan waves and that sound fields produced by the loudspeakers are plane waves at the listening position. These systems can be physical impractical as the sources and loudspeakers must be placed far from the listener. This assumption can be removed using spherical harmonic techniques as discussed below.

Using spherical harmonic techniques, each loudspeaker can be modeled as a point source. This system has no theoretical limit on the order of spherical harmonic reproduction. Such systems have also been used in the development of beamforming microphone arrays [8]. A comprehensive analysis of current sound field representation and reconstruction techniques along with a discussion of substantial psycho-acoustical sound reproduction experiments is presented in [4].

Wave field synthesis can also be used to solve holographic reproduction systems. Impulse responses are recorded along an array of microphones to obtain a sound field within a hall. The sound field is reproduced by feeding the impulse responses into an array of loudspeakers.

Following the approach with an array of loudspeakers, the loudspeaker weights can be calculated using least squares method []. This method may lead to errors or even lack of a solution if the matrix is poorly conditioned. In order to avoid matrix inversion, concept of theoretical continuous loudspeakers on a circle can be used to obtain the loudspeaker aperture function (a function of space variables and frequency) [9]. The aperture function is then approximated by a number of discrete loudspeakers to obtain the desired soundfield. This approach can be applied for three-dimensional sound field reproduction with spherical harmonics analysis using a spherical array [10]. However, this approach, to my understanding, provides an approximation rather than an absolute solution and thus specific zone calculations are difficult to implement.

Another approach to derive the loudspeaker signals required to reproduce the sound field is based on the Kirchhoff-Helmholtz (KH) integral formula. This method represents sound field produced within a volume of space by an indefinite density of monopole and normally oriented dipole sources on the surface of that volume with no exterior field. It requires the knowledge of pressure and normal velocity on the surface in order to reproduce the sound field. This was introduced by Berkhout in 1993 [11] and formed the basis of wave field synthesis (WFS).

Factors affecting the precision of sound reproduction include the wavelength and the size of the region over which reproduction is desired. In the 2D case, the number of required loudspeakers for wave number k and reproduction radius r is approximated by [12] $L_2 \approx 2kr + 1$, where $k = 2\pi f/c$, c is the speed of sound propagation of 340 m/s. This means a significantly large number of loudspeakers will be required for the reproduction over considerable areas. Significantly higher number of loudspeakers is required in the 3D case, approximately $L_3 \approx (kr+1)^2$ [13]. For reproduction over 0.1 m radius at 18 kHz requires 68 loudspeakers in the 2D case, while 1175 loudspeakers would be required in the 3D case. This renders the physical implementation of the method to be outright impossible.

As discussed in Section 1.2, production of reverberant field is also a prominent difficulty in sound reproduction systems. Following the KH approach, the exterior field can be diminished below the spatial Nyquist frequency [14] using variable directivity[1] loudspeakers. The specified frequency for 2D case can be expressed approximately as

$$f_1 = \frac{c(L-1)}{4\pi r_L}$$

where r_L is the radius of the loudspeaker array. At this particular frequency, the loudspeakers are approximately half a wavelength apart. However, exterior cancellation is only possible at low frequencies as the spatial Nyquist frequency is kept low to maintain a practical number of loudspeakers. If only fixed-directivity loudspeakers are offered, the aforementioned effect of reverberation can be decreased as the directivity to reverberation can be increased [15-18]. Conversely, digital pre-composition of the loudspeaker signals can reduce the reverberation effect as well.

Using loudspeakers with up to Nth order directivity allows reproduction over N times the bandwidth and produces a significantly attenuated exterior sound field [19]. Reproduction is feasible over approximately $2N$ times the bandwidth if the

[1] Directivity is the characteristic of how a loudspeaker sends sounds in different directions.

constraint on exterior cancellation of the field is removed. Thus a significant decrease in the number of loudspeakers units is feasible with the use of high order loudspeakers, at the expense of increased complexity in each unit.

1.4 Commercial Opportunities

In this section, the referred sound system is compared to existing sound reproduction technology and few of the many applications of this system are also included.

1.4.1 Current sound reproduction technology

Most of the current commercially available sound reproduction devices and technology are owned by Dolby Laboratories [20]. Dolby Digital, also known as AC-3, was introduced in 1992. They provided with a six-channel system, a '5.1' system comprising of five discrete full frequency range channels: left, centre, right, left surround and right surround along with a '.1' channel for low frequency effects. In addition, two channels have been added to provide the system with rear-left and rear-right surround channels.

Further abbreviations related to surround sound technology are DTS and THX. Like Dolby Digital, DTS is another multi-channel "lossy"[2] compression encoding system created by DTS Technology that contest with Dolby Digital. THX is a set of criteria that applies to the design of equipment used in sound reproduction systems. In order to be THX certified movies, audio decoders and amplifiers, loudspeakers and any sound reproduction technology must comply with these standards.

Current commercial reproduction systems provide the technology to store up to eight channels of digital data to be played through eight spatially located loudspeakers. These systems do not provide the audio producer with tools to record a spatial sound field or specify what audio should be heard through each loudspeaker. Current systems also assume that the loudspeakers are placed in pre-determined positions. This is generally not the case for maximum home installations.

A number of discrete channels are required by the sound reproduction system in order to efficiently describe the sound field. Although existing Dolby Digital and DTS technology could encode and store the data, enhanced processing power is necessary to calculate the loudspeaker audio signals. Recent research has led to systems that require a description of the sound field to be stored rather than audio

[2] A lossy compression is a data encoding scheme that compresses data by losing some of the data.

signals [21]. In such systems, the sound field description is used in conjunction with the details of the listeners' individual environment (the geometry and the number of loudspeakers) to produce the loudspeaker audio signals.

Although this system provides an optimal listening experience individualized for each listener, the number of loudspeakers required for sound field reproduction at high frequencies become physical unfeasible.

1.4.2 Applications

The applications of this system are aimed to provide listeners with individual sound field environments without using any physical barriers or using headphones. A few specified applications of this spatial sound field reproduction system are included.

One of the major applications of the project is simultaneous car entertainment system. In this case, driver can listen to one form of entertainment while the passengers sitting behind can indulge themselves in a different mode of entertainment. This functionality can be achieved without using any physical partitions or use of headphones.

In shared office spaces, one can use their own personal loudspeaker system within their cubicle without affecting other colleagues. Surround sound systems can be introduced in exhibition centers, where multiple speeches can be presented simultaneously without interference.

1.5 Thesis Outline

The objective of the thesis along with the motivation and applications has already been discussed in this chapter.

Chapter 2 presents the theoretical knowledge that forms the basis of the thesis. The definition of co-ordinate system, sound field and its multizone perspective along with a description of higher order sound sources with appropriate sources are available. The problem formulation is strictly defined at the end of the chapter.

Chapter 3 evolves in order to provide a solution to the problem surrounding the thesis. The two-dimensional sound reproduction system is responsible for placing sound around the listener. Initially a theoretical background on 2D sound fields and its representation in polar coordinates is detailed using appropriate equations with respect to both interior and exterior reproduced sound field cases. A description on how to model non-translated and translated higher order sources is then included, followed by method used for the reproduction of a desired sound field. Least square approach is used to derive the required loudspeaker weights and numerical simulations are conducted to verify the results as described in Chapter 4. An additional description of sound field reproduction using theoretical continuous loudspeakers is incorporated at the end.

Chapter 4 presents the numerical solutions and results obtained from implementing the equations developed in the previous chapter. Initially, numerical simulations of multizone sound field reproduction using higher order loudspeakers are implemented followed by relevant comparison of results. Variation of wavenumber is also implemented and the subsequent comparison of results is attached. Finally, numerical simulations and corresponding comparison of results are included.

Chapter 5 provides a summary of the results presented in the thesis. A list of few ideas for potential further research has been identified with a short description for each.

Chapter 2 Background Theory

This chapter presents the theoretical knowledge that forms the basis of the thesis. The definition of co-ordinate system, sound field and its multizone perspective along with a description of higher order sound sources with appropriate sources are available. The problem formulation is strictly defined at the end of the chapter.

2.1 THEORY OF ACOUSTICS

Acoustical theorems are formulated from the science of sound or vibrations through various media such as solids, liquids and gas. Variations in air pressure vibrating the eardrum in our ears leads to the sense of hearing. An arbitrary sound field can be described by the classical wave equation, which defines the spatial and temporal characteristics of a wave equation. The classical wave equation is [22, p 14]

$$\nabla^2 p(x,t) = \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2} \tag{2.1}$$

where

$$\text{div grad} = \nabla^2 = \sum_{j=1}^{D} \frac{\partial}{\partial x_j^2} \tag{2.2}$$

∇^2 is the *Laplacian* expressed in a chosen coordinate system, $p(\boldsymbol{x},t)$ is the scalar wave field, $\boldsymbol{x}$ is the position in $\mathbb{R}^D$, t is the time and c is the speed of sound. The wave equation thus essentially affirms that the amount of matter flowing into an elementary volume produces a corresponding change in pressure or density.

The wave equation can be expressed in numerous numbers of ways. Throughout the thesis the equation is expressed in terms of modes. **Modes** are orthogonal functions of spatial coordinates obtaining mathematical properties facilitating the analysis of arbitrary sound fields. The modal representation expresses a sound field in terms mathematical series. Truncation of such series provides a definite and necessary bound on the clarity of the sound field; extension of the distance from the center of the sound field can cause loss in clarity.

2.2 COORDINATE SYSTEM

Two-dimensional (2D) coordinate systems have been used throughout the thesis. The following coordinate system defines a point in 2D space from the origin r, the rise from the vertical axis θ. This system can be illustrated as

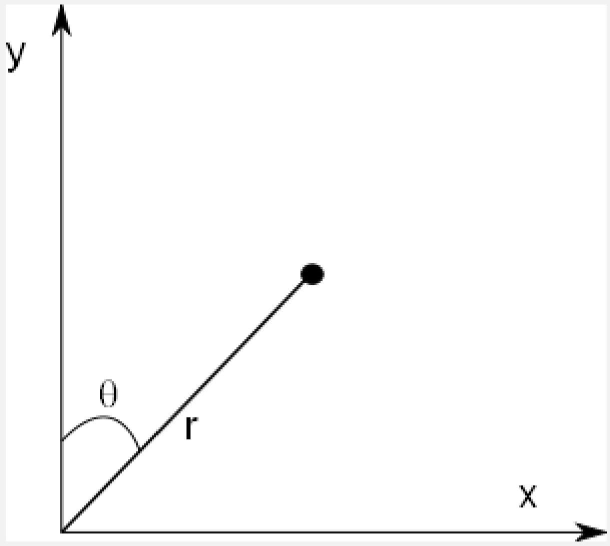

Figure 2-1 2D coordinate system

Conversion of spherical coordinates to Cartesian coordinates requires the usage of the following equations:

$$x = r\sin(\theta)$$
$$y = r\cos(\theta)$$

2.3 SOUND FIELD

As discussed in section 1.3, pressure and pressure gradient on the surface enclosing the volume can be used to reproduce a sound field within a source-free region.
The sound field is defined by $f(\boldsymbol{x}\,;k)$ and the pressure gradient can be formulated by can be calculated by taking a derivative of the sound field with respect to the distance x. The only term from (2.6) that depends on x is $h_n(x)$.

Thus, the derivative can be written as

$$\frac{d}{dx}\big(h_n(x)\big) = h_{n-1}(x) - \frac{n+1}{x}h_n(x) \tag{2.3}$$

The equation states that the derivative depends only on the lower order functions of the same kind. We can also deduce that the pressure gradient can be reproduced to the Nth order if a sound field is reproduced to the Nth order. Thus sound reproduction to Nth order at radius x_0 ensures that reproduced sound field inside the sphere at the same order.

2.4 2D MULTIZONE SOUND FIELD

2D multizone system forms the basis for the thesis over which the reproduced sound field is investigated. Assuming there are N non-overlapping spatial zones and corresponding height-invariant sound fields to reconstructed, a mutizone sound field system can be modelled in the following way.

