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DECOMPOUNDING: AN ESTIMATION PROBLEM FOR POISSON RANDOM SUMS

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Given a sample from a compound Poisson distribution, we consider estimation of the corresponding rate parameter and base distribution. This has applications in insurance mathematics and queueing theory. We propose a plug-in type estimator that is based on a suitable inversion of the compounding operation. Asymptotic results for this estimator are obtained via a local analysis of the decomposing functional.

1. Introduction. The statistical problem to be discussed in this paper is motivated by applications from insurance mathematics and queueing theory. In the standard model of risk theory [see, e.g., Beard, Pentikäinen and Pesonen (1991) or Grandell (1991)], claims of random size X_1, X_2, X_3, \dots arrive at random times $T_1, T_1 + T_2, T_1 + T_2 + T_3, \dots$. The random variables $X_1, X_2, X_3, \dots, T_1, T_2, T_3, \dots$ are assumed to be independent, the $X_k, k \in \mathbb{N}$, have distribution P and interarrival times $T_k, k \in \mathbb{N}$, are exponentially distributed with parameter λ . In particular, the claim arrival times are given by the points of a Poisson process with constant intensity λ . For all $t \geq 0$,

$$(1) \quad S_t = \sum_{k: T_1 + \dots + T_k \leq t} X_k$$

is the total claim amount up to and including time t . Similarly, in a queueing context as discussed, for example, in Asmussen (1987), if customers arrive at a service point in bulks of size X_1, X_2, \dots at the time points of a Poisson process then (1) gives the total number of customers that arrive in the time interval $(0, t]$.

The assumptions imply that the distribution Q of S_t can be written as a convolution series,

$$(2) \quad Q = \Psi(\lambda, P) \quad \text{with } \Psi(\lambda, P) = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} P^{*k}.$$

Q is the compound Poisson distribution with rate λ and base (or claim size or bulk size) distribution P . (Unfortunately, Poisson distributions with a random parameter, i.e., mixed Poisson distributions, are often called compound in the literature.)

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