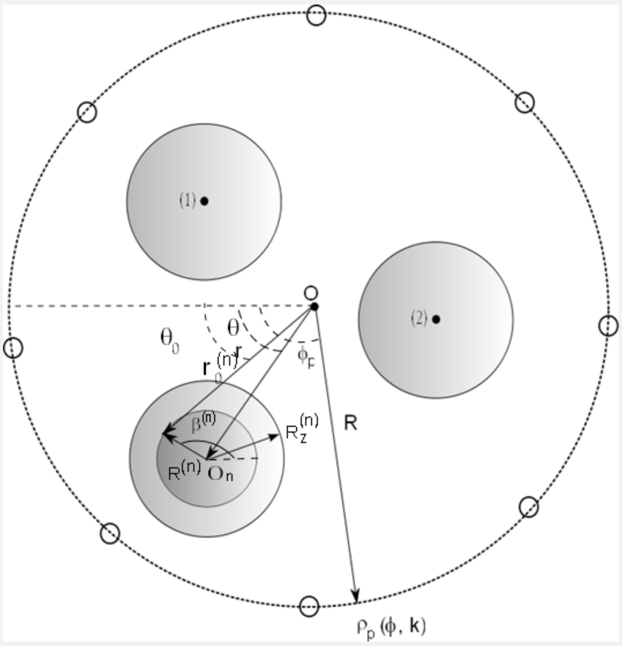

Figure 2-2 Multizone sound reproduction system

From the above figure, the radius of the nth spatial zone is $R_z^{(n)}$ with origin O_n located at $r_0^{(n)}$ from a global origin O. We can define an arbitrary observation point within this circular spatial zone as $(R^{(n)}, \beta^{(n)})$ located at a distance r away from the global origin. The loudspeakers can be placed on a circular array at a radius of $R_p \geq r$ from the global origin. The loudspeaker weight at angle φ is designated as $\rho_p(\phi, k)$ where k is the wavenumber, f is the frequency and c is the speed of propagation.

2.5 SOUND SOURCES: LOUDSPEAKERS

During the course of the thesis, it will be assumed that the loudspeakers are point sources located at various locations along the array. The loudspeakers discussed in this section are normal loudspeakers. Higher order loudspeakers and sound field properties are shown in the following section.

The equation to describe a point source at location z is [12]

$$f(\boldsymbol{x};k) = \frac{z\, e^{ikz} e^{ik\|\boldsymbol{z}-\boldsymbol{x}\|}}{\|\boldsymbol{z}-\boldsymbol{x}\|}, \boldsymbol{x} \neq \boldsymbol{z} \tag{2.4}$$

As the equation of the point source includes a normalisation term ($|z_l| \rightarrow \infty$), the point source field transforms to a plane wave field,

$$f(\boldsymbol{x};k) \rightarrow e^{ikx(\hat{\boldsymbol{z}}^{\boldsymbol{T}}.\hat{\boldsymbol{x}})} \tag{2.5}$$

Loudspeakers can be considered plane wave sources according to the ambisonics solution. This assumes that loudspeakers are located sufficiently far away from respective neighbouring loudspeakers.

Using spherical harmonics, the equation for the point source and plane wave fields are [22, p30]

$$\frac{z\, e^{ikz} e^{ik\|\boldsymbol{z}-\boldsymbol{x}\|}}{\|\boldsymbol{z}-\boldsymbol{x}\|} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} 4\pi X_n(kx)\, R_n(kz) Y_{nm}(\hat{\boldsymbol{x}}) Y_{nm}^{*}(\hat{\boldsymbol{z}}), \boldsymbol{x} < \boldsymbol{z} \tag{2.6}$$

and

$$e^{ikx(\hat{\boldsymbol{z}}^{\boldsymbol{T}}.\hat{\boldsymbol{x}})} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} 4\pi X_n(kx)\, Y_{nm}(\hat{\boldsymbol{x}}) Y_{nm}^{*}(\hat{\boldsymbol{z}}) \tag{2.7}$$

where

$$X_n(kx) = i^n j_n(kx) \tag{2.8}$$

and

$$R_n(kz) = ikze^{ikz} i^{-n} h_n^{(1)}(kz) \tag{2.9}$$

2.6 Higher Order Sound Sources

Higher order loudspeakers are used throughout the thesis. In [23], higher (Nth) order loudspeakers are shown capable of producing polar responses up to $\cos(n\emptyset)$ and $\sin(n\emptyset)$ for $n \in [01, \ldots, N]$. Higher order loudspeakers increase the bandwidth of precise reproduction by an order equal to the order. Subsequently, the radius of accurate reproduction is increased by the same factor. As mentioned in section 1.3, calibration of higher order sound reproduction system can be used to diminish reverberant field and thus reproduce a more precise sound field [24].

A 2D high order source located at the origin formulated in [19] as

$$p_n(r, \phi; k) = H_n(kr)e^{in\phi} \qquad (2.10)$$

where n is the order of the source.

Higher order loudspeakers are capable of radiating sound with far-field polar responses of the form $\cos(n\phi)$ and $\cos(n\phi)$. For investigation, we can consider a source at the origin, of radius R, with radial velocity

$$v_r(\phi) = V_0 \sum_{m=-\infty}^{\infty} \alpha_m e^{im\phi} \qquad (2.11)$$

where α_m are the Fourier series coefficients.

A more detailed expansion on practical high order sources has been included in [19].

2.7 PROBLEM FORMULATION

In the thesis, sound field reproduction is to be implemented over various different zones within the interior and exterior field using higher order sources (loudspeakers). The primary solution to the problem comprises of attaining the mathematical descriptions of arbitrary sound field using plane wave sources and the sound field produced by the loudspeakers. The desired sound field is then compared to the actual sound field in terms of the loudspeaker signals obtained. It is highly appropriate for the sound fields to complement each other in being similar. These signals can be utilised in the loudspeakers, thus leading to the creation of desired sound field.

Sound field produced by a 2D higher order source located at the origin can be given by

$$p_n(r,\phi;k) = w_n H_n^{(1)}(kr)e^{in\phi} \tag{3.5}$$

For the creation of the sound field with no exterior control, the weighted sum of the higher order fields is required to produce the desired field with arbitrary expansion coefficient $A_q^{(1,2)}$ described in greater detail in section 3.4.

In the following chapters, the problem formulated is solved assuming that no loudspeaker has been placed inside the spatial zone under consideration. Verification of the devised solution is provided as numerical simulation, under varying physical factors theoretically.

Chapter 3 Sound Field Reconstruction

This chapter evolves in order to provide a solution to the problem surrounding the thesis. The two-dimensional sound reproduction system is responsible for placing sound around the listener. Initially a theoretical background on 2D sound fields and its representation in polar coordinates is detailed using appropriate equations with respect to both interior and exterior reproduced sound field cases. A description on how to model non-translated and translated higher order sources is then included, followed by method used for the reproduction of a desired sound field. Least square approach is used to derive the required loudspeaker weights and numerical simulations are conducted to verify the results as described in Chapter 4. An additional description of sound field reproduction using theoretical continuous loudspeakers is incorporated at the end.

3.1 Expansion of 2D sound fields using polar coordinates

A solution for the wave equation can be determined using polar coordinates (r, ϕ), with radius $r = \sqrt{x^2 + y^2}$. For a homogenous region free of sources, $r < r_0$ the interior expansion of the sound field is given by

$$p(r, \phi; k) = \sum_{m=-\infty}^{\infty} A_m(k)\, J_m(kr)\, e^{im\phi} \tag{3.1}$$

where $A_m(k)$ are internal modal coefficients, $J_m(\cdot)$ is the mth order Bessel function and $i^2 = 1$.

The corresponding expansion of 2D sound field for $r > r_0$ exterior to sound sources can be represented as

$$p(r, \phi; k) = \sum_{m=-\infty}^{\infty} B_m(k)\, H_m^{(1)}(kr)\, e^{im\phi} \tag{3.2}$$

where $B_m(k)$ are external modal coefficients, $H_m^{(1)}(\cdot)$ is the mth order Hankel order of the first kind, assuming a harmonic time dependence $\exp(-i\omega t)$.

We can now describe the 2D sound field production by loudspeakers with loudspeaker position $(r_s,\ \phi_s)$.

For a normal zeroth order loudspeaker, the sound field produced is

$$p(r,\phi;k) = \ w\, H_0^{(1)}(k\, \|\boldsymbol{r} - \boldsymbol{r_s}\|)$$

$$= \ w \sum_{m=-\infty}^{\infty} J_m(kr)\, H_m^{(1)}(kr_s)\, e^{im\phi}\, e^{-im\phi_s} \tag{3.3}$$

for $r < r_s$, where w is the driving weight applied to the loudspeaker

In case of $r > r_s$, the sound field can be written as

$$p(r,\phi;k) = \ w \sum_{m=-\infty}^{\infty} J_m(kr_s)\, H_m^{(1)}(kr)\, e^{im\phi}\, e^{-im\phi_s} \tag{3.4}$$

3.2 HIGHER ORDER LOUDSPEAKERS

3.2.1 Modelling Higher Order Loudspeakers

Assuming that no loudspeaker has been placed inside the spatial zone under consideration, sound field produced by a 2D higher order source located at the origin can be given by

$$p_n(r,\phi;k) = \ w_n H_n^{(1)}(kr) e^{in\phi} \tag{3.5}$$

where order of the source index n can be distinguished from the expansion coefficient m. Although the aforementioned form has been used to generate sound fields, the magnitude of the equation tends to infinity as kr tends to zero and correspondingly physical implementation of such a sound field becomes unreasonable.

A more detailed expansion on practical high order sources has been included in [19]. It has been deduced in the paper that low magnitude of the high order responses at low frequencies can be ignored since higher order modes are only required at high frequencies while low frequencies can be implemented well using sound reproduction systems with omnidirectional loudspeakers. Moreover, we have to note the possibility of aliasing while implementing practical higher order source using discrete drivers beyond a finite frequency.

3.2.2 Modelling Translated Higher Order Sources

The sound field produced by an ideal higher order source positioned at (r_s, ϕ_s) can be expressed relative to the origin using the cylindrical addition theorem [25, Property 19],

$$p_n(r, \phi, r_s, \phi_s; k) = w_n H_n^{(1)}(k\boldsymbol{r_0}) e^{in\varphi_s}$$
$$= \begin{cases} w_n \sum_{m=-\infty}^{\infty} J_m(kr) H_{m+n}(kr_s) e^{im\phi} e^{-im\phi_s}, & r < r_s \\ w_n \sum_{m=-\infty}^{\infty} J_{m+n}(kr_s) H_m(kr) e^{im\phi} e^{-im\phi_s}, & r > r_s \end{cases} \quad (3.6)$$

where $\boldsymbol{r_0} = \|\boldsymbol{r} - \boldsymbol{r_s}\|$ and φ_s is the angle measured between $\boldsymbol{r_0}$ and source vector $\boldsymbol{r_s}$,

$$\varphi_s = \arctan\left[\frac{y_{rot}}{\boldsymbol{r_s} - x_{rot}}\right] \quad (3.7)$$

In the above equation,

$$x_{rot} = x\cos(\phi_0) + y\sin(\phi_0)$$
$$y_{rot} = -x\sin(\phi_0) + y\cos(\phi_0)$$

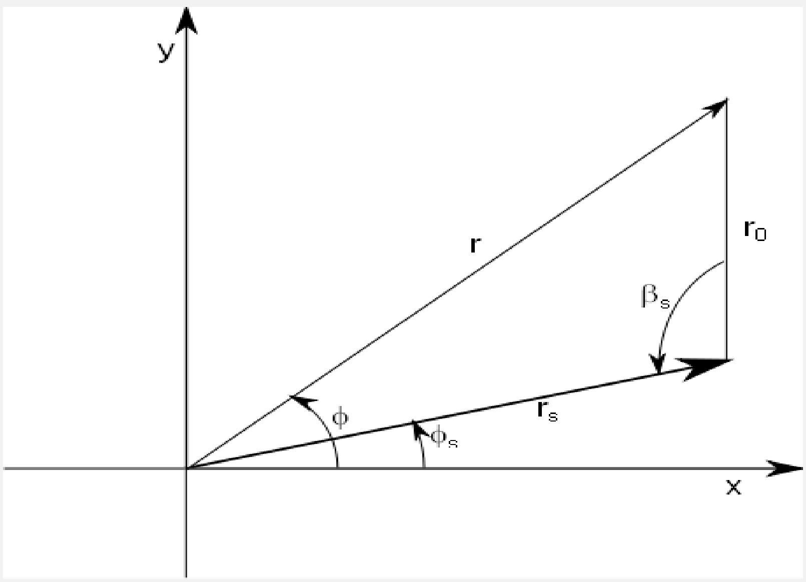

Figure 3-1 Higher order source geometry for source vector r_s and field vector r

From Eq. (3.6), the single higher order source has an expansion to the origin represented by

$$p_N(r,\phi,r_s,\phi_s;k) = \sum_{n=-N}^{N} w_n \begin{cases} \sum_{m=-\infty}^{\infty} J_m(kr)H_{m+n}(kr_s)\, e^{im\phi}\, e^{-im\phi_s}], r < r_s \\ \sum_{m=-\infty}^{\infty} J_{m+n}(kr_s)H_m(kr)\, e^{im\phi}\, e^{-im\phi_s}], r > r_s \end{cases} \tag{3.8}$$

The coefficient translation theorem and triangular identity used in this section are as follows.

Theorem 1

Suppose $\left\{\alpha_n^{(1)}(k)\right\}$ is a set of coefficients of a sound field in a coordinate system with origin at $(r_0,\theta^{(12)})$ with respect to another coordinate system, where both the coordinate systems have the same orientation. The sound field coefficients with respect to second coordinate system can be expressed as

$$\alpha_n^{(2)}(k) = \alpha_n^{(1)}(k) * T_n^{(12)}\left(r_0,\theta^{(12)};k\right) \tag{3.9}$$

where $T_n^{(12)}\left(r_0,\theta^{(12)};k\right) \triangleq J_n(kr_0)e^{-jn\theta^{(12)}}$ defines the translation course from the first zone with respect to the coordinate system of the second zone. '*' denotes convolution.

The proof is attached in Appendix A.

Theorem 2

Triangular Translation Identity [33] can be derived for the three co-ordinate system as follows:

Suppose $T_m^{(21)}$, $T_m^{(20)}$ and $T_m^{(01)}$ are the translational operators between co-ordinate systems 0, 1 and 2 as defined by Theorem 1.

$$T_m^{(21)} = T_m^{(20)} * T_m^{(01)} \tag{3.10}$$

According to Translational Theorem 1,

$$T_m^{(20)} = J_m\left(kr_{z_2}\right)e^{-jm\theta_{z_2}} \tag{3.11}$$

$$T_m^{(01)} = J_m\left(kr_{z_1}\right)e^{-jm(\theta_{z_1}+\pi)} \tag{3.12}$$

$$T_m^{(21)} = J_m\left(kr_{z_0}\right)e^{-jm(\theta^{(21)}+\pi)} = J_m(kr_0)e^{jm(\theta^{(21)})} \tag{3.13}$$

Since

$$T_m^{(20)} * T_m^{(01)} = \sum_{m=-\infty}^{\infty} J_m\left(kr_{z_2}\right)e^{-jm\theta_{z_2}}J_{n-m}\left(kr_{z_1}\right)e^{-j(n-m)(\theta_{z_1}+\pi)} \tag{3.14}$$

Using the Translation Identity [25] ,

$$T_m^{(20)} * T_m^{(01)} = J_m(kr_0)e^{-jm\theta^{(12)}} = J_m(kr_0)e^{jm\theta^{(21)}} = T_m^{(21)} \tag{3.15}$$

3.3 NORMALISED TRUNCATION ERROR

Near the source radius $r = r_s$, high mode orders compared to the aforementioned ones are required to describe the sound field in both interior and exterior cases. The order required is greater at higher orders n. This can be verified using the angle averaged normalized truncation error of Eq. 3.8

$$\bar{\varepsilon}_M = \frac{\int_0^{2\pi} |p_n(r,\phi,r_s,\phi_s;k) - p_{nM}(r,\phi,r_s,\phi_s;k)|^2 d\phi}{\int_0^{2\pi} |p_n(r,\phi,r_s,\phi_s;k)|^2 d\phi} \tag{3.16}$$

where $p_{nM}(r,\phi,r_s,\phi_s;k)$ is the Mth order truncation form of Eq. 3.8.

The truncation error can be thus be represented as

$$\bar{\varepsilon}_M = \begin{cases} 1 - \dfrac{\sum_{m=-M}^{M} J_m^2(kr)\ |H_{m+n}(kr_s)|^2}{\sum_{m=-\infty}^{\infty} J_m^2(kr)\ |H_{m+n}(kr_s)|^2}, & r < r_s \\ 1 - \dfrac{\sum_{m=-M}^{M} J_m^2(kr)\ |H_{m+n}(kr)|^2}{\sum_{m=-\infty}^{\infty} J_m^2(kr)\ |H_{m+n}(kr)|^2}, & r > r_s \end{cases} \tag{3.17}$$

Further investigation and verification is available in [19].

3.4 2D SOUND FIELD REPRODUCTION

3.4.1 Movable Sweetspot

Methods of sound field reproduction over an extended receiver area generally use a large number of loudspeakers surrounding the receiver area.

Sweet spot generally refers to the optimal physical listening area in a sound field where all the outputs of the loudspeaker system overlap and the sound field reproduction is most effective. Both numerical [26] and analytical [27] approaches can be implemented to optimize the sound field reproduction.

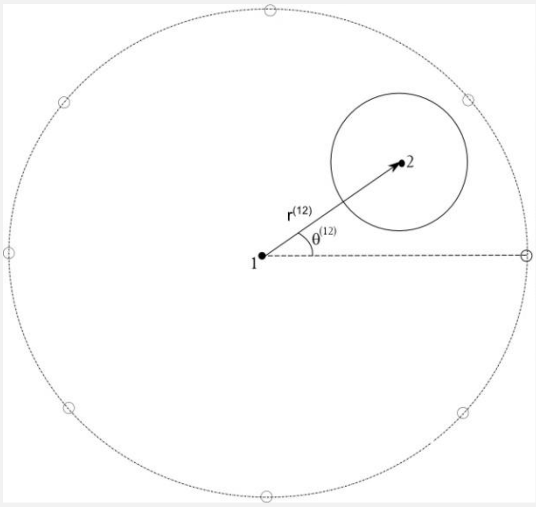

Figure 3-2 Moveable Sweetspot with higher order loudspeakers

With the implementation of moveable sweet spots, listeners will be able to relocate the location of reproduced sound field without physically repositioning of the loudspeaker system. Thus, this provides flexibility of sensor placement with some amount of freedom.

In case of no translation, the sound field produced can be represented as

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty} J_m(kr)H_{m+n}(kr_s)e^{im(\phi-\phi_l)}$$
$$= \sum_{v=-V}^{V} A_v(k)\, J_v(kr)\, e^{iv\phi} \tag{3.18}$$

With translation, the corresponding sound field becomes

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty} J_m\left(kr^{(1)}\right)H_{m+n}(kr_s)e^{im(\phi^{(1)}-\phi_l)}$$
$$= \sum_{v=-V}^{V} A_v^{(2)}(k)\, J_v\left(kr^{(2)}\right)\, e^{iv\phi^{(2)}} \tag{3.19}$$

Using triangular translation identity and coefficient translation theorem, this can be written as

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty} J_p\left(kr^{(12)}\right)e^{-ip\theta^{(12)}} J_{m+p}(kr^{(2)})e^{i(m+p)\theta^{(2)}} H_{m+n}(kr_s)e^{-im\phi_l} \tag{3.20}$$

Let’s substitute $m + p = q$, the sound field becomes

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} J_q\left(kr^{(2)}\right) e^{iq\theta^{(2)}} J_{q-m}\left(kr^{(12)}\right)e^{-i(q-m)\theta^{(12)}} H_{m+n}(kr_s)e^{-im\phi_l}$$
$$= \sum_{v=-V}^{V} A_v^{(2)}(k)\, J_v\left(kr^{(2)}\right)\, e^{iv\phi^{(2)}} \tag{3.21}$$

Simplifying using Fourier Transform properties (Theorem 3):

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\sum_{m=-\infty}^{\infty} J_{q-m}\left(kr^{(12)}\right)e^{-i(q-m)\theta^{(12)}}H_{m+n}(kr_s)e^{-im\phi_l} = A_q^{(2)}(k) \quad (3.22)$$

Theorem 3

When $\sum_q a_q e^{iq\theta} = \sum_z b_z e^{iz\phi}$, then $a_q = b_q$

Proof

Suppose the sum of the Fourier transform coefficients are equal

$$\sum_q a_q e^{iq\theta} = \sum_z b_z e^{iz\phi}$$

$$\Rightarrow \int_0^{2\pi}\sum_q a_q e^{iq\theta}\, e^{-il\theta}d\theta = \int_0^{2\pi}\sum_z b_z e^{iz\phi}\, e^{-il\theta}d\theta$$

$$= \begin{cases} a_l \; if \; l = q \\ 0 \; if \; l \neq q \end{cases} = \begin{cases} b_l \; if \; l = z \\ 0 \; if \; l \neq z \end{cases}$$

$$\therefore a_l = b_l$$

Thus, after further simplification

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, J_{n,l}^{(2)}(q,k) = A_q^{(2)}(k) \quad (3.23)$$

where

$$J_{n,l}^{(2)}(q,k) = \sum_{m=-\infty}^{\infty} J_{q-m}\left(kr^{(12)}\right)e^{-i(q-m)\theta^{(12)}}H_{m+n}(kr_s)e^{-im\phi_l} \quad (3.24)$$

We can solve the equation using matrix formation

$$\boldsymbol{J}\, w = \boldsymbol{a} \quad (3.25)$$

where $\boldsymbol{J}$ is $2M + 1$ by $(2N + 1)L$ matrix, w is a $(2N + 1)L$ by one vector of weights, and $\boldsymbol{a}$ is a $2M + 1$ vector of interior coefficients. The solution for the equation can be written in the form

$$w = \boldsymbol{J}^H \,[\boldsymbol{J}\boldsymbol{J}^H + \lambda \boldsymbol{I}]^{-1} \boldsymbol{a} \tag{3.26}$$

where $\boldsymbol{I}$ is the $2M + 1$ by $2M + 1$ identity matrix, exists for $2M + 1 \le (2N + 1)L$.

The sound field can thus be reproduced by Eq (3.8)

$$p_N(r, \phi, r_s, \phi_s; k) = \sum_{n=-N}^{N} w_n \begin{cases} \displaystyle\sum_{m=-\infty}^{\infty} J_m(kr) H_{m+n}(kr_s)\, e^{im\phi}\, e^{-im\phi_s}], r < r_s \\ \displaystyle\sum_{m=-\infty}^{\infty} J_{m+n}(kr_s) H_m(kr)\, e^{im\phi}\, e^{-im\phi_s}], r > r_s \end{cases}$$

3.4.2 Multizone Reproduction

As there are sufficient equations and information available to reproduce sound fields from desired sound fields, we can consider reproduction with multiple zones. In this case, interior field reproduction with no exterior control has been investigated.

In order to achieve such a field, the weighted sum of the higher order source fields is required to produce a desired interior field with arbitrary interior expansion coefficient A_q.

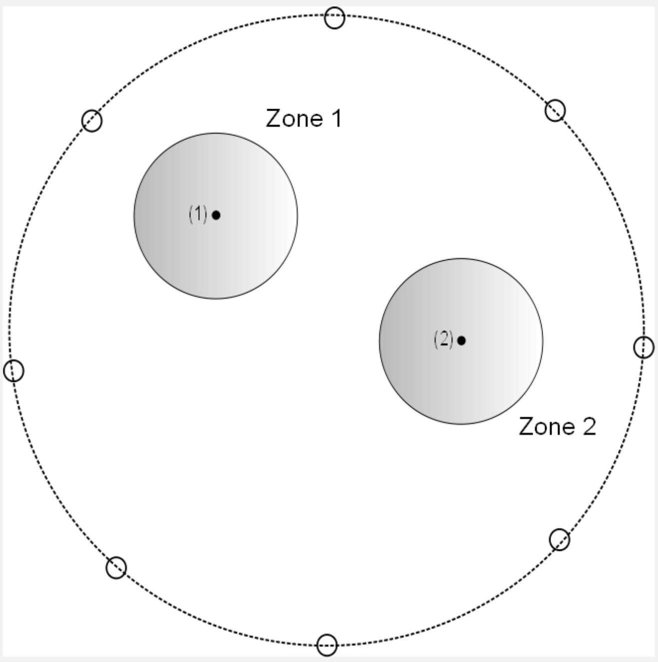

Figure 3-3 Multizone sound field with higher order loudspeakers

Instead of moving to a global sound field, the equation is put as a set of simultaneous satisfying

(1)

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, J_{n,l}^{(2)}(q,k) = A_q^{(2)}(k), \qquad -Q^{(2)} < q < Q^{(2)} \tag{3.27}$$

(2)

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, J_{n,l}^{(3)}(q,k) = A_q^{(3)}(k), \qquad -Q^{(3)} < q < Q^{(3)} \tag{3.28}$$

We can solve the equation using matrix formation as Eq (3.25)

$$\boldsymbol{J}\,w = \boldsymbol{a} \tag{3.25}$$

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}^{(\boldsymbol{2})} \\ \boldsymbol{J}^{(\boldsymbol{3})} \end{bmatrix}, \qquad \boldsymbol{a} = \begin{bmatrix} \boldsymbol{a}^{(\boldsymbol{2})} \\ \boldsymbol{a}^{(\boldsymbol{3})} \end{bmatrix} \tag{3.29}$$

where $\boldsymbol{J}^{(\boldsymbol{n})}$ is $2M + 1$ by $(2N + 1)L$ matrix, w is a $(2N + 1)L$ by one vector of weights, and $\boldsymbol{a}^{(\boldsymbol{n})}$ is a $2M + 1$ vector of interior coefficients. The solution for each separate can be written in the form

$$\boldsymbol{w} = \begin{bmatrix} \boldsymbol{w}^{(2)} \\ \boldsymbol{w}^{(3)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}^{(2)H}\,[\boldsymbol{J}^{(2)}\boldsymbol{J}^{(2)H} + \lambda\boldsymbol{I}]^{-1}\boldsymbol{a}^{(2)} \\ \boldsymbol{J}^{(3)H}\,[\boldsymbol{J}^{(3)}\boldsymbol{J}^{(3)H} + \lambda\boldsymbol{I}]^{-1}\boldsymbol{a}^{(3)} \end{bmatrix} \tag{3.30}$$

where $\boldsymbol{I}$ is the $2M + 1$ by $2M + 1$ identity matrix, exists for $2M + 1 \ \leq (2N + 1)L$.

This equation will have a solution as long as $\left(Q^{(2)} + \ Q^{(3)}\right) < NL$ and the matrix $\boldsymbol{J}$ has at least the rank

$$rank\ (\boldsymbol{J}) \geq 2\left(Q^{(2)} + \ Q^{(3)}\right) + 1$$

The sound field can thus be reproduced by Eq (3.8)

$$\boldsymbol{p}_N(r, \phi, r_s, \phi_s; k) = \sum_{n=-N}^{N} \boldsymbol{w}_n \begin{cases} \displaystyle\sum_{m=-\infty}^{\infty} J_m(kr) H_{m+n}(kr_s)\, e^{im\phi}\, e^{-im\phi_s}], r < r_s \\ \displaystyle\sum_{m=-\infty}^{\infty} J_{m+n}(kr_s) H_m(kr)\, e^{im\phi}\, e^{-im\phi_s}], r > r_s \end{cases}$$

3.4.3 Multiple-array based Multizone Reproduction

The investigation can be further extended to multiple array based loudspeaker systems. This means exterior field of respective loudspeaker arrays and their subsequent effects on the other sound field need to be considered, especially the interference effects of a particular sound field on the other.

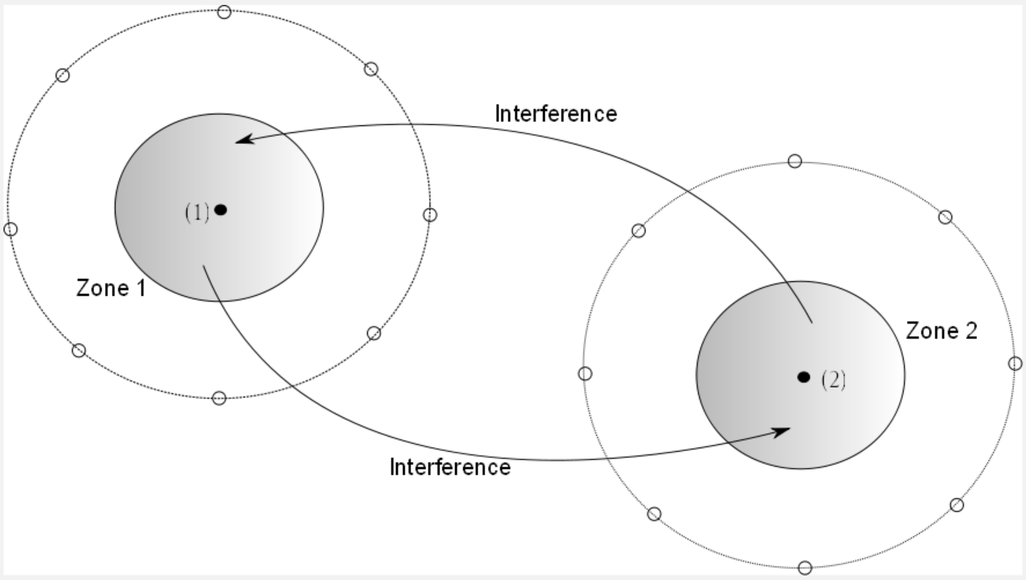

Figure 3-4 Multiple Array Based Multizone Reproduction

In order to investigate the required sound fields, process similar to the section 3.4.2 can be implemented.

The weighted sum of the higher order source fields is required to produce a desired field with arbitrary interior expansion coefficient A_q.

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, J_{n,l}^{(1)}(q,k) = A_q^{(1)}(k) \tag{3.31}$$

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, J_{n,l}^{(2)}(q,k) = A_q^{(2)}(k) \tag{3.32}$$

Considering the exterior field and using triangular translation identity and coefficient translation theorem, the sound field for the Zone 1 with respect to above figure can be written as

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty} J_m\left(kr^{(1)}\right)H_{m+n}(kr_s)e^{im(\phi^{(1)}-\phi_l)}, \qquad r > r_s \qquad (3.33)$$

Similarly, considering translation theorems with respect to Zone 1 and Zone 2, the sound field can be expressed as

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty} J_{m+n}(kr_s) \sum_{p=-\infty}^{\infty} J_p\left(kr^{(12)}\right) H_{m+p}\left(kr^{(2)}\right)e^{-ip\theta^{(12)}}e^{i(m+p)\theta^{(2)}}e^{-im\phi_l} \quad (3.34)$$

Let's substitute $m + p = q$, the sound field becomes

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l} \sum_{m=-\infty}^{\infty} J_{m+n}(kr_s) \sum_{q=-\infty}^{\infty} J_{q-m}\left(kr^{(12)}\right) H_q^{(1)}\left(kr^{(2)}\right)e^{-i(q-m)\theta^{(12)}}e^{iq\theta^{(2)}}e^{-im\phi_l}$$
$$= \sum_{v=-\infty}^{\infty} B_v(k)H_v^{(1)}\left(kr^{(12)}\right)e^{iv\theta^{(2)}} \qquad (3.35)$$

Using theorem 3, as described in section 3.4.1, the equation can be further simplified

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, J_{q-m}\left(kr^{(12)}\right)e^{-i(q-m)\theta^{(12)}}e^{-im\phi_l} = B_q(k) \qquad (3.36)$$

$$\sum_{l=1}^{L}\sum_{n=-N}^{N} w_{n,l}\, H_{n,l}(q,k,\phi_l) = B_q \qquad (3.37)$$

where

$$H_{n,l}(q,k,\phi_l) = \sum_{m=-\infty}^{\infty} J_{m+n}(kr_0)J_{q-m}\left(kr^{(12)}\right)e^{-im\phi_l}e^{-i(q-m)\theta^{(12)}} \quad (3.38)$$

We can solve the equation using matrix formation

$$\boldsymbol{J}\, w = \boldsymbol{a}$$

where $\boldsymbol{J}$ is $2M + 1$ by $(2N + 1)L$ matrix, w is a $(2N + 1)L$ by one vector of weights, and $\boldsymbol{a}$ is a $2M + 1$ vector of interior coefficients.

The solution for the equation can be written in the form

$$w = \boldsymbol{J}^H \, [\boldsymbol{J}\boldsymbol{J}^H + \lambda \boldsymbol{I}]^{-1} \boldsymbol{a}$$

where $\boldsymbol{I}$ is the $2M + 1$ by $2M + 1$ identity matrix, exists for $2M + 1 \le (2N + 1)L$.

The sound field can thus be reproduced by Eq (3.8)

$$p_N(r, \phi, r_s, \phi_s; k) = \sum_{n=-N}^{N} w_n \begin{cases} \sum_{m=-\infty}^{\infty} J_m(kr) H_{m+n}(kr_s)\, e^{im\phi}\, e^{-im\phi_s}], r < r_s \\ \sum_{m=-\infty}^{\infty} J_{m+n}(kr_s) H_m(kr)\, e^{im\phi}\, e^{-im\phi_s}], r > r_s \end{cases}$$

This theoretical implementation is outside the bounds of the thesis goals and thus, numerical simulation was not executed for this section. Nonetheless, potential for further research on multiple-array based sound field reproduction can be highly valuable for real world applications as interference patterns concerning exterior field is considered for this phenomenon.

3.5 REPRODUCTION USING THEORETICAL CONTINUOUS LOUDSPEAKERS

During the initial stage of the project, the solution of the formulated problem as discussed in 2.7 was being sought through the implementation of theoretically continuous loudspeakers based on [9]. The techniques and the theory were mostly based on normal loudspeakers so it was later decided to consider polar responses instead as demonstrated in sections 3.1 – 3.4. The formulation behind the theoretically continuous loudspeakers design has been briefly described below

All the parameters used in the following section have been defined above and consistency is maintained throughout the entire document.

3.5.1 Desired Sound Field

At a point $\boldsymbol{x} \equiv (\|\boldsymbol{x}\|, \phi)$ from origin O, the desired sound field can be expressed as

$$S^d(\boldsymbol{x}, k) = \sum_{m=-M}^{M} \alpha_m^{(d)}(k) J_m(k\|\boldsymbol{x}\|) e^{im\phi_x} \tag{3.39}$$

3.5.2 Actual Sound Field

Circular continuous loudspeakers function using an aperture function $\rho(\phi, k)$. If the loudspeakers have been placed with radius R, with none of them inside the spatial zone under consideration, the sound field reproduced within the circular zone can be written as

$$S^a(\boldsymbol{x}, k) = \int \rho(\phi, k)\ \frac{i}{4}\ H_0^{(1)}(k\|R\widehat{\boldsymbol{\phi}} - \boldsymbol{x}\|) d\phi_x \tag{3.40}$$

Given that $R > \|\boldsymbol{x}\|$, the addition theorem for cylinder harmonics [34] can be used to derive

$$H_0^{(1)}(k\|R\widehat{\boldsymbol{\phi}} - \boldsymbol{x}\| = \sum_{m=-\infty}^{\infty} H_m^{(1)}(k\|R\|) e^{-im\phi} J_m(k\|\boldsymbol{x}\|) e^{im\phi_x} \tag{3.41}$$

Fourier series expansion can be used to express

$$\rho(\phi, k) = \sum_{m=-\infty}^{\infty} \beta_m(k)\, e^{im\phi} \tag{3.42}$$

where $\beta_m(k)$ are Fourier coefficients.

Therefore, the sound field can be expressed as

$$S^a(\boldsymbol{x},k) = \sum_{m=-\infty}^{\infty} \beta_m(k)\frac{i}{2}\pi\, H_m^{(1)}(k\|R\|)\, J_m(k\|\boldsymbol{x}\|)e^{im\phi_x} \qquad (3.43)$$

3.5.3 Designing the Continuous Loudspeaker System

In order to derive the loudspeaker aperture function, the desired sound field is equated to the actual sound field.

Using (3.40) and (3.41), we can develop the Fourier coefficient

$$\beta_m(k) = \frac{2\alpha_m^{(d)}(k)}{i\pi H_m^{(1)}(k\|R\|)}\,, m = -M, \dots, M \qquad (3.44)$$

The aperture function of the theoretical continuous loudspeakers can thus be written as

$$\rho(\phi,k) = \frac{2\alpha_m^{(d)}(k)}{i\pi H_m^{(1)}(k\|R\|)}e^{im\phi} \qquad (3.45)$$

The aperture function can be used to reproduce the sound field. Numerical solutions derived from the above theoretical evaluation are implemented in section 4.3.

Chapter 4 Simulation and Results

This chapter presents the numerical solutions and results obtained from implementing the equations developed in Chapter 3. The simulations were executed using Mathworks MATLAB. The codes of the scripts used to achieve the simulations below are included in the Appendix. Initially, numerical simulations of multizone sound field reproduction using higher order loudspeakers are implemented followed by relevant comparison of results. Variation of wavenumber is also implemented and the subsequent comparison of results is attached. Finally, numerical simulations and corresponding comparison of results are included.

4.1 Reproduction of 2D sound field using higher order loudspeakers

Initially, the sound fields, to be reproduced using higher order loudspeakers, need to be created. Figure 4.1 and 4.2 illustrates the two sound fields that are going to be reproduced. The angles mentioned throughout this section are with respect to the horizontal x-axis. During the series of simulations in this section, it has been assumed an array of L = 8 higher order loudspeakers. The loudspeakers are placed in a circular with a radius of 1.5m.

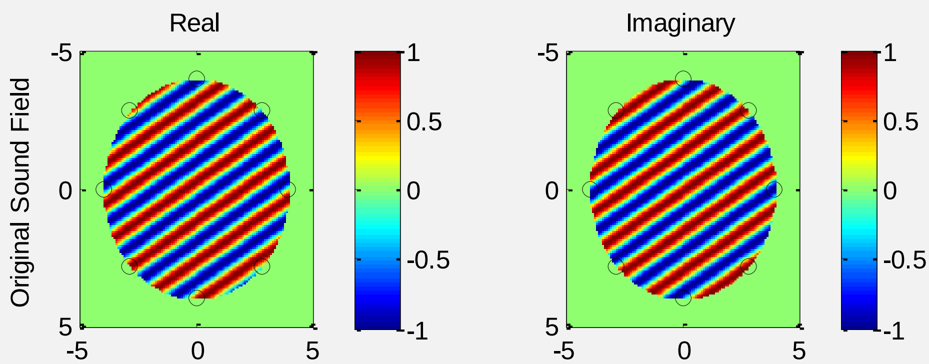

Figure 4-1 Sound field produced using higher order loudspeakers at 600 angle, f = 270 Hz, k=5, radius 4m

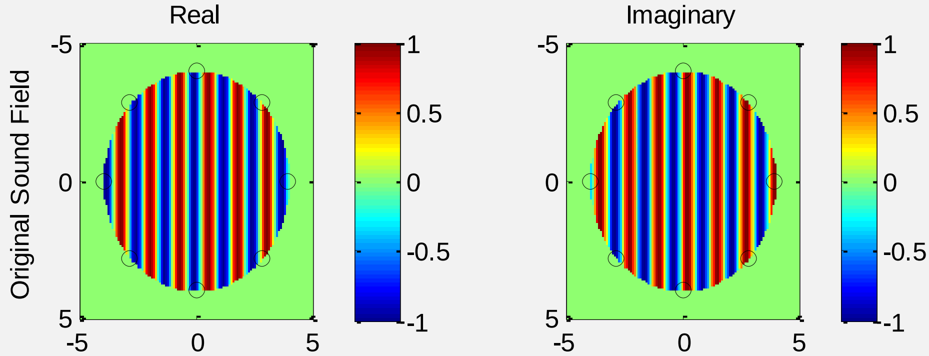

Figure 4-2 Sound field produced using higher order loudspeakers at 900 angle, f = 270 Hz, k=5, radius 4m

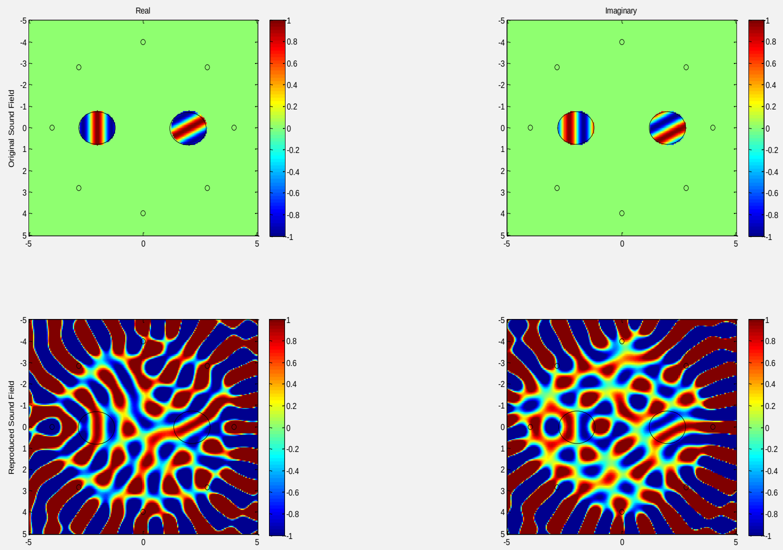

Figure 4-3 Reproduction of a 2-D two-zone sound field using higher order loudspeakers, k = 5, f= 270 Hz, radius = 4m

As the entire reproduced sound field is redundant, the respective zones which need to be reproduced for the desired effect are extracted. The resulting sound field has been shown in Figure 4.4 and subsequent comparison has been made to the desired sound field.

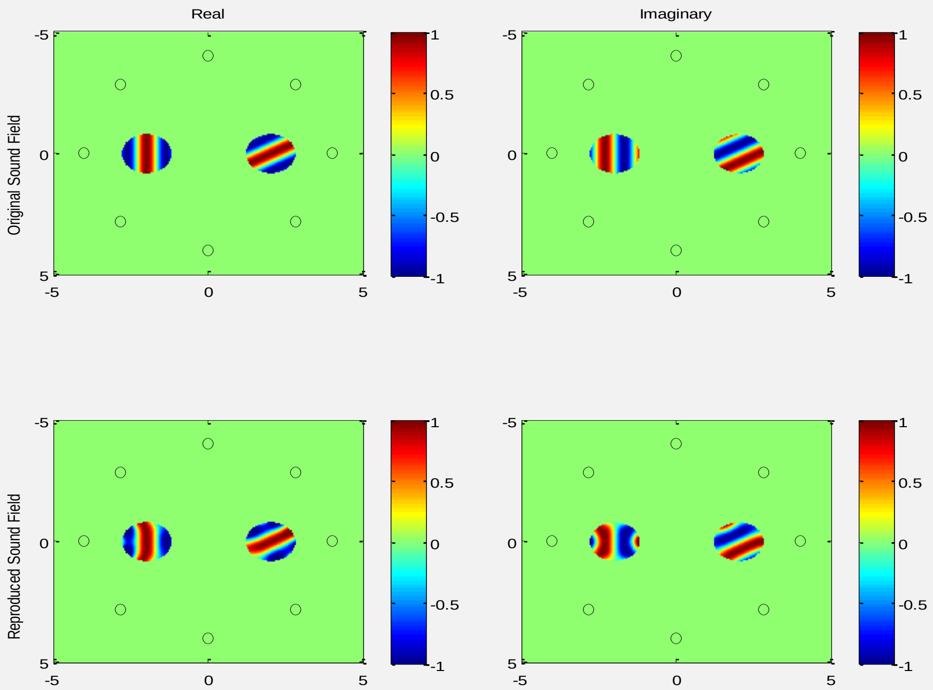

Figure 4-4 Reproduction of a 2-D two-zone sound field using higher order loudspeakers, k = 5, f= 270 Hz, radius = 4m

4.1.1 Comparison of simulations

From the simulations, it can be ascertained that successful reproduction of multizone sound field can be brought about using higher order loudspeakers. Reproduction of multiple independent sound fields in isolated zones is evident in this case without unpremeditated interference within a zone from other zones.

4.2 SIMULATION WITH VARIATION WITH WAVENUMBER

Further analysis can be performed by investigating the alteration of the reproduced sound field with respect to the original sound field. The variation of wavenumber is displayed with every simulation in this case.

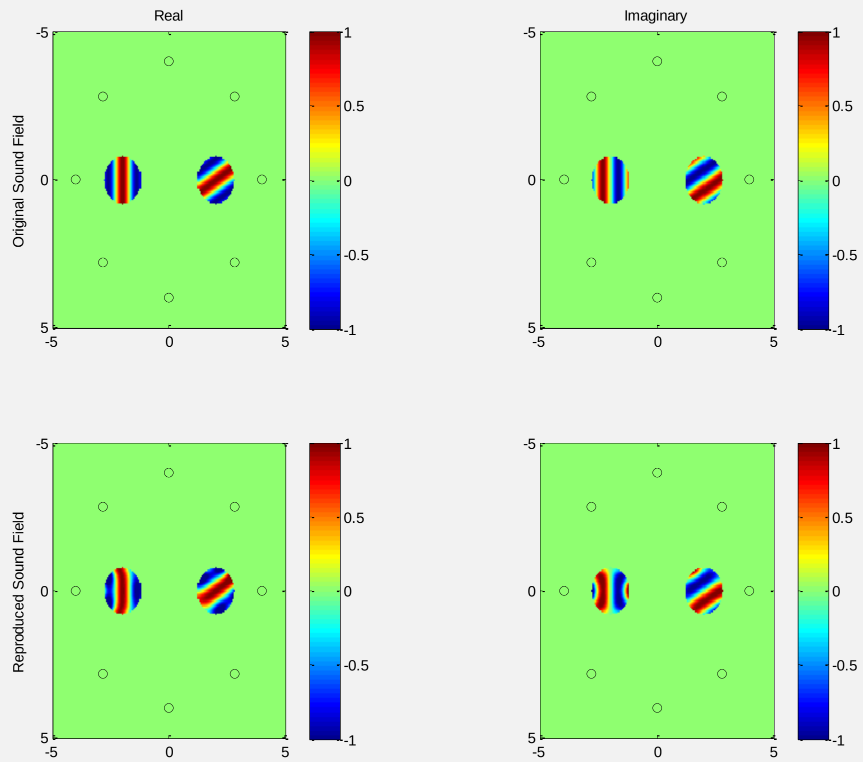

Figure 4-5 Reproduction of a 2-D two-zone sound field using higher order loudspeakers, k = 5

Figure 4.3 illustrates the sound field discussed in section 4.1. Keeping the other factors affecting the reproduction of sound field constant (e.g. number of loudspeakers), the wavenumber k is varied.

In this case, the wavenumber is only decreased to a value of 3 as clear observations can be made at that point and further decrease in wavenumber would potentially defeat the purpose of reproduction of high frequency sound field, in which case, the concept of using higher order loudspeakers would become superfluous.

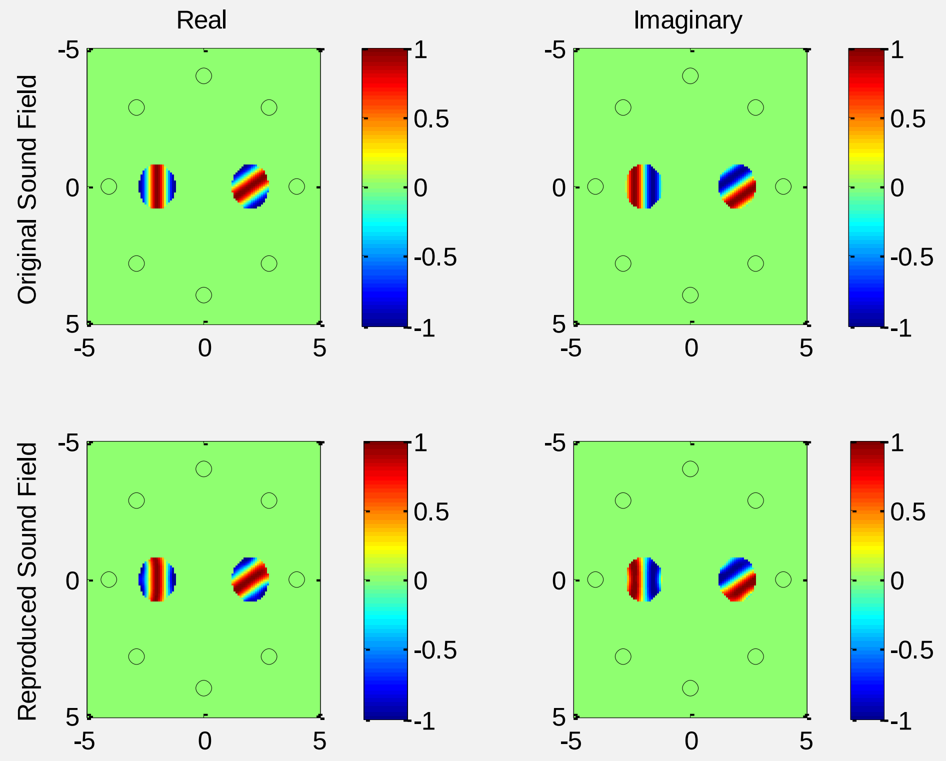

Figure 4-6 Reproduction of a 2-D two-zone sound field using higher order loudspeakers, k = 4

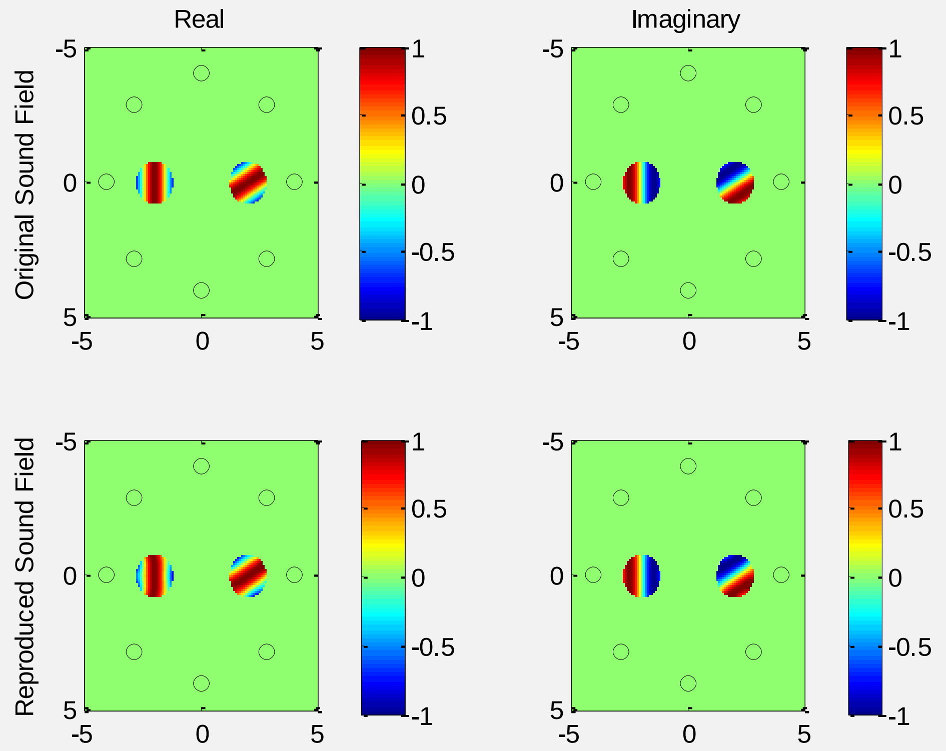

Figure 4-7 Reproduction of a 2-D two-zone sound field using higher order loudspeakers, k = 3

4.2.1 Comparison of simulations

Comparison can be made between the reproduced sound fields as the wavenumber is varied. It can be easily observed that the sound field reproduction becomes more prominent with the decrease in value of the wavenumber. This means the reproduced sound field begins to replicate the original sound field more conspicuously as illustrated by Figure 4.6.

Use of a higher wavenumber, $k = 4$, actually leads to production of a sufficiently operational sound field, and may not require further optimisation.

4.3 SIMULATIONS USING THEORETICAL CONTINUOUS LOUDSPEAKER METHOD

Sound field reproduction using theoretical continuous loudspeakers has been discussed in section 3.5. The following simulations compare the reproduction of the sound field using the aforementioned technique with variation in the radius of reproduction zone.

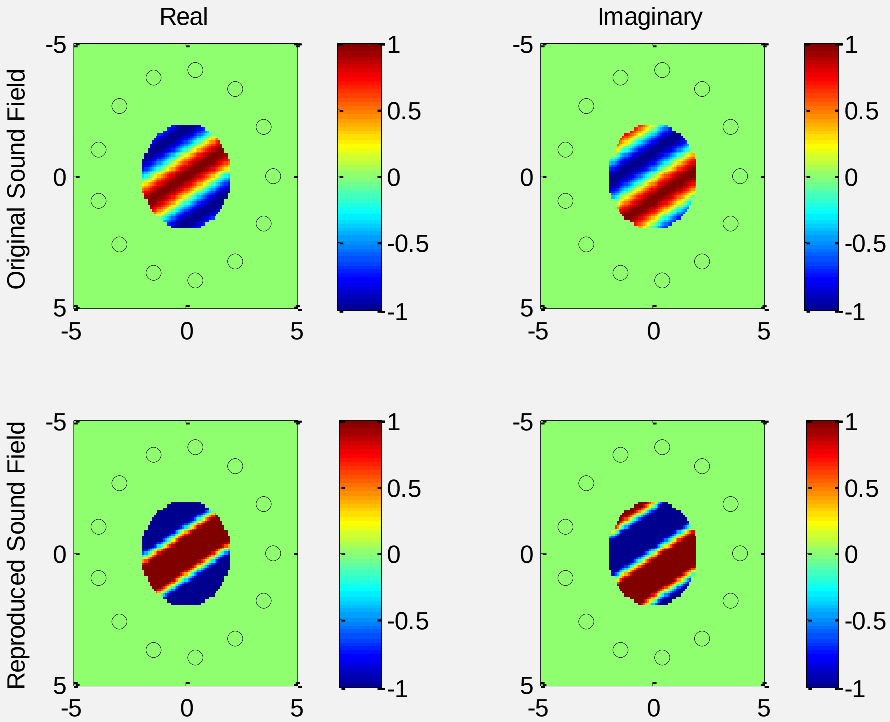

Figure 4-8 Reproduction of a 2-D two-zone sound field using theoretical continuous loudspeakers, k = 3, radius 2m

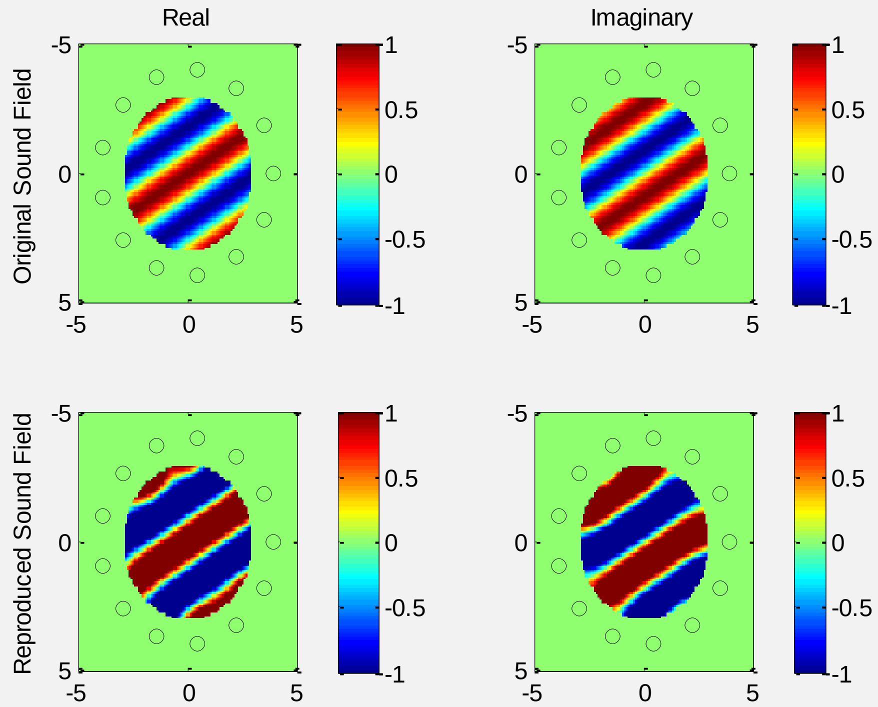

Figure 4-9 Reproduction of a 2-D two-zone sound field using theoretical continuous loudspeakers, k = 3, radius 3m

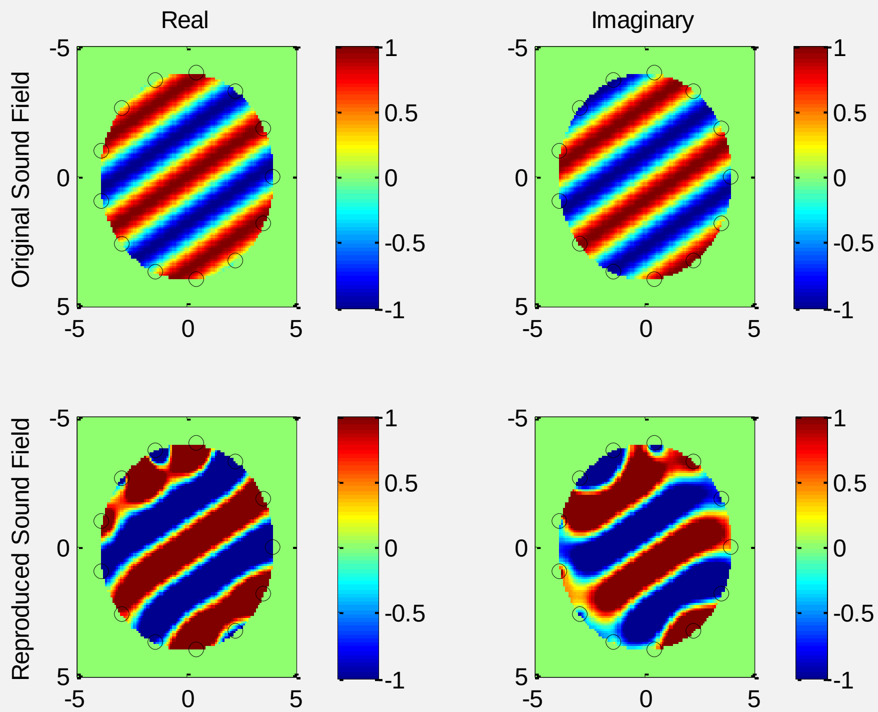

Figure 4-10 Reproduction of a 2-D two-zone sound field using theoretical continuous loudspeakers, k = 3, radius 4m

4.3.1 Summary of Results

Theoretical continuous loudspeakers can thus successfully reproduce a 2D sound field as observed from the simulations presented above. Moreover, from the simulations it is evident that increasing radius of the reproduction zone leads to distortion at higher values of radius. The reproduced field corresponds well with the desired field in the centre of the zone, and a clear distortion is noticed where the boundary of the reproduction area is indicated in the circle.

Chapter 5 Conclusions and Further Work

5.1 CONCLUSIONS

The performance of multizone sound reproduction systems that make use of higher order loudspeakers has been investigated in this thesis. The thesis succeeded in showing that higher order loudspeakers, although increase the complexity the loudspeakers themselves, can effectively produce accurate reproduction of desired sound field. For the case of 2D sound fields, it has been verified that Nth order loudspeakers placed in a circular array can successfully reproduce desired sound field as multiple independent sound fields in isolated zones without unpremeditated interference within a zone from other zones.

The systems presented in the thesis are described in terms of frequency content of the sound field, and number and order of the loudspeaker sources. Accurate loudspeaker weights were calculated for the higher order soruces, which were then used to reproduce appropriate sound fields. The study and investigation undertaken during the course of the research has made progress in the theoretical concept of sound field reconstruction systems using recently developed complex high order sources. This method proves to demonstrate a high potential commercial addition to the current off the shelf technologies.

The sound field reproduction systems investigated in the thesis were verified using numerical simulations. Necessary comparisons were executed to find the relation between variation of wavenumber and corresponding changes in the reproduced sound field. Variation of the sound field with different higher order sources were considered for the simulations as well.

The 2D system was successful in demonstrating numerical affirmation through simulation of the equations and theories developed throughout the thesis. This led to achieve the objective of the thesis. The integrity of the simulations throughout the investigation further added to achieving the goal.

In summary, the thesis presented has been effective in representing the following aspects of the sound field reproduction system:

- Progress in the study of Sound Field Reproduction – The theoretical foundation developed has definitely enhanced the field of acoustics and signal processing, provided useful material and made way for further research to be conducted for improvisations and improvements in the area.
- Viable Real World Application – The systems developed provide potential for improvements on related sound reproduction products currently available in the commercial market.
- Mathematical and Theoretical Basis – Equations derived and verified are in agreement with respect to theory as presented in the document.

5.2 FURTHER RESEARCH

During the course of conducting research and developing the sound reproduction system, a list of items have been identified as potential research prospects that would augment the field of higher order sound reproduction. These items are listed below

- Reproduction error due to multizone sound field reproduction with respect to changing wavenumbers, radius of the spatial zone and number of loudspeakers at particular frequency.
- The reproduction error can be compared to find a relationship between the performance measure and human spatial sensitivity.
- Throughout the thesis, theoretical approach has been executed to investigate and verify the use of higher order loudspeakers. Further analysis as how the theory reflects on physical implementation would be extremely interesting as this would form the bridge between theory and practicality of the thesis at hand.
- Normalised truncation and expansion error were discussed in the sound field reproduction process of Section 3.3. Implementation of further investigation is necessary to associate and find the relation between these errors and the sound field being reproduced.
- Section 3.4.3 presented with a theoretical background on how to investigate multizone sound field reconstruction based on the loudspeakers being placed in multiple arrays. Numerical simulation, error analysis and corresponding physical implementation are necessary to verify the theory and subsequent relationship between the sound field reproduction factors.
- Using higher order loudspeakers can prove to be an extremely complex task in terms of physical implementation even though the functionalities may be perfect with respect to the theoretical investigation. Physical implementation is therefore extremely vital before carrying out further theory based investigations.

Appendix A Mathematical Proofs

PROOF 1 COEFFICIENT TRANSLATION THEOREM 1[3]

Consider two coordinate systems with same orientation which have been displaced with a predefined translation. Suppose the origin of the first coordinate system with respect to the second coordinate system is (r_0, θ_0). If an arbitrary point is defined to have coordinates (R, Ω) and (r, θ) with respect to first and second coordinate system respectively. The sound field can be defined as

$$S(R, \Omega; \mathrm{k}) = \sum_{n=-\infty}^{\infty} \alpha_n^{(1)}(k)\, J_n(kR) e^{jn\Omega}$$

where $\alpha_n^{(1)}$ denotes the sound field coefficients with respect to the first coordinate system.

Using the translation theorem [25],

$$J_n(kR) e^{jn\omega} = \sum_{m=-\infty}^{\infty} T_m(r_0, \theta_0; k)\, J_{n+m}(kr) e^{j(n+m)\theta}$$

where $T_m\left(r_0, \theta^{(12)}; k\right) \triangleq J_m(kr_0) e^{-jm\theta_0}$

Thus, the sound field can be written as

$$S(R, \Omega; \mathrm{k}) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \alpha_n^{(1)}(k)\, T_m(r_0, \theta_0; k)\, J_{n+m}(kr)\, e^{j(n+m)\theta}$$

If we substitute $m + n = l$, the sound field becomes

$$S(R, \Omega; \mathrm{k}) = \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \alpha_{l-m}^{(1)}(k)\, T_m(r_0, \theta_0; k)\, J_l(kr)\, e^{jl\theta}$$

Since $S(r, \theta; k) \equiv S(R, \Omega; \mathrm{k})$,

---

[3] The theorem has been adapted from [31].

$$S(r,\theta;k) = \sum_{l=-\infty}^{\infty} \alpha_l^{(2)}(k)\, J_l(kr) e^{jl\Omega}$$

$$= \sum_{l=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} \alpha_{l-m}^{(1)}(k)\, T_m(r_0,\theta_0;k)\, J_l(kr)\, e^{jl\theta} \right]$$

where $\alpha_l^{(2)}(k)$ is the sound field coefficients in the second coordinate system. Thus,

$$\alpha_l^{(2)}(k) = \sum_{m=-\infty}^{\infty} \alpha_{l-m}^{(1)}(k)\, T_m(r_0,\theta_0;k) \; = \; \alpha_n^{(1)}(k) * \; T_m(r_0,\theta_0;k)$$

PROOF 2 2D SOUND FIELD REPRODUCTION

Considering a sound field resultant from a sum of L randomly placed loudspeakers, the sound field obtained can be expressed as

$$F(\boldsymbol{x};k) = \sum_{l=1}^{L} \frac{w_l y_l e^{iky_l} e^{-ik\|\boldsymbol{y}_l - \boldsymbol{x}\|}}{\|\boldsymbol{y}_l - \boldsymbol{x}\|}$$

and from [19]

$$H_0^{(1)}(k\,\|\boldsymbol{y}-\boldsymbol{x}\|) = \sum_{m=-\infty}^{\infty} H_m^{(1)}(ky) J_m(kx) e^{im\phi} e^{-im\xi}$$

where ϕ is the azimuth of $\boldsymbol{x}$ and ξ is the azimuth of $\boldsymbol{y}$.
Using Eq 2.1 and Eq 3.61 from [19]

$$\frac{e^{ik\|\boldsymbol{y}-\boldsymbol{x}\|}}{\|\boldsymbol{y}-\boldsymbol{x}\|} = \pi\, i\, H_0^{(1)}(k\,\|\boldsymbol{y}-\boldsymbol{x}\|)$$

the sound field can be expressed as

$$F(\boldsymbol{x};k) = \pi\, i \sum_{l=1}^{L} w_l y_l e^{iky_l} \sum_{m=-\infty}^{\infty} H_m^{(1)}(ky) J_m(kx) e^{im\phi} e^{-im\xi_l}, \boldsymbol{y_l} \neq \boldsymbol{x}$$

Thus a 2D sound field can be expressed as

$$F\,(\boldsymbol{x};k) = \sum_{m=-\infty}^{\infty} \alpha_m(k)\; J_m(kx) e^{im\phi}$$

where

$$\alpha_m(k) = \pi\, i \sum_{l=1}^{L} w_l y_l e^{iky_l} H_m^{(1)}(ky) e^{-im\xi_l}$$

PROOF 3 2D NORMALISED TRUNCATION ERROR

The 2D normalised truncation error can be defined as

$$\epsilon_M(kx) \triangleq \frac{\int \left|F(\boldsymbol{x};k) - \hat{F}_M(\boldsymbol{x};k)\right|^2 d\phi}{\int |F(\boldsymbol{x};k)|^2 d\phi}$$

given that

$$\hat{F}_M(\boldsymbol{x};k) = \sum_{m=-M}^{M} \alpha_m(k)\ J_m(kx)e^{im\phi}$$

$$F(\boldsymbol{x};k) = \sum_{m=-\infty}^{\infty} \alpha_m(k)\ J_m(kx)e^{im\phi}$$

Calculation of the numerator:

$$F(\boldsymbol{x};k) - \hat{F}_M(\boldsymbol{x};k) = \sum_{m=-\infty}^{-M-1} \alpha_m(k)\, J_m(kx)e^{im\phi} + \sum_{m=M+1}^{\infty} \alpha_m(k)\ J_m(kx)e^{im\phi}$$

$$\begin{aligned}
&\int \left|F(\boldsymbol{x};k) - \hat{F}_M(\boldsymbol{x};k)\right|^2 d\phi \\
&\quad = \int \sum_{m=-\infty}^{-M-1} \sum_{p=-\infty}^{-M-1} \alpha_m(k)\alpha_p(k)\, J_m(kx)J_p(kx)e^{im\phi}e^{-ip\phi}\, d\phi \\
&\quad + \int \sum_{m=M+1}^{\infty} \sum_{p=M+1}^{\infty} \alpha_m(k)\alpha_p(k)\, J_m(kx)J_p(kx)e^{im\phi}e^{-ip\phi}\, d\phi \\
&\quad + 2\int \sum_{m=-\infty}^{-M-1} \sum_{p=-\infty}^{\infty} \alpha_m(k)\alpha_p(k)\, J_m(kx)J_p(kx)e^{im\phi}e^{-ip\phi}\, d\phi
\end{aligned}$$

Using the orthogonality property of exponential Fourier series

$$\begin{aligned}
&\int \left|F(\boldsymbol{x};k) - \hat{F}_M(\boldsymbol{x};k)\right|^2 d\phi \\
&\quad = 2\pi \sum_{m=-\infty}^{-M-1} \alpha_m(k)^2\, J_m(kx)^2 + 2\pi \sum_{m=M+1}^{\infty} \alpha_m(k)^2\, J_m(kx)^2 \\
&\quad = 2\pi \left(\sum_{m=-\infty}^{-M-1} |\alpha_m J_m(kx)|^2 + \sum_{m=M+1}^{\infty} |\alpha_m J_m(kx)|^2 \right)
\end{aligned}$$

The total energy of the sound field can be calculated as

$$\int |F(\boldsymbol{x};k)|^2 d\phi = 2\pi \sum_{m=-\infty}^{\infty} |\alpha_m\,(k)\,J_m(kx)|^2$$

$$\int \left|F(\boldsymbol{x};k) - \hat{F}_M(\boldsymbol{x};k)\right|^2 d\phi = \int |F(\boldsymbol{x};k)|^2 d\phi - 2\pi \sum_{m=-\infty}^{\infty} |\alpha_m\,(k)\,J_m(kx)|^2$$

Therefore, 2D normalised truncation error is

$$\epsilon_M(kx) = \frac{\int |F(\boldsymbol{x};k)|^2 d\phi - 2\pi \sum_{m=-\infty}^{\infty} |\alpha_m\,(k)\,J_m(kx)|^2}{\int |F(\boldsymbol{x};k)|^2 d\phi}$$
$$= 1 - \frac{2\pi \sum_{m=-\infty}^{\infty} |\alpha_m\,(k)\,J_m(kx)|^2}{\int |F(\boldsymbol{x};k)|^2 d\phi}$$

Appendix B Matlab Code

SUMMARY OF THE MATLAB CODES

File name	Description
higherOrderRep.m Created by: Debashish Chakraborty	The MATLAB script uses the theoretical background of sound field reproduction using higher order loudspeakers in a circular array. Section 3.4 describes the theory behind the script. It also creates a plot of the original sound field against the reproduced sound field, real and imaginary components in both cases.
theoContLoudspeakerRep.m Created by: Debashish Chakraborty	The MATLAB script uses the theoretical background of continuous loudspeaker for sound field reproduction in a circular array. The theory is included in section 3.5. It also creates a plot of the original sound field against the reproduced sound field, real and imaginary components in both cases.

MATLAB CODE LISTING

5.2.1 higherOrderRep.m

```
clc
clear all

% defining the x-axis and y-axis
x = -5: 0.1 : 5;
y = -5: 0.1 : 5;

% Transform the x and y domain into arrays X and Y
[X,Y] = meshgrid ( x, y);;

% calculating polar coordinates in terms of angle and radius from
% cartesian coordinates
[THETA, RHO] = cart2pol (X, Y);;

% specify angle between the loudspeakers
THETA_L = linspace(0, 2 * pi-(2*pi/ 8), 8);
THETA_L=[THETA_L];

% Radius of the loudspeakers
R_L = [4.* ones(1,8)];

% transformation back to cartesian coordinates
[XL, YL] = pol2cart(THETA_L, R_L);

%
k=5;

%specifying various directions to be used in placement of the zones
dir1=pi/3; dir2=5*pi; dir3=pi;
N=3;

Norder=-4:4; % defining the order

modes=-24:24; % defining the modes

%form the first set of coefficients
%radius of 1.5m
%M=kr=2*1.5=3 we will take as 4
%first A_q coefficients are

% deriving the Aq coefficients as described in Chapter 3
mm1=-4:4;
Aq1=1i.^mm1.*exp(-1i*mm1*dir1);

mm2=-4:4;
Aq2=1i.^mm2.*exp(-1i*mm2*dir3);

% Combining the Aq coefficients in one vector in order to reproduce
the
% the complete soundfield with both the zones in consideration
```

```
A=[Aq1 Aq2];

Jmat=zeros(length(A), 8*7);
indexJ=1;

qq=-50:50;

% define and calculate the sound field
S=zeros(101,101);
for ii=1:length(x)
  for jj=1:length(y)

      if ((X(ii,jj)-2)^2+((Y(ii,jj))^2))<0.8^2
      S(ii,jj)=exp(1i*k*RHO(ii,jj)*cos(dir1-
THETA(ii,jj)))*exp(1i*k*2*cos(dir1-pi));
      elseif ((X(ii,jj)+2)^2+Y(ii,jj)^2)<0.8^2
      S(ii,jj)=exp(1i*k*RHO(ii,jj)*cos(dir3-
THETA(ii,jj)))*exp(1i*k*2*cos(dir3));
      else
      S(ii,jj)=0;
      end

  end
end

% Plot of the desired sound field
% Real part
figure(); subplot(2,2,1)
imagesc(x,y,real(S))
hold on
plot( XL, YL, 'ko' );
title('Real');
ylabel('Original Sound Field');
hold on
colormap('jet');
caxis([-1 1]); colorbar;
hold off

% Imaginary Part
subplot(2,2,2)
imagesc(x,y,imag(S))
hold on
plot( XL, YL, 'ko' );
title('Imaginary');
hold on
colormap('jet');
caxis([-1 1]); colorbar;
hold off


trans1=pi;
for mcount=mm1
    indexy=1;

    for lcount=1:8
       for ncount=Norder
```

```
            Jmat(indexJ, indexy)=sum(besselj(qq-mcount,
k*2).*exp(1i*(qq-mcount)*(pi-trans1)).*besselh(ncount+qq,k*4).*exp(-
1i*qq*THETA_L(lcount)));
            indexy=indexy+1;
        end
    end
    indexJ=indexJ+1;
end

% Using predefined translation, the soundfield can be reproduced by
forming
% matrix where where J is 2M+1 by (2N+1)L matrix, w is a (2N+1)L by
one
% vector of weights, and a is a 2M+1 vector of interior coefficients.
trans2=0;
for mcount=mm2
    indexy=1;
    for lcount=1:8
       for ncount=Norder
            Jmat(indexJ, indexy)=sum(besselj(qq-mcount, k*2).*exp(-
1i*(qq-mcount)*(pi-trans2)).*besselh(ncount+qq,k*4).*exp(-
1i*qq*THETA_L(lcount)));
            indexy=indexy+1;
        end
    end
    indexJ=indexJ+1;
end

% calculation of weights using pseudo inverse
weights=pinv(Jmat)*A.';


Jindex=1;
for mm=modes
   newM=besselh(Norder+mm,k*4).'*exp(-1i*mm*THETA_L);
   sz=size(newM);
   Jmat2(Jindex,:)=reshape(newM, 1,sz(1)*sz(2));
   Jindex=Jindex+1;
end

% Calculating Fourier coefficients
Aq=Jmat2*weights;

for ii=1:length(x)
  for jj=1:length(y)
     if ((X(ii,jj)-2)^2+((Y(ii,jj))^2))<0.8^2
        [thetai, rhoi]=cart2pol(X(ii,jj), Y(ii,jj));
        Sout(ii,jj)=(besselj(modes,k*rhoi).*exp(1i*modes*thetai))*Aq;
     elseif ((X(ii,jj)+2)^2+((Y(ii,jj))^2))<0.8^2
        [thetai, rhoi]=cart2pol(X(ii,jj), Y(ii,jj));
        Sout(ii,jj)=(besselj(modes,k*rhoi).*exp(1i*modes*thetai))*Aq;
     else
        Sout(ii,jj)=0;
     end
  end
end
```

```
% Plot of the reproduced sound field
% Real part
subplot(2,2,3)
imagesc(x,y,real(Sout))
hold on
plot( XL, YL, 'ko' );
ylabel('Reproduced Sound Field');
hold on
colormap('jet');
caxis([-1 1]); colorbar;
hold off

% Imaginary part
subplot(2,2,4)
imagesc(x,y,imag(Sout))
hold on
plot( XL, YL, 'ko' );
hold on
colormap('jet');
caxis([-1 1]); colorbar;
hold off
```

5.2.2 theoContLoudspeakerRep.m

```
clc
clear all
close all

% defining the x-axis and y-axis
x = -5: 0.1 : 5;
y = -5: 0.1 : 5;

% Transform the x and y domain into arrays X and Y
[X,Y] = meshgrid ( x, y);

% calculating polar coordinates in terms of angle and radius from
% cartesian coordinates
[THETA, RHO] = cart2pol (X, Y);

% specify angle between the loudspeakers
THETA_L = linspace (0, 2*pi - ( 2*pi/13) , 13);

% Radius of the loudspeakers
R_L = [4.* ones(1,13)];

% transformation back to cartesian coordinates
[XL, YL] = pol2cart(THETA_L, R_L);


S = zeros(101,101);
k = 2;
dir = pi/3;

for ii=1 : length(x)
  for jj=1 : length(y)
      if ((X(ii,jj))^2 + (Y(ii,jj))^2)< 3^2
      S (ii,jj)=exp (1i*k*RHO(ii,jj) *cos (dir-THETA(ii,jj)));
      else
          S(ii,jj) = 0;
      end
  end
end

% Plot of the desired sound field
% Real part
figure();
title ('Field')
subplot(2,2,1)
imagesc(x,y,real(S))
hold on
plot( XL, YL, 'ko' );
title('Real');
ylabel('Original Sound Field');
hold on
colormap('jet');
caxis([-1 1]); colorbar;
hold off

% Imaginary part
```

```
subplot(2,2,2)
imagesc(x,y,imag(S))
hold on
plot( XL, YL, 'ko' );
title('Imaginary');
hold on
colormap('jet');
caxis([-1 1]); colorbar;
hold off

% Defining the Fourier coefficients
alpha = zeros(13,1);
beta = zeros(13,1);

weights = zeros(13,1);

for n = -6:1:6;
        alpha(n+7) = (1i.^n)*exp(-1i*n*dir);

        beta(n+7) = (2*alpha(n+7))./(1i*pi*besselh(n,k*4));

    for tl = 1:13
        weights(tl) = weights(tl) + beta(n+7)*exp(1i*n*THETA_L(tl));
    end

end

S_a = zeros(101,101);

for ii=1:101
  for jj=1:101
      for nl = 1:13
          if ((X(ii,jj))^2 + (Y(ii,jj))^2)< 3^2
            dist = sqrt(((X(ii,jj)-XL(nl)).^2 + (Y(ii,jj)-
YL(nl)).^2));
            S_a (ii,jj)= S_a (ii,jj) + weights (nl).*(1i/4) * besselh
(0,k*dist);
          else
            S_a (ii,jj) = 0;
          end
      end
  end
end

% Plot of the reproduced sound field
% Real part
subplot (2, 2, 3)
imagesc (x, y, real(S_a))
hold on
plot( XL, YL, 'ko' );
ylabel ('Reproduced Sound Field');
hold on
colormap ('jet');
caxis ([-1 1]); colorbar;
hold off

% Imaginary part
```

```
subplot (2, 2, 4)
imagesc (x, y, imag (S_a))
hold on
plot( XL, YL, 'ko' );
hold on
colormap ('jet');
caxis ([-1 1]); colorbar;
hold off
```

Bibliography

[1] Sashida, T. and T. Kenjo (1993). Introduction to ultrasonic motors.

[2] Brian Bowers, "Scanning Our Past from London: Bell and the Telephone – the 125th Anniversary," *Proceedings of the IEEE*, vol. 89, no. 6, pp. 984–986, June 2001.

[3] J. Hull, "Surround Sound Past, Present and Future," Dolby Laboratories Inc, Tech.

Rep., 1999.

[4] G. Dickins, "Soundfield representation, reconstruction and perception," Master's thesis, Department of Telecommunications Engineering, Research School of Information Science and Engineering, March 2003.

[5] M. Gerzon, "With-height sound reproduction," *J. Audio Eng. Soc.*, vol. 21, no. 1, pp. 2–10, Jan/Feb 1972.

[6] M. J. Leese. (1998) Ambisonic Surround Sound FAQ. [Online]. Available:

http://members.tripod.com/martin_leese/Ambisonic/faq_latest.html

[7] J. S. Bamford, "An analysis of Ambisonic sound systems of first and second order," Master's thesis, University of Waterloo, 1995.

[8] R. A. Kennedy, T. D. Abhayapala, and D. B.Ward, "Broadband nearfield beamforming using a radial beampattern transformation," *IEEE Transactions on Signal Processing*, vol. 46, no. 8, pp. 2147–2156, August 1998.

[9] Y J. Wu and T. D. Abhayapala, "Soundfield Reproduction Using Theoretically Continuous Loudspeakers," in *Proc. IEEE Int. Conf. Acoust, Speech, Signal Process.*, vol. 17, no. 1, pp. 107-116, Jan. 2009

[10] Y. J. Wu and T. D. Abhayapala, "Theory and Design of Soundfield Reproduction using continuous loudspeaker concept," in *Proc. IEEE Int. Conf. Acoust, Speech, Signal Process. (ICASSP'08)*, Las Vegas, NV, Mar. 2008, pp. 377–380.

[11] A. J. Berkhout, D. De Vries, and P. Vogel, "Acoustic control by wave field synthesis," *J. Acoust. Soc. Amer.*, vol. 93, pp. 2764–2778, May 1993.

[12] D. B Ward and T. D. Abhayapala, "Reproduction of a plane wave sound field using an array of loudspeakers," *IEEE Trans. Speech Audio Proc.*, 9 (6), pp 697 – 707, 2001

[13] M. A. Poletti, "Three dimensional Surround Sound Systems based on spherical harmonics," *J. Audio Engin. Soc.*, 53 (11), pp 1004-1025, November 2005

[14] M. Unser, “Sampling-50 years after Shannon,” *Proc. IEEE.*, 88(4), pp 569-587, 2000

[15] M. A. Poletti, F. M. Fazi, and P. A. Nelson, “Sound-field Reproduction Systems using fixed-directivity loudspeakers,” *J. Acoust. Soc. Am.*, 127(6), pp. 3590-3601, 2010.

[16] D. de Vries, “Sound Reinforcement by wave synthesis: Adaptation of the synthesis operator to the loudspeaker directivity characteristics,” *J. Audio. Eng. Soc.*, 44(12) pp. 1120-1131, 1996.

[17] J. Aherns and S. Spors, “Sound field reproduction employing non-omnidirectional loudspeakers,” *126th AES Convention*, Munich, Germany, May 7-10, 2009.

[18] M. M. Boone and O. J. Ouweltjes, “Design of a loudspeaker system with a low frequency cardiolike radiation pattern,” *J. Audio. Eng. Soc.*, 45(9) pp. 702-707, 1997.

[19] M. A. Poletti, T. D. Abhayapala and P. Samarasinghe, “Interior and exterior sound field control using two dimensional higher-order variable-directivity sources,” *J. Acoust. Soc. Am.*, 131(5), May 2012.

[20] Dolby Laboratories, “The Evolution of Dolby Film Sound,” Dolby Laboratories, Tech. Rep

[21] R. Loyola (2002, January) DVD audio overview. [Online]. Available: http://www.techtv.com/screensavers/howto/story/0,24330,2334015,00.html

[22] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, 2nd ed. New York: Springer, 1998.

[23] M. A. Poletti and T. D. Abhayapala, “Spatial Sound Reproduction Systems using Higher Order Loudspeakers,” *IEEE Int. Conf. on Acoust,.Speech and Sig. Process.*, ICASSP2011, Prague, May 22-27, 2011

[24] T. Betleham, C. Anderson and M. A. Poletti, “A directional loudspeaker array for surround sound in reverberant rooms,” in *Proceedings of the International Conference on Acoustics*, Sydney, Australia, August 23-27, 2010.

[25] *More Properties of Hankel and Bessel Functions*. Columbus: The Ohio State Univ. [Online]. Available:

http://www.math.ohio-state.edu/gerlach/math/BVtypset/node127.html

[26] O. Kirkeby and P.A. Nelson, “Reproduction of plane wave sound fields,” *JASA*, vol. 94, no. 5, pp. 2992–3000, Nov. 1993.

[27] J. Ahrens and S. Spors. “An analytical approach to sound field reproduction with a movable sweet spot using circular distributions of loudspeakers,” in *Proceedings of the 2009 IEEE International Conference on Acoustics, Speech and Signal Processing* (ICASSP '09). IEEE Computer Society, Washington, DC, USA, 273-276, 2009.

[29] D. Excell, “Reproduction of a 3D Sound Field Using an array of loudspeakers,” *Australian National University,* Australia, June 2003

[30] M. A. Poletti and T. D. Abhayapala, “Interior and exterior sound field control using general two-dimensional first order sources,” J. Acoust. Soc. Am. 129(1), pp. 234-244, 2011.

[31] Y. J. Wu and T. D. Abhayapala, “Spatial Multizone Soundfield Reproduction,” in *Proc. IEEE Int. Conf. Acoust, Speech, Signal Process. (ICASSP’09),* Taipei, pp. 93-99, April 2009

[32] M. Poletti, “An investigation of 2D surround sound multizone surround sound systems,” *in Proc. AES 125th Convention. Audio Engineering Society*, San Francisco, USA, Oct 2008

[33] Y. J. Wu and T. D. Abhayapala, “Multizone 2D Soundfield Reproduction via Spatial Band Stop Filters” *in Workshop IEEE on Applications of Signal Processing to Audio and Acoustics*, New Paltz, NY, October 18-21, 2009

[34] D. Colton and R. Kress, “*Inverse Acoustic and Electromagnetic Scattering Theory,*” Springer, New York, second edition, 1998.